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OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

NAMANGAN MUHANDISLIK-QURILISH INSTITUTI

Oliy matematika kafedrasi

OLIY ALGEBRA ELEMENTLARI

Barcha texnik yo‘nalishlar talabalari uchun

USLUBIY KO‘RSATMA

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"Oliy matematika" kafedrasi uslubiy seminarida ko'rib chiqilib, institutning ilmiy uslubiy kengashiga ko'rib chiqish uchun tavsiya etilgan.

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Namangan muhandislik-qurilish instituti ilmiy-uslubiy kengashi tomonidan tasdiqlangan va chop etishga tavsiya etilgan.

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SO‘Z BOSHI

Oliy o‘quv yurtlarida yuqori malakali muhandislar, tayyorlashda Oliy matematika fanini o`qitishga katta e’tibor berib kelinmoqda. Chunki muhandislik fanlari, mexanika, fizika, ximiya va boshqa tabiiy fanlar shu fan asosida o`rganiladi.

Oliy matematika fanidan chuqur bilim olishlari uchun talabalarga berilayotgan nazariy bilimlar yetarli bo‘lmaydi, ayniqsa muhandislar matematika tadbiqlari bo`yicha chuqur bilimga ega bo‘lishlari lozim.

Ushbu metodik ko’rsatma ham yuqorida ko`zlangan maqsadga yo`naltirilgan bo‘lib, u O “Oliy matematika fani o`quv dastur”ining Oliy algebra moduli asosida tuzilgan. Metodik ko’rsatmada masala va misollarni yechish uchun asosiy formulalar, misol va masalalarni yechishning namunalari hamda darsda mustaqil yechish uchun topshiriqlar keltirilgan. To‘plamni yozishda o`zbek, rus va xorijiy tillardagi mavjud adabiyotlardan ijodiy foydalanildi.

Bu metodik ko‘rsatmalar oliy texnika o`quv yurtlarining talabalari amaliy mashg`ulot darslarida foydalanishlari uchun mo`ljallangan.

Metodik ko‘rsatmalarda yo‘l qo‘yilgan kamchiliklarga o‘z munosabatlarini bildiruvchilarga oldindan minnatdorchilik bildiramiz.

Muallif

1-§. Ikkinchi va uchinchi tartibli determinantlar

Ikkinchi tartibli determinant.

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ ko'rinishidagi ifoda **ikkinchi tartibli determinant**, $\Delta = a_{11}a_{22} - a_{12}a_{21}$ ayirma esa uning son qiymati deyiladi.

1. $\begin{vmatrix} -3 & 5 \\ -2 & -1 \end{vmatrix}$ determinantni hisoblang.

Yechish. $\begin{vmatrix} -3 & 5 \\ -2 & -1 \end{vmatrix} = -3\cdot(-1) - 5(-2) = 3 + 10 = 13$

2. $\begin{vmatrix} x & 6 \\ 1 & -3 \end{vmatrix} = 8$ tenglamani yeching.

Yechish. $-3x - 6 = 8, \quad -3x = 14, \quad x = -\dfrac{14}{3} = -4\dfrac{2}{3}$.

Uchinchi tartibli determinant.

Quyidagicha

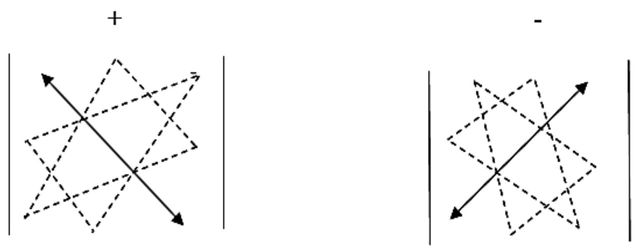
$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

belgilanadigan va son qiymati

$$\Delta = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} \qquad (1)$$

ga teng bo'lgan ifodaga **3-tartibli determinant** deyiladi.

3-tartibli determinantni hisoblash uchun uchburchak qoidasi:



Uchinchi tartibli determinantni hisoblashning Sarrius usuli:

$$\begin{array}{c} (+)\quad (+)\quad (+) \\ \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix} \\ (-)\quad (-)\quad (-) \end{array}$$

Bu usulda uchinchi tartibli determinant jadvali yoniga 1- va 2- ustunlar takroriy yoziladi, 1- asosiy diagonalga parallel uchta diagonal elementlarni o’zaro ko’paytirib, ularni yig’indisi olinadi. 2-yordamchi diagonal va unga parallel uchta diagonal elementlari ko’paytirilib ularning ayirmalari olinadi. Natijada (1) formula hosil bo’ladi.

3. $\begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 3 & 5 & -2 \end{vmatrix}$ determinantni hisoblang.

Yechish.

$$\begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 3 & 5 & -2 \end{vmatrix} = 1\cdot 1\cdot(-2) + 2\cdot(-1)\cdot 3 + 0\cdot 5\cdot 4 - 3\cdot 1\cdot 4 + 5\cdot(-1)\cdot 1 - 0\cdot 2\cdot(-2) =$$

$$= -2 - 6 + 0 - 12 + 5 + 0 = -20 + 5 = -15$$

Determinantning xossalari.

Determinatning xossalari ularning tartibiga bog’liq bo’lmagani uchun, xossalarni asosan uchinchi tartibli determinantlar uchun keltiramiz.

1^0. Determinatning mos satrlari va ustunlari o’rinlari almashtirilganda uning qiymati o’zgarmaydi.

2^0 Agar determinantning ikki satr (ustun) elementlari o'zaro almashtirilsa, uning qiymati o'zgarmaydi, ishorasi esa qarama-qarshisiga almashadi.

3^0. Agar determinant ikkita bir xil elementli satrga (ustunga) ega bo'lsa, u nolga teng.

4^0. Determinantning biror satr (ustun) elementlarini ixtiyoriy α songa ko'paytirish determinantni shu songa ko'paytirishga teng kuchlidir.

5^0. Agar determinant nollardan iborat satrga (ustunga) ega bo'lsa, u nolga teng.

6^0. Agar determinantning ikkita satr (ustun) elementlari o'zaro proporsional bo'lsa, u nolga teng.

7^0. Agar determinantning biror satrining (ustunining) har bir elementi ikkita qo'shiluvchining yig'indisidan iborat bo'lsa, u holda bu determinant (quyidagi ko'rinishdagi) ikkita determinantlar yig'indisidan iborat bo'ladi.

Masalan:

$$\begin{vmatrix} a_{11}+b_1 & a_{12} & a_{13} \\ a_{21}+b_2 & a_{22} & a_{23} \\ a_{31}+b_3 & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

8^0. Agar biror satr (ustun) elementalarini istalgan $\lambda \neq 0$, umumiy ko'paytuvchiga ko'paytirib boshqa satrning (ustunning) mos elementlariga qo'shilsa, determinant qiymati o'zgarmaydi.

Ikkinchi tartibli determinantlarni hisoblang.

4. $\begin{vmatrix} -3 & 2 \\ 5 & -1 \end{vmatrix}$

5. $\begin{vmatrix} 4 & -2 \\ 3 & -5 \end{vmatrix}$

6. $\begin{vmatrix} \frac{1}{4} & -\frac{1}{5} \\ 2\frac{1}{2} & -4 \end{vmatrix}$

7. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}$

8. $\begin{vmatrix} \sin 1^\circ & \sin 89^\circ \\ -\cos 1^\circ & \cos 89^\circ \end{vmatrix}$

9. $\begin{vmatrix} \log_b a & 1 \\ 1 & \log_a b \end{vmatrix}$

10. $\begin{vmatrix} ab & ac \\ bd & cd \end{vmatrix}$

11. $\begin{vmatrix} 1 & -tg\alpha \\ ctg\alpha & 1 \end{vmatrix}$;

Tenglamalarni yeching.

12. $\begin{vmatrix} x & x+1 \\ -4 & x+1 \end{vmatrix} = 4$

13. $\begin{vmatrix} x & 3x \\ 7 & 4 \end{vmatrix} = 34$

14. $\begin{vmatrix} x & -7 \\ 2 & x \end{vmatrix} = 23$

15. $\begin{vmatrix} \cos 8x & \sin 5x \\ -\sin 8x & \cos 5x \end{vmatrix} = 0$

16. $\begin{vmatrix} x+3 & x+2 \\ 6-2x & x+2 \end{vmatrix} = 0$

17. $\begin{vmatrix} 2x-1 & x+1 \\ x+2 & x-1 \end{vmatrix} = -6$

Uchinchi tartibli determinantlarni hisoblang.

18. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix}$

19. $\begin{vmatrix} 1 & -2 & 1 \\ 4 & 3 & 2 \\ 5 & 0 & 1 \end{vmatrix}$

20. $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 6 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

21. $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

22. $\begin{vmatrix} 18 & 9 & 27 \\ 6 & 12 & 12 \\ 13 & 26 & 39 \end{vmatrix};$

23. $\begin{vmatrix} 1 & a & 1 \\ a & a & 0 \\ a & 0 & -a \end{vmatrix}$

24. $\begin{vmatrix} x & -1 & x \\ 1 & x & -1 \\ x & 1 & x \end{vmatrix}$

25. $\begin{vmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{vmatrix}$

26. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

27. $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$

Tenglamalarni yeching.

28. $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 4 & 9 \\ x & 2 & 3 \end{vmatrix} = 0$

29. $\begin{vmatrix} 6 & 3 & x-1 \\ 4 & x+2 & 2 \\ 2x & 1 & 0 \end{vmatrix} = 0$

2-§. Yuqori tartibli determinantlar.

n-tartibli (n≥4) determinantlar va ularni hisoblashni o'rganish uchun avvalo, quyidagi yordamchi tushunchalar kiritamiz.

Algebraik to'ldiruvchi va minorlar.

Determinantning biror elementining **minori** deb, shu element turgan satrini va ustunini o'chirishdan qolgan elementlardan hosil bo'lgan determinantga aytiladi.

Determinant a_{ik} elementining minori $M_{ik},\ (i,k=1,2,3)$ bilan belgilanadi.

Determinant a_{ij} elementining **algebraik to'ldiruvchisi** deb $A_{ij}=(-1)^{i+j}M_{ij}$ songa aytiladi.

Masalan, quyidagi uchinchi tartibli determinantni olaylik.

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

a_{11} elementining algebraik to'ldiruvchisi

$$A_{11}=(-1)^{1+1}M_{11}=\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \text{ bo'ladi,}$$

a_{32} elementining algebraik to'ldiruvchisi esa

$$A_{32}=(-1)^{3+2}M_{32}=-\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \text{ bo'ladi.}$$

Bu kiritilgan tushunchalar yordamida quyidagi xossani isbotlash mumkin (xossalar tartibini saqlab qolamiz).

9°. Determinantning biror qatoridagi barcha elementlarni mos algebraik to'ldiruvchilari bilan ko'paytmasidan tashkil topgan yig'indi shu determinantning qiymatiga teng.

Yuqori tartibli determinantlar.

n ta satr va *n* ta ustundan tashkil topgan ushbu

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \tag{2}$$

determinantga **n -tartibli determinant** deyiladi. To’rtinchi tartibli determinantni qaraymiz.

Determinantlarning yuqorida keltirilgan 9°-xossasini qo’llab, ya’ni biror satr yoki ustun elementlari bo’yicha yoyish usuli bilan 4-tartibli determinantni biror ustun yoki satr elementlari bo’yicha yoyilganda hosil bo’ladigan determinantlar 3-tartibli bo’ladi. 3-tartibli determinant tushunchasi esa bizga ma’lum. n-tartibli determinantlar uchun yuqorida aytilgan barcha xossalar, jumladan, determinantning biror qator elementlari bo’yicha yoyish formulasi ham o’rinli bo’ladi.

Istalgan tartibli determinantni hisoblash quyidagi usullardan biri orqali bajarilishi mumkin:

a) algebraik to’ldiruvchilar yordamida satr yoki ustun bo’yicha yoyish usulidan foydalanish;

b) biror satrdagi (ustundagi) bittadan boshqa barcha elementlarni nolga aylantirib, so‘ngra shu satr (ustun) bo‘yicha yoyib, ya’ni tartibini pasaytirib;

c) bosh (yordamchi) diagonaldan bir tomonda yotuvchi barcha elementlarni nolga aylantiriladi, ya’ni uchburchak ko‘rinishga keltiriladi.

30. Determinant hisoblansin.

$$\Delta = \begin{vmatrix} 2 & 1 & 4 & 3 \\ 5 & 0 & -1 & 0 \\ 2 & -1 & 6 & 3 \\ 1 & 5 & -1 & 2 \end{vmatrix}$$

Yechish. Determinantni hisoblash uchun uni ikkinchi satr elementlari bo’yicha yoyib chiqamiz. U holda

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} + a_{24}A_{24} =$$

$$= -5\begin{vmatrix} 1 & 4 & 3 \\ -1 & 6 & 3 \\ 5 & -1 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 4 & 3 \\ 2 & 6 & 3 \\ 1 & -1 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 & 3 \\ 2 & -1 & 3 \\ 1 & 5 & 2 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 & 4 \\ 2 & -1 & 6 \\ 1 & 5 & -1 \end{vmatrix} =$$

$$= -5(12 + 60 + 3 - 90 + 3 + 8) + (-4 + 3 + 30 + 3 - 30 - 4) = 20 - 2 = 18$$

bo’ladi. Demak, yuqori tartibli determinantni hisoblash, determinant tartibini ketma-ket pasaytirish yo’li bilan amalga oshiriladi.

31. $\Delta = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ -2 & 0 & 3 & -4 \\ 1 & 1 & 0 & -2 \end{vmatrix}$

determinantni tartibini pasaytirish usuli bilan hisoblang.

Yechish. Buning uchun ikkita elementi nolga teng bo‘lgan uchinchi ustunni tanlaymiz va uning ikkinchi satrida joylashgan elementidan boshqa barcha elementlarini nolga aylantiramiz. Buning uchun ikkinchi satr elementlarini 3 ga ko‘paytirib, uchunchi satrning mos elementlariga qo‘shamiz va hosil bo‘lgan determinantni uchinchi ustun elementlari bo‘yicha yoyamiz:

$$\Delta = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ -2 & 0 & 3 & -4 \\ 1 & 1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 & 4 \\ 4 & 2 & -1 & 3 \\ 10 & 6 & 0 & 5 \\ 1 & 1 & 0 & -2 \end{vmatrix} = (-1)\cdot(-1)^{2+3} \begin{vmatrix} 2 & -1 & 4 \\ 10 & 6 & 5 \\ 1 & 1 & -2 \end{vmatrix}.$$

Hosil qilingan uchinchi tartibli determinantda birinchi ustunning uchinchi satri elementidan yuqorida joylashgan elementlarini nolga aylantiramiz. Buning uchun avval uchinchi satrni (-2) ga ko‘paytirib, birinchi satrga qo‘shamiz, keyin uchinchi satrni (-10) ga ko‘paytirib, ikkinchi satrga qo‘shamiz, hosil bo‘lgan determinantni birinchi ustun elementlari bo‘yicha yoyamiz va hosil bo‘lgan ikkinchi tartibli determinantni hisoblaymiz:

$$\Delta = \begin{vmatrix} 0 & -3 & 8 \\ 0 & -4 & 25 \\ 1 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -3 & 8 \\ -4 & 25 \end{vmatrix} = -75 + 32 = -43.$$

32. $\Delta = \begin{vmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix}$

determinantni uchburchak ko‘rinishga keltirib, hisoblang.

Yechish. Determinant ustida quyidagi almashtirishlarni bajaramiz:

- birinchi ustunni o‘zidan o‘ngda joylashgan ustunlar bilan ketma-ket 3 ta (toq) o‘rin almashtirib, to‘rtinchi ustunga o‘tkazamiz;

birinchi ustunning birinchi satridan pastda joylashgan elementlarini nolga aylantiramiz;

ikkinchi ustunning ikkinchi satridan pastda joylashgan elementlarini nolga aylantiramiz;

uchinchi ustunning to‘rtinchi satrida joylashgan elementini nolga aylantiramiz;

(-1) ko‘paytuvchi bilan hosil bo‘lgan uchburchak ko‘rinishdagi determinantning bosh diagonalda joylashgan elementlarini ko‘paytiramiz.

$$\Delta = \begin{vmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & -2 \end{vmatrix} =$$

$$= -\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 2 & -2 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 8 \end{vmatrix} = (-1)\cdot 1\cdot 1\cdot 1\cdot 8 = -8.$$

Determinantni ikkinchi satr elementlari bo’yicha yoyib hisoblang.

33. $\begin{vmatrix} 3 & 0 & -1 & -1 \\ \mathrm{a} & \mathrm{b} & c & d \\ 1 & 1 & 1 & 1 \\ -1 & -3 & -2 & -4 \end{vmatrix}$ **34.** $\begin{vmatrix} 3 & 0 & -1 & -1 \\ \mathrm{a} & \mathrm{b} & c & d \\ 1 & 1 & 1 & 1 \\ -1 & -3 & -2 & -4 \end{vmatrix}$

Determinantni 3-ustun elementlari bo’yicha yoyib hisoblang.

35. $\begin{vmatrix} 5 & 1 & x & 8 \\ -4 & -1 & y & -5 \\ 8 & -1 & z & 12 \\ 4 & -1 & t & 7 \end{vmatrix}$ **36.** $\begin{vmatrix} 1 & -1 & a & -1 \\ -1 & -2 & b & -1 \\ -2 & 0 & c & 1 \\ 0 & 1 & d & 0 \end{vmatrix}$

Determinantni hisoblang.

37. $\begin{vmatrix} 1 & 3 & 0 & 4 \\ 0 & -7 & 1 & -5 \\ 0 & -1 & 1 & -2 \\ 0 & 6 & 2 & 8 \end{vmatrix}$

38. $\begin{vmatrix} 0 & 3 & 0 & 1 \\ 7 & 1 & 2 & -2 \\ 5 & -5 & 0 & 0 \\ -4 & -6 & 0 & -2 \end{vmatrix}$

39. $\begin{vmatrix} 0 & 0 & 1 & -1 \\ 3 & 0 & 8 & 0 \\ -2 & -5 & 3 & 4 \\ 3 & 0 & 7 & 3 \end{vmatrix}$

40. $\begin{vmatrix} 7 & 0 & 1 & 0 \\ 3 & 3 & 0 & 0 \\ 2 & 10 & -2 & 3 \\ 1 & 6 & -1 & 0 \end{vmatrix}$

41. $\begin{vmatrix} 3 & 2 & 0 & 1 \\ -3 & 5 & 0 & 4 \\ 0 & 3 & 0 & 3 \\ 2 & -4 & 2 & 0 \end{vmatrix}$

42. $\begin{vmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & -4 & 7 \\ -2 & -5 & 7 & 5 \\ -2 & -5 & 2 & 3 \end{vmatrix}$

43. $\begin{vmatrix} -1 & 3 & 1 & 2 \\ -5 & 8 & 2 & 7 \\ 4 & -5 & 3 & -2 \\ -7 & 8 & 4 & 5 \end{vmatrix}$

44. $\begin{vmatrix} 1 & 5 & 7 & 2 \\ 0 & 6 & 3 & 7 \\ -2 & -8 & -7 & -3 \\ -1 & -6 & -5 & -4 \end{vmatrix}$

45. $\begin{vmatrix} 2 & 0 & 3 & 1 \\ -1 & -3 & 1 & 0 \\ 3 & 0 & 4 & 1 \\ 3 & 2 & 2 & 2 \end{vmatrix}$

46. $\begin{vmatrix} 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \end{vmatrix}$

a ning qanday qiymatlarida tenglik o’rinli bo’ladi.

47. $\begin{vmatrix} 0 & 3 & 0 & 1 \\ 7 & 1 & a+3 & -2 \\ 5 & -5 & 0 & 0 \\ -4 & -6 & 0 & -2 \end{vmatrix} = 40$

48. $\begin{vmatrix} 0 & 0 & 1 & -1 \\ 3 & 0 & 8 & 0 \\ -2 & a+4 & 3 & 4 \\ 3 & 0 & 7 & 3 \end{vmatrix} = -30$

49. $\begin{vmatrix} 7 & 0 & 1 & 0 \\ 3 & 3 & 0 & 0 \\ 2 & 10 & -2 & a-2 \\ 1 & 6 & -1 & 0 \end{vmatrix} = 18$

50. $\begin{vmatrix} 3 & 2 & 0 & 1 \\ -3 & 5 & 0 & 4 \\ 0 & 3 & 0 & 3 \\ 2 & -4 & 4-a & 0 \end{vmatrix} = -36$

Quyidagi yuqori tartibli determinantlarni hisoblang:

51. $\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$

52. $\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$

53. $\begin{vmatrix} 10 & 2 & 0 & 0 & 0 \\ 12 & 10 & 2 & 0 & 0 \\ 0 & 12 & 10 & 2 & 0 \\ 0 & 0 & 12 & 10 & 2 \\ 0 & 0 & 0 & 12 & 10 \end{vmatrix}$

54. $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$

55. $\begin{vmatrix} 1 & 2 & 2 & \dots & 2 & 2 \\ 2 & 2 & 2 & \dots & 2 & 2 \\ 2 & 2 & 3 & \dots & 2 & 2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 2 & 2 & 2 & \dots & n-1 & 2 \\ 2 & 2 & 2 & \dots & 2 & n \end{vmatrix}$

56. 1370, 1644, 2055 va 3245 sonlari 137 ga qoldiqsiz bo'linadi. $\begin{vmatrix} 1 & 3 & 7 & 0 \\ 1 & 6 & 4 & 4 \\ 2 & 0 & 5 & 5 \\ 3 & 4 & 2 & 5 \end{vmatrix}$ determinant ham 137 ga qoldiqsiz bo'linishini isbotlang.

57*. Determinant xossalaridan foydalanib quyidagi n-tartibli determinantlarni ko'rsatilgan qiymatlarga tengligini isbotlang:

a) $\begin{vmatrix} 1 & 1 & 1 \dots 1 \\ 1 & 0 & 1 \dots 1 \\ 1 & 1 & 0 \dots 1 \\ \dots & \dots & \dots \dots \\ 1 & 1 & 1 \dots 0 \end{vmatrix} = (-1)^{n-1};$

b) $\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ -1 & -2 & -3 & \dots & n \\ \dots & \dots & \dots & \dots & \dots \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}$=n!

3-§. **Chiziqli tenglamalar sistemasini Kramer qoidasi bilan yechish**.

Ikkita x_1 va x_2 noma'lumli chiziqli tenglamadan iborat ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \qquad (3)$$

sistema **ikki noma'lumli chiziqli tenglamalar sistemasi** deyiladi, bunda a_{11}, a_{12}, a_{21}, a_{22} - (3) sistemaning koeffitsientlari, b_1, b_2 - ozod hadlardir.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

asosiy determinant,

$$\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}, \quad \Delta_{x_2} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}.$$

yordamchi determinantlar deb nomlanadi. Agar $\Delta \neq 0$ bo'lsa, (3) tenglamalar sistemasining yechimi quyidagicha topiladi:

$$x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta} \tag{4}$$

Xuddi shuningdek, uchta x_1, x_2 va x_3 noma'lumli chiziqli tenglamalardan iborat

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \tag{5}$$

sistema **uch noma'lumli chiziqli tenglamalar sistemasi** deyiladi.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

asosiy determinant,

$$\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_{x_2} = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_{x_3} = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix},$$

yordamchi determinatlar deb nomlanadi, agar $\Delta \neq 0$ bo'lsa, (5) tenglamalar sistemasining yechimi quyidagicha topiladi:

$$x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta}, \quad x_3 = \frac{\Delta_{x_3}}{\Delta} \tag{6}$$

(4) va (6) formulalar (3) va (5) tenglamalar sistemasini yechishning Kramer formulasi deyiladi. $\Delta \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi.

$\Delta = 0$ hamda Δx_1, Δx_2, Δx_3 lardan hech bo'lmaganda bittasi noldan farqli bo'lsa sistemani yechimi mavjud emas.

$\Delta = 0$ va $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0$ bo'lsa sistema cheksiz ko'p yechimga ega bo'ladi.

58. $\begin{cases} x_1 + 3x_2 = -1 \\ 2x_1 - x_2 = 5 \end{cases}$ chiziqli tenglamalar sistemasi yechilsin.

Yechish. Asosiy va yordamchi determinantlarni hisoblaymiz.

$$\Delta = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1 - 6 = -7$$

$$\Delta_{x_1} = \begin{vmatrix} -1 & 3 \\ 5 & -1 \end{vmatrix} = 1 - 15 = -14, \quad \Delta_{x_2} = \begin{vmatrix} 1 & -1 \\ 2 & 5 \end{vmatrix} = 5 + 2 = 7,$$

U holda, Kramer formulasiga asosan,

$$x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{-14}{-7} = 2, \quad x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{7}{-7} = -1.$$

Demak, sistemaning yechimi $(2;-1)$.

59. $\begin{cases} x_1 + 2x_2 = 3 \\ 4x_1 + x_2 = 1 \end{cases}$ sistemani yeching.

Yechish.

$$\Delta = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 0, \quad \Delta_{x_1} = \begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} = 16, \quad \Delta_{x_2} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -8.$$

Demak, berilgan sistemaning yechimi mavjud emas.

60. Quyidagi sistema yechilsin $\begin{cases} 2x_1 + 3x_2 = 1 \\ 4x_1 + 6x_2 = 2 \end{cases}$

Yechish. Bu holda asosiy va yordamchi determinantlar nolga teng:

$$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0, \quad \Delta_{x_1} = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0, \quad \Delta_{x_2} = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$$

Demak, ixtiyoriy $\left(t; \frac{1-2t}{3}\right)$ ko'rinishdagi $(t \in R)$ sonlar juftligi sistemaning yechimi bo'ladi, ya'ni cheksiz ko'p yechim majud.

61. $\begin{cases} 2x_1 - x_2 + x_3 = 4 \\ 3x_1 + 2x_2 - x_3 = 1 \\ x_1 + x_2 - 2x_3 = -3 \end{cases}$ sistema yechilsin.

Yechish. Asosiy va yordamchi determinantlarni hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 1 & -2 \end{vmatrix} = -10, \quad \Delta_{x_1} = \begin{vmatrix} 4 & -1 & 1 \\ 1 & 2 & -1 \\ -3 & 1 & -2 \end{vmatrix} = -10, \quad \Delta_{x_2} = \begin{vmatrix} 2 & 4 & 1 \\ 3 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 0,$$

$$\Delta_{x_3} = \begin{vmatrix} 2 & -1 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = -20$$

U holda, Kramer formulasidan

$$x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{-10}{-10} = 1, \quad x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{0}{-10} = 0, \quad x_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{-20}{-10} = 2.$$

Demak, $(1;0;2)$ sistemaning yechimi bo'ladi.

Chiziqli tenglamalar sistemasini Kramer qoidasi bilan yeching.

62. $\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ 2x_1 + 5x_2 - 6x_3 = 1 \\ 3x_1 + 8x_2 - 10x_3 = 1 \end{cases}$

63. $\begin{cases} x_1 - x_2 + 3x_3 = 7 \\ 2x_1 + x_2 - 4x_3 = -3 \\ 3x_1 + x_2 - 3x_3 = 1 \end{cases}$

64. $\begin{cases} 4x_1 + x_2 - x_3 = 1 \\ 5x_1 + 3x_2 - 2x_3 = 2 \\ 3x_1 + 2x_2 - 3x_3 = 0 \end{cases}$

65. $\begin{cases} 5x_1 + 2x_2 + 5x_3 = 4 \\ 3x_1 + 5x_2 - 3x_3 = -1 \\ -2x_1 - 4x_2 + 3x_3 = 1 \end{cases}$

66. $\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ 3x_1 + 2x_2 + x_3 = 10 \\ 4x_1 + 3x_2 - 2x_3 = 4 \end{cases}$

67. $\begin{cases} 4x_1 + 2x_2 + 3x_3 = -2 \\ 3x_1 + 8x_2 - x_3 = 8 \\ 9x_1 + x_2 + 8x_3 = 0 \end{cases}$

68. $\begin{cases} -2x_1 + x_2 - x_3 = 7 \\ 4x_1 + 5x_2 - 3x_3 = -5 \\ x_1 + 3x_2 - 2x_3 = 1 \end{cases}$

69. $\begin{cases} x + y - 3z = -1 \\ 2x - y + z = 2 \\ 3x + 2y - 4z = 1 \end{cases}$

70. $\begin{cases} x_1+3x_2-4x_3=-1 \\ x_1-5x_2+x_3=7 \\ 2x_1+x_2-3x_3=3 \end{cases}$

71. $\begin{cases} 3x_1+x_2-x_3=2 \\ 2x_1-3x_2+x_3=-1 \\ x_1-x_2+2x_3=5 \end{cases}$

72. $\begin{cases} 2x_1+2x_2-x_3+x_4=4 \\ 4x_1+3x_2-x_3+2x_4=6 \\ 8x_1+5x_2-3x_3+4x_4=12 \\ 3x_1+3x_2-2x_3+2x_4=6 \end{cases}$

73. $\begin{cases} 2x_1+5x_2+4x_3+x_4=20 \\ x_1+3x_2+2x_3+x_4=11 \\ 2x_1+10x_2+9x_3+7x_4=40 \\ 3x_1+8x_2+9x_3+2x_4=37 \end{cases}$

74. $\begin{cases} 2x-y-6z+3t=-1 \\ 7x-4y+2z-15t=-32 \\ x-2y-4z+9t=5 \\ x-y+2z-6t=-8 \end{cases}$

75. $\begin{cases} 2x-4y+3z=1 \\ x-2y+4z=3 \\ 3x-y+5z=2 \end{cases}$

76. $\begin{cases} 2x-y+z=2 \\ 3x+2y+2z=-2 \\ x-2y+z=1 \end{cases}$

77. $\begin{cases} x+2y+3z=5 \\ 2x-y-z=1 \\ x+3y+4z=6 \end{cases}$

78. $\begin{cases} 2x_1-x_2+x_3=4 \\ 3x_1+2x_2-x_3=1 \\ x_1+x_2-2x_3=-3 \end{cases}$

4-§. **Matritsalar. Matritsalar ustida amallar.**

Quyidagi

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \tag{7}$$

ko'rinishdagi jadval $(m\times n)$**-tartibli matritsa** deyiladi.

a_{ik}-element xuddi determinantdagi kabi i-satr, k-ustunga joylashgan bo'ladi. Ba'zan (7) yozuv, qisqalik uchun, $\|a_{ik}\|$, $\left(i=\overline{1,m},\ k=\overline{1,n}\right)$ ko'rinishda yoki $A=\|a_{ik}\|$ ko'rinishda ham belgilanadi. Ravshanki, (7) matritsa m ta satr va n ta ustundan iborat.

Barcha elementlari nolga teng bo'lgan matritsa **nol matritsa** deyiladi.

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Xususiy holda, $m = n$ bo'lganda,

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \qquad (8)$$

ko'rinishidagi matritsa **kvadrat matritsa** deyiladi.

$a_{11}, a_{22}, \dots\dots, a_{nn}$ (8) matritsaning **bosh diagonal elementlari** deyiladi. Agar (8) matritsada bosh diagonalda turgan elementlardan boshqa barcha elementlari nol bo'lsa, uni **diagonal matritsa** deyiladi:

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}. \qquad (9)$$

(9) matritsada $a_{11} = a_{22} = \dots\dots = a_{nn} = 1$ bo'lsa, yani,

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

birlik matritsa deb ataladi.

Kvadrat matritsa (8) ning elementlaridan tashkil topgan determinant A matritsaning determinanti deyiladi va $\det(A)$ yoki $|A|$ kabi belgilanadi. Shu o'rinda eslatib o'tamiz: matritsa sonlarning tartibli jadvali, determinant esa elementlarning ma'lum kombinatsiyasidan hosil qilingan birgina sondir.

$$A=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Agar $\det(A)=0$ bo'lsa, bu holda A matritsa **xos matritsa**, $\det(A)\neq 0$ bo'lsa, A **xosmas matritsa** deyiladi. Kvadrat (8) matritsaning satr elementlarini mos ustun elementlari bilan almashtirishdan hosil bo'lgan

$$\begin{pmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{pmatrix}$$

matritsa **transponirlangan matritsa** deyiladi va A^T kabi belgilanadi. Determinantning 1° xossasiga asosan $\det(A)=\det(A^T)$

Ikkita

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad B=\begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}, \tag{10}$$

matritsalar berilgan bo'lib, A matritsaning har bir elementi B matritsaning mos elementiga teng, ya'ni $a_{ik}=b_{ik}$ bo'lsa, u holda A va B o'zaro teng matritsalar deyiladi va $A=B$ kabi yoziladi. Ta'rif bo'yicha ikkita (10) ko'rinishdagi $(m\times n)$ tartibli matritsalarning yig'indisi va ayirmasi mos ravshida $A\pm B$ kabi belgilanib,

$$A\pm B=\begin{pmatrix} a_{11}\pm b_{11} & a_{12}\pm b_{12} & \dots & a_{1n}\pm b_{1n} \\ a_{21}\pm b_{21} & a_{22}\pm b_{22} & \dots & a_{2n}\pm b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1}\pm b_{n1} & a_{n2}\pm b_{n2} & \dots & a_{nn}\pm b_{nn} \end{pmatrix},$$

qoida bo'yicha hisoblanadi, ya'ni mos elementlari qo'shiladi yoki ayiriladi. Matritsalarni qo'shish quyidagi xossalarga ega:

1°. $A+0=A$

2°. $A+B=B+A$.

A matritsani $\alpha \neq 0$ songa ko'paytmasi deb, uning har bir elementini α songa ko'paytirishdan hosil bo'lgan

$$\alpha A=\begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \dots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \dots & \alpha a_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha a_{n1} & \alpha a_{n2} & \dots & \alpha a_{nn} \end{pmatrix}$$

matritsaga aytiladi. Matritsani songa ko'paytirish quyidagi xossalarga ega:

3°. $\alpha(\beta A)=(\alpha\beta)A$

4°. $\alpha(A+B)=\alpha A+\alpha B$

5°. $(\alpha+\beta)A=\alpha A+\beta A$

79. Agar $A=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ bo'lsa, $A+B,\ A-B,\ 2A-3B$ matritsalar topilsin.

Yechish.

$$A+B=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}=\begin{pmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{pmatrix}=\begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{pmatrix},$$

$$A-B=\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}-\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}=\begin{pmatrix} 2-0 & 4-2 & 1-1 \\ -1-1 & 0-1 & 2-2 \end{pmatrix}=\begin{pmatrix} 2 & 2 & 0 \\ -2 & -1 & 0 \end{pmatrix},$$

$$2A-3B=2\begin{pmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}-3\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}=\begin{pmatrix} 4 & 8 & 2 \\ -2 & 0 & 4 \end{pmatrix}-\begin{pmatrix} 0 & 6 & 3 \\ 3 & 3 & 6 \end{pmatrix}=$$
$$=\begin{pmatrix} 4-0 & 8-6 & 2-3 \\ -2-3 & 0-3 & 4-6 \end{pmatrix}=\begin{pmatrix} 4 & 2 & -1 \\ -5 & -3 & -2 \end{pmatrix}.$$

Endi ikki matritsa ko'paytmasi tushunchasini kiritamiz. Bunda ko'paytiriladigan matritsalar birinchisining ustunlar soni ikkinchisining satrlar soniga teng bo'lishi talab qilinadi.

$(m\times n)$ tartibli A matritsaning $(n\times k)$ tartibli B matritsaga ko'paytmasi deb $(m\times k)$ tartibli shunday C matritsaga aytiladiki, uning c_{ij} elementi A matritsa i-

satri elementlarini B matritsa j-ustinining mos elementlariga ko'paytmalari yig'indisiga teng, ya'ni

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{ik}b_{kj}\ .$$

matritsalar ko'paytmasi $C = A\cdot B$, ko'rinishda belgilanadi.

80. Matritsalar ko'paytmasi topilsin.

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix},\ B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

Yechish. $A\cdot B$ mavjud, chunki A matritsa ikkita ustundan B matritsa esa ikkita satrdan iborat.

$$A\cdot B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 4 - 3\cdot 2 & 1\cdot 1 - 1\cdot 1 \\ 2\cdot 2 + 1\cdot 1 & 2\cdot 1 + 1\cdot 1 \\ 2\cdot 2 + 1\cdot 1 & 1\cdot 1 + 1\cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 5 & 3 \\ 3 & 2 \end{pmatrix}.$$

ko'paytma mavjud emas, chunki B matritsada 2 ta ustun, A matritsada esa 3 ta satr mavjud.

Agar A va B matritsalar bir xil tartibli kvadrat matritsalar bo'lsa $A\cdot B$ va $B\cdot A$ ni hisoblash mumkin.

81. $A = \begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -3 \\ 2 & 6 \end{pmatrix}$ bo'lsa, $A\cdot B$ va $B\cdot A$ topilsin.

Yechish.

$$A\cdot B = \begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}\cdot\begin{pmatrix} 4 & -3 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 2\cdot 4 - 3\cdot 2 & -2\cdot 3 - 3\cdot 6 \\ 5\cdot 4 + 1\cdot 2 & -5\cdot 3 + 1\cdot 6 \end{pmatrix} = \begin{pmatrix} 2 & -24 \\ 22 & -9 \end{pmatrix}$$

$$B\cdot A = \begin{pmatrix} 4 & -3 \\ 2 & 6 \end{pmatrix}\cdot\begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 4\cdot 2 - 3\cdot 5 & -4\cdot 3 - 3\cdot 1 \\ 2\cdot 2 + 6\cdot 5 & -2\cdot 3 + 6\cdot 1 \end{pmatrix} = \begin{pmatrix} -7 & -15 \\ 34 & -0 \end{pmatrix}.$$

Bu misoldan ko'rinadiki, umuman olganda, $A\cdot B \neq B\cdot A$.

Matritsalarni ko'paytirish quyidagi xossalarga ega:

1°. $(A\cdot B)\cdot C = A\cdot(B\cdot C)$

$2°. (A+B)\cdot C = A\cdot C + B\cdot C$

$3°. \ (\lambda \cdot A)\cdot B = \lambda (A\cdot B)$

$4°. \ A\cdot E = E\cdot A = A$

$5°. \ A\cdot 0 = 0\cdot A = 0$

82. Agar $A=\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ bo'lsa, $f(t)=t^2-2t+3$ ni hisoblang.

Yechish. $f(A)=\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}-2\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}+3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Berilgan matritsalar ustida amallarni bajaring.

83. $A=\begin{pmatrix} 1 & 7 & 2 \\ -3 & 4 & -2 \\ 1 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 4 & 0 \\ 2 & 3 & 1 \\ -1 & 0 & 4 \end{pmatrix}$, $C=2A-3B$

84. $A=\begin{pmatrix} 3 & -2 & 1 & 6 \\ 8 & 3 & 4 & 1 \\ -5 & 7 & 0 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 1 & 9 & 0 \\ 2 & 6 & 4 & 1 \\ -3 & 4 & 5 & 2 \end{pmatrix}$, $C=A-2B$

85. $A=\begin{pmatrix} 1 & 5 \\ 2 & -4 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$, $2A-B=?$

86. A=$\begin{pmatrix} 1 & -1 & -3 \\ 2 & 1 & 5 \end{pmatrix}$, B=$\begin{pmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \end{pmatrix}$, $3A-2B=?$

87. $A=\begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}$, $A=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$, $A+B=?$

88. $\begin{pmatrix} 7 & 0 \\ 3 & 1 \\ -1 & 2 \end{pmatrix}-3\begin{pmatrix} 2 & \sqrt{2} \\ 1 & -1 \\ -1 & 0 \end{pmatrix}+\begin{pmatrix} 1 & \sqrt{18} \\ 4 & -5 \\ 3 & 1 \end{pmatrix}$

89. A=$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, B=$\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$, $AB=?$

90. $A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \\ -4 & 5 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 4 & 1 \\ 0 & 2 & 5 \\ 1 & -1 & 4 \end{pmatrix}$, $AB=?$

A va B matritsalarni ko'paytiring.

91. $A=(1\ 2\ 3)$, $B=\begin{pmatrix} 4 & -3 \\ 1 & 2 \\ 0 & 2 \end{pmatrix}$

92. $A=\begin{pmatrix} 1 & 5 \\ 3 & -2 \end{pmatrix}$, $B=\begin{pmatrix} -3 & 1 \\ 5 & 2 \end{pmatrix}$;

93. $A=\begin{pmatrix} 2 & -1 \\ 7 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 0 & -3 \\ -1 & 5 & 3 \end{pmatrix}$,

94. $A=\begin{pmatrix} 1 & 8 \\ 3 & -2 \\ 2 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix}$;

95. $A=\begin{pmatrix} 0 & 0 & -2 \\ 1 & -2 & 4 \\ 2 & -2 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 5 & 4 \\ 2 & 6 \\ 3 & 1 \end{pmatrix}$;

96. $A=\begin{pmatrix} 1 & 3 & 0 \\ 2 & -1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 0 \\ -1 & 5 \\ 4 & 6 \end{pmatrix}$;

97. $A=\begin{pmatrix} -2 & 4 & 1 \\ 3 & -1 & 7 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 2 & -2 \\ 3 & -2 & 1 \\ 1 & -2 & 2 \end{pmatrix}$;

98. $A=\begin{pmatrix} 2 & 3 & -2 \\ -1 & -1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 3 & 0 \\ 2 & -1 & 3 \\ 1 & -2 & 3 \end{pmatrix}$;

99. $A=(1\ \ 2\ \ -2)$, $B=\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$;

100. $A=\begin{pmatrix}2\\-1\\4\end{pmatrix},\quad B=\begin{pmatrix}1 & 2 & -3\end{pmatrix};$

101. $A=\begin{pmatrix}5 & 2 & 4\\1 & 1 & -3\\1 & 0 & 3\end{pmatrix},\quad B=\begin{pmatrix}3\\-2\\5\end{pmatrix};$

102. $A=(1\ \ 2\ \ -5)\,,\quad B=\begin{pmatrix}5 & -2 & 2\\7 & 0 & 1\\5 & 3 & 1\end{pmatrix};$

103. $A=\begin{pmatrix}2 & 0 & 1\\-2 & 3 & 2\\4 & -1 & 5\end{pmatrix},\quad B=\begin{pmatrix}-3 & 1 & 0\\0 & 2 & 1\\0 & -1 & 3\end{pmatrix}$

104. $A=\begin{pmatrix}1 & -1 & 3\\2 & 1 & 5\end{pmatrix},\quad B=\begin{pmatrix}1\\2\\3\end{pmatrix}$

105. $A=\begin{pmatrix}1 & 3 & -1\\2 & 1 & 2\\0 & 1 & 0\end{pmatrix},\quad B=\begin{pmatrix}1 & 1\\2 & 3\\1 & 0\end{pmatrix}$

106. $A=\begin{pmatrix}1 & -3 & 0\\2 & 5 & 1\end{pmatrix},\quad B=\begin{pmatrix}0 & -1 & 3\\3 & 5 & 2\\4 & -2 & 1\end{pmatrix}$

107. $A=\begin{pmatrix}1 & 3 & 1\\2 & 0 & 4\\1 & 2 & 3\end{pmatrix},\quad B=\begin{pmatrix}2 & 1 & 0\\1 & -1 & 2\\3 & 2 & 1\end{pmatrix},\quad AB=?\ \ BA=?$

A va B matritsalar berilgan. Quyidagi tenglamani qanoatlantiruvchi X matritsani toping:

108. $A+2X-4B=0,\quad A=\begin{pmatrix}2 & -4 & 0\\6 & -2 & 4\\0 & 8 & 2\end{pmatrix},\ B=\begin{pmatrix}1 & 3 & 7\\-2 & 0 & 5\\4 & 5 & 3\end{pmatrix};$

109. $5A+3X-B=0,\quad A=\begin{pmatrix} 7 & 2 & 1 & 5 \\ 3 & -2 & 4 & -3 \\ 2 & 1 & 1 & 1 \end{pmatrix},\ B=\begin{pmatrix} 5 & 4 & 3 & 0 \\ 2 & 3 & -2 & 1 \\ 1 & 0 & 2 & 4 \end{pmatrix}$

A matritsaning berilgan n-darajasini hisoblang.

110. $A=\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix},\quad n=3;$

111. $A=\begin{pmatrix} 4 & -1 \\ 5 & -2 \end{pmatrix},\quad n=5;$

112. $A=\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix},\quad n=10,\ n=15;$

113. $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},\quad n=10;$

114. $A=\begin{pmatrix} \alpha & 1 \\ 0 & \alpha \end{pmatrix},\quad n\in N.$

115. A=$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, $A^2=?$

Matritsalar ustida amallarni bajaring.

116. A=$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$, E-birlik matrisa $2A^2+3A+5E=?$

117. A=$\begin{pmatrix} 3 & 4 & 2 \\ 1 & 0 & 5 \end{pmatrix}$, B=$\begin{pmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 5 \end{pmatrix}$, C=$\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$ AB-C^2=?

118. A=$\begin{pmatrix} 1 & 2 & -3 \\ 1 & 0 & 2 \\ 4 & 5 & 3 \end{pmatrix}$, B=$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, C=$\begin{pmatrix} 2 & 0 & 5 \end{pmatrix}$, E-birlik matritsa A×B×C-3E=?

119. $A=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix},\quad B=\begin{pmatrix} 5 & 7 \\ -1 & 2 \end{pmatrix},\quad A^2-AB+2BA=?$

120. Quyidagi matritsalar berilgan: A=$\begin{pmatrix} 3 & 4 \\ 5 & 1 \end{pmatrix}$, B=$\begin{pmatrix} 8 & 1 \\ 2 & 3 \end{pmatrix}$, C=$\begin{pmatrix} 2 & -7 \\ -8 & 1 \end{pmatrix}$.

Agar λ_1=2, λ_2=-1 λ_3=1 bo'lsa, D=λ_1A+λ_2B+λ_3C matritsani toping.

Berilganlar bo'yicha $f(A)$ ni toping.

121. $f(x)=x^2-3x,\ A=\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

122. $f(x)=x^2-5x+3,\quad$ A=$\begin{pmatrix} 2 & -1 \\ -3 & 3 \end{pmatrix}$

123. $f(x)=x^2-x+1,\quad$ A=$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$

124. $f(x)=x^2-2x+3,\quad$ A=$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

125. $f(x)=x^3-2x^2+x+4,\quad$ A=$\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$

126. $f(x)=3x^2-4x+1,\quad$ A=$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

127. $f(x)=x^4-2x^2+3x-5,\quad$ A=$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

128. Agar A=$\begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$, $f(x)=x^3-x^2+5x+4,\quad g(x)=x^2-2x+11$ bo'lsa, $2f(A)-3g(A)$ ni toping.

129. Agar B=$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$, $f(x)=x^2-2x+1,$ $g(x)=3x+5$ bo'lsa, $f(B)-2g(B)$ ni toping.

130. Agar A=$\begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 3 \\ 0 & -1 & -1 \end{pmatrix}$, $f(x)=x+1$ bo'lsa, $(f(A))^2$ ni toping.

B matritsa A va X matritsalarning ko'patmasidan iborat bo'lsa ($B=AX$ yoki $B=XA$), X matritsani aniqlang.

131*. $A=\begin{pmatrix} 342 & 211 & 645 \\ 457 & 992 & 719 \\ 123 & 403 & 842 \end{pmatrix},\quad B=\begin{pmatrix} 645 & 211 & 342 \\ 719 & 992 & 457 \\ 842 & 403 & 123 \end{pmatrix}$

132*. $A=\begin{pmatrix} 342 & 211 & 645 \\ 457 & 992 & 719 \\ 123 & 403 & 842 \end{pmatrix}$, $B=\begin{pmatrix} 457 & 992 & 719 \\ 342 & 211 & 645 \\ 123 & 403 & 842 \end{pmatrix}$

133*. $A=\begin{pmatrix} 332 & 211 & 123 \\ 457 & 992 & 719 \\ 123 & 403 & 842 \end{pmatrix}$, $B=\begin{pmatrix} 996 & 633 & 369 \\ 457 & 992 & 719 \\ 123 & 403 & 842 \end{pmatrix}$

134*. $A=\begin{pmatrix} 111 & 203 & 343 \\ 209 & 121 & 514 \\ 221 & 106 & 678 \end{pmatrix}$, $B=\begin{pmatrix} 333 & 812 & 343 \\ 627 & 484 & 514 \\ 663 & 424 & 678 \end{pmatrix}$

5-§. Teskari matritsa

Ushbu kvadrat matritsani qaraylik:

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}$$

Agar $A\cdot B=B\cdot A=E$ bo'lsa, B matritsa A matritsaga teskari matritsa deb ataladi. A matritsaga teskari matritsani A^{-1} kabi belgilash qabul qilingan. $A\cdot B$ kvadrat matritsaga teskari A^{-1}matritsani topish quyidagicha amalga oshiraladi:

1. $\det(A)$ hisoblanadi. (Bu o'rinda $\Delta=\det(A)\neq 0$ bo'lishi kerakligini eslatib o'tamiz, aks holda teskari matritsa mavjud bo'lmaydi).

2. A matritsa determinantining har bir a_{ij}, $(i,j=1,2,3,...,n)$ elementi algebraik to'ldiruvchisi A_{ij} ni hisoblaymiz va A^{-1} matritsani quyidagicha tuzamiz:

$$A^{-1}=\frac{1}{\det(A)}\begin{pmatrix} A_{11} & A_{21} & ... & A_{n1} \\ A_{12} & A_{22} & ... & A_{n2} \\ ... & ... & ... & ... \\ A_{1n} & A_{2n} & ... & A_{nn} \end{pmatrix}. \qquad (11)$$

Ishonch hosil qilish uchun $A\cdot A^{-1}=A^{-1}\cdot A=E$ ni tekshirib ko'rish yetarli.

135. $A=\begin{pmatrix}1 & 0 & -2\\ 3 & 1 & 0\\ -1 & 2 & 4\end{pmatrix}$ matritsaga teskari matritsa topilsin.

Yechish.

$$\Delta=\begin{vmatrix}1 & 0 & -2\\ 3 & 1 & 0\\ -1 & 2 & 4\end{vmatrix}=-10\neq 0$$

Demak, teskari matritsa A^{-1} mavjud. Matritsa determinantining barcha elementlari algebraik to'ldiruvchilarini hisoblaymiz:

$$A_{11}=\begin{vmatrix}1 & 0\\ 2 & 4\end{vmatrix}=4,\quad A_{12}=-\begin{vmatrix}3 & 0\\ -1 & 4\end{vmatrix}=-12,\quad A_{13}=\begin{vmatrix}3 & 1\\ -1 & 2\end{vmatrix}=7,$$

$$A_{21}=-\begin{vmatrix}0 & -2\\ 1 & 4\end{vmatrix}=-4,\quad A_{22}=\begin{vmatrix}1 & -2\\ -1 & 4\end{vmatrix}=2,\quad A_{23}=-\begin{vmatrix}1 & 0\\ -1 & 2\end{vmatrix}=-2,$$

$$A_{31}=\begin{vmatrix}0 & -2\\ 1 & 0\end{vmatrix}=2,\quad A_{32}=-\begin{vmatrix}1 & -2\\ 3 & 0\end{vmatrix}=-6,\quad A_{33}=\begin{vmatrix}1 & 0\\ 3 & 1\end{vmatrix}=1,$$

Topilganlarni (11) ga qo'ysak:

$$A^{-1}=\frac{1}{\Delta}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}=-\frac{1}{10}\begin{pmatrix}4 & -4 & 2\\ -12 & 2 & -6\\ 7 & -2 & 1\end{pmatrix}$$

Tekshirish.

$$A^{-1}\cdot A=-\frac{1}{10}\begin{pmatrix}4 & -4 & 2\\ -12 & 2 & -6\\ 7 & -2 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & -2\\ 3 & 1 & 0\\ -1 & 2 & 4\end{pmatrix}=$$

$$\frac{-1}{10}\begin{pmatrix}4-12-2 & 0-4+4 & -8+0+8\\ -12+6+6 & 0+2-12 & 24+0-24\\ 7-6-1 & 0-2+2 & -14+0+4\end{pmatrix}=\frac{-1}{10}\begin{pmatrix}-10 & 0 & 0\\ 0 & -10 & 0\\ 0 & 0 & -10\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Demak, A^{-1} matritsa, A matritsaga teskari matritsa ekanligi kelib chiqdi.

A matritsaga teskari matritsani toping.

136. $A=\begin{pmatrix} 3 & 6 \\ 4 & 9 \end{pmatrix}$

137. $A=\begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$

138. $A=\begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$

139. $A=\begin{pmatrix} 3 & -4 \\ 5 & -8 \end{pmatrix}$

140. $A=\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$

141. $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 4 \end{pmatrix}$

142. $A=\begin{pmatrix} 4 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$

143. $A=\begin{pmatrix} 4 & -1 & 2 \\ 1 & 1 & -2 \\ 0 & -1 & 3 \end{pmatrix}$

144. $A=\begin{pmatrix} 1 & 4 & 1 \\ 3 & 2 & 1 \\ 6 & -2 & 1 \end{pmatrix}$

145. $A=\begin{pmatrix} 3 & 1 & 3 \\ 5 & -2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$

Berilgan matritsaga teskari matritsani toping va ko'paytirish bilan tekshiring.

146. $\begin{pmatrix} -3 & 5 \\ -1 & 2 \end{pmatrix}$

147. $\begin{pmatrix} 1 & -4 \\ 2 & -3 \end{pmatrix}$

148. $\begin{pmatrix} 15 & -5 \\ 3 & -1 \end{pmatrix}$

149. $\begin{pmatrix} -5 & 4 \\ 0 & 3 \end{pmatrix}$

150. $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 3 \end{pmatrix}$

151. $\begin{pmatrix} 3 & 1 & 1 \\ 1 & -2 & 2 \\ 4 & -3 & -1 \end{pmatrix}$

152. $\begin{pmatrix} 2 & -1 & -1 \\ 3 & 4 & -2 \\ 3 & -2 & 4 \end{pmatrix}$

153. $\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$

154. $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

155. $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{12} & 0 & 0 & 0 \end{pmatrix}$

156. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{pmatrix}$

157. $\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

a, b va c parametrlarning qanday qiymatlarida A va B matritsalar o'zaro teskari matritsalar bo'ladi?

158*. $A = \begin{pmatrix} a-1 & -2 & 3 \\ 0 & -1 & c-2 \\ 4 & b & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 \\ -8 & 3 & -6 \\ -4 & 2 & -3 \end{pmatrix}$

159*. $A = \begin{pmatrix} a-3 & 3 & 5 \\ 0 & c & 3 \\ -5 & -1 & b-4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & -1 \\ -15 & 29 & 12 \\ 10 & -19 & -8 \end{pmatrix}$

160*. $A = \begin{pmatrix} a-2 & 0 & 1 \\ -8 & b+4 & -6 \\ -4 & 2 & c \end{pmatrix}, \quad B = \begin{pmatrix} -3 & -2 & 3 \\ 0 & -1 & 2 \\ 4 & 2 & -3 \end{pmatrix}$

161*. $A = \begin{pmatrix} a & -2 & -1 \\ -15 & b+20 & 12 \\ 10 & -19 & 2c \end{pmatrix}, \quad B = \begin{pmatrix} -4 & 3 & 5 \\ 0 & 2 & 3 \\ -5 & -1 & -1 \end{pmatrix}$

162*. $A = \begin{pmatrix} 2 & -3 & 1 \\ 4 & a & 2 \\ 5 & -7 & c \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & -1 \\ b & 1 & 0 \\ -3 & -1 & 2 \end{pmatrix}$

163.* A=$\begin{pmatrix} 1 & 0 & -4 \\ 3 & 1 & 1 \\ -1 & 0 & \mathrm{a} \end{pmatrix}$, B=$\begin{pmatrix} 5 & 0 & 4 \\ \mathrm{b}-20 & 1 & \mathrm{c}-10 \\ 1 & 0 & 1 \end{pmatrix}$

a ning qanday qiymatlarida berilgan matritsaga teskari matritsa mavjudligini aniqlang.

164. $\begin{pmatrix} 2 & 4 & -1 \\ a-2 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

165*. $\begin{pmatrix} 2-a & 1 & 1 \\ 1 & 2-a & 1 \\ 1 & 1 & 2-a \end{pmatrix}$

166. $\begin{pmatrix} -1 & 2 & a \\ a-1 & 5 & a \\ a & 3 & 0 \end{pmatrix}$

167. $\begin{pmatrix} a & 0 & a^2-1 \\ 1 & 0 & a \\ 3 & 2 & 4 \end{pmatrix}$

6-§. Chiziqli tenglamalar sistemasini yechishning matritsa usuli

Bizga quyidagi uch noma'lumli uchta tenglamalar sistemasi berilgan bo'lsin

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} . \tag{12}$$

Tenglamalar sistemasi koeffitsientlari, $a_{i\,j},\ \ (i, j = 1,2,3)$, $x_1,\ x_2,\ x_3$- noma'lumlar va $b_i,\ (i = 1,2,3)$ ozod hadlardan

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},\quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

matritsalarni tuzamiz.

Matritsalarni ko'paytirish amaliga asosan, (12) sistemani, quyidagicha yozish mumkin

$$A \cdot X = B. \tag{13}$$

Bu tenglamalar sistemasining matritsalar ko'rinishida yozilishidir. Aytaylik A matritsaga teskari A^{-1} matritsa mavjud bo'lsin. (13) tenglikning har ikki tomonini A^{-1} ga chapdan ko'paytirib,

$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$ ni hosil qilamiz.

$$A^{-1}\cdot A=E \quad \text{va} \quad E\cdot X=X \text{ ekanini e'tiborga olsak,}$$

$$X=A^{-1}\cdot B \qquad (14)$$

hosil bo'ladi.

(14) formula (12) tenglamalar sistemasini teskari matritsa yordamida yechish formulasidir.

168. Kramer usuli bilan yechilgan 61-misoldagi tenglamalar sistemasini teskari matritsa usulida yechilsin.

Yechish.

$$A=\begin{pmatrix}2 & -1 & 1\\ 3 & 2 & -1\\ 1 & -1 & -2\end{pmatrix},\quad X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix},\quad B=\begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix},$$

$$\Delta=\det(A)=-10\neq 0$$

$$A_{11}=\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}=-3,\quad A_{12}=-\begin{vmatrix}3 & -1\\ 1 & -2\end{vmatrix}=5,\quad A_{13}=\begin{vmatrix}3 & 2\\ 1 & 1\end{vmatrix}=1,$$

$$A_{21}=-\begin{vmatrix}-1 & 1\\ 1 & -2\end{vmatrix}=-1,\quad A_{22}=\begin{vmatrix}2 & 1\\ 1 & -2\end{vmatrix}=-5,\quad A_{23}=-\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix}=-3,$$

$$A_{31}=\begin{vmatrix}-1 & 1\\ 2 & -1\end{vmatrix}=-1,\quad A_{32}=-\begin{vmatrix}2 & 1\\ 3 & -1\end{vmatrix}=5,\quad A_{33}=\begin{vmatrix}2 & -1\\ 3 & 2\end{vmatrix}=7,$$

(1) formulaga asosan

$$A^{-1}=-0{,}1\begin{pmatrix}-3 & -1 & -1\\ 5 & -5 & 5\\ 1 & -3 & 7\end{pmatrix}$$

teskari matritsani topamiz. Bundan (14) formulaga binoan

$$X=A^{-1}\cdot B=-0{,}1\cdot\begin{pmatrix}-3 & -1 & -1\\ 5 & -5 & 5\\ 1 & -3 & 7\end{pmatrix}\cdot\begin{pmatrix}4\\ 1\\ -3\end{pmatrix}=-0{,}1\cdot\begin{pmatrix}-12-1+3\\ 20-5-15\\ 4-3-21\end{pmatrix}=-0{,}1\begin{pmatrix}-10\\ 0\\ -20\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 2\end{pmatrix}$$

Javob (1; 0; 2)

Chiziqli tenglamalar sistemasini matritsa usulida yeching.

169. $\begin{cases} x_1 - x_2 + x_3 = -2 \\ 2x_1 + x_2 - 2x_3 = 6 \\ x_1 + 2x_2 + 3x_3 = 2 \end{cases}$

170. $\begin{cases} 3x_1 + x_2 + x_3 = 2 \\ x_1 - 2x_2 + 2x_3 = -1 \\ 4x_1 - 3x_2 - x_3 = 5 \end{cases}$

171. $\begin{cases} 3x_1 + x_2 + x_3 = 15 \\ 5x_1 + x_2 + 2x_3 = 20 \\ 6x_1 - x_2 + x_3 = 10 \end{cases}$

172. $\begin{cases} 3x_1 + 4x_2 - x_3 = 2 \\ 3x_1 - 2x_2 + x_3 = 24 \\ 2x_1 - x_2 + x_3 = 8 \end{cases}$

173. $\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_1 - x_2 - 1 = 0 \\ 3x_1 + 2x_2 - x_3 - 4 = 0 \end{cases}$

174. $\begin{cases} x_1 - 2x_2 + 2x_3 = -1 \\ 3x_1 + x_2 + x_3 = 2 \\ 4x_1 - 3x_2 - x_3 = 5 \end{cases}$

175. $\begin{cases} x + y - 3z = -1 \\ 2x - y + z = 2 \\ 3x + 2y - 4z = 1 \end{cases}$

176. $\begin{cases} x_1 + 3x_2 - 4x_3 = -1 \\ x_1 - 5x_2 + x_3 = 7 \\ 2x_1 + x_2 - 3x_3 = 3 \end{cases}.$

177. $\begin{cases} 3x_1 + x_2 - x_3 = 2 \\ 2x_1 - 3x_2 + x_3 = -1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$

178*. $\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4 \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6 \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12 \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6 \end{cases}$

7-§. Matritsaning rangi.

Biror $(m\times n)$-tartibli $A=\|a_{mn}\|$ matritsa berilgan bo'lsin. A matritsaning ixtiyoriy k ta satrini va ixtiyoriy k ta ustunini olib$\left(k\le \min(m,n)\right)$, $(k\times k)-$ tartibli kvadrat matritsa tuzamiz. Bu kvadrat matritsaning determinanti A matritsaning k-tartibli **minori** deyiladi.

179. Quyidagi 4x5-tartibli

$$\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & 1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}$$

matritsani qaraylik. Ushbu

$$\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix}=0,\quad \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}=-3,\quad \begin{vmatrix} -4 & 3 & 0 \\ 1 & 1 & 1 \\ -7 & 4 & 5 \end{vmatrix}=-40,\quad \begin{vmatrix} 2 & -4 & 1 & 0 \\ 1 & -2 & -4 & 2 \\ 0 & 1 & 3 & 1 \\ 4 & -7 & -4 & 5 \end{vmatrix}=0,$$

determinantlar qaralayotgan matritsaning mos ravishda ikkinchi, uchinchi hamda to’rtinchi tartibli minorlaridir.

A matritsa yordamida hosil qilish mumkin bo’lgan barcha minorlar orasida noldan farqli bo’lgan eng yuqori tartibli minorni topish muhimdir.

Shuni ta’kidlash kerakki, agar A matritsaning barcha k-tartibli $\left(k=\min\left(m,n\right)\right)$ minorlari nolga teng bo’lsa, undan yuqori tartibli bo’lgan barcha minorlari ham nolga teng bo’ladi.

Ta’rif. A matritsaning noldan farqli minorlarining eng yuqori tartibi uning **rangi** deyiladi va $rang\,A$ kabi belgilanadi.

180. $A=\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$ matritsaning rangini toping.

Yechish.

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}=0, \qquad \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}=-1, \qquad \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{vmatrix}=0$$

Demak, A matritsaning noldan farqli minorlarining eng katta tartibi 2 ga teng ekan. Bundan $rang\,A=2$. Ta’rif bo’yicha 0 matritsaning rangi 0 deb olinadi.

Matritsalarning rangini topish ko’p hollarda murakkab bo’ladi. Chunki unda bir qancha turli tartibdagi determinantlarni hisoblashga to'g’ri keladi.

Quyidagi elementar almashtirishlar natijasida matritsaning rangi o’zgarmaydi:

1) ikki qator elementlarini o’zaro almashtirish.
2) biror qatorni o’zgarmas songa ko’paytirish.
3) biror qatorga boshqa qatorni o’zgarmas songa ko’paytirib qo’shish.

Bu tasdiqlar determinantlarning xossalaridan kelib chiqadi.

Agar $(m\times n)$- tartibli A matritsaning $a_{11},\ a_{22},\ a_{33},...,a_{ss},\ \left(0\le s\le\min\left(m,n\right)\right)$ elementlarining har biri noldan farqli bo’lib, qolgan barcha elementlari nolga teng bo’lsa, u holda A diagonal ko’rinishdagi matritsa deyiladi. Ravshanki, bunday diagonal ko’rinishdagi matritsaning rangi s ga teng bo’ladi. Aynan shu va yuqorida aytilgan elementar almashtirishlardan foydalanib, matritsani rangini topishning ikkinchi usulini bayon qilamiz.

Bizga quyidagi A matritsa berilgan bo'lsin:

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

va uning rangini topish talab etilsin. Bu matritsaning rangini yuqorida aytilgan elementar almashtirishlar yordamida diagonal ko'rinishli matritsaga keltirib topamiz.

A matritsaning hech bo'lmaganda bitta elementi noldan farqli bo'lsin. Bu elementning satrlari va ustunlarini o'zaro almashtirish yordamida birinchi satr birinchi ustunga chiqaramiz va birinchi satr elementlarini shu elementga bo'lib, ushbu

$$\begin{pmatrix} 1 & a'_{12} & \dots & a'_{1n} \\ a'_{21} & a'_{22} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots \\ a'_{m1} & a'_{m2} & \dots & a'_{mn} \end{pmatrix} \qquad (15)$$

matritsani hosil qilamiz. (1) matritsaning birinchi ustunini $-a'_{12}$ ga ko'paytirib, ikkinchi ustunga qo'shsak, so'ng $-a'_{13}$ ga ko'paytirib uchunchi ustunga qo'shsak va h. k. Birinchi ustunni $-a'_{1n}$ ga ko'paytirib n-ustunga qo'shsak, natijada hosil bo'lgan matritsaning birinchi satrdagi elementi $a'_{11}=1$, qolgan elemetlari esa nollar bo'lib qoladi.

Xuddi shunga o'xshash (15) matritsaning birinchi ustundagi $a'_{11}=1$ dan boshqa elementlari nolga aylantiriladi. Natijada

$$A_1=\begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & a'_{22} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & a'_{m2} & \dots & a'_{mn} \end{pmatrix}$$

matritsaga ega bo'lamiz. Bundan $rang\, A = rang\, A_1$ kelib chiqadi.

A_1 matritsaga yuqoridagi elementar almashtirishlarni bir necha marta qo'llash natijasida u diagonal ko'rinshidagi matritsaga keladi. Bu diagonal ko'rinishli matritsaning rangi berilgan A matritsaning rangi bo'ladi.

Matritsalarning rangini aniqlang.

181. $\begin{pmatrix} 2 & 5 & 1 \\ 3 & 8 & 2 \\ 1 & 2 & 0 \end{pmatrix}$

182. $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 9 & 3 & 6 \end{pmatrix}$

183. $\begin{pmatrix} 2 & 1 & 4 & -3 & 7 \\ 4 & 15 & 8 & 7 & 1 \\ 2 & 17 & 4 & 13 & -9 \end{pmatrix}$

184. $\begin{pmatrix} 3 & 1 & 1 & 0 & -2 \\ 1 & 5 & 0 & 2 & -1 \\ 0 & 1 & 3 & 3 & -1 \end{pmatrix}$

185. $\begin{pmatrix} 3 & 4 & 3 \\ 1 & 3 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

186. $\begin{pmatrix} 2 & 1 & -3 \\ 1 & 5 & -2 \\ 4 & 11 & -7 \\ 1 & -1 & -1 \end{pmatrix}$

187*. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 6 & -3 & 3 \end{pmatrix}$

188. $\begin{pmatrix} 1 & 0 & 0 & -5 \\ -1 & -3 & 0 & 1 \\ -2 & -9 & 4 & -2 \\ 5 & 18 & -8 & -1 \end{pmatrix}$

189. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$

190. $\begin{pmatrix} -1 & 1 & 1 & -2 \\ 1 & 1 & 3 & 0 \\ 2 & -1 & 0 & 3 \\ 3 & 1 & 5 & 2 \end{pmatrix}$

Matritsalar rangini ko'rsatuvchi eng yuqori tartibli minorni aniqlang.

191. $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$

192. $\begin{pmatrix} 1 & 2 \\ 2 & 5 \\ -1 & 3 \end{pmatrix}$

193. $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

194. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

195. $\begin{pmatrix} -2 & -1 & 3 \\ 4 & 2 & -6 \\ 2 & 1 & -3 \end{pmatrix}$

196. $\begin{pmatrix} 1 & 3 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 \end{pmatrix}$

197. $\begin{pmatrix} 4 & 2 & 0 \\ 20 & 10 & -40 \\ 10 & -30 & 40 \end{pmatrix}$

198. $\begin{pmatrix} 1 & -2 & 4 & 0 \\ -1 & 3 & 5 & 1 \\ 2 & -1 & 4 & 0 \end{pmatrix}$

Matritsalar rangini eng yuqori tartibli minor yordamida aniqlang.

199. $\begin{pmatrix} 1 & -2 & 1 & 0 \\ 5 & 1 & 6 & 11 \\ 4 & 3 & 7 & 11 \end{pmatrix}$

200. $\begin{pmatrix} 1 & -1 & 1 & -1 \\ 2 & 0 & 1 & 3 \\ 3 & -1 & 2 & 2 \end{pmatrix}$

201*. $\begin{pmatrix} 1 & 2 & -3 & 4 \\ 2 & 4 & -6 & 1 \\ -4 & -8 & 12 & 1 \\ -1 & -2 & 3 & 6 \\ 3 & 6 & -9 & 12 \end{pmatrix}$

202. $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix}$

Elementar almashtirishlar yordamida matritsani rangini aniqlang.

203*. $\begin{pmatrix} -1 & 0 & 2 & 4 \\ 2 & 1 & 3 & -1 \\ 1 & 1 & 5 & 3 \\ -4 & -2 & -6 & 2 \\ 0 & 1 & 7 & 7 \end{pmatrix}$

204*. $\begin{pmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & -1 & 0 \\ 1 & 3 & 1 & 1 \\ 2 & 5 & 0 & 1 \end{pmatrix}$

205. $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 0 \\ 1 & 8 & 6 \end{pmatrix}$

206. $\begin{pmatrix} 0 & 1 & 2 & -1 \\ 0 & 2 & 4 & -2 \\ 1 & 3 & -2 & 4 \\ 1 & 6 & 4 & 1 \end{pmatrix}$

207. $\begin{pmatrix} 24 & 14 & 17 & 15 \\ 23 & 13 & 16 & 14 \\ 47 & 27 & 33 & 29 \\ 3 & 0 & 0 & 5 \end{pmatrix}$

208. γ ning qanday qiymatida $\begin{pmatrix} \gamma & 1 \\ 1 & 2 \end{pmatrix}$ matritsaning rangi $r=1$ ga teng bo'ladi.

A matritsa rangi $r=2$ ga teng bo'ladigan γ ning qiymatlarini toping.

209. $A=\begin{pmatrix} 1 & 3 & -4 \\ \gamma & 0 & 1 \\ 4 & 3 & -3 \end{pmatrix}$

210. $A=\begin{pmatrix} \gamma & 2 & 3 \\ 0 & \gamma-2 & 4 \\ 0 & 0 & 7 \end{pmatrix}$

211. $A=\begin{pmatrix} 2 & -1 & 4 & 0 \\ -4 & 2 & 0 & -\gamma \\ 8 & -4 & 8 & \gamma \\ 6 & -3 & 12 & \gamma \end{pmatrix}$

212. $A=\begin{pmatrix} \gamma & 0 & 1 \\ 3 & 4 & 1 \\ 1 & -1 & 2 \end{pmatrix}$

Elementar almashtirishlar yordamida matritsaning rangini aniqlang.

213*. $\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$

214*. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 86 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$

8-§. **Chiziqli tenglamalar sistemasini yechishning Gauss usuli**

n ta noma'lumli m ta tenglamalar sistemasini qaraymiz

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases} \tag{16}$$

Agar chiziqli tenglamalar sistemasi yechimga ega bo'lsa, u birgalikda, agar yechimga ega bo'lmasa, u birgalikda emas deyiladi. Quyidagi elementar almashtirishlar natijasida tenglamalar sistemasi o'ziga teng kuchli sistemaga almashadi:

1) Istalgan ikki tenglamani o’rinlarini almashtirilsa;
2) Tenglamalardan istalgan birini ikkala tomonini noldan farqli songa ko’paytirilsa;
3) Tenglamalardan birini istalgan haqiqiy songa ko’paytirib, boshqa tenglamaga qo’shilsa.

Agar $n > m$ bo’lsa, $n - m$ ta bir xil noma’lumli hadlarni tengliklarning o’ng tomoniga olib o’tib, o’ng tomonidagi noma’lumlarga ixtiyoriy qiymatlarni qabul qiladi deb, tenglamalar sistemasini $n = m$ holga keltirib olish mumkin. Shuni e’tiborga olib, (16) sistemani $n = m$ holi uchun yechamiz.

Gauss usulining mohiyati noma’lumlarni ikkinchi tenglamadan boshlab, ketma-ket yo’qotib oxirgi teglamada bitta no’malum qolguncha davom ettiriladi va oxirgi tenglamadan yuqoriga qarab noma’lumlarni ketma-ket topib, yechim hosil qilinadi.

1-qadam. (16) sistemada birinchi tenglamaning har ikki tomonini a_{11} ga bo’lib, teng kuchli ushbu sistemani hosil qilamiz:

$$\begin{cases} x_1 + \dfrac{a_{12}}{a_{11}} x_2 + \ldots + \dfrac{a_{1n}}{a_{11}} x_n = \dfrac{b_1}{a_{11}} \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \qquad (17)$$

Birinchi tenglamani a_{21} ga ko’paytirib ikkinchi tenglamadan, a_{31} ga ko’paytirib uchinchi tenglamadan va xokazo a_{n1} ga ko’paytirib, n-tenglamadan ayiramiz. Natijada yana berilgan sistemaga teng kuchli ushbu yangi sistemani hosil qilamiz:

$$\begin{cases} x_1 + a'_{12}x_2 + \ldots + a'_{1n}x_n = b'_1 \\ a'_{22}x_2 + \ldots + a'_{2n}x_n = b'_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a'_{n2}x_2 + \ldots + a'_{nn}x_n = b'_n \end{cases} \qquad (18)$$

Bu sistemada quyidagicha belgilashlar kiritilgan:

$$a'_{1k} = \frac{a_{1k}}{a_{11}},\ a'_{ik} = a_{ik} - \frac{a_{1k}}{a_{11}}a_{i1},\ b'_1 = \frac{b_1}{a_{11}},\ b'_i = b_i - \frac{b_1}{a_{11}}a_{i1,}\ \ i,k = 2,3,\ldots,n.$$

Agar (4) sistemada biror tenglama chap tomonidagi barcha koeffitsentlar nolga teng, o’ng tomoni esa noldan farqli bo’lsa, ya’ni

$$0x_1 + 0x_2 + ... + 0x_n = b_k \tag{19}$$

ko’rinishdagi tenglama hosil bo’lsa, sistema birgalikda emas bo’ladi va ishni shu yerda yakunlanadi.

Agar (19) ko’rinishdagi tenglama hosil bo’lmasa keyingi qadamga o’tiladi.

2-qadam. Ikkinchi tenglamani a'_{22} koefitsentga bo’lamiz, hosil bo’lgan sistemaning ikkinchi tenglamasini ketma-ket $a'_{32},...,a'_{n2}$ ga ko’paytirib uchinchi, to’rtinchi va hokazo tenglamalardan ayiramiz.

Biz bu jarayonni oxirgi tenglamada x_n noma’lum qolguncha davom ettirsak, dastlabki sistemaga teng kuchli

$$\begin{cases} x_1 + a'_{12}x_2 + ... + a'_{1n}x_n = b'_1 \\ x_2 + ... + a''_{2n}x_n = b''_2 \\ .... \\ x_{n-1} + a''_{n-1n}x_n = b''_{n-1} \\ x_n = b''_n \end{cases} \tag{20}$$

ko’rinishdagi sistemaga ega bo’lamiz. $x_n = b''_n$ qiymatini (n-1) tenglamaga qo’yib x_{n-1} ni topamiz va hokazo, bu ishni x_1 topilgunga qadar davom ettiramiz.

215. Quyidagi tenglamalar sistemasi yechilsin .

$$\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ 3x_1 + 2x_2 - 2x_3 = 1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

Yechish. Birinchi tenglamaning barcha hadlarini $x_{11} = 2$ ga bo’lib,

$$\begin{cases} x_1 + 0,5x_2 - 0,5x_3 = 0,5 \\ 3x_1 + 2x_2 - 2x_3 = 1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

sistemani hosil qilamiz. Birinchi tenglamani 3 ga ko’paytirib ikkinchi tenglamadan, so’ngra uchinchi tenglamadan birinchi tenglamani ayiramiz:

$$\begin{cases} x_1 + 0,5x_2 - 0,5x_3 = 0,5 \\ 0,5x_2 - 0,5x_3 = -0,5 \\ -1,5x_2 + 2,5x_3 = 4,5 \end{cases}$$

Ikkinchi tenglamani 0.5 ga bo'lib ,so'ngra uni -1.5 ga ko'paytirib , uni uchinchi tenglamadan ayiramiz.

Natijada

$$\begin{cases} x_1 + 0,5x_2 - 0,5x_3 = 0,5 \\ x_2 - x_3 = -1 \\ x_3 = 3 \end{cases}$$

hosil bo'ladi. Bundan ketma-ket $x_3 = 3,\ x_2 = -1 + 3 = 2,\ x_1 = 0,5 - 0,5x_2 + 0,5x_3 = 1$ larni topamiz. Shunday qilib, berilgan sistemaning yechimi $x_1 = 1,\ x_2 = 2,\ x_3 = 3$ dan iborat ekan.

216. $\begin{cases} x_1 + 2x_2 + 4x_3 - x_4 + 3x_5 = 7 \\ 2x_1 + x_3 + x_5 = 4 \\ x_2 + 2x_4 - x_5 = 6 \end{cases}$ sistema yechilsin.

Yechish. Bu sistemada uchta tenglama beshta noma'lum bo'lganligi uchun, x_4 va x_5 larni o'ng tomonga olib o'tamiz.

$$\begin{cases} x_1 + 2x_2 + 4x_3 = 7 + x_4 - 3x_5 \\ 2x_1 + x_3 = 4 - x_5 \\ x_2 = 6 - 2x_4 + x_5 \end{cases}$$

Misol uchun, $x_4 = 2,\ x_5 = 1$ qiymatlarni qo'ysak

$$\begin{cases} x_1 + 2x_2 + 4x_3 = 6 \\ 2x_1 + x_3 = 3 \\ x_2 = 3 \end{cases}$$

sistema hosil bo'ladi. $x_2 = 3$ ekanini e'tiborga olsak,

$$\begin{cases} x_1 + 4x_3 = 0 \\ 2x_1 + x_3 = 3 \end{cases}$$

sistemaga ega bo'lamiz. Birinchi tenglamani 2 ga ko'paytirib, undan ikkinchi tenglamani ayirsak

$$\begin{cases} x_1 + 4x_3 = 0 \\ 7x_3 = -3 \end{cases}$$

hosil bo'ladi. Bundan $x_3 = -\dfrac{3}{7},\ x_2 = 3,\ x_1 = \dfrac{12}{7}.$

Bu sistemada x_4 va x_5 noma'lumlarga boshqa qiymatlar berib, yangi yechim hosil qilish mumkin ekanini, boshqacha aytganda $n > m$ bo'lganda yechim yagona bo'lmay cheksiz ko'p bo'lishini eslatib o'tamiz.

Tenglamalar sistemasini birgalikda bo'lish-bo'lmasligini, uni yechmasdan turib aniqlash usuli bilan tanishamiz.

(16) tenglamalar sistemasini koefitsientlaridan tuzilgan $n = m$ tartibli hamda $m \times (n+1)-$ tartibli kengaytirilgan

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, \quad A' = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m1} & \dots & a_{m1} & b_m \end{pmatrix}$$

matritsalarni tuzib olamiz.

Teorema. (Kroneker-Kapelli teoremasi)

(16) tenglamalar sistemasi birgalikda bo'lishi uchun A va A' matritsalarning ranglari teng bo'lishi, ya'ni $rang\,A = rang\,A'$ bo'lish zarur va yetarli.

Keltirilgan teoremadan quyidagi xulosalar kelib chiqadi:

1. Agar $rang\,A > rang\,A'$ bo'lsa, (16) sistema yechimga ega bo'lmaydi.
2. Agar $rang\,A = rang\,A' = k$ bo'lsa, (16) sistema yechimga ega bo'lib,
   a) $k < n$ bo'lganda, tenglama cheksiz ko'p yechimga ega bo'ladi;

   b) $k = n$ bo'lsa, sistema yagona yechimga ega bo'ladi.

217. $\begin{cases} 7x_1 + 3x_2 = 2 \\ x_1 - 2x_2 = 3 \\ 4x_1 + 9x_2 = 11 \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish. Bu yerda $n=2, m=3$, ya'ni $m>n$.

$$A=\begin{pmatrix}7 & 3\\ 1 & -2\\ 4 & 9\end{pmatrix},\quad A'=\begin{pmatrix}7 & 3 & 2\\ 1 & -2 & -3\\ 4 & 9 & 11\end{pmatrix},$$

$rangA=2$ chunki

$$\begin{vmatrix}7 & 3\\ 1 & -2\end{vmatrix}=-17\neq 0,\qquad \begin{vmatrix}7 & 3 & 2\\ 1 & -2 & -3\\ 4 & 9 & 11\end{vmatrix}=0$$

bo'lishini e'tiborga olsak $rangA'=2$, demak bu sistemaning yechimi mavjud. Berilgan sistemaning birinchi ikki tenglamasini birgalikda yechsak

$x_1=-\dfrac{5}{17},\ x_2=\dfrac{23}{17}$, kelib chiqadi. Bu sonlar uchinchi tenglamani ham qanoatlantiradi.

$$4x_1+9x_2=4\left(-\frac{5}{17}\right)+9\cdot\frac{23}{17}=11.$$

Demak, $\left(-\dfrac{5}{17};\dfrac{23}{17}\right)$ sistemaning yechimi bo'ladi.

Chiziqli tenglamalar sistemasini Gauss usulida yeching.

218. $\begin{cases}x_1+2x_2-x_3=3\\ 2x_1+5x_2-6x_3=1\\ 3x_1+8x_2-10x_3=1\end{cases}$

220. $\begin{cases}4x_1+2x_2-x_3=1\\ 5x_1+3x_2-2x_3=2\\ 3x_1+2x_2-3x_3=0\end{cases}$

219. $\begin{cases}x_1-x_2+3x_3=7\\ 2x_1+x_2-x_3=-3.\\ 3x_1+x_2-3x_3=1\end{cases}$

221. $\begin{cases}5x_1+2x_2+5x_3=4\\ 3x_1+5x_2-3x_3=-1\\ -2x_1-4x_2+3x_3=1\end{cases}$

Tenglamalar sistemasini birgalikda yoki birgalikda emasligini tekshiring.

222. $\begin{cases}x_1+2x_2-3x_3+x_4=1\\ 2x_1-x_2+2x_4=3\\ 5x_2-6x_3+4x_4=5\end{cases}$

223. $\begin{cases}3x_1+x_2+5x_3+x_4=2\\ -x_1+4x_2+x_3+3x_4=1\\ -5x_1+7x_2-3x_3+5x_4=2\end{cases}$

224*. $\begin{cases} 4x_1 + x_2 + 3x_3 + 2x_4 + x_5 = 3 \\ -2x_1 + x_2 + 2x_4 + 3x_5 = 0 \\ x_1 - 4x_2 + 3x_3 + x_4 = 2 \\ 3x_1 - 2x_2 + 6x_3 + 5x_4 + 4x_5 = 4 \end{cases}$

a parametrning qanday qiymatlarida tenglamalar sistemasi birgalikda bo'ladi.

225. $\begin{cases} x_1 - x_2 + 2x_3 = 3 \\ 2x_1 + 5x_3 = 7 \\ x_1 + x_2 + (a+9)x_3 = 6 \end{cases}$

226. $\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ 2x_1 + 6x_2 - 5x_3 = 7 \\ x_1 + 6x_2 - 7x_3 = a + 3 \end{cases}$

227. $\begin{cases} x_1 + 3x_2 - 2x_3 = 1 \\ 2x_1 + x_2 - 4x_3 = 4 \\ x_1 + 8x_2 + (a-7)x_3 = 5 \end{cases}$

228. $\begin{cases} 2x_1 - x_3 + 3x_4 = 10 \\ 3x_1 - x_2 + 2x_3 + x_4 = 8 \\ 8x_1 - 2x_2 + 3x_3 + 4x_4 = 18 \\ 3x_1 - x_2 + 2x_3 = a \end{cases}$

229. $\begin{cases} x_1 + 3x_2 - 2x_3 + 4x_4 = 1 \\ -x_1 + 2x_2 + x_3 + x_4 = 2 \\ 3x_1 + 2x_2 = 1 \\ 3x_1 + 5x_2 + x_3 + ax_4 = 5 \end{cases}$

230. $\begin{cases} x_1 + 5x_2 - 2x_3 + 3x_4 = 2 \\ 4x_1 + x_2 - x_3 - 3x_4 = 2 \\ -11x_1 + 2x_2 + x_3 + 12x_4 = a \end{cases}$

231. $\begin{cases} -x_1 - 4x_2 + 2x_3 + 3x_4 = 3 \\ -4x_1 + 3x_2 + 2x_3 + x_4 = -2 \\ x_1 - 15x_2 + 4x_3 + 8x_4 = a \end{cases}$

232. $\begin{cases} x_1 - 3x_2 + 4x_3 + 2x_4 = 5 \\ -3x_1 + 2x_2 + 3x_3 + x_4 = -4 \\ 8x_1 - 10x_2 + 2x_3 + 2x_4 = a \end{cases}$

233. $\begin{cases} -x_1 + 3x_2 - 2x_3 + 4x_4 = 5 \\ 2x_1 + 3x_2 - 4x_3 + 3x_4 = -4 \\ 4x_1 + 15x_2 - 16x_3 + 17x_4 = a \end{cases}$

Kroneker-Kapelli teoremasi yordamida tekshiring va birgalikda bo'lganlarini yeching.

234. $\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1 \\ 3x_1 - x_2 + 2x_4 = 0 \\ x_1 - 2x_2 + x_3 + x_4 = 2 \end{cases}$

235. $\begin{cases} x_1 - x_2 + 3x_3 = 1 \\ 4x_1 + x_2 + 4x_3 = 4 \\ 2x_1 + 3x_2 - 2x_3 = 2 \end{cases}$.

236. $\begin{cases} 2x_1 + 3x_2 - x_3 = 4 \\ x_1 - 2x_2 + 2x_3 = 1 \\ 3x_1 + x_2 + 3x_3 = 7 \\ 2x_1 - x_2 + 2x_3 = 2 \end{cases}$

237. $\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ 2x_1 + 3x_2 - x_3 = 1 \\ 3x_1 + 5x_2 + 2x_3 = 3 \end{cases}$

238. $\begin{cases} x_1+3x_2-x_3=3 \\ 2x_1+x_2+2x_3=5 \\ 3x_1+4x_2-7x_3=0 \\ x_1-x_2+3x_3=3 \end{cases}$

239. $\begin{cases} x_1+2x_2+x_3=-1 \\ 2x_1+3x_2+4x_3=3 \\ 3x_1+5x_2+5x_3=0 \end{cases}$

240. $\begin{cases} x_1-2x_2+3x_3+4x_4=-2 \\ 2x_1-4x_2+x_3-x_4=3 \end{cases}$

241. $\begin{cases} x_1-3x_2+4x_3-4x_4=2 \\ 2x_1+3x_2+x_3+5x_4=1 \\ 3x_1+5x_3+4x_4=10 \end{cases}$

242. $\begin{cases} x_1-5x_2+3x_3-x_4=1 \\ 2x_1-10x_2+3x_4=0 \\ 4x_1-20x_2+6x_3+x_4=2 \end{cases}$

243.
$\begin{cases} x_1+2x_2-x_3+4x_4+x_5=1 \\ 2x_1-3x_2+2x_3+x_4-x_5=3 \end{cases}$

9-§. n ta noma'lumli chiziqli tenglamalar sistemasi.

$$\begin{cases} a_{11}x_1+a_{12}x_2+...+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+...+a_{2n}x_n=b_2 \\ .... \\ a_{n1}x_1+a_{n2}x_2+...+a_{nn}x_n=b_n \end{cases} \qquad (21)$$

(21) tenglamalar sistemasi n **ta noma'lumli chiziqli tenglamar sistemasi** deyiladi.

(21) tenglamalar sistemasini yechishning Kramer va matritsalar usullarini ko'ramiz. Noma'lumlar oldidagi koeffitsientlardan asosiy determinantni

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{vmatrix}$$

va uch noma'lumli chiziqli tenglamalar sistemasidagiga o'xshash yordamchi $\Delta x_1, \Delta x_1, ..., \Delta x_n$ determinantlarni tuzamiz. Ular yuqori tartibli determinantlarni hisoblash usuli bilan hisoblanadi. Bu yerda ham agar $\Delta \neq 0$ bo'lsa, Kramer formulasiga asosan

$$x_1=\frac{\Delta x_1}{\Delta}, \quad x_2=\frac{\Delta x_2}{\Delta}, \quad x_n=\frac{\Delta x_n}{\Delta}.$$

yechimni hosil qilamiz. Bu yechim yagona yechim bo'ladi. Agar (21) sistemaning koeffitsientlari va noma'lumlaridan A, B va X matritsalar

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix},\quad B=\begin{pmatrix} b_1 \\ b_2 \\ ... \\ b_n \end{pmatrix},\quad X=\begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix}$$

ko'rinishida tuzilsa (21) sistemasini $AX=B$ holda yozish mumkin.

Agar $\Delta \neq 0$ bo'lsa, A ga teskari A^{-1} matritsa mavjud bo'ladi va berilgan sistemani matritsalar ko'rinishidagi yechimi $X=A^{-1}B$ bo'ladi.

Agar $\Delta = 0$ bo'lib, $\Delta x_1, \Delta x_2, ..., \Delta x_n$ lardan aqalli birortasi noldan farqli bo'lsa, (21) sistema yechimga ega bo'lmaydi.

Agar $\Delta = 0$ bo'lib, $\Delta x_1 = \Delta x_2 = ... = \Delta x_n = 0$ bo'lsa, (21) sistema cheksiz ko'p yechimga ega bo'ladi.

244. $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish. Sistemani Kramer usulida yechamiz.

$$\Delta = \begin{vmatrix} 2 & 1 & -5 & 1 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 1 & 4 & -7 & 6 \end{vmatrix} = 2\cdot\begin{vmatrix} -3 & 0 & -6 \\ 2 & -1 & 2 \\ 4 & -7 & 6 \end{vmatrix} - \begin{vmatrix} 1 & -5 & 1 \\ 2 & -1 & 2 \\ 4 & -7 & 6 \end{vmatrix} - \begin{vmatrix} 1 & -5 & 1 \\ -3 & 0 & -6 \\ 2 & -1 & 2 \end{vmatrix} = 27$$

Xuddi shu usul bilan hisoblashni davom ettirib, quyidagilarni topamiz:

$\Delta x_1 = 81,\ \Delta x_2 = -108,\ \Delta x_3 = -27,\ \Delta x_4 = 27$.

Demak, sistema yagona yechimga ega, chunki $\Delta \neq 0$. Bu yechim esa

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{81}{27} = 3,\quad x_2 = \frac{\Delta x_2}{\Delta} = \frac{-108}{27} = -4,\quad x_3 = \frac{\Delta x_3}{\Delta} = \frac{-27}{27} = -1,\quad x_4 = \frac{\Delta x_4}{\Delta} = \frac{27}{27} = 1$$

.

(3;-4;-1;1) bo'ladi.

(21) tenglamalar sistemasini $b_1 = b_2 = ... = b_n = 0$ bo'lgan holini ko'ramiz.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0 \\ .... \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = 0 \end{cases} \qquad (22)$$

(22) tenglamalar sistemasini bir jinsli, chiziqli tenglamalar sistemasi deyiladi .

Osonlik bilan ishonch hosil qilish mumkinki, $x_1 = x_2 = ... = x_n = 0$ (22) sistemaning yechimlari bo'ladi va bu yechimni trivial yechim deb ataladi. Agar (22) bir jinsli sistemanig asosiy determinanti Δ noldan farqli bo'lsa, bu sistema faqat trivial yechimga ega bo'ladi. Chunki bu holda $\Delta x_1 = \Delta x_2 = ... = \Delta x_n = 0$ va Kramer formulasiga asosan $x_1 = x_2 = ... = x_n = 0$ bo'ladi .

Demak, (22) sistemaning notrivial yani noldan farqli yechimi mavjud bo'lishi uchun $\Delta = 0$ bo'lishi zarur ekan.

245. $\begin{cases} -x_1 + x_2 = 0 \\ x_1 - x_2 = 0 \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish. $x_1 = x_2 = 0$ trivial yechim ekani ravshan.

$$\Delta = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1 - 1 = 0$$

bundan ko'rinadiki, sistemaning notrivial yechimi bo'lishi mumkin. Haqiqatdan ham $x_1 = x_2 = t$ (t -ixtiyoriy haqiqiy son) sistemaning notrivial yechimi bo'ladi.

Chiziqli tenglamalar sistemasini yeching.

246. $\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 + 4x_3 = 0 \\ 3x_1 + 4x_2 + 5x_3 = 0 \end{cases}$

247. $\begin{cases} x_1 + x_2 + x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \\ 3x_1 - x_2 - x_3 = 0 \end{cases}$

248. $\begin{cases} 6x_1 - 8x_2 + 2x_3 + 3x_4 = 0 \\ 3x_1 - 4x_2 + x_3 - x_4 = 0 \end{cases}$

249. $\begin{cases} x_1 - 2x_2 + 3x_3 - x_4 = 0 \\ x_1 + x_2 - x_3 + 2x_4 = 0 \\ 4x_1 - 5x_2 + 8x_3 + x_4 = 0 \end{cases}$

250. $\begin{cases} x_1 + 4x_2 - 7x_3 = 0 \\ 3x_1 - 2x_2 + x_3 = 0 \\ 2x_1 + x_2 - 3x_3 = 0 \end{cases}$

251. $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ 3x_1 - x_2 + 2x_3 + x_4 = 0 \\ 5x_1 - x_2 - x_4 = 0 \end{cases}$

252. $\begin{cases} x_1 + x_2 + x_3 = 0 \\ 5x_1 - x_2 - x_3 = 0 \\ 3x_1 + x_2 + x_3 = 0 \\ 2x_1 + 3x_2 + 2x_3 = 0 \end{cases}$

253. $\begin{cases} x_1 + x_2 + 3x_3 + x_4 = 0 \\ 2x_1 - x_2 + x_3 + 3x_4 = 0 \\ 4x_1 + x_2 + 7x_3 + 5x_4 = 0 \\ 5x_1 - x_2 + 5x_3 + 7x_4 = 0 \end{cases}$

254. $\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0, \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 = 0, \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0. \end{cases}$

255*. $\begin{cases} 2x_1 - x_2 + 3x_3 - 2x_4 + 4x_5 = 0, \\ 4x_1 - 2x_2 + 5x_3 + x_4 + 7x_5 = 0, \\ 2x_1 - x_2 + x_3 + 8x_4 + 2x_5 = 0. \end{cases}$

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I BOB

4.-7 **5.**14 **6.**-0,5 **7.** $4\sqrt{ab}$ **8.**1 **9.**0 **10.** 0 **11.** 2 **12.** 0;3 **13.** -2 **14.** -3;3

15. $\frac{\pi}{2}+\frac{n\pi}{3}$ **16.** (-2;1) **17.** (1;5) **18.** 40 **19.** -24 **20.** -23 **21.** 14 **22.** 1

23. $a^2(a-2)$ **24.** $4x$ **25.** 0 **26.** $\Delta=(y-x)(z-x)(z-y)$

27. $\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)$ **28.**(2;3) **29.** (-4;1;2) **35.** 8x+15y+12z-19t

36. 3a-b+2c+d **37.** -48 **38.**40 **39.**-30 **40.**18 **41.**-36 **42.**-40 **43.**-150 **44.**-10 **45.** 5

46. -720 **47.**a=-1 **48.** a=-9 **49.**a=5 **50.**a=2 **51.**900 **52.**12 **53.**39520 **54.** a^2b^2

55.-2(n-2)! **62.** $x_1=-1\ x_2=3\ x_3=2$ **63.** $x_1=2\ x_2=1\ x_3=2$ **64.** $x_1=-1\ x_2=3\ x_3=1$**65.** $x_1=-7\ x_2=7\ x_3=5$ **66.** (1;2;3) **67.** $\Delta=0,\ \Delta_1\neq 0$
68.cheksiz ko'p **69.**(1;1;1) **70.**(2;-1;0) **71.**(1;2;3) **72.** $x_1=x_2=1;\quad x_3=x_4=-1$
73. $x_1=1;\ x_2=x_3=2;\ x_4=0$ **74**. x=-3, y=0, z=-0.5 t=2/3 **75.** (-1;0;1)

76. (2;-1;-3) **77.** (1;-1;2) **78.**(1;0;2) **83.**C= $\begin{pmatrix}-7 & 2 & 4\\ -12 & -1 & -7\\ 5 & 2 & -8\end{pmatrix}$

84.C= $\begin{pmatrix}7 & -4 & -17 & 6\\ 4 & -9 & -4 & -1\\ 1 & -1 & -10 & 0\end{pmatrix}$ **85.** $\begin{pmatrix}-1 & 8\\ 0 & -9\end{pmatrix}$ **86.** $\begin{pmatrix}3 & -9 & -13\\ 8 & -5 & 13\end{pmatrix}$

87. $\begin{pmatrix}4 & 7 & 11\\ 4 & 2 & -2\\ 3 & 3 & 3\end{pmatrix}$ **88.** $\begin{pmatrix}2 & 0\\ 4 & -1\\ 5 & 3\end{pmatrix}$ **89**. $\begin{pmatrix}1 & -1\\ 0 & 1\\ -1 & 0\end{pmatrix}$ **90.** $\begin{pmatrix}5 & 0 & 4\\ 10 & 10 & 33\\ -11 & -7 & 25\end{pmatrix}$ **91**.(6 7)

92. $\begin{pmatrix}22 & 11\\ -19 & -1\end{pmatrix}$ **93**. $\begin{pmatrix}9 & -5 & -9\\ 25 & 15 & -12\end{pmatrix}$ **94**. $\begin{pmatrix}14 & 22\\ 17 & -9\\ 16 & 8\end{pmatrix}$ **95**. $\begin{pmatrix}-6 & -2\\ 13 & -4\\ 21 & 1\end{pmatrix}$ **96**. $\begin{pmatrix}-1 & 15\\ 17 & 13\end{pmatrix}$

97. $\begin{pmatrix}11 & -14 & 10\\ 7 & -6 & 7\end{pmatrix}$ **98.** $\begin{pmatrix}6 & 7 & 3\\ 0 & -8 & 6\\ 0 & -6 & 6\end{pmatrix}$ **99**. (-12) **100.** $\begin{pmatrix}2 & 4 & -6\\ -1 & -2 & 3\\ 4 & 8 & -12\end{pmatrix}$

101. $\begin{pmatrix} 31 \\ -14 \\ 18 \end{pmatrix}$ **102.** (-6 -17 -1). **103.** $\begin{pmatrix} -6 & 1 & 3 \\ 6 & 2 & 9 \\ -12 & -3 & 14 \end{pmatrix}$

104. $\begin{pmatrix} 8 \\ 19 \end{pmatrix}$ **105.** $\begin{pmatrix} 6 & 10 \\ 6 & 5 \\ 2 & 3 \end{pmatrix}$ **106.** $\begin{pmatrix} -9 & -16 & -3 \\ 19 & 21 & 17 \end{pmatrix}$ **107.** $\begin{pmatrix} 8 & 0 & 7 \\ 16 & 10 & 4 \\ 13 & 5 & 7 \end{pmatrix}$; $\begin{pmatrix} 4 & 6 & 6 \\ 1 & 7 & 3 \\ 8 & 11 & 14 \end{pmatrix}$

108. X= $\begin{pmatrix} 1 & 8 & 14 \\ -7 & 1 & 8 \\ 8 & 6 & 5 \end{pmatrix}$ **109.** X= $\frac{1}{3} \cdot \begin{pmatrix} -30 & -6 & -2 & -25 \\ -13 & 13 & -22 & 16 \\ -9 & -5 & -3 & -1 \end{pmatrix}$ **110.** $A^3=\begin{pmatrix} 13 & -14 \\ 21 & -22 \end{pmatrix}$

111. $A^5=\begin{pmatrix} 304 & -61 \\ 305 & -62 \end{pmatrix}$ **112.** $A^{10}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A^{15}=\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$ **113.** $A^{10}=\begin{pmatrix} 1 & 10 \\ 0 & 1 \end{pmatrix}$

114. $A=\begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$ **115.** $\begin{pmatrix} 11 & 14 \\ 7 & 18 \end{pmatrix}$ **116.** $\begin{pmatrix} 28 & 15 & 16 \\ 19 & 36 & 15 \\ 30 & 19 & 28 \end{pmatrix}$ **117.** $\begin{pmatrix} 9 & 7 \\ 2 & 9 \end{pmatrix}$

118. $\begin{pmatrix} 1 & 0 & 10 \\ 6 & -3 & 15 \\ 34 & 0 & 82 \end{pmatrix}$ **119.** $\begin{pmatrix} 73 & 25 \\ 1 & -11 \end{pmatrix}$ **120.** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **121.** $f(A)=\begin{pmatrix} 2 & 8 \\ 4 & 6 \end{pmatrix}$ **122.** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

123. $\begin{pmatrix} 5 & 1 & 3 \\ 8 & 0 & 3 \\ -2 & 1 & -2 \end{pmatrix}$ **124.** $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ **125.** $\begin{pmatrix} 16 & -10 \\ 0 & -6 \end{pmatrix}$ **126.** $\begin{pmatrix} 0 & 6 & -2 \\ 0 & -2 & 4 \\ 0 & -4 & -2 \end{pmatrix}$ **127.** $\begin{pmatrix} -9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -5 \end{pmatrix}$

128. $\begin{pmatrix} -18 & 40 \\ 0 & 62 \end{pmatrix}$ **129.** $\begin{pmatrix} -8 & -6 \\ -6 & -20 \end{pmatrix}$ **130.** $\begin{pmatrix} 3 & 2 & 2 \\ 2 & -1 & -2 \\ -2 & 0 & -3 \end{pmatrix}$ **131.** X= $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, B=A*X

132. X= $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B=X*A **133.** X= $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B=X*A **134.** X= $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B=X*A

136. $\frac{1}{3}\begin{pmatrix} 9 & -6 \\ -4 & 3 \end{pmatrix}$ **137.** $\frac{1}{2}\begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$ **138.** $\frac{1}{2}\begin{pmatrix} 5 & -3 \\ -6 & 4 \end{pmatrix}$ **139.** $\frac{1}{4}\begin{pmatrix} 8 & -4 \\ 5 & -3 \end{pmatrix}$

140. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$ **141.** $\begin{pmatrix} -1 & 1 & 0 \\ 1 & -5 & 3 \\ 0 & 3 & -2 \end{pmatrix}$ **142.** $\frac{1}{3}\begin{pmatrix} 1 & -4 & -3 \\ 1 & -7 & -3 \\ -1 & 10 & 6 \end{pmatrix}$

143. $\frac{1}{5}\begin{pmatrix} 1 & 1 & 0 \\ -3 & 12 & 10 \\ -1 & 4 & 5 \end{pmatrix}$ **144.** $\begin{pmatrix} -2 & 3 & -1 \\ -1{,}5 & 2{,}5 & -1 \\ 9 & -13 & 5 \end{pmatrix}$ **145.** $\begin{pmatrix} -10 & 3 & 8 \\ -11 & 3 & 9 \\ 14 & -4 & -11 \end{pmatrix}$

146. $\begin{pmatrix}-2 & 5\\ -1 & 3\end{pmatrix}$ **147.** $\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ -\frac{2}{5} & \frac{1}{5}\end{pmatrix}$ **148.**mavjud emas **149.** $\begin{pmatrix}-\frac{1}{5} & \frac{4}{15}\\ 0 & \frac{1}{3}\end{pmatrix}$

150. $\frac{1}{18}\begin{pmatrix}7 & 5 & 1\\ -8 & 2 & 4\\ 3 & -3 & 3\end{pmatrix}$ **151**. $\frac{1}{38}\begin{pmatrix}8 & -2 & 4\\ 9 & -7 & -5\\ 5 & 13 & -7\end{pmatrix}$ **152.** $\frac{1}{60}\begin{pmatrix}12 & 6 & 6\\ -18 & 11 & 1\\ -18 & 1 & 11\end{pmatrix}$

153. $\frac{1}{6}\begin{pmatrix}-6 & -2 & 4\\ 0 & -4 & 2\\ 6 & 3 & -3\end{pmatrix}$ **154.** $\frac{1}{5}\begin{pmatrix}1 & 3 & 2\\ -3 & 1 & 1\\ 1 & -2 & 3\end{pmatrix}$ **155.** $\begin{pmatrix}0 & 0 & 0 & 12\\ 0 & 0 & 6 & 0\\ 0 & 4 & 0 & 0\\ 3 & 0 & 0 & 0\end{pmatrix}$ **156.** $\begin{pmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0\\ 0 & 0 & 1 & 1\end{pmatrix}$

157. $\begin{pmatrix}1 & 3 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & -1 & 2\end{pmatrix}$ **158.** a=-2, b=2, c=4 **159.**a=-1, b=3, c=2

160.a=3, b=-1, c=-3 **161.**a=1, b=9, c=-4 **162.** a=-5, b=-2, c=3

163. a=5, b=5, c=-3 **164.** a≠-3 **165.** a≠1, a≠4 **166.** a ning bunday qiymati mavjud emas **167.** har qanday a larda **169.**(1, 2, -1) **170.** (1, 0, -1) **171.**(3, 7, -1) **172.**(5, -11, -13) **173.** (1, 1, 1) **174.** (1, 0, -1) **175.** (1;1;1) **176.**(2;-1;0) **177.**(1;2;3) **178.** $x_1 = x_2 = 1;\ \ x_3 = x_4 = -1$ **181.** 2 **182.** 1 **183.** 2 **184.** 3 **185.** 3 **186.** 2 **187.** 3 **188.** 3. **189.**3 **190.** 2 **191.**r=2, M=$\begin{vmatrix}2 & 0\\ 0 & -1\end{vmatrix}$, **192**.r=2, M=$\begin{vmatrix}1 & 2\\ 2 & 5\end{vmatrix}$

193. r=1, M=$|2|$ **194.** r=2, M=$\begin{vmatrix}1 & 0\\ 0 & 2\end{vmatrix}$ **195.** r=1, M=$|4|$, **196.** r=3, M=$\begin{vmatrix}1 & 3 & 0\\ 2 & 4 & 0\\ 0 & 0 & 2\end{vmatrix}$ **197.** r=3, M=$\begin{vmatrix}4 & 2 & 0\\ 20 & 10 & -40\\ 10 & -30 & 40\end{vmatrix}$ **198.** r=3, M=$\begin{vmatrix}1 & -2 & 0\\ -1 & 3 & 1\\ 2 & -1 & 0\end{vmatrix}$ **199.** r=2,

200. r=2, **201**.r=3 **202.** r=5 **203.** r=2 **204.** r=3, **205.** r=2, **206.** r=2, **207.** r=3

208. γ=0,5 **209.** $\gamma = 3$ **210.** $\gamma_1 = 0, \gamma_2 = 2$ **211.** har qanday γ **212**. $\gamma = \frac{7}{9}$

213. r=3 **214.** r=2 **218.** $x_1 = -1, x_2 = 3, x_3 = 2$, **219.** $x_1 = 2, x_2 = 1, x_3 = 2$, **220.** $x_1 = -1, x_2 = 3, x_3 = 1$, **221.** $x_1 = -7, x_2 = 7, x_3 = 5$. **222.**birgalikda **223.** birgalikda emas **224.**birgalikda emas. **225.** a≠-6, **226.** a=2. **227.** a≠5 **228.** a=0 **229.** a≠5 **230.** a=-4 **231.** a=11 **232**. a=18 **233.** a=-2 **234.** birgalikda emas **235.** $\left(\frac{5-7x_3}{5}, \frac{8x_3}{5}, x_3\right)$, $x_3 \in R$, **236.** (1,1,1) **237.** (11x_3-4.3-7x_3, x_3), $x_3 \in R$ **238.** (1,1,1) **239.** birgalikda emas **240.** (x_1,(5x_1-4x_4-11)/10), (-7-3x_4)/5,

x_4), $x_1, x_4 \in R$ **241.** birgalikda emas **242.** $(x_1, x_2, (3 - 5x_1 + 25x_2)/9)$, $(10x_2\text{-}2x_1)/3)$, $x_1, x_2 \in R$ **243.** $((9\text{-}x_3\text{-}14x_4\text{-}x_5)/7, ((\text{-}1\text{+}4x_3\text{-}x_4\text{-}3x_5)/7$, $x_3, x_4, x_5))$, где $x_3, x_4, x_5 \in R$ **246.** $(c, -2c, c), c \in R$ **247.** $(0,0,0)$

248. $\left(\frac{4c_1 - c_2}{3}, c_1, c_2, 0\right)$, $c_1, c_2 \in R$ **249.** $\left(-\frac{1}{4}c, c, \frac{3}{4}c, 0\right)$,c$\in R$ **250.** $(5c, c, 7c)$,c$\in R$ **251.** $\left(-\frac{1}{4}c_1, \frac{5}{4}c_1 + c_2, c_1, c_2\right)$, $c_1, c_2 \in R$ **252.** $(0,0,0)$ **253**. $\left(-\frac{4}{5}(c_1 + c_2, \frac{1}{3}(c_2 - 5c_1), c_1, c_2\right)$, $c_1, c_2 \in R$ **254.** $x_3 = -\frac{5}{2}x_1 + 5x_2,\ x_4 = \frac{7}{2}x_1 - 7x_2$.

255. $x_2 = x_1 - 13x_4 + x_5,\ x_3 = 5x_4 - x_5$.

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