**V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

MATEMATIKA INSTITUTI

OLIMOV UMRBEK RASHIDOVICH

DISKRET VAQTLI CHEKSIZ O‘LCHAMLI RATSIONAL DINAMIK SISTEMALAR

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT - 2025 yil

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar aksariyat hollarda cheksiz o‘lchamli ratsional dinamik sistemalarni tadqiq qilish masalalariga keltiriladi. Dinamik sistemalar nazariyasining ahamiyati sistemaning hozirgi holatini bilgan holda kelajakda qanday holatlar kelib chiqishini baholashdan iborat bo‘lib, statistik mexanika, fizika, biologiya, tibbiyot, iqtisodiyot va boshqa masalalarni hal etishda muhim rol o‘ynaydi. Bu esa insonlarga murakkab sistemalarni tushunish va optimal qarorlar qabul qilishda yordam beradi. Ta’kidlash joizki, dinamik sistemalar odatda ayirmali tenglamalar, differensial tenglamalar hamda differensial-ayirmali tenglamalar orqali ifodalanadi. Statistik mexanika va kvant fizikasi sohalaridagi tadqiqotlarning asosiy obyektlaridan biri cheksiz o‘lchamli ratsional operatorlarning dinamikasi hisoblanadi. Shuningdek, cheksiz o‘lchamli ratsional dinamik sistemalar fizikada zarrachalar o‘rtasidagi geometrik ta’sirlarni modellashtirish va tizimlarning termodinamik xususiyatlarini o‘rganishda muhim rol o'ynaydi. Shu sababli, cheksiz o‘lchamli ratsional akslantirishlarning dinamikasini tadqiq etish dinamik sistemalar nazariyasidagi muhim va dolzarb vazifalardan biri bo‘lib qolmoqda.

Hozirgi vaqtda cheksiz o‘lchamli ratsional dinamik sistemalar nazariyasi ko‘plab amaliy masalalarning xarakterini tushunishda, tahlil qilishda hamda ratsional yechimini topishda asosiy vosita sifatida qo‘llanilmoqda. Jumladan, dinamik sistemalar nazariyasi uchun trayektoriyaning limit nuqtalar to‘plamini tavsiflash, qo‘zg‘almas va davriy nuqtalarning mavjudligini hamda ularning turini aniqlash, invariant to‘plamlarni topish, bifurkatsiya sodir bo‘lishini tekshirish, sistemaning regulyar yoki xaotik xarakterga egaligini aniqlash kabi muammolar ko‘plab amaliy masalalarni tahlil qilishda keng qo‘llanilmoqda. Bu borada diskret vaqtli cheksiz o‘lchamli ratsional operatorlar dinamikasi turli sohalarda sistemalarning vaqt bo‘yicha o‘zgarishini tahlil qilishda maqsadli ilmiy tadqiqotlardan hisoblanadi. Matematikadan fundamental ilmiy natijalar uchun ta’sis etilgan Fields mukofoti sovrindorlarining yettitasi aynan dinamik sistemalar nazariyasi mutaxasislari ekanligi, sohaning dolzarbligidan dalolatdir.

Mamlakatimizda so‘nggi yillarda fundamental tadqiqotlarni jahonda yetakchi o‘rinlarga chiqarishga katta e’tibor berilayotganligi[1] tufayli diskret vaqtli cheksiz o‘lchamli ratsional operatorlarning dinamikasini o‘rganish bo‘yicha salmoqli natijalarga erishildi. Xalqaro matematika jamiyati e’tiborida turgan sohalardan biri bo‘lgan dinamik sistemalar nazariyasi bo‘yicha tatqiqotlarni yurtimizda yanada kuchaytirish, jumladan diskret vaqtli cheksiz o‘lchamli ratsional operatorlarning dinamikasini tadqiq qilish muhim ahamiyatga ega. Mazkur dissertatsiya ishida olib borilgan ilmiy tadqiqotlar shu masalalar turkumiga mansub natijalardan hisoblanadi.

[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash shuningdek,O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika institute faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi № PQ-4387-sonli qarori.

Oʻzbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha harakatlar strategiyasi toʻgʻrisida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga moʻljallangan Yangi Oʻzbekistonning Taraqqiyot strategiyasi toʻgʻrisida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek Oʻzbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari toʻgʻrisida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yoʻnalishlariga bogʻliqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo’nalishi doirasida bajarilgan.

Muammoning oʻrganilganlik darajasi. Dinamik sistemalarga oid dastlabki tushunchalar qadimgi yunon olimlari tomonidan kiritilgan boʻlib, unga tabiatdagi harakat va muvozanat qonunlarini oʻrganish natijasida asos solingan. Isaak Nyutonning 1687-yilda chop etilgan “The Mathematical Principles of Natural-Philosophy” asari dinamik sistemalarni oʻrganishda burilish nuqtasi boʻlgan. Nyutonning uchta harakat va gravitatsiya qonunlari dinamik sistemalarning matematik modellarini tuzishga zamin yaratdi. XVIII-XIX asrlarda Leonard Eyler, Pyer-Simon Laplas, Karl Fridrix Gauss, Andrey Lyapunov, Anri Puankare kabi matematiklar dinamik sistemalar nazariyasini yanada chuqurroq rivojlantirdi.

Dinamik sistemalar nazariyasi boʻyicha olib borilgan ilmiy tadqiqotlar natijasida bir qancha dolzarb masalalar muvaffaqiyatli hal qilindi. Jumladan, Puankare tomonidan dinamik sistemalarning asimptotik xatti-harakatlarini oʻrganish davomida analitik yechimini topish mumkin boʻlmagan sistemalar uchun holatlar fazosi tushunchasining kiritilishi davriy va deyarli davriy orbitalarni aniqlashda asosiy geometrik gʻoyalarni taqdim etdi. Shuningdek, Andronov-Pontryaginlaring ishida dagʻal tushunchasining kiritilishi esa sistema tuzilishining turgʻunligi uchun zaruriy va yetarli shartlar topilishiga olib keldi. Boshqa tomondan, 60-yillarda Lorenz tomonidan atmosferadagi konveksiyaning soddalashtirilgan modelida boshlangʻich holatga nisbatan yuqori sezuvchanlik borligining kashf etilishi, chekli oʻlchamli fazolarda aniqlangan deterministik sistemalarda ham xaotik xatti-harakatlarning mavjudligini koʻrsatdi. Yana bir muhim ishlardan biri bu Sharkovskiy tomonidan bir oʻlchamli uzluksiz akslantirishlar uchun davriy orbitalar tartibi aniqlanishi boʻldi. Sharkovskiy teoremasiga koʻra akslantirish davri uchga teng orbitaga ega boʻlsa, u holda istalgan davrli davriy orbitalarga ega boʻlishi kelib chiqadi. Bu esa, dinamikaning murakkabligini bildiradi. Bunday hollarda dinamikani ehtimollar nuqtai nazaridan oʻrganish qulay. Ya’ni, sistemaning fizik (makroskopik) xossalarini ifodalovchi ehtimollik o‘lchovlarini qurish va o‘rganish muhim ahamiyatga ega. Bunday o‘lchovlarga Gibbs o‘lchovlarini misol keltirish mumkinki, u sistemaning mikroskopik (molekulyar dinamika) xossalarini makroskopik xossalari (bosim, temperatura va h.k.) bilan bog‘laydi. Shu sababdan, Gibbs o‘lchovlari statistik fizika va dinamik sistemalarda muhim o‘rin egallaydi. Oxirgi yillarda, gradiyent Gibbs o‘lchovlari keng o‘rganilmoqda. Bu o‘lchovlar, sanoqli spin qiymatlari to‘plamiga ega modellar uchun gradiyent potentsiallar bilan bog‘liq bo‘lgan Gibbs o‘lchovlari bo‘lib, ularni tadqiq etish cheksiz o‘lchamli operatorlarga keltiriladi. Keli daraxtidagi gradiyent potentsiallar bilan bog‘liq gradiyent Gibbs o‘lchovlarini batafsil tahlil etish kvant fizikasi uchun muhim ahamiyat kasb etadi. F.Henning, K.Kulske, A.Le Ny, P.Schriever va o‘zbek matematiklari N.N. G‘anixo‘jayev, U.A.Rozikov, N.M.Xatamov, R.M.Xakimov, F.H.Haydarov, M.T.Maxammadaliyev va boshqalar cheksiz o‘lchamli operatorlarning qo‘zg‘almas va davriy nuqtalarini tadqiq etishga katta hissa qo‘shib kelmoqdalar. Ularning ishlari asosan limit Gibbs o‘lchovlarining tahliliga, davriy Gibbs o‘lchovlariga va Gibbs o‘lchovlarining turli sohalarda, jumladan, biologiya, tibbiyot va iqtisodiyotda qo‘llanishiga qaratilgan. Ular turli modellar uchun Gibbs o‘lchovlar to‘plamining bo‘sh bo‘lmasligi, fazaviy o‘tishlarning mavjudligiga oid usullarni ishlab chiqdilar va rivojlantirdilar.

Hozirda tadqiqotchilar diskret vaqtli dinamik sistemaga ko‘p e’tibor qaratmoqdalar. R.N.G‘anixo‘jayev, U.U.Jamilov, D.B.Eshmamatova va boshqalar Volterra kvadratik stoxastik operatorlari va N.N.G‘anixo‘jayev, U.A.Rozikov, U.U.Jamilov, F.M.Muxamedovlar esa novolterra kvadratik opertorlar dinamikasi bo‘yicha ilmiy izlanishlar olib bormoqdalar. U.A.Rozikov va Z.S.Boxonovlarning ishlarida ikki o‘lchamli ratsional funksiyalar dinamikasi qaralgan. N.N.G‘anikhodjaev, U.A.Rozikov, N.M.Xatamov, R.M.Xakimov, F.H.Haydarov, M.T.Maxammadaliyevlar SOS va HC modellari uchun Gibbs o‘lchovi masalasini diskret vaqtli cheksiz o‘lchamli ratsional operatorlariga keltirib, shu operatorning qo‘zg‘almas va davriy nuqtalarini o‘rganishgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish instituti ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutining FA-2021-425 raqamli “Panjarali sistemalarda gradiyent ehtimollik o‘lchovlari” (2021-2025 yy) va “Noassotsiativ algebralar strukturaviy nazariyasi va uning biologik sistemalardagi dinamik sistemalarni tadqiq qilishdagi tatbiqi” (2020-2023 yy) nomli ilmiy yo‘nalish doirasida bajarilgan.

Tadqiqot maqsadi spin qiymatlari natural bo‘lgan *k*-tartibli Keli daraxtida aniqlangan fizik jarayonlarga mos keluvchi cheksiz o‘lchamli diskret vaqtli ratsional operatorning dinamikasi, qo‘zg‘almas nuqtalari va shu nuqtalarga mos keluvchi Gibbs o‘lchovlarini tadqiq qilishdan iborat.

Tadqiqotning vazifalari:

cheksiz o‘lchamli operatorning yagona qo‘zg‘almas nuqtaga ega bo‘ladigan shartlarni topish;

davriy nuqtalar to‘plamini tavsiflash va invariant to‘plamlarni aniqlash;

cheksiz o‘lchamli va ikki o‘lchamli operatorlar orasida bog‘lanish topish;

ikki o‘lchamli operator dinamikasini taqdiq qilish orqali cheksiz o‘lchamli operator dinamikasini tavsiflash;

spin qiymatlari natural bo‘lgan HC modeli uchun davriy chegaraviy qonunga mos gradiyent va limit Gibbs o‘lchovlari to‘plamini tadqiq qilish.

Tadqiqot ob’ekti: qisqartirib akslantirish, cheksiz o‘lchamli operatorlarning dinamikasi, Yakobi matritsasi, Gamiltonian, gradiyent Gibbs o‘lchovlari.

Tadqiqot predmeti. Matematik analiz, funksional analiz, nochiziqli diskret vaqtli dinamik sistemalar nazariyasi.

Tadqiqot usullari. Tadqiqot ishida matematik analiz, funksional analiz, operatrolar nazariyasi, chiziqli algebra va diskret vaqtli dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

berilgan cheksiz o‘lchamli operatorning yagona qo‘zg‘almas nuqtaga ega bo‘lishi uchun parametrlarga shart topilgan;

cheksiz o‘lchamli ratsional dinamik sistema ikki o‘lchamli dinamik sistemaga keltirilgan va uning qo‘zg‘almas nuqtalari to‘plami hamda traektoriyalarning limit nuqtalari tavsiflangan;

Keli daraxtida berilgan sanoqli spin qiymatga ega HC modeliga mos translyatsion-invariant Gibbs o‘lchovlari soni yettitagacha bo‘lishi isbotlangan.

Tadqiqotning amaliy natijalari. Olingan natijalar va dissertatsiyada qo‘llanilgan usullar oliy o‘quv yurtlarida magistratura talabalari, tayanch doktorantlar uchun o‘quv kurs sifatida o‘qitilishi mumkin. Shuningdek, u matematik fizika sohasida fazaviy o‘tish muammolarini hal qilishda qo‘llanilishi mumkin.

Tadqiqot natijalarining ishonchliligi. Tadqiqot natijalarida matematik va funksional analiz usullari, diskret vaqtli dinamik sistemalar nazariyasidan foydalanilgan. Olingan natijalar qat’iy matematik mulohazalarga asoslanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati olingan natijalar diskret vaqtli dinamik sistemalar nazariyasi rivojiga hissa qo‘shishi bilan asoslanadi.

Tadqiqot natijalarining amaliy ahamiyati statistik fizikaning HC modellari uchun faza almashishlari ro‘y beradigan kritik haroratlar mavjudligi ko‘rsatilgani bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Biologiya va fizikada diskret vaqtli cheksiz o‘lchamli ratsional akslantirishlarning dinamik sistemalari bo‘yicha olingan natijalarga asosida:

cheksiz o‘lchamli dinamik sistemalarning limit to‘plamlari haqidagi natijalardan G00003447 raqamli “Kvant genetik algebralari va ularning qo‘llanilishi” mavzusidagi xorijiy loyihasida sanoqli spin qiymatlarga ega HC modeli uchun fazaviy o‘tishlarni tadqiq qilishda foydalanilgan (Birlashgan Arab Amirliklari universitetining 2025 yil 24-martdagi ma’lumotnomasi, BAA). Ilmiy natijani qo‘llanishi sanoqli spin qiymatlarga ega HC modellari uchun termodinamik xususiyatlar va Gibbs o‘lchovlarini to‘plamini tavsiflash, shuningdek, fazalar o‘tishlari mavjudligini aniqlash imkonini bergan;

cheksiz o‘lchamli nochiziqli operatorlar dinamikasini tadqiq qilish va Keli daraxtidagi HC modeliga mos Gibbs o‘lchovlarini tavsiflash metodologiyasidan FRGS21-230-0839 raqamli “Chegaralangan o‘lchovli ortogonal saqlovchi kubik stoxastik operatorlarning dinamikasi” mavzusidagi xorijiy loyihada cheksiz o‘lchamli nochiziqli stoxastik operaotrlarning qo‘zg‘almas nuqtalarini tavsiflashda foydalanilgan (Malaziya xalqaro islom universiteti ma’lumotnomasi, 2025 yil 24 mart, Malaziya). Ilmiy natijani qo‘llanishi ortogonallikni saqlovchi nochiziqli stoxastik operatorlar dinamikasini local ma’noda tavsiflash imkonini berdi.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 5 ta ilmiy-amaliy anjumanlarda, jumladan 5 ta xalqaro ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo‘yicha jami 10 ta ilmiy ish chop etilgan, shulardan O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 112 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Diskret vaqtli dinamik sistemalar: Asosiy tushunchalar**” deb nomlanuvchi birinchi bobida dissertatsiya mavzusini to‘la yoritish uchun zarur bo‘lgan asosiy ta’riflar va muhim tushunchalar keltirilgan. Shuningdek, ushbu ishga tegishli bo‘lgan ratsional dinamik sistema bo‘yicha so‘ngi natijalarga qisqacha sharh berilgan.

Diskret vaqtli dinamik sistemani aniqlash uchun $f: X \to X, \quad X \subseteq \mathbb{R}$ funksiyani qaraylik. f akslantirishning o‘z-o‘ziga *n* marta kompozitsiyasi $f^n(x)$ kabi belgilanadi, ya’ni

$$f^n(x) = \underbrace{f(f(f...(f}_{n\ marta}(x))...).$$

1-**ta’rif.** *Berilgan* $x_0 \in X$ *va* $f: X \to X$ *diskret vaqtli dinamik sistema uchun*

$$x_0, x_1 = f(x_0), x_2 = f^2(x_0), x_3 = f^3(x_0), \dots \qquad (1)$$

nuqtalar ketma-ketligi x_0 nuqtaning musbat orbitasi yoki musbat trayektoriyasi deyiladi.

2-**ta'rif.** *Agar $x \in X$ nuqta $f(x) = x$ tenglamani qanoatlantirsa, u holda x nuqta $f : X \to X$ akslantirishning qo'zg'almas nuqtasi deyiladi. Agar x nuqta $f^p(x) = x$ tenglamani qanoatlantirsa, u holda x nuqta f funksiyaning p davrli davriy nuqtasi deyiladi. $f^p(x) = x$ tenglikni qanoatlantiruvchi eng kichik musbat p soniga x nuqtaning asosiy davri deyiladi.*

Barcha qo'zg'almas va p davrli davriy nuqtalar to'plamini mos ravishda $Fix(f)$ va $Per_p(f)$ orqali belgilanadi.

Ushbu

$$x'_k = F_i(x_1, x_2, \ldots, x_m),\ k = 1, 2, \ldots, m, \quad (2)$$

nochiziqli

$$F : x = (x_1, x_2, ..., x_m) \in \mathbb{R}^m \to x' = F(x) = (x'_1, x'_2, ..., x'_m) \in \mathbb{R}^m$$

akslantirishni qaraylik, bu yerda $F_i : \mathbb{R}^m \to \mathbb{R},\ i = 1, \ldots, m$ uzluksiz differensial-lanuvchi bir qiymatli fuksiyalardir.

F akslantirish mos J_F Yakobi matritsasining xos qiymatlari chiziqli bo'lmagan dinamik sistemaning lokal xarakterini aniqlaydi.

3-**ta'rif.** *x^* nuqta F akslantirishning qo'zg'almas nuqtasi bo'lsin.*

- *agar F akslantirishning x^* nuqtadagi Yakobi matritsasi J_F birlik aylanada xos qiymatlarga ega bo'lmasa x^* giperbolik nuqta deyiladi;*
- *agar $J_F(x^*)$ Yakobi matritsaning barcha xos qiymatlarining absolyut qiymati 1 dan kichik bo'lsa, x^* nuqtaga tortuvchi qo'zg'almas nuqta deyiladi;*
- *agar $J_F(x^*)$ Yakobi matritsaning barcha xos qiymatlarining absolyut qiymati 1 dan katta bo'lsa, x^* nuqtaga itaruvchi qo'zg'almas nuqta deyiladi;*
- *qolgan barcha giberbolik nuqtalar uchun x^* egar nuqta deyiladi.*

Ushbu $t \in \{a, b\}$ va $0 < b < 1 < a$ lar uchun $f_t : [0,1] \to [0,1]$ funksiyani qaraylik:

$$f_t(x) = \begin{cases} (2x)^t & agar\ \ x \in \left[0, \frac{1}{2}\right], \\ 2x - 1 & agar\ \ x \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

Har bir iteratsiya bosqichida mustaqil ravishda f_a ni ehtimol p_a bilan va f_b ni ehtimol $p_b = 1 - p_a$ bilan tanlash orqali hosil bo'lgan tasodifiy dinamik sistemani ko'rib chiqamiz.

1-teorema. Agar $ab \le 1$ va $p_a < p_b$ yoki $ab < 1$ va $p_a \le p_b$ bo'lsa, u holda shunday $C, c > 0$ sonlar topilib, barcha $n \ge 1$ uchun $\mathbb{E}(x_n(\omega)) \le Ce^{-cn}$ bajariladi.

Shuningdek, shunday $n_1:\Omega\to\mathbb{N}$ funksiya va $\bar{C}>0$ sonlari topilib, barcha $n>n_1(\omega)$ lar uchun $x_n(\omega)\le Ce^{-cn/2}$ va $\mathbb{P}\{\omega|n_1(\omega)>n\}\le\bar{C}e^{-cn/2}$ tengsizliklar bajariladi.

Dissertatsiyaning **“Gibbs o‘lchovlariga bog‘liq cheksiz o‘lchamli operatorning qo‘zg‘almas nuqtalari”** deb nomlangan ikkinchi bobida cheksiz o‘lchamli operatorni aniqlaymiz va uni yagona qo‘zg‘almas nuqtaga ega bo‘lishi uchun yetarli topamiz. Bundan tashqari, cheksiz o‘lchamli operatorni muayyan shartlar asosida ikki o‘lchamli operatorga qisqartirib uning qo‘zg‘almas nuqtalarini topamiz va $\mathbb{R}^2_+$ ning bissektrisasidagi dinamikasini to‘liq tekshiramiz.

Quyidagicha belgilash kiritamiz:

$$\ell^1_+=\left\{x=(x_1,x_2,...,x_n,...):\ \ x_i>0,\ \ \|x\|=\sum_{j=1}^{\infty}x_j<\infty\right\}.$$

Tartibi $k\ge 2$ bo‘lgan Keli daraxtida Hard-core (HC) modelining invariant Gibbs o‘lchovini tasvirlash uchun $F_0:\ell^1_+\to\ell^1_+$ operatorning qo‘zg’almas nuqtalarini o‘rganishimiz kerak. Bu F_0 operator quyidagicha aniqlangan,

$$F_0:\ \ x_i'=\lambda_i\left(\frac{1+\sum\limits_{j=1}^{\infty}a_{ij}x_j}{1+\sum\limits_{j=1}^{\infty}a_{1j}x_j}\right)^k,$$

bu yerda $k,i\in\mathbb{N},\ \lambda(\lambda_1,\lambda_2,...)\in\ell^1_+$ va $a_{ij}\in\{0,1\}$ berilgan parametrlar.

Biz quyida ixtiyoriy $j\in\mathbb{N}$ uchun $a_{1j}=1$ bo‘lgan holni qaraymiz. U holda,

$$F:\ \ x_i'=\lambda_i\left(\frac{1+\sum\limits_{j=1}^{\infty}a_{ij}x_j}{1+\|x\|}\right)^k, \qquad (3)$$

operator hosil bo‘ladi. Bu yerda $k,i\in\mathbb{N},\ \lambda_i>0.$

Aytaylik, $\lambda=(\lambda_1,\lambda_2,...,\lambda_n,...)\in\ell^1_+$ bo‘lsin. Quyidagi belgilash kiritamiz:

$$A_k=\left\{x\in\ell^1_+:\lambda_1+\frac{\|\lambda\|-\lambda_1}{(1+\|\lambda\|)^k}\le\|x\|\le\|\lambda\|\right\}.$$

2-teorema. *Ixtiyoriy $\lambda\in\ell^1_+$ uchun quyidagi tasdiqlar o‘rinli.*

1) $F:\ell^1_+\to\ell^1_+$ operator uchun A_k invariant to‘plam bo‘ladi, ya’ni $F(A_k)\subset A_k$.

2) Shunday $\hat{\lambda}>0$, topilib, ixtiyoriy $\|\lambda\|<\hat{\lambda}$ uchun $F:A_k\to A_k$ qisqartirib *akslantirish bo'ladi. Xususan, (3) operator* z^* *yagona qo'zg'almas nuqtaga ega bo'lib, ixtiyoriy* $z^{(0)} \in A_k$ *uchun* $\lim_{n\to\infty} F^n\left(z^{(0)}\right) = z^*$ *bo'ladi.*

Quyida yagona qo'zg'almas nuqtaga ega bo'lgan F operatorga doir bir nechta misollar keltirib o'tamiz.

1) Aytaylik $k=2$ bo'lsin. Barcha i, j lar uchun $a_{1j}=1, a_{i1}=1$ va qolgan $a_{ij}=0$ bo'lsa, u holda quyidagi operator hosil bo'ladi

$$x_1'=\lambda_1,\ x_i'=\lambda_i\left(\frac{1+\lambda_1}{1+\|x\|}\right)^2, i\ge 2.$$

2) Agar $a_{11}=1$, $a_{i1}=0,(i\ge 2)$, va barcha $j>1$ va ixtiyoriy i lar uchun $a_{ij}=1$ bo'lsa, u holda operator quyidagi ko'rinishga keladi

$$x_1'=\lambda_1,\ x_i'=\lambda_i\left(\frac{1+\|x\|-\lambda_1}{1+\|x\|}\right)^2, i\ge 2.$$

Quyida muayyan shartlar ostida cheksiz o'lchamli operatorni o'rganishni ikki o'lchamli operatorni tadqiq qilishga keltirish mumkinligini ko'rsatamiz.

Aytaylik, $a_{1j}=1,\ (j\ge 1),\ a_{i1}=0,\ (i>1),\ a_{22}=1$ bo'lsin. Quyidagi $E_1=E_1(\vec{a})=\{i:\ a_{2i}=1\}$ va $E_2=\{i>2 \mid i\in E_1^c\}$ to'plamlarni aniqlaymiz. Ma'lumki: $E_1\bigcup E_2=N_1=\mathbb{N}\setminus\{1\}$, $\|x\|=\sum_{j\ge 1}x_j=x_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j$, $E_1\ne\varnothing$ va $E_2\ne\varnothing$.

Quyidagi sinfni qaraymiz:

$$F:\begin{cases} x_i'=\lambda_i\left(\dfrac{1+\sum_{j\in E_1}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2, & agar\ \ i\in E_1,\\ x_i'=\lambda_i\left(\dfrac{1+\sum_{j\in E_1}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2, & agar\ \ i\in E_2.\end{cases}$$

Bu yerda $E_1\ne\varnothing$, $E_2\ne\varnothing$ bo'lishi muhim shart hisoblanadi. $E_1\ne\varnothing$ emasligi aniq chunki bizda $a_{22}=1$. $E_2=\varnothing$ bo'lsa operator yagona qo'zg'almas nuqtaga ega bo'ladi. Bu shartlar asosida operator quyidagi ko'rinishga keladi:

$$x_1'=\lambda_1,\ \ x_i'=\lambda_i\left(\frac{1+\sum_{j\in E_1}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2,\ \ agar\ \ \ i\in E_1,$$

$$x_i'=\lambda_i\left(\frac{1+\sum_{j\in E_2}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2,\ \ agar\ \ \ i\in E_2.$$

Soddalik uchun quyidagi holatini qaraymiz: $a_{1j}=1\ ,(j\geq 1)$ va $a_{i1}=0,(i>1)$ qolgan barcha i va j lar uchun

$$a_{ij}=\begin{cases}1, & agar\quad i+j\quad juft,\\ 0, & agar\quad i+j\quad toq.\end{cases}\tag{4}$$

U holda operator quyidagi ko‘rinishga keladi:

$$F_1: x'_{2n}=\lambda_{2n}\left(\frac{1+\sum_{j=1}^{\infty}x_{2j}}{1+\|x\|}\right)^2, x'_{2n+1}=\lambda_{2n+1}\left(\frac{1+\sum_{j=1}^{\infty}x_{2j+1}}{1+\|x\|}\right)^2, n\geq 1.$$

Ushbu $\lambda_1>0$ va $L_1,L_2>0$ sonlar uchun $W:\ \mathbb{R}_+^2\to\mathbb{R}_+^2$ ikki o‘lchamli operatorni qaraylik,

$$W\ : x'=L_1\left(\frac{1+x}{1+\lambda_1+x+y}\right)^2,\ y'=L_2\left(\frac{1+y}{1+\lambda_1+x+y}\right)^2.\tag{5}$$

1-lemma. *F_1 operatorning yordamida hosil qilingan cheksiz o‘lchamli dinamik sistema W operatorning ikki o‘lchamli dinamikasi orqali to‘liq ifodalanadi.*

Yuqoridagi lemmaga ko‘ra, W dinamikasini tadqiq qilish yetarli. Avvalo W operatorning qo‘zg‘almas nuqtalarini o‘rganamiz.

Quyidagi to‘plamlarni aniqlaymiz:

$$M_-=\{(x,y)\in\mathbb{R}_+^2: x<y\},$$
$$M_0=\{(x,y)\in\mathbb{R}_+^2: x=y\},$$
$$M_+=\{(x,y)\in\mathbb{R}_+^2: x>y\}.$$

2-lemma. *Agar $L_1=L_2=L$ bo‘lsa, u holda $M_\varepsilon,\varepsilon\in\{-,0,+\}$ to‘plamlar W operator uchun invariant bo‘ladi, ya’ni $W(M_\varepsilon)\subset M_\varepsilon$.*

M_0 to‘plamda operatorning ko‘rinishi quyidagicha:

$$x'=f(x):=L\left(\frac{1+x}{1+\lambda_1+2x}\right)^2.$$

Quyidagicha belgilash kiritamiz:

$$\hat{L}_1=\frac{2\lambda_1^2+76\lambda_1-142-(2\lambda_1-34)\sqrt{\lambda_1^2-18\lambda_1+17}}{16},$$

$$\hat{L}_2=\frac{2\lambda_1^2+76\lambda_1-142+(2\lambda_1-34)\sqrt{\lambda_1^2-18\lambda_1+17}}{16}.$$

M_0 to‘plamda operatorning qo‘zg‘almas nuqtalar soni quyidagicha bo‘ladi.

1) Agar $\lambda_1 \in (0,17], L > 0$ yoki $\lambda_1 > 17, L \notin [\hat{L}_1, \hat{L}_2]$ bo‘lsa, u holda $f(x)$ yagona x_1^* qo‘zg’almas nuqtaga ega;

2) Agar $\lambda_1 > 17$ va $L = \hat{L}_1$ yoki $L = \hat{L}_2$ bo‘lsa, u holda $f(x)$ ikkita qo‘zg‘almas $x_1^*, x_2^* \ (x_1^* < x_2^*)$ nuqtaga ega bo‘ladi;

3) Agar $\lambda_1 > 17, L \in (\hat{L}_1, \hat{L}_2)$ bo‘lsa, u holda $f(x)$ uchta qo‘zg‘almas $x_1^*, x_2^*, x_3^* \ (x_1^* < x_2^* < x_3^*)$ nuqtaga ega.

Agar $\lambda_1 = 1$ bo‘lsa, $f(x) = \dfrac{L}{4}$ bo‘ladi. Shuning uchun ham biz $\lambda_1 \neq 1$ holatni qaraymiz.

3-lemma. *Qo‘zg‘almas nuqtalarni tiplari quyidagicha*

1) *Yagona qo‘zg‘almas nuqta*

$$x_1^* = \begin{cases} tortuvchi, \quad agar \quad \lambda_1 > 17,\ L \notin [\hat{L}_1, \hat{L}_2], \\ egar, \quad agar \quad \lambda_1 = 17,\ L = 108, \\ tortuvchi, \quad agar \quad \lambda_1 = 17,\ L \neq 108\ \ yoki\ \lambda_1 \in (0,1) \cup (1,17). \end{cases}$$

2) *Agar $\lambda_1 > 17$ va $L = \hat{L}_1$ ($L = \hat{L}_2$ mos ravishda) bo‘lsa, u holda $f(x)$ funksiya ikkita qo‘zg‘almas nuqtaga ega $x_1^* < x_2^*$ va x_1^* egar va x_2^* tortuvchi (mos ravishda x_1^* tortuvchi va x_2^* egar) bo‘ladi.*

3) *Agar $\lambda_1 > 17, L \in (\hat{L}_1, \hat{L}_2)$ bo‘lsa, u holda $f(x)$ funksiya uchta $x_1^* < x_2^* < x_3^*$ qo‘zg‘almas nuqtalarga ega. Bundan tashqari x_1^* va x_3^* tortuvchi va x_2^* esa itaruvchi bo‘ladi.*

4-lemma. *Invariant to‘plam M_0 da (5) operator davriy nuqtalarga ega emas.*

3-teorema. *Quyidagi tasdiqlar o‘rinli.*

1) *Agar $\lambda_1 \in (0,17],\ L > 0$ yoki $\lambda_1 > 17,\ L \notin [\hat{L}_1, \hat{L}_2]$ bo‘lsa, u holda ixtiyoriy $x \in (0, +\infty)$ uchun quyidagi tenglik o‘rinli: $\lim\limits_{n\to\infty} f^n(x) = x_1^*$.*

2) *Agar $\lambda_1 > 17$ va $L = \hat{L}_1$ (mos ravishda $L = \hat{L}_2$) bo‘lsa, u holda*

$$\lim_{n\to\infty} f^n(x) = \begin{cases} x_1^*, \quad if \quad x \in (0, x_1^*], \\ x_2^*, \quad if \quad x \in (x_1^*, +\infty), \end{cases}$$

$$\left(mos\,ravishda \quad \lim_{n\to\infty} f^n(x) = \begin{cases} x_1^*, & if \quad x \in (0, x_2^*), \\ x_2^*, & if \quad x \in [x_2^*, +\infty), \end{cases} \right).$$

3) Agar $\lambda_1 > 17,\ L \in \left(\hat{L}_1, \hat{L}_2\right)$ *bo'lsa, u holda*

$$\lim_{n\to\infty} f^n(x) = \begin{cases} x_1^*, & if \quad x \in (0, x_2^*), \\ x_2^*, & if \quad x = x_2^*, \\ x_3^*, & if \quad x \in (x_2^*, +\infty). \end{cases}$$

Dissertatsiyaning **"Cheksiz o'lchamli no-chiziqli operatorning dinamik sistemalari"** deb nomlangan uchinchi bobida, ikki o'lchamli operator hosil qilgan dinamik sistema tadqiq qilingan. Bundan tashqari dinamikani o'rganish maqsadida tartib tushunchasi kiritiladi va ikki o'lchamli operatorning qo'zg'almas nuqtalariga mos keluvchi Gibbs o'lchovlari soni haqidagi teorema keltiriladi.

Invariant to'plam $\mathbb{R}_+^2 \setminus M_0$ dagi W operatorni qaraymiz. Quyidagi $A_{i,j}$ to'plamlardagi i soni M_0 dagi qo'zg'almas nuqtalar sonini, j soni esa $\mathbb{R}_+^2 \setminus M_0$ dagi qo'zg'almas nuqtalar sonini ifodalaydi.

$$A_{1,0} = \left\{ (\lambda_1, L) : L \le \frac{(\lambda_1+3)^2}{4},\ 0 < \lambda_1 \le 5 \right\} \cup \left\{ (\lambda_1, L) : L \le 4(\lambda_1 - 1),\ \lambda_1 > 5 \right\},$$

$$A_{1,2} = \left\{ (\lambda_1, L) : L > \frac{(\lambda_1+3)^2}{4},\ 0 < \lambda_1 \le 17 \right\} \cup$$

$$\cup \left\{ (\lambda_1, L) : \frac{(\lambda_1+3)^2}{4} \le L < \hat{L}_1,\ 17 < \lambda_1 \le 9 + 8\sqrt{2} \right\} \cup$$

$$\cup \left\{ (\lambda_1, L) : L > \hat{L}_2,\ \lambda_1 > 17 \right\} \cup \left\{ (\lambda_1, L) : L = 4(\lambda_1 - 1),\ \lambda_1 > 5 \right\},$$

$$A_{2,2} = \left\{ (\lambda_1, L) : L = \hat{L}_2,\ \lambda_1 > 17 \right\} \cup \left\{ (\lambda_1, L) : L = \hat{L}_1,\ 17 < \lambda_1 < 9 + 8\sqrt{2} \right\},$$

$$A_{1,4} = \left\{ (\lambda_1, L) : 4(\lambda_1 - 1) < L < \frac{(\lambda_1+3)^2}{4},\ 5 < \lambda_1 \le 9 + 8\sqrt{2} \right\} \cup$$

$$\cup \left\{ (\lambda_1, L) : 4(\lambda_1 - 1) < L < \hat{L}_1,\ \lambda_1 > 9 + 8\sqrt{2} \right\},$$

$$A_{3,2}=\left\{(\lambda_1,L):\hat{L}_1<L<\hat{L}_2,\,17<\lambda_1\le 9+8\sqrt{2}\right\}\cup$$

$$\cup\left\{(\lambda_1,L):\frac{(\lambda_1+3)^2}{4}<L<\hat{L}_2,\,\lambda_1>9+8\sqrt{2}\right\},$$

$$A_{2,4}=\left\{(\lambda_1,L):L=\hat{L}_1,\,\lambda_1>9+8\sqrt{2}\right\}$$

$$A_{3,4}=\left\{(\lambda_1,L):\hat{L}_1<L<\frac{(\lambda_1+3)^2}{4},\,\lambda_1>9+8\sqrt{2}\right\}.$$

5-lemma. *Quyidagi tasdiqlar o‘rinli*

1) *Agar* $(\lambda_1,L)\in A_{1,0}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*\}$,

2) *Agar* $(\lambda_1,L)\in A_{1,2}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_1,p_2\}$,

3) *Agar* $(\lambda_1,L)\in A_{2,2}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_2^*,p_1,p_2\}$,

4) *Agar* $(\lambda_1,L)\in A_{1,4}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_1,p_2,p_3,p_4\}$,

5) *Agar* $(\lambda_1,L)\in A_{3,2}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_2^*,p_3^*,p_1,p_2\}$,

6) *Agar* $(\lambda_1,L)\in A_{2,4}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_2^*,p_1,p_2,p_3,p_4\}$,

7) *Agar* $(\lambda_1,L)\in A_{3,4}$ *bo‘lsa, u holda* $Fix(W)=\{p_1^*,p_2^*,p_3^*,p_1,p_2,p_3,p_4\}$,

bu yerda $p_i^*=(x_i^*,x_i^*),\quad i=1,2,3,$

$$p_1=(x_1,x_2),\;p_2=(x_2,x_1),\;p_3=(x_3,x_4),\;p_4=(x_4,x_3),$$

$$x_1=\frac{\sqrt{L}+\sqrt{L-4(\lambda_1-1)}+\sqrt{2L-4\lambda_1+2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_2=\frac{\sqrt{L}+\sqrt{L-4(\lambda_1-1)}-\sqrt{2L-4\lambda_1+2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_3=\frac{\sqrt{L}-\sqrt{L-4(\lambda_1-1)}+\sqrt{2L-4\lambda_1-2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_4=\frac{\sqrt{L}-\sqrt{L-4(\lambda_1-1)}-\sqrt{2L-4\lambda_1-2\sqrt{L^2-4(\lambda_1-1)L}}}{2}.$$

4-teorema. Agar $L_1=L_2=L$ *bo‘lsa, u holda* F_1 *operatorning qo‘zg‘almas nuqtalari quyidagicha:*

$$Fix(F)=\begin{cases}\{P_1\}, & if\ \ (\lambda_1,L)\in A_{1,0}\\ \{P_1,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{1,2}\\ \{P_1,P_2,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{2,2}\\ \{P_1,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{1,4}\\ \{P_1,P_2,P_3,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{3,2}\\ \{P_1,P_2,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{2,4}\\ \{P_1,P_2,P_3,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{3,4}\end{cases}$$

bu yerda

$$P_1=\left(\lambda_1,\frac{x_1^*}{L}\lambda_2,\frac{x_1^*}{L}\lambda_3,\frac{x_1^*}{L}\lambda_4,...\right),P_2=\left(\lambda_1,\frac{x_2^*}{L}\lambda_2,\frac{x_2^*}{L}\lambda_3,\frac{x_2^*}{L}\lambda_4,...\right),$$

$$P_3=\left(\lambda_1,\frac{x_3^*}{L}\lambda_2,\frac{x_3^*}{L}\lambda_3,\frac{x_3^*}{L}\lambda_4,...\right),P_4=\left(\lambda_1,\frac{x_2}{L}\lambda_2,\frac{x_1}{L}\lambda_3,\frac{x_2}{L}\lambda_4,...\right)$$

$$P_5=\left(\lambda_1,\frac{x_1}{L}\lambda_2,\frac{x_2}{L}\lambda_3,\frac{x_1}{L}\lambda_4,...\right),P_6=\left(\lambda_1,\frac{x_4}{L}\lambda_2,\frac{x_3}{L}\lambda_3,\frac{x_4}{L}\lambda_4,...\right)$$

$$P_7=\left(\lambda_1,\frac{x_3}{L}\lambda_2,\frac{x_4}{L}\lambda_3,\frac{x_3}{L}\lambda_4,...\right).$$

$\mathbb{R}^2$ da birinchi kvadrantda Shimoli-Sharqiy (NE) tartib quyidagicha aniqlanadi: agar $x_1\le x_2$ va $y_1\le y_2$ bo‘lsa, u holda $(x_1,y_1)\preceq_{NE}(x_2,y_2)$. Janubi-Sharqiy (SE) tartib esa quyidagicha aniqlanadi: agar $x_1\le x_2$ va $y_1\ge y_2$ bo‘lsa, u holda $(x_1,y_1)\preceq_{SE}(x_2,y_2)$. $\mathbb{R}_+^2$ ni quyidagi to‘plamlarga ajratamiz: $\mathbb{R}_+^2=A_{SE}^{<}\cup A_{SE}^{>}\cup A_{NE}^{<}\cup A_{NE}^{>}$. Bu to‘plamlar quyidagicha:

$$A_{SE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ x\ge\psi(y),\ y\le\psi(x)\},$$
$$A_{SE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ x\le\psi(y),\ y\ge\psi(x)\},$$
$$A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ x\le\psi(y),\ y\le\psi(x)\},$$
$$A_{NE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ x\ge\psi(y),\ y\ge\psi(x)\},$$

bu yerda $\psi(x)=\sqrt{L}\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)-(1+\lambda_1+x)$.

1-tasdiq. $\mathbb{R}_+^2$ *ning qism to‘plamlari uchun quyidagilar o‘rinlidir:*

$$A_{SE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ (x,y)\preceq_{SE}W(x,y)\},\ A_{SE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ W(x,y)\preceq_{SE}(x,y)\},$$

$$Fix(W)=\{(x,y)\in\mathbb{R}_+^2:\ x=\psi(y),\ y=\psi(x)\},$$

$$A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ (x,y)\preceq_{NE}W(x,y)\},\ A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ W(x,y)\preceq_{NE}(x,y)\}.$$

$p\in Fix(W)$ qo‘zg‘almas nuqta uchun biz quyidagicha belgilash kiritamiz:

$$B^{<}(p)=\{(x,y)\in M_{-}:\ (x,y)\prec_{SE}W(x,y)\preceq_{SE}p\},$$
$$B^{>}(p)=\{(x,y)\in M_{-}:\ p\ \preceq_{SE}W(x,y)\prec_{SE}(x,y)\},$$
$$R^{>}(p)=\{(x,y)\in M_{-}:\ W(x,y)\prec_{SE}(x,y)\preceq_{SE}\ p\ \},$$
$$R^{>}(p)=\{(x,y)\in M_{-}:\ p\ \preceq_{SE}(x,y)\prec_{SE}W(x,y)\}.$$

Yuqoridagilardan foydalanib quyidagicha teorema olamiz.

Boshlang‘ich nuqta $x^{(0)}=\left(x_1^{(0)},x_2^{(0)},x_3^{(0)},...\right)$ uchun quyidagicha belgilaymiz:

$$v_1^{(0)}=\sum_{j=1}^{\infty}x_{2j+1}^{(0)},v_2^{(0)}=\sum_{j=1}^{\infty}x_{2j}^{(0)} \tag{6}$$

5-teorema. *Boshlang‘ich nuqta* $v^{(0)}\in\mathbb{R}_+^2\setminus M_+$ *uchun quyidagi tasdiq o‘rinli*

$$\lim_{n\to\infty}W^n\left(v^{(0)}\right)=$$

$$\begin{cases} p_1^*, \quad agar \quad (\lambda_1,L)\in A_{1,0} \\ p_2, \quad agar \quad (\lambda_1,L)\in A_{1,2}, \qquad \forall v^{(0)}\in M_{-} \\ p_2,\ agar\ (\lambda_1,L)\in A_{1,4},\ \forall v^{(0)}\in\left(B^{<}(p_2)\cup B^{>}(p_2)\right)\cup\left(R^{<}(p_4)\cup R^{>}(p_4)\right); \\ p_4,\ agar\ (\lambda_1,L)\in A_{1,4},\ \forall v^{(0)}\in\left(B^{<}(p_4)\cup B^{>}(p_4)\right)\cup\left(R^{<}(p_2)\cup R^{>}(p_2)\right); \\ p,\ agar\ (\lambda_1,L)\in\bigcup_{\substack{i=2,3\\ j=2,4}}A_{i,j}, \qquad p\in Fix(W), \quad \forall v^{(0)}\in\left(B^{<}(p)\cup B^{>}(p)\right). \end{cases}$$

6-eorema. *Ixtiyoriy boshlang‘ich* $x^{(0)}=\left(x_1^{(0)},x_2^{(0)},...\right)\in\ell_+^1$ *uchun*

$$\lim_{m\to\infty}W^m\left(v_1^{(0)},v_2^{(0)}\right)=(a,b)=p\in Fix(W)$$

bo‘lsa, u holda (6)

$$\lim_{m\to\infty}F^m\left(x^{(0)}\right)=\frac{1}{L}(\lambda_1L,\lambda_2b,\lambda_3a,\lambda_4b,...)\ \text{bo‘ladi.}$$

Uchlari $\mathbb{N}$ natural sonlar to‘plamida bo‘lgan cheksiz G grafni qaraylik. Agar σ konfiguratsiya va x,y qo‘shni uchlar uchun $\{\sigma(x),\sigma(y)\}$ juftlik G grafning qirrasi bo‘lsa, u holda konfiguratsiya G-joiz konfiguratsiya deyiladi.

Barcha joiz konfiguratsiyalar oilasi Ω^G ko‘rinishda belgilanadi. $\lambda:G\mapsto\mathbb{R}_+$ chegaralangan funksiya berilgan bo‘lsin va λ_i lar λ funksiyaning $i\in\mathbb{N}$ uchdagi “faolligi” deyiladi.

Berilgan G graf va λ ga mos HC-modeli quyidagicha aniqlanadi

$$H_G^{\lambda}(\sigma)=\begin{cases}\sum_{x\in V}\ln\lambda_{\sigma(x)}, & if \quad \sigma\in\Omega^G, \\ +\infty, & if \quad \sigma\notin\Omega^G.\end{cases} \tag{7}$$

4-ta'rif. *1) Ushbu* $l=\left\{l_{xy}\right\}_{\langle x,y\rangle\in L}$, $l_{xy}=\left\{l_{xy}(i):\ i\in\mathbb{N}\right\}\in(0,\infty)^{\mathbb{N}}$ *vektorlar oilasi ixtiyoriy* $\langle x,y\rangle\in L$ *qirra va barcha* $i\in\mathbb{N}$ *uchun*

$$l_{xy}(i)=c_{xy}\prod_{z\in\partial x\setminus y}\sum_{j\in\mathbb{N}}a_{ij}\lambda_j l_{zx}(j)$$

tenglikni qanoatlantiradigan shunday $c_{xy}>0$ *mavjud bo'lsa, u holda (7) model uchun chegaraviy qonun deyiladi.*

2) Agar l *chegaraviy qonun va barcha* $x\in V$ *uchlar uchun*

$$\sum_{i\in\mathbb{N}}\left(\prod_{z\in\partial x}\sum_{j\in\mathbb{N}}a_{ij}\lambda_j l_{zx}(j)\right)<\infty$$

munosabat o'rinli bo'lsa, u holda ushbu chegaraviy qonun normallashgan deyiladi.

6-lemma. *Agar* $\lambda\in\ell_{+}^{1}$ *bo'lsa, u holda* F_1 *shakldagi har qanday yechim nomallashgan bo'ladi.*

Yuqoridagilardan foydalangan holda quyidagi Gibbs o'lchovlar soni haqidagi teorema ega bo'lamiz.

7-teorema. *Agar* $\mathcal{N}_G$ *-(7) Gamiltonian uchun translatsion-invariant Gibbs o'lchovlarining soni bo'lsa va u (4) bilan aniqlangan* G *grafga mos kelsa, u holda*

$$\mathcal{N}_G=\begin{cases}1, & agar\ (\lambda_1,L)\in A_{1,0},\\ 3, & agar\ (\lambda_1,L)\in A_{1,2},\\ 4, & agar\ (\lambda_1,L)\in A_{2,2},\\ 5, & agar\ (\lambda_1,L)\in A_{1,4},\\ 5, & agar\ (\lambda_1,L)\in A_{3,2},\\ 6, & agar\ (\lambda_1,L)\in A_{2,4},\\ 7, & agar\ (\lambda_1,L)\in A_{3,4}.\end{cases}$$

XULOSA

Dissertatsiya diskret vaqtli cheksiz o'lchamli operatorlar dinamik sistemalarni tavsiflashga bag'ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Diskret vaqtli cheksiz o'lchamli ratsional operator yagona qo'zg'almas nuqtaga ega bo'lishi uchun parametrlarga shart topilgan.
2. Cheksiz o'lchamli operatorni dinamikasini o'rganish masalasini ba'zi shartlar asosida ikki o'lchamli operatorni dinamikasini o'rganish masalasiga keltirilgan.
3. Cheksiz o'lchamli operatorni ikki o'lchamli operatorga keltirish jarayonida cheksiz o'lchamli operatorni bitta ikki o'lchamli operatorga keltirish va natijani yana cheksiz o'lchamli operatorga qaytarish imkoni yaratilgan.

4. Cheksiz o‘lchamli operator uchun olingan natijalar Hard-Core modeli uchun translatsion Gibbs o‘lchovlariga tadbiqi ko‘rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

OLIMOV UMRBEK RASHIDOVICH

DISCRETE-TIME INFINITE-DIMENSIONAL RATIONAL DYNAMICAL SYSTEMS

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the of Ministers of Higher education, Science and Innovations of the Republic of Uzbekistan under number №B2024.2.PhD/FM1050.

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INTRODUCTION

Actuality and demand of the theme of the dissertation. In the world, numerous scientific and applied research works are often directed toward the study of infinite-dimensional rational dynamic systems. The importance of the theory of dynamic systems is in evaluating the future states of a system based on its current state, playing a significant role in solving problems in statistical mechanics, physics, biology, medicine, economics, and other fields. This helps people understand complex systems and make optimal decisions. It should be noted that dynamic systems are typically represented by difference equations, differential equations, and differential-difference equations. One of the main objects of research in statistical mechanics and quantum physics is the dynamic systems of infinite-dimensional rational operators. Additionally, infinite-dimensional rational dynamic systems play a crucial role in physics for modeling geometric interactions between particles and studying the thermodynamic properties of systems. Therefore, the research of dynamic systems of infinite-dimensional rational mappings remains one of the important and urgent tasks in the theory of dynamic systems.

Currently, the theory of infinite-dimensional rational dynamic systems is used as the main tool in understanding the nature of many practical problems, analyzing them, and finding optimal solutions. Specifically, in dynamic systems theory, problems such as describing the set of limit points of the trajectory, determining the existence of fixed and periodic points and their types, finding invariant sets, checking for the occurrence of bifurcations, and studying whether the system is stable or chaotic are widely used in the analysis of many practical issues. In this regard, the dynamics of discrete-time infinite-dimensional rational operators are considered targeted scientific studies for analyzing the temporal evolution of systems in various fields. In recent years, the fact that seven of the recipients of the prestigious Fields Medal, awarded for significant scientific results in mathematics, are specialists in the theory of dynamic systems indicates the relevance of this field.

In our country, significant progress has been made in the study of fixed points and dynamics of discrete-time infinite-dimensional rational operators, driven by the recent emphasis on bringing fundamental research to leading global positions[1]. The theory of dynamic systems, a field of interest for the International Mathematical Society, holds great importance, particularly in enhancing research on the dynamics of discrete-time infinite-dimensional rational operators. The scientific research conducted in this dissertation is part of this important area of study.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 "On the strategy of action for the further development Of the

[1]Decree № PQ-4387 of the President of the Republic of Uzbekistan dated July 9, 2019, "On Further State Support for the Development of Mathematics Education and Sciences, as well as Measures for the Comprehensive Improvement of the Activities of the V.I. Romanovskiy Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan.".

Republic of Uzbekistan”, UP-60 dated January 28, 2022 “Development Strategy of New Uzbekistan for the period of 2022-2026”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, PP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, and PP-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic.

This research was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. The initial concepts related to dynamical systems were introduced by ancient Greek scholars and were founded as a result of studying the laws of motion and equilibrium in nature. A turning point in the study of dynamical systems was Isaac Newton's publication of “The Mathematical Principles of Natural Philosophy” in 1687. Newton's three laws of motion and the laws of gravitation provided the a theoretical framework for deriving mathematical models of dynamical systems. Eminent mathematicians of the 18th and 19th centuries, mathematicians like Leonard Euler, Pierre-Simon Laplace, Carl Friedrich Gauss, Andrei Lyapunov and Henri Poincaré further deepened the study of dynamical systems.

Scientific research in dynamic systems has successfully addressed several critical problems. Henri Poincaré introduced the concept of phase space to study the asymptotic behavior of dynamic systems that do not have analytic solutions. His work laid the foundation for understanding periodic and almost-periodic orbits through key geometric ideas. Later, Andronov and Pontryagin introduced the concept of structural stability and formulated necessary and sufficient conditions for the stability of a system’s structure in two-dimensions, which lead to the development of theory of hyperbolic dynamical systems. In the 1960s, Edward Lorenz discovered sensitive dependence on initial conditions in a simplified model of atmospheric convection. This discovery demonstrated that even deterministic systems in finite-dimensional spaces could exhibit chaotic behavior. Another interting achievement was Sharkovskii’s theorem, which established the ordering of periodic orbits in one-dimensional continuous maps. He proved that if a map has a periodic orbit of period three, it must also have periodic orbits of all other periods, highlighting the complexity of dynamic behavior. In complex cases, it is useful to study dynamics from a probabilistic perspective. This involves constructing and analyzing probability measures that describe the physical (macroscopic) properties of a system. A prime example is Gibbs measures, which connect the microscopic properties of a system (e.g., molecular dynamics) to its macroscopic characteristics (e.g., pressure, temperature). In recent years, gradient

Gibbs measures have been extensively studied. These measures are associated with gradient potentials in models with an infinite set of spin values and can be analyzed through infinite-dimensional operators. A detailed study of gradient potentials on Cayley trees is particularly significant for quantum physics. Prominent mathematicians contributing to the study of fixed and periodic points of infinite-dimensional operators include F. Henning, C. Kulske, A.Le Ny, P. Schriever, N.N. G‘anikhodjaev, U.A. Rozikov, N.M. Khatamov, R.M. Khakimov, F.H. Haydarov, M.T. Makhammadaliev, and others.

Their research primarily focuses on: the analysis of limit Gibbs measures, the study of periodic Gibbs measures, the application of Gibbs measures in various fields, including biology, medicine, and economics. These researchers have developed and advanced theories and methods proving the existence of Gibbs measures in different models and investigating phase transitions in complex systems.

Currently, researchers are paying considerable attention to discrete-time dynamical systems. Mathematicians like R.N. Ganikhodjaev, U.U. Jamilov, and D.B. Eshmamatova are investigating Volterra quadratic stochastic operators, while N.N. Ganikhodjaev, U.A. Rozikov, U.U. Jamilov, and F.M. Mukhamedov are exploring the dynamics of non-Volterra quadratic operators. Studies by U.A. Rozikov and Z.S. Boxonov have focused on the dynamics of two-dimensional rational functions. Additionally, N.N. Ganikhodjaev, U.A. Rozikov, N.M. Khatamov, R.M. Khakimov, F.H. Haydarov, and M.T. Makhammadaliev have examined Gibbs measures for SOS and HC models, relating them to discrete-time infinite-dimensional rational operators, and analyzed their fixed and periodic points.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research FA-2021-425 “Gradient probability measures in lattice” (2021-2025) and “Structural theory of non-associative algebras and its application in the study of dynamic systems in biological systems” (2020-2023) at the V.I. Romanovskiy Institute of Mathematics.

The aim of the research work is to investigate the dynamics, fixed points and Gibbs measures corresponding to these points of an infinite-dimensional discrete rational operator corresponding to physical processes defined in the k-th order Kelly tree with natural spin values.

Research problems:

finding conditions on the parameters have been identified under which the infinite-dimensional operator has a unique fixed point;

describing the set of peroidic points and determining invariant sets;

finding the relation between two-dimensional and infinite-dimensional operators;

describing dynamics of infinite-dimensional operator, by investigating dynamics of two dimensional operator;

investigating gradient and limit Gibbs measure that correspond to the periodic boundary law for the HC (Hard Core) model with natural spin values.

The research object: contraction mappings, dynamics of infinite-dimensional operators, Jacobian matrix, Hamiltonian, gradient Gibbs measures.

The research subject: Mathematical analysis, functional analysis, theory of nonlinear discrete-time dynamical systems.

Research methods: The research uses methods from mathematical analysis, functional analysis, operator theory, linear algebra, and the theory of discrete-time dynamical systems.

Scientific novelty of the research work consists of the following:

a condition on the parameters has been established to ensure that the given infinite-dimensional operator possesses a unique fixed point;

the infinite-dimensional rational dynamical system has been reduced to a two-dimensional dynamical system, the set of its fixed points and limit points of trajectories has been described;

it has been proved that the number of translation-invariant Gibbs measures corresponding to the HC model with a countable set of spin values defined on the Cayley tree can be at most seven.

Practical results of the research. The results obtained and the methods used in the dissertation can be used as a training course for undergraduates and basic doctoral students in higher education institutions. It can also be used to solve phase transition problems in mathematical physics.

The reliability of the results of the study. The research results are based on methods from mathematical and functional analysis, as well as discrete-time dynamical systems theory. The obtained results have been proven based on rigorous mathematical reasoning.

Scientific and practical significance of the research results. The scientific significance of the research results is based on the contribution to the functional analysis, mathematical analysis, theory of discrete-time dynamical systems.

The practical significance of the research results is explained by the demonstration of the existence of critical temperatures at which phase transitions occur in the HC models of statistical physics.

Implementation of the research results. Based on the obtained results on the dynamical systems of rational mappings in biology and physics:

The results on the limit sets of infinite-dimensional dynamical systems were used in the foreign project No. G00003447 titled “Quantum Genetic Algebras and Their Applications” to study phase transitions in the HC model with finitely many spin values (based on the reference from the United Arab Emirates University dated March 24, 2025, UAE). The application of this scientific result made it possible to describe the set of thermodynamic properties and Gibbs measures for HC models with finitely many spin values, as well as to determine the existence of phase transitions;

the methodology for studying the dynamics of infinite-dimensional nonlinear operators and describing the Gibbs measures corresponding to the HC model on a

Cayley tree was used in the foreign project No. FRGS21-230-0839 titled “Dynamics of Bounded Measure-Preserving Orthogonal Cubic Stochastic Operators” to describe fixed points of infinite-dimensional nonlinear stochastic operators (based on the reference from the International Islamic University Malaysia, dated March 24, 2025, Malaysia). The application of this scientific result made it possible to locally describe the dynamics of orthogonality-preserving nonlinear stochastic operators.

Approbation of the research results. The main results of the research have been discussed at 5 international scientific conferences.

Publications of the research results. On the topic of the dissertation 10 research papers have been published in the scientific journals, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 2 of them were published in international journals and 3 paper published in national mathematical journals.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 112 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** of the thesis includes the motivation of the research, the relevance of the research to the priorities of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem, the aim, research problems, object and subject of research, scientific novelty and practical results, theoretical and practical significance of the results, published works and information on the structure of the thesis.

In the first chapter of the thesis, titled **“Discrete-time dynamical systems: Preliminaries”** we give main definitions and important notions that are necessary to state the problems and results of the current dissertation. In this chapter we also include a technical result for a random dynamical system.

Let $X \subseteq \mathbb{R}$. To define a discrete-time dynamical system we consider a function $f: X \to X$. For $x \in X$ denote by $f^n(x)$ the n-fold composition of f with itself (i.e. n time iteration of f to x):

$$f^n(x) = \underbrace{f(f(f\ldots(f}_{n\ \text{times}}(x))\ldots).$$

Definition 1. *For a given $x_0 \in X$ and $f: X \to X$ the discrete-time dynamical system (also called forward orbit or trajectory of x_0) is the sequence of points*

$$x_0,\ x_1 = f(x_0),\ x_2 = f^2(x_0),\ x_3 = f^3(x_0), \ldots . \qquad (1)$$

Definition 2. *A point $x \in X$ is called a fixed point for $f: X \to X$ if $f(x) = x$. The point x is a periodic point of period p if $f^p(x) = x$. The least positive p for which $f^p(x) = x$ is called the prime period of x.*

Denote the set of all fixed points by $Fix(f)$ and the set of all periodic points of (not necessarily prime) period p by $\mathrm{P}er_p(f)$.

Consider a mapping

$$F: x=(x_1,x_2,\dots,x_m)\in\mathbb{R}^m \to x'=F(x)=(x_1',x_2',\dots,x_m')\in\mathbb{R}^m$$

given by

$$x_k' = F_i(x_1,x_2,\dots,x_m),\ \ k=1,2,\dots,m, \qquad (2)$$

where $F_i:\mathbb{R}^m\to\mathbb{R}$, $i=1,\dots,m$ is a continuously differentiable function.

Thus, the eigenvalues of the Jacobian matrix J_F determine the local behavior of the nonlinear dynamical system. Below we shall do this point more explicit:

Definition 3. *A fixed point x^* of a mapping F is called*

- *hyperbolic point if its Jacobian J_F at x^* has no eigenvalues on the unit circle;*
- *attracting point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are less than 1 in absolute value;*
- *repelling point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are greater than 1 in absolute value;*

a saddle point otherwise, i.e., the remaining hyperbolic fixed points.

Next, we consider a random dynamical system formed by compositions of different maps. To model such a situation fix $0<b<1<a$ and for $t\in\{a,b\}$ define $f_t:[0,1]\to[0,1]$ by

$$f_t(x)=\begin{cases}(2x)^t & if\ \ x\in\left[0,\dfrac{1}{2}\right],\\[2mm] 2x-1 & if\ \ x\in\left(\dfrac{1}{2},1\right].\end{cases}$$

We consider a random dynamical system formed by choosing f_a with probability p_a and f_b with probability $p_b=1-p_a$ independently at each step of iteration.

Theorem 1. Suppose that either $ab\le 1$ and $p_a<p_b$ or $ab<1$ and $p_a\le p_b$. Then there exists $C,c>0$ such that $\mathbb{E}(x_n(\omega))\le Ce^{-cn}$ for all $n\ge 1$. In particular, there exists $n_1:\Omega\to\mathbb{N}$ such that $x_n(\omega)\le Ce^{-cn/2}$ for all $n_1(\omega)\ge n$ and $\mathbb{P}\{\omega | n_1(\omega)>n\}\le \bar{C}e^{-cn/2}$ for some $\bar{C}>0$.

The second chapter of the dissertation entitled “**Fixed points of an infinite-dimensional operator related to Gibbs measures**”. Introduce

$$\ell_+^1=\left\{x=(x_1,x_2,\dots,x_n,\dots):\ \ x_i>0,\ \ \|x\|=\sum_{j=1}^{\infty}x_j<\infty\right\}.$$

To describe translation invariant Gibbs measures of hard-core (HC) models on a Cayley tree of order $k\ge 2$ one has to study fixed points of operator

$F_0 : \ell^1_+ \to \ell^1_+$ defined by

$$F_0: \quad x_i' = \lambda_i \left(\frac{1+\sum_{j=1}^{\infty} a_{ij}x_j}{1+\sum_{j=1}^{\infty} a_{1j}x_j} \right)^k,$$

where $k, i \in \mathbb{N}$, $\lambda = (\lambda_1, \lambda_2, ...) \in \ell^1_+$, and $a_{ij} \in \{0,1\}$ are given parameters.

We are going to study fixed points of F_0, in the case when $a_{1j} = 1$ for any $j \in \mathbb{N}$. In this case the operator takes a simpler form:

$$F: \quad x_i' = \lambda_i \left(\frac{1+\sum_{j=1}^{\infty} a_{ij}x_j}{1+\|x\|} \right)^k, \tag{3}$$

where $k, i \in \mathbb{N}$, $\lambda_i > 0$. Let $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n, ...) \in \ell^1_+$. Denote

$$A_k = \left\{ x \in \ell^1_+ : \lambda_1 + \frac{\|\lambda\| - \lambda_1}{(1+\|\lambda\|)^k} \le \|x\| \le \|\lambda\| \right\}.$$

Theorem 2. 1) *For any* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n, ...) \in \ell^1_+$ *then* A_k *is an invariant with respect to operator (3),* $F : \ell^1_+ \to \ell^1_+$, *i.e.,* $F(A_k) \subset A_k$.

2) *There exists* $\hat{\lambda} > 0$ *such that if* $\|\lambda\| < \hat{\lambda}$ *then the operator (3) is a contraction* A_k *and hence has unique fixed point* $z^* \in A_k$ *and for any initial point* $z^{(0)} \in A_k$ *we have* $\lim_{n\to\infty} F^n(z^{(0)}) = z^*$.

Notice that the condition in item 2) of the above theorem is a sufficient condition. Below we show to non-trivial examples, which violate the condition in 2) but each it has a unique fixed point.

1) Let $k = 2$. If for all i, j we have $a_{1j} = 1, \quad a_{i1} = 1$ and remaining $a_{ij} = 0$ then the operator becomes

$$x_1' = \lambda_1, \quad x_i' = \lambda_i \left(\frac{1+\lambda_1}{1+\|x\|} \right)^2, \; i \ge 2.$$

2) In this example we take $a_{i1} = 0, \; i \ge 2$ and remaining $a_{ij} = 1$. Then corresponding fixed point equation is

$$x_1 = \lambda_1, \quad x_i = \lambda_i \left(\frac{1+\|x\| - \lambda_1}{1+\|x\|} \right)^2, \quad i \ge 2.$$

We demonstrate that, under certain conditions, the analysis of an infinite-dimensional operator can be reduced to the study of a two-dimensional operator.

Let, $a_{1j}=1,\ (j\ge 1),\ a_{i1}=0,\ (i>1),\ a_{22}=1$. We define the following sets $E_1=E_1(\vec{a})=\{i:\ a_{2i}=1\}$, and $E_2=\{i>2 \mid i\in E_1^c\}$. It is known that: $E_1\cup E_2=N_1=\mathbb{N}\setminus\{1\}$, $\|x\|=\sum_{j\ge 1}x_j=x_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j$, $E_1\ne\varnothing$, and $E_2\ne\varnothing$.

We consider the following class:

$$F:\begin{cases} x_i'=\lambda_i\left(\dfrac{1+\sum_{j\in E_1}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2, & if\ \ i\in E_1,\\[2ex] x_i'=\lambda_i\left(\dfrac{1+\sum_{j\in E_2}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2, & if\ \ i\in E_2.\end{cases}$$

Where, $E_1\ne\varnothing,\ E_2\ne\varnothing$ is an important condition. It is clear, that $E_1\ne\varnothing$ does not hold, because we have $a_{22}=1$. If $E_2=\varnothing$ holds, then the operator has a unique fixed point. Based on these conditions, the operator takes the following form:

$$x_1'=\lambda_1,\ \ x_i'=\lambda_i\left(\frac{1+\sum_{j\in E_1}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2,\ if\quad i\in E_1,$$

$$x_i'=\lambda_i\left(\frac{1+\sum_{j\in E_2}x_j}{1+\lambda_1+\sum_{j\in E_1}x_j+\sum_{j\in E_2}x_j}\right)^2,\ if\quad i\in E_2.$$

For simplicity, we consider the following case: $a_{1j}=1\ (j\ge 1)$, and $a_{i1}=0$ $(i>1)$ for all other i and j

$$a_{ij}=\begin{cases}1, & if\quad i+j\ \ even,\\ 0, & if\quad i+j\quad odd.\end{cases} \tag{4}$$

In that case, the operator takes the following form:

$$F_1: x_{2n}'=\lambda_{2n}\left(\frac{1+\sum_{j=1}^{\infty}x_{2j}}{1+\|x\|}\right)^2,\ x_{2n+1}'=\lambda_{2n+1}\left(\frac{1+\sum_{j=1}^{\infty}x_{2j+1}}{1+\|x\|}\right)^2,\ n\ge 1.$$

Define two-dimensional operator $W:\mathbb{R}_+^2\to\mathbb{R}_+^2$ by

$$W\ : x'=L_1\left(\frac{1+x}{1+\lambda_1+x+y}\right)^2,\ y'=L_2\left(\frac{1+y}{1+\lambda_1+x+y}\right)^2, \tag{5}$$

where $\lambda_1>0$ and $L_i>0$ are parameters.

Lemma 1. The infinite dimensional DS generated by the operator F is fully represented by the two-dimensional DS generated by the operator W.

Invariant sets. Denote

$$M_- = \{(x,y) \in \mathbb{R}_+^2 : \ x < y\},$$
$$M_0 = \{(x,y) \in \mathbb{R}_+^2 : \ x = y\},$$
$$M_+ = \{(x,y) \in \mathbb{R}_+^2 : \ x > y\}.$$

Lemma 2. If $L_1 = L_2 = L$ then the sets M_ε, $\varepsilon = -, 0, +$ are invariant with respect to operator W, i.e., $W(M_\varepsilon) \subset M_\varepsilon$.

On the invariant set M_0, the operator W defined by (5) reduces to

$$x' = f(x) := L\left(\frac{1+x}{1+\lambda_1+2x}\right)^2 .$$

Introduce,

$$\hat{L}_1 = \frac{2\lambda_1^2 + 76\lambda_1 - 142 - (2\lambda_1 - 34)\sqrt{\lambda_1^2 - 18\lambda_1 + 17}}{16},$$
$$\hat{L}_2 = \frac{2\lambda_1^2 + 76\lambda_1 - 142 + (2\lambda_1 - 34)\sqrt{\lambda_1^2 - 18\lambda_1 + 17}}{16}.$$

Based on the aforementioned, the following result is derived

1) If $\lambda_1 \in (0,17]$, $L > 0$ or $\lambda_1 > 17$, $L \notin [\hat{L}_1, \hat{L}_2]$, then $f(x)$ has unique fixed points, denoted by x_1^*;
2) If $\lambda_1 > 17$ and $L = \hat{L}_1$ or $L = \hat{L}_2$ then $f(x)$ has two fixed points, denoted by x_1^*, x_2^* with $x_1^* < x_2^*$;
3) If $\lambda_1 > 17$, $L \in (\hat{L}_1, \hat{L}_2)$ then $f(x)$ has three fixed points x_i, $i = 1,2,3$ with $x_1^* < x_2^* < x_3^*$.

Lemma 3. *The types of fixed points are as follows*

1) The unique fixed points

$$x_1^* = \begin{cases} attracting, & if \quad \lambda_1 > 17, \ L \notin [\hat{L}_1, \hat{L}_2], \\ saddle, & if \quad \lambda_1 = 17, \ L = 108, \\ attracting, & if \quad \lambda_1 = 17, \ L \neq 108 \ \ or \ \ \lambda_1 \in (0,1) \cup (1,17). \end{cases}$$

2) If $\lambda_1 > 17$ and $L = \hat{L}_1$ (resp. $L = \hat{L}_2$) then the function f has two fixed points $x_1^* < x_2^*$ and x_1^* is saddle and x_2^* is attracting (resp. x_1^* is attracting and x_2^* is saddle).

3) If $\lambda_1 > 17$, $L \in (\hat{L}_1, \hat{L}_2)$ then f has three fixed points with $x_1^* < x_2^* < x_3^*$. Moreover, x_1^* and x_3^* are attracting and x_2^* is repelling.

Lemma 4. *The operator (5) does not have periodic points in the invariant set* M_0.

Theorem 3. *The following assertions hold*

1) *If* $\lambda_1 \in (0,17],\ L>0$ *or* $\lambda_1>17,\ L\notin[\hat{L}_1,\hat{L}_2]$ *then for any* $x\in(0.+\infty)$ *the following equality holds*

$$\lim_{n\to\infty} f^n(x)=x_1^*.$$

2) *If* $\lambda_1>17$ *and* $L=\hat{L}_1$ *(resp.* $L=\hat{L}_2$*) then*

$$\lim_{n\to\infty} f^n(x)=\begin{cases} x_1^*, & if \quad x\in(0,x_1^*],\\ x_2^*, & if \quad x\in(x_1^*,+\infty),\end{cases}$$

$$\left(resp. \quad \lim_{n\to\infty} f^n(x)=\begin{cases} x_1^*, & if \quad x\in(0,x_2^*),\\ x_2^*, & if \quad x\in[x_2^*,+\infty),\end{cases}\right).$$

3) *If* $\lambda_1>17,\ L\in(\hat{L}_1,\hat{L}_2)$ *then*

$$\lim_{n\to\infty} f^n(x)=\begin{cases} x_1^*, & if \quad x\in(0,x_2^*),\\ x_2^*, & if \quad x=x_2^*,\\ x_3^*, & if \quad x\in(x_2^*,+\infty).\end{cases}$$

In the third chapter of the dissertation entitled **"Dynamical systems of an infinite-dimensional non-linear operator"** and contains results on the dynamics of a rational dynamical system in infinite-dimensional space, as well as its applications in the construction of translation invariant Gibbs measures for HC model.

Consider operator W on the invariant set $\mathbb{R}_+^2\setminus M_0$. Denoting $A_{i,j}$ above means that the number of fixed points on M_0 is i and number of fixed points outside M_0 is j. Thus we proved the following:

$$A_{1,0}=\left\{(\lambda_1,L): L\le\frac{(\lambda_1+3)^2}{4},\ 0<\lambda_1\le5\right\}\cup\left\{(\lambda_1,L): L\le4(\lambda_1-1),\ \lambda_1>5\right\},$$

$$A_{1,2}=\left\{(\lambda_1,L): L>\frac{(\lambda_1+3)^2}{4},\ 0<\lambda_1\le17\right\}\cup$$

$$\cup\left\{(\lambda_1,L): \frac{(\lambda_1+3)^2}{4}\le L<\hat{L}_1,\ 17<\lambda_1\le9+8\sqrt{2}\right\}\cup$$

$$\cup\left\{(\lambda_1,L): L>\hat{L}_2,\ \lambda_1>17\right\}\cup\left\{(\lambda_1,L): L=4(\lambda_1-1),\ \lambda_1>5\right\},$$

$$A_{2,2}=\left\{(\lambda_1,L): L=\hat{L}_2,\ \lambda_1>17\right\}\cup\left\{(\lambda_1,L): L=\hat{L}_1, 17<\lambda_1<9+8\sqrt{2}\right\},$$

$$A_{1,4}=\left\{(\lambda_1,L):4(\lambda_1-1)<L<\frac{(\lambda_1+3)^2}{4},\ 5<\lambda_1\le 9+8\sqrt{2}\right\}\cup$$

$$\cup\left\{(\lambda_1,L):4(\lambda_1-1)<L<\hat{L}_1,\ \lambda_1>9+8\sqrt{2}\right\},$$

$$A_{3,2}=\left\{(\lambda_1,L):\hat{L}_1<L<\hat{L}_2,\ 17<\lambda_1\le 9+8\sqrt{2}\right\}\cup$$

$$\cup\left\{(\lambda_1,L):\frac{(\lambda_1+3)^2}{4}<L<\hat{L}_2,\ \lambda_1>9+8\sqrt{2}\right\},$$

$$A_{2,4}=\left\{(\lambda_1,L):L=\hat{L}_1,\ \lambda_1>9+8\sqrt{2}\right\},$$

$$A_{3,4}=\left\{(\lambda_1,L):\hat{L}_1<L<\frac{(\lambda_1+3)^2}{4},\ \lambda_1>9+8\sqrt{2}\right\}$$

Lemma 5. *Thee following assertions hold*

1) If $(\lambda_1,L)\in A_{1,0}$ *then* $Fix(W)=\{p_1^*\}$,

2) If $(\lambda_1,L)\in A_{1,2}$ *then* $Fix(W)=\{p_1^*,p_1,p_2\}$,

3) If $(\lambda_1,L)\in A_{2,2}$ *then* $Fix(W)=\{p_1^*,p_2^*,p_1,p_2\}$,

4) If $(\lambda_1,L)\in A_{1,4}$ *then* $Fix(W)=\{p_1^*,p_1,p_2,p_3,p_4\}$,

5) If $(\lambda_1,L)\in A_{3,2}$ *then* $Fix(W)=\{p_1^*,p_2^*,p_3^*,p_1,p_2\}$,

6) If $(\lambda_1,L)\in A_{2,4}$ *then* $Fix(W)=\{p_1^*,p_2^*,p_1,p_2,p_3,p_4\}$,

7) If $(\lambda_1,L)\in A_{3,4}$ *then* $Fix(W)=\{p_1^*,p_2^*,p_3^*,p_1,p_2,p_3,p_4\}$,

where $p_i^*=(x_i^*,x_i^*),\quad i=1,2,3,$

$$p_1=(x_1,x_2),\ p_2=(x_2,x_1),\ p_3=(x_3,x_4),\ p_4=(x_4,x_3)$$

with

$$x_1=\frac{\sqrt{L}+\sqrt{L-4(\lambda_1-1)}+\sqrt{2L-4\lambda_1+2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_2=\frac{\sqrt{L}+\sqrt{L-4(\lambda_1-1)}-\sqrt{2L-4\lambda_1+2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_3=\frac{\sqrt{L}-\sqrt{L-4(\lambda_1-1)}+\sqrt{2L-4\lambda_1-2\sqrt{L^2-4(\lambda_1-1)L}}}{2},$$

$$x_4=\frac{\sqrt{L}-\sqrt{L-4(\lambda_1-1)}-\sqrt{2L-4\lambda_1-2\sqrt{L^2-4(\lambda_1-1)L}}}{2}.$$

Theorem 4. *Fixed points of operator* F_1 *for* $L_1=L_2=L$ *are as follows:*

$$Fix(F)=\begin{cases}\{P_1\}, & if\ \ (\lambda_1,L)\in A_{1,0},\\ \{P_1,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{1,2},\\ \{P_1,P_2,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{2,2},\\ \{P_1,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{1,4},\\ \{P_1,P_2,P_3,P_4,P_5\}, & if\ \ (\lambda_1,L)\in A_{3,2},\\ \{P_1,P_2,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{2,4},\\ \{P_1,P_2,P_3,P_4,P_5,P_6,P_7\}, & if\ \ (\lambda_1,L)\in A_{3,4},\end{cases}$$

where

$$P_1=\left(\lambda_1,\frac{x_1^*}{L}\lambda_2,\frac{x_1^*}{L}\lambda_3,\frac{x_1^*}{L}\lambda_4,\ldots\right),P_2=\left(\lambda_1,\frac{x_2^*}{L}\lambda_2,\frac{x_2^*}{L}\lambda_3,\frac{x_2^*}{L}\lambda_4,\ldots\right)$$

$$P_3=\left(\lambda_1,\frac{x_3^*}{L}\lambda_2,\frac{x_3^*}{L}\lambda_3,\frac{x_3^*}{L}\lambda_4,\ldots\right),P_4=\left(\lambda_1,\frac{x_2}{L}\lambda_2,\frac{x_1}{L}\lambda_3,\frac{x_2}{L}\lambda_4,\ldots\right)$$

$$P_5=\left(\lambda_1,\frac{x_1}{L}\lambda_2,\frac{x_2}{L}\lambda_3,\frac{x_1}{L}\lambda_4,\ldots\right),P_6=\left(\lambda_1,\frac{x_4}{L}\lambda_2,\frac{x_3}{L}\lambda_3,\frac{x_4}{L}\lambda_4,\ldots\right)$$

$$P_7=\left(\lambda_1,\frac{x_3}{L}\lambda_2,\frac{x_4}{L}\lambda_3,\frac{x_3}{L}\lambda_4,\ldots\right)$$

Consider the North-East ordering (NE) on $\mathbb{R}^2$ for which the positive cone is the first quadrant, i.e., this partial ordering is defined by $(x_1,y_1)\preceq_{NE}(x_2,y_2)$ if $x_1\le x_2$ and $y_1\le y_2$. The South-East (SE) ordering defined as $(x_1,y_1)\preceq_{SE}(x_2,y_2)$ if $x_1\le x_2$ and $y_1\ge y_2$.

Consider the following partitions of $\mathbb{R}_+^2$:

$$\mathbb{R}_+^2=A_{SE}^{<}\cup A_{SE}^{>}\cup A_{NE}^{<}\cup A_{NE}^{>},$$

where

$$A_{SE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ x\ge\psi(y),\ y\le\psi(x)\},$$
$$A_{SE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ x\le\psi(y),\ y\ge\psi(x)\},$$
$$A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ x\le\psi(y),\ y\le\psi(x)\},$$
$$A_{NE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ x\ge\psi(y),\ y\ge\psi(x)\}.$$

Proposition 1. We have

$$A_{SE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ (x,y)\preceq_{SE}W(x,y)\},\ A_{SE}^{>}=\{(x,y)\in\mathbb{R}_+^2:\ W(x,y)\preceq_{SE}(x,y)\},$$
$$Fix(W)=\{(x,y)\in\mathbb{R}_+^2: x=\psi(y),\ y=\psi(x)\},$$
$$A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ (x,y)\preceq_{NE}W(x,y)\},\ A_{NE}^{<}=\{(x,y)\in\mathbb{R}_+^2:\ W(x,y)\ \preceq_{NE}(x,y)\}.$$

For a fixed point $p\in Fix(W)$, we denote

$$B^{<}(p)=\{(x,y)\in M_-:\ (x,y)\prec_{SE}W(x,y)\preceq_{SE}p\},$$
$$B^{>}(p)=\{(x,y)\in M_-:\ p\ \preceq_{SE}W(x,y)\prec_{SE}(x,y)\},$$

$$R^{>}(p)=\{(x,y)\in M_{-}:\ W(x,y)\prec_{SE}(x,y)\preceq_{SE}\ p\ \},$$
$$R^{>}(p)=\{(x,y)\in M_{-}:\ p\ \preceq_{SE}(x,y)\prec_{SE}W(x,y)\}.$$

For an initial point $x^{(0)}=\left(x_1^{(0)},x_2^{(0)},...\right)\in\ell_{+}^{1}$, introduce

$$v_1^{(0)}=\sum_{j=1}^{\infty}x_{2j+1}^{(0)},\ v_2^{(0)}=\sum_{j=1}^{\infty}x_{2j}^{(0)}\ . \tag{6}$$

Theorem 5. *For initial points* $v^{(0)}\in\mathbb{R}_{+}^{2}\setminus M_{+}$, *the following hold*

$$\lim_{n\to\infty}W^{n}\left(v^{(0)}\right)$$
$$=\begin{cases} p_1^{*}, & if\quad (\lambda_1,L)\in A_{1,0}\,, \\ p_2, & if\quad (\lambda_1,L)\in A_{1,2}, \qquad \forall v^{(0)}\in M_{-}\ , \\ p_2, & if\quad (\lambda_1,L)\in A_{1,4},\ \forall v^{(0)}\in\left(B^{<}(p_2)\cup B^{>}(p_2)\right)\cup\left(R^{<}(p_4)\cup R^{>}(p_4)\right), \\ p_4, & if\quad (\lambda_1,L)\in A_{1,4},\ \forall v^{(0)}\in\left(B^{<}(p_4)\cup B^{>}(p_4)\right)\cup\left(R^{<}(p_2)\cup R^{>}(p_2)\right), \\ p, & if\quad (\lambda_1,L)\in\bigcup_{\substack{i=2,3\\ j=2,4}}A_{i,j}\,,\qquad p\in Fix(W),\quad \forall v^{(0)}\in\left(B^{<}(p)\cup B^{>}(p)\right). \end{cases}$$

Theorem 6. *If initial point* $x^{(0)}=\left(x_1^{(0)},x_2^{(0)},...\right)\in\ell_{+}^{1}$ *is such that*

$$\lim_{m\to\infty}W^{m}\left(v_1^{(0)},v_2^{(0)}\right)=(a,b)=p\in Fix(W)$$

for corresponding values (6) then

$$\lim_{m\to\infty}F^{m}\left(x^{(0)}\right)=\frac{1}{L}(\lambda_1L,\lambda_2b,\lambda_3a,\lambda_4b,...).$$

For the hard-core (HC) model with a countable number of states on the Cayley tree define configuration $\sigma=\{\sigma(x)|\ x\in V\}$ as a function from V to the set of natural numbers $\mathbb{N}$.

Consider the set $\mathbb{N}$ as the set of vertices of some infinite graph G. Using the graph G. A configuration σ is called $G-$admissible on a Cayley tree if $\{\sigma(x),\sigma(y)\}$ is an edge of the graph G for any nearest neighbors x,y from V.

The set of $G-$admissible configurations is denoted by Ω^{G}.

The activity set for the graph G is the bounded function $\lambda:G\mapsto\mathbb{R}_{+}$ (where $\mathbb{R}_{+}$ is the set of positive real numbers). Define HC model for the Hamiltonian G as

$$H_G^{\lambda}(\sigma)=\begin{cases}\sum_{x\in V}\ln\lambda_{\sigma(x)}, & if\quad \sigma\in\Omega^{G}, \\ +\infty, & if\quad \sigma\notin\Omega^{G}\end{cases} \tag{7}$$

The edges of the graph G is denoted by $L(G)$. Denote by $A\equiv A^{G}\equiv\left(a_{ij}\right)_{i,j\in\mathbb{Z}}$ the adjacency matrix of G, i.e.,

$$a_{ij}=a_{ij}^{G}=\begin{cases}1, & if\quad \{i,j\}\in L(G), \\ 0, & if\quad \{i,j\}\notin L(G).\end{cases}$$

Definition 4. *A family* $l=\{l_{xy}\}_{\langle x,y\rangle\in L}$ *with* $l_{xy}=\{l_{xy}(i):\ i\in\mathbb{N}\}\in(0,\infty)^{\mathbb{N}}$ *is* *called the boundary law for the Hamiltonian* (7) *if*

1) *For each* $\langle x,y\rangle \in L$ *there exists a constant* $c_{xy} > 0$ *such that the consistency*

equation

$$l_{xy}(i) = c_{xy} \prod_{z\in\partial x\setminus y} \sum_{j\in\mathbb{N}} a_{ij}\lambda_j l_{zx}(j)$$

holds for any $i \in \mathbb{N}$, *where* ∂x *is the set of nearest neighbors of* x.

2) *The boundary law* l *is said to be normalisable if*

$$\sum_{i\in\mathbb{N}} \left(\prod_{z\in\partial x} \sum_{j\in\mathbb{N}} a_{ij}\lambda_j l_{zx}(j) \right) < \infty$$

for all $x \in V$.

Lemma 6. If $\lambda \in \ell^1_+$ then any solution of the form F_1 is normalisable.

Using the above, we obtain the following theorem about the number of Gibbs measures.

Theorem 7. Let $\mathcal{N}_G$ be the number of translation invariant Gibbs measure for Hamiltonian (7), corresponding to graph G defined by (4), then

$$\mathcal{N}_G = \begin{cases} 1, & if \quad (\lambda_1, L) \in A_{1,0}, \\ 3, & if \quad (\lambda_1, L) \in A_{1,2}, \\ 4, & if \quad (\lambda_1, L) \in A_{2,2}, \\ 5, & if \quad (\lambda_1, L) \in A_{1,4}, \\ 5, & if \quad (\lambda_1, L) \in A_{3,2}, \\ 6, & if \quad (\lambda_1, L) \in A_{2,4}, \\ 7, & if \quad (\lambda_1, L) \in A_{3,4}. \end{cases}$$

CONCLUSION

The dissertation is dedicated to the characterization of dynamical systems of infinite-dimensional operators in discrete time.

The main results of the study are as follows:

1. A condition on the parameters has been established for a discrete-time infinite-dimensional rational operator to have a unique fixed point.
2. The problem of studying the dynamics of an infinite-dimensional operator has been reduced, under certain conditions, to the study of the dynamics of a two-dimensional operator.
3. In the process of reducing the infinite-dimensional operator to a two-dimensional one, a method has been developed to map infinite-dimensional operators into a single two-dimensional operator and then revert the result back to the infinite-dimensional operator.
4. The results obtained for the infinite-dimensional operator have been applied to translational Gibbs measures in the Hard-Core model.

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ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**

ИНСТИТУТ МАТЕМАТИКИ

ОЛИМОВ УМРБЕК РАШИДОВИЧ

РАЦИОНАЛЬНЫЕ ДИНАМИЧЕСКИЕ СИСТЕМЫ С ДИСКРЕТНЫМ ВРЕМЕНЕМ В БЕСКОНЕЧНОМЕРНЫХ ПРОСТРАНСТВАХ

01.01.01 –Математический анализ

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ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является изучение динамики, неподвижных точек и мер Гиббса, соответствующих этим точкам, бесконечномерного рационального оператора с дискретным временем, соответствующего физическим процессам, определенным в дереве Келли k-го порядка с естественными значениями спина.

Объект исследования: сжимающие отображения, динамика бесконечномерных операторов, матрица Якоби, гамильтониан, градиентные меры Гиббса.

Научная новизна исследования заключается в следующем:

найдены условия на параметры, при которых бесконечномерный оператор имеет единственную неподвижную точку;

бесконечномерная рациональная динамическая система сведена к двумерной динамической системе и описано множество ее неподвижных точек;

доказано, что число трансляционно-инвариантных мер Гиббса, соответствующих модели HC со счетным значением спина, заданным в дереве Келли, не превосходит семи

Внедрение результатов исследования. На основе полученных результатов по динамическим системам дискретного времени бесконечномерных рациональных отображений в биологии и физике:

Результаты, касающиеся предельных множеств динамических систем бесконечной размерности, были использованы в зарубежном проекте № G00003447 на тему «Квантовые генетические алгебры и их применение» для исследования фазовых переходов в модели HC с конечным числом значений спина (справка Университета Объединённых Арабских Эмиратов от 24 марта 2025 года, ОАЭ). Применение научного результата позволило описать совокупность термодинамических свойств и мер Гиббса для моделей HC с конечным числом значений спина, а также установить существование фазовых переходов.

Методология исследования динамики бесконечномерных нелинейных операторов и описания мер Гиббса, соответствующих модели HC на дереве Кэли, была использована в рамках зарубежного проекта № FRGS21-230-0839 на тему «Динамика ограниченно-размерных ортогонально сохраняющих кубических стохастических операторов» для описания неподвижных точек бесконечномерных нелинейных стохастических операторов (справка Международного исламского университета Малайзии от 24 марта 2025 года, Малайзия). Применение научного результата позволило локально описать динамику нелинейных стохастических операторов, сохраняющих ортогональность.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Общий объем диссертации составляет 112 страниц.

E’LON QILINGAN ILMIY ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part 1; часть 1)

1. Olimov U.R., Rozikov U.A., Fixed points of an infinite dimensional operator related to Gibbs measures. // *Theoretical and Mathematical Physics,* 214(2), 2023. pp.282-295 (3.Scopus. IF=0.325).

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3. Olimov U.R., Description of fixed points of an infinite dimensional operator, // *Bulletin of the Institute of Mathematics*, 2024, № 2, pp.41-48.

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II bo‘lim (part 2; часть 2)

6. Olimov U.R., Rozikov U.A., Contraction of an infinite dimensional operator related to Gibbs measures. "Operator algebras non-associative structures and related problems" *International scientific conference,* September 14-15, 2022, Tashkent, pp. 301-302.

7. Olimov U.R., Rozikov U.A., Dynamical system of an infinite-dimensional operator in an invariant set. "Abstracts of the international conference" *International scientific conference*, 18-22 September,2023, Tashkent and Samarkand,pp.82-83.

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9. Olimov U.R., Dynamical system of the nonlinear operator on an invariant set. "Actual problems of modern geometry and topology" *International scientific conference*, 27-28 October, 2024, Tashkent, pp.109-110.

10. Olimov U.R., Rozikov U.A., Dynamical systems of an infinite dimensional operator. "VII World Congress of the Turkic World Mathematicians" *International scientific conference*, 20-23 September, 2023, Kazakhstan, pp.207

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