MIRZO ULUG‘BEK NOMIDAGI O‘ZBEKISTON MILLIY UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/30.12.2019.FM.01.01 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

QUDAYBERGENOV ALLAMBERGEN KENGESBAEVICH

ELLIPTIK TENGLAMA UCHUN KOSHI MASALASINING YECHILISH SHARTLARI TO‘G‘RISIDA

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PHD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT – 2025

UDK: 917.956.2

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

MIRZO ULUG‘BEK NOMIDAGI O‘ZBEKISTON MILLIY UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/30.12.2019.FM.01.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2024.3.PhD/FM1141 raqam bilan ro'yxatga olingan.

Dissertatsiya Mirzo Ulug'bek nomidagi O'zbekiston Milliy universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus(rezyume)) Ilmiy kengash veb-sahifasi (http://ik-fizmat.nuu.uz) va "ZiyoNet" ta'lim axborot tarmog'iga (http://www.ziyonet.uz/) joylashtirilgan.

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Dissertatsiya avtoreferati 2025 yil "24" mart kuni tarqatildi.

(2025 yil "24" mart dagi 2 -raqamli reestr bayonnomasi).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI MIRZO ULUG'BEK NOMIDAGI O'ZBEKISTON MILLIY UNIVERSITETI

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda ilmiy va amaliy ahamiyatga ega fizika, mexanika, kimyo, biofizika, biotexnologiya va ishlab chiqarish bo‘yicha olib borilayotgan ko‘plab izlanishlar matematik fizika tenglamalari bilan ifodalangan masalalar markaziy o‘rinlardan birini egallaydi. Tabiiyki, fizika va texnologiyaning ba‘zi matematik modellarida parametr qiymatlarini olishda qandaydir xatolik bo‘lishi mumkin va bu holda modelda kichik xatolikka yo‘l qo‘yilmaydi, ya’ni berilganlarning kichik o‘zgarishi yechimning kichik o‘zgarishiga mos kelmasligi mumkin. Boshqacha qilib aytganda, turg‘unlik shartlari bajarilmasligi mumkin. Biz bunday masalalarni nokorrekt qo‘yilgan masalalar sinfiga kiritamiz. Masalan, elliptik tenglamalar uchun Koshi masalasi, giperbolik tenglamalar uchun Dirixle masalasi, manfiy vaqtli parabolik tenglama uchun Koshi masalasi. Elliptik tenglamalar uchun Koshi masalasida chegaraning qaysidir qismida masala berilgan va biz yechimni butun sohada qurish masalasini o‘rganamiz. Shuning uchun ham nokorrekt masalalar nazariy va amaliy jihatdan muhim ahamiyatga ega hisoblanadi.

Jahon amaliyotida nokorrekt qo‘yilgan masalalarni o‘rganish funksiyalarning Furye almashtirishi va Furye qatori bilan bog‘liq. Chunki, masalani yechishda biz Furye almashtirishi va Furye qatorlarini o‘rganishga olib boramiz. So‘nggi paytlarda matematik fizikaning nokorrekt qo‘yilgan masalalari jadal rivojlanmoqda. Elliptik tenglamalar uchun Koshi masalasi amaliyotda turli sohalarda temperatura sohaning bir qismida berilsa, butun sohadagi temperaturani topish imkonini beradi. Shuningdek, geologiya va geofizika sohalarida ham qo‘llaniladi, bu yerda turli sharoitlarda tog‘ jinslari va yer ostidagi har xil qazilma boyliklar miqdorini va manbasini aniqlash uchun foydalaniladi. Bu sohada: chegaralanmagan yo‘lakchada, halqasimon sohada, silindrik va to‘rtburchak sohadalarda elliptik tenglamalar uchun Koshi masalasini yechish muhim ilmiy izlanishlar hisoblanadi.

Mamlakatimizda, ayniqsa, oxirgi yillarda ilmiy va amaliy ahamiyatga ega bo‘lgan fundamental fanlarga, ular bilan bir qatorda geologiya va geofizika masalalarida juda muhim amaliy ahamiyatga matematik fizikaning nokorrekt qo‘yilgan masalalarini yechishga alohida e’tibor kuchaytirildi. Oddiy differensial tenglamalar, xususiy hosilali differensial tenglamalar va ularning sistemalari uchun nokorrekt qo‘yilgan masalalar bo‘yicha ajoyib natijalarga erishildi. “Differensial tenglamalar va matematik fizika” fanining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy-tadqiqot ishlarini olib borish matematikaning asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi[1]. Shuning uchun farmonning bajarilishini ta’minlashda differensial tenglamalar va matematik fizika masalalarini hal etish muhim ahamiyatga ega. Bularning barchasi qaralayotgan masalalarning naqadar dolzarbligi va zarurligini ko‘rsatadi.

Ushbu dissertatsiya ishining predmeti va tadqiqot obyektlari O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi PF-4947-sonli farmoni, 2017-yil 17-fevraldagi “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot

[1] O‘zbekiston Respublikasi Prezidentining 2020-yil 7-maydagi «Matematika sohasidagi ta`lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘grisida»gi PQ-4708-son qarori

ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789-sonli qarori, 2018-yil 27-apreldagi "Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliy joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida"gi PQ-3682-sonli qarori va 2019 -yil 9-iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V. I. Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-4387 sonli Prezident qarori hamda 2020-yil 7-maydagi "Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida"gi PQ-4708 sonli Prezident qarori, bundan tashqari, mazkur faoliyatga tegishli barcha normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur dissertatsiya ishi respublika fan va texnologiyalar rivojlanishining IV “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. En birinchi 1923 yilda Hadamard Laplas tenglamasi uchun Koshi masalasining yechimi uzluksiz ravishda dastlabki ma’lumotlarga bog‘liq emasligini ko‘rsatadigan fundamental misol keltirdi. H.Han va H.J.Reynhardt o‘z maqolalarida silindrik sohadagi elliptik tenglamalar uchun Koshi masalalarini ko‘rib chiqdilar. Sobolev fazolarida Koshi masalalari uchun bir qator turg‘unlik baholari olingan bo‘lib, ular optimal bo‘lib chiqadi. Turg‘unlikni baholashdan xatoliklarni baholash mumkin bo‘lgan bir nechta tartibga solish usullari taklif etiladi. Regulyarizatsiya usullaridan klassik Hadamard misolida ko‘rsatiladigan raqamli yaqinlashishlarni hisoblash uchun foydalanish mumkin.

Atoqli sovet matematigi, akademiklar A.N.Tixonov va M.M.Lavrent’ev, ularning shogirdlari Koshi masalasi Laplas tenglamasi uchun nokorrekt qo‘yilgan va boshqa masalalar uchun korrekt qo‘yilganligini isbotladilar. Shuningdek, ular ushbu nokorrekt masalalarni yechishda regulyarizatsiya metodini taklif qilishdi. Shuningdek, A.N.Tixonov matematik fizikaning nokorrekt qo‘yilgan masalalarini sonli yechish usullarini o‘rgangan. Bundan tashqari, masala yechimining turg‘unligi va va regulyarizatsiya metodi Kabanikhin, Mizoxata, Ladijenskaya monografiyalarida uchraydi.

Keyinchalik fransuz olimlari R.Lates va J.L.Lions o‘z ishlariga kvaziteskarilanish usulini kiritdilar va berilgan tenglamani to‘rtinchi tartibli differensial tenglamaga keltirdilar va Laplas tenglamasini yechishda kvazi-yechim tushunchasini ko‘rsatdilar. O.E. Yaremko bu $r<|z|<1$ halqada Laplas tenglamasi uchun Koshi masalasini yechishning mavjudligi va yagonaligini isbotladi. Koshi masalasini yechishning integral tasviri topildi. Shuningdek, G.Alessandrini, L.Rondi, E.Rosset va S.Vessella keng umumiylikda va deyarli minimal hatolik ostida mohiyatan optimal turg‘unlik natijalarini taqdim etdilar. O‘zlarining argumentlarida umumiy sxema sifatida ular bunday turg‘unlikning barcha natijalarini yolg‘iz qavsli tuzilishdan va uch sferali tengsizlikdan foydalanish orqali olish mumkinligini ko‘rsatadi. T.Kalmenov, U.Iskakova aralash Koshi masalasi va bigarmonik tenglamaning yechilish shartlarini ko‘rib chiqdilar. Ushbu dissertatsiyadagi masalalarimiz bilan bog‘liq so‘nggi ilmiy natijalar Sh. Yarmuhamedov ishlarida ko‘rish mumkin. Ilmiy ishlarida davom ettirish formulasini va Laplas tenglamasi uchun Koshi masalasini yechish uchun regulyarizatsiya metodini ko‘rsatdi. Shuningdek, A.M.Muhamedjanov, R.Yarmuxamedov, Sh.Yarmuxamedov Laplas tenglamasi Koshi masalasini yechish asosida reaksiyalar kesmalarini ekstrapolyatsiya qilish usulini taklif qildilar. Yadro cho‘qqi konstantalarini topish usulini amaliy qo‘llash imkoniyatlari muhokama qildi. A.B.Xasanov, F.R.Tursunov Karleman funksiyasini samarali qurish Koshi masalasing regulyarlashtirilgan yechimini qurishga ekvivalent ekanligini ko‘rsatadi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim yoki ilmiy-tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti O‘zbekiston Milliy universiteti ilmiy tadqiqot ishlari va V.I. Romanovskiy nomidagi matematika instituti F-FA-2021-424–sonli “Butun va kasr tartibli xususiy hosilali differensial tenglamalar uchun chegaraviy masalalarning yechimi” mavzusidagi loyihasi doirasida bajarilgan.

Tadqiqot ishining maqsadi tekislikda berilgan to'rtburchak, qatlam, halqa va silindrsimon sohadagi haroratni topish va tegishli xususiy hosilali differensial tenglamaning yechimi mavjudligini ko‘rsatishdan iborat**.**

Tadqiqotning vazifalari quyidagilardan iborat:

Laplas tenglamasi uchun Koshi masalasining yechimi va to‘rtinchi tartibli xususiy hosilali differensial tenglamaning berilgan silindrik sohada kvaziteskarilanish usulida topilgan yechimi o‘rtasidagi bahoni topish;

berilgan soha uchun elliptik tenglama uchun Koshi masalasining yechimi mavjud bo‘ladigan funksiyalar sinfini topish;

elliptik tenglama uchun Koshi masalasining chegaralanmagan yo‘lakda yechimining mavjudligi va yagonaligini isbotlash;

konsentrik silindrda elliptik tenglama uchun Koshi masalasi yechimining mavjudligi va yagonaligini isbotlash.

Tadqiqotning obyekti. Elliptik tipdagi tenglama, to‘rtinchi tartibli xususiy hosilali differensial tenglama, o‘zgaruvchi koeffitsientli ikkinchi tartibli differensial tenglama, Eyler tenglamasi, xos qiymat masalasi.

Tadqiqotning predmeti. Elliptik tenglama uchun Koshi masalasi.

Tadqiqotning usullari. Tadqiqot ishida matematik fizikaning zamonaviy usullari, o‘zgaruvchilarni ajratish usuli nazariyasi (Furye usuli), Furye almashtirishi va Furye qatorlaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat**:**

Laplas tenglamasi va to‘rtinchi tartibli xususiy hosilali differensial tenglama uchun Koshi masalasining yechimlari topilgan va yordamchi tenglama yechimining berilgan tenglama yechimiga intilishi isbotlangan;

elliptik tenglama uchun Koshi masalasining yechimi berilgan sohada mavjud bo‘ladigan funksiyalar sinfi yordamida topilgan, hamda undan muhim tengsizliklar keltirib chiqarilgan;

elliptik tenglama uchun Koshi masalasining chegaralanmagan yo‘lakdagi yechimining mavjudligi va yagonaligi haqidagi teoremalar maxsus lemmalar yordamida isbotlangan;

konsentrik silindrda Elliptik tenglama uchun Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi teoremalar va muhim aprior baholar uchun maxsus lemmalar isbotlangan;

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Olingan natijalar va dissertatsiyada qo‘llanilgan metodlar oliy ta’lim muassasalari bakalavriat va magistratura talabalari uchun maxsus kurs sifatida o‘qitilishi mumkin. Bundan tashqari, ma’lumki teskari vaqtli parabolik tenglama uchun Koshi masalasi ham nokorrekt masala hisoblanib, olingan natijalardan ushbu masala yechimining mavjudligini va yagonaligini isbotlashda qo‘llanilishi mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalar spektral nazariya usullari, o‘zgaruvchilarni ajratish usuli (Furye usuli) va matematik tahlil usuli yordamida olingan. Olingan barcha natijalar matematik jihatdan to‘g‘ri. Bundan tashqari, olingan dissertatsiya natijalari yuqori impakt-faktorga ega va nufuzli ilmiy jurnallarda chop etilgan hamda Xalqaro va Respublika konferensiyalarida ma’ruza qilinganligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundaki, dissertatsiya ishida olingan ilmiy natijalardan teskari vaqtli parabolik tenglama uchun Koshi masalasini chuqurroq tadqiq qilishda foydalanish mumkin. Xususan, ushbu tadqiqot ishida olingan aprior baholar va ba‘zi muhim tengsizliklar teskari vaqtli parabolik tenglama uchun Koshi masalasining yechimining mavjudligi va yagonaligini isbotlash uchun ishlatilishi mumkin. Bundan tashqari, olingan natijalardan ba‘zi bir nokorrekt qo‘yilgan masalalarni yechimida qator giperbolik kosinus yoki sinusdek cheksizlikka intilsa, ushbu yechimni yaqinlashtirish usullarini o‘rganish uchun foydalanish mumkin.

Tadqiqotning amaliy ahamiyati soha chegarasining bir qismidagi harorat va uning oqimini bilish orqali butun sohadagi haroratni topish masalasini matematik modellashtirishda qo‘llash imkoniyati bilan belgilanadi. Bundan tashqari, agar suv ustidagi temperatura va uning oqimi ma’lum bo‘lsa, u holda suv tagidagi temperaturani topish imkoniyatini beradi.

Tadqiqot natijalarining joriy qilinishi. Berilgan sohada elliptik tenglama uchun qo‘yilgan Koshi masalasi bo‘yicha olingan natijalar asosida:

silindrik sohada va cheksiz yo‘lakda elliptic tenglama uchun qo‘yilgan Koshi masalasining yechimidan Uzb-Ind-2021-87-sonli “Li simmetriyasi tahlili, giperbolik sistemalarning Lyapunov bo‘yicha turg‘unligini tahlil qilish va modellashtirish” mavzusidagi xalqaro amaliy loyihada elliptik tenglama uchun Koshi masalasining yechimini topishda foydalanilgan (O‘zbekiston Milliy universitetining 2024-yil 26-sentyabrdagi 04/11-7840-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanishi konsentrik silindrda elliptik tenglama uchun Koshi masalasi yechimini mavjudligi va yagonaligini ko‘rsatish imkonini bergan;

Laplas tenglamasi uchun Koshi masalasi yechimi bilan bog‘liq olingan natijalardan № AP08855810 “Xususiy hosilali nolokal differensial tenglamalar uchun chegaraviy va boshlang‘ich-chegaraviy masalalarning yechilish masalalari” 2020-2022-y mavzusidagi xorijiy loyihada elliptik tenglama uchun Koshi masalasining yechimining yagonaligi va mavjudligini ko‘rsatishda foydalanilgan (Xoja Axmet Yassawi nomidagi Xalqaro qozoq-turk universitetining 2024-yil 3-oktyabrdagi 04/2685-sonli ma’lumotnomasi, Qozog‘iston). Ilmiy natijaning qo‘llanishi berilgan boshlang‘ich ma’lumotlardan foydalanib aprior baholar olish imkonini bergan;

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiya tadqiqoti natijalari O‘zbekiston milliy universiteti Differensial tenglamalar va matematik fizika kafedrasining “Differensial tenglamalar va matematik fizikaning zamonaviy muammolari” ilmiy seminarida, V.I.Romanovskiy nomidagi Matematika institutida M.S. Salohitdinov va T.D. Juraev nomidagi differensial tenglamalar va matematik fizika bo‘yicha ilmiy seminarida, Toshkent shahridagi Moskva davlat universiteti “Matematik fizikaning zamonaviy muammolari” nomli seminarida va bir qancha ilmiy-amaliy konferensiyalarda, jumladan, 5 ta xalqaro va 3 ta respublika konferensiyalarida muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 15 ta ilmiy ishlar chop etilgan bo‘lib, shundan 7 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktorlik dissertatsiyalarining asosiy ilmiy natijalarini chop etish uchun tavsiya etilgan ilmiy nashrlarda chop etilgan, 2 tasi SCOPUS ma’lumotlar bazalarida indekslangan jurnallarda chop etilgan, 5 tasi respublika jurnallarida chop etilgan maqolalar va 8 tasi tezisdir.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 77 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalarining rivojlanishini ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Chegaralangan sohada elliptik tenglam uchun Koshi masalasining yechilish shartlari**” deb nomlangan birinchi bobida konsentrik silindrda Elliptik tenglama uchun qo‘yilgan Koshi masalasining yechimining mavjudligi va yagonaligi aprior baholar yordamida ko‘rsatilgan. A_σ funksiyalar sinfi kiritilib, Koshi masalasi yechimini ushbu funksiyalar sinfida mavjudligi va yagonaligini isbotlash imkonini beradigan yordamchi tengsizliklar isbotlandi. Va $k(r) = c r^{-2\alpha}$ bo‘lgan hol uchun yechimning aniq ko‘rinishi topilib, oldingi paragrafdagi teoremalarni qanoatlantirishi ko‘rsatilgan.

Birinchi bobdagi ba’zi ta’riflar va tasdiqlarni keltirib o‘tamiz.

Faraz qilayliq issiqlik silindrsimon bir jinsli $\Omega\times\mathbb{R}$ soha ichiga joylashtirilgan. Konsentrik silindrning ichki yuzasidagi tmeperatura va issiqlik oqimining qiymati ma’lum bolsin. Silindrik sohaning tashqi chegarasidagi temperaturaning qiymatini topish talab qilinsin.

Aytaylik Ω ikki o‘lchovli chegaralangan qavariq soha bo‘lsin. x^0 Ω soha ichidan olingan ixtiyoriy nuqta bolsin va $\rho < \mathrm{dis}\{x^0, \partial\Omega\}$.

Quyidagi belgilashni kiritamiz

$$\Omega_\rho = \{(x_1, x_2) \in \Omega : (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 > \rho^2\}.$$

Ma’lumki, $\partial\Omega_\rho = \Gamma_\rho \cup \partial\Omega$ bu yerda

$$\Gamma_\rho = \{(x_1, x_2) \in \mathbb{R}^2 : (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 = \rho^2\}.$$

Quyidagi silindrik sohada issiqliq tarqalish jarayonini qaraymiz

$$D=\Omega_\rho\times\mathbb{R}=\{(x_1,x_2,x_3)\in\mathbb{R}^3:(x_1,x_2)\in\Omega_\rho,\quad -\infty<x_3<+\infty\}.$$

Ichki silindrsimon sirt $\Gamma_\rho\times\mathbb{R}$ - bu issiqlik egallab turgan $\{(x_1-x_1^0)^2+(x_2-x_2^0)^2<\rho^2, x_3\in\mathbb{R}\}$ sohaning chegarasi va temperatura vaqtga bog‘liq emas.

Aytayliq, $u(x_1,x_2,x_3,t)$ $(x_1,x_2,x_3)\in D$ nuqtadagi va $t\geq 0$ vaqt momentidagi temperatura bo‘lsin. Issiqliq tarqalish jarayoni quyidagi tenglama bilan tasvirlanadi

$$\frac{\partial u}{\partial t}-\operatorname{div}[k(x)\operatorname{grad}u]=0,\quad x\in D,\quad t>0, \tag{1.1.1}$$

bu yerda $k(x)$-issiqlik o‘tkazuvchanlik koeffitsienti.

Berilgan sohaning ichki chegarasida quyidagi shartlar o‘rinli bo‘lsin

$$u(x)=\phi(x),\quad \frac{\partial u(x)}{\partial r}=\psi(x),\quad x\in\Gamma_\rho, \tag{1.1.2}$$

bu yerda $r=\sqrt{(x_1-x_1^0)^2+(x_2-x_2^0)^2}$.

ϕ va ψ funksiyalar ma’lum, $u(x)$ funksiyaning berilgan sohaning tashqi chegarasi $\partial\Omega$ dagi qiymatini topishimiz kerak.

Biz tenglamaning vaqtga bog‘liq bo‘lmagan yechimini qidiramiz, ya’ni $u=u(x)$. U holda berilgan (1.1.1) tenglama quyidagi ko‘rinishga keladi

$$Lu(x)\equiv\operatorname{div}[k(x)\operatorname{grad}u]=0.$$

Faraz qilaylik berilgan boshlang‘ich shartlar x_3 ga bog‘liq bo‘lmasin, ya’ni $u=u(x_1,x_2)$.

(r,θ) polyar koordinatasi bilan tanishamiz va markazi x_0 nuqtada bo‘lgan. Shuningdek faraz qilaylik $k=k(r)$ funksiya $C^2[0,+\infty)$ sinfga tegishli va $k(r)>0$ shartni qanoatlantirsin.

Ω sohaning chegarasining tenglamasi polyar koordinatalar sistemasida quyidagicha yozilishi mumkin $r=S(\theta),-\pi\leq\theta\leq\pi$. Bu yerda $S(\theta)$ 2π-davrli silliq funksiya.

Bu holda tenglama quyidagicha ko‘rinishga ega bo‘ladi

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk(r)\frac{\partial u}{\partial r}\right)+\frac{k(r)}{r^2}\frac{\partial^2 u}{\partial\theta^2}=0. \tag{1.1.3}$$

(1.1.2) shartlar esa quyidagicha

$$u(\rho,\theta)=\phi(\theta),\quad \frac{\partial u(\rho,\theta)}{\partial r}=\psi(\theta),\quad \theta\in[-\pi,\pi]. \tag{1.1.4}$$

Bu yerda ϕ va ψ θ argumentning 2π-davrli funksiyasi. U holda faraz qilaylik ϕ va ψ funksiyalar $L_2[-\pi,\pi]$ fazoga tegishli bo‘lsin.

(1.1.3)-(1.1.4) tenglamaning yechimi $u\in C^2(\Omega)$, Ω_ρ sohada (1.1.3) tenglamani qanoatlantiradi va (1.1.4) chegaraviy shartlarni quyidagi manoda qanoatlantiradi:

$$\lim_{r\to\rho^+}\int_{-\pi}^{\pi}|u(r,\theta)-\phi(\theta)|^2d\theta=0, \tag{1.1.5}$$

va

$$\lim_{r\to\rho^+}\int_{-\pi}^{\pi}\left|\frac{\partial u}{\partial r}(r,\theta)-\psi(\theta)\right|^2 d\theta=0. \qquad (1.1.6)$$

A Masala. *Berilgan $\phi\in L_2[-\pi,\pi]$ va $\psi\in L_2[-\pi,\pi]$ funksiyalar uchun (1.1.3)-(1.1.4) masalaning yechimi $u(r,\theta)$ ning $\partial\Omega$ chegaradagi qiymatini toping.*

A masalaning yechimi quyidagi shartni qanoatlantiradi

$$\lim_{\epsilon\to 0}\int_{-\pi}^{\pi}\left|u(S(\theta)-\epsilon,\theta)-\chi(\theta)\right|^2 d\theta=0. \qquad (1.1.7)$$

bu yerda $\chi\in L_2(\partial\Omega)$.

A_σ orqali quyidagi berilgan yo'lakda holomorf bo‘lgan $f(z)$ funksiyalar sinfini belgilaymiz

$$S_\sigma=\{z\in\mathbb{C}:|\operatorname{Im}z|\le\sigma\} \qquad (1.2.1)$$

va quyidagi shartni qanoatlantirsin:

$$f(z+2\pi)=f(z),\quad z=x+iy\in S_\sigma, \qquad (1.2.2)$$

va

$$\|f\|_\sigma^2=\sup_{|y|\le\sigma}\int_{-\pi}^{\pi}|f(x+iy)|^2dx<+\infty. \qquad (1.2.3)$$

Quyidagi Furye qatorini qaraymiz

$$f(x)=\sum_{k=-\infty}^{\infty}f_k e^{ikx}.$$

Teorema 1.2.1. *Ixtiyoriy $f\in A_\sigma$ funksiyalar uchun quyidagi tengsizliklar*

$$\frac{1}{4\pi}\|f\|_\sigma^2\le\sum_{k=-\infty}^{\infty}|f_k|^2\cosh 2\sigma k\le\frac{1}{2\pi}\|f\|_\sigma^2 \qquad (1.2.4)$$

o‘rinli

Quyidagicha belgilash kiritamiz

$$\sigma_1=\ln\frac{R_1}{\rho},\quad \sigma_2=\ln\frac{R_2}{\rho}. \qquad (1.3.3)$$

Ma’lumki, $\sigma_2\ge\sigma_1>0$. $S(\theta)=R=const$ bo‘lgan holda esa quyidagiga ega bo‘lamiz $\sigma_1=\sigma_2=\ln\frac{R}{\rho}$.

Teorema 1.3.1. *Faraz qilaylik berilgan ϕ va ψ funksiyalar uchun A masalaning yechimi mavjud bo‘lsin. U holda*

$$f(\theta)=\phi(\theta)-P\psi(\rho,\theta) \qquad (1.3.4)$$

funksiya ixtiyoriy $\sigma<\sigma_1$ uchun A_σ sinfga tegishli.

Teorema 1.3.2. *Aytaylik (1.3.4) funksiya biror $\sigma>\sigma_2$ uchun A_σ sinfga tegishli bolsin. U holda A masalaning yechimi mavjud va yagona bo‘ladi.*

Ω soha markazi x^0 nuqtada bo‘lgan aylana bo‘lgan holni qaraymiz. U holda quyidagi teorema o‘rinli.

Teorema 1.3.3. *Faraz qilaylik* $S(\theta)=R=const$ *bo‘lsin. U holda A masalaning yechimi mavjud bo‘lishi uchun (1.3.4) funksiyaning* A_σ *sinfga tegishli bo‘lishi zarur va yetarli, bu yerda* $\sigma=\ln\dfrac{R}{\rho}$.

Dissertatsiyaning “**Chegaralanmagan sohada elliptik tenglama uchun Koshi masalasining yechilish shartlari**” deb nomlangan ikkinchi bobida chegaralanmagan yo‘lakda elliptik tenglama uchun qo‘yilgan Koshi masalasining yechimining mavjudligi va yagonaligi aprior baholar yordamida ko‘rsatilgan. A_σ funksiyalar sinfi kiritilib, Koshi masalasi yechimini ushbu funksiyalar sinfida mavjudligi va yagonaligini isbotlash imkonini beradigan yordamchi tengsizliklar isbotlangan. Va oxirgi paragrafida berilgan funksiyaning A_σ sinfga tegishli bo‘lishi haqidagi misol ko‘rsatilgan.

Ikkinchi bobdagi muhim ta’rif va tasdiqlar bilan tanishamiz.

$P\subset\mathbb{R}^3$ qatlamda issiqlik tarqalish jarayonini qaraymiz

$$P=\{(x_1,x_2)\in\mathbb{R}^2,\quad -h(x_1,x_2)<x_3<0\}.$$

Bu yerda h o‘lchovli funksiya va

$$0<\inf_{(x_1,x_2)\in\mathbb{R}^2} h(x_1,x_2)\le \sup_{(x_1,x_2)\in\mathbb{R}^2} h(x_1,x_2)<+\infty.$$

Qatlamning yuqori asosi $\{x_3=0\}$ da temperatura va issiqlik oqimining qiymati ma’lum. Qatlamning quyi asosi $\{x_3=-h(x_1,x_2)\}$ dagi temperaturaning qiymatini topishimiz kerak.

Aytaylik $u(x_1,x_2,x_3,t)$ funksiya (x_1,x_2,x_3) nuqtadagi $t\ge 0$ vaqt momentidagi temperature bo‘lsin va $k(x_1,x_2,x_3)$ issiqlik tarqalish koeffitsienti. Issiqlik tarqalish jarayoni quyidagi tenglama bilan ifodalanadi.

$$\frac{\partial u}{\partial t}-\operatorname{div}[k(x)\operatorname{grad}u]=0. \tag{2.1.1}$$

chegaraviy shartlar

$$u(x_1,x_2,0,t)=\phi(x),\quad (x_1,x_2)\in\mathbb{R}^2, \tag{2.1.2}$$

va

$$\frac{\partial u(x_1,x_2,0,t)}{\partial x_3}=\psi(x),\quad (x_1,x_2)\in\mathbb{R}^2. \tag{2.1.3}$$

Quyi chegaradagi $u(x_1,x_2,-h(x_1,x_2),t)$ temperaturani toping.

Tenglamaning vaqtga bog‘liq bo‘lmagan yechimini qidiramiz. Shuningdek, faraz qilaylik $h(x_1)=h=const$ va issiqlik tarqalish koeffitsienti $k=k(x_3)$ faqat balandlikdan bog‘liq bo‘lsin.

$x=x_1$ va $y=-x_3$ belgilashlarni kiritamiz, u holda quyidagi masalaga kelamiz:

$$\frac{\partial^2 u}{\partial x^2}+\frac{1}{k(y)}\frac{\partial}{\partial y}\left[k(y)\frac{\partial u}{\partial y}\right]=0,\quad x\in\mathbb{R},\quad 0<y<h, \tag{2.1.4}$$

va quyidagi chegaraviy shartlar bilan

$$u(x,0)=\phi(x), \quad \frac{\partial u(x,0)}{\partial y}=\chi(x), \quad -\infty<x<+\infty. \tag{2.1.5}$$

Bu masalani quyidagi sohada qaraymiz

$$\Omega=\{(x,y)\in\mathbb{R}^2 \mid 0<y<h, x\in\mathbb{R}\}.$$

(2.1.4)-(2.1.5) masaladan quyidagi Koshi masalasiga oson kelish mumkin:

$$\operatorname{div}[k(y)\operatorname{grad}v(x,y)]=0, \quad x\in\Omega, \tag{2.1.9}$$

$$v(x,0)=\phi(x), \quad \frac{\partial v(x,0)}{\partial y}=0, \tag{2.1.10}$$

A Masala. *Berilgan $\phi\in L_2(\mathbb{R})$ funksiya uchun (2.1.9)-(2.1.10) masalaning yechimi $v(x,y)$ funksiyaning Ω sohaning yuqori chegarasidagi qiymatini toping.*

U holda A masalaning yechimi $u(x,y)$ funksiya (2.1.9) va (2.1.10) masalani qanoatlantiradi va shunday $\psi(x)$ funksiya mavjud bo‘lib quyidagi shartni qanoatlantiradi

$$\lim_{y\to h-0}\int_{-\infty}^{\infty}|u(x,y)-\psi(x)|^2\,dx=0 \tag{2.1.11}$$

Berilgan $\sigma>0$ uchun A_σ orqali $f\in L_2(\mathbb{R})$ va quyidagi yo‘lakda analitik davom ettirilgan funksiyalar sinfini belgilaymiz

$$S_\sigma=\{z\in\mathbb{C}:|\operatorname{Im}z|<\sigma\}, \tag{2.2.1}$$

va yana $f(z)$ funksiyaning analitik davomi quyidagi shartni qanoatlantirsin

$$\|f\|_\sigma^2=\sup_{|y|<\sigma}\int_{-\infty}^{\infty}|f(x+iy)|^2dx<+\infty. \tag{2.2.2}$$

Furye almashtirishini qaraymiz

$$\widehat{f}(\xi)=\int_{-\infty}^{\infty}f(x)e^{-i\xi x}dx. \tag{2.2.3}$$

Teorema 2.2.1. *Ixtiyoriy $f\in A_\sigma$ funksiyalar uchun quyidagi tengsizliklar*

$$\pi\|f\|_\sigma^2\le\int_{-\infty}^{\infty}|\widehat{f}(\xi)|^2\cosh 2\sigma\xi\,d\xi\le 2\pi\|f\|_\sigma^2 \tag{2.2.4}$$

o‘rinli.

Ushbu paragrafda berilgan masala yechimining mavjudligi va yagonaligi o‘rganilgan. Quyidagi teoremalar o‘rinli.

Teorema 2.3.1. *Aytaylik ϕ funksiya $\sigma=h$ uchun A_σ sinfga tegishli bolsin. U holda berilgan A masalaning yechimi mavjud va yagona.*

Theorem 2.3.2. *Faraz qilaylik berilgan $\phi\in L_2(\mathbb{R})$ funksiya uchun A masalaning yechimi mavjud bo‘lsin. U holda ϕ funksiya $\sigma=h$ uchun A_σ sinfga tegishli.*

Dissertatsiyaning “**Laplas operatori uchun Koshi masalasining regulyarizatsiyasi haqida**” deb nomlangan uchinchi bobida Laplas tenglamasi uchun qo‘yilgan Koshi masalasi yechimi uchun regulyarizatsiya metodini qaraymiz. Yordamchi tenglama uchun Koshi masalasi qaralib yordamchi masala yechimining berilgan masala yechimiga intilishi isbotlangan.

Uchinchi bobdagi muhim ta’rif va tasdiqlar bilan tanishamiz.

Quyidagi silindrik sohada issiqlik tarqalish jarayonini qaraymiz(ingichka bir jinsli truba)

$$S=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2=R^2,\, 0\le z\le h\}.$$

Silindrning quyi asosida temperaturaning qiymati beriladi va issiqlik oqimi yoq. Silindrning yuqori asosidagi temperaturani topishimiz kerak.

Quyidagi silindrik koordinatalar sistemasi bilan tanishamiz

$$r=\sqrt{x^2+y^2},\quad \theta=\frac{y}{x},\quad z=z.$$

U holda

$$S=\{(r,\theta,z): r=R, -\pi\le\theta\le\pi, 0\le z\le h,\}$$

Issiqlik tarqalish jarayoni quyidagi tenglama bilan ifodalanadi

$$\frac{\partial u}{\partial t}=\Delta u.$$

Demak, $u(\theta,z,t)$ temperatura uchun quyidagi tenglamaga kelamiz

$$\frac{\partial u}{\partial t}=\frac{k(z)}{R^2}\frac{\partial^2 u}{\partial\theta^2}+k(z)\frac{\partial^2 u}{\partial z^2}+k'(z)\frac{\partial u}{\partial z}. \tag{3.1.1}$$

va quyidagi chegaraviy shartlar bilan

$$u(-\pi,z,t)=u(\pi,z,t),\quad \frac{\partial u}{\partial\theta}(-\pi,z,t)=\frac{\partial u}{\partial\theta}(\pi,z,t),\quad 0\le z\le h, \tag{3.1.2}$$

$$\frac{\partial u(\theta,0,t)}{\partial z}=0,\quad -\pi\le\theta\le\pi, \tag{3.1.3}$$

and

$$u(\theta,0,t)=\phi(\theta),\quad -\pi<\theta<\pi. \tag{3.1.4}$$

Silindrning $z=h$ yuqori asosidagi $u(\theta,h,t)$ temperaturani topishimiz kerak.

Berilgan tenglamaning vaqtga bo‘g‘liq bolmagan $u=u(\theta,z)$ yechimini qidiramiz. U holda (3.1.1) tenglama quyidagi korinishga ega boladi

$$\frac{k(z)}{R^2}\frac{\partial^2 u}{\partial\theta^2}+k(z)\frac{\partial^2 u}{\partial z^2}+k'(z)\frac{\partial u}{\partial z}=0.$$

O‘z navbatida $R=1$ deb faraz qilsak, quyidagi tenglamani qaraymiz

$$k(z)\frac{\partial^2 u}{\partial\theta^2}+k(z)\frac{\partial^2 u}{\partial z^2}+k'(z)\frac{\partial u}{\partial z}=0,\quad (\theta,z)\in S, \tag{3.1.6}$$

chegaraviy shartlar

$$u(\theta,0)=\phi(\theta),\quad \frac{\partial u(\theta,0)}{\partial z}=0,\quad -\pi\le\theta\le\pi. \tag{3.1.7}$$

$D(S)$ orqali berilgan S silindrda cheksiz marta differensiallanuvchi va $z=h$ asosiga yaqin no‘lga aylanadigan funksiyalar sinfini belgilaymiz.

Ta'rif 3.1.1. *Aytaylik* $\phi \in L_2[-\pi,\pi]$ *bo'lsin.* $L_2(S)$ *fazodan olingan* $u(\theta,z)$ *funksiya uchun (3.1.6)-(3.1.7) masalaning yechimi mavjud boladi, agarda ixtiyoriy* $v \in D(S)$ *funksiya uchun quyidagi tenglik*

$$\int_S u(\theta,z)\left(k(z)\frac{\partial^2 v}{\partial\theta^2}+k(z)\frac{\partial^2 v}{\partial z^2}+k'(z)\frac{\partial v}{\partial z}=0\right)d\theta dz=\int_{-\pi}^{\pi}\phi(\theta)\frac{\partial v}{\partial z}(\theta,0)d\theta \tag{3.1.8}$$

o'rinli bo'lsa.

Tasdiq 3.1.1. *Agar (3.1.6)-(3.1.7) masalaning yechimi mavjud bo'lsa, u holda bu yechim yagona bo'ladi.*

Quyidagi tasdiqni oson isbot qilish uchun $k(z)=1$ deb tanlab olamiz. U holda yechimni quyidagi ko'rinishda qidiramiz

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}c_k(z)e^{ik\theta}.$$

$c_k(z)=\phi_k\cosh kz$ koeffitsientlar va izlangan yechim quyidagicha ko'rinishda boladi

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}\phi_k\cosh kz\, e^{ik\theta}.$$

Quyidagi belgilash kiritamiz

$$\psi(\theta)=u(\theta,h). \tag{3.1.23}$$

ψ - funksiyaning yoyilmasi quyidagicha boladi

$$\psi(\theta)=\sum_{k=-\infty}^{\infty}\psi_k e^{ik\theta}.$$

Quyidagi yordamchi masalani qaraymiz

$$\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial\theta^2}+\alpha\frac{\partial^4 u}{\partial\theta^4}=0,\quad -\pi<\theta<\pi,\quad 0<z<h, \tag{3.1.25}$$

(3.1.7) bilan bir xil chegaraviy shartlarga ega.

Yechimni quyidagi ko'rinishda qidiramiz

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}c_k(z)e^{ik\theta}$$

noma'lum $c_k(z)$ koeffitsient bilan birga.

$c_k(z)=\phi_k\cos\mu_k z$ koeffitsientlarni ushbu korinishda topamiz va izlangan yechim quyidagi ko'rinishda bo'ladi

$$u_\alpha(x,t)=\sum_{k=-\infty}^{\infty}\phi_k\cos\mu_k z\, e^{ik\theta}, \tag{3.1.26}$$

bu yerda

$$\mu_k=\mu_k(\alpha)=\sqrt{\alpha k^4-k^2}. \tag{3.1.27}$$

Faraz qilaylik chegaraviy qiymatlar biror $\delta>0$ xatolik bilan berilsin. Buning ma'nosi shundan iboratki ϕ funksiyaning o'rniga ϕ_δ funksiyani olamiz

$$\| \phi(\theta) - \phi_\delta(\theta) \|_{L_2(\mathbb{T})} \le \delta. \qquad (3.2.1)$$

Aytaylik $u_{\delta,\alpha}(\theta, z)$ funksiya (3.1.25) tenglamaning quyidagi chegaraviy shartlardagi yechimi bo‘lsin

$$u_{\delta,\alpha}(\theta, 0) = \phi_\delta(\theta). \qquad (3.2.2)$$

Agar izlanayotgan funktsiya faqat $L_2(\mathbb{T})$ fazosiga tegishli ekanligi ma‘lum bo‘lsa, quyidagi yaqinlashish natijasi yaqinlashish tezligi bahosini hisobga olmasdan o‘rinli bo‘ladi.

Teorema 3.2.1. *Faraz qilaylik $\psi(\theta) = u(\theta, h)$ funksiya mavjud va $L_2(\mathbb{T})$ fazoga tegishli bolsin. Quyidagicha belgilash kiritamiz*

$$\alpha = \alpha(\delta) = \frac{\lambda^2}{\ln^2 \delta}, \quad 0 < \delta < 1, \qquad (3.2.9)$$

bu yerda $\lambda > h$.

U holda

$$\lim_{\delta \to 0^+} \| u_{\delta,\alpha}(\theta, h) - \psi(\theta) \| = 0. \qquad (3.2.10)$$

Teorema 3.2.2. *Faraz qilaylik $\psi \in L_2^\beta(\mathbb{T})$ bo‘lsin, bu yerda $0 < \beta \le 3$, va quyidagi shart o‘rinli bo‘lsin*

$$\| \psi \|_\beta \le 1. \qquad (3.2.13)$$

Aytaylik $\alpha = \alpha(\delta)$ (3.2.9) tenglik bilan aniqlangan bo‘lsin. U holda $0 < \delta < 1$ intervaldan olingan inxtiyoriy δ uchun quyidagi baho

$$\| u_{\delta,\alpha}(\theta, h) - \psi(\theta) \| \le C(h, \beta) \left(\ln \frac{1}{\delta} \right)^{-2\beta/3}, \quad 0 < \delta < 1, \qquad (3.2.14)$$

o‘rinli.

XULOSA

Dissertatsiyada o‘zgaruvchilarni ajratish usuli (Furye usuli) va Furye almashtirishi yordamida yo‘lakda va silindrdagi elliptik tenglama uchun Koshi masalasini o‘rganishga bag‘ishlangan. Ko‘rib chiqilayotgan masalalar yechimining mavjudligi va yagonaligi aprior baholar yordamida ko‘rsatilgan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Laplas tenglamasi va to‘rtinchi tartibli qisman differensial tenglama uchun Koshi masalasining yechimlari topildi va yordamchi tenglamaning yechimi berilgan tenglamaga yondashishi isbotlangan;

2. Elliptik tenglama uchun Koshi masalasining qaysi yechimi berilgan sohada mavjud bo‘lishi mumkin bo‘lgan funksiyalar sinfi topildi, muhim tengsizliklar isbotlangan;

3. Konsentrik silindrda elliptik tenglama uchun Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi teoremalar isbotlangan;

4. Cheksiz yo‘lakda elliptik tenglama uchun Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi teoremalar isbotlangan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES
DSc.03/30.12.2019.FM.01.01 NATIONAL UNIVERSITY OF UZBEKISTAN
NATIONAL UNIVERSITY OF UZBEKISTAN

QUDAYBERGENOV ALLAMBERGEN KENGESBAEVICH

ON THE SOLVABILITY CONDITIONS OF THE CAUCHY PROBLEM FOR THE ELLIPTIC EQUATION

01.01.02 – Differential Equations and Mathematical Physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PHD)
ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number № B2024.3.PhD/FM1141

Dissertation has been prepared at National university of Uzbekistan named after Mirzo Ulugbek.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://ik-fizmat.nuu.uz and in the website of "ZiyoNet" Information and educational portal (http://www.ziyonet.uz).

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Defense will take place " 8 " april 2025 at 15[00] at the meeting of Scientific Council number DSc.03/30.12.2019.FM.01.01 at National university of Uzbekistan named after Mirzo Ulugbek (Address: University str. 4, Almazar region, Tashkent, 100174, Uzbekistan, Tel: (+99871) 246-53-21, Fax (+99871) 246 53 21, e-mail: nauka@nuu.uz).

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Abstract of dissertation sent out on "24" march 2025 year

(Mailing report № 2 on "24" march 2025 year)

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INTRODUCTION

Actuality and demand of the theme of the dissertation. In a global scale, many scientific and practical researches in physics, mechanics, chemistry, biophysics, biotechnology and production conducted are brought to the ill-posed problem which expressed by equations of mathematical physics occupy a central place. Naturally, in some mathematical models of physics and technology, there maybe some error in the taking of parameter values and in this case, model is not allow a small error, i.e small change of givens may not correspond of small change of solution. In other words, stability conditions may not fulfilled. We include such problems in the class of ill-posed problems. For instance, Cauchy problem for elliptic equations, Dirichlet problem for hyperbolic equations, Cauchy problem for parabolic equation with negative time. In heat conducting process, we know the temperature at the currently time in some domain and stability problem is studied in problem of finding the temperature before the currently time and we need to find conditions which provide the stability. Problem is given on some part of boundary in Cauchy problem for Elliptic equations and we study problem of building the solution on whole domain. But, in there stability conditions are broken and it leads to difficulty to find the solution. That is why, ill-posed problems are considered important from both theoretical and practical sides

In world practise, studying ill-posed problems are related to the Fourier transform of functions and Fourier series of functions. Because, we lead to studying of Fourier transform and Fourier series while solving the problem. In recent times, ill-posed problems of mathematical physics have been rapidly developed. The Cauchy problem for elliptic equations is used in practice in various fields, where the temperature in one part of the sphere is given, and the temperature in the entire sphere can be found. It is also used in the fields of geology and geophysics, where it is used to determine the amount and source of various rocks and minerals under different conditions. In this field: solving the Cauchy problem for elliptic equations in unbounded stripe, ring-type domain, cylinder and rectangle-type domain is accounted scientific researches.

In our country, in particular, in recent years, special attention has been paid to fundamental sciences of scientific and practical importance, as well as to solving incorrectly posed problems of mathematical physics, which have very important practical significance in the issues of geology and geophysics. Remarkable results were obtained by ill-posed problem of ordinary differential equations, partial differential equations and their systems. Conducting scientific research at the level of international standards in the priority areas of “Differential Equations and Mathematical Physics”[2] has been defined as the main tasks and areas of activity of the mathematics. Therefore, it is important to solve problem of differential equations and mathematical physics in ensuring the execution of decree. All of this shows how urgent and necessary the issues under consideration are.

[2] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292.

The subject and object of this dissertation work is has been chosen according to actual directions, mentioned in the Decree of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On measures to further improve of the activities of the Academy of Sciences, organization, management and financing of research activities”, PP-3682 dated April 27, 2018 “On measures to further improve the system of practical implementation of innovative ideas, technologies and projects” and PP-4387 from July 9, 2019 “On measures to further development of mathematical education and sciences, total improvement of the activity of the Uzbekistan Academy of Sciences V. I. Romanovsky Institute of Mathematics” and also PD-4708 dated May 7, 2020 “On measures to Improve the quality of education and research in mathematics”. In addition, this dissertation work could be used to extent in the implementation of the tasks defined in all normative legal documents related to this activity.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. First of all, Hadamard in 1923 provided a fundamental example which shows that a solution of a Cauchy problem for Laplace’s equation does not depend continuously upon the data. H. Han and H.J. Reinhardt considered Cauchy problems for elliptic equations on a cylindrical domain in R^{n+1} in their paper. A series of stability estimates for the Cauchy problems in Sobolev spaces are derived, which turn out to be optimal. From the stability estimates, several regularization methods are proposed for which error estimates are available. The regularization methods can be used for computing numerical approximations which will be demonstrated by the classical Hadamard’s example.

The prominent soviet mathematician, academicians A.N.Tikhonov and M.M.Lavrentyev, their disciples and followers proved that the Cauchy problem is continuously ill-posed for the Laplace equation and well-posed for other problems. And also they suggested the regularization of these ill-posed problems. Also, A.N.Tikhonov studied numerical solving methods of ill-posed problem of mathematical physics. Furthermore, stability of solution of problem and regularization methods meet in monography of Kabanikhin, Mizohata, Ladyzhenskaya.

Later, french scientists R.Lates and J.L.Lions included quasi-inversion method in their works and they brought given equation to fourth order differential equation and shown that the quasi-solution approaches to solution of Laplace equation. O.E.Yaremko has proved the existence and uniqueness of the solution of the Cauchy problem for Laplace equation in this $r<|z|<1$ ring. Integral representation for the solution of the Cauchy problem was found. Also, G.Alessandrini, L.Rondi, E.Rosset, and S.Vessella have provided essentially optimal stability results, in wide generality and under substantially minimal assumptions. As a general scheme in their arguments, they show that all such stability results can be derived by the use of a single building brick, three-spheres inequality. T.Kal’menov, U.Iskakova have considered solvabilities of Mixed Cauchy problem and Biharmonic equation. Latest scientific results which related to our issues in this dissertation have been investigated by Sh.Yarmukhamedov . He showed an explicit continuation formula and a regularization procedure for a solution to the Cauchy problem fot the Laplace equation. Also, A.M.Mukhamedzhanov, R.Yarmukhamedov, Sh.Yarmukhamedov, proposed a method for extrapolating the cross sections of reactions on the basis of solution of the Laplace equation Cauchy problem. The posibility of practical application of the method for finding nuclear vertex constants was discussed. A.B.Khasanov, F.R.Tursunov have showed that the effective construction of the Carleman function is equivalent to the construction of a regularized solution to the Cauchy problem.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This PhD dissertation was carried out scientific research works of the National University of Uzbekistan and as part of the program “Solution of the boundary value problem for the fractional order and whole order partial differential equations” in accordance with the research plan of V. I. Romanovskiy Institute of Mathematics.

The aim of research work consists of finding temperature at the given rectangle, layer, ring and cylindrical domain on plane and showing existence of the solution of respectively partial differential equation.

Problems of the research consist of the following**:**

to find the estimate between solution of Cauchy problem for Laplace equation and solution of fourth order partial differential equation getting by quasi-inversion method on given cylindrical domain;

to find the class of function which to be existent of solution of Cauchy problem for elliptic equation on given domain;

to prove the existence and uniqueness of the solution of Cauchy problem for the elliptic equation on concentric cylinder;

to prove the existence and uniqueness of the solution of Cauchy problem for the elliptic equation in unbounded stripe;

The research object. Elliptic type equation, fourth order partial differential equation, second order differential equation with variable cofficients, Euler equation, eigenvalue problem.

The research subject. Cauchy problem for elliptic type equations

Research methods. Modern methods of mathematical physics, theory of separation method (Fouririer method), Fourier transform and Fourier series are used in the research work.

Scientific novelty of the research work consists of the following:

solutions of the Cauchy problem for Laplace euqation and fourth order partial differential eqaution were found and it was proved that the solution of auxiliary equation approaches to the solution of the given equation ;

class of function A_σ was found which solution of the Cauchy problem for elliptic equation can be exist on given domain, important inequalities were proved;

theorems were proved about existence and uniqueness of solution of the Cauchy problem for the elliptic equation in unbounded stripe;

theorems were proved about existence and uniqueness of solution of the Cauchy problem for the elliptic equation on concentric cylinder and important lemmas about aprior estimates were proved and special lemmas were proved for important a priori estimates;

Practical results of the research consists of the following:

The obtained results and the methods used in the dissertation can be taught as a special course for bachelor students and master's students at higer education institutions. In addition, obtained results can be applied on solving Cauchy problem for parabolic equation with negative time.

The reliability of the results of the study. The results were obtained using by the methods of spectral theory, method of separation of variables (the Fourier method) and method of mathematical analysis. All obtained results are mathematically correct. Also, the results of the dissertation are supported by the fact that they have been published in prestigious scientific journals with a high impact factor and presented at international and republican conferences.

Scientific and practical significance of the research results. The scientific significance of the research results is that the scientific results obtained in the dissertation work can be used in more in-depth research of Cauchy problem for parabolic equation with negative time. In particular, a priori estimates and some important inequalities obtained in this research work can be used to prove the existence and uniqueness of the solution of the Cauchy problem for parabolic equation with negative time. Also, the results obtained can be used to study methods for approximating the solution of some ill-posed problems when a series of hyperbolic cosine or sine tends to infinity.

The practical significance of the research is determined by the possibility of applying the problem of finding the temperature in the entire field, knowing the temperature in a part of the boundary of the field and its flow, in mathematical modeling. In addition, if the temperature above the water and its flow are known, it allows you to find the temperature below the water.

Implementation of the research results. The results obtained on Cauchy problem for elliptic equation on given domain were applied in the following research projects:

from the solution of the Cauchy problem for the elliptic equation in the cylindrical domain and unbounded stripe, in the international practical project "Analysis of Lie symmetry, analysis and modeling of Lyapunov stability of hyperbolic systems" No. UZB-Ind-2021-87 was used to find a solution to the Cauchy problem for the elliptic equation (reference number 04/11-7840 of the National University of Uzbekistan dated September 26, 2024). The application of the scientific result made it possible to show the existence and uniqueness of the solution of the Cauchy problem for the elliptic equation in the concentric cylinder;

from the results obtained related to the solution of the Cauchy problem for the Laplace equation No. AR08855810 "Problems of solving boundary and initial-boundary problems for nonlocal differential equations with particular derivatives" in the 2020-2022 foreign project, the uniqueness and existence of the solution of the Cauchy problem for the elliptic equation used to show (International Kazakh-Turkish named after Khoja Akhmet Yassawi reference number 04/2685 dated October 3, 2024 of the University, Kazakhstan). The application of the scientific result made it possible to obtain a priori estimates using the given initial data;

Approbation of the research results. The results of this research were discussed at scientifis seminar of National University of Uzbekistan which called "Modern problem of differential equations and mathematics physics" and at scientific seminar of Institute of Mathematics named after V.I. Romanovskiy by Differential equations and mathematics physics, at the seminar of Moscow State University named after M.V. Lomonosov in Tashkent which called “Modern problems of mathematical physics” also, at 5 international and 3 national scientific conferences.

Publications of the research results. On the topic of the dissertation 16 research papers were published in the scientific journals, 8 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, 2 of them are included the Scopus, 8 theses.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The total volume of the dissertation is 77 pages.

MAIN CONTENT OF THE DISSERTATION

The **Introduction** substantiates the actuality and demand of the theme of the dissertation, determines the correspondence of the study to priority areas of development of science and technology, provides an overview of foreign scientific research on the dissertation topic and the degree of study of the problem, formulates goals and objectives, identifies the object and subject of the study, outlines the scientific novelty and practical results of the study, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on published works and information on the structure of the dissertation.

The first chapter of the dissertation titled “**Solvability conditions of the Cauchy problem for the elliptic equation in bounded domain**”, the existence and uniqueness of the solution of the Cauchy problem for the elliptic equation in a concentric cylinder is shown using a priori estimates. A_σ class of functions is introduced and auxiliary inequalities are proved that allow proving the existence and uniqueness of the solution of the Cauchy problem in this class of functions. And it was shown that the exact form of the solution for the case $k(r)=cr^{-2\alpha}$ satisfies the theorems in the previous paragraph.

Let's go over some definitions and assertions from the first chapter.

Suppose that a rod radiating heat is placed inside a cylindrical non-homogeneous domain $\Omega\times\mathbb{R}$. The temperature on the surface of the rod and the heat flow through this surface are known. It is required to determine the temperature at the outer boundary of the cylindrical domain.

Let Ω be a two-dimensional convex bounded domain, x^0 be an arbitrary point of Ω and $\rho<\operatorname{dis}\{x^0,\partial\Omega\}$.

Set

$$\Omega_\rho=\{(x_1,x_2)\in\Omega:(x_1-x_1^0)^2+(x_2-x_2^0)^2>\rho^2\}.$$

It is clear that $\partial\Omega_\rho=\Gamma_\rho\cup\partial\Omega$ where

$$\Gamma_\rho=\{(x_1,x_2)\in\mathbb{R}^2:(x_1-x_1^0)^2+(x_2-x_2^0)^2=\rho^2\}.$$

Consider the process of heating the following cylindrical domain

$$D = \Omega_\rho \times \mathbb{R} = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : (x_1, x_2) \in \Omega_\rho, \quad -\infty < x_3 < +\infty\}.$$

The inner cylindrical surface $\Gamma_\rho \times \mathbb{R}$ is the boundary of the heater occupying the region $\{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 < \rho^2, x_3 \in \mathbb{R}\}$, and having a temperature independent of time.

Let $u(x_1, x_2, x_3, t)$ be the temperature at point $(x_1, x_2, x_3) \in D$ at moment $t \geq 0$ The process of heating is described by equation

$$\frac{\partial u}{\partial t} - \operatorname{div}[k(x)\operatorname{grad} u] = 0, \quad x \in D, \quad t > 0, \tag{1.1.1}$$

where $k(x)$ is the thermal conductivity.

On the surface of the rod the following conditions

$$u(x) = \phi(x), \quad \frac{\partial u(x)}{\partial r} = \psi(x), \quad x \in \Gamma_\rho, \tag{1.1.2}$$

are fulfilled. Here $r = \sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2}$.

Knowing ϕ and ψ, it is necessary to find the values of $u(x)$ at the outer boundary $\partial\Omega$.

We are looking for a stationary solution, i. e. a time-independent solution $u = u(x)$. Then equation (1.1.1) takes the form

$$Lu(x) \equiv \operatorname{div}[k(x)\operatorname{grad} u] = 0.$$

We assume that the data of the problem does not depend on x_3, i. e. $u = u(x_1, x_2)$.

Introduce polar coordinates (r, θ) centered at x_0. We also assume that $k = k(r)$ is a function which belongs to $C^2[0, +\infty)$ and satisfies condition $k(r) > 0$

The equation of the boundary of the domain Ω in polar coordinates will be written in the form $r = S(\theta), -\pi \leq \theta \leq \pi$. Here $S(\theta)$ is a smooth 2π-periodic function,

In this case, the equation in question takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk(r)\frac{\partial u}{\partial r}\right) + \frac{k(r)}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0. \tag{1.1.3}$$

The boundary conditions (1.1.2) are

$$u(\rho, \theta) = \phi(\theta), \quad \frac{\partial u(\rho, \theta)}{\partial r} = \psi(\theta), \quad \theta \in [-\pi, \pi]. \tag{1.1.4}$$

Here ϕ and ψ are 2π-periodic functions of the argument θ. In what follows, we assume that the functions ϕ and ψ belong to the space $L_2[-\pi, \pi]$.

The solution of the problem (1.1.3)-(1.1.4) is the function $u \in C^2(\Omega)$, which satisfies in the domain Ω_ρ equation (1.1.3), and satisfies the boundary conditions (1.1.2) in the following sense:

$$\lim_{r \to \rho^+} \int_{-\pi}^{\pi} |u(r, \theta) - \phi(\theta)|^2 d\theta = 0, \tag{1.1.5}$$

and

$$\lim_{r\to\rho^+}\int_{-\pi}^{\pi}\left|\frac{\partial u}{\partial r}(r,\theta)-\psi(\theta)\right|^2 d\theta=0. \qquad (1.1.6)$$

Problem A. *For a given functions $\phi\in L_2[-\pi,\pi]$ and $\psi\in L_2[-\pi,\pi]$, find the values on $\partial\Omega$ of the solution $u(r,\theta)$ to the problem (1.1.3)-(1.1.4).*

The solution of the problem A is the function $\chi\in L_2(\partial\Omega)$ satisfying the condition

$$\lim_{\epsilon\to 0}\int_{-\pi}^{\pi}\left|u(S(\theta)-\epsilon,\theta)-\chi(\theta)\right|^2 d\theta=0. \qquad (1.1.7)$$

Denote by A_σ the set of functions $f(z)$ which are holomorphic on the stripe

$$S_\sigma=\{z\in\mathbb{C}:|\operatorname{Im}z|\le\sigma\} \qquad (1.2.1)$$

and satisfy conditions:

$$f(z+2\pi)=f(z),\quad z=x+iy\in S_\sigma, \qquad (1.2.2)$$

and

$$\|f\|_\sigma^2=\sup_{|y|\le\sigma}\int_{-\pi}^{\pi}|f(x+iy)|^2dx<+\infty. \qquad (1.2.3)$$

Consider the Fourier series

$$f(x)=\sum_{k=-\infty}^{\infty}f_k e^{ikx}.$$

Theorem 1.2.1. *For any function $f\in A_\sigma$ the following inequalities*

$$\frac{1}{4\pi}\|f\|_\sigma^2\le\sum_{k=-\infty}^{\infty}|f_k|^2\cosh 2\sigma k\le\frac{1}{2\pi}\|f\|_\sigma^2 \qquad (1.2.4)$$

are valid.

Denote

$$\sigma_1=\ln\frac{R_1}{\rho},\quad \sigma_2=\ln\frac{R_2}{\rho}. \qquad (1.3.3)$$

It is clear that $\sigma_2\ge\sigma_1>0$. Note that in case where $S(\theta)=R=const$ we have $\sigma_1=\sigma_2=\ln\dfrac{R}{\rho}$.

Theorem 1.3.1. *Suppose that for the given functions ϕ and ψ there exists a solution to problem A. Then the function*

$$f(\theta)=\phi(\theta)-P\psi(\rho,\theta) \qquad (1.3.4)$$

belongs to class A_σ for any $\sigma<\sigma_1$.

Theorem 1.3.2. *Let the function (1.3.4) belong to class A_σ for some $\sigma>\sigma_2$. Then the solution of the problem A exists and is unique.*

In the case where Ω is a circle centered at point x^0, the following statement is true.

Theorem 1.3.3. *Assume that* $S(\theta)=R=const$. *Then a necessary and sufficient condition for the existence of a solution to problem A is that the function (1.3.4) belongs to class* A_σ, *where* $\sigma=\ln\dfrac{R}{\rho}$.

The second chapter of the dissertation titled "**Solvability conditions of the Cauchy problem for the elliptic equation in unbounded domain**", the existence and uniqueness of the solution of the Cauchy problem for the elliptic equation in the stripe is shown using a priori estimates. A_σ class of functions is introduced and auxiliary inequalities are proved that allow proving the existence and uniqueness of the solution of the Cauchy problem in this class of functions. And in the last paragraph, an example was shown which about the given function belonging to the A_σ class of functions.

Let's go over some definitions and assertions from the second chapter.

Consider the process of temperature distribution in the layer $P\subset\mathbb{R}^3$, which has the form

$$P=\{(x_1,x_2)\in\mathbb{R}^2,\quad -h(x_1,x_2)<x_3<0\}.$$

Here h is measurable function such that

$$0<\inf_{(x_1,x_2)\in\mathbb{R}^2} h(x_1,x_2)\le \sup_{(x_1,x_2)\in\mathbb{R}^2} h(x_1,x_2)<+\infty.$$

The temperature at the upper part $\{x_3=0\}$ of the boundary and the heat flow through this part are assumed to be known. We need to find the temperature at the lower border $\{x_3=-h(x_1,x_2)\}$ of the layer.

Let $u(x_1,x_2,x_3,t)$ be the temperature at point (x_1,x_2,x_3) at moment $t\ge 0$, and $k(x_1,x_2,x_3)$ be coefficient of thermal conductivity. The process of heating is described by equation

$$\frac{\partial u}{\partial t}-\operatorname{div}[k(x)\operatorname{grad}u]=0. \tag{2.1.1}$$

The boundary conditions are

$$u(x_1,x_2,0,t)=\phi(x),\quad (x_1,x_2)\in\mathbb{R}^2, \tag{2.1.2}$$

and

$$\frac{\partial u(x_1,x_2,0,t)}{\partial x_3}=\psi(x),\quad (x_1,x_2)\in\mathbb{R}^2. \tag{2.1.3}$$

It is necessary to find the temperature $u(x_1,x_2,-h(x_1,x_2),t)$ on the lower border.

We are looking for a stationary solution, i. e. a time-independent solution. In addition, we assume that $h(x_1)=h=const$, and the coefficient of thermal conductivity depends only on the depth: $k=k(x_3)$.

Introducing the notation $x=x_1$ and $y=-x_3$, we come to the next simplified problem:

$$\frac{\partial^2 u}{\partial x^2}+\frac{1}{k(y)}\frac{\partial}{\partial y}\left[k(y)\frac{\partial u}{\partial y}\right]=0,\quad x\in\mathbb{R},\quad 0<y<h, \tag{2.1.4}$$

with boundary conditions

$$u(x,0)=\phi(x), \quad \frac{\partial u(x,0)}{\partial y}=\chi(x), \quad -\infty<x<+\infty. \tag{2.1.5}$$

We consider this problem in the domain

$$\Omega=\{(x,y)\in\mathbb{R}^2 \mid 0<y<h, x\in\mathbb{R}\}.$$

We can reduce the problem (2.1.4)-(2.1.5) to the following Cauchy problem:

$$\operatorname{div}[k(y)\operatorname{grad}v(x,y)]=0, \quad x\in\Omega, \tag{2.1.9}$$

$$v(x,0)=\phi(x), \quad \frac{\partial v(x,0)}{\partial y}=0, \tag{2.1.10}$$

Problem A. *For a given functions $\phi\in L_2(\mathbb{R})$ find the values of the solution $v(x,y)$ to the problem (2.1.9)-(2.1.10) on the upper border of Ω .*

Then the solution of the problem A is the function $u(x,y)$, which satisfies (2.1.9) and (2.1.10), and the function $\psi(x)$ satisfying the condition

$$\lim_{y\to h-0}\int_{-\infty}^{\infty}|u(x,y)-\psi(x)|^2\,dx=0 \tag{2.1.11}$$

For a given $\sigma>0$, we denote by the symbol A_σ the class of functions $f\in L_2(\mathbb{R})$, which analytically continues into the stripe

$$S_\sigma=\{z\in\mathbb{C}:|\operatorname{Im}z|<\sigma\}, \tag{2.2.1}$$

moreover, the analytical continuation of $f(z)$ satisfies the condition

$$\|f\|_\sigma^2=\sup_{|y|<\sigma}\int_{-\infty}^{\infty}|f(x+iy)|^2dx<+\infty. \tag{2.2.2}$$

Consider the Fourier transform

$$\widehat{f}(\xi)=\int_{-\infty}^{\infty}f(x)e^{-i\xi x}\,dx. \tag{2.2.3}$$

Theorem 2.2.1. *For any function $f\in A_\sigma$ the following inequalities*

$$\pi\|f\|_\sigma^2\le\int_{-\infty}^{\infty}|\widehat{f}(\xi)|^2\cosh 2\sigma\xi\,d\xi\le 2\pi\|f\|_\sigma^2 \tag{2.2.4}$$

are valid.

In this paragraph, we will study existence and uniqueness of the given problem. And the following theorems are true.

Theorem 2.3.1. *Let the function ϕ belongs to class A_σ for $\sigma=h$. Then the solution of the Problem A exists and is unique.*

Theorem 2.3.2. *Suppose that for a given function $\phi\in L_2(\mathbb{R})$ there is a solution to Problem A. Then the function ϕ belongs to class A_σ for $\sigma=h$.*

The third chapter of the dissertation titled "**On the regularization of the Cauchy problem for Laplace operator**", we consider the regularization method for the solution of the Cauchy problem for the Laplace equation. By considering the Cauchy problem for the auxiliary equation, it has been proved that the solution of auxiliary equation approaches to the solution of the given.

Let's get acquainted with important definitions and assertions in the third chapter

Consider the process of heating the following cylindrical surface (thin-walled pipe)

$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = R^2, 0 \le z \le h\}.$$

At the lower base of the cylinder, the given temperature is maintained and there is no heat flow. We need to find the temperature at the top base of the cylinder.

We introduce the cylindrical coordinates

$$r = \sqrt{x^2 + y^2}, \quad \theta = \frac{y}{x}, \quad z = z.$$

Then

$$S = \{(r, \theta, z) : r = R, -\pi \le \theta \le \pi, 0 \le z \le h,\}$$

The process of heating is described by equation

$$\frac{\partial u}{\partial t} = \Delta u.$$

Hence, we have for the temperature $u(\theta, z, t)$ equation

$$\frac{\partial u}{\partial t} = \frac{k(z)}{R^2}\frac{\partial^2 u}{\partial \theta^2} + k(z)\frac{\partial^2 u}{\partial z^2} + k'(z)\frac{\partial u}{\partial z}. \tag{3.1.1}$$

The boundary conditions are

$$u(-\pi, z, t) = u(\pi, z, t), \quad \frac{\partial u}{\partial \theta}(-\pi, z, t) = \frac{\partial u}{\partial \theta}(\pi, z, t), \quad 0 \le z \le h, \tag{3.1.2}$$

$$\frac{\partial u(\theta, 0, t)}{\partial z} = 0, \quad -\pi \le \theta \le \pi, \tag{3.1.3}$$

and

$$u(\theta, 0, t) = \phi(\theta), \quad -\pi < \theta < \pi. \tag{3.1.4}$$

It is necessary to find the temperature $u(\theta, h, t)$ on the top base $z = h$.

We are looking for a stationary solution, i. e. a time-independent solution $u = u(\theta, z)$. In this case equation (3.1.1) takes the form

$$\frac{1}{R^2}k(z)\frac{\partial^2 u}{\partial \theta^2} + k(z)\frac{\partial^2 u}{\partial z^2} + k'(z)\frac{\partial u}{\partial z} = 0.$$

In what follows we assume that $R = 1$. Hence, we consider equation

$$k(z)\frac{\partial^2 u}{\partial \theta^2} + k(z)\frac{\partial^2 u}{\partial z^2} + k'(z)\frac{\partial u}{\partial z} = 0, \quad (\theta, z) \in S, \tag{3.1.6}$$

with boundary conditions

$$u(\theta, 0) = \phi(\theta), \quad \frac{\partial u(\theta, 0)}{\partial z} = 0, \quad -\pi \le \theta \le \pi. \tag{3.1.7}$$

Let us denote by the symbol $D(S)$ the class of functions infinitely differentiable on the cylindrical surface S and vanishing near the upper base $z = h$.

Definition 3.1.1. *Let* $\phi \in L_2[-\pi,\pi]$. *We say that the function* $u(\theta,z)$ *from* $L_2(S)$ *is a solution to the problem (3.1.6)-(3.1.7) if for any function* $v \in D(S)$ *the following equation*

$$\int_S u(\theta,z)\left(k(z)\frac{\partial^2 v}{\partial\theta^2}+k(z)\frac{\partial^2 v}{\partial z^2}+k'(z)\frac{\partial v}{\partial z}\right)d\theta dz=\int_{-\pi}^{\pi}\phi(\theta)\frac{\partial v}{\partial z}(\theta,0)d\theta$$

(3.1.8)

is valid.

Proposition 3.1.1. *If the problem (3.1.6)-(3.1.7) has a solution, then this solution is unique.*

To proof easily the following statements we choose as $k(z)=1$. Then we are looking for the solution in the form

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}c_k(z)e^{ik\theta}.$$

The coefficients are $c_k(z)=\phi_k\cosh kz$ and required solution is

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}\phi_k\cosh kz\, e^{ik\theta}. \qquad (3.1.22)$$

Set

$$\psi(\theta)=u(\theta,h). \qquad (3.1.23)$$

Consider the expansion of ψ:

$$\psi(\theta)=\sum_{k=-\infty}^{\infty}\psi_k e^{ik\theta}.$$

Consider the auxiliary equation

$$\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial\theta^2}+\alpha\frac{\partial^4 u}{\partial\theta^4}=0,\quad -\pi<\theta<\pi,\quad 0<z<h, \qquad (3.1.25)$$

with the same boundary values (3.1.7).

We can look for the solution in the form

$$u(\theta,z)=\sum_{k=-\infty}^{\infty}c_k(z)e^{ik\theta}$$

with unknown coefficients $c_k(z)$.

The coefficients are $c_k(z)=\phi_k\cos\mu_k z$ and required solution is

$$u_\alpha(x,t)=\sum_{k=-\infty}^{\infty}\phi_k\cos\mu_k z\, e^{ik\theta}, \qquad (3.1.26)$$

where

$$\mu_k=\mu_k(\alpha)=\sqrt{\alpha k^4-k^2}. \qquad (3.1.27)$$

Assume that the boundary value is measured with some error $\delta>0$. It means that instead of ϕ we have the function ϕ_δ such that

$$\| \phi(\theta) - \phi_\delta(\theta) \|_{L_2(\mathbb{T})} \le \delta. \tag{3.2.1}$$

Let $u_{\delta,\alpha}(\theta,z)$ be a solution to the equation (3.1.25) with boundary condition

$$u_{\delta,\alpha}(\theta,0) = \phi_\delta(\theta). \tag{3.2.2}$$

In the case when the sought function is known only that it belongs to space $L_2(\mathbb{T})$, the following convergence result is valid, without estimating the convergence rate.

Theorem 3.2.1. *Assume that the function* $\psi(\theta) = u(\theta,h)$ *exists and belongs to* $L_2(\mathbb{T})$. *Set*

$$\alpha = \alpha(\delta) = \frac{\lambda^2}{\ln^2 \delta}, \quad 0 < \delta < 1, \tag{3.2.9}$$

where $\lambda > h$.

Then

$$\lim_{\delta \to 0^+} \| u_{\delta,\alpha}(\theta,h) - \psi(\theta) \| = 0. \tag{3.2.10}$$

Theorem 3.2.2. *Assume that the function* $\psi \in L_2^\beta(\mathbb{T})$, *where* $0 < \beta \le 3$, *satisfies condition*

$$\| \psi \|_\beta \le 1. \tag{3.2.13}$$

Let $\alpha = \alpha(\delta)$ *is defined by equation (3.2.9). Then for any* δ *from the interval* $0 < \delta < 1$ *the estimate*

$$\| u_{\delta,\alpha}(\theta,h) - \psi(\theta) \| \le C(h,\beta) \left(\ln \frac{1}{\delta} \right)^{-2\beta/3}, \quad 0 < \delta < 1, \tag{3.2.14}$$

is valid.

CONCLUSION

The dissertation was devoted to the study of Cauchy problem for Elliptic equation on stripe and cylinder using the method of separation of variables (the Fourier method) and Fourier transform. The uniqueness and existence of the solution of the considered problems were shown using apriori estimates.

The main results of the research are as follows:

1. Solutions of the Cauchy problem for Laplace euqation and fourth order partial differential eqaution were found and it was proved that the solution of auxiliary equation approaches to the solution of the given equation ;
2. Class of function A_σ was found which solution of the Cauchy problem for elliptic equation can be exist on given domain, important inequalities were proved;
3. Theorems were proved about existence and uniqueness of solution of the Cauchy problem for the elliptic equation on concentric cylinder;
4. Theorems were proved about existence and uniqueness of solution of the Cauchy problem for the elliptic equation in unbounded stripe and special lemmas were proved for important a priori estimates;

**НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.01.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
НАЦИОНАЛЬНОМ УНИВЕРСИТЕТЕ УЗБЕКИСТАНА
ИМЕНИ МИРЗО УЛУГБЕКА**

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА

КУДАЙБЕРГЕНОВ АЛЛАМБЕРГЕН КЕНГЕСБАЕВИЧ

ОБ УСЛОВИЯХ РАЗРЕШИМОСТИ ЗАДАЧИ КОШИ ДЛЯ ЭЛЛИПТИЧЕСКОГО УРАВНЕНИЯ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ
диссертации доктора философии (PhD) по физико-математическим наукам

Ташкент – 2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Висшего образования, Науки и Инноваций Республики Узбекистан за № B2024.3.PhD/FM1141.

Диссертация выполнена в Национальном университете Узбекистана имени М. Улугбе

Автореферат диссертации на трех языках (узбекский, английский, русский (резюм- размещен на веб-странице по адресу http://ik-fizmat.nuu.uz и на информационно-образовате портале «ZiyoNet» по адресу http://www.ziyonet.uz.

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Защита диссертации состоится « 8 » апреля 2025 года в 15^{00} на заседании Научного совета DSc.30.09.2019.FM.01.01 при Национальном университете Узбекистана (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 4. Тел.: (+99871) 227-12-24, факс: (+99871) 246-53-21 e-mail: nauka@nuu.uz).

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Автореферат диссертации разослан «24» марта 2025 года.

(протокол рассылки № 2 от «24» марта 2025 года).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования состоит из найти температуру в заданном прямоугольнике, слое, кольце и цилиндрической области на плоскости и показать существование решения соответственно уравнения в частных производных.

Объектом исследования. Уравнение эллиптического типа, уравнение в частных производных четвертого порядка, дифференциальное уравнение второго порядка с переменными коэффициентами, уравнение Эйлера, проблема собственных значений..

Научная новизна исследования состоит в следующем:

найдены решения задачи Коши для уравнения Лапласа и уравнения в частных производных четвертого порядка и доказано, что решение вспомогательного уравнения приближается к решению данного уравнения;

найден класс функций, решение задачи Коши для эллиптического уравнения которого может существовать в заданной области, доказаны важные неравенства;

доказаны теоремы о существовании и единственности решения задачи Коши для эллиптического уравнения на концентрическом цилиндре;

доказаны теоремы о существовании и единственности решения задачи Коши для эллиптического уравнения в неограниченной полосе и для важных априорных оценок были доказаны специальные леммы;

Внедрение результатов исследования. Результаты, полученные в диссертационной работе, внедрены в практику по следующим направлениям:

из решения задачи Коши для эллиптического уравнения в цилиндрической области и неограниченной полосе, в международном практическом проекте «Анализ лиевой симметрии, анализ и моделирование устойчивости по Ляпунову гиперболических систем» № УЗБ-Инд-2021-87 использован для поиска решения задачи Коши для эллиптического уравнения (номер ссылки Национального университета Узбекистана 04/11-7840 от 26 сентября 2024 г.). Применение научного результата позволило показать существование и единственность решения задачи Коши для эллиптического уравнения в концентрическом цилиндре;

из полученных результатов, связанных с решением задачи Коши для уравнения Лапласа № AR08855810 «Задачи решения краевых и начально-краевых задач для нелокальных дифференциальных уравнений с частными производными» в зарубежном проекте 2020-2022 гг., установлена уникальность и существование решение задачи Коши для использованного эллиптического уравнения (Международный казахско-турецкий имени Ходжи Ахмета Яссави, номер 04/2685 от 3 октября 2024 года Университета, Казахстан). Применение научного результата позволило получить априорные оценки по заданным исходным данным;

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка литературы. Общий объем диссертации составляет 77 страниц.

E’LON QILINGAN ISHLAR ROʻYXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I boʻlim (part 1; 1 часть)

1. Alimov Sh.A., Qudaybergenov A.K. On the determining of the stationar temperature in the unbounded stripe//Differential equations., 2024, VOL 60, No8, 1049-1062. **(Scopus IF= 0,8)**

2. Alimov Sh. A, Qudaybergenov A.K. Determination of temperature at the outer boundary of a body// Journal of Mathematical sciences, Vol.274, No.2, August, 2023.**(Scopus IF= 0,36)**

3. Alimov Sh.A., Qudaybergenov A.K. On the Cauchy Problem for Laplace Equation// “Uzbek Mathematical Journal” 2023, Volume 67, Issue 2, pp.5-16. **(01.00.00. №6)**

4. Alimov Sh.A., Qudaybergenov A.K. On the solvability of the Cauchy problem for Laplace equation// “Uzbek Mathematical Journal” 2022, Issue 4, pp.5-14. **(01.00.00. №6)**

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6. Qudaybergenov A.K. On the solvability of the Cauchy problem for Laplace equation in the stripe// Fanlar Akademiyasi Maruzalari,2024 3-son, pp.14-19. **(01.00.00. №7)**

7. Qudaybergenov A.K. On the solvability of the special Cauchy problem for elliptic equation// Fanlar Akademiyasi Maruzalari,2023 3-son, pp.16-22. **(01.00.00. №7)**

II boʻlim (part 2; 2 часть)

8. Alimov Sh.A, Qudaybergenov A.K. On the Cauchy problem for Laplace equation// Actual Problems of Stochastic Analysis. Feb.20-21. 2021. Tashkent.

9. Qudaybergenov A.K. On approximation of the Cauchy problem for Laplace equation// Традиционная международная апрельская математическая конференция в честь Дня работников науки Республики Казахстан, посвященная 75-летию академика НАН РК Тынысбека Шариповича Кальменова Алматы, 5-8 апреля 2021 г.

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11. Qudaybergenov A.K. Sharipova S.A. The existence of the solution of the Cauchy Problem for Laplace equation// Алгебра ва анализнинг долзарб масалалари мавзусидаги республика илмий-амалий анжумани, 2022 йил 18-19 ноябрь, Termiz.

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Avtoreferat “Oʻ‘zMU Xabarlari” jurnali tahririyatida tahrirdan oʻtkazilib, oʻzbek, ingliz va rus tillaridagi matnlar oʻzaro muvofiqlashtirildi.

№ 10-3279

Bosishga ruxsat etildi: 20.03.2025.
Bichimi: 60x84 $^{1/16}$ «Times New Roman»
garniturada raqamli bosma usulda bosildi.
Shartli bosma tabogʻi 2,3. Adadi 100. Buyurtma: № 61
Tel: (99) 832 99 79; (77) 300 99 09
Guvohnoma reestr № 10-3279
«IMPRESS MEDIA» MChJ bosmaxonasida chop etildi.
Manzil: Toshkent sh., Yakkasaroy tumani, Qushbegi koʻchasi, 6-uy.