SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/30.12.2019.FM.02.01 RAQAMLI ILMIY KENGASH

BUXORO DAVLAT UNIVERSITETI

ATOYEV DILSHOD DILMURODOVICH

INTEGRO – DIFFERENSIAL ISSIQLIK TARQALISH TENGLAMASI UCHUN NOLOKAL TESKARI MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI

Buxoro – 2025

UDK: 517.958

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/30.12.2019.FM.02.01 RAQAMLI ILMIY KENGASH

BUXORO DAVLAT UNIVERSITETI

ATOYEV DILSHOD DILMURODOVICH

INTEGRO – DIFFERENSIAL ISSIQLIK TARQALISH TENGLAMASI UCHUN NOLOKAL TESKARI MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI

Buxoro – 2025

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2023.2.PhD/FM863 raqam bilan ro‘yxatga olingan.

Dissertatsiya Buxoro davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, ingliz, rus(rezyume)) Ilmiy kengash veb sahifasi (www.samdu.uz) va “ZiyoNet” ta’lim axborot tarmog‘iga http://www.ziyonet.uz/ joylashtirilgan.

Ilmiy rahbar:	**Durdiyev Durdimurod Qalandarovich** fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Ishankulov Tolib** fizika-matematika fanlari doktori, professor
	Safarov Jo‘rabek Shakarovich fizika-matematika fanlari doktori
Yetakchi tashkilot:	**Urganch davlat universiteti**

Dissertatsiya himoyasi Sharof Rashidov nomidagi Samarqand davlat universiteti huzuridagi DSc.03/30.12.2019.FM.02.01 raqamli Ilmiy kengashning 2025 yil “ __ ” __ soat ___ dagi majlisida bo‘lib o‘tadi. (Manzil: 140104, Samarqand sh., Universitet xiyoboni, 15-uy. Tel: (+99866) 231–06–32, faks: (+99866) 235–19–38, e–mail: patent@samdu.uz).

Dissertatsiya bilan Sharof Rashidov nomidagi Samarqand davlat universitetining Axborot-resurs markazida tanishish mumkin (___-raqami bilan ro‘yxatga olingan). (Manzil: 140104, Samarqand sh., Universitet xiyoboni, 15-uy. Tel: (+99866) 231–06–32, faks: (+99866) 235–19–38, e–mail: patent@samdu.uz).

Dissertatsiya avtoreferati 2025 yil “__ ” ____ kuni tarqatildi.

(2025 yil “ ___ ” _____dagi ___-raqamli reestr bayonnomasi).

A.S. Soliyev
Ilmiy darajalar beruvchi Ilmiy kengash raisi, fizika-matematika fanlar doktori, professor

A.M. Xalxo’jayev
Ilmiy darajalar beruvchi Ilmiy kengash ilmiy kotibi, fizika-matematika fanlari doktori

A.B. Xasanov
Ilmiy darajalar beruvchi Ilmiy kengash huzuridagi Ilmiy seminar raisi, fizika-matematika fanlar doktori, professor

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar xususiy hosilali differensial va integro-differensial tenglamalar, ular uchun qo‘yilgan to‘g‘ri va teskari masalalarni o‘rganishga olib kelinadi. Teskari masalalar issiqlik fizikasiga, kvant mexanikasi va optika sohasiga, tibbiyotga, kompyuter grafikasi va sun’iy intellektga, shu bilan birga zamonaviy ilm-fanning barcha sohalariga kirib bordi. Matematik fizikada to‘g‘ri masalalarning yechimini topish uchun tenglama koeffitsiyentlarini, soha chegarasini, boshlang‘ich va chegaraviy shartlarni berish lozim. Ammo amaliyotda, tenglama koeffitsiyentlarini har doim berib bo‘lmaydi, bundan tashqari masalaning yechimi nafaqat boshlang‘ich va chegaraviy shartlarga balki muhitning ichidagi boshqa ba’zi bir xususiyatlariga ham bog‘liq bo‘ladi. Bunday holda, masalani yechish integro-differensial issiqlik tarqalish tenglamalari uchun nolokal teskari masalalarni tadqiq etishga olib kelinadi. Bunday masalalarni yechish to‘la o‘rganilmaganligi bois, ushbu yo‘nalishdagi tadqiqotlarga alohida e’tibor qaratilmoqda.

Jahonda matematik fizikaning jadal suratlarda rivojlanayotgan yo‘nalishlaridan biri bo‘lgan integro-differensial issiqlik tarqalish tenglamalari va ularga qo‘yilgan teskari masalalar bilan modellashtiriladigan jarayonlarni tadqiq qilishga qaratilgan ilmiy tadqiqotlar olib borilmoqda. Ushbu yo‘nalishda nolokal boshlang‘ich-chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasi uchun to‘g‘ri va teskari masala yechimining mavjudligi va yagonaligini isbotlashga oid tadqiqotlar ustuvor hisoblanmoqda. Shuningdek, issiqlik tarqalish jarayonlarini boshqarish hamda issiqlikning butun muhit bo‘ylab teng taqsimlanishini ta’minlash muhimdir, bu jarayonlar esa nolokal shartli issiqlik tarqalish tenglamasi yordamida modellashtiriladi. Shuning uchun nolokal boshlang‘ich-chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasi uchun qo‘yilgan to‘g‘ri va teskari masalalarning korrektligini o‘rganishga oid tadqiqotlarni rivojlantirish dolzarb vazifalardan hisoblanmoqda.

Respublikamizda amaliy va fundamental fanlar orqali ham nazariy, ham amaliy ahamiyatga ega bo‘lgan tadqiqotlar yuzasidan keng qamrovli izlanishlar olib borilmoqda, xususan nolokal shartli integro-differensial issiqlik tarqalish tenglamasi uchun to‘g‘ri va teskari masalalar yechimi mavjudligi hamda yagonaligini isbotlash va olingan natijalarni amaliyotda qo‘llash bo‘yicha keng ko‘lamli chora-tadbirlar amalga oshirilmoqda. “Algebra va uning tatbiqlari, differensial tenglamalar va uning tatbiqlari, chiziqsiz tizimlar, dinamik tizimlar va ularning tatbiqlarini matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi” [1] fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilangan. Ushbu vazifani amalga oshirishda o‘ng

[1] O‘zbekiston Respublikasi Prezidentining 2019 yil 9 iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-4387-son qarori.

tomoni oʻrama koʻrinishdagi issiqlik tarqalish tenglamalari uchun nolokal teskari masalalarning korrektlilik shartlarini topish muhim ilmiy ahamiyatga ega hisoblanadi.

Oʻzbekiston Respublikasi Prezidentining 2017-yil 7-fevral PF-4947-sonli "Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha harakatlar strategiyasi toʻgʻrisida"gi, 2022-yil 28-yanvar PF-60 sonli "2022-2026- yillarga moʻljallangan Yangi Oʻzbekistonning taraqqiyot strategiyasi toʻgʻrisida"gi, 2023-yil 11-sentabrdagi PF-158-son "Oʻzbekiston–2030" strategiyasi toʻgʻrisidagi farmonlari, 2017-yil 17-fevral PQ-2789-sonli "Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari toʻgʻrisida"gi, 2020 yil 7 maydagi PQ-4708-son "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari toʻgʻrisida"gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yoʻnalishlariga mosligi. Mazkur tadqiqot Oʻzbekiston Respublikasida fan va texnologiyalar rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yoʻnalishi doirasida bajarilgan.

Muammoning oʻrganilganlik darajasi. Matematik fizikaning teskari masalalari nazariyasini rivojlantirishga M.M. Lavrentev, A.S. Alekseev, A.L. Bukhgeim, N.Ya. Beznoshchenko, A.I. Prilepko, V.G. Romanov, A. Lorenzi, J. Janno, G.V. Dyatlov, A. Favaron, H. Grabmueller va boshqalar oʻz hissalarini qoʻshganlar. Parabolik tipdagi tenglamalar uchun teskari masalalar A.L. Bukhgeim, J. Janno, L.v. Wolfersdorf, V.G. Romanov, F. Colombo, A.I. Prilepko, A.D. Iskenderov, M. Grasselli, V.A. Dedok, N.Ya. Beznoshchenko, A. Boumenir, O. Danilkina, M.E. Gurtin, M.I. Ivanchov, A.C. Pipkin L. Yang va J. Jannolar tomonidan qoʻyilgan va tadqiq qilingan. Parabolik tipdagi integro-differensial tenglamalar uchun teskari masalalarni tadqiq etishning turli usullari A. Lorenzi, D. Lesnic, F. Colombo, M. Grasselli, K. Karuppiah, D.Q. Durdiyev, J.J. Jumaev va boshqalarning ishlarida taklif etilgan va rivojlantirilgan. Nolokal shartli xususiy hosilali differensial tenglamalarni oʻrganishning turli usullari D.G. Gordeziani, G.A. Avalishvili, E.I. Azizbayov, Y.T. Mehraliyev, S.V. Kirichenko, D. Lesnic, M.I. Ismailov, F. Kanca va boshqalarning ishlarida tadqiq qilingan.

Jumladan, E.I. Azizbayov va Y.T. Mehraliyevlarning[2] ishlarida ikkinchi tartibli parabolik tipdagi nolokal boshlangʻich shartli tenglama uchun koeffitsiyentni aniqlashning teskari masalasi tadqiq qilingan. Ushbu tadqiqot ishida qoʻyilgan masalaga nisbatan ekvivalent masala olinib, ekvivalent masala yechimining mavjudligi va yagonaligi qisqartirib akslantirishlar prinsipi yordamida isbotlangan. M.S. Hussein, D. Lesnic, M.I. Ismailovlarning[3] maqolasida bir oʻlchamli issiqlik

---

[2]E.I. Azizboyev, Y.T. Mehraliyev, Solvability of nonlocal inverse boundary-value problem for second order parabolic equation with integral condition, Electronic journal of differential eqautions, no. 125,1-14, 2017.

[3] M.S. Hussein, D. Lesnic va M.I. Ismailov, An inverse problem of finding the time-dependent diffusion coefficient from an integral condition, Mathematical Methods in the Applied Sciences, 39 (5), 963-980, 2015.

tarqalish tenglamasidan nolokal qo‘shimcha shart orqali koeffitsiyentni aniqlashning teskari masalasi tadqiq qilingan. Dastlab, masala klassik yechimining mavjudligi, yagonaligi va yechimning berilganlarga nisbatan uzluksiz bog‘liqligi ko‘rsatilgan. So‘ngra tenglama koeffitsiyentlarini aniqlashning analitik va sonli usullari taklif etilgan. D.Q. Durdiyev, J.J. Jumayev[4] o‘z tadqiqot ishlarida bir o‘lchamli integro-differensial issiqlik o‘tkazuvchanlik tenglamasidan yadroni aniqlashning teskari masalalari tadqiq qilgan va yechimning mavjudligi, yagonaligi isbotlangan.

Ushbu dissertatsiya ishida integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich va Dirixle chegaraviy shartli, shuningdek, nolokal boshlang‘ich va nolokal chegaraviy shartlar bilan to‘g‘ri va teskari masalalar o‘rganilgan.

Dissertatsiya ishi E.I. Azizbayov, Y.T. Mehraliyev, M.S. Hussein, D. Lesnic, M.I. Ismailov, D.Q. Durdiyev va J.J. Jumayevlarning ilmiy tadqiqotlariga yaqin bo‘lib, masalalarni tadqiq etishda ular tomonidan tavsiya etilgan usullardan foydalanilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Buxoro davlat universitetining 2020-2024-yillarga mo‘ljallangan M-02.2018 “Matematik-fizikaning teskari masalalari” mavzusidagi ilmiy-tadqiqot yo‘nalishi doirasida bajarildi.

Tadqiqotning maqsadi. Integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich va nolokal chegaraviy masalalardan yadroni aniqlash usullarni qurish va bu teskari masalalar yechimlarining mavjudligi, yagonaligini isbotlashdan iborat.

Tadqiqotning vazifalari:

chegaralangan sohada integro-differensial issiqlik tarqalish tenglamasi uchun qo‘yilgan nolokal boshlang‘ich-chegaraviy masala yechimining mavjud va yagonaligi haqidagi yetarlilik shartlarini topish;

integro-differensial issiqlik tarqalish tenglamasi uchun integral had qatnashgan nolokal boshlang‘ich-chegaraviy masala klassik yechimining lokal mavjud va yagonalik shartlari o‘zgarmas koeffitsiyentlarga bog‘liq ravishda isbotlash;

integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich-chegaraviy, integral ko‘rinishdagi qo‘shimcha shartli masaladan vaqtga bog‘liq yadroni aniqlashning teskari masalasining lokal bir qiymatli yechiluvchanligini isbotlash;

nolokal boshlang‘ich va nolokal chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasidan yadroni aniqlash teskari masalaning bir qiymatli yechiluvchanligini, yechimining global mavjudligi va yagonaligini isbotlash.

Tadqiqotning obyekti ikkinchi tartibli integro-differensial issiqlik tarqalish tenglamalardan iborat.

[4] D.K. Durdiev, J.J. Jumaev, One-dimensional inverse problems of finding the kernel of integro-differential heat equation in a boundedd domain, Ukrain. Math. J., 73(3), 1723-1740, 2021.

Tadqiqotning predmeti nolokal shartli integro-differensial issiqlik tarqalish tenglamasi uchun to‘g‘ri va teskari masalalar.

Tadqiqotning usullari. Dissertatsiya ishida integral tenglamalar va integral tengsizliklar nazariyasi, shuningdek, Volterra-Fredgolm tipidagi chiziqli bo‘lmagan integral tenglamalar sistemasini yechish, funksional analiz metodlaridan xususan, Furye metodi, Shauder prinsipi va Banax teoremasidan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

chegaralangan sohada integro-differensial issiqlik tarqalish tenglamasi uchun qo‘yilgan nolokal boshlang‘ich-chegaraviy masala yechimining mavjud va yagonaligi haqidagi yetarlilik shartlari topilgan;

integro-differensial issiqlik tarqalish tenglamasi uchun integral had qatnashgan nolokal boshlang‘ich-chegaraviy masala klassik yechimining lokal mavjud va yagonalik shartlari o‘zgarmas koeffitsiyentlarga bog‘liq ravishda isbotlangan;

integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich-chegaraviy, integral ko‘rinishdagi qo‘shimcha shartli masaladan vaqtga bog‘liq yadroni aniqlashning teskari masalasining lokal bir qiymatli yechiluvchanligi isbotlangan;

nolokal boshlang‘ich va nolokal chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasidan yadroni aniqlash teskari masalaning bir qiymatli yechilishi, yechimining global mavjudligi va yagonaligi isbotlangan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

parabolik tipdagi integro-differensial tenglamalar uchun yadroni aniqlash nolokal teskari masalasining bir qiymatli yechiluvchanligi isbotlangan;

chegaralangan sohada issiqlik tarqalish tenglamalari uchun o‘rtacha haroratni kiritish orqali xotira funksiyasining mavjudlik va yagonalik shartlari topilgan.

Tadqiqot natijalarining ishonchliligi. Tadqiqotdagi natijalar va matematik isbotlar funksional analiz, differensial va integral tenglamalar nazariyasi, teskari masalalar nazariyasi, Furye usuli, matematik analiz usullari qo‘llanilganligi, hamda, integral tengsizliklar nazariyasi va matematik mulohazalarning qat’iyligi bilan asoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati matematik fizikaning integro-differensial tenglamalar uchun nolokal teskari masalalar nazariyasini yanada rivojlantirishi, yadroni aniqlash usullari qurilganligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati seysmologiyada, neft va gaz konlarini qidirishda, issiqlik tarqatuvchi xotirali muhitlarda issiqlik tarqalish jarayonlarini tekshirishda tatbiq etilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Integro-differensial issiqlik tarqalish tenglamasi uchun nolokal teskari masalalarga oid olingan ilmiy natijalaridan:

O‘zbekiston Respublikasi Fanlar akademiyasi Mexanika va inshootlar seysmik mustahkamligi institutida IL-21071166 (2022-2024 yy.) “Shamolning past tezligi uchun mo‘ljallangan vertikal o‘qli shamol turbinasini yaratish” mavzusidagi innovatsion loyihani bajarishda foydalanilgan (2024 yil 17-dekabrdagi 1563-3-son ma’lumotnomasi). Jumladan, dissertatsiya ishida parabolik tipdagi integro-differensial tenglamalar uchun yadroni aniqlash nolokal teskari masalalarining bir qiymatli yechiluvchanligi isbotlangan. Ushbu natijalardan loyihada shamol turbinalarining samarali ishlashi maqsadida tuzilgan matematik model tenglamalarining korrektligini tekshirishda foydalanilgan;

№122041100096-4. “Sotsiologiyada, geofizikada va muhandislik fanlarida matematik modellashtirish” mavzusidagi xorijiy fundamental loyihada teskari masalalarni tadqiq etishning taklif etilgan usuli qo‘llanilgan, qovushqoq-elastiklik tenglamalar sistemasidan o‘rama ko‘rinishidagi yadroni aniqlash teskari masalalarida foydalanilgan (Южный Математический Институт филиал ФГНБУ ФНЦ “Владикавказский научный центр РАН”, 2024 yil 2-dekabr 141-son ma’lumotnomasi). Ilmiy natijaning qo‘llanilganligi qovushqoq-elastiklik tenglamalar sistemasidan nolokal shartlar yordamida o‘rama ko‘rinishidagi yadroni aniqlash va teskari masala yechimining mavjudlik va yagonaligini isbotlash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqot natijalari 6 ta ilmiy-amaliy anjumanlarda, jumladan 4 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 11 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlar ro‘yxatida 5 ta maqola, jumladan, 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 98 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma‘lumotlar keltirilgan.

Dissertatsiyaning birinchi bobi **“Integral hadli nolokal boshlang‘ich-chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasi uchun to‘g‘ri va teskari masala”** deb nomlangan bo‘lib, integro-differensial issiqlik tarqalish tenglamasi uchun nolokal shartli to‘g‘ri va teskari masala yechimining mavjudligi va yagonaligi o‘rganilgan. Bu bobning birinchi paragrafida dissertatsiya ishida foydalanilgan funksional analiz va matematik fizika tenglamalarining ayrim asosiy tushuncha va ta’riflari keltirilgan. Birinchi bobning ikkinchi paragrafda integro-differensial issiqlik tarqalish tenglamasi uchun integral hadli nolokal boshlang‘ich-chegaraviy masala yechimining mavjudligi va yagonaligi o‘rganilgan.

Quyidagi integro-differensial issiqlik tarqalish tenglamasini

$$u_t - u_{xx} = \int_0^t k\,(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_T, \qquad (1)$$

nolokal boshlang‘ich va chegaraviy shartlar

$$u(x,0) + \delta u(x,T) + \int_0^T p\,(\tau)u(x,\tau)d\tau = \varphi(x), \;\; x \in [0,l], \qquad (2)$$

$$u|_{x=0} = u|_{x=l} = 0, \;\; t \in [0,T], \qquad (3)$$

bilan qaraymiz. (1)-(3) masaladan $u(x,t)$ funksiyani aniqlaymiz: bu yerda $D_T = \{(x,t): 0 < x < l, 0 < t \le T\}\,(T > 0,\; l > 0$ o‘zgarmas sonlar), $\delta \ge 0$, $\varphi(x)$, $p(t)(p(t) > 0)$lar berilgan yetarlicha silliq funksiyalar.

(1)-(3) masalani qanoatlantiruvchi $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$ funksiyani topish masalasiga to‘g‘ri masala deyiladi.

(1)-(3) masala yechimi quyidagi integral tenglamani qanoatlantiradi:

$$u(x,t) = \Phi(x,t) + \int_0^t \int_0^l G\,(x,\xi,t-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$
$$-\delta \int_0^T \int_0^l G_0\,(x,\xi,t+T-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$
$$-\int_0^T \int_0^\mu \int_0^l p\,(\mu)G_0(x,\xi,t+\mu-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta d\mu, \qquad (4)$$

bu yerda

$$\Phi(x,t) = \int_0^l \varphi\,(\xi)G_0(x,\xi,t)d\xi,$$

$$G(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty} e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n\xi}{l} sin\frac{\pi n x}{l},$$

$$G_0(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty} \frac{1}{1+\delta e^{-\left(\frac{\pi n}{l}\right)^2 T} + \int_0^T p\,(\tau)e^{-\left(\frac{\pi n}{l}\right)^2 \tau} d\tau} e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n\xi}{l} sin\frac{\pi n x}{l}.$$

Banax prinsipidan foydalanib, (4) Volterra-Fredgolm integral tenglamasi yechimi mavjud va yagonaligi ko‘rsatilgan. Buning uchun quyidagi lemma o‘rinli:

1-lemma. *Faraz qilaylik,* $\varphi(x) \in C[0,l]$, $\varphi'(x) \in L_2(0,l)$, $\{p(t),k(t)\} \in C[0,T]$, $p(t) > 0$, $\varphi(0) = \varphi(l) = 0$ *bo‘lsin. U holda* $0 < T \le T_1$ *larda (4) integral tenglamaning* $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$ *sinfda aniqlangan yagona yechimi mavjud, bu yerda* T_1 *ushbu* $p_0 k_0 T^3 + 3(1+\delta)k_0 T^2 - 3 = 0$ *tenglamaning musbat ildizi va* $k_0 = \max_{t\in[0,T]}|k(t)|$, $p_0 = \max_{t\in[0,T]}|p(t)|$.

Uchinchi paragrafda (1)-(3) masala va

$$u(x_0,t) = h(t), \quad x_0 \in (0,l) \qquad (5)$$

qo‘shimcha shart yordamida $u(x,t)$, $k(t)$ funksiyalarni aniqlash teskari masalasi qaraladi. Bu yerda $h(t)$ yetarlicha silliq funksiya.

1-teorema. *Faraz qilaylik, ushbu shartlar bajarilsin:* $\varphi(x)\in C^4[0,l]$, $p(t)\in C[0,T]$, $p(t)>0$, $h(t)\in C^2[0,T]$, $\varphi(0)=\varphi(l)=\varphi''(0)=\varphi''(l)=0$, $h(0)\neq 0$, $\delta\geq 0$. *U holda shunday yetarlicha kichik* $T^*\in(0,T)$ *son topilib, (1)-(3), (5) teskari masalaning* $u(x,t)\in C^{4,1}(D_{T^*})\cap C(\overline{D}_{T^*})$ *va* $k(t)\in C[0;T^*]$ *yagona yechimi mavjud, bu yerda* $D_{T^*}=\{(x,t)|x\in(0,l),t\in(0,T^*]\}$.

Dissertatsiyaning ikkinchi bobi “**Integro-differensial issiqlik tarqalish tenglamasi uchun nolokal to‘g‘ri va teskari masalalar**” deb nomlangan bo‘lib, bobning birinchi paragrafida integro-differensial parabolik tipdagi tenglama uchun nolokal boshlang‘ich chegaraviy masala yechimining mavjudligi va yagonaligi tadqiq qilingan.

Ushbu

$$u_t-u_{xx}=\int_0^t k\,(t-\tau)u(x,\tau)d\tau,\quad (x,t)\in D_T, \tag{6}$$

$$u(x,0)+\delta u(x,T)=\varphi(x),\;\; x\in[0,l], \tag{7}$$

$$u|_{x=0}=u|_{x=l}=0,\;\; t\in[0,T],\quad \varphi(0)=\varphi(l)=0, \tag{8}$$

masaladan $u(x,t)$ funksiyani aniqlashni qaraymiz, bu yerda $\delta\geq 0$ berilgan son, $\varphi(x)$ va $k(t)$ berilgan yetarlicha silliq funksiyalar.

1-ta’rif. *Agar* $u(x,t)$ *funksiya*

1. D_T *sohada* $u(x,t)$ *funksiya va uning hosilalari* u_t,u_x,u_{xx} *uzluksiz bo‘lsin;*
2. *(6) tenglama va (7), (8) shartlarni klassik ma’noda qanoatlantirsin*

u holda bu funksiya (6)-(8) masalaning klassik yechimi deyiladi.

(6)-(8) masalaning yechimini

$$u(x,t)=\sum_{n=1}^{\infty}X_n\,(x)u_n(t) \tag{9}$$

ko‘rinishda izlaymiz, bu yerda $X_n(x)=\sqrt{\frac{2}{l}}\sin\lambda_n x$, $\lambda_n=\frac{\pi n}{l}$, $n=1,2,3,...$

(6) va (7) tengliklarni $X_n(x)$ ga ko‘paytirib $(0,l)$ da integrallab quyidagi masalani olamiz.

$$u'_n(t)+\lambda_n^2u_n(t)=\int_0^t k\,(t-\alpha)u_n(\alpha)d\alpha, \tag{10}$$

$$u_n(0)+\delta u_n(T)=\varphi_n,\quad n=1,2,\ldots, \tag{11}$$

bu yerda $\varphi_n=\sqrt{\frac{2}{l}}\int_0^l\varphi\,(x)\sin\lambda_n xdx$.

Har bir tayinlangan $n\in\mathbb{N}$ da (10)-(11) masalaning yechimi quyidagi Volterra-Fredgolm integral tenglamasiga olib kelinadi

$$u_n(t)=\frac{e^{-\lambda_n^2t}}{\psi_n(T)}\varphi_n-\frac{\delta e^{-\lambda_n^2t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\int_0^\tau k\,(\tau-\alpha)u_n(\alpha)d\alpha d\tau+$$

$$+ e^{-\lambda_n^2 t}\int_0^{t} e^{\lambda_n^2 \tau}\int_0^{\tau} k\,(\tau-\alpha)u_n(\alpha)d\alpha d\tau, \qquad (12)$$

bu yerda

$$\psi_n(T) := 1 + \delta e^{-\lambda_n^2 T}.$$

2-lemma. *Ixtiyoriy* $0 < t \le T$ *da quyidagi tengsizliklar o'rinli:*

$$|u_n(t)| \le C_1 e^{-\lambda_n^2 t}|\varphi_n|, \qquad t \in [0,T],$$

$$|u'_n(t)| \le C_1 e^{-\lambda_n^2 t}|\varphi_n|(\lambda_n^2 + T \parallel k \parallel),$$

bu yerda $0 < t \le T$, $\|k\| = \max\limits_{t\in[0,T]}|k(t)|$, $C_1(\|k\|, T, \delta)$.

2-teorema. *Faraz qilaylik,* $\varphi(x) \in C[0,l]$, $\varphi'(x) \in L_2(0,l)$, $k(t) \in C[0,T]$, $\varphi(0) = \varphi(l) = 0$ *shartlar bajarilsin. U holda*

$$\frac{\delta\|k\|T^2}{2}\left(1 + \|k\|e^{\frac{\|k\|T^2}{2}}\right) < 1$$

tengsizlikni qanoatlantiruvchi T *lar uchun (6)-(8) to'g'ri masalaning* $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$ *yagona yechimi mavjud.*

Ikkinchi bobning ikkinchi paragrafida integro-differensial issiqlik tarqalish tenglamasi uchun qo'yilgan teskari masala yechimining mavjud va yagonaligi o'rganilgan. Bunda,

$$\int_0^l \omega\,(x)u(x,t)dx = h(t), \qquad (13)$$

qo'shimcha shartdan foydalanib, $u(x,t)$ va $k(t)$ funksiyalarni aniqlash teskari masalasi tadqiq etiladi. Bu yerda $\omega(x)$, $h(t)$ berilgan yetarlicha silliq funksiyalar.

(6)-(8), (13) masaladan $u(x,t)$ va $k(t)$ funksiyalarni topish masalasiga teskari masala deb ataladi.

(9), (12) ga ko'ra (6)-(8) masalani quyidagi Volterra-Fredgolm integral tenglamasi shaklida yozamiz:

$$u(x,t) = \Phi(x,t) -$$

$$-\delta\int_t^{T+t}\int_0^l G_0\,(x,\xi,\tau)\int_0^{T+t-\tau} k\,(\alpha)u(\xi, T+t-\tau-\alpha)d\alpha d\xi d\tau +$$

$$+\int_0^t\int_0^l G\,(x,\xi,\tau)\int_0^{t-\tau} k\,(\alpha)u(\xi, t-\tau-\alpha)d\alpha d\xi d\tau. \qquad (14)$$

bu yerda

$$\Phi(x,t) = \int_0^l G_0\,(x,\xi,t)\varphi(\xi)d\xi,$$

$$G_0(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty}\frac{1}{1+\delta e^{-\left(\frac{\pi n}{l}\right)^2 T}}\,e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n\xi}{l} sin\frac{\pi n x}{l}.$$

(14) tenglamani t bo'yicha differensiallab, quyidagiga ega bo'lamiz:

$$u_t(x,t)=\Phi_t(x,t)+\delta\int_0^l G_0\,(x,\xi,t)\int_0^T k\,(\alpha)u(\xi,T-\alpha)d\alpha d\xi-$$
$$-\delta\int_t^{T+t}\int_0^l G_0\,(x,\xi,\tau)k(T+t-\alpha)u(\xi,0)d\alpha d\xi d\tau-$$
$$-\delta\int_t^{T+t}\int_0^l G_0\,(x,\xi,\tau)\int_0^{T+t-\tau} k\,(\alpha)u_t(\xi,T+t-\tau-\alpha)d\alpha d\xi d\tau+ \qquad (15)$$
$$+\int_0^t\int_0^l G\,(x,\xi,\tau)k(t-\tau)u(\xi,0)d\xi d\tau+$$
$$+\int_0^t\int_0^l G\,(x,\xi,\tau)\int_0^{t-\tau} k\,(\alpha)u_t(\xi,t-\tau-\alpha)d\alpha d\xi d\tau.$$

Faraz qilaylik, $\omega(0)=\omega(l)=0$ shartlar bajarilsin. (6) tenglikni ikkala tomonini $\omega(x)$ ga ko'paytirib, x bo'yicha 0 dan l gacha integrallaymiz. Sodda almashtirishlardan so'ng, quyidagi integral tenglamani olamiz:

$$k(t)=\frac{h''(t)}{|h(0)|}-\frac{1}{|h(0)|}\int_0^l \omega''(x)u_t(x,t)\,dx-\frac{1}{|h(0)|}\int_0^t k\,(\tau)h_t(t-\tau)d\tau. \qquad (16)$$

(14) – (16) birgalikda $\big(u(x,t),u_t(x,t),\ k(t)\big)$ noma'lumlarga nisbatan yopiq integral tenglamalar sistemasini tashkil etadi. Masala yechimining mavjud va yagonaligi uchun quyidagi teorema o'rinli:

3-teorema. *Faraz qilaylik,*

1. $\varphi(x)\in C^2[0,l],\ \varphi'''(x)\in L_2(0,l), \varphi(0)=\varphi(l)=\varphi''(0)=\varphi''(l)=0,$

2. $h(t)\in C^2[0,T], h(0)\neq 0,\ \delta\geq 0,$

3. $\omega(x)\in C^2[0,l], \int_0^l \omega\,(x)\varphi(x)dx=h(0)+\delta h(T),\quad \omega(0)=\omega(l)=0$

shartlar bajarilsin. U holda shunday yetarlicha kichik $T_0>0, l_0>0$ *sonlari topilib,* $T\in(0,T_0), l\in(0,l_0)$ *da (6) – (8) va (13) masalaning* $u(x,t)\in C^{2,1}(D_T)\cap C\big(\overline{D}_T\big)$, $k(t)\in C[0,T]$ *yagona yechimi mavjud.*

Uchinchi paragrafda (6)-(8) masala va ushbu

$$u(x_0,t)=h(t) \qquad (17)$$

qo'shimcha shartdan foydalanib, $u(x,t)$ va $k(t)$ funksiyalarni aniqlash teskari masalasi tadqiq etilgan.

4-teorema. *Faraz qilaylik,* $\varphi(x)\in C^4[0,l], h(t)\in C^2[0,T], h(0)\neq 0, \delta\geq 0$ *bo'lsin va* $\varphi(0)=\varphi(l)=\varphi''(0)=\varphi''(l)=0,\quad h(0)+\delta h(T)=\varphi(x_0)$ *kelishuvchanlik shartlari bajarilsin. U holda shunday yetarlicha kichik* $T^*\in(0,T)$ *son topilib, (6) – (8), (17) teskari masalaning* $u(x,t)\in C^{4,1}(D_{T^*})\cap C(\overline{D}_{T^*})$, $k(t)\in C[0,T^*]$ *yagona klassik yechimi mavjud.*

Dissertatsiya ishining uchinchi bobi "**Nolokal boshlang'ich va nolokal chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasi uchun teskari masala"** deb nomlangan bo'lib, to'g'ri va teskari masala yechimi uchun mavjudlik va yagonalik shartlari olingan.

Ushbu

$$u_t - u_{xx} = \int_0^t k(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_T, \tag{18}$$

$$u(x,0) + \delta u(x,T) = \varphi(x), \ \ x \in [0,l], \tag{19}$$

$$u_x(0,t) - hu(0,t) = \psi_1(t), \ \ u_x(l,t) + hu(l,t) = \psi_2(t), \ \ t \in [0,T], \tag{20}$$

$$\int_0^l \omega(x)u(x,t)dx = p(t), \tag{21}$$

masaladan $u(x,t)$, $k(t)$ funksiyalarni aniqlash masalasini qaraymiz, bu yerda $\delta \geq 0$, $h > 0$ berilgan sonlar, $\varphi(x), \omega(x), p(t), \psi_1(t), \psi_2(t)$ berilgan yetarlicha silliq funksiyalar.

2-ta'rif. *Agar* $\big(u(x,t); k(t)\big)$ *funksiyalar quyidagi:*

1. D_T *sohada* $u(x,t)$ *funksiya va uning hosilalari* u_t, u_x, u_{xx} *uzluksiz bo'lsin;*
2. *(18) tenglama va (19)-(21) shartlarni klassik ma'noda qanoatlantirsin;*
3. $k(t)$ *funksiya* $[0,T]$ *oraliqda uzluksiz bo'lsin*

shartlarni qanoatlantirsa, u holda $\big(u(x,t); k(t)\big)$ *funksiyalar juftligiga (18)-(21) masalaning klassik yechimi deyiladi.*

(18)-(21) teskari masala quyidagi

$$\vartheta_t = \vartheta_{xx} + \int_0^t k(t-\tau)\vartheta(x,\tau)d\tau + \int_0^t k(t-\tau)\nu(x,\tau)d\tau - \nu_t(x,t), \tag{22}$$

$$\vartheta(x,0) + \delta\vartheta(x,T) = \phi(x) \quad 0 \leq x \leq l, \tag{23}$$

$$\vartheta_x(0,t) - h\vartheta(0,t) = 0, \ \ \vartheta_x(l,t) + h\vartheta(l,t) = 0, \tag{24}$$

$$\int_0^l \omega(x)\vartheta(x,t)dx = p(t) - \int_0^l \nu(x,t)\omega(x)dx, \tag{25}$$

yordamchi masalaga olib kelinib, $\vartheta(x,t), k(t)$ funksiyalarni topish talab qilinadi. Bu yerda $u(x,t) = \vartheta(x,t) + \nu(x,t)$, $\nu(x,t) = (\alpha_1 x + \beta_1)\psi_1(t) + (\alpha_2 x + \beta_2)\psi_2(t))$, $\alpha_1 = \frac{1}{2+hl}$, $\beta_1 = -\frac{1+hl}{(2+hl)h}$, $\alpha_2 = \frac{1}{2+hl}$, $\beta_2 = \frac{1}{h(2+hl)}$, $\phi(x) = \varphi(x) - \nu(x,0) - \delta\nu(x,T)$.

Uchinchi bobning ikkinchi paragrafida (22)-(24) to'g'ri masalaning $\vartheta(x,t) \in C^{2,1}(D_T) \cap C^{1,0}\big(\overline{D}_T\big)$ klassik yechimini topish masalasi tadqiq qilingan.

(22)-(24) masala yechimni

$$\vartheta(x,t) = \sum_{n=0}^{\infty} X_n(x)\vartheta_n(t) \tag{26}$$

ko'rinishda izlaymiz, bu yerda, $X_n(x)$ xos funksiyalar va ular $L_2(0,l)$ da to'la ortonormal sistema tashkil etadi, $\vartheta_n(t) = (\vartheta, X_n)$.

(22), (23) tengliklarni $X_n(x)$ ga ko'paytirib $(0,l)$ da integrallab quyidagi masalani olamiz:

$$\vartheta'_n(t) + \lambda_n^2\vartheta_n(t) = f_n(t), \tag{27}$$

$$\vartheta_n(0) + \delta\vartheta_n(T) = \phi_n, \quad n = 1,2,\ldots, \tag{28}$$

bu yerda

$$\phi_n = \big(\phi, X_n(x)\big), \quad f_n = \big(f, X_n(x)\big),$$

$$f(x,t)=\int_0^t k(t-\tau)\vartheta(x,\tau)d\tau+\int_0^t k\,(t-\tau)\nu(x,\tau)d\tau-\nu_t(x,t).$$

Har bir tayinlangan $n\in\mathbb{N}$ da (27), (28) masalaning yechimi quyidagi Volterra-Fredgolm integral tenglamasiga olib kelinadi

$$\vartheta_n(t)=\frac{e^{-\lambda_n^2 t}}{\psi_n(T)}\phi_n+e^{-\lambda_n^2 t}\int_0^t e^{\lambda_n^2\tau}\left(\int_0^\tau k(\tau-\alpha)\nu_n(\alpha)\,d\alpha-\nu_n'(\tau)\right)d\tau-$$

$$-\frac{\delta e^{-\lambda_n^2 t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\left(\int_0^\tau k(\tau-\alpha)\nu_n(\alpha)\,d\alpha-\nu_n'(\tau)\right)d\tau+$$

$$+e^{-\lambda_n^2 t}\int_0^t e^{\lambda_n^2\tau}\int_0^\tau \vartheta_n(\alpha)\,k(\tau-\alpha)d\alpha d\tau-$$

$$-\frac{\delta e^{-\lambda_n^2 t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\int_0^\tau \vartheta_n(\alpha)\,k(\tau-\alpha)d\alpha d\tau.$$

3-lemma. *Ixtiyoriy $0<t\le T$ da quyidagi tengsizliklar o‘rinli:*

$$|\vartheta_n(t)|\le C_1e^{-\lambda_n^2 t}(|\phi_n|+\|\nu_n\|+\|\nu_n'\|),\ \ t\in[0,T],$$

$$|\vartheta'_n(t)|\le \lambda_n^2e^{-\lambda_n^2 t}C_1(|\phi_n|+\|\nu_n\|+\|\nu_n'\|),$$

bu yerda

$$C_1=\left(1+\|k\|T^2e^{\|k\|T^2}\right)\left(1+\frac{\delta\|k\|T^2}{1-p_2}\left(1+\|k\|T^2e^{\|k\|T^2}\right)\right),$$

$$\|k\|=\max_{t\in[0,T]}|k(t)|.$$

Ushbu paragrafning asosiy natijasi (22)-(24) masala yechimining mavjud va yagonalik teoremasidan iborat.

5-teorema. *Faraz qilaylik, $\varphi(x)\in C[0,l]$, $\varphi'(x)\in L_2(0,l)$, $k(t)\in C[0,T]$, $\psi_1(t),\ \psi_2(t)\in C^1[0,T]$, $\delta\ge 0$, $\varphi'(0)-h\varphi(0)=\psi_1(0)+\delta\psi_1(T)$, $\varphi'(l)+h\varphi(l)=\psi_2(0)+\delta\psi_2(T)$ shartlar bajarilsin. U holda*

$$\delta\|k\|T^2\left(1+\|k\|T^2e^{\|k\|T^2}\right)<1$$

tengsizlikni qanoatlantiruvchi T lar uchun (22)-(24) to‘g‘ri masalaning $\vartheta(x,t)\in C^{2,1}(D_T)\cap C^{1,0}(\overline{D}_T)$ yagona yechimi mavjud.

Uchinchi bobning uchinchi paragrafida (22)-(25) teskari masala yechimining mavjudligi va yagonaligi isbotlangan.

6-teorema. *Faraz qilaylik,*

1. $\varphi(x)\in C^2[0,l],\varphi'''(x)\in L_2(0,l)$;

2. $\psi_1(t),\ \psi_2(t)\in C^1[0,T],\varphi'^{(0)}-h\varphi(0)=\psi_1(0)+\delta\psi_1(T)$,
$\varphi'(l)+h\varphi(l)=\psi_2(0)+\delta\psi_2(T)$;

3. $p(t)\in C^2[0,T],p(0)\neq 0$;

4. $\omega(x)\in C^2[0,l],\delta\ge 0,\omega'(0)=h\omega(0),\ \ \omega'(l)=-h\omega(l)$,
$p(0)+\delta p(T)=\int_0^l\omega\,(x)\varphi(x)dx$

shartlar bajarilsin. U holda (22) – (25) teskari masalaning $\vartheta(x,t)\in C^{2,1}(D_T)\cap C^{1.0}(\overline{D}_T)$, $k(t)\in C[0,T]$ yagona yechimi mavjud.

XULOSA

Dissertatsiyada integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich-chegaraviy shartli to‘g‘ri va teskari masalalar tadqiq qilingan. To‘g‘ri masalada integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich-chegaraviy, nolokal boshlang‘ich-nolokal chegaraviy shartli masalalarning korrektligi o‘rganilgan. Teskari masalada o‘ng tomoni o‘rama ko‘rinishda bo‘lgan issiqlik tarqalish tenglamasidan yadroni aniqlash masalalari ko‘rib chiqilgan. Ushbu masalalar ikkinchi tur Volterra-Fredgolm tipidagi integral tenglamalar sistemasiga keltirib tadqiq qilingan. To‘g‘ri va teskari masalalarning yechimi mavjud va yagonaligi haqidagi teoremalar Shauder prinsipi, qisqartirib akslantirishlar prinsipi va integral tengsizliklar nazariyasidagi teoremalar orqali isbotlangan. Asosiy tadqiqot natijalari quyidagilardan iborat:

chegaralangan sohada integro-differensial issiqlik tarqalish tenglamasi uchun qo‘yilgan nolokal boshlang‘ich-chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan;

integro–differensial issiqlik tarqalish tenglamasi uchun integral had qatnashgan nolokal boshlang‘ich va chegaraviy masala yechimining mavjud va yagonaligi ko‘rsatilgan;

integro-differensial issiqlik tarqalish tenglamasi uchun nolokal boshlang‘ich-chegaraviy masaladan yadroni aniqlashning teskari masalasi bir qiymatli yechiluvchanligi isbotlangan;

nolokal boshlang‘ich va nolokal chegaraviy shartli integro-differensial issiqlik tarqalish tenglamasidan yadroni aniqlash teskari masalasi yechimining mavjud va yagonaligi isbotlangan.

SCIENTIFIC COUNCIL FOR AWARDING SCIENTIFIC DEGREES DSc.03/30.12.2019.FM.02.01 AT SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

BUKHARA STATE UNIVERSITY

ATOEV DILSHOD DILMURODOVICH

NONLOCAL INVERSE PROBLEMS FOR AN INTEGRO-DIFFERENTIAL HEAT EQUATION

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION
for the doctor of philosophy (PhD) on physical and mathematical sciences

Bukhara – 2025

The theme of dissertation of doctor of philosophy (PhD) was registered at the Supreme Attestation Commission at the Cabinet of Ministers of the Republic of Uzbekistan under number B2023.4PhD/FM946.

The dissertation was performed at the Bukhara State University.

The abstract of dissertation is posted in three languages (Uzbek, English and Russian (resume)) on the website (www.samdu.uz) and in the website «ZiyoNet» Information and educational portal (www.ziyonet.uz).

Scientific superviser:	**Durdiev Durdimurod Kalandarovich** Doctor of Physical and Mathematical Sciences, Professor
Official opponents:	**Ishankulov Tolib** Doctor of Physical and Mathematical Sciences, Professor
	Safarov Jurabek Shakarovich Doctor of Physical and Mathematical Sciences
Leading organization:	**Urganch State University**

Defense will take place «____» ____________2025 at _____ at the meeting of Scientific Council number DSc.03/30.12.2019.FM.02.01 at Samarkand State University named after Sharof Rashidov. (Address: University Boulevard 15, Samarkand city, 140104, Uzbekistan, Phone number: (+99866)231-06-32, fax: (+99866)235-19-38, e-mail: patent@samdu.uz).

Dissertation is possible to review in Information-resource center at Samarkand State University named after Sharof Rashidov (is registered №___) (Address: University Boulevard 15, Samarkand city, 140104, Uzbekistan, Phone number: (+99866)231-06-32).

Abstract of dissertation sent out on «____» _______________2025 year

(Mailing report № ____ on «____» ___________ 2025 year)

A.S.Soliev
Chairman of scientific council on award of scientific degree, Doctor of Physical and Mathematical Sciences, professor

A.M.Khalkhuzhaev
Scientific secretary of scientific council on award of scientific degree, Doctor of Physical and Mathematical Sciences

A.B.Khasanov
Chairman of Scientific seminar under scientific council on award of scientific degree, Doctor of Physical and Mathematical Sciences, professor

INTRODUCTION (Abstract of Doctor of Philosophy (PhD) dissertation)

Actuality and demand of the theme of the dissertation. In the world, many scientific and applied researches conducted globally lead to the investigation of integro-differential partial differential equations, as well as direct and inverse problems formulated for them. Inverse problems have permeated fields, including thermal physics, quantum mechanics and optics, medicine, computer graphics, and artificial intelligence, as well as in all areas of modern science. To find the solution to a direct problem in mathematical physics, it is necessary to give the coefficients of the equation, the domain boundary, and the initial and boundary conditions. However, in practice, the coefficients of the equation are not always available. Additionally, the solution to the problem depends not only on the initial and boundary conditions but also on some other properties within the medium. In this case, solving the problem leads to the study of nonlocal inverse problems for integro-differential heat propagation equations. As the methods of solving such problems are not fully formed, for this reason research in this direction is one of the important actual tasks.

In the world scientific research is underway worldwide aimed at studying processes modeled by integro-differential equations of heat spread and inverse problems, which are one of the rapidly developing fields of mathematical physics. In this area, priority is given to research aimed at proving the existence and uniqueness of a correct and inverse solution to the problem for the integro-differential heat spread equation with non-local initial boundary conditions. It is also important to control the processes of heat spread and ensure an even distribution of heat throughout the environment, while these processes are modeled using a non-local equation of conditional heat spread. Therefore, the development of research on the correctness of the correct and inverse problems posed for the integro-differential heat spread equation with a non-local initial boundary condition is considered one of the urgent tasks.

In our republic, through applied and fundamental sciences, large-scale research is being conducted, which has both theoretical and practical significance, in particular, large-scale measures are being taken to prove the existence and uniqueness of solutions to correct and inverse problems for the nonlocal conditional integro-differential equation of heat spread and the application of the results obtained in practice. The main tasks and activities of mathematical science are defined as conducting scientific research at the level of international standards in the priority areas of the disciplines “Algebra and its applications, differential equations and their applications, nonlinear systems, dynamical systems and their applications, mathematical modeling, stochastic analysis, biomedical informatics, computational mathematics[1].” When performing this task, it is of great scientific importance to find

[1] Resolution PQ-4387 of the President of the Republic of Uzbekistan, dated July 9, 2019, titled "On State Support for the Further Development of Mathematics Education and Sciences, as well as Measures to Fundamentally Improve the Activities of the V.I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan.

the conditions for the correctness of non-local inverse problems for heat spread equations in the form of a right winding.

Decree of the President of the Republic of Uzbekistan on February 7, 2017 No. PF-4947 “On the strategy of action for the further development of the Republic of Uzbekistan”, on January 28, 2022 No. PF-60 “On the new development strategy of Uzbekistan for 2022-2026”, on September 11, 2023 No. PF-158 Decrees on the strategy “Uzbekistan–2030” on February 17, 2017 No. PP-2789 " on the activities of the Academy of Sciences, “On measures to further improve the organization, management and financing of scientific research”, PP-4708 on May 7, 2020 “On measures to improve the quality of education and the development of scientific research in the field of mathematics” and other regulatory legal acts related to this area of activity as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This work was performed in accordance with the priority areas of science and technology development in the Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. The inverse problems of mathematical physics were developed by M.M. Lavrentev, A.S. Alekseev, A.L. Bukhgeim, N.Ya. Beznoshchenko, A.I. Prilepko, V.G. Romanov, A. Lorenzi, J. Janno, G.V. Dyatlov, A. Favaron, H. Grabmueller and etc. Inverse problems for parabolic-type equations were first posed and studied by A.L. Bukhgeim, J. Janno, L.v. Wolfersdorf, V.G. Romanov, F. Colombo, A.I. Prilepko, A.D. Iskenderov, M. Grasselli, V.A. Dedok, N.Ya. Beznoshchenko, A. Boumenir, O. Danilkina, M.E. Gurtin, M.I. Ivanchov, A.C. Pipkin L. Yang and J. Jannos. Different ways of studying the issue were proposed and developed in the works of A. Lorenzi, D. Lesnic, F. Colombo, M. Grasselli, K. Karuppiah, D.Q. Durdiyev, J.J. Jumaev and others. Various methods for studying nonlocal conditional partial differential equations have been investigated in the works of D.G. Gordeziani, G.A. Avalishvili, E.I. Azizbayov, Y.T. Mehraliyev, S.V. Kirichenko, D. Lesnic, M.I. Ismailov, F. Kanca and etc.

In particular, in the work of E.I. Azizbaev and Y.T. Mehraliev[2], the inverse problem of determining the coefficient for a second-order parabolic-type equation with a nonlocal initial condition was studied. In this research, an equivalent problem to the original problem was formulated, and the existence and uniqueness of the solution to the equivalent problem were proven using the contraction mapping principle. In the article by M.S. Hussein, D. Lesnic, M.I. Ismailov[3], the inverse problem of determining the coefficient from the one-dimensional heat conduction equation with a nonlocal additional condition is studied. First, the existence and uniqueness of a classical solution to the problem, and the continuous dependence of the solution for the given functions were shown. Besides, analytical and numerical

---

[2] E.I. Azizboyev, Y.T. Mehraliyev, Solvability of nonlocal inverse boundary-value problem for second order parabolic equation with integral condition, Electronic journal of differential eqautions, no. 125, 1–14, 2017.

[3] M.S. Hussein, D. Lesnic va M.I. Ismailov, An inverse problem of finding the time-dependent diffusion coefficient from an integral condition, Mathematical Methods in the Applied Sciences, 39 (5), 963-980, 2015.

methods for determining the coefficients of the equation were proposed. In D.Q. Durdiyev, J.J. Jumayev[4] works studied the inverse problems of determining the kernel for the one-dimensional integro-differential heat conduction equation and proved the existence and uniqueness of the solution.

In this dissertation, inverse problems for the integro-differential heat equation with nonlocal initial and Dirichlet boundary conditions, as well as nonlocal initial and nonlocal boundary conditions, are studied.

The dissertation work is close to the above scientists E.I. Azizbayov, Y.T. Mehraliyev, M.S. Hussein, D. Lesnic, M.I. Ismailov, D.Q. Durdiev and J.J. Jumaevs' research, and the methods recommended by them were used in analysis.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation was conducted within the framework of the scientific research direction "Inverse Problems of Mathematical Physics" (M-02.2018) planned for 2020-2024 at Bukhara State University.

The aim of research work. The main purpose of this dissertation is to construct methods for determining the kernel from the nonlocal initial-boundary problems for one-dimensional integro-differential heat spread equations and to proof the existence, and uniqueness of solutions of these inverse problems.

Tasks of the research:

to find sufficient conditions for the existence and uniqueness of a solution to a non-local initial boundary value problem posed for the integro-differential equation of heat dissipation in a limited domain;

to prove that the locally accessible and only terms of the classical solution of the nonlocal initial boundary value problem with integral terms for the Integro–differential equation of heat dissipation depend on the invariant coefficients;

to prove the local unambiguous solvability of the inverse problem of determining the time-dependent kernel from an additional conditional problem in the form of an integral initial limit, an integral representation for the Integro-differential equation of heat dissipation;

to define the core from the integro-differential equation of heat dissipation with a non-local initial and non-local boundary condition is an unambiguous solution to the inverse problem, proof of the global existence and uniqueness of the solution.

The research object is a second-order integro-differential heat spread equation.

The research subject is the direct and inverse problems for the integro-differential heat spread equation with nonlocal conditions.

Research methods: The research used the theories of integral equations and integral inequalities, the second type of nonlinear closed integral equations of the

[4] D.K. Durdiev, J.J. Jumaev, One-dimensional inverse problems of finding the kernel of integro-differential heat equation in a bounded domain, Ukraine. Math. J., 73(3), 1723-1740, 2021.

Volterra-Fredholm type, methods of functional analysis particularly the Fourier method, the Schauder principle, and the Banach theorem.

The scientific novelty of the research is as follows:

sufficient conditions for the existence and uniqueness of the solution of a non-local initial boundary value problem posed for the Integro-differential equation of heat dissipation in a limited domain are found;

it is proved that the locally existing and only conditions for the classical solution of a nonlocal initial boundary value problem with integral terms for the Integro–differential equation of heat dissipation depend on invariant coefficients;

the local unambiguous solvability of the inverse problem of determining the time-dependent kernel from an additional conditional problem in the form of an integral initial limit, an integral for the Integro-differential equation of heat dissipation is proved;

the definition of the core from the Integro-differential equation of heat dissipation with a non-local initial and non-local boundary condition is an unambiguous solution to the inverse problem, the global existence and uniqueness of the solution are proved.

Practical results of the research:

for integro-differential equations of parabolic type, the one-valued solvability of the null inverse problem of kernel recognition has been proven;

by introducing the mean temperature for heat spread equations in the bounded domain, the conditions of existence and singularity of the memory function have been found.

The reliability of the results of the study. Our results have been obtained by using the methods of ffunctional analysis, the theory of differential and integral equations, the theory of inverse problems, the Fourier method, and the application of mathematic analysis methods. The obtained results are mathematically strongly proved.

Scientific and practical significance of research results. The scientific significance of the research results is explained by the further development of the theory of nonlocal inverse problems for the integro-differential equations of mathematical physics and the construction of methods for determining kernels.

The practical significance of the research results lies in their application to seismology, the exploration of oil and gas fields, and the study of heat propagation processes in heat-conducting media with memory.

Implementation of the research results. Based on scientific results on nonlocal inverse problem for integro-differential heat conduction equation:

the results of this dissertation were utilized in the implementation of the innovative project IL-21071166 (2022-2024 yy.) “Development of a Vertical-Axis Wind Turbine Designed for Low Wind Speeds”**,** carried out at the Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan (reference No. 1563-3, dated November 17, 2024). In particular, the dissertation proves the unique solvability of nonlocal inverse problems for identifying the kernel in parabolic-type integro-differential equation.

These results were applied in the project to verify the correctness of the mathematical model equations developed to ensure the efficient operation of wind turbines;

the proposed method for studying inverse problems has been applied in the international fundamental project №122041100096-4 “Mathematical Modeling in Sociology, Geophysics, and Engineering Sciences” for solving inverse problems involving the identification of convolution-type kernels in the system of visco-elasticity equations (reference No. 141, dated December 2, 2024, Yujniy Matematicheskiy Institut filial FGNBU FNTs «Vladikavkazskiy nauchniy sentr RAN”). The application of the scientific result viscoelasticity a wrapper kernel using non-local conditions from a system of equations made it possible to prove the existence and uniqueness of a solution to the inverse problem.

The approbation of the research results. The main results of the research have been discussed at 4 international and 2 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 11 research papers have been published in scientific journals, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the Doctor of Philosophy thesis, in addition, 2 of them were published in international journals of mathematics and physics and 3 paper published in a national mathematical journal.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion, and references. The volume of the dissertation is 98 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction the motivation of the research theme and correspondence to the priority research areas of science and technology of the Republic is given, we present a review of international research on the theme of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state the scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works, and the structure of the dissertation.

The first chapter of the thesis is called **“The direct and inverse problems for an integro-differential heat conduction equation with nonlocal initial-boundary and integral form additional conditions”** and The direct and inverse problems for the heat conduction integro-differential equation with nonlocal initial-boundary and integral form additional conditions. The first section introduces essential concepts, including foundational information on functional analysis and mathematical physics. The second section of the first chapter studies the direct problem of the integro-differential heat equation in a bounded domain and proves its unique solvability.
The following Integro-differential equation of heat spread:

$$u_t - u_{xx} = \int_0^t k\,(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_T, \tag{1}$$

nonlocal initial condition

$$u(x,0) + \delta u(x,T) + \int_0^T p\,(\tau)u(x,\tau)d\tau = \varphi(x), \ \ x \in [0,l], \tag{2}$$

nonlocal initial and final conditions

$$u|_{x=0} = u|_{x=l} = 0, \ \ t \in [0,T], \tag{3}$$

look at. From problem (1) - (3), we define the function $u(x,t)$: here $D_T = \{(x,t): 0 < x < l, 0 < t \le T\}$ ($T > 0,\ l > 0$ invariant numbers), $\delta \ge 0$, $\varphi(x),\ p(t)(p(t) > 0)$ are sufficiently smooth functions given.

The problem of finding a function $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$ satisfying problem (1) -(3) is called the direct problem.

The solution of the problem (1) - (3) satisfies the following integral equation:

$$u(x,t) = \Phi(x,t) + \int_0^t \int_0^l G\,(x,\xi,t-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$

$$-\delta \int_0^T \int_0^l G_0\,(x,\xi,t+T-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$

$$-\int_0^T \int_0^\mu \int_0^l p\,(\mu)G_0(x,\xi,t+\mu-\beta) \int_0^\beta k\,(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta d\mu, \tag{4}$$

here

$$\Phi(x,t) = \int_0^l \varphi\,(\xi)G_0(x,\xi,t)d\xi,$$

$$G(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty} e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n\xi}{l} sin\frac{\pi n x}{l},$$

$$G_0(x,\xi,t)$$

$$= \frac{2}{l}\sum_{n=1}^{\infty} \frac{1}{1+\delta e^{-\left(\frac{\pi n}{l}\right)^2 T} + \int_0^T p\,(\tau)e^{-\left(\frac{\pi n}{l}\right)^2 \tau} d\tau}\ e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n\xi}{l} sin\frac{\pi n x}{l}.$$

Using the Banach principle, it is shown that the solution of the Volterra-Fredholm integral equation (4) exists and is unique. The following Lemma is appropriate for this (4).

Lemma 1. Suppose that $(x) \in C[0,l], \varphi'(x) \in L_2(0,l), \{p(t),k(t)\} \in C[0,T]$, $p(t) > 0$, $\varphi(0) = \varphi(l) = 0$. Then in $0 < T \le T_1$s (4) there is the only solution of the integral equation defined in Class $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$, where T_1 is the positive root of this $p_0k_0T^3 + 3(1+\delta)k_0T^2 - 3 = 0$ equation and $k_0 = \max_{t\in[0,T]}|k(t)|,\ p_0 = \max_{t\in[0,T]}|p(t)|$.

In the third paragraph (1)-(3) the matter and

$$u(x_0,t) = h(t), \quad x_0 \epsilon (0,l), \tag{5}$$

the inverse problem of defining functions $u(x,t)$, $k(t)$ using an additional condition is considered. Where $h(t)$ is a sufficiently smooth function.

Theorem 1. *Let conditions:* $\varphi(x) \in C^4[0,l]$, $p(t) \in C[0,T]$, $p(t) \geq 0$, $h(t) \in C^2[0,T]$, $\varphi(0) = \varphi(l) = \varphi''(0) = \varphi''(l) = 0$, $h(0) \neq 0$, $\delta \geq 0$ *be satisfied. Then there exists sufficiently small numbers* $T^* \in (0,T)$ *that the solution to the integral equations (1)-(3), (5) in the class of functions* $u(x,t) \in C^{4,1}(D_{T^*}) \cap C(\overline{D}_{T^*})$ *and* $k(t) \in C[0;T^*]$ *exist and unique, where* $D_{T^*} = \{(x,t) | x \in (0,l^*), t \in (0,T^*]\}$.

The second chapter is called **"The problem of determining kernel of the integro-differential heat equation"** and in the first paragraph of the chapter, the existence and uniqueness of the solution to the nonlocal initial-boundary problem for the integro-differential parabolic-type equation is investigated. Consider the nonlocal initial-boundary problem for the heat conduction equation with an integral term of convolution type on the right-hand side:

$$u_t - u_{xx} = \int_0^t k\,(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_T, \tag{6}$$

$$u(x,0) + \delta u(x,T) = \varphi(x), \;\; x \in [0,l], \tag{7}$$

$$u|_{x=0} = u|_{x=l} = 0, \;\; t \in [0,T], \quad \varphi(0) = \varphi(l) = 0, \tag{8}$$

where $\delta \geq 0$ -given number, $\varphi(x)$ and $k(t)$ are given smooth functions.

Definition 1. *If a function u(x,t):*

1. In the D_T *field, let the function* $u(x,t)$ *and its derivatives* u_t, u_x, u_{xx} *be continuous;*

2. Let equation (6) and conditions (7), (8) satisfy in the classical sense then this function is called the classical doing of the (6)-(8) sum.

(6)-(8) doing of the sum

$$u(x,t) = \sum_{n=1}^{\infty} X_n\,(x)u_n(t), \tag{9}$$

looking for, where $X_n(x) = \sqrt{\frac{2}{l}}\sin\lambda_n x$, $\lambda_n = \frac{\pi n}{l}$, $n = 1,2,3, ...$

Multiplying the equality (6) and (7) by $X_n(x)$ and integrating at $(0,l)$, we get the following problem.

$$u'_n(t) + \lambda_n^2 u_n(t) = \int_0^t k\,(t-\alpha)u_n(\alpha)d\alpha, \tag{10}$$

$$u_n(0) + \delta u_n(T) = \varphi_n, \quad n = 1,2,\ldots, \tag{11}$$

here $\varphi_n = \sqrt{\frac{2}{l}}\int_0^l \varphi\,(x)\sin\lambda_n x dx$.

For each assigned $n \in \mathbb{N}$ the problem (20)-(21) is to the following Volterra-Fredholm integral equation:

$$u_n(t) = \frac{e^{-\lambda_n^2 t}}{\psi_n(T)}\varphi_n - \frac{\delta e^{-\lambda_n^2 t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\int_0^\tau k\,(\tau-\alpha)u_n(\alpha)d\alpha d\tau +$$

$$+ e^{-\lambda_n^2 t}\int_0^{t} e^{\lambda_n^2 \tau}\int_0^{\tau} k\,(\tau-\alpha)u_n(\alpha)d\alpha d\tau \qquad (12)$$

where

$$\psi_n(T) := 1 + \delta e^{-\lambda_n^2 T}.$$

Lemma 2. *For any* $0 < t \le T$ *and for sufficiently large n, the estimates are valid:*

$$|u_n(t)| \le C_1 e^{-\lambda_n^2 t}|\varphi_n|, \quad t \in [0,T],$$
$$|u'_n(t)| \le C_1 e^{-\lambda_n^2 t}|\varphi_n|(\lambda_n^2 + T \parallel k \parallel).$$

Here $0 < t \le T$, $\|k\| = \max_{t\in[0,T]}|k(t)|$, $C_1(\|k\|, T, \delta)$.

Theorem 2. *Imagine that,*$\varphi(x) \in C[0,l]$, $\varphi'(x) \in L_2(0,l)$, $k(t) \in C[0,T]$, $\varphi(0) = \varphi(l) = 0$ *are satisfied. Then,*

$$\frac{\delta \parallel k \parallel T^2}{2}\left(1 + \parallel k \parallel e^{\frac{\|k\|T^2}{2}}\right) \le 1$$

such that for T ∈ (0, T] the solution to the direct problem (6)–(8) in the class of functions* $u(x,t) \in C^{2,1}(D_T) \cap C(\overline{D}_T)$ *exists and it is unique.*

The second paragraph of the second chapter explores the existence and uniqueness of the inverse problem solution posed for the integro-differential heat spread equation. In this,

$$\int_0^l \omega\,(x)u(x,t)dx = h(t), \qquad (13)$$

using the additional condition, the inverse problem of defining functions $u(x,t)$ and $k(t)$ is investigated. Where $\omega(x)$, $h(t)$ are given sufficiently smooth functions.

(6)-(8), (13) from the problem,it is called the inverse problem to the question of finding functions $u(x,t)$ and $k(t)$.

According to (9), (12), we write the problem(6)-(8) in the form of the following Volterra-Fredgolm integral equation:

$$u(x,t) = \Phi(x,t) -$$
$$-\delta\int_t^{T+t}\int_0^l G_0\,(x,\xi,\tau)\int_0^{T+t-\tau} k\,(\alpha)u(\xi, T+t-\tau-\alpha)d\alpha d\xi d\tau +$$
$$+\int_0^t\int_0^l G\,(x,\xi,\tau)\int_0^{t-\tau} k\,(\alpha)u(\xi, t-\tau-\alpha)d\alpha d\xi d\tau. \qquad (14)$$

where

$$\Phi(x,t) = \int_0^l G_0\,(x,\xi,t)\varphi(\xi)d\xi,$$

$$G_0(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty}\frac{1}{1+\delta e^{-\left(\frac{\pi n}{l}\right)^2 T}} e^{-\left(\frac{\pi n}{l}\right)^2 t} sin\frac{\pi n}{l}\xi sin\frac{\pi n}{l}x.$$

Differentiating equation (14) with respect to t, we obtain the following:

$$u_t(x,t) = \Phi_t(x,t) + \delta\int_0^l G_0(x,\xi,t)\int_0^T k(\alpha)u(\xi,T-\alpha)d\alpha d\xi -$$
$$-\delta\int_t^{T+t}\int_0^l G_0(x,\xi,\tau)k(T+t-\alpha)u(\xi,0)d\alpha d\xi d\tau -$$
$$-\delta\int_t^{T+t}\int_0^l G_0(x,\xi,\tau)\int_0^{T+t-\tau} k(\alpha)u_t(\xi,T+t-\tau-\alpha)d\alpha d\xi d\tau + \quad (15)$$
$$+\int_0^t\int_0^l G(x,\xi,\tau)k(t-\tau)u(\xi,0)d\xi d\tau +$$
$$+\int_0^t\int_0^l G(x,\xi,\tau)\int_0^{t-\tau} k(\alpha)u_t(\xi,t-\tau-\alpha)d\alpha d\xi d\tau.$$

Suppose $\omega(0) = \omega(l) = 0$ conditions are satisfied. (6) multiplying both sides of the equality by $\omega(x)$ and integrating over *x* from 0 to *l*. After simple substitutions, we obtain the following integral equation:

$$k(t) = \frac{h''(t)}{|h(0)|} - \frac{1}{|h(0)|}\int_0^l \omega''(x)u_t(x,t)\,dx - \frac{1}{|h(0)|}\int_0^t k(\tau)h_t(t-\tau)d\tau. \quad (16)$$

(14) - (16) together $\left(u(x,t), u_t(x,t),\ k(t)\right)$form a system of closed integral Equations with respect to unknowns. For the existence and uniqueness of the solution to the problem, the following theorem holds:

Theorem 3. *If we imagine,*

1. $\varphi(x) \in C^2[0,l], \varphi'''(x) \in L_2(0,l), \varphi(0) = \varphi(l) = \varphi''(0) = \varphi''(l) = 0,$

2. $h(t) \in C^2[0,T], h(0) \neq 0,\ \delta \geq 0,$

3. $\omega(x) \in C^2[0,l], \int_0^l \omega(x)\varphi(x)dx = h(0) + \delta h(T), \quad \omega(0) = \omega(l) = 0$

are met, then there exists sufficiently small numbers $T_0 > 0, l_0 > 0$ *that the solution to the problem (6) – (8) and (13) in the class of functions* $u(x,t) \in C^{2,1}(D_T) \cap C\left(\overline{D}_T\right)$, $k(t) \in C[0,T]$ *exists is unique.*

In the third paragraph (6)-(8) issue and this

$$u(x_0,t) = h(t) \quad (17)$$

It is assumed that $h(t), t \in [0,T]$ is given sufficiently smooth function.

Theorem 4. *Let conditions:* $\varphi(x) \in C^4\ h(t) \in C^2[0,T], \varphi(0) = \varphi(l) = \varphi''(0) = \varphi''(l) = 0,\ h(0) + \delta h(T) = \varphi(x_0)\ ,\ h(0) \neq 0,\ \delta \geq 0$ *are satisfied. Then there exists sufficiently small numbers* $T^* \in (0,T)$ *that the solution to the integral equations (6)-(8), (17) in the class of functions* $u(x,t) \in C^{4,1}(D_{T^*}) \cap C\left(\overline{D}_{T^*}\right), k(t) \in C[0,T^*]$, *exist and unique.*

The third chapter of the dissertation work was called the **"Inverse problem for the nonlocal initial and nonlocal boundary condition integro-differential heat conduction equation"**, obtaining the conditions of existence and uniqueness for the solution of the right and inverse problem.

This

$$u_t - u_{xx} = \int_0^t k(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_T, \quad (18)$$

$$u(x,0) + \delta u(x,T) = \varphi(x), \;\; x \in [0,l], \tag{19}$$
$$u_x(0,t) - hu(0,t) = \psi_1(t), \;\; u_x(l,t) + hu(l,t) = \psi_2(t), \;\; t \in [0,T], \tag{20}$$
$$\int_0^l \omega\,(x)u(x,t)dx = p(t), \tag{21}$$
from the problem,we look at the question of defining functions $u(x,t), \; k(t)$, where $\delta \geq 0, \;\; h > 0$ given numbers, $\varphi(x), \omega(x), p(t), \psi_1(t), \psi_2(t)$ given sufficiently smooth functions.

Definition 2. *If* $\big(u(x,t); k(t)\big)$ *the functions are:*

1. *the function* $u(x,t)$ *and its derivatives* u_t, u_x, u_{xx} *are continuous in the domain* D_T
2. *equation (18) and conditions (19)-(21) are satisfied in the classical sense;*
3. *the function* $k(t)$ *is continuous on the interval [0, T].*

if the conditions are satisfied,then the pair of functions$\big(u(x,t); k(t)\big)$is called the classical solution of the(18)-(21) problem.

(18) - (21) the inverse problem follows
$$\vartheta_t = \vartheta_{xx} + \int_0^t k\,(t-\tau)\vartheta(x,\tau)d\tau + \int_0^t k\,(t-\tau)\nu(x,\tau)d\tau - \nu_t(x,t), \tag{22}$$
$$\vartheta(x,0) + \delta\vartheta(x,T) = \phi(x) \quad 0 \leq x \leq l, \tag{23}$$
$$\vartheta_x(0,t) - h\vartheta(0,t) = 0, \;\; \vartheta_x(l,t) + h\vartheta(l,t) = 0, \tag{24}$$
$$\int_0^l \omega\,(x)\vartheta(x,t)dx = p(t) - \int_0^l \nu\,(x,t)\omega(x)dx, \tag{25}$$
where $\vartheta(x,t) = u(x,t) - \nu(x,t)$,
$$\nu(x,t) = (\alpha_1 x + \beta_1)\psi_1(t) + (\alpha_2 x + \beta_2)\psi_2(t)),$$
$$\alpha_1 = \frac{1}{2+hl}, \; \beta_1 = -\frac{1+hl}{(2+hl)h}, \; \alpha_2 = \frac{1}{2+hl}, \; \beta_2 = \frac{1}{h(2+hl)},$$
$$\phi(x) = \varphi(x) - \nu(x,0) - \delta\nu(x,T).$$
the auxiliary is brought to the issue, and it is required to find $\vartheta(x,t), k(t)$functions. Where $u(x,t) = \vartheta(x,t) + \nu(x,t)$, $\nu(x,t) = (\alpha_1 x + \beta_1)\psi_1(t) + (\alpha_2 x + \beta_2)\psi_2(t))$, $\alpha_1 = \frac{1}{2+hl}$, $\beta_1 = -\frac{1+hl}{(2+hl)h}$, $\alpha_2 = \frac{1}{2+hl}$, $\beta_2 = \frac{1}{h(2+hl)}$, $\phi(x) = \varphi(x) - \nu(x,0) - \delta\nu(x,T)$.

The second paragraph of the third chapter (22)-(24) explores the question of finding the classical solution of the right problem $\vartheta(x,t) \in C^{2,1}(D_T) \cap C^{1,0}\big(\overline{D}_T\big)$

(22)-(24) problem solution
$$\vartheta(x,t) = \sum_{n=0}^{\infty} X_n\,(x)\vartheta_n(t) \tag{26}$$
We look for, here, $X_n(x)$ are specific functions and they form a complete orthonormal system in $L_2(0,l), \vartheta_n(t) = (\vartheta, X_n)$.

By multiplying the (22), (23) equalities by $X_n(x)$ and integrating at $(0,l)$ we get the following problem
$$\vartheta'_n(t) + \lambda_n^2\vartheta_n(t) = f_n(t), \tag{27}$$

$$\vartheta_n(0) + \delta\vartheta_n(T) = \phi_n, \qquad n = 1,2,\ldots, \tag{28}$$

here

$$\phi_n = (\phi, X_n(x)), \qquad f_n = (f, X_n(x)),$$

$$f(x,t) = \int_0^t k(t-\tau)\vartheta(x,\tau)d\tau + \int_0^t k\,(t-\tau)\nu(x,\tau)d\tau - \nu_t(x,t).$$

For each assigned $n \in N$ the problem (46)-(47) is equivalent to the following Volterra-Fredholm integral equation:

$$\vartheta_n(t) = \frac{e^{-\lambda_n^2 t}}{\psi_n(T)}\phi_n + e^{-\lambda_n^2 t}\int_0^t e^{\lambda_n^2 \tau}\left(\int_0^\tau k(\tau-\alpha)\nu_n(\alpha)\,d\alpha - \nu_n'(\tau)\right)d\tau -$$

$$-\frac{\delta e^{-\lambda_n^2 t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\left(\int_0^\tau k(\tau-\alpha)\nu_n(\alpha)\,d\alpha - \nu_n'(\tau)\right)d\tau +$$

$$+e^{-\lambda_n^2 t}\int_0^t e^{\lambda_n^2\tau}\int_0^\tau \vartheta_n(\alpha)\,k(\tau-\alpha)d\alpha d\tau +$$

$$-\frac{\delta e^{-\lambda_n^2 t}}{\psi_n(T)}\int_0^T e^{-\lambda_n^2(T-\tau)}\int_0^\tau \vartheta_n(\alpha)\,k(\tau-\alpha)d\alpha d\tau$$

Lemma 3. *For any $0 < t \le T$ and for sufficiently large n, the estimates are valid:*

$$|\vartheta_n(t)| \le C_1 e^{-\lambda_n^2 t}(|\phi_n| + \|\nu_n\| + \|\nu_n'\|), \;\; t \in [0,T],$$

$$|\vartheta'_n(t)| \le \lambda_n^2 e^{-\lambda_n^2 t} C_1(|\phi_n| + \|\nu_n\| + \|\nu_n'\|),$$

where $C_1 = \left(1 + \| k \| T^2 e^{\|k\|T^2}\right)\left(1 + \frac{\delta\|k\|T^2}{1-p_2}\left(1+\| k \| T^2 e^{\|k\|T^2}\right)\right)$,

$\|k\| = \max_{t\in[0,T]}|k(t)|$.

Theorem 5. *Let condtions of $\varphi(x) \in C[0,l]$, $\varphi'(x) \in L_2(0,l)$, $k(t) \in C[0,T]$, $\psi_1(t),\ \psi_2(t) \in C^1[0,T]$, $\delta \ge 0$, $\varphi'(0) - h\varphi(0) = \psi_1(0) + \delta\psi_1(T)$, $\varphi'(l) + h\varphi(l) = \psi_2(0) + \delta\psi_2(T)$ are satisfied. Then,*

$$\delta \| k \| T^2\left(1+\| k \| T^2 e^{\|k\|T^2}\right) < 1$$

for inequality-satisfying T 's (22) - (24) of the right issue there exists a unique solution $\vartheta(x,t) \in C^{2,1}(D_T) \cap C^{1,0}(\overline{D}_T)$.

The third paragraph of the third chapter (22)-(25) proves the existence and uniqueness of the solution to the inverse problem.

Theorem 6. *Let conditions:*

1. $\varphi(x) \in C^2[0,l], \varphi'''(x) \in L_2(0,l)$;

2. $\psi_1(t),\ \psi_2(t) \in C^1[0,T], \varphi'^{(0)} - h\varphi(0) = \psi_1(0) + \delta\psi_1(T)$,
$\varphi'(l) + h\varphi(l) = \psi_2(0) + \delta\psi_2(T)$;

3. $p(t) \in C^2[0,T], p(0) \ne 0$;

4. $\omega(x) \in C^2[0,l], \delta \ge 0, \omega'(0) = h\omega(0),\ \ \omega'(l) = -h\omega(l)$,
$p(0) + \delta p(T) = \int_0^l \omega\,(x)\varphi(x)dx$

let the conditions be met. Then there exists a unique solution of the(22) - (25) inverse problem $\vartheta(x,t) \in C^{2,1}(D_T) \cap C^{1.0}(\overline{D}_T)$, $k(t) \in C[0,T]$.

CONCLUSION

In the dissertation, the direct and inverse problems for the integro-differential heat conduction equation with nonlocal initial-boundary conditions were investigated. In the direct problem, the nonlocal initial-boundary and nonlocal initial-nonlocal boundary problems for the integro-differential heat conduction equation were studied. In the inverse problem, the kernel identification problems for the integro-differential heat conduction equation with a convolution-type integral operator on the right-hand side were considered. These problems were reduced to an type system of second-kind Volterra-Fredholm integral equations and analyzed. Theorems on the existence and uniqueness of solutions for direct and inverse problems were proven using the Schauder fixed-point principle, the contraction mapping principle, and theorems from the theory of integral inequalities.

The main results of the study are as follows:

the existence and uniqueness of the solution for the nonlocal initial-boundary value problem of the integro-differential heat conduction equation in a bounded domain have been proven;

the existence and uniqueness of the solution for the one-dimensional integro-differential heat conduction problem with nonlocal initial and boundary conditions involving an integral term have been showed;

the unique solvability of the inverse problem for identifying the kernel in the nonlocal initial-boundary value problem of the integro-differential heat conduction equation has been proven;

the existence and uniqueness of the solution for the inverse problem of identifying the kernel in the integro-differential heat conduction equation with nonlocal initial and nonlocal boundary conditions have been established.

**НАУЧНЫЙ СОВЕТ DSc.03/30.12.2019.FM.02.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
САМАРКАНДСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ
ИМЕНИ ШАРОФА РАШИДОВА**

БУХАРСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

АТОЕВ ДИЛШОД ДИЛМУРОДОВИЧ

НЕЛОКАЛЬНЫЕ ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ РАСПРОСТРАНЕНИЕ ТЕПЛА

01.01.02 – Дифференциальные уравнения и математическая физика

А В Т О Р Е Ф Е Р А Т
диссертации доктора философии (PhD) по физико-математическим наукам

Бухара – 2025

Тема диссертации доктора философии (PhD) зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за № B2023.4PhD/FM946.

Диссертация выполнена в Бухарском государственном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.samdu.uz) и на Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz)

Научный консультант:	**Дурдиев Дурдимурод Каландарович** доктор физико-математических наук, профессор
Официальные оппоненты:	**Ишанкулов Толиб** доктор физико-математических наук, профессор
	Сафаров Журабек Шакарович доктор физико-математических наук
Ведущая организация:	**Ургенчский государственный университет**

Защита диссертации состоится «____» __________2025 года в «____» часов на заседании Научного совета DSc.03/30.12.2019.FM.02.01 при Самаркандском государственном университете имени Шарофа Рашидова. (Адрес: 140104, г. Самарканд, Университетский бульвар, 15. Тел.: (+99866)231-06-32, факс: (+99866) 235-19-38, e-mail: patent@samdu.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Самаркандского государственного университета имени Шарофа Рашидова (зарегистрирована за №___). (Адрес: 140104, г. Самарканд, Университетский бульвар, 15. Тел.: (+99866)231-06-32, факс: (+99866) 235-19-38).

Автореферат диссертации разослан «____» ______________2025 года.

(протокол рассылки № ____ от «___» _______________2025 года).

А.С. Солиев
Председатель научного совета по присуждению научных степеней, доктор физико-математических наук, профессор

А.М. Халхужаев
Ученый секретарь научного совета по присуждению научных степеней, доктор физико-математических наук

А.Б. Хасанов
Председатель научного семинара при научном совете по присуждению научных степеней, доктор физико-математических наук, профессор

ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования. Разработка методов определения ядра для нелокальных начальных и нелокальных краевых задач для одномерного интегро-дифференциального уравнения теплопроводности и доказательство существования и единственности решений данных обратных задач.

Объект исследования. Интегро-дифференциальные уравнения теплопроводности второго порядка.

Научная новизна исследования состоит из следующих:

Найдены достаточные условия существования и единственности решения нелокальной начально-краевой задачи для интегро-дифференциального уравнения теплопроводности в ограниченной области;

доказаны локальные условия существования и единственности классического решения нелокальной начально-краевой задачи, содержащей интегральный член для интегро-дифференциального уравнения теплопроводности, в зависимости постоянных коэффициентов;

доказана локальная однозначная разрешимость обратной задачи определения нестационарного ядра из нелокальной начально-краевой задачи для интегро-дифференциального уравнения теплопроводности, где дополнительное условие в интегральной форме;

доказана однозначность решения обратной задачи определения ядра из интегро-дифференциального уравнения теплопроводности с нелокальными начальными и нелокальными граничными условиями, а также глобальное существование и единственность решения.

Внедрение результатов исследования. Полученные в диссертации результаты по нелокальныем обратныем задачам для интегро-дифференциального уравнения распространение тепла были использованы в следующих научно-исследовательских проектах:

результаты данной диссертации были использованы при выполнении инновационного проекта ИЛ-21071166 (2022-2024 гг.) «Создание ветротурбины с вертикальной осью, предназначенной для низкой скорости ветра», выполненного в Институте механики и сейсмостойкости сооружений Академии наук Республики Узбекистан (справка № 1563-3 от 17 ноября 2024 года). В частности, в диссертационной работе доказана однозначная разрешимость нелокальных обратных задач определения ядра для интегро-дифференциальных уравнений параболического типа. Данные результаты были использованы в проекте при проверке корректности уравнений математической модели, разработанной с целью эффективной работы ветротурбин;

предложенный метод исследования обратных задач был использован в зарубежном фундаментальном проекте 122041100096-4 «Математическое моделирование в социологии, геофизике и инженерных науках» в обратных задачах определения сверточного ядра из системы уравнений вязкоупругости (Южный математический институт - филиал ФГНБУ ФНЦ «Владикавказский научный центр РАН», справка № 141 от 2 декабря 2024 года). Применение научного результата позволило определить сверточное ядро из системы уравнений вязкоупругости с помощью нелокальных условий и доказать его существование и единственность.

Структура и объем диссертации.

Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации составляет 98 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (Part I; Часть I)

1. Durdiev D.K., Jumayev J.J., Atoev D.D. Kernel determination problem in an integro-differential equation of parabolic type with nonlocal condition // *Vestnik Udmurtskogo Universiteta*, 33(1) (2023), 90–102. (Scopus, IF=0.807)

2. Durdiev D.K., Jumayev J.J., Atoev D.D. Convolution kernel determining problem for an integro-differential heat equation with nonlocal initial boundary and overdetermination conditions //*Journal of Mathematical Sciences*, 27(2023),56–65. 10723374 (Scopus, IF=0.302)

3. Jumaev J.J, Atoev D.D. Inverse problem of determining the kernel in an integro-differential equation of parabolic type with nonlocal conditions // *Bulletin of the Institute of Mathematics*, 6(4) (2023), 46–56. (01.00.00; №6).

4. Jumaev J.J., Atoev D.D. Investigation of the integro-differential equation of parabolic type with nonlocal condition // *Scientific Reports of Bukhara State University*, 98(4) (2023), 15–19. (01.00.00; № 3).

5. Atoev D.D. Solvability of an integro-differential heat equation with nonlocal initial-boundary condition // *Scientific Reports of Bukhara State University*, 111(6) (2024), 7–12. (01.00.00; № 3).

II bo‘lim (Part II; Часть II)

6. Durdiev D.K., Jumayev J.J., Atoev D.D. Inverse problem of determining the kernel in an integro-differential equation of parabolic type with nonlocal condition // *International Scientific-Practical Conference on Modern Problems of Applied Mathematics and Information Technology*, May 11–12, 2022, 149–150.

7. Atoev D.D. Inverse problem for integro-differential equation of parabolic type with nonlocal initial-boundary condition // *International Scientific-Practical Conference on Actual Problems of Physics, Mathematics, and Mechanics*, Bukhara, Uzbekistan, May 24–25, 2023, 118–119.

8. Atoev D.D. Inverse problem of determining the kernel in an integro-differential equation of parabolic type with nonlocal conditions // *International Conference on Ali Qushchi and Mirzo Ulugbek Scientific Legacy*, Samarkand, September 21–22, 2023, 167–170.

9. Atoev D.D. An inverse problem for the integro-differential parabolic equation in the case of nonlocal initial-boundary and overdetermination conditions // *International Scientific Conference on Nonclassical Equations of Mathematical Physics and Their Applications*, Tashkent, October 24–26, 2024, 60-61.

10. Jumaev J.J., Atoev D.D. Investigation in an integro-differential equation of parabolic type with nonlocal condition // *Ajiniyoz Nukus State Pedagogical Institute Conference Proceedings*, Nukus, April 12–13, 2023, 91–94.

11. Atoyev D.D. Integro-differential heat conduction equation for a nonlocal initial boundary condition problem // *Conference Proceedings of Modern Analysis and Mathematical Physics Issues*, Samarkand, September 16–17, 2024, 211–213.

Avtoreferat “Durdona” nashriyotida tahrirdan o‘tkazildi hamda o‘zbek, rus va ingliz tillaridagi matnlarning mosligi tekshirildi.

Bosishga ruxsat etildi: 25.04.2025 yil. Bichimi 60x84 1/16 , «Times New Roman» garniturada raqamli bosma usulida bosildi.
Shartli bosma tabog‘i 2,5 Adadi: 100 nusxa. Buyurtma №235.

Guvohnoma AI №178. 08.12.2010.
“Sadriddin Salim Buxoriy” MChJ bosmaxonasida chop etildi.
Buxoro shahri, M.Iqbol ko‘chasi, 11-uy. Tel.: 65 221-26-45