**SAMARQAND DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/30.12.2019.FM.02.01 RAQAMLI ILMIY KENGASH**

**SHAROF RASHIDOV NOMIDAGI
SAMARQAND DAVLAT UNIVERSITETI**

AXMADOVA MUXAYYO OLIMJON QIZI

BIR VA IKKI O‘LCHAMLI PANJARALARDAGI IKKI ZARRACHALI SHREDINGER OPERATORLARI XOS QIYMATLARI SONI VA O‘RNI

01.01.01–Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Samarqand – 2024

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Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurligi. Jahonda olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlarda optik panjaradagi kvant zarrachalar sistemasiga mos diskret Shredinger operatorlari qattiq jismlar fizikasi, yuqori energiyali sistemalar fizikasi va ultrasovuq optik sistemalar fizikasi kabi turli sohalarga oid eksperiment natijalarini nazariy jihatdan asoslashda asosiy mexanizmlardan biri bo‘lib kelmoqda. Hozirgi kunda rivojlangan mamlakatlarda zarrachalarning erkin harakatini tavsiflovchi Tyoplitz o‘rama tipidagi operatorlar hamda zarrachalarning juft-jufti bilan o‘zaro ta’siri yoki potensial maydon ta’sirini tavsiflovchi ko‘paytirish operatorlari yig‘indisidan tashkil topgan operatorlar spektral nazariyasini o‘rganish muhim o‘rin tutmoqda. Bu borada zarrachalar sistemasiga mos gamiltonianlar xos funksiyalar yordamida to‘liq tavsiflanganligi uchun gamiltonianlarning spektral xossalari bilan bog‘liq masalalarni tadqiq qilishga alohida e’tibor qaratilmoqda.

Jahonda kichik o‘lchamli panjaralarda ikki zarrachali sistemalarga mos diskret Shredinger operatorlarining spektral xossalarini o‘rganishga qaratilgan ilmiy tadqiqotlar olib borilmoqda. Ushbu yo‘nalishda rivojlangan davlatlarning ilmiy-tadqiqot institutlarida ultrasovuq atomlar uchun Feshbax rezonanslarini o‘rganishga oid tadqiqotlar ustuvor hisoblanmoqda. Shuningdek bo‘sag‘a hodisasi matematik fizikadagi o‘ta muhim hodisalardandir. Efimov (uch o‘lchamli fazoda) hamda super Efimov effektlari (ikki o‘lchamli antisimmetrik fazolarda) bunga asos bo‘ladi. Bu borada bir va ikki o‘lchamli panjaralarda Shredinger operatorlari uchun ta’sir doimiysining bo‘sag‘a qiymatlari hamda muhim spektrdan yuqorida yotuvchi xos qiymatlari mavjudligini isbotlash, xos qiymatlarning aniq sonini toppish va ta’sir doimiysining bo‘sag‘a qiymatlari atrofida yaqinlashuvchi yoyilmalar olish, operatorning bo‘sag‘a xos qiymatlari hamda bo‘sag‘a rezonanslarini o‘rganish bo‘yicha tadqiqotlarni rivojlantirish dolzarb vazifalardan hisoblanmoqda.

Mamlakatimizda bir va ikki o‘lchamli panjaralardagi ikki zarrachali sistemalarga mos diskret Shredinger operatorlarining xossalarini o‘rganishga alohida e’tibor qaratilmoqda. “Algebra va funksional analiz, differensial tenglama va matematik fizika, dinamik tizimlar nazariyasi, geometriya va topologiya, ehtimollik nazariyasi va matematik statistika, amaliy matematika va matematik modellashtirish[1]” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalar va faoliyat yo‘nalishlari etib belgilangan. Ushbu vazifalarni amalga oshirish, xususan bir va ikki o‘lchamli panjaralardagi ikki zarrachali Shredinger operatorlarining spektral nazariyasini rivojlantirish muhim ilmiy ahamiyatga ega hisoblanadi.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi Farmoni, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi Farmoni, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ–huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yo'nalishi doirasida bajarilgan.

Muammoning o'rganilganlik darajasi.

So'nggi yillarda Yu.Kondratiyev, O.Kutoviy, S.Pirogov, S.Molchanov va E.Jijina kabi olimlar tomonidan olib borilgan tadqiqotlar shuni ko'rsatadiki, ba'zi stoxastik modellardagi diffuziya hodisalari uzluksiz nolokal Shredinger operatorlarining diskret spektri sifatida namoyon bo'ladi. Shuningdek, S.Albeverio, S.Laqayev, K.Makarov, P.Faria de Veyga, A.Xalxo'jayev va Sh.Xolmatov kabi olimlarning ishlarida bir va ikki o'lchamli panjaralardagi ikkita ixtiyoriy zarrachali sistemalarni o'rganishda diskret Shredinger operatori qaralganligi va uning spektral xossalari tadqiq qilinganliginini ko'rish mumkin.

Ikki va uch zarrachali diskret Shredinger operatorlari xos qiymatlarining mavjudligi va soni masalasi S.Albeverio, K.Makarov, P.Faria de Veyga, S.Laqayev, A.Xalxo'jayev, Z.Mo'minov, E.O'zdemir, I.Bozorov va Sh.Xolmatov kabi olimlarning bir nechta ishlarida tadqiq qilingan. O'lchamlari bir, ikki va uch bo'lgan panjaralarda tortishuvchi ikkita kvant zarrachali sistemaga mos Shredinger operatorlarining muhim spektrdan quyida yagona xos qiymati mavjudligi S.Laqayev, Sh.Xolmatov, A.Xalxo'jayev va Sh.Laqayevlarning ishlarida isbotlangan.

Bir, ikki va uch o'lchamli panjaralarda ta'sirlashuvchi ikkita bir xil bozonlar sistemasiga mos Shredinger operatorining xos qiymatlari soni va joylashgan o'rni S.Laqayev, E.O'zdemir, I.Bozorov, Sh.Hamidov, S.Abduhakimov, I.Alladustova va F.Almuratovlar tomonidan parametrlar va panjaraning o'lchamiga bog'liq holda o'rganilgan. Xususan, S.Lakaev, E.O'zdemir and I.Alladustovalarning ishlarida bir o'lchamli panjarada nuqtada va qo'shni tugunlarda ta'sirlashuvchi ikkita bir xil bozonlar sistemasiga mos Shredinger operatorining xos qiymatlari soni, o'rni va xos qiymatlar dinamikasi parametrlarga bog'liq holda aniqlangan. S.Lakaev, Sh.Xolmatov, Sh.Hamidov va S.Abduhakimovlarning ishlarida ikki o'lchamli panjaradagi ikki zarrachali Shredinger operatorlarining xos qiymatlari soni kvaziimpulsning barcha nolmas qiymatlarida tadqiq qilingan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim yoki ilmiy tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiya tadqiqoti Samarqand davlat universiteti ilmiy-tadqiqot ishlari rejasiga muvofiq №SMat-05 raqamli "Matematik analiz va uning zamonaviy matematik fizikaga tadbiqi" mavzusida olib borilayotgan majmuali ishlar doirasida bajarilgan.

Tadqiqotning maqsadi panjaradagi ikki zarrachali Shredinger operatorlarining muhim va diskret spektrini tadqiq qilishdan iborat.

Tadqiqotning vazifalari:

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soni va joylashgan o‘rnini aniqlash;

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soniga kvaziimpulsning barcha nolmas qiymatlarida aniq baho olish;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi bog‘langan holatlari mavjud bo‘lishi uchun yetarli shartlar topish;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining soni uchun kvaziimpulsning barcha nolmas qiymatlarida baho olish.

Tadqiqotning ob’yekti bir va ikki o‘lchamli panjaralardagi ikki zarrachali sistemaga mos energiya operatorlaridan iborat.

Tadqiqotning predmeti panjaradagi ikki zarrachali Shredinger operatorlari spektral tahlilidan iborat.

Tadqiqotning usullari. Dissertatsiya ishida matematik analiz, kompleks o‘zgaruvchili funksiyalar nazariyasi, funksional analiz, matematik fizika, o‘z-o‘ziga qo‘shma operatorlarning spektral nazariyasi, va Fredgolm determinantini analitik davom ettirish usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soni va joylashgan o‘rni aniqlangan;

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soniga kvaziimpulsning barcha nolmas qiymatlarida aniq baho olingan;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi bog‘langan holatlari mavjud bo‘lishi uchun yetarli shartlar topilgan;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining soni uchun kvaziimpulsning barcha nolmas qiymatlarida baho olingan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

bir va ikki o‘lchamli panjaralardagi ikki bozonli sistemaga mos Shredinger operatori xos qiymatlari xossalari Laplasning nolokal operatori uchun ba’zi chegaraviy masalalarning xos qiymat va xos funksiyalarni qurish usullarini tadqiq etish imkonini bergan;

ikki zarrachali diskret Shredinger operatorining bog‘langan holatlari mavjudligi neytronga boy yaxshi deformatsiyalangan lantanid yadrolarining qo‘zg‘algan holatlari dinamikasini simulyatsiya qilish imkonini beradi.

Tadqiqot natijalarining ishonchliligi matematik analiz, funksional analiz, zamonaviy matematik fizika va o‘z-o‘ziga qo‘shma operatorlarning spektral nazariyasi metodlaridan foydalangan holda aniq matematik tahlillar va isbotlashlar bilan izohlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati o‘z-o‘ziga qo‘shma operatorlar spektral nazariyasida, kvant mexanikasi va qattiq jismlar fizikasida panjaradagi ikki va uch zarrachali sistema energiya operatorlari spektrlari hamda xos qiymati mavjudligini ko‘rsatish bilan bog‘liq masalalarni hal etishda foydalanish mumkinligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati olingan ilmiy natijalarning qattiq jismlar fizikasi, elastiklik nazariyasi va kvant mexanikasida eksperimental tadqiqotlar o‘tkazish va qo‘llashga nazariy asos sifatida xizmat qilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Bir va ikki o‘lchamli panjaralardagi ikki zarrachali Shredinger operatorlari xos qiymatlari soni va o‘rniga oid olingan natijalar asosida:

bir va ikki o‘lchamli panjaralarda ikki bozonli sistemaga mos Shredinger operatorlarining keng sinfi uchun kvaziimpulsning nolmas qiymatlarida Shredinger operatorining muhim spektrdan quyida xos qiymatlari mavjudligini ko‘rsatish usullaridan “Elliptik tenglamalar va ularning kasr tartibli analogi uchun klassik va klassik bo‘lmagan chegaraviy masalalarni yechish usullarini ishlab chiqish” mavzusidagi AP09259074 raqamli xorijiy loyihada (Xo‘ja Ahmad Yassaviy nomli xalqaro qozoq-turk universitetining 2024 yil 04 yanvardagi № 0121PK00198 sonli ma’lumotnomasi) foydalanilgan. Panjaradagi ikki bozonli sistemaga mos Shredinger operatori xos qiymatlari Laplasning nolokal operatori uchun ba’zi chegaraviy masalalarning xos qiymat va xos funksiyalarni qurish usullarini tadqiq etish imkonini bergan;

Ikki zarrachali diskret Shredinger operatorining bog‘langan holatlari mavjudligi neytronga boy yaxshi deformatsiyalangan lantanid yadrolarining qo‘zg‘algan holatlari dinamikasini simulyatsiya qilish imkonini beradi. Olingan natijalardan “Neytronga boy yaxshi deformatsiyalangan lantanid yadrolarining qo‘zg‘algan holatlari dinamikasi” mavzusidagi FRGS19–039–0647 raqamli xorijiy loyihada (Malayziya Muhandislik Xalqaro Islom universitetining 2024 yil 11 yanvardagi № 003–KOE–24 sonli ma’lumotnomasi) foydalanilgan.

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiyaning asosiy natijalari 3 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida, jami 7 ta ilmiy-amaliy anjumanlarda muhokamadan o‘tgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 12 ta ilmiy ish chop etilgan, shulardan Oliy attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta, jumladan, 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat bo‘lib, 82 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqotning maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiya ishining **«Boshlang‘ich ma’lumotlar va panjaradagi ikki zarrachali Hamiltonianlar»** deb nomlanuvchi birinchi bobida asosiy natijalarni isbotlashda muhim o‘rin tutadigan o‘z-o‘ziga qo‘shma va kompakt operatorlarning spektri haqidagi ba’zi teoremalar keltirilgan. Shuningdek, panjaradagi bir va ikki zarrachali diskret Shredinger operatorlarining koordinata va impuls tasvirlari aniqlangan va bu operatorlarning chegaralangan va o‘z-o‘ziga qo‘shma ekanligi ko‘rsatilgan.

Dissertatsiya ishining **“Bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatorlarining xos qiymatlari soni va o‘rni”** deb nomlangan ikkinchi bobida, ikki zarrachali diskret Shredinger operatorlarining muhim spektrdan tashqarida joylashgan xos qiymatlari soni va o‘rni o‘rganilgan.

Faraz qilaylik $\mathbb{T}$ bir o‘lchamli tor, $L^{2,e}(\mathbb{T})$ esa $\mathbb{T}$ da aniqlangan kvadrati bilan jamlanuvchi juft funksiyalarning Hilbert fazosi bo‘lsin.

Ikkita bir xil bozon zarrachali sistemaga mos $H_{\gamma\lambda\mu}(K)$, $K \in \mathbb{T}$ diskret Shredinger operatori $L^{2,e}(\mathbb{T})$ Hilbert fazosida quyidagicha aniqlanadi:

$$H_{\gamma\lambda\mu}(K) := H_0(K) + V_{\gamma\lambda\mu},$$

bu yerda qo‘zg‘almas operator

$$\mathcal{E}_K(p) := 2\left(1 - \cos\frac{\mathrm{K}}{2}\cos p\right)$$

funksiyaga ko‘paytirish operatori va $V_{\gamma\lambda\mu}$ qo‘zg‘alish operatori quyidagicha aniqlanadi:

$$V_{\gamma\lambda\mu} f(p) = \frac{1}{\pi}\int_{\mathbb{T}} (\gamma + \lambda\cos p\cos q + \mu\cos 2p\cos 2q)\, f(q)\mathrm{d}q.$$

$V_{\gamma\lambda\mu}$ chekli rangli operator bo‘lganligi uchun Veyl teoremasiga ko‘ra barcha $K \in \mathbb{T}^2$ lar uchun $H_{\gamma\lambda\mu}(K)$ ning $\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(K)\right)$ muhim spektri $H_0(K)$ ning spektri bilan ustma-ust tushadi, ya’ni

$$\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(K)\right) = \sigma\left(H_0(K)\right) = [\mathcal{E}_{\min}(K), \mathcal{E}_{\max}(K)],$$

bu yerda

$$\mathcal{E}_{\min}(K) := \quad \min_{p\in\mathbb{T}} \mathcal{E}_K(p) = 2\left(1-\cos\frac{K}{2}\right) \geq 0 = \mathcal{E}_{\min}(0),$$
$$\mathcal{E}_{\max}(K) := \quad \max_{p\in\mathbb{T}} \mathcal{E}_K(p) = 2\left(1+\cos\frac{K}{2}\right) \leq 4 = \mathcal{E}_{\max}(0).$$

Faraz qilaylik $K \in \mathbb{T}$ va $n_+\left(H_{\gamma\lambda\mu}(K)\right)$, mos ravishda $n_-\left(H_{\gamma\lambda\mu}(K)\right)$ $H_{\gamma\lambda\mu}(K)$ operatorning muhim spektrdan yuqoridagi, mos ravishda quyidagi xos qiymatlari soni bo'lsin.

1-teorema. *Aytaylik* $\gamma, \lambda, \mu \in \mathbb{R}$ *bo'lsin. Agar* $H_{\gamma\lambda\mu}(0)$ *operator muhim spektrdan quyida* n_- *va yuqorida* n_+ *ta xos qiymatlarga ega bo'lsa, u holda istalgan* $K \in \mathbb{T}$ *uchun* $H_{\gamma\lambda\mu}(K)$ *Gamiltonian muhim spektrdan quyida kamida* n_- *ta va yuqorida kamida* n_+ *ta xos qiymatlarga ega bo'ladi. Boshqacha aytganda,*

$$n_-\left(H_{\gamma\lambda\mu}(K)\right) \geq n_-\left(H_{\gamma\lambda\mu}(0)\right)$$

va

$$n_+\left(H_{\gamma\lambda\mu}(K)\right) \geq n_+\left(H_{\gamma\lambda\mu}(0)\right)$$

tengsizliklar o'rinli.

Ushbu

$$C^{\pm}(\gamma, \lambda, \mu) := \mp(\gamma + \lambda + \mu + \gamma\lambda\mu) + \gamma\lambda + 2\gamma\mu + \lambda\mu$$

formula yordamida aniqlangan uch o'zgaruvchining $C^{\pm}$ ko'phadini qaraylik.

1-lemma. $\mathbb{R}^3$ *fazoning* $C^{\pm}(\gamma, \lambda, \mu) = 0$ *tenglamani qanoatlantiruvchi nuqtalari to'plami*

$$\gamma^{\pm}(\lambda, \mu) = -\frac{Q_0^{\pm}(\lambda, \mu)}{Q_1^{\pm}(\lambda, \mu)},$$

funksiyaning grafigi bilan ustma-ust tushadi, bu yerda $Q_0^{\pm}(\lambda, \mu)$ *va* $Q_1^{\pm}(\lambda, \mu)$ *lar quyidagicha aniqlanadi*

$$Q_0^{\pm}(\lambda, \mu) = \mp(\lambda + \mu) + \lambda\mu,$$
$$Q_1^{\pm}(\lambda, \mu) = \mp(1 + \lambda\mu) + \lambda + 2\mu.$$

2-lemma. $\mathbb{R}^2$ *fazoning* $Q_1^{\pm}(\lambda, \mu) = 0$ *tenglamani qanoatlantiruvchi nuqtalari to'plami*

$$\lambda^{\pm}(\mu) = -\frac{2\mu \mp 1}{1 \mp \mu}$$

funksiyaning grafigi bilan ustma-ust tushadi.

$\mu = -1$ va $\mu = 1$ to'g'ri chiziqlar (λ, μ)-tekislikdagi $\lambda^-(\cdot)$ va $\lambda^+(\cdot)$ funksiyaning grafigini ikkita (turli) $\{\tau_1^-, \tau_2^-\}$ va $\{\tau_1^+, \tau_2^+\}$ uzluksiz chiziqlarga ajratadi.

$$\tau_1^- = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = -\frac{1+2\mu}{1+\mu},\ \mu > -1\},$$
$$\tau_2^- = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = -\frac{1+2\mu}{1+\mu},\ \mu < -1\}$$

va

$$\tau_1^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = \frac{1-2\mu}{1-\mu},\ \mu < 1\},$$
$$\tau_2^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = \frac{1-2\mu}{1-\mu},\ \mu > 1\}.$$

$\{\tau_1^-, \tau_2^-\}$ va $\{\tau_1^+, \tau_2^+\}$ chiziqlar (λ,μ)- tekislikni mos ravishda $\lambda^-(\cdot)$ va $\lambda^+(\cdot)$ funksiyalarning aniqlanish sohalari D_1^-, D_2^-, D_3^- va D_1^+, D_2^+, D_3^+ ga ajratadi:

$$D_1^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) > 0,\ \mu > -1\},$$
$$D_2^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) < 0\},$$
$$D_3^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) > 0,\ \mu < -1\}$$

va

$$D_1^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) > 0,\ \mu < 1\},$$
$$D_2^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) < 0\},$$
$$D_3^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) > 0,\ \mu > 1\}.$$

Ta'kidlash joizki, $\gamma^{\pm}(\cdot,\cdot)$ funksiyaning aniqlanish sohasi ushbu

$$\mathbb{R}^2 \backslash \left(\tau_1^{\pm} \cup \tau_2^{\pm}\right) = D_1^{\pm} \cup D_2^{\pm} \cup D_3^{\pm}$$

ochiq to‘plamdan iboratdir. $\mathbb{R}^2$ dagi $\tau_1^{\pm}$ va $\tau_2^{\pm}$ chiziqlar $\mathbb{R}^3$ da mos ravishda $\Upsilon_j^{\pm} \subset \mathbb{R}^3$, $j = 1,2$ sirtlarni aniqlaydi:

$$\Upsilon_j^{\pm} := \left\{(\gamma,\lambda,\mu) \in \mathbb{R}^3,\ (\lambda,\mu) \in \tau_j^{\pm}\right\}.$$

Shuning uchun, $\mathbb{R}^3$ dagi $\Upsilon_1^{\pm}$ va $\Upsilon_2^{\pm}$ sirtlar $\gamma^{\pm}(\cdot,\cdot)$ funksiyaning grafigini uchta turli uzluksiz (bog‘langan) $\Gamma_1^{\pm}$, $\Gamma_2^{\pm}$ va $\Gamma_3^{\pm}$ sirtlarga ajratadi:

$$\Gamma_j^{\pm} = \left\{(\gamma,\lambda,\mu) \in \mathbb{R}^3 :\ \gamma = -\frac{Q_0^{\pm}(\lambda,\mu)}{Q_1^{\pm}(\lambda,\mu)},\ (\lambda,\mu) \in D_j^{\pm}\right\}, j = 1,2,3.$$

Shunday qilib, ushbu lemma o‘rinlidir.

3-lemma. $\mathbb{R}^3$ *fazodagi* $\gamma^{\pm}(\cdot,\cdot)$ *funksiyaning grafigi uchta* $\Gamma_1^{\pm}$, $\Gamma_2^{\pm}$ *va* $\Gamma_3^{\pm}$ *uzluksiz sirtlardan iboratdir.* $\Gamma_1^{-}, \Gamma_2^{-}, \Gamma_3^{-}$, *mos ravishda* $\Gamma_1^{+}, \Gamma_2^{+}, \Gamma_3^{+}$ *sirtlar uch o'lchamli* $\mathbb{R}^3$ *fazoni to'rtta* $\mathbb{G}_0^-$, $\mathbb{G}_1^-$, $\mathbb{G}_2^-$, $\mathbb{G}_3^-$, *mos ravishda* $\mathbb{G}_0^+$, $\mathbb{G}_1^+$, $\mathbb{G}_2^+$, $\mathbb{G}_3^+$ *bog'langan komponentalarga ajratadi:*

$$\mathbb{G}_0^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^-(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in D_1^-\},$$
$$\mathbb{G}_1^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^-(\gamma,\lambda,\mu\} < 0,\ (\lambda,\mu) \in \overline{D_1^-} \cup D_2^-\},$$
$$\mathbb{G}_2^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^-(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in \overline{D_2^-} \cup D_3^-\},$$
$$\mathbb{G}_3^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^-(\gamma,\lambda,\mu) < 0,\ (\lambda,\mu) \in D_3^-\}$$

va

$$\mathbb{G}_0^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^+(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in D_1^+\},$$
$$\mathbb{G}_1^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^+(\gamma,\lambda,\mu\} < 0,\ (\lambda,\mu) \in \overline{D_1^+} \cup D_2^+\},$$
$$\mathbb{G}_2^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^+(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in \overline{D_2^+} \cup D_3^+\},$$
$$\mathbb{G}_3^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3 \colon C^+(\gamma,\lambda,\mu) < 0,\ (\lambda,\mu) \in D_3^+\}.$$

2-teorema. *Faraz qilaylik* $\mathcal{C}^-$ *yuqoridagi* $\mathbb{G}_\alpha^-$, $\alpha = 0,1,2,3$ *bog'langan komponentalarning biri bo'lsin. U holda, istalgan* $(\gamma,\lambda,\mu) \in \mathcal{C}^-$ *uchun* $H_{\gamma\lambda\mu}(0)$ *ning* $\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(0)\right)$ *muhim spektrdan quyida yotuvchi xos qiymatlari soni o'zgarmaydi.*

Shuningdek, aytaylik $\mathcal{C}^+$ *yuqoridagi* $\mathbb{G}_\alpha^+$, $\alpha = 0,1,2,3$ *bog'langan komponentalarning biri bo'lsin. U holda, istalgan* $(\gamma,\lambda,\mu) \in \mathcal{C}^+$ *uchun* $H_{\gamma\lambda\mu}(0)$ *ning* $\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(0)\right)$ *muhim spektrdan yuqorida yotuvchi xos qiymatlari soni o'zgarmaydi.*

3-teorema. *Istalgan* $\alpha,\beta = 0,1,2,3$ *uchun quyidagi tasdiqlar o'rinlidir:*

i) *agar* $(\gamma,\lambda,\mu) \in \mathbb{G}_\alpha^-$ *bo'lsa, u holda* $n_-\left(H_{\gamma\lambda\mu}(0)\right) = \alpha$*;*

ii) *agar* $(\gamma,\lambda,\mu) \in \mathbb{G}_\beta^+$ *bo'lsa, u holda* $n_+\left(H_{\gamma\lambda\mu}(0)\right) = \beta$.

Quyidagicha belgilash kiritaylik:

$$\mathcal{C}_{\alpha\beta} = \mathbb{G}_\alpha^- \cap \mathbb{G}_\beta^+,\ \alpha,\beta = 0,1,2,3.$$

4-teorema. *Faraz qilaylik* $K \in \mathbb{T}$ *va* $(\gamma,\lambda,\mu) \in \mathbb{R}^3$. $\alpha + \beta \leq 3$ *shartni qanoatlantiruvchi barcha* $\alpha,\beta = 0,1,2,3$ *lar uchun quyidagi munosabatlar o'rinlidir:*

i) agar $(\gamma,\lambda,\mu) \in \mathcal{C}_{\alpha\beta}$ va $\alpha + \beta < 3$ *bo'lsa, u holda*

$$n_-\left(H_{\gamma\lambda\mu}(K)\right) \geq \alpha,\ n_+\left(H_{\gamma\lambda\mu}(K)\right) \geq \beta;$$

ii) agar $(\gamma,\lambda,\mu) \in \mathcal{C}_{\alpha\beta}$ va $\alpha + \beta = 3$ *bo'lsa, u holda*

$$n_-\left(H_{\gamma\lambda\mu}(K)\right)=\alpha,\ n_+\left(H_{\gamma\lambda\mu}(K)\right)=\beta.$$

“Panjaradagi ikkita bozonli sistema Gamiltaniani bog‘langan holatlari mavjudligi ” deb nomlangan uchinchi bobda, ikki o‘lchamli panjara Z^2 da nuqtada, eng yaqin qo‘shni va ikkinchi eng yaqin qo‘shni tugunlarda mos ravishda $\gamma\in\mathbb{R}$, $\lambda\in\mathbb{R}$ va $\mu\in\mathbb{R}$ sonlar yordamida o‘zaro ta’sirlashuvchi ikkita bir xil bozonlar sistemasiga mos $H_{\gamma\lambda\mu}(K), K\in T^2$ Shredinger operatorlari oilasi o‘rganilgan. $H_{\gamma\lambda\mu}(K)$ operatorning biri muhim spektrning quyi chegarasidan chapda, ikkinchisi yuqori chegarasidan o‘ngda joylashgan ikkita xos qiymatga ega bo‘lishi uchun yetarli shartlar topilgan.

Faraz qilaylik $\mathbb{T}^2$ ikki o‘lchamli tor, $L^{2,e}(\mathbb{T}^2)$ esa $\mathbb{T}^2$ da aniqlangan kvadrati bilan jamlanuvchi juft funksiyalarning Hilbert fazosi bo‘lsin.

Ikkita bir xil bozonlar sistemasiga mos $H_{\gamma\lambda\mu}(K)$, $K\in\mathbb{T}^2$ diskret Shredinger operatori $L^{2,e}(\mathbb{T}^2)$ Hilbert fazosida quyidagicha aniqlanadi:

$$H_{\gamma\lambda\mu}(K):=H_0(K)+V_{\gamma\lambda\mu},$$

bu yerda qo‘zg‘almas operator

$$e_K(p):=2\sum_{i=1}^{2}(1-\cos\frac{K_i}{2}\cos p_i)$$

Funksiyaga ko‘paytirish operatori va $V_{\gamma\lambda\mu}$ qo‘zg‘alish operatori quyidagicha aniqlanadi:

$$V_{\gamma\lambda\mu}f(p)=\frac{\gamma}{4\pi^2}\int\limits_{\mathbb{T}^2}f(q)\mathrm{d}q+\frac{\lambda}{4\pi^2}\sum_{i=1}^{2}\cos p_i\int\limits_{\mathbb{T}^2}\cos q_i f(q)\mathrm{d}q+$$
$$\frac{\mu}{4\pi^2}\sum_{i=1}^{2}\cos 2p_i\int\limits_{\mathbb{T}^2}\cos 2q_i f(q)\mathrm{d}q+$$
$$\frac{\mu}{2\pi^2}\cos p_1\cos p_2\int\limits_{\mathbb{T}^2}\cos q_1\cos q_2 f(q)\mathrm{d}q+$$
$$\frac{\mu}{2\pi^2}\sin p_1\sin p_2\int\limits_{\mathbb{T}^2}\sin q_1\sin q_2 f(q)\mathrm{d}q,\qquad f\in L^{2,e}(\mathbb{T}^2).$$

$\gamma,\lambda,\mu\in\mathbb{R}$ parametrlarga bog‘liq ravishda $V_{\gamma\lambda\mu}$ ning rangi o‘zgaradi, ammo yettidan oshmaydi. Shuning uchun, klassik Veyl teoremasiga ko‘ra barcha $K\in\mathbb{T}^2$ lar uchun $H_{\gamma\lambda\mu}(K)$ ning $\sigma_{\text{ess}}\left(H_{\gamma\lambda\mu}(K)\right)$ muhim spektri $H_0(K)$ ning spektri bilan ustma-ust tushadi:

$$\sigma_{\text{ess}}\left(H_{\gamma\lambda\mu}(K)\right)=\sigma\left(H_0(K)\right)=[e_{min}(K),e_{max}(K)],$$

bunda

$$e_{min}(K) := \min_{p\in\mathbb{T}^2} e_K(p) = 2\sum_{i=1}^{2}(1-\cos\frac{K_i}{2}) \geq e_{min}(0) = 0,$$
$$e_{max}(K) := \max_{p\in\mathbb{T}^2} e_K(p) = 2\sum_{i=1}^{2}(1+\cos\frac{K_i}{2}) \leq e_{max}(0) = 8.$$

Aytaylik

$$L^{2,e,s}(\mathbb{T}^2) := \{f \in L^{2,e}(\mathbb{T}^2):\ f(p_1,p_2) = f(p_2,p_1),\ p_1,p_2 \in \mathbb{T}\}$$

va

$$L^{2,e,a}(\mathbb{T}^2) := \{f \in L^{2,e}(\mathbb{T}^2):\ f(p_1,p_2) = -f(p_2,p_1),\ p_1,p_2 \in \mathbb{T}\}.$$

lar mos ravishda $L^2(\mathbb{T}^2)$ dagi juft-simmetrik va juft-antisimmetrik funksiyalar qism fazosi bo‘lsin. Eslatib o‘tamizki

$$\sigma\left(H_{\gamma\lambda\mu}(0)\right) = \sigma\left(H^s_{\gamma\lambda\mu}(0)\right) \cup \sigma\left(H^a_{\gamma\lambda\mu}(0)\right),$$

bunda $H^s_{\gamma\lambda\mu}(0)$ va $H^a_{\gamma\lambda\mu}(0)$ lar $H_{\gamma\lambda\mu}(0)$ operatorning $L^{2,e,s}(\mathbb{T}^2)$ va $L^{2,e,a}(\mathbb{T}^2)$ fazolardagi qismi.

Quyidagi teoremada $H^a_{\lambda\mu}(0)$ ning muhim spektrdan tashqarida aniq ikkita xos qiymatga ega bo‘lishi uchun yetarli shartlar bayon etilgan.

5-teorema. *Faraz qilaylik $\lambda,\mu \in \mathbb{R}$ bo‘lsin. Quyidagi munosabatlar o‘rinlidir:*

i) agar $\lambda > 10, \mu > 10$ bo‘lsa, u holda $H^a_{\lambda\mu}(0)$ energiyasi muhim spektrdan yuqorida yotuvchi ikkita bog‘langan holatlarga ega bo‘ladi;

ii) agar $\lambda < -10, \mu > 10$ yoki $\lambda > 10, \mu < -10$ bo‘lsa, u holda $H^a_{\lambda\mu}(0)$ energiyalarining biri muhim spektrdan yuqorida, ikkinchisi esa quyida yotuvchi bog‘langan holatlarga ega bo‘ladi;

iii) agar $\lambda < -10, \mu < -10$ bo‘lsa, u holda $H^a_{\lambda\mu}(0)$ energiyasi muhim spektrdan quyida yotuvchi ikkita bog‘langan holatlarga ega bo‘ladi.

6-teorema. *Faraz qilaylik $\lambda,\mu \in \mathbb{R}$ bo‘lsin. Barcha $K \in \mathbb{T}^2$ lar uchun ushbu munosabatlar o‘rinlidir:*

i) agar $\lambda > 10, \mu > 10$ bo‘lsa, u holda $\mathrm{n}_+\left(H_{\gamma\lambda\mu}(K)\right) \geq 2;$

ii) agar $\lambda < -10, \mu > 10$ yoki $\lambda > 10, \mu < -10$ bo‘lsa, u holda

$$\mathrm{n}_-\left(H_{\gamma\lambda\mu}(K)\right) \geq 1 \quad \text{va} \quad \mathrm{n}_+\left(H_{\gamma\lambda\mu}(K)\right) \geq 1;$$

iii) agar $\lambda < -10, \mu < -10$ bo‘lsa, u holda $n_-\left(H_{\gamma\lambda\mu}(K)\right) \geq 2.$

Xulosa

Dissertatsiya ishi bir va ikki o‘lchamli panjaradagi ikki zarrachali sistemaga mos Shredinger operatorlarining muhim va diskret spektrlarini tadqiq qilishga bag‘ishlangan.

Ilmiy izlanishlarning asosiy natijalari quyidagilardan iborat:

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soni va joylashgan o‘rni aniqlangan;

bir o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining aniq soniga kvaziimpulsning barcha nolmas qiymatlarida aniq baho olingan;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi bog‘langan holatlari mavjud bo‘lishi uchun yetarli shartlar topilgan;

ikki o‘lchamli panjaradagi ikki zarrachali Shredinger operatori o‘zaro ta’sir energiyasi $\gamma, \lambda, \mu \in \mathbb{R}$ parametrlar orqali berilganda kvaziimpulsning nol qiymatida uning muhim spektrdan tashqaridagi xos qiymatlarining soni uchun kvaziimpulsning barcha nolmas qiymatlarida baho olingan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.03/30.12.2019.FM.02.01 AT SAMARKAND STATE UNIVERSITY

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

AKHMADOVA MUKHAYYO OLIMJON QIZI

THE NUMBER AND LOCATION OF EIGENVALUES OF TWO-PARTICLE SCHRÖDINGER OPERATORS ON ONE AND TWO-DIMENSIONAL LATTICES

01.01.01-Mathematical analysis

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

Samarkand – 2024

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Dissertation is possible to review in Information-resource centre at Samarkand State University (is registered № 64) (Address: University Boulevard 15, Samarkand, 140104, Uzbekistan, Ph.:(+99866) 231-06-32).

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O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI • SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI • O'ZBEKISTON

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INTRODUCTION (Annotation of Doctor of Philosophy (PhD) dissertation)

Actuality and demand of the theme of dissertation. In the world, special attention are being paid to the research of spectral properties of discrete Schrödinger operators (Hamiltonians) suitable for the system of (quantum) particles in the optical lattice. One the main mechanisms in the theoretical justification of experimental results in various fields such as physics of solid bodies, physics of high-energy systems, and physics of ultracold optical systems. Currently, in developed countries, the spectral properties of operators consisting of set of Toeplitz-type convolution operator and multiplication operator into account only the zero-range and one-range interactions are important place in a wide class of physical models. In this regard, the particles systems of the energy operator on the lattice serve as the theoretical basis for experimental observations.

In the world, the spectral properties of Schrödinger operators corresponding to two-particle systems on one and two-dimensional lattices are important. In this direction, research on the study of Feshbax resonances for ultracold atoms is considered a priority in research institutes of developed countries. Nowadays, these resonances are the only means of studying ultracold atoms. The threshold effect be the basis in mathematical physics for the Efimov effect (in three-dimensional space) and the super-Efimov effect (in two-dimensional antisymmetric spaces). In particular, the development of the research such as to prove the coupling constant threshold phenomena, to find the exact number and location of eigenvalues above the essential spectrum in one and two dimensions, to obtain a convergent expansions for absorbing eigenvalues in one and two dimensions at the corresponding coupling constant thresholds, to identify the threshold resonances and threshold eigenvalues in two dimensions are very demanding.

In our country, special attention is paid to the study of the properties of the two-particle Schrödinger operator in one-dimensional and two-dimensional lattices. For the energy operators associated with systems of two particles moving on one and two-dimensional lattices, number of interesting results were obtained on the existence of bound states which is located outside of the essential spectrum and for their number under various conditions. Conducting research at the level of international standards in the priority areas of "Algebra and its applications, differential equations and their applications, linear systems, dynamic systems and mathematical modeling of their applications, stochastic analysis, medical-biological informatics, computational mathematics"[1] is identified as one of the main tasks of the scientific researches. To ensure the implementation of the decision, it is important to develop theory of quantum field, in particular, the development of the spectral theory of two-particle discrete Schrödinger operators in one- and two-dimensional lattices.

[1] Decree of President of the Republic of Uzbekistan at the "On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan" PQ-4387 dated July 9, 2019.

The subject and object of our dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, "On the strategy of action for the further development of the Republic of Uzbekistan", PQ-4387 dated July 9, 2019 "On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan" and PQ-4708 of May 7, 2020 "On measures to improve the quality of education and research in the field of mathematics" as well as in other regulations related to this activity.

Connection of research to priority directions of development of science and technologies of the Republic. This study was accomplished in conformance with the priority fields of science and technology of the Republic of Uzbekistan, IV, "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem In recent years, studies by scientists such as Yu.Kondratiev, O.Kutovoi, S.Pirogov, S.Molchanov and E.Zhizhina shown that diffusion phenomena in certain stochastic models can be described by a discrete spectrum of continuous non-local Schrödinger operators. Also, these phenomena associated with the Hamiltonian of two arbitrary or identical particle systems in one- and two-dimensional lattices, as studied by S.Albeverio, S.Lakaev, K.Makarov, P.Faria de Veiga, A.Khalkhuzhaev and Sh.Kholmatov can also be viewed as a one-particle discrete Schrödinger operator parameterized by a quasi-momentum.

The existence of eigenvalues of discrete Schrödinger operators was studied by such scientists as S.Albeverio, K.Makarov, P.Faria de Veiga, S.Lakaev, A.Khalkhuzhaev, Z.Muminov, E.Özdemir, I.Bozorov and Sh.Kholmatov. The existence of a unique eigenvalue below the essential spectrum of the Schrödinger operators associated to the system of two quantum particles acting on lattices of dimensions one, two, and three through the attractive contact potential was proved in the works of S.Lakaev, Sh.Kholmatov, A.M.Khalkhuzhaev and Sh.Lakaev.

The problem of finding the number of eigenvalues of the discrete Schrödinger operator and determining its location depending on the parameters and the dimension of lattice was studied in the works of S.Lakaev, E.Özdemir, I.Bozorov, Sh.Hamidov, S.Abduhakimov, I.Alladustova and F.Almuratov. In particular, the works of S.Lakaev, E.Özdemir and I.Alladustova the number and location of eigenvalues of the Schrödinger operator for a system of two identical bosons at a point and near neighboring sites on a one-dimensional lattice are found, and the dynamics of these eigenvalues is shown. In the works of S.Lakaev, Sh.Kholmatov, Sh.Hamidov and S.Abduhakimov, the number of eigenvalues of the two-particle Schrödinger operator on a two-dimensional lattice was obtained for all non-zero values of the quasi-momentum outside the essential spectrum.

Connection of the topic of the dissertation with the research work of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research Samarkand State University within the framework of the complex scientific work

No. SMat-05 "Mathematical analysis and its application in modern mathematical physics".

The aim of research work is to investigate essential and discrete spectra of two particle Schrödinger operator on a lattice.

Tasks of the research:

to find the exact number and location of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum depending on the parameters $\gamma, \lambda, \mu \in \mathbb{R}$;

to find the exact estimate of the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator;

to find the sufficiently conditions for the existence of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice depending on the potential $\lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum;

to find the estimate for the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator.

The research object is energy operators associated to a system of two identical particles on the one and two-dimensional lattices

The research subject is spectral study of two-particle discrete Schrödinger operators on a lattice.

Research methods: The research uses the methods of mathematical analysis, complex analysis, functional analysis, mathematical physics, theory of the self-adjoint operators and analytical continuity of the Fredholm determinant.

The scientific novelty of the research is as follows**:**

the exact number and location of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum depending on the parameters $\gamma, \lambda, \mu \in \mathbb{R}$ was shown;

the exact estimate of the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator was obtained.

the sufficiently conditions for the existence of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice depending on the potential $\lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum was found;

the estimate for the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator was obtained.

Practical results of the research:

the properties of the eigenvalues of the two-particle Schrödinger operator corresponding to a two-boson system in one and two-dimensional lattices allowed the study of methods for constructing the eigenvalues and eigenfunctions of some boundary value problems for the nonlinear operator Laplace;

the existence of bound states of the two-particle discrete Schrödinger operator enabled us to simulating the dynamics of excited states of neutron rich well deformed lanthanide nuclei dynamics which leads to a strong coupling between nuclear motion and electronic states. This serves as a theoretical basis for experimental observations carried out in the calculation of energy levels, B(E2) values and potential energy surface for even-even 170–178Yb isotopes using the computer simulation program.

The reliability of the results of the study. The results were obtained by using the methods of mathematical analysis, complex analysis, mathematical physics, functional analysis, theory of the self-adjoint operators, analytical continuity of the Fredholm determinant, strict mathematical proofs and the application of rigorous mathematical considerations.

Scientific and practical significance of research results. The scientific value of the results of the study lies in the fact that obtained results can be used in the spectral theory of self-adjoint operators, quantum mechanics, solid state physics, quantum field theory, in particular, solutions of problems related to the spectrum of Hamiltonians of systems of two and three particles on a lattice. Our work is fully fundamental and the obtained results are applicable in different fields of science such as soled state physics and quantum mechanics.

Implementation of the research results. Based on the results obtained on the number and location of eigenvalues of the discrete Schrodinger operators corresponding to a system of two identical particles in the one and two-dimensional lattices:

the existence of eigenvalues below the threshold of the essential spectrum is proved for a nonzero value of the quasi-momentum for a wide class of two-particle Schrödinger operators corresponding to a system of two bosons on one and two-dimensional lattices, were used in the foreign grant project AP09259074 on the topic "Development of methods for solving classical and non-classical boundary value problems for elliptic equations and their fractional analogues" (International Kazakh-Turkish University named after Khoja Ahmed Yasawi, certificate dated January 04, 2024, № 0121PK00198). The properties of the eigenvalues of two-particle Schrödinger operators corresponding to a system of two bosons on one and two-dimensional lattices made it possible to study methods for constructing eigenfunctions and eigenvalues of some boundary value problems for the nonlocal Laplace operator;

the existence of bound states of the two-particle discrete Schrödinger operator enabled us to simulating the dynamics of excited states of neutron rich well deformed lanthanide nuclei dynamics which leads to a strong coupling between nuclear motion and electronic states. The results have been used in the foreign research project № FRGS19–039–0647, entitled “Dynamics of Excited

States of Neutron-Rich Well Deformed Lanthanide Nuclei” of the International Islamic University Malaysia, approved by Ministry of Higher Education of Malaysia (Proceeding of Kulliyyah of Engineering International Islamic University Malaysia, 2024, January 11, № 003–KOE–24).

Approbation of the research results. The main results of the research have been discussed in 3 international and 4 republican scientific conferences.

Publications of the research results. On the topic of the dissertation 12 scientific works have been published, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the Doctor of Philosophy thesis, in addition 2 of them was published in international journal mathematics and physics indexed in Scopus and Web of Science, and also 3 papers published in national mathematical journals.

The volume and structure of the dissertation. The dissertation consists of introduction, three main chapters, conclusion and bibliography. The volume of the thesis is 82 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction is given the actuality and relevance of the thesis topics are described, the appropriate research priority areas of science and technology of the Republic are determined. Moreover, we give a review of international research on the theme of the dissertation and the degree of scrutiny of the problem, formulate goals and objectives, identify the object and subject of study. The scientific novelty and practical results of the research, the theoretical and practical importance of the obtained results, information on the implementation of the research results about the published works and the structure of dissertation are also presented in this chapter.

In the first chapter of the thesis, titled **"Preliminary notations and two particle Hamiltonian on lattices"**, some theorems on the spectrum of bounded self-adjoint and compact operators are formulated, which play an important role in the proof of the main results. Moreover, we defined the coordinate and momentum representations of one- and two-particle discrete Schrödinger operators on a lattice, and we showed boundedness, self-adjointness of these operators.

In the second chapter, titled **"The number and location of eigenvalues for the two-particle Schrödinger operators on one-dimensional lattices"** we establish the number and location of eigenvalues of the two particle discrete Schrödinger operators lying outside the essential spectrum.

Let $\mathbb{T}$ be the one-dimensional torus and $L^{2,e}(\mathbb{T})$ be the Hilbert space of square-integrable even functions defined in $\mathbb{T}$.

The discrete Schrödinger operator $H_{\gamma\lambda\mu}(K)$, $K \in \mathbb{T}$, associated to a system of two identical bosons is defined in $L^{2,e}(\mathbb{T})$ as

$$H_{\gamma\lambda\mu}(K) := H_0(K) + V_{\gamma\lambda\mu},$$

where the unperturbed operator $H_0(K)$ is the multiplication operator by the function

$$\mathcal{E}_K(p) := 2\left(1 - \cos\frac{K}{2}\cos p\right)$$

and the perturbation $V_{\gamma\lambda\mu}$ is defined as

$$V_{\gamma\lambda\mu} f(p) = \frac{1}{\pi}\int_{\mathbb{T}} (\gamma + \lambda \cos p \cos q + \mu \cos 2p \cos 2q)\, f(q) \mathrm{d}q.$$

Since $V_{\gamma\lambda\mu}$ is a finite rank operator, by well known Weyl's Theorem for any $K \in \mathbb{T}$ the essential spectrum $\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(K)\right)$ of $H_{\gamma\lambda\mu}$ coincides with the spectrum of $H_0(K)$, i.e.,

$$\sigma_{\mathrm{ess}}\left(H_{\gamma\lambda\mu}(K)\right) = \sigma\big(H_0(K)\big) = [\mathcal{E}_{\min}(K), \mathcal{E}_{\max}(K)],$$

where

$$\mathcal{E}_{\min}(K) := \min_{p\in\mathbb{T}} \mathcal{E}_K(p) = 2\left(1-\cos\frac{K}{2}\right) \geq 0 = \mathcal{E}_{\min}(0),$$
$$\mathcal{E}_{\max}(K) := \max_{p\in\mathbb{T}} \mathcal{E}_K(p) = 2\left(1+\cos\frac{K}{2}\right) \leq 4 = \mathcal{E}_{\max}(0).$$

Let $K \in \mathbb{T}$ and $n_+\left(H_{\gamma\lambda\mu}(K)\right)$ resp. $n_-\left(H_{\gamma\lambda\mu}(K)\right)$ be the number of eigenvalues of the operator $H_{\gamma\lambda\mu}(K)$ above resp. below its essential spectrum.

Theorem 1. *Assume that $\gamma,\lambda,\mu \in \mathbb{R}$. If the operator $H_{\gamma\lambda\mu}(0)$ has n_- eigenvalues below and n_+ eigenvalues above the essential spectrum then, for any $K \in \mathbb{T}$ the fiber Hamiltonian $H_{\gamma\lambda\mu}(K)$ has at least n_- eigenvalues below the essential spectrum and at least n_+ above it. In other words, the following inequalities hold:*

$$n_-\left(H_{\gamma\lambda\mu}(K)\right) \geq n_-\left(H_{\gamma\lambda\mu}(0)\right)$$

and

$$n_+\left(H_{\gamma\lambda\mu}(K)\right) \geq n_+\left(H_{\gamma\lambda\mu}(0)\right).$$

Let us consider the cubic polynomial $C^{\pm}$ of three-variable defined by

$$C^{\pm}(\gamma,\lambda,\mu) := \mp(\gamma+\lambda+\mu+\gamma\lambda\mu) + \gamma\lambda + 2\gamma\mu + \lambda\mu.$$

Lemma 1. *The set of points $\mathbb{R}^3$ satisfying the equation $C^{\pm}(\gamma,\lambda,\mu) = 0$ coincides with the graph of function*

$$\gamma^{\pm}(\lambda,\mu) = -\frac{Q_0^{\pm}(\lambda,\mu)}{Q_1^{\pm}(\lambda,\mu)},$$

where $Q_0^{\pm}(\lambda,\mu)$ and $Q_1^{\pm}(\lambda,\mu)$ are defined as

$$Q_0^{\pm}(\lambda,\mu) = \mp(\lambda+\mu) + \lambda\mu,$$
$$Q_1^{\pm}(\lambda,\mu) = \mp(\lambda+2\mu) + 1 + \lambda\mu.$$

Lemma 2. *The set of points of $\mathbb{R}^2$ satisfying the equation $Q_1^{\pm}(\lambda,\mu) = 0$ coincides with the graph of the function*

$$\lambda^{\pm}(\mu) = -\frac{2\mu \mp 1}{1 \mp \mu}.$$

The straight lines $\mu = -1$ and $\mu = 1$ separate the graph of the functions $\lambda^-(\cdot)$ and $\lambda^+(\cdot)$ on the (λ,μ)-plane into two (different) continuous curves $\{\tau_1^-,\tau_2^-\}$ and $\{\tau_1^+,\tau_2^+\}$:

$$\tau_1^- = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = -\frac{1+2\mu}{1+\mu},\ \mu > -1\},$$
$$\tau_2^- = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = -\frac{1+2\mu}{1+\mu},\ \mu < -1\}$$

and

$$\tau_1^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = \frac{1-2\mu}{1-\mu},\ \mu < 1\},$$
$$\tau_2^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : \lambda = \frac{1-2\mu}{1-\mu},\ \mu > 1\}.$$

The curves $\{\tau_1^-, \tau_2^-\}$ and $\{\tau_1^+, \tau_2^+\}$ divide the (λ,μ)- plane into the domains D_1^-, D_2^-, D_3^- and D_1^+, D_2^+, D_3^+ of the functions $\lambda^-(\cdot)$ and $\lambda^+(\cdot)$, respectively:

$$D_1^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) > 0,\ \mu > -1\},$$
$$D_2^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) < 0\},$$
$$D_3^- = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^-(\lambda,\mu) > 0,\ \mu < -1\}$$

and

$$D_1^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) > 0,\ \mu < 1\},$$
$$D_2^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) < 0\},$$
$$D_3^+ = \{(\lambda,\mu) \in \mathbb{R}^2 : Q_1^+(\lambda,\mu) > 0,\ \mu > 1\}.$$

Recall that, the domain of the function $\gamma^\pm(\cdot,\cdot)$ is an open set

$$\mathbb{R}^2 \backslash \left(\tau_1^\pm \cup \tau_2^\pm\right) = D_1^\pm \cup D_2^\pm \cup D_3^\pm.$$

The curves $\tau_1^\pm$ and $\tau_2^\pm$ in $\mathbb{R}^2$ are define the corresponding surfaces $\Upsilon_j^\pm \subset \mathbb{R}^3$, $j = 1,2$:

$$\Upsilon_j^\pm := \left\{(\gamma,\lambda,\mu) \in \mathbb{R}^3,\ (\lambda,\mu) \in \tau_j^\pm\right\}.$$

Further, the surfaces $\Upsilon_1^\pm$ and $\Upsilon_2^\pm$ will separate the graph of the function $\gamma^\pm(\cdot,\cdot)$ into three different continuous (connected) surfaces $\Gamma_1^\pm$, $\Gamma_2^\pm$ and $\Gamma_3^\pm$ in $\mathbb{R}^3$:

$$\Gamma_j^\pm = \left\{(\gamma,\lambda,\mu) \in \mathbb{R}^3 : \gamma = -\frac{Q_0^\pm(\lambda,\mu)}{Q_1^\pm(\lambda,\mu)},\ (\lambda,\mu) \in D_j^\pm\right\}, j = 1,2,3.$$

Thus, the following lemma is true:

Lemma 3. *The graph of the function $\gamma^\pm(\cdot,\cdot)$ into three different continuous surfaces $\Gamma_1^\pm$, $\Gamma_2^\pm$ and $\Gamma_3^\pm$ in $\mathbb{R}^3$. The surfaces $\Gamma_1^-, \Gamma_2^-, \Gamma_3^-$, resp $\Gamma_1^+, \Gamma_2^+, \Gamma_3^+$ divides the three dimensional space $\mathbb{R}^3$ into four separated connected components $\mathbb{G}_0^-$, $\mathbb{G}_1^-$, $\mathbb{G}_2^-$, $\mathbb{G}_3^-$, resp $\mathbb{G}_0^+$, $\mathbb{G}_1^+$, $\mathbb{G}_2^+$, $\mathbb{G}_3^+$:*

$$\mathbb{G}_0^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^-(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in D_1^-\},$$
$$\mathbb{G}_1^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^-(\gamma,\lambda,\mu\} < 0,\ (\lambda,\mu) \in \overline{D_1^-} \cup D_2^-\},$$
$$\mathbb{G}_2^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^-(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in \overline{D_2^-} \cup D_3^-\},$$
$$\mathbb{G}_3^- := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^-(\gamma,\lambda,\mu) < 0,\ (\lambda,\mu) \in D_3^-\}$$

and

$$\mathbb{G}_0^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^+(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in D_1^+\},$$
$$\mathbb{G}_1^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^+(\gamma,\lambda,\mu\} < 0,\ (\lambda,\mu) \in \overline{D_1^+} \cup D_2^+\},$$
$$\mathbb{G}_2^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^+(\gamma,\lambda,\mu) > 0,\ (\lambda,\mu) \in \overline{D_2^+} \cup D_3^+\},$$
$$\mathbb{G}_3^+ := \{(\gamma,\lambda,\mu) \in \mathbb{R}^3\colon C^+(\gamma,\lambda,\mu) < 0,\ (\lambda,\mu) \in D_3^+\}.$$

Theorem 2. *Let $\mathcal{C}^-$ be one of the above connected components $\mathbb{G}_\alpha^-$, $\alpha = 0,1,2,3$, of the partition of the (γ,λ,μ)-space. Then for any $(\gamma,\lambda,\mu) \in \mathcal{C}^-$ the number $n_-\left(H_{\gamma\lambda\mu}(0)\right)$ of eigenvalues of $H_{\gamma\lambda\mu}(0)$ lying below the essential spectrum $\sigma_{\mathrm{ess}}(H_{\gamma\lambda\mu}(0))$ remains constant.*

Analogously, let $\mathcal{C}^+$ be one of the above connected components $\mathbb{G}_\alpha^+$, $\alpha = 0,1,2,3$, of the partition of the (γ,λ,μ)-space. Then for any $(\gamma,\lambda,\mu) \in \mathcal{C}^+$ the number $n_+\left(H_{\gamma\lambda\mu}(0)\right)$ of eigenvalues of $H_{\gamma\lambda\mu}(0)$ lying above $\sigma_{\mathrm{ess}}(H_{\gamma\lambda\mu}(0))$ remains constant.

Theorem 3. *For any $\alpha,\beta = 0,1,2,3$ the following statements are true:*

i) *if $(\gamma,\lambda,\mu) \in \mathbb{G}_\alpha^-$, then $n_-\left(H_{\gamma\lambda\mu}(0)\right) = \alpha$;*

ii) *if $(\gamma,\lambda,\mu) \in \mathbb{G}_\beta^+$, then $n_+\left(H_{\gamma\lambda\mu}(0)\right) = \beta$.*

We set

$$\mathcal{C}_{\alpha\beta} = \mathbb{G}_\alpha^- \cap \mathbb{G}_\beta^+,\ \alpha,\beta = 0,1,2,3.$$

Theorem 4. *Let $K \in \mathbb{T}$ and $(\gamma,\lambda,\mu) \in \mathbb{R}^3$. For all $\alpha,\beta = 0,1,2,3$ satisfying the condition $\alpha+\beta \le 3$ the following relations are true :*

i) *if $(\gamma,\lambda,\mu) \in \mathcal{C}_{\alpha\beta}$ and $\alpha+\beta < 3$, then*
$$n_-\left(H_{\gamma\lambda\mu}(K)\right) \ge \alpha,\ n_+\left(H_{\gamma\lambda\mu}(K)\right) \ge \beta;$$

ii) if $(\gamma,\lambda,\mu) \in \mathcal{C}_{\alpha\beta}$ and $\alpha+\beta = 3$, *then*
$$n_-\left(H_{\gamma\lambda\mu}(K)\right) = \alpha,\ n_+\left(H_{\gamma\lambda\mu}(K)\right) = \beta.$$

In the third chapter, titled **“The existence of bound states of the Hamiltonian of a lattice two-boson system”,** we study the family $H_{\gamma\lambda\mu}(K)$, $K \in \mathbb{T}^2$, of discrete Schrödinger operators, associated to the Hamiltonian of a system of two identical bosons on the two-dimensional lattice $\mathbb{Z}^2$, interacting through on one site, nearest-neighbour sites and next-nearest-neighbour sites with interaction magnitudes γ, λ and μ, respectively. We obtain conditions for the existence of two eigenvalues of the operator $H_{\gamma\lambda\mu}(K)$, one of them situated below the bottom of the essential spectrum and the other one above its top.

Let $\mathbb{T}^2$ be the two-dimensional torus and $L^{2,e}(\mathbb{T}^2)$ be the Hilbert space of square-integrable even functions defined in $\mathbb{T}^2$.

The discrete operator $H_{\gamma\lambda\mu}(K)$, $K \in \mathbb{T}^2$, associated to a system of two identical bosons is defined in $L^{2,e}(\mathbb{T}^2)$ as

$$H_{\gamma\lambda\mu}(K) := H_0(K) + V_{\gamma\lambda\mu},$$

where the unperturbed operator $H_0(K)$ is the multiplication operator by the function

$$e_K(p) := 2\sum_{i=1}^{2}(1 - \cos\frac{K_i}{2}\cos p_i)$$

and the perturbation $V_{\gamma\lambda\mu}$ is defined as

$$V_{\gamma\lambda\mu}f(p) = \frac{\gamma}{4\pi^2}\int_{\mathbb{T}^2} f(q)\mathrm{d}q + \frac{\lambda}{4\pi^2}\sum_{i=1}^{2}\cos p_i\int_{\mathbb{T}^2}\cos q_i f(q)\mathrm{d}q +$$

$$+\frac{\mu}{4\pi^2}\sum_{i=1}^{2}\cos 2p_i\int_{\mathbb{T}^2}\cos 2q_i f(q)\mathrm{d}q +$$

$$+\frac{\mu}{2\pi^2}\cos p_1\cos p_2\int_{\mathbb{T}^2}\cos q_1\cos q_2 f(q)\mathrm{d}q +$$

$$+\frac{\mu}{2\pi^2}\sin p_1\sin p_2\int_{\mathbb{T}^2}\sin q_1\sin q_2 f(q)\mathrm{d}q, \qquad f \in L^{2,e}(\mathbb{T}^2).$$

Depending on $\gamma, \lambda, \mu \in \mathbb{R}$ the rank of $V_{\gamma\lambda\mu}$ varies but never exceeds seven. Hence, by the classical Weyl theorem for any $K \in \mathbb{T}^2$ the essential spectrum $\sigma_{\text{ess}}\left(H_{\gamma\lambda\mu}(K)\right)$ of $H_{\gamma\lambda\mu}(K)$ coincides with the spectrum of $H_0(K)$:

$$\sigma_{\text{ess}}\left(H_{\gamma\lambda\mu}(K)\right) = \sigma\left(H_0(K)\right) = [e_{min}(K), e_{max}(K)],$$

where

$$e_{min}(K) := \min_{p\in\mathbb{T}^2} e_K(p) = 2\sum_{i=1}^{2}(1 - cos\frac{K_i}{2}) \geq e_{min}(0) = 0,$$

$$e_{max}(K) := \max_{p\in\mathbb{T}^2} e_K(p) = 2\sum_{i=1}^{2}(1 + cos\frac{K_i}{2}) \leq e_{max}(0) = 8.$$

Let

$$L^{2,e,s}(\mathbb{T}^2) := \{f \in L^{2,e}(\mathbb{T}^2):\ f(p_1,p_2) = f(p_2,p_1),\ p_1,p_2 \in \mathbb{T}\}$$

and

$$L^{2,e,a}(\mathbb{T}^2):=\{f\in L^{2,e}(\mathbb{T}^2):\ f(p_1,p_2)=-f(p_2,p_1),\ p_1,p_2\in\mathbb{T}\}.$$

be the (Hilbert) subspaces of (essentially) even-symmetric and even-antisymmetric functions in $L^2(\mathbb{T}^2)$, respectively.
Recall that

$$\sigma\left(H_{\gamma\lambda\mu}(0)\right)=\sigma\left(H^s_{\gamma\lambda\mu}(0)\right)\cup\sigma\left(H^a_{\gamma\lambda\mu}(0)\right),$$

where $H^s_{\gamma\lambda\mu}(0)$ and $H^a_{\gamma\lambda\mu}(0)$ are the restrictions of $H_{\gamma\lambda\mu}(0)$ onto $L^{2,e,s}(\mathbb{T}^2)$ and $L^{2,e,a}(\mathbb{T}^2)$.

The result below concerns a sufficiently conditions for the existence of two eigenvalues of the Hamiltonian $H^a_{\lambda\mu}(0)$ lying outside the essential spectrum.

Theorem 5. *Let $\lambda,\mu\in\mathbb{R}$. The following relations hold:*

i) *if $\lambda>10,\mu>10$, then $H^a_{\lambda\mu}(0)$ has two antisymmetric bound states with energy lying above the essential spectrum;*

ii) *if $\lambda<-10,\mu>10$ or $\lambda>10,\mu<-10$, then $H^a_{\lambda\mu}(0)$ has two antisymmetric bound states with energy lying both as below the esential spectrum, as well as above;*

iii) *if $\lambda<-10,\mu<-10$, then $H^a_{\lambda\mu}(0)$ has two antisymmetric bound states with energy lying below the essential spectrum.*

Theorem 6. *Let $\boldsymbol{\lambda},\boldsymbol{\mu}\in\mathbb{R}$. For all $K\in\mathbb{T}^2$ the following assertions hold:*

i) *if $\lambda>10,\mu>10$, then* $\mathrm{n}_-\left(H_{\gamma\lambda\mu}(K)\right)\geq 2$;

ii) *if $\lambda<-10,\mu>10$ or $\lambda>10,\mu<-10$, then*

$$\mathrm{n}_-\left(H_{\gamma\lambda\mu}(K)\right)\geq 1\quad\text{and}\quad \mathrm{n}_+\left(H_{\gamma\lambda\mu}(K)\right)\geq 1;$$

iii) *if $\lambda<-10,\mu<-10$, then* $\mathrm{n}_-\left(H_{\gamma\lambda\mu}(K)\right)\geq 2$.

CONCLUSION

The dissertation is devoted to study essential and discrete spectrum of the two particle Schrödinger operator on one and two dimensional lattices.

The main results of the research are as follows:

1. The exact number and location of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum depending on the parameters $\gamma, \lambda, \mu \in \mathbb{R}$ was shown;

2. The exact estimate of the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the one-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator was obtained.

3. The sufficiently conditions for the existence of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice depending on the potential $\lambda, \mu \in \mathbb{R}$, for zero value of the quasi-momentum was found;

4. The estimate for the number of eigenvalues outside the essential spectrum of the two-particle Schrödinger operator on the two-dimensional lattice with interaction energy $\gamma, \lambda, \mu \in \mathbb{R}$, for all non-zero values of the quasi-momentum of the operator was obtained.

НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.02.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ САМАРКАНДСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

АХМАДОВА МУХАЙЁ ОЛИМЖОН ҚИЗИ

ЧИСЛО И МЕСТОЛОЖЕНИЕ СОБСТВЕННЫХ ЗНАЧЕНИЙ ДВУХЧАСТИЧЕСКИХ ОПЕРАТОРОВ ШРЕДИНГЕРА НА ОДНО И ДВУМЕРНЫХ РЕШЕТКАХ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования - является исследование существенного и дискретного спектров двухчастичного оператора Шредингера на решетке.

Объектом исследования оператор энергии соответсвующих системе двух идентичных частиц на одно и двумерной решетке

Научная новизна исследовани ключается в следующем:

найдено точное количество и гоположение собственных значений лежащих вне существенного спектра, в зависимости от всех параметров, одночастичных операторов Шредингера на одномерной решетке с энергией взаимодействия $\gamma, \lambda, \mu \in R$, для нулевого значения квазиимпульса, зависящего от параметров $\gamma, \lambda, \mu \in R$;

получено точная оценка для числа собственных значений вне существенного спектра двухчастичного оператора Шрёдингера на одномерной решетке с энергией взаимодействия $\gamma, \lambda, \mu \in R$, для всех ненулевых значений квазиимпульса оператора;

найдено достаточное условия существования связанных состояний вне существенного спектра двухчастичного оператора Шредингера на двумерной решетке, зависящих от потенциала $\lambda, \mu \in R$, при нулевом значении квазиимпульса;

получено оценка для числа собственных значений вне существенного спектра двухчастичного оператора Шредингера на двумерной решетке с энергией взаимодействия $\gamma, \lambda, \mu \in R$ для всех ненулевых значений квазиимпульса.

Внедрение результатов исследования. По результатам, полученным по количеству и местоположению собственных значений дискретных операторов Шредингера, соответсвующих системе двух идентичных частиц на одно и двумерной решетке:

из методов доказательств существования собственных значений ниже порога существенного спектра при ненулевом значении квазиимпульса для широкого класса двухчастичных операторов Шредингера, соответствующих системе двух фермионов на целочисленной кубической решетке $\mathbb{Z}^d, d \geq 1$ использованы в зарубежном гранте AP09259074 на тему «Разработка методов решения классических и неклассических краевых задач для эллиптических уравнений и их дробных аналогов» (Международный казахско-турецкий университет имени Ходжа Ахмеда Ясави, справка от 04 января 2024 г., № 0121PK00198). Свойства собственных значений двухчастичного оператора Шредингера, соответствующего системе двух фермионов на решетке, позволило исследовать методы построения собственных функций и собственных значений некоторых краевых задач для нелокального оператора Лапласа;

из результатов о существования связанных состояний двух и трехчастичного дискретного оператора Шредингера позволили нам смоделировать динамику возбужденных состояний богатых нейтронами хорошо деформированных ядер лантаноидов, что приводит к сильной связи между движением ядра и электронными состояниями. Результаты использованы в научно-исследовательском проекте № FRGS19-039-0647 «Динамика возбужденных состояний нейтронно-богатых хорошо деформированных ядер лантанидов» Международного исламского университета Малайзии, одобренном Министерством высшего образования Малайзии (Справка Международного инженерного исламского университета Малайзии, 11 января 2024 г., № 003–KOE–24).

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации составляет 82 страницы.

E’LON QILINGAN ISHLAR RO’YHATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo’lim (Part I; Часть I)

1. Lakaev S.N., Akhmadova M.O., Alladustova I.U. On the number of eigenvalues of the discrete Schrödinger operator on lattices. Lobachevskii Journal of Mathematics–Russia, 2023. Vol.44 – №6.–P.1091–1099. (3.Scopus. IF=1.01).
2. Lakaev S.N., Akhmadova M.O., The number and location of eigenvalues for the two-particle Schrödinger operators on lattices. Complex analyses and operator theory-USA, 2023,–№17. – P. 963-972. (3.Scopus. IF=0.88).
3. Akhmadova M.O. The number of eigenvalues of the discrete Schrödinger operator.// Scientific journal of Samarkand University. – Samarkand, 2023. – №3. – P. 67-71. (01.00.00; №02).
4. Akhmadova M.O. The existence of bound states of the Hamiltonian of a lattice two-boson system.// Scientific journal of Samarkand University. – Samarkand, 2023. – №5. – P. 79-83. (01.00.00; №02).
5. Akhmadova M.O. On the existence of zeros of matrix-type functions, estimates for the zeros and their number. // Scientific journal of Samarkand University. – Samarkand, 2022. – №3. – P. 47-50. (01.00.00; №02).

II bo’lim (Part II; Часть II)

6. Lakaev S.N., Akhmadova M.O., Alladustova I.U. On the number of eigenvalues of the discrete Schrödinger operator on lattices.// “Mathematical analysis and its applications in modern mathematical physics” International conference. – Samarkand, 2022. –P. 195–197.
7. Lakaev S.N., Akhmadova M.O. On the number of eigenvalues of the discrete Schrödinger operator on the two-dimensional lattices “Mathematical analysis and its applications in modern mathematical physics” International conference. – Samarkand, 2022. –P. 76-78
8. Akhmadova M.O. The number of eigenvalues of the discrete Schrödinger operator.// “Pedagogical sciences and teaching methods” International scientific conference , - Copenhagen:2023. part 19 – 165p.
9. S.N.Lakaev, Sh.I.Khamidov, M.O.Akhmadova “Spectral properties of two-particle Hamiltonians with interactions up to next-neighboring sites”, “Ali Qushji – an outstanding ambassador of the scientific school of Ulughbeg” celebrating the 620th anniversary of ali qushji’s birth international conference, Samarkand-2023, 43-46.
10. S.N.Lakaev, M.O.Akhmadova “Spectral properties of discrete Schrödinger operators with interactions up to next-neighboring sites”, “Ali Qushji – an outstanding ambassador of the scientific school of Ulughbeg” celebrating the 620th anniversary of ali qushji’s birth international conference, Samarkand-2023, 46-50.

11. Abduxakimov S.X., Axmadova M.O. Panjaradagi ikki fermionli sistemaga mos diskret Schrödinger operatori xos qiymatlari.// “Новые результаты математики и их приложения” республиканская научная конференция.-Samarkand, 2018-P. 62-64.
12. Lakaev S.N., Axmadova M.O. The existence of bound states in a system of two fermions in an optical lattice.// “Актуальные проблемы дифференциальных уравнений и их приложения”,республиканская научная конференция.-Toshkent, 2017-P. 154-155.

Avtoreferat “Samarqand davlat universiteti tahririy-nashriyot bo’limi” tahririyatida tahrirdan o’tkazildi va o’zbek, ingliz, rus (rezyume) tillardagi matnlari mosligi tekshirildi(11.07.2024 y.).

04.02.2024 yilda bosishga ruxsat etildi.
Shartli bosma tabog’i 2,5. Qog‘oz bichimi 60x84 $^{1}/_{16.}$
“Times” garniturasi. Adadi 60 nusxa. Buyurtma №8/1.

“Sardor poligraf” OK bosmaxonasida chop etildi.
Manzil: Samarqand viloyati, Samarqand tumani “Xishrav” MFY