V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

ARALOVA KAMOLA AKBAR QIZI

NOVOLTERRA KVADRATIK STOXASTIK OPERATORLARNING SUPERPOZITSIYASI DINAMIKASI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT - 2024 yil

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

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Ilmiy rahbar:	**Jamilov Uyg‘un Umurovich** fizika-matematika fanlari doktori
Rasmiy opponentlar:	**G‘anixo‘jayev Nosir Nabiyevich** fizika-matematika fanlari doktori, professor
	Xamrayev Axror Yusupovich fizika-matematika fanlari nomzodi, dotsent
Yetakchi tashkilot:	**O‘zbekiston Milliy universiteti**

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U.A. Roziqov
Ilmiy darajalar beruvchi
Ilmiy kengash raisi,
f.-m.f.d., akademik

J.K. Adashev
Ilmiy darajalar beruvchi
Ilmiy kengash ilmiy kotibi,
f.-m.f.d., katta ilmiy xodim

A.A.Raximov
Ilmiy darajalar beruvchi
Ilmiy kengash huzuridagi
Ilmiy seminar rais o‘rinbosari,
f.-m.f.d., professor

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Dunyo miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar aksariyat hollarda nochiziqli dinamik sistemalarni tadqiq qilish masalalariga keltiriladi. Dinamik sistemalar nazariyasining ahamiyati sistemaning dastlabki ma’lum holatini bilgan holda kelajakda qanday holatlar kelib chiqishini baholashdan iborat bo‘lib, u fizika, biologiya, tibbiyot, iqtisodiyot sohalaridagi murakkab jarayonlarni tushunish va optimal qarorlar qabul qilishda muhim ahamiyat kasb etadi. Shu sababli, diskret vaqtli nochiziqli dinamik sistemalarni, xususan, matematik biologiyada keng qo‘llanuvchi kvadratik stoxastik operatorlar va ularning superpozitsiyalari dinamikasini tadqiq etish dinamik sistemalar nazariyasidagi muhim va dolzarb vazifalardan biri bo‘lib qolmoqda.

Hozirgi vaqtda dunyo bo‘ylab ko‘plab amaliy masalalarning xarakterini tushunishda, tahlil qilishda hamda optimal yechimini topishda asosiy vosita sifatida qo‘llaniladigan nochiziqli dinamik sistemalarga doir muntazam ilmiy izlanishlar olib borilmoqda. Shu jumladan, berilgan dinamik sistema uchun mazkur sistemaning invariant to‘plamlarini aniqlash, barcha davriy nuqtalarni topish va ularning tipini aniqlash hamda eng muhim bo‘lgan, berilgan ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalar to‘plamini tavsiflash kabi masalalarni o‘rganishga alohida e’tibor qaratilmoqda.

Mamlakatimizda so‘nggi yillarda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan tibbiyot, biologiya, matematika va fizika fanlariga e’tibor kuchaytirildi. Jumladan, biologiya, tibbiyot, mexanika, elektronika, iqtisodiyot va optimal boshqaruv nazariyalarida keng tatbiqiga ega bo‘lgan nochiziqli dinamik sistemalar nazariyasini rivojlantirishga alohida e’tibor berildi va bu borada salmoqli natijalarga erishildi. “Algebra, funksional analiz va dinamik tizimlar nazariyasi” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi[1]. Bu qaror ijrosini ta’minlashda ilmiy natijalarni fanning turdosh sohalarida qo‘llash maqsadida diskret vaqtli dinamik sistemalar nazariyasini rivojlantirish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim

[1] O’zbekiston Respublikasi Vazirlar mahkamasining 2017-yil 18-maydagi “O’zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqotlar muassasalari faoliyatini tashkil etish to’g‘risida”gi 292-sonli qarori.

sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. So‘nggi yillarda biologik yoki fizik sistemalardagi tabiiy jarayonlarni tushunishda nochiziqli dinamik sistemalarni matematik modellar sifatida qo‘llash va ularni tadqiq etishga bo‘lgan qiziqish ortib bormoqda. Nochiziqli dinamik sistemalarning eng sodda holi bo‘lgan, kvadratik stoxastik operatorlar yordamida hosil qilingan diskret vaqtli dinamik sistemalar populyatsiya dinamikasi, fizika, iqtisodiyot kabi turli sohalardagi tadqiqotlarning muhim obyektlaridan biri bo‘lib kelmoqda. Kvadratik stoxastik operatorlar dastlab 1924-yilda S. Bernshteyn ishlarida matematik genetikadagi biror bir irsiy qonuniyatga asoslangan gen chastotalari dinamikasini aniqlovchi “evolyutsion operatorlar” sifatida kiritilgan bo‘lib S. Ulam, Yu.I. Lyubich, G. Kesten, S.S. Vallander, R. Jenks, E. Akin, V. Losert va o‘zbek matematiklari T.A. Sarimsoqov, R.N. G‘anixo‘jayev, N.N. G‘anixo‘jayev, O‘.A. Roziqov, F.M. Muxamedov, U.U. Jamilov, O.N. Hakimovlarning ishlarida bu kabi operatorlarning dinamikasini o‘rganish masalasi rivojlantirilgan.

Umumiy holda, kvadratik stoxastik operatorning asimptotik xossalarini o‘rganish murakkab masala bo‘lib, hattoki ikki o‘lchamli simpleksda ham ochiq turibdi. Bu asosiy masala Volterra kvadratik stoxastik operatorlari uchun R.N. G‘anixo‘jayev tomonidan nisbatan to‘liqroq hal etilgan. Hozirgi kunda N.N. G‘anixo‘jayev, O‘.A. Roziqov, F.M. Muxamedov, A. Zada, U.U. Jamilov, D.B.Eshmamatova kabi matematiklar tomonidan novolterra kvadratik operatorlar dinamikasi bo‘yicha ilmiy izlanishlar olib borilmoqda.

Novolterra kvadratik operatorlarning superpozitsiyasi dinamikasini o‘rganish polinomial stoxastik operatorlar nazariyasi nuqtai nazaridan ham qiziqarli bo‘lib, bu nazariya M. Scheutzow va M. Wilke Berenguer ishlarida rivojlantirilgan. Ma’lumki, eng sodda polynomial stoxastik operatorlar bu kvadratik va kubik stoxastik operatorlardir. Xususan, kubik stoxastik operatorlar dinamikasi M. Ladra, U.A. Roziqov, F.M. Muxamedov, A.Yu. Hamrayev, U.U. Jamilovlarning ishlarida tadqiq qilingan.

D.B. Eshmamatova, Sh.J. Seytov va N.B. Narziyevlarning ishlarida ikki o‘lchamli simpleksda aniqlangan turlicha xarakteristikaga ega bo‘lgan Volterra operatorlarning superpozitsiyasi bilan hosil qilingan dinamik sistemalar o‘rganilgan va superpozitsiya operator uchun ixtiyoriy orbitaning asimptotik xossalari dastlabki berilgan operatorlar orbitalari xossalaridan mutlaq farq qilishi ko‘rsatilgan. Xususan, turli harakat yo‘nalishiga ega bo‘lgan noergodik operatorlar superpozitsiyasidan hosil qilingan operator regular ekanligi isbotlangan.

Ta’kidlab o‘tish joizki, ko‘p sonli ilmiy izlanishlarga qaramasdan, ixtiyoriy kvadratik stoxastik operatorlar va ularning superpozitsiyasi orqali hosil qilingan operatorlar uchun orbitaning limit nuqtalari to‘plamini to‘la tavsifi olinmagan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish instituti ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutining OT-F4-87 raqamli “Evklid va psevdo-Evklid fazolaridagi egri chiziqlar va sirtlarning global invariantlari nazariyasi va uning mexanikaga tatbiqlari” (2017-2020 yy), OT-F4-82 raqamli “Operatorlar va noassotsiativ algebralarda lokal differensiallash va avtomorfizmlar, nochiziqli dinamik sistemalarda faza almashishlar va xaos” (2017-2020 yy) mavzusidagi ilmiy tadqiqot loyihalari va “Noassotsiativ algebralar strukturaviy nazariyasi va uning biologik sistemalardagi dinamik sistemalarni tadqiq qilishdagi tatbiqi” (2020-2023 yy) nomli ilmiy yo‘nalish doirasida bajarilgan.

Tadqiqot maqsadi novolterra kvadratik stoxastik operatorlari superpozitsiyasi orqali hosil qilingan diskret vaqtli dinamik sistemalar ixtiyoriy orbitasining limit nuqtalari to‘plamini to‘la tavsiflashdan iborat.

Tadqiqotning vazifalari:

superpozitsiya operatoriga nisbatan invariant to‘plamlarni aniqlash;

superpozitsiya operatorining davriy nuqtalar to‘plamini tavsiflash va ularning turlarini aniqlash;

superpozitsiya operatori uchun Lyapunov funksiyalarini qurish;

superpozitsiya operatori uchun orbitalarning limit nuqtalari to‘plamini tavsiflash.

Tadqiqot ob’ekti: novolterra operatorlarning superpozitsiyasi, Volterra va o‘rin almashtirilgan Volterra operatorlari superpozitsiyasi.

Tadqiqot predmeti. Matematik analiz, funksional analiz, nochiziqli dinamik sistemalar nazariyasi.

Tadqiqot usullari. Tadqiqot ishida matematik analiz, funksional analiz, algebra va dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

ikki va uch o‘lchamli simpleksda aniqlangan turli novolterra operatorlar superpozitsiyasi orqali hosil qilingan operatorlarning davriy nuqtalari va bu operatorlarga nisbatan invariant to‘plamlar topilgan bo‘lib, operatorlar uchun ixtiyoriy orbitaning qo‘zg‘almas nuqtaga yoki davriy orbitaga yaqinlashishi isbotlangan;

regular va ergodik novolterra kvadratik stoxastik operatorlar superpozitsiyasi uchun deyarli barcha orbitalar qo‘zg‘almas nuqtaga yaqinlashishi isbotlangan;

ikki o‘lchamli simpleksda aniqlangan har qanday ekstremal Volterra va uning o‘rin almashtirishlari orqali hosil qilingan operator bilan superpozitsiyasi uchun barcha qo‘zg‘almas, davriy nuqtalari topilgan hamda ixtiyoriy orbitaning limit nuqtalari to‘plami tavsiflangan.

Tadqiqotning amaliy natijalari. Dissertatsiya yangi qiziqarli natijalarni o‘z ichiga oladi va ular diskret vaqtli nochiziqli dinamik tizimlar nazariyasida qo‘llanilishi mumkin. Shuningdek, dissertatsiyada qo‘llanilgan usullar va olingan natijalar oliy o‘quv yurtlari magistrantlari va doktorantlari uchun maxsus kurs uchun material sifatida qo‘llanilishi mumkin.

Tadqiqot natijalarining ishonchliligi matematik va funksional analiz, Lyapunov funksiyalari nazariyasi va dinamik sistemalar nazariyasi usullari, shuningdek, qat’iy matematik mulohazalar orqali isbotlanganligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati ularning stoxastik operatorlar nazariyasida, matematik biologiya muammolarini hal qilishda qo‘llanilishi mumkinligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati matematik biologiyaning turli modellarida qo‘zg‘almas nuqtalar va limit nuqtalar to‘plamining tavsifi populyatsiyaning kelajakdagi evolyutsiyasi haqida ma’lumotlarni berishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Novolterra kvadratik stoxastik operatorlarning superpozitsiya dinamikasi bo‘yicha olingan natijalar quyidagi yo‘nalishlarda amaliyotga joriy qilingan:

novolterra kvadratik stoxastik operatorlarning superpozitsiyasi orqali hosil hosil qilingan operatorning invariant to‘plamlari va davriy nuqtalari to‘plamidan G00003447 raqamli “Kvant genetik algebralar va ularning tatbiqlari” mavzusidagi xorijiy loyihada nochiziqli stoxastik operatorlarning dinamikasini tahlil qilishda foydalanilgan (Birlashgan Arab Amirliklari universitetining 2024 yil 19-sentabrdagi ma’lumotnomasi, BAA). Ilmiy natijaning qo‘llanilishi biologik sistema holati o‘zgarishini ta’minlaydigan nochiziqli stoxastik operatorlarning davriy orbitalarini tavsiflash imkonini bergan;

Volterra va o‘rin almashtirilgan Volterra kvadratik stoxastik operatorlarining superpozitsiyasi orqali hosil qilingan operator uchun orbitalarning limit nuqtalari tavsifidan PID2020-115155GB-I00 raqamli “Assotsiativ bo‘lmagan gruppalar va algebralarda gomologiyalar, gomotopiyalar va kategorik invariantlar” mavzusidagi xorijiy loyihada evolyutsion operatorlarning musbat orbitalarini tahlil qilishda foydalanilgan (Santyago de Kompostela universitetining 2024 yil 23-sentyabrdagi ma’lumotnomasi, Ispaniya). Ilmiy natijaning qo‘llanilishi evolyutsion algebra, matematik biologiya va populyatsion genetika sohalarida foydalaniladigan nochiziqli operatorlar uchun orbitalarning limit nuqtalari to‘plamini yuqoridan baholash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 16 ta ilmiy-amaliy anjumanlarda, jumladan 6 ta xalqaro va 10 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo‘yicha jami 22 ta ilmiy ish chop etilgan, shulardan O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan 3 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 97 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asosli ravishda tahlil qilingan bo‘lib tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlari bilan bog‘liqligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan. Shuningdek, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining amaliy joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi haqida ham batafsil ma’lumotlar taqdim etilgan.

Dissertatsiyaning **“Ikkita ergodik novolterra kvadratik operatorlarning superpozitsiyasi”** deb nomlanuvchi birinchi bobida, kvadrat stoxastik operatorlar nazariyasi va dinamik sistemalar nazariyasidan zaruriy ta’riflar va tushunchalar keltirilgan. Shuningdek, ikki va uch o‘lchamli simplekslarda bir oilaga mansub ergodik novolterra kvadrat stoxastik operatorlarning superpozitsiyalari orqali hosil qilingan operatorlarning dinamikasi o‘rganilgan.

Aytaylik $E=\{1,\ldots,m\}$ bo‘lsin. U holda E to‘plamdagi barcha ehtimollik taqsimotlari $(m-1)$- o‘lchamli simpleksni aniqlaydi va

$$S^{m-1}=\left\{\mathbf{x}=(x_1,x_2,\ldots,x_m)\in\mathbb{R}_+^m:\ \sum_{i\in E}x_i=1\right\}$$

kabi ifodalanadi.

Agar $V:\mathbb{R}_+^m\to\mathbb{R}_+^m$ operator uchun $V(S^{m-1})\subset S^{m-1}$ bo‘lsa, V operatorga stoxastik operator(SO) deyiladi.

Kvadratik stoxastik operator(KSO) deb S^{m-1} simpleksni o‘zini-o‘ziga akslantiruvchi quyidagi akslantirishga aytiladi:

$$V: x'_k=\sum_{i,j=1}^{m}p_{ij,k}x_ix_j,\quad k\in E, \tag{1}$$

bu yerda $p_{ij,k}$ koeffisientlar

$$p_{ij,k}=p_{ji,k}\geq 0,\quad \sum_{k=1}^{m}p_{ij,k}=1,\quad i,j,k\in E \tag{2}$$

shartlarni qanoatlantiradi.

Ixtiyoriy $\mathbf{x}^{(0)}\in S^{m-1}$ boshlang‘ich nuqtaning orbitasi $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$ ketma-ketlik

$$\mathbf{x}^{(n+1)}=V(\mathbf{x}^{(n)})=V^{n+1}(\mathbf{x}^{(0)}),\ n=0,1,2,\ldots$$

kabi aniqlanadi.

$\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$ orbitaning limit nuqtalari to‘plamini $\omega_V(\mathbf{x}^{(0)})$ bilan belgilaymiz.

Dinamik sistemalarning asosiy masalasi ixtiyotiy $\mathbf{x}^{(0)} \in S^{m-1}$ boshlang‘ich nuqta va berilgan V operator uchun $\omega_V(\mathbf{x}^{(0)})$ to‘plamni tavsiflashdan iboratdir.

Agar har bir $\mathbf{x} \in S^{m-1}$ uchun $\lim\limits_{n\to\infty} V^n(\mathbf{x})$ limit mavjud bo‘lsa, V stoxastik operator *regulyar* deyiladi.

Agar har bir $\mathbf{x} \in S^{m-1}$ uchun

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1} V^k(\mathbf{x})$$

limit mavjud bo‘lsa, V stoxastik operator *ergodik* deyiladi.

1-ta’rif. *V_1, V_2 SOlar berilgan bo‘lsin. $V_2 = H^{-1}V_1H$ tenglikni qanoatlantiruvchi H gomeomorfizm mavjud bo‘lsa, biz V_1 operator V_2 operatorga topologik qo‘shma deymiz va $V_1 \sim V_2$ kabi belgilaymiz.*

Dissertatsiya ishida quyidagi belgilashlardan foydalanilgan.

$\partial S^{m-1} = \{\mathbf{x} \in S^{m-1} :$ kamida bitta $i \in E$ uchun $x_i = 0\}$ - simpleksning chegarasi;

$\Gamma_\alpha = \{\mathbf{x} \in S^{m-1} : x_i = 0,\ \forall i \notin \alpha\},\ \alpha \subset E$ - simpleksning yoqi;

$\mathrm{int}S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1x_2\cdots x_m > 0\}$ - simpleksning ichi;

$\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \ldots, \delta_{mi}) \in S^{m-1},\ i = 1,\ldots,m$ - simpleksning uchlari, bu yerda δ_{ij} - Kroneker deltasi;

O‘rin almashtirishlarga mos keluvchi S^{m-1} simpleksda aniqlangan KSOlar oilasi quyidagi ko‘rinishda berilgan bo‘lsin:

$$V_\pi : \begin{cases} x'_k = 2x_m x_{\pi(k)}, & k = 1,\ldots,m-1 \\ x'_m = 2x_m^2 - 2x_m + 1, & \end{cases} \tag{3}$$

bu yerda π - $\{1,\ldots,m-1\}$ to‘plamning biror o‘rin almashtirishi.

Dastlab S^2 simpleksdagi V_π KSO larning superpozitsiyasini qaraymiz. Ma’lumki, $\{1,2\}$ to‘plamning $\pi_1 = Id,\ \pi_2 = (21)$ o‘rin almashtirishlari mavjud.

$V_i = V_{\pi_1} \circ V_{\pi_i},\ i = 1,2$ deb belgilaymiz. U holda

$$V_1 = V_{\pi_1} \circ V_{\pi_1} = V_{\pi_2} \circ V_{\pi_2}, \qquad V_2 = V_{\pi_1} \circ V_{\pi_2} = V_{\pi_2} \circ V_{\pi_1}.$$

1-teorema. *V_1 va V_2 operatorlar uchun quyidagi tasdiqlar o‘rinli:*

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}}$ *uchun* $V_i(\mathbf{x}^{(0)}) = \mathbf{e}_3,\ i = 1,2$;
- *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\})$ *uchun* $\omega_{V_1}(\mathbf{x}^{(0)}) = \{\hat{\mathbf{x}}\}$;
- $\omega_{V_2}(\mathbf{x}^{(0)}) = \begin{cases} \{\hat{\mathbf{x}}\}, & agar\ \ \mathbf{x}^{(0)} \in M_1 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\}), \\ \{\hat{\mathbf{x}}, V_2(\hat{\mathbf{x}})\}, & agar\ \ \mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup M_1\}), \end{cases}$

bu yerda $\hat{\mathbf{x}} = \dfrac{1}{2}\left(\dfrac{x_1^{(0)}}{x_1^{(0)} + x_2^{(0)}}, \dfrac{x_2^{(0)}}{x_1^{(0)} + x_2^{(0)}}, 1\right)$ va $M_1 = \{\mathbf{x} \in S^2 : x_1 = x_2\}$.

Endi S^3 simpleksda (3) tenglik bilan aniqlangan KSOlarning superpozitsiyasi yordamida hosil qilingan $V_{\pi_i} \circ V_{\pi_j} : S^3 \to S^3$ operatorlarni qaraymiz.

Ma'lumki $\{1,2,3\}$ to'plamning barcha o'rin almashtirishlari quyidagicha:

$$\pi_1 = Id,\ \pi_2 = (12),\ \pi_3 = (13), \pi_4 = (23),\ \pi_5 = (123),\ \pi_6 = (132).$$

U holda, bizda 36 ta superpozitsiya operatorlari hosil bo'ladi. Tekshirib ko'rish mumkinki, bu operatorlarning har biri $W_i = V_{\pi_1} \circ V_{\pi_i},\ 1 \le i \le 6$ operatorlardan biriga teng bo'ladi.

1-lemma. $W_3 \sim W_4 \sim W_2,\ W_6 \sim W_5.$

2-teorema. $W_i,\ i = 1,2,5$ *operatorlar uchun quyidagi tasdiqlar o'rinli:*

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2,3\}}$ *uchun* $W_i(\mathbf{x}^{(0)}) = \mathbf{e}_4,\ i = 1,2,5$*;*
- *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\})$ *uchun* $\omega_{W_1}(\mathbf{x}^{(0)}) = \{\tilde{\mathbf{x}}\}$*;*
- $\omega_{W_2}(\mathbf{x}^{(0)}) = \begin{cases} \{\tilde{\mathbf{x}}\}, & agar\ \mathbf{x}^{(0)} \in M_2 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\}), \\ \{\tilde{\mathbf{x}}, W_2(\tilde{\mathbf{x}})\}, & agar\ \mathbf{x}^{(0)} \in S^2 \setminus (M_2 \cup \Gamma_{\{1,2,3\}}), \end{cases}$
- $\omega_{W_5}(\mathbf{x}^{(0)}) = \begin{cases} \{\tilde{\mathbf{x}}\}, & agar\ \mathbf{x}^{(0)} \in M_5 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\}), \\ \{\tilde{\mathbf{x}}, W_5(\tilde{\mathbf{x}}), W_5^2(\tilde{\mathbf{x}})\}, & agar\ \mathbf{x}^{(0)} \in S^2 \setminus (M_5 \cup \Gamma_{\{1,2,3\}}), \end{cases}$

bu yerda $\tilde{\mathbf{x}} = \frac{1}{2}\left(\frac{x_1^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, \frac{x_2^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, \frac{x_3^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, 1 \right),$

$$M_2 = \{\mathbf{x} \in S^3 : x_1 = x_2\}, \quad M_5 = \{\mathbf{x} \in S^3 : x_1 = x_2 = x_3\}.$$

Dissertatsiyaning **"Regulyar va ergodik novolterra kvadratik operatorlarning superpozitsiyasi"** nomli ikkinchi bobida, ikki o'lchamli simpleksda aniqlangan regulyar va ergodik novolterra KSOlarning superpozitsiyasi orqali hosil qilingan dinamik sistemalar tadqiq qilingan. Bu operatorlar quyidagi ko'rinishlarga ega:

$$W_0 : \begin{cases} x_1' = (1+\alpha)^2 x_3^2 x_2^2 + 2(1+\alpha)\big(1-(1+\alpha)x_3(1-x_3)\big)x_3 x_1, \\ x_2' = (1+\alpha)^2 x_3^2 x_1^2 + 2(1+\alpha)\big(1-(1+\alpha)x_3(1-x_3)\big)x_3 x_2, \\ x_3' = \big(1-(1+\alpha)x_3(1-x_3)\big)^2 + 2(1+\alpha)^2 x_3^2 x_1 x_2, \end{cases}$$

$$W_1 : \begin{cases} x_1' = (1+\alpha)(x_3^2 + 2x_1x_2)(x_2^2 + 2x_1x_3), \\ x_2' = (1+\alpha)(x_3^2 + 2x_1x_2)(x_1^2 + 2x_2x_3), \\ x_3' = 1-(1+\alpha)(x_3^2 + 2x_1x_2)(1-(x_3^2 + 2x_1x_2)), \end{cases}$$

bu yerda $\alpha \in [-1,1]$.

Quyidagi belgilashlarni kiritamiz:

$$t_\alpha = \sqrt[3]{(\alpha+1)\Big((2-\alpha)^2 + \sqrt{6\alpha^2 - 32\alpha + 43}\Big)},$$

$$x_\alpha^* = \frac{1}{3}\left(\frac{t_\alpha}{1+\alpha} + \frac{\alpha-3}{t_\alpha} + 1\right),\ \mathbf{x}_\alpha^* = \left(\frac{1-x_\alpha^*}{2}, \frac{1-x_\alpha^*}{2}, x_\alpha^*\right),$$

$$l_\alpha = \sqrt[3]{118 - 44\alpha + 18\sqrt{6\alpha^2 - 32\alpha + 43}},$$

$$x_\alpha^{**} = \frac{l_\alpha}{9\sqrt[3]{(\alpha+1)}} - \frac{2\sqrt[3]{(\alpha+1)}}{9l_\alpha} + \frac{1}{9},\ \mathbf{x}_\alpha^{**} = \left(\frac{1-x_\alpha^{**}}{2}, \frac{1-x_\alpha^{**}}{2}, x_\alpha^{**}\right).$$

3-teorema. W_0 *operator uchun quyidagi tasdiqlar o'rinli:*

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\}$ *uchun* $W_0(\mathbf{x}^{(0)}) = \mathbf{e}_3$ *;*
- *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\})$ *uchun*

$$\omega_{W_0}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & agar\ \alpha \le -1/2, \\ \{\mathbf{x}_\alpha^*\}, & agar\ \alpha > -1/2. \end{cases}$$

4-teorema. W_1 *operator uchun quyidagi tasdiqlar o'rinli:*

- ixtiyoriy $\mathbf{x}^{(0)} \in \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ *uchun* $W_1(\mathbf{x}^{(0)}) = \mathbf{e}_3$ *;*
- *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ *uchun*

$$\omega_{W_1}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & agar\ \alpha \le -1/2, \\ \{\mathbf{x}_\alpha^{**}\}, & agar\ \alpha > -1/2. \end{cases}$$

Agar operator regulyarlik xususiyatiga ega bo'lsa, unda u ergodik gipotezani qanoatlantiradi va 3- va 4- teoremalardan W_0 va W_1 SOlar ergodikdir, bundan biz quyidagi natijani olishimiz mumkin.

1-natija. W_0 *va* W_1 *stoxastik operatorlar ergodikdir.*

Dissertatsiyaning **"Kvadratik gomeomorfizmlarning superpozitsiyasi"** nomli uchinchi bobida, ikki o'lchamli simpleksda aniqlangan ekstremal Volterra va o'rin almashtirilgan Volterra KSOlarining superpozitsiyasi orqali hosil qilingan dinamik sistemalar tadqiq qilingan.

S^{m-1} simpleksda aniqlangan V Volterra KSO berilgan bo'lsin. Ma'lumki, Volterra KSOlar uchun $p_{ii,i} = 1$ va barcha $i, k \in E,\ i \ne k$ larda $p_{ik,k} + p_{ik,i} = p_{ki,k} + p_{ki,i} = 1$tengliklar o'rinli hamda bunday operatorlar quyidagi umumiy ko'rinishga ega:

$$V: x_k' = x_k\left(1 + \sum_{i=1}^m a_{ki} x_i\right)$$

bu yerda $A = \left(a_{ij}\right)_1^m$ elementlari $i \ne k$ uchun $a_{ki} = 2p_{ik,k} - 1$ bo'lib, $a_{ii} = 0$ va $|a_{ij}| \le 1$ bo'lgan koso-simmetrik matritsadir. Bu yerda $i, j \in E$.

2-ta'rif. *Agar* V *KSO ga mos keluvchi koso-simmetrik matritsa elementlari uchun barcha* $k \ne i$ *larda* $a_{ki} = -1$ *yoki* $a_{ki} = 1$ *tenglik bajarilsa, bunday operator ektremal Volterra operatorlari deyiladi.*

E to‘plamning π o‘rin almashtirishi berilgan bo‘lsin. $T_\pi : S^{m-1} \to S^{m-1}$ akslantirishni quyidagicha aniqlaymiz: $T_\pi(\mathbf{x}) = \left(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(m)}\right)$.

R.N. G‘anixo‘jayev va D.B.Eshmamatova ishlarida simpleksning ixtiyoriy kvadratik gomeomorfizmi T_π o‘rin almashtirish va V_0 Volterra operatori superpozitsiyasi ko‘rinishida, ya’ni $T_\pi \circ V_0$ kabi ifodalanishi isbotlangan.

S^2 dagi har qanday Volterra KSO quyidagi ko‘rinishda ifodalanadi:

$$V_{(a,b,c)} : \begin{cases} x_1' = x_1\left(1 + ax_2 - bx_3\right), \\ x_2' = x_2\left(1 - ax_1 + cx_3\right), \\ x_3' = x_3\left(1 + bx_1 - cx_2\right), \end{cases}$$

bu yerda $a,b,c \in [-1,1]$.

$\{1,2,3\}$ to‘plamning o‘rin almashtirishi π bo‘lsin. Ma’lumki:

$$\pi_1 = Id,\ \pi_2 = (12),\ \pi_3 = (13),\ \pi_4 = (23),\ \pi_5 = (123),\ \pi_6 = (132).$$

$\tilde{V}_k = T_{\pi_k} \circ V$, $k = 1,\ldots,6$ operatorlarni aniqlaymiz. Biz KSOlar superpozitsiyasi orqali hosil qilingan $V \circ \tilde{V}_k$, $k = 1,\ldots,6$ SOlarning dinamikasini o‘rganamiz. $\tilde{V}_k \circ V \sim V \circ \tilde{V}_k$ ekanligini tekshirib ko’rish mumkin. Bu masala umumiy holda murakkab bo‘lganligi sababli biz dinamikani ekstremal Volterra KSOlar sinfi uchun tadqiq qilamiz.

1-ta’rifga ko‘ra S^2 da 8ta ekstremal Volterra KSOlar mavjud:

$$V_1 = V_{(-1,1,-1)};\ V_2 = V_{(-1,1,1)};\ V_3 = V_{(1,1,-1)};\ V_4 = V_{(1,-1,-1)};$$

$$V_5 = V_{(-1,-1,1)};\ V_6 = V_{(1,-1,1)};\ V_7 = V_{(1,1,1)};\ V_8 = V_{(-1,-1,-1)}.$$

2-ta’rifda keltirilgan ekvivalentlik munosabatidan ekstremal Volterra operatorlari $K_1 = \{V_1,\ldots,V_6\}$ va $K_2 = \{V_7, V_8\}$ ekvivalent sinflarga ajraladi. Volterra operatorlari nazariyasidan ma’lumki, K_1 sinfdan olingan har qanday Volterra KSO regulyar va K_2 sinfdan olingan har qanday Volterra KSO noergodikdir.

$V \in K_1$ bo‘lsin, aytaylik $V = V_1$. KSOlar superpozitsiyasi orqali hosil qilingan $W_k = V \circ \tilde{V}_k$, $k = 1,\ldots,6$ SOlar dinamikasini tadqiq qilamiz.

5-teorema. *W_1 SO uchun quyidagi tasdiqlar o‘rinli:*

- $\mathrm{F}ix(W_1) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$;
- *agar* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \setminus \{\mathbf{e}_1\}$ *bo‘lsa, u holda* $\lim\limits_{n\to\infty} W_1^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_2$;
- *agar* $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{1,2\}}$ *bo‘lsa, u holda* $\lim\limits_{n\to\infty} W_1^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_3$.

$\mathbf{x}^* = \left(0, x^*, 1 - x^*\right)$ kabi belgilash kiritamiz, bu yerda x^* qiymat $x^4 - 2x^2 - x + 1 = 0$ tenglamaning $[0,1]$ dagi haqiqiy yechimi va $x^* \approx 0.525$.

6-teorema. *W_2 SO uchun quyidagi tasdiqlar o‘rinli:*

- $\mathrm{F}ix(W_2) = \{\mathbf{e}_1, \mathbf{x}^*\}$;

• *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}}$ *uchun*

$$\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{x}^*\}, & agar \quad x_2^{(0)} = x^*, \\ \{\mathbf{e}_2, \mathbf{e}_3\}, & boshqa\ hollarda. \end{cases}$$

• *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{\{1,3\}} \cup \Gamma_{\{1,2\}}$ *uchun* $\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_2, \mathbf{e}_3\}$.

• *ixtiyoriy* $\mathbf{x}^{(0)} \in \mathrm{int}S^2$ *uchun* $\omega_{W_2}\left(\mathbf{x}^{(0)}\right) \subset \Gamma_{23}$. *Bundan tashqari,* $\mathbf{x}^*$ *nuqtadan o'tuvchi shunday* ζ *invariant chiziq mavjud va* $\varepsilon = \sqrt{\frac{2}{3}}r$ *uchun shunday* $n_0 \in \mathbb{N}$ *mavjudki* $\forall n > n_0$ *uchun* $0 \le x_1^{(n)} < \varepsilon$ *o'rinli va*

$$\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{x}^*\}, & agar \ \ \mathbf{x}^{(n_0)} \in \zeta, \\ \{\mathbf{e}_2, \mathbf{e}_3\}, & boshqa\ hollarda, \end{cases}$$

bu yerda r qiymat ζ chiziq va $\Gamma_{\{2,3\}}$ qirra orasidagi eng katta masofa.

7-teorema. W_3 *SO uchun quyidagi tasdiqlar o'rinli:*

• $\mathrm{F}ix(W_3) = \{\mathbf{e}_2, \mathbf{y}^*\}$*;*

• *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_2, \mathbf{y}^*\}$ *uchun*

$$\omega_{W_3}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2\}, & agar \quad x_1^{(0)} < x^*, x_3^{(0)} < 1 - x^*, \\ \{\mathbf{x}^*, \mathbf{z}^*\}, & agar \ \ x_1^{(0)} = x^* \ \ yoki \ \ x_3^{(0)} = 1 - x^*, \\ \{\mathbf{e}_1, \mathbf{e}_3\}, & boshqa\ \ hollarda, \end{cases}$$

bu yerda $\mathbf{y}^* = \left(x^*, 0, 1 - x^*\right)$ va $\mathbf{z}^* = \left(x^*, 1 - x^*, 0\right)$.

8-teorema. W_4 *SO uchun quyidagi tasdiqlar o'rinli:*

• $\mathrm{F}ix(W_4) = \{\mathbf{e}_3, \mathbf{z}^*\}$*;*

• *ixtiyoriy* $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{z}^*\}$ *uchun*

$$\omega_{W_4}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & agar \ \ x_3^{(0)} > 0, \\ \{\mathbf{z}^*\}, & agar \ \ x_1^{(0)} = x^*, x_3^{(0)} = 0, \\ \{\mathbf{e}_1, \mathbf{e}_2\}, & agar \ \ x_1^{(0)} \ne x^*, x_3^{(0)} = 0. \end{cases}$$

$\widehat{\mathbf{x}} = \left(\widehat{x}^2(2 - \widehat{x})^2, \frac{1 - \sqrt{1 - \widehat{x}}}{\widehat{x}^2\left(2 - \widehat{x}\right)^2 - \widehat{x} + 1}, \widehat{x}\right)$ belgilash kiritamiz, bu yerda $\widehat{x} \approx 0.389$ va bu quyidagi tenglamaning yechimi

$$x^2\left(2 - x\right)^2 + \frac{1 - \sqrt{1 - x}}{x^2\left(2 - x\right)^2 - x + 1} + x = 1, \ \ x \in [0,1].$$

Quyidagicha funksiyalarni qaraylik: $\alpha(x) = x^2(2 - x)^2$, $\beta(x) = x^2\left(2 - x^2\right)$, $\gamma(x) = x^4$, $\lambda(x) = \alpha(\beta(\gamma(x)))$, $x \in [0,1]$.

$$\hat{X}=\left\{\left(\hat{x},1-\hat{x},0\right),\left(0,\gamma\left(\hat{x}\right),1-\gamma\left(\hat{x}\right)\right),\left(1-\beta\left(\gamma\left(\hat{x}\right)\right),0,\beta\left(\gamma\left(\hat{x}\right)\right)\right)\right\}$$

to‘plamni belgilaymiz, bu yerda $\hat{x}\approx 0.932$ va bu $\lambda(x)=x$ ning $(0,1)$ dagi yechimi.

9-teorema. *W_5 SO uchun quyidagi tasdiqlar o‘rinli:*

- $\mathrm{F}ix(W_5)=\{\hat{\mathbf{x}}\}$*;*
- *ixtiyoriy* $\mathbf{x}^{(0)}\in\partial S^2$ *uchun*

$$\omega_{W_5}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}, & agar\quad \mathbf{x}^{(0)}\notin\hat{X},\\ \hat{X} & agar\quad \mathbf{x}^{(0)}\in\hat{X};\end{cases}$$

$\breve{\mathbf{x}}=\left(\breve{x},1-\breve{x}-\breve{x}^2\left(2-\breve{x}^2\right),\breve{x}^2\left(2-\breve{x}^2\right)\right)$ nuqtani belgilab olamiz, bu yerda $\breve{x}\approx 0.314$ va bu $x=\left(\left(1-x^2\left(2-x^2\right)\right)^2-x^2\right)^2$ tenglamaning $(0,1)$ intervaldagi haqiqiy yechimi.

$$\bar{X}=\left\{\left(\bar{x},1-\bar{x},0\right),\left(1-\beta\left(\bar{x}\right),0,\beta\left(\bar{x}\right)\right),\left(0,\alpha\left(\beta\left(\bar{x}\right)\right),1-\alpha\left(\beta\left(\bar{x}\right)\right)\right)\right\},$$

to‘plamni belgilab olamiz, bu yerda $\bar{x}\approx 0.754$ va bu $\gamma(\alpha(\beta(x)))=x$ tenglamaning $(0,1)$ intervaldagi haqiqiy yechimi .

10-teorema. *W_6 SO uchun quyidagi tasdiqlar o‘rinli :*

- $\mathrm{F}ix(W_6)=\{\breve{\mathbf{x}}\}$*;*
- *ixtiyoriy* $\mathbf{x}^{(0)}\in\partial S^2$ *uchun*

$$\omega_{W_6}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}, & agar\quad \mathbf{x}^{(0)}\notin\bar{X},\\ \bar{X}, & agar\quad \mathbf{x}^{(0)}\in\bar{X}.\end{cases}$$

Kompyuter dasturi yordamidagi hisoblashlarga asoslanib quyidagi gipotezani keltirishimiz mumkin.

1-gipoteza. *Ixtiyoriy* $\mathbf{x}^{(0)}\in\mathrm{int}S^2$ *uchun*

$$\omega_{W_5}\left(\mathbf{x}^{(0)}\right)=\omega_{W_6}\left(\mathbf{x}^{(0)}\right)=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}.$$

$V\in K_2$ bo‘lsin, aytaylik $V=V_7$. Quyidagi KSOlarni aniqlaymiz $\tilde{V}_k=T_{\pi_k}\circ V$, $k=1,\ldots,6$. V va $\tilde{V}_k$, $k=1,\ldots,6$ KSOlarning superpozitsiyasi orqali hosil qilingan $\mathcal{W}_k=V\circ\tilde{V}_k$, $k=1,\ldots,6$ operatorlarning dinamikasini tadqiq qilamiz.

11-teorema. *$\mathcal{W}_1$ SO uchun quyidagi tasdiqlar o‘rinli:*

- $\Gamma_{12},\Gamma_{23},\Gamma_{13}$ *qirralar invariant to‘plamlardir;*
- $\mathrm{F}ix(\mathcal{W}_1)=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3,\mathbf{c}\}$*, bu yerda* $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ *egar va* $\mathbf{c}$ *itaruvchi;*
- *ixtiyoriy* $\mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{c}\}$ *uchun* $\omega_{\mathcal{W}_1}\left(\mathbf{x}^{(0)}\right)\subset\partial S^2$*. Bundan tashqari, ixtiyoriy* $\mathbf{x}^{(0)}\in\mathrm{int}S^2\setminus\{\mathbf{c}\}$ *uchun har qanday orbita va orbitalarning o‘rtachalaridan hosil qilingan* $\frac{1}{n}\sum_{k=0}^{n-1}\mathcal{W}_1^k\left(\mathbf{x}^{(0)}\right)$ *ketma-ketlik uzoqlashuvchi.*

2-lemma. $\mathcal{W}_3 \sim \mathcal{W}_4 \sim \mathcal{W}_2$, $\mathcal{W}_6 \sim \mathcal{W}_5$.

1-tasdiq. $\mathcal{W}_2^2$ *operator uchun qo'zg'almas nuqtalarning tiplari quyidagicha:*

- $\mathbf{c}$ *itaruvchi;*
- $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ *tortuvchi;*
- $\mathbf{x}^*, \mathbf{y}^*, \mathbf{t}^* = (1-x^*, x^*, 0)$ *egar.*

Dinamik sistemalar umumiy nazariyasiga ko'ra $\mathbf{e}_1 \in U_1$, $\mathbf{e}_2 \in U_2$, $\mathbf{e}_3 \in U_3$ bo'lgan U_1, U_2, U_3 ochiq to'plamlar mavjudki, ixtiyoriy $\mathbf{x}^{(0)} \in U_1 \cap \mathrm{int}S^2$ uchun $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_1\}$ va $\mathbf{x}^{(0)} \in (U_2 \cup U_3) \cap \mathrm{int}S^2$ bo'lganda $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_2, \mathbf{e}_3\}$.

r bilan markazlari $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ nuqtalarda bo'lgan va mos ravishda U_1, U_2, U_3 to'plamlarda yotuvchi aylanalar radiuslarining minimumini belgilab olamiz.

$\mathbf{x}^* \in \mathcal{V}_1$, $\mathbf{y}^* \in \mathcal{V}_2$, $\mathbf{t}^* \in \mathcal{V}_3$ bo'lgan $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_3$ ochiq to'plamlar va bu ochiq to'plamlarda shunday $\zeta_1, \zeta_2, \zeta_3$ invariant chiziqlar mavjudki, ixtiyoriy $\mathbf{x}^{(0)} \in \zeta_1 \cap \mathrm{int}S^2$ (mos rav. $\mathbf{x}^{(0)} \in \zeta_2 \cap \mathrm{int}S^2$, $\mathbf{x}^{(0)} \in \zeta_3 \cap \mathrm{int}S^2$) uchun $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{x}^*\}$ (mos rav. $\omega_{\mathcal{W}_2^2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{y}^*\}$, $\omega_{\mathcal{W}_2^2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{z}^*\}$).

d_1 (mos rav. d_2, d_3) bilan ζ_1 (mos rav. ζ_2, ζ_3) chiziq va $\Gamma_{\{2,3\}}$ (mos rav. $\Gamma_{\{1,3\}}$, $\Gamma_{\{1,2\}}$). qirra orasidagi eng katta masofani belgilab olamiz.

12-teorema. $\mathcal{W}_2$ *SO uchun quyidagi tasdiqlar o'rinli:*

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{23}$ *uchun*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & agar \quad \mathbf{x}^{(0)} \in \Gamma_{23} \setminus \{\mathbf{x}^*\}, \\ \{\mathbf{x}^*\}, & agar \quad \mathbf{x}^{(0)} = \mathbf{x}^*. \end{cases}$$

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{13}$ *uchun*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & agar \quad 0 \le x_1^{(0)} < x^*, \\ \{\mathbf{y}^*, \mathbf{t}^*\}, & agar \quad x_1^{(0)} = x^*, \\ \{\mathbf{e}_1\}, & agar \quad x^* < x_1^{(0)} \le 1. \end{cases}$$

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \Gamma_{12}$ *uchun*

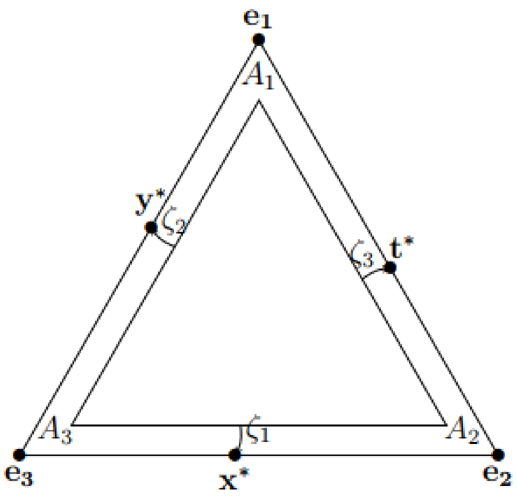
$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & agar \quad 0 \le x_1^{(0)} < 1 - x^*, \\ \{\mathbf{y}^*, \mathbf{t}^*\}, & agar \quad x_1^{(0)} = 1 - x^*, \\ \{\mathbf{e}_1\}, & agar \quad x^* < x_1^{(0)} \le 1, \end{cases}$$

- *ixtiyoriy* $\mathbf{x}^{(0)} \in \mathrm{int}S^2$ *uchun* $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) \subset \partial S^2$. *Bundan tashqari,* $\varepsilon = \min\left\{\sqrt{2}d_1/\sqrt{3}, \sqrt{2}d_2/\sqrt{3}, \sqrt{2}d_3/\sqrt{3}, r/3\right\}$ *uchun shunday* $n_0 \in \mathbb{N}$ *topiladiki,* $\forall n > n_0$ *uchun* $\mathbf{x}^{(n)} \in \partial S_\varepsilon^2$ *va*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_1\}, & agar \quad \mathbf{x}^{(n_0)}\in A_1,\\ \{\mathbf{e}_2,\mathbf{e}_3\}, & agar \quad \mathbf{x}^{(n_0)}\in A_2\cup A_3,\\ \{\mathbf{x}^*\}, & agar \quad \mathbf{x}^{(n_0)}\in \zeta_1,\\ \{\mathbf{y}^*,\mathbf{t}^*\}, & agar \quad \mathbf{x}^{(n_0)}\in \zeta_2\cup\zeta_3,\end{cases}$$

bu yerda

$$\partial S_\varepsilon^2=\left\{\mathbf{x}:\ dist(\mathbf{x},\partial S^2)<\sqrt{\frac{3}{2}\varepsilon}\right\}.$$



1-chizma: $\partial S_\varepsilon^2 = A_1\cup A_2\cup A_3\cup\{\zeta_1\}\cup\{\zeta_2\}\cup\{\zeta_3\}$.

13-teorema. $\mathcal{W}_5$ *SO uchun quyidagi tasdiqlar o'rinli:*

- $\mathrm{F}ix(\mathcal{W}_5)=\{\mathbf{c}\}$;
- *ixtiyoriy* $\mathbf{x}^{(0)}\in\partial S^2$ *uchun* $\omega_{\mathcal{W}_5}\left(\mathbf{x}^{(0)}\right)=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$;
- *ixtiyoriy* $\mathbf{x}^{(0)}\in\mathrm{int}S^2$ *uchun* $\omega_{\mathcal{W}_5}\left(\mathbf{x}^{(0)}\right)$ *to'plam* ∂S^2 *ning cheksiz qism to'plamidir.*

XULOSA

Dissertatsiya ishi novolterra KSOlar va kvadratik gomeomorfizmlar orqali hosil qilingan nochiziqli dinamik sistemalarni tadqiq etishga bag‘ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Ikki o‘lchamli simpleksda aniqlangan o‘rin almashtirishlarga mos keluvchi novolterra KSOlarning superpozitsiyasi orqali hosil qilingan diskret vaqtli dinamik sistemalarning orbitasi ixtiyoriy boshlang‘ich nuqta uchun yoki qo‘zg‘almas nuqtaga yoki davri ikkiga teng bo‘lgan davriy orbitaga yaqinlashishi ko‘rsatilgan;
2. Uch o‘lchamli simpleksda aniqlangan o‘rin almashtirishlarga mos keluvchi novolterra KSOlarning superpozitsiyasi asimptotik xossalari to‘liq o‘rganilgan bo‘lib, ixtiyoriy boshlang‘ich nuqta uchun orbita qo‘zg‘almas nuqtaga yoki davri ikkiga teng bo‘lgan yoki davri uchga teng bo‘lgan davriy orbitaga yaqinlashishi ko‘rsatilgan;
3. Regular va ergodik novolterra KSOlar superpozitsiyasi orqali hosil qilingan stoxastik operatorning deyarli barcha orbitalari qo‘zg‘almas nuqtaga yaqinlashishi isbotlangan;
4. Ikki o‘lchamli simpleksda aniqlangan regular ekstremal Volterra KSO orqali hosil qilingan kuartik gomeomorfizmlar uchun har qanday orbita yoki qo‘zg‘almas nuqtaga yoki davriy orbitaga yaqinlashishi ko‘rsatilgan;
5. Noergodik ekstremal Volterra KSO orqali hosil qilingan kvartik gomeomorfizmlar uchun har qanday orbita yoki qo‘zg‘almas nuqtaga yoki davriy orbitaga yaqinlashishi yoki uzoqlashuvchi bo‘lishi ko‘rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

ARALOVA KAMOLA AKBAR QIZI

DYNAMICS OF SUPERPOSITION OF NON-VOLTERRA QUADRATIC STOCHASTIC OPERATORS

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2024

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the of Ministers of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2023.4.PhD/FM932.

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Scientific supervisor:	**Jamilov Uygun Umurovich** doctor of physical and mathematical sciences
Official opponents:	**Ganikhodjaev Nosir Nabiyevich** doctor of physical and mathematical sciences, professor
	Khamraev Akhror Yusupovich candidate of physical and mathematical sciences, docent
Leading organization:	**National University of Uzbekistan**

Defense will take place 7 January 2025 at 16:00 at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I. Romanovskiy (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertation is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovskiy (is registered №193). (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871)-207-91-40).

Abstract of the thesis sent out on 20 December 2024 year
(Mailing report № 2 on 20 December 2024 year)

U.A. Rozikov
Chairman of Scientific Council
on award of scientific degrees,
D.F.-M.S., academician

J.K. Adashev
Scientific secretary of Scientific Council
on award of scientific degrees,
D.F.-M.S., senior researcher

A.A.Raximov
Deputy chairman of Scientific seminar under
Scientific Council on award of scientific degrees,
D.F.-M.S., professor

INTRODUCTION

Actuality and demand of the theme of the dissertation. Many scientific and practical researches conducted on a global scale are mostly related to the investigation of nonlinear dynamical systems. The importance of the theory of dynamical systems is to assess what situations will arise in the future, knowing the initial known state of the system, and it is important for better understanding complex processes in the fields of physics, biology, medicine, and economics. Therefore, the study of discrete-time nonlinear dynamical systems, in particular, the dynamics of quadratic stochastic operators and their superpositions, which are widely used in mathematical biology, remains one of the important and pressing areas of research in the theory of dynamical systems.

Currently, scientific research on nonlinear dynamical systems is being conducted worldwide, as these systems are essential tools for understanding the nature of many practical problems, analyzing them and finding the optimal solution. Including, for a given dynamical system, special attention is paid to the study of issues such as determining the invariant sets of this system, finding all periodic points and determining their types, and most importantly, describing the set of limit points of the orbit for any initial point.

In recent years, in our country, attention has been paid to medicine, biology, mathematics and physics, which have scientific and practical application of fundamental sciences. In particular, special attention was paid to the development of the theory of nonlinear dynamic systems, which has wide application in biology, medicine, mechanics, electronics, economics and optimal control theories, and significant results were achieved in this regard. Conducting scientific research at the level of international standards in the priority areas of "Algebra, functional analysis and theory of dynamic systems" was defined as the main tasks and areas of activity of mathematics[1]. It is important to develop the theory of discrete-time dynamic systems in order to use scientific results in related fields of science to ensure the performance of these tasks.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development Of the Republic of Uzbekistan”, UP-60 dated January 28, 2022 “Development Strategy of New Uzbekistan for the period of 2022-2026”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, PP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, and PP-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic science.

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” No. 292.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. In recent years, there has been an increasing interest in the use of nonlinear dynamical systems as mathematical models for understanding natural processes in biological or physical systems and their research. Discrete-time dynamical systems generated by quadratic stochastic operators, which are the simplest case of nonlinear dynamical systems, are one of the important objects of research in various fields such as population dynamics, physics, economics. Quadratic stochastic operators were first introduced in 1924 in the works of S. Bernstein as “evolutionary operators” that determine the dynamics of gene frequencies based on some hereditary law in mathematical genetics. In the works of S. Ulam, Yu.I. Lyubich, G. Kesten, S.S. Wallander, R. Jenks, E. Akin, V. Losert and Uzbek mathematicians T.A. Sarimsakov, R.N. Ganikhodzhaev, N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, U.U. Jamilov, O. N. Hakimov, the problem of studying the dynamics of such operators was developed.

In general, the study of the asymptotic properties of a quadratic stochastic operator is a somewhat complicated problem, even on the two-dimensional simplex. This basic problem was relatively more completely solved by R.N. Ganikhodzhaev for Volterra quadratic stochastic operators. Currently, mathematicians such as N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, A. Zada, U.U. Jamilov, S. Nazir, D.B. Eshmamatova are conducting scientific research on the dynamics of novolterra quadratic operators.

The study of the dynamics of the superposition of novolterra quadratic operators is also interesting from the point of view of the theory of polynomial stochastic operators, which was developed in the works of M. Scheutzow and M. Wilke Berenguer. It is known that quadratic and cubic stochastic operators are simplest cases of polynomial operators. In particular, the dynamics of cubic stochastic operators was investigated in the works of M. Ladra, U.A. Rozikov, F.M. Mukhamedov, A.Yu. Khamraev, U.U. Jamilov.

In the work of D.B.Eshmamatova, Sh.J.Seytov and N.B.Narziyev, dynamical systems generated by the superposition of Volterra operators with different characteristics defined on the two-dimensional simplex were studied, and it is shown that the asymptotic properties of an arbitrary orbit for the superposition operator are absolute different from the properties of orbits of the initial operators. In particular, it was proved that the operator generated by the superposition of nonergodic operators with different directions of motion has the property of regularity.

It should be noted that, despite the fact that many scientific researches of dynamical systems generated by quadratic stochastic operators and their superposition have been carried out, a complete description of the set of limit points for dynamical systems formed by superpositions of quadratic stochastic operators has not yet been obtained.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research OT-F4-87 "The theory of global invariants of curves and surfaces in Euclidean and pseudo-Euclidean spaces and its applications in mechanics" (2017-2020), OT-F4-82 "Local derivations and automorphisms of operator and nonassociative algebras, phase transitions and chaos in nonlinear dynamical systems" (2017-2020) and scientific research "YoFA-Ftex-2018-78, Dynamical and thermodynamical systems on non-amenable graphs" (2018-2019 y) and "Structural theory of non-associative algebras and its application in the study of dynamic systems in biological systems" (2020-2023) at the V.I. Romanovskiy Institute of Mathematics.

The aim of research work is to investigate asymptotic behavior of discrete-time dynamical systems generated by superposition of non-Volterra quadratic stochastic operators (superposition operator).

Research problems:

to determine invariant sets for a superposition operator;

to describe the set of periodic points of superposition operators and determine their types;

to construct Lyapunov functions for superposition operators;

to study the limit points of orbits for superposition operators.

The research object: superposition of non-Volterra operators, superposition of Volterra and permuted Volterra operators.

The research subject: Mathematical analysis, functional analysis, theory of nonlinear dynamical systems.

Research methods: In the research the methods of mathematical analysis, functional analysis, linear algebra and theory of dynamical systems are used.

Scientific novelty of the research work consists of the following:

for the superposition of different non-Volterra operators defined on the two-dimensional simplex, periodic points and invariant sets were found and it was proven that the orbits of such operators always converge to either a fixed point or a periodic orbit;

for the superposition of a regular and an ergodic non-Volterra operators defined on the two-dimensional it was shown that almost all orbits for the operator converge to a fixed point;

for the superposition operator of any extremal Volterra quadratic operator and its permutation defined on the two-dimensional simplex it was found the set of all fixed points and described the set of limit points of a orbit.

Practical results of the research. The dissertation contain new interesting results and they can be used in the theory of discrete-time nonlinear dynamical systems. Also, used methods and obtained results in the dissertation can be used as materials for a special course for master's students and doctoral students in higher educational institutions.

The reliability of the results of the study is justified by the use of methods of mathematical and functional analysis, the theory of Lyapunov functions and the theory of dynamical systems, as well as the rigor of mathematical reasoning.

Scientific and practical significance of the research results. The scientific significance of these research results stems from their applicability to both the theory of stochastic operators and the resolution of problems in mathematical biology.

The practical significance of this thesis lies in its demonstration that, in various models of mathematical biology, analyzing the fixed points and limit points of orbits can provide valuable insights into the future dynamics of a population.

Implementation of the research results. The results related to dynamics of superposition of non-Volterra quadratic stochastic operators were used in the following research projects:

invariant sets and periodic points of an operator generated by superposition of non-Volterra quadratic stochastic operators have been used in the research project "Quantum genetic algebras and their applications" with the reference number G0003447 for analyzing the dynamics of nonlinear stochastic operators (Reference of United Arab Emirates University dated September 19, 2024, UAE). The application of scientific result made it possible to classify periodic orbits of nonlinear stochastic operators, which is crucial for understanding the state of biological systems;

for operator generated by superposition of Volterra and permuted Volterra quadratic stochastic operators, the limit points of orbits have been used in the research project "Homology, homotopy and categorical invariants in groups and non-associative algebras", with reference number PID2020-115155GB-I00 for analyzing the orbits of nonlinear stochastic operators (Reference of University of Santiago de Compostela dated September 23, 2024, Spain). The application of scientific result made it possible to upper-bound the set of limit points for nonlinear operators used in evolution algebra, mathematical biology and population genetics.

Approbation of the research results. The main results of the research have been discussed at 6 international and 10 national scientific conferences.

Publications of the research results. On the topic of the dissertation 20 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 3 of them were published in international journals and 3 papers published in national mathematical journals.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 97 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** of the thesis includes the underlying motivation for the research, its significance in relation to current priorities in science and technology, the review of international research related to the topic, the degree of scrutiny of the problem, the aim, research problems, object and subject of research, scientific novelty and practical results, theoretical and practical significance of the research results , the approbation of research results, published works and information on the structure of the thesis.

In the first chapter of the thesis, titled **“Superposition of two ergodic non-Volterra quadratic operators”** we recall the necessary definitions and notions from the theory of quadratic stochastic operators and the theory of dynamical systems. Also therein we studied the dynamics of the operators generated by superpositions of non-Volterra quadratic stochastic operators on two-dimensional and three-dimensional simplices.

Let $E=\{1,\dots,m\}$. Then the set of all probability distributions on E is given by

$$S^{m-1}=\{\mathbf{x}=(x_1,x_2,\dots,x_m)\in\mathbb{R}_+^m:\ \sum_{i\in E}x_i=1\}$$

which clearly represents an $(m-1)$-dimensional simplex.

A map $V:\mathbb{R}_+^m\to\mathbb{R}_+^m$ is called stochastic operator(SO) if $V(S^{m-1})\subset S^{m-1}$.

A *quadratic stochastic operator(QSO)* is a mapping $V:S^{m-1}\to S^{m-1}$ given by

$$V: x'_k=\sum_{i,j=1}^{m}p_{ij,k}x_ix_j,\ k\in E \tag{1}$$

and the coefficients $p_{ij,k}$ satisfy

$$p_{ij,k}=p_{ji,k}\geq 0,\ \sum_{k=1}^{m}p_{ij,k}=1,\quad i,j,k\in E. \tag{2}$$

The orbit $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$ of any point $\mathbf{x}^{(0)}\in S^{m-1}$ is defined by

$$\mathbf{x}^{(n+1)}=V(\mathbf{x}^{(n)})=V^{n+1}(\mathbf{x}^{(0)}),\quad n=0,1,2,\dots$$

Denote by $\omega_V(\mathbf{x}^{(0)})$ the set of limit points of the orbit $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$.

The main problem in dynamical system is the description of the set $\omega_V(\mathbf{x}^{(0)})$ for any initial point $\mathbf{x}^{(0)}\in S^{m-1}$ and for a given SO V.

A SO V is called *regular* if the limit $\lim_{n\to\infty}V^n(\mathbf{x})$ exists for any $\mathbf{x}\in S^{m-1}$.

A SO is called *ergodic* if the limit

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}V^k(\mathbf{x})$$

exists for any initial $\mathbf{x}\in S^{m-1}$.

Definition 1. *Let* V_1, V_2 *be SOs. We will say that the operator* V_1 *is topologically conjugate to* V_2 *and write* $V_1 \sim V_2$ *if there exists a homeomorphism* H *such that* $V_2 = H^{-1}V_1H$.

The following notations will be used throughout this thesis.

The *boundary of the simplex* S^{m-1} be let the set $\partial S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_i = 0 \text{ for at least one } i \in E\}$;

a *face of the simplex* S^{m-1} be the set $\Gamma_\alpha = \{\mathbf{x} \in S^{m-1} : x_i = 0,\ \forall i \notin \alpha\},\ \alpha \subset E$;

the interior of the simplex S^{m-1} be the set $\text{int}S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1 x_2 \cdots x_m > 0\}$;

let $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \ldots, \delta_{mi}) \in S^{m-1},\ i = 1, \ldots, m$, denote the *vertices* of the simplex S^{m-1}, where δ_{ij} is the Kronecker delta;

Let the family of QSOs corresponding to permutation defined on the simplex S^{m-1} consists from the QSOs in the form

$$V_\pi : \begin{cases} x'_k = 2x_m x_{\pi(k)},\ k = 1, \ldots, m-1 \\ x'_m = x_m^2 + (x_1 + \cdots + x_{m-1})^2, \end{cases} \tag{3}$$

where π is a permutation of the set $\{1, \ldots, m-1\}$.

First we consider the superposition of QSOs V_π on the simplex S^2. Evidently that there are $\pi_1 = Id,\ \pi_2 = (21)$ permutations of the set $\{1,2\}$.

Denote $V_i = V_{\pi_1} \circ V_{\pi_i},\ i = 1,2$.

$$V_1 = V_{\pi_1} \circ V_{\pi_1} = V_{\pi_2} \circ V_{\pi_2}, \qquad V_2 = V_{\pi_1} \circ V_{\pi_2} = V_{\pi_2} \circ V_{\pi_1}.$$

Theorem 1. *For the operators* V_1 *and* V_2, *the following statements are hold:*

- *For* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}}$, *we have* $V_i(\mathbf{x}^{(0)}) = \mathbf{e}_3,\ i = 1,2$;
- *For any* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\})$, *we have* $\omega_{V_1}(\mathbf{x}^{(0)}) = \{\hat{\mathbf{x}}\}$;
- $\omega_{V_2}(\mathbf{x}^{(0)}) = \begin{cases} \{\hat{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in M_1 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\}), \\ \{\hat{\mathbf{x}}, V_2(\hat{\mathbf{x}})\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup M_1\}), \end{cases}$

where $\hat{\mathbf{x}} = \frac{1}{2}\left(\frac{x_1^{(0)}}{x_1^{(0)} + x_2^{(0)}}, \frac{x_2^{(0)}}{x_1^{(0)} + x_2^{(0)}}, 1 \right)$ *and* $M_1 = \{\mathbf{x} \in S^2 : x_1 = x_2\}$.

We consider the operators $V_{\pi_i} \circ V_{\pi_j} : S^3 \to S^3$ defined by superposition of QSOs (3). It is known that permutations of the set $\{1,2,3\}$ are

$$\pi_1 = Id,\ \pi_2 = (12),\ \pi_3 = (13), \pi_4 = (23),\ \pi_5 = (123),\ \pi_6 = (132).$$

Then we have 36 superposition operators. It can be verified that each of these operators is equal to one of the operators $W_i = V_{\pi_1} \circ V_{\pi_i},\ 1 \le i \le 6$.

Lemma 1. $W_3 \sim W_4 \sim W_2,\ W_6 \sim W_5$.

Theorem 2. *For the operators* $W_i,\ i = 1,2,5$, *the following statements are hold:*

• *for any* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2,3\}}$, *we have* $W_i(\mathbf{x}^{(0)}) = \mathbf{e}_4$, $i = 1,2,5$;

• *for any* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\})$, *we have* $\omega_{W_1}(\mathbf{x}^{(0)}) = \{\tilde{\mathbf{x}}\}$;

• $\omega_{W_2}(\mathbf{x}^{(0)}) = \begin{cases} \{\tilde{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in M_2 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\}), \\ \{\tilde{\mathbf{x}}, W_2(\tilde{\mathbf{x}})\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus (M_2 \cup \Gamma_{\{1,2,3\}}), \end{cases}$

• $\omega_{W_5}(\mathbf{x}^{(0)}) = \begin{cases} \{\tilde{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in M_5 \setminus (\Gamma_{\{1,2,3\}} \cup \{\mathbf{e}_4\}), \\ \{\tilde{\mathbf{x}}, W_5(\tilde{\mathbf{x}}), W_5^2(\tilde{\mathbf{x}})\}, & \text{if } \mathbf{x}^{(0)} \in S^2 \setminus (M_5 \cup \Gamma_{\{1,2,3\}}), \end{cases}$

where

$$\tilde{\mathbf{x}} = \frac{1}{2}\left(\frac{x_1^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, \frac{x_2^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, \frac{x_3^{(0)}}{x_1^{(0)} + x_2^{(0)} + x_3^{(0)}}, 1 \right),$$

$$M_2 = \left\{ \mathbf{x} \in S^3 : x_1 = x_2 \right\}, \quad M_5 = \left\{ \mathbf{x} \in S^3 : x_1 = x_2 = x_3 \right\}.$$

In the second chapter of the dissertation titled **"Superposition of regular and ergodic non-Volterra quadratic operators"** we studied dynamical systems generated by SOs which are superpositions of regular and ergodic non-Volterra QSOs defined on the two-dimensional simplex. These operators have the following forms:

$$W_0 : \begin{cases} x_1' = (1+\alpha)^2 x_3^2 x_2^2 + 2(1+\alpha)\big(1-(1+\alpha)x_3(1-x_3)\big)x_3 x_1, \\ x_2' = (1+\alpha)^2 x_3^2 x_1^2 + 2(1+\alpha)\big(1-(1+\alpha)x_3(1-x_3)\big)x_3 x_2, \\ x_3' = \big(1-(1+\alpha)x_3(1-x_3)\big)^2 + 2(1+\alpha)^2 x_3^2 x_1 x_2, \end{cases}$$

$$W_1 : \begin{cases} x_1' = (1+\alpha)(x_3^2 + 2x_1x_2)(x_2^2 + 2x_1x_3), \\ x_2' = (1+\alpha)(x_3^2 + 2x_1x_2)(x_1^2 + 2x_2x_3), \\ x_3' = 1-(1+\alpha)(x_3^2 + 2x_1x_2)(1-(x_3^2 + 2x_1x_2)), \end{cases}$$

where $\alpha \in [-1,1]$.

Denote

$$t_\alpha = \sqrt[3]{(\alpha+1)\left((2-\alpha)^2 + \sqrt{6\alpha^2 - 32\alpha + 43}\right)},$$

$$x_\alpha^* = \frac{1}{3}\left(\frac{t_\alpha}{1+\alpha} + \frac{\alpha - 3}{t_\alpha} + 1 \right), \quad \mathbf{x}_\alpha^* = \left(\frac{1-x_\alpha^*}{2}, \frac{1-x_\alpha^*}{2}, x_\alpha^* \right).$$

Theorem 3. *For the operator* W_0 *the following statements are hold:*

• $W_0(\mathbf{x}^{(0)}) = \mathbf{e}_3$ *for any* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}}$;

• *for any* $\mathbf{x}^{(0)} \in S^2 \setminus (\Gamma_{\{1,2\}} \cup \{\mathbf{e}_3\})$, *we have*

$$\omega_{W_0}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & \text{if } \alpha \le -1/2, \\ \{\mathbf{x}_\alpha^*\}, & \text{if } \alpha > -1/2. \end{cases}$$

Denote

$$l_\alpha = \sqrt[3]{118 - 44\alpha + 18\sqrt{6\alpha^2 - 32\alpha + 43}},$$

$$x_\alpha^{**}=\frac{l_\alpha}{9\sqrt[3]{(\alpha+1)}}-\frac{2\sqrt[3]{(\alpha+1)}}{9l_\alpha}+\frac{1}{9},\ \mathbf{x}_\alpha^{**}=\left(\frac{1-x_\alpha^{**}}{2},\frac{1-x_\alpha^{**}}{2},x_\alpha^{**}\right).$$

Theorem 4. *For the operator W_1 the following statements are hold:*

- $W_1(\mathbf{x}^{(0)})=\mathbf{e}_3$ *for any* $\mathbf{x}^{(0)}\in\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$;
- *for any* $\mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$, *we have*

$$\omega_{W_1}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_3\}, & if\ \alpha\le -1/2,\\ \{\mathbf{x}_\alpha^{**}\}, & if\ \alpha>-1/2.\end{cases}$$

If an operator has the property of being regular, then it satisfies the ergodic hypothesis, and by Theorem 3 the SO W_0 and by Theorem 4 the SO W_1 are regular transformations, so we have the following corollary.

Corollary 1. *The SOs W_0 , W_1 are ergodic.*

In the third chapter of the dissertation entitled **"Superposition of quadratic homeomorphisms"**, we studied dynamical systems generated by SOs which are superpositions of extremal Volterra and permuted Volterra QSOs defined on the two-dimensional simplex.

Let V be a Volterra QSO on S^{m-1}. Evidently, for any Volterra QSO it hold

$$p_{ii,i}=1,\ p_{ik,k}+p_{ik,i}=p_{ki,k}+p_{ki,i}=1 \text{ forall } i,k\in E\ i\ne k.$$

A Volterra QSO V defined on S^{m-1} has the form

$$V: x_k'=x_k\left(1+\sum_{i=1}^m a_{ki}x_i\right)$$

where $A=\left(a_{ij}\right)_1^m$ is a skew-symmetric matrix with $a_{ki}=2p_{ik,k}-1$ for $i\ne k$, $a_{ii}=0$ and $|a_{ij}|\le 1$. Here $i,j\in E$.

Definition 2. *The QSO V is called extremal Volterra if the elements of the corresponding skew-symmetric matrix satisfy $a_{ki}=-1$ or $a_{ki}=1$ for all $k\ne i$.*

Let π be a permutation of the set E. Define a map $T_\pi:S^{m-1}\to S^{m-1}$ by the formula $T_\pi(\mathbf{x})=\left(x_{\pi(1)},x_{\pi(2)},\dots,x_{\pi(m)}\right)$.

In the works of R.N. Ganikhodzhaev and D.B. Eshmamatova, it has been proven that any quadratic homeomorphism of the simplex is a superposition of the permutation T_π and the Volterra operator V_0, that is $T_\pi\circ V_0$.

Let $m=3$. Then any Volterra QSO has the form:

$$V_{(a,b,c)}:\begin{cases}x_1'=x_1\left(1+ax_2-bx_3\right),\\ x_2'=x_2\left(1-ax_1+cx_3\right),\\ x_3'=x_3\left(1+bx_1-cx_2\right),\end{cases}$$

where $a,b,c\in[-1,1]$.

Let π be a permutation of the set $\{1,2,3\}$. It is easy to see that

$$\pi_1=Id,\ \pi_2=(12),\ \pi_3=(13),\ \pi_4=(23),\ \pi_5=(123),\ \pi_6=(132).$$

Let V be a Volterra KSO. We define the following operators $\tilde{V}_k = T_{\pi_k} \circ V$, $k = 1,\dots,6$. We interested to investigate the dynamics of the following superposition of SOs $V \circ \tilde{V}_k$, $k = 1,\dots,6$. It can be check that $\tilde{V}_k \circ V \sim V \circ \tilde{V}_k$. In general this problem was difficult and consequently we shall study the dynamics this problem in the case V is an extremal Volterra QSO.

By Definition 1 there are 8 extremal Volterra QSOs defined on S^2:

$$V_1 = V_{(-1,1,-1)};\ V_2 = V_{(-1,1,1)};\ V_3 = V_{(1,1,-1)};\ V_4 = V_{(1,-1,-1)};$$
$$V_5 = V_{(-1,-1,1)};\ V_6 = V_{(1,-1,1)};\ V_7 = V_{(1,1,1)};\ V_8 = V_{(-1,-1,-1)}.$$

Due to the equivalence relation given in Definition 2, the extremal Volterra operators is split into two equivalence classes $K_1 = \{V_1,\dots,V_6\}$ and $K_2 = \{V_7,V_8\}$. From the theory of Volterra QSOs one has that any Volterra QSO from the class K_1 is a regular operator and any Volterra QSO from the class K_2 is a non-ergodic transformation.

Let $V \in K_1$, namely $V = V_1$. We study the dynamics of the following SOs, which are superposition SOs $W_k = V \circ \tilde{V}_k$, $k = 1,\dots,6$.

Theorem 5. *For the SO W_1, the following assertions are true:*

- $\mathrm{Fix}(W_1) = \{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$;
- *if* $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \setminus \{\mathbf{e}_1\}$, *then* $\lim_{n\to\infty} W_1^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_2$;
- *if* $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{1,2\}}$, *then* $\lim_{n\to\infty} W_1^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_3$.

Denote $\mathbf{x}^* = \left(0, x^*, 1 - x^*\right)$, where x^* is a real solution of the quartic equation $x^4 - 2x^2 - x + 1 = 0$ in [0,1] and $x^* \approx 0.525$.

Theorem 6. *For the SO W_2, the following assertions are true:*

- $\mathrm{Fix}(W_2) = \{\mathbf{e}_1,\mathbf{x}^*\}$;
- *for any* $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}}$, *we have*
$$\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{x}^*\}, & \textit{if} \quad x_2^{(0)} = x^*, \\ \{\mathbf{e}_2,\mathbf{e}_3\}, & \textit{otherwise.} \end{cases}$$
- *for any* $\mathbf{x}^{(0)} \in \Gamma_{\{1,3\}} \cup \Gamma_{\{1,2\}}$, *we have* $\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_2,\mathbf{e}_3\}$.
- *for any* $\mathbf{x}^{(0)} \in \mathrm{int}S^2$, *it holds* $\omega_{W_2}\left(\mathbf{x}^{(0)}\right) \subset \Gamma_{23}$. *Moreover, there is an invariant curve ζ which passes through $\mathbf{x}^*$ and for the $\varepsilon = \sqrt{\frac{2}{3}r}$ there is $n_0 \in \mathbb{N}$ such that $\forall n > n_0$ it holds $0 \le x_1^{(n)} < \varepsilon$ and*
$$\omega_{W_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{x}^*\}, & \textit{if} \quad \mathbf{x}^{(n_0)} \in \zeta, \\ \{\mathbf{e}_2,\mathbf{e}_3\}, & \textit{otherwise,} \end{cases}$$

where r is the greatest distance between the curve ζ and the face $\Gamma_{\{2,3\}}$.

Theorem 7. *For the SO W_3 the following assertions are hold:*

- $\mathrm{F}ix(W_3)=\{\mathbf{e}_2,\mathbf{y}^*\}$;
- *for any $\mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_2,\mathbf{y}^*\}$, we have*

$$\omega_{W_3}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_2\}, & if \quad x_1^{(0)}<x^*, x_3^{(0)}<1-x^*,\\ \{\mathbf{x}^*,\mathbf{z}^*\}, & if \quad x_1^{(0)}=x^* \text{ or } x_3^{(0)}=1-x^*,\\ \{\mathbf{e}_1,\mathbf{e}_3\}, & otherwise,\end{cases}$$

where $\mathbf{y}^*=\left(x^*,0,1-x^*\right)$ and $\mathbf{z}^*=\left(x^*,1-x^*,0\right)$.

Theorem 8. *For the So W_4 the following assertions are true:*

- $\mathrm{F}ix(W_4)=\{\mathbf{e}_3,\mathbf{z}^*\}$;
- *for any $\mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{z}^*\}$, we have*

$$\omega_{W_4}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_3\}, & if \quad x_3^{(0)}>0,\\ \{\mathbf{z}^*\}, & if \quad x_1^{(0)}=x^*, x_3^{(0)}=0,\\ \{\mathbf{e}_1,\mathbf{e}_2\}, & if \quad x_1^{(0)}\neq x^*, x_3^{(0)}=0.\end{cases}$$

Denote $\widehat{\mathbf{x}}=\left(\widehat{x}^2(2-\widehat{x})^2,\dfrac{1-\sqrt{1-\widehat{x}}}{\widehat{x}^2\left(2-\widehat{x}\right)^2-\widehat{x}+1},\widehat{x}\right)$, where $\widehat{x}\approx 0.389$ is a real solution of the equation

$$x^2\left(2-x\right)^2+\frac{1-\sqrt{1-x}}{x^2\left(2-x\right)^2-x+1}+x=1,\ x\in[0,1].$$

Consider the following functions $\alpha(x)=x^2(2-x)^2$, $\beta(x)=x^2\left(2-x^2\right)$, $\gamma(x)=x^4$, $\lambda(x)=\alpha(\beta(\gamma(x)))$, $x\in[0,1]$. Denote the set

$$\hat{X}=\left\{\left(\hat{x},1-\hat{x},0\right),\left(0,\gamma\left(\hat{x}\right),1-\gamma\left(\hat{x}\right)\right),\left(1-\beta\left(\gamma\left(\hat{x}\right)\right),0,\beta\left(\gamma\left(\hat{x}\right)\right)\right)\right\}.$$

where $\hat{x}$ is a real solution of the equation $\lambda(x)=x$ in $(0,1)$ and $\hat{x}\approx 0.932$.

Theorem 9. *For the SO W_5, the following assertions are hold:*

- $\mathrm{F}ix(W_5)=\{\widehat{\mathbf{x}}\}$;
- *for any $\mathbf{x}^{(0)}\in\partial S^2$ we have*

$$\omega_{W_5}\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}, & if \quad \mathbf{x}^{(0)}\notin\hat{X},\\ \hat{X} & if \quad \mathbf{x}^{(0)}\in\hat{X};\end{cases}$$

Denote $\breve{\mathbf{x}}=\left(\breve{x},1-\breve{x}-\breve{x}^2\left(2-\breve{x}^2\right),\breve{x}^2\left(2-\breve{x}^2\right)\right)$, where $\breve{x}$ is a real solution of the equation $x=\left(\left(1-x^2\left(2-x^2\right)\right)^2-x^2\right)^2$ in $(0,1)$ and $\breve{x}\approx 0.314$.

Denote the set

$$\bar{X} = \left\{ (\bar{x}, 1-\bar{x}, 0), (1-\beta(\bar{x}), 0, \beta(\bar{x})), \left(0, \alpha(\beta(\bar{x})), 1-\alpha(\beta(\bar{x}))\right) \right\},$$

where $\bar{x}$ is a real solution of the equation $\gamma(\alpha(\beta(x))) = x$ in $(0,1)$ and $\bar{x} \approx 0.754$.

Theorem 10. *For the SO W_6, the following assertions are hold:*

- $\mathrm{F}ix(W_6) = \{\breve{\mathbf{x}}\}$*;*
- *for any* $\mathbf{x}^{(0)} \in \partial S^2$*, we have*

$$\omega_{W_6}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}, & if \quad \mathbf{x}^{(0)} \notin \bar{X}, \\ \bar{X}, & if \quad \mathbf{x}^{(0)} \in \bar{X}. \end{cases}$$

Using simulations and some calculations suggest the following conjecture.

Conjecture 1. *For any* $\mathbf{x}^{(0)} \in \mathrm{int}S^2$*, we have*

$$\omega_{W_5}\left(\mathbf{x}^{(0)}\right) = \omega_{W_6}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}.$$

Let $V \in K_2$, namely $V = V_7$ and we define the following QSOs $\tilde{V}_k = T_{\pi_k} \circ V$, $k = 1, \ldots, 6$. We shall study the dynamics of the operators $\mathcal{W}_k = V \circ \tilde{V}_k$, $k = 1, \ldots, 6$ which are superposition of QSOs V and $\tilde{V}_k$, $k = 1, \ldots, 6$.

Theorem 11. *For the SO $\mathcal{W}_1$, the following assertions are true:*

- *the faces* $\Gamma_{12}, \Gamma_{23}, \Gamma_{13}$ *are invariant sets;*
- $\mathrm{F}ix(\mathcal{W}_1) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{c}\}$*, here* $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ *are saddles and* $\mathbf{c}$ *is repelling;*
- *for any* $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{c}\}$*, we have* $\omega_{\mathcal{W}_1}\left(\mathbf{x}^{(0)}\right) \subset \partial S^2$*. Moreover, any orbit and the sequence of averages* $\frac{1}{n}\sum_{k=0}^{n-1} \mathcal{W}_1^k\left(\mathbf{x}^{(0)}\right)$ *diverge for any* $\mathbf{x}^{(0)} \in \mathrm{int}S^2 \setminus \{\mathbf{c}\}$.

Lemma 2. $\mathcal{W}_3 \sim \mathcal{W}_4 \sim \mathcal{W}_2$, $\mathcal{W}_6 \sim \mathcal{W}_5$.

Proposition 1. *The type of the fixed points for $\mathcal{W}_2^2$ are as follows:*

- *the center* $\mathbf{c}$ *is a repelling;*
- *the vertexes* $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ *are attracting;*
- $\mathbf{x}^*, \mathbf{y}^*, \mathbf{t}^* = (1-x^*, x^*, 0)$ *are saddle.*

From general theory of dynamical systems it follows that there exist open sets U_1, U_2, U_3 such that $\mathbf{e}_1 \in U_1$, $\mathbf{e}_2 \in U_2$, $\mathbf{e}_3 \in U_3$ and we have that if $\mathbf{x}^{(0)} \in U_1 \cap \mathrm{int}S^2$ then $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_1\}$ and $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_2, \mathbf{e}_3\}$ when $\mathbf{x}^{(0)} \in (U_2 \cup U_3) \cap \mathrm{int}S^2$.

Denote by r the minimum of the radiuses of the circles whose centers are at the vertices $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ and which are subsets of U_1, U_2, U_3, respectively.

Similarly, there exist open sets $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_3$ such that $\mathbf{x}^* \in \mathcal{V}_1$, $\mathbf{y}^* \in \mathcal{V}_2$, $\mathbf{t}^* \in \mathcal{V}_3$ and invariant curves $\zeta_1, \zeta_2, \zeta_3$ such that $\zeta_1 \in \mathcal{V}_1$, $\zeta_2 \in \mathcal{V}_2$, $\zeta_3 \in \mathcal{V}_3$ and for them we have that if $\mathbf{x}^{(0)} \in \zeta_1 \cap \mathrm{int} S^2$ (resp. $\mathbf{x}^{(0)} \in \zeta_2 \cap \mathrm{int} S^2$, $\mathbf{x}^{(0)} \in \zeta_3 \cap \mathrm{int} S^2$), then $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{x}^*\right\}$ (resp. $\omega_{\mathcal{W}_2^2}\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{y}^*\right\}$, $\omega_{\mathcal{W}_2^2}\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{z}^*\right\}$).

Denote by d_1 (resp. d_2, d_3) the greatest distance between the curve ζ_1 (resp. ζ_2, ζ_3) and the face $\Gamma_{\{2,3\}}$ (resp. $\Gamma_{\{1,3\}}$, $\Gamma_{\{1,2\}}$).

Theorem 12. *For the SO $\mathcal{W}_2$ the following statements are true:*

• *for any $\mathbf{x}^{(0)} \in \Gamma_{23}$, we have that*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & if \quad \mathbf{x}^{(0)} \in \Gamma_{23} \setminus \{\mathbf{x}^*\}, \\ \{\mathbf{x}^*\}, & if \quad \mathbf{x}^{(0)} = \mathbf{x}^*. \end{cases}$$

• *for any $\mathbf{x}^{(0)} \in \Gamma_{13}$, we have that*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & if \quad 0 \le x_1^{(0)} < x^*, \\ \{\mathbf{y}^*, \mathbf{t}^*\}, & if \quad x_1^{(0)} = x^*, \\ \{\mathbf{e}_1\}, & if \quad x^* < x_1^{(0)} \le 1. \end{cases}$$

• *for any $\mathbf{x}^{(0)} \in \Gamma_{12}$, we have that*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2, \mathbf{e}_3\} & if \quad 0 \le x_1^{(0)} < 1 - x^*, \\ \{\mathbf{y}^*, \mathbf{t}^*\}, & if \quad x_1^{(0)} = 1 - x^*, \\ \{\mathbf{e}_1\}, & if \quad x^* < x_1^{(0)} \le 1, \end{cases}$$

• *for any $\mathbf{x}^{(0)} \in \mathrm{int} S^2$, it hold $\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) \subset \partial S^2$. Moreover, for the $\varepsilon = \min\left\{\sqrt{2}d_1/\sqrt{3}, \sqrt{2}d_2/\sqrt{3}, \sqrt{2}d_3/\sqrt{3}, r/3\right\}$ there is $n_0 \in \mathbb{N}$ such that $\forall n > n_0$ it holds $\mathbf{x}^{(n)} \in \partial S_\varepsilon^2$ and*

$$\omega_{\mathcal{W}_2}\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_1\}, & if \quad \mathbf{x}^{(n_0)} \in A_1, \\ \{\mathbf{e}_2, \mathbf{e}_3\}, & if \quad \mathbf{x}^{(n_0)} \in A_2 \cup A_3, \\ \{\mathbf{x}^*\}, & if \quad \mathbf{x}^{(n_0)} \in \zeta_1, \\ \{\mathbf{y}^*, \mathbf{t}^*\}, & if \quad \mathbf{x}^{(n_0)} \in \zeta_2 \cup \zeta_3, \end{cases}$$

where

$$\partial S_\varepsilon^2 = \left\{\mathbf{x} : \ dist(\mathbf{x}, \partial S^2) < \sqrt{\frac{3}{2}}\varepsilon\right\}.$$

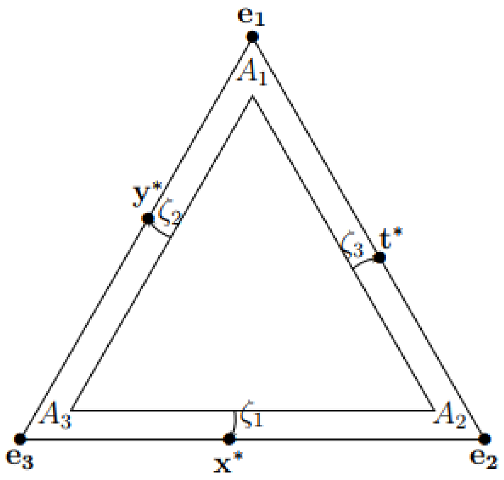

Figure 1: The set $\partial S^2_\varepsilon = A_1 \cup A_2 \cup A_3 \cup \{\zeta_1\} \cup \{\zeta_2\} \cup \{\zeta_3\}$.

Theorem 13. *For the SO $\mathcal{W}_5$ the following statements are true:*

- $\mathrm{F}ix(\mathcal{W}_5) = \{\mathbf{c}\}$. *The center* $\mathbf{c}$ *is a repelling fixed point;*
- *if* $\mathbf{x}^{(0)} \in \partial S^2$, *then we have* $\omega_{\mathcal{W}_5}\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$;
- *if* $\mathbf{x}^{(0)} \in \mathrm{int} S^2$, *then we have that* $\omega_{\mathcal{W}_5}\left(\mathbf{x}^{(0)}\right)$ *is an infinite subset of the* ∂S^2.

CONCLUSION

The dissertation work is devoted to investigation of the nonlinear dynamical systems generated by the superpositions of non-Volterra QSOs and quadratic homeomorphisms.

Main results of the research are as follows:

1. For a specific family of discrete-time dynamical systems generated by superposition of non-Volterra QSOs corresponding to permutations defined on the two-dimensional simplex we showed that for any initial point the orbit converges to either a fixed point or a 2-periodic orbit;
2. The asymptotical behaviour of superposition of non-Volterra QSOs corresponding to permutation defined on the three-dimensional simplex was fully studied and we showed that for any initial point the orbit converges to either a fixed point or a 2-periodic orbit or 3-periodic orbit;
3. It was proved that almost all orbits of a stochastic operator which is a superpositions of regular and ergodic non-Volterra QSOs converges to a fixed point;
4. For the quartic homeomorphisms generated by a regular extremal Volterra QSO defined on the two-dimensional simplex we showed that any orbit converges to either a fixed point or a periodic orbit;
5. For the quartic homeomorphisms generated by a non-ergodic extremal Volterra QSO defined on the two-dimensional simplex we showed that any orbit converges to either a fixed point or a periodic orbit or diverges.

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ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**

ИНСТИТУТ МАТЕМАТИКИ

АРАЛОВА КАМОЛА АКБАР КИЗИ

ДИНАМИКА СУПЕРПОЗИЦИИ НЕВОЛЬТЕРРОВСКИХ КВАДРАТИЧНЫХ СТОХАСТИЧЕСКИХ ОПЕРАТОРОВ

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**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

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Научный руководитель:	**Жамилов Уйгун Умурович** доктор физико-математических наук
Официальные оппоненты:	**Ганиходжаев Носир Набиевич** доктор физико-математических наук, профессор
	Хамраев Ахрор Юсупович кандидат физико- математических наук, доцент
Ведущая организация:	**Национальный университет Узбекистана**

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У.А. Розиков
Председатель Научного
совета по присуждению ученых
степеней, д.ф.-м.н., академик

Ж.К. Адашев
Ученый секретарь Научного
совета по присуждению ученых
степеней, д.ф.-м.н., старший
научный сотрудник

А.А. Рахимов
Заместитель председателя Научного семинара
при Научном совете по присуждению
ученых степеней, д.ф.-м.н., профессор

ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является изучение асимптотического поведения орбит дискретных динамических систем, порожденных суперпозициями невольтерровских квадратичных стохастических операторов.

Объект исследования: суперпозиция невольтерровских операторов, суперпозиция вольтерровских операторов и переставленных операторов Вольтерра.

Научная новизна исследования состоит в следующем:

для суперпозиции различных невольтерровских операторов, определенных на двумерном симплексе, были найдены периодические точки и инвариантные множества, доказано, что орбиты таких операторов сходятся либо к неподвижной точке, либо к периодической орбите;

для суперпозиции регулярного и эргодического невольтерровских операторов, определенных в двумерном симплексе, показано, что почти все орбиты оператора сходятся к неподвижной точке;

для суперпозиции любого крайнего квадратичного оператора Вольтерра и его перестановки, определенных на двумерном симплексе, найдено множество всех неподвижных точек и описано множество предельных точек орбит.

Внедрение результатов исследования. Результаты, связанные с динамикой суперпозиции квадратичных стохастических невольтерровских операторов, были использованы в следующих исследовательских проектах:

инвариантные множества и периодические точки оператора суперпозиции квадратичных стохастических невольтерровских операторов были использованы в исследовательском проекте «Квантовые генетические алгебры и их приложения» с номером G0003447 для анализа динамики нелинейных стохастических операторов (Справка от Университета Объединенных Арабских Эмиратов, 19 сентября 2024 г., ОАЭ). Применение научного результата позволило классифицировать периодические орбиты нелинейных стохастических операторов, что имеет важное значение для понимания состояния биологических систем;

предельные точки орбит оператора суперпозиции вольтерровских и переставленных вольтерровских операторов были использованы в исследовательском проекте «Гомология, гомотопия и категориальные инварианты в группах и неассоциативных алгебрах» с номером PID2020-115155GB-I00 для анализа орбит нелинейных стохастических операторов (Справка от Университета Сантьяго-де-Компостела, 23 сентября 2024 г., Испания). Применение научного результата позволило устоновить верхнюю границу множества предельных точек для нелинейных операторов, используемых в эволюционной алгебре, математической биологии и популяционной генетики.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 97 страниц.

E'LON QILINGAN ILMIY ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo'lim (part 1; часть 1)

1. Aralova K.A., Jamilov U.U. On the dynamics of superposition of non-Volterra quadratic stochastic operators // *Bulletin of the Institute of Mathematics*, 2020 No.3, p. 1-14. (01.00.00; № 6)

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6. Aralova K.A., Jamilov U.U. On a superposition of Volterra and permuted Volterra quadratic stochastic operators // *Lobachevskii Journal of Mathematics,* 2024, Vol. 45, No. 3, p. 922-937. (3. Scopus, SJR=0.453)

II bo'lim (part 2; часть 2)

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11. Jamilov U.U., Aralova K.A. The dynamics of superposition of non-Volterra quadratic stochastic operators // New theorems of young mathematicians-2022, *Republican scientific conference*, May 13-14, 2022, Namangan, p. 42-43.

12. Aralova K.A. The dynamics of superposition of non-Volterra quadratic stochastic operators // *Newsletter for young scientists*. N-2(4), 2022, p. 10-13.

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15. Aralova K.A., Xoliqova F.Q. Regular dynamics of superposition of non-volterra quadratic stochastic operators on S^2 // Actual problems of physics, mathematics and mechanics, *International scientific conference*, May 24-25, 2023, Bukhara, p. 13-14.

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