MIRZO ULUG‘BEK NOMIDAGI O‘ZBEKISTON MILLIY UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/30.12.2019.FM.01.01 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

QUCHQAROVA SARVINOZ ATAMURATOVNA

CHEKSIZ IKKI BLOK DIFFERENSIAL TENGLAMALAR SISTEMASI BILAN YOZILADIGAN DIFFERENSIAL O‘YINLAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA
FALSAFA DOKTORI (PHD) DISSERTATSIYASI
AVTOREFERATI

Toshkent – 2025

UDK: 517.977.8

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

MIRZO ULUG‘BEK NOMIDAGI O‘ZBEKISTON MILLIY UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/30.12.2019.FM.01.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2022.2.PhD/FM707 raqam bilan ro'yxatga olingan.

Dissertatsiya Mirzo Ulug'bek nomidagi O'zbekiston Milliy universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus(rezyume)) Ilmiy kengash veb-sahifasi (http://ik-fizmat.nuu.uz) va "ZiyoNet" ta'lim axborot tarmog'iga (http://www.ziyonet.uz/) joylashtirilgan.

Ilmiy rahbarlar:	**Ibragimov G'ofurjon Ismoilovich** fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Mamadaliyev Numanjon** fizika-matematika fanlari doktori, professor
	Ibaydullayev To'lanboy Tursunboyevich fizika-matematika fanlari nomzodi, dotsent
Yetakchi tashkilot:	**Namangan Davlat Universiteti**

Dissertatsiya himoyasi Mirzo Ulug'bek nomidagi O'zbekiston Milliy universiteti huzuridagi ilmiy darajalar beruvchi DSc.03/30.12.2019.FM.01.01 raqamli Ilmiy kengashning 2025 yil "8." "04" soat 14.00 dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 4-uy. Tel.: (+99871) 227-12-24, faks: (+99871) 246-53-21, e-mail: nauka@nuu.uz).

Dissertatsiya bilan Mirzo Ulug'bek nomidagi O'zbekiston Milliy universitetining Axborot-resurs markazida tanishish mumkin (37-raqami bilan ro'yxatga olingan). Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 4-uy. Tel.: (+99871) 246-02-24.

Dissertatsiya avtoreferati 2025 yil "24" mart kuni tarqatildi.

(2025 yil "24" mart dagi 2-raqamli reestr bayonnomasi).

A.Sadullayev
Ilmiy darajalar beruvchi Ilmiy kengash raisi, f.-m.f.d., akademik

R.M.Jo'rayev
Ilmiy darajalar beruvchi Ilmiy kengash ilmiy kotibi, f.-m.f.f.d., PhD

Sh.A.Alimov
Ilmiy darajalar beruvchi Ilmiy kengash huzuridagi Ilmiy seminar raisi, f.-m.f.d., akademik

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Bugungi kunda jahon miqyosida ilmiy-texnik taraqqiyotning jadallik bilan rivojlanishi matematik boshqaruv nazariyasining, shu jumladan, differensial o‘yinlar nazariyasining yangi sohalarini rivojlantirish va olingan natijalarning amaliyotga tadbiq qilishni talab etmoqda. Matematikadagi amaliyot talablaridan kelib chiqadigan bir qancha masalalar optimal boshqarish va differensial tenglamalar masalalariga keltiriladi. Matematika, differensial o‘yinlar va iqtisodiyotning turli sohalarida ikki blok (ko‘p hollarda qo‘sh blok deb qaraladi) tushunchasining tadbiqlari amalda qo‘llanilmoqda. Shuning uchun ham cheksiz ikki blok differensial tenglamalar sistemasi bilan tavsiflanuvchi ziddiyatli holarlatning boshqaruv parametrlariga geometrik, integral va turli chegaralar qo‘yilgan hollarda tadqiq qilish hozirgi kunning dolzarb muammolaridan hisoblanadi.

Hozirgi kunda jahonda olib borilayotgan ilmiy tadqiqotlarda blokli differensial tenglamalar cheksiz sistemasi bilan tavsiflanuvchi ziddiyatli muammolarni hal etish muhim ahamiyatlardan biri sanaladi. Cheksiz ikki blok differensial tenglamalar sistemasi bilan tavsiflanuvchi differensial o‘yinlar nazariyasi va uning tatbiqlari bo‘yicha birinchi ustuvor yo‘nalishlarda qator ilmiy-tadqiqot ishlari olib borilmoqda. O‘yinchilarning boshqaruvlariga turli chegaralar qo‘yilgan cheksiz ikki blok differensial tenglamalar sistemasi bilan tavsiflanuvchi quvish-qochish masalalari, amaliyotda ko‘p uchraydigan ziddiyatli sharoitlardan kelib chiquvchi fizik jarayonlarga mos keladi. Ammo natijalar shuni ko‘rsatadiki, boshqaruv parametrlariga turli chegaralar qo‘yilgan xususiy hosilali differensial tenglamalarda quvish masalalarini yechish jarayonlarini tadbiq etish uchun turli xil usullar talab qilinadi. Shuning uchun ham ikki blok differensial tenglamalar sistemasi yechimlari fazosining funksional va algebraik xossalarini o‘rganish maqsadli ilmiy tadqiqotlardan biri hisoblanadi.

Keyingi paytlarda mamlakatimizda fundamental fanlarning amaliy tatbiqqa ega bo‘lgan dolzarb ilmiy yo‘nalishlariga muntazam ravishda e’tibor qaratib kelinmoqda. Ilm-fan rivojlanishida fundamental tadqiqotlarni amaliyotga yaqinlashtirish asosiy vazifalardan biri etib belgilangan. Differensial o‘yinlar nazariyasi ziddiyatli vaziyatlarda jarayonlarni boshqarish muammolarini hal etishda eng samarali sohalardan biridir. Xususan, “Differensial tenglamalar nazariyasi, algebra va uning tatbiqlari, funksional analiz va uning tatbiqlari, dinamik sistemalar nazariyasi, matematik statistika, matematik modellashtirish” kabi ustuvor yo‘nalishlar bo‘yicha muhim vazifalar belgilab berilgan[1]. Bu vazifalarni amalga oshirishda dinamik sistemalar, differensial tenglamalar, matematik boshqaruv nazariyasi va dinamik o‘yinlar nazariyasida turli chegaralarga ega differensial o‘yin masalalari uchun optimal yechimlarni ta’minlaydigan boshqaruv modellarini yaratish muhim ahamiyat kasb etadi.

[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi qarori.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljal-langan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlar hamda mazkur faoliyatga tegishli boshqa nor-mativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustivor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi.

Differensial o‘yinlar tushunchasini R.Ayzeks kiritgan, garchi Ayzeksdan avval ham ba’zi differensial o‘yinlar ko‘rib chiqilgan edi. Ushbu mavzuga oid oldingi adabiyotlarning aksariyati chekli o‘lchamli fazoda differensial o‘yinlarga bag‘ishlangan. Differensial o‘yinlar L.D. Bercovitz, W.H. Fleming, L.S. Pontryagin, N.N. Krasovskiy, A. Bryson, O. Hajek, A. Friedman, R.J. Elliot, N.J. Kalton, S. Baron, E.F. Mishenko, Y.S. Osipov, A.B. Kurjanskiy, B.N. Pshenichnyi, L.A. Petrosyan, A.I. Subbotin, Chikrii, N.Yu. Satimov, A.A.Azamov, A.Sh. Kuchkarov, M. To‘xtasinov, N.Mamadaliev, B.T.Samatov, G‘.I.Ibragimov va boshqalar tomonidan rivojlantirilgan.

Cheksiz o‘lchamli fazolar bilan bog‘liq ko‘plab ilmiy ishlar bor. A.A.Azamovning ishida cheksiz o‘lchamli chiziqli oddiy differensial tenglamalar sistemai uchun koeffitsiyentlar matritsasining diagonal elementi λ va yuqori diagonal elementlari 1 ga teng. $\lambda \le -1$ barqarorligi va sistemaning boshqariluvchanligi o‘rganilgan.

Xususiy hosilali differensial tenglamalar bilan tavsiflangan differensial o‘yinlar masalalarini birinchi marta J.L.Lions va Yu.S.Osipov tomonidan tadqiq etilgan. Xususiy hosilali differensial tenglamalar bilan tavsiflangan sistemalar uchun boshqaruv va/yoki differensial o‘yin masalalarini o‘rganishda asosiy usullardan biri Fure usulidir. Bu usuldan xususiy hosilali differensial tenglamalar bilan tavsiflangan differensial o‘yin masalalarini cheksiz oddiy differensial tenglamalar sistemasi bilan tavsiflangan differensial o‘yin masalalariga keltirish uchun foydalanishimiz mumkin.

N.Yu.Satimov va M.To‘xtasinovning ishida parabolik tipdagi xususiy hosilali tenglama bilan ifodalangan quvish va qochish differensial o‘yinlari o‘rganilgan. Turli xil boshqaruv chegaralari (integral, geometrik) holatlari tahlil qilingan. O‘yinchilarning boshlang‘ich holatlari ma’lum bir to‘plamlarga tegishli bo‘lganda quvlovchi uchun tutish, qochuvchi uchun esa qochish shartlari aniqlangan.

G‘.I.Ibragimov tomonidan l_2 Gilbert fazosida ko‘p quvuvchi ishtirokidagi o‘yinda, o‘yin davomiyligi berilgan. O‘yin qiymati uchun formula topilib, o‘yinchilarning optimal strategiyalari aniq ko‘rinishda qurilgan. M.Salimi va M.Ferraraning ishida bir yoki chekli sondagi quvuvchilarning bir qochuvchiga optimal yaqinlashish masalasi o‘rganilgan. Ushbu ishda o‘yin qiymati va o‘yinchilarning optimal strategiyalari uchun formula taklif qilingan. A. A. Azamov va M.B.Ruziboevning ishida sistema holatini boshlang‘ich holatda koordinata boshiga vaqt bo‘yicha optimal o‘tkazish masalasi o‘rganilgan va optimal vaqt yuqoridan baholangan.

H.O.Fattorini tomonidan parabolik tipdagi xususiy hosilali differensial tenglamalar uchun vaqt bo‘yicha optimal boshqaruv masalasi birinchi marta o‘rganilgan. A.V.Fursikovning ishida taqsimlangan parametrli sistemalardagi optimal boshqaruv masalalari keng o‘rganilgan. S.Albeverio va Sh.A.Alimovning ishi parabolik differensial tenglama uchun vaqt bo‘yicha optimal boshqaruv masalasida, boshqaruv funksiyasi chegarada berilgan holda muhim natijalar olingan. Shuningdek, Chaves-Silva tomonidan xotira tarkibiy qismlariga ega evolyutsion tenglamalarning nol boshqariluvchanligi va Filippe Martin tomonidan Sobolev fazosida nol boshqariluvchanlik mavjudligi uchun ba’zi struktura bo‘yicha to‘lqin tenglamalari o‘rganilgan.

Q.Wangning ishida Apolloniy doirasi nazariyasi asosida bir nechta quvuvchilar tomonidan tezkor qochuvchini ushlash uchun boshlang‘ich holat taqsimoti taklif etilgan. R.Dengning ilmiy ishlarida tekislikda ko‘p quvlovchi va bir qochuvchi o‘rtasidagi o‘yin o‘rganilgan. E.Garkia, D.W.Kasbeer va M.Pachter tomonidan o‘rganilgan ishda uch o‘lchamli, ikki quvlovchi va bir qochuvchidan iborat differensial o‘yin tekshirilgan, bunda tezligi bir xil bo‘lgan quvlovchilar belgilangan hududni qochuvchidan himoya qilish uchun hamkorlik qiladilar. Mualliflar ushbu o‘yinning to‘liq yechimini taqdim etishgan. E.Garkianing ilmiy ishida ko‘p quvlovchi va ko‘p qochuvchili differensial o‘yin o‘rganiladi. O‘yinning qiymat funksiyasi doimiy ravishda farqlanishi aniqlanadi. Bundan tashqari, o‘yinchilar uchun optimal strategiyalar olinadi.

R.Yanning ilmiy ishlari ikki qarama-qarshi jamoa o‘rtasidagi ko‘p o‘yinchi ishtirok etuvchi quvish-qochish differensial o‘yinlarini tadqiq etishga bag‘ishlangan. A.Sh.Kuchkarovning ilmiy ishlarida ko‘p quvlovchi va bir qochuvchidan iborat differensial o‘yini o‘rganilgan, unda o‘yinchilarning dinamikasi bir xil turdagi chiziqli sistemalar bilan tavsiflangan. Mazkur ishda o‘yin davomiyligi aniqlangan, quvlovchilar va qochuvchi o‘rtasidagi minimal masofani hisoblash uchun to‘lov funksiyasi qurilgan va bu funksiya uchun baho olingan.

Agar modellashtirish boshqaruv jarayonlari uchun energiya, yoqilg‘i, resurslar va boshqalar kabi tugaydigan resurslar cheklangan bo‘lsa, boshqaruv funksiyalari integral cheklovlar bilan cheklanadi. G‘.I.Ibragimov va Y.Salleh ishida ko‘p quvlovchilar va bir qochuvchili qochish differensial o‘yini o‘rganilgan, bunda qochuvchi quvlovchilarga nisbatan boshqaruv resursida ustunlikka ega. G‘.I. Ibragimov ishida boshqaruvga integral chegara qo‘yilganda ko‘p quvlovchili va bir qochuvchili chiziqli differensial o‘yini tadqiq etilgan va boshlang‘ich holat uchun qochish strategiyasi qurilgan hamda qochish mumkinligi ko‘rsatilgan.

A.Sh.Kuchkarov ishida Evklid fazosidagi ko‘pyoqlikda ko‘p quvlovchi va bir qochuvchining oddiy harakatli differensial o‘yini o‘rganilgan. Barcha o‘yinchilar bir xil dinamik imkomiyatlarga ega. Agar ma’lum shart bajarilsa, quvish yakunlanishi yoki aks holda qochish mumkinligi ko‘rsatilgan.

Bir nechta quvlovchili o‘yinlarning amaliy qo‘llanilishi bilan bog‘liq mihim tadqiqot sohalaridan biri ko‘p karrali tutish o‘yiniga bag‘ishlangan masalalar hisoblanadi. Quvlovchilar guruhi qochuvchilarni $m, m \geq 1$ marta tutishni maqsad qiladi. Agar m dan kam quvlovchilar qochuvchiga yetib borsa, ularni qochuvchi yo‘q qiladi.

A.I.Blagodatskih va N.N.Petrov bir xil imkoniyatlarga ega bo‘lgan bir guruh qat’iy koordinatadagi qochuvchilar va ko‘p quvlovchilarning bir vaqtda ko‘p karrali tutish masalasini o‘rgangan. Ushbu ishda quvlovchilar bo‘laklarga bo‘lingan qarshi strategiyalarni qo‘llagan holda bir vaqtda ko‘p karrali tutish uchun zarur va yetarli shartlar olingan. N.N.Petrov tomonidan o‘yinchilar dinamikasi kasr tartibli hosilaga ega differensial tenglamalar bilan tavsiflangan holatda ko‘p karrali tutish masalasini hal qilish uchun ba’zi shartlar olingan va barcha o‘yinchilar teng imkoniyatlarga ega ekanligi ko‘rsatilgan.

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti O‘zbekiston Milliy universitetining ilmiy-tadqiqot rejasi doirasida “Differensial tenglamalar va ular bilan bog‘liq matematik sohalarning markaziy muammolari” dasturi asosida amalga oshirildi.

Tadqiqotning maqsadi l_2 Gilbert fazosida cheksiz 2 blok differensial tenglamalar sistemasining yechimi mavjud va yagonaligini isbotlash, kafolatlangan tutish va qochish vaqtlarini topish, optimal tutish vaqtini topish, quvlovchi va qochuvchi uchun optimal strategiya qurish, hamda $\mathbb{R}^n$ fazoda ko‘p karrali tutish masalalarini yechishdan iborat.

Tadqiqotning vazifalari quyidagilardan iborat:

differensial o‘yinda o‘yinchilar boshqaruvlari bo‘yicha aralash cheklovlarga ega bo‘lgan holda ko‘p karrali tutish shartlarini aniqlash;

l_2 Gilbert fazosida o‘yinchilar boshqaruvlariga integral cheklovlar qo‘yilgan differensial o‘yinni ifodalovchi cheksiz 2 blok differensial tenglamalar sistemasi yechimining mavjudligi va yagonaligini isbotlash;

o‘yinchilarning boshqaruvlariga geometrik cheklovlar qo‘yilgan quvish differensial o‘yinida kafolatlangan quvish vaqtini va qochish differensial o‘yinida kafolatlangan qochish vaqtini aniqlash;

o‘yinchilarning boshqaruvlariga integral cheklovlar qo‘yilgan differensial o‘yinda quvlovchi va qochuvchi uchun optimal strategiyalar qurish;

o‘yinchilarning boshqaruvlariga integral cheklovlar qo‘yilgan differensial o‘yinda optimal quvish vaqtini aniqlash.

Tadqiqotning obyekti. l_2 Gilbert fazosida boshqaruvlariga integral va geometrik cheklovlar qo‘yilgan differensial o‘yinlar, vaqt bo‘yicha optimal boshqaruv masalasi.

Tadqiqotning predmeti. Kafolatlangan tutish va kafolatlangan qochish vaqtlarini aniqlash, integral va geometrik cheklovlarda optimal quvish vaqtini aniqlash va o‘yinchilar uchun optimal strategiyalar qurish.

Tadqiqotning usullari. Tadqiqot ishida oddiy differensial tenglamalar nazariyasi, funksional analiz, optimal boshqaruv nazariyasi va differensial o‘yinlar nazariyalari usullari qo‘llanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

$\mathbb{R}^n$ fazoda o‘yinchilar boshqaruvlariga aralash cheklovlar qo‘yilgan differensial o‘yinda ko‘p karrali tutish uchun zarur va yetarli shartlar tengsizliklarni yechish yordamida aniqlangan;

l_2 Gilbert fazosida o‘yinchilar boshqaruvlari uchun integral cheklovlar qo‘yilgan differensial o‘yinni ifodalovchi cheksiz 2 blok differensial tenglamalar sistemasi yechimining mavjudligi va yagonaligi bo‘yicha intuitiv yechimlarning yetarli shartlari keltirilgan;

l_2 Gilbert fazosida o‘yinchilar boshqaruvlariga geometrik cheklovlar qo‘yilgan quvish differensial o‘yinida kafolatlangan tutish hamda qochish differensial o‘yinida kafolatlangan qochish vaqtini aniqlash bo‘yicha zaruriy shartlar topilgan;

l_2 Gilbert fazosida o‘yinchilarning boshqaruvlari integral cheklovli differensial o‘yinda optimal tutish vaqtini aniqlash uchun yangi yetarli shartlar aniqlanib, o‘yinchilar uchun to‘la sistema aniqlaydigan optimal strategiyalar qurilgan.

Tadqiqotning amaliy natijasi. Ushbu dissertatsiya tadqiqotida ziddiyatli boshqaruv jarayonlarini ifodalovchi matematik modellar sifatida quvish-qochish muammolari hal qilingan. Olingan natijalar va taklif etilgan usullar, qarama-qarshi maqsadlar bilan boshqariladigan jarayonlar nazariyasini tadqiq etish bo‘yicha tadqiqotlarda qo‘llanilishi mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalarning ishonchliligi oddiy differensial tenglamalar nazariyasi, funksional analiz, dinamik sistemalar nazariyasi, optimal boshqaruv metodlarini qo‘llagam holda matematikada qabul qilingan deduktiv xulosalarga, jumladan teorema va tasdiqlarning qat’iy isbotlanganligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati.

Tadqiqot natijalarining ilmiy ahamiyati ziddiyatli vaziyatlarda boshqariladigan jarayonlar nazariyasini yanada rivojlantirish hamda qaralayotgan sohada boshqaruv masalalarini bevosita optimallashtirish bilan baholanadi.

Tadqiqot natijalarining amaliy ahamiyati ziddiyatli sharoitlarda va axborot noaniqligida konstruktiv boshqaruv usullarini ishlab chiqishda qo‘llanilishi va ulardan uchuvchisiz uchish apparatlarini boshqarish muammolarida foydalanish imkoniyatlari mavjudligidan iborat.

Tadqiqot natijalarining joriy qilinishi.

Turli chegarali boshqaruvlarga ega cheksiz ikki blok differensial tenglamalar sistemasi bo‘yicha olingan natijalar asosida:

l_2 Gilbert fazosida o‘yinchilar boshqaruvlariga geometrik cheklovlar qo‘yilgan quvish differensial o‘yinida kafolatlangan tutish hamda qochish differensial o‘yinida kafolatlangan qochish vaqtini aniqlash bo‘yicha topilgan yangi yetarli shartlar 01-01-20-2295FR raqamli “Geometrik struktura ichidagi qavariq giperfazoda quvish differensial o‘yini” mavzusidagi ilmiy loyihada, cheksiz sistemalar bilan ifodalangan differensial o‘yin masalalarini tadqiq etishda qo‘llanilgan (Putra Malayziya universitetining 2022-yil 31-oktabrdagi ma’lumotnomasi, Malayziya). Ilmiy natijalarning qo‘llanilishi, qaralgan differensial o‘yinda kafolatlangan quvish va kafolatlangan qochish vaqtlarini aniqlashga imkon bergan.

l_2 Gilbert fazosida o‘yinchilarning boshqaruvlari integral cheklovli differensial o‘yinda optimal tutish vaqtini aniqlash bo‘yicha topilgan yangi yetarli shartlar OT-F-4-(36+32) raqamli “Matematik fizika va optimal boshqaruv masalalarini yechishning zamonaviy usullarini ishlab chiqish. Toq tartibli xususiy hosilali tenglamalar uchun noklassik boshlang‘ich va spektral masalalar va ularning tatbiqlari” mavzusidagi ilmiy loyihada, optimal boshqaruv masalalarini tadqiq etishda qo‘llanilgan (O‘zbekiston Milliy universitetining 2024-yil 17-oktabrdagi №04/11-9008-sonli ma’lumotnomasi). Ilmiy natijalarning qo‘llanilishi, optimal boshqaruv funksiyasini qurishga imkon bergan.

Tadqiqot natijalarining aprobatsiyasi.

Dissertatsiya tadqiqoti natijalari V.I. Romanovskiy nomidagi Matematika institutidagi “Dinamik va boshqariluvchi sistemalar” nomli ilmiy seminarida, Sharof Rashidov nomidagi Samarqand davlat universiteti huzuridagi SMat-01 “Differensial operatorlar spektral nazariyasining nochiziqli evolutsion tenglamalarga tatbiqlari va zamonaviy matematik fizikaning nokorrekt masalalari” nomli differensial tenglamalar kafedrasi ilmiy seminarida, V.I. Romanovskiy nomidagi Matematika institutidagi “Differensial tenglamalar va ularning qo‘llanmalari” nomli ilmiy seminarida va bir qancha ilmiy-amaliy konferensiyalarda, jumladan 4 ta xalqaro va 3 ta respublika miqyosidagi konferensiyalarida taqdim etilgan va muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi.

Dissertatsiya mavzusi bo‘yicha jami 12 ta ilmiy ish chop etilgan, shulardan, 5 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan falsafa fanlari doktori (PhD) ilmiy darajasini olish uchun dissertatsiyalarda taqdim etilgan asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda, jumladan 2 ta ilmiy maqola “Scopus” va “Web of Science Core Collection” ma’lumotlar bazasida indekslangan ilmiy jurnallarda va 3 ta respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat bo‘lib, umumiy hajmi 84 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida tadqiqot mavzusining dolzarbligi va zaruriyligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi ko'rsatilgan, muammoning o'rganilganlik darajasi yoritilgan, tadqiqotning maqsad va vazifalari, obyekti va predmeti ko'rsatilgan, tadqiqot natijalarining ilmiy yangiligi ochib berilgan, olingan natijalarning nazariy va amaliy ahamiyati ko'rsatilgan, tadqiqot natijalarining tatbiqi, shuningdek nashr etilgan ilmiy ishlar va dissertatsiyaning tuzilishi haqida ma'lumotlar keltirilgan.

Birinchi bob "**Boshqaruvlariga geometrik va integral chegara bo'lganda ko'p karrali tutish"** deb nomlanadi va *d*-karrali tutishni amalga oshirish uchun yetarli shartlar qo'yiladi va quvlovchi uchun strategiya quriladi.

O'yinchilari m ta quvlovchilar $x_1, x_2, ..., x_m$ va bitta qochuvchi y orqali ifodalanuvchi tenglamalar sistemasini qaraymiz

$$\begin{aligned} &\dot{x}_i = u_i,\ x_i(0) = x_i^0,\ i = 1, 2, ..., m, \\ &\dot{y} = v,\ y(0) = y^0, \end{aligned} \tag{1}$$

bu yerda, $x_i, x_i^0, y, y^0, u_i, v \in \mathbb{R}^n$, $x_i^0 \neq y^0$, $u_i = \left(u_{i1}, ..., u_{in}\right)$ va $v = \left(v_1, ..., v_n\right)$lar mos ravishda x_i quvlovchilar va y qochuvchining boshqaruv parametrlari.

Quvlovchilar va qochuvchining joiz boshqaruvlari quyidagi shartlarni qanoatlantiradigan $u_i(t)$ va $v(t)$ funksiyalar sifatida aniqlanadi:

$$\begin{aligned} &\sum_{j=1}^{k} u_{ij}^2(t) \le \rho_{i1}^2, && t \ge 0, \quad i = 1, 2, ..., m, \\ &\int_0^\infty \sum_{j=k+1}^{n} u_{ij}^2(t)dt \le \rho_{i2}^2, && t \ge 0, \quad i = 1, 2, ..., m, \\ &\sum_{j=1}^{k} v_j^2(t) \le \sigma_1^2, && t \ge 0, \\ &\int_0^\infty \sum_{j=k+1}^{n} v_j^2(t)dt \le \sigma_2^2, && t \ge 0, \end{aligned} \tag{2}$$

bunda $\rho_{i1}, \rho_{i2}, i = 1, 2, ..., m; \sigma_1, \sigma_2$ berilgan musbat sonlar, $k, k < n$, berilgan butun musbat son.

1-ta'rif. Biz $U_i\left(t, x_i, y, v\right)$ funksiyani x_i quvlovchining strategiyasi deymiz, agar qochuvchining ixtiyoriy $v(t), t \ge 0$ joiz boshqaruvida (1) masala yagona $(x_i(t), y(t))$ yechimga ega bo'lsa, va bu yechim bo'ylab quyidagi shartlarni qanoatlantirsa:

$$\sum_{j=1}^{k} U_{ij}^2\left(t, x_i(t), y(t), v(t)\right) \le \rho_{i1}^2,$$

$$\int_0^\infty \sum_{j=k+1}^{n} U_{ij}^2\left(t, x_i(t), y(t), v(t)\right) \le \rho_{i2}^2,\ t \ge 0,\ i = 1, ..., m.$$

$1 \le d \le m$ bo'lsin.

2-ta'rif. Biz (1)-(2) o'yinda d-karrali tutish mumkin deymiz, agar quvlovchining shunday strategiyasi topilib, qochuvchining ixtiyoriy joiz boshqaruvida, biror o'zaro farqli $i_1, i_2, ..., i_d \in \{1, 2, ..., m\}$ sonlar va $0 < \tau_1 < \tau_2 < ... < \tau_d$ vaqtlarda

$$x_{i_1}(\tau_1) = y(\tau_1), \quad x_{i_2}(\tau_2) = y(\tau_2), \ ..., \quad x_{i_d}(\tau_d) = y(\tau_d)$$

tengliklar o'rinli bo'lsa.

1-teorema.

$$Q = \{i \mid \rho_{i1} > \sigma_1, i \in \{1, 2, ..., m\}\}$$

to'plamning kesishmaydigan $Q_1, Q_2, ..., Q_d$ qism to'plamlari mavjud bo'lib, har bir $j = 1, 2, ..., d$ uchun

$$\sum_{i \in Q_j} \rho_{i2}^2 > \sigma_2^2$$

tengsizlik o'rinli bo'lsin. U holda, har qanday $(y_0, x_{10}, ..., x_{m0})$ boshlang'ich holat uchun, (1)-(2) o'yinda d-karrali tutish mumkin.

Ikkinchi paragrafda $v_1(t), ..., v_n(t), \ u_{i1}(t), ..., u_{in}(t)$ boshqaruvlar quyidagi shartlarni qanoatlantiradi:

$$\begin{aligned}
&\|u_{iK}(\cdot)\|^2 = \int_0^\infty \sum_{j \in K} |u_{ij}(t)|^2 \, dt \le \rho_{iK}^2, \quad i = 1, ..., m, \ K \subset \{1, ..., n\} = N, \\
&\|u_{ij}(\cdot)\|^2 = \int_0^\infty |u_{ij}(t)|^2 \, dt \le \rho_{ij}^2, \qquad i = 1, ..., m, \ \ j \in N \setminus K, \\
&\|v_K(\cdot)\|^2 = \int_0^\infty \sum_{j \in K} |v_j(t)|^2 \, dt \le \sigma_K^2, \quad K \subset \{1, ..., n\} = N, \\
&\|v_j(\cdot)\|^2 = \int_0^\infty |v_j(t)|^2 \, dt \le \sigma_j^2, \qquad j \in N \setminus K,
\end{aligned} \tag{3}$$

ya'ni, K to'plamni tashkil etuvchi barcha o'yinchilarning koordinatalari uchun umumiy integral chegara, qolganlari uchun esa koordinatali integral chegara qo'yiladi.

3-ta'rif. Biz (1), (3) o'yinda d-karrali tutish mumkin deymiz, agar quvlochining shunday $U_i(t, x_i, y, v)$ strategiyasi topilib, qochuvchining ixtiyoriy $v(t), t \ge 0$ joiz boshqaruvi uchun, $x_{i_s}(\tau_s) = y(\tau_s)$ tengliklar biror o'zaro farqli $i_1, i_2, ..., i_d \in \{1, 2, ..., m\}$ sonlar va $\tau_s > 0, s = 1, 2, ..., d$ vaqtlarda bajarilsa.

2-teorema. Agar quyidagi

$$\sigma_K < \rho_{iK}, \ \sigma_j < \rho_{ij},$$
$$i = 1, ..., d, \quad j \in N \setminus K,$$

tengsizliklar o'rinli bo'lsa, (1), (3) o'yinda d-karrali tutish mumkin.

3-teorema. Agar

$$\rho_{iK} > \sigma_K, \quad \rho_{ij} > \sigma_j,$$

$$\rho_{1s}^2 + \rho_{2s}^2 + \ldots + \rho_{ds}^2 > \sigma_s^2,$$

$$i = 1, 2, \ldots, d, \ j \in N \setminus K \setminus \{s\},$$

bo'lsa, quvlash (1) o'yinda yakunlanadi. Bunda K to'plam N to'plamning qism to'plami va $s \in N \setminus K$.

Ikkinchi bob **"Boshqaruvlariga geometrik chegara bo'lganda kafolatlangan tutish va kafolatlangan qochish vaqti"** deb nomlanadi va cheksiz differensial tenglamalar sistemaning yechimi yagona va mavjudligi isbotlangan, hamda kafolatlangan tutish va qochish vaqtlarini hisoblash uchun formulalar aniqlangan.

Bu bobda, biz l_2 Gilbert fazosida quyidagi ko'rinishda ifodalanuvchi differensial tenglamalar sistemasini qaraymiz:

$$\begin{cases} \dot{x}_i = -\alpha_i x_i - \beta_i y_i + w_{i1}, & x_i(0) = x_{i0}, \\ \dot{y}_i = \beta_i x_i - \alpha_i y_i + w_{i2}, & y_i(0) = y_{i0}, \end{cases} \quad i = 1, 2, \ldots \tag{4}$$

bunda α_i, β_i lar haqiqiy sonlar, $x_0 = (x_{10}, x_{20}, \ldots)$, $y_0 = (y_{10}, y_{20}, \ldots) \in l_2$. Faraz qilaylik, α_i chegaralangan manfiy son: $a \le \alpha_i \le 0$, bunda a berilgan son.

$w(t)$ funksiyalarni qaraymiz. Uning $w_i(t) = \left(w_{i1}(t), w_{i2}(t)\right)$, $0 \le t \le T$, $i = 1, 2, \ldots$ o'lchovli funksiyalari ushbu

$$\sum_{i=1}^{\infty} \int_0^T \left(w_{i1}^2(s) + w_{i2}^2(s)\right) ds \le \rho_0^2$$

tengsizlikni qanoatlantiradi. Biz bu funksiyalar sinfini $S(\rho_0)$ bilan belgilaymiz, bunda ρ_0 berilgan musbat son, T yetarlicha katta son deb faraz qilamiz.

Ma'lumki,

$$z(t) = \left(z_1(t), z_2(t), \ldots\right) = \left(x_1(t), y_1(t), x_2(t), y_2(t), \ldots\right),$$

$$z_i(t) = \left(x_i(t), y_i(t)\right), \quad |z_i| = \sqrt{x_i^2 + y_i^2},$$

$$z_0 = (z_{10}, z_{20}, \ldots) = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots).$$

4-ta'rif. $w(\cdot) \in S(\rho_0)$ bo'lsin. $z_i(t)$ uzluksiz koordinatalari ushbu $z_i(t_0) = z_{i0}$, $i = 1, 2, \ldots,$ boshlangich shartni qanoatlantiruvchi $z(t) = (z_1(t), z_2(t), \ldots)$, $0 \le t \le T$, funksiyaga (4) sistemaning yechimi deyiladi, agar u $[t_0, T]$ ning deyarli hamma joyida differensiallansa va $[t_0, T]$ ning deyarli hamma joyida (4) sistemani qanoatlantirsa.

4-teorema. Agar $w(\cdot) \in S(\rho_0)$ va $a \le \alpha_i \le 0$ bo'lsa, har qanday berilgan $T > 0$ uchun $C(0, T; l_2)$ fazoda (4) cheksiz differensial tenglamalar sistemasining yechimi mavjud va u yagona.

Bu bobning ikkinchi paragrafida, l_2 Gilbert fazosida quyidagi differensial o‘yinni o‘rganamiz

$$\begin{aligned} \dot{x}_i &= -\alpha_i x_i - \beta_i y_i + u_{i1} - v_{i1}, \quad x_i(0) = x_{i0}, \\ \dot{y}_i &= \beta_i x_i - \alpha_i y_i + u_{i2} - v_{i2}, \quad y_i(0) = y_{i0}, \end{aligned} \quad i = 1, 2, \ldots \tag{5}$$

bunda α_i, β_i haqiqiy sonlar, $\alpha_i \geq 0$, $(x_{10}, x_{20}, \ldots)$, $(y_{10}, y_{20}, \ldots) \in l_2$, $u = (u_1, u_2, ...)$ quvlovchining boshqaruv parametri va $v = (v_1, v_2, ...)$ qochuvchining boshqaruv parametri, $u_i = (u_{i1}, u_{i2})$ va $v_i = (v_{i1}, v_{i2})$, $i = 1, 2, ...$, ikki olchovli vektordan iborat. Faraz qilamiz, $0 \leq t \leq T$, bunda T yetarlicha katta son va $z_0 = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots) \neq 0$.

5-ta’rif. $u(t) = (u_1(t), u_2(t), \ldots)$, $t \in [0, T]$ funksiyaga quvlovchining joiz boshqaruvi deymiz, agar uning $u_i(t)$ koordinatalari o‘lchovli funksiya bo‘lib, quyidagi shartni qanoatlantirsa

$$\sum_{i=1}^{\infty} \left(u_{i1}^2(t) + u_{i2}^2(t) \right) \leq \rho^2, \quad 0 \leq t \leq T.$$

6-ta’rif. $v(t) = (v_1(t), v_2(t), \ldots)$, $t \in [0, T]$ funksiyaga qochuvchining joiz boshqaruvi deymiz, agar uning $v_i(t)$ koordinatalari o‘lchovli funksiya bo‘lib, quyidagi shartni qanoatlantirsa

$$\sum_{i=1}^{\infty} \left(v_{i1}^2(t) + v_{i2}^2(t) \right) \leq \sigma^2, \quad 0 \leq t \leq T.$$

Bunda $\rho > \sigma$ deb faraz qilinadi.

7-ta’rif. Quyidagi ko‘rinishdagi funksiyaga quvlovchining strategiyasi deymiz

$$U(t, v) = U^0(t) + v = (U_1^0(t) + v_1, U_2^0(t) + v_2, \ldots),$$
$$U_i^0(t) = \left(U_{i1}^0(t), U_{i2}^0(t) \right),$$

agar $U^0(t) = (U_1^0(t), U_2^0(t), \ldots)$ ning $U_i^0(t)$, $0 \leq t \leq T$, koordinatalari o‘lchovli bo‘lib, ushbu tengsizlikni qanoatlantirsa

$$\sum_{i=1}^{\infty} \left(\left(U_{i1}^0(t) \right)^2 + \left(U_{i2}^0(t) \right)^2 \right) \leq (\rho - \sigma)^2, \quad 0 \leq t \leq T.$$

8-ta’rif. Biz θ sonini (5) o‘yinda kafolatlangan tutish vaqti deymiz, agar quvlovchining biror U strategiyasi va qochuvchining har qanday joiz boshqaruvi uchun, biror t', $0 \leq t' \leq \theta$ da $z(t') = 0$ bo‘lsa, bunda $z(t) = (z_1(t), z_2(t), \ldots)$ boshlang‘ich (5) masalaning yechimi.

9-ta’rif. τ sonini kafolatlangan qochish vaqti deymiz, agar ixtiyoriy τ', $0 \leq \tau' < \tau$ son uchun qochuvchining $v_0(t)$ joiz boshqaruvini qurish mumkin bo‘lib, quvlovchining har qanday joiz boshqaruvi uchun barcha $0 \leq t \leq \tau'$ larda $z(t) \neq 0$ bo‘lsa.

1-masala. (5) o‘yinda θ kafolatlangan tutish vaqti va τ kafolatlangan qochish vaqtini topish.

5-teorema. Ushbu

$$\sum_{\alpha_i>0}\frac{\alpha_i^2\left|z_{i0}\right|^2}{\sinh^2(\alpha_i\theta)}+\frac{1}{\theta^2}\sum_{\alpha_i=0}\left|z_{i0}\right|^2=\left(\rho-\sigma\right)^2$$

tenglamani qanoatlantiradigan θ soni (5) o‘yinda kafolatlangan tutish vaqti bo‘ladi.

6-teorema. Har qanday $z_0=(z_{10},z_{20},\cdots)\neq 0$ boshlang‘ich holat uchun,

$$\tau=\sup_i\tau_i,\quad \tau_i=\begin{cases}\dfrac{1}{\alpha_i}\ln\left(\dfrac{\alpha_i\left|z_{i0}\right|}{\rho-\sigma}+1\right), & \alpha_i>0,\\ \dfrac{\left|z_{i0}\right|}{\rho-\sigma}, & \alpha_i=0\end{cases}$$

soni (5) o‘yinda kafolatlangan qochish vaqti bo‘ladi.

To‘rtinchi paragrafda, quvish masalasi uchun kafolatlangan tutish vaqtini topamiz.

l_2 Gilbert fazosida ushbu sistemani qaraymiz

$$\begin{aligned}\dot{x}_i&=\alpha_i x_i-\beta_i y_i+u_{i1}-v_{i1}, \quad x_i(0)=x_{i0},\\ \dot{y}_i&=\beta_i x_i+\alpha_i y_i+u_{i2}-v_{i2}, \quad y_i(0)=y_{i0},\end{aligned}\qquad i=1,\,2,\,... \tag{6}$$

bunda α_i, β_i haqiqiy sonlar va $0\le\alpha_i\le a$.

2-masala. (6) masala uchun θ kafolatlangan tutish vaqtni topish.

7-teorema. Quyidagi

$$\sum_{\alpha_i>0}\frac{\alpha_i^2e^{2\alpha_i t}}{\sinh^2(\alpha_i t)}|z_{i0}|^2+\frac{1}{t^2}\sum_{\alpha_i=0}|z_{i0}|^2=(\rho-\sigma)^2$$

tenglama $t=\theta$ yechimga ega bo‘lsin. Bu θ son (6) o‘yinda kafolatlangan tutish vaqti bo‘ladi.

Uchinchi bob **“Boshqaruvlariga integral chegara bo‘lganda optimal boshqaruv va differensial o‘yinlar masalalari”** deb nomlanadi va cheksiz ikki blok differensial tenglamalar sistemasi bilan ifodalanadigan bitta quvlovchi va bitta qochuvchidan iborat quvish-qochish differensial o‘yin o‘rganilgan:

$$\begin{aligned}\dot{x}_i&=\alpha_i x_i-\beta_i y_i+u_{i1}-v_{i1}, \qquad x_i(0)=x_{i0},\\ \dot{y}_i&=\beta_i x_i+\alpha_i y_i+u_{i2}-v_{i2}, \qquad y_i(0)=y_{i0},\end{aligned} \tag{7}$$

bunda $\alpha_i\ge 0$.

10-ta’rif. Ushbu ko‘rinishdagi

$$U(t,v)=(U_1(t,v),U_2(t,v),\ldots)$$

$$U_i(t,\,v)=(U_{i1}(t,\,v),U_{i2}(t,\,v)),\;\; U:[0,T]\times l_2\to l_2,$$

funksiyani quvlovchining strategiyasi deymiz, agar $U_i(t,\,v)=v_i+w_i(t)$ komponentalari $v_i(t)=(v_{i1}(t),\,v_{i2}(t))$ va $w_i(t)=(w_{i1}(t),\,w_{i2}(t))$ bo‘lgan, hamda qochuvchining har qanday $v(\cdot)\in S(\sigma)$ joiz boshqaruvi uchun quyidagi shartni qanoatlantirsa

$$\sum_{i=1}^{\infty}\int_0^T |U_i(t,v(t))|^2\,dt \le \rho^2,$$

$$|U_k(t,\,v(t))|^2 = U_{k1}^2(t,\,v(t)) + U_{k2}^2(t,\,v(t)),$$

bunda $w(\cdot) = (w_1(\cdot),\, w_2(\cdot),\, \ldots) \in S(\rho - \sigma)$.

11-ta'rif. Quvlovchining $U = U(t,\,v)$ strategiyasi $\theta(U)$ vaqt ichida quvlashni yakunlashni kafolatlaydi deymiz, agar biror $t' \in [0,\,\theta(U)]$ da $z(t') = 0$ bo'lsa. Shuningdek, $\theta(U)$ soni kafolatlangan tutish vaqti deyiladi.

Shubhasiz, har qanday $\theta',\, \theta' \ge \theta(U)$ sonini, U strategiyaga mos keladigan tutish vaqti sifatida ko'rish mumkin. $\theta(U)$ sonlarning eng katta quyi chegarasini $\hat{\theta}(U)$ bilan belgilaymiz. Agar $\hat{\theta}(U^0) = \inf\limits_U \hat{\theta}(U)$ bo'lsa, U^0 strategiya quvlovchining optimal strategiyasi deyiladi.

Ushbu

$$\mu(t) = \left(\rho^2 - \int_0^t \|u(s)\|^2\,ds\right)^{1/2},\quad \nu(t) = \left(\sigma^2 - \int_0^t \|v(s)\|^2\,ds\right)^{1/2}$$

belgilashlarni olamiz. Bunda $\mu^2(t)$ va $\nu^2(t)$ quvlovchi va qochuvchining t vaqtdagi qoldiq resurslari.

12-ta'rif. Ushbu ko'rinishdagi

$$(t,\,z,\,\mu,\,\nu,\,u(t-\varepsilon)) \to V(t,\,z,\,\mu,\,\nu,\,u(t-\varepsilon)),$$
$$V : [0,\,T] \times l_2 \times [0,\,\rho] \times [0,\,\sigma] \times l_2 \to l_2,$$

funksiyani qochuvchining strategiyasi deymiz, agar quyidagi shartlarni qanoatlantirsa

(i) $V = V_0(t)$ qachonki $\mu > \nu$, bu yerda $V_0(t)$ qochuvchining joiz boshqaruvi,

(ii) $v = 0$ qachonki $\mu \le \nu$ va $(t,\,z) \in G_1$, bu yerda $G_1 \subset [0,\,T] \times l_2$,

(iii) $v = V(t,\,z,\,\mu,\,\nu,\,u(t-\varepsilon))$ qachonki $\mu \le \nu$ va $(t,\,z) \in G_2$, bu yerda ε musbat son, $u(\cdot)$ quvlovchining joiz boshqaruvi, $G_2 \subset [0,\,T] \times l_2$,

(iv) $V(\cdot,\,z,\,\mu,\,\nu,\,\cdot)$ o'lchovli funksiya.

13-ta'rif. Qochuvchining V strategiyasi $[0,\,\theta(V))$ oraliqda qochishni kafolatlaydi deymiz, agar quvlovchining har qanday $u(t),\, t \in [0,\,T]$ joiz boshqaruvi uchun $z(t) \ne 0,\quad t \in [0,\,\theta(V))$ bo'lsa. Bunda $\theta(V)$ sonini kafolatlangan qochish vaqti deymiz.

$\theta(V)$ sonlarning eng kichik yuqori chegarasini $\breve{\theta}(V)$ bilan belgilaymiz. Agar $\breve{\theta}(V^0) = \sup\limits_V \breve{\theta}(V)$ bo'lsa, V^0 strategiyani qochuvchining optimal strategiyasi deymiz.

Agar $\hat{\theta}(U^0) = \breve{\theta}(V^0)$ bo'lsa, bu son o'yinning optimal tutish vaqti deyiladi.

3-masala. (6) sistema holatini berilgan $z(0) = z_0$ boshlang'ich holatdan koordinata boshiga o'tkazuvchi optimal vaqtni topish.

Bu masalani yechish uchun quyidagi belgilash olamiz

$$\phi_i(t)=\int_0^t e^{-2\alpha_i t}dt=\begin{cases}\dfrac{1-e^{-2\alpha_i t}}{2\alpha_i}, & \alpha_i\neq 0,\\ t, & \alpha_i=0.\end{cases}$$

Ko‘rish mumkin, $t\to 0^+$ har bir $i=1,2,...$ da $\dfrac{1}{\phi_i(t)}\to+\infty$. $z_0\neq 0$, shuning uchun ushbu tenglamaning chap tomondagi yig‘indining hech bo‘lmaganda bitta hadi nolga teng bo‘lmagan z_{i0} ga mos keladi

$$\sum_{i=1}^{\infty}\frac{|z_{i0}|^2}{\phi_i(t)}=\rho_0^2, \tag{8}$$

$t\to 0^+$ bo‘lganda $+\infty$ ga yaqinlashadi. Bundan tashqari, (8) ning chap qismi $t, t>0$ ning kamayuvchi funksiyasi, chunki $\phi_i(t)$ funksiya o‘suvchi.

Agar

$$\sum_{i=1}^{\infty}2\alpha_i|z_{i0}|^2<\rho_0^2, \tag{9}$$

bo‘lsa, (8) tenglama yagona $t=\theta$ ildizga ega.

8-teorema. Agar $z_0\neq 0$ (9) tengsizlikni qanoatlantirsa, (8) tenglamaning yagona ildizi bo‘lgan θ son, vaqt-optimal boshqaruv masalasi uchun optimal o‘tkazish vaqti bo‘ladi.

Quyidagi tenglamaning chap tomonidagi yig‘indining hech bo‘lmaganda bitta hadi 0 ga teng emas.

$$\sum_{i=1}^{\infty}\frac{|z_{i0}|^2}{\phi_i(t)}=(\rho-\sigma)^2, \tag{10}$$

chunki $z_0\neq 0$.

Agar $\sum_{i=1}^{\infty}2\alpha_i|z_{i0}|^2<(\rho-\sigma)^2$ bo‘lsa, (10) tenglama yagona $t=\theta_1$ yechimga ega.

9-teorema. θ_1 soni (7) o‘yin uchun optimal tutish vaqti bo‘ladi.

XULOSA

Dissertatsiyada, $\mathbb{R}^n$ fazoda ko‘p karrali tutish differensial o‘yinlari, hamda l_2 Gilbert fazosida ikki blok differensial tenglamalar sistemasi uchun quvish va qochish masalalari ustida tadqiqotlar bajarildi.

Tadqiqotda quyidagi asosiy natijalarga erishildi:

1. Bir nechta quvlovchi va bitta qochuvchili differensial o‘yin uchun ko‘p karrali tutish aniqlangan. Bunda o‘yinchilarning boshqaruv parametrlarining ba’zilariga geometrik, qolganlariga esa integral chegara qo‘yilgan.

2. Cheksiz 2-blok differensial tenglamalar sistemasi uchun yechimning yagona va mavjudligi isbotlangan.

3. Cheksiz 2-blok differensial tenglamalar sistemasi uchun quvish va qochish differensial o‘yinlari uchun kafolatlangan tutish vaqti, hamda kafolatlangan qochish vaqti aniqlangan.

4. Quvlovchi uchun aniq strategiya yaratilgan, bu o‘yinni kafolatlangan tutish vaqtida tugallanishini ta’minlaydi.

5. Vaqt-optimal masalasi uchun optimal o‘tkazish vaqti uchun tenglama topilgan. Hamda, ushbu masala uchun optimal boshqaruv qurilgan.

6. Differensial o‘yin uchun optimal tutish vaqti aniqlangan va o‘yinchilar uchun optimal strategiyalar qurilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES
DSc.03/30.12.2019.FM.01.01 NATIONAL UNIVERSITY OF UZBEKISTAN
NATIONAL UNIVERSITY OF UZBEKISTAN

KUCHKAROVA SARVINOZ ATAMURATOVNA

DIFFERENTIAL GAMES DESCRIBED BY INFINITE SYSTEMS OF TWO BLOCK DIFFERENTIAL EQUATIONS

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION
OF THE DOCTOR OF PHILOSOPHY (PhD)
ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2022.2.PhD/FM707.

Dissertation has been prepared at National university of Uzbekistan named after Mirzo Ulugbek.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://ik-fizmat.nuu.uz and in the website of "ZiyoNet" Information and educational portal (http://www.ziyonet.uz).

Scientific supervisors:	**Ibragimov Gafurjan Ismailovich** Doctor of Physical and Mathematical Sciences, Professor
Official opponents:	**Mamadaliev Numanjon** Doctor of Physical and Mathematical Sciences, Professor
	Ibaydullayev Tulanboy Tursunbaevich Candidate of Physical and Mathematical Sciences, Associate Professor
Leading organization:	**Namangan State University**

Defense will take place " 8 " 04 2025 at 14:00 at the meeting of Scientific Council number DSc.03/30.12.2019.FM.01.01 at National university of Uzbekistan named after Mirzo Ulugbek (Address: University str. 4, Almazar area, Tashkent, 100174, Uzbekistan, Tel: (+99871) 246-53-21, Fax ; (+99871) 246 53 21, e-mail: nauka@nuu.uz).

Dissertation is possible to review in Information-resource centre at National university of Uzbekistan named after Mirzo Ulugbek (is registered № 37) (Address: University str. 4, Almazar area, Tashkent, 100174, Uzbekistan, Ph.: (+99871) 246-02--24).

Abstract of dissertation sent out on " 24 " march 2025 year

(Mailing report № 2 on " 24 " march 2025 year)

A.Sadullaev
Chairman of Scientific Council on award of scientific degrees, D.F.-M.S., Academician

R.M.Juraev
Scientific secretary of Scientific Council on award of scientific degrees, PhD

Sh.A.Alimov
Chairman of Scientific seminar under Scientific Council on award of scientific degrees, D.F.-M.S., Academician

INTRODUCTION (abstract of the PhD thesis)

Actuality and demand of the theme of dissertation. The theory of differential games is a branch of mathematical control theory that studies control problems in conflict situations when the movements of objects are described by differential equations. The formulation of problems that make up the content of the theory of differential games presupposes the presence of opposing sides-players, each of which can influence the change in the state of the controlled system. Some functional is given on the motions of this system. The task of each player is to determine such a method of forming control actions that, for any admissible behavior of the opponent, the player guarantees the smallest (or largest) possible value of the given functional. In the theory of differential games, optimal control problems are also considered, complicated by various conditions for control, lack of information, and other reasons. The relevance of these problems the theory of differential games, start to develop rapidly since the middle of the 20th century. The solution of many practical and theoretical problems arising as a result of numerous scientific and practical research conducted worldwide is often reduced to the study of conflicting controlled processes described differential equations. Therefore, the study of conflicting situations described by an infinite system of two-block differential equations in cases where geometric, integral and various constraints are imposed on the control parameters is one of the actual problems of today.

In scientific research conducted in the world, solving conflicting problems described by an infinite system of block differential equations is of great importance. A number of research works are being carried out in the following priority areas on the theory of differential games described by an infinite system of two-block differential equations and its applications: pursuit-evader problems described by infinite system of two-block differential equations with various constraints on the players' controls, application of methods for solving pursuit problems in partial differential equations with various constraints on the control parameters, which adequately reflect the physical processes arising from conflict situations that are often encountered in practice; solving group pursuit-evader problems in a differential game described by an infinite system of two-block differential equations; development of methods for numerically solving differential game problems described by differential equations, etc.

In our country, has paid increased attention is regularly to the current scientific directions of fundamental sciences with practical application. Bringing fundamental research closer to practice has been identified as one of the main tasks in the development of science. Differential game theory is one of the most effective areas for solving problems of process control in conflict situations. As a result of the development of this direction, significant results have been achieved in solving problems in conflict situations characterized by differential equations arising from problems in mathe-matical physics, mechanics, mathematical biology, and economics. In particular, essential tasks[2] in the priority directions such as "the theory of differential equations, algebra and its applications, functional analysis and its applications, the theory of dynamical systems, mathematical statistics, mathematical modeling" have been defined. In the implementation of these tasks, it is momentous to build control models, which provide optimal solutions for differential game problems with various restrictions, in the progress of dynamical systems, differential equations, the theories of mathematical control and dynamical game.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan of February 7, 2017, UP-4947, "On the strategy of action for the further development of the Republic of Uzbekistan", PP-60 of January 28, 2022 "On the Development Strategy of the New Uzbekistan for 2022-2026", UP-2789 dated February 17, 2017 "On measures to further improvement of the activities of the Academy Sciences, organization, management and financing of research activities", UP-4708 of May 7, 2020 «On measures to improve the quality of education and research in the field of mathematics» as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This research was carried out in connection with the priority areas of science and technology in the Republic of Uzbekistan: IV. "Mathematics, Mechanics and Informatics".

The degree of scrutiny of the problem. The notion of differential games was introduced by R.Isaacs, although some differential games were considered before R.Isaacs. The majority of the past literature on this subject is devoted to finite-dimensional space differential games. The theory differential games were developed by The theory differential games were developed by L.D.Bercovitz, W.H.Fleming, L.S.Pontryagin, N.N.Krasovskii, A.Bryson, O.Hajek, A.Friedman, R.J.Elliot, N.J.Kalton, S.Baron, E.F.Mishenko, Y.S.Osipov, A.B.Kurjanskiy, B.N.Pshenichnyi, L.A.Petrosyan, A.I.Subbotin, Chikrii, N.Yu.Satimov, A.A.Azamov, A.Sh. Kuchkarov, M. Tukhtasinov, N.Mamadaliev, B.T. Samatov, G.I. Ibragimov and others.

In the paper of A.A.Azamov et al., for an infinite system of linear ordinary differential equations, the principle diagonal of the coefficient matrix is λ and the upper diagonal entries of the matrix are 1 s. The stability of $\lambda \le -1$ and the controllability of the system were studied.

In the paper of N.Yu.Satimov and M.Tukhtasinov pursuit and evasion differential games were studied for the parabolic equation. Various cases of control constraints (integral, geometric) were analyzed. Two sets were specified such that pursuer wins if the initial state belongs to the first set, and evader wins if the initial state belongs to the second set.

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[2] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 № PQ-4387 "On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan".

In the game with countably many pursuers studied by G.I.Ibragimov et. al. in the Hilbert space l_2, the duration of the game is prescribed. A formula for the value of the game was found and optimal strategies of the players were constructed explicitly. In the work of M.Salimi and M.Ferrara, an optimal approach of a finite or denumerable pursuers to one evader is studied. In that paper a formula for the value of the game and optimal strategies of players are proposed. A time-optimal problem of transition of the state of system into the origin was studied by A.A.Azamov and M.B.Ruziboev. The main result of that paper is the estimate of the optimal time from above.

The work by H.O.Fattorini is the first paper on time-optimal control problem for the parabolic type partial differential equations. The optimal control problems in systems with distributed parameters is widely studied in the paper Fursikov. Interesting results were obtained by S.Albeverio and Sh.A.Alimov for a time-optimal control problem for the parabolic differential equation where the control function is defined on the boundary, and by Chaves-Silva et. al. for the null controllability of evolution equations with memory terms, and by Philippe Martin et. al. for the structurally damped wave equation where the null controllability holds in some suitable Sobolev space and after a fixed positive time independent of the initial conditions.

Differential game problems described by partial differential equations are considered for the first time in the works by J.L.Lions and Yu.S.Osipov. One of the main tools in studying control or/and differential game problems for the systems described by partial differential equations is the method of Fourier.

The work by Q.Wang et al. proposes the initial state distribution to capture a fast evader by multiple pursuers based on the theory of Apollonius circle. The article by R.Deng et al. examines a multiple-pursuer and single-evader reach-avoid planar game under a constant flow field. The work E.Garcia, D.W.Casbeer, A.Von Moll and M.Pachter investigates a three-dimensional, two-pursuer, one-evader reach-avoid differential game. The pursuers collaborate to defend a designated region against an evader with identical speed. The authors present a complete solution to this game. The paper E.Garcia et. al studies a multi-pursuer versus multi-evader differential game. It was established that the value function of the game is continuously differentiable. In addition, the optimal, strategies for the players are obtained.

The paper by R.Yan et al. is devoted to the survey of the recent works on multi-person reach-avoid differential games of two adversarial teams. A differential game of optimal approach of many pursuers and one evader was studied in the work by A.Sh.Kuchkarov et al. The dynamics of players are described by linear systems of the same type. Control functions of players are subject to integral constraints. The duration of the game is fixed, and the payoff functional of the game is the minimum of the distances between the evader and the pursuers when the game terminates. An estimate for the payoff functional of the game was obtained.

If exhaustible resources such as energy, fuel, resources, etc. are restricted for the modeling control processes, then control functions are restricted by integral constraints. In the paper by G.I.Ibragimov and Y.Salleh, an evasion differential game of many pursuers and one evader was considered. The evader has an advantage over pursuers in control resource. The work by Ibragimov et. al. was devoted to a linear differential game of many pursuers with integral constraints. For any initial position, an evasion strategy was constructed and it was shown that evasion is possible.

In the paper A.Sh.Kuchkarov et al. a simple motion differential game of many pursuers and one evader was studied on manifolds with Euclidean metric. All players have equal dynamic capabilities. It was obtained a condition, and if this condition is satisfied, then pursuit can be completed, otherwise evasion is possible.

One of the interesting research areas of differential games of multiple pursuers with significant practical applications is multiple capture game problems. A group of defenders (pursuers) aims to capture attacker(s) (evader(s)) $m, m \geq 1$, times. The defenders get destroyed by the attacker if fewer than m defenders reach the attacker.

A.I.Blagodatskikh and N.N.Petrov studied a simultaneous multiple capture problem of multiple pursuers and a group of rigidly coordinated evaders with equal capabilities. In that paper, necessary and sufficient conditions for simultaneous multiple capture were obtained when pursuers use piecewise-program counter strategies. Using the method of resolving functions N.N.Petrov obtained some conditions to solve the problem of multiple capture when the dynamics of players are given by differential equations with fractional derivatives. All the players have equal capabilities.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. Dissertation research was carried within the framework of the program “Differential Equations and central problems of mathematical fields related to them” in view of the scientific-research plan of National University of Uzbekistan.

The aim of the research is to prove the existence and uniqueness of the solution of an infinite system of 2-block of differential equations in the Hilbert space l_2, to find guaranteed pursuit and evasion times, to determine the optimal pursuit time, to construct the optimal strategies of pursuer and evader, to solve multiple capture in a differential game in space $\mathbb{R}^n$.

Problems of the research:

to find conditional of multiple capture in a differential game with mixed constraints on controls of players in space $\mathbb{R}^n$;

to prove the uniqueness and existence of the solution for the system of infinite 2-block differential equations with integral constraints on controls of players in Hilbert space l_2;

to find a guaranteed pursuit time for the pursuit differential game and a guaranteed evasion time for the evasion differential game with geometric constraints on controls of players in Hilbert space l_2;

to construct the optimal strategies of pursuer and evader in a differential game with integral constraints on controls of players in Hilbert space l_2;

to determine the optimal pursuit time in a differential game with integral constraints on controls of players in Hilbert space l_2.

The object of the research. Differential games with integral and geometric constraints on controls, time-optimal control problem in Hilbert space l_2.

The subject of the research. Determine of guaranteed pursuit and guaranteed evasion time, optimal pursuit time, with integral and geometric constraints on control parameters and construct the optimal strategies for the players.

Methods of the research. In the research work, the theory of ordinary differential equations, functional analysis, mathematical control theory, differential game theories are used.

Scientific novelty of the research is composed of the following:

new sufficient conditions, of multiple capture in a differential game with mixed constraints on controls of players in space $\mathbb{R}^n$ are determined;

new sufficient conditions, of the uniqueness and existence of the solution for the system of infinite 2-block differential equations with integral constraints on controls of players in Hilbert space l_2 are proved;

new formula for a guaranteed pursuit time for the pursuit differential game and a guaranteed evasion time for the evasion differential game with geometric constraints on controls of players in Hilbert space l_2 are found;

new formula for the optimal strategies of pursuer and evader in a differential game with integral constraints on controls of players in Hilbert space l_2 are constructed;

new equation for the optimal pursuit time in a differential game with integral constraints on controls of players in Hilbert space l_2 are determined.

Practical results of the research. In this dissertation research, the pursuit-escape problems are solved as mathematical models representing conflicting control processes. The results obtained and the proposed methods can be used in research on the theory of control processes with conflicting goals.

The reliability of the results of the research. The reliability of the results is based on deductive conclusions accepted in mathematics, including the rigorous proof of theorems and assertions, using the theory of ordinary differential equations, functional analysis, dynamical systems theory, and optimal control methods.

The scientific and practical significance of the research results.

The scientific significance of the results of the dissertation is that its results can serve to further develop the theory of controlled processes in conflict situations.

The practical significance of the results of the dissertation is that its results can be used in the development of constructive control methods in conflict situations and information uncertainty, and they can also be used in the problems of controlling unmanned aerial vehicles.

Implementation of the research results. Based on the results obtained for a system of infinite two-block differential equations with different constraint controls:

the new sufficient conditions found for determining guaranteed pursuit time in a pursuit differential game and the guaranteed evasion time in a evasion differential game with geometric constraints on the players' controls in the l_2 Hilbert space were used in the scientific project 01-01-20-2295FR on the title "Pursuit differential game in a convex hyperspace within a geometric structure" to study differential game problems represented by infinite systems (Reference of University Putra Malaysia dated October 31, 2022, Malaysia). The application of the scientific results made it possible to determine the guaranteed capture and guaranteed escape times in the considered differential game.

the new sufficient conditions found for determining the optimal pursuit time in a differential game with integral constraints on the player' controls in the l_2 Hilbert space were used in the research of optimal control problems in the scientific project OT-F-4-(36+32) on the title "Development of modern methods of solving mathematical physics and optimal management problems. Noclassical initial and spectral problems for odd-order partial differential equations and their applications" (Reference of National University of Uzbekistan dated October 17, 2024, №04/11-9008). The application of scientific results allowed to construct an optimal control function.

Approbation of the research results. The results of the dissertation research were presented and discussed at the scientific seminar "Dynamic and Controlled Systems" at the V.I. Romanovsky Institute of Mathematics, the scientific seminar at the Samarkand State University named after Sharof Rashidov SMat-01 "Applications of the Spectral Theory of Differential Operators to Nonlinear Evolutionary Equations and Irregular Problems of Modern Mathematical Physics", the scientific seminar "Differential Equations and Their Applications" at the V.I. Romanovsky Institute of Mathematics, and a number of scientific and practical conferences, including 4 international and 3 national conferences.

Publications of the research results. On the topic of the dissertation 12 research papers were published in the scientific journals, 5 of which are included in the list of scientific publications suggested by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of these of the Doctor of Philosophy (PhD), including 2 scientific articles in scientific journals indexed in the "Scopus" and "web of Science Core Collection" databases, 3 of them in national scientific journals.

The structure and volume of the dissertation. The dissertation is composed of the introduction, three chapters, conclusion and bibliography, with a total volume of 84 pages.

THE MAIN CONTENT OF THE DISSERTATION

In introduction the motivation of the research and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

The first Chapter is entitled "**Multiple capture with geometric and integral constraints**", and it is the eslablishment of sufficient conditions to achieve *d*-multiple capture, and the devolopment of construction strategies for the pursuers to realize *d*-multiple capture.

The dynamics of m pursuers $x_1, x_2, ..., x_m$ and one evader y are described by the following differential equations

$$\begin{aligned} &\dot{x}_i = u_i,\ x_i(0) = x_i^0,\ i = 1, 2, ..., m, \\ &\dot{y} = v,\ y(0) = y^0, \end{aligned} \tag{1}$$

where $x_i,\ x_i^0,\ y,\ y^0,\ u_i,\ v \in \mathbb{R}^n,\ x_i^0 \neq y^0,\ u_i = (u_{i1}, ..., u_{in})$ and $v = (v_1, ..., v_n)$ are the control parameters of the pursuer x_i and evader y.

The admissible control functions of the pursuers and evader are defined as the measurable functions $u_i(t)$ and $v(t)$ that satisfy the following constraints:

$$\begin{aligned} &\sum_{j=1}^{k} u_{ij}^2(t) \leq \rho_{i1}^2, && t \geq 0, \quad i = 1, 2, ..., m, \\ &\int_0^\infty \sum_{j=k+1}^{n} u_{ij}^2(t)dt \leq \rho_{i2}^2, && t \geq 0, \quad i = 1, 2, ..., m, \\ &\sum_{j=1}^{k} v_j^2(t) \leq \sigma_1^2, && t \geq 0, \\ &\int_0^\infty \sum_{j=k+1}^{n} v_j^2(t)dt \leq \sigma_2^2, && t \geq 0, \end{aligned} \tag{2}$$

where $\rho_{i1}, \rho_{i2}, i = 1, 2, ..., m; \sigma_1, \sigma_2$ are given positive numbers, $k, k < n$, is a given positive integer.

Definition 1. We call the function $U_i(t, x_i, y, v)$ strategy of the pursuer x_i if, for an arbitrary admissible control of the evader $v(t), t \geq 0,$ the initial value problem (1) has a unique solution $(x_i(t), y(t))$ at $u_i = U_i(t, x_i, y, v)$ and $v = v(t)$, and along this solution the following constraints

$$\sum_{j=1}^{k} U_{ij}^2(t, x_i(t), y(t), v(t)) \leq \rho_{i1}^2,$$

$$\int_0^{\infty} \sum_{j=k+1}^{n} U_{ij}^2\left(t,\, x_i(t),\, y(t),\, v(t)\right) \le \rho_{i2}^2,\ t \ge 0,\ i = 1, ..., m,$$

are satisfied.

Let $1 \le d \le m$.

Definition 2. We say that *d*-multiple capture is possible in game (1)-(2), if there exist strategies of pursuers such that, for an admissible control of the evader, we have

$$x_{i_1}(\tau_1) = y(\tau_1), ..., x_{i_d}(\tau_d) = y(\tau_d)$$

for some pairwise distinct numbers $i_1, i_2, ..., i_d \in \{1, 2, ..., m\}$ and times $0 < \tau_1 < \tau_2 < ... < \tau_d$.

Theorem 1. Let there exist disjoint subsets $Q_1, Q_2, ..., Q_d$ of the set

$$Q = \left\{ i \mid \rho_{i1} > \sigma_1,\, i \in \{1, 2, ..., m\} \right\}$$

such that

$$\sum_{i \in Q_j} \rho_{i2}^2 > \sigma_2^2$$

for each $j = 1, 2, ..., d$. Then, for any initial position $\left(y_0, x_{10}, ..., x_{m0}\right)$, *d*-multiple capture is possible in game (1)-(2).

In second paragraph, the controls $v_1(t), ..., v_n(t),\ u_{i1}(t), ..., u_{in}(t)$ satisfy the following conditions:

$$\begin{aligned}
&\|u_{iK}(\cdot)\|^2 = \int_0^{\infty} \sum_{j \in K} |u_{ij}(t)|^2\, dt \le \rho_{iK}^2, \quad i = 1, ..., m,\ K \subset \{1, ..., n\} = N,\\
&\|u_{ij}(\cdot)\|^2 = \int_0^{\infty} |u_{ij}(t)|^2\, dt \le \rho_{ij}^2, \qquad i = 1, ..., m,\ \ j \in N \setminus K,\\
&\|v_K(\cdot)\|^2 = \int_0^{\infty} \sum_{j \in K} |v_j(t)|^2\, dt \le \sigma_K^2, \quad K \subset \{1, ..., n\} = N,\\
&\|v_j(\cdot)\|^2 = \int_0^{\infty} |v_j(t)|^2\, dt \le \sigma_j^2, \qquad j \in N \setminus K,
\end{aligned} \tag{3}$$

that is, the coordinates of all players that form the set K is given by common integral constraints, and for the rest coordinate-wise integral constraints.

Definition 3. We say that *d*-multiple capture is possible in game (1), (3) if there exist strategies of pursuers $U_i\left(t,\, x_i,\, y,\, v\right)$ of pursuers such that, for arbitrary control $v(t), t \ge 0$, of the evader, $x_i(\tau_s) = y(\tau_s)$, for some $i_1, i_2, ..., i_d \in \{1, 2, ..., m\}$ and $\tau_s > 0,\ s = 1, 2, ..., d$.

Theorem 2. If the following inequalities are satisfied

$$\sigma_K < \rho_{iK},\ \sigma_j < \rho_{ij},$$
$$i = 1, ..., d, \quad j \in N \setminus K,$$

then d-multiple capture is possible in the game.

Theorem 3. If

$$\rho_{iK} > \sigma_K, \quad \rho_{ij} > \sigma_j,$$
$$\rho_{1s}^2 + \rho_{2s}^2 + \ldots + \rho_{ds}^2 > \sigma_s^2,$$
$$i = 1, 2, \ldots, d, \;\; j \in N \setminus K \setminus \{s\},$$

where K is a subset of N, $s \in N \setminus K$, then pursuit can be completed in game (1).

The second Chapter is titled "**Guaranteed pursuit time and guaranteed evasion time when geometric constraints on controls** and it is proved existence and uniqueness of solution of an infinite system and obtained a formula for the guaranteed pursuit time and guaranteed evasion time.

In this Chapter, we considered the following infinite system of differential equations:

$$\begin{cases} \dot{x}_i = -\alpha_i x_i - \beta_i y_i + w_{i1}, & x_i(0) = x_{i0}, \\ \dot{y}_i = \beta_i x_i - \alpha_i y_i + w_{i2}, & y_i(0) = y_{i0}, \end{cases} \quad i = 1, 2, \ldots \tag{4}$$

in Hilbert space l_2, where α_i, β_i are real numbers, $x_0, \; y_0 \in l_2$. It is assumed that α_i are bounded negative numbers: $a \le \alpha_i \le 0$, where a is a given number.

The class of functions $w(t)$ with measurable coordinates $w_i(t) = \left(w_{i1}(t), \, w_{i2}(t)\right), \; 0 \le t \le T$, $i = 1, 2, \ldots$ satisfying the condition

$$\sum_{i=1}^{\infty} \int_0^T \left(w_{i1}^2(s) + w_{i2}^2(s) \right) ds \le \rho_0^2$$

we denote by $S(\rho_0)$, where ρ_0 is a given positive number, T is assumed to be sufficiently large number.

Let

$$z(t) = \left(z_1(t), \, z_2(t), \ldots\right) = \left(x_1(t), \, y_1(t), \, x_2(t), \, y_2(t), \ldots\right),$$

$$z_i(t) = \left(x_i(t), \, y_i(t)\right), \quad |z_i| = \sqrt{x_i^2 + y_i^2},$$

$$z_0 = (z_{10}, z_{20}, \ldots) = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots).$$

Definition 4. Let $w(\cdot) \in S(\rho_0)$. A function $z(t) = \left(z_1(t), \, z_2(t), \ldots\right), \;\; 0 \le t \le T$, with continuous coordinates $z_i(t)$ satisfying the initial conditions $z_i(t_0) = z_{i0}$, $i = 1, 2, \ldots,$ is said to be the solution of the system (4) if it is differentiable almost everywhere on $[t_0, T]$ and satisfies almost everywhere on $[t_0, T]$ the system (4).

Theorem 4. If $w(\cdot) \in S(\rho_0)$ and $a \le \alpha_i \le 0$, then for any given $T > 0$ there exists a unique solution of the infinite system of differential equations (4) in the space $C(0,T;l_2)$.

In the second paragraph in this Chapter, we studied a differential game for the following system

$$\begin{aligned} \dot{x}_i &= -\alpha_i x_i - \beta_i y_i + u_{i1} - v_{i1}, \qquad x_i(0) = x_{i0}, \\ \dot{y}_i &= \beta_i x_i - \alpha_i y_i + u_{i2} - v_{i2}, \qquad y_i(0) = y_{i0}, \end{aligned} \quad i = 1, 2, \ldots \tag{5}$$

in Hilbert space l_2, where α_i, β_i are real numbers, $\alpha_i \ge 0$, $(x_{10}, x_{20}, \ldots)$, $(y_{10}, y_{20}, \ldots) \in l_2$, pursuer's control parameter $u = (u_1, u_2, \ldots)$ and evader's control parameter $v = (v_1, v_2, \ldots)$, consist of 2-vectors $u_i = (u_{i1}, u_{i2})$ and $v_i = (v_{i1}, v_{i2})$, $i = 1, 2, \ldots$, respectively. We assumed that $0 \le t \le T$, where T is a sufficiently large number, and $z_0 = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots) \ne 0$.

Definition 5. An admissible control of pursuer is a function $u(t) = (u_1(t), u_2(t), \ldots)$, $t \in [0, T]$, whose coordinates $u_i(t)$ are measurable functions and satisfy the condition

$$\sum_{i=1}^{\infty} \left(u_{i1}^2(t) + u_{i2}^2(t) \right) \le \rho^2, \quad 0 \le t \le T.$$

Definition 6. An admissible control of evader is a function $v(t) = (v_1(t), v_2(t), \ldots)$, $t \in [0, T]$, whose coordinates $v_i(t)$ are measurable and satisfy the condition

$$\sum_{i=1}^{\infty} \left(v_{i1}^2(t) + v_{i2}^2(t) \right) \le \sigma^2, \quad 0 \le t \le T.$$

It is assumed that $\rho > \sigma$.

Definition 7. A strategy of pursuer is a function of the form

$$U(t, v) = U^0(t) + v = (U_1^0(t) + v_1, U_2^0(t) + v_2, \ldots),$$
$$U_i^0(t) = \left(U_{i1}^0(t), U_{i2}^0(t) \right),$$

where $U^0(t) = (U_1^0(t), U_2^0(t), \ldots)$ has measurable coordinates $U_i^0(t)$, $0 \le t \le T$, that satisfy the condition

$$\sum_{i=1}^{\infty} \left(\left(U_{i1}^0(t) \right)^2 + \left(U_{i2}^0(t) \right)^2 \right) \le (\rho - \sigma)^2, \quad 0 \le t \le T.$$

Definition 8. We call the number θ a guaranteed pursuit time in game (5) if for some strategy of pursuer U and for any admissible control of the evader, $z(t') = 0$ at some t', $0 \le t' \le \theta$, where $z(t) = (z_1(t), z_2(t), \ldots)$ is the solution of the initial value problem (1). The pursuer is interested in minimizing the guaranteed pursuit time.

Definition 9. A number τ is called a guaranteed evasion time if for any number τ', $0 \le \tau' < \tau$, we can construct an admissible control $v_0(t)$ such that, for the evader such that for any admissible control of the pursuer, we have $z(t) \ne 0$ for all $0 \le t \le \tau'$. The evader is interested in minimizing the guaranteed evasion time.

Problem 1. Find an equation for a guaranteed pursuit time θ and a guaranteed evasion time τ in game (5).

Theorem 5. The number θ that satisfy the equation

$$\sum_{\alpha_i>0}\frac{\alpha_i^2|z_{i0}|^2}{\sinh^2(\alpha_i\theta)}+\frac{1}{\theta^2}\sum_{\alpha_i=0}|z_{i0}|^2=(\rho-\sigma)^2$$

is a guaranteed pursuit time in game (5).

Theorem 6. For any initial state $z_0=(z_{10},z_{20},\cdots)\neq 0$, the number

$$\tau=\sup_i\tau_i,\quad \tau_i=\begin{cases}\dfrac{1}{\alpha_i}\ln\left(\dfrac{\alpha_i|z_{i0}|}{\rho-\delta}+1\right), & \alpha_i>0,\\ \dfrac{|z_{i0}|}{\rho-\delta}, & \alpha_i=0\end{cases}$$

is a guaranteed evasion time in game (5).

In the fourth paragraph, we found a guaranteed pursuit time for the pursuit differential game.

The differential game is given by following system

$$\begin{aligned}\dot{x}_i&=\alpha_i x_i-\beta_i y_i+u_{i1}-v_{i1}, & x_i(0)&=x_{i0},\\ \dot{y}_i&=\beta_i x_i+\alpha_i y_i+u_{i2}-v_{i2}, & y_i(0)&=y_{i0},\end{aligned}\tag{6}$$

in the Hilbert space l_2, where α_i, β_i are real numbers, $0\le\alpha_i\le a$ for some $a>0$.

Problem 2. Find an equation for a guaranteed pursuit time θ in game (6).

Theorem 7. We let the following equation

$$\sum_{\alpha_i>0}\frac{\alpha_i^2e^{2\alpha_i t}}{\sinh^2(\alpha_i t)}|z_{i0}|^2+\frac{1}{t^2}\sum_{\alpha_i=0}|z_{i0}|^2=(\rho-\sigma)^2$$

have a root $t=\theta$. Then θ is a guaranteed pursuit time in game (6).

The third Chapter is entitled "**Problems of optimal control and differential games with integral constraints on controls**", and it is researched a pursuit-evasion differential game of one pursuer and a single evader governed by an infinite system of two-block differential equation

$$\begin{aligned}\dot{x}_i&=\alpha_i x_i-\beta_i y_i+u_{i1}-v_{i1}, & x_i(0)&=x_{i0},\\ \dot{y}_i&=\beta_i x_i+\alpha_i y_i+u_{i2}-v_{i2}, & y_i(0)&=y_{i0},\end{aligned}\tag{7}$$

under the condition $\alpha_i\ge 0$.

Definition 10. A function

$$U(t,v)=(U_1(t,v),U_2(t,v),\ldots),$$
$$U_i(t,v)=(U_{i1}(t,v),U_{i2}(t,v)),\ U:[0,T]\times l_2\to l_2,$$

with the components U_j of the form $U_i(t,v)=v_i(t)+w_i(t)$, with

$$v_i(t)=(v_{i1}(t),v_{i2}(t))\ \text{and}\ w_i(t)=(w_{i1}(t),w_{i2}(t))$$

that satisfy, for any $v(\cdot)\in S(\sigma)$, the condition

$$\sum_{i=1}^{\infty}\int_0^T|U_i(t,v(t))|^2dt\le\rho^2,$$
$$|U_i(t,v(t))|^2=U_{i1}^2(t,v(t))+U_{i2}^2(t,v(t)),$$

is called a strategy of the pursuer, where $w(\cdot)=(w_1(\cdot),w_2(\cdot),\ldots)\in S(\rho-\sigma)$.

Definition 11. We say that the strategy $U = U(t, v)$ of the pursuer guarantees completion of pursuit starting from the initial state z_0 for the time $\theta(U)$, if $z(t') = 0$ at some $t' \in [0, \theta(U)]$ for the solution $z(t)$ of initial value problem 3 generated by $u(t) = U(t, v(t))$ and any admissible control of evader $v(t), 0 \le t \le \theta(U)$. Also, the number $\theta(U)$ is called a guaranteed pursuit time.

Obviously, any number $\theta', \theta' \ge \theta(U)$, can also be considered as a guaranteed pursuit time corresponding to the same strategy U. Let $\hat{\theta}(U)$ denote the greatest lower bound of the numbers $\theta(U)$.

The strategy U^0 is called the optimal strategy for the pursuer if $\hat{\theta}(U^0) = \inf\limits_U \hat{\theta}(U)$.

Let

$$\mu(t) = \left(\rho^2 - \int_0^t \|u(s)\|^2 \, ds \right)^{1/2}, \quad \nu(t) = \left(\sigma^2 - \int_0^t \|v(s)\|^2 \, ds \right)^{1/2}.$$

Note that $\mu^2(t)$ and $\nu^2(t)$ express the quantities of pursuer's and evader's control resources, respectively, remained at the time t.

Definition 12. We call the function

$$(t, z, \mu, \nu, u(t-\varepsilon)) \to V(t, z, \mu, \nu, u(t-\varepsilon)),$$
$$V : [0, T] \times l_2 \times [0, \rho] \times [0, \sigma] \times l_2 \to l_2,$$

the evader's strategy, if it satisfies the following conditions

(i) $V = V_0(t)$ whenever $\mu > \nu$, where $V_0(t)$ is an admissible control of evader,

(ii) $v = 0$ whenever $\mu \le \nu$ and $(t, z) \in G_1$, where G_1 is a subset of $[0, T] \times l_2$,

(iii) $v = V(t, z, \mu, \nu, u(t-\varepsilon))$ whenever $\mu \le \nu$ and $(t, z) \in G_2$, where ε is a positive number, $u(\cdot)$ is an admissible control of pursuer, G_2 is a subset of $[0, T] \times l_2$,

(iv) $V(\cdot, z, \mu, \nu, \cdot)$ is a measurable function.

Definition 13. We say that the strategy V of the evader guarantees evasion on the interval $[0, \theta(V))$ from the initial state $z_0 \in l_2$ if $z(t) \ne 0, \quad t \in [0, \theta(V))$, for any pursuer's admissible control $u(t), t \in [0, T]$. We call the number $\theta(V)$ guaranteed evasion time.

Let $\breve{\theta}(V)$ be the least upper bound of the numbers $\theta(V)$ that correspond to the strategy V. If $\breve{\theta}(V^0) = \sup\limits_V \breve{\theta}(V)$, we call the strategy V^0 the optimal strategy of the evader.

If $\hat{\theta}(U^0) = \breve{\theta}(V^0)$, this number is called the optimal pursuit time.

Problem 3. Find the optimal time to transfer the state of the system (7) from the given initial state $z(0) = z_0$ to the origin of l_2.

To solve this problem, we let

$$\phi_i(t) = \int_0^t e^{-2\alpha_i t} dt = \begin{cases} \dfrac{1-e^{-2\alpha_i t}}{2\alpha_i}, & \alpha_i \neq 0, \\ t, & \alpha_i = 0. \end{cases}$$

Observe $\dfrac{1}{\phi_i(t)} \to +\infty$ as $t \to 0^+$ for each $i = 1,2,\ldots$. Since $z_0 \neq 0$, therefore at least one term of the sum in the left-hand side of the equation

$$\sum_{i=1}^{\infty} \frac{|z_{i0}|^2}{\phi_i(t)} = \rho_0^2, \tag{8}$$

which corresponds to non zero z_{i0}, approaches $+\infty$ as $t \to 0^+$. Consequently, left-hand side of the equation (8) approaches $+\infty$ as $t \to 0^+$. Moreover, the left part of (8) is decreasing function of t, $t > 0$, because the functions $\phi_i(t)$ are increasing.

If

$$\sum_{i=1}^{\infty} 2\alpha_i |z_{i0}|^2 < \rho_0^2, \tag{9}$$

then equation (8) has a unique root $t = \theta$.

Theorem 8. If $z_0 \neq 0$ satisfies inequality (9), then the number θ, the unique root of equation (8), is the optimal transfer time for the time-optimal control problem.

Since $z_0 \neq 0$, therefore at least one term of the sum in the left-hand side of the equation

$$\sum_{i=1}^{\infty} \frac{|z_{i0}|^2}{\phi_i(t)} = (\rho - \sigma)^2 \tag{10}$$

is not equal to 0.

If $\sum_{i=1}^{\infty} 2\alpha_i |z_{i0}|^2 < (\rho - \sigma)^2$, then equation (10) has the only solution $t = \theta_1$.

Theorem 9. The number θ_1 is the optimal pursuit time in the game (7).

CONCLUSION

In the dissertation, research was conducted on differential games with multiple captures in space $\mathbb{R}^n$, as well as pursuit and evasion problems for an infinite system of two-block differential equations in the Hilbert space l_2.

The following key results were achieved in the research:

1. Multiple capture was determined for a differential game with multiple pursuers and one evader. In this case, some of the players' control parameters are subject to geometric constraints, while others are constrained by integral constraints.

2. The existence and uniqueness of the solution for the system of infinite 2-block differential equations were proved.

3. Guaranteed pursuit and evasion times for differential pursuit and evasion games described by the infinite 2-block differential equations system were determined.

4. An explicit strategy for the pursuer was developed, ensuring that the game is completed within the guaranteed pursuit time.

5. An equation for the optimal transfer time was found for the time-optimal problem, and the optimal control for this problem was constructed.

6. The optimal pursuit time for the differential game was determined, and optimal strategies for the players were constructed.

**НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.01.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
НАЦИОНАЛЬНОМ УНИВЕРСИТЕТЕ УЗБЕКИСТАНА
ИМЕНИ МИРЗО УЛУГБЕКА**

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА

КУЧКАРОВА САРВИНОЗ АТАМУРАТОВНА

ДИФФЕРЕНЦИАЛЬНЫЕ ИГРЫ, ОПИСЫВАЕМЫЕ БЕСКОНЕЧНОЙ СИСТЕМОЙ ДВУХБЛОЧНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

01.01.02 – Дифференциальные уравнения и математическая физика

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

ТАШКЕНТ – 2025

Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образования, Науки и Инноваций Республики Узбекистан за № B2022.2.PhD/FM707.

Диссертация выполнена в Национальном университете Узбекистана имени М. Улугбека.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме)) размещен на веб-странице по адресу http://ik-fizmat.nuu.uz и на информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

Научные руководители:	**Гафуржан Ибрагимов** доктор физико-математических наук, профессор
Официальные оппоненты:	**Нуманжан Мамадалиев** доктор физико-математических наук, профессор
	Туланбай Ибайдуллаев кандидат физико-математических наук, доцент
Ведущая организация:	**Наманганский государственный университет**

Защита диссертации состоится « 8 » 04 2025 года в 14:00 на заседании Научного совета DSc.03/30.12.2019.FM.01.01 при Национальном университете Узбекистана. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 4. Тел.: (+99871) 227-12-24, факс: (+99871) 246-53-21 e-mail: nauka@nuu.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Национального университета Узбекистана (зарегистрирована за № 37). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 4. Тел.: (+99871) 246-02-24).

Автореферат диссертации разослан «24» марта 2025 года.

(протокол рассылки № 2 от « 24 » марта 2025 года).

А. Садуллаев
Председатель Научного совета по присуждению ученых степеней, д.ф.-м.н., академик

Р.М. Жураев
Ученый секретарь Научного совета по присуждению ученых степеней, PhD

Ш.А.Алимов
Председатель Научного семинара при Научном совете по присуждению ученых степеней, д.ф.-м.н., академик

ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является доказательство существования и единственности решения для бесконечной системы 2-блочных дифференциальных уравнений, нахождение гарантированных времен преследования и убегания, определение оптимального времени преследования, построение оптимальных стратегий преследователя и убегающего в играх, описываемых бесконечными системами дифференциальных уравнений в Гильбертовом пространстве l_2, решение задачи многократной поимки в дифференциальной игре в пространстве $\mathbb{R}^n$.

Объектом исследования Дифференциальные игры с интегральными и геометрическими ограничениями на управление, задача оптимального управления по времени в Гильбертовом пространстве l_2.

Научная новизна исследования состоит в следующем:

определены новые достаточные условия для многократного поимка в дифференциальной игре с смешанными ограничениями на управление игроков в пространстве $\mathbb{R}^n$;

доказаны новые достаточные условия для существования и единственность решения для системы бесконечных 2-блочных дифференциальных уравнений с интегральными ограничениями на управление игроков в Гильбертовом пространстве l_2;

найдены новые формулы для гарантированного времени поимка в дифференциальной игре преследования и гарантированного времени убегания в дифференциальной игре убегания с геометрическими ограничениями на управление игроков в Гильбертовом пространстве l_2;

построены новые формулы для оптимальных стратегий преследователя и убегающего в дифференциальной игре с интегральными ограничениями на управления игроков в Гильбертовом пространстве l_2;

определены новые уравнения для оптимального времени преследования в дифференциальной игре с интегральными ограничениями на управление игроков в Гильбертовом пространстве l_2.

Внедрение результатов исследования. На основе результатов, полученных для системы бесконечных двухблочных дифференциальных уравнений с различными ограничными управлениями:

найденные новые достаточные условия для определения гарантированного времени преследования в дифференциальной игре преследования и гарантированного времени убегания в дифференциальной игре убегания с геометрическими ограничениями на управления игроков в Гильбертовом пространстве l_2 были использованы в научном проекте 01-01-20-2295FR на тему «Дифференциальная игра преследования в выпуклом гиперпространстве в рамках геометрической структуры» для исследования задач дифференциальной игры, представленных бесконечными системами (Справка Университета Путра Малайзия от 31 октября 2022 г., Малайзия).

Применение научных результатов позволило определить гарантированные времена поимки и гарантированного убегания в рассматриваемой дифференциальной игре.

найденные новые достаточные условия для определения оптимального времени преследования в дифференциальной игре с интегральными ограничениями на управления игроков в Гильбертовом пространстве l_2 были использованы при исследовании задач оптимального управления в научном проекте ОТ-Ф-4-(36+32) на тему «Развитие современных методов решения задач математической физики и оптимального управления. Неклассические начальные и спектральные задачи для дифференциальных уравнений нечётного порядка и их применения» (Справка Национального университета Узбекистана от 17 октября 2024 г. №04/11-9008). Применение научных результатов позволило построить оптимальную функцию управления.

Структура и объём диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы, общим объемом 84 страницы.

E’lon qilingan ishlar ro‘yxati
List of published works
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1-bo‘lim (part 1; часть 1)

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Avtoreferat “O‘zMU Xabarlari” jurnali tahririyatida tahrirdan o‘tkazilib, o‘zbek, ingliz va rus tillaridagi matnlar o‘zaro muvofiqlashtirildi.

№ 10-3279

Bosishga ruxsat etildi: 20.03.2025.
Bichimi: 60x84 1/16 «Times New Roman»
garniturada raqamli bosma usulda bosildi.
Shartli bosma tabog‘i 2,5. Adadi 100. Buyurtma: № 60
Tel: (99) 832 99 79; (77) 300 99 09
Guvohnoma reestr № 10-3279
«IMPRESS MEDIA» MChJ bosmaxonasida chop etildi.
Manzil: Toshkent sh., Yakkasaroy tumani, Qushbegi ko‘chasi, 6-uy.