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OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

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OLIY MATEMATIKA KAFEDRASI

OLIY MATEMATIKA FANIDAN

KOMPLEKS SONLAR USTIDA AMALLAR

mavzularini o’z ichiga olgan

ATT ta’lim yo’nalishi

talabalari uchun

USLUBIY QO’LLANMA

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So’z boshi

Uslubiy qo’llanma oliy matematika fanining «Kompleks sonlar ustida amallar» bo’limini o’z ichiga olgan. Unda nazariy materiallar, namunaviy masalalar yechimlari va mustaqil yechish uchun mashqlar keltirilgan.

Qo’llanma oliy ta’lim muassasalarining “Axbor tizimlari va texnologiyalari” yo’nalishi bo’yicha bakalavriat bosqichi tatalabalari uchun uslubiy qo’llanma sifatida tavsiya qilinadi.

Mundarija

1§. Kompleks sonlar tushunchasi

Ma’lumki kvadrat tenglamalarni yechishda ba’zan diskriminant manfiy sondan iborat bo’lib qoladi: $D = b^2 - 4ac < 0.$

Bu holda berilgan kvadrat tenglama haqiqiy ildizga ega bo’lmaydi. Chunki, manfiy haqiqiy sonlardan kvadrat ildiz chiqarish ma’noga ega emas.

Diskriminanti manfiy sondan iborat bo’lgan kvadrat tenglamani yechish uchun sonlar tushunchasini kengaytirish lozim bo’ladi. Bunday holda haqiqiy sonlar to’plamiga kvadrati -1 ga teng bo’lgan yangi ***i*** sonini kiritish maqsadga muvoffiq bo’ladi. Bu sonni ***mavhum birlik*** deb atash qabul qilingan. U holda quyidagi tenglik o’rinli bo’ladi:

$$i^2=-1$$

i soni ***bi*** ko’rinishdagi ko’paytma va ***a+ ib*** yig’indini kiritish imkoniyatini beradi.

Ta’rif: ***a+bi*** ko’rinishdagi ifodaga ***kompleks son*** deyiladi. Bunda ***a*** va ***b*** ixtiyoriy haqiqiy sonlar, ***i-*** mavhum birlik.

a soni ***a+bi*** kompleks sonning ***haqiqiy qismi, bi*** ko’paytma esa ***mavhum qismi*** deb ataladi, ***b*** soni ***mavhum qismning koeffisiyenti*** deyiladi.

Masalan, 5+2***i*** kompleks son uchun 5 soni haqiqiy qism, 2***i*** esa mavhum qism bo’ladi, uning koeffisiyenti 2 dan iborat; ***0+7i*** sonning haqiqiy qismi 0, mavhum qismi 7***i*** , mavhum qismning koeffisiyenti 7 dan iborat; 6-0***i*** sonning haqiqiy qismi 6, mavhum qismi 0***i,*** mavhum qismning koeffisiyenti 0 dan iboratdir.

Kompleks sonlar kiritilgach algebra, nazariy fizikaning gidrodinamika, elementar zarralar nazariyasi va hokazolardagi fikrlar hamda tushunchalar soddalashdi.

Ta’rif: Ikkita kompleks sonning haqiqiy qismlari teng va mavhum qismlarining koeffisiyentlari ham teng bo’lsa, bu sonlar o’zaro teng deyiladi, ya’ni ***a=c*** va b=***d*** bo’lsa, quyidagi tenglik o’rinli bo’ladi:

$$a+bi=c+di$$

Ikkita kompleks sonlar orasida tartib («katta» yoki «kichik») munosabatlarni aniqlab bo’lmaydi.

Kompleks sonlar uchun quyidagi qoidalar o’rinli:

1. ***a+bi=c+di.*** (agar ***a=b, c=d*** bo’lsa).

2. ***(a±bi)+(c±di)= (a±c)+(b±d)I*** (kompleks sonlarni qo’shish va ayirish).

3. $(a+bi)(c+di)=(ac-bd)+(ad+bc)i$ (kompleks sonlarni ko'paytirish).

4. $(a+bi)(a-bi)=a^2+b^2$ (o'zaro qo'shma kompleks sonlar ko'paytmasi).

5. $a+0i=a$ (haqiqiy son bilan mavhum qism koeffisiyenti 0 bo'lgan kompleks son).

6. $0+0i=0$ (har qanday kompleks sonning 0 bilan ko'paytmasi).

2§. Komplek sonlarni qo'shish va ayirish

Ta'rif: $a+bi$ va $c+di$ ikkita kompleks sonlar yig'indisi deb $(a+c)+(b+d)i$ songa aytiladi, ya'ni:

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

Misollar.

1) $(6+5i)+(4+3i)=(6+4)+(5+3)i=10+8i;$

2) $(9-11i)+(4+3i)=(9+4)+(-11+3)i=13-8i;$

3) $(0-6i)+(8-5i)=(0+8)+(-6-5)i=8-11i.$

Ta'rif: $z_1=a+bi$ va $z_2=c+di$ kompleks ***sonlarning ayirmasi*** deb shunday $z_3=x+yi$ kompleks songa aytiladiki, bu sonning z_2 bilan yig'indisi z_1 dan iborat bo'ladi, ya'ni:

$$z_1-z_2=z_3 \text{ dan } z_2+z_3=z_1$$

Yoki $(a+bi)-(c+di)=x+yi$ dan $(c+di)+(x+yi)=(c+x)+(d+y)i$

U holda, $(c+x)+(d+y)i=a+bi$ bo'ladi. Bu hol faqatgina $c+x=a$ va $d+y=b$ bo'lgandagina o'rinli bo'ladi.

Misollar.

1) $(2+3i)-(1+2i)=(2-1)+(3-2)i=1+i.$
2) $(7+i)-(5+2i)=(7-5)+(1-2)i=2-i.$
3) $(3+4i)-(5+4i)=(3-5)+(4-4)i=-2+0i.$
4) $(5+8i)-(5+3i)=(5-5)+(8-3)i=0+5i.$

Mustaqil yechish uchun mashqlar

№1. Agar $a+bi$ va $c+di$ berilgan bo'lsa,

a) qaysi holda $a+bi=c+di$ bo'ladi;

b) qaysi holda $a+bi\neq c+di$ bo'ladi?

№2. Kompleks sonlarni qo'shing:

a) $(7-6i)+(7+6i)$; b) $(8-7i)+(7-6i)$;

v) $(0+2i)+(3+0i)$; g) $(-3+4i)+(2+2i)$;

d) (1+0*i*)+(0-2*i*); ye) (2-3*i*)+(0+0*i*).

№3. (x-y)+(3x+y)***i***=3-3***i*** tenglamadan ***x*** va y larni toping.

№4. Berilgan tenglamalardan ***x*** va y sonlarni toping:

a) (***x***-5y)+(2***x-y***)***i***=6+3***i***;

b) (***x+2y***)+(3***x-2y***)***i***=1+ ***i***.

№5. Quyidagi tenglamalardan ***x*** va y haqiqiy sonlarni toping:

a) (2***x***-13***yi***)+(2***x-yxi***)=3-***i***;

b) (5***x***+3***yi***)+(2***y***- ***xi***)=12+4***yi***;

v) (0***x***-2***yi***)+(6***x+yi***)=4+7***yx.***

№6. Kompleks sonlarni ayiring:

a) (3+4***i***)-(3-***i***); b) (7-***i***)-(7-***i***);

v) (5+6***i***)-(3+7***i***); g) (12-3***i***)-(6-9***i***).

№7. Tenglamalarni yeching:

a) $\left(-3y+\frac{1}{2}xi\right)-\left(-8x+5yi\right)=-2+12i;$

b) $\left(0+3xi\right)-\left(10x+2yi\right)=-5y+3i;$

v) $\left(\frac{3}{4}x-2yi\right)-\left(\frac{1}{3}y+6xi\right)=0+21i.$

№8. Tenglamalardan ***x*** va ***y*** haqiqiy sonlarni toping:

a) (***x+y***)+(***x-y***)***i***=2+4***i***;

b) (***x+y***)+(***x-y***)***i***=4***i***;

v) (***x+y***)+(***x-y***)***i***=2;

g) $\left(x+\frac{3}{2}y\right)+\left(2x+3y\right)i=13i.$

№9. Quyidagi tenglamalardan ***u*** va ***v*** sonlarni toping:

a) $\left(3u+4vi\right)+\left(4-v\right)=3+2i;$

b) $(2u-0vi)+(u-3i)=5i.$

№10. Hisoblang:

a) $\left[i(2-i)\right]^2$; b) $\left[2i(3-4i)\right]^2$.

3§. Kompleks sonlarni ko'paytirish va bo'lish

Ikkita ***a+bi*** va c+***di*** kompleks sonlarni ko'paytirish 1**§** dagi 3-qoida asosida bajariladi, ya'ni birinchi va ikkinchi ko'paytuvchi kompleks sonlar hadma-had ko'paytiriladi:

$$(a+bi)\cdot(c+di) = ac + adi + bci + bdi^2 = ac + (ad+bc)i + bdi^2.$$

Bundan ***i*** ^{2}=-1 bo'lganligi sababli, $bdi^2 = -bd$.

Demak, $$(a+bi)\cdot(c+di) = (ac-bd)+(ad+bc)i.$$

Ta'rif: $a+bi$ va $c+di$ ***kompleks sonlarning ko'paytmasi*** deb

$$(ac-bd)+(ad+bc)i$$

kompleks songa aytiladi.

Har qanday $a+bi$ ko'rinishdagi kompleks sonning nol 0+0***i***=0 songa ko'paytmasi noldan iborat bo'ladi, ya'ni

$$(a+bi)(0+0i) = 0+0i$$

Har qanday $a+bi$ kompleks sonning ***n=n+0i*** haqiqiy songa ko'paytmasi quyidagidan iborat:

$$(a+bi)(n+0i) = na + nbi.$$

Misollar.

a) $(3+4i)(5+0i) = (3\cdot5-4\cdot0)+(4\cdot5+3\cdot0)i = (15-0)+(120+0)i = 15+20i$

b) $(2+0i)(6-5i) = (2\cdot6-0\cdot5)+(2\cdot(-5)+0\cdot6)i = (12-0)+(-10+0)i = 12-10i.$

Ikkita ***z₁=a+bi*** va ***z₂=s+di*** kompleks sonlarni bo'lishda ***z₃=x+yi*** kompleks son hosil bo'ladi, ya'ni

$$z_3 = \frac{z_1}{z_2} \quad \left(yoki \quad x+yi = \frac{a+вi}{s+di} \right). \qquad (1)$$

buni $z_1 = z_2 \cdot z_3$ kabi yozish ham mumkin.

Ta'rif: z_1 kompleks sonning z_2 kompleks songa bo'linmasi deb, shunday z_3 ga aytiladiki, bu sonni z_2 ga ko'paytirganda z_1 hosil bo'ladi.

Kasrlarning xossasiga asosan $\frac{a+bi}{c+di}$ nisbat $c+di \neq 0$ shart bajarilgan taqdirda o'rinli bo'ladi.

Agar $c+di \neq 0+0i$ bo'lsa

$$(x+yi)(c+di)=a+bi. \qquad (2)$$

Kompleks sonlarni ko'paytirish qoidasiga asosan

$$(x+yi)(c+di)=(xc-yd)+(xd+yc)i.$$

U holda, (2) ni quyidagicha yozish mumkin:

$$(xc-yd)+(xd+yc)i=a+bi. \qquad (3)$$

(3) tenglik

$$\begin{cases} xc-yd=a \\ xd+yc=b \end{cases} \qquad (4)$$

bo'lgandagina o'rinli bo'ladi.

(4) dan ***x*** va ***y*** larni topamiz:

$$x=\frac{ac+bd}{c^2+d^2}, \qquad y=\frac{bc-ad}{c^2+d^2}. \qquad (5)$$

U holda,

$$\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i \qquad (6)$$

tenglik hosil bo'ladi.

1-misol. $\frac{8-5i}{3-2i}$ nisbatni toping.

Yechilishi: $\frac{8-5i}{3-2i}=x+yi$ deb belgilaymiz. U holda,

$$(x+yi)(3-2i)=8-5i,$$
$$3x-2xi+3yi-2yi^2=8-5i,$$
$$(3x+2y)+(-2x+3y)i=8-5i,$$

Bundan,

$$\begin{cases} 3x+2y=8, \\ -2x+3y=-5 \end{cases}$$

Sistemani yechib, $x=\frac{34}{13}$ va $y=\frac{1}{13}$ ni topamiz.

U holda,

$$\frac{8-5i}{3-2i}=\frac{34}{13}+\frac{1}{13}i.$$

2-misol. $\frac{1+i}{2-i}$ nisbatni toping.

Yechilishi: Kompleks sonlar nisbatini topish uchun kasrning surat va maxrajini ***2-i*** ning qo'shmasi ***2+i*** ga ko'paytiramiz:

$$\frac{1+i}{2-i}=\frac{(1+i)(2+i)}{(2-i)(2+i)}=\frac{2+i+2i+i^2}{4-i^2}=\frac{1+3i}{4+1}=\frac{1+3i}{5}=\frac{1}{5}+\frac{3}{5}i.$$

3-misol. Kompleks sonlarning nisbatini toping: $\frac{-2-3i}{1-2i}$

Yechilishi: Berilgan nisbatning surat va maxrajini ***1+2i*** ga ko'paytiramiz:

$$\frac{-2-3i}{1-2i}=\frac{(-2-3i)(1+2i)}{(1-2i)(1+2i)}=\frac{-2-4i-3i-6i^2}{1-(2i)^2}=\frac{-2-7i+6}{1-4i^2}=\frac{4-7i}{1-4\cdot(-1)}=\frac{4-7i}{5}=\frac{4}{5}-\frac{7}{5}i.$$

4-misol. Hisoblang: $\frac{a+вi}{в+ai}-\frac{в-ai}{a+вi}i.$

Yechilishi:

$$\frac{(a+вi)(в-ai)}{(в+ai)(в-ai)}-\frac{(в-ai)(a-вi)}{(a+вi)(a-вi)}i=\frac{aв-a^2i+в^2i-aвi^2}{в^2-(ai)^2}-\frac{aв-в^2i-a^2i+aвi^2}{a^2-(вi)^2}i=$$

$$=\frac{(aв+aв)+(-a^2+в^2)i}{в^2+a^2}-\frac{(aв-aв)+(-в^2-a^2)i}{a^2+в^2}\cdot i=\frac{\left[2aв+(-a^2+в^2)i\right]-\left[-(a^2+в^2)i\right]}{a^2+в^2}\cdot i=$$

$$=\frac{2aв-a^2i+в^2i+a^2i+в^2i}{a^2+в^2}=\frac{2aв+2в^2i}{a^2+в^2}=\frac{2aв}{a^2+в^2}+\frac{2в^2}{a^2+в^2}\cdot i$$

Mustaqil yechish uchun mashqlar

№11. Kompleks sonlar ko'paytmasini toping:

a) (-1-2)(-2+2***i***); b) (2+3***i***)(3+2***i***).

№12. Quyidagi kompleks sonlarni ko'paytiring:

a) (3,5-***i***)(7-2***i***); b) (5+***i***)(15-3***i***).

№13. Kompleks sonlar ko'paytmasini toping:

a) (4-***i***)(3+2***i***); b) (-7+2***i***)(1-***i***).

№14. Komplek sonlar ko'paytmasini toping:

a) $\left(\sqrt{3}+i\right)\left(\sqrt{3}-i\right)$;   b) $\left(1-\sqrt{2}i\right)\left(\sqrt{2}+3i\right)$;

v) $\left(a-вi\right)\left(a+вi\right)$;   g) $\left(2a-i\right)\left(-a-вi\right)$;

d) $\left(\sqrt{3}+4i\right)\left(3-\sqrt{3}i\right)$;   e) $\left(1-\sqrt{5}i\right)\left(2+\sqrt{3}i\right)$.

№15. Kompleks sonlar nisbatini toping:

a) $\frac{2+i}{2-i}$;   b) $\frac{0+4i}{1+i}$; v) $\frac{5+0i}{-4+3i}$.

№16. Kompleks sonlarni bo'ling:

a) $\frac{4+6i}{1-i}$;   b) $\frac{10-i}{1+i}$;   v) $\frac{1-2i}{3+2i}$;   g) $\frac{-2-3i}{1+2i}$.

№17. Kompleks sonlarni bo'ling:

a) $\frac{2+i}{3-i}$;   b) $\frac{6-i}{3+4i}$;   v) $\frac{13+4i}{1+i}$;   g) $\frac{3+4i}{7-2i}$.

№18. Quyidagi nisbatlar tengligini isbot qiling:

a) $\frac{2+i}{3-i}=\frac{13+4i}{17-9i}$;   b) $\frac{6-i}{3+4i}=\frac{13+41i}{-25+25i}$.

№19. Kvadrat tenglamalarni yeching:

a) $x^2-4x+5=0$;   b) $x^2-8x+7=0$;

v) $3x^2+10x+9=0$; g) $4x^2-14x+13=0$.

№20. Tenglamani yeching:

a) $\left(i-z\right)\left(1-2i\right)+\left(1-zi\right)\left(3-4i\right)=1+7i.$

№21. Tenglamalar sistemasini yeching: $\begin{cases} z_1+2z_2=1+i \\ 3z_1+iz_2=2-3i \end{cases}$

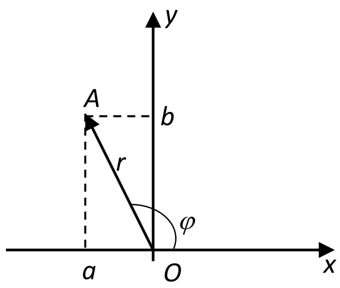
4§. Kompleks sonning trigonometrik shakli, moduli va argumenti

a+bi kompleks songa koordinatalari (***a,b***) bo'lgan $\overrightarrow{OA}$ vektor mos kelsin. $\overrightarrow{OA}$ vektor uzunligini ***r***, uning ***x*** o'qi bilan hosil qiladigan burchagini φ bilan belgilaymiz. U holda, chizmadan quyidagi tengliklar o'rinli bo'ladi:

$$\frac{a}{r} = \cos\varphi, \ \frac{b}{r} = \sin\varphi \qquad (1)$$

(1)dan,

$$a = r \cdot \cos\varphi, \quad b = r \cdot \sin\varphi. \qquad (2)$$



U holda, $a + bi$ kompleks sonni quyidagi ko'rinishda yozish mumkin bo'ladi:

$$a + bi = r\cos\varphi + ir\sin\varphi = r\left(\cos\varphi + i\sin\varphi\right) \qquad (3)$$

Chizmadan $$r^2 = a^2 + b^2 \qquad (4)$$

Shuning uchun ixtiyoriy ***a+bi*** kompleks sonni

$$a + bi = r(\cos\varphi + i\sin\varphi) \qquad (5)$$

ko'rinishda ifodalash mumkin. Bunda $r = \sqrt{a^2 + b^2}$ va φ burchak qo'yidagi shartlarda topiladi:

$$\begin{cases} \sin\varphi = \dfrac{b}{\sqrt{a^2 + b^2}}, \\ \cos\varphi = \dfrac{a}{\sqrt{a^2 + b^2}}. \end{cases} \qquad (6)$$

(5) tenglamadan ***r*** soni ***a+bi kompleks sonning moduli,*** φ burchak esa ***kompleks sonning argumenti*** deb ataladi.

Agar $a+bi\neq 0$ bo’lsa, uning moduli musbat, $a+bi=0$ bo’lsa, $a=b=0$ va $r=0$ bo’ladi.

Agar $a+bi\neq 0$ bo’lsa, uning argumenti (6) formulalar yordamida 2π ga karrali bo’lgan burchakgacha aniqlikda topiladi. $a+bi=0$ bo’lgan holda $a=b=0$ va $r=0$ bo’ladi.

Har qanday kompleks sonning modulini $|z|$, argumentini esa $\arg z$ kabi belgilash ham mumkin.

1-misol. ***1+i*** kompleks sonni trigonometrik shaklda ifodalang.

Yechilishi: Berilgan ***1+i*** kompleks sonning ***r*** moduli va φ argumentini topamiz:

$$r=\sqrt{a^2+b^2}=\sqrt{1^2+1^2}=\sqrt{2}.$$

U holda, $$\sin\varphi=\frac{b}{\sqrt{a^2+b^2}}=\frac{1}{\sqrt{2}};\quad \cos\varphi=\frac{a}{\sqrt{a^2+b^2}}=\frac{1}{\sqrt{2}}.$$

Bo’lardan, $$\varphi=\frac{\pi}{4}+2n\pi.$$

Demak, $1+i=\sqrt{2}\left[\cos\left(\frac{\pi}{4}+2n\pi\right)+i\sin\left(\frac{\pi}{4}+2n\pi\right)\right]$ yoki $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right).$

2-misol. $\sqrt{3}-i$ kompleks sonning trigonometrik shaklini aniqlang.

Yechilishi: Berilgan kompleks sonning moduli ***r*** ni topamiz:

$$r=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{3+1}=2.$$

Endi kompleks sonning argumenti φ ni (6) formulalar yordamida topamiz:

$$\sin\varphi=\frac{-1}{2}=-\frac{1}{2},\quad \cos\varphi=\frac{\sqrt{3}}{2}.$$

U holda, $\varphi=\frac{11}{6}\pi$ bo’ladi.

Demak, berilgan kompleks sonning trigonometrik shakli quyidagidan iborat bo’ladi:

$$\sqrt{3-i}=2\left(\cos\frac{11}{6}\pi+\sin\frac{11}{6}\pi\right).$$

Mustaqil yechish uchun mashqlar

№22. ***1+0i*** kompleks sonni trigonometrik shaklda ifodalang.

№23. ***-1+0i*** kompleks sonni trigonometrik shaklda ifodalang.

№24. **0+*i*** kompleks sonni trigonometrik ko'rinishda yozing.

№25. ***1-i*** kompleks sonni trigonometrik ko'rinishda yozing.

№26. 3 kompleks sonning trigonometrik shaklini yozing.

№27. -5 kompleks sonni trigonometrik ko'rinishda yozing.

№28. Quyidagi berilgan kompleks sonlarning modullari ***r*** va argumentlari φ larni toping hamda ularning trigonometrik shakllarini yozing:

a) 6-6***i***; d) -2***i;***

b) 12***i***-5; e) 3***i***-4;

v) 25; yo) $\sqrt{3}+i$;

g) 3***i***; j) $2+2\sqrt{3}i$.

№29. $1+\cos\alpha+i\sin\alpha$ kompleks sonning trigonometrik shaklini yozing.

№30. Quyidagi $2\left(\cos 20^0 - i\sin 20^0\right)$ kompleks sonning trigonometrik ko'rinishini yozing.

№31. $3\left(-\cos 15^0 - i\sin 15^0\right)$ kompleks sonni trigonometrik ko'rinishda ifodalang.

5§. Trigonometrik shaklda berilgan kompleks sonlar ustida amallar. Komplek sonlarni ko'paytirish va darajaga ko'tarish. Muavr formulasi.

Ikkita trigonometrik shakldagi z_1 va z_2 kompleks sonlar, ya'ni

$$z_1 = r_1(\cos\alpha_1 + i\sin\alpha_1) \text{ va } z_2 = r_2(\cos\alpha_2 + i\sin\alpha_2)$$

berilgan bo'lsin, z_1 va z_2 kompleks sonlarning ko'paytmasini topamiz:

$$z_1 \cdot z_2 = r_2(\cos\alpha_1 + i\sin\alpha_1) \cdot r_2(\cos\alpha_2 + i\sin\alpha_2) =$$
$$= r_1 \cdot r_2((\cos\alpha_1 \cdot \cos\alpha_2 - \sin\alpha_1 \cdot \sin\alpha_2) + i(\cos\alpha_1 \cdot \sin\alpha_2 + \sin\alpha_1 \cdot \cos\alpha_2)) =$$
$$= r_1 \cdot r_2(\cos(\alpha_1 + \alpha_2) + i\sin(\alpha_1 + \alpha_2)).$$

$$r_1(\cos\alpha_1 + i\sin\alpha_1) \cdot r_2(\cos\alpha_2 + i\sin\alpha_2) = r_1 \cdot r_2(\cos(\alpha_1 + \alpha_2) + i\sin(\alpha_1 + \alpha_2)) \qquad (1)$$

Demak, ikkita trigonometrik shaklda berilgan kompleks sonlarni ko'paytirish uchun ularning r_1 va r_2 modullari o'zaro ko'paytiriladi, $\alpha_1$va α_2 argumentlari esa o'zaro qo'shiladi, ya'ni:

$$|z_1| \cdot |z_2| = r_1 \cdot r_2 = |r_1| \cdot |r_2|, \quad \arg(z_1 \cdot z_2) = \alpha_1 + \alpha_2. \qquad (2)$$

Agar ***n*** ta trigonometrik shakldagi kompleks sonlar berilgan bo'lsa, ularning ko'paytmasi quyidagicha bo'ladi:

$$z_1 \cdot z_2 ... z_n = r_1 \cdot r_2 ... r_n(\cos(\alpha_1 + \alpha_2 + ... + \alpha_n) + i\sin(\alpha_1 + \alpha_2 + ... + \alpha_n)). \qquad (3)$$

1-misol.
$z_1 = 5(\cos160^0 + i\sin160^0)$, $z_2 = 2(\cos45^0 + i\sin45^0)$, $z_3 = 4(\cos100^0 + i\sin100^0)$,
$z_4 = 6(\cos55^0 + i\sin55^0)$ berilgan. Ularning ko'paytmasini toping.

Yechilishi: Berilgan kompleks sonlarni ko'paytirish uchun (3) formuladan foydalanamiz:

$$z_1z_2z_3z_4 = 5 \cdot 2 \cdot 4 \cdot 6(\cos(160^0 + 45^0 + 100^0 + 55^0) + i\sin(160^0 + 45^0 + 100^0 + 55^0)) =$$
$$= 240(\cos360^0 + i\sin360^0) = 240(1 + i \cdot 0) = 240.$$

Agar ***n*** ta ko'paytuvchi kompleks sonlar o'zaro teng, ya'ni

$$z_1 = z_2 = ... = z_n = r\left(\cos\alpha + i\sin\alpha\right) \tag{4}$$

bo’lsa, (3) formula quyidagi ko’rinishga keladi:

$$z_1 \cdot z_2 ... z_n = \left(r\left(\cos\varphi + i\sin\varphi\right)\right)^n = r^n\left(\cos n\varphi + i\sin n\varphi\right). \tag{5}$$

Bu formulaga ***trigonometrik shakldagi kompleks sonni n -darajaga ko’tarish*** yoki ***Muavr formulasi*** deyiladi.

2-misol. $\left(4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^3$ ni darajaga ko’taring va hisoblang.

Yechilishi: Berilgan trigonometrik shakldagi kompleks sonni darajaga ko’tarish uchun (5) formula, ya’ni Muavr formulasidan foydalanmiz:

$$\left(4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^3 = 4^3\left(\cos\frac{\pi}{3}\cdot 3 + i\sin\frac{\pi}{3}\cdot 3\right) = 64\left(\cos\pi + i\sin\pi\right) = 64\left(-1 + i\cdot 0\right) = -64.$$

6§. Trigonometrik shakldagi kompleks sonlarni bo'lish

Ikkita z_1 va z_2 kompleks sonlar trigonometrik shaklda berilgan bo'lsin, ya'ni:

$$z_1 = r_1(\cos\alpha_1 + i\sin\alpha_1), \quad z_2 = r_2(\cos\alpha_2 + i\sin\alpha_2). \tag{6}$$

Ularning nisbati quyidagicha topiladi:

$$\frac{z_1}{z_2} = \frac{r_1(\cos\alpha_1 + i\sin\alpha_1)}{r_2(\cos\alpha_2 + i\sin\alpha_2)} = \frac{r_1(\cos\alpha_1 + i\sin\alpha_1)(\cos\alpha_2 - i\sin\alpha_2)}{r_2(\cos\alpha_2 + i\sin\alpha_2)(\cos\alpha_2 - i\sin\alpha_2)} =$$

$$= \frac{r_1(\cos(\alpha_1 - \alpha_2) + \sin(\alpha_1 - \alpha_2))}{r_2(\cos^2\alpha_2 + \sin^2\alpha_2)} = \frac{r_1}{r_2}(\cos(\alpha_1 - \alpha_2) + \sin(\alpha_1 - \alpha_2)). \tag{7}$$

Demak, ikkita trigonometrik shaklda berilgan kompleks sonning nisbatini topishda ularning modullari bo'linadi, argumentlari esa o'zaro ayriladi, ya'ni:

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{r_1}{r_2}, \quad \arg\left(\frac{z_1}{z_2}\right) = arc\, z_1 - arc\, z_2 = \alpha_1 - \alpha_2. \tag{8}$$

3-misol. $z_1 = \sqrt{2}(\cos 45^0 + i\sin 45^0)$ va $z_2 = 2(\cos 15^0 + i\sin 15^0)$ lar berilgan bo'lsin. $\frac{z_1}{z_2}$ nisbatni toping.

Yechilishi: **(7)** formuladan foydalanib, trigonometrik shakldagi kompleks sonlar nisbatini topamiz:

$$\frac{z_1}{z_2} = \frac{\sqrt{2}(\cos 45^0 + i\sin 45^0)}{2(\cos 15^0 + i\sin 15^0)} = \frac{\sqrt{2}}{2}\cos(45^0 \cdot 15^0) + i\sin 45^0 - 15^0)) =$$

$$= \frac{\sqrt{2}}{2}(\cos 30^0 - i\sin 30^0) = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{\sqrt{2}}{4}(\sqrt{3} + 1).$$

4-misol. $z_1 = \cos 70^0 + i\sin 70^0$ va $z_2 = \cos 100^0 + i\sin 100^0$ kompleks sonlar nisbatini toping.

Yechilishi: **(7)** formuladan foydalanamiz:

$$\frac{z_1}{z_2} = \frac{\cos 70^0 + i\sin 70^0}{\cos 100^0 + i\sin 100^0} = \cos\left(70^0 - 100^0\right) + i\sin\left(70^0 - 100^0\right) =$$

$$\cos\left(-30^0\right) + i\sin\left(-30^0\right) = \cos 30^0 - i\sin 30^0 = \frac{\sqrt{3}}{2} - \frac{1}{2}i = \frac{1}{2}\left(\sqrt{3} - i\right).$$

7§. Trigonometrik shakldagi kompleks sondan ildiz chiqarish

$r(\cos\varphi + i\sin\varphi)$ kompleks sonning ***n***-darajali ildizi quyidagicha bo'lsin:

$\rho(\cos\theta + i\sin\theta)$ U holda, quyidagi tenglik o'rinli bo'ladi:

$$r(\cos\varphi + i\sin\varphi) = \left(\rho(\cos\theta + i\sin\theta)\right)^n.$$

Muavr formulasiga asosan: $r(\cos\varphi + i\sin\varphi) = \rho^n(\cos n\theta + i\sin n\theta)$. (9)

Agar ikkita kompleks son o'zaro teng bo'lsa, ularning modullari teng, argumentlari esa bir-biridan 2π ga karrali burchakka farq qiladi. Shuning uchun $\rho^n = r$ hamda $n\theta = \varphi + 2k\pi$ yoki $\rho = \sqrt[n]{r}$ va $\theta = \dfrac{\varphi + 2k\pi}{n}$, $k \in Z$, $n \in N$. (10)

ρ va θ larning topilgan qiymatlarini (9) ga qo'yamiz:

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right). \quad (11)$$

5-misol. $\sqrt[n]{1}$ kompleks sondan ildiz chiqaring.

Yechilishi: Berilgan ildiz ostidagi 1 sonini trigonometrik ko'rinishga keltiramiz:

$$1 = \cos 0^0 + i\sin 0^0).$$

Ildiz chiqarish formulasi (11) dan foydalanamiz:

$$z_k = \sqrt[n]{1} = \sqrt[n]{\cos 0^0 + i\sin 0^0} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}.$$

Bunda $k = 0,1,2,...,n-1$ dan iborat.

8§. Kompleks son uchun Eyler formulasi

Kompleks ko'rsatkichli e^z funksiyani qaraylik. Bunda $z = x + iy$, **"*e*"** esa

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \text{ dan iborat.}$$

U holda, ***e^z*** ni quyidagicha yozish mumkin bo'ladi:

$$e^z = e^x\left(\cos y + i\sin y\right) \text{ yoki} \tag{1}$$

$$e^{x+iy} = e^x\left(\cos y + i\sin y\right). \tag{2}$$

Agar ***x=0*** bo'lsa, (2) tenglik

$$e^{iy} = \cos y + i\sin y \tag{3}$$

ko'rinishga ega bo'ladi. (3) tenglikka ***Eyler formulasi*** deyiladi.

Kompleks ko'rsatkichli funksiyaning davri $2\pi i$ ga teng. Agar uning davri hisobga olinsa, e^z ko'rsatkichli funksiyani

$$e^{z+2\pi i} = e^z \tag{4}$$

ko'rinishda ifodalash mumkin. (4) da ***z=0*** bo'lsa,

$$e^{2\pi i} = 1 \tag{5}$$

munosabat o'rinli bo'ladi.

$z = r\left(\cos\varphi + i\sin\varphi\right)$- trigonometrik ko'rinishdagi kompleks sonni ko'rsatkichli shaklda quyidagicha ifodalash mumkin:

$$z = re^{\varphi i}. \tag{6}$$

(6) ga ***kompleks sonning ko'rsatkichli ko'rinishi*** deyiladi.

Kompleks ko'rsatkichli funksiyalar uchun ko'paytirish, bo'lish, darajaga ko'tarish va ildiz chiqarish amallarini bajarish mumkin.

Faraz qilaylik, $z_1 = r_1 e^{i\varphi_1}$ va $z_2 = r_2 e^{i\varphi_2}$ bo'lsin. U holda,

$$z_1 \cdot z_2 = r_1 e^{i\varphi_1} \cdot r_2 e^{i\varphi_2} = r_1 \cdot r_2 e^{i(\varphi_1+\varphi_2)}, \tag{7}$$

$$\frac{z_1}{z_2}=\frac{r_1e^{i\varphi_1}}{r_2e^{i\varphi_2}}=\frac{r_1}{r_2}\cdot e^{i(\varphi_1-\varphi_2)} \quad . \tag{8}$$

$z=re^{i\varphi}$ bo'lsin. U holda, z^n ni qo'yidagi ko'rinishda ifodalash mumkin:

$$z^n=\left(re^{i\varphi}\right)^n=r^ne^{in\varphi}, \tag{9}$$

bundan, $$\sqrt[n]{z}=\sqrt[n]{re^{i\varphi}}=\sqrt[n]{r}\cdot e^{\frac{i(\varphi+2\pi k)}{n}}, \ (k=0,1,2,...n-1).$$

Agar (3) dagi ***y*** ni φ va -φ lar bilan almashtirilsa, qo'yidagilar hosil bo'ladi:

$$e^{\varphi i}=\cos\varphi+i\sin\varphi, \quad e^{-\varphi i}=\cos\varphi-i\sin\varphi. \tag{10}$$

(10) dagi tengliklarni qushib, ayiramiz hamda $\cos\varphi$ va $\sin\varphi$ larni topamiz:

$$\cos\varphi=\frac{e^{\varphi i}+e^{-\varphi i}}{2}, \tag{11}$$

$$\sin\varphi=\frac{e^{\varphi i}-e^{-\varphi i}}{2} \tag{12}$$

(11) va (12) lar trigonometrik funksiyalarni ko'rsatkichli funksiyalar orqali ifodalaydi, hamda ular ham Eyler formulalari deb nomlanadi.

1-misol. $z=1+i$ bo'lsa, ***e*** sonni ***z*** darajaga ko'taring.

Yechilishi: ***e*** sonni ***z*** darajaga ko'tarish uchun $z=x+iy$ va (2) formuladan foydalanamiz. Berilganga ko'ra ***x=1, y=1.*** U holda,

$$e^z=e^{x+iy}=e^{1+i}=e(\cos1+i\sin1).$$

2-misol. ***e*** sonni $z=i\frac{\pi}{2}$ darajaga ko'taring.

Yechilishi: (1) yoki (2) formulalardan birini qo'llaymiz:

$$e^z=e^{x+iy}=e^{i\cdot\frac{\pi}{2}}=e^0\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=1\cdot(0+i\cdot1)=i\,.$$

4-misol. $z=-\sqrt{3}-i$ sonni ko'rsatkichli ko'rinishda ifodalang.

Yechilishi: $r=|z|=2,\ \arg z=\frac{7\pi}{6}.$ U holda, $z=-\sqrt{3}-i=2e^{\frac{7\pi}{6}i}$.

5-misol. $z=e^{-3+4i}$ sonni algebraik ko'rinishda ifodalang.

Yechilishi: $z = e^{-3+4i} = e^{-3} \cdot e^{4i} = e^{-3}\left(\cos 4 + i\sin 4\right).$

6-misol. $z = (-1+i)^5$ kompleks sonni ko'rsatkichli ko'rinishda ifodalang.

Yechilishi: Berilgan kompleks sonni ko'rsatkichli ko'rinishga keltirish uchun

$$z^n = \left(re^{i\varphi}\right)^n = r^n e^{in\varphi}$$ formuladan ifodalaymiz:

$$(-1+i)^4 = \left(\sqrt{2}\, e^{i\cdot\frac{3\pi}{4}}\right)^5 = 4\sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}\cdot 5} = 4\sqrt{2}\cdot e^{\frac{15\pi}{4}\cdot i} = 4\sqrt{2}\cdot e^{-\frac{\pi}{4}i}$$

7-misol. $z_1 = e^{5-6i}$ va $z_2 = e^{-2+5i}$ kompleks sonlar berilgan. $z_1 \cdot z_2$ va $\frac{z_1}{z_2}$ larni toping. Natijalarni trigonometrik shaklda ifodalang.

Yechilishi: (7) va (8) formulalarni qo'llaymiz:

$$e^{3-i} = e^3\left(\cos(-1) + i\sin(-1)\right) = e^3\left(\cos 1 - i\sin 1\right).$$

Endi $\frac{z_1}{z_2}$ nisbatni topamiz va natijani trigonometrik shaklda ifodalaymiz:

$$\frac{z_1}{z_2} = \frac{e^{5-6i}}{e^{-2+5i}} = e^{7-11i} = e^7\left(\cos(-11) + i\sin(-11)\right) = e^7\left(\cos 11 - i\sin 11\right).$$

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№32. Kompleks sonni ko'rsatkichli ko'rinishda ifodalang:

$$z = \sqrt{3} - i.$$

№33. $z = -1 - \sqrt{3}i$ kompleks sonni ko'rsatkichli shaklga keltiring.

№34. Quyidagi kompleks sonlarni ko'rsatkichli shaklda ifodalang:

a) $z = -3$; b) $z = 2i$; v) $z = -1 - i$; g) $z = -\sqrt{2} + \sqrt{6}$.

№35. Kompleks sonlarning ko'paytmasi va nisbatini toping. Natijani algebraik ko'rinishda ifodalang:

a) $z_1 = \frac{3}{2}e^{\frac{1}{7}i}$ va $z_2 = \frac{1}{7}e^{\frac{17}{10}i}$;

b) $z_1 = e^{-\frac{1}{7}+3i}$ va $z_2 = e^{\frac{3}{2}+2i}$.

№36. $z = -\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}$ kompleks soni ko'rsatkichli ko'rinishga keltiring.

№37. $z = -\sqrt{12} - 2i$ kompleks sonni ko'rsatkichli shaklda ifodalang.

№38. Quyidagilarni ko'rsatkichli va algebraik shakllarda yozing:

a) $5e^{\frac{\pi}{4}i} \cdot 0{,}2e^{\frac{\pi}{6}i}\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$; b) $\left(\frac{1}{2}e^{\frac{\pi}{12}\cdot i}\right)^{-3}$; v) $\left(\sqrt{3} - i\right)^6$.

№39. $\sqrt[n]{re^{i\varphi}} = \sqrt[n]{r} \cdot e^{\frac{i(\varphi+2\pi k)}{n}}$ dan foydalanib,

a) $w = 1,\ \ n = 3;$ b) $w = -1,\ \ n = 4;$

v) $w = -4 + \sqrt{48}i,\ \ n = 3;$ g) $w = -1 - \sqrt{3}i,\ \ n = 4.$

bo'lganda $\sqrt[n]{w}$ ning barcha qiymatlarini toping.

9§. Kompleks sonlarga oid nostandart masalalani yechish

1. Kompleks sonlarni darajaga oshiring.

1) $(1+i)^{10}$ 2) $\left(1-i\sqrt{3}\right)^{6}$ 3) $\left(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{4}$

Yechish. 1) $(1+i)^{10}$ $=> x=1\,,\ y=1$ $\begin{cases} r=\sqrt{1^2+1^2}=\sqrt{2} \\ \cos\varphi=\frac{1}{\sqrt{2}} \\ \sin\varphi=\frac{1}{\sqrt{2}} \end{cases}$ $=>$ $\begin{cases} r=\sqrt{2} \\ \varphi=\frac{\pi}{4}+2\pi k \end{cases}$ $z=\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

$z^n=r^n(\cos(n\varphi)+i\sin(n\varphi))$ Muavr formulasidan

$$z^{10}=\sqrt{2}^{10}\left(\cos\frac{10\pi}{4}+i\sin\frac{10\pi}{4}\right)=32\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)=32i$$

2) $\left(1-i\sqrt{3}\right)^{6}$ $=> x=1\,,\ y=-\sqrt{3}$ $\begin{cases} r=\sqrt{1^2+(-\sqrt{3})^2}=2 \\ \cos\varphi=\frac{1}{2} \\ \sin\varphi=\frac{-\sqrt{3}}{2} \end{cases}$ $=>$ $\begin{cases} r=2 \\ \varphi=\frac{5\pi}{3}+2\pi k \end{cases}$ $z=2(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3})$

Muavr formulasidan $z^6=2^6\left(\cos\frac{6\cdot 5\pi}{3}+i\sin\frac{6\cdot 5\pi}{3}\right)=64\left(\cos 10\pi+i\sin 10\pi\right)=64$ kelib chiqadi.

3) $\left(1+\cos\frac{\pi}{3}+i\sin\left(\frac{\pi}{3}\right)\right)^{6}$. Umumiy holni qaraymiz . Ya'ni $(1+\cos\alpha+i\sin\alpha)^n$ ni hisoblaymiz.

$$(1+\cos\alpha+i\sin\alpha)^n=\left(2\cos^2\frac{\alpha}{2}+2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)^n=2^n\cos^n\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)^n=2^n\cos^n\frac{\alpha}{2}\left(\cos\frac{n\alpha}{2}+i\sin\frac{n\alpha}{2}\right)$$

$\alpha=\frac{\pi}{4}$ $,n=4$ bo'lganda

$$\left(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^4=2^4\cos^4\frac{\pi}{8}\left(\cos\frac{4\pi}{8}+i\sin\frac{4\pi}{8}\right)=4i\left(1+\cos\frac{\pi}{4}\right)^2=4i\left(1+\frac{1}{\sqrt{2}}\right)^2=$$

$$=4i(\frac{3}{2}+\sqrt{2})=2i(3+2\sqrt{2})$$

2. Tenglamalarni yeching.

1) $x^3+8=0$ 2) $x^4+4=0$

Yechish. 1) $x^3+8=0$ => $x^3=-8$ => $x=\sqrt[3]{-8}$

$\sqrt[n]{1}=\cos\frac{2\pi k}{n}+i\sin\frac{2\pi k}{n}=\alpha_k$ $k=0,1,...n-1$, u_0 z ning n- ildizlaridan biri bo'lsin.

$u_k=u_0\cdot\alpha_s$ $s,k=0,1,...n-1$ bo'ladi. Bu formuladan quyidagilarni olamiz:

$\alpha_0=\cos 0+i\sin 0=1$ $\alpha_1=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$

$\alpha_2=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

$x_0=-2$ $x_k=x_0\cdot\alpha_s$ $s=0,1,2$, $x_0=(-2)\cdot 1=-2$, $x_1=(-2)\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=1-i\sqrt{3}$

$x_1=(-2)\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=1+i\sqrt{3}$

2) $x^4+4=0$ => $x^4=-4$ => $x=\sqrt[4]{-4}$ $z=-4$ $x_1=-4$, $y_1=0$ $\begin{cases} r=\sqrt{(-4)^2+0^2}=4 \\ \cos\varphi=\frac{-4}{4}=-1 \\ \sin\varphi=\frac{0}{4}=0 \end{cases}$

$\begin{cases} r=4 \\ \varphi=\pi+2\pi k \end{cases}$ $z=4(\cos\pi+i\sin\pi)$ Ildiz chiqarish uchun ushbu Muavr formulasidan

$u_k=\sqrt[n]{z}=\sqrt[n]{r}(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n})$ $k=0,1,...,n-1$ quyidagini olamiz.

$u_k=\sqrt[4]{4}(\cos\frac{\pi+2\pi k}{4}+i\sin\frac{\pi+2\pi k}{4})$ $k=0,1,2,3$

$k=0$ $u_0=\sqrt[4]{4}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})=\sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=1+i$

$k=1$ $u_1=\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})=\sqrt{2}\cdot\left(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=-1+i$

$k=2$ $u_2=\sqrt{2}(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4})=\sqrt{2}\cdot\left(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)=-1-i$

$k=3$ $u_3=\sqrt{2}(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4})=\sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)=1-i$

3. 1) $\sin x+\sin 2x+\sin 3x+...+\sin nx$ 2) $\cos x+\cos 2x+\cos 3x+...+\cos nx$ **yig'indilarni hisoblang.**

Yechish. $\sin x+\sin 2x+\sin 3x+...+\sin nx=T$, $\cos x+\cos 2x+\cos 3x+...+\cos nx=S$ bo'lsin.

.U holda

$$S+iT=\alpha^2+\alpha^4+\alpha^6+...+\alpha^{2n}=\frac{\alpha^2\left(\alpha^{2n}-1\right)}{\alpha^2-1}=\alpha^2\cdot\frac{\alpha^n\left(\alpha^n-\alpha^{-n}\right)}{\alpha\left(\alpha-\alpha^{-1}\right)}=\alpha\cdot\frac{\alpha^n\cdot 2i\sin\frac{nx}{n}}{2i\sin\frac{x}{2}}=\alpha^{n+1}\cdot\frac{\sin\frac{nx}{n}}{\sin\frac{x}{2}}=$$

$$=\frac{\sin\frac{nx}{n}}{\sin\frac{x}{2}}\left(\cos\frac{n+1}{2}x+i\sin\frac{n+1}{2}x\right)=\frac{\sin\frac{nx}{n}\cdot\cos\frac{n+1}{2}x}{\sin\frac{x}{2}}+i\frac{\sin\frac{nx}{n}\cdot\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}$$

Bundan $\cos x+\cos 2x+\cos 3x+...+\cos nx=\dfrac{\sin\frac{nx}{n}\cdot\cos\frac{n+1}{2}x}{\sin\frac{x}{2}}$

$\sin x+\sin 2x+\sin 3x+...+\sin nx=\dfrac{\sin\frac{nx}{n}\cdot\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}$ kelib chiqadi.

4. 1) $\cos 5x$ ni $\cos x$ va $\sin x$ **orqali ifodalang.**

2) $tg5x$ ni tgx **orqali ifodalang.**

Yechish. Muavr formulasidan $\left(\cos x+i\sin x\right)^5=\cos 5x+i\sin 5x$ bo'ladi.

Nyuton binomi formulasidan

$$\left(\cos x+i\sin x\right)^5=\cos^5 x+5\cos^4 x(i\sin x)+10\cos^3 x\cdot i^2\sin^2 x+$$
$$+10\cos^2 x\cdot i^3\sin^3 x+5\cos x\cdot i^4\sin^4 x+i^5\sin^5 x=$$
$$=\cos^5 x+5i\cos^4 x\sin x-10\cos^3 x\cdot\sin^2 x-10i\cos^2 x\cdot\sin^3 x+5\cos x\cdot\sin^4 x+i\sin^5 x$$

Bundan $\cos 5x=\cos^5 x-10\cos^3 x\cdot\sin^2 x+5\cos x\cdot\sin^4 x$,
$\sin 5x=5\cos^4 x\sin x-10\cos^2 x\cdot\sin^3 x+\sin^5 x$ ekani kelib chiqadi.

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№40. Quyidagi misollar Muavr formulasi bilan yechilsin.

1) $(1-i)^6$ 2) $(2+i)^5$ 3) $\left(1+\cos\frac{\pi}{3}+i\sin\left(\frac{\pi}{3}\right)\right)^6$

№41. Ushbu 1) $\sqrt[4]{-1}$ va 2) $\sqrt[5]{1}$ ildizlarning barcha qiymatlari toping hamda ular radius-vektorlar bilan tasvirlang.

№42. $\sin^3 x$, $\sin^4 x$, $\cos^5 x$ x ga karrali burchakning trigonometrik funksiyalari orqali ifodalang.

№43. 1 ning barcha n- ildizlarini tekislikdagi tasviri muntazam n burchakning uchlari bo’lishini isbotlang.

№44. 1 ning barcha n- ildizlari geometrik progerssiya hosil qilishini isbotlang.

Asosiy formulalar

1. Kompleks son va kompleks sonlar to'plami.

$$z = x + iy \ (x + yi),\ i^2 = -1\ ,\ x = \operatorname{Re} z\ ,\ y = \operatorname{Im} z \qquad (1)$$

$$C = \{x + iy \mid x, y \in R, i^2 = -1\} \qquad (2)$$

2. Kompleks sonning moduli.

$$|z| = \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2} = \sqrt{x^2 + y^2} \qquad (3)$$

3. Qo'shma kompleks son.

$$\bar{z} = x - iy \qquad (4)$$

4. Algebraik shakldagi kompleks sonlar ustida amallar.

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

1) Tenglik munosabati. $z_1 = z_2 \Leftrightarrow \begin{cases} \operatorname{Re} z_1 = \operatorname{Re} z_2 \\ \operatorname{Im} z_1 = \operatorname{Im} z_2 \end{cases}$ yoki $\begin{cases} x_1 = x_2 \\ y_1 = y_2 \end{cases}$ (5)

2) Qo'shish va ayirish. $z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$ (6)

3) Ko'paytirish.

$$z_1 \cdot z_2 = (x_1 \cdot x_2 - y_1 \cdot y_2) + i(x_1 \cdot y_2 + y_1 \cdot x_2) \qquad (7)$$

$i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$ => $i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$

4) Bo'lish.

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \cdot \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2} \quad (z_2 \neq 0) \qquad (8)$$

5. Kompleks sonlarning xossalari

1^0 $z_1 + z_2 = z_2 + z_1$, $z_1 \cdot z_2 = z_2 \cdot z_1$ (kommutativlik)

2^0 $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ $(z_1 \cdot z_2) \cdot z_3 = z_1 \cdot (z_2 \cdot z_3)$ (assotsativlik)

3^0 $(z_1 + z_2) \cdot z_3 = z_1 \cdot z_3 + z_2 \cdot z_3$ (distributivlik)

4^0 $\overline{(\bar{z})} = z$ 5^0 $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

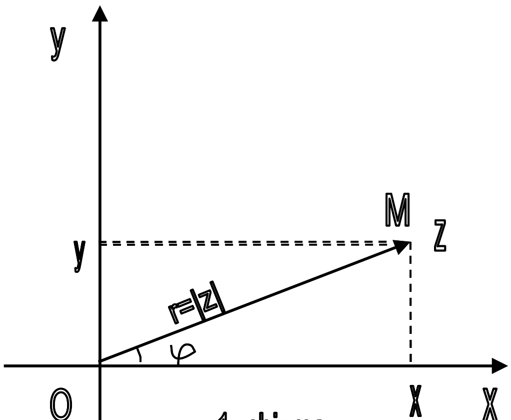
6^0 $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$ 7^0 $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ 8^0 $z \cdot \bar{z} = |z^2| = |z|^2 = |\bar{z}|^2 = x^2 + y^2$

$9^0 \quad \operatorname{Re} z = \frac{z+\bar{z}}{2} \qquad 10^0 \quad \operatorname{Im} z = \frac{z-\bar{z}}{2i}$

6. Kompleks sonning trigonometrik shakli.

$|\overrightarrow{OM}| = r = |z| = \sqrt{x^2+y^2} \geq 0$ z kompleks sonning modulini .

$\varphi = \angle(MOX)$ burchak z kompleks sonning **argumenti** deyiladi.



1- chizma

Kompleks sonning algebraik ko'rinishdan trigonometrik ko'rinishiga o'tish.

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \cos\varphi = \frac{x}{r} \\ \sin\varphi = \frac{y}{r} \end{cases} \tag{9}$$

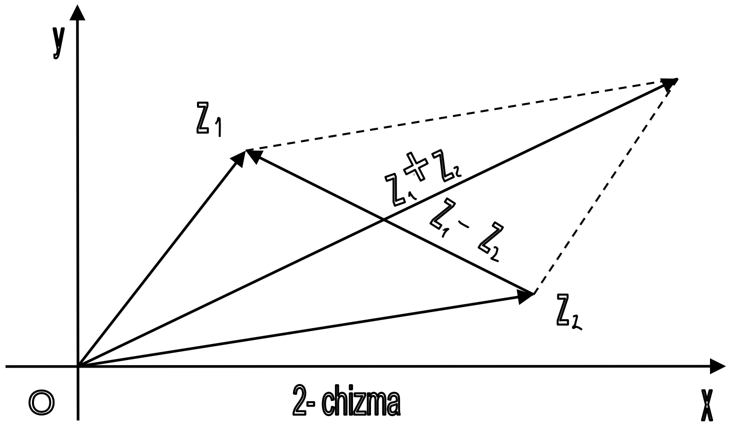
$$z = r(\cos\varphi + i\sin\varphi) \tag{10}$$

Kompleks sonning trigonometrik ko'rinishidan algebraik ko'rinishdan o'tish.

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \tag{11}$$

7. Trigonometrik shakldagi kompleks sonlar ustida amallar.

Kompleks sonlar ustida qo'shish va ayirish vektorlarni qo'shish va ayirish kabi hisoblanadi.(2-chizma)



2- chizma

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),\, z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

$$z_1 z_2 = r_1 r_2(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)) \tag{12}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)) \tag{13}$$

$$z_1 \cdot z_2 \cdot \ldots \cdot z_n = r_1 \cdot r_2 \cdot \ldots \cdot r_n(\cos(\varphi_1 + \varphi_2 + \ldots + \varphi_n) + i\ \sin(\varphi_1 + \varphi_2 + \ldots + \varphi_n)) \tag{14}$$

8. Trigonometrik shakldagi kompleks sonni darajaga ko'tarish va ildiz chiqarish. Muavr formulalari.

$$z = r(\cos\varphi + i\sin\varphi) \quad z^n = r^n(\cos(n\varphi) + i\sin(n\varphi)) \tag{15}$$

$$u_k = \sqrt[n]{z} = \sqrt[n]{r}(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}) \quad k = 0,1,\ldots,n-1 \tag{16}$$

9. Birning n-ildizi.

$$\sqrt[n]{1} = \cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n} = \alpha_k \quad k = 0,1,\ldots n-1 \tag{17}$$

u_0 z ning n- ildizlaridan biri bo'lsin.

$$u_k = u_0 \cdot \alpha_s \quad s,k = 0,1,\ldots n-1 \tag{18}$$

$$\alpha_s^{\ k} = \alpha_k \quad k = 0,1,\ldots n-1 \tag{19}$$

α_s **birning boshlang'ich ildizi.**

Masalan: $\alpha_1 = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$

10. Eyler formulasi.

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \tag{20}$$

$$e^{i\pi} = -1 \tag{21}$$

$$z = re^{i\varphi} \tag{22}$$

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