**V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

MATEMATIKA INSTITUTI

XUDOYBERDIYEV XAYOTJON OCHILTOSH O‘G‘LI

NOVOLTERRA KVADRATIK OPERATORLAR DINAMIKASI: O‘RIN ALMASHTIRISHLI VA EPIDEMIK MODELLAR

01.01.01 – Matematik analiz

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
DISSERTATSIYASI AVTOREFERATI**

TOSHKENT - 2025

UDK: 517.98

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestasiya komissiyasida B2024.4.PhD/FM1176 raqam bilan ro‘yxatga olingan.

Dissertatsiya Matematika institutida bajarilgan.
Dissertatsiya avtoreferati uch tilda (o‘zbek, ingliz, rus (rezyume)) Ilmiy kengash veb-sahifasi (https://kengash.mathinst.uz) va "ZiyoNet" ta’lim axborot tarmog‘ida (http://www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya himoyasi V.I. Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2025-yil "01" iyul kuni soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+998 71) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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Dissertatsiya avtoreferati 2025-yil "11" iyun kuni tarqatildi.
(2025-yil "11" iyundagi 2- raqamli reestr bayonnomasi).

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar aksariyat hollarda nochiziqli dinamik sistemalarning xossalarini o‘rganishni taqozo etadi. Dinamik sistemalar xossalaridan matematik biologiya, genetika va tibbiyot sohalarida duch keladigan prognozlash masalalarini hal qilishda keng foydalaniladi. Dinamik sistemalar ikki turga bo‘linadi: uzluksiz vaqtli dinamik sistemalar va diskret vaqtli dinamik sistemalar. Biologik populyatsiyaning evolyutsiyasini ifodalovchi operatorlar hosil qilgan dinamik sistemalar matematik biologiyaning asosiy tadqiqot obyektlaridan biri hisoblanadi. Shu sababli, evolyutsion operatorlarning muhim sinfi bo‘lgan kvadratik stoxastik operatorlar dinamikasini tadqiq qilish nochiziqli dinamik sistemalar nazariyasida dolzarb vazifalardan biri bo‘lib qolmoqda.

Hozirgi kunda Volterra kvadratik stoxastik operatorlarining dinamik xossalari atroflicha o‘rganilgan. Nochiziqli dinamik sistemalar nazariyasidagi muhim muammolardan biri bo‘lgan novolterra kvadratik stoxastik operatorlari orbitalarining asimptotikasini tavsiflash borasida ko‘plab ilmiy tadqiqotlar olib borilmoqda. Ushbu tadqiqotlarning asosida kvadratik stoxastik operatorlardan hosil qilingan nochiziqli dinamik sistemalarning invariant to‘plamlarini aniqlash, davriy nuqtalarining aniq ko‘rinishini topish hamda ularning tipini aniqlash, dinamik sistema uchun Lyapunov funksiyalarini topish va dinamik sistema orbitalarining asimptotik xarakterlarini tadqiq qilish kabi masalalar yotadi. Ta’kidlash joizki, umumiy holdagi nochiziqli dinamik sistemalar uchun yuqorida sanab o‘tilgan masalalar to‘liq hal qilinmagan. Shu tufayli o‘rin almashtirishlarga va epidemik modellarga bog‘liq kvadratik stoxastik operatorlarning dinamik xossalarini tadqiq etish maqsadli ilmiy tadqiqotlardan biri hisoblanadi.

Mamlakatimizda so‘nggi yillarda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan matematika, fizika, kimyo va biologiya fanlariga e’tibor kuchaytirildi. Jumladan, biologiya, tibbiyot, iqtisodiyot sohalarida keng qo‘llaniladigan nochiziqli dinamik sistemalar nazariyasini rivojlantirishga alohida e’tibor berildi va bu borada salmoqli natijalarga erishildi. “Algebra, funksional analiz va dinamik tizimlar nazariyasi” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi[1]. Bu qaror ijrosini ta’minlash maqsadida ilmiy natijalarni fanning turdosh sohalariga tatbiq etish borasida kvadratik stoxastik operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalar nazariyasini rivojlantirish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini

[1] O‘zbekiston Respublikasi Vazirlar Mahkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqotlar muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning Respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Tabiatda ro‘y beradigan hodisa va jarayonlarni tushunish va o‘rganish uchun turli matematik modellardan, jumladan, dinamik sistemalar nazariyasi usullaridan keng foydalaniladi. Shu sababli diskret vaqtli dinamik sistemalarga tadqiqotchilar tomonidan qiziqish ortib bormoqda. Kvadratik stoxastik operatorlar matematik biologiyadagi irsiyat qonunlari bilan bog‘liq masalalarni hal qilish uchun kiritilgan. Bunday operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalar nazariyasidan biologiya, ekologiya, tibbiyot, iqtisodiyot va axborot texnologiyalari sohalaridagi ayrim murakkab jarayonlarni modellashtirishda foydalanib kelinmoqda. Kvadratik stoxastik operatorlar 1924-yilda S. Bernshteyn tomonidan ilk bora matematik biologiya va populyatsion genetikada gen chastotalari evolyutsiyasini ifodalovchi operatorlar sifatida fanga kiritilgan. Bu kabi operatorlar fizika, kimyo, iqtisodiyot kabi sohalarda ham tatbiqiga ega bo‘lgani uchun kvadratik stoxastik operatorlarning dinamik xossalari katta qiziqish bilan tadqiq etila boshlandi. Kvadratik operatorlar nazariyasi S. Ulam, H. Kesten, Yu.I. Lyubich, S.S. Vallander, R. Jenks, E.Akin, V. Losert, T.A. Sarimsoqovlar tomonidan rivojlantirilgan va bugungi kunda M.I. Zaharevich, R.N. G‘anixo‘jayev, N.N. G‘anixo‘jayev, O‘.A. Roziqov, F.M. Muxamedov, U.U. Jamilov va O.N. Hakimovlar tomonidan tadqiqotlar davom ettirilib kelinmoqda.

Shuni ham ta’kidlash joizki, umumiy holda kvadratik stoxastik operatorning asimptotik xarakterini o‘rganish asosiy masala bo‘lib hisoblanadi. Ushbu masala bir o‘lchamli simpleksda aniqlangan kvadratik operatorlar uchun Yu.I. Lyubich tomonidan to‘la hal qilingan. Ammo, umumiy holda bu masala ikki va undan yuqori o‘lchamli simplekslarda aniqlangan kvadratik operatorlar uchun to‘liq hal qilinmagan. Volterra kvadratik stoxastik operatorlari uchun asosiy masala R.N. G‘anixo‘jayev va uning shogirdlari tomonidan chuqur tadqiq qilinib kelinmoqda. Bugungi kunda N.N. G‘anixo‘jayev, O‘.A. Roziqov, F.M. Muxamedov, A. Zada, M. Ladra, U.U. Jamilov, O.N. Hakimov, A.Yu. Xamrayev, S.K. Shoyimardonov, S.S. Xudayarovlar novolterra kvadratik stoxastik operatorlar dinamikasini tadqiq qilish bo‘yicha ko‘plab ilmiy izlanishlar olib borishmoqda.

R.N. G‘anixo‘jayev va D.B. Eshmamatova ishlarida chekli o‘lchamli simpleksda aniqlangan kvadratik gomeomorfizmlari tavsiflangan va ularning aniq ko‘rinishi topilgan. Ular tomonidan Volterra kvadratik stoxastik operatorlari va ularning tatbiqlari bo‘yicha ilmiy tadqiqotlar olib borilmoqda. Okean ekosistemalarining ba’zi modellari tomonidan hosil qilingan diskret vaqtli dinamik sistemalarni O‘.A. Roziqov va S.K. Shoyimardonovlarning ishlarida tadqiq qilingan. Kvadratik stoxastik operatorlar yordamida kompyuterlarda tarqaladigan ba’zi internet viruslari dinamikasini F.T. Adilova, U.U. Jamilov va A. Reynfeldslar ishlarida tadqiq etilgan. Yuqorida ta’kidlanganidek, hozirgi kungacha, kvadratik stoxastik operatorlar ustida ko‘plab ilmiy tadqiqotlar olib borilishiga qaramasdan, ixtiyoriy kvadratik stoxastik operator uchun asosiy masala to‘la hal qilinmagan. Shu nuqtai nazardan dissertatsiya ishida qaralayotgan novolterra kvadratik stoxastik operatorlari dinamikasini tadqiq etish muhim bo‘lib hisoblanadi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish instituti ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutining OT-F4-87 raqamli “Yevklid va psevdo-Yevklid fazolaridagi egri chiziqlar va sirtlarning global invariantlari nazariyasi va uning mexanikaga tatbiqlari” (2017-2020 yy), OT-F4-82 raqamli “Operatorlar va noassotsiativ algebralarda lokal differensiallash va avtomorfizmlar, nochiziqli dinamik sistemalarda faza almashishlar va xaos” (2017-2020 yy) mavzusidagi ilmiy tadqiqot loyihalari va “Noassotsiativ algebralar strukturaviy nazariyasi va uning biologik sistemalardagi dinamik sistemalarni tadqiq qilishdagi tatbiqi” (2020-2023 yy) nomli ilmiy yo‘nalish doirasida bajarilgan.

Tadqiqot maqsadi o‘rin almashtirishlarga bog‘liq (I operator) va qayta yuquvchi diskret vaqtli SIRD epidemik modelga mos keluvchi (II operator) novolterra kvadratik stoxastik operatorlar dinamik xossalarini tadqiq qilishdan iborat.

Tadqiqotning vazifalari:

I va II operatorlarga nisbatan invariant to‘plamlarni aniqlash;

I va II operatorlarning davriy nuqtalarini va ularning tiplarini topish;

I va II operatorlar uchun Lyapunov funksiyalarini qurish;

I va II operatorlar uchun ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalari to‘plamini tavsiflash.

Tadqiqot obyekti: Chekli o‘lchamli simpleksda aniqlangan novolterra kvadratik stoxastik operatorlari.

Tadqiqot predmeti. Kvadratik stoxastik operatorlar nazariyasi va diskret dinamik sistemalar nazariyasi.

Tadqiqot usullari. Tadqiqot ishida matematik analiz, funksional analiz, algebra, ehtimollar nazariyasi va dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

ikki o‘lchamli simpleksda aniqlangan novolterra kvadratik stoxastik operatorning davriy nuqtalari to‘plami tavsiflangan. Shuningdek, ixtiyoriy boshlang‘ich nuqtaning orbitasi ko‘pi bilan ikkita limit nuqtaga ega bo‘lishi isbotlangan;

I operatorning davriy nuqtalar to‘plami tavsiflangan hamda ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalar to‘plami yagona nuqtadan, yoki cheklita nuqtalardan iborat bo‘lishi ko‘rsatilgan;

II operator simpleksni invariant to‘plam sifatida saqlashi uchun operator parametrlariga zarur va yetarli shartlari topilgan;

II operator uchun Lyapunov funksiyasi topilib va u yordamida ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalar to‘plami tavsiflangan.

Tadqiqotning amaliy natijalari. Olingan yangi natijalardan va dissertatsiyada qo‘llanilgan usullardan diskret vaqtli dinamik sistemalar nazariyasida foydalanishi mumkin. Shuningdek, ushbu natijalardan oliy o‘quv yurtlari magistrantlari va tayanch doktorantlar uchun maxsus kurslarda manba sifatida foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi. Barcha natijalar matematik analiz va funksional analiz, Lyapunov funksiyalari nazariyasi va diskret dinamik sistemalar nazariyasi usullaridan foydalanib olingan. Shuningdek, tadqiqotda olingan natijalar qat’iy matematik mulohazalarga tayanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati kvadratik stoxastik operatorlar nazariyasida, matematik biologiya va epidemiologiya sohalaridagi muammolarni hal qilishda qo‘llanilishi mumkinligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati I operatorga mos orbitalarning limit nuqtalar to‘plamining tavsifi populyatsiyaning evolyutsiyasi haqida ma’lumotlar berishi bilan izohlanadi. II operatorga mos orbitalarning asimptotik xarakteri epidemiologiyada kasallikning tarqalishi ssenariylarini ifodalashi bilan asoslanadi.

Tadqiqot natijalarining joriy qilinishi. I va II operatorlar dinamikasiga oid olingan natijalar quyidagi yo‘nalishlarda amaliyotga joriy etilgan:

qayta yuquvchi diskret vaqtli SIRD epidemik modeli uchun orbitalarning limit nuqtalari to‘plami tavsifidan G0003447 raqamli “Kvant genetik algebralari va ularning tatbiqlari” mavzusidagi xorijiy loyihada nochiziqli stoxastik operatorlarning regulyarlik xossalarini tahlil qilish uchun foydalanilgan (Birlashgan Arab Amirliklari universiteti 2025-yil 1-maydagi ma’lumotnomasi, BAA). Ilmiy natijaning qo‘llanilishi biologik sistemalarning kelajagini tushunish uchun muhim ahamiyatga ega bo‘lgan kvadratik stoxastik operatorlar uchun orbitalarning yaqinlashishini aniqlashga imkon bergan;

o‘rin almashtirishga mos novolterra kvadratik stoxastik operatorlarining davriy nuqtalar to‘plami va limit nuqtalari to‘plamidan PID2020-115155GB-I00 raqamli “Assotsiativ bo‘lmagan gruppalar va algebralarda gomologiyalar, gomotopiyalar va kategorik invariantlar” mavzusidagi xorijiy loyihada evolyutsion algebralarning muvozanat holatlarini tahlil qilishda foydalanilgan (Santyago de Kompostela universitetining 2025 yil 6-maydagi ma’lumotnomasi, Ispaniya). Ushbu ilmiy natijani qo‘llash evolyutsiya algebralaridagi idempotent va absolyut nilpotent elementlarni tavsiflash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 9 ta ilmiy-amaliy anjumanlarda, jumladan 4 ta xalqaro va 5 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo‘yicha jami 15 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan 5 tasi xorijiy va 1 tasi respublika jurnalida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 103 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asosli ravishda tahlil qilingan bo‘lib tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlari bilan mosligi ko‘rsatilgan, muammoning o‘rganilganligi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti bayon etilgan, tadqiqotda olingan natijalarning ilmiy yangiligi va amaliy ahamiyati tavsiflangan, olingan natijalarning nazariy va amaliy ahamiyati ko‘rsatilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi haqida ham batafsil ma’lumotlar taqdim etilgan.

Dissertatsiyaning **“Kvadratik stoxastik operatorlarning diskret vaqtli dinamik sistemalari”** deb nomlanuvchi birinchi bobida, kvadratik stoxastik operatorlar nazariyasidagi asosiy ta’riflar va tushunchalar eslatib o‘tilgan. Shuningdek, ikki o‘lchamli simpleksda aniqlangan $(\alpha,\beta)-$kvadratik stoxastik operatorning dinamikasi tadqiq qilingan.

Aytaylik, $E_m=\{1,\dots,m\}$ chekli to‘plam berilgan bo‘lsin. E_m to‘plamda aniqlangan barcha ehtimollik taqsimotlari

$$S^{m-1}=\left\{\mathbf{x}\in R_+^m:\ \sum_{i=1}^m x_i=1\right\}$$

kabi aniqlangan $(m-1)$- o‘lchamli simpleksni tashkil etadi. S^{m-1} simpleksni o‘zini-o‘ziga o‘tkazuvchi V akslantirish ixtiyoriy $\mathbf{x}\in S^{m-1}$ uchun

$$(V\mathbf{x})_k=\sum_{i,j=1}^m p_{ij,k}x_ix_j,\ k\in E_m \qquad (1)$$

ko‘rinishga ega bo‘lsa, V akslantirishga kvadratik stoxastik operator (KSO) deyiladi. Bunda $p_{ij,k}$ koeffitsientlar quyidagi

$$
\begin{aligned}
&p_{ij,k} = p_{ji,k} \geq 0, \quad \text{barcha } i,j,k \in E_m; \\
&\sum_{k=1}^{m} p_{ij,k} = 1, \qquad \text{barcha } i,j \in E_m
\end{aligned}
\tag{2}
$$

shartlarni qanoatlantiradi.

Ixtiyoriy $\mathbf{x}^{(0)} \in S^{m-1}$ boshlang‘ich nuqtaning V operator ta’siridagi orbitasi (trayektoriyasi) deb $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$ ketma-ketlikka aytiladi va bu ketma-ketlikning elementlari barcha $n = 0,1,2,\ldots$ uchun $\mathbf{x}^{(n+1)} = V\left(\mathbf{x}^{(n)}\right) = V^{n+1}\left(\mathbf{x}^{(0)}\right)$ qonuniyat bilan aniqlanadi. $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$ orbitaning barcha limit nuqtalari to‘plami $\omega_V\left(\mathbf{x}^{(0)}\right)$ kabi belgilanadi. Odatda bu to‘plamga orbitaning ω-limit nuqtalari to‘plami deyiladi. Matematik biologiyaning asosiy masalalaridan biri berilgan operator uchun orbitalarning asimptotik xarakterini o‘rganishdan iborat. Boshqacha aytganda, berilgan V KSO uchun ixtiyoriy $\mathbf{x}^{(0)} \in S^{m-1}$ boshlang‘ich nuqta orbitasining ω-limit nuqtalari to‘plamini tavsiflashdan iborat.

1-ta’rif. *Agar $V^n(\mathbf{x}) = \mathbf{x}$ tenglikni qanoatlantiruvchi natural n soni mavjud bo‘lsa, $\mathbf{x} \in S^{m-1}$ nuqtaga V operatorning davriy nuqtasi deyiladi. Yuqoridagi shartni qanoatlantiruvchi n sonlarining eng kichigiga $\mathbf{x}$ nuqtaning asosiy davri yoki eng kichik davri deyiladi. Davri birga teng bo‘lgan nuqta V operatorning qo‘zg‘almas nuqtasi deyiladi.*

Barcha qo‘zg‘almas nuqtalar to‘plamini $\mathrm{Fix}(V)$ bilan va barcha asosiy davri n $(n \geq 2)$ ga teng davriy nuqtalar to‘plamini $\mathrm{Per}_n(V)$ bilan belgilaymiz.

2-ta’rif. *Ixtiyoriy $\mathbf{x} \in S^{m-1}$ uchun $\lim\limits_{n\to\infty} V^n\left(\mathbf{x}\right)$ limit mavjud bo‘lsa, V KSO regulyar operator deyiladi.*

3-ta’rif. *Ixtiyoriy $\mathbf{x} \in S^{m-1}$ uchun*

$$
\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} V^k(\mathbf{x})
$$

limit mavjud bo‘lsa, V KSO ergodik operator deyiladi.

S^{m-1} simpleksning uchlarini $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \ldots, \delta_{mi}) \in S^{m-1},\ i \in E_m$ kabi belgilaymiz, bu yerda δ_{ij} Kroneker simvoli. Shuningdek, quyidagi to‘plamlarni aniqlaymiz

$$
\mathrm{int}S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1 x_2 \cdots x_m > 0\},
$$

$$
\partial S^{m-1} = S^{m-1} \setminus \mathrm{int}\, S^{m-1},
$$

$$
\Gamma_I = \{\mathbf{x} \in S^{m-1} : x_i = 0,\ \forall i \notin I\},\ \forall I \subset E.
$$

Ikki o‘lchamli simpleksda aniqlangan quyidagi $(\alpha, \beta)-$ kvadratik stoxastik operatorini qaraymiz

$$V:\begin{cases} x_1' = x_1^2 + (x_2+x_3)^2 + (1-\alpha)x_1x_{\pi(3)} + (1-\beta)x_1x_{\pi(2)}, \\ x_2' = (1+\beta)x_1x_{\pi(2)}, \\ x_3' = (1+\alpha)x_1x_{\pi(3)}, \end{cases} \tag{3}$$

bu yerda $\alpha,\beta \in [-1,1]$ va π esa $\{2,3\}$ to‘plamning biror o‘rin almashtirishi.

Berilgan $\alpha,\beta \in (0,1]$ sonlar uchun quyidagi nuqtalarni belgilab olamiz:

$$\mathbf{x}_\alpha = \left(\frac{1}{1+\alpha}, 0, \frac{\alpha}{1+\alpha}\right), \quad \mathbf{x}_\beta = \left(\frac{1}{1+\beta}, \frac{\beta}{1+\beta}, 0\right),$$

$$\mathbf{y}_{\alpha,\mathbf{x}^{(0)}} = \left(\frac{1}{1+\alpha}, \frac{\alpha x_2^{(0)}}{\left(1-x_1^{(0)}\right)(1+\alpha)}, \frac{\alpha x_3^{(0)}}{\left(1-x_1^{(0)}\right)(1+\alpha)}\right), \quad \forall \mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1\}.$$

1-teorema. *$\pi = Id$ bo‘lsin. (3) (α,β)-KSO uchun quyidagi tasdiqlar o‘rinli:*

i) agar $\alpha,\beta \in [-1,1]$ bo‘lsa, u holda ixtiyoriy $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}}$ uchun $V\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$;

ii) agar $\alpha,\beta \in [-1,0]$ bo‘lsa, u holda barcha $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{2,3\}}$ uchun $\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$;

iii) ixtiyoriy $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \setminus \{\mathbf{e}_1, \mathbf{e}_2\}$ uchun

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_1, & agar\ \beta \in [-1,0], \\ \mathbf{x}_\beta, & agar\ \beta \in (0,1]; \end{cases}$$

iv) ixtiyoriy $\mathbf{x}^{(0)} \in \Gamma_{\{1,3\}} \setminus \{\mathbf{e}_1, \mathbf{e}_3\}$ uchun

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_1, & agar\ \alpha \in [-1,0], \\ \mathbf{x}_\alpha, & agar\ \alpha \in (0,1]; \end{cases}$$

v) ixtiyoriy $\mathbf{x}^{(0)} \in \operatorname{int} S^2$ uchun

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{y}_{\alpha,\mathbf{x}^{(0)}}, & agar\ \alpha = \beta \in (0,1], \\ \mathbf{x}_\alpha, & agar\ \alpha \in (0,1],\ \beta \in [-1,0]\ yoki\ \alpha > \beta,\ \alpha,\beta \in (0,1], \\ \mathbf{x}_\beta, & agar\ \alpha \in [-1,0],\ \beta \in (0,1]\ yoki\ \alpha < \beta,\ \alpha,\beta \in (0,1]. \end{cases}$$

Ixtiyoriy $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \cup \Gamma_{\{1,3\}} \setminus \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ uchun quyidagi belgilashlarni kiritamiz

$$\mathbf{y}_{\alpha\beta} = \left(x_{\alpha\beta}, 0, 1-x_{\alpha\beta}\right) \text{ va } \mathbf{z}_{\alpha\beta} = \left(f_\beta(x_{\alpha\beta}), 1-f_\beta(x_{\alpha\beta}), 0\right),$$

bu yerda

$$x_{\alpha\beta} = \frac{\theta(\alpha,\beta)}{6\sqrt[3]{(1+\alpha)(1+\beta)^2}} + \frac{2\sqrt[3]{(1+\alpha)}(\beta-2)}{3\theta(\alpha,\beta)\sqrt[3]{(1+\beta)}} + \frac{1}{3},\ \alpha,\beta \in (-1,1] \text{ va}$$

$$\theta(\alpha,\beta) = \sqrt[3]{4\beta(1+\alpha)(2\beta-5) - 4(7\alpha-20) + 12\kappa(\alpha,\beta)},$$

$$\kappa(\alpha,\beta) = \sqrt{3(4-\alpha\beta)(2-\alpha)(2-\beta) + 9(\alpha-\beta)^2},$$

$$f_\beta(x) = (1+\beta)x^2 - (1+\beta)x + 1,\ x \in [0,1].$$

Har qanday $\mathbf{x}^{(0)} \in \operatorname{int} S^2 \setminus Fix(V)$ uchun quyidagi belgilashlarni kiritib olamiz

$$\hat{\mathbf{x}}_{\delta\gamma}=\left(\hat{x}_1,\hat{x}_2,\hat{x}_3\right),\ \hat{x}_1=x_{\delta\gamma},\ \hat{x}_2=\frac{x_2^{(0)}\left(1-x_{\delta\gamma}\right)}{x_2^{(0)}+x_3^{(0)}},\ \hat{x}_3=\frac{x_3^{(0)}\left(1-x_{\delta\gamma}\right)}{x_2^{(0)}+x_3^{(0)}},$$

bu yerda

$$x_{\delta\gamma}=\frac{\theta(\delta,\gamma)}{6\sqrt[3]{(1+\delta)(1+\gamma)^2}}+\frac{2\sqrt[3]{(1+\delta)}(\gamma-2)}{3\theta(\delta,\gamma)\sqrt[3]{(1+\gamma)}}+\frac{1}{3},$$

$$\theta(\delta,\gamma)=\sqrt[3]{4\gamma(1+\delta)(\gamma-5)-4(7\delta-20)+12\sqrt{\kappa(\delta,\gamma)}},$$

$$\kappa(\delta,\gamma)=3(4-\delta\gamma)(2-\delta)(2-\gamma)+9(\delta-\gamma)^2$$

$$\gamma=\frac{\alpha x_2^{(0)}+\beta x_3^{(0)}}{x_2^{(0)}+x_3^{(0)}},\ \delta=\frac{(1+\alpha)\beta x_2^{(0)}+(1+\beta)\alpha x_3^{(0)}}{(1+\alpha)x_2^{(0)}+(1+\beta)x_3^{(0)}}.$$

$$\tilde{\mathbf{x}}_{\delta\gamma}=\left(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3\right),\ \tilde{x}_1=1-\tilde{x}_2-\tilde{x}_3,\ \tilde{x}_2=(1+\beta)\hat{x}_1\hat{x}_3,\ \tilde{x}_3=(1+\alpha)\hat{x}_1\hat{x}_2.$$

Shuningdek, quyidagi belgilashni kiritib olamiz

$$\mathbf{x}^*=\left(\frac{1}{\sqrt{(1+\alpha)(1+\beta)}},\frac{\sqrt{1+\beta}}{\sqrt{1+\alpha}}x_3^*,x_3^*\right),\ \text{bu yerda}\ x_3^*=\frac{\sqrt{(1+\alpha)(1+\beta)}-1}{\sqrt{1+\beta}\left(\sqrt{1+\alpha}+\sqrt{1+\beta}\right)},\ \alpha,\beta\in(-1,1].$$

2-teorema. $\pi\neq Id$ *bo‘lsin. (3)* (α,β)*-KSO uchun quyidagi tasdiqlari o‘rinli:*

i) agar $\alpha,\beta\in[-1,1]$ *bo‘lsa, u holda har qanday* $\mathbf{x}^{(0)}\in\Gamma_{\{2,3\}}$ *uchun* $V\left(\mathbf{x}^{(0)}\right)=\mathbf{e}_1$*;*

ii) agar $\alpha=-1$ *yoki* $\beta=-1$ *bo‘lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^2\setminus\Gamma_{\{2,3\}}$ *uchun* $V^2\left(\mathbf{x}^{(0)}\right)=\mathbf{e}_1$;

iii) agar $\alpha+\beta+\alpha\beta>0$, $\alpha,\beta\in(-1,1]$ *bo‘lsa, u holda barcha* $\mathbf{x}^{(0)}\in\left(\Gamma_{\{1,2\}}\cup\Gamma_{\{1,3\}}\right)\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\begin{cases}\mathbf{y}_{\alpha\beta}, & agar\ n=2k,\\ \mathbf{z}_{\alpha\beta}, & agar\ n=2k+1,\end{cases}\quad k=0,1,2,...$$

iv) agar $\alpha+\beta+\alpha\beta>0$, $\alpha,\beta\in(-1,1]$ *bo‘lsa, u holda har qanday* $\mathbf{x}^{(0)}\in\operatorname{int}S^2\setminus\{\mathbf{x}^*\}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\begin{cases}\hat{\mathbf{x}}_{\delta\gamma}, & agar\ n=2k,\\ \tilde{\mathbf{x}}_{\delta\gamma}, & agar\ n=2k+1,\end{cases}\quad k=0,1,2,...$$

v) agar $\alpha+\beta+\alpha\beta\leq 0$, $\alpha,\beta\in(-1,1]$ *bo‘lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^2\setminus\left(\Gamma_{\{2,3\}}\cup\{\mathbf{e}_1\}\right)$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\mathbf{e}_1.$$

Dissertatsiyaning **“O‘rin almashtirishlarga mos novolterra kvadratik operatorlar”** nomli ikkinchi bobida, chekli o‘lchamli simpleksda aniqlangan o‘rin almashtirishlarga va mutatsiyalarga mos novolterra kvadratik stoxastik operatorlarning dinamik xossalari o‘rganilgan.

Ikkinchi bobning birinchi paragrafida chekli o‘lchamli simpleksda aniqlangan, o‘rin almashtirishga mos novolterra kvadratik stoxastik operatorlarining invariant to‘plamlari, davriy nuqtalari topilgan va ikkinchi bobning ikkinchi paragrafida shu operatorlarning limit nuqtalar to‘plami tavsiflangan.

Ikkinchi bobning birinchi paragrafida quyidagi operator qaralgan

$$V:\begin{cases} x_k^{'}=(1+\alpha)x_{\pi(k)}x_m, & k\in E_{m-1}, \\ x_m^{'}=x_m^2+\left(\sum_{i=1}^{m-1}x_i\right)^2+(1-\alpha)x_m\left(\sum_{i=1}^{m-1}x_i\right), \end{cases} \qquad (4)$$

bu yerda $\alpha\in[-1,1]$ va π esa E_{m-1} to‘plamning biror o‘rin almashtirishi.

Quyidagi belgilashlarni kiritib olamiz $s=lcm\left(ord(\tau_1),...,ord(\tau_q)\right)$,

$$\Delta_{\tau_l}=\left\{\mathbf{x}\in S^{m-1}: x_u=x_v,\, u,v\in\operatorname{supp}(\tau_l)\right\},\ \Omega_1=\{1,2,...,r\}\subset\{1,2,...,q\},$$

$\Omega_2=\{1,2,...,q\}\setminus\Omega_1$, $\sigma=lcm\left(ord(\tau_{i_1}),...,ord(\tau_{i_l})\right)$, $i_l\in\Omega_2$ va $\alpha>0$, $x_m^{(0)}\neq 1$ uchun

$$\overline{\mathbf{x}}:=\overline{\mathbf{x}}_{\alpha,\mathbf{x}^{(0)}}=\left(\frac{\alpha x_1^{(0)}}{(1+\alpha)\left(1-x_m^{(0)}\right)},...,\frac{\alpha x_{m-1}^{(0)}}{(1+\alpha)\left(1-x_m^{(0)}\right)},\frac{1}{(1+\alpha)}\right),$$

$$M=S^{m-1}\setminus\left(Fix(V)\cup\Gamma\right) \text{ va } N=\bigcap_{i=1}^{q}\Delta_{\tau_i}\setminus\left(Fix(V)\cup\Gamma\right).$$

Keyingi teoremada (4) KSOga mos orbitalarning limit nuqtalari to‘plami tavsiflangan.

3-teorema. *(4) KSO uchun quyidagi tasdiqlar o‘rinli:*

i) agar $\mathbf{x}^{(0)}\in\Gamma=\left\{\mathbf{x}\in S^{m-1}: x_m=0\right\}\cup\{\mathbf{e}_m\}$ *bo‘lsa, u holda* $\omega_V\left(\mathbf{x}^{(0)}\right)=\{\mathbf{e}_m\}$*;*

ii) agar $\alpha\in[-1,0]$ *bo‘lsa, u holda* $\forall\mathbf{x}^{(0)}\in S^{m-1}\setminus\Gamma$ *uchun* $\omega_V\left(\mathbf{x}^{(0)}\right)=\{\mathbf{e}_m\}$*;*

iii) agar $\alpha\in(0,1]$ *va* $\pi=\mathrm{Id}$ *bo‘lsa, u holda* $\forall\mathbf{x}^{(0)}\in M$ *uchun* $\omega_V\left(\mathbf{x}^{(0)}\right)=\{\overline{\mathbf{x}}_M\}$*;*

iv) agar $\alpha\in(0,1]$ *va* $\pi\neq\mathrm{Id}$ *bo‘lsa, u holda* $\forall\mathbf{x}^{(0)}\in N$ *uchun* $\omega_V\left(\mathbf{x}^{(0)}\right)=\{\overline{\mathbf{x}}_N\}$*;*

v) agar $\alpha\in(0,1]$ *va* $\pi\neq\mathrm{Id}$ *bo‘lsa, u holda* $\forall\mathbf{x}^{(0)}\in\bigcap_{i\in\Omega_1}\Delta_{\tau_i}\setminus\left(Fix(V)\cup\Gamma\cup\bigcup_{j\in\Omega_2}\Delta_{\tau_j}\right)$ *uchun* $\overline{\mathbf{x}}=\lim_{n\to\infty}V^{\sigma n}\left(\mathbf{x}^{(0)}\right)$ *limit mavjud va* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\overline{\mathbf{x}},\,\overline{\mathbf{x}}^{(1)},...,\overline{\mathbf{x}}^{(\sigma-1)}\right\}$*;*

vi) agar $\alpha\in(0,1]$ *va* $\pi\neq\mathrm{Id}$ *bo‘lsa, u holda* $\forall\mathbf{x}^{(0)}\in S^{m-1}\setminus\left(Fix(V)\cup\Gamma\cup\bigcup_{i=1}^{q}\Delta_{\tau_i}\right)$ *uchun* $\overline{\mathbf{x}}=\lim_{n\to\infty}V^{sn}\left(\mathbf{x}^{(0)}\right)$ *limit mavjud va* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\overline{\mathbf{x}},\,\overline{\mathbf{x}}^{(1)},...,\overline{\mathbf{x}}^{(s-1)}\right\}$.

Ikkinchi bobning uchinchi paragrafida chekli o‘lchamli simpleksda aniqlangan mutatsiyalarga mos novolterra kvadratik stoxastik operatorlari qurilgan va ularning invariant to‘plamlari, davriy nuqtalari topilgan. Shuningdek, ikkinchi bobning to‘rtinchi paragrafida shu operatorning limit nuqtalar to‘plami tavsiflangan.

Ikkinchi bobning uchinchi paragrafida quyidagi operator qaralgan

$$V:\begin{cases} x_k^{'} = \alpha x_{\pi(k)}^2 + \dfrac{2\alpha}{m-1}\sum\limits_{\substack{i,l=1:\\ i<l}}^{m-1} x_i x_l + (1+\alpha) x_{\pi(k)} x_m,\ k\in E_{m-1}, \\ x_m^{'} = x_m^2 + (1-\alpha)\left(\sum\limits_{i=1}^{m-1} x_i\right)^2 + (1-\alpha) x_m \left(\sum\limits_{i=1}^{m-1} x_i\right), \end{cases} \tag{5}$$

bu yerda $\alpha\in[0,1]$ va π esa E_{m-1} to‘plamning biror o‘rin almashtirishi.

Shuni ta’kidlash kerakki, agar $\alpha=0$ bo‘lsa, (5) operator (4) operator bilan bir xil bo‘ladi. Shuning uchun quyida $\alpha\in(0,1]$ hol tadqiq qilinadi.

$\alpha>0$ son uchun quyidagi belgilashlarni kiritamiz:

$$\mathbf{y}_\alpha=(\alpha,0,1-\alpha),\ z_\alpha=(0,\alpha,1-\alpha),\ \mathbf{x}_{\alpha,m}=\left(\frac{\alpha}{m-1},\dots,\frac{\alpha}{m-1},1-\alpha\right) \text{ va}$$

$$\hat{\mathbf{x}}=\left(\frac{\alpha x_1^{(0)}}{x_1^{(0)}+x_2^{(0)}},\frac{\alpha x_2^{(0)}}{x_1^{(0)}+x_2^{(0)}},1-\alpha\right),\quad \tilde{\mathbf{x}}=\left(\frac{\alpha x_2^{(0)}}{x_1^{(0)}+x_2^{(0)}},\frac{\alpha x_1^{(0)}}{x_1^{(0)}+x_2^{(0)}},1-\alpha\right).$$

Keyingi teoremada $m=3$ uchun (5) KSOga mos orbitalarning limit nuqtalar to‘plami tavsiflangan.

4-teorema. *$m=3$ bo‘lsin. (5) KSO uchun quyidagi tasdiqlar o‘rinli:*

i) agar $\pi=Id$ bo‘lsa, u holda

$$\omega_V\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_3\}, & agar\ \mathbf{x}^{(0)}=\mathbf{e}_3,\\ \{\mathbf{y}_\alpha\}, & agar\ \mathbf{x}^{(0)}\in\Gamma_{\{1,3\}}\setminus\{\mathbf{e}_3\},\\ \{\mathbf{z}_\alpha\}, & agar\ \mathbf{x}^{(0)}\in\Gamma_{\{2,3\}}\setminus\{\mathbf{e}_3\},\\ \{\hat{\mathbf{x}}\}, & agar\ \mathbf{x}^{(0)}\in S^2\setminus\left(\Gamma_{\{1,3\}}\cup\Gamma_{\{2,3\}}\right);\end{cases}$$

ii) agar $\pi\neq Id$ bo‘lsa, u holda

$$\omega_V\left(\mathbf{x}^{(0)}\right)=\begin{cases}\{\mathbf{e}_3\}, & agar\ \mathbf{x}^{(0)}=\mathbf{e}_3,\\ \{\mathbf{x}_{\alpha,3}\}, & agar\ \mathbf{x}^{(0)}\in M_{=}^{(12)}\setminus\{\mathbf{e}_3\},\\ \{\mathbf{y}_\alpha,\mathbf{z}_\alpha\}, & agar\ \mathbf{x}^{(0)}\in\left(\Gamma_{\{1,3\}}\cup\Gamma_{\{2,3\}}\right)\setminus\{\mathbf{e}_3\},\\ \{\tilde{\mathbf{x}},\hat{\mathbf{x}}\}, & agar\ \mathbf{x}^{(0)}\in S^2\setminus\left(\Gamma_{\{1,3\}}\cup\Gamma_{\{2,3\}}\cup M_{=}^{(12)}\right).\end{cases}$$

$\alpha>0$ uchun quyidagi belgilashlarni kiritib olamiz $\mathbf{x}_\xi=(\xi_1,\xi_2,\dots,\xi_{m-1},1-\alpha)$, bu yerda $\xi_u=\alpha\delta_{k,u}$, $k,u\in\operatorname{supp}(\tau_i)$ va $C=C_1\cup C_2$, bu yerda

$$C_1=\bigcup_{u\in E_{m-1}\setminus\operatorname{supp}(\pi)}\Gamma_{\{u,m\}}\ \ va\ C_2=\bigcup_{u\in\operatorname{supp}(\pi)}\Gamma_{\{u,m\}}.$$

$$\hat{B}_\alpha=\left\{\mathbf{x}_\ell\in S^{m-1}:\mathbf{x}_\ell=\left(\alpha\delta_{1,\ell},\dots,\alpha\delta_{m-1,\ell},1-\alpha\right),\ \ell\in\operatorname{supp}(\pi)\right\},$$

$$\tilde{B}_\alpha=\left\{\mathbf{x}_\ell\in S^{m-1}:\mathbf{x}_\ell=\left(\alpha\delta_{1,\ell},\dots,\alpha\delta_{m-1,\ell},1-\alpha\right),\ \ell\in E_{m-1}\setminus\operatorname{supp}(\pi)\right\},$$

bu yerda $\delta_{i,j}$ Kroneker deltasi, $B_\alpha=\tilde{B}_\alpha\cup\hat{B}_\alpha$.

Keyingi teoremada $m>3$ hol uchun (5) KSOga mos orbitalarning limit nuqtalari to‘plami tavsiflangan.

5-teorema. $m>3$ *bo‘lsin.* (5) *KSO uchun quyidagi tasdiqlar o‘rinli:*

i) har qanday π *va* $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus C$ *uchun* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\mathbf{x}_{\alpha,m}\right\}$;

ii) agar $\pi=\mathrm{Id}$ *bo‘lsa, u holda* $\forall \mathbf{x}^{(0)} \in C \setminus Fix(V)$ *uchun* $\exists\, u \in E_{m-1}$ *topilib,* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *va* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\mathbf{x}_u\right\}$*;*

iii) agar $\pi \neq \mathrm{Id}$ *bo‘lsa, u holda* $\forall \mathbf{x}^{(0)} \in C_1 \setminus Fix(V)$ *uchun* $\exists\, u \in E_{m-1} \setminus \operatorname{supp}(\pi)$ *topilib,* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *va* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\mathbf{x}_u\right\}$*;*

iv) agar $\pi \neq \mathrm{Id}$ *bo‘lsa, u holda* $\forall \mathbf{x}^{(0)} \in C_2 \setminus Fix(V)$ *uchun* $\exists\, u \in \operatorname{supp}(\tau_i)$ *topilib,* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *o‘rinli bo‘ladi. Bundan tashqari* $\mathbf{x}_\xi=\lim_{n\to\infty} V^{nt_i}\left(\mathbf{x}^{(0)}\right)$, $t_i=ord\left(\tau_i\right)$ *limit mavjud va* $\omega_V\left(\mathbf{x}^{(0)}\right)=\left\{\mathbf{x}_\xi, \mathbf{x}_\xi^{(1)},...,\mathbf{x}_\xi^{(t_i-1)}\right\}$.

Dissertatsiyaning **“Qayta yuquvchi diskret vaqtli SIRD epidemik modeli dinamikasi”** nomli uchinchi bobida, qayta yuquvchi diskret vaqtli SIRD (Susceptible-Infected-Recovered-Died) epidemik modelining dinamik xossalari tadqiq etilgan. Bobning birinchi paragrafida epidemik modellar haqida dastlabki ma’lumotlar berilgan, klassik SIRD epidemik modelini o‘zgartirishdan hosil bo‘lgan qayta yuquvchi SIRD epidemik modelining o‘zgaruvchilari va parametrlari tavsiflangan.

Uchinchi bobning ikkinchi paragrafida quyidagi qayta yuquvchi SIRD

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI + \mu R, \\ \dfrac{dI}{dt} = \beta SI - (\gamma_1+\gamma_2)I, \\ \dfrac{dR}{dt} = \gamma_1 I - \mu R, \\ \dfrac{dD}{dt} = \gamma_2 I, \end{cases} \tag{6}$$

epidemik modelining diskret muqobili bo‘lgan (7) kvadratik operator qaralgan

$$V: \begin{cases} x_1' = x_1 + \mu x_3 - \beta x_1 x_2, \\ x_2' = (1-\gamma_1-\gamma_2)x_2 + \beta x_1 x_2, \\ x_3' = (1-\mu)x_3 + \gamma_1 x_2, \\ x_4' = x_4 + \gamma_2 x_2. \end{cases} \tag{7}$$

Yuqoridagi (7) kvadratik operator simpleksni invariant saqlashi uchun operator parametrlariga zarur va yetarli shartlar topilgan. Bundan tashqari operatorning invariant to‘plamlari, qo‘zg‘almas nuqtalari to‘plami tavsiflangan va har bir qo‘zg‘almas nuqtaning tiplari aniqlangan.

1-tasdiq. *(7) operator* S^3 *simpleksni o'zini o'ziga akslantirishi uchun uning parametrlari*

$$\mu,\ \gamma_1,\ \gamma_2,\ \gamma_1+\gamma_2\in[0,1],\ \ \gamma_1+\gamma_2-1\le\beta\le\left(1+\sqrt{\gamma_1+\gamma_2}\right)^2 \qquad (8)$$

shartlarni qanoatlantirishi zarur va yetarli.

Bobning oxirgi paragrafida (7) kvadratik operatorga mos orbitalarning limit nuqtalari to'plami tavsiflangan.

6-teorema. *(7) operator uchun quyidagi tasdiqlar o'rinli:*

i) agar $\mathbf{x}^{(0)}\in\Gamma_{\{1,3,4\}}\setminus Fix(V)$ *bo'lsa, u holda* $\lim\limits_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right)$*;*

ii) agar $\beta=\gamma_1=\gamma_2=0,\ \mu>0$ *bo'lsa, u holda barcha* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,2,4\}}\cup\Gamma_{\{1,3,4\}}\right)$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_2^{(0)}-x_4^{(0)},x_2^{(0)},0,x_4^{(0)}\right);$$

iii) agar $\beta=\gamma_1=\mu=0,\ \gamma_2>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,x_3^{(0)},1-x_1^{(0)}-x_3^{(0)}\right);$$

iv) agar $\beta=\gamma_2=\mu=0,\ \gamma_1>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,1-x_1^{(0)}-x_4^{(0)},x_4^{(0)}\right);$$

v) agar $\gamma_1=\gamma_2=\mu=0,\ \beta>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,3,4\}}\right)$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(0,1-x_3^{(0)}-x_4^{(0)},x_3^{(0)},x_4^{(0)}\right)\ ;$$

vi) agar $\gamma_1=\gamma_2=\mu=0,\ \beta<0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,3,4\}}\right)$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_3^{(0)}-x_4^{(0)},0,x_3^{(0)},x_4^{(0)}\right);$$

vii) agar $\beta=\gamma_1=0,\ \gamma_2\mu>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,0,1-x_1^*\right);$$

viii) agar $\beta=\gamma_2=0,\ \gamma_1\mu>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right);$$

ix) agar $\beta=\mu=0,\ \gamma_1\gamma_2>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,x_3^*,1-x_1^{(0)}-x_3^*\right);$$

x) agar $\gamma_1=\gamma_2=0,\ \beta\mu>0$ *bo'lsa, u holda* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,4\}}\right)$ *uchun*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(0,1-x_4^{(0)},0,x_4^{(0)}\right);$$

xi) agar $\gamma_1=\gamma_2=0,\ \beta\mu<0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,4\}}\right)$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right);$$

xii) agar $\gamma_1=\mu=0,\ \beta\gamma_2\neq 0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,x_3^{(0)},1-x_1^*-x_3^{(0)}\right);$$

xiii) agar $\gamma_2=\mu=0,\ \beta\gamma_1\neq 0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,1-x_1^*-x_4^{(0)},x_4^{(0)}\right);$$

xiv) agar $\beta=0,\ \gamma_1\gamma_2\mu>0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,0,1-x_1^*\right);$$

xv) agar $\gamma_1=0,\ \beta\gamma_2\mu\neq 0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^*,0,0,x_4^*\right);$$

xvi) agar $\gamma_2=0,\ \beta\gamma_1\mu\neq 0,\ \beta\leq\gamma_1$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right);$$

xvii) agar $\mu=0,\ \gamma_1\gamma_2>0,\ \beta\neq 0$ *bo'lsa, u holda har qanday* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,x_3^*,1-x_1^*-x_3^*\right);$$

xviii) agar $\beta\gamma_1\gamma_2\mu\neq 0$ *bo'lsa, u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *uchun*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^*,0,0,x_4^*\right).$$

1-gipoteza. *Agar* $\gamma_2=0,\ \beta>\gamma_1>0,\ \mu>0$ *bo'lsa u holda ixtiyoriy* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *boshlang'ich nuqta uchun*

$$\lim_{n\to\infty}\mathbf{x}^{(n)}=\begin{cases}\tilde{\mathbf{y}}, & agar\quad 0\leq x_4^{(0)}<\dfrac{\beta-\gamma_1}{\beta},\\[2ex] \mathbf{x}^*\in\Gamma_{\{1,4\}}, & agar\quad \dfrac{\beta-\gamma_1}{\beta}\leq x_4^{(0)}<1,\end{cases}$$

bu yerda $\tilde{\mathbf{y}}=\left(\dfrac{\gamma_1}{\beta},\dfrac{\mu\left(\beta-\gamma_1-\beta x_4^{(0)}\right)}{\beta\left(\mu+\gamma_1\right)},\dfrac{\gamma_1\left(\beta-\gamma_1-\beta x_4^{(0)}\right)}{\beta\left(\mu+\gamma_1\right)},x_4^{(0)}\right),\ 0\leq x_4^{(0)}\leq\dfrac{\beta-\gamma_1}{\beta}.$

XULOSA

Ushbu dissertatsiya tadqiqoti o‘rin almashtirishlarga mos novolterra kvadratik stoxastik operatorlari va qayta yuquvchi diskret vaqtli SIRD epidemik modeli tomonidan hosil qilgan nochiziqli dinamik sistemalarni tadqiq etishga bag‘ishlangan.

Tadqiqot ishining asosiy natijalari quyidagilardan iborat:

1. Ikki o‘lchamli simpleksda aniqlangan (α,β)-KSO uchun ixtiyoriy boshlang‘ich nuqtaning orbitasi operatorning qo‘zg‘almas nuqtasiga yoki davri ikkiga teng bo‘lgan davriy orbitasiga yaqinlashishi isbotlangan;
2. Chekli o‘lchamli simpleksda aniqlangan o‘rin almashtirishlarga mos novolterra KSOning barcha invariant to‘plamlari, davriy nuqtalari topilgan. Shu bilan birga ixtiyoriy boshlang‘ich nuqta orbitasi operatorning qo‘zg‘almas nuqtasiga yoki davriy orbitasiga yaqinlashishi ko‘rsatilgan;
3. Chekli o‘lchamli simpleksda aniqlangan mutatsiyalarga mos novolterra KSOning invariant to‘plamlari, davriy nuqtalar to‘plami topilgan va ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalari to‘plami chekli bo‘lishi isbotlangan;
4. Qayta yuquvchi SIRD epidemik modelining diskret muqobili bo‘lgan kvadratik operator uch o‘lchamli simpleksni invariant saqlashi uchun operator parametrlariga zarur va yetarli shartlar topilgan. Operatorga mos Lyapunov funksiyasi qurilgan. Bundan tashqari, ixtiyoriy boshlang‘ich nuqtaning orbitasi operatorning qo‘zg‘almas nuqtalaridan biriga yaqinlashishi ko‘rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

KHUDOYBERDIEV KHAYOTJON OCHILTOSH OGLI

DYNAMICS OF NON-VOLTERRA QUADRATIC OPERATORS: PERMUTED AND EPIDEMIC MODELS

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the of Ministers of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2024.4.PhD/FM1176.

Thesis has been prepared at Institute of Mathematics.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://kengash.mathinst.uz and in the website of "ZiyoNet" Information and educational portal http://www.ziyonet.uz/.

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Defense will take place on "01" July 2025 at 16:00 at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I. Romanovskiy. (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

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Abstract of the thesis sent out on "11" June 2025 year
(Mailing report № 2 on "11" June 2025 year)

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INTRODUCTION

Actuality and demand of the theme of the dissertation. Many scientific and applied research studies conducted worldwide often involve examining the properties of nonlinear dynamical systems. These properties are widely used in forecasting problems in fields such as mathematical biology, genetics, and medicine. Dynamical systems are classified into two types: continuous-time dynamical systems and discrete-time dynamical systems. In mathematical biology, a primary focus of research is on dynamical systems generated by operators that describe the evolution of biological populations. Consequently, the study of the dynamics of quadratic stochastic operators, which are an important class of evolutionary operators, remains a topical problem in the theory of nonlinear dynamical systems.

Currently, the dynamical properties of Volterra quadratic stochastic operators have been extensively studied. However, a key unsolved problem in the theory of nonlinear dynamical systems the investigation of the asymptotic behavior of orbits generated by non-Volterra quadratic stochastic operators. Fundamental to these studies are problems such as identifying invariant sets of nonlinear dynamical systems generated by quadratic stochastic operators, determining the exact forms and types of periodic points, constructing Lyapunov functions, and analyzing the asymptotic behavior of orbits. It should be noted that these problems have not yet been fully resolved in the general theory of nonlinear dynamical systems. Therefore, one of the key directions of scientific research is the study of the dynamical properties of non-Volterra quadratic stochastic operators associated with permutations and epidemic models.

In recent years, in our country, increased attention has been given to medicine, biology, mathematics and physics, fields that represent the scientific and practical application of fundamental sciences. In particular, special focus has been placed on developing the theory of nonlinear dynamical systems, which has broad applications in biology, medicine, and economics, leading to significant achievements in this area. Conducting scientific research at an international standard in the priority fields of “Algebra, functional analysis and theory of dynamical systems” has been established as a key objective in mathematics[1]. To ensure the implementation of this decision, it is important to develop the theory of discrete-time dynamical systems generated by quadratic stochastic operators, enabling the application of scientific results to related fields.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development Of the Republic of Uzbekistan”, UP-60 dated January 28, 2022 “Development Strategy of New Uzbekistan for the period of 2022-2026”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, UP-2789 dated April

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” No. 292.

20, 2017 “On measures to further develop the system of higher education”, PP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I. Romanovskiy Institute of Mathematics”, and PP-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. Various mathematical models, including methods from dynamical systems theory, are widely used to understand and analyze natural phenomena and processes. Consequently, researchers’ interest in discrete-time dynamical systems is increasing. Quadratic stochastic operators were originally introduced to solve problems related to the laws of heredity in mathematical biology. The theory of discrete-time dynamical systems generated by such operators has been used to model various complex processes in the fields of biology, ecology, medicine, economics, and information technology. Quadratic stochastic operators were first introduced in 1924 by S. Bernstein as mathematical models to represent the evolution of gene frequencies in biology and population genetics. Since these operators also have applications in fields such as physics, chemistry, and economics, their dynamical properties have been studied with great interest. The theory of quadratic operators was developed by S. Ulam, H. Kesten, Yu.I. Lyubich, S.S. Vallander, R. Jenks, E.Akin, V. Losert, and T.A. Sarimsakov. In recent years, research in this area has been actively continued by M.I. Zakharevich, R.N. Ganikhodzhaev, N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, U.U. Jamilov, and O.N. Khakimov.

It is also worth noting that, in general, studying the asymptotical behaviour of a quadratic stochastic operator is a main problem. Yu.I. Lyubich completely solved this problem for quadratic operators defined on the one-dimensional simplex. However, for quadratic operators defined on the two-dimensional and higher-dimensional simplexes, the main problem remains unsolved. The main problem of Volterra quadratic stochastic operators is being extensively studied by R.N. Ganikhodzhaev and his students. Nowadays, N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, A. Zada, M. Ladra, U.U. Jamilov, O.N. Khakimov, A.Yu. Khamraev, S.K. Shoyimardonov, S.S. Xudayarov are conducting extensive scientific research on the dynamics of non-Volterra quadratic stochastic operators.

In the works of R.N. Ganikhodzhaev and D.B. Eshmamatova, quadratic homeomorphisms defined on a finite-dimensional simplex were described, and their exact form was provided. Scientific research on Volterra quadratic stochastic operator and their applications is being conducted by them. Discrete-time dynamical systems generated by certain ocean ecosystem models have been studied of U.A. Rozikov and S.K. Shoyimardonov. The dynamics of worm propagation using quadratic stochastic operators were examined by F.T. Adilova, U.U. Jamilov, and A. Reinfelds. As noted above, despite the extensive scientific research on discrete-time dynamical systems generated by quadratic stochastic operators, the main problem remains unresolved. In this context, studying the dynamics of the non-Volterra quadratic stochastic operators examined in this dissertation is important.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research OT-F4-87 "The theory of global invariants of curves and surfaces in Euclidean and pseudo-Euclidean spaces and its applications in mechanics" (2017-2020), OT-F4-82 "Local derivations and automorphisms of operator and nonassociative algebras, phase transitions and chaos in nonlinear dynamical systems" (2017-2020) and "Structural theory of non-associative algebras and its application in the study of dynamical systems in biological systems" (2020-2023) at the V.I. Romanovskiy Institute of Mathematics.

The aim of research work is to investigate the dynamical properties of the non-Volterra quadratic stochastic operators corresponding to permutations (operator I) and to discrete-time SIRD reinfection epidemic model (operator II).

Research problems:

identifying invariant sets with respect to operators I and II;

find periodic points of the operators I and II and their types;

construct Lyapunov functions for the operators I and II;

describe the set of limit sets of orbits of arbitrary initial points for the operators I and II.

The research object: non-Volterra quadratic stochastic operators defined on a finite-dimensional simplex.

The research subject: The theory of quadratic stochastic operators and the theory of discrete-time dynamical systems.

Research methods: The research employs methods from mathematical analysis, functional analysis, algebra, theory of probability and theory of dynamical systems.

Scientific novelty of the research work consists of the following:

the set of periodic points of the non-Volterra quadratic stochastic operator defined on the two-dimensional simplex is described. Furthermore, it is proved that the orbit of any initial point has at most two limit points;

the set of periodic points of the operator I is described. Besides, it is shown that the set of limit points of the orbit of any initial point consists either of a single point or finite points;

necessary and sufficient conditions on the parameters of the operator II for preserving the simplex as an invariant set have been established;

a Lyapunov function is found for the operator II, and using it, the set of limit points of the orbit of an arbitrary initial point has been described.

Practical results of the research. The novel results and methodologies developed in this dissertation are applicable to the theory of discrete-time dynamical systems. Furthermore, these results can also be used as a resource in special courses for graduate students and basic doctoral students of higher education institutions.

The reliability of the results of the study. New results were obtained using methods of mathematical analysis and functional analysis, theory of Lyapunov functions, and the theory of discrete-time dynamical systems. Additionally, every result obtained in the study is supported by rigorous mathematical reasoning.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that they can be used in the theory of quadratic stochastic operators and in solving problems in the fields of mathematical biology and epidemiology.

The practical significance of the research results is explained by the fact that the description of the set of limit points of orbits for operator I provides information about the evolution of the population. The asymptotical behavior of orbits for operator II allows one to describe scenarios of disease spread in epidemiology.

Implementation of the research results. The results obtained on the dynamics of operators I and II have been applied in the following areas:

the description of the set of limit points of orbits for a discrete-time SIRD reinfection epidemic model has been used in the research project "Quantum Genetic Algebras and Their Applications" with the reference number G0003447 for analyzing the regularity property of nonlinear stochastic operators (reference of United Arab Emirates University dated May 1, 2025, UAE). The application of the scientific result made it possible to establish the convergence of orbits for quadratic stochastic operators, which is crucial for understanding the future of biological systems;

for the quadratic stochastic operators corresponding to permutations, the set of periodic points and the set of limit points have been used in the research project "Homologies, homotopies, and categorical invariants in non-associative groups and algebras", with reference number PID2020-115155GB-I00 for analyzing the equilibrium state for the evolution algebras (Reference of University of Santiago de Compostela dated May 6, 2025, Spain). The application of this scientific result made it possible to describe the idempotent and absolutely nilpotent elements within evolution algebras.

Approbation of the research results. The main results of the research have been discussed at 4 international and 5 national scientific conferences.

Publications of the research results. On the topic of the dissertation 15 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 5 of them were published in international journals and 1 paper published in a national journal.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 103 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** provides a comprehensive analysis of the relevance and necessity of the dissertation topic, highlighting its alignment with the priority areas of scientific and technological development in the republic. It presents the research problem, outlines the purpose and objectives, and defines the object and subject of the study. Additionally, it describes the scientific novelty and practical significance of the findings, emphasizing both their theoretical and practical implications. The introduction also includes detailed information on the practical implementation of the research results, published works, and the structure of the dissertation.

In the firs chapter of the thesis, titled "**Discrete-time dynamical systems of quadratic stochastic operators**", the fundamental definitions and consepts of the theory of quadratic stochastic operators are introduced. Additionally, the dynamics of the quadratic stochastic operator, defined on the two-deminsional simplex, are examined.

Let $E_m=\{1,\dots,m\}$ be a finite set. The set of all probability distributions on E_m is then given by

$$S^{m-1}=\left\{\mathbf{x}\in R_+^m:\ \sum_{i=1}^m x_i=1\right\}$$

be the $(m-1)$- dimensional simplex.

A map V of S^{m-1} into itself is called a quadratic stochastic operator (QSO) if

$$(V\mathbf{x})_k=\sum_{i,j=1}^m p_{ij,k}x_ix_j, \tag{1}$$

for any $\mathbf{x}\in S^{m-1}$ and for all $k\in E_m$, where

$$\begin{aligned} &p_{ij,k}=p_{ji,k}\geq 0,\ \text{for all } i,j,k\in E_m;\\ &\sum_{k=1}^m p_{ij,k}=1,\qquad \text{for all } i,j\in E_m \end{aligned} \tag{2}$$

Assume that $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$ is the orbit (trajectories) of the any initial point $\mathbf{x}^{(0)}\in S^{m-1}$, where $\mathbf{x}^{(n+1)}=V\left(\mathbf{x}^{(n)}\right)=V^{n+1}\left(\mathbf{x}^{(0)}\right)$ for all $n=0,1,2,\dots$ We denote by $\omega_V\left(\mathbf{x}^{(0)}\right)$ the set of all ω- limit points of the orbit $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$. One of the main problems in dynamical system consists in the study of the asymptotical behavior of these orbits. In other words, for a given V QSO, the problem consists in describing the set of ω- limit points of the orbit generated by any initial point $\mathbf{x}^{(0)} \in S^{m-1}$.

Definition 1. *A point $\mathbf{x} \in S^{m-1}$ is called a periodic point of V if there exists an n so that $V^n(\mathbf{x}) = \mathbf{x}$. The smallest positive integer n satisfying the above is called the prime period or least period of the point $\mathbf{x}$. A period-one point is called a fixed point of V.*

Denote the set of all fixed points by $\mathrm{Fix}(V)$ and the set of all periodic points of prime period n $(n \geq 2)$ by $\mathrm{Per}_n(V)$.

Definition 2. *A QSO V is called regular if for any initial point $\mathbf{x} \in S^{m-1}$, the limit $\lim_{n\to\infty} V^n(\mathbf{x})$ exists.*

Definition 3. *A QSO V is said to be ergodic if the limit*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} V^k(\mathbf{x})$$

exists for any $\mathbf{x} \in S^{m-1}$.

Let $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, ..., \delta_{mi}) \in S^{m-1}$, $i \in E_m$ denote the vertices of the simplex S^{m-1}, where δ_{ij} is the Kronecker symbol. We now define the following sets

$$\mathrm{int} S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1 x_2 \cdots x_m > 0\};$$

$$\partial S^{m-1} = S^{m-1} \setminus \mathrm{int}\, S^{m-1};$$

$$\Gamma_I = \{\mathbf{x} \in S^{m-1} : x_i = 0,\ \forall i \notin I\},\ \forall I \subset E.$$

Consider the $(\alpha, \beta)-$ quadratic stochastic operator, defined on the two-dimensional simplex, which is given by

$$V: \begin{cases} x_1' = x_1^2 + (x_2 + x_3)^2 + (1-\alpha) x_1 x_{\pi(3)} + (1-\beta) x_1 x_{\pi(2)}, \\ x_2' = (1+\beta) x_1 x_{\pi(2)}, \\ x_3' = (1+\alpha) x_1 x_{\pi(3)}, \end{cases} \tag{3}$$

where $\alpha, \beta \in [-1,1]$ and π is a permutation on the set $\{2,3\}$.

For the given numbers $\alpha, \beta \in (0,1]$, we denote the following points

$$\mathbf{x}_\alpha = \left(\frac{1}{1+\alpha}, 0, \frac{\alpha}{1+\alpha}\right),\quad \mathbf{x}_\beta = \left(\frac{1}{1+\beta}, \frac{\beta}{1+\beta}, 0\right),$$

$$\mathbf{y}_{\alpha,\mathbf{x}^{(0)}} = \left(\frac{1}{1+\alpha}, \frac{\alpha x_2^{(0)}}{\left(1-x_1^{(0)}\right)(1+\alpha)}, \frac{\alpha x_3^{(0)}}{\left(1-x_1^{(0)}\right)(1+\alpha)}\right),\ \forall \mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1\}.$$

Theorem 1. *Let $\pi = Id$. Then for the $(\alpha,\beta)-$QSO (3) the following statements are hold:*

i) if $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}}$ then for all $\alpha, \beta \in [-1,1]$ it holds $V\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$;

ii) if $\alpha, \beta \in [-1, 0]$ then for all initial point $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{2,3\}}$ we have $\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$;

iii) if $\mathbf{x}^{(0)} \in \Gamma_{\{1,2\}} \setminus \{\mathbf{e}_1, \mathbf{e}_2\}$ then we have

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\begin{cases}\mathbf{e}_1, & if\ \ \beta\in[-1,0],\\ \mathbf{x}_\beta, & if\ \ \beta\in(0,1];\end{cases}$$

iv) if $\mathbf{x}^{(0)}\in\Gamma_{\{1,3\}}\setminus\{\mathbf{e}_1,\mathbf{e}_3\}$ *then we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\begin{cases}\mathbf{e}_1, & if\ \ \alpha\in[-1,0],\\ \mathbf{x}_\alpha, & if\ \ \alpha\in(0,1];\end{cases}$$

v) if $\mathbf{x}^{(0)}\in\operatorname{int}S^2$ *then we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right)=\begin{cases}\mathbf{y}_{\alpha,\mathbf{x}^{(0)}}, & if\ \ \alpha=\beta\in(0,1],\\ \mathbf{x}_\alpha, & if\ \ \alpha\in(0,1],\ \beta\in[-1,0]\ \ or\ \alpha>\beta,\quad \alpha,\beta\in(0,1],\\ \mathbf{x}_\beta, & if\ \ \alpha\in[-1,0],\ \beta\in(0,1]\ \ or\ \alpha<\beta,\quad \alpha,\beta\in(0,1].\end{cases}$$

For any point $\mathbf{x}^{(0)}\in\Gamma_{\{1,2\}}\cup\Gamma_{\{1,3\}}\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ we denote

$$\mathbf{y}_{\alpha\beta}=\left(x_{\alpha\beta},0,1-x_{\alpha\beta}\right)\ \text{and}\ \mathbf{z}_{\alpha\beta}=\left(f_\beta(x_{\alpha\beta}),1-f_\beta(x_{\alpha\beta}),0\right),$$

where

$$x_{\alpha\beta}=\frac{\theta(\alpha,\beta)}{6\sqrt[3]{(1+\alpha)(1+\beta)^2}}+\frac{2\sqrt[3]{(1+\alpha)}(\beta-2)}{3\theta(\alpha,\beta)\sqrt[3]{(1+\beta)}}+\frac{1}{3},\ \alpha,\beta\in(-1,1],$$

$$\theta(\alpha,\beta)=\sqrt[3]{4\beta(1+\alpha)(2\beta-5)-4(7\alpha-20)+12\kappa(\alpha,\beta)},$$

$$\kappa(\alpha,\beta)=\sqrt{3(4-\alpha\beta)(2-\alpha)(2-\beta)+9(\alpha-\beta)^2}\ \text{and}$$

$$f_\beta(x)=(1+\beta)x^2-(1+\beta)x+1,\ x\in[0,1].$$

For any point $\mathbf{x}^{(0)}\in\operatorname{int}S^2\setminus Fix(V)$ we denote

$$\hat{\mathbf{x}}_{\delta\gamma}=\left(\hat{x}_1,\hat{x}_2,\hat{x}_3\right),\ \hat{x}_1=x_{\delta\gamma},\ \hat{x}_2=\frac{x_2^{(0)}\left(1-x_{\delta\gamma}\right)}{x_2^{(0)}+x_3^{(0)}},\ \hat{x}_3=\frac{x_3^{(0)}\left(1-x_{\delta\gamma}\right)}{x_2^{(0)}+x_3^{(0)}},$$

where

$$x_{\delta\gamma}=\frac{\theta(\delta,\gamma)}{6\sqrt[3]{(1+\delta)(1+\gamma)^2}}+\frac{2\sqrt[3]{(1+\delta)}(\gamma-2)}{3\theta(\delta,\gamma)\sqrt[3]{(1+\gamma)}}+\frac{1}{3},$$

$$\theta(\delta,\gamma)=\sqrt[3]{4\gamma(1+\delta)(\gamma-5)-4(7\delta-20)+12\sqrt{\kappa(\delta,\gamma)}},$$

$$\kappa(\delta,\gamma)=3(4-\delta\gamma)(2-\delta)(2-\gamma)+9(\delta-\gamma)^2$$

$$\gamma=\frac{\alpha x_2^{(0)}+\beta x_3^{(0)}}{x_2^{(0)}+x_3^{(0)}},\ \delta=\frac{(1+\alpha)\beta x_2^{(0)}+(1+\beta)\alpha x_3^{(0)}}{(1+\alpha)x_2^{(0)}+(1+\beta)x_3^{(0)}}\ \text{and}$$

$$\tilde{\mathbf{x}}_{\delta\gamma}=\left(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3\right),\ \tilde{x}_1=1-\tilde{x}_2-\tilde{x}_3,\ \tilde{x}_2=(1+\beta)\hat{x}_1\hat{x}_3,\ \tilde{x}_3=(1+\alpha)\hat{x}_1\hat{x}_2.$$

Also, we denote

$$\mathbf{x}^*=\left(\frac{1}{\sqrt{(1+\alpha)(1+\beta)}},\frac{\sqrt{1+\beta}}{\sqrt{1+\alpha}}x_3^*,x_3^*\right),\ \text{where}\ x_3^*=\frac{\sqrt{(1+\alpha)(1+\beta)}-1}{\sqrt{1+\beta}\left(\sqrt{1+\alpha}+\sqrt{1+\beta}\right)},\ \alpha,\beta\in(-1,1].$$

Theorem 2. *Let* $\pi \neq Id$. *Then for the* $(\alpha,\beta)-QSO$ *(3) the following statements are hold:*

i) if $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}}$ *then for any* $\alpha,\beta \in [-1,1]$ *we have* $V\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$*;*

ii) if either $\alpha = -1$ *or* $\beta = -1$ *then for any* $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{2,3\}}$ *we have* $V^2\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1$*;*

iii) if $\pi \neq Id$ *and* $\alpha+\beta+\alpha\beta > 0$, $\alpha,\beta \in (-1,1]$ *then for any* $\mathbf{x}^{(0)} \in \left(\Gamma_{\{1,2\}} \cup \Gamma_{\{1,3\}}\right) \setminus \{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{y}_{\alpha\beta}, & if\ n = 2k, \\ \mathbf{z}_{\alpha\beta}, & if\ n = 2k+1, \end{cases} \quad k = 0,1,2,...$$

iv) if $\alpha+\beta+\alpha\beta > 0$, $\alpha,\beta \in (-1,1]$ *then for any initial point* $\mathbf{x}^{(0)} \in \operatorname{int} S^2 \setminus \{\mathbf{x}^*\}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \hat{\mathbf{x}}_{\delta\gamma}, & if\ n = 2k, \\ \tilde{\mathbf{x}}_{\delta\gamma}, & if\ n = 2k+1, \end{cases} \quad k = 0,1,2,...$$

v) if $\alpha+\beta+\alpha\beta \leq 0$, $\alpha,\beta \in (-1,1]$ *then for any initial point* $\mathbf{x}^{(0)} \in S^2 \setminus \left(\Gamma_{\{2,3\}} \cup \{\mathbf{e}_1\}\right)$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_1 .$$

In the second chapter of the thesis, titled "**Non-Volterra quadratic operators corresponding to permutations**", we study the dynamical properties of non-Volterra quadratic stochastic operators corresponding to permutations and mutations which defined on a finite-dimensional simplex.

In the first section of this chapter, the invariant sets and periodic points of the non-Volterra quadratic stochastic operators coressponding to permutation, are identified. In the second section, the set of limit points of orbits for such operators is described.

In the first section of the chapter, we considered an operator in the form

$$V: \begin{cases} x_k' = (1+\alpha)\, x_{\pi(k)} x_m, & k \in E_{m-1}, \\ x_m' = x_m^2 + \left(\sum\limits_{i=1}^{m-1} x_i\right)^2 + (1-\alpha)\, x_m \left(\sum\limits_{i=1}^{m-1} x_i\right), \end{cases} \tag{4}$$

where $\alpha \in [-1,1]$ and π is a permutation on the set E_{m-1}.

Denote

$$s = lcm\left(ord\left(\tau_1\right),...,ord\left(\tau_q\right)\right),$$

$$\Delta_{\tau_l} = \left\{\mathbf{x} \in S^{m-1} : x_u = x_v,\ u,v \in \operatorname{supp}\left(\tau_l\right)\right\}$$

$$\Omega_1 = \{1,2,...,r\} \subset \{1,2,...,q\},\ \Omega_2 = \{1,2,...,q\} \setminus \Omega_1,$$

$$\sigma = lcm\left(ord\left(\tau_{i_1}\right),...,ord\left(\tau_{i_l}\right)\right),\ i_l \in \Omega_2,\ \text{for } x_m^{(0)} \neq 1$$

$$\overline{\mathbf{x}} := \overline{\mathbf{x}}_{\alpha,\,\mathbf{x}^{(0)}} = \left(\frac{\alpha x_1^{(0)}}{(1+\alpha)\left(1-x_m^{(0)}\right)}, ..., \frac{\alpha x_{m-1}^{(0)}}{(1+\alpha)\left(1-x_m^{(0)}\right)}, \frac{1}{(1+\alpha)}\right),$$

$$M = S^{m-1} \setminus \left(Fix(V) \cup \Gamma\right) \text{ and } N = \bigcap_{i=1}^{q} \Delta_{\tau_i} \setminus \left(Fix(V) \cup \Gamma\right).$$

The next theorem provides a description of the set of limit points of the orbits of the QSO (4).

Theorem 3. *For the operator V (4) the following statements are hold:*

i) if $\mathbf{x}^{(0)} \in \Gamma = \left\{\mathbf{x} \in S^{m-1} : x_m = 0\right\} \cup \{\mathbf{e}_m\}$ *then* $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_m\}$ *;*

ii) if $\alpha \in [-1,0]$ *then* $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{e}_m\}$ *for any* $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \Gamma$*;*

iii) if $\alpha \in (0,1]$ *and* $\pi = \mathrm{Id}$ *then* $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\overline{\mathbf{x}}_M\}$ *for any* $\mathbf{x}^{(0)} \in M$ *;*

iv) if $\alpha \in (0,1]$ *and* $\pi \neq \mathrm{Id}$ *then* $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\overline{\mathbf{x}}_N\}$ *for any* $\mathbf{x}^{(0)} \in N$*;*

v) if $\alpha \in (0,1]$ *and* $\pi \neq \mathrm{Id}$ *then for any* $\mathbf{x}^{(0)} \in \bigcap_{i \in \Omega_1} \Delta_{\tau_i} \setminus \left(Fix(V) \cup \Gamma \cup \bigcup_{j \in \Omega_2} \Delta_{\tau_j} \right)$ *there is* $\overline{\mathbf{x}} = \lim_{n\to\infty} V^{\sigma n}\left(\mathbf{x}^{(0)}\right)$ *and we have* $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\overline{\mathbf{x}}, \overline{\mathbf{x}}^{(1)}, \ldots, \overline{\mathbf{x}}^{(\sigma-1)}\right\}$;

vi) if $\alpha \in (0,1]$ *and* $\pi \neq \mathrm{Id}$ *then for any* $\mathbf{x}^{(0)} \in S^{m-1} \setminus \left(Fix(V) \cup \Gamma \cup \bigcup_{i=1}^{q} \Delta_{\tau_i} \right)$ *there is* $\overline{\mathbf{x}} = \lim_{n\to\infty} V^{sn}\left(\mathbf{x}^{(0)}\right)$ *and we have* $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\overline{\mathbf{x}}, \overline{\mathbf{x}}^{(1)}, \ldots, \overline{\mathbf{x}}^{(s-1)}\right\}$.

In the third section of the second chapter, non-Volterra quadratic stochastic operators corresponding to mutations which defined on a finite-dimensional simplex are constructed, and their invariant sets and periodic points are identified. Moreover, in the fourth section, the set of limit points of orbits for such operators is described.

In the third section of the chapter, we consider the following the operator

$$V: \begin{cases} x_k' = \alpha x_{\pi(k)}^2 + \dfrac{2\alpha}{m-1} \sum\limits_{\substack{i,l=1:\\ i<l}}^{m-1} x_i x_l + (1+\alpha) x_{\pi(k)} x_m, \ k \in E_{m-1}, \\ x_m' = x_m^2 + (1-\alpha)\left(\sum\limits_{i=1}^{m-1} x_i \right)^2 + (1-\alpha) x_m \left(\sum\limits_{i=1}^{m-1} x_i \right), \end{cases} \tag{5}$$

where $\alpha \in [0,1]$ and π is a permutation on the set E_{m-1}.

It should be noted that when $\alpha = 0$, the operator (5) coincides with operator (4). Therefore, we focus on the case $\alpha \in (0,1]$ in below.

For any $\alpha > 0$ we denote

$$\mathbf{y}_\alpha = (\alpha, 0, 1-\alpha), \quad z_\alpha = (0, \alpha, 1-\alpha),$$

$$\mathbf{x}_{\alpha,m} = \left(\frac{\alpha}{m-1}, \ldots, \frac{\alpha}{m-1}, 1-\alpha \right),$$

$$\hat{\mathbf{x}} = \left(\frac{\alpha x_1^{(0)}}{x_1^{(0)} + x_2^{(0)}}, \frac{\alpha x_2^{(0)}}{x_1^{(0)} + x_2^{(0)}}, 1-\alpha \right),$$

$$\text{and } \tilde{\mathbf{x}} = \left(\frac{\alpha x_2^{(0)}}{x_1^{(0)} + x_2^{(0)}}, \frac{\alpha x_1^{(0)}}{x_1^{(0)} + x_2^{(0)}}, 1-\alpha \right).$$

The next theorem describes the set of limit points of orbits for the QSO (5) when $m = 3$.

Theorem 4. *Let* $\alpha \in (0,1]$ *and* $m = 3$. *Then for the operator* V *(5) the following statements are hold:*

i) if $\pi = Id$ *then*

$$\omega_V\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & if \quad \mathbf{x}^{(0)} = \mathbf{e}_3, \\ \{\mathbf{y}_\alpha\}, & if \quad \mathbf{x}^{(0)} \in \Gamma_{\{1,3\}} \setminus \{\mathbf{e}_3\}, \\ \{\mathbf{z}_\alpha\}, & if \quad \mathbf{x}^{(0)} \in \Gamma_{\{2,3\}} \setminus \{\mathbf{e}_3\}, \\ \{\hat{\mathbf{x}}\}, & if \quad \mathbf{x}^{(0)} \in S^2 \setminus \left(\Gamma_{\{1,3\}} \cup \Gamma_{\{2,3\}}\right); \end{cases}$$

ii) if $\pi \neq Id$ *then*

$$\omega_V\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_3\}, & if \quad \mathbf{x}^{(0)} = \mathbf{e}_3, \\ \{\mathbf{x}_{\alpha,3}\}, & if \quad \mathbf{x}^{(0)} \in M_=^{(12)} \setminus \{\mathbf{e}_3\}, \\ \{\mathbf{y}_\alpha, \mathbf{z}_\alpha\}, & if \quad \mathbf{x}^{(0)} \in \left(\Gamma_{\{1,3\}} \cup \Gamma_{\{2,3\}}\right) \setminus \{\mathbf{e}_3\}, \\ \{\tilde{\mathbf{x}}, \hat{\mathbf{x}}\}, & if \quad \mathbf{x}^{(0)} \in S^2 \setminus \left(\Gamma_{\{1,3\}} \cup \Gamma_{\{2,3\}} \cup M_=^{(12)}\right). \end{cases}$$

For all $\alpha > 0$ we denote

$\mathbf{x}_\xi = \left(\xi_1, \xi_2, \ldots, \xi_{m-1}, 1-\alpha\right)$, where $\xi_u = \alpha\delta_{k,u}$, $u, k \in \operatorname{supp}\left(\tau_i\right)$.

$$C = C_1 \cup C_2,$$

where

$$C_1 = \bigcup_{u \in E_{m-1} \setminus \operatorname{supp}(\pi)} \Gamma_{\{u,m\}} \text{ and } C_2 = \bigcup_{u \in \operatorname{supp}(\pi)} \Gamma_{\{u,m\}}.$$

$$\hat{B}_\alpha = \left\{\mathbf{x}_\ell \in S^{m-1} : \mathbf{x}_\ell = \left(\alpha\delta_{1,\ell}, \ldots, \alpha\delta_{m-1,\ell}, 1-\alpha\right), \ell \in \operatorname{supp}(\pi)\right\},$$

$$\tilde{B}_\alpha = \left\{\mathbf{x}_\ell \in S^{m-1} : \mathbf{x}_\ell = \left(\alpha\delta_{1,\ell}, \ldots, \alpha\delta_{m-1,\ell}, 1-\alpha\right), \ell \in E_{m-1} \setminus \operatorname{supp}(\pi)\right\},$$

where $\delta_{i,j}$ Kronecker symbol and $B_\alpha = \tilde{B}_\alpha \cup \hat{B}_\alpha$.

The next theorem describes the set of limit points of orbits for the QSO (5) when $m > 3$.

Theorem 5. *Let* $\alpha \in (0,1]$ *and* $m = 3$. *Then for the operator* V *(5) the following statements are hold:*

i) if $\mathbf{x}^{(0)} \in S^{m-1} \setminus C$ *then for any* π *we have* $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{x}_{\alpha,m}\right\}$;

ii) if $\pi = \mathrm{Id}$ *then for any* $\mathbf{x}^{(0)} \in C \setminus Fix(V)$ *there exists* $u \in E_{m-1}$ *such that* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *and we have* $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{x}_u\right\}$;

iii) if $\pi \neq \mathrm{Id}$ *then for any* $\mathbf{x}^{(0)} \in C_1 \setminus Fix(V)$ *there exists* $u \in E_{m-1} \setminus \mathrm{supp}(\pi)$ *such that* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *and we have* $\omega_V\left(\mathbf{x}^{(0)}\right) = \{\mathbf{x}_u\}$*;*

iv) if $\pi \neq \mathrm{Id}$ *then for any* $\mathbf{x}^{(0)} \in C_2 \setminus Fix(V)$ *there exists* $u \in \mathrm{supp}(\tau_i)$ *such that* $\mathbf{x}^{(0)} \in \Gamma_{\{u,m\}}$ *and we have there is the limit* $\mathbf{x}_\xi = \lim_{n\to\infty} V^{nt_i}\left(\mathbf{x}^{(0)}\right)$ *and* $\omega_V\left(\mathbf{x}^{(0)}\right) = \left\{\mathbf{x}_\xi, \mathbf{x}_\xi^{(1)}, ..., \mathbf{x}_\xi^{(t_i-1)}\right\}$*, where* $t_i = ord(\tau_i)$*.*

In the third chapter of the dissertation, titled "**Discrete-time dynamics of the SIRD reinfection epidemic model**", the dynamical properties of the discrete-time reinfection SIRD (Susceptible-Infected-Recovered-Died) epidemic model are examined. The first section introduces the fundamental concepts of epidemic models, detailing the variables and parameters of the SIRD reinfection epidemic model, which is derived by modifying the classical SIRD epidemic model.

In the second section of the third chapter, we investigate a quadratic operator in form

$$V: \begin{cases} x_1' = x_1 + \mu x_3 - \beta x_1 x_2, \\ x_2' = (1-\gamma_1-\gamma_2)x_2 + \beta x_1 x_2, \\ x_3' = (1-\mu)x_3 + \gamma_1 x_2, \\ x_4' = x_4 + \gamma_2 x_2, \end{cases} \tag{6}$$

which is a discrete analogue of the SIRD reinfection epidemic model

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI + \mu R, \\ \dfrac{dI}{dt} = \beta SI - (\gamma_1 + \gamma_2) I, \\ \dfrac{dR}{dt} = \gamma_1 I - \mu R, \\ \dfrac{dD}{dt} = \gamma_2 I. \end{cases} \tag{7}$$

Necessary and sufficient conditions for the parameters operator (6) to preserve the simplex were determined. Additionally, the invariant sets, fixed points, and their types of these operator were identified.

Propsition 1. *The operator (6) maps* S^3 *to itself if only if*

$$\mu, \gamma_1, \gamma_2, \gamma_1+\gamma_2 \in [0,1], \ \gamma_1+\gamma_2-1 \leq \beta \leq \left(1+\sqrt{\gamma_1+\gamma_2}\right)^2. \tag{8}$$

The last theorem of the chapter describes the set of limit points of orbits for quadratic operator (6).

Theorem 6. *For the operator* V *(6) the following statements are true:*

i) if $\mathbf{x}^{(0)} \in \Gamma_{\{1,3,4\}} \setminus Fix(V)$ *then we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \left(1-x_4^{(0)}, 0, 0, x_4^{(0)}\right);$$

ii) if $\beta=\gamma_1=\gamma_2=0,\ \mu>0$ *then for any* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,2,4\}}\cup\Gamma_{\{1,3,4\}}\right)$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_2^{(0)}-x_4^{(0)},x_2^{(0)},0,x_4^{(0)}\right);$$

iii) if $\beta=\gamma_1=\mu=0,\ \gamma_2>0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,x_3^{(0)},1-x_1^{(0)}-x_3^{(0)}\right);$$

iv) if $\beta=\gamma_2=\mu=0,\ \gamma_1>0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,1-x_1^{(0)}-x_4^{(0)},x_4^{(0)}\right);$$

v) if $\gamma_1=\gamma_2=\mu=0,\ \beta>0$ *then for any* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,3,4\}}\right)$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(0,1-x_3^{(0)}-x_4^{(0)},x_3^{(0)},x_4^{(0)}\right)\ ;$$

vi) if $\gamma_1=\gamma_2=\mu=0,\ \beta<0$ *then for any* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,3,4\}}\right)$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_3^{(0)}-x_4^{(0)},0,x_3^{(0)},x_4^{(0)}\right);$$

vii) if $\beta=\gamma_1=0,\ \gamma_2\mu>0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,0,1-x_1^*\right);$$

viii) if $\beta=\gamma_2=0,\ \gamma_1\mu>0$ *then for any intial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right);$$

ix) if $\beta=\mu=0,\ \gamma_1\gamma_2>0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^{(0)},0,x_3^*,1-x_1^{(0)}-x_3^*\right);$$

x) if $\gamma_1=\gamma_2=0,\ \beta\mu>0$ *then for any* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,4\}}\right)$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(0,1-x_4^{(0)},0,x_4^{(0)}\right);$$

xi) if $\gamma_1=\gamma_2=0,\ \beta\mu<0$ *then for any* $\mathbf{x}^{(0)}\in S^3\setminus\left(\Gamma_{\{1,3,4\}}\cup\Gamma_{\{2,4\}}\right)$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(1-x_4^{(0)},0,0,x_4^{(0)}\right);$$

xii) if $\gamma_1=\mu=0,\ \beta\gamma_2\neq0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,x_3^{(0)},1-x_1^*-x_3^{(0)}\right);$$

xiii) if $\gamma_2=\mu=0,\ \beta\gamma_1\neq0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,1-x_1^*-x_4^{(0)},x_4^{(0)}\right)\ ;$$

xiv) if $\beta=0,\ \gamma_1\gamma_2\mu>0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty}V^n\left(\mathbf{x}^{(0)}\right)=\left(x_1^*,0,0,1-x_1^*\right);$$

xv) if $\gamma_1=0,\ \beta\gamma_2\mu\neq0$ *then for any initial point* $\mathbf{x}^{(0)}\in S^3\setminus\Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \left(1 - x_4^*, 0, 0, x_4^*\right);$$

xvi) if $\gamma_2 = 0,\ \beta\gamma_1\mu \neq 0,\ \beta \leq \gamma_1$ *then for any initial point* $\mathbf{x}^{(0)} \in S^3 \setminus \Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \left(1 - x_4^{(0)}, 0, 0, x_4^{(0)}\right);$$

xvii) if $\mu = 0,\ \gamma_1\gamma_2 > 0,\ \beta \neq 0$ *then for any initial point* $\mathbf{x}^{(0)} \in S^3 \setminus \Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \left(x_1^*, 0, x_3^*, 1 - x_1^* - x_3^*\right);$$

xviii) if $\beta\gamma_1\gamma_2\mu \neq 0$ *then for any initial point* $\mathbf{x}^{(0)} \in S^3 \setminus \Gamma_{\{1,3,4\}}$ *we have*

$$\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \left(1 - x_4^*, 0, 0, x_4^*\right).$$

Conjecture 1. *If* $\gamma_2 = 0,\ \beta > \gamma_1 > 0,\ \mu > 0$ *then there is the limit*

$$\lim_{n\to\infty} \mathbf{x}^{(n)} = \begin{cases} \tilde{\mathbf{y}}, & \text{if} \quad 0 \leq x_4^{(0)} < \dfrac{\beta - \gamma_1}{\beta}, \\ \mathbf{x}^* \in \Gamma_{\{1,4\}}, & \text{if} \quad \dfrac{\beta - \gamma_1}{\beta} \leq x_4^{(0)} < 1, \end{cases}$$

for any $\mathbf{x}^{(0)} \in S^3 \setminus \Gamma_{\{1,3,4\}}$, *where*

$$\tilde{\mathbf{y}} = \left(\frac{\gamma_1}{\beta}, \frac{\mu\left(\beta - \gamma_1 - \beta x_4^{(0)}\right)}{\beta\left(\mu + \gamma_1\right)}, \frac{\gamma_1\left(\beta - \gamma_1 - \beta x_4^{(0)}\right)}{\beta\left(\mu + \gamma_1\right)}, x_4^{(0)}\right), \quad 0 \leq x_4^{(0)} \leq \frac{\beta - \gamma_1}{\beta}.$$

CONCLUSION

This dissertation research focuses on the study of nonlinear dynamical systems generated by non-Volterra quadratic stochastic operators corresponding to permutation and the discrete-time SIRD reinfection epidemic model.

The main results of this research are as follows:

1. For any (α,β)-QSO defined on the two-dimensional simplex, it is proved that the orbit of an arbitrary initial point converges either to a fixed point of the operator or to a periodic orbit of period two;
2. The invariant sets and periodic points of the non-Volterra QSO corresponding to permutations defined on a finite-dimensional simplex are identified. Moreover, it is shown that the orbit of an arbitrary initial point converges to either a fixed point or to a periodic orbit of the operator;
3. For the non-Volterra QSO corresponding to mutations defined on a finite-dimensional simplex, the invariant sets and the set of periodic points are identified. Moreover, it is proven that the set of limit points of the orbit generated by an arbitrary initial point is finite;
4. For a discrete analogue of the SIRD reinfection epidemic model to preserve the three-dimensional simplex under a quadratic operator, necessary and sufficient conditions on the parameters have been established. A Lyapunov function corresponding to the operator is constructed. Furthermore, it is shown that the orbit of any initial point converges to a fixed point of the operator.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

ИНСТИТУТ МАТЕМАТИКИ

ХУДОЙБЕРДИЕВ ХАЁТЖОН ОЧИЛТОШ УГЛИ

ДИНАМИКА НЕВОЛЬТЕРРОВСКИХ КВАДРАТИЧНЫХ ОПЕРАТОРОВ: ПЕРЕСТАВЛЕННЫЕ И ЭПИДЕМИЧЕСКИЕ МОДЕЛИ

01.01.01 –Математический анализ

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ-2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образования, Науки и Инноваций Республики Узбекистан за №. B2024.4.PhD/FM1176.

Диссертация выполнена в Институте Математики.

Автореферат диссертации на трех языках (узбекский, английский, русский, (резюме)) размещен на веб-странице по адресу http://kengash.mathinst.uz и на Информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

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Защита диссертации состоится « 01 » июля 2025 года в 16:00 на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

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Автореферат диссертации разослан « 11 » июня 2025 года.
(протокол рассылки № 2 от « 11 » июня 2025 года).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования работы является изучение динамических свойств невольтерровских квадратичных стохастических операторов, зависящих от перестановок (оператор I) и операторов, соответствующих рецидивирующей эпидемиологической модели SIRD с дискретным временем (оператор II).

Объект исследования: Невольтерровские квадратичные стохастические операторы, определенные на конечномерном симплексе.

Научная новизна исследования состоит в следующем:

Описано множество периодических точек невольтеровского квадратичного стохастического оператора, заданного на двумерном симплексе. Кроме того, доказано, что орбита произвольной начальной точки имеет не более двух предельных точек;

описано множество периодических точек оператора I. Более того, показано, что множество предельных точек орбиты произвольной начальной точки состоит либо из одной точки, либо из конечного числа точек;

для оператора II найдены необходимые и достаточные условия на параметры, при которых симплекс сохраняется как инвариантное множество;

для оператора II построены функции Ляпунова и с их помощи описано множество предельных точек орбит произвольных начальных точек.

Внедрение результатов исследования. Полученные результаты по динамике операторов I и II были применены в следующих направлениях:

описание множества предельных точек орбит для дискретной эпидемиологической модели SIRD с рецидивом заражения было использовано в исследовательском проекте «Квантовые генетические алгебры и их приложения» (регистрационный номер G0003447) для анализа свойства регулярности нелинейных стохастических операторов (справка Университет Объединённых Арабских Эмиратов, 1 мая 2025 г., ОАЭ). Применение полученного научного результата позволило установить сходимость орбит квадратичных стохастических операторов, что имеет ключевое значение для понимания будущего биологических систем;

для квадратичных стохастических операторов, соответствующих перестановкам, множества периодических и предельных точек были использованы в рамках исследовательского проекта «Гомологии, гомотопии и категориальные инварианты в не ассоциативных группах и алгебрах» (номер проекта: PID2020-115155GB-I00) для анализа равновесных состояний эволюционных алгебр (справка Университет Сантьяго-де-Компостела, 6 мая 2025 г., Испания). Применение данного научного результата позволило описать идемпотентные и абсолютно нильпотентные элементы в эволюционных алгебрах.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка литературы. Общий объём диссертации составляет 103 страницы.

E’LON QILINGAN ILMIY ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part 1; часть 1)

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2. Jamilov U.U., Khudoyberdiev Kh.O. An $(\alpha,\beta)-$quadratic stochastic operator acting in S^2 // *Journal of Applied Nonlinear Dynamics,* 2022, Vol. 11, No. 4, p.777-788. (3. Scopus, IF=0.211)
3. Eshmatov F.F., Jamilov U.U., Khudoyberdiev Kh.O. Discrete-time dynamics of a SIRD reinfection model // *International Journal of Biomathematics*, 2023, Vol. 16, No. 5, 2250104 (22 pages). (3. Scopus, IF=0.592)
4. Jamilov U.U., Khudoyberdiev Kh.O. On the dynamics of non-Volterra quadratic operators corresponding to permutations // *Journal of Difference Equations and Applications*, 2024, Vol. 30, No. 3, p.336-360. (3. Scopus, IF=0.549)
5. Khudoyberdiev Kh.O. On dynamics of a non-Volterra quadratic operator // *Reports of the Academy of Sciences of the Republic of Uzbekistan*, 2024, No 4, p.110-113. (01.00.00. No.7)
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II bo‘lim (part 2; часть 2)

7. Khudoyberdiev Kh.O. On the discrete-time SIRD reinfection model // The bulletin of young scientists, Vol. 4, No. 2, p.45-51 (2023).
8. Khudoyberdiev Kh.O. The behavior of trajectories of a quadratic operator // The bulletin of young scientists, No. 4, p.24-26 (2023).
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Avtoreferat "Oʻzbekiston matematika jurnali" tahririyatida
2025-yil 26-mayda tahrirdan oʻtkazilib, oʻzbek, rus va ingliz tillaridagi matnlar
oʻzaro muvofiqlashtirildi.

Bosmaxona litsenziyasi:

9338

Bichimi: 84x60 $^{1}/_{16}$. «Times New Roman» garniturasi.
Raqamli bosma usulda bosildi.
Shartli bosma tobogʻi: 2,5. Adadi 100 dona. Buyurtma № 72/23.

Guvohnoma № 851684.
«Tipograff» MChJ bosmaxonasida chop etilgan.
Bosmaxona manzili: 100011, Toshkent sh., Beruniy koʻchasi, 83-uy.