V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

ESHIMBETOV MUZAFFAR REYIMBAYEVICH

τ-SILLIQ IDEMPOTENT O‘LCHOVLARNING TAVSIFI

01.01.01 – Matematik analiz

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PHD)
DISSERTATSIYASI AVTOREFERATI**

TOSHKENT – 2024

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestasiya komissiyasida B2023.3.PhD/FM908 raqam bilan ro‘yxatga olingan.

Dissertatsiya V.I.Romanovskiy nomidagi Matematika institutida bajarilgan.

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Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2024 yil 3-dekabr kuni soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I. Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (190-raqami bilan ro‘yhatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlarda idempotent o‘lchovlar nazariyasining keng doiradagi muammolarni tafsivlash va tahlil qilishga kuchli vosita ekanligi ayon bo‘lib kelmoqda. Idempotent o‘lchovlar nazariyasiga yondashuvning juda yaxshi rivojlangan sifatli geometrik usullari mavjud bo‘lib, bu yondashuv odatda idempotent o‘lchovlar orqali aniqlanadigan matematik fizika, kompyuter ilmlari, optimallashtirish va boshqa muammolarga tayanib, fizika, informatsion texnologiya va iqtisodiyotdagi masalalarni hal qilishda muhim rol o‘ynaydi. Undan tashqari τ-silliq idempotent ehtimollik o‘lchovlar nazariyasida olingan natijalar ham nazariy, ham amaliy ahamiyatga ega ekanligidan idempotent o‘lchovlar nazariyasini tadqiq etish zamonaviy matematika sohalaridagi muhim va dolzarb vazifalardan biri ekanligi ko‘rinadi.

Hozirgi vaqtda ko‘pgina mualliflar limit og‘ishlarini idempotent jarayonlarning taqsimoti sifatida izohlab keladilar va natijalarni yarim martingallarning yarim maksingallarga taqsimoti bo‘yicha katta og‘ish yaqinlashishi ko‘rinishida ifodalaydilar. Masalan, A.Puhalskii kichik diffuziya hadlari bilan diffuziya jarayonlari uchun katta og‘ish prinsipini idempotent diffuziyaga taqsimotini katta og‘ishlarga yaqinlashishi sifatida ifodalaydi. Bugungi kunda, Markov jarayonlarining katta og‘ishlarga yaqinlashishi va keyingi sistemalarda hosil bo‘ladigan jarayonlar ko‘plab amaliy masalalarni tahlil qilishda keng qo‘llanilmoqda. Shu munosabat bilan τ-silliq idempotent ehtimollik o‘lchovlarini tavsiflash, bu o‘lchovlar fazolarining geometrik va topologik xossalarini o‘rganish maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda so‘ngi yillarda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan tabiiy va aniq fanlarga e’tibor kuchaytirilmoqda. Idempotent o‘lchovlar nazariyasi juda yaxshi rivojlangan bo‘lib, xususan, yangi integral nazariya, yangi chiziqli algebra, spektral nazariya va funksional tahlilni o‘z ichiga oladi. Bu sahoda ko‘p me’zonli qarorlar qabul qilish, grafiklarda optimallashtirish, katta parametrli diskret optimallashtirish, kompyuter sistemalaridagi optimal loyiha va kompyuter media, ma’lumotlarni parallel qayta ishlashni optimal tashkil qilish, dinamik dasturlash, diskret hodisalar sistemalari, kompyuter ilmlari, diskret matematika, matematik mantiq va boshqalar kabi turli xil optimallashtirish masalalarini o‘z ichiga oladigan fanlarida keng tatbiqiga ega bo‘lgan idempotent o‘lchovlarni rivojlantirishga alohida e’tibor qaratilmoqda. Bu borada τ-silliq idempotent ehtimollik o‘lchovlar bilan bog‘liq muhim natijalarga erishildi. “Funksional analiz, matematik fizika va statistik fizika” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi[1]. Bu vazifalar ijrosini ta’minlashda ilmiy natijalardan ilm-fanning turdosh sohalarida foydalanish maqsadida τ-silliq idempotent o‘lchovlar nazariyasini rivojlantirish muhim ahamiyatga ega.

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[1] O‘zbekiston Respublikasi Vazirlar mahkamasi 2017-yil 18 maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqotlar muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

Oʻzbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi “Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha harakatlar strategiyasi toʻgʻrisida”gi PF-4947-son Farmoni, 2019 yil 9 iyuldagi “Matematika taʼlimi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek, Oʻzbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida”gi PQ-4387-son Qarori va 2020 yil 7 maydagi “Matematika sohasidagi taʼlim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari toʻgʻrisida”gi PQ-4708-sonli Qarori hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yoʻnalishlariga bogʻliqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yoʻnalishi doirasida bajarilgan.

Muammoning oʻrganilganlik darajasi. Idempotent oʻlchovlar nazariyasi matematika fanlarining bir tarmogʻi boʻlib, bu soha rivojlanishining muhim bosqichi J.Gunavardena muharrirligi ostida nashr qilingan “Idempotentlik” kitobida keltirilgan. Idempotent va tropik matematika rivojlanishining keyingi bosqichlari G.L.Litvinov va V.P.Maslov muharrirligi ostida nashr qilingan “Idempotent matematika va Matematik fizika” kitobida keltirilgan. Idempotent matematika odatdagi arifmetik amallarni yangi asosiy amallar toʻplami bilan almashtirishga, yaʼni sonli maydonlarni idempotent yarim xalqalar va yarim maydonlar bilan almashtirishga asoslangan. Bunga max-plus algebrasi deb ataluvchi $\mathbb{R}_{\max}$ yarim maydon oddiy misol boʻladi. Koʻplab mualliflar (S.K.Klin, S.-N.N.Pandit, N.N.Vorobyov, B.A.Carré, R.A.Kuningem-Green, K.Zimmermann, U.Zimmermann, M.Gondran, F.L.Bakkelli, G.Kohen, S.Gaubert, G.J.Olsder, J.-P.Quadrat, V.N.Kolokolsov va boshqalar) S.K.Klinning klassik maqolasidan soʻng kompyuter ilmlari va diskret matematikaning baʼzi amaliy masalalarini yechish uchun ushbu yarim halqalar ustida idempotent yarim halqalar va matritsalardan foydalanish keng rivojlandi.

Idempotent ehtimollik oʻlchovlar fazosi nisbatan yangi obʼekt hisoblanib, M.Zarichniy 2010 yilda idempotent ehtimollik oʻlchovlar fazosining kategorik xossalarini tadqiq qildi. Aslida idempotent ehtimollik oʻlchovlari avvalroq Oʻlchovlar nazariyasi, Funksional analiz, Ehtimollar nazariyasi, Topologiya va Kategoriyalar nazariyasida har xil nuqtai nazardan bevosita yoki bilvosita oʻrganilganini eʼtirof etish joiz. Idempotent ehtimollik oʻlchovlari fazolarini oʻrganish uni kompakt Hausdorf fazolarning sinfiga nisbatan kengroq topologik fazolar sinflarida, xususan, Tixonov fazolar sinfida oʻrganish masalasiga olib keladi. T.O.Banax va T.N.Radul Tixonov fazolarida ehtimollik oʻlchovlar boʻyicha tizimli tatqiqot oʻtkazgan. Ular oʻzlarining tatqiqotlarida ehtimollik oʻlchovlarning chiziqliligidan samarali foydalangan. Sh.Ayupov, A.Zaitov ishlarida τ-silliq kuchsiz additiv funksionallar nazariyasi ishlab chiqilgan va baʼzi kategorik xossalari oʻrganilgan. Koʻpgina mualliflar tomonidan olingan natijalar shuni koʻrsatadiki, idempotent ehtimollik oʻlchovlar fazosining “yaxshi” xossalarini oʻrganish uchun klassik usullardan juda farq qiladigan usullar talab qilinadi. τ-silliq idempotent ehtimollik oʻlchovlari fazosini o‘rganish klassik masalalarning yangi talqinini vujudga keltiradi. Masalan, Chex ma’nosida to‘la fazolar Eduard Chex tomonidan 1937 yilda Berning kategoriyaviy teoremasini isbotlash uchun kiritilgan edi. O‘z-o‘zidan τ-silliq idempotent ehtimollik o‘lchovlari fazosining Chex ma’nosida to‘la bo‘lishi haqida savol tug‘iladi.

1909 yilda Vengriyalik matematik Friges Riss barcha uzluksiz chiziqli funksionallarni birlik kesmada Riman-Stilt’es integrallari bilan ifodalash mumkinligini isbotladi. Riss kiritgan belgilashdan foydalanib, Riss teoremasini quyidagicha yozish mumkin:

$$A[f(\omega)] = \int_0^1 f(\omega)\,d\alpha(\omega),$$

bu yerda α birlik kesmada o‘zgarishi chegaralangan funksiya. Yuqoridagi tenglik Rissning ifodalash haqidagi teoremasi sifatida nomlanadi. 1938 yilda rossiyalik matematik Andrey Markov Rissning bu yutug‘ini kompakt bo‘lmagan intervallarga davom ettirdi. Shundan uch yil o‘tgach, 1941 yilda Yapon-Amerikalik matematik Shizuo Kakutani mazkur teoremani abstrakt kompakt Hausdorf fazolarda isbotladi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish instituti ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya ishi V.I.Romanovskiy nomidagi Matematika institutining ilmiy-tadqiqot ishlari rejalari doirasida bajarilgan.

Tadqiqot maqsadi funksional tilda aniqlangan τ-silliq idempotent o‘lchovlarni to‘plam funksiya tilida aniqlangan τ-silliq idempotent o‘lchovlar orqali tavsiflashdan iborat.

Tadqiqotning vazifalari:

an’anaviy (chiziqli) o‘lchovlarning τ-silliqligi (yoki τ-additivligi) me’zoniga o‘xshash idempotent o‘lchovlarning τ-silliqligi me’zonini aniqlash;

τ-silliq idempotent o‘lchovlarning funksional tavsifini topish;

to‘plam funksiyasi sifatida idempotent o‘lchovlar fazosi va uzluksiz funksiyalarning max-plus chiziqli fazosining max-plus qo‘shma fazosi orasidagi izomorfizm bo‘yicha Riss ifodalashining muqobilini o‘rnatishdan iborat.

Tadqiqot ob’ekti: τ-silliq idempotent o‘lchov, τ-maksitiv idempotent o‘lchov, o‘lchovli funksiyalar, Chex ma’nosida to‘la fazolar.

Tadqiqot predmeti. Matematik analiz, funksional analiz, o‘lchovlar nazariyasi, topologiya, idempotent analiz.

Tadqiqot usullari. Tadqiqot ishida matematik analiz, funksional analiz, o‘lchovlar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

berilgan to‘plam funksiyasi uchun uning davomi va cheklanishi idempotent o‘lchov bo‘lishi va bu amallarning kompozitsiyasi ayniy akslantirish bo‘lishi isbotlangan;

berilgan kompakt Hausdorf fazosining ochiq to‘plamlaridan foydalanib idempotent ehtimollik o‘lchovlar to‘plamida biror topologiyaning bazasi bo‘ladigan sistema aniqlangan va kompakt Hausdorf fazo uchun hosil bo‘lgan idempotent ehtimollik o‘lchovlarning topologik fazosi ham kiritilgan bu topologiyaga nisbatan kompakt Hausdorf fazo bo‘lishi isbotlangan;

τ-silliq idempotent ehtimollik o‘lchovlarning fazosi Chex ma’nosida to‘la bo‘lishi uchun berilgan Tixonov fazosi Chex ma’nosida to‘la bo‘lishi zarur va yetarli ekanligi isbotlangan;

to‘plam funksiyasi sifatida idempotent o‘lchovlar fazosi va uzluksiz funksiyalarning max-plus chiziqli fazosining max-plus qo‘shma fazosi orasidagi izomorfizm bo‘yicha Riss ifodalash teoremasining muqobili isbotlangan va Tixonov fazosidagi τ-silliq idempotent o‘lchovlarning to‘plam funksiya sifatidagi tavsifi olingan.

Tadqiqotning amaliy natijalari. Dissertatsiyada olingan natijalar va foydalanilgan usullarni oliy ta’lim muassasalari magistrantlari va tayanch doktorantlari uchun maxsus kurslarda o‘qitish mumkin. Shuningdek, olingan natijalar iqtisodiy masalalarni hal qilish imkonini beradi.

Tadqiqot natijalarining ishonchliligi. Matematik analiz, funksional analiz, haqiqiy o‘zgaruvchili funksiyalar nazariyasi va o‘lchovlar nazariyasi usullaridan foydalanilgan. Olingan natijalar qat’iy matematik mulohazalarga tayanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati τ-silliq idempotent o‘lchovlarning ba’zi kategorik xossalarini tekshirishda foydalanish mumkinligi bilan izohlanadi.

Dissertatsiyaning amaliy ahamiyati τ-silliq idempotent o‘lchovlarning ba’zi kategorik xossalarini tavsiflash imkonini bergani bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. τ-silliq idempotent o‘lchovlarning tavsifi bo‘yicha olingan natijalar asosida:

berilgan to‘plam funksiyasining davomi va cheklanishi idempotent o‘lchov va bu amallarning kompozitsiyasi ayniy akslantirish bo‘lishi tavsifidan A-OT-2021-108 raqamli “Orolbo‘yi mintaqasining atrof-muhit va ekologik holatini monitoring qilish va prognozlashning axborot-tahliliy dasturiy mahsulotini yaratish” nomli ilmiy loyihada atmosferada zararli aralashmalarning tarqalish jarayonining matematik modeli va hisoblash algoritmlarini matematik tahlil qilishga oid masalalarni yechishda foydalanilgan (Raqamli texnologiyalar va sun’iy intellektni rivojlantirish ilmiy-tadqiqot institutining 2023 yil 24-avgustdagi ma’lumotnomasi, O‘zbekistan). Dissertatsiya natijalari grant doirasida paydo bo‘lgan optimallashtirish va optimal boshqarish masalalarini tahlil etishda yuzaga kelgan muammolarini hal qilishga xizmat qilgan;

τ-silliq idempotent ehtimollik o‘lchovlarning fazosi Chex ma’nosida to‘la bo‘lishidan Rossiya ilmiy Jamg‘armasining 24-21-00278-sonli “Qavariq geometriya muammolarida operator usullari” granti tomonidan qo‘llab quvvatlangan ilmiy tadqiqotlar doirasida foydali texnik vosita sifatida ishlatilgan. (K.L.Xetagurov nomidagi Shimoliy Osetiya davlat universitetining 2024 yil 3 oktyabrdagi ma’lumotnomasi, Rossiya). Idempotent ehtimollik o‘lchovlari fazosining topologik xossalari Hausdorf topologik fazosi hosil qilgan kompaktda uzluksiz funksiyalar fazosida aniqlangan chiziqli va additiv ortogonal operatorlarni analitik tasvirlash masalasini hal qilish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 3 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami

10 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan, 1 tasi xorijiy va 4 tasi respublika jurnallarida, shuningdek 5 ta ma’ruza tezislari ilmiy konferensiya materiallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, o‘nta paragrafga bo‘lingan uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 84 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan bo‘lib tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning **“Muhim tushunchalar”** deb nomlangan birinchi bobi uchta paragrafdan iborat bo‘lib, unda dissertatsiya mavzusini to‘la yoritish uchun zarur bo‘lgan asosiy ta’riflar va muhim faktlar keltirilgan.

Birinchi paragrafda τ-algebra va τ-silliq idempotent o‘lchovlar uchun asosiy ta’riflar va faktlar keltirilgan.

Ω ixtiyoriy to‘plam bo‘lib, $\mathcal{E}$ sistema $\varnothing$ ni o‘z ichiga olgan Ω ning qism to‘plamlari sistemasi, $\mathcal{P}(\Omega)$ esa Ω ning barcha qism to‘plamlari sistemasi va $\overline{\mathbb{R}}_+ = [0,+\infty) \cup \{+\infty\} = [0,+\infty]$ bo‘lsin. $\mathfrak{A}$ orqali yo‘naltirilgan to‘plamni, Δ orqali esa ixtiyoriy indekslar to‘plamini belgilaymiz.

1-ta’rif. *Agar* $\mu : \mathcal{P}(\Omega) \to \overline{\mathbb{R}}_+$ *to‘plam funksiya*

1. $\mu(\varnothing) = 0$*;*

2. ixtiyoriy $A, B \in \mathcal{P}(\Omega)$ *uchun* $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}$*;*

3. Ω *to‘plamning qism to‘plamlarining ixtiyoriy* $\{A_\alpha : \alpha \in \mathfrak{A}\}$ *o‘suvchi to‘ri uchun*

$$\mu\Big(\bigcup_{\alpha\in\mathfrak{A}} A_\alpha\Big) = \sup_{\alpha\in\mathfrak{A}}\{\mu(A_\alpha)\}$$

shartlar bajarilsa, u holda μ *to‘plam funksiyasi* Ω *da idempotent o‘lchov deb ataladi.*

Agar $\mu(\Omega) = 1$ *bo‘lsa, u holda* μ *idempotent o‘lchov* Ω *da idempotent ehtimollik o‘lchov deb ataladi.*

2-ta’rif. *Agar 1-ta’rifning shartlariga qo‘shimcha ravishda* $\mathcal{E}$ *sistema elementlarining har bir* $\{F_\alpha : \alpha \in \mathfrak{A}\}$ *kamayuvchi to‘ri uchun*

$$\mu\Big(\bigcap_{\alpha\in\mathfrak{A}} F_\alpha\Big) = \inf_{\alpha\in\mathfrak{A}}\{\mu(F_\alpha)\}$$

shart o‘rinli bo‘lsa, u holda μ idempotent o‘lchov Ω da $\mathcal{E}$ ga nisbatan τ-silliq (yoki τ-additiv) idempotent o‘lchov yoki qisqalik uchun, $\mathcal{E}$-idempotent o‘lchov deyiladi.

Ikkinchi paragrafda o‘lchovli funksiyaning va idempotent integralning ta’riflari va ularning xossalari berilgan. Bundan tashqari, idempotent integralning asosiy xossalarining to‘liq isboti ham bayon qilingan.

Endi idempotent fazolardagi o‘lchovli funksiyalarni qaraylik. $\mathcal{E}_1$ and $\mathcal{E}_2$ sistemalar mos ravishda, ixtiyoriy Ω_1 va Ω_2 to‘plamlarning qism to‘plamlari sistemasi bo‘lib, bu sistemalarning har biri $\varnothing$ ni o‘z ichiga olsin. $f:\Omega_1 \to \Omega_2$ funksiya uchun Ω_1 ning qism to‘plamlari sistemasini

$$f^{-1}(\mathcal{E}_2) = \left\{ f^{-1}(B) : B \in \mathcal{E}_2 \right\}$$

ko‘rinishda aniqlaymiz.

3-ta’rif. *Agar har bir $A \in \mathcal{E}_2$ uchun $f^{-1}(A) \in \mathcal{E}_1$ bo‘lsa, u holda $f:\Omega_1 \to \Omega_2$ funksiya $\mathcal{E}_1 / \mathcal{E}_2$-o‘lchovli deb ataladi.*

$f:\Omega_1 \to \Omega_2$ funksiya uchun Ω_2 to‘plamdagi μ' to‘plam funksiyasini $\mu'(A) = \mu\left(f^{-1}(A)\right)$, $A \subset \Omega_2$ kabi aniqlaymiz. μ' to‘plam funksiyasi f funksiyaga nisbatan μ ning obrazi deb ataladi va $\mu \circ f^{-1}$ kabi belgilanadi.

Endi idempotent o‘lchovga nisbatan idempotent integral tushunchasini keltiramiz. (Ω, μ) juftlik idempotent o‘lchovli fazo bo‘lib, buning uchun $\mu(\Omega) < \infty$ bo‘lsin. Biz $\infty \cdot 0 = 0$ deb qabul qilamiz.

4-ta’rif. O*‘lchovli $f:\Omega \to \overline{\mathbb{R}}_+$ funksiya uchun μ ga nisbatan f funksiyaning idempotent integralini*

$$\int_{\Omega}^{\oplus} f d\mu = \sup_{t \in \overline{\mathbb{R}}_+} \left\{ t \cdot \mu\left\{ \omega \in \Omega : f(\omega) \geq t \right\} \right\}$$

orqali aniqlaymiz. $A \subset \Omega$ uchun $\int_{A}^{\oplus} f d\mu = \int_{\Omega}^{\oplus} f \chi_A d\mu$ deb olamiz.

Uchinchi paragrafda ba’zi umumiy topologik tushunchalar va teoremalar keltirilgan.

Agar Ω Tixonov fazosi va $\beta\Omega \setminus \Omega$ to‘ldirma $\beta\Omega$ Stoun-Chex kompaktifikatsiyada F_σ-to‘plam bo‘lsa, u holda Ω topologik fazo Chex ma’nosida to‘la deyiladi. Lokal kompakt fazolar Chex ma’nosida to‘la bo‘ladi, ammo teskarisi o‘rinli emas. To‘g‘ri chiziqning qism fazosidagi topologiya bilan barcha irratsional sonlar fazosi lokal kompakt bo‘lmagan Chex ma’nosida to‘la fazoga misol bo‘ladi. Chex-to‘lalik G_δ-qism to‘plamlarga va yopiq qism to‘plamlarga nisbatan bog‘langan.

Dissertatsiyaning **“Idempotent o‘lchovlarning davom ettirishlari va cheklanishlari”** deb nomlangan ikkinchi bobi uchta paragrafdan tashkil topgan. Birinchi paragrafda τ-silliq idempotent o‘lchovlarning ayrim kengaytmalari uchun ularning τ-silliq va τ-maksitiv bo‘lishlari isbotlangan. So‘ngra, Luzin ma’nosidagi o‘lchovli ixtiyoriy funksiyaning teskarisi va har bir τ-silliq idempotent o‘lchovning kompozitsiyasi yana τ-silliq idempotent bo‘lishi ko‘rsatilgan.

Faraz qilaylik, ξ_i, $i=1,2,\ldots$ sistemalar Ω ning qism to‘plamlari sistemalari bo‘lib, ularning har biri $\varnothing$ ni o‘z ichiga olsin. Ω ning qism to‘plamlaridan tuzilgan yangi ζ sistemani quyidagicha aniqlaymiz:

$$\zeta=\overset{\infty}{\underset{i=1}{\oplus}}\xi_i=\left\{\bigcup_{i=1}^{\infty}A_i : A_i\in\xi_i\right\}.$$

1-teorema. *μ idempotent o‘lchov $\zeta=\overset{\infty}{\underset{i=1}{\oplus}}\xi_i$ ga nisbatan τ-silliq bo‘lishi uchun μ idempotent o‘lchov har bir ξ_i, $i\in\mathbb{N}$ ga nisbatan τ-silliq bo‘lishi zarur va yetarli.*

Endi $\mathcal{E}_i$ va $\mathcal{E}'_i$ sistemalar mos ravishda, har biri $\varnothing$ ni o‘z ichiga olgan Ω va Ω' to‘plamlarning qism to‘plamlari sistemalari bo‘lsin, bu yerda $i\in\mathbb{N}$. So‘ngra, $\mathcal{E}$ va $\mathcal{E}'$ sistemalarni quyidagicha aniqlaymiz:

$$\mathcal{E}=\overset{\infty}{\underset{i=1}{\oplus}}\mathcal{E}_i=\left\{\bigcup_{i=1}^{\infty}A_i : A_i\in\mathcal{E}_i\right\} \qquad \text{va} \qquad \mathcal{E}'=\overset{\infty}{\underset{i=1}{\oplus}}\mathcal{E}'_i=\left\{\bigcup_{i=1}^{\infty}A'_i : A'_i\in\mathcal{E}'_i\right\}.$$

2-teorema. *Agar $\mu:\mathcal{E}\to\overline{\mathbb{R}}_+$ to‘plam funksiyasi τ-maksitiv bo‘lsa, u holda $\mu':\mathcal{E}'\to\overline{\mathbb{R}}_+$ to‘plam funksiyasi ham τ-maksitiv bo‘ladi.*

3-teorema. *$\mathcal{T}$ sistema μ – $\mathcal{E}$-idempotent o‘lchov uchun siqilish bo‘lsin. Agar $f:\Omega\to\Omega'$ funksiya Luzin ma’nosida $(\mathcal{E},\mathcal{T})/\mathcal{E}'$-o‘lchovli bo‘lsa, u holda Ω' to‘plamda μ' ham $\mathcal{E}'$-idempotent o‘lchov bo‘ladi. Agar $T\in\mathcal{T}$ uchun $f(T)\cap F'\in\mathcal{E}'$ va $F'\in\mathcal{E}'$ bo‘lsa, u holda $f(T)$ sistema μ' uchun siqilish bo‘ladi.*

Ikkinchi paragrafda τ-maksitiv to‘plam funksiyalarning davom ettirishlari va cheklanishlari o‘rganilgan. So‘ngra davom ettirish va cheklash amallarining kompozitsiyasi ayniy akslartirish bo‘lishi isbotlangan.

Ω ixtiyoriy to‘plam, Ω_0 esa Ω ning qism to‘plami va $\mathcal{E}$ sistema Ω da τ-algebra bo‘lsin. Quyidagi sistemani olamiz:

$$\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}.$$

Faraz qilaylik, $\mu:\mathcal{P}(\Omega_0)\to\overline{\mathbb{R}}_+$ idempotent o‘lchov berilgan bo‘lsin. Uning davomini $e_{\Omega}^{\Omega_0}(\mu):\mathcal{P}(\Omega)\to\overline{\mathbb{R}}_+$ quyidagi qoida bo‘yicha aniqlaymiz:

$$e_{\Omega}^{\Omega_0}(\mu)(B)=\mu(B\cap\Omega_0),\ B\subset\Omega.$$

$\mathcal{E}$ sistema Ω da τ-algebra va $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$, μ esa Ω da $\mathcal{E}$-idempotent o‘lchov bo‘lsin. μ ning $r_{\Omega_0}^{\Omega}(\mu)$ cheklanishini quyidagi qoida bo‘yicha aniqlaymiz:

$$r_{\Omega_0}^{\Omega}(\mu)(A)=\inf\{\mu(C) : C\in\mathcal{E}, C\cap\Omega_0=A\},\ \ A\in\mathcal{E}_0.$$

Endi $IM(\Omega)$ orqali Ω to‘plamning barcha qism to‘plamlar sistemasi $\mathcal{P}(\Omega)$ da aniqlangan barcha idempotent o‘lchovlar to‘plamini va $M^{\tau}(\Omega)$ orqali esa $\mathcal{E}$ – τ-algebrada aniqlangan barcha τ-maksitiv to‘plam funksiyalarini belgilaymiz.

1-tasdiq. *$\mathcal{E}$ sistema Ω da τ-algebra, Ω_0 esa Ω ning qism to'plami va $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$ bo'lsin. $r^{\Omega}_{\Omega_0}\circ e^{\Omega_0}_{\Omega} : M^{\tau}(\Omega_0)\to M^{\tau}(\Omega_0)$ kompozitsiya $M^{\tau}(\Omega_0)$ da ayniy akslantirish, ya'ni*

$$r^{\Omega}_{\Omega_0}\circ e^{\Omega_0}_{\Omega} = id_{IM(\Omega_0)}.$$

1-izoh. *$\mathcal{E}$ sistema Ω da τ-algebra, Ω_0 esa Ω ning qism to'plami, va $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$ bo'lsin. $e^{\Omega_0}_{\Omega}\circ r^{\Omega}_{\Omega_0} : M^{\tau}(\Omega)\to M^{\tau}(\Omega)$ kompozitsiya $e^{\Omega_0}_{\Omega}\circ r^{\Omega}_{\Omega_0} = id_{IM(\Omega)}$ tenglikni qanoatlantirmaydi.*

Masalan, $\Omega=\{0,1,2\}$ va $\Omega_0=\{0,1\}$ bo'lsin. $\mathcal{E}=\mathcal{P}(\Omega)$ orqali Ω ning barcha qism to'plamlari sistemasini belgilaymiz. Biz $\mu:\mathcal{P}(\Omega)\to\overline{\mathbb{R}}_+$ to'plam funksiyani quyidagicha aniqlaymiz: agar $2\in B$ bo'lsa, $\mu(B)=1$ va agar $2\notin B$ bo'lsa, $\mu(B)=0$. Endi $B=\{0,2\}\subset\Omega$ deb olaylik. U holda

$$e^{\Omega_0}_{\Omega}\left(r^{\Omega}_{\Omega_0}(\mu)\right)(B) = r^{\Omega}_{\Omega_0}(\mu)(B\cap\Omega_0) = \inf\{\mu(C) : C\in\mathcal{E}, C\cap\Omega_0 = B\cap\Omega_0=\{0\}\} =$$
$$=\inf\{\mu(\{0\})\} = \mu(\{0\}) = 0 \neq 1 = \mu(B).$$

Demak, $e^{\Omega_0}_{\Omega}\circ r^{\Omega}_{\Omega_0} \neq id_{IM(\Omega)}$.

$\mathcal{E}$ sistema Ω da τ-algebra, Ω_0 esa Ω ning qism to'plami, va $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$ bo'lsin. Quyidagi to'plamni qaraylik:

$$M^{*}_{\Omega_0}(\Omega) = \{\mu\in M^{\tau}(\Omega) : K\subset\Omega\setminus\Omega_0 \text{ bo'ladigan ixtiyoriy } K\in\mathcal{E} \text{ uchun } \mu(K)=0\}.$$

4-teorema. *$\mathcal{E}$ sistema Ω to'plamda τ-algebra, Ω_0 esa Ω ning qism to'plami, va $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$ bo'lsin. U holda*

1. $e^{\Omega_0}_{\Omega}\left(M^{\tau}(\Omega_0)\right)\subset M^{*}_{\Omega_0}(\Omega)$.

2. $e^{\Omega_0}_{\Omega}\circ\left(r^{\Omega}_{\Omega_0}\mid_{M^{*}_{\Omega_0}(\Omega)}\right) : M^{*}_{\Omega_0}(\Omega)\to M^{*}_{\Omega_0}(\Omega)$ kompozitsiya $M^{*}_{\Omega_0}(\Omega)$ ayniy akslantirish, ya'ni

$$e^{\Omega_0}_{\Omega}\circ\left(r^{\Omega}_{\Omega_0}\mid_{M^{*}_{\Omega_0}(\Omega)}\right) = id_{M^{*}_{\Omega_0}(\Omega)}.$$

Uchinchi paragrafda τ-maksitiv idempotent o'lchovlarning davom ettirishlari va cheklanishlari, idempotent o'lchovlar fazolari orasidagi akslantirishlar o'rganilgan.

Ixtiyoriy Ω_1 va Ω_2 to'plamlarni qaraylik. $\mathcal{E}_2$ sistema Ω_2 ning qism to'plamlari sistemasi bo'lsin. $f:\Omega_1\to\Omega_2$ akslantirish uchun quyidagi sistemani olamiz:

$$f^{-1}(\mathcal{E}_2)=\{f^{-1}(B) : B\in\mathcal{E}_2\}.$$

$f:\Omega_1\to\Omega_2$ akslantirish uchun $IM(\Omega_1)$ to'plamdan $IM(\Omega_2)$ to'plamga akslantirishni $M(f)$ orqali belgilab, uni quyidagi qoida bo'yicha aniqlaymiz:

$$M(f)(\mu)(B)=\mu\left(f^{-1}(B)\right),\ B\subset\Omega_2.$$

Ω_2' to‘plam Ω_2 ning qism to‘plami, $\Omega_1=f^{-1}\left(\Omega_2'\right)$ va $\mu\in IM\left(\Omega_1\right)$ bo‘lsin.

Ushbu

$$e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right)(A)=\mu\left(f^{-1}\left(A\cap\Omega_2'\right)\right),\ A\subset\Omega_2$$

formula orqali aniqlangan $e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right):\mathcal{P}(\Omega_2)\to\overline{\mathbb{R}}_+$ to‘plam funksiyasini qaraymiz.

$\mathcal{K}=\left\{B\cap\Omega_2' : B\in\mathcal{E}_2\right\}$ bo‘lsin. U holda $f^{-1}(\mathcal{K})=f^{-1}(\mathcal{E}_2)$.

2-tasdiq. *Agar μ idempotent o‘lchov Ω_1 da $f^{-1}(\mathcal{E}_2)$ ga nisbatan τ-silliq bo‘lsa, u holda $e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right)$ to‘plam funksiyasi Ω_2 da $\mathcal{E}_2$ ga nisbatan τ-silliq idempotent o‘lchov bo‘ladi.*

$\mu\in IM\left(\Omega_1\right)$ idempotent o‘lchov berilgan bo‘lsin. Ushbu

$$r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right):\mathcal{P}\left(\Omega'\right)_2\to\overline{\mathbb{R}}_+$$

to‘plam funksiyasini

$$r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right)(A)=\inf\left\{\mu\left(f^{-1}(C)\right):C\in\mathcal{E}_2, C\cap f\left(\Omega_1\right)=A\right\}$$

qoida bo‘yicha aniqlaymiz, bu yerda $A\subset\Omega'_2=f\left(\Omega_1\right)$.

3-tasdiq. *Agar μ idempotent o‘lchov Ω_1 da $f^{-1}(\mathcal{E}_2)$ ga nisbatan τ-silliq idempotent o‘lchov bo‘lsa, u holda $r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right)$ to‘plam funksiyasi Ω_2' da $\mathcal{K}$ ga nisbatan τ-silliq idempotent o‘lchov bo‘ladi.*

5-teorema. *$\mu\in M^{\tau}\left(\Omega_1\right)$ bo‘lishi uchun $r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right)\in M^{\tau}\left(f\left(\Omega_1\right)\right)$ bo‘lishi zarur va yetarli.*

Dissertatsiyaning **“τ-silliq idempotent o‘lchovlarning tavsifi”** deb nomlangan uchinchi bobi to‘rtta paragrafdan iborat.

Uchinchi bobning birinchi paragrafida berilgan to‘plamda aniqlangan idempotent o‘lchovni uning havzasida aniqlangan idempotent o‘lchov deb qarash mumkinligi isbotlangan. Berilgan kompakt Hausdorf fazosining ochiq to‘plamlaridan foydalanib idempotent ehtimollik o‘lchovlar to‘plamida biror topologiyaning bazasi bo‘ladigan sistema aniqlangan. Undan tashqari, Ω kompakt Hausdorf fazo uchun hosil bo‘lgan $I(\Omega)$ da topologik fazo ham kiritilgan bo‘lib, uning kompakt Hausdorf fazo bo‘lishi isbotlangan.

Ω kompakt Hausdorf fazo va $\mathfrak{B}(\Omega)$ esa Ω ning Borel qism to‘plamlar sistemasi bo‘lsin. Quyidagicha tushuncha kiritamiz.

5-ta’rif. *Agar $\mu:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ to‘plam funksiyasi*

1. $\mu(\varnothing)=0$*;*

2. ixtiyoriy $A,B\in\mathfrak{B}(\Omega)$ *uchun* $\mu(A\cup B)=\max\left\{\mu(A),\mu(B)\right\}$*;*

3. $\bigcup_{\alpha\in\mathfrak{A}} A_\alpha \in \mathfrak{B}(\Omega)$ *bo'ladigan ixtiyoriy* $\{A_\alpha : \alpha\in\mathfrak{A}\}\subset\mathfrak{B}(\Omega)$ *o'suvchi to'r uchun*

$$\mu\Big(\bigcup_{\alpha\in\mathfrak{A}} A_\alpha\Big)=\sup_{\alpha\in\mathfrak{A}}\{\mu(A_\alpha)\}$$

shartlarni qanoatlantirsa, u holda μ *to'plam funksiyasi* Ω *da idempotent o'lchov deyiladi.*

Biz Ω to'plamdagi barcha idempotent o'lchovlar to'plamini $IM(\Omega)$ orqali belgilaymiz va quyidagicha belgilash kiritamiz:

$$I(\Omega)=I_{\mathfrak{B}}(\Omega)=\{\mu\in IM(\Omega):\mu(\Omega)=1\}.$$

1-misol. $\Omega=[0,1]$ *bo'lsin.* $\mu:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ *to'plam funksiyasini (CH da)*

$$\mu(A)=\begin{cases}0, \; agar \; |A|\le\aleph_0,\\ 1, \; agar \; |A|=c\end{cases}$$

kabi aniqlasak, u idempotent o'lchov bo'ladi.

Endi $I(\Omega)$ da topologiya kiritamiz. Ω kompakt Hausdorf fazo va $\mathcal{B}$ esa Ω da baza bo'lsin, $U_i\in\mathcal{B}$, $i=1,\dots,n$, va $\varepsilon>0$. Ixtiyoriy $\mu\in I(\Omega)$ idempotent ehtimollik o'lchov uchun quyidagi to'plamni aniqlaymiz:

$$\langle\mu;U_1,\dots,U_n;\varepsilon\rangle=\{\nu\in I(\Omega):|\nu(U_i)-\mu(U_i)|<\varepsilon, i=1,\dots,n\}.$$

Barcha shunday to'plamlarni yig'ib

$$\mathcal{B}(\mu)=\{\langle\mu;U_1,\dots,U_n;\varepsilon\rangle:U_i\in\mathcal{B}, i=1,\dots,n;\varepsilon>0\},\quad \mu\in I(\Omega)$$

oilani quramiz va quyidagicha belgilash kiritamiz:

$$\mathcal{B}_{I(\Omega)}=\bigcup_{\mu\in I(\Omega)}\mathcal{B}(\mu).$$

4-tasdiq. $\mathcal{B}_{I(\Omega)}$ *qurilgan oila* $I(\Omega)$ *da topologik nuqtali yaqinlashish uchun baza (yoki atroflar sistemasi) tashkil qiladi.*

6-teorema. Ω *kompakt Hausdorf fazo uchun* $I(\Omega)$ *topologik fazo kompakt Hausdorf fazo bo'ladi.*

$f:\Omega_1\to\Omega_2$ akslantirish uchun Ω_1 va Ω_2 kompakt Hausdorf fazolarning $I(f):I(\Omega_1)\to I(\Omega_2)$ akslantirishini

$$I(f)(\mu)(B)=\mu\big(f^{-1}(B)\big),\qquad B\in\mathfrak{B}(\Omega_2)$$

qoida bo'yicha aniqlaymiz.

Uchinchi bobning ikkinchi paragrafida Ω Tixonov fazosi uchun Ω dagi τ-silliq idempotent ehtimollik o'lchovlarning $I_\tau(\Omega)$ fazosi o'rganilgan.

Ω Tixonov fazosi bo'lsin. Quyidagi to'plamni aniqlaymiz:

$$I_{\mathfrak{B}\tau}(\Omega)=\{\mu\in I(\beta\Omega):\forall F\in\mathfrak{B}(\beta\Omega), F\subset\beta\Omega\setminus\Omega \text{ uchun } \mu(F)=0\}.$$

Qulaylik uchun $I_{\mathfrak{B}\tau}(\Omega)$ to'plamni $I_\tau(\Omega)$ orqali belgilaymiz.

$I_\tau(\Omega)$ ning elementlari τ-silliq idempotent ehtimollik o'lchov deyiladi.

Endi har bir $\mu\in I_\tau(\Omega)$ uchun Ω ning barcha Borel qism to'plamlarining $\mathfrak{B}(\Omega)$ sistemasida aniqlangan $\tilde{\mu}:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ to'plam funksiyasini

$$\tilde{\mu}(A)=\mu^*(A)=\inf\{\mu(B):B\in\mathfrak{B}(\beta\Omega),B\supset A\},\ \ A\in\mathfrak{B}(\Omega)$$

formula bo‘yicha aniqlaymiz.

1-lemma. *$\tilde{\mu}$ kattalik Ω da idempotent ehtimollik o‘lchov bo‘ladi.*

2-lemma. *Ω Tixonov fazo bo‘lsin. Agar $\mu\in I_\tau(\Omega)$ bo‘lsa, u holda $A\cap\Omega=B\cap\Omega$ tenglik o‘rinli bo‘ladigan ixtiyoriy ikkita $A,B\subset\beta\Omega$ Borel qism to‘plamlari uchun $\mu(A)=\mu(B)$ bo‘ladi.*

Ω_1, Ω_2 Tixonov fazolari va $f:\Omega_1\to\Omega_2$ uzluksiz akslantirish uchun quyidagi akslantirishni aniqlash mumkin

$$I_\tau(f)=I(\beta f)|_{I_\tau(\Omega_1)}:I_\tau(\Omega_1)\to I_\tau(\Omega_2).$$

7-teorema. *$f:\Omega_1\to\Omega_2$ akslantirishga $I_\tau(f):I_\tau(\Omega_1)\to I_\tau(\Omega_2)$ akslantirishni mos qo‘yuvchi I_τ amali mukammal akslantirishlar sinfini saqlaydi.*

8-teorema. *$f:\Omega_1\to\Omega_2$ akslantirishga $I_\tau(f):I_\tau(\Omega_1)\to I_\tau(\Omega_2)$ akslantirishni mos qo‘yuvchi I_τ amali joylashtirishlar sinfini saqlaydi.*

Uchinchi bobning uchinchi paragrafida idempotent ehtimollik o‘lchovlarning $I_\tau(\Omega)$ fazosi Chex ma’nosida to‘la bo‘lishi uchun berilgan Ω Tixonov fazosi Chex ma’nosida to‘la bo‘lishi zarur va yetarli ekanligi isbotlangan.

9-teorema. *I_τ amali Tixonov fazolarining Chex-to‘laligini saqlaydi, boshqacha aytganda, agar Ω Chex-to‘la Tixonov fazosi bo‘lsa, u holda $I_\tau(\Omega)$ ham Chex-to‘la Tixonov fazosi bo‘ladi.*

10-teorema. *I_τ amali Borel to‘plamlarining aslini saqlaydi, ya’ni Tixonov fazolarining ixtiyoriy $f:\Omega_1\to\Omega_2$ uzluksiz akslantirishi va ixtiyoriy $A\subset\Omega_2$ Borel qism to‘plami uchun $I_\tau(f)^{-1}(I_\tau(A))=I_\tau(f^{-1}(A))$ tenglik o‘rinli bo‘ladi.*

11-teorema. *Ω_1 Tixonov fazosi va ixtiyoriy $A,\ B\subset\Omega_1$ qism to‘plamlari bo‘lib, ularning kamida bittasi Borel to‘plami bo‘lsin. U holda $I_\tau(A\cap B)=I_\tau(A)\cap I_\tau(B)$ o‘rinli.*

Uchinchi bobning to‘rtinchi paragrafida Ω kompakt Hausdorf fazo uchun Ω dagi Borel to‘plamlarning σ-algebrasida aniqlangan Ω dagi barcha idempotent ehtimollik o‘lchovlarning $I_{\mathfrak{B}}(\Omega)$ fazosi qurilgan. Shuningdek, Ω dagi uzluksiz funksiyalarning idempotent amallar kiritilgan to‘plamida aniqlangan barcha normalangan max-plus chiziqli funksionallarning $I_C(\Omega)$ fazosi qurilgan. Bu fazolar qurilishi natijasida Ω Tixonov fazosidagi τ-silliq idempotent o‘lchovlarning to‘plam funksiya sifatidagi tavsifi, ya’ni Riss ifodalanishining muqobili o‘rinli ekanligi isbotlangan.

Ω kompakt Hausdorf fazo va $\mathfrak{B}(\Omega)$ esa Ω ning Borel qism to‘plamlari sistemasi bo‘lsin.

6-ta’rif. *Agar $\nu:C(\Omega)\to\mathbb{R}$ funksional*

1. ixtiyoriy $\varphi,\psi\in C(\Omega)$ uchun $\nu(\varphi\oplus\psi)=\nu(\varphi)\oplus\nu(\psi)$;

2. har bir $c\in\mathbb{R}$ *va* $\varphi\in C(\Omega)$ *uchun* $\nu(c\odot\varphi)=c\odot\nu(\varphi)$

shartlarni qanoatlantirsa, u holda $\nu:C(\Omega)\to\mathbb{R}$ *funksional max-plus chiziqli funksional deyiladi.*

$C(\Omega)$ dagi barcha max-plus chiziqli funksionallar to'plamini $C(\Omega)^{\oplus}$ orqali belgilaymiz. $\nu\in C(\Omega)^{\oplus}$ max-plus chiziqli funksional uchun ushbu

$$\langle\nu;\varphi_1,\ldots,\varphi_n;\varepsilon\rangle=\left\{\nu'\in C(\Omega)^{\oplus}:|\nu'(\varphi_i)-\nu(\varphi_i)|<\varepsilon, i=1,\ldots,n\right\}$$

to'plamlar sistemasi μ nuqtada Ω to'plamning bazasini tashkil qiladi. Bu yerda $\varphi_i\in C(\Omega)$, $i=1,\ldots,n$ va $\varepsilon>0$.

Quyidagi belgilashni kiritamiz:

$$I_C(\Omega)=\left\{\nu\in C(\Omega)^{\oplus}:\nu(0_\Omega)=0\right\}.$$

$I_C(\Omega)$ ni $C(\Omega)^{\oplus}$ ning qism fazosi sifatida qaraylik.

12-teorema. *Har bir normallangan max-plus chiziqli funksional* $\nu:C(\Omega)\to\mathbb{R}$ *uchun quyidagi qoida bo'yicha aniqlangan*

$$\mu(A)=\inf\left\{\nu(\varphi):\varphi\in C(\Omega),\varphi\geq\chi_A\right\},\quad A\in\mathfrak{B}(\Omega)$$

$\mu:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ *to'plam funksiyasi* Ω *da idempotent ehtimollik o'lchov bo'ladi.*

Quyidagi teorema dissertatsiyaning asosiy natijasidir.

13-teorema. Ω *kompakt Hausdorf fazo va normallangan max-plus chiziqli funksional* $\nu:C(\Omega)\to\mathbb{R}$ *uchun* $\mathfrak{B}(\Omega)$ *da shunday yagona* $\mu(\Omega)\neq 0$ *idempotent o'lchov mavjudki, quyidagi tenglik o'rinli:*

$$\nu(\varphi)=\ln\left(\frac{1}{\mu(\Omega)}\int_{\Omega}^{\oplus}e^{\varphi(\omega)}d\mu\right),\quad \varphi\in C(\Omega).$$

1-natija. *Har bir* Ω *kompakt Hausdorf fazo uchun* $I_{\mathfrak{B}}(\Omega)$ *va* $I_C(\Omega)$ *fazolar gomeomorf bo'ladi.*

Endi bu masalani kengroq fazolarda qaraylik. Ω Tixonov fazosi bo'lsin va $C_b(\Omega)$ orqali Ω aniqlangan haqiqiy-qiymatli chegaralangan uzluksiz funksiyalar to'plamini belgilaymiz.

14-teorema. Ω *Tixonov fazosi bo'lsin. Agar* $\tilde{\mu}(\Omega)\neq 0$ *to'plam funksiyasi* $\mathfrak{B}(\Omega)$ *da* τ*-silliq idempotent o'lchov bo'lsa, u holda ushbu integral*

$$\tilde{\varphi}\mapsto\ln\left(\frac{1}{\tilde{\mu}(\Omega)}\int_{\Omega}^{\oplus}e^{\tilde{\varphi}(\omega)}d\tilde{\mu}\right)$$

$C_b(\Omega)$ *fazoda normallangan max-plus chiziqli funksional bo'ladi. Aksincha, ixtiyoriy normallangan max-plus chiziqli funksional* $\tilde{\nu}:C_b(\Omega)\to\mathbb{R}$ *uchun* $\mathfrak{B}(\Omega)$ *da shunday yagona* $\tilde{\mu}(\Omega)\neq 0$ *bo'lgan* τ*-silliq idempotent o'lchov mavjudki, quyidagi tenglik o'rinli:*

$$\tilde{\nu}(\tilde{\varphi}) = \ln\left(\frac{1}{\tilde{\mu}(\Omega)}\int\limits_{\Omega}^{\oplus} e^{\tilde{\varphi}(\omega)} d\tilde{\mu}\right), \quad \tilde{\varphi} \in C_b(\Omega).$$

Sh.Ayupov, A.Zaitov ishlarida qisman tartibni saqlovchi τ-silliq funksionallar to‘plami $O_\tau(\Omega)$ kiritilgan edi. Ravshanki, $I_C(\beta\Omega)\bigcap O_\tau(\Omega) \neq \varnothing$.

Quyidagicha belgilash kiritamiz: $I_{C\tau}(\Omega) = I_C(\beta\Omega)\bigcap O_\tau(\Omega)$.

2-natija. *Ixtiyoriy* Ω *Tixonov fazosi uchun* $I_{C\tau}(\Omega)$ *va* $I_{\mathfrak{B}\tau}(\Omega)$ *fazolar gomeomorf bo‘ladi.*

XULOSA

Dissertatsiya ishi asosan τ-silliq idempotent ehtimollik o‘lchovlarning tavsifini o‘rganishga bag‘ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. τ-silliq idempotent o‘lchov uchun uning τ-silliq idempotent o‘lchov bo‘ladigan davomi topilgan. Ixtiyoriy berilgan to‘plam funksiyasi uchun uning davomi va cheklanishi τ-maksitiv bo‘lishi uchun to‘plam funksiyasining o‘zi τ-maksitiv bo‘lishi zarur va yetarli ekanligi isbotlangan. Shuningdek, davom ettirish va cheklash amallarining kompozitsiyasi ayniy akslantirish bo‘lishi ko‘rsatilgan.
2. Berilgan kompakt Hausdorf fazosining ochiq to‘plamlaridan foydalanib idempotent ehtimollik o‘lchovlar to‘plamida biror topologiyaning bazasi bo‘ladigan sistema aniqlangan. Undan tashqari, Ω kompakt Hausdorf fazo uchun hosil bo‘lgan $I(\Omega)$ topologik fazo ham kiritilgan bo‘lib, uning kompakt Hausdorf fazo ekanligi isbotlangan. Ω Tixonov fazosi uchun Ω to‘plamdagi τ-silliq idempotent ehtimollik o‘lchovlari $I_\tau(\Omega)$ fazosining topologik xossalari tadqiq qilingan.
3. Idempotent ehtimollik o‘lchovlarning $I_\tau(\Omega)$ fazosi Chex ma’nosida to‘la bo‘lishi uchun berilgan Ω Tixonov fazosi Chex ma’nosida to‘la bo‘lishi zarur va yetarliligi isbotlangan.
4. To‘plam funksiyasi sifatida idempotent o‘lchovlar fazosi va uzluksiz funksiyalarning max-plus chiziqli fazosining max-plus qo‘shma fazosi orasidagi izomorfizm bo‘yicha Riss teoremasining muqobili isbotlangan. Shuningdek, Ω Tixonov fazosidagi τ-silliq idempotent o‘lchovlarning to‘plam funksiya sifatidagi tavsifi olingan.

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INSTITUTE OF MATHEMATICS

ESHIMBETOV MUZAFFAR REYIMBAYEVICH

DESCRIPTION OF τ-SMOOTH IDEMPOTENT MEASURES

01.01.01 – Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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INTRODUCTION

Actuality and demand of the theme of the dissertation. In the world, numerous scientific and applied research works are reduced to the study theory of idempotent measures has been proven to be a powerful tool for analyzing and understanding the behavior of a wide range of problems. There is a well-developed qualitative, geometric approach to theory idempotent measures that usually relies on the problems of mathematical physics, computer sciences, optimization and other as determined by the idempotent measures and it plays an important role in solving problems in physics, information technology and economics. Therefore, the results obtained in the theory of τ-smooth idempotent probability measures have both theoretical and practical significance and the theory of idempotent measures remains one of the important and urgent tasks in the aeras of modern mathematics.

At present, many authors interpret the limit deviabilities as distributions of idempotent processes and state the results in the form of large deviation convergence in distribution of semimartingales to semimaxingales. For example, A.Puhalskii formulate the large deviation principle for diffusion processes with small diffusion terms as large deviation convergence in distribution to an idempotent diffusion. Currently, to large deviation convergence of Markov processes and processes arising in queueing systems are widely used in analysis of many practical issues. In this regard, the study of geometric and topological properties of spaces of τ-smooth idempotent probability measures is considered a purposeful scientific research.

In our country much attention has been paid to natural and exact sciences, which have scientific and practical applications of fundamental sciences. The theory of idempotent measures is well advanced and includes, in particular, new integration theory, new linear algebra, spectral theory and functional analysis. In this area, special attention is paid to the development of the idempotent measures which has wide application in optimization problems such as multicriteria decision making, optimization on graphs, discrete optimization with a large parameter (asymptotic problems), optimal design of computer systems and computer media, optimal organization of parallel data processing, dynamic programming, discrete event systems, computer science, discrete mathematics, mathematical logic. In this regard, important results related to the τ-smooth idempotent probability measures, have been achieved. Conducting scientific research at the level of international standards in the priority areas of “Functional analysis, mathematical physics and statistical physics” was defined as the main tasks and areas of activity of mathematics[1]. It is important to develop the theory of τ-smooth idempotent measures in order to use scientific results in related fields of science to ensure the performance of these tasks.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development of the Republic of Uzbekistan”, PD-2789 dated February 17, 2017 “On measures to further

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[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” № 292 dated May 17, 2017.

improvement of the activities of the Academy of Sciences, organization, management and financing of research activities", PD-4387 from July 9, 2019 "On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics", and also PD-4708 from May 7, 2020 "On measures to improve the quality of education and research in mathemtics" as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem. Theory of idempotent measures is a branch of mathematical sciences, rapidly developing and gaining popularity over the last four decades. An important stage of development of the subject was presented in the book "Idempotency" edited by J.Gunawardena. The next stage of development of idempotent and tropical mathematics was presented in the book Idempotent Mathematics and Mathematical Physics edited by G.L.Litvinov and V.P.Maslov. Idempotent mathematics is based on replacing the usual arithmetic operations with a new set of basic operations, i. e., on replacing numerical fields by idempotent semirings and semifields. Typical example is the so-called max-plus algebra $\mathbb{R}_{\max}$. Many authors (S.C.Kleene, S.-N.N.Pandit, N.N.Vorobjev, B.A.Carré, R.A.Cuninghame-Green, K.Zimmermann, U.Zimmermann, M.Gondran, F.L.Baccelli, G.Cohen, S.Gaubert, G.J.Olsder, J.-P.Quadrat, V.N.Kolokoltsov and others) used idempotent semirings and matrices over these semirings for solving some applied problems in computer science and discrete mathematics, starting from the classical paper by S.C.Kleene.

M.Zarichnyi in 2010 investigated categorical properties of the space of idempotent probability measures. So, the space of idempotent probability measures is a new object. But it already is studied from different points of view in Measure Theory, Functional Analysis, Probability Theory, Topology and Category Theory. The study of spaces of idempotent probability measures leads to the problem its investigations on wider classes of topological classes than the class of compact Hausdorff spaces, in particular, the class of Tychonoff spaces. T.O.Banakh and T.N.Radul carried out a systematic study on probability measures on Tychonoff spaces. In their studies, they fruitfully used the linearity of probability measures. In works of Sh.Ayupov, A.Zaitov the theory of order-preserving functionals was developed and some categorical properties of τ-smooth weakly additive functionals were established. The obtained results by many authors show that in order to establish "good" properties of the space of idempotent probability measures, methods are required that are very different from classical methods. For example, Čech-complete spaces were introduced by Eduard Čech in 1937 to prove the Baire category theorem. A question arises whether the space of τ-smooth idempotent probability measures is Čech-complete.

In 1909 the Hungarian mathematician Frigyes Riesz proved a theorem stating that all continuous linear functionals can be represented by Riemann-Stieltjes integrals on the unit interval. Using Riesz original notation it looked like this:

$$A[f(\omega)] = \int_0^1 f(\omega)\,d\alpha(\omega),$$

where α is a function of bounded variation on the unit interval. This has become known as the Riesz representation theorem. In 1938 the Russian mathematician Andrey Markov extended Riesz's result to some non-compact spaces and three years after that, in 1941, the Japanese-American mathematician Shizuo Kakutani proved a theorem regarding compact Hausdorff spaces.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation work was carried out within the framework of the research plans of the Institute of Mathematics named after V.I.Romanovsky.

The aim of research work consists of discription τ-smooth idempotent measures defined in the functional language using τ-smooth idempotent measures defined in the language of set functions.

Research problems:

to determine a criterion for τ-smoothness of an idempotent measure similar to the criterion for τ-smoothness of a traditional measure;

to description a τ-smooth idempotent measure in a functional language;

to establish an analog of the Riesz theorem on isomorphism between the spaces of idempotent measures as a set function and the max-plus conjugate space of the max-plus of the linear space of continuous functions.

The research object: τ-smooth idempotent measure, τ-maxitive idempotent measure, measurable functions, Čech-complete spaces.

The research subject: Mathematical analysis, Functional analysis, Measure Theory, Topology, Idempotent analysis.

Research methods: In the research work the methods of mathematical analysis, functional analysis, measure theory are applied.

Scientific novelty of the research work consists of the following:

the given set function, for its extension and restriction, is a idempotent measure, and it is proved that the composition of these operations is an identity map;

using open sets of a given compact Hausdorff space, on the set of idempotent probability measures it defines a system that is a base of some topology and it is proved that for a compact Hausdorff space of the idempotent probability measures topological space is also a compact Hausdorff space;

it is got that the space τ-smooth of idempotent probability measures is Čech-complete if and only if the given Tychonoff space is Čech-complete;

an analog of the Riesz theorem has been established regarding the isomorphism between the space of idempotent measures as set functions and the max-plus conjugate space of the max-plus linear space of continuous functions. Additionally, a characterization of τ-smooth idempotent measures in the Tychonoff space has been obtained as set functions.

Practical results of the research. The obtained results and used methods in the dissertation can be taught as a graduate course for masters and doctoral students of higher educational institutions. In addition, it allows to solve economic issues.

The reliability of the results of the study. The results have been obtained by using the methods of mathematical and functional analysis, real analysis and measure theory. The obtained results are proved mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by their applicability in investigating some categorical properties of τ-smooth idempotent measures.

The practical significance of the research results is determined by description some categorical properties of τ-smooth idempotent measures.

Implementation of the research results. Based on the results of the description of τ-smooth idempotent measures:

from the description that the extension and restriction of a given set function are idempotent measures, and that the composition of these operations is an identity map, this framework has been applied in the scientific research under the State Grant A-OT-2021-108 "Development of an information and analytical system for monitoring and forecasting the ecological state of the environment of the Aral Sea region" Mathematical modeling of hydrodynamic processes of mass transfer in the atmosphere. (Reference of the Digital Technologies and Artificial Intelligence Research Institute of dated August 24, 2023, Uzbekistan). The results of the dissertation gave possibility to solve the problems that arose in the analysis of optimization and optimal management issues within the framework of the grant;

the results obtained in the space τ-smooth of idempotent probability measures is Čech-complete were used as a useful technical tool in the framework of scientific research supported by the Russian Science Foundation grant number 24-21-00278 "Operator methods in problems of convex geometry." (Reference of the North Ossetian State University named after K. L. Khetagurov of dated October 3, 2024, Russia). Topological properties of the space of idempotent probability measures allowed studying the problem of the analytical representation of linear and orthogonally additive operators defined on the space of continuous functions on a compactly generated Hausdorff topological space.

Approbation of the research results. The main results of the research have been discussed at 3 international and 2 national scientific conferences.

Publications of the research results. On the topic of the dissertation 10 scientific papers have been published, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of theses of the Doctor of Philosophy, in addition 1 of them were published in international journal and 4 in national scientific journals and 5 abstracts.

The structure and volume of the dissertation. The dissertation consists of the introduction, three chapters divided into ten paragraphs, conclusion and bibliography. The total volume of the thesis is 84 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction besides the motivation of research theme and correspondence to the priority research areas of science and technology of the Republic, we present a review of international research on the theme of the dissertation and the degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we reduce the theoretical and practical importance of the obtained results, and give information on the implementation of the research results, the published works and the structure of dissertation.

The first chapter of the thesis, titled **"Basic notions"** consists of three sections, in that, we give main definitions and important facts necessary to cover the dissertation and research the subject.

In the first section for τ-algebras and τ-smooth idempotent measures basic definitions and facts are provided.

Let Ω be a set and $\mathcal{E}$ a system of subsets of Ω, which contains $\varnothing$. Let $\mathcal{P}(\Omega)$ denote the powerset of Ω and $\overline{\mathbb{R}}_+ = [0,+\infty) \cup \{+\infty\} = [0,+\infty]$. The symbol $\mathfrak{A}$ denotes a directed set, and Δ an arbitrary index set.

Definition 1. *A set function $\mu : \mathcal{P}(\Omega) \to \overline{\mathbb{R}}_+$ is an idempotent measure on Ω if the following conditions hold:*

1. $\mu(\varnothing) = 0$;

2. $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}$ for any $A, B \in \mathcal{P}(\Omega)$;

3. $\mu\left(\bigcup_{\alpha\in\mathfrak{A}} A_\alpha\right) = \sup_{\alpha\in\mathfrak{A}}\{\mu(A_\alpha)\}$ for all increasing net $\{A_\alpha : \alpha \in \mathfrak{A}\}$ of subsets of Ω.

If $\mu(\Omega) = 1$, the idempotent measure μ is called an idempotent probability measure on Ω.

Definition 2. *If, in addition to conditions of definition 1,*

$$\mu\left(\bigcap_{\alpha\in\mathfrak{A}} F_\alpha\right) = \inf_{\alpha\in\mathfrak{A}}\{\mu(F_\alpha)\},$$

for every decreasing net $\{F_\alpha : \alpha \in \mathfrak{A}\}$ of elements of $\mathcal{E}$, then idempotent measure μ is called τ-smooth (or τ-additive) idempotent measure with respect to $\mathcal{E}$ on Ω or, for short, is an $\mathcal{E}$-idempotent measure.

In the second section definitions of measurable functions and idempotent integrations and their properties are presented. In addition to, a complete proof of the main properties of the idempotent integral is also stated.

Let us consider measurable functions on idempotent spaces. Let Ω_1 and Ω_2 be sets. Further, $\mathcal{E}_1$ and $\mathcal{E}_2$ are systems of subsets of Ω_1 and Ω_2, respectively, containing $\varnothing$. For the function $f : \Omega_1 \to \Omega_2$ we define the system

$$f^{-1}(\mathcal{E}_2) = \{f^{-1}(B) : B \in \mathcal{E}_2\}$$

subsets of Ω_1.

Definition 3. *If $f^{-1}(A) \in \mathcal{E}_1$ for each $A \in \mathcal{E}_2$, then the function $f : \Omega_1 \to \Omega_2$ is called $\mathcal{E}_1 / \mathcal{E}_2$-measurable.*

For a function $f : \Omega_1 \to \Omega_2$, a set-function μ' on Ω_2 is defined as $\mu'(A) = \mu(f^{-1}(A))$, $A \subset \Omega_2$. The set-function μ' is called the image of μ with respect to the function f and is denoted by $\mu \circ f^{-1}$.

We shall present the idempotent integral with respect to idempotent measure. Let (Ω, μ) be an idempotent measure space such that $\mu(\Omega) < \infty$. We adopt the convention that $\infty \cdot 0 = 0$.

Definition 4. *For a measurable function $f : \Omega \to \overline{\mathbb{R}}_+$ we define the idempotent integral of f with respect to μ by*

$$\int_{\Omega}^{\oplus} f d\mu = \sup_{t \in \overline{\mathbb{R}}_+} \{ t \cdot \mu \{ \omega \in \Omega : f(\omega) \geq t \} \}.$$

For $A \subset \Omega$, we let $\int_{A}^{\oplus} f d\mu = \int_{\Omega}^{\oplus} f \chi_A d\mu$.

In the third section some general topological concepts and theorems are given.

A topological space Ω is Čech-complete if Ω is a Tychonoff space and the remainder $\beta\Omega \setminus \Omega$ is an F_σ-set in the Stone-Čech compactification $\beta\Omega$. Locally compact spaces are Čech-complete, but the inverse is not take place. The space of all irrational numbers with the topology of a subspace of the real line is an example of a Čech-complete space that is not locally compact.

Čech-completeness is hereditary with respect to closed subsets and with respect to G_δ-subsets.

The second chapter of the dissertation entitled **"Extensions and restrictions of idempotent measures"**, consists of three section. In the first section for some extensions of τ-smooth idempotent measures it is proved that they are τ-smooth idempotent measures. Further, for each idempotent τ-smooth measure any Luzin measurable function established an τ-smooth idempotent measure.

Now let ξ_i, $i = 1,2,\ldots$ – be systems of subsets of Ω, which contain $\varnothing$. The system ζ of subsets of Ω is defined as follows:

$$\zeta = \bigoplus_{i=1}^{\infty} \xi_i = \left\{ \bigcup_{i=1}^{\infty} A_i : A_i \in \xi_i \right\}.$$

Theorem 1. *An idempotent measure μ is τ-smooth with respect to $\zeta = \bigoplus_{i=1}^{\infty} \xi_i$ if and only if for of each $i \in \mathbb{N}$, the idempotent measure μ is τ-smooth with respect to each ξ_i.*

Let now $\mathcal{E}_i$ and $\mathcal{E}'_i$ be systems of subsets, respectively, Ω and Ω' containing $\varnothing$, $i \in \mathbb{N}$. The systems $\mathcal{E}$ and $\mathcal{E}'$ are defined as follow:

$$\mathcal{E}=\mathop{\oplus}_{i=1}^{\infty}\mathcal{E}_i=\left\{\bigcup_{i=1}^{\infty}A_i : A_i\in\mathcal{E}_i\right\} \qquad \text{and} \qquad \mathcal{E}'=\mathop{\oplus}_{i=1}^{\infty}\mathcal{E}'_i=\left\{\bigcup_{i=1}^{\infty}A'_i :\ A'_i\in\mathcal{E}'_i\right\}.$$

Theorem 2. *If a set-function* $\mu:\mathcal{E}\to\overline{\mathbb{R}}_+$ *is* τ*-maxitive, then* $\mu':\mathcal{E}'\to\overline{\mathbb{R}}_+$ *is also* τ*-maxitive.*

Theorem 3. *Let the system* $\mathcal{T}$ *be a tightening for the* $\mathcal{E}$*-idempotent measure* μ*. If the function* $f:\Omega\to\Omega'$ *is* $(\mathcal{E},\mathcal{T})/\mathcal{E}'$*-measurable in the sense of Luzin, then* μ' *is an* $\mathcal{E}'$*-idempotent measure on the set* Ω'*. If* $f(T)\cap F'\in\mathcal{E}'$ *for* $T\in\mathcal{T}$ *and* $F'\in\mathcal{E}'$*, then* $f(T)$ *is a tightening for* μ'*.*

In the second section we studied τ-maxitive idempotent measures and their extensions and restrictions. Then we showed that the composition of the extension operation and the restriction operation is an identity mapping.

Let Ω be a set, Ω_0 a subset of Ω and $\mathcal{E}$ a τ-algebra on Ω. We put

$$\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}.$$

Assume an idempotent measure $\mu:\mathcal{P}(\Omega_0)\to\overline{\mathbb{R}}_+$ is given. Its extension $e_{\Omega}^{\Omega_0}(\mu):\mathcal{P}(\Omega)\to\overline{\mathbb{R}}_+$ we will determine by the rule:

$$e_{\Omega}^{\Omega_0}(\mu)(B)=\mu(B\cap\Omega_0),\ B\subset\Omega.$$

Let $\mathcal{E}$ be a τ-algebra on Ω and $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$, μ be an $\mathcal{E}$-idempotent measure on Ω. We will define a restriction $r_{\Omega_0}^{\Omega}(\mu)$ of μ in this case by the rule

$$r_{\Omega_0}^{\Omega}(\mu)(A)=\inf\{\mu(C) : C\in\mathcal{E}, C\cap\Omega_0=A\},\ \ A\in\mathcal{E}_0.$$

Now let us denote by $IM(\Omega)$ a set of all idempotent measures μ defined on the powerset $\mathcal{P}(\Omega)$, and by $M^{\tau}(\Omega)$ a set of all τ-maxitive idempotent measures on a τ-algebra $\mathcal{E}$.

Proposition 1. *Let* $\mathcal{E}$ *be a* τ*-algebra on a set* Ω *and* Ω_0 *a subset of* Ω*, and* $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$*. The composition* $r_{\Omega_0}^{\Omega}\circ e_{\Omega}^{\Omega_0}:M^{\tau}(\Omega_0)\to M^{\tau}(\Omega_0)$ *is the identity mapping of* $M^{\tau}(\Omega_0)$*, i. e*

$$r_{\Omega_0}^{\Omega}\circ e_{\Omega}^{\Omega_0}=id_{IM(\Omega_0)}.$$

Remark 1. *Let* $\mathcal{E}$ *be a* τ*-algebra on a set* Ω *and* Ω_0 *a subset of* Ω*, and* $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$*. The composition* $e_{\Omega}^{\Omega_0}\circ r_{\Omega_0}^{\Omega}:M^{\tau}(\Omega)\to M^{\tau}(\Omega)$ *should not satisfy the equality* $e_{\Omega}^{\Omega_0}\circ r_{\Omega_0}^{\Omega}=id_{IM(\Omega)}$*.*

For instance, let $\Omega=\{0,1,2\}$ and $\Omega_0=\{0,1\}$. Let $\mathcal{E}=\mathcal{P}(\Omega)$ denote the powerset of Ω. We define $\mu:\mathcal{P}(\Omega)\to\overline{\mathbb{R}}_+$ as following: $\mu(B)=1$ if $2\in B$, and $\mu(B)=0$ if $2\notin B$. Take $B=\{0,2\}\subset\Omega$. Then

$$e_{\Omega}^{\Omega_0}\left(r_{\Omega_0}^{\Omega}(\mu)\right)(B)=r_{\Omega_0}^{\Omega}(\mu)(B\cap\Omega_0)=\inf\left\{\mu(C):C\in\mathcal{E},C\cap\Omega_0=B\cap\Omega_0=\{0\}\right\}=$$
$$=\inf\left\{\mu(\{0\})\right\}=\mu(\{0\})=0\neq 1=\mu(B).$$

So, $e_{\Omega}^{\Omega_0}\circ r_{\Omega_0}^{\Omega}\neq id_{IM(\Omega)}$.

Let $\mathcal{E}$ be a τ-algebra on a set Ω and Ω_0 a subset of Ω, and $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$. Now we define a set

$$M_{\Omega_0}^{*}(\Omega)=\left\{\mu\in M^{\tau}(\Omega):\mu(K)=0 \text{ for any } K\in\mathcal{E} \text{ such that } K\subset\Omega\setminus\Omega_0\right\}.$$

Theorem 4. *Let $\mathcal{E}$ be a τ-algebra on a set Ω and Ω_0 a subset of Ω, and $\mathcal{E}_0=\{B\cap\Omega_0 : B\in\mathcal{E}\}$. Then*

1. $e_{\Omega}^{\Omega_0}\left(M^{\tau}(\Omega_0)\right)\subset M_{\Omega_0}^{*}(\Omega)$.

2. The composition $e_{\Omega}^{\Omega_0}\circ\left(r_{\Omega_0}^{\Omega}|_{M_{\Omega_0}^{*}(\Omega)}\right):M_{\Omega_0}^{*}(\Omega)\to M_{\Omega_0}^{*}(\Omega)$ is an identity mapping on $M_{\Omega_0}^{*}(\Omega)$ i. e.

$$e_{\Omega}^{\Omega_0}\circ\left(r_{\Omega_0}^{\Omega}|_{M_{\Omega_0}^{*}(\Omega)}\right)=id_{M_{\Omega_0}^{*}(\Omega)}.$$

In the third section we investigated mappings between idempotent measures spaces, τ-maxitive and their extensions and restrictions.

Consider any sets Ω_1 and Ω_2, and a system $\mathcal{E}_2$ of subsets of Ω_2. For a mapping $f:\Omega_1\to\Omega_2$ we put $f^{-1}(\mathcal{E}_2)=\left\{f^{-1}(B):B\in\mathcal{E}_2\right\}$.

For a mapping $f:\Omega_1\to\Omega_2$ let $M(f)$ denote a mapping from the set $IM(\Omega_1)$ into the set $IM(\Omega_2)$ defined by the rule

$$M(f)(\mu)(B)=\mu\left(f^{-1}(B)\right),\ B\subset\Omega_2.$$

Let Ω_2' be a subset of Ω_2, $\Omega_1=f^{-1}(\Omega_2')$ and $\mu\in IM(\Omega_1)$. Consider a set function $e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right):\mathcal{P}(\Omega_2)\to\overline{\mathbb{R}}_+$ defined as

$$e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right)(A)=\mu\left(f^{-1}(A\cap\Omega_2')\right),\ A\subset\Omega_2.$$

Let $\mathcal{K}=\left\{B\cap\Omega_2':B\in\mathcal{E}_2\right\}$. Then $f^{-1}(\mathcal{K})=f^{-1}(\mathcal{E}_2)$.

Proposition 2. *If an idempotent measure μ is τ-smooth with respect to $f^{-1}(\mathcal{E}_2)$ on Ω_1 then the set function $e_{\Omega_2}^{\Omega_2'}\left(M(f)(\mu)\right)$ is an idempotent measure, moreover it is τ-smooth with respect to $\mathcal{E}_2$ on Ω_2.*

Let an idempotent measure $\mu\in IM(\Omega_1)$ be given. A set function $r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right):\mathcal{P}(\Omega')_2\to\overline{\mathbb{R}}_+$ we define by

$$r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right)(A)=\inf\left\{\mu\left(f^{-1}(C)\right):C\in\mathcal{E}_2,C\cap f(\Omega_1)=A\right\},\ A\subset\Omega_2'=f(\Omega_1)$$

Proposition 3. *If μ is a τ-smooth idempotent measure according to the system $f^{-1}(\mathcal{E}_2)$ on Ω_1 then a set function $r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right)$ is also a τ-smooth idempotent measure according to the system $\mathcal{K}$ on Ω_2'.*

Theorem 5. $\mu \in M^{\tau}(\Omega_1)$ *if and only if* $r_{\Omega_2'}^{\Omega_2}\left(M(f)(\mu)\right) \in M^{\tau}\left(f(\Omega_1)\right)$.

The third chapter of the dissertation entitled **"Description of τ-smooth idempotent measures"** consists of four sections.

In the first section of the third chapter for a support of the given idempotent measure μ, we proved some lemmas. Also, in the set of idempotent probability measures, the base of the product topology is introduced and we showed that for a compact Hausdorff space Ω the topological space $I(\Omega)$ is a compact Hausdorff space.

Let Ω be a compact Hausdorff space and $\mathfrak{B}(\Omega)$ the family of Borel subsets of Ω. We enter the following notion.

Definition 5. *A set function $\mu:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ is said to be an idempotent measure on Ω if the following conditions hold*

1. $\mu(\varnothing)=0$*;*

2. $\mu(A\cup B)=\max\{\mu(A),\mu(B)\}$ *for any* $A,B\in\mathfrak{B}(\Omega)$*;*

3. $\mu\left(\bigcup_{\alpha\in\mathfrak{A}} A_\alpha\right)=\sup_{\alpha\in\mathfrak{A}}\{\mu(A_\alpha)\}$ *for every increasing net* $\{A_\alpha:\alpha\in\mathfrak{A}\}\subset\mathfrak{B}(\Omega)$ *such that* $\bigcup_{\alpha\in\mathfrak{A}} A_\alpha\in\mathfrak{B}(\Omega)$.

The set of all idempotent measure on Ω we denote by $IM(\Omega)$. We denote

$$I(\Omega)=I_{\mathfrak{B}}(\Omega)=\{\mu\in IM(\Omega):\mu(\Omega)=1\}.$$

Example 1. *Let* $\Omega=[0,1]$. *A set function $\mu:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ (in CH) defined as*

$$\mu(A)=\begin{cases}0, \text{ if } |A|\le\aleph_0,\\ 1, \text{ if } |A|=c\end{cases}$$

is an idempotent measure.

Now we introduce the topology in $I(\Omega)$. Let Ω be a compact Hausdorff space, $\mathcal{B}$ a base in Ω, $U_i\in\mathcal{B}$, $i=1,\ldots,n$, and $\varepsilon>0$. For an idempotent probability measure $\mu\in I(\Omega)$ we define a set

$$\langle\mu;U_1,\ldots,U_n;\varepsilon\rangle=\{\nu\in I(\Omega):|\nu(U_i)-\mu(U_i)|<\varepsilon, i=1,\ldots,n\}.$$

Gathering all of such sets construct a family

$$\mathcal{B}(\mu)=\{\langle\mu;U_1,\ldots,U_n;\varepsilon\rangle:U_i\in\mathcal{B}, i=1,\ldots,n;\varepsilon>0\},\quad \mu\in I(\Omega),$$

and put

$$\mathcal{B}_{I(\Omega)}=\bigcup_{\mu\in I(\Omega)}\mathcal{B}(\mu).$$

The following proposition is important.

Proposition 4. *The built family* $\mathcal{B}_{I(\Omega)}$ *forms a base (or, a neighbourhoods system) for the point-wise convergence topology in* $I(\Omega)$.

Theorem 6. *For a compact Hausdorff space* Ω *the topological space* $I(\Omega)$ *is also a compact Hausdorff space.*

For a mapping $f:\Omega_1 \to \Omega_2$ of compact Hausdorff spaces Ω_1 and Ω_2 we define a mapping $I(f):I(\Omega_1)\to I(\Omega_2)$ by the rule

$$I(f)(\mu)(B)=\mu\left(f^{-1}(B)\right), \quad B\in\mathfrak{B}(\Omega_2).$$

In the second section of the third chapter for a Tychonoff space Ω, we studied the space $I_\tau(\Omega)$ of idempotent probability τ-smooth measures on Ω.

Let Ω be a Tychonoff space. We determine the following set:

$$I_{\mathfrak{B}\tau}(\Omega)=\left\{\mu\in I(\beta\Omega):\mu(F)=0 \text{ for every } F\in\mathfrak{B}(\beta\Omega), F\subset\beta\Omega\setminus\Omega\right\}.$$

For breivity the set $I_{\mathfrak{B}\tau}(\Omega)$ is denoted by $I_\tau(\Omega)$.

Elements of $I_\tau(\Omega)$ is said to be τ-smooth idempotent probability measures.

Now, for each $\mu\in I_\tau(\Omega)$ we define a set function $\tilde{\mu}:\mathfrak{B}(\Omega)\to\overline{\mathbb{R}}_+$ on the family $\mathfrak{B}(\Omega)$ of all Borel subsets of Ω by the formula

$$\tilde{\mu}(A)=\mu^*(A)=\inf\left\{\mu(B):B\in\mathfrak{B}(\beta\Omega), B\supset A\right\}, \ A\in\mathfrak{B}(\Omega).$$

Lemma 1. $\tilde{\mu}$ *is an idempotent probability measure on* Ω.

Lemma 2. *Let* Ω *be a Tychonoff space. If* $\mu\in I_\tau(\Omega)$, *then* $\mu(A)=\mu(B)$ *for any two Borel subsets* $A, B\subset\beta\Omega$ *such that* $A\cap\Omega=B\cap\Omega$.

For Tychonoff spaces Ω_1, Ω_2 and a continuous mapping $f:\Omega_1\to\Omega_2$ we can determine the following mapping:

$$I_\tau(f)=I(\beta f)|_{I_\tau(\Omega_1)}:I_\tau(\Omega_1)\to I_\tau(\Omega_2).$$

Theorem 7. *The operation* I_τ *putting the mapping* $I_\tau(f):I_\tau(\Omega_1)\to I_\tau(\Omega_2)$ *in correspondence with the mapping* $f:\Omega_1\to\Omega_2$, *preserves the class of perfect mappings.*

Theorem 8. *The operation* I_τ *putting the mapping* $I_\tau(f):I_\tau(\Omega_1)\to I_\tau(\Omega_2)$ *in correspondence with the mapping* $f:\Omega_1\to\Omega_2$, *preserves the class of embeddings.*

Theorem 9. *The operation* I_τ *preserves preimages of Borel sets, i. e. for every continuous mapping* $f:\Omega_1\to\Omega_2$ *of Tychonoff spaces and any Borel subset* $A\subset\Omega_2$ *the equality* $I_\tau(f)^{-1}(I_\tau(A))=I_\tau\left(f^{-1}(A)\right)$ *holds.*

Theorem 10. *Let* Ω_1 *be a Tychonoff space and* A, $B\subset\Omega_1$ *any subsets such that at least one of them is Borel. Then* $I_\tau(A\cap B)=I_\tau(A)\cap I_\tau(B)$ *holds.*

In the third section of the this chapter for a Tychonoff space Ω, we studied the space $I_\tau(\Omega)$ of τ-smooth idempotent probability measures on Ω is Čech-complete.

Theorem 11. *The operation I_τ preserves the Čech-completeness of Tychonoff spaces, in the other words, if Ω is a Čech-complete Tychonoff space then $I_\tau(\Omega)$ is also a Čech-complete Tychonoff space.*

In the fourth section of the third chapter for a compact Hausdorff space Ω, we constructed a space $I_{\mathfrak{B}}(\Omega)$ of all idempotent probability measures on Ω, which define on the σ-algebra of Borel sets in Ω. Also, constructed a space $I_C(\Omega)$ of all normed max-plus linear functionals on the set of all continuous functions on Ω, equipped with idempotent operations. As a result of the construction of these spaces a description of τ-smooth idempotent measures in Tychonoff space Ω as a set function i. e. it has been proven that an anolog of the Riesz representation theorem.

Let Ω be a compact Hausdorff space and $\mathfrak{B}(\Omega)$ the family of Borel subsets of Ω.

Definition 6. *A functional $\nu: C(\Omega) \to \mathbb{R}$ is said to be a max-plus-linear functional, if it has the following properties*
1. $\nu(\varphi \oplus \psi) = \nu(\varphi) \oplus \nu(\psi)$ for any $\varphi, \psi \in C(\Omega)$;
2. $\nu(c \odot \varphi) = c \odot \nu(\varphi)$ for every $c \in \mathbb{R}$ and $\varphi \in C(\Omega)$.

The set of all max-plus-linear functionals on $C(\Omega)$ we denote by $C(\Omega)^{\oplus}$. For a max-plus-linear functional $\nu \in C(\Omega)^{\oplus}$ a system of sets

$$\langle \nu; \varphi_1, \ldots, \varphi_n; \varepsilon \rangle = \left\{ \nu' \in C(\Omega)^{\oplus} : | \nu'(\varphi_i) - \nu(\varphi_i) | < \varepsilon, i = 1, \ldots, n \right\}$$

forms a base of Ω at μ. Here $\varphi_i \in C(\Omega)$, $i = 1, \ldots, n$, and $\varepsilon > 0$.

Put

$$I_C(\Omega) = \left\{ \nu \in C(\Omega)^{\oplus} : \nu(0_\Omega) = 0 \right\}.$$

Consider $I_C(\Omega)$ as a subspace of $C(\Omega)^{\oplus}$.

Theorem 12. *For each normed max-plus-linear functional $\nu: C(\Omega) \to \mathbb{R}$ a set-function $\mu: \mathfrak{B}(\Omega) \to \overline{\mathbb{R}}_+$ defined by the rule*

$$\mu(A) = \inf \left\{ \nu(\varphi) : \varphi \in C(\Omega), \varphi \geq \chi_A \right\}, \quad A \in \mathfrak{B}(\Omega),$$

is an idempotent probability measure on Ω.

This result is the main result of the dissertaion.

Theorem 13. *For a compact Hausdorff space Ω and for any normed max-plus functional $\nu: C(\Omega) \to \mathbb{R}$ there exist a unique idempotent measure $\mu(\Omega) \neq 0$ on $\mathfrak{B}(\Omega)$ such that*

$$\nu(\varphi) = \ln \left(\frac{1}{\mu(\Omega)} \int_{\Omega}^{\oplus} e^{\varphi(\omega)} d\mu \right), \quad \varphi \in C(\Omega).$$

Corollary 1. *For every compact Hausdorff space Ω the spaces $I_{\mathfrak{B}}(\Omega)$ and $I_C(\Omega)$ are homeomorphic.*

Now we will consider wider case. Let Ω be a Tychonoff space, and $C_b(\Omega)$ denote the set of real-valued bounded continuous functions on Ω.

Theorem 14. *Let Ω be a Tychonoff space. If $\widetilde{\mu}(\Omega)\neq 0$ is τ-smooth idempotent measure on $\mathfrak{B}(\Omega)$, then integration*

$$\tilde{\varphi}\mapsto \ln\left(\frac{1}{\tilde{\mu}(\Omega)}\int_{\Omega}^{\oplus} e^{\tilde{\varphi}(\omega)}d\tilde{\mu}\right)$$

is a normed max-plus linear functional on $C_b(\Omega)$. Conversely, for any normed max-plus linear functional $\tilde{\nu}:C_b(\Omega)\to\mathbb{R}$ there exists a unique τ-smooth idempotent measure $\widetilde{\mu}(\Omega)\neq 0$ on $\mathfrak{B}(\Omega)$ such that

$$\tilde{\nu}(\tilde{\varphi})=\ln\left(\frac{1}{\tilde{\mu}(\Omega)}\int_{\Omega}^{\oplus} e^{\tilde{\varphi}(\omega)}d\tilde{\mu}\right),\quad \tilde{\varphi}\in C_b(\Omega).$$

In works Sh.Ayupov, A.Zaitov there was introduced a set $O_\tau(\Omega)$ of order-preserving τ-smooth functionals. Clearly, $I_C(\beta\Omega)\cap O_\tau(\Omega)\neq\varnothing$ and let $I_{C\tau}(\Omega)=I_C(\beta\Omega)\cap O_\tau(\Omega)$.

Corollary 2. *For any Tychonoff space Ω the spaces $I_{C\tau}(\Omega)$ and $I_{\mathfrak{B}\tau}(\Omega)$ are homeomorphic.*

CONCLUSION

The dissertation work is mainly devoted to the study of discription τ-smooth idempotent measures.

The main results of the study are:

1. For some extensions of idempotent τ-smooth measures it is proved that they are idempotent τ-smooth measures. Also, for a set function we proved that its extension and restriction are τ-maxitive if and only if the given set function is τ-maxitive. Then we showed that the composition of the extension operation and the restriction operation is an identity map.
2. Using the open sets of a given compact Hausdorff space, idempotent probability set of measures defines a system that is the base of a topology. We showed that for a compact Hausdorff space Ω the topological space $I(\Omega)$ is also a compact Hausdorff space. For a Tychonoff space Ω, we studied the topologic properties of space $I_\tau(\Omega)$ of idempotent probability τ-smooth measures on Ω.
3. We got that the space $I_\tau(\Omega)$ of idempotent probability measures is Čech-complete if and only if the given Tychonoff space Ω is Čech-complete.
4. An analog of the Riesz theorem has been established regarding the isomorphism between the space of idempotent measures as set functions and the max-plus conjugate space of the max-plus linear space of continuous functions. Additionally, a characterization of τ-smooth idempotent measures in the Tychonoff space Ω has been obtained as set functions.

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ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**

ИНСТИТУТ МАТЕМАТИКИ

ЭШИМБЕТОВ МУЗАФФАР РЕЙИМБАЕВИЧ

Описание τ -гладких идемпотентных мер

01.01.01 – Математический анализ

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ТАШКЕНТ – 2024

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является описание τ-гладких идемпотентных мер, определенных на функциональном языке, с помощью τ-гладких идемпотентных мер, определенных на языке функций-множеств.

Объект исследования: τ-гладкая идемпотентная мера, τ-макситивная мера, измеримые функции, полные по Чеху пространства.

Научная новизна исследования состоит в следующем:

Для заданной функции функция множества, ее расширения и ограничения является идемпотентной мерой, и доказано, что композиция этих операций является тождественным отображением;

Используя открытые множества данного компактного хаусдорфово пространства, на множестве идемпотентных вероятностных мер определена система, и доказано, что она является базой некоторой топологии на множестве мер и сосикателем установлено, что для компактного хаусдорфово пространства топологическое пространство идемпотентных вероятностных мер с введенной также является компактным Хаусдорфовым пространством;

Получено, что пространство τ-гладких идемпотентных вероятностных мер является полным по Чеху тогда и только тогда, когда данное тихоновское пространство является полным по Чеху;

Установлен аналог теоремы Рисса об изоморфизме между пространствами идемпотентных мер (как функции множества) и max-plus сопряженным пространством max-plus линейного пространства непрерывных функций и получено описание функционалов, являющихся τ-гладкими идемпотентными мерами на языке max-plus-интегралов по функции-множества, являющейся τ-гладкими идемпотентными мерами.

Внедрение результатов исследования. На основе полученных результатов о описание τ-гладких идемпотентных мер:

из описания, что продолжение и ограничение данной функции множества являются идемпотентными мерами, и что композиция этих операций является тождественным отображением, следует, что эта структура была применена в научных исследованиях по Государственному гранту А-ОТ-2021-108 «Разработка информационно-аналитической системы мониторинга и прогнозирования экологического состояния окружающей среды региона Приаралья» Математическое моделирование гидродинамических процессов массопереноса в атмосфере. (Справка Научно-Исследовательский институт развития Цифровых Технологий и Искусственного Интеллекта от 24 августя 2023 г., Узбекистан). Результаты диссертации применялись при решении задач оптимизации и оптимального управления, возникающие в гранте;

полученные результаты в полученные в пространстве τ-гладких идемпотентных вероятностных мер, полно по Чеху, использовались в качестве полезного технического инструмента в рамках научных исследований, поддержанных грантом Российского научного фонда номер 24-21-00278 «Операторные методы в задачах выпуклой геометрии». (Справка Северо-Осетинского государственного университета имени К. Л. Хетагурова от 3 октября 2024 г., Россия). Топологические свойства пространства идемпотентных вероятностных мер позволили изучить проблему аналитического представления линейных и ортогонально аддитивных операторов, заданных на пространстве непрерывных функций на компактно порожденном хаусдорфовом топологическим пространстве.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, разбитых на десять параграфов, заключения и списка использованной литературы. Объем диссертации составляет 84 страниц.

E'LON QILINGAN ILMIY ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
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