O‘ZBEKISTON MILLIY UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/30.12.2019.FM.01.09 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

OTAJONOV SHERZOD RUSTAMOVICH

KVANT FLUKTUATSIYALARI TA’SIRI OSTIDA BOZE-EYNSHTEYN KONDENSATIDAGI MATERIYA TO‘LQINLARI

01.04.02 – Nazariy fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DARAJASINI OLISH UCHUN YOZILGAN DISSERTATSIYASI AVTOREFERATI

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Atomlarni (bozonlarni) absolyut nol kelvinga yaqin haroratgacha sovitilsa ular bir xil energetik sathlarni egallay boshlaydi. Bu hodisaga Bose-Einstein kondensati deyiladi. Kondensat birinchi bo‘lib Bose va Einstein tomonidan 1925-yillarda bashorat qilingan bo‘lsada, o‘sha paytdagi mavjud uskunalar yordamida atomlarni bu haroratgacha yaqinlikda sovitish imkoniyati mavjud bo‘lmagan. Keyinchalik, 1995-yilda ikkita alohida gruppalar Bose-Einstein kondensatini tajribada kuzatishgani uchun 2001-yilda bu tajribalar uchun Nobel mukofoti bilan taqdirlanishdi.

Hozirgi vaqtda xalqaro ilmiy hamjamiyat Bose-Einstein kondensatlarida materiya to‘lqinlarining xususiyatlarini o‘rganishga qaratilgan ko‘plab nazariy va eksperimental tadqiqotlar olib bormoqda. Ushbu tadqiqotlar katta ahamiyatga ega, chunki ular zamonaviy fizikaning asosiy savollariga javob topishga imkon beradi. Bose-Einstein kondensatlaridagi atom gazlari tashqi magnit va optik maydonlar orqali samarali boshqarilishi mumkin, bu turli tajribalar va aniq nazariy modellar uchun asos yaratishga imkonini beradi. Bose-Einstein kondensati asosida tajribalarda quyidagi qurilmalarni yaratishda foydalanish mumkin: materiya-to‘lqin interferometrlari, atom-to‘lqinli lazerlarni ishlab chiqarishda, atom soatlarida, kvant komputerlarida kubit yoki hotira sifatida yoki kvant kriptografiyalarida sonni tub sonlarga ajratishda va boshqa maqsadlarda.

O‘rtacha maydon nuqtai nazaridan ikki- va uch-o‘lchamli, tortishuvchi ikki-atom o‘zaro-ta’siriga ega bo‘lgan kondensatlar nostabil hisoblanadi. Kondensatni stabil qilishning bir nechta usullari mavjud, jumladan tashqi tuzoqlar yordamida, uch-atom o‘zaro tasirini hisobga olish orqali, dipolyar ta'sirlashuvlar orqali, sochilish uzunliklarini davriy o‘zgartirishlar.

Yaqin yillarda nazariy metodlar orqali ikki- va uch-o‘lchamli nostabil kondensatlarni kvant fluktuatsiyalarni hisobga olish orqali stabil holatga keltirish mumkinligi va bunday kondensatlarda siqilmaydigan suyuqlik tabiatiga ega bo‘lgan kvant tomchilari hosil bo‘lishi nazariy yo‘llar bilan ko‘rsatib berildi [1]. Ma’lum bo‘lishicha, kvant fluktuatsiyalari yuqori darajada siyraklashgan suyuqlik hususiyatiga ega bo‘lgan holatlarning paydo bo'lishiga olib keladi. Bunday holatning o’z-o’zidan bog’langan tuzilishi kvant tomchilari deb nomlanadi [1]. Ushbu ishdan ko‘p o‘tmay, bunday sistemalarning stabil bo‘lishi va kvant tomchilarining mavjudligi eksperimental tarzda namoyish qilindi. Ushbu tadqiqotlar natijasida Bose-Einstein kondensati sohasidagi tadqiqotchilar uchun yangi yo‘nalish ochdi.

Yuqorida ko‘rsatilganlardan kelib chiqadiki, hozirgi vaqtda Bose-Einstein kondensatidagi materiya to‘lqinlari fizikasi jadal suratda rivojlanib bormoqda va amaliy qo‘llash uchun katta istiqbolga ega bo‘lgan fanning dolzarb sohalaridir. Mamlakatimizda so‘nggi o‘n yilliklarda fanni, ayniqsa, nazariy fizika sohalarini rivojlantirishga, fundamental tadqiqotlar uchun yuqori darajada sharoit yaratishga katta e’tibor qaratilmoqda.

Mazkur dissertatsiya ishi hukumatning me’yoriy hujjatlari va O‘zbekiston Respublikasi Prezidentining “Fanlar Akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789-son qarori, 2017-yil 7-fevraldagi PF-4947-sonli “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi qarori, 2021-yil 19-martdagi PQ-5032-sonli “Fizika sohasidagi ta'lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlarida nazarda tutilgan vazifalarni hayotga tatbiq etish maqsadida bajarildi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot ishi O‘zbekiston Respublikasi fan va texnologiyalar rivojlantirishning II. «Fizika, astronomiya, energetika va mashinasozlik» ustuvor yo‘nalishlariga muvofiq amalga oshirildi.

Muammoning o‘rganilganlik darajasi. Bose-Einstein kondensatida kvant fluktuatsiyalarlari natijasida hosil bo‘ladigan kvant tomchilarining xususiyatlarini o‘rganish birinchi nazariy maqoladan [1] va tajribada kuzatilganidan beri bu yo‘nalish olimlar orasida juda katta qiziqish uyg‘otmoqda. Dunyodagi ko‘plab tadqiqotchilar materiya to‘lqinlarining kvant fluktuatsiyalarlari ta'siri ostida turli xil xususiyatlarini o‘rganmoqdalar. Jumladan, uch-o‘lchamli kondensatdan ikki- va bir-o‘lchamli kondensatga o‘tish havola [2] da keltirilgan. Bir-o‘lchamli kondensat dinamikasi [3] da o‘rganilgan. Ikki-o‘lchamli kvant tomchilari va uyurmalar sonli metodlar yordamida havola [4] da o‘rganilgan. Uch-o‘lchamli kvant tomchilari va uyurmalarning zarrachalar soniga bog‘liq holda turg‘un sohalari havola [5] da e'lon qilingan.

Kvant tomchilari eksperimentda birinchi bo‘lib dipolar kondensatda [6], keyinchalik bozonlar aralashmasida amalga oshirilgan [7-8]. Ular kvant tomchilarining xossalarini tushuntiradigan turli xil modellarni ishlab chiqdilar va asosiy eksperimental natijalarni oldilar. Biroq, kvant fluktuatsiyalarlarining materiya to‘lqinlariga ta'sirining ko‘p jihatlari hali ham yetarlicha o‘rganilmagan bo‘lib, o‘z yechimini kutayotgan ko‘plab masalalar mavjud. Ular qatoriga quyidagilarni kiritish mumkin: bir- va ko‘p-o‘lchamli kvant tomchilari va uyurmalarning dinamikasi, sistema parametrlarining vaqt bo‘yicha davriy o‘zgaruvchi bo‘lgan holatlari, modulyatsion noturg‘unlik hodisasi natijasida yassi to‘lqinlardan kvant tomchilari hosil bo‘lishi va ularning o‘zaro ta'siri. Ushbu ishda biz shu savollarni ko'rib chiqamiz.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim yoki ilmiy-tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi.

Tadqiqot O‘zbekiston Respublikasi Fanlar Akademiyasi Fizika-Texnika Institutining ilmiy-tadqiqot ishlari rejasiga, shu jumladan O‘zbekiston Respublikasi Innovatsion Rivojlanish Vazirligining FA-F2-004 “Kvant va dissipativ sistemalarda nochiziqli mujassamlashgan to‘lqinlarning dinamikasi va o‘zaro ta’siri” nomli ilmiy loyihasiga muvofiq amalga oshirildi, 2017-2020 yillar. “Kvant gazlari va nochiziqli optik muhitlarda mujassamlashgan to‘lqinlar dinamikasini o‘rganish” mavzusidagi davlat byudjeti loyihasi doirasida amalga oshirildi, 2020-2021 yillar.

Tadqiqotning maqsadi kvant fluktuatsiyalarlari ta'sirini hisobga oluvchi ko‘p-o‘lchamli ikki-komponentali Bose-Einstein kondensatlarida materiya to‘lqinlarining xususiyatlarini o‘rganish va ushbu sistemalarda mujassamlashgan holatlarni tushuntirish uchun nazariy metodlar ishlab chiqishdir.

Tadqiqotning vazifalari: Bir va ko‘p o‘lchamli Bose-Einstein kondensatlarida kvant tomchilari, uyurmalar va yassi to‘lqinlarning xossalarini tushuntirish uchun matematik modellar ishlab chiqish. Ko‘p o‘lchamli Bose-Einstein kondensatlarida o‘zaro ta’sir konstantalari vaqt bo‘yicha davriy o‘zgaruvchi bo‘lgan holatlarni tadqiq qilish.

Modulyatsion noturg‘unlik hodisasi tufayli birjinsli zichlik taqsimotiga ega bo‘lgan Bose-Einstein kondensatlarining kvant tomchilariga aylanishi shartlarini ochib berish. Parametrlari fazosida noturg‘unlik sohalarini aniqlash va hosil bo‘lgan kvant tomchilarining dinamikasini o‘rganish.

Gross-Pitaevskii yoki nochiziqli Schrodinger turidagi bir- va ko‘p- o‘lchamli tenglamalarni va variatsion yaqinlashish va chiziqli turg‘unlik taxlili modelida yuqoridagi fizik jarayonlarni sonli simulyatsiyalar qilish. Variatsion usulning va chiziqli turg‘unlik taxlili yordamida olingan natijalarning to‘g‘riligini asoslash uchun har bir holatni alohida o‘zaro batafsil taqqoslash.

Tadqiqotning obyekti. Bir va ko‘p o‘lchamli Bose-Einstein kondensatlarida kvant tomchilari, uyurmalar va yassi to‘lqinlar.

Tadqiqotning predmeti bo‘lib mujassamlashgan holatlar parametrlarini zarrachalar soniga bog‘liqligi, rezonans hodisasi, kichik tebranishlar davri, kvant tomchilari va uyurmalarning turg‘unlik shartlari, yassi to‘lqinlarning modulyatsion noturg‘unligi va kvant tomchilarining hosil bo‘lish xususiyatlari hisoblanadi.

Tadqiqotning usullari. Ushbu tadqiqotda biz murakkab fizik sistemalar dinamikasini o'rganishda analitik va sonli tadqiqot usullarining kombinatsiyasidan foydalanamiz. Tadqiqotning analitik qismi uchun biz variatsion yaqinlashish, Tomas-Fermi yaqinlashuvi va chiziqli turg‘unlik tahlilidan foydalanamiz. Ushbu usullar bizga chuqur nazariy bilimlarni olish va sistemaning xatti-harakatlarini boshqaradigan asosiy tamoyillarni chuqurroq tushunish imkonini beradi.

Analitik yondashuvlardan tashqari, biz hisob-kitoblar va simulyatsiyalarni amalga oshirish uchun turli xil sonli usullardan foydalanamiz. Xususan, tadqiqot uchun asosiy bo‘lgan Gross-Pitayevskii tipidagi tenglamalarni yechishga e’tibor qaratamiz. Ushbu tadqiqotda ishlatiladigan sonli usullar qatoriga to'rtinchi tartibli Runge-Kutta usuli va Crank-Nicolson usuli kiradi, ularning ikkalasi ham nochiziqli hususiy hosilali differensial tenglamalarni yechishda aniqligi va barqarorligi bilan mashhur.

Bundan tashqari, biz sonli tahlillarning bir qismi sifatida rasmga qayta ishlov berishning usullaridan foydalanamiz, bu bizga eksperimental ma'lumotlardan qimmatli ma’lumotlarni olish va sistemaning xatti-harakatlarini tasavvur qilish imkonini beradi. Ushbu analitik va sonli tadqiqot usullarini birlashtirib, biz tekshirilayotgan murakkab fizik hodisalarni har tomonlama va ishonchli tahlil qilishni maqsad qilganmiz. Bu yondashuv nazariy tushuntirish va empirik ma’lumotlar o‘rtasidagi tafovutni bartaraf etish va shu orqali sistema dinamikasini to'liqroq tushunish imkonini beradi va bu sohadagi ilmiy bilimlarning rivojlanishiga hissa qo'shadi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Kvant fluktuatsiyalarlarini hisobga oladigan bir va ko‘p o‘lchamli Bose-Einstein kondensatlarida kvant tomchilarini va ikki-o‘lchamli holatda uyurmalarni xarakterlash uchun super-Gaussian funksiyasi yordamida variatsion metod ishab chiqildi.

Barcha o‘lchamlarda kvant tomchilarining energiyasi, effektiv potensiali, kichik tebranishlar chastotalari va kimyoviy potensiallarining analitik tenglamalari hisoblandi va kvant tomchilarining turg‘unligi Vakhitov-Kolokolov kriteriyasi orqali tekshirildi.

Ikki- va uch-o‘lchamli Bose-Einstein kondensatida o‘zaro ta’sir konstantalari davriy o‘zgaruvchi bo‘lgan holatlarda kvant tomchilarining resonans tebranishlari va tashqi modulyatsiya amplitudasiga bog‘liq ravishda dinamikasining turli regimlari aniqlandi.

Kvant fluktuatsiyalarlari ta’sirini hisobga oladigan ikki-o‘lchamli ikki komponentali Bose-Einstein kondensatida g‘alayonlangan yassi to‘lqinlarning eksponensial o‘sish shartlari va parametrlar fazosida stabil va nostabil sohalari topildi. Modulyatsion noturg‘unlikning nochiziqli bosqichida kvant tomchilari hosil bo‘lishi, ularning kamayishi qonuni va kamayish tezliklari aniqlandi. Bu qonuniyatlarni hisoblashda rasmlarga ishlov berish texnikasining Bose-Einstein kondensati dinamikasiga qo‘llash mumkinligi ko‘rsatildi.

Tadqiqotning amaliy natijalari. Gross-Pitayevskii tipidagi bir va ikki komponentli tenglamalar fazoning turli xil o‘lchamlarda analitik va sonli yechiladi. Bose-Einstein kondensatlari kogorentlik xususiyatlariga ko‘ra kvant ma’lumotlarini qayta ishlash uchun potentsial nomzodlardir. Turli geometriyalardagi kondensatlarning dinamikasini tushunish ularning kvant hisoblashlaridagi muayyan muammolarni hal qilishga imkon beradi. Bunday sistemalarda Gross-Pitayevskiy tenglamalarini yechish bizga potentsial texnologik ilovalar uchun ushbu holatlarni loyihalash va boshqarishga yordam beradi.

Tadqiqot natijalarining ishonchliligi. Olingan natijalarga ularning ishonchliligini ta’minlaydigan bir nechta omillar tufayli ishonish mumkin:

Birinchidan, tadqiqot qat'iy nazariy fizika, matematika va yuqori aniqlikdagi sonli usullar va algoritmlarni qo'llashga tayanadi. Ushbu yondashuv ishonchli va aniq ma’lumotlarni tahlil qilishni ta’minlaydi, potentsial xatolar va noaniqliklarni minimallashtiradi.

Ikkinchidan, olingan natijalar va aniq yechimlar, statsionar holatlarda Thomas-Fermi chegaralari bilan va sonli simulyatsiyalar o'rtasidagi aniq moslilik ularning ishonchliligini yanada oshiradi. Turli metodlar bir hil natijalar berganda, natijalarning haqiqiyligiga ishonchni oshiradi, bu esa qo'llaniladigan usullarning ishonchliligini ko'rsatadi.

Uchinchidan, ba’zi natijalarning boshqa olimlar tomonidan muqobil usullar va haqiqiy tajribalar yordamida mustaqil ravishda olingan natijalarga mos kelishi natijalarning ishonchliligini oshiradi. Bir nechta manbalardan olingan natijalarning bunday mos kelishi tadqiqot natijasida olingan natijalar ishonchliligini ko‘rsatadi.

Bundan tashqari, haqiqiy fizik parametrlarni baholash Bose-Einstein kondensatlarida o'tkazilgan odatiy tajribalar bilan yaxshi mos keladi. Eksperimental ma’lumotlar bilan ushbu muvofiqlik tadqiqotning amaliy qo'llash haqiqatiga asoslanganligini ta’minlaydi va natijalarning ishonchliligini yanada tasdiqlaydi.

Ushbu jihatlarni hisobga olish va tadqiqot jarayonida qo'llanilgan yondoshuvlar orqali olingan natijalar ishonchlilikka ega bo'ladi va natijalarning aniqligi va asosliligiga ishonchni uyg'otadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati quyidagilardan iborat:

Olingan natijalar kvant fluktuatsiyalarlari ta’siri ostida bir va ko‘p o‘lchamli Bose-Einstein kondensatlarida materiya to‘lqinlarining bir qator muhim (siqilmaydigan suyuqlik hususiyati va h.k.) hususiyatlarini ochib beradi va batafsil tushuntiradi. Ushbu fundamental tadqiqotlar Bose-Einstein kondensatlarida kvant fluktuatsiyalari effektining to‘liq nazariyasini ishlab chiqishga zamin yaratadi. Shuningdek, bu natijalar yordamida kondensatdagi atomlar soni, kritik harorat, kondensatning yashash vaqti va shu kabi boshqa haqiqiy tajriba parametrlarini baholashda foydalanish mumkin. Bundan tashqari, dissertatsiya natijalari turli xil ta’sirlar mavjud bo‘lganda ham, kvant tomchilarining parametrlarining vaqt bo‘yicha dinamikasini o‘rganishda ham foydali bo‘lishi mumkin. Bu ayniqsa, tashqi tuzoq tomonidan hosil qilingan Bose-Einstein kondensatlarining dastlabki taqsimoti aniq yechimlar bilan ozgina farq qiladigan tajribalar uchun juda muhimdir. Tadqiqot natijalarining tajribalar bilan mos kelishi Bose-Einstein kondensatlarida materiya to‘lqinlarining tabiatini biz taklif qilgan metodlar orqali o‘rganish muhim ahamiyat kasb etishini ko‘rsatadi.

Tadqiqot natijalarining joriy qilinishi. Dissertatsiya ishida olingan natijalarga scopus.com saytlaridagi ma’lumotlarga ko‘ra umumiy 58 ta (2023-yil noyabr) havola mavjud.

Jumladan dissertatsiya ishining birinchi bobi bo‘yicha chop qilingan maqolaga 27 ta havola mavjud, quyida bir nechtasi keltirilgan: Physical Review Letters, 126, 244101, 2021, IF: 9.185; Chaos, Solitons & Fractals, 152, 111313, 2021, IF: 9.922; Scientific Reports, 12, 6904, 2022, IF: 4.997; Physical Review A, 103, 053302, 2021, IF: 2.971;

Dissertatsiya ishining ikkinchi bobi bo‘yicha chop qilingan 2 ta maqolaga 9 va 18 ta jami 27 ta havola mavjud, quyida bir nechtasi keltirilgan: Chaos, 33, 033141, 2023, IF: 3.741; Physics Letters A, 480, 128987, 2023, IF: 2.6; Physical Review A, 105, 063328, 2022, IF: 2.971;

Dissertatsiya ishining uchinchi bobi bo‘yicha chop qilingan maqolaga 4 ta havola mavjud, quyida bir nechtasi keltirilgan: Chaos, Solitons & Fractals, 164, 112665, 2022, IF: 9.922; Physical Review A, 108, 033312, 2023, IF: 2.971; Physical Review A, 106, 033309, 2022 IF: 2.971.

Tadqiqot natijalarining aprobatsiyasi. Ushbu dissertatsiyaning asosiy natijalari bir qator xalqaro va Respublika konferensiyalarida va ilmiy seminarlarda muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha Scopus ma’lumotlar bazasiga kiradigan xalqaro jurnallarda 4 ta ilmiy maqola nashr qilingan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosalar va adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 81 betdan iborat.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida tadqiqot mavzusining dolzarbligi va zarurati, dissertatsiyaning O‘zbekiston Respublikasi fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga muvofiqligi asoslangan. Tadqiqotning maqsad va vazifalari shakllantirilib, tadqiqot ob’ekti va predmeti ko‘rsatilgan, olingan natijalarning ilmiy yangiligi va amaliy natijalari ta’kidlangan, ularning ishonchliligi asoslangan, natijalarning ilmiy va amaliy ahamiyati ochib berilgan, amaliyotga joriy qilinganligi, chop qilingan nashrlar va dissertatsiya tuzilishi haqida ma’lumotlar berilgan.

Dissertatsiyaning “**Bir o‘lchamli Bose-Einstein kondensatida kvant tomchilari**” deb nomlangan birinchi bobida bir o‘lchamli kvant tomchilarining statik va dinamik xususiyatlari analitik va sonli metodlar yordamida o‘rganildi. Kvant fluktuatsiyalari ta’sirida ikki komponentli Bose-Einstein kondensatini ko‘rib chiqaylik. Komponentalar simmetrik bo‘lgan holda bu sistema bir-o‘lchamli Gross-Pitaevskii tenglamasi bilan ifodalanadi:

$$i\Psi_t + \frac{1}{2}\Psi_{xx} + \gamma|\Psi|^2\Psi + \delta|\Psi|\Psi = 0, \tag{1}$$

bu yerda tenglama o‘lchamsiz holatda yozilgan. Oxirgi kvadratik nochiziqli had kvant fluktuatsiyalari ta’siriga mos keladi va γ va δ lar o‘zaro ta’sir konstantalari. Statsionar holatda o‘zaro ta’sir konstantalarining ixtiyoriy qiymatlari uchun (1) tenglamaning umumlashgan aniq yechimi aniqlandi.

Tajribalarda tashqi tuzoqlar tomonidan hosil qilingan Bose-Einstein kondensatining dastlabki zichlik taqsimoti aniq yechim bilan mos kelmasligi mumkin. Shuning uchun kvant tomchilari parametrlarining vaqt bo‘yicha o‘zgarish dinamikasini o‘z ichiga oladigan va turli xil ta`sirlarni hisobga oluvchi yondashuvlarni ishlab chiqish juda muhimdir.

Kvant tomchilari parametrlari uchun dinamik tenglamalarni topish uchun biz Lagrange formalizmidan foydalandik. (1) tenglamaning Lagrange zichligi quyidagicha:

$$\mathcal{L} = \frac{i}{2}(\Psi\Psi_t^* - \Psi^*\Psi_t) + \frac{1}{2}|\Psi_x|^2 - \frac{\gamma}{2}|\Psi|^4 - \frac{2\delta}{3}|\Psi|^3 . \tag{2}$$

Quyidagi super-Gauss funksiyasidan foydalangan holda variatsion yaqinlashish metodi bilan kvant tomchilarining xossalari o‘rganildi:

$$\psi(r,t) = A\exp\left(-\frac{1}{2}\left(\frac{x}{w}\right)^{2m} + ibr^2 + i\varphi\right), \tag{3}$$

bu yerda $A(t), w(t), b(t)$ va $\varphi(t)$ lar variatsion parametrlar bo‘lib, mos ravishda amplituda, kenglik, chirp va boshlang‘ich fazalardir. Super-Gauss indeksi m vaqtga bog‘liq emas deb olindi, uning qiymati statsionar tenglamaning yechimidan topiladi $m = m_s$. Super-Gaussian funksiyasining tanlashning avzalligi u zarrachalar soni kam bo‘lganda “qo‘ng‘iroqcha” shakldagi yechimlarni va zarrachalar soni katta bo‘lgandagi "yassi-ustli" ko‘rinishdagi yechimlarni ifodalash imkonini beradi. "Yassi-ustli" ko‘rinishdagi yechimlar kvant tomchilarining fundamental xossasi hisoblanadi. Oddiy Gaussian funksiyasi bilan bunday yechimlarni ifodalab bo‘lmaydi.

Euler-Lagrange tenglamalaridan foydalangan holda kvant tomchilari parametrlari uchun harakat tenglamalari topildi,

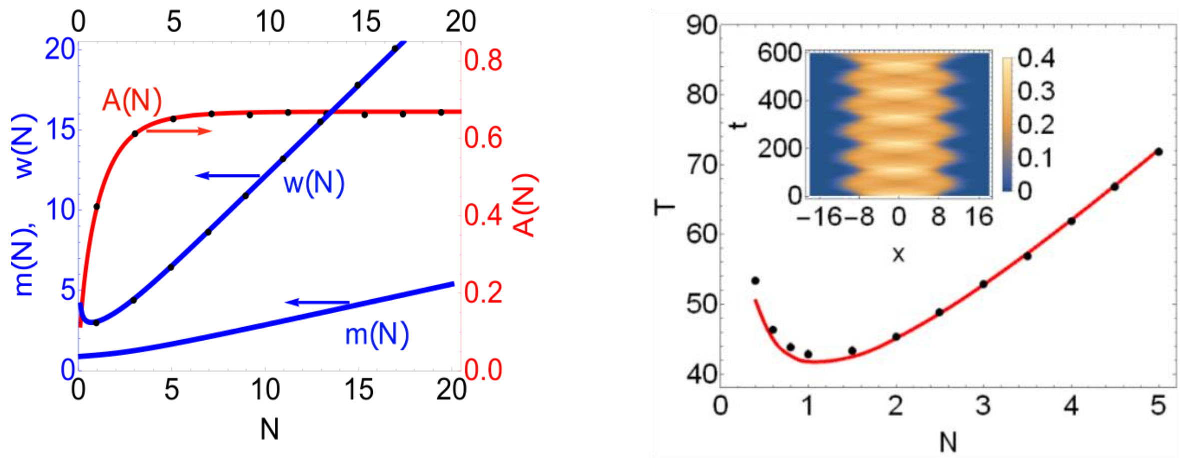
$$b_t = -\frac{1}{8\Gamma(1+3s)}\left[\frac{-3\Gamma(2-s)}{sw^4} + \frac{3\gamma}{2^s}\frac{N}{w^3} + \frac{2^{3/2+s}\delta\sqrt{N\Gamma(s+1)}}{3^s w^{5/2}} + 16\Gamma(3s+1)b^2\right], \tag{4}$$

$$w_t = 2wb.$$

Kimyoviy potensialning ko‘rinishi quyidagich:

$$\mu = \frac{-3\gamma N}{2^{s+3}\Gamma(s+1)w_s} - \frac{5\cdot 2^{s-1}\delta N^{1/2}}{3^{s+1}[2\Gamma(s+1)w_s]^{1/2}}. \tag{5}$$

Parametrlarning har xil qiymatlarida statsionar kvant tomchilari yechimlari topildi, yechimlarning turg‘unligi Vakhitov-Kolokolov kriteriyasi va sonli hisoblashlar orqali tekshirildi (1- rasm chap panel).



1-rasm: Chap panel: Kvant tomchisi kengligi w, super-Gaussian indeksi m (chap o‘q) va amplituda A ning (o‘ng o‘q) normaga bog‘liqligini ifodalaydi. O‘ng panel: Kvant tomchisi kengligining kichik tebranishlar davri T ning N ga bog‘liqligi ko‘rsatadi. Ichki grafikda kvant tomchisining zichlik taqsimoti $|\Psi(x,t)|^2$ ning dinamikasi ko‘rsatilgan, $N=5$ uchun. Har ikkala grafikda chiziqlar variatsion yaqinlashuvdan, nuqtalar esa (1) tenglamaning sonli simulyatsiyalaridan olingan. Boshqa parametrlar $\gamma=-1$ va $\delta=1$.

Harakat tenglamalaridan foydalanib parametrlarning har xil qiymatlari uchun effektiv potensial ko‘rinishi aniqlandi. Potensialning minimumi kvant tomchilarining statsionar kengligiga mos keladi. Statsionar holatidan siljitilgan kvant tomchilari muvozanat nuqtasi atrofida tebranma harakat qiladi, 1-rasm o‘ngdagi ichki grafik. Zarrachalar sonining har xil qiymatlari uchun kichik tebranishlarning davri variatsion yaqinlashishda va sonli hisoblashlar yordamida aniqlandi, 1- rasm o‘ng panel.

Dissertatsiyaning **“Ikki o‘lchamli Bose-Einstein kondensatida kvant tomchilari”** deb nomlangan ikkinchi bobida**,** o‘rtacha maydondan keyingi yaqinlashuvda ikki o‘lchamli ikki-komponentali Bose-Einstein kondensatida g‘alayonlangan yassi to‘lqinlarning modulyatsion noturg‘unligi va kvant tomchilari hosil bo‘lishi, shuningdek kvant tomchilari va uyurmalarning statik va dinamik xususiyatlari o‘rganildi. Bunday sistemani quyidagi Gross-Pitaevskii tenglamalari orqali ifodalash mumkin:

$$i\frac{\partial\psi_j}{\partial t}+\frac{1}{2}\nabla^2\psi_j-\frac{\psi_j}{2\sqrt{\sigma_1\sigma_2}}\left(\sqrt{\sigma_j/\sigma_{3-j}}\,|\psi_j|^2-|\psi_{3-j}|^2\right)-\sqrt{\sigma_j/\sigma_{3-j}}\,\psi_j\,p\ln(p)=0, \tag{6}$$

bu yerda, tenglama o‘lchamsiz holarda yozilgan, $j=1,2$, $\nabla^2=\partial_x^2+\partial_y^2$, σ_1 va σ_2 lar sochilish parametrlariga bog‘liq konstantalar, $p=(\sigma_1|\psi_1|^2+\sigma_2|\psi_2|^2)/(2\sqrt{\sigma_1\sigma_2})$. Oxirgi logarifmik had kvant fluktuatsiyalari ta’sirini ifodalaydi.

G‘alayonlangan yassi to‘lqin quyigagi ko‘rinishda olindi:

$$\psi_j=(A_j+\delta\psi_j)\exp(-i\mu_j t),\qquad \mu_j=\frac{A_j^2}{2\sigma_{3-j}}-\frac{A_{3-j}^2}{2\sqrt{\sigma_j\sigma_{3-j}}}+\sqrt{\sigma_j/\sigma_{3-j}}\,p_0\ln(p_0), \tag{7}$$

bu yerda $\delta\psi_j \ll A_j$, $p_0=(\sigma_1A_1^2+\sigma_2A_2^2)/(2\sqrt{\sigma_1\sigma_2})$, μ_j – kimyoviy potensial, A_j – yassi to‘lqin amplitudasi, $\delta\psi_j$ – kichik g‘alayonni ifodalaydi, $\delta\psi_j=0$ da (7) ifoda (6) ning statsionar yassi to‘lqin yechimini beradi. Chiziqli turg‘unlik taxlili yordamida g‘alayonlangan yassi to‘lqinlar amplitudasi $\delta\psi_j$ uchun quyidagi chiziqli tenglamalar olindi:

$$i\frac{\partial\delta\psi_j}{\partial t}+\frac{1}{2}(\delta\psi_{jxx}+\delta\psi_{jyy})-c_j(\delta\psi_j^*+\delta\psi_j)-c_3(\delta\psi_{3-j}^*+\delta\psi_{3-j})=0, \tag{8}$$

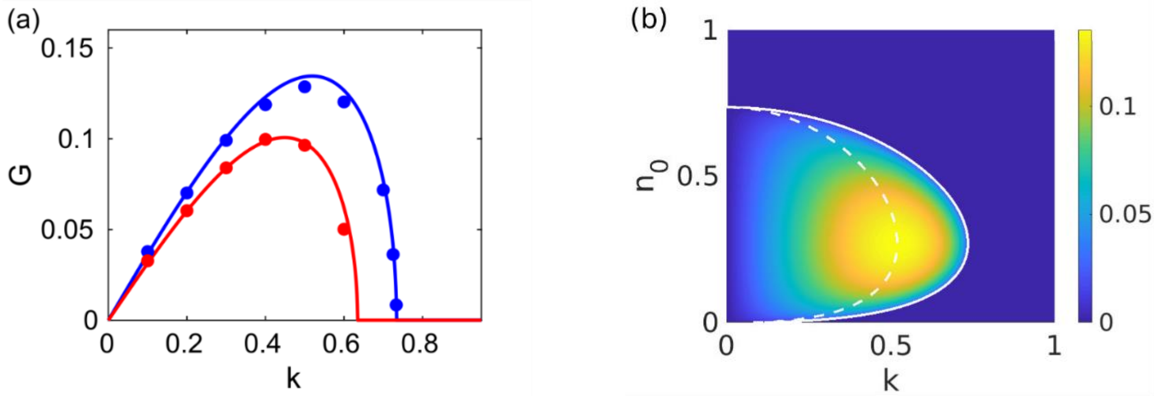
bu yerda $j=1,2$,

$$c_j\equiv\frac{A_j^2}{2\sigma_{3-j}}+\frac{A_j^2\sigma_j}{2\sigma_{3-j}}\ln\left(ep_0\right),\qquad c_3\equiv\frac{A_1A_2}{2}\left[-\frac{1}{\sqrt{\sigma_1\sigma_2}}+\ln\left(ep_0\right)\right], \tag{9}$$

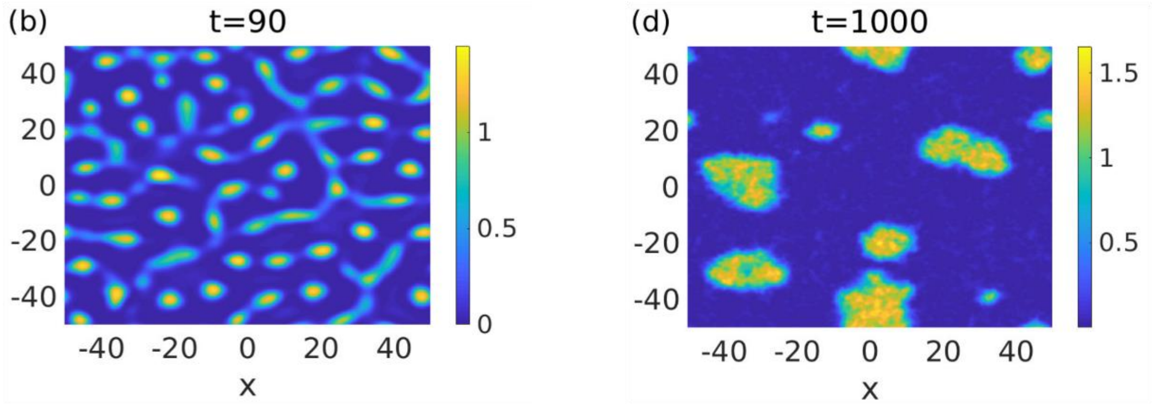
G‘alayonni $\delta\psi_j=u_j+iv_j$ ko‘rinishda olamiz va (8) tenglamani haqiqiy va mavhum qismlarga ajratamiz, hosil bo‘lgan differensial tenglamani yechimini $(u_j,v_j)\sim\exp(\lambda t+ik_x x+ik_y y)$ ko‘rinishda qidiramiz va algebraik tenglamalar sistemasini olamiz. Bu tenglamalarni taxlil qilish orqali quyidagi natijalarni topildi: G‘alayon amplitudasining keskin oshish shartlari $ep_0<1, |k|<k_{cr}$ aniqlandi. Modulyatsion noturg‘unlikning maksimal o'sish sur'ati, $G_{max}=k_{max}^2/2=k_{cr}^2/4$, unga mos keladigan to‘lqin soni $k_{max}=k_{cr}/\sqrt{2}$, va to‘lqin sonining kritik

$$|k|<k_{cr}\equiv\sqrt{2}\sqrt{-(c_1+c_2)+\sqrt{(c_1-c_2)^2+4c_3^2}}$$

qiymatlari aniqlandi. G‘alayonlangan yassi to‘lqinning amplitudasi to‘lqin sonining kritik qiymatidan ($k<k_{cr}$) kichik qiymatlarida noturg‘un, ($k\geq k_{cr}$) katta qiymatlarida esa turg‘unligi aniqlandi, 2 (a)- rasm.



2-rasm: (a) Modulyatsion noturg‘unlik o‘sish tezligi G ning to‘lqin soni k ga bog‘liqligi. Yuqori ko‘k chiziq simmetrik holat uchun $n_{10} = n_{20} = 0.3$ va pastki qizil chiziq assimetrik holat uchun $n_{10} = 0.3$ va $n_{20} = 0.1$. Nuqtalar to‘g‘ridan-to‘g‘ri sonli simulyatsiyalaridan topiladi. (b) Simmetrik holat uchun o‘sish tezligi G, k va n larning funksiyasi sifatida. Ranglar paneli G ning qiymatlarini ifodalaydi. Uzluksiz chiziq modulyatsion noturg‘unlik chegarasini ko'rsatadi, uzlukli chiziq esa o‘sish tezligining maksimal qiymati G_{max} ni ko'rsatadi. O‘zaro ta’sir parametrlari $\sigma_1 = \sigma_2 = 0.1$.



3-rasm: Zichlik taqsimotining odatiy dinamikasi quyidagi vaqt momentlarida (b), $t = 90$, (d) $t = 1000$. Boshlang‘ich zichlik $n_0 = 0.2$, va $\sigma_1 = \sigma_2 = 0.1$. Ranglar paneli $n = |\psi_1|^2 + |\psi_2|^2$ ni ifodalaydi.

Parametrlarning har xil qiymatlarida turg‘unlik va noturg‘unlik sohalari topildi, 2 (b)-rasm. Olingan natijalariga asoslangan holda tajribalarda modulatsion noturg‘unlikni kuzatishning quyidagi usulini taklif qilindi. Birinchi navbatta yuqori zichlikli ikki komponentli kondensat hosil qilish kerak. Bu holat uchun kondensat turg‘un. Keyin kondensat zichligini kamaytirish, masalan, tashqi tuzoqni kengaytirish orqali modulatsion noturg‘unlikni kuzatish mumkin.

Modulyatsion noturg‘unlikning keyingi nochiziqli bosqichida kvant tomchilari hosil bo‘ladi. Bu bosqich sonli hisoblashlar yordamida o‘rganildi. Hosil bo‘lgan kvant tomchilarining soni vaqt o‘tishi bilan ularning bir-biri bilan ta’sirlashib bir-biriga birikishi orqali keskin kamayadi. Parametrlarning har xil qiymatlarida kvant tomchilarining soni, kamayish qonuniyati va kamayish tezliklari aniqlandi, 3-rasm. Hosil bo‘lgan kvant tomchilarining sonini hisoblashda MATLABning rasmga ishlov berish texnikasidan foydalanildi.

Bobning ikkinchi qismida kvant fluktuatsiyalari ta'siri ostida Bose-Einstein kondensatida ikki o'lchamli kvant tomchilari va uyirmalarning statik va dinamik xususiyatlari nazariy va sonli metodlar yordamida o'rganildi. Kvant tomchilari va uyirmalarning xususiyatlarini o'rganishda biz zarrachalar soni, massasi va o'zaro ta'sir konstantalari teng bo'lgan simmetrik Bose-Einstein kondensatini ko'rib chiqdik. Kvant fluktuatsiyalarini hisobga oladigan bunday sistema quyidagi Gross-Pitaevskii tenglamasi orqali ifodalanadi:

$$i\partial_t\Psi+\frac{1}{2}\nabla^2\Psi+\delta|\Psi|^2\ln(|\Psi|^2)\Psi=0, \tag{10}$$

bu yerda, tenglama o'lchamsiz holatda yozilgan. $\nabla^2=\partial_x^2+\partial_y^2$ ikki o'lchamli Laplace operatori. (10) ning analitik ko'rinishdagi aniq yechimi mavjud emas, shuning uchun biz variatsion yaqinlashish metodidan foydalandik. Kvant tomchisi va uyurmalarni ifodalashda quyidagi super-Gaussian funksiyasidan foydalandik:

$$\Psi(r,\theta,t)=Ar^S\exp\left[-\frac{1}{2}\left(\frac{r}{w}\right)^{2m}+i(br^2\pm S\theta+\varphi)\right], \tag{11}$$

bu yerda $A(t)$, $w(t)$, $b(t)$, va $\varphi(t)$ lar variatsion o'zgaruvchilar bo'lib, ular mos ravishda amplituda parametri, kenglik, chirp va boshlang'ich fazalarni ifodalaydi. Profil shaklini ifodalaydigan super-Gaussian parametri m, statsionar yechimdan topiladi. Topologik zaryad $S\geq 0$, butun qiymatlarni qabul qiladi. Agar $S=0$ bo'lsa, (11) tenglama kvant tomchilarini ifodalaydi, $S=1,2,3,...$ bo'lgan holatlar uyurmalarni ifodalaydi.

Euler-Lagrange tenglamalari orqali kvant tomchilari va uyurmalarni ifodalaydigan parametrlar uchun dinamik tenglamalar olindi. Kenglik uchun tenglamani ko'rinishi quyidagicha:

$$w_{tt}=-\frac{\Gamma(M(S+1))}{\Gamma(M(S+2))}\frac{\partial G}{\partial w}, \tag{12}$$

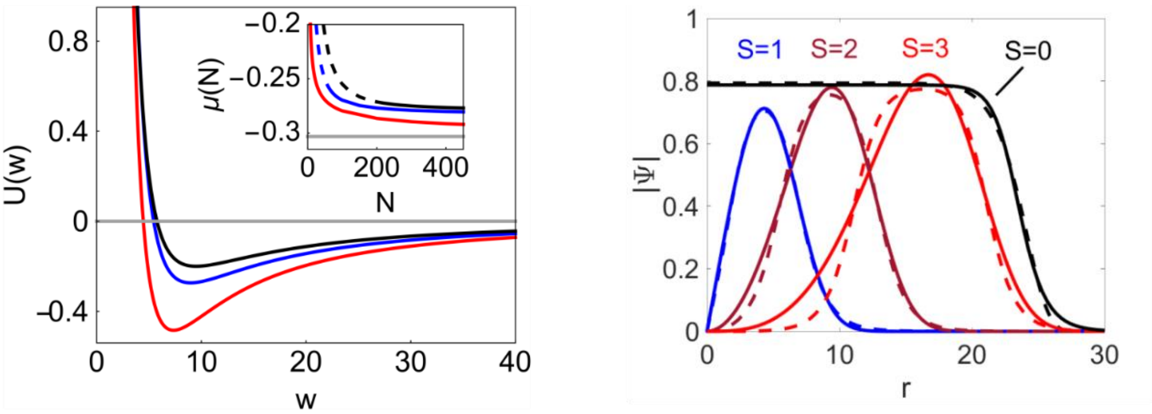
bu yerda

$$\begin{aligned}G(w,N,M)&=\frac{\Gamma(MS+2)}{2M^2\Gamma(M(S+1))w^2}+\frac{N\Gamma(M(2S+1))}{2^{2+M(2S+1)}\pi M\Gamma^2(M(S+1))w^2}\\&\times\delta\{1+M+2MS\left[1-\psi(M(2S+1))\right]-2\log\left[\frac{N}{2^{MS}\pi Mw^2\Gamma(M(S+1))}\right]\},\end{aligned} \tag{13}$$

$\psi(z)=d\ln\Gamma(z)/dz$ – digamma funksiya, $N=\pi MA^2w^{2(S+1)}\Gamma(M(S+1))$.

Bu tenglamalardan foydalanib effektiv potensial, kichik tebranishlar chastotalari va kimyoviy potensiallar aniqlandi. Effektiv potensialning ko'rinishi 4-rasm chap tomonda keltirilgan. Kimyoviy potensialning ko'rinishi $\mu=\partial E_s/\partial N$ dan aniqlanadi, bu yerda $E_s=NG$ – kvant tomchisining statsionar energiyasi.

Kvant tomchilari va uyurmalarning turg'unligi Vakhitov-Kolokolov kriteriyasi (4-rasm chap paneldagi ichki grafik) va sonli metodlar orqali tekshirildi, (4- rasm o'ng panel).



4-rasm: Chap panel: (S, N, m) ning turli qiymatlari uchun effektiv potensialning ko‘rinishi. Pastki (qizil), o‘rta (ko‘k) va yuqori (qora) chiziqlar mos ravishda $(0,100,2,537)$, $(1,200,1,971)$ va $(2,200,2,073)$ uchun. Ichki chizilgan grafik, kimyoviy potentsial μ ning norma N ga bog‘liqligini ifodalaydi. Ichki grafikda kulrang to‘g‘ri chiziq Tomas-Fermi chegarasini ifodalaydi. O‘ng panel: Kvant tomchilari va uyurmalarning profili $|\Psi|$ ko‘rinishi, $S=1,2,3,0$, va normalar mos ravishda $N=60,200,510,1000$ bo‘lgan holatlar uchun uzluksiz chiziqlar variatsion metoddan topilgan, uzlukli chiziqlar esa sonli metodlardan aniqlangan.

Uyurmalar uchun zarrachalar soni (norma) ga bog‘liq holda turg‘un va noturg‘un sohalari aniqlandi. Noturg‘un sohada topologik zaryadi S bo‘lgan uyurmalarning odatda $S+1$ ta fragmentlarga parchalanishi aniqlandi. Kvant tomchilarining dinamikasi o‘zarota’sir konstantalari vaqt bo‘yicha davriy o‘zgaruvchi bo‘lgan holatlarda sonli hisoblashlar orqali o‘rganildi. Bunda kvant tomchilarining resonans tebranishlari va tashqi modulyatsiya amplitudasiga bog‘liq ravishda adiabatik tebranishlari, bug‘lanishi va parchalanish holatlari aniqlandi.

Dissertatsiyaning “**Uch o‘lchamli Bose-Einstein kondensatida kvant tomchilari**” deb nomlangan uchinchi bobida kvant fluktuatsiyalarini hisobga oladigan ikki-komponentali Bose-Einstein kondensatida, uch o‘lchamli kvant tomchilarining statik va dinamik xususiyatlari, shuningdek o‘zaro ta’sir konstantalari vaqt bo‘yicha davriy o‘zgaruvchi bo‘lgan hollari analitik va sonli metodlar yordamida o‘rganildi. Hisoblashlar komponentalarning simmetrik bo‘lgan, ya’ni atom massalari, atomlar soni va o‘zaro ta’sir konstantalari bir xil bo‘lgan Bose gazlari uchun amalga oshirildi. Bu sistemani quyidagi Gross-Pitayevskii tenglamasi orqali ifodalash mumkin:

$$i\psi_t + \frac{1}{2}\nabla^2\psi + \alpha|\psi|^2\psi - \beta|\psi|^3\psi = 0, \tag{14}$$

bu yerda tenglama (14) o‘lchamsiz holda keltirilgan. α va β parametrlar ixtiyoriy tanlanishi mumkin. Ular o‘lchamsiz tenglamada o‘rtacha-maydon va kvant fluktuatsiyalari o‘zaro ta’sirini ifodalaydi. (14) tenglamada, $g_{12}=g$ deb olish orqali ikki-atom o‘zaro ta’sirini tenglamada hisobga olmaslik mumkin ($\alpha=\delta g=0$). U holda itaruvchi xarakterga ega kvant fluktuatsiyalari tashqi potensial yordamida muvozanatlashtiriladi. Bunday sistema Lee-Huang-Yang suyuqligi deb yuritiladi va birinchi marta nazariy jihatdan [9] havolada va eksperimental jihatdan [10] havolada o‘rganilgan.

Zichlik taqsimotini bir- va ikki-o‘lchamlarda hisoblanganidek super-Gaussian funksiyasi ko‘rinishida ta’savvur qilamiz, ya’ni (3) tenglamada $x=r$ deb olish kerak. Super-Gaussian indeksi m ning qiymati bir- va ikki-o‘lchamlardagidek statsionar tenglamaning yechimidan topiladi. Norma $N=4\pi A^2 w^3\Gamma(1+3M)/3$, ($M=1/2m$) saqlanuvchi kattalik bo‘lib, kvant tomchisidaga atomlar soniga proporsional. $\Gamma(z)$ – Gamma funksiya. Sistemaning Lagrange funksiyasi quyidagi $L=4\pi\int_0^\infty r^2\mathcal{L}dr$ integral yordamida topiladi, bu yerda $\mathcal{L}$ (14)- tenglamaning Lagrange zichligi:

$$\frac{L}{N}=\varphi_t+\frac{\Gamma(5M)}{\Gamma(3M)}w^2(2b^2+b_t)+G(M,w,N), \tag{15}$$

bu yerda

$$G(M,w,N)=\frac{3\Gamma(M+2)}{8M\Gamma(3M+1)w^2}-\frac{G_1\alpha}{w^3}+\frac{G_2\beta}{w^{9/2}},$$

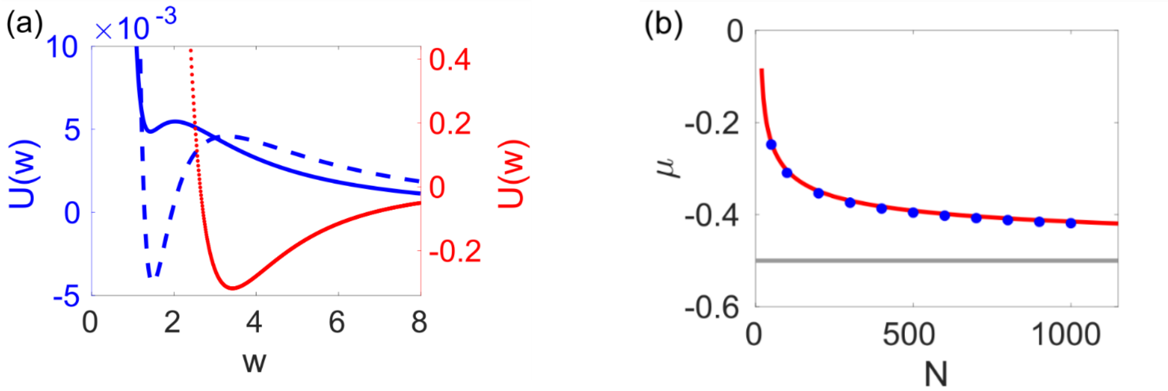
$$G_1\equiv\frac{3\cdot 2^{-3(M+1)}N}{\pi\Gamma(3M+1)},\quad G_2\equiv\frac{3\sqrt{3}2^{3M-2}}{5^{3M+1}}\left(\frac{N}{\pi\Gamma(3M+1)}\right)^{3/2}. \tag{16}$$

Euler-Lagrange tenglamalaridan foydalangan holda kvant tomchilari parametrlari uchun harakat tenglamalari olindi. Kvant tomchisining kengligi uchun harakat tenglamasi quyidagicha:

$$w_{tt}\equiv-\frac{\partial U(w)}{\partial w},\quad U\left(w\right)=\frac{\Gamma(3M)}{\Gamma(5M)}G, \tag{17}$$

bu yerda $U(w)$ – effektiv potensial, kichik tebranishlar chastotasi effektiv potensialdan kenglik bo‘yicha ikkinchi tartibli hosila sifatida aniqlanadi, $\Omega_0^2=\partial^2U(w)/\partial w^2$.

Potensialning ko‘rinishi zarrachalar sonining har xil qiymatlari uchun 5-rasmda (a) panelda keltirilgan. O‘zaro ta’sir konstantalarining berilgan qiymatlari uchun, zarrachalar sonining kritik qiymatidan N_{cr} boshlab kvant tomchilari paydo bo‘ladi, kritik qiymatdan chegaraviy qiymatlar oralig‘ida $N_{cr}<N<N_{th}$ kvant tomchilari metastabil bo‘ladi, chegaraviy qiymatidan katta holatlarda $N>N_{th}$ kvant tomchilari stabil bo‘ladi. O‘lchamsiz tenglamada o‘zaro ta’sir konstantalari $(\alpha,\beta)=(3,5/2)$ bo‘lganda, havola [1] da Bogoliubov-de Gennes tenglamasini sonli yechish orqali normaning kritik va chegaraviy qiymatlari $18.65\le N<22.55$ intervalda bo‘lishi ko‘rsatilgan. Havola [11] ga ko‘ra bu qiymatlar haqiqiy eksperimentlarda [7, 8] mavjudligi va o‘zaro ta’sir konstantalarining aynan shu qiymatlarida shu intervalga mos kelishi e'lon qilingan. Shuning uchun biz variatsion yaqinlashish metodi orqali olingan natijalarni boshqa yo‘l bilan olingan [1] va eksperimentlar [7, 8] bilan solishtirish maqsadida hisoblashlarda o‘zaro ta’sir konstantalarining shu qiymatlarini tanladik. Variatsion yaqinlashuvda normaning kritik va chegaraviy qiymatlari $19.5\le N\le 23.05$ intervalga mos kelishi ko‘rsatildi. Bulardan kvant tomchilarining xususiyatlarini biz taklif qilgan variatsion yaqinlashish yordamida o‘rganish haqiqiy eksperiment orqali olingan natijalar bilan mos kelishini ko‘rish mumkin. Kvant tomchilarining turg‘unligi Vakhitov-Kolokolov kriteriyasi va sonli hisoblashlar orqali tekshirildi. 5-rasm (b) panelda kimyoviy potensialning normaga bog‘lanishi keltirilgan. Vakhitov-Kolokolov stabillik kriteriyasi bo‘yicha $d\mu/dN<0$ hosilaning manfiy bo‘lishi kvant tomchilarining turg‘unligini bildiradi.



5-rasm: (a) N ning turli qiymatlari uchun effektiv potentsiallar shakli. Uzluksiz chiziq, uzuq chiziq va nuqtalar (o‘ng o‘q) mos ravishda 20, 25 va 200 lar uchun. (b) Kimyoviy potentsial μ normaning N funksiyasi sifatida, $(\alpha,\beta)=(3,5/2)$ parametrlar uchun. Qizil chiziq variatsion yaqinlashishdan olingan va nuqtalar mavhum vaqt simulyatsiyasidan topilgan. Kulrang chiziq Tomas-Fermi chegarasini ifodalaydi.

Kvant tomchilari dinamikasi o‘zaro ta’sir konstantalari $\alpha(t)=\alpha_0[1+\epsilon_1\sin(\omega t)]$, $\beta=\beta_0[1+\epsilon_2\sin(\omega t)+\theta]$ davriy o‘zgaruvchi bo‘lgan hollardayam o‘rganildi. Bu yerda $(\alpha_0,\beta_0)=(3,5/2)$, $\epsilon_1,\epsilon_2\ll 1$, ω_m va θ lar mos ravishda modulyatsiya amplitudasi, chastotasi va boshlang‘ich fazalari. Eksperimentlarda bunday modulyatsiyalar Feshbach rezonans texnikasi orqali amalga oshiriladi. Tashqi periodik modulyatsiya natijasida kvant tomchisining modulyatsiya amplitudasiga bog‘liq ravishda ikki hil regimi aniqlandi, kichikroq amplitudalarda adiabatik tebranish va kattaroq amplitudalarda kvant tomchisining bug‘lanishi aniqlandi.

Sonli hisoblashlarda, tashqi modulyatsiya chastotasi erkin tebranish chastotasiga teng bo‘lganda rezonans biyeniya hodisasi kuzatildi. Sonli hisoblashlarda rezonans chastotasi amplituda farqining tashqi modulyatsiya chastotasiga bog‘liqligidan aniqlash mumkin. Kvant tomchisi kengligini ifodayldigan (17) tenglamani, statsionar kenglik atrofida kichik siljish bo‘yicha qatorga yoyilmasidan $w=w_s+\xi$, bu yerda $\xi\ll w_s$ quyidagi chiziqli tenglamani olamiz:

$$\ddot{\xi}+\Omega_0^2\xi=\left(-\frac{3\Gamma(3M)\alpha_0G_1\epsilon_1}{\Gamma(5M)w_s^4}+\frac{9\Gamma(3M)G_2\beta_0\epsilon_2}{2\Gamma(5M)w_s^{11/2}}\right)\sin(\omega_m t), \tag{18}$$

bu yerda $\ddot{\xi}$ - ξ dan vaqt bo‘yicha ikkinchi tartibli hosilani bildiradi, G_1 va G_2 lar (16) tenglamadan topiladi. (18) tenglamaning o‘ng tomoni nol bo‘lish shartidan modulyatsiya amplitudalarining quyidagi munosabatini olamiz $\epsilon_2=2\alpha_0G_1w_s^{3/2}\epsilon_1/3\beta_0G_2$. Agar ϵ_1 va ϵ_2 lar bu shartni qanoatlantirsa modulyatsiya chastotasining har qanday qiymatida kvant tomchisining dinamikasi vaqt bo‘yicha o‘zgarmaydi. Agar bu shart bajarilmasa kvant tomchisining dinamikasi modulyatsiya chastotasining qiymatiga bog‘liq bo‘ladi. Tashqi modulyatsiya chastotasi ω_m sistemaning erkin tebranish chastotasi Ω_0 ga teng bo‘lsa, sistemada rezonans hodisasi kuzatiladi. Rezonans chastotasi yaqinida biyeniya va rezonans kengligidan tashqarida tebranishlar kuzatiladi. Bundan tashqi modulyatsiyalar boshlang‘ich fazaga bog‘liq bo‘lishini ko‘rish mumkin.

Haqiqiy tajribalarda biz taklif qilgan modelning parametrlarini ^{39}K atomining har xil holatlari uchun baholaylik. ^{39}K atomining massasi $m = 6.49\times10^{-26}$ kg ga teng. O‘zaro ta’sir konstantalari $|\delta g| \ll g$ shartni qanoatlantirishi kerak, shuning uchun sochilish parametrlarini quyidagicha tanlab olamiz $a_{11} = a_{22} = a = 50a_0$, $a_{12} = -42a_0$ bu yerda a_0 - Bohr radiusi. $|\alpha| = 3$ va $|\beta| = 5/2$ uchun sistemaning xarakteristik parametrlari quyidagicha $r_s \approx 0.413$ μm, $t_s \approx 0.11$ ms, $N_s \sim \psi_s^2 r_s^3 \approx 460$. O‘lchamsiz $t = 1000$, 105 ms ga mos keladi. Sonli simulyatsiyalarda biz integrallash domenini $d = 80$ deb olgan edik, bu fizik birliklarda ≈ 33 μm ga mos keladi. $N = 1000$ uchun kvant tomchisidagi atomlar soni va tomchining o‘lchami mos ravishda $4.65\cdot10^5$ and 3 μm ga teng. Bu parametrlar odatiy eksperimentdagi parametrlar intervallari bilan mos keladi.

XULOSALAR

Ushbu dissertatsiya ishi markaziy nazariy vosita sifatida super-Gaussian funksiyasiga asoslangan variatsion yondashuvdan foydalangan holda bir-, ikki- va uch-o‘lchamli geometriyalarda kvant tomchilarining yagona tadqiqotini taqdim etadi. Tadqiqot bir o‘lchamli Bose-Einstein kondensatida statsionar holatdagi kvant tomchilari parametrlarini bashorat qilishda variatsion metodning ishonchliligini ko‘rsatishdan boshlanadi, I bobga qarang. Nazariy natijalar va sonli simulyatsiyalar o‘rtasidagi kuzatilgan moslilik, kvant tomchisi dinamikasi va shakli tebranishlari haqida qimmatli ma’lumotlarni olishda super-Gaussian funksiyasining samaradorligini ko‘rsatadi. Ushbu funksiyaga qo’shimcha parametrlarni kiritish jarayonni texnik jihatdan murakkablashtiradi. Shunga qaramay, ushbu yondashuv kvant tomchisining xarakteristikalari uchun aniq munosabatlarni olish imkonini beradi.

Bir o‘lchamli geometriyadan tashqari, tadqiqot ikki o‘lchamli ikki-komponentali Bose-Einstein kondensatlarida modulyatsion noturg‘unlikni o‘rganadi (§ 2.1-bo‘limga qarang), kichik zichliklarda noturg‘unlik saqlanib qoladigan chegaraviy zichlikning qiymatlari aniqlanadi. Modulyatsion noturg‘unlik tufayli kvant tomchilarning paydo bo‘lishi sonli simulyatsiyalar bilan tasdiqlanadi, bular tomchilarning shakllanishida kvant fluktuatsiyalarining rolini har tomonlama to‘liq tushunishga zamin yaratadi. Ikki o‘lchamli kvant tomchisining xususiyatlari batafsil tahlil qilish § 2.2 bo‘limda amalga oshirilgan. Kvant tomchilarning turg‘unligi va ularning tashqi vaqtga bog‘liq modulyatsiyaga ta’siri ham o‘rganilgan.

Tadqiqotni uch o‘lchamga kengaytirgada ham super-Gaussian funksiyasiga asoslangan variatsion yaqinlashish statsionar uch o‘lchamli kvant tomchilarini tavsiflash uchun ishonchli yaqinlashish bo‘lib xizmat qilishda davom etadi, III bobga qarang. Variatsion yondashuv ishonchli ravishda kvant tomchilarining dinamikasini aks ettiradi, kichik tebranishlarning chastotalarini aniqlaydi va tashqi modulyatsiyalar ostida kvant tomchilari turli hil rejimlarini tavsiflaydi.

Tadqiqot davomida super-Gaussian funksiyasi Bose-Einstein kondensatidagi mujassamlashgan to‘lqinlar shaklini tavsiflash uchun universal vosita sifatida ishlatiladi. Barcha o‘lchamlarda, nazariy natijalarning sonli simulyatsiyalar bilan mos kelishi, kvant tomchilarining hosil bo‘lishi, turg‘unligi va dinamikasini o‘rganishda variatsion metodning samaradorligini ko‘rsatadi. Bularga, qo‘shimcha ravishda, tadqiqot Bose-Einstein kondensatlarining zichlik taqsimotini tahlil qilishda tasvirni qayta ishlash metodlaridan foydalanish mumkinligini ko‘rsatadi. Ushbu izchil tadqiqot barcha o‘lchamlarda olingan natijalarning turli jihatlarini birlashtirib, kvant tomchilarini yaxlit ravishda o‘rganishni taklif qiladi.

Dissertatsiya ishi bo‘yicha olib borilgan tadqiqotlarda quyida asosiy natijalar olingan:

1. Super-Gaussian funksiyasidan foydalangan holda variatsion yondashuv ishlab chiqilgan. Ushbu funksiya siqilmaydigan suyuqlik tabiatiga ega bo‘lgan yassiustli zichlik taqsimoti ko‘rinishidagi mujassamlashgan holatlarni yaxshi tavsiflaydi. Ishlab chiqilgan yondashuv asosida bir va ko‘p o‘lchamli Bose-Einstein kondensatlarida kvant tomchilarini tavsiflovchi parametrlar uchun, shuningdek, ikki o‘lchamda uyurmalar uchun dinamik tenglamalar topiladi.

2. Barcha o‘lchamlarda super-Gaussian funksiyasiga asoslangan variatsion metoddan foydalangan holda statsionar kvant tomchilarini ifodalaydigan parametrlarning atomlar soniga bog‘liqligi aniqlandi. Xuddi shunday, ikki-o‘lchamda uyurmalarni ifodalaydigan parametrlarning atomlar soniga bog‘liqligi aniqlandi. Bularga qo‘shimcha ravishda, effektiv potensial va kichik tebranishlar chastotalari aniqlandi.

3. Kvant tomchilari va uyurmalarning turg‘unlik sohalari aniqlandi. Noturg‘unlikning rivojlanish ssenariylari sonli metodlar orqali tahlil qilindi.

4. Bir-o‘lchamli Bose-Einstein kondensatida o‘zaro ta’sir konstantalarining ixtiyoriy qiymatlari uchun statsionar kvant tomchilarining umumlashgan aniq yechimi topildi.

5. Ikki-o‘lchamli Bose-Einstein kondensatida g‘alayonlangan yassi to‘lqinlarning eksponensial o‘sish shartlari aniqlandi. Parametrlar fazosida g‘alayonlangan yassi to‘lqinlarning stabil va nostabil sohalari topildi. Modulyatsion noturg‘unlikning nochiziqli bosqichida kvant tomchilari hosil bo‘lishi va hosil bo‘lgan tomchilarning kamayishi qonunini aniqlandi.

6. O‘zaro ta’sir konstantalari davriy o‘zgaruvchi bo‘lgan uch- va ikki o‘lchamli Bose-Einstein kondensatida kvant tomchilarining resonans tebranishlari, tashqi modulyatsiya amplitudasiga bog‘liq ravishda adiabatik tebranishlar va kvant tomchilarining bug‘lanishlari aniqlandi. Shuningdek, uch o‘lchamli kvant tomchilari uchun tomchi shaklining ikki chastotali tashqi modulyatsiyaga javobi fazalar farqiga bog'liqligi ko'rsatilgan.

7. Uch o‘lchamli kvant tomchilari uchun Lagrange formalizmidan foydalanib,

tomchi mavjudligi uchun zarur bo‘lgan atomlarning kritik soni aniqlanadi. Turg‘un tomchilar hosil bo‘lishi uchun atomlar sonining chegaraviy qiymati ham aniqlangan. Mavjudlik va turg‘unlikning ushbu sohalari boshqa mualliflarning sonli ravishda olgan natijalariga, shuningdek, uch o‘lchamli kvant tomchilarida o‘tkazilgan haqiqiy tajribalarda olingan natijalar bilan mos kelishi ko‘rsatildi.

8. Kvant fluktuatsiyasi ta’siri ostidagi bir-, ikki-, va uch-o‘lchamli Bose-Einstein kondensati dinamikasini modellashtirish uchun sonli kodlar yaratildi. Bundan tashqari, rasmga ishlov berish metodining Bose-Einstein kondensati zichlik taqsimotini tahlil qilishda foydalanish mumkinligi ko‘rsatildi.

SCIENTIFIC COUNCIL DSc.03/30.12.2019.FM.01.09
ON AWARDING OF THE SCIENTIFIC DEGREES AT THE
NATIONAL UNIVERSITY OF UZBEKISTAN

OTAJONOV SHERZOD RUSTAMOVICH

MATTER WAVES IN A BOSE-EINSTEIN CONDENSATE UNDER THE ACTION OF QUANTUM FLUCTUATIONS

01.04.02 - Theoretical Physics

**Abstract of the Dissertation for the Degree of Doctor of Philosophy (Ph.D.)
in Physical and Mathematical Sciences**

Tashkent-2024

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The Abstract of the dissertation is posted in three languages (Uzbek, English, Russian) on the website "ZiyoNet" Information and educational portal http://www.ziyonet.uz/.

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The text of the Dissertation is available at the Information-resource Centre at the National University of Uzbekistan (is registered No   ) (Address: University str. 4, Almazar area, Tashkent, 100174, Uzbekistan, Ph.: (99871) 227-12-24).

This Abstract of the Dissertation has been sent out on "____" ______________ 202__ year
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INTRODUCTION (abstract of the PhD dissertation)

Topicality and demand of the theme of the dissertation. When atoms (bosons) are cooled to almost absolute zero, they gather in the energy level, forming what is known as a Bose-Einstein condensate (BEC). In 1995, advancements in the experimental methods allowed two separate research groups to finally observe the Bose-Einstein condensate experimentally, leading to them being awarded the Nobel Prize in 2001.

Presently, the international scientific community conducts numerous theoretical and experimental studies focused on exploring properties of matter-waves in Bose-Einstein condensates. These investigations are of utmost importance as they address fundamental issues in modern physics. By subjecting atomic gases in Bose-Einstein condensates to external magnetic and optical fields, researchers can effectively manipulate them, forming the basis for various experiments and precise theoretical models. The potential applications of Bose-Einstein condensates are diverse, encompassing matter-wave interferometers, atomic-wave lasers, atomic clocks, qubits or memory units in quantum computers, quantum cryptography for factoring numbers into primes, and other purposes.

From the mean-field perspective, two- and three-dimensional condensates in Bose-Einstein condensates with attractive two-body interactions are inherently unstable, resulting in their collapse. There are several viable methods to stabilize the condensate, such as employing external traps, accounting for three-atom interactions, dipolar interactions, and periodically altering scattering lengths.

Recently, it was demonstrated theoretically that two- and three-dimensional unstable condensates can achieve stability by accounting for quantum fluctuations [1]. It turns out that quantum fluctuations lead to a formation of ultra-dilute liquid-like state of matter. Self-bound structure of such a state is known as quantum droplets [1]. Shortly after this breakthrough, experimental evidence verified the stability of such systems and a formation of quantum droplets (QDs). In present work, density distribution and dynamical properties of QDs in BECs are studied in all three geometrical dimensions.

The description above shows that the physics of matter waves in Bose-Einstein condensates is rapidly developing. In our State, during the last decades, great attention has been paid to the development of science, especially theoretical physics, to provide conditions at the higher international level for basic research.

This work fulfils the tasks is stipulated by the regulatory documents of the government and the Decree of the President of the Republic of Uzbekistan “On measures of the further improvement of the activities of the Academy of Sciences, organization, management, and financing of scientific research works” No. PQ-2789, Decree No. PF-4947, dated February 7, 2021, “On the strategy of actions for the further development of the Republic of Uzbekistan”, and Decree No. PQ-5032, dated March 19, 2021 “On measures to improve the quality of education and improve scientific research in physics”.

Conformity of research to priority directions of development of science and technologies of the Republic of Uzbekistan. The research of the present thesis has been carried out in accordance with the priority fields of science and technology development of the Republic of Uzbekistan: II “Physics, Astronomy, Energy and Mechanical Engineering”.

Degree of study of the problem. Since its pioneering work [1] and subsequent experimental realization, the study of quantum droplets formed due to quantum fluctuations in the Bose-Einstein condensate has garnered significant interest among scientists. Numerous researchers have explored various aspects of matter waves affected by quantum fluctuations, including dimensional reductions from three dimensions to two and one dimensions [2], the dynamics of one-dimensional quantum droplets [3], and the investigation of two-dimensional quantum droplets and vortices through numerical methods [4]. Additionally, the stability of three-dimensional quantum droplets and vortices has been examined in relation to the number of particles [5].

The experimental realization of quantum droplets was first achieved in a dipolar condensate [6] and later in bosonic mixtures [7-8]. Mathematical models have been developed, and essential experimental evidence has been obtained, shedding light on the properties of quantum droplets. However, many aspects of the impact of quantum fluctuations on matter waves still need to be studied, and many questions await their solutions. These include the dynamics of one-dimensional and multidimensional quantum droplets and vortices, the response of the system to the periodic variation in time of the parameters, and the formation of quantum droplets from perturbed plane waves due to the phenomenon of modulation instability and their interaction. We address these questions in this work.

Connection of the dissertation research with the plans of research works of the scientific institution, where the dissertation was performed. The research was done in accordance with the plan of research activities of the Physical-Technical Institute of the Academy of Sciences of the Republic of Uzbekistan, including the project under the grant of the Ministry of Innovation Development of the Republic of Uzbekistan FA-F2-004 “Dynamics and interaction of nonlinear localized waves in quantum and dissipative systems”, 2017 - 2020, “Investigation of the dynamics of localized waves in quantum gases and nonlinear optical media”, 2020 - 2021.

The aim of the research is to study the properties of matter waves in multi- dimensional two-component Bose-Einstein condensates in the presence of quantum fluctuations, and to develop a theoretical formalism for description of localized structures in this system. In particular, stationary parameters and dynamical behavior of quantum droplets are considered.

The tasks of the research are: To develop mathematical models that can elucidate the characteristics of quantum droplets, vortices and plane waves in one- and multi-dimensional Bose-Einstein condensates.

To study cases when interaction parameters are periodic time variables in multidimensional Bose-Einstein condensates.

To reveal the conditions for transforming Bose-Einstein condensates with a uniform density distribution into quantum droplets due to modulation instability, determining the regions of instability in the parameter space, and studying the dynamics of generated quantum droplets.

To perform numerical simulations of the one- and multi-dimensional Gross-Pitaevskii for the aforementioned physical processes to justify the accuracy of approximate theoretical methods.

The objects of the research are quantum droplets, vortices and plane waves in one- and multi- dimensional Bose-Einstein condensates.

The subjects of the research are the parameters of localized states with the different number of particles, the period of small oscillations, the splitting of localized states due to resonance oscillations, and stability conditions of plane waves, quantum droplets and vortices.

The methods of the research. In this study, we employ a combination of analytical and numerical research methods to investigate the dynamics of complex physical systems. For the analytical part of our research, we use variational approximation (VA), Thomas-Fermi approximation, and linear stability analyses. These methods enable us to derive deep theoretical insights and gain a proper understanding of the underlying principles governing the system's behavior.

To complement our analytical approach, we employ various numerical techniques for performing calculations and simulations. Specifically, we focus on solving Gross-Pitaevsky type equations, which are fundamental to our investigation. The numerical methods employed in this study include the fourth-order Runge-Kutta method and the Crank-Nicolson method, both of which are widely acknowledged for their accuracy and stability in solving nonlinear partial differential equations.

Furthermore, we leverage image processing techniques as a part of our numerical analysis, enabling us to extract valuable information from experimental data and visualize the system's behavior. By combining these analytical and numerical research methods, we aim to provide a comprehensive and robust analysis of the complex physical phenomena under investigation. This approach allows us to bridge the gap between theoretical predictions and empirical evidence, thus yielding a more complete understanding of the system's dynamics and contributing to the advancement of scientific knowledge in this field.

The scientific novelty of the dissertation research.

A variational method based on the super-Gaussian function has been developed to characterize quantum droplets in one- and multi-dimensional Bose-Einstein condensates in the presence of quantum fluctuations, and also for vortices in two-dimensional cases.

In all three space geometries, analytical equations for parameters of quantum droplets have been calculated, these parameters include the stationary width and amplitude, the energy, the chemical potential, the frequency of small oscillations of the droplet shape. The stability of quantum droplets was checked by the Vakhitov-Kolokolov criterion.

In two- and three-dimensional Bose-Einstein condensates, resonance oscillations of quantum droplets and different regimes of its dynamics depending on the external modulation amplitude were determined when the coupling constants vary periodically in time.

Conditions of the exponential growth of modulations of a plane wave have been found for two-dimensional binary Bose-Einstein condensates in the presence of quantum fluctuations. In the nonlinear stage of modulation instability, the formation of quantum droplets, the law of their reduction, and the decreasing rate of the droplet number have been determined. It is demonstrated that the image processing techniques can be applied to analyze important patterns of the Bose-Einstein condensate density distribution.

Practical results of the investigation. One and two-component Gross-Pitaevskii type equations are solved analytically and numerically in different space geometries. Bose-Einstein condensates are potential candidates for quantum information processing tasks due to their coherence properties. Understanding the dynamics of condensates in different geometries can shed light on their suitability for specific quantum computation and quantum communication tasks. Solving the Gross-Pitaevskii equations in such scenarios can help us design and control these states for potential technological applications.

Reliability of the obtained results. One can place trust in the obtained results due to several factors that ensure their reliability:

Firstly, the investigation relies on the application of rigorous theoretical physics, mathematics, and high-precision numerical methods and algorithms. This approach guarantees a robust and accurate analysis of the data, minimizing potential errors and uncertainties.

Furthermore, the excellent agreement between the obtained results and exact solutions, Thomas-Fermi limit for stationary case, as well as numerical simulations, further bolsters their reliability. When different methods produce consistent outcomes, it enhances confidence in the validity of the findings, indicating the robustness of the applied techniques.

Moreover, a correspondence of some results to those obtained independently by other scientists, using alternative methods and realistic experiments reinforces the credibility of the results. This concordance of results from multiple sources adds weight to the conclusions drawn from the investigation.

Additionally, the estimates of real physical parameters derived from the obtained results align well with typical experiments conducted on Bose-Einstein condensates. This correspondence with experimental data ensures that the investigation remains grounded in the reality of practical applications and further validates the reliability of the findings.

By considering these aspects and the meticulous approach applied throughout the research process, the obtained results gain credibility and instill trust in their accuracy and validity.

The scientific and practical significance of the research results are as follows:

The results obtained reveal and explain in detail several essential (such as a self-bound liquid-like property) features of matter waves in one- and multi-dimensional Bose-Einstein condensates under the action of quantum fluctuations. These fundamental studies lay the groundwork for developing a complete theory of the quantum fluctuation effect in Bose-Einstein condensates. Also, these results can be used to estimate the number of atoms in a condensate, the critical temperature, and the lifetime of the condensate. In addition, the research findings may also be helpful for the prediction of variations of quantum droplet parameters over time, in the presence of different perturbations. This is particularly significant for experiments where the initial density distribution of Bose-Einstein condensates formed by an external trap differs from the exact solutions. The agreement of the research results with the experiments shows that it is appropriate to study the nature of matter waves in Bose-Einstein condensates using the proposed methods.

Application of the results of the dissertation. According to the information on scopus.com there are 58 citations (November, 2023) to the author's published articles on the dissertation topic.

Including 27 citetions to the published article on the first Chapter of the dissertation, some of them are listed below: Physical Review Letters, 126, 244101, 2021, IF: 9.185; Chaos, Solitons & Fractals, 152, 111313, 2021, IF: 9.922; Scientific Reports, 12, 6904, 2022, IF: 4.997; Physical Review A, 103, 053302, 2021, IF: 2.971;

There are two articles published in second Chapter, they have 9 and 18 citetions, total of 27 citetions, below are some of them: Chaos, 33, 033141, 2023, IF: 3.741; Physics Letters A, 480, 128987, 2023, IF: 2.6; Physical Review A, 105, 063328, 2022, IF: 2.971;

The published article on the third Chapter of the dissertation, includes 4 citations, a few of them outlined below: Chaos, Solitons & Fractals, 164, 112665, 2022, IF: 9.922; Physical Review A, 108, 033312, 2023, IF: 2.971; Physical Review A, 106, 033309, 2022 IF: 2.971.

Approbation of the work. The main results of this dissertation were presented and discussed at international and national conferences, and scientific seminars.

Publication of the results. In the field of a dissertation theme, 4 papers are published in international scientific journals included in the Scopus database.

The structure and volume of dissertation. The dissertation consists of Introduction, three Chapters, Conclusions and List of references. The volume of the dissertation is 81 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the **Introduction**, we justify the relevance and necessity of the research topic and its alignment with the priority directions of science and technology development in Uzbekistan. We also formulate the research goals and tasks, identify the study's object and subject, and highlight the scientific novelty and practical results of our findings. Moreover, we emphasize the reliability of our results and reveal their scientific and functional significance. Lastly, we provide information about the implementation of our results in practice and the structure of the dissertation.

In the first Chapter of the thesis entitled "**Quantum droplets in a one-dimensional Bose-Einstein condensate**", the stationary and dynamical properties of one-dimensional quantum droplets are studied using analytical and numerical methods. Let us consider the two-component Bose-Einstein condensate under the action of quantum fluctuations. To facilitated our calculations, we consider a symmetrical system consisting of two Bose gases with identical atomic masses, numbers of atoms, and interaction constants. In this case, the system is described by the one-dimensional Gross-Pitaevskii equation:

$$i\Psi_t + \frac{1}{2}\Psi_{xx} + \gamma |\Psi|^2 \Psi + \delta |\Psi| \Psi = 0, \quad (1)$$

written in a dimensionless form. The last quadratic nonlinear term corresponds to the effect of quantum fluctuations, and γ and δ are coupling constants. An exact solution of equation (1) was found for arbitrary values of coupling constants.

In experiments, the initial density distribution may not align with the exact solution. Consequently, it becomes important to develop an approach that can effectively describe the evolution of quantum droplet parameters over time, while accounting for the presence of various perturbations.

We used the Lagrange formalism to find the dynamical equations for the quantum droplet parameters. The Lagrange density of Eq. (1) is:

$$\mathcal{L}=\frac{i}{2}(\Psi\Psi_t^*-\Psi^*\Psi_t)+\frac{1}{2}|\Psi_x|^2-\frac{\gamma}{2}|\Psi|^4-\frac{2\delta}{3}|\Psi|^3. \qquad (2)$$

In the variational approach, the density distribution in a quantum droplet is approximated by the following super-Gaussian trial function:

$$\psi(r,t)=A\exp\left(-\frac{1}{2}\left(\frac{x}{w}\right)^{2m}+ibr^2+i\varphi\right), \qquad (3)$$

where $A(t), w(t), b(t)$ and $\varphi(t)$ are the amplitude, width, chirp and initial phases, respectively. The index m is taken as independent of time, its value is found from the solution of the stationary equation $m=m_s$. The advantage of choosing the super-Gaussian function is that it describes both a “bell-shaped” profile and a “flat-top” profile. “Flat-top” profiles are a fundamental property of quantum droplets. Such solutions cannot be represented by the standard Gaussian function.

The equations of motion for the parameters of quantum droplets are found using the Euler-Lagrange equations:

$$b_t=-\frac{1}{8\Gamma(1+3s)}\left[\frac{-3\Gamma(2-s)}{sw^4}+\frac{3\gamma}{2^s}\frac{N}{w^3}+\frac{2^{3/2+s}\delta\sqrt{N\Gamma(s+1)}}{3^s w^{5/2}}+16\Gamma(3s+1)b^2\right], \qquad (4)$$

$$w_t=2wb.$$

The chemical potential of a droplet is found as:

$$\mu=\frac{-3\gamma N}{2^{s+3}\Gamma(s+1)w_s}-\frac{5\cdot 2^{s-1}\delta N^{1/2}}{3^{s+1}[2\Gamma(s+1)w_s]^{1/2}}. \qquad (5)$$

Approximate solutions for stationary quantum droplets are found for different values of the norm., The stability of the solutions is checked by the Vakhitov-Kolokolov criterion and numerical simulations (see Fig. 1, left panel).

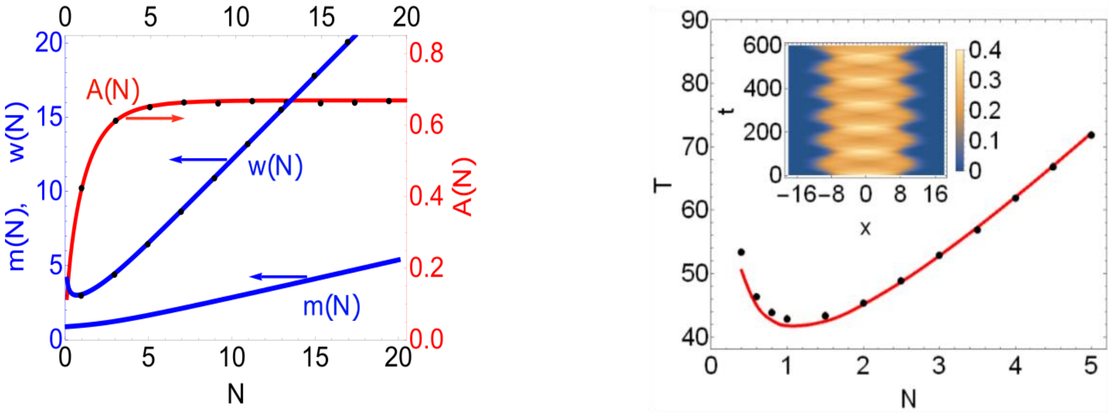

Fig. 1. Left panel: Dependencies of droplet width w, the power exponent m (the left axis), and the droplet amplitude A (the right axis) on N. Right panel: The period of small oscillations T of the QD width as a function of N. The inset shows a map plot of $|\Psi(x,t)|^2$ for $N=5$. In both plots, the lines are found from the variational approximation, and the points are found from numerical simulations of Eq. (1). Other parameters are $\gamma=-1$ and $\delta=1$.

By using the equations of motion (4), the effective potential for w is determined. The minimum of the potential corresponds to the stationary width of the quantum droplet. A small displacement from the the stationary value leads to oscillations around the equilibrium point, see the inset graph in Fig. 1. The period of small oscillations for different values of the number of atoms was determined by the variational approximation and numerical simulations, see the right panel of Fig. 1.

In the second Chapter of the dissertation, entitled "**Quantum droplets in two-dimensional Bose-Einstein condensate**", the modulation instability of slightly perturbed plane waves and the formation of quantum droplets was studied in two-dimensional binary Bose-Einstein condensate. Such a system can be represented in a dimensionless form by the following Gross-Pitaevskii equations:

$$i\frac{\partial\psi_j}{\partial t}+\frac{1}{2}\nabla^2\psi_j-\frac{\psi_j}{2\sqrt{\sigma_1\sigma_2}}\left(\sqrt{\sigma_j/\sigma_{3-j}}\,|\psi_j|^2-|\psi_{3-j}|^2\right)-\sqrt{\sigma_j/\sigma_{3-j}}\psi_j p\ln(p)=0, \qquad (6)$$

where $j=1,2$, $\nabla^2=\partial_x^2+\partial_y^2$, σ_1 and σ_2 are modified coupling constants depending on scattering parameters, $p=(\sigma_1|\psi_1|^2+\sigma_2|\psi_2|^2)/(2\sqrt{\sigma_1\sigma_2})$. The last logarithmic term represents the effect of quantum fluctuations.

The perturbed plane wave solution is taken as follows:

$$\psi_j=(A_j+\delta\psi_j)\exp(-i\mu_j t), \qquad \mu_j=\frac{A_j^2}{2\sigma_{3-j}}-\frac{A_{3-j}^2}{2\sqrt{\sigma_j\sigma_{3-j}}}+\sqrt{\sigma_j/\sigma_{3-j}}\,p_0\ln(p_0), \qquad (7)$$

where $\delta\psi_j \ll A_j$, $p_0=(\sigma_1A_1^2+\sigma_2A_2^2)/(2\sqrt{\sigma_1\sigma_2})$, μ_j is the chemical potential, A_j is the plane wave amplitude, $\delta\psi_j$ represents a small perturbation. Using the linear stability analysis, the following dynamical equations were obtained for $\delta\psi_j$:

$$i\frac{\partial\delta\psi_j}{\partial t}+\frac{1}{2}(\delta\psi_{jxx}+\delta\psi_{jyy})-c_j(\delta\psi_j^*+\delta\psi_j)-c_3(\delta\psi_{3-j}^*+\delta\psi_{3-j})=0, \qquad (8)$$

where $j=1,2$,

$$c_j\equiv\frac{A_j^2}{2\sigma_{3-j}}+\frac{A_j^2\sigma_j}{2\sigma_{3-j}}\ln(ep_0), \qquad c_3\equiv\frac{A_1A_2}{2}\left[-\frac{1}{\sqrt{\sigma_1\sigma_2}}+\ln(ep_0)\right], \qquad (9)$$

We take the perturbation in the form $\delta\psi_j=u_j+iv_j$ and divide the equation (8) into real and imaginary parts, we search the characteristic solution of the resulting differential equation in the form $(u_j,v_j)\sim\exp(\lambda t+ik_xx+ik_yy)$ and get a system of algebraic equations. By analyzing these equations, the following results were found:
Conditions for a sharp increase in the perturbation amplitude $ep_0<1, |k|<k_{cr}$ is defined. The maximum growth rate of modulation instability, $G_{max}=k_{max}^2/2=k_{cr}^2/4$, the corresponding wave number $k_{max}=k_{cr}/\sqrt{2}$. This mode experiences the higest amplification among all possible perturbations. The value of critical wave number

$$|k|<k_{cr}\equiv\sqrt{2}\sqrt{-(c_1+c_2)+\sqrt{(c_1-c_2)^2+4c_3^2}}$$

corresponds to the threshold value where the modulational instability gain spectrum exhibits transitions from unstable behaviour to stability. It was found that the amplitude of the perturbed plane wave is unstable at values smaller than the critical value of the wave number ($k<k_{cr}$), and stable at values greater than ($k\geq k_{cr}$), see Fig. 2 (a).

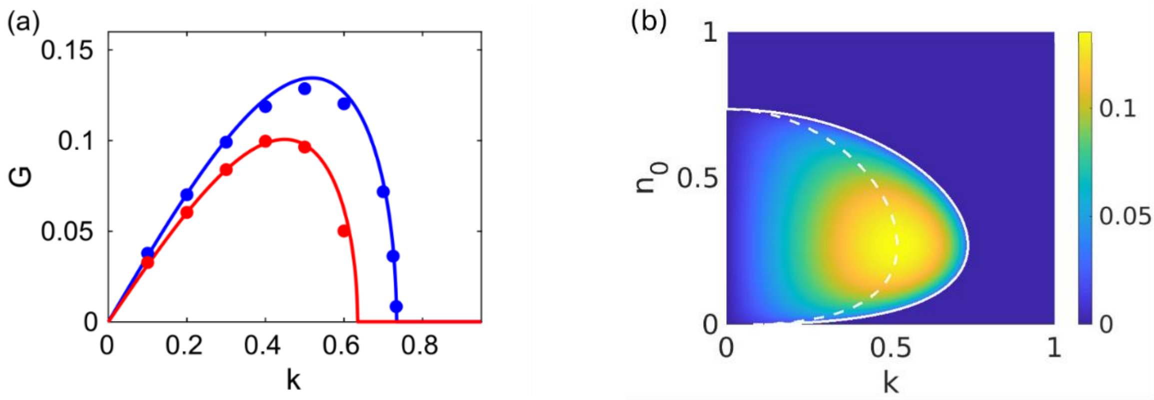

Fig. 2. (a) The MI growth rate G vs k. The top blue line is for the symmetric case $n_{10} = n_{20} = 0.3$ and the bottom red line is for the asymmetric case $n_{10} = 0.3$ and $n_{20} = 0.1$. Points are found from direct numerical simulations. (b) The growth rate G as a function of k and n for the symmetric case. The color bar represents the values of G. The solid line shows the MI boundary and the dashed line represents G_{max}. The interaction parameters are $\sigma_1 = \sigma_2 = 0.1$.

Stability and instability regions were found at different values of the parameters, Fig. 2 (b). Based on the results obtained, the following method of observation of modulation instability in experiments was proposed. First of all, it is necessary to form a high-density two-component condensate. In this case, the condensate is stable. Then, by reducing the density of the condensate, for example, by expanding the external trap, modulation instability can be observed.

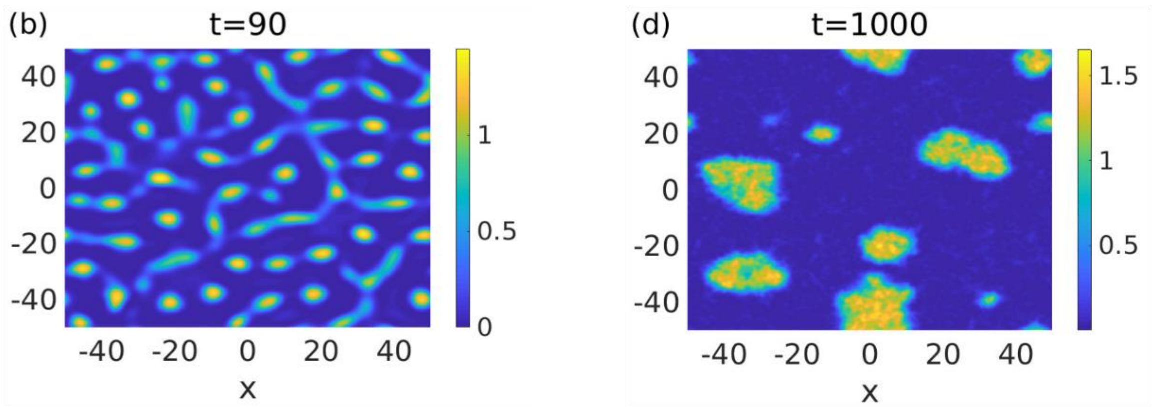

Fig. 3. The typical dynamics of the density distribution at (b) $t = 90$, and (d) $t = 1000$. The initial densities are $n_0 = 0.2$, and $\sigma_1 = \sigma_2 = 0.1$. The color bar represents the density $n = |\psi_1|^2 + |\psi_2|^2$.

In the late, nonlinear, stage of modulation instability, quantum droplets are formed. This step was studied using numerical simulations. The number of generated quantum droplets decreases over time by interacting and merging with each other. The number of quantum droplets, the law of decrease and the decreasing rate was determined for different values of the parameters, see Fig. 3. The image processing technique of Matlab was used to calculate the number of generated quantum droplets.

In the second part of this Chapter, the stationary and dynamical properties of two-dimensional quantum droplets and vortices in the presence of quantum fluctuations are studied by using analytical and numerical methods. We consider a symmetric Bose-Einstein condensate with identical distribution of atoms in the components. Such a systemis described in a dimensional form by the following Gross-Pitaevsky equation:

$$i\partial_t\Psi + \frac{1}{2}\nabla^2\Psi + \delta|\Psi|^2\ln(|\Psi|^2)\Psi = 0, \qquad (10)$$

where $\nabla^2 = \partial_x^2 + \partial_y^2$ is a two-dimensional Laplace operator. Equation (10) does not have an exact analytical solution, so we develop the variational approximation. We employ the following super-Gaussian trial function to represent the quantum droplets and vortices:

$$\Psi(r,\theta,t) = Ar^S\exp\left[-\frac{1}{2}\left(\frac{r}{w}\right)^{2m} + i(br^2 \pm S\theta + \varphi)\right], \qquad (11)$$

where $A(t)$, $w(t)$, $b(t)$, and $\varphi(t)$ are variational parameters, which are the amplitude, width, chirp, and initial phase, respectively. The super-Gaussian parameter m represents the shape of the profile, and it is determined from the stationary solution. The topological charge $S \ge 0$ takes integer values. If $S = 0$, Eq. (11) represents a quantum droplet, the cases $S = 1, 2, 3, \ldots$ represent vortices.

Via the Euler-Lagrange equations, the dynamical equations for the parameters of quantum droplets and vortices were derived. The equation for width is

$$w_{tt} = -\frac{\Gamma(M(S+1))}{\Gamma(M(S+2))}\frac{\partial G}{\partial w}, \qquad (12)$$

where

$$G(w,N,M) = \frac{\Gamma(MS+2)}{2M^2\Gamma(M(S+1))w^2} + \frac{N\Gamma(M(2S+1))}{2^{2+M(2S+1)}\pi M\Gamma^2(M(S+1))w^2}$$
$$\times\delta\{1 + M + 2MS\left[1-\psi(M(2S+1))\right] - 2\log\left[\frac{N}{2^{MS}\pi M w^2\Gamma(M(S+1))}\right]\}, \qquad (13)$$

$\psi(z) = d\ln\Gamma(z)/dz$ – digamma function, $N = \pi MA^2 w^{2(S+1)}\Gamma(M(S+1))$.

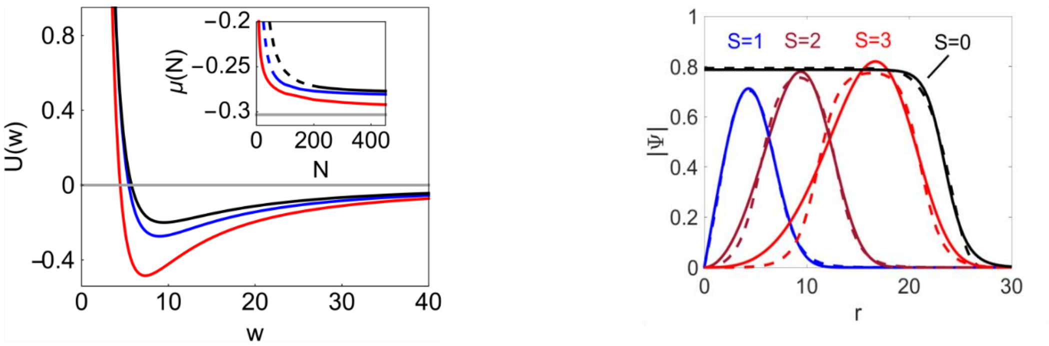

Fig. 4. Left panel: The shape of effective potentials for different parameters of (S,N,m). The bottom (red), middle (blue) and top (black) lines are for $(0,100,2.537)$, $(1,200,1.971)$, and $(2,200,2.073)$, respectively. The inset shows the chemical potential μ as a function of norm N. Right panel: Profiles $|\Psi|$ of QDs, found from the variational approach (solid lines) and from the imaginary-time method (dashed lines) for $S = 0,1,2$ and 3, $N = 1000, 60, 200$ and 510, respectively.

Using these equations, the effective potential, frequencies of small oscillations and the chemical potential are determined. The potential curves are shown on the left side of Fig. 4. The chemical potential is defined as $\mu = \partial E_s / \partial N$, where $E_s = NG$ is the stationary energy of a quantum droplets. The inset of Fig. 4 shows dependence of the chemical potential on number of atoms. From bottom to top curves corresponds to quantum droplet (*S=0*) and vortices for *S=1* and *2*, respectively. The stationary profiles of quantum droplets and vortices exhibit significant congruence in their comparisons with numerically found solutions, see right panel in Fig. 4.

The dependence of vortices' stability on the number of atoms is also checked. The dashed line in the inner plot of Fig. 4 represents unstable vortices. It is found numerically that unstable vortices with topological charge S usually break up into $S+1$ fragments. The behavior of quantum droplets under the action of periodic variations of the coupling constants was also examined. It is obtained that periodic modulations of the strength of quantum fluctuations can actuate resonance oscillations of the quantum droplets, an emission of waves and a splitting of quantum droplets into smaller droplets.

The third Chapter of the dissertation, entitled "**Quantum droplets in a three-dimensional Bose-Einstein condensate**", is devoted to the study of stationary and dynamical properties of three-dimensional quantum droplets due to quantum fluctuations in a two-component Bose-Einstein condensate. Our calculations were based on a symmetrical system of Bose gases with the same atomic mass, number of atoms, and interaction constants. This system is described in a dimensional form by the following Gross-Pitaevskii equation:

$$i\psi_t + \frac{1}{2}\nabla^2\psi + \alpha|\psi|^2\psi - \beta|\psi|^3\psi = 0. \tag{14}$$

The parameters α and β represent the strength of the mean-field and quantum fluctuation interaction terms, respectively. By choosing the intra- and inter-species coupling constants equal modulo the two-body interaction term can be ignored $(\alpha = 0)$ from the Eq. (14). In this case, the repulsive quantum fluctuation can be balanced with an external trap. Such a system is known as LHY fluid and was first studied theoretically in Ref. [9] and experimentally in Ref. [10].

We took the density distribution of quantum droplets in the form of a super-Gaussian function as in one- and two-dimensions, that is, $x = r$ should be taken in equation (3). The value of the super-Gaussian index m is found from the solution of the stationary equation. Norm $N = 4\pi A^2 w^3 \Gamma(1+3M)/3$, $(M = 1/2m)$ is the conserved quantity of Eq. (14), it is proportional to the number of atoms in a quantum droplet. $\Gamma(z)$ is the Gamma function. The Lagrangian of the system is found using the following $L = \int\int_{-\infty}^{\infty}\int \mathcal{L} dxdydz$ integral, where $\mathcal{L}$ is the Lagrangian density of equation (14):

$$\frac{L}{N} = \varphi_t + \frac{\Gamma(5M)}{\Gamma(3M)} w^2(2b^2 + b_t) + G(M, w, N), \tag{15}$$

where

$$G(M, w, N) = \frac{3\Gamma(M+2)}{8M\Gamma(3M+1)w^2} - \frac{G_1\alpha}{w^3} + \frac{G_2\beta}{w^{9/2}},$$

$$G_1 \equiv \frac{3\cdot 2^{-3(M+1)}N}{\pi\Gamma(3M+1)},\quad G_2 \equiv \frac{3\sqrt{3}2^{3M-2}}{5^{3M+1}}\left(\frac{N}{\pi\Gamma(3M+1)}\right)^{3/2}. \tag{16}$$

The equations for variations of the quantum droplet's parameters were derived via the Euler-Lagrange equations. Specifically, the equation governing the width of a quantum droplet can be expressed as:

$$w_{tt} \equiv -\frac{\partial U(w)}{\partial w},\quad U(w) = \frac{\Gamma(3M)}{\Gamma(5M)}G, \tag{17}$$

where $U(w)$ is the effective potential. The frequency of small oscillations is determined as the second-order derivative of the effective potential, $\Omega_0^2 = \partial^2 U(w)/\partial w^2$.

Typical shapes of the effective potential are plotted in Fig.5 (a) for the different values of N. For the given values of the coupling constant, quantum droplets exist, starting from the critical value of the number of atoms N_{cr}, within the range of $N_{cr} < N < N_{th}$, the quantum droplets are metastable. However, when the norm N exceeds the threshold value N_{th}, the quantum droplets become stable. Numerical solutions of the Bogoliubov-de Gennes equation presented in reference [1] demonstrate that, for the dimensionless equation with coupling constants $(\alpha, \beta) = (3, 5/2)$, the norm exhibits critical and threshold values within the range $18.65 \le N < 22.55$. It is stated in Ref. [11] that, this threshold value of N was also confirmed in experiments [7, 8]. That is why we use the same parameters for comparison of VA with the results of Ref. [1, 7, 8]. It was shown that the critical and threshold values of the norm in the variational approximation correspond to the interval $19.5 \le N \le 23.05$. Thus, it is evident that our proposed variational approach for investigating the properties of quantum droplets aligns with the findings derived from actual experiments.

The stability of quantum droplets was checked by the Vakhitov-Kolokolov criterion as well as numerical calculations. In Fig. 5 (b) we show the dependencies of norm N on a chemical potential μ. According to the Vakhitov-Kolokolov stability criterion, the negative values of the derivative $d\mu/dN < 0$ indicate the stability of quantum droplets.

The behavior of quantum droplets was analyzed when the coupling constants $\alpha(t) = \alpha_0[1+\epsilon_1 \sin(\omega t)]$, $\beta(t) = \beta_0[1+\epsilon_2 \sin(\omega t)+\theta]$ are periodic variables, where (α_0, β_0), $\epsilon_1, \epsilon_2 \ll 1$, ω_m and θ are the modulation amplitude, frequency and initial phase, respectively. In experiments, such modulations can be created by using the Feshbach resonance technique. External periodic modulations are found to result in a dual regime for quantum droplets, dependent upon the amplitude of modulation. At lower amplitudes, the droplets undergo adiabatic oscillation, while at higher amplitudes, they undergo evaporation.

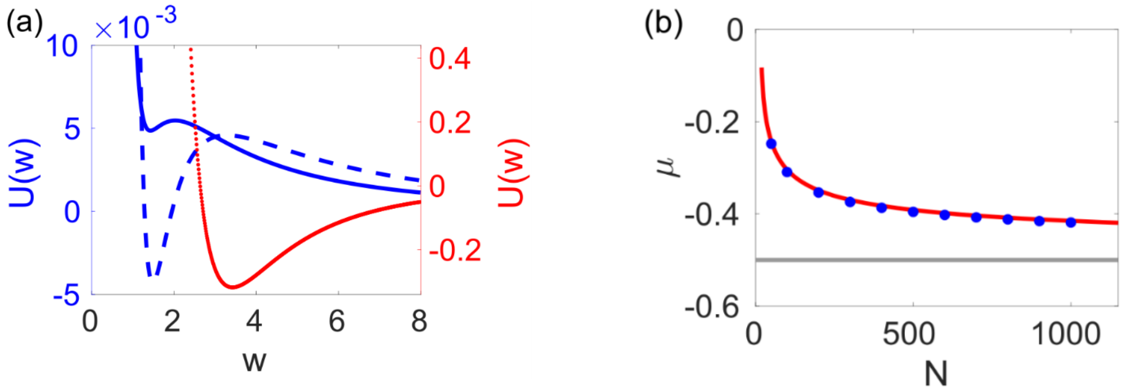

Fig. 5. (a) The shape of effective potentials for different values of N. The solid line dashed line, and points (right axes) are for 20, 25, and 200, respectively. (b) The chemical potential μ as a function of norm N, for the parameters $(\alpha,\beta)=(3,5/2)$. The red line is found from the variational approach, and points are found from imaginary time simulations. The gray line represents the Thomas-Fermi limit.

In numerical calculations, when the frequency of external modulations is equal to the eigen-frequency of the droplet shape, the resonance beating was observed. In numerical calculations, the resonance frequency can be determined from the dependence of the amplitude difference on the external modulation frequency. Let us expand Eq. (17) by a series of small deviation ξ from the equilibrium width w_s, $w=w_s+\xi$, where $\xi \ll w_s$. The dynamics of ξ are described by the following linearized equation:

$$\ddot{\xi}+\Omega_0^2\xi=\left(-\frac{3\Gamma(3M)\alpha_0 G_1\epsilon_1}{\Gamma(5M)w_s^4}+\frac{9\Gamma(3M)G_2\beta_0\epsilon_2}{2\Gamma(5M)w_s^{11/2}}\right)\sin(\omega_m t), \tag{18}$$

where $\ddot{\xi}$ represents the second derivative of ξ with respect to t, and $B=-\frac{3c\alpha_0 G_1\epsilon_1}{w_s^4}+\frac{9cG_2\beta_0\epsilon_2}{2w_s^{11/2}}$, $c\equiv\frac{5\Gamma(3M+1)}{3\Gamma(5M+1)}$, the coefficients G_1 and G_2 are found from Eq.(16).

By setting the right side of equation (18) to zero, we can obtain the following expression for the modulation amplitudes $\epsilon_2=2\alpha_0 G_1 w_s^{3/2}\epsilon_1/3\beta_0 G_2$. If ϵ_1 and ϵ_2 fulfil this condition, the modulations totally cancel each other. This condition is valid for any values of modulation frequency ω_m. In this case, the dynamics of QD parameters do not change over time. The accuracy of this relation is confirmed in the dynamics of QD in VA and as well as in numerical simulations. These linear analyses show, if the modulation amplitudes ϵ_1 and ϵ_2 do not fulfil the above condition, dynamics of ξ depend on the modulation frequency ω_m. When the frequency ω_m is equal to eigenfrequency Ω_0, the system is under resonance oscillations. Near resonance frequency, we observe a beating. It can be inferred that the external modulations depend on the initial phase.

The parameters of quantum droplets are estimated for realistic experiments, considering potatsium atoms. Values of droplet widths, the number of atoms in them, the frequency of resonance are in in a range of typical experiments.

CONCLUSIONS

This dissertation work presents a unified investigation of QDs across one-, two-, and three-dimensional geometries, employing the variational approach with the super-Gaussian function as a central theoretical tool. The investigation begins by establishing the reliability of the VA in predicting the stationary parameters of QDs in one-dimensional BECs, see Chapter I. The observed agreement between theoretical predictions and numerical simulations underscores the effectiveness of the super-Gaussian function, offering valuable insights into QD dynamics and shape oscillations. An introduction of additional parameters in this function makes the procedure more involved technically. Nevertheless, such an approach allows us to obtain explicit relations for QD characteristics.

Moving beyond one-dimensional geometry, the study delves into modulational instability in two-dimensional binary Bose-Einstein condensates, see Section § 2.1, revealing a threshold density belove which instability is maintained. The emergence of droplets due to modulational instability is confirmed through numerical simulations, providing a comprehensive understanding of a role of quantum fluctuations in the droplet formation. A detailed analysis of characteristics of two-dimensional QDs have been performed in Section § 2.2. Also, the stability of QDs, and their response to external time-dependent perturbation have been investigated.

Extending the exploration to three dimensions, the super-Gaussian function continues to serve as a robust approximation for describing stationary 3D quantum droplets, see Chapter III. The VA, consistently reliable throughout, captures the dynamics of QDs, identifying frequencies of small oscillations and characterizing different regimes of QD behavior under external modulations.

Throughout the study, the super-Gaussian function emerges as a versatile tool for approximating the shape of localized waves in the system. The close alignment between theoretical predictions and numerical simulations attests to the efficacy of the VA in elucidating the complexities of QD stability, formation, and dynamics. Apart from the theoretical advancements, the study introduces practical applications by employing image processing techniques to analyze the density distribution of Bose-Einstein condensates. This cohesive exploration unites the different dimensions of the study, offering a holistic perspective on quantum droplets.

The main obtained results of the dissertation work can be formulated as follows:

1. The variational approach has been developed by using the super-Gaussian trial function. Such a trial function describes well flat-top localized density distributions, corresponding to incompressible liquid. Based on the developed approach we find the dynamical equations for parameters characterizing quantum droplets in one- and multi-dimensional Bose-Einstein condensates, as well as for vortices in two-dimensional case.

2. In all dimensions, the dependence of the stationary parameters of quantum droplets on the number of atoms has been determined, using the variational approach based on the super-Gaussian trial functions. Similarly, in two-dimensional case the impact of the number of atoms on the parameters characterizing vortices has also been examined. Additionally, the effective potential and frequencies of small oscillations have been obtained.

3. The regions of stability of droplets and vortices has been found. Scenarios of the development of instability has been analyzed numerically.

4. An exact generalized solution of stationary quantum droplets has been found in one-dimensional Bose-Einstein condensate for the arbitrary signs and values of coupling constants.

5. Conditions of the exponential growth of a slightly perturbed uniform state in two-dimensional Bose-Einstein condensate have been determined. Stable and unstable regions of perturbed plane waves have been found in the parameter space. In the nonlinear stage of the instability the formation of quantum droplets due to modulational instability and decreasing rate of the generated droplets are determined.

6. Different regimes of resonance oscillations of quantum droplets, adiabatic oscillations and evaporation of quantum droplets have been determined depending on the amplitude of external modulation in two- and three-dimensional Bose-Einstein condensates when coupling constants vary in time. Also, for three-dimensional quantum droplets, it has been shown that the response of the droplet shape to the two-frequencies external modulations depend on the phase difference.

7. By using the Lagrange formalism for three-dimensional quantum droplets, the critical the number of atoms for the droplet existence has been determined. Also, the threshold number of atoms for stable droplets has been obtained. It is shown that these regions of existence and stability are consistent with the numerical results of other authors work, and with the results obtained in actual experiments conducted on three-dimensional quantum droplets.

8. Numerical codes for modelling of the BEC dynamics under the action of quantum fluctuations for 1D, 2D, and 3D are developed. Also, image processing techniques are applied for analysis of BEC distributions.

НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.01.09 ПО ПРИСУЖДЕНИЮ УЧЁНЫХ СТЕПЕНЕЙ ПРИНАЦИОНАЛЬНОМ УНИВЕРСИТЕТЕ УЗБЕКИСТАНА НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ

ОТАЖОНОВ ШЕРЗОД РУСТАМОВИЧ

ВОЛНЫ МАТЕРИИ В КОНДЕНСАТЕ БОЗЕ-ЭЙНШТЕЙНА ПОД ДЕЙСТВИЕМ КВАНТОВЫХ ФЛУКТУАЦИЙ

01.04.02 – Теоретическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ НА СОИСКАНИЕ УЧЕНОЙ СТЕПЕНИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент – 2024

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ВВЕДЕНИЕ (аннотация кандидатской диссертации)

Актуальность и востребованность темы диссертации.

Когда атомы (бозоны) охлаждаются почти до абсолютного нуля, они собираются на энергетическом уровне, образуя так называемый конденсат Бозе-Эйнштейна (КБЭ). В 1995 году достижения в экспериментальных методах позволили двум отдельным исследовательским группам наконец экспериментально наблюдать конденсат Бозе-Эйнштейна, что привело к их присуждению Нобелевской премии в 2001 году.

В настоящее время мировое научное сообщество проводит многочисленные теоретические и экспериментальные исследования, направленные на изучение свойств волн материи в бозе-эйнштейновских конденсатах. Эти исследования имеют огромное значение, поскольку решают фундаментальные проблемы современной физики. Подвергая атомарные газы в бозе-эйнштейновских конденсатах внешним магнитным и оптическим полям, исследователи могут эффективно манипулировать ими, создавая основу для различных экспериментов и точных теоретических моделей. Потенциальные применения конденсатов Бозе-Эйнштейна разнообразны, включая интерферометры материи-волны, атомно-волновые лазеры, атомные часы, кубиты или блоки памяти в квантовых компьютерах, квантовую криптографию для разложения чисел в простые числа и другие цели.

С точки зрения среднего поля, двух- и трехмерные конденсаты в конденсатах Бозе-Эйнштейна с притягивающими двухчастичными взаимодействиями по своей природе нестабильны, что приводит к их коллапсу. Существует несколько жизнеспособных методов стабилизации конденсата, таких как использование внешних ловушек, учет трехатомных взаимодействий, диполярных взаимодействий и периодическое изменение длин рассеяния.

Недавно было теоретически продемонстрировано, что двумерные и трехмерные нестабильные конденсаты могут достигать стабильности за счет учета квантовых флуктуаций [1]. Оказывается, квантовые флуктуации приводят к образованию сверхразбавленного жидкоподобного состояния вещества. Самосвязанная структура такого состояния известна как квантовые капли [1]. Вскоре после этого прорыва экспериментальные данные подтвердили стабильность таких систем и образование квантовых капель (КК). В настоящей работе распределение плотности и динамические свойства КК в КБЭ изучаются во всех трех геометрических измерениях.

Из приведенного выше описания видно, что физика волн материи в бозе-эйнштейновских конденсатах быстро развивается. В нашем государстве в последние десятилетия большое внимание уделялось развитию науки, особенно теоретической физики, с целью создания условий на более высоком международном уровне для фундаментальных исследований.

Настоящая диссертационная работа выполняет задачи, предусмотренные нормативными документами правительства и Указом Президента Республики

Узбекистан “О мерах по дальнейшему совершенствованию деятельности Академии наук, организации, руководства и финансирования научно-исследовательских работ”. №PQ-2789, Постановление №ПФ-4947 от 7 февраля 2021 года «О стратегии действий по дальнейшему развитию Республики Узбекистан», Постановление №PQ-5032 от 19 марта 2021 года “О мерах по повышению качества образования и совершенствованию научных исследований в области физики”.

Соответствие исследования приоритетным направлениям развития науки и технологий в Республике. Данное исследование выполнено в соответствии с приоритетным направлением развития науки и технологий Республики Узбекистан - II. “Физика, астрономия, энергетика и машиностроение”.

Степень изученности проблемы. С момента пионерской работы [1] и последующей экспериментальной реализации исследование квантовых капель, образующихся за счет квантовых флуктуаций в бозе-эйнштейновском конденсате, вызвало значительный интерес среди ученых. Многочисленные исследователи исследовали различные аспекты волн материи, на которые влияют квантовые флуктуации, включая уменьшение размеров из трех измерений в два и одно измерения [2], динамику одномерных квантовых капель [3] и исследование двумерных квантовых капель. и вихри численными методами [4]. Кроме того, была исследована стабильность трехмерных квантовых капель и вихрей в зависимости от числа частиц [5].

Экспериментальная реализация квантовых капель была впервые достигнута в диполярном конденсате [6], а затем в бозонных смесях [7-8]. Разработаны математические модели и получены важные экспериментальные данные, проливающие свет на свойства квантовых капель. Однако многие аспекты воздействия квантовых флуктуаций на волны материи еще требуют изучения, и многие вопросы ждут своего решения. К ним относятся динамика одномерных и многомерных квантовых капель и вихрей, реакция системы на периодическое изменение во времени параметров, а также образование квантовых капель из возмущенных плоских волн за счет явления модуляционной неустойчивости и их взаимодействия. Эти вопросы мы рассматриваем в данной работе.

Связь темы диссертации с научно-исследовательскими работами учреждения, где выполняется диссертация. Исследования выполнены в соответствии с планом НИР ФТИ АН РУз, в том числе по проекту по гранту Министерства инновационного развития РУз FA-F2- 004 “Динамика и взаимодействие нелинейных локализованных волн в квантовых и диссипативных системах”, 2017-2020 гг., “Исследование динамики локализованных волн в квантовых газах и нелинейно-оптических средах”, 2020 - 2021 гг.

Цель исследования изучить свойства волн материи в многомерных двухкомпонентных Бозе-Эйнштейновских конденсатах при наличии квантовых флуктуаций и разработать теоретический формализм для описания локализованных структур в этой системе. В частности, рассматриваются стационарные параметры и динамическое поведение квантовых капель.

Задачи исследования: Разработать математические модели, которые смогут объяснить характеристики квантовых капель, вихрей и плоских волн в одномерных и многомерных Бозе-Эйнштейновских конденсатах.

Изучить случаи, когда параметры взаимодействия являются периодическими переменными времени в многомерных бозе-эйнштейновских конденсатах.

Выявить условия превращения бозе-эйнштейновских конденсатов с однородным распределением плотности в квантовые капли за счет модуляционной неустойчивости, определить области неустойчивости в пространстве параметров и изучить динамику генерируемых квантовых капель.

Провести численное моделирование одно- и многомерного Гросса-Питаевского для вышеупомянутых физических процессов для обоснования точности приближенных теоретических методов.

Объектом исследования являются квантовые капли, вихри и плоские волны в одно- и многомерных конденсатах Бозе-Эйнштейна.

Предметом исследования являются параметры локализованных состояний с различным числом частиц, период малых колебаний, расщепление локализованных состояний за счет резонансных колебаний, а также условия устойчивости плоских волн, квантовых капель и вихрей.

Методы исследования. В этом исследовании мы используем сочетание аналитических и численных методов исследования для изучения динамики сложных физических систем. Для аналитической части нашего исследования мы используем вариационную аппроксимацию, аппроксимацию Томаса-Ферми и анализ линейной устойчивости. Эти методы позволяют нам получить глубокие теоретические знания и правильно понять основные принципы, управляющие поведением системы.

В дополнение к нашему аналитическому подходу мы используем различные численные методы для выполнения расчетов и моделирования. В частности, мы концентрируемся на решении уравнений типа Гросса-Питаевского, которые являются фундаментальными для нашего исследования. Численные методы, использованные в этом исследовании, включают метод Рунге-Кутты четвертого порядка и метод Кранка-Николсона, оба из которых широко известны своей точностью и стабильностью при решении нелинейных уравнений в частных производных.

Кроме того, мы используем методы обработки изображений как часть нашего численного анализа, что позволяет нам извлекать ценную информацию из экспериментальных данных и визуализировать поведение системы. Объединив эти аналитические и численные методы исследования, мы стремимся обеспечить всесторонний и надежный анализ сложных исследуемых физических явлений. Такой подход позволяет нам преодолеть разрыв между теоретическими предсказаниями и эмпирическими данными, тем самым давая более полное понимание динамики системы и способствуя развитию научных знаний в этой области.

Научная новизна диссертационного исследования, следующая:

Разработан вариационный метод на основе функции супергаусса для характеристики квантовых капель в одно- и многомерных бозе-эйнштейновских конденсатах при наличии квантовых флуктуаций, а также для вихрей в двумерных случаях.

Во всех трех геометриях пространства рассчитаны аналитические уравнения для параметров квантовых капель, к этим параметрам относятся стационарная ширина и амплитуда, энергия, химический потенциал, частота малых колебаний формы капли. Устойчивость квантовых капель проверялась по критерию Вахитова-Колоколова.

В двумерных и трехмерных бозе-эйнштейновских конденсатах определены резонансные колебания квантовой капли и различные режимы ее динамики в зависимости от амплитуды внешней модуляции при периодическом изменении констант связи во времени.

Найдены условия экспоненциального роста модуляций плоской волны для двумерных бинарных бозе-эйнштейновских конденсатов при наличии квантовых флуктуаций. На нелинейной стадии модуляционной неустойчивости установлены процессы образования квантовых капель, закон их уменьшения и скорость уменьшения количества капель. Показано, что методы обработки изображений можно применять для анализа важных закономерностей распределения плотности бозе-эйнштейновского конденсата.

Практические результаты исследования. Одно- и двухкомпонентные уравнения типа Гросса-Питаевского решаются аналитически и численно в различных геометриях пространства. Конденсаты Бозе-Эйнштейна являются потенциальными кандидатами для задач квантовой обработки информации благодаря своим свойствам когерентности. Понимание динамики конденсатов в различной геометрии может пролить свет на их пригодность для конкретных задач квантовых вычислений и квантовой связи. Решение уравнений Гросса-Питаевского в таких сценариях может помочь нам спроектировать и контролировать эти состояния для потенциальных технологических приложений.

Достоверность результатов исследования. Доверять полученным результатам можно благодаря нескольким факторам, обеспечивающим их достоверность:

Во-первых, исследование опирается на применение строгой теоретической физики, математики и высокоточных численных методов, и алгоритмов. Такой подход гарантирует надежный и точный анализ данных, сводя к минимуму потенциальные ошибки и неопределенности.

Кроме того, отличное согласие между полученными результатами и точными решениями, пределом Томаса-Ферми для стационарного случая, а также численным моделированием еще больше повышает их надежность. Когда разные методы дают согласованные результаты, это повышает уверенность в достоверности результатов, указывая на надежность применяемых методов.

Более того, соответствие некоторых результатов тем, которые получены независимо другими учеными, с использованием альтернативных методов и реалистичных экспериментов, усиливает достоверность результатов. Такое совпадение результатов из нескольких источников придает весомость выводам, сделанным в результате расследования.

Кроме того, оценки реальных физических параметров, полученные на основе полученных результатов, хорошо согласуются с типичными экспериментами, проводимыми на бозе-эйнштейновских конденсатах. Такое соответствие экспериментальным данным гарантирует, что исследование остается основанным на реальности практического применения, и дополнительно подтверждает надежность результатов.

Благодаря учету этих аспектов и тщательному подходу, применяемому на протяжении всего процесса исследования, полученные результаты приобретают достоверность и вселяют уверенность в их точности и достоверности.

Научная и практическая значимость результатов исследования заключается в следующем. Полученные результаты выявляют и подробно объясняют некоторые существенные (такие как свойство самосвязанной жидкости) особенности волн материи в одно- и многомерных бозе-эйнштейновских конденсатах под действием квантовых флуктуаций. Эти фундаментальные исследования закладывают основу для разработки полной теории квантового флуктуационного эффекта в бозе-эйнштейновских конденсатах. Также эти результаты можно использовать для оценки количества атомов в конденсате, критической температуры и времени жизни конденсата. Кроме того, результаты исследования также могут быть полезны для прогнозирования изменений параметров квантовых капель с течением времени при наличии различных возмущений. Это особенно важно для экспериментов, в которых начальное распределение плотности бозе-эйнштейновских конденсатов, образованных внешней ловушкой, отличается от точных решений. Согласие результатов исследований с экспериментами показывает целесообразность изучения природы волн вещества в бозе-эйнштейновских конденсатах с использованием предложенных методов.

Внедрение результатов исследования. По данным сайта scopus.com имеется 58 цитирований (ноябрь 2023 г.) на опубликованные статьи автора по теме диссертации.

В том числе 27 цитат к опубликованной статье по первой главе диссертации, некоторые из них приведены ниже: Physical Review Letters, 126, 244101, 2021, IF: 9.185; Chaos, Solitons & Fractals, 152, 111313, 2021, IF: 9.922; Scientific Reports, 12, 6904, 2022, IF: 4.997; Physical Review A, 103, 053302, 2021, IF: 2.971;

Во второй главе опубликованы две статьи, они имеют 9 и 18 цитирований, всего 27 цитирований, ниже приведены некоторые из них: Chaos, 33, 033141, 2023, IF: 3.741; Physics Letters A, 480, 128987, 2023, IF: 2.6; Physical Review A, 105, 063328, 2022, IF: 2.971;

Опубликованная статья по третьей главе диссертации включает 4 цитаты, некоторые из них приведены ниже: Chaos, Solitons & Fractals, 164, 112665, 2022, IF: 9.922; Physical Review A, 108, 033312, 2023, IF: 2.971; Physical Review A, 106, 033309, 2022 IF: 2.971.

Апробация результатов исследования. Основные результаты диссертации были представлены и обсуждены на международных и республиканских конференциях, научных семинарах.

Публикация результатов исследования. По теме диссертации опубликовано 4 научных работы в международных научных журналах, входящих в базу данных Scopus.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка литературы. Объем диссертации составляет 81 страницы.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
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I bo'lim (I часть; part I)

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II bo'lim (II часть; part II)

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