“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI HUZURIDAGI FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/31.03.2022 T/FM.10.04 RAQAMLI ILMIY KENGASH

FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI

TURIMOV BOBUR VALENTINOVICH

EYNSTEYN-MAKSVELL-SKALYAR MAYDON TENGLAMALARINING ANIQ YECHIMLARI

01.03.01-Astronomiya
01.04.02 – Nazariy fizika
(fizika-matematika fanlari)

E’lon qilingan ilmiy ishlar bo’yicha dissertatsiyasiz falsafa doktori (PhD) ilmiy darajasini olish uchun
TAQDIMNOMA

Toshkent – 2024

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Fizika-matematika bo’yicha falsafa doktori (PhD) Taqdimnomasi mundarijasi

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.1.PhD/FM999 raqami bilan ro'yxatga olingan.

Taqdimnoma "TIQXMMI" Milliy tadqiqot universiteti huzuridagi fundamental va amaliy tadqiqotlar institutida bajarilgan.

Taqdimnoma uch tilda (o'zbek, ingliz, rus (rezyume)) Ilmiy kengashning internet sahifasida (www.ifar.uz) va "Ziyonet" axborot-ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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Taqdimnoma 2024-yil "___" __________ kuni tarqatildi.

(2024-yil "___"_________dagi ___ raqamli reestr bayonnomasi)

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KIRISH (Falsafa doktori (PhD) taqdimnoma annotatsiyasi)

Mavzuning dolzarbligi va zarurligi. LIGO-VIRGO hamkorligida yaqin tizimlarda qo‘shaloq qora tuynuklar va qo‘shaloq neytron yulduzlarining to’qnashuvidan hosil bo’lgan gravitatsion to‘lqinlarni bevosita aniqlash, Messier 87 (M87) elliptik galaktika markazida joylashgan o‘ta massiv qora tuynukning birinchi tasvirini kuzatish. Event Horizon teleskopi va SgrA* o‘ta massiv qora tuynukning yaqin muhitidagi issiq bulutlarni va GRAVITY bilan hamkorlikda S2 yulduzi dinamikasini muqobil gravitatsiya nazariyalari effektlarini kuzatish uchun kuzatuv ma'lumotlarini taqdim etadi. Boshqa tomondan, muqobil gravitasiya nazariyalarining yangi aniq va taxminiy yechimlariga, shu jumladan, qo'shimcha sohalarga katta qiziqish mavjud. Ya’ni, skalyar maydonlar, qorong'u materiya va qorong'u energiya kabi zamonaviy kosmologik modellardagi materiyaning yangi shakllarini tushuntirishni misol qilib keltirishimiz mumkin. Nihoyat, muqobil gravitasiya nazariyalarining aniq yechimlari gravitatsion kompakt ob'ektlari markazidagi cheksiz egrilik tufayli hosil bo’ladigan fizik singularlikning yo’qolishida o'ziga xos ahamiyatga ega hisoblanadi.

Umumiy nisbiylik nazariyasining muhim muammolaridan biri Eynshteyn maydon tenglamalarining yangi aniq analitik yechimlarini topishdan iboratdir. 1915-yilda Eynshteyn umumiy nisbiylik nazariyasini yaratganidan keyin gravitatsiaon maydoni tenglamalarining yangi yechimlarini topish uchun ko‘plab mukammal matematik usullar ishlab chiqilgan. Eynshteyn maydon tenglamalarining aniq yechimlari astrofizik ahamiyatga ega, shu jumladan tashqi vakuum yechimlar statik va aylanadigan qora tuynuklar uchun mos ravishda Shvartzshild va Kerr tomonidan topilgan. Xususan, Hartl-Torn metrikasi har qanday sekin aylanadigan astrofizik ob'ekt tashqi vakuum fazo-vaqtini relativistik yulduz sifatida tasvirlaydi. Undan tashqari, turli mualliflar tomonidan Eynshteyn maydon tenglamalarining juda ko'p aniq analitik yechimlari olingan.

Astronomik ob'ektlar turli sabablarga ko'ra deformatsiyalanishi mumkin va shuning uchun deformatsiyalangan gravitatsion kompakt ob’ektlarning fazo-vaqtini o'rganish astrofizik nuqtaiy nazardan qiziq va muhim masalalardan biridir. Hozirgi kunga qadar Eynshteyn tenglamalarining deformatsiyalangan gravitatsion kompakt ob’ektlar uchun aniq aksial-simmetrik statik vakuum yechimlari olingan. Shunday yechimlar tashqi kvadrupol momentga ega bo'lgan metrika deb ataladi. Deformatsiyalangan ob’ektlar atrofidadi fazo-vaqt uchun Eynshteyn tenglamalarining yana bir aniq yechimi q-metrika deb ataladi. Bu yechimlar Veyl tipidagi yechimlar sinfiga kiradi. Bizga ma'lumki, massiv skalyar maydonning sekin aylanuvchi neytron yulduz atrofidagi gravitatsion maydonga massasiz skalyar maydonga nisbatan ancha katta ta’sir qiladi. Eynshteyn tenglamalarining skalyar maydonga ega yumronqoziq ini(wormhole) uchun aniq yechimi ham turli mualliflar tomonidan keng o‘rganilgan. Biroq, Eynshteyn maydon tenglamalarining gravitatsion skalyar maydonlarni hisobga olgan holda aksial-simmetrik va statik yechimlari o'rganilmagan. Shu paytgacha Veyl va prolat koordinatalaridagi Eynshteyn maydon tenglamalarining aniq analitik yechimlari olinmagan.

Ta’kidlash joizki, keyingi yillarda mamlakatimizda fundamental va amaliy tadqiqotlarning dolzarb yo‘nalishlarini rivojlantirishga tobora ko‘proq e’tibor qaratilmoqda. Xususan, istiqbolli yo‘nalishlardan biri bo‘lgan nazariy astrofizik tadqiqotlarni rivojlantirish bugungi kunning muhim masalasidir. Mamlakatimizda ilm-fanni muvaffaqiyatli rivojlantirish uchun fundamental tadqiqot va ishlanmalarning asosiy yo‘nalishlari va ularni amaliyotda qo‘llash 2022-2026-yillarda O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasida o‘z ifodasini topgan. Shuning uchun plazma muhitida gravitatsion linzalarning ta'sirini o'rganish fundamental tadqiqotlar sohasidagi dolzarb masalalardan biri bo'lib qolmoqda.

Mazkur ilmiy tadqiqot ishi quyidagi davlat me’yoriy hujjatlari bilan belgilangan vazifalarga mos keladi: O‘zbekiston Respublikasi Prezidentining 2017-yil 07-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi PD-4947-son Farmoni, O‘zbekiston Respublikasi Prezidentining 2017-yil 18-fevraldagi “Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy-tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789-son qarori va boshqalar.

Tadqiqotning Respublika fan va texnikasini rivojlantirishning asosiy ustuvor yo‘nalishlariga muvofiqligi. Tadqiqot O‘zbekiston Respublikasi fan va texnikaning ustuvor yo‘nalishlariga muvofiq amalga oshirildi: II. "Quvvat, energiya va resurslarni tejash".

Muammoni bilish darajasi. Eynshteyn-skalyar maydon tenglamalarining sferik-simmetrik statik yechimi va shunga o‘xshash yechimlar bir qancha mualliflar (A. I. Janis, E. T. Nyuman, J. Vinikur, R. Penni, M. Uayman, A. G. Agnese, M. La Camera) tomonidan o‘rganilgan. , T. Damour, K. S. Virbhadra, A. Bhadra, K. K. Nandi, N. Dadhich, N. Banerji, S. Abdolrahimi, A. A. Shoom. Biroq, Eynshteyn-Maksvell- skalyar maydon tenglamalarining sferik-simmetrik va statik yechimi tizimli ravishda amalga oshirilmagan.

Eynshteyn tenglamalarining ba'zi taxminiy statik yechimlari bir nechta mualliflar (Z.-Y. Fan, X. Lu) tomonidan konformal maydon nazariyasi yondashuvi asosida o'rganilgan. Statik va aylanuvchi qora tuynuklarning fazo vaqtidagi skalyar maydonning hissasi ham o‘rganilgan (K. S. Virbxadra, N. Dadxich, N. Banerji, L. Errera, G. Magli, D. Malafarina, C. A. R. Gerdeiro va E. Radu, C. Erices va C. Martinez, A. Sen). Biroq, Eynshteyn maydon tenglamalarining gravitatsion skalyar maydonining ta'sirini hisobga olgan holda aksial-simmetrik va statik yechimlari hali o'rganilmagan.

Dissertatsiya mavzusini ushbu mavzuda dissertatsiya olib borilayotgan oliy o‘quv yurtlari va ilmiy-tadqiqot muassasalarining ilmiy ishlari bilan bog’lash. Dissertatsiya Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan ilmiy loyihalar doirasida bajarilgan. F-FA-2021-510 "Modifikatsiyalangan gravitatsiya nazariyasi doirasida neytron yulduzlardagi yadro moddalarini tadqiq etish".

Tadqiqotning maqsadi Eynshteyn-Maksvell-skalyar maydon tenglamalarining aniq analitik aksial-simmetrik va statik yechimlarini topishdir.

Tadqiqot vazifalari:

Eynshteyn maydon tenglamalarining aksial-simmetrik va statik vakuum yechimlarining fazo-vaqt xossalarida skalyar maydonning ta'sirini o'rganish;

qo'shimcha parametrli gamma-metrikaning umumlashtirilgan shaklini va Eres-Rozen metrikasining umumlashtirilgan shaklini olish;

gravitasion skalyar maydon uchun energiya-impuls tenzori komponentlarining analitik ifodalarni olish;

gravitatsion skalyar maydon tomonidan hosil bo’lgan umumlashtirilgan gamma-metrika va kvadrupol momentli metrika parametrlarining fazo-vaqtidagi sinov zarralarining harakatini o'rganish;

metrik parametrlar bo'yicha eng ichki barqaror aylana orbitalari (ISCO) radiusi, energiyaning kritik qiymatlari va sinov zarrachalarining impuls momentining aniq analitik ifodalarini olish;

Eynshteyn-Maksvell-skalyar maydon tenglamalarining aniq analitik yechimini uch xil maydonlar, ya’ni gravitatsion, vektor va massasiz skalyar maydon bir-biri bilan o‘zaro ta’sir qilmagan holda deb hisoblash;

Richchi skalyar, Richchi kvadrati va Kretshman skalyari kabi fazoning egrilik invariant kattaliklarini hisoblash.

Tadqiqot obyekti astrofizik kompakt obyektlar, zarralar dinamikasi, Maksvell va skalyar maydonlar hisoblanadi.

Tadqiqot predmeti bu maydon tenglamalarining aniq analitik yechimlari, skalyar maydon ta’sirida gravitatsion kompakt obyektlar yaqinidagi zarralar dinamikasini o‘rganishning nazariy modellari, differensial tenglamalarni yechishning sonli va analitik usullari hisoblanadi.

Tadqiqot metodi bu hisoblash matematikasi metodlari, umumumiy nisbilik nazariyasi matematik apparati, nazariy fizika va astrofizika usullari, matematik fizikaning zamonaviy metodlari, maydon va zarrachalar harakati uchun differentsial tenglamalarni hisoblashning analitik va raqamli metodlaridan iborat.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Qo'shimcha gravitatsion skalyar maydonning ta'sirini hisobga olgan holda Eynshteyn maydon tenglamalarining aksial-simmetrik va statik yechimlari olingan. Bular Veyl yechimlar sinfiga tegishli bo'lgan ikki xil yechimlardir, (i) modifikatsiyalangan gamma-metrik va (ii) modifikatsiyalangan kvadrupol moment metrikasidir;

Eynshteyn maydon tenglamalarining aksial-simmetrik va statik vakuum yechimlarining fazo-vaqt xossalariga skalyar maydonning ta'siri o'rganildi;

Qo'shimcha parametrlarga ega bo'lgan gamma-metrikaning umumlashtirilgan shakli va gravitatsion skalyar maydonida hosil bo'lgan kvadrupol momentini o'z ichiga olgan Eres-Rozen metrikasining umumlashtirilgan shakli olindi;

Gravitatsion skalyar maydon uchun energiya-impuls tenzorining komponentlari uchun analitik ifodalar olindi. Fantom maydon holatida yechim nol energiya shartini (NEC) qanoatlantirishi ko'rsatilgan;

Gamma va kvadrupol parametrlari sinov zarrachasining birlik massadagi energiyasi va impuls momentiga hissa qo'shmasligi va natijada ekvatorial tekislikda harakatlanayotgan zarraning trayektoriyasiga ta'sir qilmasligi ko'rsatilgan;

Eng ichki barqaror aylana orbita radiusi, birlik massadagi energiyaning kritik qiymatlari va sinov zarrachalarining impuls momentining aniq analitik ifodalari olindi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Gamma parametr qiymatlarining mos keladigan diapazoni uchun ISCO radiusi va foton sferasi ortishi ko'rsatilgan.

Eynshteyn-Maksvell-skalyar maydon tenglamalarining analitik yechimi topildi. Shuningdek, Richchi skalyar, Richchi tenzori kvadrati va Kretchman skalyari kabi egrilik invariantlari ham hisoblab chiqildi.

Barcha fazoning egrilik invariant kattaliklari uchta yagona nuqtaga ega ekanligi ko'rsatilgan. Olingan yangi yechimlarda voqealar gorizonti mavjud emas degan xulosa olindi.

Yangi sferik-simmetrik kompakt ob’ekt yechimi bilan ifodalanadigan fazoda sinov zarralari uchun energiya samaradorligi $6\% \lesssim \eta \lesssim 8\%$ oraliqda ekanligi aniqlandi.

Tadqiqot natijalarining ishonchliligi bu matematik fizika, hisoblash matematikasi va relyativistik astrofizikaning zamonaviy tasdiqlangan usullarini qo‘llash orqali ta’minlanadi. Natijalar qat’iy ravishda umumiy nisbiylik va nazariy fizikaning matematik apparati doirasida olingan. Hisoblashning zamonaviy raqamli va analitik usullari ham qo‘llaniladi va natijalar mavjud kuzatuv ma’lumotlari va boshqa mualliflarning natijalari bilan taqqoslanadi. Ishning tuzilgan xulosalari kompakt obyektlar astrofizikasining asosiy qoidalariga mos keladi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, maydon tenglamalarining olingan yechimlari oldingi yechimlarni umumlashtirishi va deformatsiyalangan kompakt ob'ektlarni tavsiflashi mumkin.

Tadqiqot natijalarining amaliy ahamiyati shundaki, ular Eynseyn-Maksvell-skalyar gravitatsiya modeli doirasida gamma va kvadrupol parametrlarining yuqori chegaralari va cheklovlarini aniqlashda muhim rol o'ynashi mumkin.

Tadqiqot natijalarini amalga oshirish. Eynshteyn-Maksvell-skalyar maydon tenglamalarining yangi analitik yechimi asosida:

zarralar harakati bo'yicha olingan ilmiy natijalar Shanxaydagi Fudan universiteti (FU) olimlari tomonidan qo'llanildi (FU, Xitoy, 2024 yil 7 mart ma'lumotnomasi);

zarralarning kompakt ob’ekt atrofidagi harakati bo'yicha natijalar xorijiy tadqiqotchilarning ishlarida, nufuzliy xorijiy jurnallarda qo'llanilgan (The European Physical Journal C, Volume 83, Issue 12, article id.1131, Web-Sc, IF: 4.4 Pramana, 97-jild, 1-son, id.29-maqola, Web-Sc, IF: 2.219; Qirollik Astronomiya Jamiyatining oylik xabarlari, 521-jild, 1-son, 474-477-bet, Web-

Sc, IF: 5.235). Kompakt ob'ektlar atrofidagi zarrachalar dinamikasiga metrik parametrlarning ta'sirini aniqlashda qo’llanilgan.

Tadqiqot natijalarini nashr etish. PhD tadqiqot natijalari O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan nufuzli Q1/Q2 tipdagi ilmiy jurnallarida chop etilgan 15 ta ilmiy maqolalarda taqdim etilgan.

ISHNING ASOSIY TARKIBI

Eynshteyn-skalyar maydon tizimi uchun harakat integrali quyidagi shaklda ifodalanadi

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(R - 2\epsilon g^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi\right) \tag{1}$$

Bu yerda g metrik tenzorning determinant hisoblanadi, R fazo egriligining Ricci skalyari va Φ massaga ega bo'magan skalyar maydonni ifodalaydi, konstanta ϵ esa skayar maydonni mos ravishda gravitatsiyon ($\epsilon = 1$) va fantom ($\epsilon = -1$) hususiyatga ega ekanligini ko'rsatadi.

(1) tenglamadagi harakatni minimallashtirgandan so'ng, gravitatsion (fantom) skalyar maydonini hisobga olgan holda Eynshteyn maydoni tenglamalari va gravitatsion (fantom) skalyar maydoni uchun Klein-Gordon tenglamasini quyidagi shaklda olish mumkin

$$R_{\mu\nu} = 2\epsilon\,\partial_\mu\Phi\,\partial_\nu\Phi, \tag{2}$$

$$\boxdot\,\Phi = 0, \tag{3}$$

Bu yerda $R_{\mu\nu}$ fazo egrilikning Ricci tenzori va $\boxdot$ to'rt o'lchamli egrilangan fazodagi d'Alember operatori hisoblandi. Ma'lumki (2)-(3) tenglamalar differensial tenglamalar sistami bo'lib, ularning yechimlarini topish hozircha oson ish emas. Bu ishda (2)-(3) maydon tenglamalarining eksenel-simmetrik va statik yechimlarini taqdim etamiz va yechimlarni ilgari adabiyotda olingan yechimlar bilan solishtiramiz.

Aksial-simmetrik and statik yechim.

Muammoni soddalashtirish maqsadida biz gravitatsion skalyar maydoni aksial-simmetrik va statsionar deb tanlab olamiz. Veyl koordinatalarida (t, ρ, ϕ, z) statik metrikaning umumiy shakli quyidagicha tasvirlanishi mumkin

$$ds^2 = -e^{2U}dt^2 + e^{-2U}[e^{2V}(d\rho^2 + dz^2) + \rho^2 d\phi^2] \tag{4}$$

Bu yerda U va V mosh ravishda ρ va z koordinatalarini funksiyalari hisoblanadi. Keyin fazo-vaqt metrikasi (4) uchun maydon tenglamalarining (2)-(3) aniq shakli quyidagicha yozilishi mumkin.

$$\Delta\Phi = \Phi_{\rho\rho} + \frac{1}{\rho}\Phi_\rho + \Phi_{zz} = 0 \tag{5}$$

$$\Delta U = U_{\rho\rho} + \frac{1}{\rho}U_\rho + U_{zz} = 0 \tag{6}$$

$$V_\rho = \rho\left(U_\rho^2 - U_z^2 + \epsilon\Phi_\rho^2 - \epsilon\Phi_z^2\right) \tag{7}$$

$$V_z = 2\rho\left(U_\rho U_z + \epsilon\Phi_\rho\Phi_z\right) \tag{8}$$

bu yerda quyi indexlar mos ravishda ρ va z koordinatalar bo'yicha olingan xosilarlarni bildiradi. Qulaylik uchun muammoni prolat koordinatalarida xususan (t,X,Y,s) qayta ko'rib chiqish mumkin, bunda fazo-vaqt metrikasi (4) quyidagi shaklda qayta yoziladi

$$ds^2 = -e^{2U}dt^2 + \sigma^2 e^{-2U}\left[e^{2V}(X^2 - Y^2)\left(\frac{dX^2}{X^2-1} + \frac{dY^2}{1-Y^2}\right) + (X^2 - 1)(1 - Y^2)d\phi^2\right], \tag{9}$$

bu yerda σ o'lchovli parametrdir, keyinchalik bu parametrning fizik ma'nosi kiritiladi.

Bu erda biz foydali belgilarni kiritishimiz mumkin, ular prolat sferoid koordinatalari (X, Y, ϕ) va Veyl koordinatalari (ρ, z, ϕ) o'rtasidagi munosabatlardir

$$\rho = \sigma\sqrt{(X^2 - 1)(1 - Y^2)}, \quad z = \sigma XY, \quad \phi = \phi. \tag{10}$$

va shunga o'xshash, ular sferik koordinatalar (r, θ, ϕ) bilan quyidagi shakldа bog'lanishi mumkin

$$X = \frac{r}{\sigma} - 1, \quad Y = \cos\theta, \quad \phi = \phi \tag{11}$$

Bu yerda koordnata sistemasining nolinchi t tashkil etuvchisi bu yerda bir xil. Natiijada, (5)-(8) maydon tenglamalari X va Y prolat koordinatalari ko'rinishida qayta yozilishi mumkin

$$[(X^2 - 1)\Phi_X]_X + [(1 - Y^2)\Phi_Y]_Y = 0 \tag{12}$$

$$[(X^2 - 1)U_X]_X + [(1 - Y^2)U_Y]_Y = 0 \tag{13}$$

$$V_X = \frac{1 - Y^2}{X^2 - Y^2}[X(X^2 - 1)U_X^2 - X(1 - Y^2)U_Y^2 - 2Y(X^2 - 1)U_X U_Y] + (U \to \epsilon\Phi), \tag{14}$$

$$V_Y = \frac{X^2 - 1}{X^2 - Y^2}[Y(X^2 - 1)U_X^2 - Y(1 - Y^2)U_Y^2 + 2X(1 - Y^2)U_X U_Y] + (U \to \epsilon\Phi). \tag{15}$$

(12) va (13) tenglamalar bir-biriga o'xshashligini osongina ko'rish mumkin, ularning yechimlarini quyidagi ajratiladigan shaklda izlash mumkin $\{\Phi, U\} = f(X)g(Y)$ va (12) va (13) tenglamalardan foydalanib $f(X)$ va $g(Y)$ funksiyalar uchun quyidagi Legendre tenglamalarini shaklda yozish mumkin

$$[(X^2 - 1)f_X]_X - l(l + 1)f = 0 \tag{16}$$

$$[(1 - Y^2)g_Y]_Y + l(l + 1)g = 0, \tag{17}$$

bu erda l- butun son qiymatlarini qabul qila oladigan ko'p kiymatli son. (16) va (17) tenglamalarning yechimlari quyidagicha bo'ladi

$$f(X) = C_{1l}P_l(X) + C_{2l}Q_l(X) \tag{18}$$

$$g(Y) = C_{3l}P_l(Y) + C_{4l}Q_l(Y) \tag{19}$$

bu yerda $P_l(X)$ Legendre ko'p had, $Q_l(Y)$ ikkinchi turdagi Legendre funksiyasi va $C_{1l} - C_{4l}$ mos ravishda integrasiya konstantalari. Fizik nuqtai nazardan, ikkala yechim ham $\{\Phi, U\}$ asimptotik tekis bo'lishi kerak, ya'ni

$$\lim_{X\to\infty} f(X) = 0, \ C_{1l} = 0 \tag{20}$$

va ular fazoning hamma joyda regular bo'lishi kerak

$$\lim_{Y\to 0} g(Y) = \text{const}, \ C_{4l} = 0. \tag{21}$$

Fizik ma'no eag bo'lgan yechimni topish uchun $\epsilon = 0, q_0 = 1$ va $q_l = 0 \ (l > 0)$ ni o'rnatish va taniqli Shvarzshild yechimini olish mumkin

$$U = \frac{1}{2}\ln\frac{X - 1}{X + 1} = \frac{1}{2}\ln\left(1 - \frac{2\sigma}{r}\right), \tag{22}$$

$$V = \frac{1}{2}\ln\frac{X^2 - 1}{X^2 - Y^2} = \frac{1}{2}\ln\frac{r^2 - 2\sigma r}{r^2 - 2\sigma r + \sigma^2\sin^2\theta}. \tag{23}$$

Bu erda o'lchovli parametr ixcham ob'ektning umumiy massasi $\sigma = M$ ekanligini osongina ko'rish mumkin.

γ-metrik uchun o'z-o'zidan tortishish skalyar maydoniga ega Eynshteyn tenglamalarining analitik yechimi

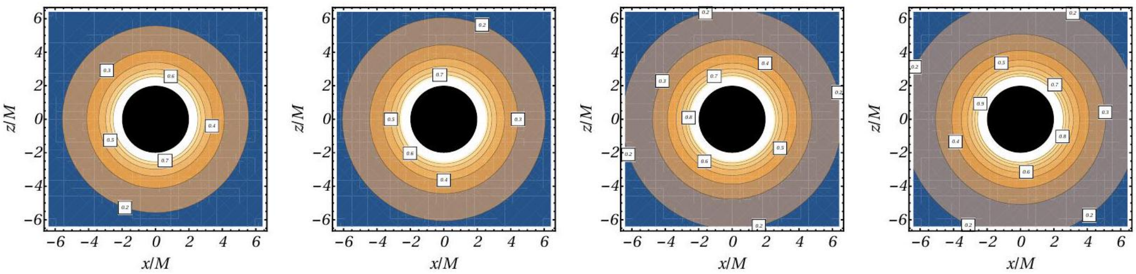
(11) ifodadagi koordinatani o'zgartirishdan foydalanib, sferik koordinatalarda γ-metrikaning umumlashtirilgan shaklini olishimiz mumkin

$$ds^2 = -\left(1-\frac{2M}{r}\right)^{\gamma} dt^2 + \left(1-\frac{2M}{r}\right)^{1-\gamma} \times \left\{\left(1-\frac{M^2\sin^2\theta}{r^2-2Mr}\right)^{1-\gamma^2-\epsilon\gamma_*^2}\left[\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2d\theta^2\right]+r^2\sin^2\theta d\phi^2\right\}, \tag{24}$$

Skalyar maydon esa quyidagi ifodaga ega

$$\Phi(r) = \frac{\gamma_*^2}{2}\ln\left(1-\frac{2M}{r}\right) \tag{25}$$

(25) ifodada skalyar funksiya $\Phi(r)$ faqat radial koordinataga bog'liqligini ko'rishimiz mumkin. 1-rasmda γ_* parametrining turli qiymatlari uchun $(x-z)$ tekislikda tortishish skalyar maydoni $\Phi(r)$ning ekvipotensial yuzasi chizilgan. γ_* parametrini oshirish bilan tortishish kuchi kuchayib borishini va 1-rasmda ko'rsatilganidek, skalyar maydon mavjudligi sababli ob'ekt atrofidagi fazo vaqti deformatsiyalanishini osongina ko'rish mumkin.



1-rasm. γ_* parametrining turli qiymatlari uchun $x-z$ tekisligida (25) tenglama bilan tasvirlangan $\Phi(r,\theta)$ skalyar maydonining shakli: γ_*=0,9, γ_*=1, γ_*=1,1 va γ_*=1,2.

Skalyar maydon uchun energiya-momentum tenzorini quyidagicha ifodalash mumkin

$$T_{\mu\nu} = \epsilon\left(\partial_\mu\Phi\,\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\,\partial_\alpha\Phi\,\partial_\beta\Phi\right) \tag{26}$$

(26) ifodadan energiya zichligi va bosim komponentlarini $\rho = T_0^0$ va $P_i = T_i^i$,, energiya zichligi va bosim komponentlarining aniq shaklini aniqlash mumkin.

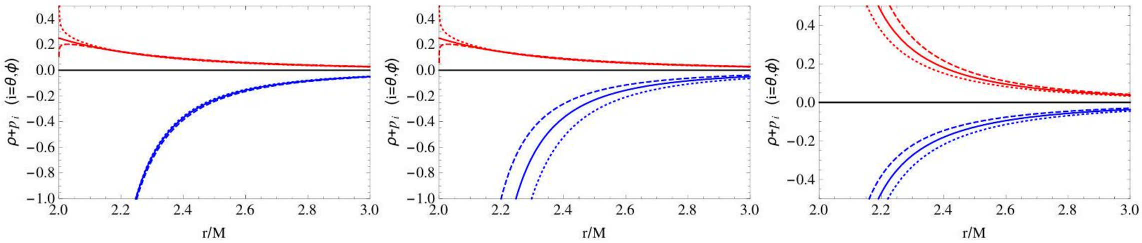
$$\rho = P_\theta = P_\phi = -P_r = -\frac{\epsilon\gamma_*^2 M^2}{2r^4}\left(1-\frac{2M}{r}\right)^{\gamma-2}\left(1-\frac{M^2\sin^2\theta}{r^2-2Mr}\right)^{\gamma^2+\epsilon\gamma_*^2-1}.$$

Nol energiya holatini (NEC) $\rho + P_i \geq 0 (i = r, \theta, \phi)$ ifodasidan (27) tenglamadan foydalanib topish mumkin

$$\rho + P_r \equiv 0, \tag{27}$$

$$\rho + P_\theta = \rho + P_\phi = -\frac{\epsilon\gamma_*^2 M^2}{r^4}\left(1-\frac{2M}{r}\right)^{\gamma-2}\left(1-\frac{M^2\sin^2\theta}{r^2-2Mr}\right)^{\gamma^2+\epsilon\gamma_*^2-1}. \tag{28}$$

NEC ning jismoniy talqini shundan iboratki, kuzatuvchi tomonidan nol egri chiziq bo'ylab o'tadigan energiya zichligi har doim ijobiy bo'ladi (hech qachon manfiy emas). Ko'rish mumkinki, (27) ifoda har doim fazo-vaqt ko'rsatkichi (24) uchun NEC sharti bilan qanoatlantiriladi, ifoda (28) esa faqat fantom maydoniga mos keladigan $\epsilon \leq 0$ bo'lganda NEC shartini qondiradi. Bu shuni anglatadiki, nol egri chiziq bo'ylab harakatlanayotgan kuzatuvchi hatto antigravitatsion fantom skalyar maydonida ham ijobiy energiyani o'lchashi mumkin. 2-rasmda $\rho + P_i(i = r, \theta, \phi)$ ning radial bog'liqligi NEC aniq ko'rsatilgan.



2-rasm. Parametrlarning turli qiymatlari uchun $\{\rho + P_i\}(i = r, \theta, \phi)$ ning radial bog'liqligi γ and γ_*. (Chap panel) Qalin chiziq $\gamma = 1$ ga, uzuq chiziq $\gamma = 0.9$ and nuqtali uzuq chiziq $\gamma = 1.1$ at $\gamma_* = 1$ va $\theta = \pi/2$. (O'rta panel) Qalin chiziq $\gamma_* = 1$, uzuq chiziq $\gamma_* = 0.9$ va nuqtali uzuq chiziq $\gamma_* = 1.1$ quyidagi qiymatlarda $\gamma = 1$ va $\theta = \pi/2$. (O'ng panel) Qalin chiziq $\gamma = 1$, uzuq chiziq $\gamma = 0.9$ va nuqtali uzuq $\gamma = 1.1$ quyidagi qiymatalarda $\gamma_* = 1$ va $\theta = 0$.

Massasi m bo'lgan sinov zarrasi uchun Gamiltonian quyidagi ko'rinishida yozilishi mumkin

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu + \frac{1}{2} m^2 \tag{29}$$

bu erda $p^\mu = m u^\mu$ kinematik to'rt impuls. Zarrachaning harakat tenglamalari

$$\frac{dx^\mu}{d\zeta} \equiv p^\mu = \frac{\partial H}{\partial p_\mu}, \ \frac{dp_\mu}{d\zeta} = -\frac{\partial H}{\partial x^\mu} \tag{30}$$

Bu yerda zarrachaning affin ζ parametri uning to'g'ri vaqtiga $\zeta = \tau/m$ munosabati bilan bog'langan. Qulaylik uchun o'ziga xos parametrlar, energiya $\mathcal{E}$ and burchak momentini $\mathcal{L}$ kiritib olamiz

$$\mathcal{E} = \frac{E}{m}, \ \mathcal{L} = \frac{L}{m}, \tag{31}$$

va (29) ifodagi Gamilnianni quyidagicha yozamiz

$$H = \frac{1}{2} g^{rr} p_r^2 + \frac{1}{2} g^{\theta\theta} p_\theta^2 + \frac{m^2}{2} g^{tt} [\mathcal{E}^2 - V_{\text{eff}}(r, \theta)], \tag{32}$$

bu yerda $V_{\text{eff}}(r, \theta)$ zarraning effektiv potensiali hisoblanadi va quyidagicha ifodalanadi

$$\begin{aligned} V_{\text{eff}}(r, \theta) &\equiv -g_{tt}\left(1 + g^{\phi\phi}\mathcal{L}^2\right) \\ &= \left(1 - \frac{2M}{r}\right)^\gamma \left[1 + \frac{\mathcal{L}^2}{r^2 \sin^2 \theta}\left(1 - \frac{2M}{r}\right)^{\gamma - 1}\right]. \end{aligned} \tag{33}$$

Zarrachalar harakati tomonidan berilgan energiya chegaralari bilan chegaralanadi

$$\mathcal{E}^2 = V_{\text{eff}}(r, \theta). \tag{34}$$

Effektiv potentsialning (33) xususiyatlari 3-rasmda ko'rsatilgan. Maksimal yoki minimal mavjud bo'lishi mumkin bo'lgan samarali potentsial $V_{eff}(r, \theta)$ funktsiyasining statsionar nuqtalari tenglamalar bilan berilgan

$$\partial_r V_{\text{eff}}(r, \theta) = 0, \ \partial_\theta V_{\text{eff}}(r, \theta) = 0. \tag{35}$$

Ekstremal tenglamalarning ikkinchisi (35) $\theta = \pi/2$ ni beradi. (35) ning birinchi ekstremal tenglamasi tenglamaning o'ziga xos burchak momenti $\mathcal{L}$ ga nisbatan kvadratik bo'lishiga olib keladi va shuning uchun aylana orbitalarni munosabatlar orqali aniqlash mumkin

$$\mathcal{L}^2 = \mathcal{L}_{\text{ext}}^2(r) \equiv \frac{\gamma M r^2}{r - M(1+2\gamma)}\left(1 - \frac{2M}{r}\right)^{1-\gamma} \qquad (36)$$

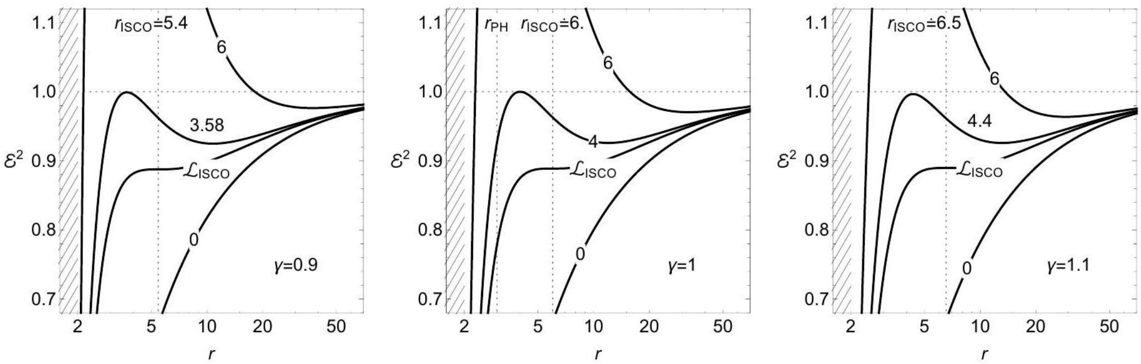
4-rasmda γ parametrning turli qiymatlari uchun $\mathcal{L}_{\text{ext}}(r)$ funksiyasi chizilgan. Xuddi shunday, tekshirilayotgan zarrachaning energiyasi ham quyidagicha ifodalanishi mumkin

$$\mathcal{E}^2 = \mathcal{E}_{\text{ext}}^2(r) \equiv \frac{r - M(1+\gamma)}{r - M(1+2\gamma)}\left(1 - \frac{2M}{r}\right)^{\gamma} \qquad (37)$$

$\mathcal{L}_{\text{ext}}(r)$ funksiyaning lokal ekstremumi quyidagi ifodaga $\partial_r^2 V_{\text{eff}}(r, \theta = \pi/2) = 0$ ekvivalent bo'ladi va ichki barqaror aylanma orbitalar (ISCO) joylashuvi quyidagi ifoda bilan beriladi

$$r_{\text{ISCO}}/M = 1 + 3\gamma + \sqrt{5\gamma^2 - 1}, \qquad (38)$$

va 65 tenglamadan γ parameter uchun $\gamma \geq 1/\sqrt{5}$ shartni topamiz.

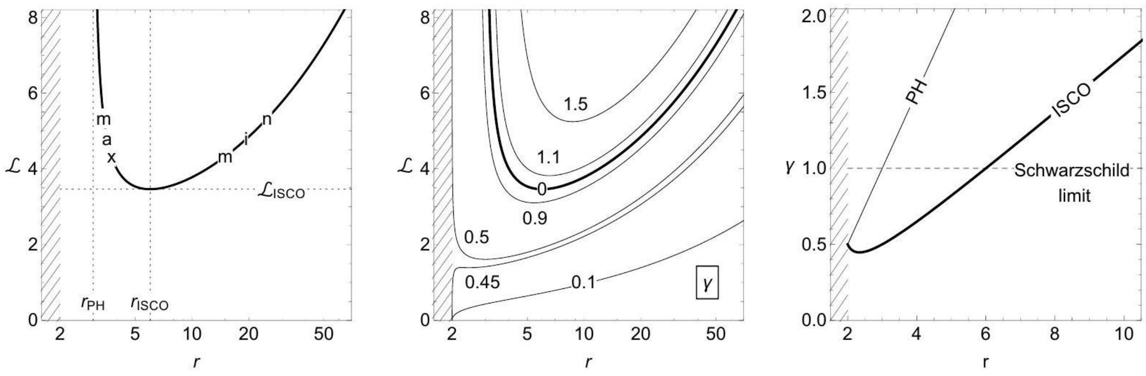


3-rasm. Impuls momentining turli qiymatlari uchun effektiv potensial ekvator tekisligida radial bog'lanishi har xil gamma parametrlar uchi ko'rsatilgan.

(33) ifodada keltirilgan effektiv potentsialning cheksiz katta qiymat qabul qiluvchi radial koordinata, beqaror aylana foton orbitasiga mos keladi va quyidagicha ifodalanadi

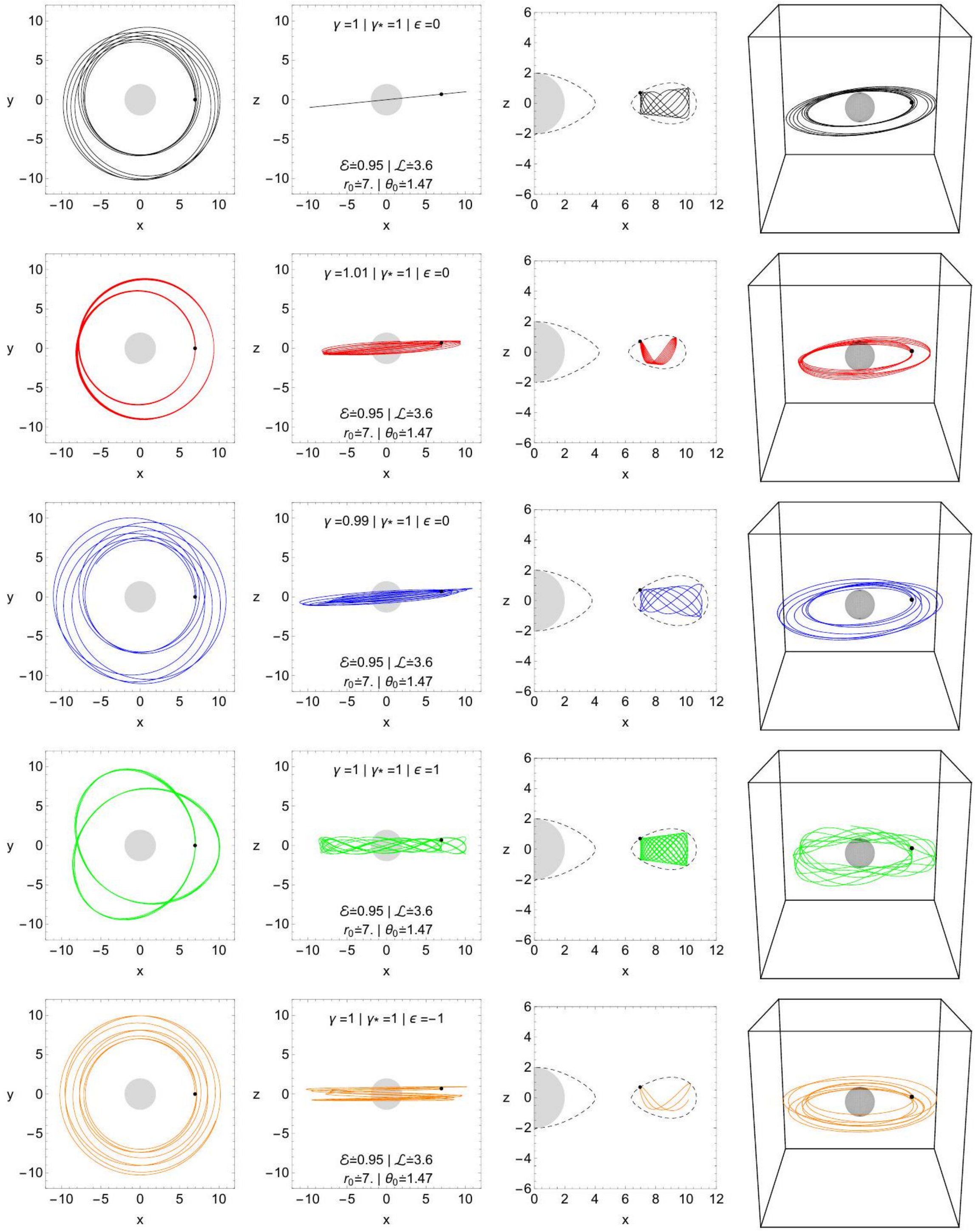
$$r_{\text{ph}}/M = 1 + 2\gamma. \qquad (39)$$

$\gamma = 1$ bo'lgan holda, biz ISCO va foton sferasi radiuslari uchun $r_{ISCO} = 6M$ va $r_{ph} = 3M$ qiymatlari Shwarzshild fazo vaqtiga mos keladi. 4-rasmda ISCO radiusi va foton sferasining turli bog'liqliklari ko'rsatilgan. $\gamma \geq 1$ qiymatlari diapazonida γ parametri ortishi bilan ISCO radiusi va foton sferasi ortib borishini, $1/\sqrt{5} \leq \gamma \leq 1$ qiymatlar oralig'ida esa umumiy nisbiylik nazariyasi bilan solishtirganda kichik ekanligini ko'rish mumkin.



4-Rasm. (Chap panel) Schwarzschild ($\gamma = 1$) fazo vaqti uchun barqaror (min) va beqaror(max) aylana orbitalarini beruvchi effektiv potentsialning ekstrimum (max, min.) joylashuvi. (markaziy panel) γ parametrining turli qiymatlari uchun effektiv potentsialning ekstrimum (max. min.) joylashuvi. (o'ng panel) ISCO va foton orbitasining γ parametriga bog'liqligi.

Impuls momenti uchun (36), (37) va (38) tenglamalar sinov zarrasining ISCO energiyasi va radiusi mos ravishda q ni o'z ichiga olmaydi, bu tortishish skalyar maydonining ekvatorial sinov zarralariga ta'sir qilmasligini anglatadi. samolyot. Raqamli hisob-kitoblar shuni ko'rsatadiki, tortishish skalyar maydonining ta'siri ekvatordan tashqari tekislikdagi zarrachalar harakatida ko'rinadi. Fazo-vaqt geometriyasini tekshirish (24) sifatida biz 5-rasmda bir nechta tekisliklarda γ, γ_* va ϵ metrik parametrlarining turli qiymatlari uchun zarracha traektoriyalarini taqdim etdik.



5-Rasm. γ, γ_* va ϵ parametrlarining turli qiymatlari uchun fazo-vaqt metrikasida (24) zarracha traektoriyalari. Birinchi va ikkinchi (shu jumladan uchinchi) ustunlarda zarrachalar traektoriyalari $x-y$ va $x-z$ tekisliklari, to'rtinchi ustunda esa 3D $x-y-z$ zarrachalar traektoriyasining namunasi ko'rsatilgan.

Erez-Rozen yechimi $q_* = 0$ bo'lganda chegaraviy holatni olish mumkin. Skalyar maydon uchun fizik ma'noli yechim topish uchun uni sferik koordinatalar shaklida yoziladi

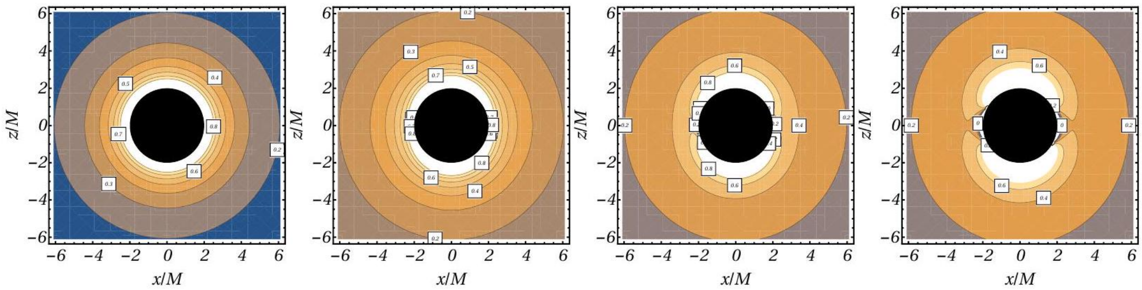
$$\Phi(r,\theta) = \frac{1}{2}\ln\left(1-\frac{2M}{r}\right) + \frac{q_*}{2}\left[\frac{3r^2-6Mr+2M^2}{4M^2}\ln\left(1-\frac{2M}{r}\right) + \frac{3(r-M)}{2M}\right](3\cos^2\theta - 1) \qquad (40)$$

kuchsiz maydon yaqinlashuvida esa 40- tenglama quyidagi shaklga ega

$$\Phi(r,\theta) \simeq -\frac{M}{r} + \frac{q_* M^3}{15 r^3}(3\cos^2\ \theta - 1). \qquad (41)$$

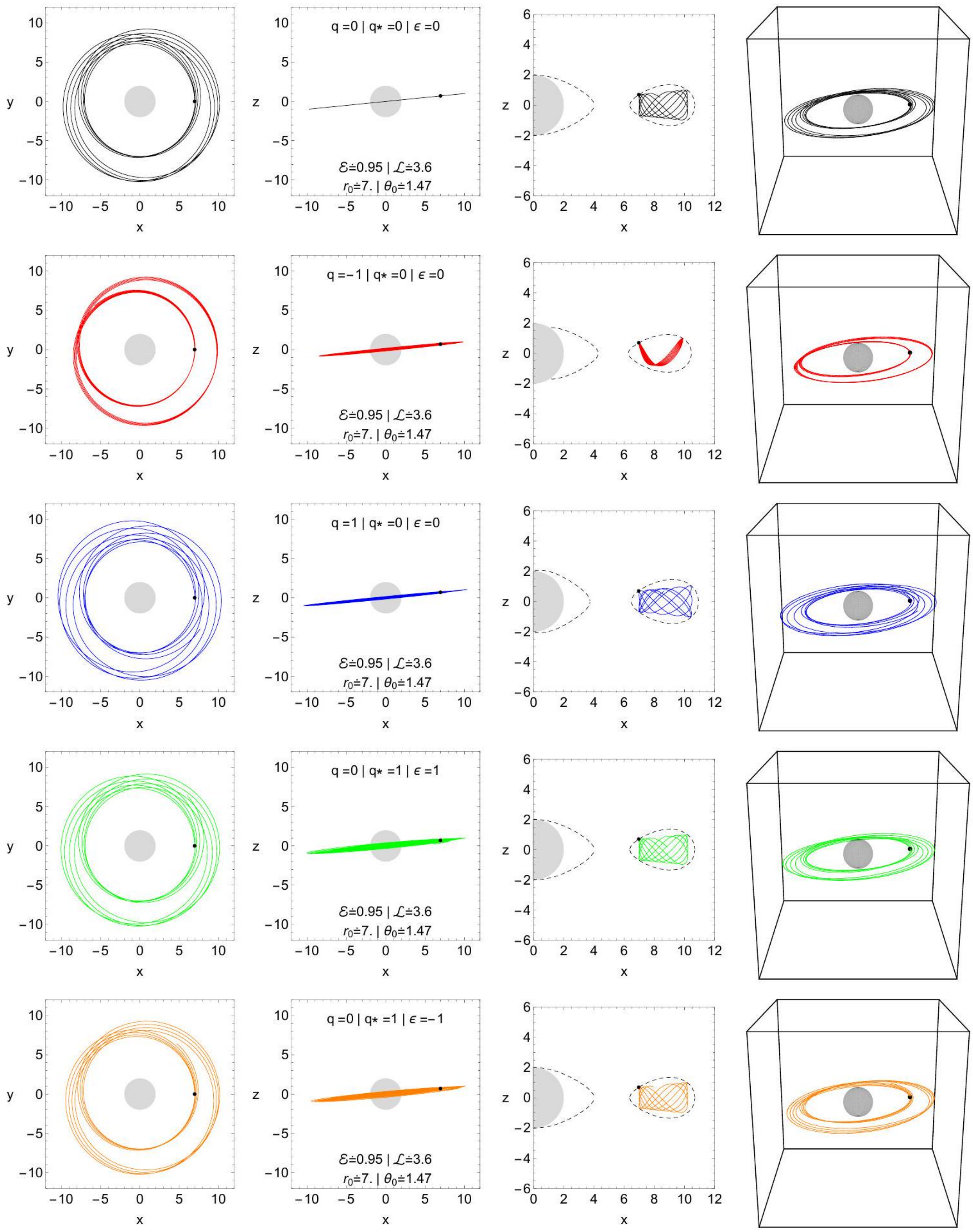
Ko'rishimiz mumkinki, tenglamaning o'ng tomonidagi birinchi chiziqli had (41) Nyuton potentsiali uchun javob beradi, ikkinchi qismi quadrupol moment potentsialiga javob beradi, bu erda q_* tortishish skalyar maydoni tomonidan ishlab chiqarilgan o'lchovsiz massali quadrupol moment.

6-rasmda quadrupol moment q_* ning turli qiymatlari uchun (40) ifoda yordamida skalyar maydonning $\Phi(r,\theta)$ ekvipotensial yuzasi tasvirlangan. q_* parametri tufayli qora tuynuk atrofidagi fazo-vaqt axial deformatsiyaga uchraganini osongina ko'rish mumkin.

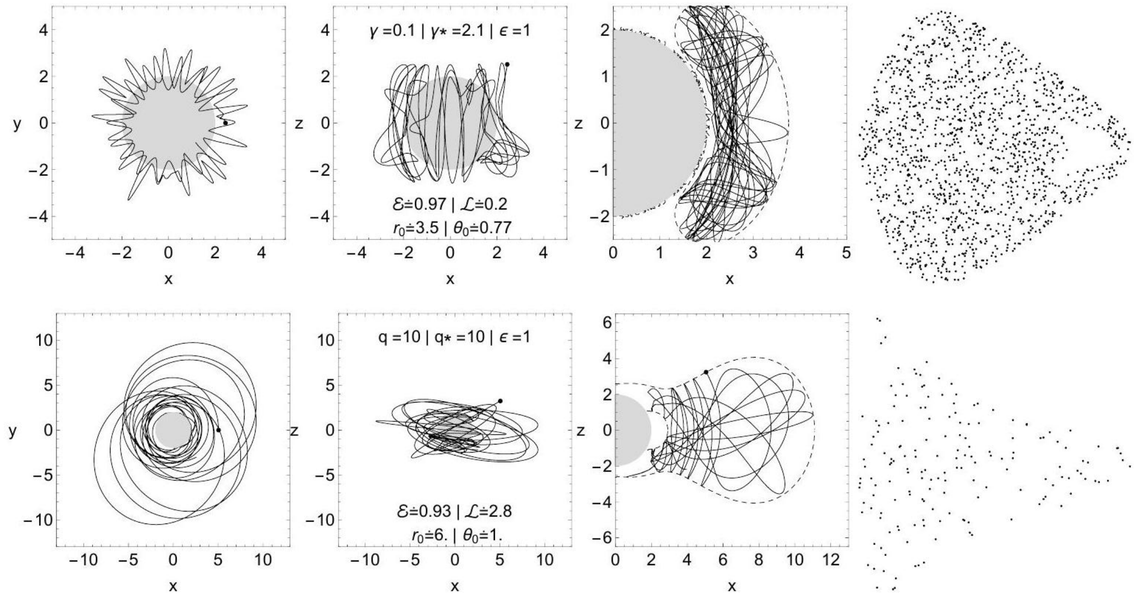


6-Rasm. Massa quadrupol momentining turli qiymatlari uchun $x - z$ tekislikdagi skalyar potentsial $\Phi(r,\theta)$ ekvipotensial yuzasi: $q_* = 0, q_* = 0.2, q_* = 0.5$ va $q_* = 1$.

Parametrlarning turli qiymatlari uchun bir necha tekisliklarda umumlashtirilgan Erez-Rozen metrikasining fazo vaqtidagi sinov zarralarining traektoriyalari 7-rasmda ko'rsatilgan. Tekshiriluvchi zarrachaning harakati muntazam bo'lib qoladi (Kerr fazo vaqtidagi kabi xaotik emas) quadrupol moment metrikasida. γ, γ_*, q va q_* deformatsiya parametrlari bilan fazoda xaotik harakatni o'rganish ham qiziq. Qora tuynuk atrofidagi xaotik harakatni tekshirish uchun biz fazo-vaqt metrikasining umumiy shaklidan foydalandik. Raqamli hisoblar shuni ko'rsatadiki, 8-rasmda ko'rsatilgandek, γ_*, q, va q_* parametrlarining katta qiymatlari uchun sinov zarralarining traektoriyasi xaotik bo'ladi.



7-Rasm. $\boldsymbol{q, q_*}$ **va** $\boldsymbol{\epsilon}$ **parametrlarning turli qiymatlari uchun sinov zarrasining traektoriyalari. Birinchi va ikkinchi (uchinchi) ustunlar zarrachaning** $\boldsymbol{x-y}$ **va** $\boldsymbol{x-z}$ **tekisliklardagi traektoriyasi, to'rtinchi ustunda esa** $\boldsymbol{x-y-z}$ **(3D) fazodagi zarra traektoriyasi ko'rsatilgan.**



8-Rasm. $\boldsymbol{\epsilon = 1}$ **bo'lgan holdagi har xil tekisliklardagi zarrachaning xaotik traektoriyalari. Birinchi va ikkinchi (uchinchi ham) ustunlar zarrachaning** $\boldsymbol{x - y}$ **va** $\boldsymbol{x - z}$ **tekisliklardagi traektoriyasi, to'rtinchi ustunda esa fazalar fazosidagi zarra traektoriyasi ko'rsatilgan.**

Einstein-Maxwell-skalyar maydonlari sistemasida harakat tenglamasini ko'raylik. Bu sistema uchun ta'sir integrali quyidagicha yoziladi

$$S = \frac{1}{16\pi}\int\, d^4x\sqrt{-g}\big(R - F_{\alpha\beta}F^{\alpha\beta} - 2\,\partial_\alpha\Phi\,\partial^\alpha\Phi\big), \tag{42}$$

bu yerda R Ricci egrilik skalyari, $g = |g_{\alpha\beta}|$ esa $g_{\alpha\beta}$ metrik tenzorning determinanti, Φ skalyar maydon va $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ bunda A_μ vektor maydon. Ko'rish mumkinki, butun sistema uchun harakat tenglamalari, xususan, Einstein maydon tenglamalari, Klein-Gordon tenglamasi va Maxwell tenglamalari quyidagi ko'rinishga ega bo'ladi

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T_{\alpha\beta}, \tag{43}$$

$$\nabla_\alpha F^{\alpha\beta} = \frac{1}{\sqrt{-g}}\partial_\alpha\big(\sqrt{-g}F^{\alpha\beta}\big) = 0, \tag{44}$$

$$\nabla_\alpha\nabla^\alpha\Phi = \frac{1}{\sqrt{-g}}\partial_\alpha\big(\sqrt{-g}\,\partial^\alpha\Phi\big) = 0, \tag{45}$$

bu yerda ∇_α kovariant xosila, $G_{\alpha\beta}$ va $R_{\alpha\beta}$ lar esa, mos ravishda, Einstein va Ricci tenzorlari. Sistemaning energiya-impuls tenzori $T_{\alpha\beta}$ ikki qismdan tashkil topadi:

$$T_{\alpha\beta} = T^S_{\alpha\beta} + T^{EM}_{\alpha\beta}, \tag{46}$$

bu yerda birinchi qism skalyar maydonga, ikkinchi qism esa vektor maydonga mos keladi.

Bizning maqsadimiz Einstein-Maxwell-scalar maydon tenglamasini yechish. Faraz qilaylik, Einstein tenglamalarining yechimi sferikal simmetrik bo'lsin. Bu vector potensialning faqat vaqt komponentasigina noldan farqli bo'lishligiga olib keladi ya'ni $A_\alpha = (A_t, 0{,}0{,}0)$. Ikkinchi soddalashtirish sifatida, skalyar maydon va vector potensialning vaqt komponentasi faqat radial koordinataga bog'liq bo'lsin deb olaylik $\Phi = \Phi(r)$ va $A_t = A_t(r)$. Sferikal simmetrik, statik fazo-vaqt metrikasi umumiy holda quyidagicha beriladi

$$ds^2 = -e^{\nu(r)}dt^2 + e^{-\nu(r)}\left[dr^2 + e^{\lambda(r)}d\Omega\right] \tag{47}$$

bu yerda $\nu(r)$ va $\lambda(r)$ radial metrik funksiyalar.
Quyidagi bo'g'lanishni osongina keltirish mumkin:

$$G_r^r + G_\theta^\theta = 0 = \frac{1}{2}e^{\nu-\lambda}\left(\frac{d^2e^\lambda}{dr^2} - 2\right), \tag{48}$$

$$G_t^t + G_r^r = G_t^t - G_\theta^\theta = 2F_{rt}F^{rt} = -e^\nu(\nu'' + \nu'\lambda'), \tag{49}$$

$$G_r^r - G_t^t = -G_t^t - G_\theta^\theta = 2\,\partial_r\Phi\,\partial^r\Phi = e^\nu\left[\nu'' - \lambda'' - \frac{1}{2}(\nu' - \lambda')^2\right]. \tag{50}$$

(48) tenglamadan osongina ko'rish mumkinki, bu tenglama manbaga bog'liq emas va uning e^λ metrik funksiyasi uchun yechimi trivial va u quyidagicha yoziladi

$$e^\lambda = r^2 + 2C_1 r + C_2, \tag{51}$$

bu yerda C_1 va C_2 lar, mos ravishda, integrallash doimiysi bo'lib ular gravitatsion obyektning massasi va zaryadi bilan bog'langan. (49) va (50) tenglamalarni yechish uchun, birinchi navbatda Maxwell va Klein-Gordon tenglamalariga e'tiborimizni qaratamiz. (44) va (45) tenglamalarni qayta yozamiz

$$F_{rt} = Q_e e^{\nu-\lambda},\ \partial_r\Phi = Ce^{-\lambda}, \tag{52}$$

bu yerda Q_e elektr zaryad va C esa skalyar zaryad. (52) dagi birinchi tenglamadan foydalanib (49) ni quyidagicha yozamiz

$$\nu'' + \nu'\lambda' = 2Q_e^2 e^{\nu-2\lambda} \tag{53}$$

(51) tenglamadagi e^λ uchun yechimdan foydalanib va uzun hisob-kitoblarni bajarib, (53) tenglama uchun quyidagicha yechim olamiz

$$e^\nu = \left(1 + \frac{2C_1}{r} + \frac{Q_e^2}{n^2r^2}\right)^n \left[\frac{r_+\left(1-\frac{r_-}{r}\right)^n - r_-\left(1-\frac{r_+}{r}\right)^n}{r_+ - r_-}\right]^{-2}, \tag{54}$$

bu yerda n integrallash doimiysi va $r_\pm$ ni quyidagicha aniqlash mumkin

$$r_\pm = -C_1 \pm \sqrt{C_1^2 - \frac{Q_e^2}{n^2}} \tag{55}$$

Bu yerda ta'kidlab o'tishimiz kerakki integrallash doimiysi C_2 zaryad bilan bog'langan $C_2 = Q_e^2/n^2$ ko'rinishda bog'langan, qolgan ikki konstantalar C_1 va n gravitatsion obyektning massasi M va skalyar zaryad C orqali ifodalanish kerak. Ularni quyidagicha tanlash mumkin $C_1 = -M/n$ va$n = M/\sqrt{M^2 + C^2}$. Natijada, Einstein-Maxwellscalar aniq yechimi quyidagicha ko'rinishda bo'ladi

$$e^\nu = \left[\frac{r_+\left(\frac{r-r_-}{r-r_+}\right)^{n/2} - r_-\left(\frac{r-r_+}{r-r_-}\right)^{n/2}}{r_+ - r_-}\right]^{-2}, \tag{56}$$

$$e^\lambda = (r - r_+)(r - r_-),$$

bunda $nr_\pm = M \pm \sqrt{M^2 - Q_e^2}$. Bunga mos keluvchi va (52) ni qanoatlantiruvchi elektr va skalyar maydonlar quyidagi ko'rinishni oladi

$$E^{\hat{r}} = \frac{Q_e}{(r-r_+)(r-r_-)}\left[\frac{r_+\left(\frac{r-r_-}{r-r_+}\right)^{n/2} - r_-\left(\frac{r-r_+}{r-r_-}\right)^{n/2}}{r_+ - r_-}\right]^{-2},$$
$$\Phi(r) = \frac{\sqrt{1-n^2}}{2}\ln\left(\frac{r-r_+}{r-r_-}\right). \tag{57}$$

bunda $nr_{\pm} = M \pm \sqrt{M^2 - Q_e^2}$. Bunga mos keluvchi va (52) ni qanoatlantiruvchi elektr va skalyar maydonlar quyidagi ko'rinishni oladi

$$E^{r^{\hat{}}} = \frac{Q_e}{(r-r_+)(r-r_-)}\left[\frac{r_+\left(\frac{r-r_-}{r-r_+}\right)^{n/2} - r_-\left(\frac{r-r_+}{r-r_-}\right)^{n/2}}{r_+ - r_-}\right]^{-2}, \ \Phi(r) = \frac{\sqrt{1-n^2}}{2} ln\ \left(\frac{r-r_+}{r-r_-}\right), \tag{57}$$

Fazo-vaqtni yanada yaxshiroq tushunish uchun, uchta parameterga ega bo'lgan kompakt obyekt atrofidagi zarracha harakatini o'rganish juda muhim. Sinov zarrasining to'rt tezligi $u^{\alpha} = x^{\cdot\alpha} = dx^{\alpha}/d\lambda$, bunda λ affin parametri, quyidagi normallashtirish shartiga bo'ysunadi: $u_{\alpha}u^{\alpha} = -1$, boshqa tomondan uni quyidagicha ifodalay olamiz: $p_{\alpha} \equiv mu_{\alpha} = \partial_{\alpha}S$.

Biz Ekvatorial tekislikdagi $\theta = \pi/2$ harakat bilan ishlaymiz va bunda sodda algebraik hisoblashlardan so'ng quyidagi tenglikni olishimiz mumkin:

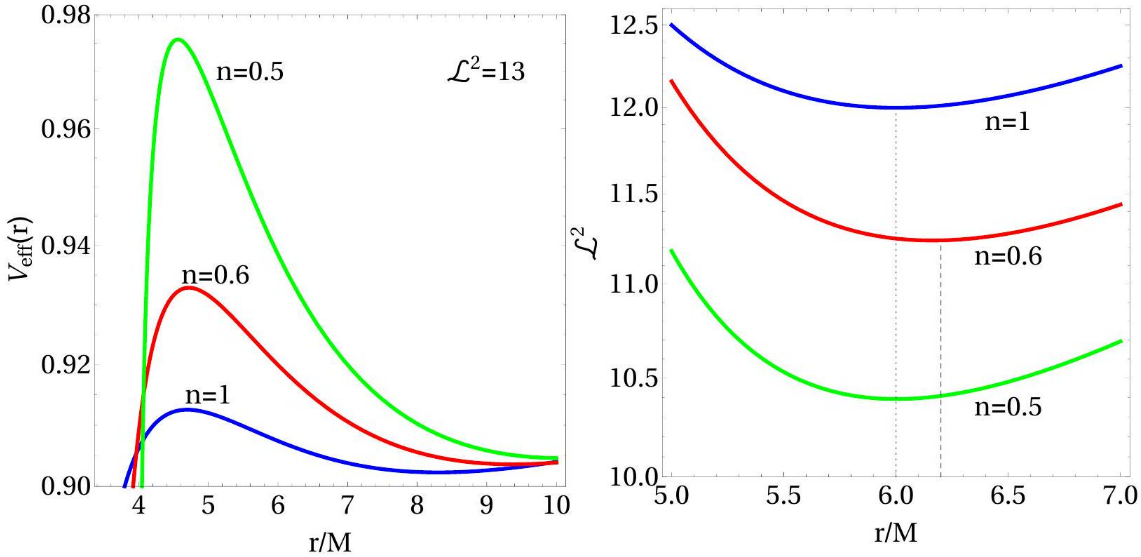
$$\dot{t} = \mathcal{E}e^{-\nu} \tag{58}$$
$$\dot{\phi} = \frac{\mathcal{L}}{\sin^2\ \theta}e^{\nu-\lambda} \tag{59}$$
$$\dot{r}^2 = f(r) = \mathcal{E}^2 - e^{\nu}\left(1 + e^{\nu-\lambda}\mathcal{L}^2\right) \tag{60}$$

bu yerda $E = E/m$ va $L = L/m$ lar, mos ravishda, cheksizlikdagi kuzatuvchiga nisbatan birlik massaga to'g'ri keluvchi keltirilgan energiya va keltirilgan impuls momenti.

Keltirilgan energiya va keltirilgan impuls momentining kritik qiymatlarini topish uchun quyidagi shartlardan foydalanish mumkin $f(r) = f'(r) = 0$, bundan foydalanib quyidagiga ega bo'lamiz

$$\mathcal{E}^2 = \mathcal{E}^2_{\text{ext}}(r) \equiv \frac{\lambda' - \nu'}{\lambda' - 2\nu'}e^{\nu}, \tag{61}$$
$$\mathcal{L}^2 = \mathcal{L}^2_{\text{ext}}(r) \equiv \frac{\nu'}{\lambda' - 2\nu'}e^{\lambda-\nu}. \tag{62}$$

9-rasm effektiv potensialning radial koordinataga bog'lanishlarining turli n lar uchun $Q = 0$ dagi tasvirini ko'rsatadi.

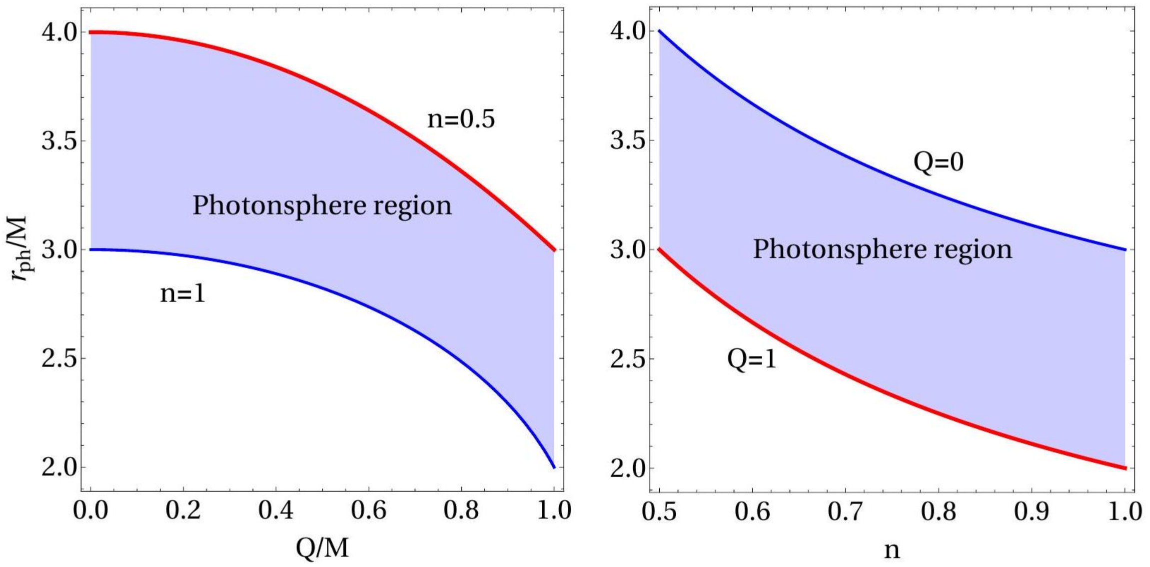


Rasm 9. ***Q=0*** **da** ***n*** **parametrning turli qiymatlari uchun effektiv potentsial (chap panel) va impuls momentining (o'ng panel) radiusga bog'liqlik grafigi.**

Massasiz sinov zarrasining energiyasi va impulsi cheksizlikka intilganligi sababli (ya'ni $E \to 0, L \to 0$), bu iboralarning maxraji nolga teng bo'lishi kerak. Fotonsferaning r_{ph} radiusini analitik yechimga ega bo'lmagan quyidagi $\lambda' = 2\nu'$ tenglamaning yechimi sifatida topish mumkin. Biroq, JNW va RN fazovaqtida fotonsfera radiusi uchun analitik ifodalarni quyidagicha olish mumkin.

$$r_{\rm ph}/M = \begin{cases} 2 + \frac{1}{n}, \ Q = 0 \\ \frac{3}{2}\left(1 + \sqrt{1 - \frac{8Q^2}{9M^2}}\right), \ n = 1 \end{cases} \tag{63}$$

Raqamli hisob-kitoblarni boshlashdan oldin shuni ta'kidlash kerakki, (56) metrikaning afzalligi shundaki, yechim parametrlari berilgan diapazonda $0 \leq Q \leq 1$ va $1/2 \leq n \leq 1$ oraliqda aniqlanadi. Bu ma'lum bir hududda fotonsfera radiusini olishga yordam beradi. Ehtiyotkorlik bilan bajarilgan raqamli hisob-kitoblar shuni ko'rsatadiki, fotonfera radiusi $2M \leq r_{ph} \leq \ 4M$ oraliqda joylashgan. 10-rasmda fotonsfera radiuslarining gravitatsion kompakt obyektning zaryad parametrlariga bog'liqligi ko'rsatilgan. Ma'lumki, Schwarzschild fazovaqtidagi fotonsferaning radiusi $r_{ph} = 3M$, ekstremal RN qora tuynuk holati uchun (ya'ni Q=M) $r_{ph} = 2M$, JNW yechimida esa $r_{ph} = 4M$ ga etadi (10-rasm ga qarang).



Rasm 10. Chap panel: *n* parametrning turli qiymatlari uchun fotonsfera radiusining *Q* zaryadiga bog'liqligi. O'ng panel: qora tuynuk zaryadining turli qiymatlari uchun fotonsfera radiusining *n* parametriga bog'liqligi.

ISCO radiusini topish uchun (61), (62) ifodalar bilan birga yana bitta $f''(r) = 0$ shartidan foydalanish kerak va oddiy algebrani bajargandan so'ng quyidagini olish mumkin. ISCO radiuslarini hisoblash uchun tenglamani olamiz:

$$\lambda' + \frac{2\nu'^2 - \lambda''}{\lambda'} - 3\nu' + \frac{\nu''}{\nu'} = 0 \tag{64}$$

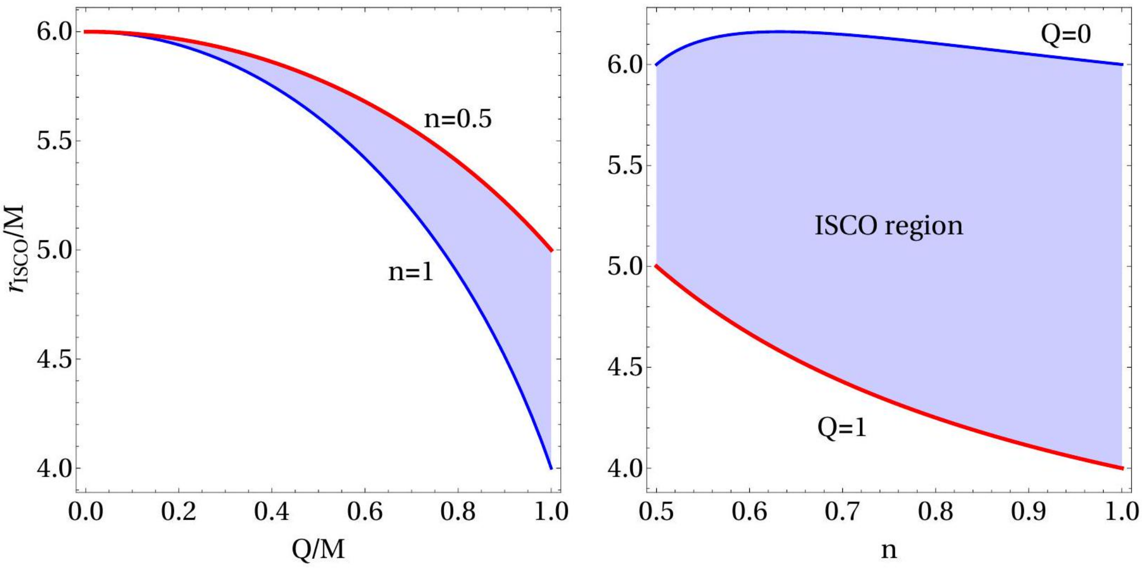
Buni analitik tarzda ishlash qilish qiyin. Shu bilan birga, JNW va RN fazo-vaqt metrikasidagi sinov zarralari uchun ISCO radiuslari uchun ifodalar quyidagicha berilgan

$$r_{\text{ISCO}}/M = \begin{cases} 3 + \frac{1}{n} + \sqrt{5 - \frac{1}{n^2}} & Q = 0, \\ 2 + 2\zeta + \zeta^{-1}\left(2 - \frac{3Q^2}{2M^2}\right), & n = 1 \end{cases} \tag{65}$$

bu yerda ζ quyidagicha ifodalanadi

$$\zeta = \sqrt[3]{1 + \frac{Q^4}{4M^4} + \frac{Q^2}{8M^2}\left[\sqrt{\left(1 - \frac{Q^2}{M^2}\right)\left(5 - \frac{4Q^2}{M^4}\right)} - 9\right]} \tag{66}$$

11-rasmda ISCO radiuslarining $0 \le Q/M \le 1$ va $1/2 \le n \le 1$ oraliqda gravitatsion manbaning elektr va skalyar zaryad parametrlariga bog‘liqligi tasvirlangan. 11-rasmdagi bo`yalgan soha ISCO radiuslarining o'zgarishini ifodalaydi. Bizning raqamli tahlilimiz shuni ko'rsatadiki, ISCO radiuslari diapazoni $4 \le r_{ISCO}/M \le 6.2$ ni tashkil qiladi. 11-rasmdan ko'rinib turibdiki, ISCO radiuslari aniq sohada berilgan.



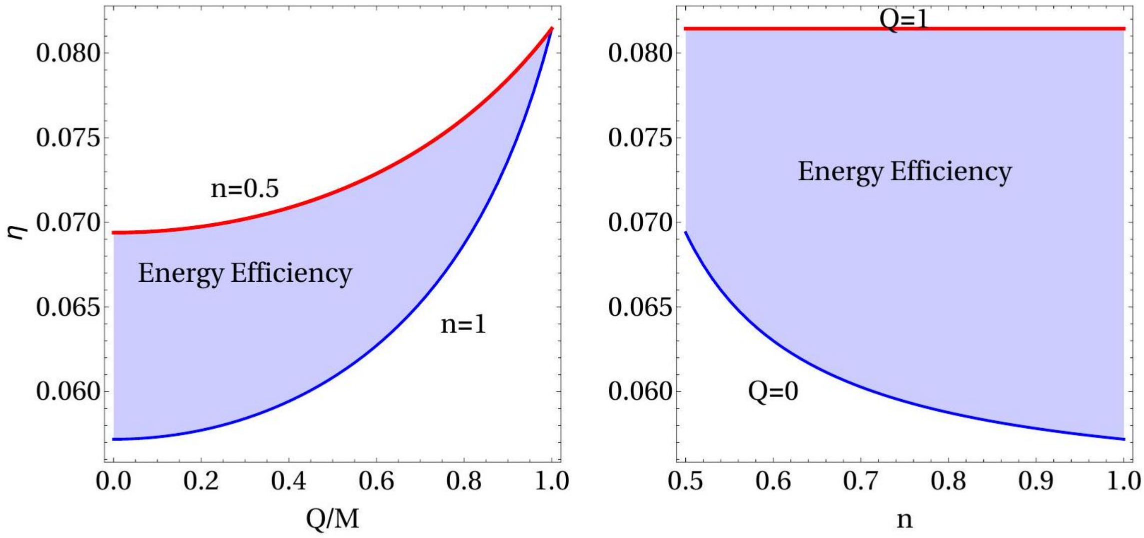
11-rasm. (Chap tomon): Har xil n parametr qiymatlari uchun ISCO radiusining zaryad Q ga bogliqligi. (O'ng tomon): Har xil zaryad Q qiymatlari uchun ISCO radiusining n parametrga bogliqligi.

Shuningdek sinov zarrachasini energiya effektivligi ham ko`rib o`taylik. Bu yerda, $E_{bind} = E_{rest} - E_{ISCO}$ bog`lanish energiyasi va $E_{rest} = mc^2$ zarraning tinchlikdagi energiyasi:

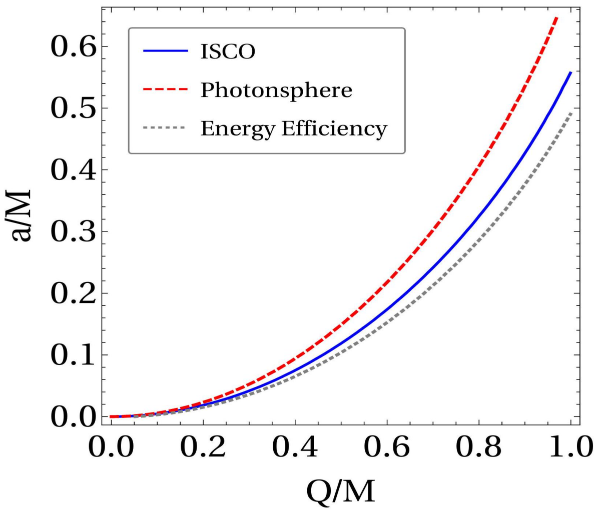
$$\eta = \frac{E_{\text{bind}}}{E_{\text{rest}}} = 1 - \mathcal{E}_{\text{ISCO}} \quad (67)$$

Ular ISCOdan sinaladigan zarralarning maxsus energiyasiga bog'liq. 12-rasmda energiya samaradorligining turli xil parametrlarga bog'liqligi ko'rsatilgan. Ma'lumki, energiya samaradorligi taxminan ≃6% ni tashkil qiladi. Biroq, JNV fazo-vaqtida 12-rasmdagi chap tomonda ko'rsatilganidek taxminan ~7% ni tashkil qiladi, ekstremal RN qora tuynuk uchun esa energiya samaradorligi o'ng tomonda ko'rsatilganidek ~8,1% gacha tashkil etadi.

Ehtiyotkorlik bilan bajarilgan sonli hisob-kitoblar shuni ko'rsatadiki, yangi olingan yechim (56) 13-rasmda ko'rsatilganidek, Kerr qora tuynugining spin parametrini $a_* \lesssim 0.6$ gacha bo`lishi mumkinligini ko`rsatadi. Biroq, astrofizik qora tuynuklarning akkretsion disklarining ichki qismini tahlil qilish shuni ko'rsatadiki, astrofizik qora tuynukning spin parametri deyarli a*≲0,99 ga teng. Bu esa olingan yangi yechim (56) tez aylanadigan astrofizik qora tuynuk uchun realistik nomzod deb hisoblash mumkin emas degan xulosaga kelishimiz mumkin.



12-rasm. Chaptomon: *n* parametrning turli qiymatlari uchun energiya samaradorligining *Q* zaryadiga bog'liqligi. O'ng tomon: *Q* zaryadining turli qiymatlari uchun energiya samaradorligining *n* parametriga bog'liqligi.



13-rasm. ISCO, fotonsfera radiuslarini va energiya samaradorligi taqqoslash orqali turli xil *n* parametrlar qiymati uchun fazo-vaqt metrikasi (56) bilan tavsiflangan kompakt obyektning *Q* zaryadi bilan spin parametri *a* o'rtasidagi bog`liqligi.

XULOSA

"Eynshteyn-Maksvel-skalyar maydon tenglamalarining aniq yechimlari" mavzusida olib borilgan tadqiqotlar asosida quyidagi xulosalar keltirildi:

1. 1. Birinchi marta gravitatsiayon skalyar maydonning ta'sirini hisobga olgan holda Eynshteyn maydon tenglamalarining aksial-simmetrik va statik yechimlari olingan. Xususan, (i) modifikatsiyalangan gamma-metrik va (ii) modifikatsiyalangan kvadrupol moment metrikasi kabi Veyl sinfiga tegishli bo'lgan Eynshteyn tenglamalarining aniq analitik yechimlari olingan.
2. Birinchi marta tashqi gravitatsiyon skalyar maydoni ta'siridan qo'shimcha parametrli gamma-metrikaning umumlashtirilgan shakli va quadrupole momenti ega Eres-Rozen metrikasining umumlashtirilgan shakli olindi.
3. Tashqi skalyar maydon uchun energiya-impuls tenzorining komponentlari uchun analitik ifodalar olinadi. Fantom maydon yechimlari mavjud bo'lganda nol energiya sharti qanoatlanishi, ammo gravitatsiyon skalyar maydon mavjud bo'lganda esa nol energiya shartini qanoatlantirmasligi ko'rsatilgan.
4. Birinchi marta eng ichki barqaror aylanma orbitalarning radiusi (ISCO), gamma-metrik fazoda sinovdan o'tadigan zarrachalarning energiyaning kritik qiymatlari va burchak momentumining aniq analitik ifodasi. Gamma-parametrning maxsus qiymatlari uchun ISCO radiusi va foton sferasi ortishi ko'rsatilgan. Bundan tashqari, quadrupol moment Shvartzshild metrikasi bilan solishtirganda kuchliroq chegaralangan aylana orbitalariga ega ekanligi ko'rsatildi.
5. Eynshteyn-Maksvell-skalyar maydon tenglamalarining analitik yechimilari birinchi marta Raisner-Nordstrom, Yanis-Nyuman-Vinikur va Shvartzshild yechimlari kabi uchta mashhur yechimlarining umumlashgan ko'rinishi hisoblanadi. Olingan yangi yechimning gorizontlari yo'qligi ko'rsatilgan va u yalang'och singulyarlikka misol bo'la oladi.
6. Tegishli vektor potentsialining ikki tomonlama yechimi maydon tenglamalari bilan ham qanoatlantirilishi aniqlandi. Elektr va skalyar zaryadlarning ta'siri tufayli sinov zarralarining ISCO ning markaziy tortishish manbai tomon siljishi ko'rsatilgan. Yangi kompakt ob'ektlar yechimi bilan tasvirlangan fazodagi sinov zarralari uchun energiya samaradorligi $6\% \lesssim \eta \lesssim 8\%$ oralig'ida ekanligi aniqlandi.

SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH "TIIAME" NATIONAL RESEARCH UNIVERSITY

INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH

TURIMOV BOBUR VALENTINOVICH

EXACT SOLUTIONS OF EINSTEIN-MAXWELL-SCALAR FIELDS EQUATIONS

01.03.01 – Astronomy
01.04.02 – Theoretical Physics
(physical and mathematical sciences)

PRESENTATION
on awarding the scientific degree of Doctor of Philosophy (PhD) on the basis of published papers without a dissertation

Tashkent – 2024

The theme of the Doctor of Philosophy (PhD) researche is registered by Supreme Attestation Commission of Higher Education, Science and Innovations of Republic of Uzbekistan under B2024.1.PhD/FM999.

The esearch work has been carried out at the Institute of Fundamental and Applied Research under “TIAMEE” National Research University.

The presentation was posted in three (Uzbek, English, Russian (resume)) languages on the website of the Scientific Council (www.ifar.uz) and on the information and education portal at "Zionet" (www.ziyonet.uz).

Scientific supervisors: **Abdujabbarov Ahmadjon Adiljanovich**
Doctor of physical and mathematical sciences, professor

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Doctor of physical and mathematical sciences, academician

Presentation of the research will be held on “__” _april 2024 at __ in the meeting of the Scientific Council No. DSc.03/31.03.2022 T/FM.10.4 at the Institute of Fundamental and Applied Research under the National Research University "TIIAME" (Address: 100000, Tashkent city, Qori Niyazov Street 39, Institute of Fundamental and Applied Research, Hall 108; tel.: 71 237-09-61.; e-mail: info@ifar.uz

The presentation can be looked through at the Information Resource Center of the Institute of Fundamental and Applied Research under the National Research University "TIIAME" (registered under № ____). (Address: 100000, Tashkent city, 39 Qori Niyazov str., Institute of Fundamental and Applied Research, hall 205; ph.: 71 237-09-61

The presentation was distributed on "___" __________, 2024.

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INTRODUCTION (presentation abstract)

Relevance and necessity of the topic. The direct detection of the gravitational waves from the co-alescence of the binary black holes and binary neutron stars in close systems by LIGO–VIRGO collaboration, observation of the first image of the supermassive black hole at the center of elliptic galaxy Messier 87 (M87) by the Event Horizon Telescope and the dynamics of the hot spots and S2 star in the close environment of the supermassive black hole SgrA* by GRAVITY collaboration provide the rich observational data from the various highly sensitive instruments for testing the alternate gravity theories in the strong field regime. On other hand there is a great interest to the new exact and approximate solutions of the extended theories of gravity including additional fields e.g. the scalar ones to explain, for example, the new forms of matter in the modern cosmological models as the dark matter and dark energy. Finally, the exact solutions of the extended theories of gravity can potentially remove the physical singularity with the infinite curvature at the center of the gravitational compact objects.

One of the important problems in general relativity is to find new exact analytical solutions of the Einstein field equations. Numerous powerful methods have been developed for the derivation of new solutions of the gravitational field equations since Einstein discovered the the- ory of general relativity in 1915. Well-known, astrophysically relevant, external vacuum solutions of Einstein field equations have been obtained by Schwarzschild and Kerr for static and rotating black holes, respectively. Particulalry, the Hartle-Thorne metric describes the interior and the vacuum spacetime outside any slowly rotating astrophysical object as relativistic star. A huge number of interesting exact solutions of the Einstein field equations have been obtained by various authors.

Astronomical objects can be deformed for various reasons and consequently it is interesting to study the space- time of the deformed compact gravitational objects. Recently an exact axisymmetric static vacuum solution of the Einstein equations in the case of a nonspherical mass distributed by a compact object has been obtained. This solution is often called the quadrupolar (quadrupole moment) metric with an external mass quadrupole moment. Another exact solution of the Einstein equations for deformed spacetime is called γ-metric. These solutions belong to the class of Weyl type solutions. Recently, it has been shown that the massive scalar field may give a much larger contribution to the gravitational field around the slowly rotating neutron star in comparison with that of the massless scalar field. The exact solution of the Einstein equations for the wormhole with the scalar field has been also extensively studied by various authors. However, the axisymmetric and static solutions of the Einstein field equations considering the effect of self gravitating scalar field have not been studied. The Einstein field equations in Weyl and prolate coordinates have not been derived.

It is worth to note that in recent years, in our country, more and more attention has been paid to the development of current directions of fundamental and applied research. In particular, the development of theoretical astrophysical research, which is one of the promising areas, is an important issue today. The main directions of fundamental research and development and their practical application for the successful development of science in our country are reflected in the Strategy[1]for the further development of the Republic of Uzbekistan from 2022-2026. Therefore, the research of effect of gravitational lemsing in the plasma environment remains one of the urgent issues in the field of fundamental research.

This research work corresponds to the tasks by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. PD-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Resolution of the President of the Republic of Uzbekistan No. PR-2789 "On measures for further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities" dated February 18, 2017 and others.

Conformity of the research to the main priorities of science and technology development of the Republic. The research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

The degree of knowledge of the problem. The spherically-symmetric static solution of the Einstein-scalar field equations and similar solutions have been studied by number of authors (A. I. Janis, E. T. Newman, J. Winicour, R. Penney, M. Wyman, A. G. Agnese, M. La Camera, T. Damour, K. S. Virbhadra, A. Bhadra, K. K. Nandi, N. Dadhich, N. Banerjee, S. Abdolrahimi, A.A.Shoom. However, spherically symmetric and static solution of the Einstein-Maxwell-scalar field equations have not been systematically studied.

Some approximate static solutions of the Einstein equations have been studied by several authors (Z.-Y. Fan, H. Lu) under the conformal field theory approach. The contribution of the scalar field in the spacetime of static and rotating black holes have been also studied (K. S. Virbhadra, N. Dadhich, N. Banerjee, L. Herrera, G. Magli, D. Malafarina, C. A. R. Herdeiro and E. Radu, C. Erices and C. Martınez, A. Sen). However, axisymmetric and static solutions of the Einstein field equations considering the effect of self gravitating scalar field have not been studied yet.

Connection of the topic of the research topic to the scientific works of higher education and research institutions, where the dissertation is carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development. F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity".

[1] Decree No. PF-60 of the President of the Republic of Uzbekistan dated January 1, 2022 "On the Development Strategy of New Uzbekistan for 2022-2026"

The aim of the research is the exact axisymmetric and static solution of the Einstein equations coupled to the gravitating scalar field and Einstein–Maxwell-scalar field equations.

The tasks of the research:

to study the influence of the scalar field in spacetime properties of axial-symmetric and static vacuum solutions of combined Einstein field equations;

to obtain a generalized form of the gamma-metric with additional parameter and generalized form of the Eres-Rosen metric;

to get the analytical expressions for the components of the energy-momentum tensor for the self-gravitating scalar field;

to study the test particle motion in the spacetime of both generalized the gamma-metric and the quadrupole moment metric and probe metric parameters produced by the gravitating scalar field into the test particle motion;

to obtain the exact analytical expression for the radius of the innermost stable circular orbits (ISCO), the critical values of the energy and the angular momentum of the test particles in terms of the metric parameters;

to obtain the analytical solution of the Einstein-Maxwell-scalar field equations assuming that three different fields, namely, gravitational, vector, and massless scalar field do not interact with each other;

to calculate the curvature invariants such as the Ricci scalar, Ricci square and Kretschman scalar.

The object of the research are astrophysical compact objects, particle dynamics, Maxwell and scalar fields.

The subject of the research are exact analytical solutions of field equations, theoretical models for studying particle dynamics near compact gravitational objects in the presence of scalar field, numerical and analytical methods for solving differential equations.

The methods of the research are methods of theoretical physics, methods of theoretical astrophysics, modern methods of mathematical physics, analytical and numerical methods of calculating differential equations for field and particle motion.

The scientific novelty of the research is the following:

The axisymmetric and static solutions of the Einstein field equations by considering the effect of an additional self-gravitating scalar field have been derived. It has been presented an exact analytical solution of the combined Einstein equations for two different modified spacetime metrics which belong to the Weyl class of solutions as (i) the modified gamma-metric and (ii) the modified quadrupole moment metric.

It has been studied the influence of the scalar field in spacetime properties of axial-symmetric and static vacuum solutions of combined Einstein field equations.

It has been obtained a generalized form of the gamma-metric with additional parameter and generalized form of the Eres-Rosen metric which includes mass quadrupole produced by the self-gravitating scalar field.

The analytical expressions for the components of the energy-momentum tensor are obtained for the self-gravitating scalar field. It has been shown that in the case of phantom field the solution satisfies the null energy condition.

It has been shown that gamma and quadrupole parameters do not contribute into the energy and angular momentum of the test particle and consequently do not affect particle trajectory at the equatorial plane.

The exact analytical expression for the radius of the ISCO, the critical values of the energy and the angular momentum of the test particles have been obtained.

The practical results of the research are the following:

It is shown that for the corresponding range of the values of the gamma parameter the radius of ISCO and the photon sphere increase.

It has been found the analytical solution of the Einstein-Maxwell-scalar field equations. It has been also calculated the curvature invariants such as the Ricci scalar, Ricci square and Kretschman scalar.

It has been shown that all three curvature invariants have three singular points. It has been concluded that the obtained new solution does not exist horizons.

The energy efficiency for the test particles in the spacetime described by the new black hole solution is found to be at the range $6\% \lesssim \eta \lesssim 8\%$.

The reliability of the research results provided by applying modern proven methods of mathematical physics, computational mathematics, and relativistic astrophysics. The results were obtained strictly within the mathematical apparatus of general relativity and theoretical physics. Modern numerical and analytical methods of calculation are also used, and the results are compared with available observational data and the results of other authors. The structured conclusions of the thesis correspond to the basic rules of astrophysics of compact objects.

The scientific and practical significance of the research results. The scientific significance of the research results is found that obtained solutions of field equations may generalize the previous solutions and describe the deformed compact objects.

The practical significance of the research results is that they can play a role in the obtaining the upper limits and constraints on the gamma and quadrupole parameters within EMS gravity model.

Implementation of the research results. Based on the new analytical solution of Einstein-Maxwell-scalar field equations:

scientific results obtained on the motion of particles have been used by scientists from Fudan University (FU) in Shanghai (FU, China, March 7, 2024 reference);

results on the particle motion around compact have been used in the works of foreign researchers, in foreign journals with a high impact factor (The European Physical Journal C, Volume 83, Issue 12, article id.1131, Web-Sc, IF: 4.4; Pramana, Volume 97, Issue 1, article id.29, Web-Sc, IF: 2.219; Monthly Notices of the Royal Astronomical Society, Volume 521, Issue 1, pp.474-477, Web-Sc, IF: 5.235) to describe the effects of metric parameters on dynamics of particles around compact objects.

Publication of research results. The results of PhD research have been presented in 15 peer-reviewed articles published in prestigious Q1/Q2 quartile scientific journals recommended by Supreme Attestation Commission at the Ministry of higer education, science and innovations of the Republic of Uzbekistan.

MAIN CONTENT OF THE WORK

The action is described by the following form

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(R - 2\epsilon g^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi\right) \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ of the arbitrary spacetime, R is the Ricci scalar of the curvature and Φ is the massless gravitating scalar field, ϵ is the constant which is responsible for the scalar field at $\epsilon = 1$ and the phantom field with value $\epsilon = -1$, respectively.

Hereafter minimizing the action in the equation (1) one can obtain equations of motion of the system which is described by the Einstein field equations taking into account of the gravitating scalar field and the Klein-Gordon equation for the gravitational scalar field in the following form.

$$R_{\mu\nu} = 2\epsilon\,\partial_\mu\Phi\,\partial_\nu\Phi, \tag{1}$$

$$\boxdot\,\Phi = 0, \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor of the curvature and $\boxdot$ is the d'Alembertian in four dimensional curved spacetime. It is well known that the equations (2)-(3) are coupled differential equations and finding their solutions is not easy task so far. In this work we present axial-symmetric and static solutions of the field equations (2)-(3) and compare the solutions with those previously obtained in the literature.

Axisymmetric and static solution.

In order to simplify the problem, we can assume that the gravitating scalar field is axially symmetric and stationary. In Weyl coordinates (t, ρ, ϕ, z) the general form of the static metric can be described by

$$ds^2 = -e^{2U}dt^2 + e^{-2U}[e^{2V}(d\rho^2 + dz^2) + \rho^2 d\phi^2] \tag{4}$$

where U and V are the functions of the coordinates ρ and z, respectively. Then the explicit form of the field equations (2)-(3) for the spacetime metric (4) can be written as

$$\Delta\Phi = \Phi_{\rho\rho} + \frac{1}{\rho}\Phi_\rho + \Phi_{zz} = 0 \tag{5}$$

$$\Delta U = U_{\rho\rho} + \frac{1}{\rho}U_\rho + U_{zz} = 0 \tag{6}$$

$$V_\rho = \rho\left(U_\rho^2 - U_z^2 + \epsilon\Phi_\rho^2 - \epsilon\Phi_z^2\right) \tag{7}$$

$$V_z = 2\rho\left(U_\rho U_z + \epsilon\Phi_\rho\Phi_z\right) \tag{8}$$

where subindices indicate the derivative with respect to the coordinates ρ and z, respectively.
For the convenience one can consider the prolate coordinates (t, X, Y, ϕ) in which the spacetime metric (4) can be rewritten in the following form

$$ds^2 = -e^{2U}dt^2 + \sigma^2 e^{-2U}\left[e^{2V}(X^2 - Y^2)\left(\frac{dX^2}{X^2-1} + \frac{dY^2}{1-Y^2}\right) + (X^2 - 1)(1 - Y^2)d\phi^2\right], \tag{9}$$

where σ is the dimensional parameter, later in the text the physical meaning of this parameter will be introduced.

Here we can introduce useful notations which are the relations between the prolate spheroidal coordinates (X, Y, ϕ) and Weyl coordinates (ρ, z, ϕ) indicated as

$$\rho = \sigma\sqrt{(X^2 - 1)(1 - Y^2)},\ z = \sigma XY, \phi = \phi. \tag{10}$$

and similarly, they can be related with the spherical coordinates (r, θ, ϕ) in the following form

$$X = \frac{r}{\sigma} - 1,\ Y = \cos\ \theta,\ \phi = \phi \tag{11}$$

Note that here zeroth (temporal) component of the coordinate t is the same in all these coordinates.
Finally, the field equations (5)-(8) can be rewritten in terms of prolate coordinates X and Y in the form

$$[(X^2 - 1)\Phi_X]_X + [(1 - Y^2)\Phi_Y]_Y = 0 \tag{12}$$

$$[(X^2 - 1)U_X]_X + [(1 - Y^2)U_Y]_Y = 0 \tag{13}$$

$$V_X = \frac{1 - Y^2}{X^2 - Y^2}[X(X^2 - 1)U_X^2 - X(1 - Y^2)U_Y^2 - 2Y(X^2 - 1)U_X U_Y] + (U \to \epsilon\Phi), \tag{14}$$

$$V_Y = \frac{X^2 - 1}{X^2 - Y^2}[Y(X^2 - 1)U_X^2 - Y(1 - Y^2)U_Y^2 + 2X(1 - Y^2)U_X U_Y] + (U \to \epsilon\Phi). \tag{15}$$

One can easily see that the equations (12) and (13) are similar to each other, one can seek their solutions in the following separable form $\{\Phi, U\} = f(X)g(Y)$ and using the equations (12) and (13) one can write the following Legendre equations for the functions $f(X)$ and $g(Y)$ in the form

$$[(X^2 - 1)f_X]_X - l(l + 1)f = 0 \tag{16}$$

$$[(1 - Y^2)g_Y]_Y + l(l + 1)g = 0, \tag{17}$$

where l is the multipole number that can take the integer value. The solutions of the equations 16 and (17) are

$$f(X) = C_{1l}P_l(X) + C_{2l}Q_l(X) \tag{18}$$

$$g(Y) = C_{3l}P_l(Y) + C_{4l}Q_l(Y) \tag{19}$$

where $P_l(X)$ is the Legendre polynomial, $Q_l(Y)$ is the Legendre function of the second kind and $C_{1l} - C_{4l}$ are the integration constants, respectively. From the physical point of view both solutions $\{\Phi, U\}$ should be asymptotically flat which means

$$\lim_{X\to\infty} f(X) = 0,\ C_{1l} = 0 \tag{20}$$

and they should be regular everywhere

$$\lim_{Y\to 0} g(Y) = \text{ const },\ C_{4l} = 0. \tag{21}$$

In order to find the physically meaningful solution one can set $\epsilon = 0, q_0 = 1$ and $q_l = 0$ $(l > 0)$ and obtain the well-known Schwarzschild solution

$$U = \frac{1}{2}\ln\frac{X-1}{X+1} = \frac{1}{2}\ln\left(1-\frac{2\sigma}{r}\right), \tag{22}$$

$$V = \frac{1}{2}\ln\frac{X^2-1}{X^2-Y^2} = \frac{1}{2}\ln\frac{r^2-2\sigma r}{r^2-2\sigma r+\sigma^2\sin^2\theta}. \tag{23}$$

Here one can easily see that the dimensional parameter σ is the total mass of the compact object $\sigma = M$.

Analytic solution of the Einstein equations with self-gravitating scalar field for the γ-metric

Using the coordinate transformation in the expression (11) we can obtain the generalized form of the γ-metric in spherical coordinates

$$ds^2 = -\left(1-\frac{2M}{r}\right)^{\gamma} dt^2 + \left(1-\frac{2M}{r}\right)^{1-\gamma} \times\left\{\left(1-\frac{M^2\sin^2\theta}{r^2-2Mr}\right)^{1-\gamma^2-\epsilon\gamma_*^2}\left[\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2d\theta^2\right]+r^2\sin^2\theta d\phi^2\right\}, \tag{24}$$

and the scalar field has a form

$$\Phi(r) = \frac{\gamma_*^2}{2}\ln\left(1-\frac{2M}{r}\right) \tag{25}$$

In the expression (25) we can see that the scalar function $\Phi(r)$ depends on the radial coordinate only. Figure 1 draws the equipotential surface of the gravitating scalar field $\Phi(r)$ in the $(x-z)$ plane for the different values of the γ_* parameter. One can easily see that with increasing the γ_* parameter the gravitational force is getting stronger and the spacetime around the object will be deformed due to the presence of the scalar field as shown in Fig. 1.

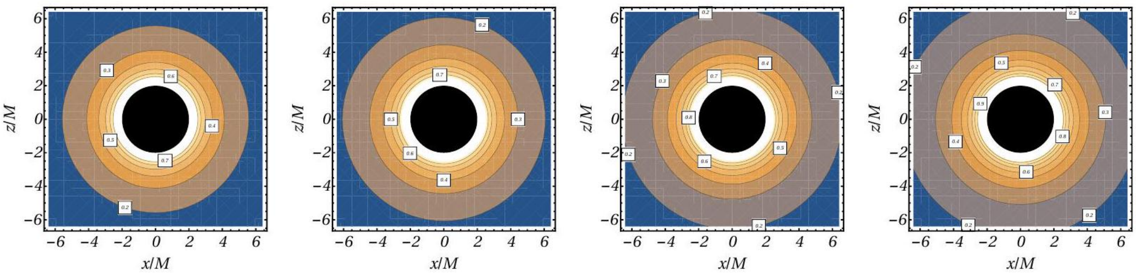

Figure 1. The shape of the scalar field $\Phi(r,\theta)$ described by the equation (25) in $x-z$ plane for the different values of γ_* parameter: $\gamma_* = 0.9, \gamma_* = 1, \gamma_* = 1.1$ and $\gamma_* = 1.2$.

The energy-momentum tensor for the scalar field can be expressed as

$$T_{\mu\nu} = \epsilon\left(\partial_\mu\Phi\,\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\,\partial_\alpha\Phi\,\partial_\beta\Phi\right) \tag{26}$$

from the expression (26) the energy density and the components of the pressure can be defined as $\rho = T_0^0$ and $P_i = T_i^i$, and the explicit form of the energy density and the components of the pressure is

$$\rho = P_\theta = P_\phi = -P_r = -\frac{\epsilon\gamma_*^2M^2}{2r^4}\left(1-\frac{2M}{r}\right)^{\gamma-2}\left(1-\frac{M^2\sin^2\theta}{r^2-2Mr}\right)^{\gamma^2+\epsilon\gamma_*^2-1}.$$

The null energy condition (NEC) can be found from the expression $\rho + P_i \geq 0 (i = r, \theta, \phi)$, using the equation (27) as

$$\rho + P_r \equiv 0, \tag{27}$$

$$\rho + P_\theta = \rho + P_\phi = -\frac{\epsilon\gamma_*^2 M^2}{r^4}\left(1 - \frac{2M}{r}\right)^{\gamma-2}\left(1 - \frac{M^2 \sin^2\theta}{r^2 - 2Mr}\right)^{\gamma^2+\epsilon\gamma_*^2-1}. \tag{28}$$

The physical interpretation of NEC is that the energy density measured by an observer traversing along null curve is always positive (never negative). One can see that the expression (27) is always satisfied by the NEC condition for the spacetime metric (24) while the expression (28) satisfies the NEC condition only in the case when $\epsilon \leq 0$ which corresponds to the phantom field. This means that the observer traversing along null curve can measure positive energy even in the case of the antigravitating phantom scalar field. Figure 2 shows the NEC precisely where the radial dependence of $\rho + P_i (i = r, \theta, \phi)$.

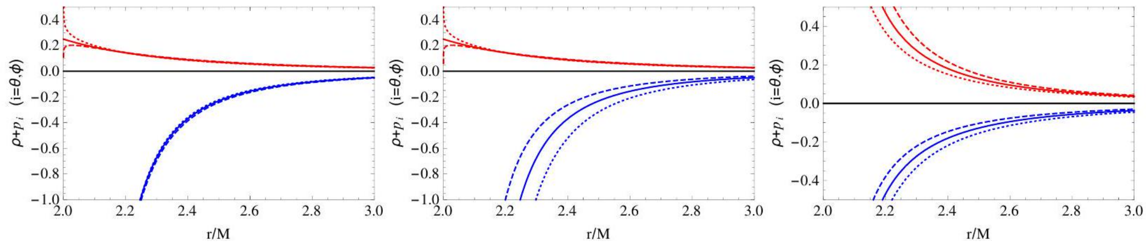

Figure 2. Radial dependence of $\{\rho + P_i\}(i = r, \theta, \phi)$ for the different values of the parameters γ and γ_*. (Left panel) Solid line corresponds to $\gamma = 1$, dashed line to $\gamma = 0.9$ and dashed line to $\gamma = 1.1$ at $\gamma_* = 1$ and $\theta = \pi/2$. (Central panel) Solid line corresponds to $\gamma_* = 1$, dashed line to $\gamma_* = 0.9$ and dashed line to $\gamma_* = 1.1$ at $\gamma = 1$ and $\theta = \pi/2$. (Right panel) Solid line corresponds to $\gamma = 1$, dashed line to $\gamma = 0.9$ and dashed line to $\gamma = 1.1$ at $\gamma_* = 1$ and $\theta = 0$.

The Hamiltonian for test particle with mass m can be written in the form

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu + \frac{1}{2} m^2 \tag{29}$$

where $p^\mu = m u^\mu$ is the kinematical four-momentum. The equations for particle motion are

$$\frac{dx^\mu}{d\zeta} \equiv p^\mu = \frac{\partial H}{\partial p_\mu}, \ \frac{dp_\mu}{d\zeta} = -\frac{\partial H}{\partial x^\mu} \tag{30}$$

Here the affine parameter ζ of the particle is related to its proper time τ by the relation $\zeta = \tau/m$.

Introducing for convenience the specific parameters, energy $\mathcal{E}$ and axial angular momentum $\mathcal{L}$

$$\mathcal{E} = \frac{E}{m}, \ \mathcal{L} = \frac{L}{m}, \tag{31}$$

one can rewrite the Hamiltonian (29) in the form

$$H = \frac{1}{2} g^{rr} p_r^2 + \frac{1}{2} g^{\theta\theta} p_\theta^2 + \frac{m^2}{2} g^{tt} [\mathcal{E}^2 - V_{\rm eff}(r, \theta)], \tag{32}$$

where $V_{\rm eff}(r, \theta)$ denotes the effective potential of the test particle which is given by the relation

$$V_{\rm eff}(r,\theta) \equiv -g_{tt}\left(1+g^{\phi\phi}\mathcal{L}^2\right)$$
$$= \left(1-\frac{2M}{r}\right)^{\gamma}\left[1+\frac{\mathcal{L}^2}{r^2\sin^2\theta}\left(1-\frac{2M}{r}\right)^{\gamma-1}\right]. \quad (33)$$

The particle motion is limited by the energetic boundaries given by
$$\mathcal{E}^2 = V_{\rm eff}(r,\theta). \quad (34)$$

The features of the effective potential (33) is represented in Fig. 3. The stationary points of the effective potential $V_{\rm eff}(r,\theta)$ function, where maxima or minima can exist, are given by the equations
$$\partial_r V_{\rm eff}(r,\theta) = 0, \ \ \partial_\theta V_{\rm eff}(r,\theta) = 0. \quad (35)$$

The second of the extrema equations (35) gives $\theta = \pi/2$. The first extrema equation of (35) leads to equation being quadratic with respect to the specific angular momentum $\mathcal{L}$ and hence the circular orbits can be determined by the relation
$$\mathcal{L}^2 = \mathcal{L}^2_{\rm ext}(r) \equiv \frac{\gamma M r^2}{r-M(1+2\gamma)}\left(1-\frac{2M}{r}\right)^{1-\gamma} \quad (36)$$

In Fig. 4 the function $\mathcal{L}_{\rm ext}(r)$ is plotted for various values of parameter γ. Similarly, the energy of the test particle can be expressed as
$$\mathcal{E}^2 = \mathcal{E}^2_{\rm ext}(r) \equiv \frac{r-M(1+\gamma)}{r-M(1+2\gamma)}\left(1-\frac{2M}{r}\right)^{\gamma} \quad (37)$$

The local extrema of $\mathcal{L}_{\rm ext}(r)$ function is equivalent to $\partial_r^2 V_{\rm eff}(r,\theta=\pi/2)=0$ condition and they determine the innermost stable circular orbits (ISCO) radius located at
$$r_{\rm ISCO}/M = 1+3\gamma+\sqrt{5\gamma^2-1}, \quad (38)$$

and from equation 65 we can find that γ parameter should be $\gamma \geq 1/\sqrt{5}$.

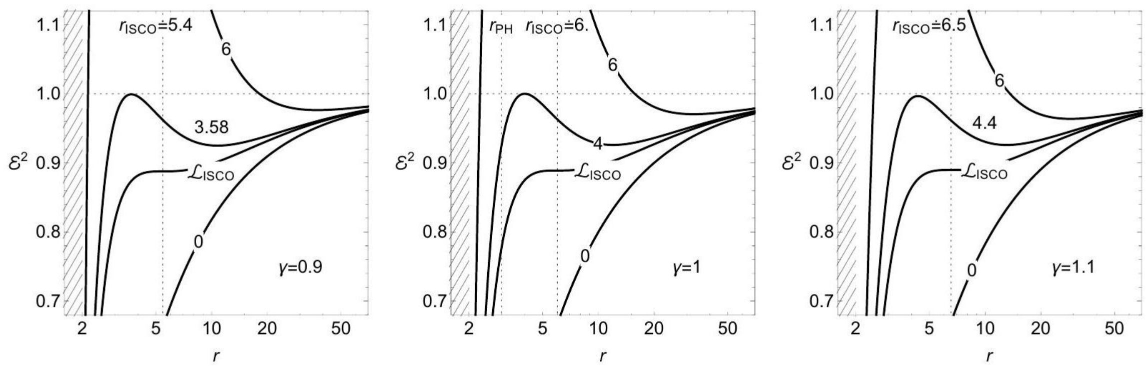

Figure 3. Radial profiles of effective potential in equatorial plane $V_{\rm eff}(r,\pi/2)$ for the various values of angular momentum L. In the plots the different values for γ parameter is used.

The unstable circular photon orbit $m = 0$ given by the divergence of the effective potential (33) will be located at
$$r_{\rm ph}/M = 1+2\gamma. \quad (39)$$

In the case when $\gamma = 1$ one can have $r_{\rm ISCO} = 6M$ and $r_{\rm ph} = 3M$ which are responsible for the radius of the ISCO and photon sphere, respectively, in the Schwarzschild spacetime.

In Fig. 4 the various dependences of the radius of the ISCO and the photon sphere are shown. In the range of the values of the $\gamma \geq 1$ one can see that with increasing the γ parameter the radius of ISCO and photon sphere increase while in the range of the values $1/\sqrt{5} \leq \gamma \leq 1$ they are small in comparison with that in general relativity.

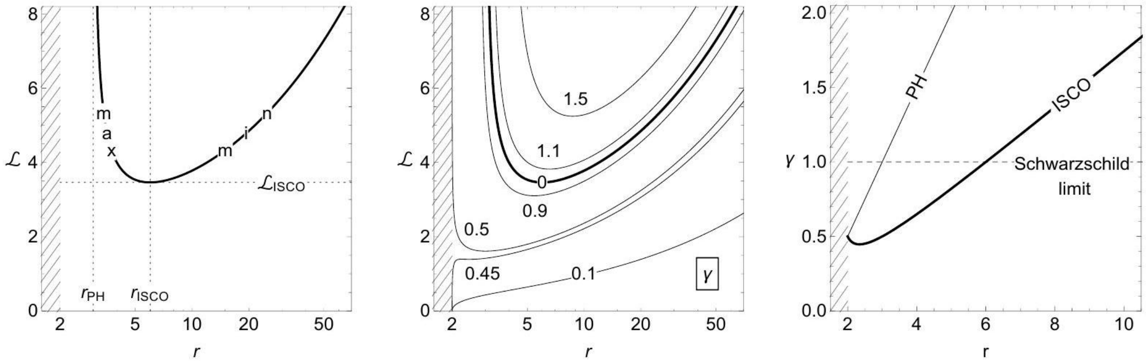

Figure 4. (Left panel) Position of extrema (max. min.) of the effective potential, giving stable $(\mathbf{min})$ and unstable $(\mathbf{max})$ circular orbits for the Schwarzschild $(\boldsymbol{\gamma} = \mathbf{1})$ spacetime. (Central panel) Position of extrema (max. min.) of effective potential for the different values of the $\boldsymbol{\gamma}$ parameter. (Right panel) Position of the ISCO and photon orbit in the dependence from the parameter the $\boldsymbol{\gamma}$.

One can easily see that the Eqs. (36), (37) and (38) for the angular momentum, the energy and radius of ISCO of the test particle, respectively, do not contain q which means that the gravitating scalar field does not act on the test particles in the equatorial plane. Numerical calculations show that the effects of the gravitating scalar field can be seen in particle motion in off-equatorial plane. As a test of the spacetime geometry (24) we have presented the particle trajectories for the different values of the metric parameters γ, γ_* and ϵ in several planes in Fig. 5.

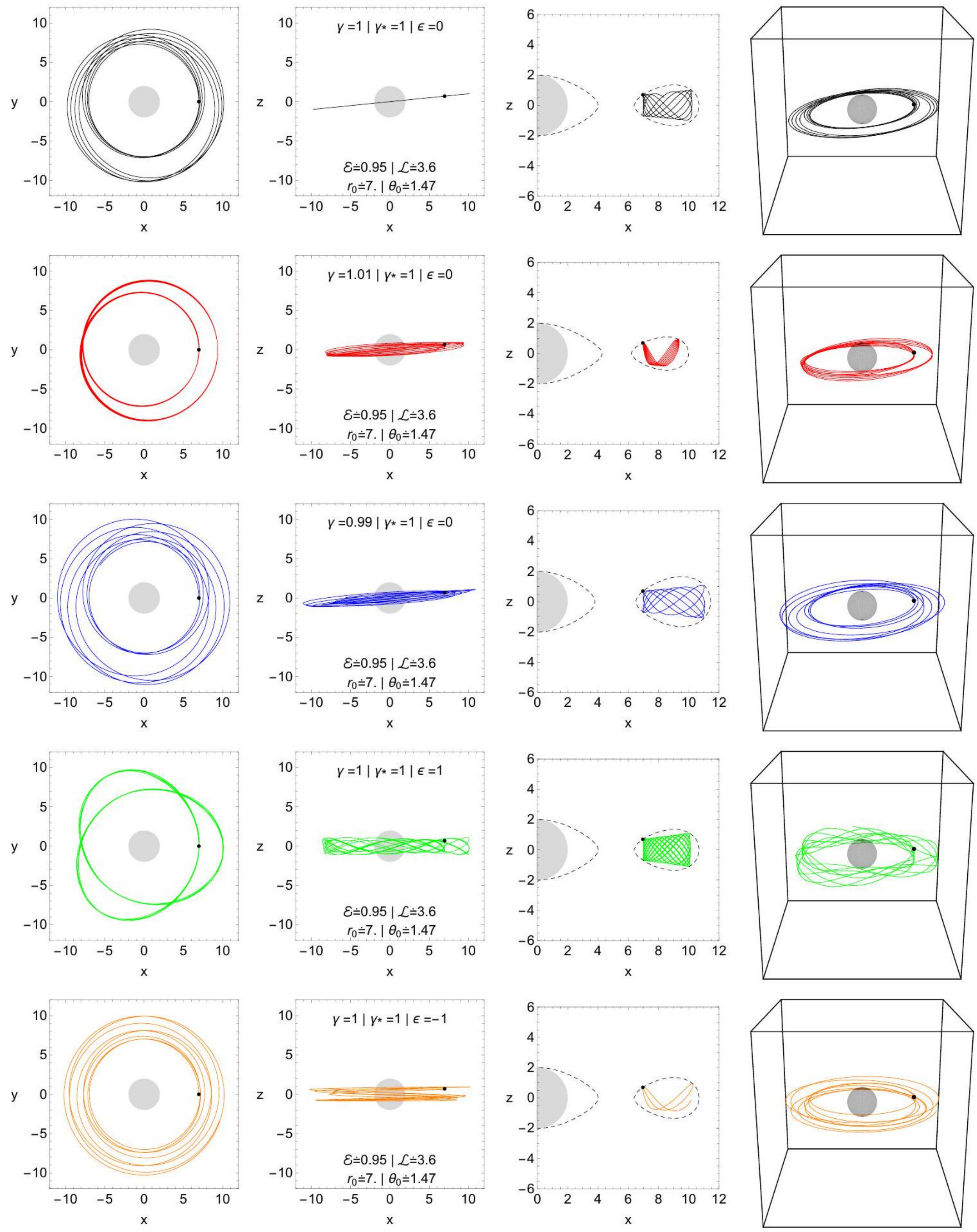

Figure 5. Test particle trajectories in the spacetime metric (24) for the different values of parameters γ, γ_* and ϵ. In first and second (including third) columns particle trajectories $x-y$ and $x-z$ planes are given while in the fourth column a 3D $x-y-z$ pattern of particle trajectory is shown.

The Erez-Rosen solution can be obtained in the limiting case when $q_* = 0$. In order to find the physically meaningful solution for the scalar field one writes it in terms of the spherical coordinates in the form

$$\Phi(r,\theta) = \frac{1}{2}\ln\left(1-\frac{2M}{r}\right) + \frac{q_*}{2}\left[\frac{3r^2-6Mr+2M^2}{4M^2}\ln\left(1-\frac{2M}{r}\right) + \frac{3(r-M)}{2M}\right](3\cos^2\theta - (\mathbb{M})) \quad (40)$$

and in the weak field approximation the equation 40 has a form

$$\Phi(r,\theta) \simeq -\frac{M}{r} + \frac{q_* M^3}{15r^3}(3\cos^2\theta - 1). \quad (41)$$

We can see that the first linear term in the right hand side of the equation (41) is responsible for Newtonian potential, the second term is responsible for the quadrupole moment potential, where q_* is dimensionless mass quadrupole moment produced by the gravitating scalar field.

In Fig 6 the equipotential surface of the scalar field $\Phi(r,\theta)$ using the expression (40) for the different values of the quadrupole moment q_* is illustrated. One can easily see that due to the q_* parameter the spacetime around the black hole is axially deformed.

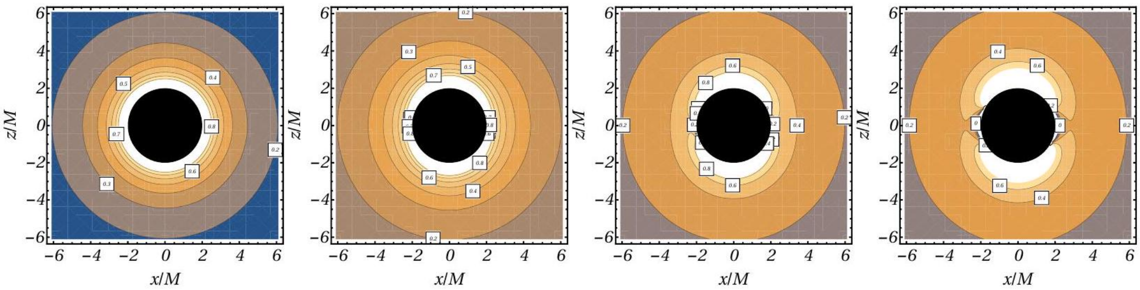

Figure 6. The equipotential surface of the scalar potential $\Phi(r,\theta)$ in $x-z$ plane for the different values of the mass quadrupole moment: $q_* = 0, q_* = 0.2, q_* = 0.5$ and $q_* = 1$.

The trajectories of the test particles in the spacetime of the generalized Erez-Rosen metric at the several planes for the different values of the parameters are shown in Fig. 7. The motion of the test particle becomes regular (not chaotic as in the Kerr spacetime) in the quadrupole moment metric.

It is also interesting to study chaotic motion in the spacetime with deformation parameters γ, γ_*, q and q_*. In order to check chaotic motion around the black hole we have used the general form of the spacetime metric. Numerical calculations show that the trajectory of test particles become chaotic for large values of the γ_*, q, and q_* parameters, as shown in Fig. 8.

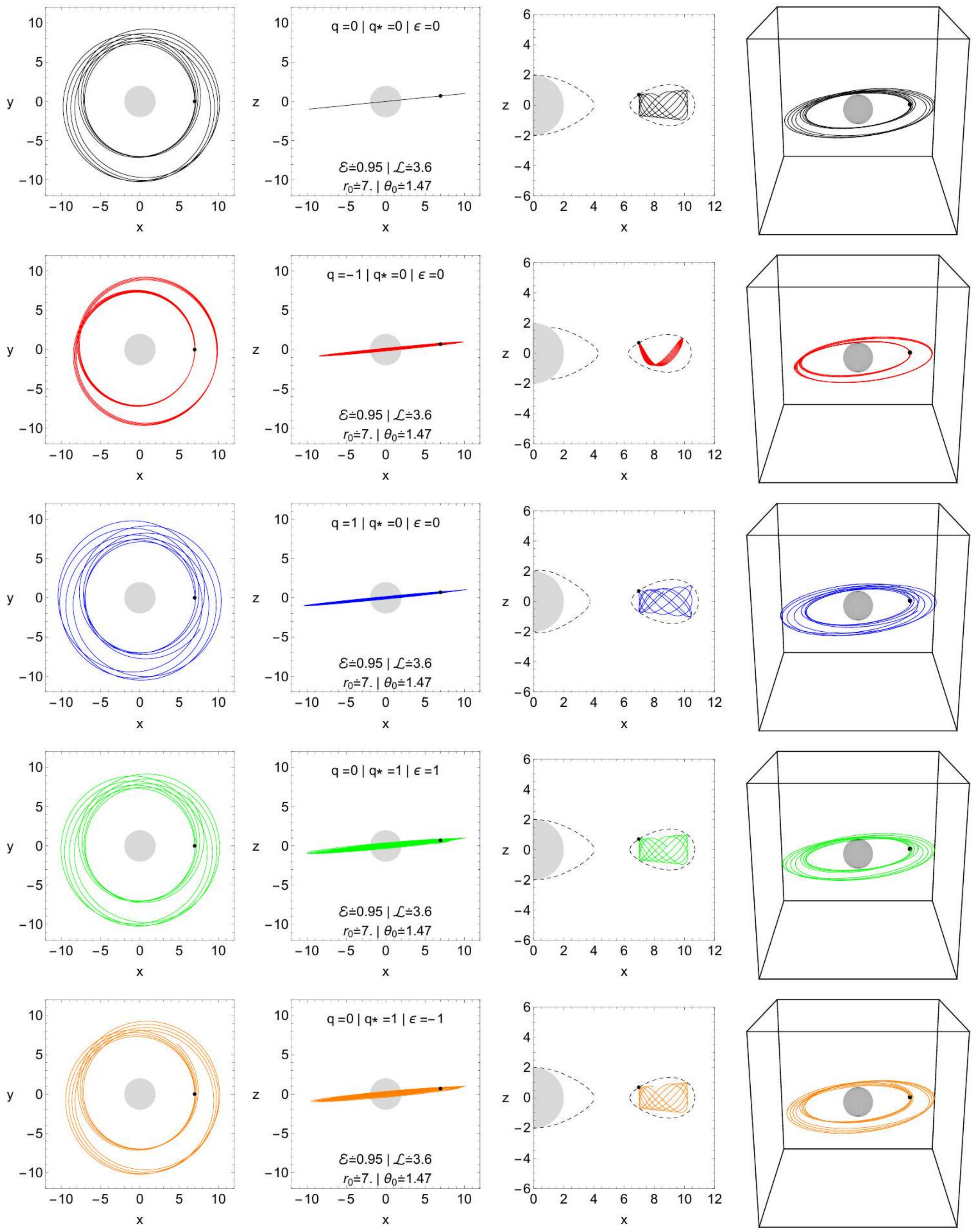

Figure 7. Test particle trajectories for the different values of parameters q, q_* and ϵ. In first and second (including third) columns particle trajectories in $x - y$ and $x - z$ planes are given, respectively, while in the fourth column 3D $x - y - z$ pattern of particle trajectory is shown.

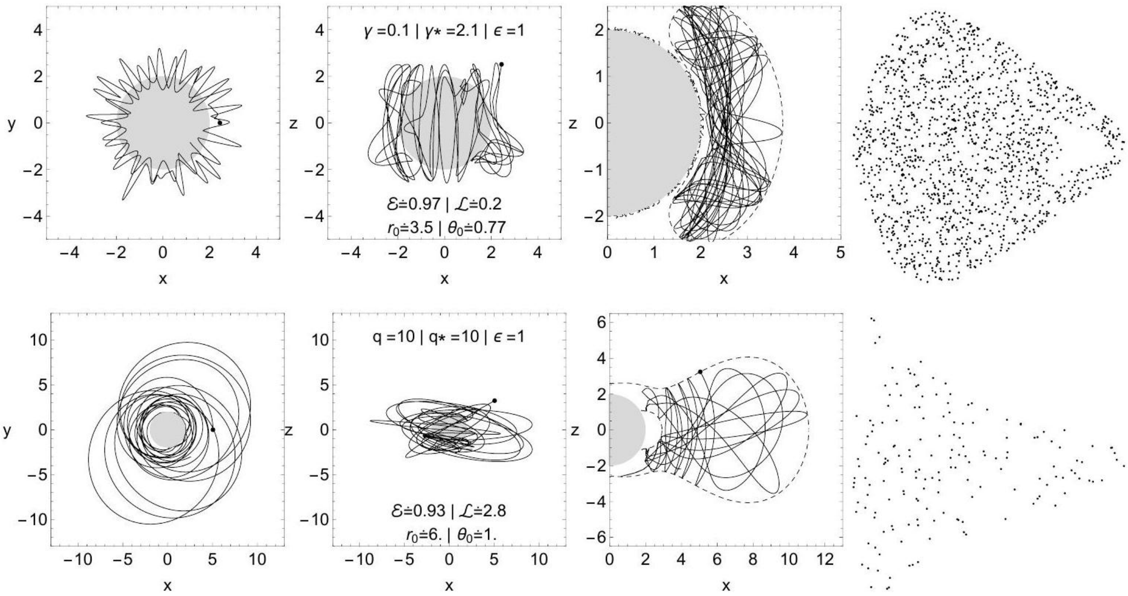

Figure 8. Chaotic trajectories of test particle in several planes in background geometry when $\epsilon = 1$. In first and second (including third) columns particle trajectories $x - y$ and $x - z$ planes are given while in the fourth column phase-space diagram of particle trajectory is shown.

Consider the equation of motion for the Einstein-Maxwell-scalar fields system. The action for the system is given by

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - F_{\alpha\beta}F^{\alpha\beta} - 2\,\partial_\alpha\Phi\,\partial^\alpha\Phi\right), \quad (42)$$

where R is the Ricci scalar of the curvature, $g = |g_{\alpha\beta}|$ is the determinant of metric tensor $g_{\alpha\beta}, \Phi$ is the scalar field and A_μ is the vector field with $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. One may obtain equations of motion for the whole system, namely, the Einstein field equations, the Klein-Gordon equation and Maxwell equations in the following form

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T_{\alpha\beta}, \quad (43)$$

$$\nabla_\alpha F^{\alpha\beta} = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}F^{\alpha\beta}\right) = 0, \quad (44)$$

$$\nabla_\alpha\nabla^\alpha\Phi = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\,\partial^\alpha\Phi\right) = 0, \quad (45)$$

where ∇_α is the covariant derivative, $G_{\alpha\beta}$ and $R_{\alpha\beta}$ are, respectively, the Einstein and Ricci tensors, respectively. The energy momentum tensor $T_{\alpha\beta}$ of the system is decomposed into two parts:

$$T_{\alpha\beta} = T^{S}_{\alpha\beta} + T^{EM}_{\alpha\beta}, \quad (46)$$

where the first one is responsible for the scalar field, while the second one for the vector field.

We are aimed to solve the Einstein-Maxwell-scalar field equations. Assume that the solution of Einstein equations to be spherical-symmetric, which ensures that only the time component of the vector potential is stored i.e. $A_\alpha = (A_t, 0,0,0)$. The second assumption is the scalar field and zeroth component of the vector field A_t depend on the radial coordinate r only $\Phi = \Phi(r)$ and $A_t = A_t(r)$. The general form of the spherically-symmetric, static spacetime metric is given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{-\nu(r)}\left[dr^2 + e^{\lambda(r)}d\Omega\right] \quad (47)$$

where $\nu(r)$ and $\lambda(r)$ are radial metric functions.
One can easily get the following relations:

$$G_r^r + G_\theta^\theta = 0 = \frac{1}{2}e^{\nu-\lambda}\left(\frac{d^2e^\lambda}{dr^2} - 2\right), \quad (48)$$

$$G_t^t + G_r^r = G_t^t - G_\theta^\theta = 2F_{rt}F^{rt} = -e^\nu(\nu'' + \nu'\lambda'), \quad (49)$$

$$G_r^r - G_t^t = -G_t^t - G_\theta^\theta = 2\,\partial_r\Phi\,\partial^r\Phi = e^\nu\left[\nu'' - \lambda'' - \frac{1}{2}(\nu' - \lambda')^2\right]. \quad (50)$$

It can be easily seen that equation (48) is independent of the sources and its solution is trivial for the radial metric function e^λ which can be written as

$$e^\lambda = r^2 + 2C_1r + C_2, \quad (51)$$

where C_1 and C_2 are, respectively, the constants of integration related to the mass and charge of the gravitational object. In order to solve equations (49) and (50) we first concentrate on Maxwell and Klein-Gordon equations. Recalling equations (44) and (45), we obtain

$$F_{rt} = Q_e e^{\nu-\lambda},\ \partial_r\Phi = Ce^{-\lambda}, \quad (52)$$

where Q_e is an electric charge and C is a scalar charge. Using the first equation in (52), the equation (49) can be expressed as

$$\nu'' + \nu'\lambda' = 2Q_e^2e^{\nu-2\lambda} \quad (53)$$

Using the solution for e^λ in equation (51) and making lengthy calculations we obtain the solution of equation (53) as

$$e^\nu = \left(1 + \frac{2C_1}{r} + \frac{Q_e^2}{n^2r^2}\right)^n\left[\frac{r_+\left(1-\frac{r_-}{r}\right)^n - r_-\left(1-\frac{r_+}{r}\right)^n}{r_+-r_-}\right]^{-2}, \quad (54)$$

where n is the constant of integration and $r_\pm$is defined as

$$r_\pm = -C_1 \pm \sqrt{C_1^2 - \frac{Q_e^2}{n^2}} \quad (55)$$

Here one have to emphasize that the integration constant C_2 in equation (51) is related to charge as $C_2 = Q_e^2/n^2$, remaining two constants C_1 and n should be expressed in terms of the mass M of the gravitational object and scalar charge C, which can be chosen as $C_1 = -M/n$, with $n = M/\sqrt{M^2 + C^2}$. Finally, exact analytical solution for Einstein-Maxwellscalar equations takes a form

$$e^\nu = \left[\frac{r_+\left(\frac{r-r_-}{r-r_+}\right)^{n/2} - r_-\left(\frac{r-r_+}{r-r_-}\right)^{n/2}}{r_+-r_-}\right]^{-2}, \quad (56)$$

$$e^\lambda = (r - r_+)(r - r_-),$$

with $nr_\pm = M \pm \sqrt{M^2 - Q_e^2}$. The associated electric field and associated scalar field take the form

$$E^{\hat{r}} = \frac{Q_e}{(r-r_+)(r-r_-)}\left[\frac{r_+\left(\frac{r-r_-}{r-r_+}\right)^{n/2} - r_-\left(\frac{r-r_+}{r-r_-}\right)^{n/2}}{r_+-r_-}\right]^{-2},$$
$$\Phi(r) = \frac{\sqrt{1-n^2}}{2}\ln\left(\frac{r-r_+}{r-r_-}\right), \tag{57}$$
which obey the equations (52).

In order to understand spacetime better, it is very important to investigate particle motion around a gravitational compact object with three parameters. The four-velocity of test particle, i.e. $u^\alpha = \dot{x}^\alpha = dx^\alpha/d\lambda$, where λ is an affine parameter, obeys the following normalization condition: $u^\alpha u_\alpha = -1$, on the other hand it can be expressed as $p_\alpha \equiv m u_\alpha = \partial_\alpha \mathcal{S}$.

We are interested in motion at the equatorial plane in which i.e. $\theta = \pi/2$, and hereafter making simple algebraic calculations one can obtain the following equations:

$$\dot{t} = \mathcal{E} e^{-\nu} \tag{58}$$

$$\dot{\phi} = \frac{\mathcal{L}}{\sin^2\theta} e^{\nu-\lambda} \tag{59}$$

$$\dot{r}^2 = f(r) = \mathcal{E}^2 - e^\nu\left(1 + e^{\nu-\lambda}\mathcal{L}^2\right) \tag{60}$$

where $\mathcal{E} = E/m$ and $\mathcal{L} = L/m$ are, respectively, the specific energy and specific angular momentum in per unit mass of the particle at infinity.

In order to get the critical values of the specific energy and specific angular momentum one can use the following conditions $f(r) = f'(r) = 0$, then one can obtain

$$\mathcal{E}^2 = \mathcal{E}^2_{\text{ext}}(r) \equiv \frac{\lambda' - \nu'}{\lambda' - 2\nu'} e^\nu, \tag{61}$$

$$\mathcal{L}^2 = \mathcal{L}^2_{\text{ext}}(r) \equiv \frac{\nu'}{\lambda' - 2\nu'} e^{\lambda-\nu}. \tag{62}$$

Figure 9 draws radial dependence of the effective potential for different values of n parameter at $Q = 0$.

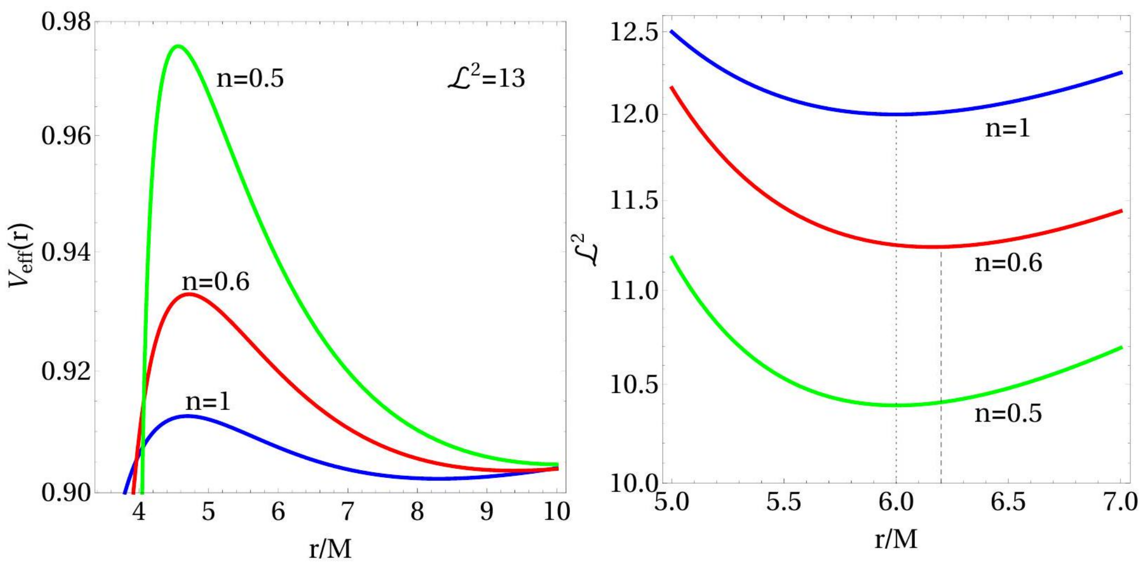

Figure 9. Radial dependence of the effective potential (left panel) and angular momentum (right panel) for different values of n parameter at $Q = 0$.

Since the specific energy and specific momentum of test particle of the massless particle tends to infinity (i.e. $\mathcal{E} \to 0, \mathcal{L} \to 0$), which requires the denominator of the expressions should be zero. The radius of the photonsphere $r_{\rm ph}$ can be found as a solution of the following equation $\lambda' = 2\nu'$ which doesn't have analytical solution. However, the analytical expressions for the radius of photonsphere in pure JNW and pure RN spacetime can be obtained as

$$r_{\rm ph}/M = \begin{cases} 2 + \frac{1}{n}, \; Q = 0 \\ \frac{3}{2}\left(1 + \sqrt{1 - \frac{8Q^2}{9M^2}}\right), \; n = 1 \end{cases} \tag{63}$$

Before starting numerical calculations, one has to emphasize that advantage of the metric (56) is that parameters of the solution are identified in the given range as $0 \leq Q \leq 1$ and $1/2 \leq n \leq 1$. So that helps to get the radius of photonsphere in a given area. Careful numerical calculations show that the radius of the photonsphere lies in the region $2M \leq r_{\rm ph} \leq 4M$. Figure 10 shows dependence of the radii of photonsphere from the charge parameters of the gravitational compact object. It is known that the radius of the photonsphere in Schwarzschild spacetime is $r_{\rm ph} = 3M$, for the extreme RN black hole case (i.e. $Q = M$) it is $r_{\rm ph} = 2M$, while it reachs up to $r_{\rm ph} = 4M$ in JNW solution as shown in Fig. 10.

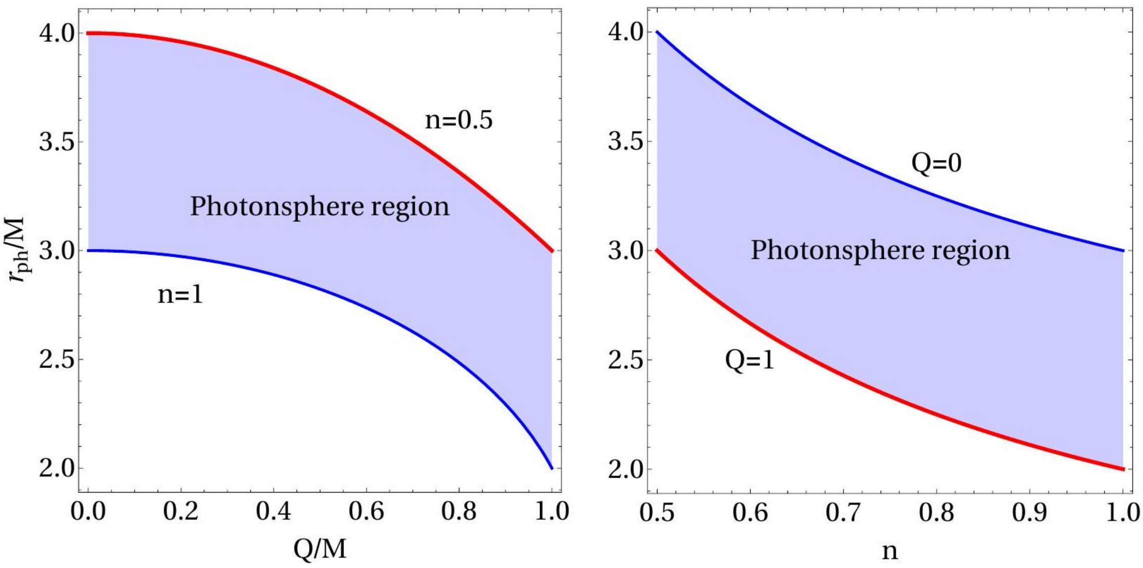

Figure 10. Left panel: Dependence of the radius of photonsphere from charge Q for different sets of n parameter. Right panel: Dependence of the radius of photonsphere from n parameter for different values of black hole charge.

In order to find the radius of innermost stable circular orbit (ISCO) one needs to use one more condition $f''(r) = 0$ along with the expressions (61), (62) and after performing simple algebra one can obtain the following equation to calculate ISCO radii:

$$\lambda' + \frac{2\nu'^2 - \lambda''}{\lambda'} - 3\nu' + \frac{\nu''}{\nu'} = 0 \tag{64}$$

which gives a complicated equation for radial coordinate. It is difficult to solve it analytically. However, again the exact expressions for ISCO radii for a test particle in JNW and RN spacetime metric are given as

$$r_{\rm ISCO}/M = \begin{cases} 3 + \frac{1}{n} + \sqrt{5 - \frac{1}{n^2}} \; Q = 0, \\ 2 + 2\zeta + \zeta^{-1}\left(2 - \frac{3Q^2}{2M^2}\right), \; n = 1 \end{cases} \tag{65}$$

where ζ is defined as

$$\zeta = \sqrt[3]{1 + \frac{Q^4}{4M^4} + \frac{Q^2}{8M^2}\left[\sqrt{\left(1 - \frac{Q^2}{M^2}\right)\left(5 - \frac{4Q^2}{M^4}\right)} - 9\right]} \tag{66}$$

Figure 11 draws dependence of ISCO radii from the electric and scalar charge parameters of the gravitational source in the following range, $0 \leq Q/M \leq 1$ and $1/2 \leq n \leq 1$. The shaved region in Fig. 11 represents variation of ISCO radii. Our numerical analysis show that range of ISCO radii are $4 \leq r_{\rm ISCO}/M \leq 6.2$. As one can see from Fig. 11 that ISCO radii is given in exact area.

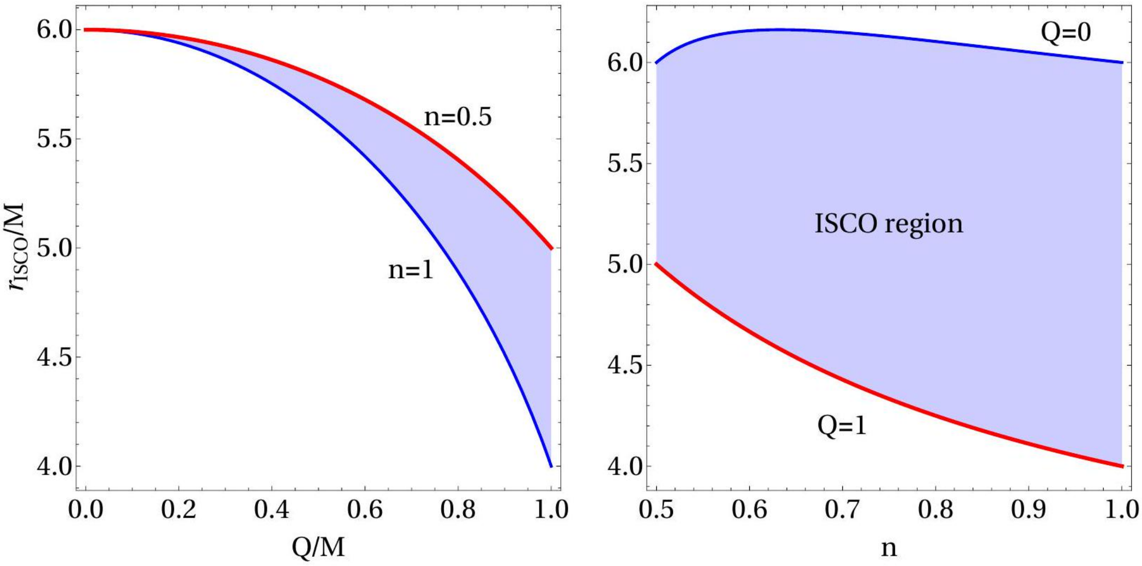

Figure 11. Left panel: Dependence of the ISCO radius from charge Q for different sets of n parameter. Right panel: Dependence of ISCO radius from n parameter for different values of charge Q.

It is also interesting to study the energy efficiency for the test particle which is ratio of the binding, $E_{\rm bind} = E_{\rm rest} - E_{\rm ISCO}$ and the rest energy, $E_{\rm rest} = mc^2$, of the test particle defined as

$$\eta = \frac{E_{\rm bind}}{E_{\rm rest}} = 1 - \mathcal{E}_{\rm ISCO} \tag{67}$$

which depends on the specific energy of test particle at ISCO. Figure 12 illustrates dependence of the energy efficiency from the different set of the parameters. It is well known that the energy efficiency is about $\eta \simeq 6\%$. However, it is approximately $\sim 7\%$ in JNW spacetime as one can see from Fig.

12 for $Q = 0$ at the left panel, while for extreme RN black hole solution the energy efficiency reaches up to $\sim 8.1\%$ as shown in the right panel of Fig. 12.

Careful numerical calculations show that newly obtained solution (56) can mimic spin parameter of the Kerr black hole up to $a_* \lesssim 0.6$ as shown in Fig. 13 . However, analysis of the inner edge of the accretion discs of the astrophysical black holes indicates that the spin parameter of astrophysical black hole almost reaches up to $a_* \lesssim 0.99$, which concludes that the obtained new solution (56) can not be considered as realistic candidate for the rapidly rotating almost extreme astrophysical black hole.

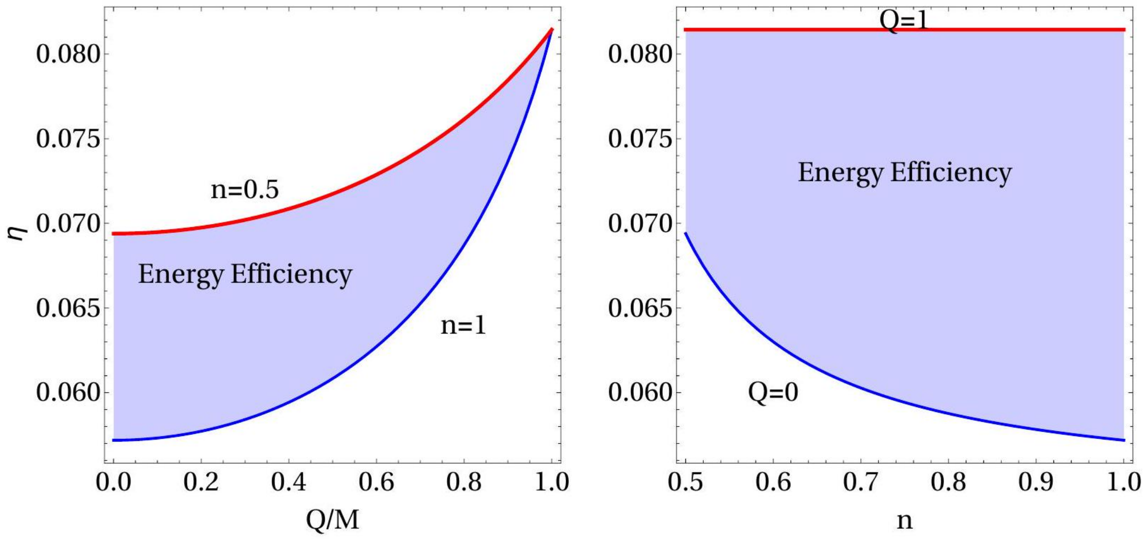

Figure 12. Left panel: Dependence of the energy efficiency from charge Q for different sets of n parameter. Right panel: Dependence of the the energy efficiency from n parameter for different values of charge Q.

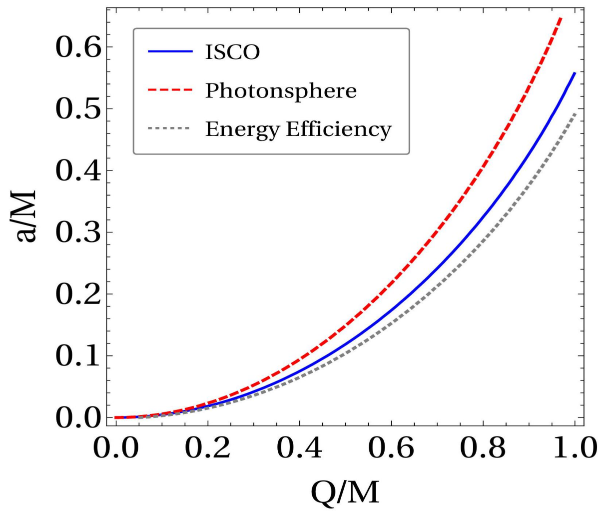

Figure 13. The possible degeneracy between the spin parameter a of the Kerr black hole with the charge Q of compact object described by the spacetime metric (56) for the different set of n parameter through the comparison of the radii of ISCO, photonsphere and energy efficiency.

CONCLUSIONS

The following conclusions have been presented on the basis of research carried out on the topic of “Exact solutions of Einstein-Maxwell-scalar fields equations”:

7. For the first time the axisymmetric and static solutions of the Einstein field equations considering the effect of an additional self-gravitating scalar field have been derived. Particularly the exact analytical solutions of the combined Einstein equations for two different modified spacetime metrics which belong to the Weyl class of solutions as (i) the modified gamma-metric and (ii) the modified quadrupole moment metric have been obtained.
8. For the first time the generalized form of the gamma-metric with additional parameter and generalized form of the Eres-Rosen metric with mass quadrupole produced by the self-gravitating scalar field have been obtained.
9. The analytical expressions for the components of the energy-momentum tensor are obtained for the self-gravitating scalar field. It has been shown that in the presence of phantom field solution satisfies the null energy condition while in the presence of gravitating scalar field it does not satisfy the null energy condition.
10. For the first time the exact analytical expression for the radius of the innermost stable circular orbits (ISCO), the critical values of the energy and the angular momentum of the test particles in the spacetime of the gamma-metric. It has been shown that for the special range of gamma parameter the radius of ISCO and the photon sphere increase. It has been also shown that the quadrupole moment has circular orbits that are more strongly bounded when compared to that in the Schwarzschild metric.
11. For the first time the analytical solution of the Einstein-Maxwell-scalar field equations which covers three well-known solutions such as Reissner-Nordstrom, Janis-Newman-Winicour and as well as Schwarzschild solutions. It has been shown that the obtained new solution does not exist horizons and it can be an example for naked singularity.
12. It has been found that the dual solution for the corresponding vector potential is also satisfied by field equations. It has been shown that due to the effect of the electric and scalar charges ISCO of test particles is shifted towards the central gravitational source. The energy efficiency for the test particles in the spacetime described by the new black hole solution is found to be at the range $6\% \lesssim \eta \lesssim 8\%$.

НАУЧНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ, «ТИИИМСХ» НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ

ТУРИМОВ БОБУР ВАЛЕНТИНОВИЧ

ТОЧНЫЕ РЕШЕНИЯ УРАВНЕНИЙ ЭЙНШТЕЙНА-МАКСВЕЛЛА-СКАЛЯРНОГО ПОЛЯ

01.03.01 – Астрономия
01.04.02 – Теоретическая физика
(физико-математические науки)

ПРЕДСТАВЛЕНИЕ
по присуждению ученой степени доктора философии (PhD) на основе научных публикаций без диссертации

Ташкент – 2024

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан под номером B2024.1.PhD/FM999.

Работа выполнена в институте фундаментальных и прикладных исследований при НИУ “ТИИМСХ”.

Представление на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ifar.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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Представление научного исследования состоится «__» апреля **2024 года в ___ часов** на заседании Научного Совета **DSc.03/31.03.2022.T/FM.10.04** по защите диссертаций на соискание ученых степеней при Институте фундаментальных и прикладных исследований, “ТИИИМСХ” Национальный Исследовательский университет по адресу: 100000, г. Ташкент, Qori Niyaziy Street 39, Институт фундаментальных и прикладных исследований, Зал 108; Тел.: 71 237-09-61; email: info@ifar.uz.

С представлением научного исследованияможно ознакомиться в Информационно-ресурсном центре при Институте фундаментальных и прикладных исследований, “ТИИИМСХ” Национальный Исследовательский университет (регистрационный номер ____) (Адрес: 100000, г. Ташкент, Qori Niyaziy Street 39, Институт фундаментальных и прикладных исследований, Зал 205; Тел.: 71 237-09-61).

Представление научного исследования разослано «____» _______ 2024 г.
(протокол рассылки № 49 от __ ________ 2024 г.).

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ВВЕДЕНИЕ (Аннотация к представлению)

Целью исследования является точное аксиально-симметричное и статическое решение уравнений Эйнштейна, связанных с гравитирующим скалярным полем, и уравнений Эйнштейна-Максвелла-скалярного поля.

Задачи исследования:

изучить влияние скалярного поля на свойства аксиально-симметричных и статических вакуумных решений уравнений Эйнштейна;

получить обобщенную форму гамма-метрики с дополнительным параметром и обобщенную форму метрики Эреса-Розена;

получить аналитические выражения для компонент тензора энергии-импульса самогравитирующего скалярного поля;

исследовать движение пробной частицы в пространстве-времени как обобщенной метрики гамма-метрики, так и метрики квадрупольного момента, а также параметров метрики, создаваемых гравитирующим скалярным полем в движении пробной частицы;

получить точное аналитическое выражение радиуса внутренних стабильных круговых орбит (ISCO), критических значений энергии и момента импульса пробных частиц через метрические параметры;

получить аналитическое решение уравнений скалярного поля Эйнштейна-Максвелла в предположении, что три различных поля: гравитационное, векторное и безмассовое скалярное поле не взаимодействуют друг с другом;

провести расчет инвариантов кривизны, таких как скаляр Риччи, квадрат тензора Риччи и скаляр Кречмана.

Объектом исследования являются астрофизические компактные объекты, динамика частиц, скалярное и элекромагнитное поля.

Предметом исследования являются точные аналитические решения уравнений поля, теоретические модели для изучения динамики частиц вблизи компактных гравитационных объектов в присутствии скалярного поля, численные и аналитические методы решения дифференциальных уравнений.

Методами исследования являются методы теоретической физики, методы теоретической астрофизики, современные методы математической физики, аналитические и численные методы расчета дифференциальных уравнений поля и движения частиц.

Научная новизна исследования заключается в следующем:

Получены аксиально-симметричные и статические решения уравнений Эйнштейна с учетом влияния дополнительного самогравитирующего скалярного поля. Получено точное аналитическое решение обобщенных уравнений Эйнштейна для двух различных модифицированных метрик пространства-времени, которые принадлежат к классу решений Вейля: (i) модифицированная гамма-метрика и (ii) модифицированная метрика квадрупольного момента.

Изучено влияние скалярного поля на пространственно-временные свойства осесимметричных и статических вакуумных решений комбинированных уравнений поля Эйнштейна.

Получены обобщенная форма гамма-метрики с дополнительным параметром и обобщенная форма метрики Эреса-Розена, включающая массовый квадруполь, создаваемый самогравитирующим скалярным полем.

Получены аналитические выражения для компонент тензора энергии-импульса самогравитирующего скалярного поля. Показано, что в случае фантомного поля решение удовлетворяет условию нулевой энергии.

Показано, что гамма- и квадрупольные параметры не дают вклада в энергию и угловой момент пробной частицы и, следовательно, не влияют на траекторию частицы в экваториальной плоскости.

Получены точные аналитические выражения для радиуса ISCO, критических значений энергии и момента импульса пробных частиц.

Практические результаты исследования, следующие:

Показано, что для соответствующего диапазона значений гамма-параметра радиус ISCO и фотонной сферы увеличиваются.

Найдено аналитическое решение скалярных уравнений поля Эйнштейна-Максвелла. Также рассчитаны инварианты кривизны, такие как скаляр Риччи, квадрат тензора Риччи и скаляр Кречмана.

Показано, что все три инварианта кривизны имеют три сингулярные точки. Сделан вывод, что полученное новое решение не имеет горизонтов.

Обнаружено, что энергетическая эффективность пробных частиц в пространстве, описываемом новым решением черной дыры, находится в диапазоне $6\% \lesssim \eta \lesssim 8\%$. .

Достоверность результатов исследований обеспечивается применением современных апробированных методов математической физики, вычислительной математики и релятивистской астрофизики. Результаты были получены строго в рамках математического аппарата общей теории относительности и теоретической физики. Также используются современные численные и аналитические методы расчета, результаты сравниваются с имеющимися данными наблюдений и результатами других авторов. Структурированные выводы диссертации соответствуют основным правилам астрофизики компактных объектов.

Научная и практическая значимость результатов исследования. Научная значимость результатов исследования заключается в том, что полученные решения уравнений поля могут обобщать предыдущие решения и описывать деформированные компактные объекты.

Практическая значимость результатов исследования заключается в том, что они могут сыграть роль в получении верхних пределов и ограничений на гамму и квадрупольные параметры в рамках гравитационной модели EMS.

Внедрение результатов исследования. На основе нового аналитического решения скалярных уравнений поля Эйнштейна-Максвелла:

научные результаты, полученные о движении частиц, были использованы учёными Фуданьского университета (FU) в Шанхае (справочник FU, Китай, 7 марта 2024 г.);

результаты по движению частиц вокруг компакта использовались в работах зарубежных исследователей, в зарубежных журналах с высоким импакт-фактором (The European Physical Journal C, Volume 83, Issue 12, article id.1131, Web-Sc, IF: 4.4; Pramana, Volume 97, Issue 1, article id.29, Web-Sc, IF: 2.219; Monthly Notices of the Royal Astronomical Society, Volume 521, Issue 1, pp.474-477, Web-Sc, IF: 5.235), для описания влияние метрических параметров на динамику частиц вокруг компактных объектов.

Публикация результатов исследований. Результаты исследования доктора философии представлены в 15 рецензируемых статьях, опубликованных в престижных научных журналах, рекомендованных Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан.

ВЫВОДЫ

На основе исследований, проведенных по теме «Точные решения уравнений Эйнштейна-Максвелла-скалярного поля», сделаны следующие выводы:

1. Впервые получены аксиально-симметричные и статические решения уравнений Эйнштейна с учетом влияния дополнительного самогравитирующего скалярного поля. В частности, были получены точные аналитические решения обобщенных уравнений Эйнштейна для двух различных модифицированных метрик пространства-времени, которые принадлежат к классу решений Вейля: (i) модифицированная гамма-метрика и (ii) модифицированная метрика квадрупольного момента.
2. Впервые получены обобщенная форма гамма-метрики с дополнительным параметром и обобщенная форма метрики Эреса-Розена с массовым квадруполем, создаваемым самогравитирующим скалярным полем.
3. Получены аналитические выражения для компонент тензора энергии-импульса самогравитирующего скалярного поля. Показано, что при наличии фантомного поля решение удовлетворяет условию нулевой энергии, а при наличии гравитирующего скалярного поля оно не удовлетворяет условию нулевой энергии.
4. Впервые получено точное аналитическое выражение для радиуса внутренней устойчивой круговой орбиты (ISCO), критических

значений энергии и момента импульса пробных частиц в пространстве-времени гамма-метрики. Показано, что для специального диапазона гамма-параметров радиус ISCO и фотонной сферы увеличиваются. Было также показано, что квадрупольный момент имеет более строго ограниченные круговые орбиты по сравнению с решнием Шварцшильда.

5. Впервые аналитическое решение скалярных уравнений поля Эйнштейна-Максвелла, которое решения Рейсснера-Нордстрема, Яниса-Ньюмана-Виникура, а также решения Шварцшильда. Показано, что полученное новое решение не имеет горизонтов и может служить примером голой сингулярности.
6. Установлено, что решение для соответствующего векторного потенциала удовлетворяет уравнение поля. Показано, что за счет воздействия электрических и скалярных зарядов ISCO пробных частиц смещается в сторону центрального источника гравитации. Обнаружено, что энергетическая эффективность пробных частиц в пространстве, описываемом новым решением черной дыры, находится в диапазоне $6\% \lesssim \eta \lesssim 8\%$.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim (part I; I часть)

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