**FARG‘ONA DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
PhD.03/30.12.2019.FM.05.04 RAQAMLI ILMIY KENGASH**

**ANDIJON DAVLAT UNIVERSITETI**

**JO‘RAYEV BAHODIRJON INOMJON O‘G‘LI**

# BOSHQARUV FUNKSIYALAR TURLI CHEGARALANISHGA EGA HOLLAR UCHUN II–STRATEGIYA VA UNING TATBIQLARI

**01.01.02 – Differensial tenglamalar va matematik fizika**

**FIZIKA–MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

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**Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences**

**Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам**

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## KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

**Dissertatsiya mavzusining dolzarbligi va zarurati.** Jahon miqyosida ilmiy-amaliy izlanishlar shiddat bilan rivojlanayotganligi sababli ko‘plab amaliy masalalar va murakkab sistemalarni boshqarishni o‘rganish, zamonaviy matematikaning muhim va dolzarb yo‘nalishlaridan biri bo‘lgan boshqaruv jarayonlarining matematik nazariyasini tadqiq etish va qo‘llash yetakchi o‘rinlardan birini egallamoqda. Bunday masalalarni o‘rganish zaruriyati matematikaning yangi sohasi, ya’ni dinamik o‘yinlar nazariyasida yangi usullarni ishlab chiqishni va qarama-qarshi maqsadli iqtisodiy va texnik jarayonlarni matematik modellarini amaliyotga joriy etishni taqozo etadi. Shu jihatdan, bu nazariya ikki katta qismga, ya’ni diskret va differensial o‘yinlar nazariyalariga bo‘lingan holda qarama-qarshi boshqaruvli jarayonlarni tadqiq qilish orqali boshqaruvlari aralash chegaralanishga ega bo‘lgan quvish-qochish masalalarini harbiy, axborot texnologiyalari, tibbiyot, sug‘urta moliyasi kabi sohalarda foydalanish muhim ahamiyatga ega hisoblanadi.

Jahonda differensial o‘yinlar nazariyasi va uning turli sohalarga qo‘llanishini kengaytirish maqsadida obyektlarning boshqaruv parametrlari turli chegaralanishlarga ega dinamik o‘yin masalalarini o‘rganishga yo‘naltirilgan ilmiy-tadqiqot ishlari olib borilmoqda. Bu borada, differensial o‘yinlar nazariyasining o‘yinchilar boshqaruvlari turli chegaralanishli quvish-qochish muammolarini boshqa sohalarning turli masalalariga tatbiq qilish, qarama-qarshi maqsadli boshqaruv jarayonlarining matematik modellarini qurish, ular bilan bog‘liq masalalarni yechish uchun o‘yinchilarning optimal strategiyalarini qurish va ular yordamida yangi yetarli yechilish shartlarini aniqlash, R.Ayzeksning “Qutulish chizig‘i” o‘yinini hal etishga alohida e’tibor berilmoqda.

Respublikamizda fundamental tadqiqotlarning ilmiy va amaliy qo‘llanishi bilan bog‘liq bo‘lgan dolzarb sohalariga e’tibor qaratish yuzasidan keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Jumladan, “dinamik sistemalar va ularning tatbiqlari, differensial tenglamalar va ularning tatbiqlari, algebra va uning tatbiqlari, chiziqsiz sistemalarni matematik modellashtirish, hisoblash matematikasi, tibbiy-biologik informatika, stoxastik analiz” kabi ustuvor yo‘nalishlari bilan bog‘liq bo‘lgan xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematik olimlarning asosiy vazifalari va faoliyat yo‘nalishlari bo‘yicha muhim vazifalar belgilab berilgan[1]. Ushbu vazifalarni amalga oshirishda, jumladan, boshqariluvchi sistemalar va dinamik o‘yinlar nazariyalarini rivojlantirish maqsadida boshqaruvlari turli tipdagi cheklovlarga ega differensial o‘yinlarda ziddiyatli masalalarning optimal yechimlarini aniqlaydigan strategiyalarni ishlab chiqish muhim ahamiyat kasb etmoqda.

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[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi №PQ-4387 qarori.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2017-yil 20-apreldagi PQ-2909-son “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2018-yil 27-apreldagi PQ-3682-son “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”gi qarorlar hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

**Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustuvor yo‘nalishlariga bog‘liqligi.** Mazkur tadqiqot O‘zbekiston Respublikasi fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

**Muammoning o‘rganilganlik darajasi.** Differensial o‘yinlar nazariyasi 1950-yillarda amerikalik olim R.Ayzeks tomonidan tizimli o‘rganila boshlangan va 1965-yilga kelib, uning “Differensial o‘yinlar” nomli monografiyasi chop etilgan. Bu monografiyada juda ko‘plab differensial o‘yin masalalari tadqiq qilingan bo‘lib, keyingi izlanishlar uchun ham bir qator yangi masalalar taklif qilingan. Bu masalalar dunyo olimlari tomonidan shu kungacha keng o‘rganilmoqda. Bundan tashqari, L.S.Pontryagin, N.N.Krasovskiy, R.E.Bellman, A.Friedman, W.H.Fleming, L.D.Berkovitz, R.J.Elliot, A.Bryson, S.Baron, N.J.Kalton, Y.Ho, E.F.Mishenko, L.A.Petrosyan, A.A.Azamov, N.Yu.Satimov, B.N.Pshenichniy, A.I.Subbotin va boshqa matematik olimlar differensial o‘yinlar nazariyasining fundamental rivojlanishida o‘zlarining katta hissalarini qo‘shishgan.

R.Ayzeks monografiyasida keltirilgan qiziqarli masalalardan biri “Qutulish chizig‘i” differensial o‘yin masalasini birinchi bo‘lib L.A.Petrosyan tomonidan sodda harakatli differensial o‘yin uchun parallel yaqinlashish usuli yordamida yechilgan va bu usul parallel quvish strategiyasi (qisqacha, П-strategiya) deb ham nomlanadi. Keyinchalik esa A.A.Azamov tomonidan ko‘p quvlovchi va bitta qochuvchiga ega differensial o‘yin uchun “Qutulish chizig‘i” masalasining analitik yechimi keltirilgan. П-strategiya g‘oyalari va L.S.Pontryaginning birinchi to‘g‘ri metodidan foydalanib, A.A.Chikriy П-strategiya usulini chiziqli sistemalar uchun umumlashtirgan. Keyinchalik, П-strategiya ko‘plab differensial o‘yinlarda quvish usullari rivojlanishining asosi bo‘lib xizmat qildi (B.N.Pshenichniy, B.B.Rixsiyev, B.T.Samatov, N.N.Petrov, A.S.Bannikov, A.I.Blagodaskix, N.L.Grigorenko va boshqalar). Differensial o‘yinlar nazariyasida boshqaruvlari geometrik, integral, Gronuoll va turli chegaralanishlar qo‘yilgan hollarda sodda, inersion va ko‘p quvlovchili differensial o‘yin masalalari ko‘rilgan (O.Hajek, N.N.Petrov, N.Yu.Satimov, A.N.Dar’in, D.V.Kornev, A.B.Kurjanskiy, N.Yu.Lukoyanov, A.Sh.Kuchkarov, M.To‘xtasinov, B.T.Samatov, N.A.Mamadaliyev, G.I.Ibragimov, M.Sh.Mamatov).

Bugungi kunda, respublikamizda N.Yu.Satimov tomonidan asos solingan differensial o‘yinlar nazariyasi ilmiy maktabiga A.A.Azamov rahbarlik qilmoqda. Ushbu ilmiy maktab vakillari B.B.Rixsiyev, M.To‘xtasinov, B.T.Samatov, A.Sh.Kuchkarov, N.A.Mamadaliyev, G.I.Ibragimov, M.Sh.Mamatovlar tomonidan optimal boshqaruv va differensial o‘yinlar nazariyalari bo‘yicha muhim natijalarga erishilmoqda. Xususan, B.T.Samatov parallel yaqinlashish strategiyasidan foydalanib o‘yinchilarning boshqaruvlariga integral, chiziqli, Gronuoll, Langenhop va aralash chegaralanishlar qo‘yilgan hollarda sodda harakatli differensial o‘yinlarni tadqiq qildi. N.A.Mamadaliyev o‘yinchilar boshqaruvlariga asosan, integral cheklovlar qo‘yilgan hollar uchun kechikish parametriga ega chiziqli differensial o‘yinlarning quvish masalalarini o‘rgangan. Hozirgi kungacha boshqaruvlari turli chegaralanishli differensial o‘yinlar yetarli darajada o‘rganilmagan bo‘lib, bu masalalarni o‘rganish muhim ahamiyat kasb etadi. Mazkur dissertatsiya ishi boshqaruv funksiyalari geometrik, Gronuoll, integral va turli chegaralanishli, sodda va inersion harakatli differensial o‘yinlarda quvish, qochish va “Qutulish chizig‘i” masalalarini o‘rganishga bag‘ishlangan.

**Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi.** Dissertatsiya tadqiqoti Andijon davlat universitetining ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial tenglamalar va uning turdosh matematik sohalarning dolzarb muammolari” dasturi doirasida bajarilgan.

**Tadqiqotning maqsadi** o‘yinchilarning boshqaruvlariga geometrik, Gronuoll, integral va turli chegaralanishlar qo‘yilgan hollarda quvish-qochish masalalari hamda R.Ayzeksning “Qutulish chizig‘i” differensial o‘yin masalasini hal qilishdan iborat.

**Tadqiqotning vazifalari** quyidagilardan iborat:

o‘yinchilar boshqaruvlariga turli chegaralanishlar (geometrik-Gronuoll, Gronuoll-geometrik, integral-Gronuoll) qo‘yilgan sodda harakatli differensial o‘yin uchun quvish-qochish masalasi yechilishini kafolatlaydigan yetarli shartlarni aniqlash;

o‘yinchilar boshqaruvlari umumlashgan tipidagi integral chegaralanishga ega sodda harakatli differensial o‘yinda quvish masalasining yechilishini kafolatlaydigan yetarli shartlarni topish;

quvlovchining boshqaruviga chiziqsiz, qochuvchining boshqaruviga esa chiziqli integral chegaralanishlar qo‘yilgan sodda harakatli differensial o‘yinda quvish-qochish masalasi yechimini aniqlash;

quvlovchining boshqaruviga integral qochuvchining boshqaruviga esa geometrik chegaralanishlar qo‘yilgan holda ikkinchi tartibli differensial o‘yin uchun quvish-qochish masalasining yechilishini kafolatlaydigan yetarli shartlarni topish hamda R.Ayzeksning “Qutulish chizig‘i” masalasini hal qilish.

**Tadqiqotning obyekti** sodda va inersion harakatli differensial o‘yinlarda quvish, qochish va “Qutulish chizig‘i” masalalaridan iborat.

**Tadqiqotning predmeti** o‘yinchilarning boshqaruvlari geometrik, Gronuoll, integral va turli chegaralarishlarga ega hollar uchun quvish, qochish va “Qutulish chizig‘i” masalalarining yechimlarini aniqlashdan iborat.

**Tadqiqotning usullari.** Dissertatsiya ishida quvish masalalarini hal qilish uchun yechim beruvchi funksiyalar metodi yordamida parallel quvish strategiyasi qo‘llanilgan, qochish masalalarini hal qilish uchun esa teskari yo‘nalish bo‘yicha qochish strategiyasi tatbiq qilingan. Shuningdek, differensial tenglamalar, optimal boshqaruv, funksional va qavariq analiz nazariyalaridan keng foydalanilgan.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

o‘yinchilar boshqaruvlariga turli chegaralanishlar (geometrik-Gronuoll, Gronuoll-geometrik, integral-Gronuoll) qo‘yilgan sodda harakatli differensial o‘yinda quvish-qochish masalalari quvlovchi uchun parallel quvish metodi bilan, qochuvchi uchun esa yo‘nalishli boshqaruv funksiyasi yordamida yechilgan;

o‘yinchilar boshqaruvlari umumlashgan tipidagi integral chegaralanishga ega sodda harakatli differensial o‘yinda parallel quvish strategiyasi yordamida quvish masalasi yechilishini kafolatlaydigan yetarlilik shartlari topilgan;

quvlovchining boshqaruviga chiziqsiz, qochuvchining boshqaruviga esa chiziqli integral chegaralanishlar qo‘yilgan differensial o‘yinlarda quvish-qochish masalasining yechimi quvlovchi uchun parallel quvish metodi bilan, qochuvchi uchun esa yo‘nalishli boshqaruv funksiyasi yordamida isbotlangan;

quvlovchining boshqaruviga integral qochuvchining boshqaruviga esa geometrik chegaralanishlar qo‘yilgan holda ikkinchi tartibli differensial o‘yin uchun quvish-qochish masalasining yechilishini kafolatlaydigan yetarlilik shartlari topilgan va R.Ayzeksning “Qutulish chizig‘i” masalasi hal qilingan.

**Tadqiqotning amaliy natijalari** quyidagilardan iborat:

optimal boshqaruv va differensial o‘yinlar nazariyasi asoslarining amaliy tatbiqlariga bevosita aloqadorligi nuqtayi nazaridan turli cheklovlarga ega qarama-qarshi maqsadli jarayonlarni boshqarishning matematik modellari sifatida boshqaruvlari geometrik, Gronuoll, integral va turli chegaralanishli quvish-qochish masalalari hal qilingan.

**Tadqiqot natijalarining ishonchliligi** oddiy differensial tenglamalar nazariyasi, funksional va matematik analiz, qarama-qarshi boshqaruvli dinamik sistemalarda quvish-qochish masalalariga doir teoremalardan foydalanilgan. Olingan natijalar zamonaviy matematikaning qat’iy yondashuvlari bo‘yicha isbotlanganligi bilan asoslanadi.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.** Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, optimal boshqaruv va differensial o‘yinlar nazariyalariga asoslanib, sodda va inersion harakatli dinamik sistemalar bilan ifodalangan ziddiyatli boshqaruv masalalarini yechishning optimal usullarini takomillashtirish va ularni amaliyotga joriy qilishga oid muammolarni samarali hal qilish bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati, dinamik o‘yinlar nazariyasida boshqaruvlarga turli chegaralanishlar qo‘yilgan hollarda optimal strategiyalarning qurilishi va resurs boshqaruvlari uchun optimal taqsimotning berilishi hamda ularning sonli algoritmlarini ishlab chiqish orqali texnik, iqtisodiy-moliyaviy va ekologik sohalarga tatbiq etish mumkinligi bilan izohlanadi.

**Tadqiqot natijalarining joriy qilinishi.** Boshqaruv funksiyalar turli chegaralanishga ega hollar uchun Π-strategiya va uning tatbiqlari bo‘yicha olingan natijalar asosida:

boshqaruvlari chiziqsiz va chiziqli integral chegaralanishlarga ega differensial o‘yinda quvish-qochish masalalarini yechishda olingan natijalardan №OT-F4-(36+32) raqamli “Matematik fizika va optimal boshqaruv masalalarini yechishning yangi usullarini ishlab chiqish. Toq tartibli xususiy hosilali tenglamalar uchun noklassik boshlang‘ich va spektral masalalar va ularning tadbiqlari” mavzusidagi fundamental loyihada boshqaruv funksiyalari turli chegaralanishli qarama-qarshi boshqaruvli dinamik sistemalarda foydalanilgan (Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetining 2024-yil 19-yanvardagi 04/11-671-sonli ma’lumotnomasi). Natijada, optimal boshqaruv masalalarini yechishning yangi usullarini ishlab chiqish va ularni sonli amalga oshirishda kelib chiqadigan chiziqsiz boshqaruv sistemalar uchun yetarli shartlar aniqlash imkonini bergan;

boshqaruvlari turli chegaralanishli sodda va inersion harakatli differensial o‘yinlarda quvish-qochish va “Qutulish chizig‘i” masalalarini yechishda olingan natijalardan UZB-Ind-2021-87-sonli “Li simmetriyasi tahlili, giperbolik sistemalarning Lyapunov bo‘yicha turg‘unligini tahlil qilish va modellashtirish” mavzusidagi fundamental loyihada turli chegaralanishli ziddiyatli jarayonlarni boshqarishda foydalanilgan (Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetining 2024-yil 26-fevraldagi 04/11-1301-sonli ma’lumotnomasi). Natijada, giperbolik tenglamalar sistemasi uchun aralash masalaning aniq yechimlari va eksponensial turg‘un sonli yechimini topish uchun Li simmetriyasi tahlili va adekvat hisoblash modelini ishlab chiqish imkonini bergan.

**Tadqiqot natijalarining aprobatsiyasi.** Dissertatsiyaning asosiy natijalari 13 ta ilmiy-amaliy anjumanlarda, jumladan 7 tasi xalqaro va 6 tasi respublika miqyosidagi anjumanlarda muhokamadan o‘tkazilgan.

**Tadqiqot natijalarining e’lon qilinganligi.** Dissertatsiya mavzusi bo‘yicha jami 21 ta ilmiy ish chop etilgan, shulardan, 8 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda, jumladan 3 tasi xorijiy va 5 tasi respublika jurnallarida nashr etilgan.

**Dissertatsiyaning tuzilishi va hajmi.** Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 104 betni tashkil etadi.

## DISSERTATSIYANING ASOSIY MAZMUNI

**Kirish** qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlarning tahlili berilgan, muammoning o‘rganilganlik darajasi yoritilgan, tadqiqotning maqsad va vazifalari, obyekti va predmeti ko‘rsatilgan, tadqiqot natijalarining ilmiy yangiligi ochib berilgan, olingan natijalarning nazariy va amaliy ahamiyati ko‘rsatilgan, tadqiqot natijalarining tatbiqi, shuningdek nashr etilgan ilmiy ishlar va dissertatsiyaning tuzilishi haqida ma’lumotlar keltirilgan.

I-bob “$GGr$, $GrG$ **va** $IGr-$**chegaralanishlarga ega hollar uchun** $\Pi-$**strategiya**“ deb nomlanadi va o‘yinchilarning boshqaruv funksiyalari geometrik, integral va Gronuoll tipidagi chegaralanishlarni qanoatlantiradigan hol uchun sodda quvish-qochish differensial o‘yinlarga bag‘ishlangan.

$\mathbb{R}^n$ fazoda mos ravishda $P$ quvlovchi va $E$ qochuvchi deb nomlangan ikkita boshqariladigan o‘yinchilar berilgan bo‘lsin. Agar $x$ vektorni quvlovchining holati va $y$ vektorni esa qochuvchining holati deb olsak, u holda, ularning harakatlari quyidagi tenglamalar bo‘yicha hosil bo‘ladi:

$$P:\dot{x}=u,\ \ x(0)=x_0, \qquad E:\dot{y}=v,\ \ y(0)=y_0, \tag{1}$$

bu yerda $x,y,x_0,y_0,u,v\in\mathbb{R}^n$, $n\geq 1$; $x_0$ va $y_0$ o‘yinchilarning boshlang‘ich holatlari va bu vektorlar uchun $x_0\neq y_0$ munosabat bajarilishi talab qilinadi; mexanik nuqtayi nazardan, $u$ va $v$ tezlik vektorlarini ifodalaydi va ular mos ravishda $u(\cdot):\left[0;\infty\right)\to\mathbb{R}^n$ va $v(\cdot):\left[0;\infty\right)\to\mathbb{R}^n$ o‘lchovli funksiyalar sifatida tanlanadigan o‘yinchilarning boshqaruv parametrlari hisoblanadi.

Ushbu bobda, asosan, $P$ quvlovchi va $E$ qochuvchining boshqaruv funksiyalariga quyidagi 4 ta tipdagi chegaralanishlarni joriy qilamiz.

1) $G$*-chegaralanish.* Bu tipdagi chegaralanish o‘yinchining dinamik imkoniyatining cheklanganlik xususiyatini ifodalaydi (masalan: quvvatga nisbatan chegaralanish). Optimal boshqaruv va differensial o‘yinlar nazariyalarida $u(\cdot)$ va $v(\cdot)$ boshqaruvlarga umumiy holda $G$-chegaralanishlar quyidagi ko‘rinishlarda beriladi:

$$\left|u(t)\right|\leq\alpha,\quad t\geq 0, \tag{2}$$

$$\left|v(t)\right|\leq\beta,\quad t\geq 0, \tag{3}$$

bu yerda $\alpha$ musbat son va $\beta$ nomanfiy son. Aytish kerakki, $\mathbb{R}^n$ fazoda (2) va (3) da va keyingi chegaralanishlarda $u(t)$ va $v(t)$ larning normalari sifatida odatdagi Yevklid normalarini nazarda tutamiz, ya’ni $\left|u(t)\right|=\sqrt{u_1^2(t)+u_2^2(t)+...+u_n^2(t)}$ va $\left|v(t)\right|=\sqrt{v_1^2(t)+v_2^2(t)+...+v_n^2(t)}$, bu yerda $u_1(t),u_2(t),...,u_n(t)$ $-$ $u(t)$ ning koordinatalari, va $v_1(t),v_2(t)...,v_n(t)$ $-$ $v(t)$ ning koordinatalaridir. Bunda va keyinchalik, $U_G$ (mos ravishda, $V_G$) orqali (2) (mos ravishda, (3)) $G$-chegaralanish bajariladigan barcha $u(\cdot)$ (mos ravishda, $v(\cdot)$) o‘lchovli funksiyalar sinfini ifodalaymiz.

2) $Gr$*-chegaralanish.* Bu turdagi chegaralanish analitik tarzda Gronuoll integral tengsizligidan hosil qilinadi va muayyan ma’noda $G$-chegaralanishning umumlashtirilgan ko‘rinishidir. Hayotiy jarayonlarda $Gr$-chegaralanish vaqtga bog‘liq holda o‘yinchining cheklangan va ortib borayotgan xossalarini ko‘rsatadi.

Differensial o‘yinlar nazariyasida $u(\cdot)$ va $v(\cdot)$ boshqaruvlarga $Gr$-chegaralanishlar quyidagi ko‘rinishlarda beriladi:

$$\left|u(t)\right|^2 \le \rho^2 + 2k\int_0^t \left|u(s)\right|^2 ds,\ \ t \ge 0, \tag{4}$$

$$\left|v(t)\right|^2 \le \sigma^2 + 2k\int_0^t \left|v(s)\right|^2 ds,\ \ t \ge 0, \tag{5}$$

bu yerda $\rho$, $\sigma$, $k$ musbat sonlar. Keyingi o‘rinlarda, (4) (mos ravishda, (5)) $Gr$-chegaralanishga mos keluvchi barcha $u(\cdot)$ (mos ravishda, $v(\cdot)$) o‘lchovli funksiyalar oilasini $U_{Gr}$ (mos ravishda, $V_{Gr}$) bilan tasvirlaymiz.

3) $I$*-chegaralanish.* Umuman olib qaraganda, $I$-chegaralanishlar $G$-chegaralanishlarga nisbatan ko‘proq o‘rganiladigan va tatbiq qilinadigan hisoblanadi. Chunki integral tipidagi chegaralanishlar bevosita o‘yinchi resursining (masalan, yoqilg‘i, quvvat) cheklanganlik xususiyatini bildiradi. Differensial o‘yin va optimal boshqaruv masalalarida $u(\cdot)$ va $v(\cdot)$ boshqaruvlarga $I$-chegaralanishlar asosan quyidagi ko‘rinishlarda bo‘ladi:

$$\int_0^\infty \left|u(s)\right|^2 ds \le \rho_0,\ \ t \ge 0, \tag{6}$$

$$\int_0^\infty \left|v(s)\right|^2 ds \le \sigma_0,\ \ t \ge 0, \tag{7}$$

bu yerda $\rho_0$ va $\sigma_0$ musbat sonlar va ularning har biri mos ravishda $u(\cdot)$ va $v(\cdot)$ boshqaruvlar uchun resurslarning maksimal miqdorini bildiradi.

Endi $u(\cdot)$ va $v(\cdot)$ boshqaruvlarga (6)-(7) $I$-chegaralanishlarning yangicha tiplari sifatida quyidagi eksponensial ko‘rinishlardagi integral chegaralanishlarni kiritaylik:

$$\int_0^t \left|u(s)\right|^2 ds \le \rho_0 e^{kt},\ t \ge 0, \tag{8}$$

$$\int_0^t \left|v(s)\right|^2 ds \le \sigma_0 e^{kt},\ \ t \ge 0, \tag{9}$$

bu yerda $k$ aynan (4)-(5) da berilgan son. (6)-(7) dan (8)-(9) ning farqli bo‘lishi uchun (8) ni (yoki (9) ni) $I_k^+$*-chegaralanish* deyiladi. Fizik nuqtayi nazardan, (8) (yoki (9)) $I_k^+$-chegaralanish har bir $t \ge 0$ vaqtdagi o‘yinchining boshlang‘ich resursi eksponensial tarzda ortib borishini bildiradi. Keyingi o‘rinlarda, $U_I^+$ (mos ravishda, $V_I^+$) (8) (mos ravishda, (9)) $I_k^+$-chegaralanishni qanoatlantiruvchi barcha $u(\cdot)$ (mos ravishda, $v(\cdot)$) o‘lchovli funksiyalar oilasini tasvirlaydi.

Aniqlik va soddalik uchun (1), (2), (5) o‘yinni “$GGr - o$‘*yin*” deb, (1), (4), (3) o‘yinni “$GrG - o$‘*yin*” deb, (1), (8), (5) o‘yinni esa “$IGr - o$‘*yin*” deb ataymiz.

**1-ta'rif.** $u(\cdot)\in U_G$ va $v(\cdot)\in V_{Gr}$ ($u(\cdot)\in U_G$ va $v(\cdot)\in U_{Gr}$ yoki $u(\cdot)\in U_I^+$ va $v(\cdot)\in V_{Gr}$) o'lchovli funksiyalarga $GGr-$o'yinda ($GrG-$o'yinda yoki $IGr-$o'yinda) mos ravishda $P$ quvlovchi va $E$ qochuvchining *joiz boshqaruvlari* deyiladi.

Faraz qilaylik, $U$ (mos ravishda, $V$) quvlovchining (mos ravishda, qochuvchining) yuqorida keltirilgan $U_G$, $U_{Gr}$, $U_I^+$ (mos ravishda, $V_G$, $V_{Gr}$, $V_I^+$) boshqaruvlar sinflaridan biri bo'lsin.

**2-ta'rif.** $(x_0, u(\cdot))$ (yoki $(y_0, v(\cdot))$) juftlik uchun, bu yerda $u(\cdot)\in U$ (yoki $v(\cdot)\in V$) (1) ning $x(t) = x_0 + \int_0^t u(s)ds$ (yoki $y(t) = y_0 + \int_0^t v(s)ds$) yechimi barcha $t\ge 0$ uchun $P$ quvlovchi (yoki $E$ qochuvchining) *harakat trayektoriyasi* deyiladi.

Bu yerda $P$ quvlovchining maqsadi $E$ qochuvchini tutish, ya'ni $x(t) = y(t)$ tenglikka erishish (quvish masalasi) va $E$ qochuvchi esa $P$ quvlovchi bilan uchrashishdan saqlanishga harakat qiladi (qochish masalasi), ya'ni barcha $t\ge 0$ uchun $x(t)\ne y(t)$ tengsizlikni saqlashga harakat qiladi va aksincha holatda esa iloji boricha uchrashish vaqtini kechiktirishga harakat qiladi.

$B(c,r)$ markazi $c$ nuqtada bo'lgan $r$ radiusli sharni ifodalasin.

**3-ta'rif.** $IGr-$o'yinda $\mathbf{u}: V\times[0,+\infty)\to U$ funksiyaga $P$ *quvlovchining strategiyasi* deyiladi agar: 1) $[0,t]$ oraliqda ixtiyoriy $v(\cdot)\in V$ uchun $\mathbf{u}(v(\cdot),t)\in U$ bajarilsa. Bunda $\mathbf{u}(v(\cdot),t)$ ni $\mathbf{u}(\cdot)$ *strategiyaning amalga oshirilishi* deyiladi; 2) ixtiyoriy $v_1(\cdot)$, $v_2(\cdot)\in V$ va har bir $t\in[0,+\infty)$ uchun $v_1(s) = v_2(s)$ tenglik $[0,t]$ oraliqdagi deyarli barcha qiymatlarda bajarilsa, u holda, $u_1(s) = u_2(s)$ munosabat $[0,t]$ oraliqdagi deyarli barcha qiymatlarda o'rinli bo'lsa, bu yerda $u_i(\cdot) = \mathbf{u}(v_i(\cdot),t)$, $i = 1,2$.

Keyingi ta'rifni bayon qilish uchun dastlab quyidagi yangi ifodalarni kiritamiz: $z = x - y$, $z_0 = x_0 - y_0$.

**4-ta'rif.** $\mathbf{v}:[0,+\infty)\to V$ funksiyaga $E$ *qochuvchining strategiyasi* deyiladi agar $\mathbf{v}(t)$ funksiya $t\ge 0$ bo'yicha Lebeg o'lchovli bo'lsa.

**5-ta'rif.** $\mathbf{u}(\cdot)$ strategiya $T(\mathbf{u})$ vaqt paytida *tutishni kafolatlaydi* deymiz, agar ixtityoriy $v(\cdot)\in V$ boshqaruv uchun biror $\tau\in[0,T(\mathbf{u})]$ vaqtda $x(\tau) = y(\tau)$ tenglik o'rinli bo'lsa.

**6-ta'rif.** $\mathbf{v}(\cdot)$ boshqaruv $[0,\infty)$ vaqt oralig'ida *qochishni kafolatlaydi* deymiz, agar ixtiyoriy $u(\cdot)\in U$ boshqaruv va barcha $t\in[0,\infty)$ uchun $x(t)\ne y(t)$ munosabat bajarilsa.

**1-lemma.** (Gronuoll) Agar $\left|\omega(t)\right|^2 \le \chi^2 + 2l\int_0^t \left|\omega(s)\right|^2 ds$ bo‘lsa, u holda, $\left|\omega(t)\right| \le \chi e^{lt}$ o‘rinli, bu yerda $\omega(t)$, $t \ge 0$, o‘lchovli funksiya va $\chi$, $l$ nomanfiy sonlar.

Quvish masalasini yechish uchun quvlovchi ayni $t \ge 0$ vaqtdagi $v(t)$ ning qiymatlarini va $x(t)$, $y(t)$ holatlarni bilishi yetarli bo‘ladi.

**7-ta’rif.** $\mathbf{u}_{GGr}(v) = v - \lambda_{GGr}(v)\xi_0$ funksiyani $[0,\theta]$ vaqt oralig‘ida $GGr$-quvish o‘yinida quvlovchining $\Pi_{GGr}$-*strategiyasi* deb ataymiz, bu yerda

$$\lambda_{GGr}(v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \alpha^2 - \left|v\right|^2},\ \xi_0 = z_0 / \left|z_0\right|,\ \theta = \frac{1}{k}\ln(\alpha / \sigma) \quad \text{va} \quad \langle v, \xi_0\rangle$$

ifoda $v$ va $\xi_0$ ning skalyar ko‘paytmasini bildiradi. $\lambda_{GGr}(v)$ funksiyani *hal qiluvchi funksiya* deyiladi.

**2-lemma.** Agar $\alpha > \sigma$, $\left|z_0\right| \le \Theta$ shartlar o‘rinli bo‘lsa, u holda, $[0,\theta]$ oraliqda $e^{kt} = At + B$ tenglamaning $t = T_{GGr}$ musbat ildizi mavjud, bu yerda

$$\theta = \frac{1}{k}\ln\frac{\alpha}{\sigma},\ \Theta = \frac{1}{k}\left(\sigma - \alpha\left(1 - \ln\frac{\alpha}{\sigma}\right)\right),\ A = \frac{\alpha k}{\sigma},\ B = 1 - \frac{\left|z_0\right|k}{\sigma}.$$

**1-teorema.** Agar $\alpha > \sigma$ va $\left|z_0\right| \le \Theta$ bo‘lsa, u holda, $GGr$-quvish o‘yinida $\Pi_{GGr}$-strategiya $[0, T_{GGr}]$ vaqt oralig‘ida qochuvchini tutishni kafolatlaydi.

$GGr$-qochish o‘yinida $\mathbf{v}_{GGr}(t) = -\sigma e^{kt}\xi_0,\ t \ge 0$ boshqaruv funksiyaga *qochuvchining strategiyasi* deymiz.

**2-teorema.** $\alpha > \sigma$ va $\left|z_0\right| \le \Theta$ bo‘lsin. U holda, $GGr$-quvish o‘yinida $\mathbf{v}_{GGr}(t)$ strategiya $[0, T_{GGr})$ vaqt oralig‘ida qochishni kafolatlaydi, bu yerda $T_{GGr}$ kafolatlangan quvish vaqti.

**3-teorema.** Quyidagi shartlardan biri o‘rinli bo‘lsin: 1) $\alpha \le \sigma$; 2) $\alpha > \sigma, \left|z_0\right| > \Theta$. U holda, $GGr$-qochish o‘yinida $\mathbf{v}_{GGr}(t)$ strategiya $[0,\infty)$ vaqt oralig‘ida qochishni kafolatlaydi va $\left|z(t)\right|$ masofa funksiyasi uchun quyidagi baholash o‘rinli: $\left|z(t)\right| \ge \left|z_0\right| + \frac{\sigma}{k}(e^{kt} - 1) - \alpha t$.

**8-ta’rif.** Agar $\rho \ge \beta$ bo‘lsa, $\mathbf{u}_{GrG}(t,v) = v - \lambda_{GrG}(t,v)\xi_0$ funksiya $GrG$-quvish o‘yinida quvlovchining $\Pi_{GrG}$-*strategiyasi* deyiladi, bu yerda

$$\lambda_{GrG}(t,v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \rho^2 e^{2kt} - |\,v\,|^2},\ \xi_0 = z_0 / \left|z_0\right|.$$

**4-teorema.** Agar $\rho \geq \beta$ shart bajarilsa, u holda, $GrG$-quvish o'yinida $\Pi_{GrG}$-strategiya $[0, T_{GrG}]$ oraliqda tutishni amalga oshirishni kafolatlaydi, bu yerda $T_{GrG}$ ushbu $e^{kt} = \frac{k\beta}{\rho}t + 1 + \frac{|z_0|k}{\rho}$ tenglamaning birinchi musbat ildizi.

**5-teorema.** Agar $\rho \geq \beta$ bo'lsa, u holda, quvlovchining ixtiyoriy boshqaruvi uchun qochuvchining $\mathbf{v}_{GrG}(t) = -\beta\xi_0$ strategiyasi $[0, T_{GrG})$ vaqt oralig'ida $x(t) \neq y(t)$ tengsizlikni kafolatlaydi.

**9-ta'rif.** Agar $\rho < \beta$ bo'lsa, quvlovchining $\Pi^*_{GrG}$-*strategiyasi* deb, ushbu

$$\mathbf{u}^*_{GrG}(t,v) = \begin{cases} 0, & \text{agar} \quad 0 \leq t < t^*, \\ v - \lambda^*_{GrG}(t,v)\xi^*_0, & \text{agar} \quad t \geq t^*, \end{cases}$$

funksiyaga aytamiz, bu yerda $\lambda^*_{GrG}(t,v) = \langle v, \xi^*_0 \rangle + \sqrt{\langle v, \xi^*_0 \rangle^2 + \rho^2 e^{2kt} - |v|^2}$,

$\xi^*_0 = z^*_0 / |z^*_0|$, $z^*_0 = x_0 - y(t^*)$, $t^* = \frac{1}{k} ln \frac{\beta}{\rho}$.

**6-teorema.** $\rho < \beta$ bo'lsin. U holda, $GrG$-quvish o'yinida quvlovchi $\Pi^*_{GrG}$-strategiya yordamida $[0, T^{**}_{GrG}]$ vaqt oralig'ida yutadi, bu yerda $T^{**}_{GrG} = t^* + T^*_{GrG}$ va $T^*_{GrG}$ ushbu $e^{kt} = \frac{k\beta}{\rho}t + \frac{\beta}{\rho} + \frac{|z_0|k}{\rho}$ tenglamaning birinchi musbat ildizi.

**7-teorema.** Agar $\rho < \beta$ bo'lsa, u holda, quvlovchining ixtiyoriy boshqaruvi uchun qochuvchining $\mathbf{v}^*(t) = -\beta\xi^*_0$ strategiyasi $[t^*, T^*_{GrG})$ vaqt oralig'ida $x(t) \neq y(t)$ tengsizlikni kafolatlaydi.

**10-ta'rif.** $\mathbf{u}_{IGr}(v,t) = v - \lambda_{IGr}(v,t)\xi_0$ boshqaruv $IGr$-quvish o'yinida $\Pi_{IGr}$-*strategiya* deb ataladi, bu yerda

$$\lambda_{IGr}(v,t) = \langle v, \xi_0 \rangle + \frac{\mu(t)}{2} + \sqrt{\left(\langle v, \xi_0 \rangle + \frac{\mu(t)}{2}\right)^2 - |v|^2},\ \xi_0 = z_0 / |z_0|,\ \mu(t) = \frac{\rho_0 e^{kt}}{|z_0|}.$$

**8-teorema.** $\rho_0 \geq 4\sigma |z_0|$ bo'lsin. U holda, $\Pi_{IGr}$-strategiya $[0, T_{IGr}]$ vaqt oralig'ida quvlovchining yutishini kafolatlaydi, bu yerda

$$T_{IGr} = \begin{cases} \frac{1}{k}\ln(\frac{k|z_0|}{\vartheta_0} + 1), & \text{agar} \quad k > 0, \\ \frac{|z_0|}{\vartheta_0}, & \text{agar} \quad k = 0, \end{cases} \qquad \vartheta_0 = \frac{\rho_0}{2|z_0|} - \sigma + \sqrt{\frac{\rho_0^2}{4|z_0|^2} - \frac{\rho_0\sigma}{|z_0|}}.$$

**3-lemma.** $\min\left(|z_0|,\dfrac{\sigma}{k}\right) > \sqrt{\dfrac{\rho_0}{ek}}$ bo‘lsin. U holda, barcha $t \in [0,\infty)$ uchun $\dfrac{\sigma}{k}e^{kt} - \sqrt{\rho_0}te^{\frac{kt}{2}} + |z_0| - \dfrac{\sigma}{k} > 0$ o‘rinli.

Qochuvchi uchun $\mathbf{v}_{IGr}(t) = -\sigma e^{kt}\xi_0$, $t \geq 0$, boshqaruvni taklif qilaylik.

**9-teorema.** $\min\left(|z_0|,\dfrac{\sigma}{k}\right) > \sqrt{\dfrac{\rho_0}{ek}}$ bo‘lsin. U holda, $\mathbf{v}_{IGr}(t)$ boshqaruv barcha $t \in [0,\infty)$ uchun qochuvchining yutishini kafolatlaydi.

$P_{IGr}(x,y,\rho)$ biror $x$ holatdan boshlab harakatlanadigan va $\rho$ resursni sarflaydigan quvlovchining biror $y$ holatdan boshlab harakatlanadigan qochuvchi bilan to‘qnashadigan barcha $p$ nuqtalar to‘plami bo‘lsin, ya’ni $P_{IGr}(x,y,\rho) = \left\{p : |p-x|^2 \geq \dfrac{\rho}{\sigma}|p-y|\right\}$ va bu to‘plamning $\rho \neq \sigma$ uchun chegarasi $\partial P_{IGr}(x,y,\rho) = \left\{p : |p-x|^2 = \dfrac{\rho}{\sigma}|p-y|\right\}$ ko‘rinishda bo‘ladi.

$P^*_{IGr}\left(x(t),y(t),\rho(t)\right) = \left\{p : |p-x(t)|^2 \geq \dfrac{\rho_0\Lambda_{IGr}(v(\cdot),t)}{\sigma}|p-y(t)|\right\}$ bo‘lsin.

**10-teorema.** $P^*_{IGr}\left(x(t),y(t),\rho(t)\right)$ ko‘p qiymatli akslantirish $t \in [0,\tau]$ ga nisbatan monoton kamayuvchidir, bu yerda $\tau \in \left[0,T_{IGr}\right]$, ya’ni agar $t_1,t_2 \in [0,\tau]$ va $t_1 < t_2$ bo‘lsa, u holda, $P^*_{IGr}\left(x(t_2),y(t_2),\rho(t_2)\right) \subset P^*_{IGr}\left(x(t_1),y(t_1),\rho(t_1)\right)$ o‘rinli.

**11-ta’rif.** $\rho_0 \geq 4\sigma|z_0|$ uchun $P_{IGr}(x_0,y_0,\rho_0)$ to‘plam $IGr$-quvish o‘yinida *obyektlarning yetishish sohasi* deyiladi.

**11-teorema.** Agar $\rho_0 \geq 4\sigma|z_0|$ bo‘lsa, u holda, $IGr$-quvish o‘yinida qochuvchi $P_{IGr}(x_0,y_0,\rho_0)$ yetishish sohasidan chiqib keta olmaydi.

II-bob “**Chiziqsiz $I-$chegaralanishlarga ega hol uchun $\Pi-$strategiya**” deb nomlanib, o‘yinchilarning sodda harakatlari bilan tasvirlangan quvish-qochish differensial o‘yinlarni yechishga qaratilgan va bunda asosiy jihatlardan biri muayyan ma’noda o‘yinchilar tezliklarining energiya sarflari uchun cheklovlarni ifodalaydigan ikkita tipdagi nostatsionar integral chegaralanishlardir.

$\mathbb{R}^n$ fazoda mos ravishda holatlari $x$ va $y$ bilan ifodalanadigan va harakatlari (1) dinamika orqali sodir bo‘ladigan ikkita boshqariladigan $P$ va $E$ o‘yinchilarga ega differensial o‘yinni ko‘rib chiqamiz.

1) Keyingi o‘rinlarda, quyidagi

$$\int_0^t |u(s)|^2\, ds \leq t\rho_1^2,\ \ \rho_1 > 0,\ \ t \geq 0, \tag{10}$$

$$\int_0^t |v(s)|^2\, ds \le t\sigma_1^2,\ \ \sigma_1 > 0,\ \ t \ge 0, \tag{11}$$

shartlarni qanoaltantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sinflari mos ravishda $U_G^{gen}$ va $V_G^{gen}$ bilan tasvirlanadi, bu yerda $U_G^{gen}, V_G^{gen} \subset L_\infty[0,+\infty)$, va $gen$ esa inglizcha “general” so‘zining qisqartmasidir hamda (10) (yoki (11)) ko‘rinishdagi integral tengsizlik *geometrik chegaralanish* (qisqacha, $G^{gen}$-*chegaralanish*) deb ataladi.

2) (10)–(11) larning umumiyroq ko‘rinishi sifatida, quyidagi

$$\int_0^t |u(s)|^2\, ds \le \rho_1^2 t + \rho_0^2,\ \ \rho_0 \ge 0,\ \ \rho_1 \ge 0,\ \ t \ge 0, \tag{12}$$

$$\int_0^t |v(s)|^2\, ds \le \sigma_1^2 t + \sigma_0^2,\ \ \sigma_0 \ge 0,\ \ \sigma_1 \ge 0,\ \ t \ge 0, \tag{13}$$

shartlarni qanoaltantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sinflari mos ravishda $U_L$ va $V_L$ bilan tasvirlanadi va shuningdek, (12) (yoki (13)) ko‘rinishdagi integral tengsizlik *chiziqli chegaralanish* (qisqacha, $L$-*chegaralanish*) deb ataladi.

3) (10)–(11) larning umumlashgan ko‘rinishi sifatida, quyidagi

$$\int_0^t |u(s)|^2\, ds \le \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right),\ \ \rho_1 > 0,\ \ k > 0,\ \ t \ge 0, \tag{14}$$

$$\int_0^t |v(s)|^2\, ds \le \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right),\ \ \sigma_1 > 0,\ \ k > 0,\ \ t \ge 0, \tag{15}$$

shartlar o‘rinli bo‘ladigan barcha $u(\cdot)$ va $v(\cdot)$ o‘lchovli funksiyalar sinflarini keltiramiz va bu boshqaruv sinflarini mos ravishda $U_{\exp}$ va $V_{\exp}$ bilan tasvirlaymiz hamda (14) (yoki (15)) ko‘rinishidagi integral tengsizlikni *noldan eksponensial chegaralanish* (qisqacha, $I_{\exp}$-*chegaralanish*) deb ataymiz.

4) Endi esa yuqorida keltirilgan barcha (10)–(15) chegaralanishlarning umulashmasi sifatida quyidagi

$$\int_0^t |u(s)|^2\, ds \le \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right) + \rho_0^2,\ \ \rho_0 > 0,\ \ \rho_1 > 0,\ \ k > 0,\ \ t \ge 0, \tag{16}$$

$$\int_0^t |v(s)|^2\, ds \le \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right) + \sigma_0^2,\ \ \sigma_0 > 0,\ \ \sigma_1 > 0,\ \ k > 0,\ \ t \ge 0 \tag{17}$$

integral tipidagi tengsizliklarni qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ boshqaruv funksiyalar sinflarini keltiramiz va bu boshqaruv sinflarini mos ravishda $U_{\exp}^{gen}$ va $V_{\exp}^{gen}$ orqali tasvirlaymiz va (16) (yoki (17)) ko‘rinishidagi integral tengsizlikni *boshlang‘ich resursga ega eksponensial chegaralanish* (qisqacha, $I_{\exp}^{gen}$-*chegaralanish*) deb ataymiz.

(12), (13), (16), (17) integral tipidagi chegaralanishlardagi $\rho_0^2$ va $\sigma_0^2$ parametrlar mos ravishda o‘yinchilarning boshlang‘ich resurslarini bildiradi.

5) (10) va (14) chegaralanishlarning umumiyligi nuqtayi nazaridan, quvlovchining boshqaruvi uchun quyidagi umumlashgan tipdagi integral chegaralanishni taklif qilamiz:

$$\int_0^t |u(s)|^2 ds \le \alpha \int_0^t \varphi(s)ds, \; t \ge 0, \tag{18}$$

bu yerda $\alpha$ musbat son va $\varphi(t)$ quyidagi xossalarga ega skalyar funksiya: $(a)$ $\varphi(\cdot)$ ushbu $[0, \infty)$ oraliqda uzluksiz va qat'iy kamayuvchi funksiya; $(b)$ $t > 0$ uchun $\varphi(0) = 1$, $\varphi(t) > 0$ va $t \to +\infty$ da $\varphi(t) \to 0$; $(c)$ $\int_0^t \varphi(s)ds \le t.$

(18) ko'rinishdagi integral tengsizlik *chiziqsiz o'sishga ega umumiy integral chegaralanish* (qisqacha, $I_{gen}$*-chegaralanish*) deyiladi. Keyingi o'rinlarda, $U_{gen}$ orqali $(a),(b),(c)$ shartlarga ega (18) chegaralanish bajariladigan barcha $u(\cdot)$ o'lchovli funksiyalar sinfini tasvirlaymiz.

**12-ta'rif.** $\mathbf{u}_{\exp}(t,v) = v - \lambda_{\exp}(t,v)\xi_0$ funksiyani $I_{\exp}^{gen}$ – quvish o'yinida $\Pi_{\exp}$-*strategiya* deb ataymiz, bu yerda

$\lambda_{\exp}(t,v) = \mu_0 + \langle v,\xi_0\rangle + \sqrt{\left(\mu_0 + \langle v,\xi_0\rangle\right)^2 + \delta_1 e^{-2kt}}$, $\mu_0 = \delta_0 / 2|z_0|$, $\delta_0 = \rho_0^2 - \sigma_0^2$, $\delta_1 = \rho_1^2 - \sigma_1^2$, $\xi_0 = z_0 / |z_0|$.

**4-lemma.** $\delta_1 > 0$ va $\delta_0 \ge 0$ bo'lsin. U holda, $\sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} = |z_0|$ tenglama $t \ge 0$ bo'yicha faqat bitta ildizga ega, bu yerda

$$\Phi(t) = t\left(\frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right) + \sigma_0^2\right), \quad \Psi(t) = \delta_0 t + \frac{\delta_1}{k^2}\left(1 - e^{-kt}\right)^2.$$

**12-teorema.** $\delta_1 > 0$ va $\delta_0 \ge 0$ bo'lsin. U holda, $\Pi_{\exp}$-strategiya $\left[0, T_{\exp}\right]$ vaqt oralig'ida tutishni amalga oshirishni kafolaydi.

**13-ta'rif.** $\mathbf{u}_{gen}(t,v) = v - \lambda_{gen}(t,v)\xi_0$ funksiya quvlovchining $\Pi_{gen}$-*strategiyasi* deb ataladi, bu yerda $\lambda_{gen}(t,v) = \langle v,\xi_0\rangle + \sqrt{\langle v,\xi_0\rangle^2 + \alpha\varphi(t) - \sigma_1}$, $\xi_0 = z_0 / |z_0|$.

**13-teorema.** Agar $\alpha > \sigma_1$ va $\Gamma_* \ge |z_0|$ bo'lsa, u holda, $\Pi_{gen}$-strategiya $[0, T_{gen}]$ vaqt oralig'ida tutishni kafolatlaydi, bu yerda

$$T_{gen} = \min\{t \in [0, T_d] : \Gamma(t) = |z_0|\}.$$

**14-ta'rif.** $I_{gen}$ – qochish o'yinida $\mathbf{v}_{gen}(t) = -\sqrt{\sigma_1}\xi_0$, $\xi_0 = z_0 / |z_0|$, funksiyaga *qochuvchining strategiyasi* deb ataladi.

**14-teorema.** $I_{gen}$ – qochish o‘yinida: 1) agar $\alpha > \sigma_1$ bo‘lsa, u holda, $\mathbf{v}_{gen}(t)$ strategiya $[0, T_o)$ vaqt oralig‘ida qochishni kafolatlaydi, bu yerda $T_o = |z_0| / \left(\sqrt{\alpha} - \sqrt{\sigma_1}\right)$; 2) agar $\alpha \le \sigma_1$ bo‘lsa, u holda $\mathbf{v}_{gen}(t)$ strategiya $[0, +\infty)$ vaqt oralig‘ida qochishni kafolatlaydi.

III-bob “$IG-$**chegaralanishlarga ega inersion o‘yinchilar uchun differensial o‘yinlar**” deb nomlanib, asosan, boshqaruvlarga integral va geometrik chegaralanishlar qo‘yilgan holda quvish-qochish differensial o‘yinlarini yechish haqida bayon qilingan.

Boshqaruv parametrlari mos ravishda $u$ va $v$, harakat dinamikasi esa

$$P: \ddot{x} = u, \ \ x(0) = x_0, \ \ \dot{x}(0) = x_1; \qquad E: \ddot{y} = v, \ \ y(0) = y_0, \ \ \dot{y}(0) = y_1$$

bilan berilgan $P$ quvlovchi va $E$ qochuvchiga ega differensial o‘yinni ko‘rib chiqamiz, bu yerda $x, y, u, v \in \mathbb{R}^n$, $n \ge 2$; $x_0$, $y_0$ o‘yinchilarning boshlang‘ich holatlari va $x_1$, $y_1$ esa mos ravishda ularning boshlang‘ich tezliklari. Bunda $x_0 \ne y_0$ va $x_1 = y_1$ deb qaraladi.

**15-ta’rif.** 1) $u(\cdot) = (u(t), t \ge 0)$ o‘lchovli funksiyani quvlovchining *joiz boshqaruvi* deb aytamiz, agar u $\int_0^t (t-s)|u(s)|^2 ds \le \rho_0, t \ge 0,$ tengsizlikni qanoatlantirsa, bu yerda $\rho_0$ berilgan musbat son. Quvlovchining barcha $u(\cdot)$ joiz boshqaruvlari to‘plamini $\mathbb{U}$ orqali belgilaymiz; 2) $v(\cdot) = (v(t), t \ge 0)$ o‘lchovli funksiyani qochuvchining *joiz boshqaruvi* deb aytamiz, agar u $|v(t)| \le \beta, t \ge 0,$ tengsizlikni qanoatlantirsa, bu yerda $\beta$ berilgan musbat son. Qochuvchining barcha $v(\cdot)$ joiz boshqaruvlari to‘plamini $\mathbb{V}$ orqali belgilaymiz.

Ushbu

$$u(v) = v - \theta(v)\xi_0 \tag{19}$$

funksiyani quvlovchining П - *strategiyasi* deb ataymiz, bu yerda

$$\theta(v) = \langle v, \xi_0 \rangle + \frac{\eta_0}{2} + \sqrt{\left(\langle v, \xi_0 \rangle + \frac{\eta_0}{2}\right)^2 - |v|^2}, \ \ \xi_0 = \frac{z_0}{|z_0|}, \ \ \eta_0 = \frac{\rho_0}{|z_0|}.$$

**15-teorema.** Agar $\rho_0 \ge 4\beta|z_0|$ bo‘lsa, u holda, quvlovchi (19) П-strategiyadan foydalanib, $[0, T]$ vaqt oralig‘ida yutadi, bu yerda $T = \sqrt{2|z_0| / \theta_1}$ va $\theta_1 = \frac{\eta_0}{2} - \beta + \sqrt{\frac{\eta_0^2}{4} - \eta_0\beta}$.

Qochuvchi quyidagi boshqaruvni qo‘llasin:

$$v^*(t) = -\beta\xi_0, \ \ \xi_0 = z_0 / |z_0|. \tag{20}$$

**16-teorema.** 1) Agar $\rho_0 \geq 4\beta|z_0|$ bo‘lsa, u holda, qochuvchi (20) boshqaruv yordamida $[0,T)$ vaqt oralig‘ida yutadi, bu yerda $T = \sqrt{2|z_0| / \theta_1}$. 2) Agar $\rho_0 < 4\beta|z_0|$ bo‘lsa, u holda, qochuvchi (20) boshqaruv yordamida $[0,\infty)$ vaqt oralig‘ida yutadi va shunigdek, $|z(t)| = |y(t) - x(t)| \geq |z_0| - \rho_0 / 4\beta$ o‘rinli bo‘ladi.

$\mathbb{R}^n$ fazoning bo‘sh bo‘lmagan va yopiq $L$ qism to‘plami berilgan bo‘lsin. Keyingi o‘rinlarda, $L$ soha *qutulish chizig‘i* deb ataladi. $L$ qutulish chizig‘i o‘yinida qochuvchi $\mathbb{R}^n \setminus L$ sohada bo‘lganda quvlovchi qochuvchi bilan ustam-ust tushishga, ya’ni chekli $t_* > 0$ vaqtda $x(t_*) = y(t_*)$ ga erishishga harakat qiladi. Qochuvchi $x(t) \neq y(t)$, $t \geq 0$ shartni saqlagan holda $L$ sohaga yetib borishni intiladi va agar bunday qilishning imkoni bo‘lmasa, u holda, qochuvchi quvlovchi bilan to‘qnashish paytini uzaytirishga harakat qiladi. Aytish kerakki, $L$ soha quvlovchining harakatini cheklamaydi hamda $x_0$ va $y_0$ boshlang‘ich holatlar uchun $x_0 \neq y_0$ va $y_0 \not\in L$ shartlar o‘rinli bo‘ladi.

**16-ta’rif.** (19) $\Pi$-strategiya $L$ qutulish chizig‘i o‘yinida $[0,T]$ vaqt oralig‘ida *yutuqli* deb aytiladi agar, qochuvchining $v(\cdot) \in \mathbb{V}$ boshqaruvi uchun shunday $t_* \in [0,T]$ vaqt topilsaki quyidagilar bajarilsa: 1) $x(t_*) = y(t_*)$ bo‘lsa; 2) har bir $t \in [0,t_*]$ da $y(t) \not\in L$ bo‘lsa.

**17-ta’rif.** Qochuvchi $L$ qutulish chizig‘i o‘yinida $v(\cdot) \in \mathbb{V}$ boshqaruv yordamida *yutadi* deymiz, agar ixtiyoriy $u(\cdot) \in \mathbb{U}$ uchun: 1) shunday $\bar{t} > 0$ vaqt topilsaki, $t \in [0,\bar{t})$ bo‘lganda $y(\bar{t}) \in L$ va $x(t) \neq y(t)$ bo‘lsa; 2) barcha $t \geq 0$ uchun $x(t) \neq y(t)$ bo‘lsa.

Endi qochuvchi ixtiyoriy $v(\cdot) \in \mathbb{V}$ boshqaruvda foydalanganda quvlovchi (19) $\Pi$-strategiyani tatbiq qilsin. U holda, har bir $\left(x(t), y(t), \rho(t)\right)$ $t \in [0,t^*]$ ($t^*$, $0 < t^* \leq T$, o‘yinchilar uchrashish vaqti) uchun quyidagi to‘plamlarni quramiz:

$$M\left(x(t), y(t), \rho(t)\right) = \{\mu : | \mu - x(t) |^2 \geq \rho(t) / \beta \, | \mu - y(t) | \},$$

$$M(x_0, y_0, \rho_0) = \{\mu : | \mu - x_0 |^2 \geq \rho_0 / \beta \, | \mu - y_0 | \}.$$

**5-lemma.** $M\left(x(t), y(t), \rho(t)\right) - tx_1$, $t \in [0,t^*]$, ko‘p qiymatli akslantirish ichma-ich joylashishga nisbatan monoton kamayuvchidir, ya’ni agar $t_1$, $t_2 \in [0,t^*]$ va $t_1 < t_2$ bo‘lsa, u holda $M\left(x(t_1), y(t_1), \rho(t_1)\right) - t_1 x_1 \supset M\left(x(t_2), y(t_2), \rho(t_2)\right) - t_2 x_1$.

$M^*(x_0, y_0, \rho_0, T) = \bigcup_{t=0}^{T} \{M(x_0, y_0, \rho_0) + tx_1\}$ to‘plamni quvish o‘yinida *qochuvchining yetishish sohasi* deb ataymiz.

**17-teorema.** Faraz qilaylik, $\rho_0 \geq 4\beta |z_0|$ va $M^*(x_0, y_0, \rho_0, T) \cap L = \varnothing$ bo‘lsin. U holda, (19) $\Pi$-strategiya $L$ qutulish chizig‘i o‘yinida $[0, T]$ vaqt oralig‘ida yutuqli bo‘ladi, bu yerda $T = \sqrt{2|z_0| / \theta_1}$ .

**18-teorema.** $\rho_0 < 4\beta |z_0|$ bo‘lsin. U holda, $L$ qutulish chizig‘i o‘yinida qochuvchi yutishini kafolatlaydigan $v(\cdot) \in \mathbb{V}$ boshqaruv mavjud.

## XULOSA

Dissertatsiyada boshqaruv funksiyalari geometrik, Gronuoll, integral va turli chegaralanishli, sodda va inersion harakatli differensial o‘yinlarda quvish, qochish va “Qutulish chizig‘i” masalalari ko‘rib chiqilgan.

Tadqiqotda quyidagi asosiy natijalarga erishildi:

1.O‘yinchilar boshqaruvlariga turli chegaralanishlar (geometrik-Gronuoll, Gronuoll-geometrik, integral-Gronuoll) qo‘yilgan hollar uchun sodda harakatli differensial o‘yinda quvish-qochish masalalari yechilishini kafolatlovchi yangi yetarli shartlar aniqlangan va o‘yinchilarning yetishish nuqtalar to‘plami qurilgan;

2.O‘yinchilar boshqaruvlari umumlashgan tipidagi integral chegaralanishga ega sodda harakatli differensial o‘yinda quvishning yakunlanishini kafolatlovchi yangi yetarli shartlar topilgan;

3.Quvlovchining boshqaruviga chiziqsiz integral chegaralanish qochuvchining boshqaruviga esa chiziqli integral chegaralanish qo‘yilgan holda sodda harakatli differensial o‘yinda quvish-qochish masalalari yechilishini kafolatlovchi yangi yetarli shartlar aniqlangan;

4.Quvlovchining boshqaruviga integral qochuvchining boshqaruviga esa geometrik chegaralanishlar qo‘yilgan holda ikkinchi tartibli differensial o‘yin uchun quvish-qochish masalalari yechilishini kafolatlovchi yangi yetarli shartlar topilgan va R.Ayzeksning “Qutulish chizig‘i” masalasi hal qilingan.

**SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES PhD.03/30.12.2019.FM.05.04 FERGANA STATE UNIVERSITY**

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**ANDIJAN STATE UNIVERSITY**

**JURAEV BAHODIRJON INOMJON UGLI**

# Π–STRATEGY FOR CASES OF VARIOUS CONSTRAINT ON CONTROL FUNCTIONS AND ITS APPLICATIONS

**01.01.02 – Differential equations and mathematical physics**

**ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES**

**Fergana – 2024**

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|---|---|
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Dissertation is possible to review in Information-resource centre at Fergana State University (is registered № 368). (Address: Murabbiylar str. 19, Fergana, Uzbekistan, 150100, Phone: (+99873) 244-44-94).

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## INTRODUCTION (abstract of the dissertation)

**Actuality and demand of the theme of dissertation.** On account of the fast progress of scientific and practical surveys on a global scale, the study of many practical issues and controlling complex systems, as well as the research and application of the mathematical theory of controlling processes, which is one of the important and relevant majors of modern mathematics, are occupying one of the leading positions. The need to study such issues requires forming novel methods in a new field of mathematics, that is, the theory of dynamic games, and implementing mathematical models of economic and technical processes with opposite targets into practice. In this respect, dividing that theory into two major parts, i.e. the theories of discrete and differential games, and it is considered that using the pursuit-evasion problems with mixed constraints on controls in the fields as military, information technologies, medicine, insurance finance by researching the conflict-controlled processes is important.

In the world, in order to expand the theory of differential games and its applications to various fields, scientific and research works, which are aimed at studying dynamic game problems with different constraints on the control parameters of objects, are being conducted. In this regard, the special attention is being paid to apply the pursuit-evasion issues of the theory of differential games, in which controls of players are subject to different constraints, to various problems of other fields, to build mathematical models of conflict-controlled processes, to find optimal strategies of players in solving problems related to them, and to determine new sufficient solvability conditions by them, and to solve the "Life-line" game of R.Isaacs.

In our republic, comprehensive measures are being performed in terms of paying attention to the current directions depending on scientific and practical applications of fundamental researches, and specific results are being achieved. In particular, the main tasks and directions of activities of mathematicians, and carrying out scientific research at the level of international standards related to priority areas such as "dynamic systems and their applications, differential equations and their applications, algebra and its applications, mathematical modeling of non-linear systems, computational mathematics, medical-biological informatics, stochastic analysis" are defined[1]. In the implementation of these tasks, in order to develop the theories of controlling systems and dynamic games, formulating optimal strategies determining solutions of the controlled problems in differential games with different types of constraints on controls is being taken an essential significance.

The subject and object of the research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 "On the strategy of action for the further development of the

[1] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 №PQ-4387 "On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan".

Republic of Uzbekistan", UP-2789 dated February 17, 2017 "On measures to further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities, UP-2909 dated April 20, 2017 "On measures to further develop the higher education system", PP-3682 from April 27, 2018 "On measures to further improve the system of practical implementation of innovative ideas, technologies and projects" and PP-4387 from July 9, 2019 "On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics", UP-4708 dated May 7, 2020 "Quality of education in the field of mathematics" PP-60 of January 28, 2022, "On the Development Strategy of New Uzbekistan for 2022-2026", and this dissertation research serves to a certain extent in the implementation of the tasks specified in other normative-law documents related to above activities.

**Connection of research to priority directions of development of science and technologies of the Republic.** This research was carried out in connection with the priority areas of science and technology of the Republic of Uzbekistan, IV. "Mathematics, Mechanics and Computer Science".

**The degree of scrutiny of the problem.** The theory of differential games was began to explore systematically by the American scientist R.Isaacs, and in 1965, his monograph entitled "Differential Games" was published. In this monograph, many differential game problems are investigated, and a number of new problems for the further research were offered. These problems have been widely studied by world scientists so far. Besides, L.S.Pontryagin, N.N.Krasovskii, R.E.Bellman, A.Friedman, W.H.Fleming, L.D.Bercovitz, R.J.Elliott, A.Bryson, S.Baron, N.J.Kalton, Y.Ho, E.F.Mishenko, L.A.Petrosyan, A.A.Azamov, N.Yu.Satimov, B.N.Pshenichnii, A.I.Subbotin, and other mathematician scientists are considered as the fundamental founders of the theory of differential games, and they made their great contributions to the development of this field.

The "Life-line" differential game problem, which is one of the intriguing problems proposed in the monograph of R.Isaacs, was first solved for a simple motion differential game by L.A.Petrosyan using the method of parallel approach, and this method is also called the parallel pursuit strategy (briefly, $\Pi$-strategy). Later on, A.A.Azamov gave an analytical solution of the "Life-line" problem for a differential game with multi-pursuer and one evader. Employing ideas of the $\Pi$-strategy and L.S.Pontryagin's first direct method A.A.Chikrii generalized the method of $\Pi$-strategy for linear systems. Subsequently, the $\Pi$-strategy served as the basis for the development of methods of pursuit in differential games with multiple pursuers (B.N.Pshenichnii, B.B.Rikhsiev, B.T.Samatov, N.N.Petrov, A.S.Bannikov, A.I.Blagodatskikh, N.L.Grigorenko, and others). In the theory of differential games, simple, inertial and multi-pursuer differential game problems were discussed in cases where geometric, integral, Grönwall and various constraints are imposed on controls (O.Hajek, N.N.Petrov, N.Yu.Satimov, A.N.Dar'in, D.V.Kornev, A.B.Kurzhanskii, N.Yu.Lukoyanov, A.Sh.Kuchkarov, M.Tukhtasinov, B.T.Samatov, N.A.Mamadaliev, G.I.Ibragimov, M.Sh.Mamatov).

In the present, in our republic A.A.Azamov is heading the scientific school on the theory of differential games founded by N.Yu.Satimov. The representatives of this scientific school, i.e. B.B.Rikhsiev, M.Tukhtasinov, B.T.Samatov, A.Sh.Kuchkarov, N.A.Mamadaliev, G.I.Ibragimov, M.Sh.Mamatov have been gaining significant results on the theories of optimal control and differential games. In particular, B.T.Samatov explored simple motion differential games with integral, linear, Grönwall, Langenhop, and mixed constraints on players' controls using the parallel approach strategy. N.A.Mamadaliev studied pursuit problem of linear differential games with a delay parameter for cases when players' controls are mainly subject to integral bounds. Until now, since differential games with various constraints on controls haven't been studied sufficiently, researching on these problems take an important significance. This dissertation work is devoted to survey on pursuit, evasion, and "Life-line" problems in simple and inertial motion differential games with geometric, Grönwall, integral, and various constraints on control functions.

**Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out.** Dissertation research was performed within the framework of the program "Actual problems of differential equations and mathematical fields related to them" according to the plan of scientific research works of Andijan State University.

**The aim of the research** is to solve pursuit-evasion problems and "Life-line" differential game problem of R.Isaacs for cases of geometric, Grönwall, integral, and various constraints on players' controls.

**Problems of the research:**

determine sufficient conditions guaranteeing that pursuit-evasion problem is solved for a simple motion differential game with various constraints (geometric-Grönwall, Grönwall-geometric, integral-Grönwall) on players' controls;

find sufficient conditions assuring the solvability of pursuit problem in a simple motion differential game with a generalized type of integral constraint on players' controls;

determine a solution of pursuit-evasion problem in a simple motion differential game with nonlinear and linear integral constraints on pursuer and evader's controls;

find new sufficient conditions ensuring the solvability of pursuit-evasion problem for a second-order differential game with integral constraint on a pursuer's control and geometric constraint on an evader's control, and solve the "Life-line" problem of R.Isaacs.

**The object of the research** consists of the pursuit, evasion, and "Life-line" problems in simple and inertial motion differential games.

**The subject of the research** is to determine solutions to pursuit, evasion, and "Life-line" problems for cases of geometric, Grönwall, integral, and various constraints on players' controls.

**Methods of the research.** In the dissertation work, the strategy of parallel pursuit by the method of resolving functions is applied for solving pursuit problems, and the strategy of evasion with inverse direction is implemented for solving evasion problems. Also, the theories of differential equations, optimal control, functional and convex analysis are widely used.

**Scientific novelty of the research** is composed of the following:

pursuit-evasion problem in a simple motion differential game with various constraints (geometric-Grönwall, Grönwall-geometric, integral-Grönwall) on players' controls has been solved for pursuer by the method of parallel pursuit and for evader by the control function with direction;

sufficient conditions assuring the solvability of pursuit problem in a simple motion differential game with a generalized type of integral constraint on players' controls have been found by the strategy of parallel pursuit;

the solution of pursuit-evasion problem in differential games with nonlinear and linear integral constraints on pursuer and evader's controls have been proved for pursuer by the method of parallel pursuit and for evader by the control function with direction;

sufficient conditions assuring the solvability of pursuit-evasion problem for a second-order differential game with integral constraint on a pursuer's control and geometric constraint on an evader's control have been found, and the "Life-line" problem of R.Isaacs has been handled.

**Practical results of the research** consists of the following:

from the standpoint of the interrelationship of fundamentals of the theories of optimal control and differential games with practical uses, as the mathematical models of controlling processes with opposite targets under various restrictions, the pursuit-evasion problems with geometric, Grönwall, integral, and various constraints on controls have been handled.

**The reliability of the results of the research.** The theory of ordinary differential equations, functional and mathematical analysis, and the theorems related to pursuit-evasion problems in the conflict-controlled dynamical systems have been utilized. The obtained results are explained by the fact that they've been substantiated on the basis of rigorous perspectives of modern mathematics.

**The scientific and practical significance of the research results.** The scientific significance of the research results consists of that, on the basis of the theories of optimal control and differential games, they are explained by improving optimal methods of solving conflict control problems given by simple and inertial motion dynamical systems, and working out effectively the issues related to implementations them into practice.

The practical importance of the research results is explained by the construction of optimal strategies in cases of various constraints on controls, and the form of optimal distribution for resource controls, and the possibility of applying to technical, economic-financial, and ecological fields through the formulation of their numerical algorithms.

**Implementation of the research results.** Relying on the results of the $\Pi-$strategy for cases of various constraints on control functions and its applications:

the results derived in solving the pursuit-evasion problems in a differential game with nonlinear and linear integral constraints on controls were employed in the conflict-controlled dynamical systems with various constraints on control functions in the fundamental project titled “Formulation of new methods for solving problems of mathematical physics and optimal control. Nonclassical initial and spectral problems for odd-order partial differential equations and their applications” of number OT-F4-(36+32) (the reference of number 04/11-671 of the National University of Uzbekistan named after Mirzo Ulugbek dated January 19, 2024). In consequence, it was possible to create new methods of solving optimal control problems, and to identify for nonlinear control systems appearing in their numerical realization;

the results derived in solving the pursuit-evasion and “Life-line” problems in simple and inertial motion differential games with various constrainst on controls were employed in controlling conflict processes with various restrictions in the fundamental project titled “Lie symmetry analysis, Lyapunov stability analysis and modeling of hyperbolic systems” of number UZB-Ind-2021-87 (the reference of number 04/11-1301 of the National University of Uzbekistan named after Mirzo Ulugbek dated February 26, 2024). In consequence, it was possible to formulate the Lee symmetry analysis and the adequate calculation model to find the exact solutions and the exponential stable numerical solution of the mixed problem for the system of hyperbolic equations.

**Approbation of the research results.** The main results of the research were discussed at 13 scientific-practical conferences, including 7 international and 6 national scientific conferences.

**Publications of the research results.** On the topic of the dissertation, 21 scientific papers were published, 8 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of theses of the Doctor of Philosophy, including 3 of them in foreign journals, 5 of them in national scientific journals.

**The structure and volume of the dissertation.** The dissertation is composed of the introduction, three chapters, conclusion and bibliography. The total volume of the dissertation is 104 pages.

## THE MAIN CONTENT OF THE DISSERTATION

**In introduction** the motivation of research topic and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

Chapter I is titled "$\Pi -$**strategy with** $GGr$, $GrG$ **and** $IGr -$**constraints**", and it is devoted to simple pursuit-evasion differential games for the cases when the control functions of players satisfy geometric, integral and Grönwall type constraints.

Let two controlled players, which are named Pursuer $P$ and Evader $E$, respectively, be given in space $\mathbb{R}^n$. If we take a vector $x$ as the state of the Pursuer and a vector $y$ as that of the Evader, then their motions take place by the equations

$$P : \dot{x} = u, \;\; x(0) = x_0, \qquad E : \dot{y} = v, \;\; y(0) = y_0, \tag{1}$$

where $x, y, x_0, y_0, u, v \in \mathbb{R}^n$, $n \geq 1$. Here, $x_0$ and $y_0$ are meant as the initial states of the players, and it is demanded that these vectors meets the relation $x_0 \neq y_0$; $u$ and $v$ imply the velocity vectors from the mechanical standpoint, and they act as the control parameters of the players that are selected as measurable functions $u(\cdot) : \left[0; \infty\right) \to \mathbb{R}^n$ and $v(\cdot) : \left[0; \infty\right) \to \mathbb{R}^n$, respectively.

In the current Chapter, we are going to primarily impose constraints of the following four types on the control functions of Pursuer $P$ and Evader $E$:

1) $G$*-constraint*. This type of constraint represents the restricted character of the dynamical opportunities of the object (player) (for instance: a restriction on the propulsion). In the theories of differential games and optimal control, $G$-constraints on controls $u(\cdot)$ and $v(\cdot)$ are generally described in the following forms:

$$\left|u(t)\right| \leq \alpha, \quad t \geq 0, \tag{2}$$

$$\left|v(t)\right| \leq \beta, \quad t \geq 0, \tag{3}$$

where $\alpha$ is a positive number, and $\beta$ is a non-negative number. Note that in (2) and (3) and in further constraints, as the norms of $u(t)$ and $v(t)$ in space $\mathbb{R}^n$, we mean the usual Euclidean norms, i.e. $\left|u(t)\right| = \sqrt{u_1^2(t) + u_2^2(t) + ... + u_n^2(t)}$ and $\left|v(t)\right| = \sqrt{v_1^2(t) + v_2^2(t) + ... + v_n^2(t)}$, where $u_1(t), ..., u_n(t)$ are the coordinates of $u(t)$, and $v_1(t), ..., v_n(t)$ are the coordinates of $v(t)$. Here and future, we refer to the family of such all measurable functions $u(\cdot)$ (respectively, $v(\cdot)$) that are consistent with $G$-constraint (2) (respectively, (3)) by $U_G$ (respectively, $V_G$).

2) $Gr$*-constraint*. This kind of constraint is analitically generated from the Grönwall integral inequality, and in a certain sense, it is a generalized form of $G$-constraint. In real-life processes, $Gr$-constraint gives the limited and accumulative properties of player's dynamical opportunities dependent on time. $Gr$-constraints on controls $u(\cdot)$ and $v(\cdot)$ in the theory of differential games are introduced and implemented in the following forms:

$$\left|u(t)\right|^2 \leq \rho^2 + 2k\int_0^t \left|u(s)\right|^2 ds, \;\; t \geq 0, \tag{4}$$

$$\left|v(t)\right|^2 \le \sigma^2 + 2k\int_0^t \left|v(s)\right|^2 ds,\ t \ge 0, \tag{5}$$

where $\rho$, $\sigma$, $k$ are positive numbers. In what follows, we designate the family of such all measurable functions $u(\cdot)$ (respectively, $v(\cdot)$) that are consonant with $Gr$-constraint (4) (respectively, (5)) by $U_{Gr}$ (respectively, $V_{Gr}$).

3) $I$*-constraint.* In a general sense, $I$-constraints are more discussing and applicable than $G$-constraints. Because integral type restrictions directly present the finite nature of player's resource (for instance: fuel, juice). In the problems of differential games and optimal control, $I$-constraints on controls $u(\cdot)$ and $v(\cdot)$ are, by and large, taken in the following forms:

$$\int_0^\infty \left|u(s)\right|^2 ds \le \rho_0,\ t \ge 0, \tag{6}$$

$$\int_0^\infty \left|v(s)\right|^2 ds \le \sigma_0,\ t \ge 0, \tag{7}$$

where $\rho_0$ and $\sigma_0$ are positive numbers, and each of them denotes the maximum amount of the resources for the controls $u(\cdot)$ and $v(\cdot)$ correspondingly.

Now, let's establish the following integral constraints of exponential forms as novel types of $I$-constraints (6)-(7) on controls $u(\cdot)$ and $v(\cdot)$:

$$\int_0^t \left|u(s)\right|^2 ds \le \rho_0 e^{kt},\ t \ge 0, \tag{8}$$

$$\int_0^t \left|v(s)\right|^2 ds \le \sigma_0 e^{kt},\ t \ge 0, \tag{9}$$

where $k$ is the same number with that in (4)-(5). To be the verification of (8)-(9) from (6)-(7), we term (8) (or (9)) as $I_k^+$*-constraint.* From the physical perspective, $I_k^+$-constraint (8) (or (9)) means that the player's initial resource at each time $t$, $t \ge 0$, grows exponentially. In the sequel, $U_I^+$ (appropriately, $V_I^+$) stands for the family of such all measurable functions $u(\cdot)$ (appropriately, $v(\cdot)$) fulfilling $I_k^+$-constraint (8) (respectively, (9)).

For clarity and simplicity, let's rename the game (1), (2), (5) as "$GGr-Game$", the game (1), (4), (3) as "$GrG-Game$", the game (1), (8), (5) as "$IGr-Game$".

**Definition 1.** It is termed that measurable functions $u(\cdot) \in U_G$ and $v(\cdot) \in V_{Gr}$ ($u(\cdot) \in U_G$ and $v(\cdot) \in U_{Gr}$ or $u(\cdot) \in U_I^+$ and $v(\cdot) \in V_{Gr}$) are the *admissible controls* of Pursuer $P$ and Evader $E$, appropriately in the $GGr-$Game (in the $GrG-$Game or in the $IGr-$Game).

Assume that $U$ (accordingly, $V$) is one of the above-stated control families $U_G$, $U_{Gr}$, $U_I^+$ (accordingly, $V_G$, $V_{Gr}$, $V_I^+$) of the Pursuer (accordingly, the Evader).

**Definition 2.** A solution $x(t) = x_0 + \int_0^t u(s)ds$ (or $y(t) = y_0 + \int_0^t v(s)ds$) of (1) for a pair $(x_0, u(\cdot))$ (or $(y_0, v(\cdot))$), where $u(\cdot) \in U$ (or $v(\cdot) \in V$), is said the *motion trajectory* of Pursuer $P$ (or Evader $E$) for all $t \geq 0$.

Here, the goal of Pursuer $P$ is capture Evader $E$, i.e. the achievement of an equality $x(t) = y(t)$ (Pursuit problem), and while Evader $E$ strives to avoid the encounter with Pursuer $P$ (Evasion problem), i.e. to achieve an inequality $x(t) \neq y(t)$ for all $t \geq 0$, and in the opposite case, to postpone the instant of the encounter as long as possible.

Let $B(c, r)$ denote the ball of radius $r$ centered at a point $c$.

**Definition 3.** In the $IGr$ – Game, a function $\mathbf{u}: V \times [0, +\infty) \to U$ is called the *strategy of Pursuer* $P$ if: 1) $\mathbf{u}(v(\cdot), t) \in U$ holds for any $v(\cdot) \in V$ on a finite time interval $[0, t]$. Here, $\mathbf{u}(v(\cdot), t)$ is called the *realization of the strategy* $\mathbf{u}(\cdot)$; 2) for any $v_1(\cdot)$, $v_2(\cdot) \in V$ and for each $t \in [0, +\infty)$, equality $v_1(s) = v_2(s)$ holds almost everywhere on $[0, t]$, then the relation $u_1(s) = u_2(s)$ is valid almost everywhere on $[0, t]$, where $u_i(\cdot) = \mathbf{u}(v_i(\cdot), t)$, $i = 1, 2$.

To state the next definition, we should first establish new denotations below:

$$z = x - y, \ z_0 = x_0 - y_0.$$

**Definition 4.** A function $\mathbf{v}: [0, +\infty) \to V$ is called the *strategy of Evader* $E$ if $\mathbf{v}(t)$ is Lebesgue measurable in $t$, $t \geq 0$.

**Definition 5.** We say that a strategy $\mathbf{u}(\cdot)$ *guarantees capture* at time moment $T(\mathbf{u})$ if at some time $\tau \in [0, T(\mathbf{u})]$, equality $x(\tau) = y(\tau)$ is satisfied for any control $v(\cdot) \in V$.

**Definition 6.** We say that a control $\mathbf{v}(\cdot)$ *guarantees evasion* on the time interval $[0, \infty)$ if for any control $u(\cdot) \in U$, the relation $x(t) \neq y(t)$ holds for all $t$, $t \in [0, \infty)$.

**Lemma 1.** (Grönwall) If $\left|\omega(t)\right|^2 \leq \chi^2 + 2l\int_0^t \left|\omega(s)\right|^2 ds$, then $\left|\omega(t)\right| \leq \chi e^{lt}$, where $\omega(t)$, $t \geq 0$, is a measurable function, and $\chi$, $l$ are non-negative numbers.

To solve the Pursuit problem, it suffices that the Pursuer knows the positions $x(t)$, $y(t)$, and the values of $v(t)$ at the current time $t$, $t \geq 0$.

**Definition 7.** We call the function $\mathbf{u}_{GGr}(v) = v - \lambda_{GGr}(v)\xi_0$ the $\Pi_{GGr}$*-strategy* of the Pursuer in the $GGr$-Game of Pursuit on the time interval $[0, \theta]$, where

$\lambda_{GGr}(v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \alpha^2 - |v|^2}$, $\xi_0 = z_0 / |z_0|$, $\theta = \frac{1}{k}\ln(\alpha / \sigma)$, and $\langle v, \xi_0 \rangle$ gives inner product of $v$ and $\xi_0$. The function $\lambda_{GGr}(v)$ is termed a *resolving function*.

**Lemma 2.** If the conditions $\alpha > \sigma$, $|z_0| \le \Theta$ are satisfied, then there exists a positive root $t = T_{GGr}$ of the equation $e^{kt} = At + B$ on the interval $[0, \theta]$, where

$$\theta = \frac{1}{k}\ln\frac{\alpha}{\sigma},\ \Theta = \frac{1}{k}\left(\sigma - \alpha\left(1 - \ln\frac{\alpha}{\sigma}\right)\right),\ A = \frac{\alpha k}{\sigma},\ B = 1 - \frac{|z_0| k}{\sigma}.$$

**Theorem 1.** If $\alpha > \sigma$ and $|z_0| \le \Theta$, then in the $GGr$-Game of Pursuit, the $\Pi_{GGr}$-strategy guarantees capture the Evader on the time interval $[0, T_{GGr}]$.

In the $GGr$-Game of Evasion, we call the control $\mathbf{v}_{GGr}(t) = -\sigma e^{kt}\xi_0$, $t \ge 0$, the strategy of the Evader.

**Theorem 2.** Let $\alpha > \sigma$ and $|z_0| \le \Theta$. Then in the $GGr$-Game of Pursuit, strategy $\mathbf{v}_{GGr}(t)$ guarantees evasion in the time interval $[0, T_{GGr})$, where $T_{GGr}$ is the guaranteed time of pursuit (see Theorem 1).

**Theorem 3.** Let one of the following conditions be valid: 1) $\alpha \le \sigma$; 2) $\alpha > \sigma, |z_0| > \Theta$. Then in the $GGr$-Game of Evasion, strategy $\mathbf{v}_{GGr}(t)$ guarantees evasion in the time interval $[0, \infty)$, and for the distance function $|z(t)|$ the following estimation is valid: $|z(t)| \ge |z_0| + \frac{\sigma}{k}(e^{kt} - 1) - \alpha t$.

**Definition 8.** If $\rho \ge \beta$, the function $\mathbf{u}_{GrG}(t, v) = v - \lambda_{GrG}(t, v)\xi_0$ is called the $\Pi_{GrG}$-*strategy* of the Pursuer in the $GrG$-Game of Pursuit, where

$$\lambda_{GrG}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \rho^2 e^{2kt} - |v|^2},\ \xi_0 = z_0 / |z_0|.$$

**Theorem 4.** If the condition $\rho \ge \beta$ holds, in the $GrG$-Game of Pursuit, then the $\Pi_{GrG}$-strategy guarantees realization of capture on $[0, T_{GrG}]$, where $T_{GrG}$ is the first positive solution of the equation $e^{kt} = \frac{k\beta}{\rho}t + 1 + \frac{|z_0| k}{\rho}$.

**Theorem 5.** If $\rho \ge \beta$, then for any control of Pursuer, the strategy of Evader $\mathbf{v}(t) = -\beta\xi_0$ guarantees inequality $x(t) \ne y(t)$ on time interval $[0, T_{GrG})$.

**Definition 9.** If $\rho < \beta$, we call the $\Pi^*_{GrG}$-*strategy* of Pursuer the function

$$\mathbf{u}^*_{GrG}(t, v) = \begin{cases} 0, & if \quad 0 \le t < t^*, \\ v - \lambda^*_{GrG}(t, v)\xi^*_0, & if \quad t \ge t^*, \end{cases}$$

where $\lambda^*_{GrG}(t,v) = \langle v,\xi^*_0\rangle + \sqrt{\langle v,\xi^*_0\rangle^2 + \rho^2 e^{2kt} - |v|^2}$, $\xi^*_0 = z^*_0 / |z^*_0|$, $z^*_0 = x_0 - y(t^*)$, $t^* = \frac{1}{k} ln \frac{\beta}{\rho}$.

**Theorem 6.** Let $\rho < \beta$. Then in the $GrG$-Game of Pursuit, Pursuer wins by means of $\Pi^*_{GrG}$-strategy on time interval $\left[0, T^{**}_{GrG}\right]$, where $T^{**}_{GrG} = t^* + T^*_{GrG}$, and $T^*_{GrG}$ is the first positive solution of equation $e^{kt} = \frac{k\beta}{\rho} t + \frac{\beta}{\rho} + \frac{|z_0| k}{\rho}$.

**Theorem 7.** If $\rho < \beta$, then for any control of Pursuer, the strategy of Evader $\mathbf{v}^*(t) = -\beta \xi^*_0$ guarantees inequality $x(t) \neq y(t)$ on time interval $[t^*, T^*_{GrG})$.

**Definition 10.** The control $\mathbf{u}_{IGr}(v,t) = v - \lambda_{IGr}(v,t)\xi_0$ is called the $\Pi_{IGr}$-*strategy* in the $IGr$-Game of the Pursuit, where

$$\lambda_{IGr}(v,t) = \langle v,\xi_0\rangle + \frac{\mu(t)}{2} + \sqrt{\left(\langle v,\xi_0\rangle + \frac{\mu(t)}{2}\right)^2 - |v|^2},\ \xi_0 = z_0 / |z_0|,\ \mu(t) = \frac{\rho_0 e^{kt}}{|z_0|}.$$

**Theorem 8.** Let $\rho_0 \geq 4\sigma |z_0|$. Then the $\Pi_{IGr}$-strategy guarantees pursuer's winning in the time interval $[0, T_{IGr}]$, where

$$T_{IGr} = \begin{cases} \frac{1}{k}\ln\left(\frac{k|z_0|}{\vartheta_0} + 1\right), & \text{if } k > 0, \\ \frac{|z_0|}{\vartheta_0}, & \text{if } k = 0, \end{cases} \qquad \vartheta_0 = \frac{\rho_0}{2|z_0|} - \sigma + \sqrt{\frac{\rho_0^2}{4|z_0|^2} - \frac{\rho_0\sigma}{|z_0|}}.$$

**Lemma 3.** Let $\min\left(|z_0|, \frac{\sigma}{k}\right) > \sqrt{\frac{\rho_0}{ek}}$. Then $\frac{\sigma}{k}e^{kt} - \sqrt{\rho_0 t e^{\frac{kt}{2}}} + |z_0| - \frac{\sigma}{k} > 0$ is true for all $t \in [0, \infty)$.

Let us propose $\mathbf{v}_{IGr}(t) = -\sigma e^{kt}\xi_0$, $t \geq 0$, for the Evader.

**Theorem 9.** Let $\min\left(|z_0|, \frac{\sigma}{k}\right) > \sqrt{\frac{\rho_0}{ek}}$. Then the control $\mathbf{v}_{IGr}(t)$ guarantees evader's winning for all $t \in [0, \infty)$.

Let $P_{IGr}(x,y,\rho)$ be the set of all points $p$ on which the pursuer moving from the position $x$ and consuming the resource $\rho$ collides with the Evader moving from the position $y$, i.e. $P_{IGr}(x,y,\rho) = \left\{p : |p-x|^2 \geq \frac{\rho}{\sigma}|p-y|\right\}$, which is bounded by $\partial P_{IGr}(x,y,\rho) = \left\{p : |p-x|^2 = \frac{\rho}{\sigma}|p-y|\right\}$ for $\rho \neq \sigma$.

Let $P^*_{IGr}\left(x(t),y(t),\rho(t)\right)=\left\{p:\left|p-x(t)\right|^2\geq\frac{\rho_0\Lambda_{IGr}(v(\cdot),t)}{\sigma}\left|p-y(t)\right|\right\}.$

**Theorem 10.** Multi-valued mapping $P^*_{IGr}\left(x(t),y(t),\rho(t)\right)$ is monotonically decreasing in respect to $t, t\in[0,\tau]$, where $\tau\in\left[0,T_{IGr}\right]$, i.e. if $t_1,t_2\in[0,\tau]$ and $t_1<t_2$, then $P^*_{IGr}\left(x(t_2),y(t_2),\rho(t_2)\right)\subset P^*_{IGr}\left(x(t_1),y(t_1),\rho(t_1)\right)$.

**Definition 11.** For $\rho_0\geq 4\sigma\left|z_0\right|$, set $P_{IGr}(x_0,y_0,\rho_0)$ is called the *attainability domain of the objects* in the $IGr$-Game of the Pursuit.

**Theorem 11.** If $\rho_0\geq 4\sigma\left|z_0\right|$, then the Evader can not leave from the attainability domain $P_{IGr}(x_0,y_0,\rho_0)$ in the $IGr$-Game of the Pursuit.

Chapter II is titled "$\Pi-$**strategy with nonlinear** $I-$**constraints**", and it is focused on solving pursuit-evasion differential games described by simple motions of players, and here, one of the main qualities is that two types of non-stationary integral constraints, in a certain sense, imply limitations for energy expenditures of the players' speed.

Consider a differential game with two controlled players $P$ and $E$, whose positions are expressed by $x$ and $y$, and whose motions are carried out by the dynamics (1), respectively in $\mathbb{R}^n$.

1) From now on, the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ such that satisfy the following conditions

$$\int_0^t\left|u(s)\right|^2 ds\leq t\rho_1^2,\ \ \rho_1>0,\ \ t\geq 0, \tag{10}$$

$$\int_0^t\left|v(s)\right|^2 ds\leq t\sigma_1^2,\ \ \sigma_1>0,\ \ t\geq 0, \tag{11}$$

are designated by $U_G^{gen}$ and $V_G^{gen}$, resp., where $U_G^{gen},V_G^{gen}\subset L_\infty[0,+\infty)$, and $gen$ is the abbreviation of the word "general", and the integral inequality of form (10) (or (11)) is termed the *geometric constraint* (briefly, the $G^{gen}$*-constraint*).

2) As the more general form of (10)–(11), the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ satisfying the following conditions

$$\int_0^t\left|u(s)\right|^2 ds\leq \rho_1^2 t+\rho_0^2,\ \ \rho_0\geq 0,\ \ \rho_1\geq 0,\ \ t\geq 0, \tag{12}$$

$$\int_0^t\left|v(s)\right|^2 ds\leq \sigma_1^2 t+\sigma_0^2,\ \ \sigma_0\geq 0,\ \ \sigma_1\geq 0,\ \ t\geq 0, \tag{13}$$

are designated by $U_L$ and $V_L$, resp., and in addition, the integral inequality of form (12) (or (13)) is termed the *linear constraint* (briefly, the $L$*-constraint*).

3) As the another generalized form of (10)–(11), we present the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ such that the following conditions

$$\int_0^t\left|u(s)\right|^2 ds\leq \frac{\rho_1^2}{2k}\left(1-e^{-2kt}\right),\ \ \rho_1>0,\ \ k>0,\ \ t\geq 0, \tag{14}$$

$$\int_0^t \left|v(s)\right|^2 ds \le \frac{\sigma_1^2}{2k}\left(1-e^{-2kt}\right),\ \ \sigma_1>0,\ \ k>0,\ \ t\ge 0, \qquad (15)$$

are valid, and these control classes are designated by $U_{\exp}$ and $V_{\exp}$, resp., and the integral inequality of form (14) (or (15)) is called the *exponential constraint from scratch* (briefly, the $I_{\exp}$*-constraint*).

4) And now, as a generalization of all the above-stated constraints (10)–(15), we bring in the classes of all control functions $u(\cdot)$ and $v(\cdot)$ fulfilling the following integral type inequalities

$$\int_0^t \left|u(s)\right|^2 ds \le \frac{\rho_1^2}{2k}\left(1-e^{-2kt}\right)+\rho_0^2,\ \ \rho_0>0,\ \ \rho_1>0,\ \ k>0,\ \ t\ge 0, \quad (16)$$

$$\int_0^t \left|v(s)\right|^2 ds \le \frac{\sigma_1^2}{2k}\left(1-e^{-2kt}\right)+\sigma_0^2,\ \ \sigma_0>0,\ \ \sigma_1>0,\ \ k>0,\ \ t\ge 0, \quad (17)$$

are satisfied, and these control classes are denoted by $U_{\exp}^{gen}$ and $V_{\exp}^{gen}$, resp., and the integral inequality of form (16) (or (17)) is termed the *exponential constraint with initial resource* (for short, the $I_{\exp}^{gen}$*-constraint*).

The parameters $\rho_0^2$ and $\sigma_0^2$ in the integral type constraints (12), (13), (16), (17) suggest the initial resources of the players correspondingly.

5) From the viewpoint of generality of the constraints (10) and (15), we offer the following generalized type of integral constraint for the Pursuer's control:

$$\int_0^t \left|u(s)\right|^2 ds \le \alpha \int_0^t \varphi(s)ds,\ \ t\ge 0, \qquad (18)$$

where $\alpha$ is a positive number, and $\varphi(t)$ is a scalar function such that:
$(a)$ is continuous and strongly decreasing on the interval $[0,\infty)$;

$(b)$ $\varphi(0)=1$, $\varphi(t)>0$ for $t>0$, and $\varphi(t)\to 0$ as $t\to+\infty$; $(c)$ $\int_0^t \varphi(s)ds \le t.$

The integral inequality of form (18) is termed the *general integral constraint with nonlinear increase* (briefly, the $I_{gen}$*-constraint*). In the sequal, by $U_{gen}$, we mean the class of all measurable functions $u(\cdot)$, for which the constraint (18) with the conditions $(a),(b),(c)$ holds.

**Definition 12.** We say the function $\mathbf{u}_{\exp}(t,v)=v-\lambda_{\exp}(t,v)\xi_0$ the $\Pi_{\exp}$-*strategy* in the $I_{\exp}^{gen}$ – Game of Pursuit, where

$$\lambda_{\exp}(t,v)=\mu_0+\langle v,\xi_0\rangle+\sqrt{\left(\mu_0+\langle v,\xi_0\rangle\right)^2+\delta_1 e^{-2kt}},\ \ \mu_0=\delta_0/2\left|z_0\right|,\ \ \delta_0=\rho_0^2-\sigma_0^2,$$

$\delta_1=\rho_1^2-\sigma_1^2$, $\xi_0=z_0/\left|z_0\right|$.

**Lemma 4.** Let $\delta_1>0$ and $\delta_0\ge 0$. Then the equation

$$\sqrt{\Phi(t)+\Psi(t)}-\sqrt{\Phi(t)}=\left|z_0\right|$$

has at least one positive root in respect to $t$, $t \geq 0$, where

$$\Phi(t)=t\left(\frac{\sigma_1^2}{2k}\left(1-e^{-2kt}\right)+\sigma_0^2\right), \quad \Psi(t)=\delta_0 t+\frac{\delta_1}{k^2}\left(1-e^{-kt}\right)^2.$$

**Theorem 12.** Let $\delta_1 > 0$ and $\delta_0 \geq 0$. Then the $\Pi_{\exp}$-strategy guarantees realization of capture on the time interval $\left[0, T_{\exp}\right]$.

**Definition 13.** The function $\mathbf{u}_{gen}(t,v)=v-\lambda_{gen}(t,v)\xi_0$ is called the $\Pi_{gen}$-*strategy* of Pursuer, where

$$\lambda_{gen}(t,v)=\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\alpha\varphi(t)-\sigma_1}, \ \xi_0=z_0 / \left|z_0\right|.$$

**Theorem 13.** If $\alpha > \sigma_1$ and $\Gamma_* \geq \left|z_0\right|$, then the $\Pi_{gen}$-strategy guarantees capture on the time interval $[0, T_{gen}]$, where $T_{gen}=\min\{t\in[0,T_d]:\Gamma(t)=\left|z_0\right|\}$.

**Definition 14.** In the $I_{gen}$ – Game of Evasion, a function $\mathbf{v}_{gen}(t)=-\sqrt{\sigma_1}\xi_0$, $\xi_0=z_0 / \left|z_0\right|$, is called the *strategy* of Evader.

**Theorem 14.** In the $I_{gen}$ – Game of Evasion: 1) if $\alpha > \sigma_1$, then strategy $\mathbf{v}_{gen}(t)$ guarantees evasion in the time interval $[0, T_o)$, where $T_o=\left|z_0\right| / (\sqrt{\alpha}-\sqrt{\sigma_1})$; 2) if $\alpha \leq \sigma_1$, then strategy $\mathbf{v}_{gen}(t)$ guarantees evasion in the time interval $[0,+\infty)$.

Chapter III is titled "**Differential games for inertial players with $IG$ – constraints**", and it is especially concerned with solving pursuit-evasion differential games for the inertial movements of players for the case of integral and geometric constraints on the controls.

Discuss a differential game of Pursuer $P$ and Evader $E$, whose control parameters are $u$ and $v$, respectively, and whose the dynamics are given by

$$P:\ddot{x}=u, \ x(0)=x_0, \ \dot{x}(0)=x_1; \qquad E:\ddot{y}=v, \ y(0)=y_0, \ \dot{y}(0)=y_1,$$

where $x,y,u,v\in\mathbb{R}^n$, $n\geq 2$; $x_0$, $y_0$ are players' initial positions, and $x_1$, $y_1$ are their initial velocities, respectively. It is assumed that $x_0 \neq y_0$ and $x_1 = y_1$.

**Definition 15.** We say that: 1) a measurable function $u(\cdot)=(u(t), t\geq 0)$ is the *admissible control* of Pursuer if it satisfies $\int_0^t (t-s)\left|u(s)\right|^2 ds \leq \rho_0$, $t\geq 0$, where $\rho_0$ is a given positive number. Denote the set of all admissible controls $u(\cdot)$ of pursuer by $\mathbb{U}$; 2) a measurable function $v(\cdot)=(v(t), t\geq 0)$ is the *admissible* *control* of Evader if it satisfies $|v(t)| \le \beta,\ t \ge 0$, where $\beta$ is a given positive number. Denote the set of all admissible controls $v(\cdot)$ of evader by $\mathbb{V}$.

We call the function

$$u(v) = v - \theta(v)\xi_0 \tag{19}$$

the $\Pi$-*strategy* of Pursuer, where

$$\theta(v) = \langle v, \xi_0\rangle + \frac{\eta_0}{2} + \sqrt{\left(\langle v, \xi_0\rangle + \frac{\eta_0}{2}\right)^2 - \left|v\right|^2},\ \ \xi_0 = \frac{z_0}{\left|z_0\right|},\ \ \eta_0 = \frac{\rho_0}{\left|z_0\right|}.$$

**Theorem 15.** If $\rho_0 \ge 4\beta\left|z_0\right|$, then Pursuer wins by using $\Pi$-strategy (19) on the time interval $[0, T]$, where $T = \sqrt{2\left|z_0\right| / \theta_1}$ and $\theta_1 = \frac{\eta_0}{2} - \beta + \sqrt{\frac{\eta_0^2}{4} - \eta_0\beta}$.

Let Evader employ the following control

$$v^*(t) = -\beta\xi_0,\ \ \xi_0 = z_0 / \left|z_0\right|. \tag{20}$$

**Theorem 16.** 1) If $\rho_0 \ge 4\beta\left|z_0\right|$, then Evader wins by control (20) in the interval $[0, T)$, where $T = \sqrt{2\left|z_0\right| / \theta_1}$; 2) If $\rho_0 < 4\beta\left|z_0\right|$, then Evader wins by control (20) in the time interval $[0, \infty)$ and $\left|z(t)\right| = \left|y(t) - x(t)\right| \ge \left|z_0\right| - \rho_0 / (4\beta)$.

Let a non-empty and closed subset $L$ of the space $\mathbb{R}^n$ be given. Here and subsequently, the area $L$ is called the *Life-line*. In the Life-line game $L$, Pursuer intends to intercept Evader, to accomplish $x(t_*) = y(t_*)$ at a finite time $t_* > 0$ when Evader is in $\mathbb{R}^n \setminus L$. Evader aims to get the area $L$ maintaining the condition $x(t) \ne y(t),\ t \ge 0$; and if there is no chance of doing this, then Evader strives to maximize the moment of the encounter with Pursuer. Note that the area $L$ doesn't limit the motion of Pursuer, and the conditions $x_0 \ne y_0$ and $y_0 \not\in L$ are satisfied for the initial positions $x_0$ and $y_0$.

**Definition 16.** $\Pi$-strategy (19) is said to be *winning* on the time interval $[0, T]$ in the Life-line game $L$, if, for Evader's arbitrary control $v(\cdot) \in \mathbb{V}$, there is an instant $t_* \in [0, T]$ such that: 1) $x(t_*) = y(t_*)$; 2) $y(t) \not\in L$ at each $t \in [0, t_*]$.

**Definition 17.** We say that Evader *wins* in the Life-line game $L$ by a control $v(\cdot) \in \mathbb{V}$ if for every $u(\cdot) \in \mathbb{U}$: 1) there is some moment $\bar{t}, \bar{t} > 0$, that $y(\bar{t}) \in L$ and $x(t) \ne y(t)$ while $t \in [0, \bar{t})$; or 2) $x(t) \ne y(t)$ for all $t \ge 0$.

Next, let Pursuer hold $\Pi$-strategy (19) while Evader employs arbitrary control $v(\cdot) \in \mathbb{V}$. Then $t^*$, $0 < t^* \le T$ is players' meeting time, i.e. $x(t^*) = y(t^*)$. Then for each triad $\left(x(t), y(t), \rho(t)\right)$, $t \in [0, t^*]$, we build the following sets:

$$M\left(x(t), y(t), \rho(t)\right) = \{\mu : |\mu - x(t)|^2 \ge \rho(t) / \beta\, |\mu - y(t)|\},$$

$$M(x_0, y_0, \rho_0) = \{\mu : |\mu - x_0|^2 \geq \rho_0 / \beta |\mu - y_0|\}.$$

**Lemma 5.** The multi-valued mapping $M\left(x(t), y(t), \rho(t)\right) - tx_1$, $t \in [0, t^*]$, is monotonically decreasing with respect to the inclusion, i.e. if $t_1$, $t_2 \in [0, t^*]$ and $t_1 < t_2$, then $M\left(x(t_1), y(t_1), \rho(t_1)\right) - t_1 x_1 \supset M\left(x(t_2), y(t_2), \rho(t_2)\right) - t_2 x_1$.

We call the set $M^*(x_0, y_0, \rho_0, T) = \bigcup_{t=0}^{T} \{M(x_0, y_0, \rho_0) + tx_1\}$ the *attainability domain of the Evader* in the Pursuit Game.

**Theorem 17.** Suppose $\rho_0 \geq 4\beta |z_0|$ and $M^*(x_0, y_0, \rho_0, T) \cap L = \varnothing$. Then $\Pi$-strategy (19) is winning on the time interval $[0, T]$ in the Life-line Game $L$, where $T = \sqrt{2|z_0| / \theta_1}$.

**Theorem 18.** Let $\rho_0 < 4\beta |z_0|$. Then there exists a control $v(\cdot) \in \mathbb{V}$ guaranteeing that the Evader wins in the Life-line Game $L$.

## CONCLUSION

In the dissertation, pursuit, evasion, and “Life-line” problems in simple and inertial motion differential games with geometric, Grönwall, integral, and various constraints on control functions.

The obtained principal results of the research include the following:

1. New sufficient conditions assuring solutions of pursuit-evasion problems for a simple motion differential game with various constraints (geometric-Grönwall, Grönwall-geometric, integral-Grönwall) on players’ controls have been determined, and a set of attainability points of the players has been constructed;

2. New sufficient conditions assuring the completion of pursuit in a simple motion differential game with a generalized type of integral constraint on players’ controls have been found;

3. New sufficient conditions assuring solutions of pursuit-evasion problems in a simple motion differential game with nonlinear integral constraint on a pursuer’s control and linear integral constraint on an evader’s control have been determined;

4. New sufficient conditions assuring solutions of pursuit-evasion problems for a second-order differential game with integral constraint on a pursuer’s control and geometric constraint on an evader’s control have been found, and the “Life-line” problem of R.Isaacs has been handled.

НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.05.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ФЕРГАНСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ

АНДИЖАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ЖУРАЕВ БАХОДИРЖОН ИНОМЖОН УГЛИ

# П–СТРАТЕГИЯ И ЕЁ ПРИЛОЖЕНИЯ ПРИ РАЗЛИЧНЫХ ОГРАНИЧЕНИЯХ НА УПРАВЛЕНИЯ ИГРОКОВ

**01.01.02 – Дифференциальные уравнения и математическая физика**

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

**Фергана – 2024**

**Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан за №B2023.3.PhD/FM910.**

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## ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

**Целью исследования** является решения задачи преследования-убегания при геометрических, Гронуоловских, интегралных и различных ограничениях на управление игроков и решения задачи с “линией жизни” Р.Айзекса.

**Объектом исследования** являются задачи преследования, убегания и с “линией жизни” в дифференциальных играх с простейшими и инерционными движениями игроков.

**Научная новизна исследования** заключается в следующем:

задачи преследования-убегания в дифференциальных играх с простейшими движениями, при различных ограничениях (геометрический-Гронуолл, Гронуолл-геометрический, интегро-Гронуолл) на управления игроков, решены при помощи метода параллелльного сближения для задачи преследования, а для задачи убегания решены при помощи метода убегания по заданному направлению;

получены достаточные условия гарантирующие решения задачи преследования при помощи стратегии параллелльного сближения в дифференциальной игре с простыми движениями при обобщенных интегральных ограничениях на управления игроков;

решения задачи преследования-убегания в дифференциальных играх при нелинейном интегральном ограничении на управления преследователя и линейном интегральном ограничении на управления убегающего, доказано для преследователя при помощи метода параллелльного сближения, а для убегающего при помощи метода убегания по направлению;

получены достаточные условия гарантирующие решения задачи преследования-убегания в дифференциальной игре, заданной с дифференциальными уравнениями второго порядка при интегральном ограничении на управления преследователя и при геометрическом ограничении на управление убегающего и решена задача с “линией жизни” Р.Айзекса.

**Внедрение результатов исследований.** На основе полученных результатов, относящихся к П-стратегии и её приложений при различных ограничениях на управления игроков:

результаты задач преследования-убегания в дифференциальных играх при нелинейных и линейных интегральных ограничениях на управления игроков были использованы в фундаментальном проекте по теме “Разработка новых методов решения задач математической физики и оптимального управления” с номером №OT-F4-(36+32), в котором рассматриваются динамические системы с противоположными управлениями с различными ограничениями (справка №04/11-671 от 19 января 2024 года Национального университета Узбекистана имени Мирзо Улугбека). В результате удалось разработать новые методы решения задач управления и определить достаточные условия для нелинейных систем, возникающих в результате их численной реализации;

результаты решения задач преследования-убегания и с “линией жизни” при различных ограничениях на управления игроков в дифференциальных играх с простейшими и инерционными движениями были использованы в фундаментальном проекте по теме “Анализ симметрии Ли, анализ устойчивости по Ляпунову и моделирование гиперболических систем” с номером UZB-Ind-2021-87, в котором рассматриваются конфликтно-управляемые процесы с различными ограничениями (справка №04/11-1301 от 26 февраля 2024 года Национального университета Узбекистана имени Мирзо Улугбека). В результате проведен анализ Лиевой симметрии и разработана адекватная расчетная модель для поиска точных решений смешанных задач и экспоненциальных стационарных решений системы гиперболических уравнений с различными ограничениями на границах.

## E’lon qilingan ishlar ro‘yxati
## List of published works
## Список опубликованных работ

### I bo‘lim (part 1; часть 1)


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Avtoreferat Farg‘ona davlat universiteti «FarDU. Ilmiy habarlar-Научный вестник. ФерГУ» ilmiy-metodik jurnal tahririyatida tahrirdan o‘tkazilib, o‘zbek, rus va ingliz tillaridagi matnlar o‘zaro muvofiqlashtirildi.

FDU “Nusxa ko‘paytirish bo‘limi”da chop etildi.
2024 yil. Nashriyot bosma tabog‘i – 2,75
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