“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI HUZURIDAGI FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/31.03.2022 T/FM.10.04 RAQAMLI ILMIY KENGASH

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

URINOV SUNNATILLO XUDOYOR O‘G‘LI

NOCHIZIQLI ELEKTRODINAMIKANING QORA TUYNUKLAR ATROFIDAGI ASTROFIZIK JARAYONLARGA TA’SIRI (MODMAX NAZARIYASI MISOLIDA)

01.04.02 – Nazariy fizika
01.03.01 – Astronomiya

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

Toshkent – 2024

UO‘K 530.12:531.51

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertasiyasi avtoreferati mundarijasi

Содержание диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida B2024.3.PhD/FM1169 raqami bilan ro‘yxatga olingan.

Dissertatsiya Sharof Rashidov nomidagi Samarqand Davlat Universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, rus, ingliz (rezyume)) Ilmiy Kengashning internet sahifasida (www.ifar.uz) va “Ziyonet” axborot-ta’lim portalida (www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya avtoreferati 2024-yil “___” __________ kuni tarqatildi.
(2024-yil “___”_________dagi ___ raqamli reestr bayonnomasi)

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Qora oʻralar atrofida sodir boʻladigan turli astrofizik hodisalar bilan bogʻliq nazariy va astrofizik modellarni takomillashtirish zamonaviy nisbiylik astrofizikasining asosiy yoʻnalishlaridan biri hisoblanadi. Soʻnggi oʻn yillikda kuzatuv astronomiyasidagi muhim kashfiyotlar nazariy va matematik asoslarning rivojlanishiga turtki berdi, bu esa ixcham gravitatsion obyektlar yaqinidagi astrofizik voqealarni tushuntirish uchun zarur. Shu bilan birga, fundamental fizika sohasida yagona nazariyaga erishish yoʻlidagi sa’y-harakatlar joriy umumiy nisbiylik paradigmasidan tashqarida boʻlgan yangi alternativ yoki modifikatsiyalangan gravitatsiya nazariyalarini oʻrganishni talab etadi. Bu nazariyalarni kuzatuv va eksperimental ma'lumotlar bilan solishtirish, shuningdek, qora oʻralar yaqinida sodir boʻladigan optik va energetik jarayonlarni matematik simulyatsiya qilish orqali maydon model parametrlarining qiymatlarini aniqlashni oʻz ichiga oladi.

Umumiy nisbiylikdagi bir qator kamchiliklar va muvofiqliklarni bartaraf etish uchun alternativ va modifikatsiyalangan gravitatsiya nazariyalari zarur. Ushbu nazariyalar umumiy nisbiylik va kvant mexanikasi oʻrtasidagi tafovutlarni bartaraf etishga, qorongʻi materiya va qorongʻi energiya kabi hodisalarni tushuntirishga hamda qora oʻralar ichida gravitatsiyaning xatti-harakatlariga oid tushunchalarni oshirishga qaratilgan. Shu bilan birga, ushbu alternativ nazariyalarni oʻrganish bizning koinot haqidagi bilimlarimizda tubdan oʻzgarishlarga olib kelishi mumkin boʻlgan vaqt va boʻshliq va gravitatsiya mohiyatini yanada chuqurroq anglashga imkon beradi. Shunday qilib, ixcham obyektlar boʻyicha astrofizik tadqiqotlar doirasida egri boʻshliqni oʻrganish astronomik kuzatuvlarni dolzarb qiladi.

Soʻnggi yillarda mamlakatimizda fundamental va amaliy tadqiqotlarni, ayniqsa, istiqbolli yoʻnalishlardan biri boʻlgan astrofizika sohasidagi izlanishlarni rivojlantirishga katta e’tibor qaratilmoqda. Mamlakatimizda ilm-fan rivojida muvaffaqiyatga erishish uchun asosiy fundamental tadqiqot yoʻnalishlari va ularning amaliy qoʻllanilishlari 2022-2026 yillarda Oʻzbekiston Respublikasini yanada rivojlantirish strategiyasida[1] belgilab qoʻyilgan. Shu sababli, modifikatsiyalangan gravitatsiya nazariyalarini oʻrganish fundamental tadqiqotlar doirasidagi dolzarb masala boʻlib qolmoqda.

Mazkur dissertatsiya ishi quyidagi davlat tartibga soluvchi hujjatlarning vazifalariga mos keladi: 2017-yil 7-fevraldagi Oʻzbekiston Respublikasi Prezidentining PF-4947-sonli "Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha Harakatlar strategiyasi toʻgʻrisida"gi Farmoni, 2017-yil 18-fevraldagi Oʻzbekiston Respublikasi Prezidentining PR-2789-sonli "Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy-tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish boʻyicha chora-tadbirlar toʻgʻrisida"gi Qarori va boshqa hujjatlar.

[1] Oʻzbekiston Respublikasi Prizidentining 2022-yil 1-yanvardagi № PF-60 son farmoni “2022-2026 yillarda moʻljallangan Yangi Oʻzbekiston taraqqiyot strategiyasi toʻgʻrisida”

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Ushbu dissertatsiya ishi O‘zbekiston Respublikasi fan va texnikasini rivojlantirishning ustuvor yo‘nalishi II. “Energiya, energiya va resurslarni tejash”.

Muammoning o‘rganilganlik darajasi.

Qora o‘ralar atrofida massali va massasiz zarralarning harakati nazariy jihatdan ko‘plab olimlar tomonidan ko‘rib chiqilgan, jumladan, ingliz (R. Wald, J. Petterson), rus (A. Zaxarov, V. Frolov, I. Novikov), nemis (C. Laemmerzahl, J. Kunz, A. Grezenbach), argentinalik (L. Amarilla, E. Eiroa), chex (M. Kolos, J. Vrba), hindistonlik (N. Dadhich, S. Ghosh, P. Joshi, M. Patil) va boshqa olimlar. Biroq, ModMax nazariyasida qora tuynuk atrofidagi zarrachaning dinamikasi hali o‘rganilmagan. Xususan, ModMax modelida qora o‘ralar atrofida massali va massasiz zarralarning dinamikasini tizimli o‘rganish amalga oshirilmagan.

Fotonnning harakati astrofizik modeli umumiy nisbiylik nazariyasi doirasida mahalliy olimlar B.J. Ahmedov, A.A. Abdujabborov, B.A. Toshmatov, F.S. Atamurotov, J.R. Rayimbaev va boshqalar tomonidan, shuningdek, xorijiy olimlar R. Wald, S.R., F. de Felice, F. Sorge, L. Rezzolla, Z. Stuchlik, C. Bambi va boshqalar tomonidan ishlab chiqilgan. Ammo markaziy obyekt ModMax nazariyasi orqali tavsiflanganda astrofizik modelni takomillashtirish masalasi ochiq qolmoqda.

Shuningdek, qora tuynuk atrofidagi astrofizik va energetik jarayonlarga ModMax parametriva zaryad parametri ta’sirining o‘rganilmagani ham haligacha dolzarb bo‘lib qolmoqda. Qora tuynuk atrofidagi zarrachaning dinamikasini tavsiflovchi nazariy modellarning rivojlanishi va takomillashtirilishi ModMax nazariyasidagi model parametrlariga nisbatan cheklovlar olish uchun qo‘llanilishi mumkin.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan F-FA-2021-510 "Modifikatsiyalangan gravitatsiyada neytron yulduzlar yadroviy materiyasini tadqiq qilish" ilmiy loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi: ModMax gravitatsiya modelida qora o‘ralar yaqinida massali va massasiz zarralar dinamikasi hamda termodinamikasini tavsiflovchi astrofizik modellarning rivojlantirilishi va takomillashtirilishi.

Tadqiqotning vazifalari:

ModMax nazariyasida qora tuynuklarning gorizont tuzilishiga zaryad va ModMax parametrining ta’sirini o‘rganish;

qora tuynuk soyasining o‘lchami va shaklini ModMax parametriga bog‘liq ravishda tahlil qilish;

qora tuynuk soyasining deformatsiya parametri qiymatiga ModMax parametrlarining ta’sirini o‘rganish;

energiyaning chiqarilish tezligini, xususan, ModMax parametrlarining maksimal samarali potensial qiymatiga ta’sirini o‘rganish;

dyonik ModMax parametrlarining, masalan, zaryad va ModMax parametrining fotonlarning og‘ish burchagiga ta’sirini tahlil qilish;

plazma mavjudligida ModMax modifikatsiyalarining foton sferasi va soya radiusiga ta’sirini o‘rganish;

ichki va tashqi gorizontlarning radiuslari bir-biriga mos keladigan joyda aylanish, ModMax parametriva zaryad parametrlarining kritik qiymatlarini aniqlash;

barqaror aylana orbitaning eng ichki radiusining ModMax parametri va zaryadga bog‘liqligini o‘rganish;

ModMax qora tuynugi atrofidagi energiya chiqarish jarayonini tahlil qilish;

ModMax nazariyasidagi qora tuynukning entalpiyasi, Gibbs erkin energiyasi, Xoking harorati va qora tuynukning termodinamikasini to‘liq tahlil qilish.

Tadqiqot obyekti: astrofizik qora tuynuklar, massaga ega bo‘lgan va bo‘lmagan sinov zarralar, ModMax maydoni.

Tadqiqot predmeti: qora o‘ralar yaqinida test zarralarning dinamikasini o‘rganish uchun nazariy yondashuvlar, differensial tenglamalarni yechishning sonli va analitik usullari.

Tadqiqot usullari: hisoblash matematikasi usullari, nazariy astrofizika usullari, zamonaviy matematik fizika usullari, maydon va zarralar harakati uchun differensial tenglamalarni hisoblashning analitik va sonli usullari.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Ilk bor, ModMax nazariyasida qora o‘ra zaryadi, gravitatsiya va radial o‘zgaruvchan plazma parametrlari foton orbitasini va qora tuynuk soyasining hajmi kamaytirishi ko‘rsatildi.

Ilk bor, kuchsiz gravitatsion linzalanish tahlili Dyonik ModMax maydoni og‘ish burchagini kamaytirishi aniqlandi.

aylanuvchi qora tuynukning ergosfera qalinligi zaryad va aylanish parametrlari hisobiga ortishi, aksincha ModMax parametri ergosferani ingichkalashtishi ko‘rsatildi.

Ilk bor, ModMax nazariyasida qora tuynuk atrofidagi ichki barqaror aylana orbita (ISCO) radiusi ortishi topildi.

Tadqiqotning amaliy natijalari quyidagilarni o‘z ichiga oladi:

Birinchi marta zaryadlangan ModMax qora tuynugining termodinamikasi, jumladan, entalpiya, Xoking harorati va Gibbs erkin energiyasi batafsil o‘rganildi. ModMax parametrini oshishi bilan entalpiya va Gibbs erkin energiyasi kamayishi, Xoking harorati esa oshishi ko‘rsatildi.

Deformatsiya parametri va soyaning o‘rtacha radiusi tahlil qilindi. Tadqiqot natijalariga ko‘ra, ModMax parametrini oshishi bilan soya radiusi kattalashishi, lekin deformatsiya parametri kamayishi aniqlandi.

Hodisalar Gorizonti Teleskopi ma’lumotlariga asosan, ModMax nazariyasida qora tuynuk soyasi radiusiga mos keladigan qora tuynuk zaryadining maksimal qiymatlari hisoblandi.

Tadqiqot natijalarining ishonchliligi. Zamonaviy tasdiqlangan matematik fizika, hisoblash matematikasi, nisbiylik nazaryasi va astrofizika usullaridan foydalanish bilan ta’minlangan. Natijalar umumiy nisbiylik nazaryasi va nazariy fizika matematik apparati doirasida olingan. Shuningdek, zamonaviy sonli va analitik hisoblash usullari qo‘llanilgan bo‘lib, natijalar mavjud kuzatish ma’lumotlari va boshqa mualliflar natijalari bilan taqqoslangan. Dissertatsiya xulosalari, ixcham obyektlarning astrofizikasi asosiy qoidalariga mos keladi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati.

Tadqiqot natijalarining ilmiy ahamiyati shundaki, ModMax gravitatsiya modelidagi qora o‘ralar yechimlarining tahlili ushbu model doirasidagi turli astrofizik jarayonlarni o‘rganish uchun foydali vosita bo‘lishi mumkin.

Tadqiqot natijalarining amaliy ahamiyati esa shundaki, ular modifikatsiyalangan gravitatsiya nazariyalarini sinab ko‘rish va qora tuynuk parametrlarini qora tuynuklarning soyasi kuzatishlari orqali cheklovlar olishda yordam berishi mumkin.

Tadqiqot natijalarining qo‘llanilishi: ModMax nazariyasida foton va zarra harakati uchun ishlab chiqilgan nazariy modellar quyidagi maqsadlarda qo‘llanildi:

Zarra va fotonlar harakati bo‘yicha olingan ilmiy natijalar Shanxaydagi Fudan universiteti (FU, Xitoy, 2024-yil 23-sentabrdagi ma’lumotnomasi) olimlari tomonidan foydalanilgan;

Birinchi marta zaryadlangan ModMax qora tuynugining termodinamikasi, jumladan, entalpiya, Xoking harorati va Gibbs erkin energiyasi batafsil o‘rganilib, ModMax parametrini oshishi bilan entalpiya va Gibbs erkin energiyasining kamayishi, Xoking haroratining oshishi aniqlangan.

Zarralarning dinamikasi bo‘yicha natijalar xorijiy tadqiqotchilar tomonidan yuqori impact faktorli xorijiy jurnallarda (Physics Letters B, Volume 854, id.138758, Web-Sc, IF-4.4; Communications in Theoretical Physics, Volume 76, Issue 8, id.085402, Web-Sc, IF-2.4; Pramana, Volume 98, Issue 3, id.92, Web-Sc, IF-2.219 va boshqalar) chop etilgan 5 dan ortiq ilmiy maqolalarda qora tuynuk atrofidagi zarra harakatining ta’sirini tasvirlash uchun qo‘llanilgan.

Bu natijalar modifikatsiyalangan gravitatsiya nazariyalari doirasida kompakt ob'ektlar atrofidagi zarra dinamikasi haqida ma’lumot olishga imkon berdi.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqotning asosiy natijalari 1 ta xalqaro va 3 ta Respublika miqyosidagi ilmiy-amaliy konferensiyalarda ma’ruzalar qilinib, tegishli muhokamalardan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Tadqiqot mavzusi bo‘yicha jami 12 ta ilmiy ish, jumladan O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish uchun tavsiya etgan ilmiy nashrlar ro‘yxatida 6 ta ilmiy maqola (1 ta xorijiy jurnalda) chop etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, to‘rt bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiya hajmi 81 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismida mavzuning dolzarbligi va zarurligi, tadqiqotning respublika ilm-fan va texnologiyalarini rivojlantirishning ustuvor yo‘nalishlariga mosligi, muammoning o‘rganilganligi darajasi, dissertatsiya o‘tkazilgan oliy ta’lim muassasasining tadqiqot rejalari bilan bog‘liqligi, tadqiqot maqsadi, vazifalari, obyekti, predmeti, usullari, ilmiy yangiligi, amaliy natijalari, natijalar ishonchliligi, natijalarining ilmiy va amaliy ahamiyati, natijalarni amaliyotga joriy etish, natijalarni tasdiqlash, natijalarni nashr etish, shuningdek, dissertatsiyaning tuzilishi va hajmi haqida qisqacha ma’lumot keltirilgan.

Birinchi bob **“Aylanuvchi zaryadlangan ModMax qora tuynukning soyasi”** deb nomlanib, zaryad (Q) va ModMax parametri(γ) parametrlarining aylanuvchi zaryadlangan ModMax qora tuynuklarining gorizont va silueti xususiyatlariga ta’siri muhokama qilinadi. Bundan tashqari, ushbu tadqiqot nol geodezikalarining xatti-harakatini o‘rganadi, bu esa qora tuynuk siluetining ko‘rinadigan shaklini yaxshiroq tushunishimizga yordam beradi, shuningdek, yuqorida qayd etilgan parametrlar ta’sir etadigan deformatsiya parametri va siluetning taxminiy radiuslarini o‘rganadi va ma’lum zaryad parametrlarining Event Horizon Telescope (EHT) ma’lumotlariga mos keluvchi qiymatlari aniqlanadi.

Shuni ta’kidlash joizki, deformatsiya parametri va siluetning taxminiy radiuslarini o‘rganish natijasida, γ oshishi siluet radiusining (R_s) ortishiga olib kelsa, shu bilan birga deformatsiya darajasining (δ_s) kamayishiga sabab bo‘ladi. Aksincha, zaryadning oshishi R_s ni kamaytiradi, buning natijasida δ_s esa oshadi. Shuningdek, qora tuynuk parametrlarining samarali potensialga va energiya chiqarishga ta’sirini tahlil qilamiz va energiya chiqarish tezligining maksimal qiymati ModMax parametri(γ) oshishi bilan kamayishini, zaryad parametrlarining (Q) yuqori qiymatlari bilan esa oshishini aniqlaymiz.

ModMax elektromagnitizmining Lagrangiani quyidagicha ifodalanadi:

$$L_{ModMax}(x,y) = -x \cosh\gamma + \sqrt{x^2 + y^2} \sinh\gamma \qquad (1)$$

bu yerda elektromagnit Lorentz invariantlari quyidagicha aniqlanadi: $x := \frac{1}{4} F_{\mu\nu}F^{\mu\nu}, y := \frac{1}{4} F_{\mu\nu}\tilde{F}^{\mu\nu}$. Elektromagnit tensor esa $F_{\mu\nu}$ va uning duali $\tilde{F}_{\mu\nu} := \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$ bilan berilgan. ModMax Lagrangianidan foydalanib, Plebański dual o‘zgaruvchisi quyidagicha aniqlanadi:

$$P_{\mu\nu} := -L_x F_{\mu\nu} - L_y \tilde{F}_{\mu\nu}$$

uning duali esa quyidagicha:

$\tilde{P}_{\mu\nu} = \left[\cosh\gamma - \frac{x}{(x^2+y^2)^{1/2}} \sinh\gamma\right] \tilde{F}_{\mu\nu} + \frac{y\sinh\gamma}{(x^2+y^2)^{1/2}} F_{\mu\nu}$#(3) Biz sferik simmetrik qora tuynuk (BH) yechimini quyidagi Boyer-Linquist koordinatalarida ifodalaymiz:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (4)$$

Bu yerda \(f(r)\) quyidagicha aniqlanadi:

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2 e^{-\gamma}}{r^2} \quad (5)$$

Bu formulalar ModMax nazariyasidagi qora tuynuklarning xususiyatlarini tushunishga yordam beradi.

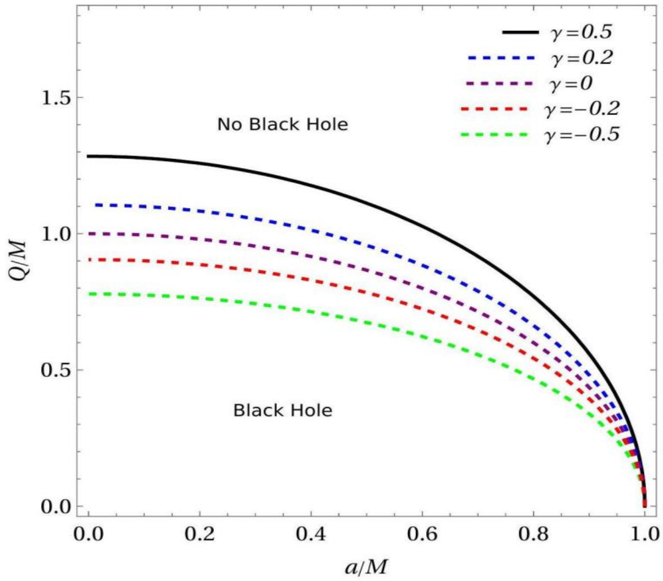
Aylanuvchi ModMax qora tuynugining (BH) atrofidagi bo‘shliq va vaqt quyidagi chiziqli element bilan ifodalanishi mumkin:

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a sin^2\theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

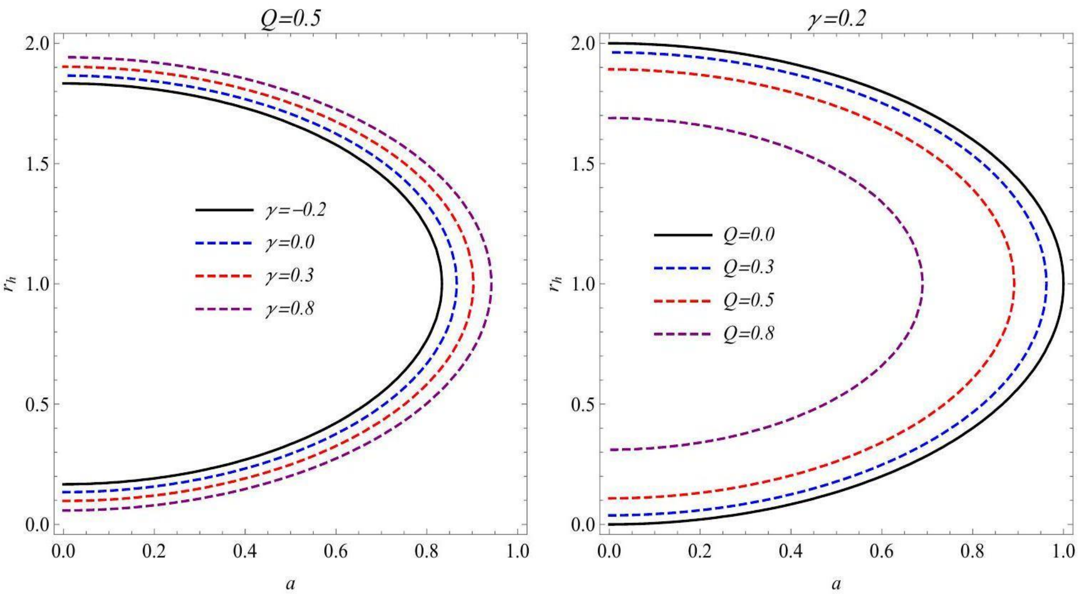
Bu yerda metrik funksiyalar quyidagicha aniqlanadi:

$$\Sigma = r^2 + a^2 cos^2\theta \#(7)\ \Delta = r^2 + a^2 - 2Mr + Q^2 e^{-\gamma} \#(8)$$

1-rasmda separatrix chiziqlari ekstremal qora o'ralar $(r_+ = r_-)$ uchun mos keladigan chegarani ko'rsatadi va qora o'ralar hududini qora o'ralar yo'q hududidan ajratadi. Rasmga ko'ra, γ parametrining oshishi bilan qora o'ralar hududi kengayadi va qora o'ralar yo'q hududi torayadi.

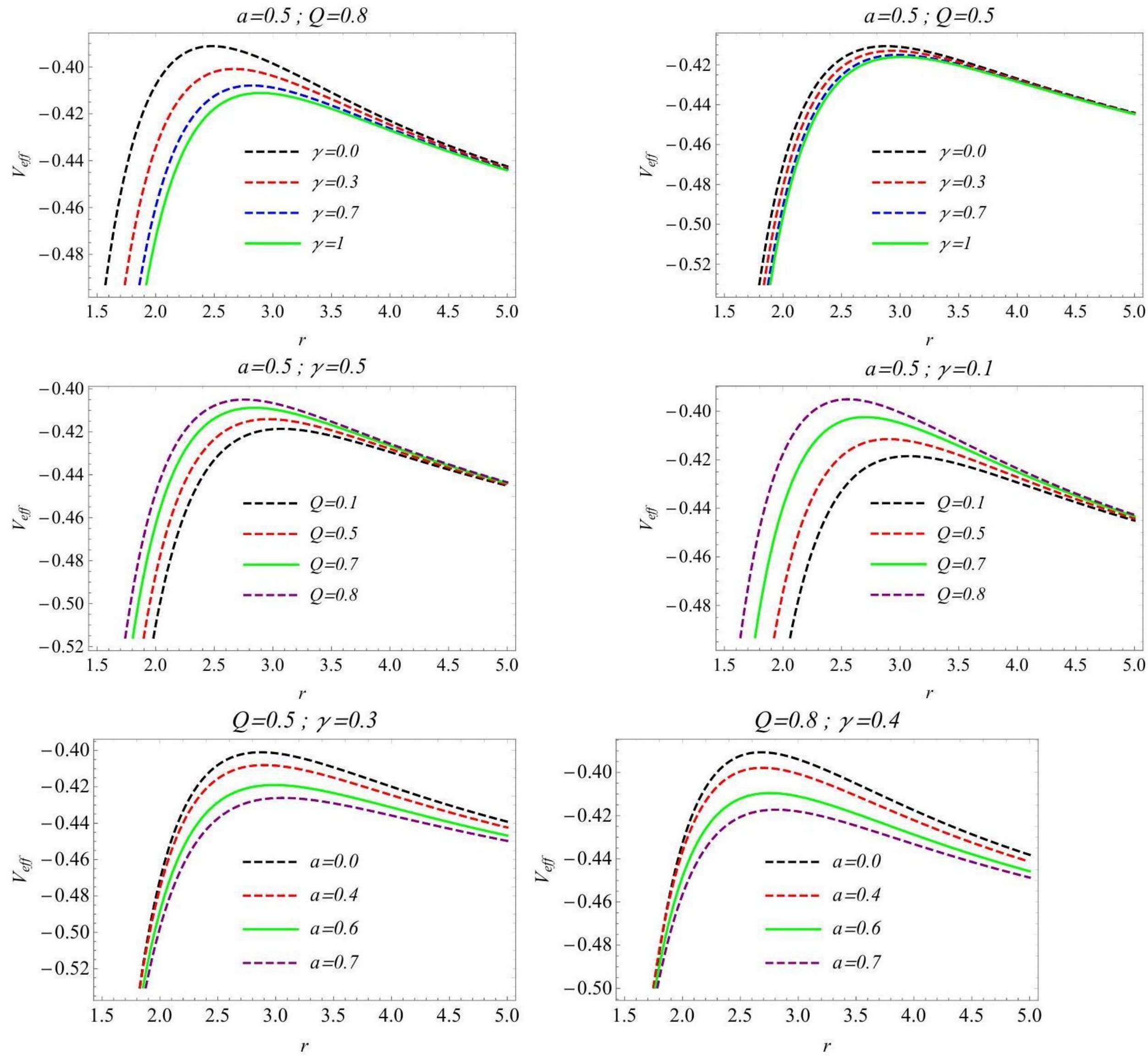


1-rasm. Separatrix chiziqlari ekstremal qora o'ralar (BH) bilan yo'q qora o'ralar (no BH) o'rtasidagi chegarani ko'rsatadi, bu esa tanlangan γ parametrining qiymatlariga mos keladi.



2-rasm. Gorizont radiusi r_h ning qora tuynuk (BH) aylanish parametr aaa ga bog'liqligi, zaryad Q va ModMax parametriγ uchun belgilangan qiymatlar asosida ko'rsatilgan. Bu yerda $M = 1$.

Qora tuynuk (BH) gorizonti uchun raqamli echimlarning grafik ifodasi Rasm 2 da ko‘rsatilgan. Chap va o‘ng panelda gorizontning ichki va tashqi radiuslari spin parametri bo‘yicha turli γ va Q qiymatlari uchun Q=0.5 va γ=0.2 parametrlarining belgilangan qiymatlari asosida ko‘rsatilgan. Rasm 2 dan ichki (Cauchy) va tashqi gorizont radiuslari haqida ma'lumot olish mumkin: Q zaryadini va γ ekranlash omilini o‘zgartirish orqali radiusning qanday o‘zgarishini ko‘rish mumkin. Rasm 2 ga ko‘ra, gorizont radiusi Q=0.5 uchun γ oshishi bilan ortadi, va γ=0.2 uchun esa radius Q parametrining oshishi bilan kamayadi.



3-rasm. Grafika radially effektiv potensial V_{eff} ning turli a, Q, va γ. qiymatlari uchun bog‘lanishini ko‘rsatadi. Bu yerda $M = 1$.

Hamilton-Jacobi formulasi yordamida ModMax nazariyasidagi BH atrofidagi fotonning harakat tenglamalarini olish mumkin. ModMax nazariyasida fotonning radial harakat tenglamasini dr/dτ ifodasidan foydalanib quyidagicha yozish mumkin:

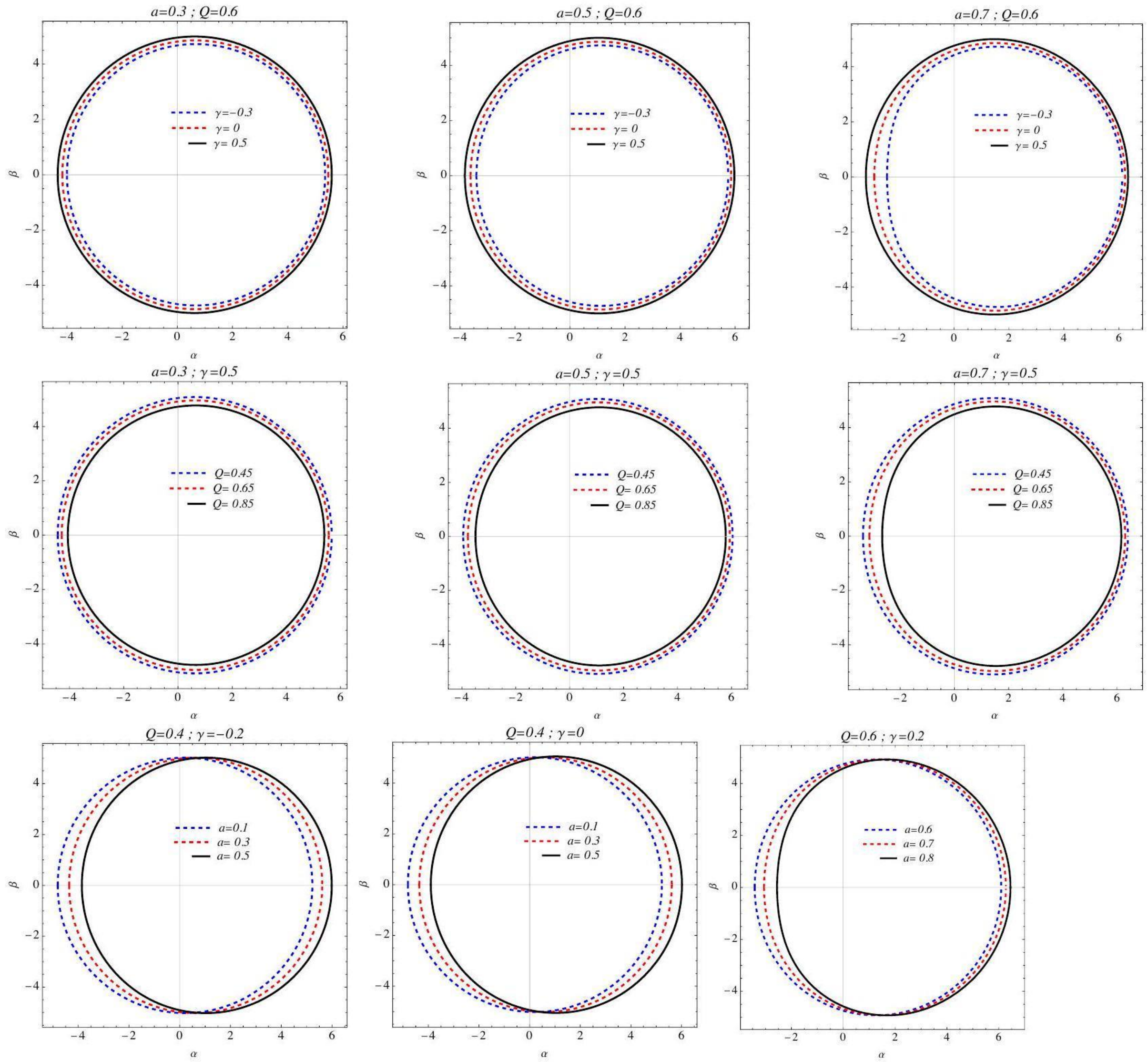
$$\left(\Sigma \frac{dr}{d\tau}\right)^2 + V_{eff} = 0\#(9)$$

Bu yerda ekvatorial tekislikda (θ=π/2) radial harakatning effektli potensiali quyidagicha ifodalanadi:

$$V_{eff} = \frac{\Delta(K + (L - aE)^2) - \left((a^2 + r^2)E - aL\right)^2}{2r^4} \#(10)$$

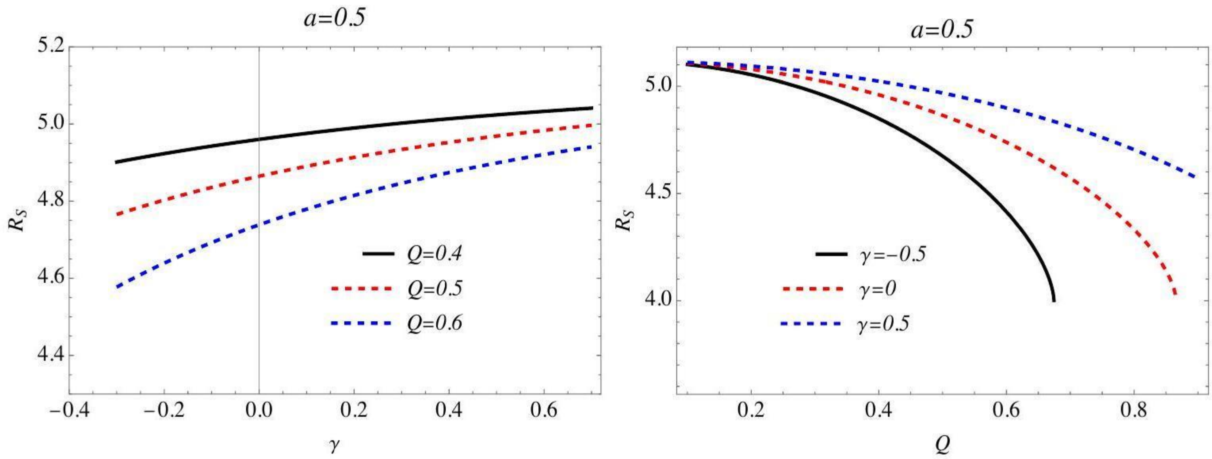
Yuqoridagi (10) ifodadan foydalanib, ModMax nazariyasida turli parametrlar a, Q va γ uchun radius koordinatasiga bog'liq effektli potensialning grafiklarini chizdik. Rasm 3 dan foton orbitlariga nisbatan effektli potensial V_{eff} ning umumiy xulq-atvori qora tuynuk parametrlarining turli qiymatlari bilan bog'liq ekanligi ko'rinadi. Shuningdek, belgilangan γ va a uchun zaryad Q oshgani sari foton orbitiga tegishli effektli potensialning eng yuqori qiymatlari o'sib, chapga siljishini ham ko'rishimiz mumkin. Natijada, belgilangan γ ikki potensial chiziq orasidagi hududni qisqartiradi. ModMax parametri γ va spin parametri a qiymatlari oshganda, effektli potensialning eng yuqori qiymati pasayadi va belgilangan a va Q, Q va γ uchun o'ngga siljishini ham ko'rish mumkin.

$$r + 2\frac{\Delta}{\Delta'^2}(\Delta' - r\Delta'') > 0\#(11)$$

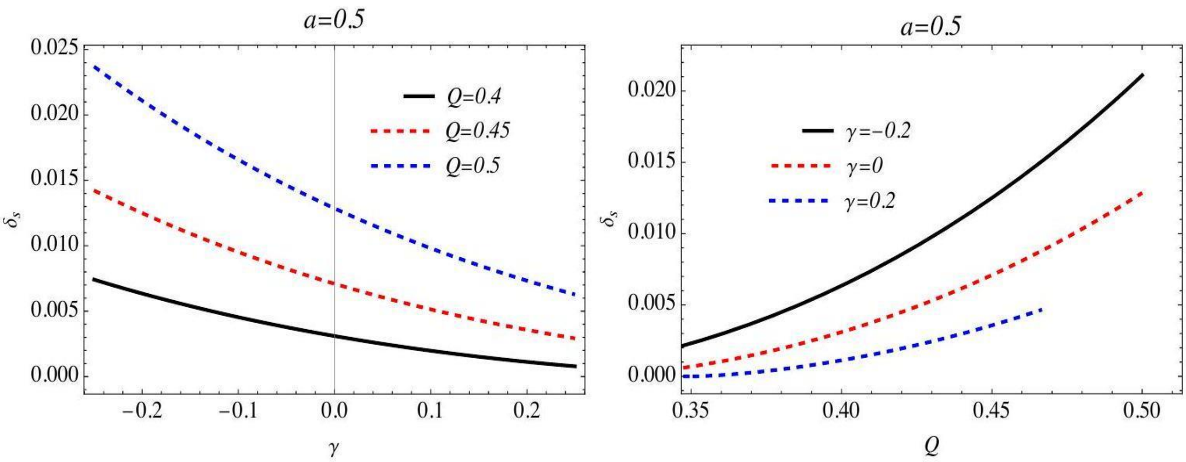


4-rasm. Grafiklar ModMax parametrlarining turli qiymatlari uchun qora tuynukning soyasini ko'rsatadi. Bu yerda *M=1*.

4-rasmda qora tuynukning soyasi zaryadlangan qora tuynukning ModMax parametrlarining belgilangan qiymatlari: aylanish a, zaryad Q va ModMax parametriγ uchun koʻrsatilgan. Olingan natijalardan koʻrinib turibdiki, qora tuynuk soyasining oʻlchami γ parametrining qiymati oshgani sari kengayadi (yuqori panel) va a va Q parametrlarining belgilangan qiymatlari uchun. Boshqa tomondan, Q parametrining oshishi bilan qora tuynuk soyasining oʻlchami kamayadi (oʻrta panelda a va γ belgilangan). Pastki panel esa qora tuynuk soyasining a spin parametrining oshishi bilan kamayishini koʻrsatadi (Q va γ belgilangan). Bundan tashqari, qora tuynuk soyasining shakli BH spin parametrining oʻsishidan oʻng tomonga deformatsiyalanadi.



5-rasm. Zaryadlangan aylanadigan ModMax qora tuynuk soyasining oʻrtacha radiusi R_s ning uning parametrlariga bogʻliqligi. Yuqori panelda ModMax parametriγ ning a parametriga va tanlangan Q ga nisbatan bogʻliqligi koʻrsatilgan. Pastki panelda esa Q parametrining a ga va tanlangan γ ga nisbatan bogʻliqligi koʻrsatilgan. Bu yerda $M = 1$.



6-rasm. Zaryadlangan aylanadigan ModMax qora tuynuk soyasining deformatsiya (ogʻish) δ_s parametrining uning parametrlariga bogʻliqligi. Yuqori panelda ModMax parametriγ ning a parametriga va tanlangan Q ga nisbatan bogʻliqligi koʻrsatilgan. Pastki panelda esa zaryad Q ning a ga va tanlangan γ ga nisbatan bogʻliqligi koʻrsatilgan. Bu yerda $M = 1$.

32 cm	$\gamma, a = 0.5$	$Q, a = 0.5$
M 87*	-0.5 0 0.5	0.46114 0.59212 0.76293
32 cm SgrA*	-0.5 0 0.5	0.672056 0.8629381 1.08034

Jadval 1: M87* va SgrA* uchun turli belgilangan γ ga nisbatan kuzatuv ma'lumotlaridan olingan minimal radius uchun ba’zi Q qiymatlari.

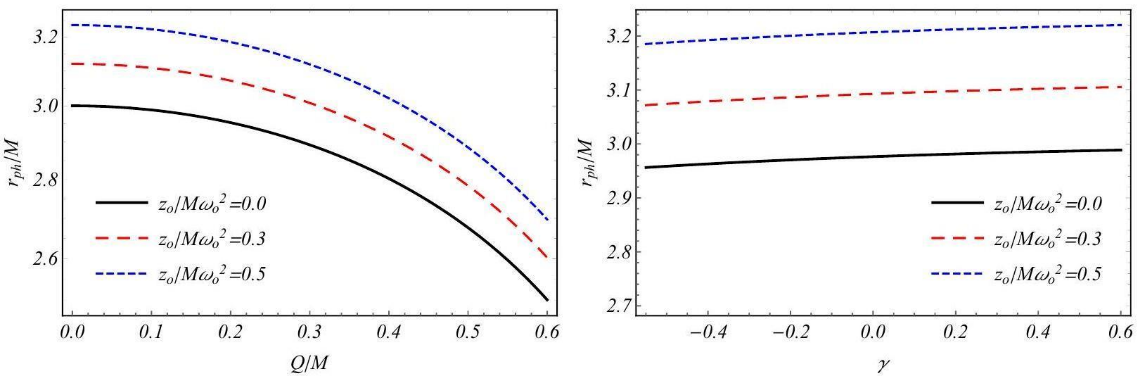
5-rasmda R_s ko‘rinadigan parametri qora tuynukning γ va zaryad Q parametrlariga bog‘liq sifatida tasvirlangan. O'rtacha radius R_s qora tuynuk soyasining ModMax parametriγ oshishi bilan ortishini (yuqori panel) va zaryad Q oshishi bilan kamayishini (pastki panel) ko‘rish mumkin. Shuningdek, 6-rasmda (yuqori panel) kamroq deformatsiyalangan bo‘lib, γ o‘sishi bilan δ_s kamayishini va (pastki panel) soyasining shakli zaryad Q oshishi bilan ko‘proq deformatsiyalanganini (Q o‘sishi bilan δ_s oshishini) kuzatamiz.

M87* va SgrA* uchun EHT hamkorligi tomonidan taqdim etilgan ma'lumotlardan foydalanib, biz metrikaga muvofiq γ ning turli belgilangan qiymatlari uchun Q zaryadi uchun yuqori chegarani topishimiz mumkin. Biz quyidagi ifodadan foydalanib, soyani diametrini aniqladik:

$$d_s = \frac{D\theta}{M} \#(12)$$

bu yerda θ, D va M qora tuynukning burchak diametri, qora tuynukning Yerdan masofasi va qora tuynuk massasi mos ravishda. M87* uchun bu qiymatlar $\theta_{M87*} = 42 \pm 3\mu\, s$, $D_{M87*} = 16.8Mpc$ va $M_{M87*} = 6.5 \pm 0.90 \times 10^9 M_{\odot}$, SgrA* uchun esa $\theta_{SgrA*} = 48{,}7 \pm 7\mu$, $D_{SgrA*} = 8277 \pm 33pc$ va $M_{SgrA*} = 4.3 \pm 0.013 \times 10^6 M_{\odot}$. Qora tuynuk soyasining diametrlarini quyidagi ma'lumotlar asosida hisobladik: $d_s^{M87*} = (11 \pm 1.5)M$ va $d_s^{SgrA*} = (9.5 \pm 1.4)M$. Biz bilamizki, qora tuynuk soyasining radiusini quyidagi ifoda yordamida topish mumkin: $d_s = 2R_s$. Hisoblangan minimal radiuslar M87* va SgrA* uchun mos ravishda $R_s^{M87*} = 4.75M$ va $R_s^{Sgr\,A*} = 4.05M$. 1-jadvalda belgilangan ekranlash faktori γ va $a = 0.5$ uchun zaryad parametri Q ning baholangan maksimal qiymatlari hisoblangan bo‘lib, ular taqdim etilgan ma'lumotlardan olingan radiuslarga mos keladi. O‘sib borayotgan γ bilan maksimal Q qiymati oshishini ko‘rish mumkin va bu 5-rasmga muvofiq keladi.

Ikkinchi bob, **"Plazmada dyonik ModMax qora tuynuki atrofidagi zaif gravitatsion linzalanish"** deb nomlangan, biz qora tuynuk atrofidagi zaif gravitatsion linzalanish ta'sirini o‘rganib, Dyonik ModMax (DM) fazosidagi soyani radiusini aniqladik. Shuningdek, biz noaniq plazma taqsimoti bo‘lgan modellarni ham ko‘rib chiqdik. Turli gravitatsion linzalanish modellari uchun biz vakuum linzalanishiga nisbatan plazmadagi gravitatsion ta'sir va plazma noaniqligi tufayli bo‘lgan tuzatishlarni taqqosladik. Issiq gaz holatida gravitatsion ta'sirni aniqlash mumkinligini ko‘rsatdik.



7-rasm. Q (chap panel) va γ (o‘ng panel) nisbatan r_{ph}/M ning o‘zgarishini tasvirlaydi, bu galaktika klasterining gravitatsion maydoni uchun $\omega_p^2 = z_0/r$ holatidir.

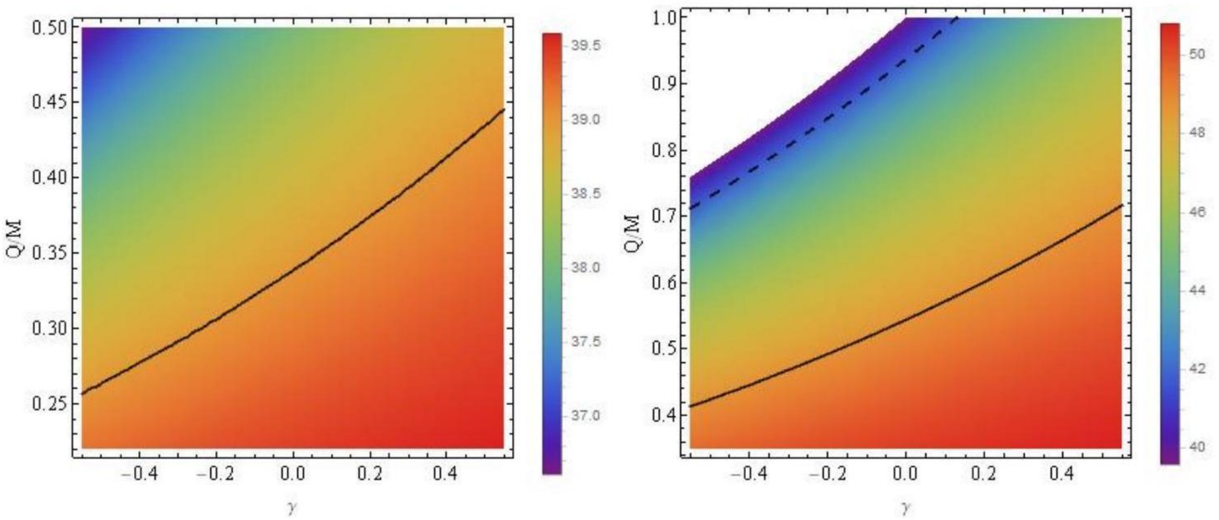
Biz DM da qora tuynuk atrofidagi fotonlar orbitalaridan boshladik. Bundan tashqari, biz bunday qora tuynuk atrofidagi soyani va gravitatsion zaif linzalanishni o‘rganamiz. EHT loyihasining M87* va SgrA* bo‘yicha kuzatuv ma'lumotlaridan foydalangan holda, biz DM gravitatsiyasida parametrni cheklab qo‘ydik. Keyinchalik kuzatuvlar bilan bog‘lash uchun, biz linzalanish natijasida shakllangan tasvirlarning kattalashtirish va joylashishini o‘rganamiz, va nihoyat turli galaktikalar yaqinidagi manbalar uchun zaif og‘ish burchagi va kattalashtirishni o‘rganamiz.

Biz bir jinsli bo‘lmagan plasma mavjudligida foton sferalarini o‘rganamiz, bunda plasma chastotasi quyidagi oddiy kuch-qonuniga mos kelishi kerak:

$$\omega_p^2(r) = \frac{z_0}{r} \#(13)$$

bu yerda z_0 - erkin parametr. Kuch-qonun modelining asosiy xususiyatlarini tahlil qilish uchun biz quyidagi holatlarga cheklanib qolamiz: $\omega_p^2(r) \sim \frac{1}{r}$ - manfiy massa divergent linzani to‘g‘ri takrorlaydigan doimiy. (4) va (13) - tenglamalardan foydalanib, biz noxil plasma muhitida foton sferasining radiusini hisoblaymiz, bu esa 7-rasmda ko‘rsatilgan. Ushbu profil, *Q/M* parametri oshgani sayin, foton sferasining radiusi kamayishini ko‘rsatadi. Biroq, γ qiymatini oshirish foton sferasining kattalashishiga olib keladi. Boshqa tomondan, plasma muhitining ta'siri $\omega_p^2(r) = z_0/r$ qonuniga amal qilgan taqdirda foton sferasining radiusini kengaytiradi, qora tuynuk atrofida plasma mavjudligi foton radiusini biroz qisqartiradi. Bundan tashqari, $\omega_p^2(r) = z_0/r$ holatidagi foton radiusi va $\omega_p^2(r) =$ const holatidagi plasma o‘rtasidagi farq yetarlicha kichik. Bu esa qora tuynuk atrofidagi g‘ovak plasma bilan bir xil plasma orasida test qilish va farqlashni juda qiyinlashtirishi mumkinligini ko‘rsatadi.

Biz supermassive qora o‘ralar M87* va SgrA* ni sferik simmetrik va statik deb hisoblaymiz. Biroq, Event Horizon Telescope (EHT)dan olingan kuzatuvlar bu taxminni to‘liq qo‘llab-quvvatlamaydi. Shunga qaramay, tadqiqot EHT loyihasi ma'lumotlaridan foydalangan holda Q va γ parametrlariga cheklovlarni o‘rganadi. Q va γ parametrlarini aniqlash uchun EHT loyihasi supermassive qora o‘ralar M87* va Sgr A* ning qora tuynuk soyalaridan olingan kuzatuv ma'lumotlaridan foydalanadi. M87 galaktikasidagi qora tuynukning soyasining burchak diametri, quyosh tizimidan masofa va qora tuynukning massasi quyidagicha: $\Omega_{M87*} = 42 \pm 3\mu as, D = 16.8 \pm 0.8Mpc,$ va $M_{M87*} = (6.5 \pm 0.7) \times 10^9 M_\odot$ mos ravishda. Sgr A* uchun EHT loyihasi tomonidan yaqinda olingan ma'lumotlar: $\Omega_{SgrA*} = 48.7 \pm 7\mu as, D = 8277 \pm 9 \pm 33pc$ va $M_{SgrA*} = (4.297 \pm 0.013) \times 10^6 M_\odot$. 8-rasmda Dyonic ModMax qora tuynukining fazo-vaqt parametrlarini kuzatishimiz mumkin.



8-rasm. M87* (chap panel) va Sgr A* (o‘ng panel) qora tuynuklarining kuzatuv soyasi burchak diametri orqali parametrlarni cheklash. Qora chiziqlar M87* uchun qora tuynuk soyasi burchak diametri $\theta_{sh} = 39\mu a$ va Sgr A* uchun $\theta_{sh} = 41.7\mu as$ (o‘ng panel) mos ravishda.

Ushbu hududlar M87* (chap panel) va Sgr A* (o‘ng panel) qora tuynuklarining soyasi 1-σ cheklovini qanoatlantiradi va ma'lum qiymatlarga ega bo‘lib, ular kuzatishlar bilan mos kelishi mumkin. Birga keltirilgan rasm ushbu qiymatlarni parametrik fazoda ko‘rsatadi. Qora chiziqlar M87* uchun qora tuynuk soyasi burchak diametri $\theta_{sh} = 39\mu a$ (chap panel) va Sgr A* uchun $\theta_{sh} = 41.7\mu a$ (o‘ng panel) mos ravishda. Bu shuni anglatadiki, $Q/M = 1$, ga yetganda, Sgr A* (o‘ng panel) uchun $\gamma \geq 0$ bo‘lishi kerak.

Uchinchi bob **"ModMax qora o‘ralar atrofidagi zarrachalar dinamikasi"** deb nomlangan bo‘lib, biz aylanayotgan ModMax qora tuynugi horizonti va ergosferasi tuzilishini o‘rganamiz. Sinov zarrachalarining harakati ichki eng barqaror aylana orbitasi (ISCO) va samarali potensialning xususiyatlarini o‘rganish orqali tahlil qilinadi. Ushbu tadqiqotda aylanayotgan ModMax qora tuynugining turli qiymatdagi screening omili γ va Q zaryadini o‘rganamiz.

Bizning asosiy natijalarimizning qisqacha bayoni:

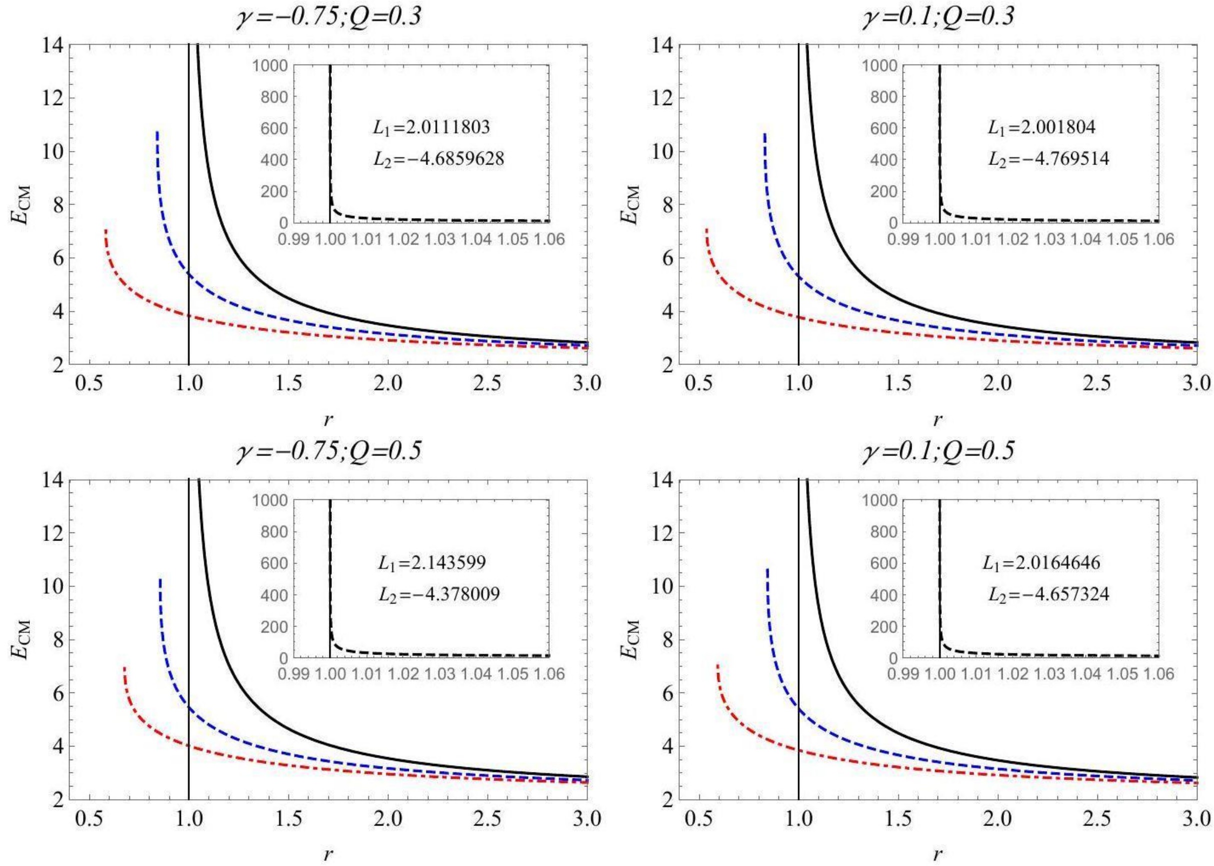
Qora tuynuk va qora tuynuk bo‘lmagan hududlar γ va Q parametrlarini ishlatib ko‘rsatildi. γ oshgani sayin, zaryadning ruxsat etilgan qiymatlari ham oshadi, Q zaryadi oshganda esa γ uchun ruxsat etilgan hudud torayadi.

Ichki va tashqi horizontlarning radiuslari bir-biriga to‘g‘ri keladigan kritik qiymatlar $a = a_\epsilon, \gamma = \gamma_\epsilon$ va $Q = Q_\epsilon$ aniqlangan. Qora tuynuk parametrlarining boshqa qiymatlarida, ikki horizont (Cauchy va voqea) mavjud yoki umuman qora tuynuk yo‘q.

Qora tuynukning ergosferasining qalinligi Q va a oshgani sayin oshadi, lekin γ oshganda, u ingichka bo‘ladi.

Qora tuynukning eng ichki barqaror aylana orbita (ISCO) radiusi γ va Q ga boshqacha ta'sir qiladi: γ oshgani sayin, ISCO radiusi oshadi, Q zaryadi oshganda esa, u kamayadi.

Umuman olganda, tadqiqotimiz zaryadlangan ModMax qora tuynukining dinamikasi va termodinamikasining turli jihatlarini yoritib, uning chuqur tushunilishiga hissa qo‘shadi.

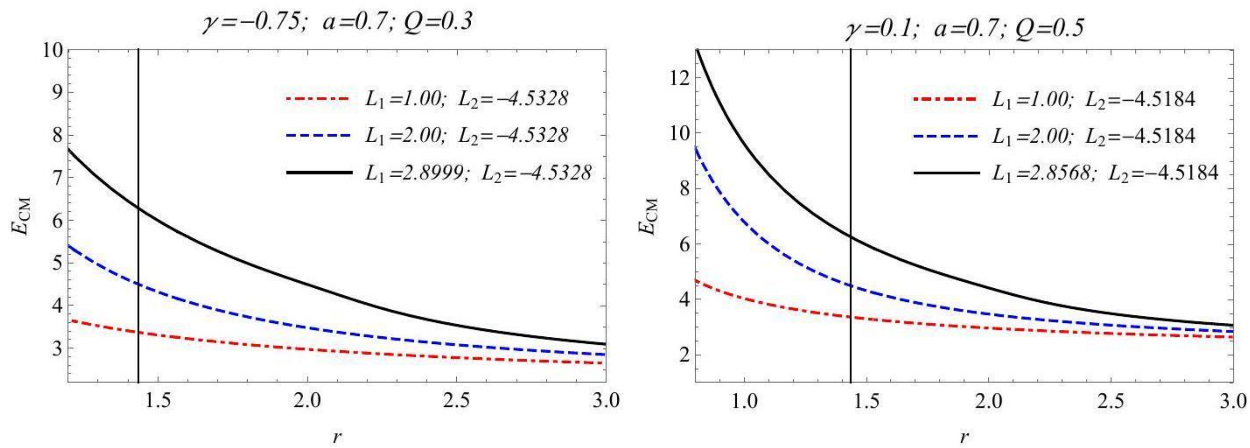


9-rasm. Ekstremal qora tuynuk uchun radial koordinata r ga bog‘liq bo‘lgan markaziy massaning energiyasi E_{CM} ning turli screening faktor γ, burchak momenti L va zaryad Q qiymatlari uchun ta'siri.

9-Rasmda markaziy massaning energiyasi E_{CM} va radial koordinat r ning screening faktori γ, burchak momenti L va zaryad Q ning turli qiymatlari uchun bog‘lanishini ko‘rsatadi. Rasmda qora tuynuk yaqinida CM energiyasi keskin oshishi va cheksizlikka borishi ko‘rinadi.

To‘rtinchi bob "ModMax qora tuynuklari atrofidagi termodinamika va energetik jarayonlar" deb nomlangan bo‘lib, qora tuynukning entalpiya, Hawking temperaturasi, Gibbs erkin energiyasi va entropiya kabi bir qancha termodinamik miqdorlarini o‘rganamiz. Shuningdek, bu qora tuynukka bog‘liq bo‘lgan ma'lum parametr qiymatlari uchun ikkita zarra (ekstremal va noekstremal holatlarda) to‘qnashuvi natijasida hosil bo‘lgan markaziy massaning energiyasini (CM) qisqacha tahlil qilamiz.

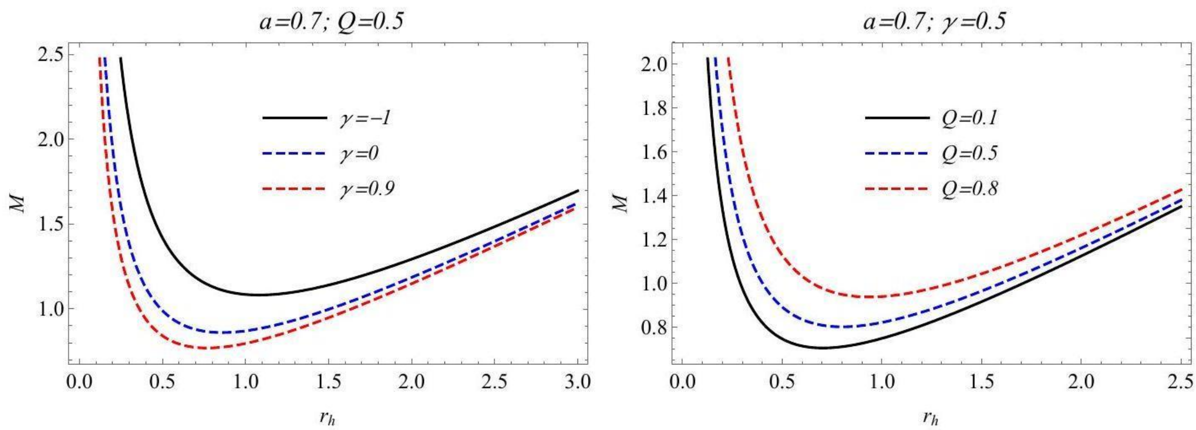
Qora o‘ralar zarralarni tezlatishi mumkinligini ikki zarra to‘qnashuvini tahlil qilib muhokama qilamiz va keyin bizning metrikamiz uchun ushbu to‘qnashuv natijasida ishlab chiqarilgan markaziy massaning energiyasini tahlil qilamiz. Chegaraviy ekstremal qora tuynuk holati yaqinida ikkita og‘ir test zarra to‘qnashganda, natijada ekvatorial tekislikda $(\theta = 0)$ ultrabaland markaziy massaning energiyasi ishlab chiqariladi.



10-rasm. Noto‘g‘ri ekstremal BH uchun markaziy massalar energiyasi E_{CM} va radial koordinata r o‘rtasidagi bog‘lanishni ko‘rsatadi.

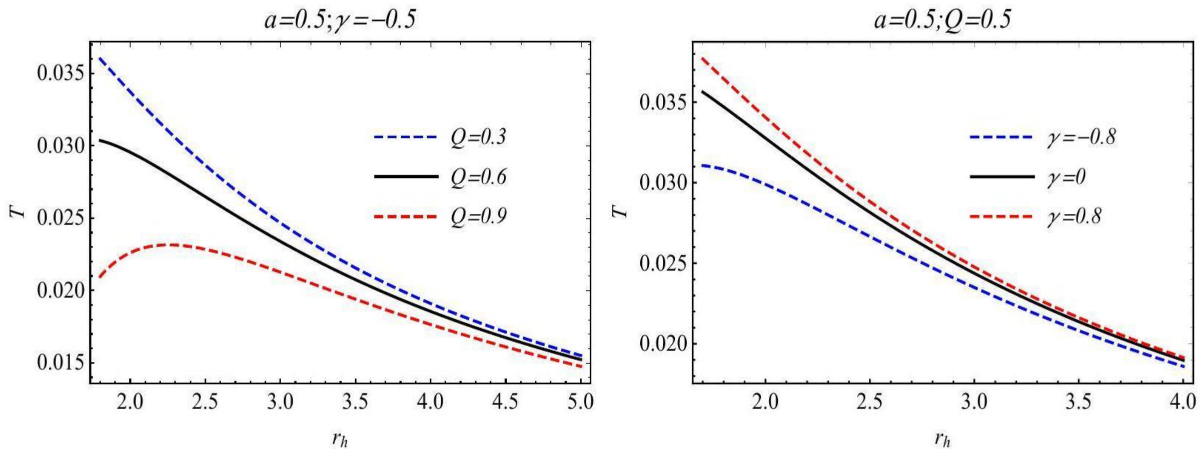
10-Rasmda esa noekstremal holat uchun markaziy massaning energiyasi E_{CM} va radial koordinat r ning bog‘lanishi ko‘rinadi.

Shuningdek, biz ModMax qora tuynukning termodinamikasini muhokama qilamiz. Hisob-kitoblarni osonlashtirish uchun jarayonlarni ekvatorial tekislikda ko‘rib chiqamiz.



11-rasm. Ufqi radiusiga nisbatan entalpiya o‘zgarishi.

11-Rasmda entalpiya H va ufq radiusi r_h ning bog‘lanishi ko‘rsatilgan. Biz ko‘rishimiz mumkinki, screening faktori γ oshgani sayin qora tuynuk (yoki massa) uchun entalpiya kamayadi, lekin zaryad Q oshganda esa u ortadi. Hawking temperaturasi kabi muhim termodinamik miqdorni topish uchun qora tuynukning burchak tezligi $\Omega = -g_{t\phi}/g_{\phi\phi}$ ifodasini olishimiz kerak.



12-rasm. Ufqi radiusiga nisbatan temperaturaning o‘zgarishi. Ushbu rasmdan ko‘rinib turibdiki, agar zaryad parametri Q oshsa, QT "soviydi", lekin γ ning oshishi uni "issitadi".

12-Rasm hodisa ufqi radiusiga nisbatan temperaturaning harakatini ko‘rsatadi. Nozichiziqli elektrodinamik maydon zaryadi hodisalar gorizonti haroratini kamaytiradi and ModMax maydon parametri esa uni orttiadi. Ammo, gorizont ro‘lchami kattalashgan sari qora o‘ra zaryadi va ModMax maydon ta\sirlari sezilarsiz bo‘lib boradi.

XULOSALAR

Falsafa doktori (PhD) dissertatsiyasi uchun “Qora o‘ralar atrofidagi astrofizik jarayonlarga nochiziqli elektrodinamikaning ta’siri (ModMax nazariyasi misolida)” mavzusida olib borilgan tadqiqotlar asosida quyidagi xulosalar taqdim etiladi:

1. ModMax parametriningoshishi qora tuynuk hududini kengaytirishi, qora tuynuksiz hudud esa torayishini ko‘rsatib o‘tildi. Shuningdek, zaryadlangan aylanayotgan ModMax qora tuynugida zaryad va ModMax parametriningo‘zgarishi bilan hodisa gorizonti tuzilishi quyidagicha o‘zgarishi aniqlandi: zaryadning belgilangan qiymatlarida ModMax parametrioshganda gorizont radiusi kattalashadi, belgilangan ekranlash omilida esa radius zaryad parametrining oshishi bilan kichrayadi.

2. Ichki (Koshi) gorizont radiusi kamayishi va tashqi gorizont esa ModMax parametriningoshishi bilan kattalashishi qayd etildi. ModMax parametriningoshishi bilan qora tuynuk soyasi kattalashishi aniqlandi. Shuningdek, ModMax parametrioshishi bilan soyaning radiusi kattalashishi, ammo soyaning deformatsiyasi kamayishi qayd etildi.

3. Energiya chiqish tezligining maksimal qiymati ModMax parametriningoshishi bilan kamayishi, qora tuynuk zaryadi oshishi bilan esa ko‘payishi ko‘rsatib o‘tildi. Dyonik ModMax parametrlarining, ya’ni zaryad va ModMax parametriningmavjudligi burilish burchagi kamayishiga olib kelishi aniqlandi. Shuningdek, plazma parametri oshishi bilan foton va soya radiusi kamayishi qayd etildi. Plazma mavjudligida burilish burchagi va tasvirlarning kattalashishi ko‘payishi ham ko‘rsatildi.

4. Ichki va tashqi gorizontlarning radiuslari bir-biriga mos keladigan aylanma, ModMax parametriva zaryad parametrlarining kritik qiymatlari aniqlangan holda hisoblandi. Qora tuynukning ergosfera qalinligi zaryad va aylanish parametrlarining oshishi bilan ko‘payishi, ModMax parametrioshishi bilan esa kamayishi aniqlandi. Eng barqaror aylanish orbitasi radiusi ModMax parametriningoshishi bilan ko‘payishi, zaryad parametrining oshishi bilan esa kamayishi qayd etildi.

5. ModMax qora tuynugi atrofidagi energiyani olish jarayonini tahlil qilish shuni ko‘rsatdiki, ekstremal holatda markaziy massaning energiyasi cheksizlikka o‘tishi mumkin. ModMax parametriningoshishi bilan entalpiya va Gibbsning erkin energiyasi kamayishi, Hawking harorati esa oshishi aniqlandi. ModMax qora tuynugida zaryadning termodinamik xususiyatlarga ta’siri qarama-qarshi ekanligi ham ko‘rsatildi.

SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH "TIIAME" NATIONAL RESEARCH UNIVERSITY

SAMARKAN STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

URINOV SUNNATILLO KHUDOYOR UGLI

EFFECTS OF NONLINEAR ELECTRODYNAMICS ON ASTROPHYSICAL PROCESSES AROUND BLACK HOLES (EXAMPLES OF MODMAX THEORY)

01.03.01 – Astronomy
01.04.02 – Theoretical Physics

Doctor of Philosophy in Physical and Mathematical Sciences (PhD)
ABSTRACT OF THE DISSERTATION

Tashkent – 2024

The theme of dissertation of the doctor of philosophy (PhD) on technical sciences was registered by Supreme Attestation Commission of the Cabinet of Ministers of the Republic of Uzbekistan under B2024.3.PhD/FM1169

The doctoral (PhD) dissertation was carried out at Samarkand State University named after Sharof Rashidov

The abstract of the dissertation was posted in three (Uzbek, Russian, English (resume)) languages on the website of the Scientific Council (www.ifar.uz) and on the information and education portal at "Zionet" (www.ziyonet.uz).

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The defense of the dissertation will be held on “___” __________ 2024 at ___ in the meeting of the Scientific Council No. DSc.03/31.03.2022 T/FM.10.4 at the Institute of Fundamental and Applied Research under the National Research University "TIIAME" (Address: 100000, Tashkent city, Qori Niyazov Street 39, Institute of Fundamental and Applied Research, Hall 108; Phone: 71 237-09-61.; e-mail: info@ifar.uz

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The Abstract of the dissertation was distributed on "___" __________, 2024.
(Registry record № ___ dated "___"____ _____, 2024)

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INTRODUCTION (Annotation of PhD dissertation)

Topicality and demand of the theme of the dissertation. The advancement and refinement of theoretical and astrophysical models related to various astrophysical phenomena around black holes are among the top priorities in modern relativistic astrophysics. Over the last decade, significant discoveries in observational astronomy have accelerated the development of theoretical and mathematical frameworks to understand astrophysical events near compact gravitational objects. Simultaneously, the pursuit of a unified theory in fundamental physics requires exploring new alternative or modified theories of gravity which are beyond the current general relativity paradigm. This involves comparing these theories with observational and experimental data, as well as determining the values of field/model parameters through mathematical simulations of optical and energetic processes occurring near black holes.

Alternative and modified gravity theories are essential to address several shortcomings and inconsistencies within general relativity. These theories aim to bridge the gaps between general relativity and quantum mechanics, explain phenomena like dark matter and dark energy, and provide insights into gravitational behavior under extreme conditions, such as within black holes. Additionally, investigating these alternative theories enhances our understanding of the fundamental nature of space-time and gravity, potentially leading to breakthroughs in our comprehension of the universe. Consequently, the study of curved space in astrophysical research concerning compact objects makes astronomical observations highly relevant.

In recent years, our country has increasingly focused on advancing fundamental and applied research, particularly in astrophysics, which is a promising area of study. The main directions for fundamental research and development, along with their practical applications for the successful progression of science in our country, are outlined in the Strategy for the Further Development of the Republic of Uzbekistan from 2022 to 2026. Therefore, studying the modified theories of gravity remains a pressing issue in fundamental research.

This dissertation work corresponds to the tasks by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. PD-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Resolution of the President of the Republic of Uzbekistan No. PR-2789 "On measures for further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities" dated February 18, 2017, and others.

Conformity of the research to the main priorities of science and technology development of the Republic. The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. “Power, energy and resource-saving”.

The degree of knowledge of the problem. The theoretical description of the massive and massless particle motion around black holes have been considered by many scientists, including British (R. Wald, J. Petterson), Russian (A. Zakharov, V. Frolov, I. Novikov), German (C. Laemmerzahl, J. Kunz, A. Grezenbach), Argentinian (L. Amarilla, E. Eiroa), Czech (M. Kolos, J. Vrba), Indian (N. Dadhich, S. Ghosh, P. Joshi, M. Patil), etc. However, the particle dynamics around black hole in ModMax theory has not been studied. Particularly, there is no systematic study of the massive and massless particle's dynamics around black holes in ModMax model.

The astrophysical model of the photon motion has been developed within the general relativity by local scientists B.J. Ahmedov, A.A. Abdujabbarov, B.A. Toshmatov, F.S. Atamurotov, J.R. Rayimbaev and others, and Foreign scientists R. Wald, S.R., F. de Felice, F. Sorge, L. Rezzolla, Z. Stuchlik, C. Bambi and others. However, the question of improving the astrophysical model for the case when the central object is described by the ModMax theory remains open.

The influence of screening factor and charge parameter on the astrophysical and energetical processes around a black hole also remains unexplored. The development and improvement of theoretical models describing the dynamics of particles around black hole may be used to obtain the constraints on model parameters of ModMax theory.

Connection of the topic of the dissertation topic to the scientific works of higher education and research institutions, where the dissertation is carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development. F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity".

The aim of the research is the development and improvement of astrophysical models for describing the dynamics of massive and massless particles and thermodynamics in the vicinity of black holes in ModMax gravity model.

The tasks of the research:

to study the effects of charge and screening factor on horizon structure of the black holes in ModMax theory;

to analyze the size and for of the black hole shadow depending on the screening factor;

to study the effect of ModMax parameters on the value of distortion parameter of shadow of black hole;

to study the energy emission rate, particularly, effects of ModMax parameter on the maximum value of the effective potential;

to analyze the effects of Dyonic ModMax parameters such as charge and screening factor on the deflection angle of photons;

to study the effects of ModMax modifications on photon-sphere and shadow radius in the presence of plasma;

to find the critical values of rotation, screening factor and charge parameters have been obtained, where the radii of the inner and outer horizons coincide;

to study the radius of innermost stable circular orbit dependence on screening factor and charge;

to analyze the energy extraction process around ModMax black hole;

to study the enthalpy, Gibbs free energy, Hawking temperature of te black hole in ModMax theory, to perform full analysis of the thermodynamics of the black hole.

The object of the research are astrophysical black holes, test particles with zero and nonzero rest mass, ModMax field.

The subject of the research are theoretical approaches for studying test particle dynamics near black holes, numerical and analytical methods for solving differential equations.

The methods of the research are methods of computational mathematics, methods of theoretical astrophysics, modern methods of mathematical physics, analytical and numerical methods of calculating differential equations for field and particle motion.

The scientific novelty of the research is the following:

for the first time, in the ModMax theory, black hole charge, gravity, and radially variable plasma parameters causes decreasing in the orbit of a photon and the size of a black hole's shadow.

for the first time, a weak gravitational lensing analysis found that the Dyonic ModMax field defines the location of the deviation.

the increase of the rotating black hole due to the high load and parameters of the ergosphere, the setting Max parameter of the ergosphere was shown to fix the ergosphere.

for the first time, the ModMax theory found an increase in the radius of the inner black circular orbit (ISO) around the hole.

The practical results of the research are the following:

For the first time the thermodynamics of the charged ModMax black hole, including enthalpy, Hawking temperature, and Gibbs free energy have been explored in detail. It has been shown that the enthalpy and Gibbs free energy decreases with the increase of screening factor, while the Hawking temperature increases.

The rate of distortion parameter and average radius of shadow have been analyzed. It has been shown that with the increase of the screening factor the radius of shadow grows, but distortion parameter reduces.

For the first time the maximum values of black hole charge which corresponds to radii of black hole shadow which came from Event Horizon Telescope data have been calculated.

The analysis shows that the peak of energy emission rate falls off with increasing screening factor and moves up with growth in the values of charge.

The reliability of the research results provided by applying modern proven methods of mathematical physics, computational mathematics, and relativistic astrophysics. The results were obtained strictly within the mathematical apparatus of general relativity and theoretical physics. Modern numerical and analytical methods of calculation are also used, and the results are compared with available observational data and the results of other authors. The structured conclusions of the thesis correspond to the basic rules of astrophysics of compact objects.

The scientific and practical significance of the research results. The scientific significance of the research results is found that the analysis of solutions for black holes in ModMax gravity model can be a useful tool for studying various astrophysical processes within this model.

The practical significance of the research results is that they can play to construct the test of the modified theories of gravity and get constraints on the black hole parameter using the observations of shadow of black holes.

Application of the research results. The developed theoretical models of the photon and particle motion in the ModMax theory have been applied to the followings:

scientific results obtained on the motion of particles and photons have been used by scientists from Fudan University (FU) in Shanghai (FU, China, September 23, 2024, reference);

For the first time the thermodynamics of the charged ModMax black hole, including enthalpy, Hawking temperature, and Gibbs free energy have been explored in detail. It has been shown that the enthalpy and Gibbs free energy decreases with the increase of screening factor, while the Hawking temperature increases. Results on the dynamics of particles have been used in the works of foreign researchers, in foreign journals with a high impact factor (Physics Letters B, Volume 854, id.138758, Web-Sc, IF-4.4; Communications in Theoretical Physics, Volume 76, Issue 8, id.085402, Web-Sc, IF-2.4; Pramana, Volume 98, Issue 3, id.92, Web-Sc, IF-2.219 and others) are used in more than 5 published scientific papers to describe the effects of particles around a black hole;

as a result, it was possible to get information about the dynamics of particles around compact objects within the modified gravity theories.

Testing of the research results. The research results were reported and discussed at 1 international conference and 3 local scientific conferences.

Publication of research results. 9 research papers were published on research results, 1 of them – in the international scientific journals recommended by Supreme Attestation Commission at the Ministry of higer education, science and innovations of the Republic of Uzbekistan.

Volume and structure of the dissertation. The dissertation consists of an introduction, four chapters, a conclusion and references, all in 81 pages.

THE MAIN CONTENT OF DISSERTATION

The introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

The first chapter entitled 'The Shadow of a Rotating Charged ModMax Black Hole' discusses the impact that the parameters of charge (Q) and screening factor (γ) have on properties of the horizon and silhouette of rotating charged ModMax black holes. Furthermore, the study explores the behavior of null geodesics, which can help us better understand the apparent shape of the black hole's silhouette, as well as the distortion parameter and approximate radii of the silhouette that are influenced by the aforementioned parameters and there are some values of parameter

Q which corresponds to data from Event Horizon Telescope (EHT). Notably, we explore the distortion parameter and approximate radii of the silhouette, revealing that while an increase in γ leads to a growth in silhouette radius (R_s), it simultaneously reduces the distortion rate (δ_s). Conversely, heightened Q charge results in a reduction of R_s accompanied by an increase in δ_s. We also analyze the effects of black hole's parameters on the effective potential and energy emission that the peak of the energy emission rate experiences a decline as the screening factor (γ) increases, while it ascends with higher values of the charge parameter (Q).

The Lagrangian of ModMax electrodynamics is given by

$$L_{ModMax}(x,y) = -x\cos h\,\gamma + (x^2+y^2)^{\frac{1}{2}}\sin h\,\gamma \qquad (1)$$

with the electromagnetic Lorentz invariants $x := \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, y := \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$, and the electromagnetic tensor is $F_{\mu\nu}$ with its dual $\tilde{F}_{\mu\nu} := \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$. The ModMax Lagrangian the Plebański dual variable are

$$P_{\mu\nu} := -L_x F_{\mu\nu} - L_y \tilde{F}_{\mu\nu} \qquad (2)$$

with its dual

$$\tilde{P}_{\mu\nu} = \left[\cosh\gamma - \frac{x}{(x^2+y^2)^{1/2}}\sinh\gamma\right]\tilde{F}_{\mu\nu} + \frac{y\sinh\gamma}{(x^2+y^2)^{1/2}}F_{\mu\nu} \qquad (3)$$

We consider spherically symmetric BH solution described by the line element written in Boyer-Linquist coordinates

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (4)$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2 e^{-\gamma}}{r^2} \qquad (5)$$

The spacetime around rotating ModMax BH with the spin parameter a can be represented using the line element

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \qquad (6)$$

where the metric functions are defined as

$$\Sigma = r^2 + a^2\cos^2\theta \quad (7) \quad \Delta = r^2 + a^2 - 2Mr + Q^2 e^{-\gamma} \quad (8)$$

Fig. 1 shows separatrix lines which are the border corresponding to extreme $BH(r_+ = r_-)$ and separates BH region from no BH region for selected values of γ. According to Fig. 1 BH region expands and non black hole region narrows with the increase of γ parameter.

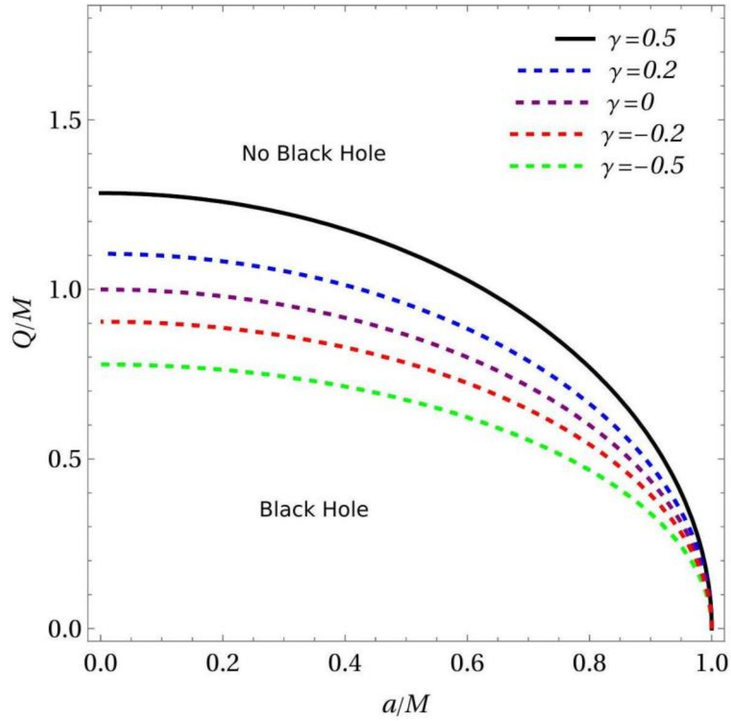

Figure 1: The separatrix lines indicate the border corresponding to extreme BHs which separates BHs from no BHs for selected values of γ parameter.

The graphical representation of the numerical solutions for the BH horizon is shown in Fig. 2. Left and right panels show the dependence of inner and outer horizons of BH from spin parameter for the different values of γ and Q for the fixed values of parameters $Q = 0.5$ and $\gamma = 0.2$, respectively. From the Fig. 2 one may get information about inner(Cauchy) and outer radius of horizon: how radius modify by varying charge Q and screening factor γ. According to Fig. 2 the horizon radius increases with rising γ for fixed $Q = 0.5$ and for fixed $\gamma = 0.2$ radius sees lowering with rising Q parameter.

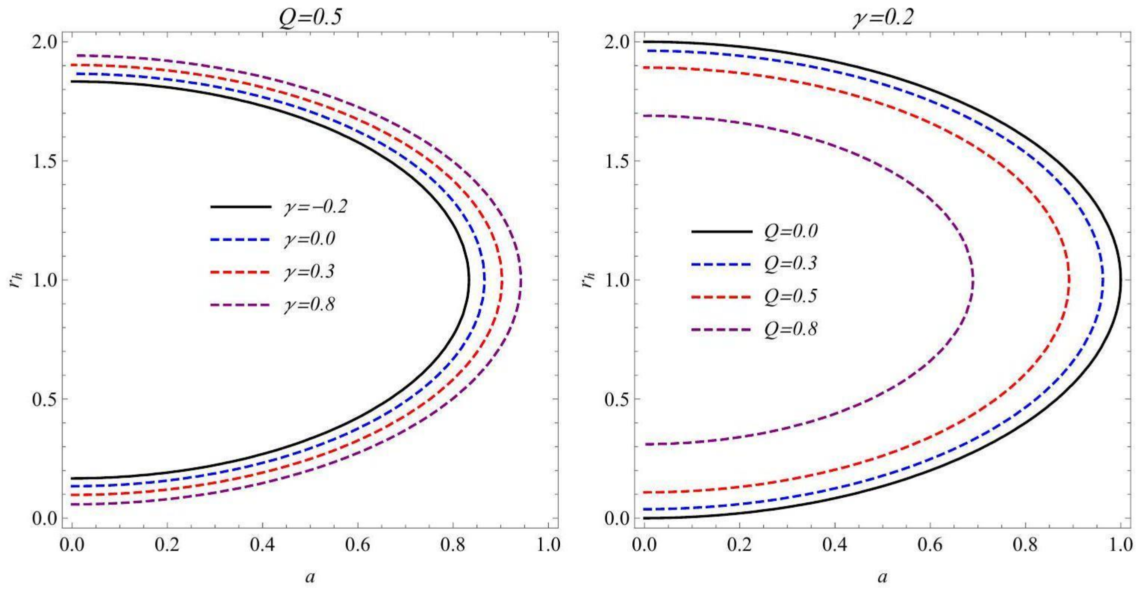

Figure 2: Dependence of horizon radius r_h on BH spin parameter a for the fixed values of parameters Q and γ. Where $M = 1$.

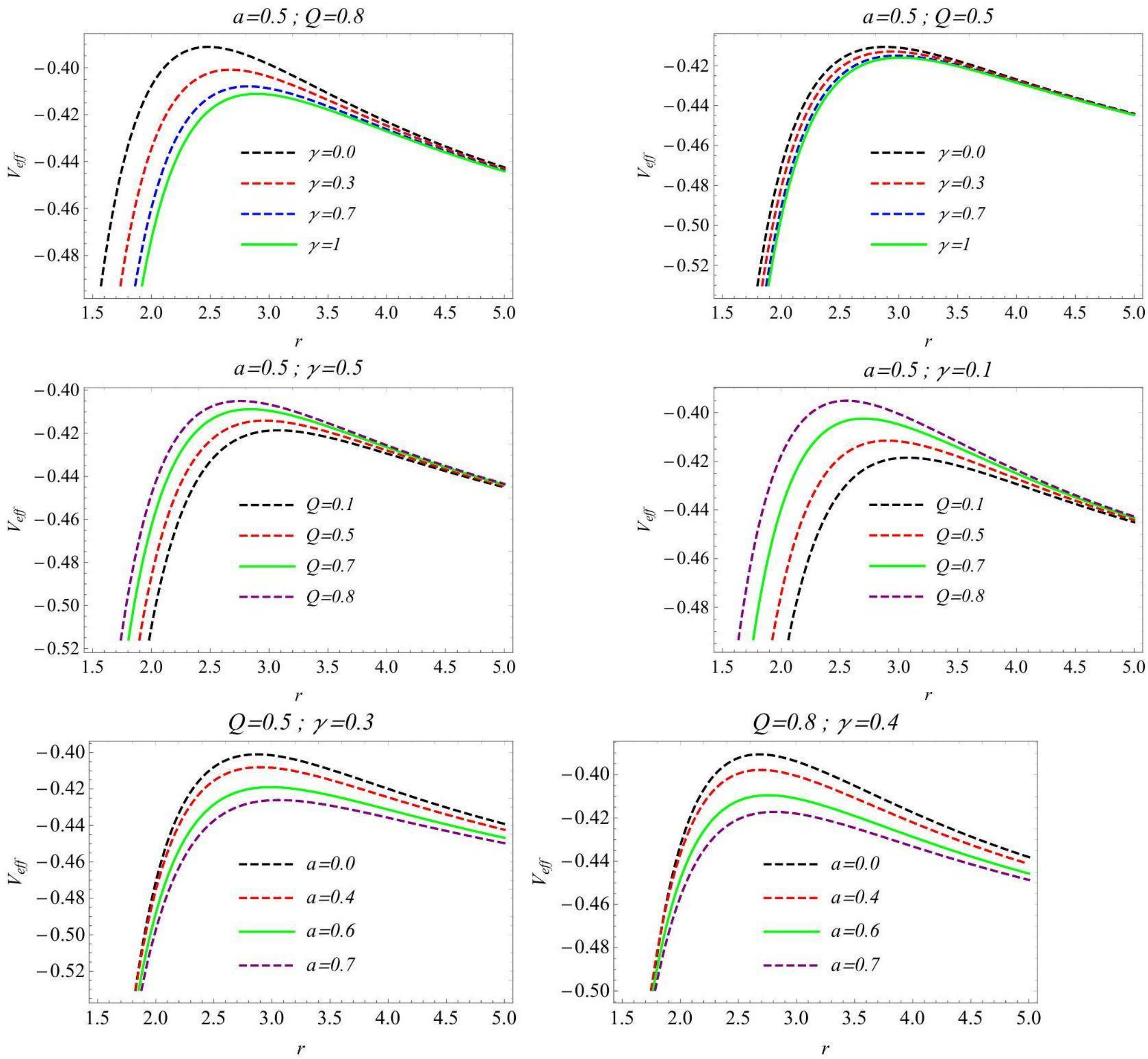

Figure 3: Graphs show the radial dependence of the effective potential V_{eff} for different values of a, Q and γ. Where $M = 1$. where M is the BH mass, Q is charge and γ is screening factor.

Using the Hamilton-Jacobi formulation one may get the equations of motion of photon around BH in ModMax gravity. One can rewrite the radial equation of motion of the photon around BH in ModMax theory using the expression $dr/d\tau$ as

$$\left(\Sigma \frac{dr}{d\tau}\right)^2 + V_{eff} = 0 \tag{9}$$

where the effective potential of radial motion in the equatorial plane $(\theta = \pi/2)$ reads as

$$V_{eff} = \frac{\Delta(K + (L - aE)^2) - \left((a^2 + r^2)E - aL\right)^2}{2r^4} \tag{10}$$

Using above expression (10) we have plotted the plots of the dependence of the effective potential on radius coordinate for different values of parameters a, Q

and γ in ModMax gravity. From Fig. 3 it can be represented that the general behaviour of the effective potential V_{eff} with refer to photon orbits for different values of black hole's parameters. And also we may see that a peak values of the effective potential which refer to photon orbit grows and shifts towards the left direction with the growing charge Q for fixed γ and a. As result fixed γ contracts region between two potential lines expands. And rise of the values screening factor γ and spin parameter a peak of effective potential fall off and there is also shifting towards right for fixed a and Q, Q and γ, respectively.

$$r + 2\frac{\Delta}{\Delta'^2}(\Delta' - r\Delta'') > 0 \tag{11}$$

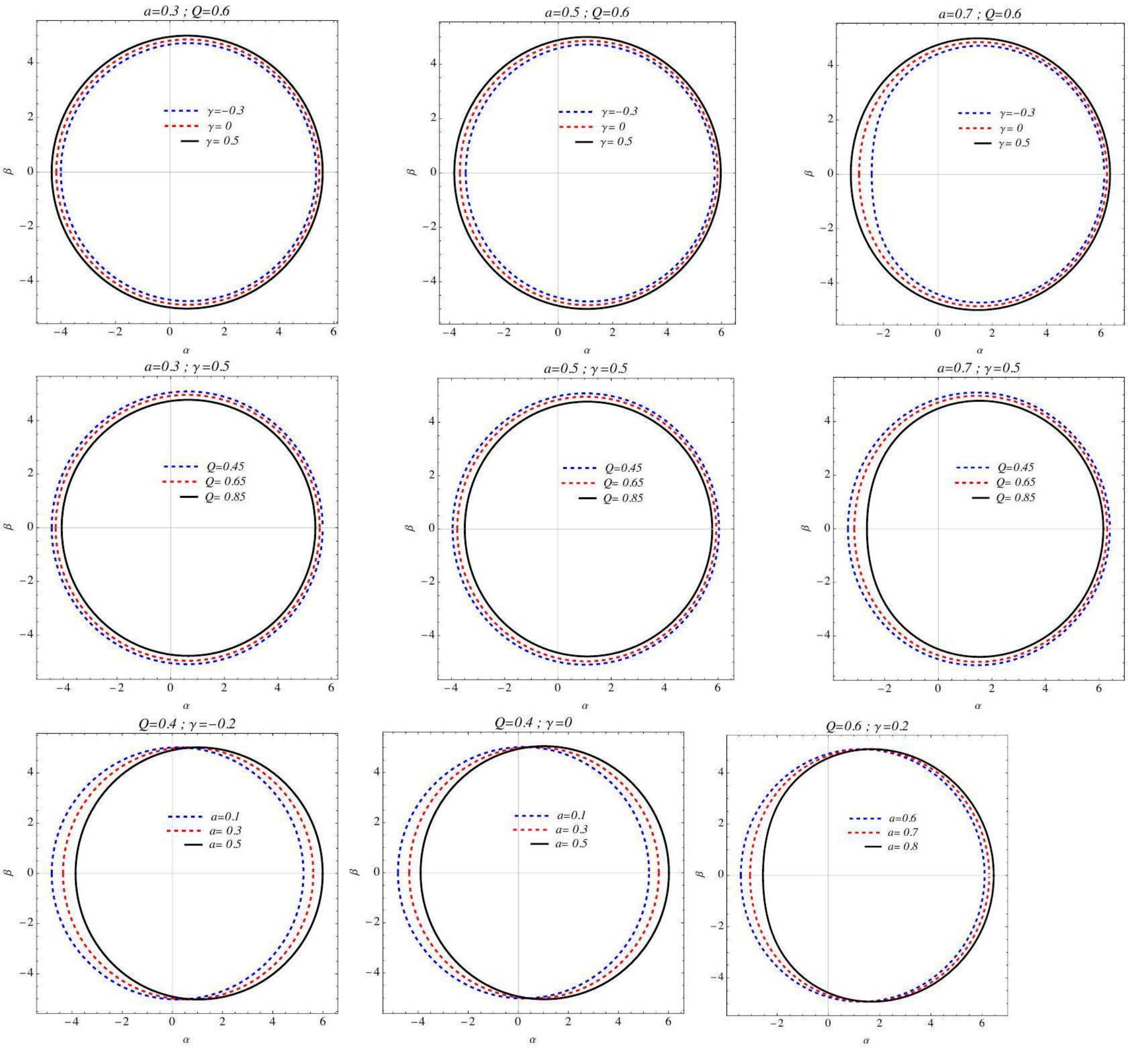

Figure 4: Plots show BH shadow for different values of ModMax parameters. Where $M = 1$.

Fig. 4 displays the shadows of the charged BH for fixed values of the ModMax BH parameter: spin a, charge Q and screening factor γ. From the obtained results one can see that the size of the BH shadow is enlarged with growth of the value of parameter γ (Upper panel) for fixed a and Q. On the other hand with rising Q parameter the size of the BH shadow reduces, for the fixed a and γ (Middle panel). Lower panel gives information about decreasing in size of BH shadow with increasing spin parameter a (for fixed Q and γ). Moreover, shape of BH shadow deforms towards right under influence of growing values of BH spin parameter a.

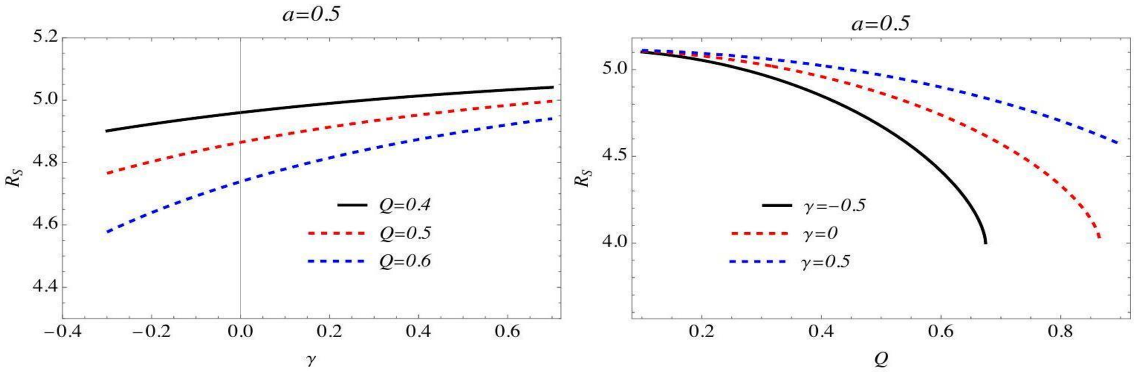

Figure 5: The dependence of average radius R_s of charged rotating ModMax black hole shadow from its parameters. Upper panel is for dependence from screening factor γ for fixed a and selected Q. Lower panel is for dependence from Q parameter for fixed a and selected γ. Where $M = 1$.

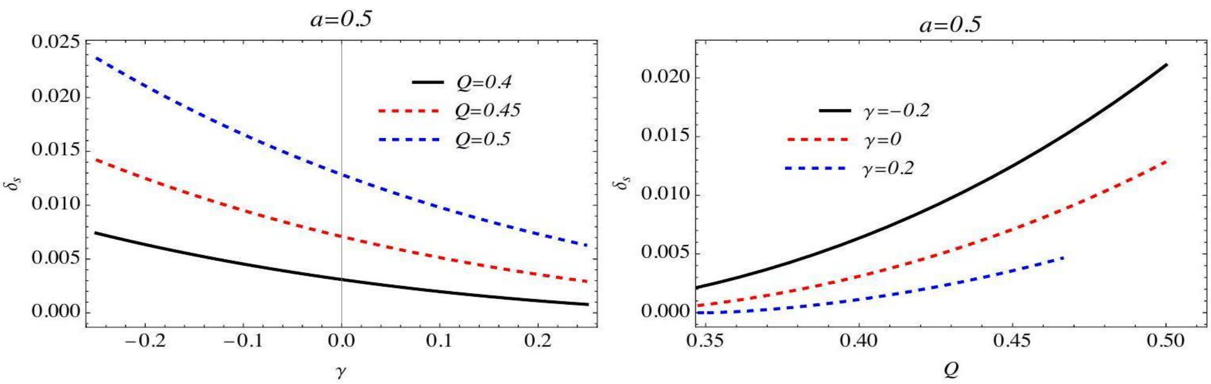

Figure 6: The dependence of the distortion(deviation) δ_s parameter of charged rotating ModMax black hole shadow from its parameters. Upper panel is for dependence from screening factor γ for fixed a and selected Q. Lower panel is for dependence from charge Q for fixed a and selected γ. Where $M = 1$.

32 cm	$\gamma, a = 0.5$	$Q, a = 0.5$
M^*7^*	-0.5	0.46114
	0	0.59212
	0.5	0.76293
32 cm SgrA*	-0.5	0.672056
	0	0.8629381
	0.5	1.08034

Table 1: Some values of Q for minimum radius of M87* and SgrA* which comes from observational data for different fixed γ

In Fig. 5, the observable parameter R_s is represented as a function of BH's parameters γ and charge Q. One may see that average radius R_s of the BH shadow rises with increasing screening factor γ (Upper) and declines with rise of charge Q (Lower). And also we observe in Fig. 6 that (Upper) less distorted with swelling γ(δ_s reduces with growth of) and (Lower panel) shape of the shadow is more distorted with rising Q(δ_s rises with growth of Q).

By using the data for M87* and SgrA* which was provided by EHT collaboration we may find upper limit for Q charge different fixed values of γ according to metric. We have learnt diameter of the shadow by using:

$$d_s = \frac{D\theta}{M} \quad (12)$$

here θ, D and M are the angular diameter of BH shadow, the distance of BH from Earth and mass of the black hole, respectively. For M87* these quantities are $\theta_{M87*} = 42 \pm 3\mu\ s$, $D_{M87*} = 16.8Mpc$ and $M_{M87*} = 6.5 \pm 0.90 \times 10^9 M_\odot$ and for $SgrA^* \theta_{SgrA*} = 48{,}7 \pm 7\mu$, $D_{SgrA*} = 8277 \pm 33pc$ and $M_{SgrA*} = 4.3 \pm 0.013 \times 10^6 M_\odot$. Diameters of BH shadow which are calculated from data $d_s^{M87*} = (11 \pm 1.5)M$ and $d_s^{SgrA*} = (9.5 \pm 1.4)M$ and we know that it can be find radius of BH shadow by following expression $d_s = 2R_s$. And from this calculated minimum radii for $M87^*$ and Sgr A^* are $R_s^{M87*} = 4.75M$ and $R_s^{Sgr\ A*} = 4.05M$, respectively. In Tab. 1 we have calculated for fixed screening factor γ and $spina = 0.5$, estimated maximum values of charge parameter Q which are conformed to radii from provided data. It can be seen that for rising γ maximum value of Q increases, and it corresponds to Fig. 5

The second chapter entitled 'Weak gravitational lensing around dyonic ModMax black hole in plasma' we have studied weak gravitational lensing effect around black hole and determine shadow radius in Dyonic ModMax(DM) spacetime. We also consider models with a nonuniform plasma distribution. For different gravitational lens models, we compare the corrections to the vacuum lensing due to the gravitational effect in plasma, and due to the plasma inhomogeneity. We have shown that the gravitational effect could be detected in the case of a hot gas in

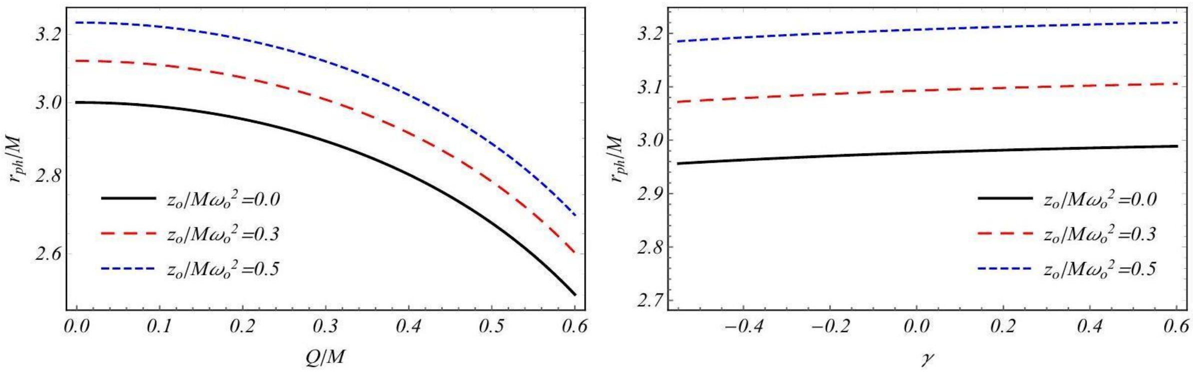

Figure 7: describes a variation of r_{ph}/M with respect to Q (left panel) and γ (right panel) for the case $\omega_p^2 = z_0/r$ the gravitational field of a galaxy cluster.

We started with orbits of photons around black hole in DM. In addition, we have studied shadow and gravitational weak lensing around such black hole. By using observational data of EHT project for the $M87*$ and $SgrA*$, we have got constrains parameter in DM gravity. Further to make a connection with observations, we investigate magnification and position of images formed as a result of lensing, and finally weak deflection angle and magnification for sources near different galaxies.

We probe photon spheres in the presence of a non-uniform plasma, where the plasma frequency must satisfy a simple power-law of the form

$$\omega_p^2(r) = \frac{z_0}{r} \tag{13}$$

where z_0 is the free parameter. To analyze the primary features of the power-law model, we restrict ourselves to the following cases: $\omega_p^2(r) \sim \frac{1}{r}$ and z_0 as a constant which reproduces the negative-mass diverging lens exactly. Using Eqs.(4) and (13), We obtain the radius of the photon sphere based on a numerical scheme for the inhomogeneous plasma medium, as illustrated in Fig. 7. This profile shows that the radius of the photon sphere decreases when the Q/M parameter increases. However, increasing the value of γ leads to photon sphere's also increasing. On the other hand, the plasma medium leads to the widening of the photon sphere radius if its distribution obeys $\omega_p^2(r) = z_0/r$ law, the presence of plasma around the black hole slightly shrinks the photon radius. Furthermore, a difference in the photon radius in the case of $\omega_p^2(r) = z_0/r$ and with $\omega_p^2(r) =$ const plasma is enough small. This suggests that testing and distinguishing the homogeneous plasma from inhomogeneous plasma around black holes based on their shadows may be quite challenging.

We consider supermassive black holes M87* and Sgr A* * as spherically symmetric and static. However, observations from the Event Horizon Telescope (EHT) do not fully support this assumption. Despite that, the study investigates constraints on the parameters Q and γ using data from the EHT project. To determine the parameters Q and γ, the EHT project utilizes observational data from the black hole shadows of supermassive black holes M87* and Sgr A*. The angular diameter of the shadow, the distance from sun system and the mass of the black hole at the centre of the galaxy M87, are $\Omega_{M87*} = 42 \pm 3\mu as, D = 16.8 \pm 0.8Mpc$ and $M_{M87*} = (6.5 \pm 0.7) \times 10^9 M_{\odot}$, respectively. For the Sgr A* the data recently obtained by the EHT project is $\Omega_{SgrA*} = 48.7 \pm 7\mu as, D = 8277 \pm 9 \pm 33pc$ and $M_{SgrA*} = (4.297 \pm 0.013) \times 10^6 M_{\odot}$.

Regions contained within it satisfy M87*(left panel) and Sgr A* (right panel) black holes shadow 1- σ bound. exhibit certain values that could potentially coincide with observations. The accompanying image represents these values in parametric space. Black lines correspond to black hole shadow angular diameter $\theta_{sh} = 39\mu a$ for M87* (left panel) and $\theta_{sh} = 41.7\mu a$ for Sgr A* (right panel), respectively. This means that As we get $Q/M = 1$, parameter $\gamma \geq 0$ for for Sgr A* (right panel)

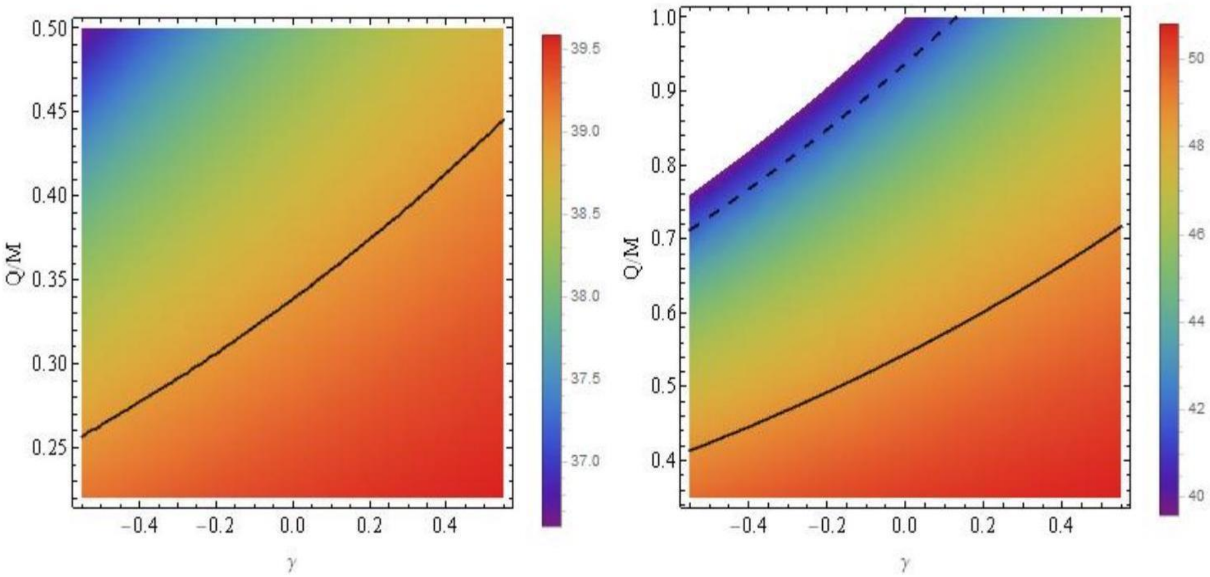

Figure 8: Constraining parameters via observational shadow angular diameter of M87* (left panel) and Sgr A* (right panel) black holes. Black lines correspond to black hole shadow angular diameter $\theta_{sh} = 39\mu a$ for M87* (left panel) and $\theta_{sh} = 41.7\mu as$ for Sgr A* (right panel), respectively.

In Fig.(8) We can observe that the spacetime parameters of the Dyonic ModMax black hole.

In the third chapter entitled 'Particle dynamics around ModMax black holes' we investigate the structure of the horizon and ergosphere of a rotating ModMax black hole. The motion of test particles is analyzed by studying the characteristics of the innermost stable circular orbit (ISCO) and the behavior of the effective potential. Different values of the screening factor γ and the Q charge of the rotating ModMax black hole are explored in this study.

The summary of our main findings:

The black hole and no black hole regions were illustrated using the γ and Q parameters. As γ increases, the allowed values of charge also rise, while growing charge Q results in a narrowing of the allowed region for γ.

Critical values $a = a_\epsilon, \gamma = \gamma_\epsilon$, and $Q = Q_\epsilon$ were identified, where the radii of the inner and outer horizons coincide. For other values of black hole parameters, either two horizons (Cauchy and event) are observed, or there is no black hole at all.

The thickness of the black hole's ergosphere increases with growing Q and a, but with rising γ, it becomes thinner.

The innermost stable circular orbital (ISCO) radius of the black hole depends on γ and Q differently: with increasing γ, the ISCO radius increases, while with growing charge Q, it decreases.

Overall, our study contributes to a deeper understanding of the charged ModMax black hole, shedding light on various aspects of its dynamics and thermodynamics.

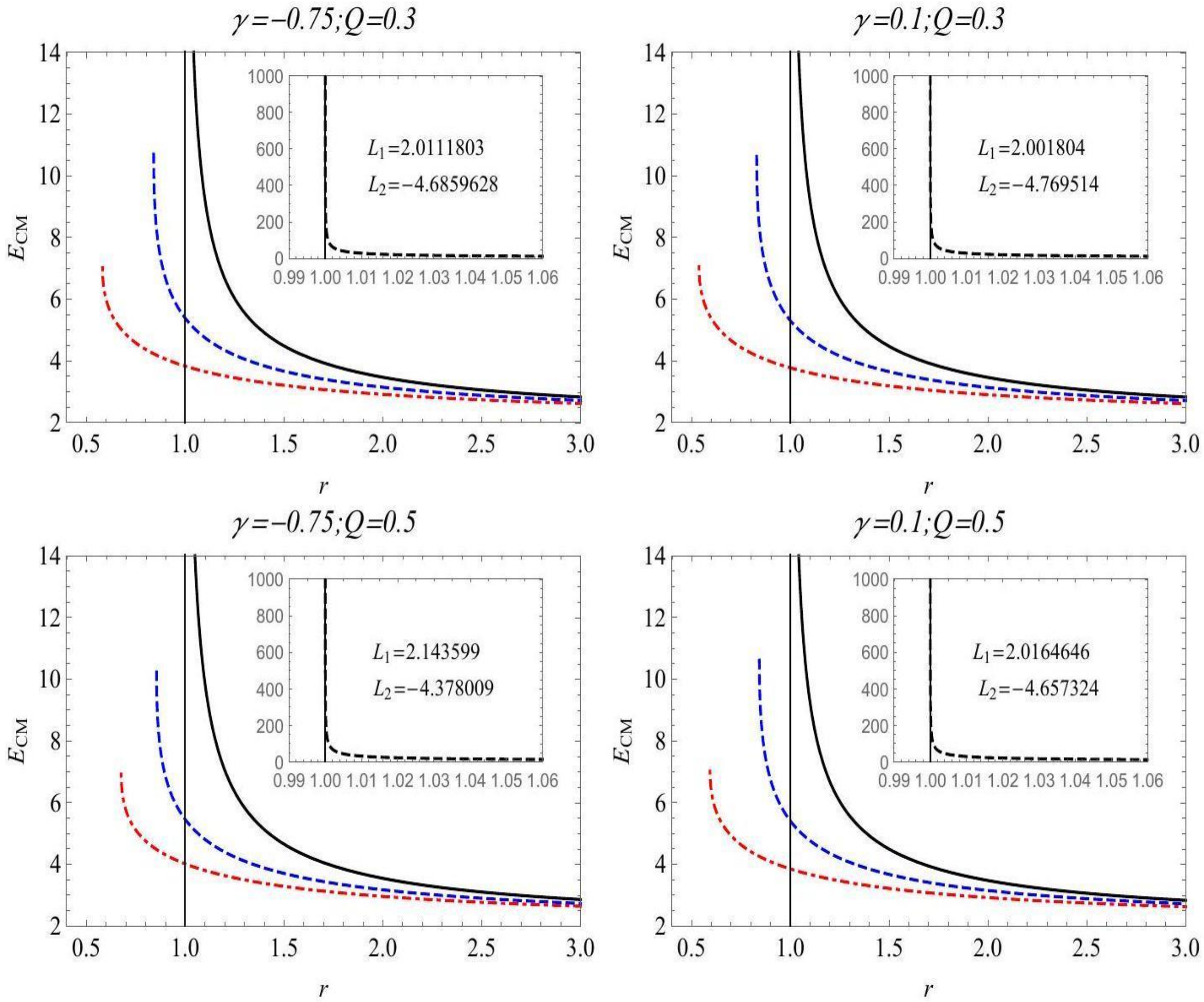

Figure 9: The center-of-mass energy E_{CM} dependence of the radialcoordinate r for an extremal BH for various values of screening factor γ, angular momentum L and charge Q.

Fig. 9 shows dependence of center-of-mass energy E_{CM} and radial coordinate r for different values of screening factor γ, angular momentum L and charge Q. From the figure it can be seen that near the horizon CM energy sharply grows and goes to infinity.

In the fourth chapter entitled 'Thermodynamics and energetic processes around ModMax black holes', we examine several thermodynamic quantities such as Enthalpy, Hawking temperature, Gibbs free energy, and entropy of the black hole. Additionally, the center-of-mass-energy (CM) generated by the collision of two particles (both in extremal and non-extremal cases) is briefly investigated for specific parameter values associated with this black hole.

We discuss how black hole can be the accelerator for the particles by analysing two-particle collision and then we analyze center of mass energy which is produced by this collision for our metric. For the limiting extremal black hole case at the near the horizon two massive test particles collide and as a result of collision here produced ultrahigh center-of-mass energy in the equatorial plane ($\theta = 0$).

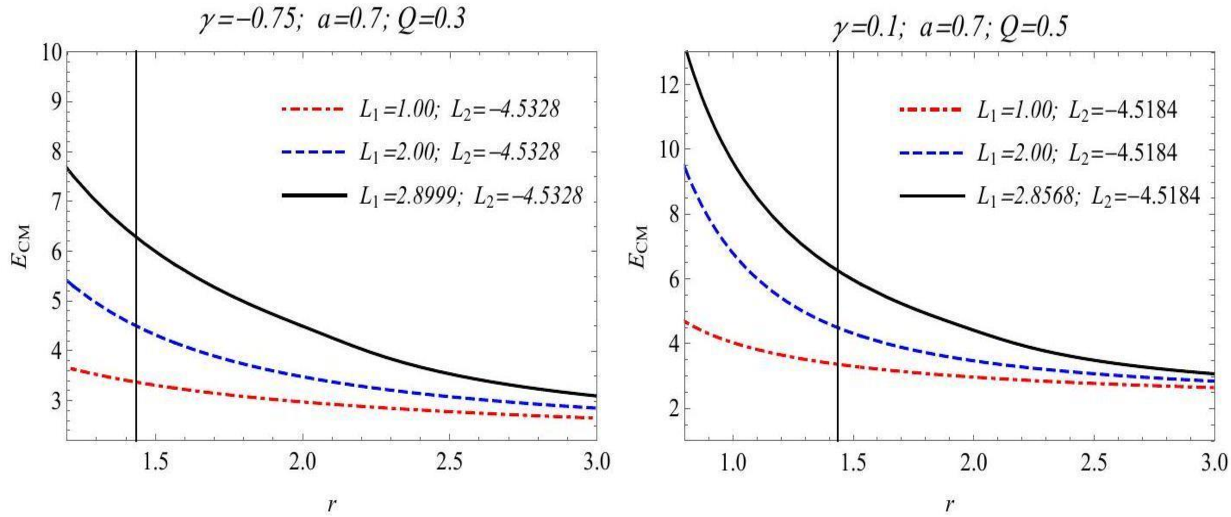

Figure 10: Plot showing the behaviour of the center-of-mass energy E_{CM} vs. the radial coordinate r for a non-extremal BH.

The dependence of the center-of-mass energy E_{CM} and radial coordinate r for non-extremal case in Fig. 10.

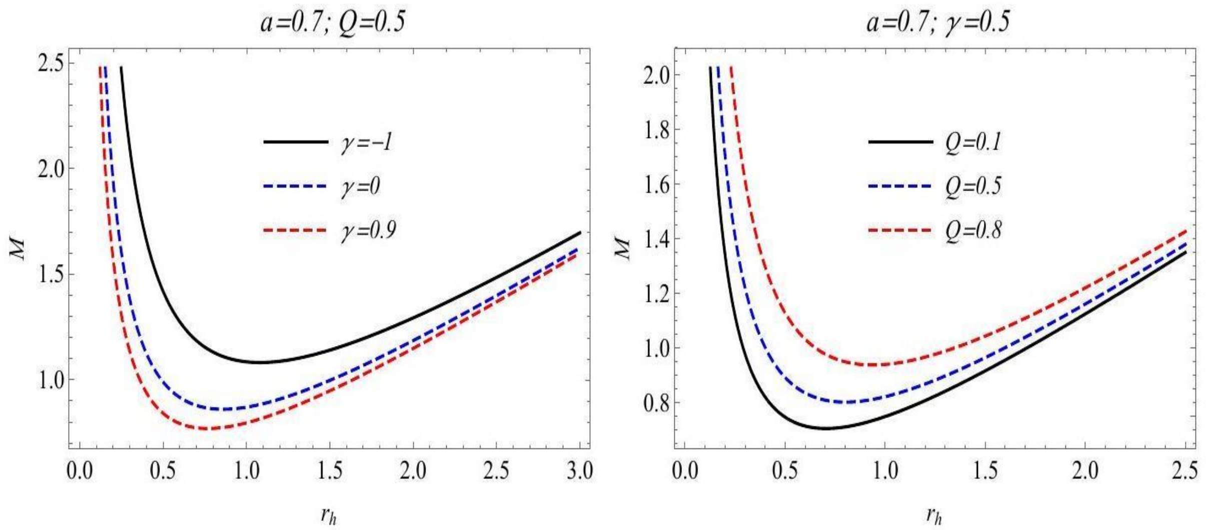

Figure 11: Enthalpy variation with horizon radius.

We also discuss thermodynamics of ModMax black hole. For making calculations easier we see processes in equatorial plane. The dependence of the enthalpy H and horizon radius r_h is illustrated in Fig.11. We can see that with increasing screening factor γ enthalpy for the black hole (or mass) decreases, but it grows when charge Q have rise. In order to find one of the crucial thermodynamical quantities, Hawking temperature we need get expression for the angular velocity of the black hole $\Omega = -g_{t\phi}/g_{\phi\phi}$.

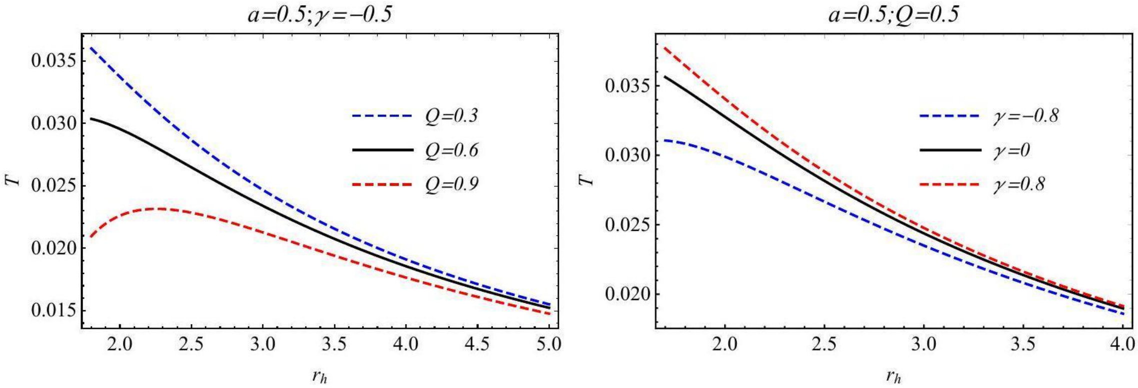

Figure 12: Variation of temperature with horizon radius different values of γ and Q. It can be noticed from this figure that if charge parameter Q sees rise it 'cools down' BH , but increasing values of γ 'warms up' it.

Figure 12 depicts behaviour of temperature with respect to the radius of event horizon for different values of the black hole charge and ModMax field parameter. The nonlinear electrodynamic field charge event horizon increases the temperature and the ModMax field parameter increases it. However, as the horizon size increases, the black hole charge and ModMax field effects become negligible.

CONCLUSIONS

The following conclusions were presented on the basis of research carried out on the topic of "Effects of Nonlinear Electrodynamics on Astrophysical Processes Around Black Holes (Examples of Modmax Theory)" for the Doctor of Philosophy (PhD) dissertation:

1. It has been demonstrated that increasing the screening factor causes the black hole region to expand while the no black hole region narrows. It has been also shown that the event horizon structure of the charged rotating ModMax black hole will be modified by varying charge and screening factor as follows: the horizon radius grows with the increase of the value of the screening factor for fixed values of charge; for fixed screening factor the radius decreases with the increase charge parameter.

2. It has been stated that inner (Cauchy) horizon radius decreases and outer horizon increases with increasing screening factor. It has been found that the size of the black hole shadow is enlarged with the increase of the screening factor. It has been also stated that with the increase of the screening factor the radius of shadow increases, however, the distortion of the shadow decreases.

3. It has been shown that the maximum value of the energy emission rate decreases with the increase of screening factor, while it increases with the increase of charge of the black hole. It has been shown that the presence of Dyonic ModMax parameters such as charge and screening factor causes the deflection angle to decrease. It has been also obtained that photon and shadow radius decreasing with increasing plasma parameter. It has been indicated that the deflection angle and magnification of the images increase in the case of the presence of plasma.

4. The numerical values of the critical values of rotation, screening factor and charge parameters have been obtained, where the radii of the inner and outer horizons coincide. It has been shown that the thickness of the black hole's ergosphere increases with the increase of the charge and rotation parameters, while it decreases with the increase of screening factor. It has been found that the radius of innermost stable circular orbit increases with the increase of screening factor, while with the increase of charge parameter the radius of the innermost stable circular orbit decreases.

5. The analysis of the energy extraction process around ModMax black hole has shown that in the extremal case, the center-of-mass energy may diverge. It has been shown that the enthalpy and Gibbs free energy decrease with the increase of screening factor, while the Hawking temperature increases. It has been shown that the effect of the charge on thermodynamical properties of ModMax black hole is opposite.

НАУЧНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ НАЦИОНАЛЬНОГО ИССЛЕДОВАТЕЛЬСКОГО УНИВЕРСИТЕТА”ТИИИМСХ”.

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

УРИНОВ СУННАТИЛЛО ХУДОЕР УГЛИ

ВЛИЯНИЕ НЕЛИНЕЙНОЙ ЭЛЕКТРОДИНАМИКИ НА АСТРОФИЗИЧЕСКИЕ ПРОЦЕССЫ ВОКРУГ ЧЁРНЫХ ДЫР (НА ПРИМЕРЕ ТЕОРИИ MODMAX)

01.03.01 – Астрономия
01.04.02 – Теоретическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ
доктора философии (PhD) по физико-математическим наукам

Ташкент – 2024

Тема диссертации доктора философии (PhD) по физикоьматематическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за номером B2024.3.PhD/FM1169

Диссертация выполнена в Самаркандском государственном университете имени Шарофа Рашидова

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ifar.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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Защита диссертации состоится «__» ________ 2024 года в ___ часов на заседании Научного совета DSc.03/31.03.2022.T/FM.10.04 при Института фундаментальных и прикладных исследований национального исследовательского университета "ТИИИМСХ". Адрес: 100000, г.Ташкент, улица Кори Ниязова 39, Тел.: +998 71 237-09-61; e-mail: info@ifar.uz

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(протокол рассылки № __ от «__» ________ 2024 г.)

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования – разработка и усовершенствование астрофизических моделей для описания динамики массивных и безмассовых частиц, а также термодинамики вблизи чёрных дыр в модели гравитации ModMax.

Задачи исследования:

Изучить влияние заряда и фактора экранирования на структуру горизонта чёрных дыр в теории ModMax;

Проанализировать размер и форму тени чёрной дыры в зависимости от фактора экранирования;

Исследовать влияние параметров ModMax на величину параметра искажения тени чёрной дыры;

Изучить скорость излучения энергии, в частности, влияние параметра ModMax на максимальное значение эффективного потенциала;

Проанализировать влияние параметров Dyonic ModMax, таких как заряд и фактор экранирования, на угол отклонения фотонов;

Изучить влияние модификаций ModMax на фотонную сферу и радиус тени в присутствии плазмы;

Найти критические значения параметров вращения, фактора экранирования и заряда, при которых радиусы внутренних и внешних горизонтов совпадают;

Исследовать зависимость радиуса самой внутренней устойчивой круговой орбиты от фактора экранирования и заряда;

Проанализировать процесс извлечения энергии вокруг чёрной дыры ModMax;

Изучить энтальпию, свободную энергию Гиббса, температуру Хокинга чёрной дыры в теории ModMax, провести полный анализ термодинамики чёрной дыры.

Объектом исследования являются астрофизические чёрные дыры, пробные частицы с нулевой и ненулевой массой покоя, а также поле ModMax.

Предметом исследования являются теоретические подходы к изучению динамики тестовых частиц вблизи чёрных дыр, численные и аналитические методы решения дифференциальных уравнений.

Методы исследования включают методы вычислительной математики, методы теоретической астрофизики, современные методы математической физики, а также аналитические и численные методы решения дифференциальных уравнений для движения полей и частиц.

Научная новизна исследования заключается в следующем:

Изучена структура горизонта и уравнения движения фотонов вблизи чёрной дыры, а также их влияние на эффективный потенциал и форму тени чёрной дыры. Кроме того, исследовано влияние параметров чёрной дыры ModMax, таких как спин, заряд и фактор экранирования, на процесс излучения энергии. Показано, что увеличение параметра фактора экранирования приводит к расширению области чёрной дыры и сужению области, где чёрной дыры нет.

Структура горизонта событий заряженной вращающейся чёрной дыры ModMax изменяется следующим образом: радиус горизонта увеличивается с ростом значения фактора экранирования и уменьшается при увеличении параметра заряда. Показано, что радиус внутреннего (Коши) горизонта уменьшается, а внешний горизонт увеличивается с ростом фактора экранирования.

Показано, что пик эффективного потенциала, соответствующий орбите фотона, увеличивается и смещается влево при увеличении заряда, а фиксированный фактор экранирования уменьшает область между двумя потенциальными линиями.

С помощью формулировки Гамильтона-Якоби и метода разделения переменных выведено уравнение движения фотона вокруг чёрной дыры. Используя уравнения геодезических, была получена видимая форма тени чёрной дыры в небесных координатах. Установлено, что размер тени чёрной дыры увеличивается с ростом значения фактора экранирования.

Наблюдается, что орбиты фотонов уменьшаются из-за наличия параметра заряда и фактора экранирования. Анализ слабого гравитационного линзирования показывает, что присутствие параметров дионической теории ModMax приводит к уменьшению угла отклонения.

Также было получено, что радиусы фотонов и тени уменьшаются при увеличении параметра плазмы в случае неоднородной плазмы.

Показано, что толщина эргосферы чёрной дыры увеличивается с ростом заряда и параметров вращения, а с увеличением фактора экранирования становится тоньше.

Радиус самой внутренней устойчивой круговой орбиты (ISCO) чёрной дыры увеличивается с ростом фактора экранирования, в то время как радиус ISCO уменьшается с увеличением заряда чёрной дыры.

Анализ процесса извлечения энергии в результате столкновения двух массивных тестовых частиц вблизи горизонта чёрной дыры показал, что в экстремальном случае энергия в центре масс может быть сколь угодно высокой, тогда как для неэкстремальных чёрных дыр это невозможно.

Практические результаты исследования заключаются в следующем:

Впервые подробно исследована термодинамика заряженной чёрной дыры ModMax, включая энтальпию, температуру Хокинга и свободную энергию Гиббса. Показано, что энтальпия и свободная энергия Гиббса уменьшаются с ростом фактора экранирования, в то время как температура Хокинга увеличивается.

Проанализированы коэффициент искажения и средний радиус тени чёрной дыры. Показано, что с увеличением фактора экранирования радиус тени увеличивается, а коэффициент искажения уменьшается.

Впервые рассчитаны максимальные значения заряда чёрной дыры, соответствующие радиусам тени чёрной дыры, полученным на основе данных проекта Event Horizon Telescope.

Анализ показывает, что пик скорости излучения энергии уменьшается с ростом фактора экранирования и увеличивается с ростом значений заряда.

Достоверность результатов исследования обеспечивается применением современных проверенных методов математической физики, вычислительной математики и релятивистской астрофизики. Результаты были получены строго в рамках математического аппарата общей теории относительности и теоретической физики. Также использованы современные численные и аналитические методы вычислений, а результаты сопоставлены с доступными наблюдательными данными и работами других авторов. Структурированные выводы диссертации соответствуют основным правилам астрофизики компактных объектов.

Научная и практическая значимость результатов исследования:

Научная значимость результатов исследования заключается в том, что анализ решений для чёрных дыр в модели гравитации ModMax может быть полезным инструментом для изучения различных астрофизических процессов в рамках данной модели.

Практическая значимость результатов исследования состоит в том, что они могут способствовать разработке тестов модифицированных теорий гравитации и получить ограничения на параметры чёрных дыр с использованием наблюдений тени чёрных дыр.

Внедрение результатов исследования. Разработанные теоретические модели движения фотонов и частиц в теории ModMax были применены к следующим задачам:

Научные результаты, полученные по движению частиц и фотонов, были использованы учеными из Университета Фудань (FU) в Шанхае (Китай, 23 сентября 2024 года, ссылка);

Впервые термодинамика заряженной чёрной дыры ModMax, включая энтальпию, температуру Хокинга и свободную энергию Гиббса, была подробно изучена. Показано, что энтальпия и свободная энергия Гиббса уменьшаются с увеличением фактора экранирования, в то время как температура Хокинга увеличивается. Результаты по динамике частиц были использованы в работах зарубежных исследователей, опубликованных в зарубежных журналах с высоким импакт-фактором (Physics Letters B, Volume 854, id.138758, Web of Science, IF-4.4; Communications in Theoretical Physics, Volume 76, Issue 8, id.085402, Web of Science, IF-2.4; Pramana, Volume 98, Issue 3, id.92, Web of Science, IF-2.219 и других). Эти результаты использовались более чем в 5 опубликованных научных статьях для описания эффектов частиц вокруг чёрной дыры;

Таким образом, удалось получить информацию о динамике частиц вокруг компактных объектов в рамках модифицированных теорий гравитации.

Апробация результатов исследования.

Результаты исследований были представлены на 3 республиканских и 1 международных научных конференциях.

Опубликованность результатов исследования.

По теме диссертационной работы опубликованы 7 научных работ в научных журналах, рекомендованных Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан для публикации основных научных результатов докторских диссертаций, в том числе 1 зарубежная.

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка литературы. Объем диссертации 81 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim(I часть; I part)

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2. S. Urinov, Magnetized particle motion around magnetized Schwarzschild-MOG black hole. // Journal of Fundamental and Applied Research, Vol 4 (3), 2024, id. 20240012 (01.00.00. No 5)
3. S. Urinov, B. Rahmatov, I. Nishonov, The motion of a particle around Hayward regular black hole // Scientific reports of Bukhara state University, 2024/8 (113) B. 116-123 (01.00.00. No 5)
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7. Sunnatillo Urinov, Ziyodulla Turakhonov, and Ahmadjon Abdujabbarov Strong Gravitational Lensing Around Dyonic ModMax Black Holes // xalqaro ilmiy-amaliy konferensiya Fizika -2024. Samarqand. B. 78-83 (01.00.00. No 5)
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Bosishga ruxsat etildi: 05.11.2024-yil.
Bichimi 60x84 1/16, «Times New Roman»
garniturada raqamli bosma usulida bosildi.
Shartli bosma tabog‘i 3,2. Adadi: 100. Buyurtma: № 21.

«DAVR MATBUOT SAVDO» MChJ
bosmaxonasida chop etildi.
Toshkent, Qo‘yliq, 4-mavze, 46-uy.