"YANGI O'ZBEKISTON" UNIVERSITETI HUZURIDAGI ILG'OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/07.07.2025.FM/T.192.01 RAQAMLI ILMIY KENGASH

"YANGI O'ZBEKISTON" UNIVERSITETI ASTRONOMIYA INSTITUTI

DAVLATALIYEV AKBARJON AKMALJON O'G'LI

SKALYAR MAYDON MAVJUDLIGIDA KOMPAKT OBYEKTLAR ATROFIDA RELYATIVISTIK ASTROFIZIK JARAYONLAR.

01.03.01 – Astronomiya
01.04.02 – Nazariy fizika

FIZIKA-MATEMATIKA FANLARI BO'YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFARATI

TASHKENT-2025

UDK: 519.688 (519.24)

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

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E'lon qilingan ishlar ro'yxati

List of published works

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TOSHKENT-2025

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.2.PhD/FM1325 raqami bilan ro'yxatga olingan.

Dissertatsiya "Yangi O'zbekiston" universiteti va O'zbekiston Respublikasi Fanlar akdemiyasi Astronomiya institutida bajarilgan. Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz (rezyume)). Ilmiy Kengashning internet sahifasida (ias.newuu.uz) va "Ziyonet" axborot-ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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KIRISH (PhD dissertatsiya annotatsiyasi)

Dissertatsiyaning dolzarbligi va talab yuqoriligi. Astronomik kuzatuvlarning aniqligi ortib borishi bilan – masalan, gravitatsion to'lqinlarni (LIGO/Virgo) qayd etish va rentgen qo'shaloq yulduzlaridan kelayotgan yuqori chastotali kvazi-davriy tebranishlarni (KDT) o'rganish – klassik umumiy nisbiylik nazariyasidan tashqariga chiquvchi nazariy asoslarga talab kuchaymoqda. Ushbu ilmiy ish skalyar maydonlar va tashqi magnit maydonlar fazo-vaqt dinamikasini qanday o'zgartirishi haqida muhim tushunchalarni taqdim etadi va kelajakdagi kuzatuvlar uchun sinovdan o'tkazilishi mumkin bo'lgan bashoratlarni taklif qiladi. Ekzotik kompakt ob'ektlarni (masalan, JNW yalang'och singulyarliklari, Ellis yumronqoziq inlari) o'rganish, shuningdek, Einstein nazariyasi chegaralarini sinab ko'rish va muqobil gravitatsiya modellarini tadqiq qilish borasidagi say-harakatlarga mos keladi. Dissertatsiyada zamonaviy relyativistik astrofizikaning dolzarb masalalari, xususan, kompakt ob'ektlar (qora o'ralar, yalang'och singulyarliklar va yumronqoziq ini) va skalyar maydonlar o'rtasidagi o'zaro ta'sir o'rganilgan.

Dissertatsiyaning skalyar-tenzor bog'lanishlari va ularning astrofizik natijalariga qaratilganligi, modifikatsiyalangan gravitatsiya, qorong'u materiya va qorong'u energiya sohasidagi so'nggi yutuqlarni hisobga olganda, juda dolzarbdir. Dissertatsiya ishi skalyar maydonlar f(R) gravitatsiyasi va torlar nazariyasidan ilhomlangan modellar kabi nazariyalarda fundamental ahamiyatga ega bo'lib, bu ishni ham nazariy, ham kuzatuv kosmologiyasi uchun ahamyati kattadir. Bundan tashqari, kvazi-rezonant modalar va kvazi-davriy tebranishlar tahlili yangi avlod teleskoplari (masalan, LISA, Athena) va rentgen observatoriyalari (masalan, NICER) ma'lumotlarini sharhlashga o'z hissasini qo'shadi. IV bobda o'rganilgan magnitlangan qora o'ralardagi perturbatsiyalarning g'ayrioddiy kechikuvchi dumlari NANOGrav tomonidan tahlil qilinganidek, pulsar vaqtlar massivi (PTA) signallaridagi anomaliyalarga oydinlik kiritishi mumkin.

Zarrachalar dinamikasi, g'alayonlar nazariyasi va kuzatuv natijalarini bog'lash orqali ushbu dissertatsiya ekstremal muhitlarda kompakt ob'ektlarni o'rganish uchun uslubiy yo'llanma bo'lib xizmat qiladi. Ishning metodologiyalari, masalan, kvazinormal modalar uchun WKB yaqinlashuvi va kvazi-davriy tebranishlar uchun MCMC tahlili, neytron yulduzlari va egzotik gorizontsiz ob'ektlar kabi boshqa kompakt ob'ektlarga ham qo'llanilishi mumkin. Shuningdek, ish umumiy nisbiylik, kvant maydon nazariyasi va raqamli axborotlarni qayta ishlash sohalarini birlashtirgan fanlararo yondashuvlarning astrofizikadagi ochiq muammolarni hal qilishda muhimdir. Ilmiy hamjamiyat umumiy nisbiylik va kvant nazariyasi o'rtasidagi ziddiyatlarni hal qilishga intilayotgan bir paytda, bunday tadqiqotlar Standart modeldan tashqaridagi yangi fizikaga yo'l ochadi. Dissertatsiyaning dolzarbligi, talabgirligi va ahamiyati uning relyativistik astrofizikadagi eng ilg'or masalalar bilan bevosita bog'liqligi, joriy va kelajakdagi kuzatuv kompaniyalarida qo'llanilishi mumkinligi hamda gravitatsiya va kosmologiyadagi nazariy paradigmalarga ta'sir qilish salohiyati bilan belgilanadi.

Bundan tashqari, dissertatsiya O'zbekistondagi va xalqaro hamjamiyatdagi strategik ilmiy ustuvorliklarga mos keladi. Ushbu dissertatsiya ishi quyidagi davlat me'yoriy hujjatlari vazifalariga muvofiqdir: O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha Harakatlar strategiyasi to'g'risidagi PF-4947-sonli Farmoni, O'zbekiston Respublikasi Prezidentining 2017-yil 18-fevraldagi "Fanlar akademiyasi faoliyatini, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to'g'risida"gi PQ-2789-sonli Qarori.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi. Dissertatsiya tadqiqoti O'zbekiston Respublikasidagi fan va texnologiyalarning ustuvor yo'nalishlariga muvofiq amalga oshirilgan, xususan: II. "Energetika, energiya va resurslarni tejash" yo'nalishi bo'yicha.

Muammoning o'rganilganlik darajasi. Umumiy nisbiylik nazariyasida (Subrahmanyan Chandrasekhar, Brandon Carter, James Bardeen, Valeri Frolov, Andrei Zelnikov, Pankaj S. Joshi, Remo Ruffini) qora o'ra fazo-vaqtlarida sinov zarrachalarining harakati keng o'rganilgan bo'lsada, skalyar maydonlarning, xususan, yalang'och singulyarliklar (JNW fazo-vaqti) va yumronqoziq ini (Ellis fazo-vaqti) kontekstida hisobga olinishi yangi murakkabliklarni keltirib chiqaradi. Ushbu tadqiqot ishi Misner, Wheeler va boshqalarning fundamental tadqiqotlariga asoslanadi, lekin ularni skalyar bog'lanishlar, nurlanish reaksiyasi effektlari va magnit maydonlarini qo'shish orqali kengaytiradi, bu fazo-vaqtlarda ular tizimli ravishda o'rganilmagan. Bu geodezik harakatning klassik tahlillaridan sezilarli darajada yuqori aniqlikni beradi.

Astrofizikada skalyar maydonlar bo'yicha oldingi tadqiqotlar asosan kosmologik qo'llanmalar yoki qora o'ra g'alayonlariga (Claudia de Rham, Thomas P. Sotiriou, Vitor Cardoso, Kostas D. Kokkotas, Toby Crisford, Hideo Kodama) qaratilgan bo'lib, ularning yalang'och singulyarliklar va yumronqoziq inlaridagi rolini tushunishdagi bo'shliqlarni to'ldirgan. Ushbu dissertatsiya bu bo'shliqlarni aniq analitik yechimlarni (masalan, JNW fazo-vaqtidagi IBAO radiuslari), zarrachalar trayektoriyalarining sonli simulyatsiyalarini va g'alayon tahlillarini (masalan, magnitlangan qora o'ralardagi kvazinormal modalar) topish orqali to'ldiradi. Skalyar-modifikatsiyalangan fazo-vaqtlarda kvazi-davriy tebranishlarni (KDT) o'rganish alohida o'ziga xoslikka ega, chunki u nazariy bashoratlarni rentgen qo'shaloq yulduzlaridan olingan kuzatuv ma'lumotlari bilan bog'laydi, bu aloqa avvalgi adabiyotlarda kamdan-kam o'rganilgan. Fazo-vaqt parametrlarining qiymatlari uchun MCMC usullarini kiritish nazariy modellarni sinash uchun zamonaviy, ma'lumotlarga asoslangan yondashuvni yana bir bor namoyish etadi.

Shunga o'xshash tadqiqotlar bilan taqqoslaganda, ushbu ish skalyar maydonlarni, elektromagnit o'zaro ta'sirlarni va nurlanish reaksiyasini bir nechta turdagi kompakt ob'ektlar uchun yagona tizimga birlashtirilganligi bilan ajralib turadi. Biroq, ba'zi kamchiliklar mavjud, masalan, zaif teskari reaksiya (zarrachalarning fazo-vaqt geometriyasiga ta'sirini e'tiborsiz qoldirish) va statik,

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sferik simmetrik yechimlar bilan cheklanish. Aylanuvchi kompakt ob'ektlar (masalan, Kerrga o'xshash yalang'och singulyarliklar) va chiziqli bo'lmagan skalyar bog'lanishlar ochiq muammoligicha qolmoqda. Kelajakdagi tadqiqotlar ushbu ishni raqamli usullai yoki yuqori o'lchovli nazariyalarni (masalan, braneworld modellari) qo'shish orqali kengaytirishi mumkin, bu esa yanada realistik astrofizik stsenariylarda skalyar maydon dinamikasini o'rganishni chuqurlashtiradi. Dissertatsiya muammoning keng qamrovli, ammo to'liq bo'lmagan tadqiqotdir, hamda kelajakdagi tadqiqotlar uchun yo'llanma bo'lib xizmat qiladi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiya Innovatsion rivojlanish agentligi tomonidan moliyalashtirilgan F-FA-2021-510 "Modifikatsiyalangan gravitatsiyada neytron yulduzlar va ularning yadroviy materiyasini tadqiq qilish" ilmiy loyihasi doirasida bajarilgan(2021-2026).

Tadqiqotning maqsadi: skalyar maydon o'zaro ta'sirlarini o'rganish uchun yangi nazariy model ishlab chiqish va kompakt ob'ektlar yaqinidagi zarralar dinamikasi bo'yicha yangi natijalarni aniqlashdir.

Tadqiqotning vazifalari:

− JNW va Ellis fazoviy-vaqtlarida skalyar maydonlar va zarra dinamikasi o'rtasidagi o'zaro ta'sirni hisobga oluvchi nazariy model ishlab chiqish;

− Skalyar maydon ta'sirida massiv zarralar harakati uchun harakat tenglamalarini ishlab chiqish;

− Skalyar o'zaro ta'sir parametrlarining IBAO (ichki barqaror aylanish orbitasi)ga ta'sirini tahlil qilish;

− Yalang'och singulyarliklar yaqinida zarra trayektoriyalariga nurlanish reaksiyasining ta'sirini o'rganish;

− KDT (kvazi-davriy tebranishlar) tadbiqlari uchun fundamental chastotalarni (orbital, radial, vertikal) hisoblash;

− Rentgen qo'shaloq yulduzlari ma'lumotlaridan foydalanib, fazo-vaqt parametrlarini cheklash uchun MCMC (Markov zanjiri Monte-Karlo usuli) tahlilini o'tkazish;

− Magnitlangan yumronqoziq ini va qora o'ra fazo-vaqtlarida zaryadlangan zarrachalar dinamikasini tadqiq qilish.

− Skalyar-modifikatsiyalangan fazo-vaqtlarda g'alayonlanish spektrlarini (kvazinormal modalar, kechikuvchi vaqt dumlarini) o'rganish.

Tadqiqot ob'ekti — skalyar maydon bilan o'zaro ta'sirlanuvchi Janis–Newman–Winicour (JNW) yalang'och singulyarlik fazoviy-vaqti; tashqi skalyar maydonlar bilan o'zaro ta'sirlanuvchi Ellis yumronqoziq ini; tashqi magnit maydonga joylashgan Schwarzschild yechimiga o'xshash aylanmaydigan qora o'ralar; va ushbu modifikatsiyalangan fazoviy-vaqt geometriyalaridagi test (neytral va zaryadlangan) zarralari.

Tadqiqot predmeti — egzotik fazoviy-vaqtlarida skalyar maydon ta'siridagi massiv zarralar dinamikasi; skalyar o'zaro ta'sirlar natijasida orbital mexanika va asosiy chastotalarning o'zgarishi; skalyar maydon bilan modifikatsiyalangan

kompakt ob'ektlarning astrofizik ahamiyati (KDTlar, IBAOlar va boshqalar); skalyar o'zaro ta'sirli kompakt ob'ektlarning buzilish spektrlari va barqarorlik xossalari.

Tadqiqot usullari —Skalyar bog'langan fazo-vaqtlarda (JNW, Ellis) zarrachalar uchun harakatning aniq tenglamalari va effektiv potensiallarini keltirib chiqarish maqsadida analitik modellashtirishdan foydalanildi. Nurlanish reaksiyasi ta'siri ostidagi geodezik tenglamalarni yechish va zarrachalar trayektoriyalarini tahlil qilish uchun sonli simulyatsiyalar (Runge-Kutta integrallash usuli) qo'llanildi. Kvazinormal modalar va g'alayonlanish spektrlarini hisoblash uchun yarim analitik WKB yaqinlashuvi hamda vaqt sohasi bo'yicha integrallash usullari tatbiq etildi. Nazariy kvazi-davriy tebranishlar (KDT) chastotalarini rentgen qo'shaloq yulduzlaridan olingan kuzatuv ma'lumotlariga moslashtirish uchun Bayesian MCMC tahlili o'tkazildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Tadqiqot, skalyar bog'lanishga ega JNW yalang'och singulyarlik fazo-vaqtida zarrachalar dinamikasining birinchi to'liq nazariy modelini yaratadi, bu esa uni qora o'ralardan farqlovchi noyob IBAO xususiyatini ochib beradi.

Yangi topilmalar skalyar maydonlarning Ellis yumronqoziq ini fizikasini qanday o'zgartirishini, shu jumladan nurlanish ta'sirida zarrachalarning qochishi va ekzotik ixcham ob'ektlarni qora o'ralardan ajratib turuvchi xarakterli KDT chastotalarini namoyish etadi.

Ushbu ish Rentgen qo'shaloq yulduzlari ma'lumotlaridan foydalangan holda skalyar maydon parametrlariga MCMC (Markov zanjiri Monte-Carlo usuli) asosidagi cheklovlarni kashf etadi va kuchli maydon rejimlarida modifikatsiyalangan gravitatsiya nazariyalarini sinash uchun yangi spektral imzolarni aniqlaydi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Skalyar bog'lanishga ega fazo-vaqtlarda zarrachalar dinamikasi uchun keltirib chiqarilgan analitik ifodalar ekzotik kompakt ob'ektlar atrofidagi akkretsion disklar tuzilmalari va nurlanish profillarini aniq modellashtirish imkonini beradi.

Hisoblangan KDT chastotalari va IBAO radiuslari rentgen qo'shaloq yulduzlari ma'lumotlaridan foydalanib, JNW yalang'och singulyarliklari va Ellis yumronqoziq inlarini qora o'ralardan farqlash uchun kuzatiladigan belgilarni taqdim etadi.

MCMC usuli yordamida moslashtirilgan skalyar maydon parametrlari ($n \sim 0.75$, $g_s \sim 0.2$) kelajakdagi gravitatsion to'lqin va ko'p tarmoqli kompaniyalar uchun sinovdan o'tkazilishi mumkin bo'lgan tahlillarni taklif qiladi.

Aniqlangan g'alayonlanish spektrlari (masalan, birlashgan Heun yechimlari, ossillyatsion dumlar) yangi avlod rasadxonalarida skalyar-tenzor gravitatsiya effektlarini aniqlash uchun etalon (benchmark) bo'lib xizmat qiladi.

Tadqiqot natijalarining ishonchliligi quyidagilardan iborat:

Analitik asos fundamental prinsiplardan qat'iy ravishda keltirib chiqarildi va barcha hisob-kitoblarda matematik izchillikni ta'minlash uchun umumiy nisbiylik nazariyasidagi tasdiqlangan natijalar bilan solishtirib tekshirildi.

Sonli simulyatsiyalarda bir nechta mustaqil usullar (Runge-Kutta integrallash usuli, WKB yaqinlashuvi) qo'llanildi va ular bir-biriga mos keluvchi natijalarni berdi, bu esa zarrachalar dinamikasi va g'alayonlanish (perturbatsiya) tahlillarining ishonchliligini tasdiqladi.

MCMC usulidagi parametrlar baholanishi yaqinlashish testlari va mavjud astrofizik cheklovlar bilan taqqoslash orqali tasdiqlandi, bu skalyar maydon bog'lanish qiymatlarining statistik ishonchliligini namoyish etdi.

Barcha nazariy bashoratlar rentgen qo'shaloq yulduzlaridan olingan kuzatuv ma'lumotlari va ma'lum bo'lgan qora o'ra fenomenologiyasi bilan mos keladi, shu bilan birga kelajakdagi tekshiruvlar uchun sinovdan o'tkazilishi mumkin bo'lgan og'ishlarni aniq belgilaydi.

Tadqiqotning ilmiy va amaliy ahamiyati:

Tadqiqot ekzotik kompakt ob'ektlardagi skalyar maydon ta'sirlarini tahlil qilish uchun keng qamrovli asosni yaratadi va klassik qora o'ra paradigmalaridan tashqarida gravitatsiya va fundamental maydonlar o'rtasidagi o'zaro ta'sir haqida yangi tushunchalarni taqdim etadi.

Keltirib chiqarilgan KDT chastotalari va IBAO modifikatsiyalari qora o'ralar, yalang'och singulyarliklar va yumronqoziq inlari o'rtasidagi muhim kuzatuv diskriminatorlari (ajratuvchilari) bo'lib xizmat qiladi va ular hozirgi rentgen teleskoplari ma'lumotlarini tahlil qilishda bevosita qo'llanilishi mumkin.

Ishlab chiqilgan MCMC-KDT tahlil usuli va g'alayonlanish metodlari astrofizik ma'lumotlardan foydalangan holda modifikatsiyalangan gravitatsiya nazariyalarini cheklash uchun kuchli yangi vositalarni taklif qiladi, bu esa gravitatsiyani yuqori aniqlikdagi sinovlardan o'tkazish imkoniyatini sezilarli darajada oshiradi.

Skalyar-tenzor o'zaro ta'sirlarining noyob spektral belgilarini aniqlash orqali, natijalar yangi avlod gravitatsion to'lqin detektorlari va ko'p tarmoqli astronomiya kampaniyalarini loyihalash talablarini shakllantirish uchun asos bo'ladi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Modifikatsiyalangan gravitatsiya nazariyalarida qora o'ralar atrofidagi zarralar dinamikasini o'rganish natijalari quyidagi yo'nalishlarda qo'llanilgan:

Nazariy tadqiqot natijalari va usullari "A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, "Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma," // *Phys. Dark Univ.* 42 (2023) 101340, https://doi.org/10.1016/j.dark.2023.101340" nomli ilmiy maqolada e'lon qilingan va A. Davlatalievning falsafa doktori (PhD) dissertatsiyasida taqdim etilgan.

Mazkur ilmiy natijalar Fudan universiteti tomonidan qo'llab-quvvatlangan dasturlar doirasida qo'llanilgan (Prof. Cosimo Bambi tomonidan taqdim etilgan rasmiy xat asosida).

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiya natijalari 3 ta xalqaro va 1 ta mahalliy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e'lon qilinganligi.

Tadqiqot natijalariga asosan 11 ta ilmiy nashr chop etilgan bo'lib, ularning barchasi xorijiy jurnallarda e'lon qilingan maqolalardir.

Dissertatsiya hajmi va tuzilishi

Dissertatsiya kirish qismi, to'rt bob, xulosa va adabiyotlar ro'yxatidan iborat bo'lib, jami 121 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismida mavzuning dolzarbligi va zaruriyati, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi, muammoning o'rganilganlik darajasi, dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi, shuningdek, maqsad, vazifalar, tadqiqot ob'ekti, predmeti, metodlari, ilmiy yangiligi, amaliy natijasi, natijalarning ishonchliligi, ilmiy va amaliy ahamiyati, natijalarning amaliyotga joriy etilishi, aprobatsiyasi, e'lon qilinganligi hamda dissertatsiyaning tuzilishi va hajmi to'g'risida qisqacha ma'lumotlar keltirilgan.

Dissertatsiyaning "JNW fazo-vaqtida massiv zarrachalar harakatiga skalyar maydonning ta'siri" deb nomlangan birinchi bobida JNW yalang'och singulyarligi atrofidagi zarrachalar dinamikasi o'rganiladi. JNW yalang'och singulyarligi Einstein tenglamalarining massasiz skalyar maydon bilan aniq yechimini ifodalaydi. Biz Einstein-skalyar maydon-massiv zarracha tizimini qisqacha ta'riflab, butun tizim uchun harakat tenglamasini keltirib chiqaramiz. Shuningdek, JNW fazo-vaqtida skalyar maydon ishtirokida massiv zarrachalarning aylanma harakatini ko'rib chiqamiz. Bundan tashqari, nurlanish reaksiyasi hadini hisobga olgan holda massiv zarrachalarning aylanma harakatini o'rganib, zarrachalar trayektoriyalarini taqdim etamiz.

Ushbu Einstein-skalyar maydon tenglamalarining eng oddiy yechimlaridan biri Janis-Newman-Winicour yalang'och singulyarligi bilan ifodalanadi. Unga mos keluvchi chiziqli element quyidagicha berilgan:

$$ds^2 = -f^n dt^2 + f^{-n} dr^2 + f^{1-n} r^2 (d\theta^2 + \sin^2\theta d\phi^2) \,,$$

$$\varphi(r) = \frac{\sqrt{1-n^2}}{2} \ln f \,, \qquad f = 1 - \frac{2M}{nr} \,, \qquad (1)$$

bu yerda M — ob'ektining massasi, n esa skalyar maydonning parametri hisoblanadi. Fazoviy-vaqtning singulyarligi $r_* = 2M/n$ nuqtada joylashgan.

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Massiv zarra uchun harakat tenglamasi quyidagicha aniq ifodalanadi:

$$\ddot{t} + \frac{2M\dot{t}\dot{r}}{fr^2}\left(1 - B\right) = 0 \ , \tag{2}$$

$$\ddot{r} + \frac{Mf^n}{fr^2}\left(f^n\dot{t}^2 - 2B\right) - \frac{M\dot{r}^2}{fr^2}\left(1 + 2B\right) \tag{3}$$

$$+ \left(\frac{M(n+1)}{n} - r\right)\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) = 0 \ ,$$

$$\ddot{\theta} - \frac{1}{2}\sin 2\theta\dot{\phi}^2 - \frac{2M\dot{\theta}\dot{r}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{4}$$

$$\ddot{\phi} + 2\cot\theta\dot{\theta}\dot{\phi} - \frac{2M\dot{r}\dot{\phi}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{5}$$

bu yerda B

$$B = \frac{2g_s\sqrt{1 - n^2}}{n(g_s\sqrt{1 - n^2}\ln f + 2)} \ . \tag{6}$$

1-rasmda massiv zarracha uchun IBAOning o'zaro ta'sir parametri g_s ning turli qiymatlarida n skalyar parametrga bog'liqligi ko'rsatilgan.

2-rasmda biz skalyar maydon mavjud bo'lganda, nurlanish reaksiyasi hadini hisobga olgan holda massiv zarrachaning trayektoriyasini ko'rsatamiz. Sonli hisoblashlardan foydalanib, biz nurlanish reaksiyasi tufayli yuzaga keladigan hissa sezilarli emasligini ko'rsatdik. Shu sababli, aylanma harakatdagi nurlanish reaksiyasi hadining ta'sirini kuzatish uchun biz so'nish vaqtini $\tau\tau_0 = 0$ s va $\tau_0 = 0.5$ s qilib belgiladik. Bu shuni anglatadiki, so'nish vaqti 1 soniyadan ancha qisqa va zarracha nurlanish reaksiyasi kuchi tufayli qochib ketgunicha JNW fazo-vaqti bilan ifodalangan kompakt ob'ekt atrofida 2-rasmda ko'rsatilganidan ancha ko'p marta aylanadi. 3-rasmda biz massiv zarrachaning trayektoriyasini ikki stsenariyda namoyish etamiz: (i) skalyar maydon ta'sirini hisobga olib, ammo nurlanish reaksiyasi hadisiz va (ii) skalyar maydon ta'sirini e'tiborsiz qoldirib, ammo nurlanish reaksiyasi hadini hisobga olgan holda. Ushbu natija shuni ko'rsatadiki, skalyar maydon haqiqatan ham tortishish effektini hosil qiladi, biroq nurlanish reaksiyasi yalang'och singulyarlik atrofida aylanayotgan massiv zarracha harakatida itarish effektini vujudga keltiradi.

Bizning natijalarimiz shuni ko'rsatadiki, skalyar maydon tortishish kuchini hosil qiladi, nurlanish reaksiyasi esa itarish kuchini vujudga keltiradi. JNW fazo-vaqtida yalang'och singulyarlik atrofidagi massiv zarrachaning trayektoriyasiga sonli simulyatsiyalarimizda ko'rsatilganidek, bu ikkala effekt ham ta'sir qiladi.

Dissertatsiyaning ikkinchi bobi — "JNW fazo-vaqtida massiv zarrachalar harakatiga skalyar maydonning ta'siri" deb nomlangan va biz zarrachalar dinamikasi natijalarini JNW kompakt ob'ektlari atrofidagi kvazi-davriy tebranishlarni (KDT) o'rganishga qo'lladik. Biz JNW fazo-vaqtida barqaror aylanma orbita yaqinidagi

massiv zarrachaning tebranma harakatini ko'rib chiqdik. Astrofizik natija sifatida biz massiv zarracha tebranma harakatining qo'sh cho'qqili KDTga tadbiqini o'rgandik va bu natijalardan foydalanib, markaziy ob'ekt massasi, skalyar parametr va bog'lanish konstantasi kabi asosiy parametrlarga cheklovlar oldik.

Gravitatsion kompakt ob'ektlar atrofida test zarralarning tebranma harakati astrofizikani tekshirishda muhim jihatlaridan biridir. Bu harakat odatda ikki turga bo'linadi: (i) radial va (ii) vertikal tebranishlar, va bu tebranishlar fazo-vaqtning tuzilishi va ob'ekt dinamikasini aniqlashda muhim ahamiyatga ega.
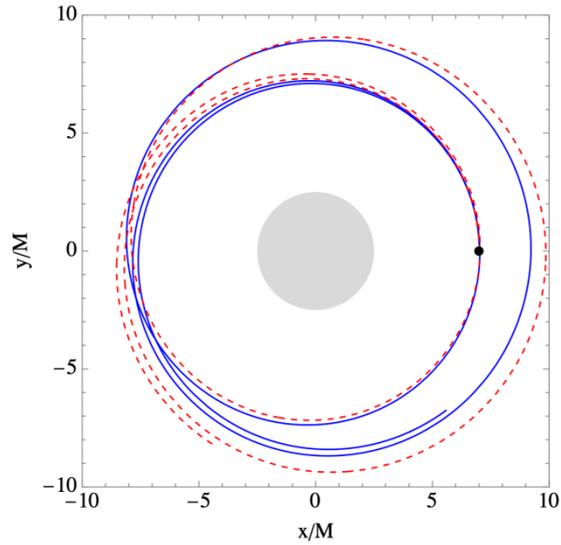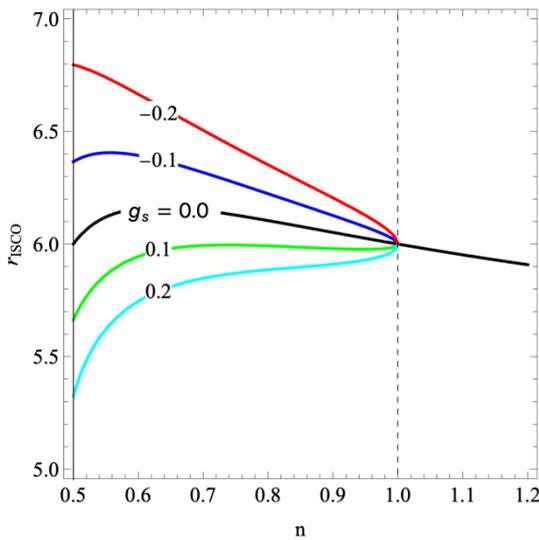
1-rasm: O'zaro ta'sir parametrining turli qiymatlarida massiv zarrachaning IBAO o'rnining n parametrga bog'liqligi.

2-rasm: $g_s = 0.03$ va $n = 0.8$ parametrlarining ma'lum bir qiymatlari uchun JNW yalang'och singulyarligi yaqinidagi zarracha nurlanish reaksiyasi hadi hisobga olinmaganda ($\tau_0 = 0$, uzluksiz ko'k chiziq) va hisobga olinganda ($\tau_0 = 1.0$, punktir qizil chiziq)gi trayektoriyalari.
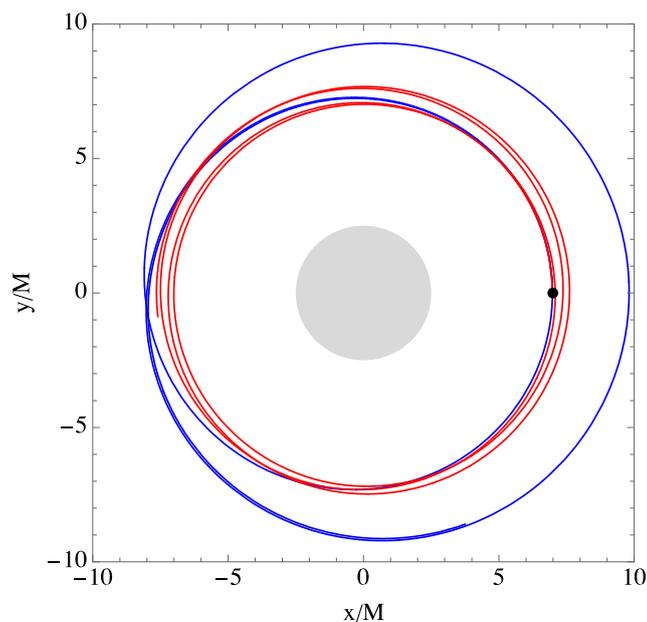
3-rasm: JNW yalang'och singulyarligi yaqinidagi tanlangan fazo-vaqt parametrlari uchun zarra trayektoriyalari: ko'k uzluksiz chiziq g_s=0.03, $\tau_0 = 0$ holatni; qizil uzluksiz chiziq esa g_s=0, τ_0=0.5 holatni ifodalaydi.

(i)   Radial tebranishlar — zarra markaziy ob'ektga nisbatan oldinga-orqaga (yaqinlashish-va-uzoqlashish) yo'nalishdagi harakatini bildiradi. Bunday tebranishlar barqaror aylana orbitalar atrofida sodir bo'ladi. Bu tebranishlarning barqarorligi va chastotasi fazoviy-vaqt geometriyasi va kompakt ob'ektning xossalariga bog'liq. Masalan, Schwarzschild qora o'rasi holatida radial tebranish chastotasi qora o'ra massasiga va zarraning undan bo'lgan radial masofasiga bog'liq bo'ladi. Qora o'raga yaqinlashgan sari tortishish kuchi ortadi va bu tebranish chastotasining oshishiga olib keladi.

(ii)   Vertikal tebranishlar — orbitaning tekisligiga nisbatan perpendikulyar yo'nalishda sodir bo'ladigan harakatdir. Bu harakatlar ba'zida latitudinal yoki epitsiklik tebranishlar deb ham yuritiladi. Ular orbitaning vertikal barqarorligi haqida ma'lumot beradi va markaziy ob'ektning burchak impulsi hamda aylanishiga bog'liq. Vertikal epitsiklik chastota — bu zarra markaziy ob'ektning ekvatorial tekisligi atrofida vertikal ravishda tebranadigan chastotadir.

Bu ikki turdagi chastotalar orbital (yoki Kepler) chastotasi bilan bog'langan bo'lib, ular birgalikda asosiy chastotalar deb yuritiladi. Schwarzschildqora o'rasi uchun bu chastotalar orasida $\Omega_\theta=\Omega_K$ bog'lanishi mavjud bo'lib, bu yerda Ω_K — Schwarzschildfazosidagi Kepler chastotasidir.

4-rasmda asosiy chastotalarning radial bog'liqligi tasvirlangan. Chap panelda skalyar bog'lanish parametri g_s ning turli qiymatlari uchun, n parametri o'zgarmas keltirilgan qiymatida grafiklar ko'rsatilgan. O'ng panelda esa g_s parametri o'zgarmas keltirilgan qiymatida n ning turli qiymatlari uchun grafiklar berilgan. Grafiklardan

ko'rinadiki, skalyar maydon bilan o'zaro ta'sir sababli asosiy chastotalar ortadi, biroq n parametri oshishi bilan ularning qiymatlari kamayadi.

Orbital, radial va vertikal tebranishlarning birgalikdagi ta'siri test zarralar uchun murakkab trayektoriyalarni hosil qiladi. Bu tebranishlar rentgen nurli qo'shaloq yulduzlarda kuzatiladigan KDT ni tushunishda muhim rol o'ynaydi. KDT hodisalari qora o'ra yoki neytron yulduz atrofidagi akkretatsion diskdagi moddaning tebranuvchi harakatiga bog'liq deb hisoblanadi. Tebranish harakati akkretatsion disklarning barqarorligi va tuzilmasiga ta'sir qiladi. Ushbu tebranishlarni o'rganish nurlanish spektrlari va disklarning o'zgaruvchanligini modellashtirishga yordam beradi. Kuzatilayotgan chastotalar va tebranish modlarini tahlil qilish qora o'ralar va neytron yulduzlarning massasi, aylanishi va atrofdagi fazoviy-vaqt geometriyasi haqida muhim ma'lumot beradi.
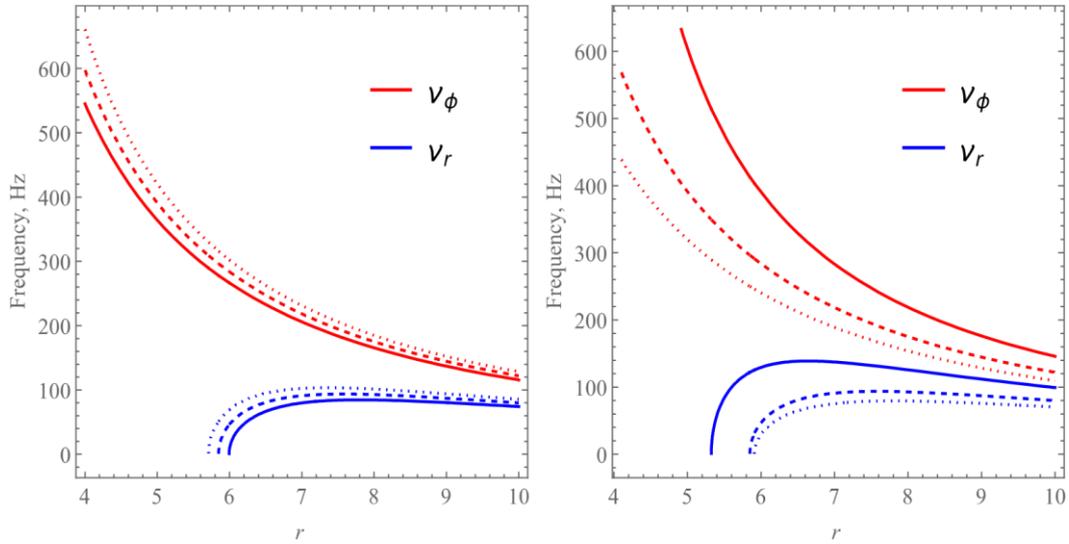

4-rasm: (Chap panel) – Massiv zarraning asosiy chastotalarining radial bog'liqligi turli bog'lanish parametrlari qiymatlari uchun: g_s=0.1— uzluksiz chiziq, g_s=0.2— uzilgan chiziq, va g_s=0.3 — nuqtali chiziq holatida; bu yerda n=0.7 dep olingan. (O'ng panel) – Massiv zarraning asosiy chastotalarining radial bog'liqligi turli n parametr qiymatlari uchun: n=0.5 — uzluksiz chiziq, n=0.7 — uzilgan chiziq, va n=0.9— nuqtali chiziq holatida; bu yerda g_s=0.2. Har ikkala holatda ham markaziy ob'ekt massasi M=10M$\odot$ deb olingan.

Relativistik pretsessiya (RP) modeli KDT hodisasini tushuntirish uchun nazariy asos bo'lib xizmat qiladi. Ushbu modelga ko'ra, KDTlar qora o'ralar va yumronqoziq inlari atrofida harakatlanayotgan zaryadlangan zarralarning radial va burchak bo'yicha kvazigarmonik tebranishlari bilan bog'liq. RP model doirasida egizak cho'qqili KDTlar quyidagicha talqin qilinadi: yuqori chastota zarra orbital chastotasiga to'g'ri keladi — bu $\nu_U = \nu_\varphi$ bilan belgilanadi; pastki chastota esa orbital va radial chastotalar farqi sifatida ifodalanadi — $\nu_L = \nu_\varphi - \nu_r$. Boshqacha aytganda, egizak cho'qqili KDTlarda yuqori chastota zarra aylanish chastotasini, pastki chastota esa radial tebranish chastotasini ayirish orqali olinadi.
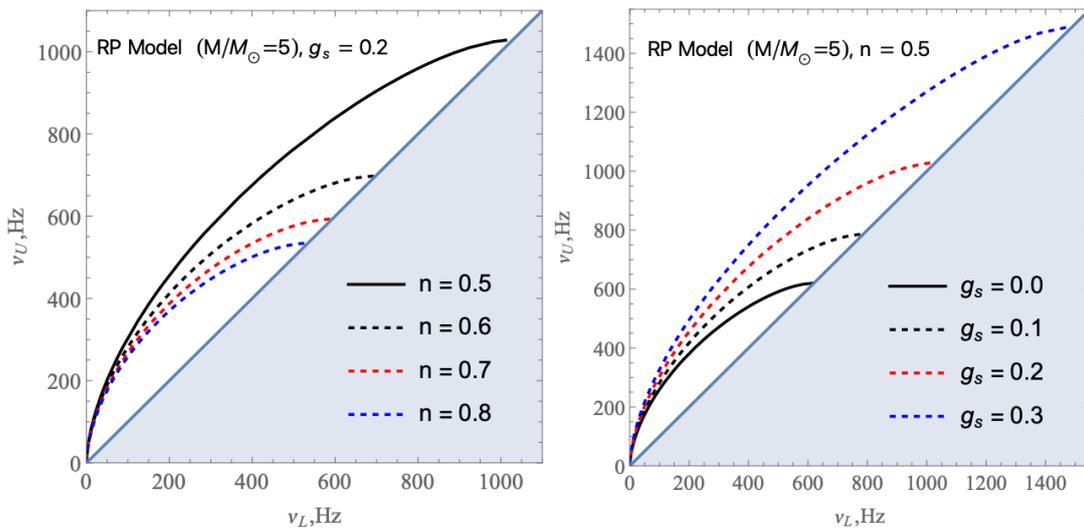
5-rasm RP modeli doirasida egizak KDTlarning yuqori (ko'k chiziq) va pastki (qora chiziq) chastotalarining radial bog'liqligini ko'rsatadi. Chap panelda uzluksiz

chiziq n=0.5 holatini, uzilgan chiziq esa n=0.7 holatini tasvirlaydi, bu yerda g_s=0.2 dep olingan. O'ng panelda esa uzluksiz chiziq g_s=0.1, uzilgan chiziq esa g_s =0.3 qiymatlariga mos keladi, bu yerda n=0.7 dep olingan. Grafiklardan ko'rinib turibdiki, 3:2 chastotalar nisbati sodir bo'ladigan radial pozitsiya n va g_s parametrlari oshgan sari yalang'och singulyarlik tomon biroz siljiydi.

6-rasmda esa turli n va g_s qiymatlari uchun egizak cho'qqili KDTlarning yuqori va pastki chastotalari orasidagi bog'liqlik ko'rib chiqilgan. Tahlil shuni ko'rsatadiki, n ortgani sari yuqori hamda pastki chastotalar kamayadi. Aksincha, g_s ortishi bu chastotalarning oshishiga olib keladi. Grafiklardan shuni ham ko'rish mumkin: 6-rasmda soya bilan belgilangan hudud ichida chastotalar kuzatib bo'lmaydi, sababi bu oraliqlar IBAO pozitsiyasining ichki sohasiga to'g'ri keladi.
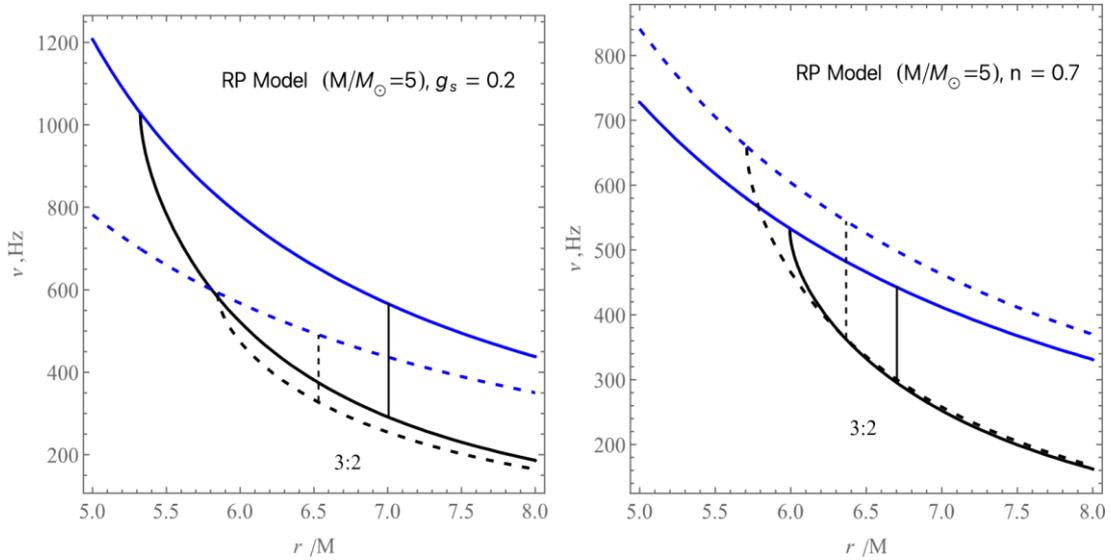
5-rasm: RP modelida 3:2 nisbatda kuzatiladigan yuqori va pastki chastotalarning radial bog'liqligi.


6-rasm: RP modelida massa bilan bog'liq egizak cho'qqili KDTlarning yuqori va pastki chastotalari orasidagi o'zaro bog'liqlik M = 5M☉

7-rasmda ilgari tanlab olingan uchta turli manba uchun JNW fazoviy-vaqtiga o'xshash parametrlar (M, g_s, n, r) uchun eng yaxshi mos keluvchi qiymatlar burchakli (corner) grafiklar orqali tasvirlangan. Bu natijalar MCMC tahlillari yordamida 1σ aniqlik darajasida olingan. Mos keluvchi eng yaxshi qiymatlar 1-jadvalda keltirilgan.

Olingan natijalar har bir mikrokvazar uchun JNW parametrlariga cheklovlar qo'yadi. Shuningdek, KDT ma'lumotlari asosida aniqlangan eng yaxshi mos parametrlar qiymatlari ham keltirilib, bu RP modelining mazkur rentgen nurli qo'shaloq yulduz tizimlarida KDT hodisasini tavsiflashdagi samaradorligini ko'rsatadi. Ushbu tadqiqot kuchli tortishish maydonlaridagi kompakt ob'ektlar dinamikasi va ularning xossalarini chuqurroq tushunishga xizmat qiladi.

1-Jadval: Tanlangan X-nurli manbalar uchun KDT asosida aniqlangan JNW fazoviy-vaqtiga mos eng yaxshi parametr qiymatlari.

	XTE J1550-564	GRO J1655-40	GRS 1915+105
$M(M_\odot)$	8.96 ± 0.50	5.31 ± 0.27	$12.72^{+0.66}_{-0.82}$
g_s	$0.251^{+0.065}_{-0.096}$	$0.209^{+0.059}_{-0.11}$	$0.138^{+0.077}_{-0.12}$
n	0.734 ± 0.083	0.755 ± 0.092	$0.779^{\pm}0.085$
r/M	6.498 ± 0.032	6.538 ± 0.069	6.700 ± 0.014
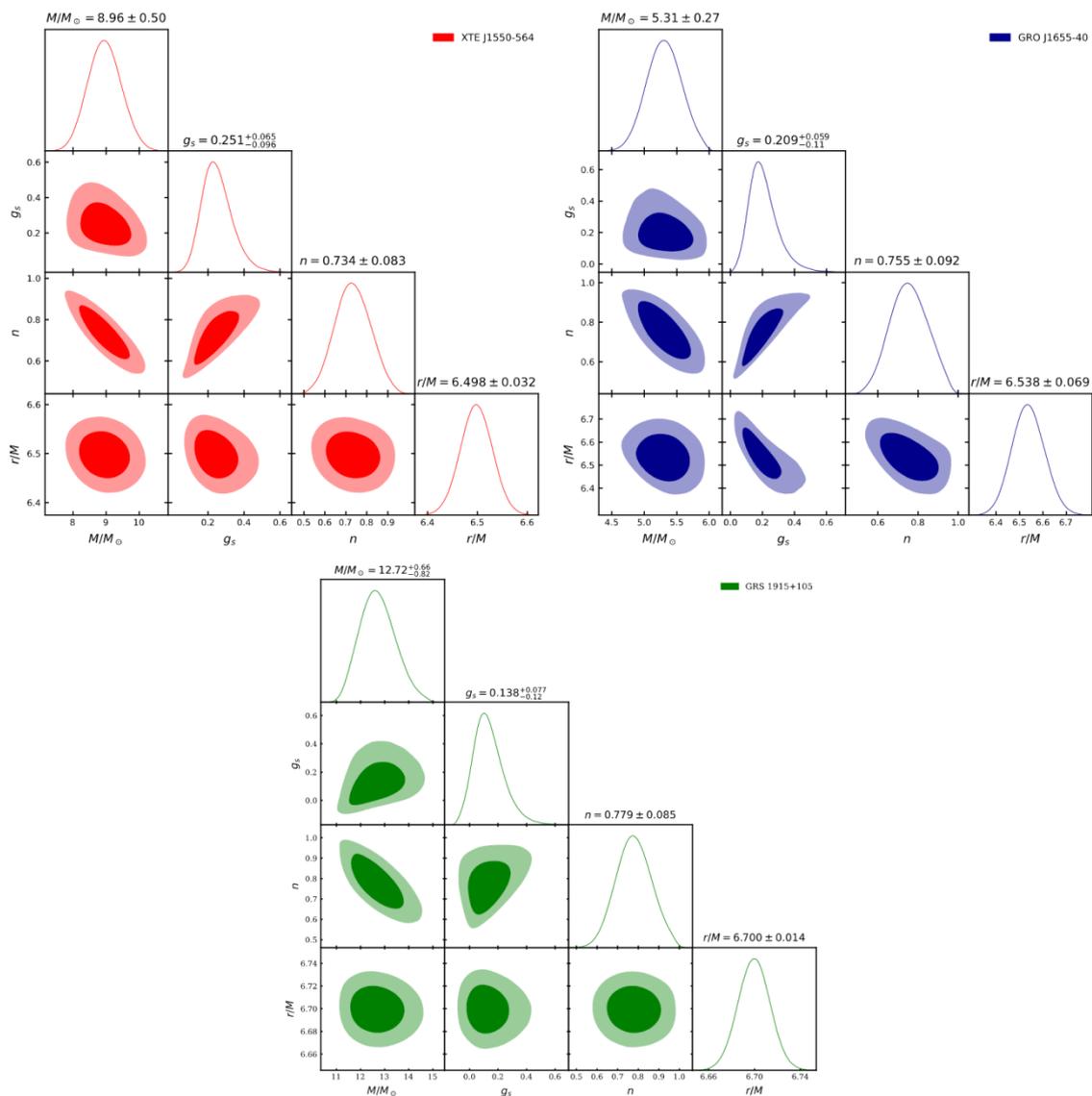
7-rasm. JNW massasiga, shuningdek**,** g_s va n parametrlariga qo'yilgan cheklovlar: XTE J1550-564 (yuqori chap), GRO J1655-40 (yuqori o'ng) va GRS 1915+105 (pastki panel) mikrokvazarlari uchun MCMC tahlili yordamida aniqlangan.

   Dissertatsiyaning uchinchi bobi — "Ellis fazo-vaqtining yangi xususiyatlarini o'rganish: skalyar maydon dinamikasiga oid qarashlar" deb nomlanadi. Ushbu bobda biz Ellis fazo-vaqtini tashqi skalyar maydon ishtirokida umronqoziq ini atrofida zarra harakatini ko'rib chiqish orqali sinovdan o'tkazdik. Tashqi skalyar maydon mavjudligida fon fazo-vaqtiga va test zarraning dinamik harakatiga tegishli asosiy tenglamalar keltirildi. Shuningdek, zarraning harakati va unga nurlanish reaksiyasi ta'siri ham o'rganildi.

   Ellis yumronqoziq ini quyidagi fazoviy-vaqt chiziqli element (line element) bilan ifodalanadi:

$$ds^2 = -dt^2 + dr^2 + (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\phi^2) \ , \qquad\qquad \text{(7)(7)}$$

tegishli skalyar maydon bilan birgalikda

$$\Phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{r}{r_0}\right) \ , \qquad\qquad \text{(8) (8)}$$

bu yerda r_0 — yumronqoziq inining bo'yin radiusi hisoblanadi. Radial koordinata r, r_0 dan boshlab cheksizlikkacha o'zgaradi, ya'ni $r_0<r<\infty$. Fazo-vaqtning skalyar invariantlari, masalan, Ritchi skalyari va Kretschmann skalyari quyidagicha ifodalash mumkin:

$$R = -\frac{2r_0^2}{(r^2 + r_0^2)^2} \ , \qquad K = \frac{12r_0^4}{(r + r_0^2)^4} \ , \qquad\qquad \text{(9)}$$

bu invariantlar $r>r_0$ bo'lgan fazoviy-vaqtning istalgan nuqtasida regulyar (singulyarsiz) hisoblanadi. 8-rasmda skalyar maydon, shuningdek, **Ricci** va **Kretschmann** invariantlarining radial bog'liqligi tasvirlangan.

Bundan tashqari, Ellis fazoviy-vaqtida massiv zarraning asosiy chastotalari, ya'ni orbital va epitsiklik chastotalari ham muhokama qilindi. 9-rasmda orbital, radial va vertikal chastotalarning radial bog'liqligi grafik tarzda ko'rsatilgan.

Tashqi magnit maydon mavjud bo'lgan holatda zaryadlangan zarra uchun Lagrange funksiyasi quyidagicha ifodalanadi:

$$L = \frac{1}{2}m_* g_{\mu\nu}u^\mu u^\nu + qA_\mu u^\mu \ , \qquad\qquad \text{(10)}$$

bu yerda q — test zarraning zaryadi. Lagrange tenglamasidan kelib chiqqan harakat tenglamasi quyidagicha hosil qilinadi:

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \ln\frac{m_*}{m} \ , \qquad\qquad \text{(11)}$$

hamda oddiy algebraik o'zgartirishlardan so'ng, radial harakat tenglamasi quyidagi ko'rinishni oladi:

$$\dot{r}^2 = \frac{1}{(1 + \mathrm{g}_s\Phi)^2}\left[\mathcal{E}^2 - \left(r^2 + r_0^2\right)\left(\frac{\mathcal{L}}{r^2 + r_0^2} - \omega\right)^2 \right] - 1 \ , \qquad \text{(12)}$$

bu yerda ω=qB/2m— magnit parametri hisoblanadi. Zaryadlangan test zarrasi uchun chegaraviy bog'langan aylana orbitaning radiusi quyidagi shartlar asosida aniqlanadi:

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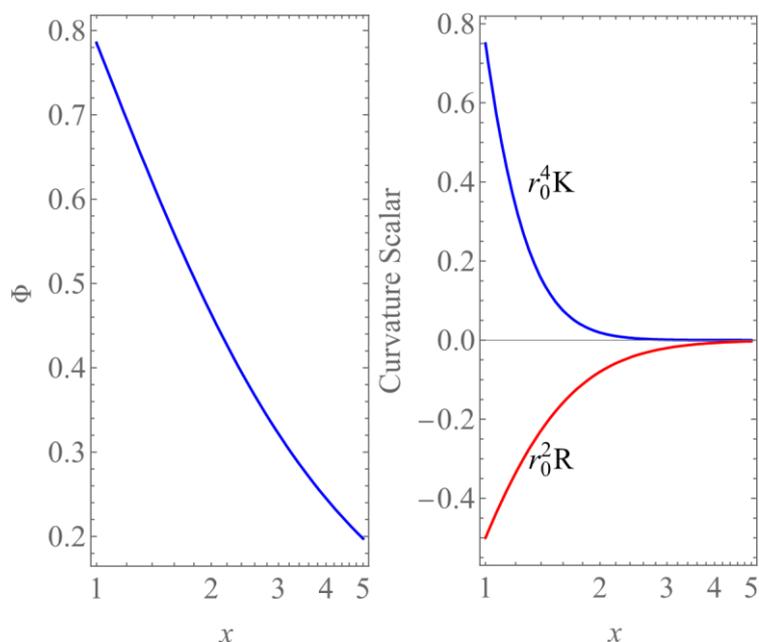
8-rasm: Skalyar maydon $\Phi(x)$ ning va egrilik invariantlari o'lchamsiz Ricci skalyari hamda o'lchamsiz Kretschmann skalyari ning radial bog'liqligi.
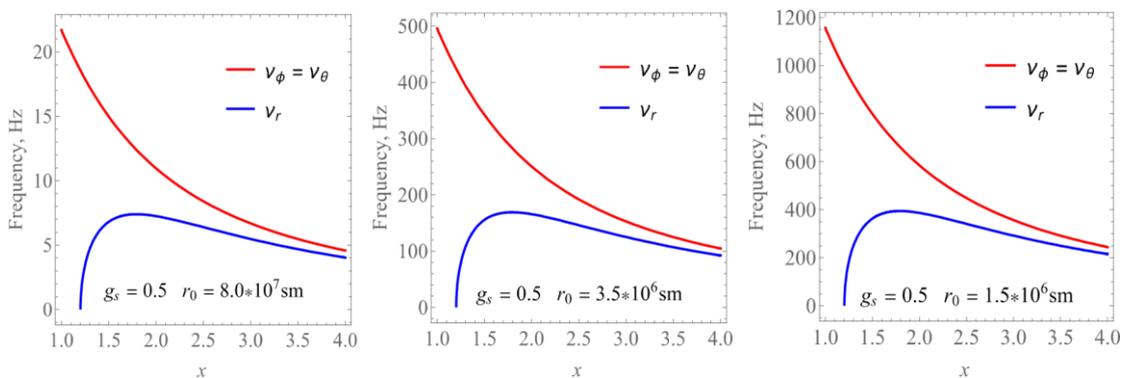
9-rasm: Massiv zarraning asosiy chastotalarining radial bog'liqligi.
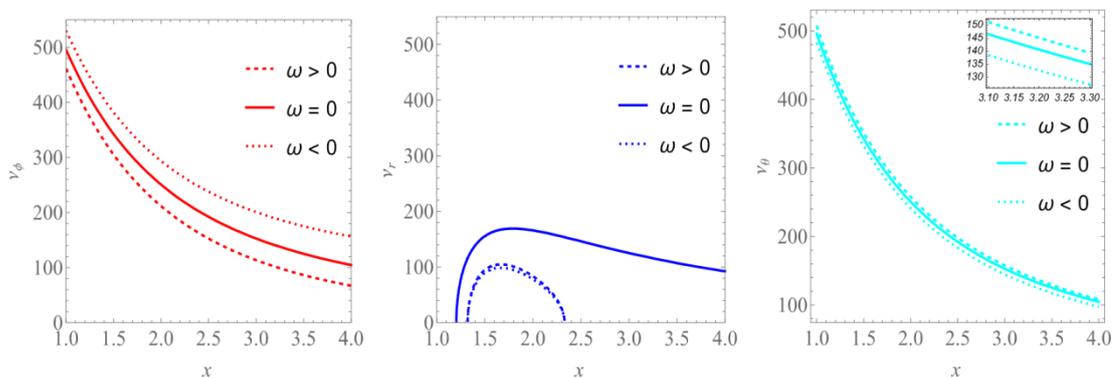
10-rasm: Zaryadlangan zarraning Ellis chuqurchasi atrofidagi asosiy chastotalari uchun $\omega=\pm0.4$ va bog'lanish parametri $g_s=0.5$ deb olingan.

$$\mathrm{g}_s x + \left(1 - x^2\right)\left(1 + \mathrm{g}_s\Phi\right) + 4\omega^2 x^3(1 + x^2) = 0 \ . \tag{13}$$

Ellis yumronqoziq ini yaqinida tashqi magnit maydon ishtirokida zaryadlangan zarraning asosiy chastotalarini (orbital, radial va vertikal) aniqlash oson. Kepler chastotasi (orbital chastota) quyidagi ifodadan hosil qilinadi:

$$\Omega^2 + \omega\Omega\sqrt{1 - \Omega^2(x^2 + 1)} = \frac{\mathrm{g}_s\left[1 - \Omega^2(x^2 + 1)\right]}{x(x^2 + 1)(1 + \mathrm{g}_s\Phi)} \ . \tag{14}$$

Xuddi shuningdek, Ellis yumronqoziq ini atrofida tashqi magnit maydon ishtirokida harakatlanuvchi zaryadlangan zarraning epitsiklik chastotalari ham aniqlanishi mumkin. Biroq, biz bu yerda batafsil hisob-kitoblarni keltirmaymiz. 10-rasmda zaryadlangan zarraning asosiy chastotalarining radial bog'liqligi ko'rsatilgan. Rasmga ko'ra, tashqi magnit maydon ta'sirida asosiy chastotalar bir-biridan ajraladi (ya'ni, chastotalarning ajralishi sodir bo'ladi). Shu bilan birga, natijalarimiz Kepler chastotalari magnit parametriga kuchli bog'liqligini ko'rsatadi.

LIGO va Virgo hamkorliklari ikkilik tizimlardan gravitatsion to'lqinlarni qayd eta boshlagandan so'ng qora o'ralardagi g'alayonlanish (perturbatsiya) hozirgi astrofizikadagi dolzarb mavzulardan biridir. Bunday perturbatsiyalar yumronqoziq ini fazo-vaqtiga ham qo'llanilishi mumkin. Bu yerda biz Ellis fazo-vaqtining perturbatsiyasini ko'rib chiqamiz.

Yuqorida ta'kidlanganidek, Ellis fazo-vaqti — bu Einstein-skalyar maydon tenglamalarining yechimidir. Shuning uchun skalyar maydon va metrik tensor quyidagicha ifodalanadi:

$$\Phi = \Phi_0 + \delta\Phi, \ \mathrm{g}_{\mu\nu} = \mathrm{g}_{\mu\nu}(0) + \delta\mathrm{g}_{\mu\nu}$$

bu yerda Φ_0 va $\mathrm{g}_{\mu\nu}(0)$ — mos ravishda skalyar maydon va fon fazo-vaqt metrikasi (7) va (8)-tenglamalarda berilgan, $\delta\Phi$ esa buzilgan (perturbatsiyalangan) skalyar maydonni bildiradi.

$$\delta\Phi = e^{-i\omega t}F(r)P_\ell(\cos\theta) \ , \tag{15}$$

hamda buzilgan metrik tenzor komponentalari quyidagicha aniqlanadi:

$$\tag{16}$$

$$h_{\mu\nu} = e^{-i\omega t}\begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & K(r^2 + r_0^2) & 0 \\ 0 & 0 & 0 & K(r^2 + r_0^2)\sin^2\theta \end{pmatrix}P_\ell(\cos\theta) \ ,$$

bu yerda $H_0(r)$, $H_1(r)$, $H_2(r)$, $K(r)$, $F(r)$ — hali aniqlanmagan radial funksiyalar, $P_\ell = P_\ell(\cos\theta)$ esa Legendre polinomi bo'lib, u quyidagi differensial tenglamani qanoatlantiradi:

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$$\frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d}{d\theta})P_\ell + \ell(\ell+1)P_\ell = 0 \ . \tag{17}$$

Biz endi radial funksiyalar uchun maydon tenglamalarining yechimlarini ko'rib chiqamiz. Keyingi tahlillarga o'tishdan oldin, quyidagicha ta'riflanuvchi o'lchamsiz chastota kiritiladi: $\omega_0=\omega_{r0}$. Bu yerda ω — fizik chastota, r_0 esa yumronqoziq inining bo'yin radiusi. F funksiyasi uchun tenglama esa quyidagicha oson hosil qilinadi:

$$\left[\left(x^2+1\right)F'\right]' + \left[\omega_0^2 x^2 + \frac{4}{x^2+1} - 4\eta\right]F = 0 \ , \tag{18}$$

bu yerda $\eta=\ell^2+\ell$— separatsiya konstantasi bo'lib, yuqoridagi tenglama uchun analitik yechim, Heun funksiyasi (ya'ni, HeunC(a,b,c,d,x) yordamida quyidagicha yozilishi mumkin:

$$F(r) = \left(x^2+1\right)\left[c_1 F_{1\ell}(x) + c_2 x F_{2\ell}(x)\right] \ , \tag{19}$$

bu yerda

$$F_{1\ell}(x) = \text{HeunC}\left[\eta - \frac{3}{2}, -\frac{\omega_0^2}{4}, \frac{1}{2}, 3, 0, -x^2\right] \ , \tag{20}$$

$$F_{2\ell}(x) = \text{HeunC}\left[\eta - 3, -\frac{\omega_0^2}{4}, \frac{3}{2}, 3, 0, -x^2\right] \ . \tag{21}$$

Biz aniqladikki, Ellis fazo-vaqtida skalyar perturbatsiyaning yechimi aniq analitik ifoda bilan – ya'ni birlashtirilgan Heun funksiyasi bilan tasvirlanadi. 11-rasmda F(x) funksiyasining radial bog'liqligi keltirilgan. Grafikdan ko'rinib turibdiki, radial profil funksiyasi F(x) tebranma harakat qilgan holda asta-sekin kamayib boradi.

Ushbu tadqiqot doirasida biz Ellis fazoviy-vaqtidagi skalyar va gravitatsion perturbatsiyalarni tahlil qildik. Faraz qilamizki, skalyar va gravitatsion to'lqinlar bir xil chastotada tarqaladi va bu to'lqinlar sferik garmonikalar asosida kengaytirilgan. Tahlil natijalariga ko'ra, skalyar profil funksiyasi uchun tenglama butunlay tenzor profil funksiyalaridan mustaqil bo'lsa-da, tenzor profil funksiyalari uchun tenglamalar Ellis fazo-vaqtida skalyar profil funksiyasiga kuchli bog'liq ekanligi ko'rsatildi.

Shuningdek, biz vaqtga bog'liq bo'lmagan yechimlar — ya'ni skalyar va gravitatsion buzilishlar uchun yechimlar kompleks argumentga ega Legendre va biriktirilgan Legendre funksiyalari orqali ifodalanishini aniqladik. Biroq to'lqin zonasi doirasida statsionar yechimlar ko'rib chiqilganda, skalyar g'alayonanishlar uchun aniq analitik yechim birlashtirilgan Heun funksiyasi yordamida olinadi.

Qayd etish joizki, gravitatsion g'alayonlanishlarni boshqaruvchi tenglamalar ancha murakkab bo'lishiga qaramay, ularni Regge–Wheeler–Zerilli tenglamasi

shakliga keltirish orqali soddalashtirish mumkin. Nihoyat, biz bu tenglama uchun radial funksiyalar bo'yicha sonli yechimlarni ham taqdim etdik.

Dissertatsiyaning to'rtinchi bobi — "Magnit maydon ishtirokidagi regular Shvartsshild-simon qora o'ralarning uzoq yashovchi kvazinormal modalar va asimptotik dumlari" deb nomlanadi. Ushbu bobda biz Schwarzschild-simon fazo-vaqtida zayadli skalyar maydon g'alayonlanishini ko'rib chiqamiz. Schwarzschild-simon qora o'raning chiziq elementi quyidagicha berilgan:

$$ds^2 = -f\, dt^2 + f^{-1}\, dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

(22)

bu yerda metrik funksiya quyidagicha ifodalanadi:

$$f = 1 - \frac{2Me^{-a/r}}{r}\,.$$

(23)

Bu yerda M qora o'ra massasini, a esa Simpson va Visser tomonidan kiritilgan og'ish (deviatsiya) parametrini bildiradi. Shuni ta'kidlash joizki, yuqorida keltirilgan fazo-vaqt metrikasi umumiy nisbiylik nazariyasidagi standart Schwarzschild qora o'rasiga a→0 limitida mos keladi.
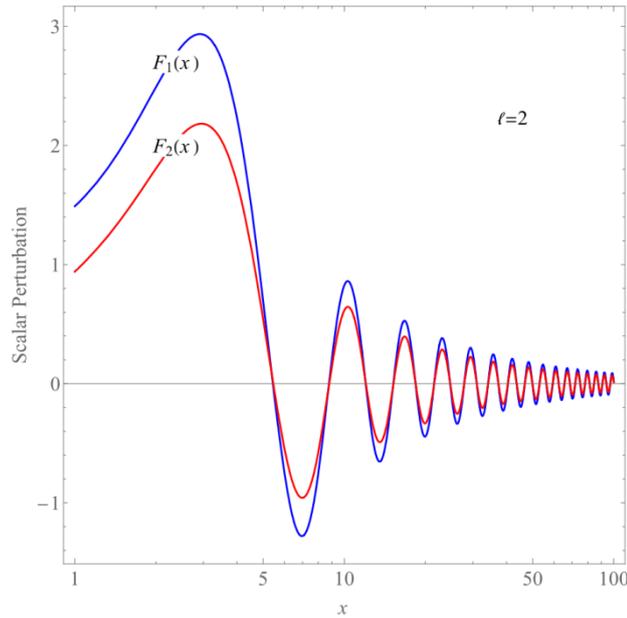
11-rasm: $\ell = 2$ holda skalyar g'alayonlanishning radial bog'liqligi.

Zaryadlangan, massiv skalyar maydon Ψ uchun elektromagnit maydon mavjudligida relativistik Klein–Gordon tenglamasi quyidagicha ifodalanadi:

$$g^{\alpha\beta}(\Delta_\alpha - iqA_\alpha)(\Delta_\beta - iqA_\beta)\Psi - \mu^2\Psi = 0\,,$$

Bu yerda μ — skalyar maydon massasi, q — skalyar va elektromagnit maydonlar orasidagi kupling konstantasi, ∇α esa kovariant hosila, *i* — mavhum birlikni bildiradi.

(23)-tenglamada o'zgaruvchilarni ajratish murakkab bo'lishiga qaramay, quyidagi fizik jihatdan asosli taxminlar orqali muammoni soddalashtirish mumkin:

- Vektor potentsial uchun Lorens kalibrlash sharti: ∇αAα = 0;
- Kuchsiz o'zaro ta'sir limitida, yuqori tartibli hadlarni, masalan, q²B², e'tiborsiz qoldirish mumkin, ya'ni, *q²B² → 0*. Shunda (23)-tenglama quyidagi ko'rinishga keladi:

$$\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta\Psi) - 2iqA^\alpha\partial_\alpha\Psi - \mu^2\Psi = 0 , \tag{25}$$

Yechimni quyidagicha yozishimiz mumkin:

$$\Psi(t,r,\theta,\phi) = e^{-i\omega t}Y_{lm}(\theta,\phi)\frac{R(r)}{r}. \tag{26}$$

Biz tashqi asimptotik jihatdan bir jinsli magnit maydonga joylashtirilgan Shvartsshildga o'xshash qora o'ra uchun kvazinormal rejimlar (QNM) bo'yicha olib borilgan raqamli hisoblash natijalarini qisqacha taqdim etamiz.

Chastota sohasidagi tahlil uchun biz yarim-analitik WKB usulidan foydalanamiz. Ushbu usulda yechim har ikki cheksizlikda WKB qatoriga kengaytiriladi va bu asimptotik kengaytmalar effektive potensialning maksimumi yaqinidagi Teylor qatoriga yoyiladi.

Yuqori tartibli WKB formulasi quyidagicha beriladi:

$$\omega^2 = V_0 + A_2(K^2) + A_4(K^2) + A_6(K^2) + \ldots$$
$$- iK\left(-2V_1 + A_3(K^2) + A_5(K^2) + A_7(K^2) + \ldots\right) \tag{27}$$

bu yerda *K = n + 1/2*, va *n = 0, 1, 2, 3, ...*

12-rasm tashqi magnit maydonda joylashgan massasiz skalyar maydon uchun WKB usuli yordamida olingan ma'lumotlarga mos keluvchi eng yaxshi polinom funksiyani ko'rsatadi. Rasmda ikki xil egri chiziq tasvirlangan: kulrang va qora – ular fazo-vaqt parametrlarining turli qiymatlariga mos keladi, ya'ni: a*/M*=0.3 (kulrang chiziq) va a*/M*=0.5 (qora chiziq). Bu parametr qiymatlari uchun kvazinormal rejim chastotasi ω ning mavhum qismlari rasmning o'ng tomonida tasvirlangan.

Har bir chiziq zaryad magnit parametri qB ning nol qiymatini quyidagi nuqtalarda kesib o'tadi: Kulrang chiziq uchun: qB≈0.611711, Qora chiziq uchun: qB≈0.651842.
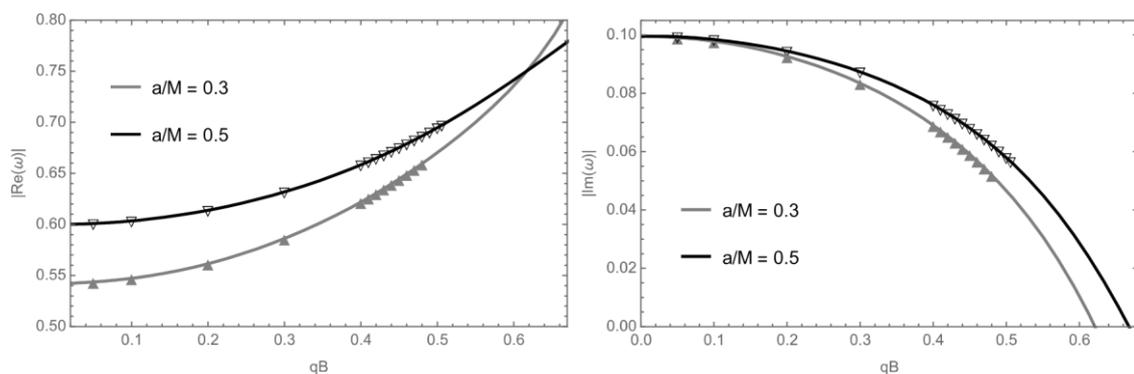
12-Rasm: Fazo-vaqt qiymatlarining turli qiymatlari (l = 2, m = −2) uchun WKB ma'lumotlariga eng yaxshi mos keluvchi polinomial funksiyalar.

Bu yerda WKB natijalari vaqt sohasidagi integrallash orqali tekshirildi. Biroq, bunday taqqoslashni to'g'ri amalga oshirish uchun ikki muhim jihatni hisobga olish zarur: $\ell = 0$ perturbatsiyalar uchun kvazinormal tebranish davri juda qisqa bo'ladi, chunki u tezda tebranma xarakterdagi kuch qonuni (power-law) quyruqlari bilan almashtiriladi (14-rasmga qarang). Shu sababli, bu holatda vaqt sohasidagi profildan chastotani aniqlik bilan ajratib olish qiyin bo'ladi. Ikkinchi muhim jihat — bu kechki vaqt oralig'ida perturbatsiyaning o'sishi (13-rasmga qarang). Bunday o'sish, ehtimol, m > 0 bo'lgan holatda katta r qiymatlarida effektiv potensialning manfiy bo'lishi tufayli yuzaga keladigan barqarorlik buzilishi (noaniqlik) deb talqin qilinishi mumkin.
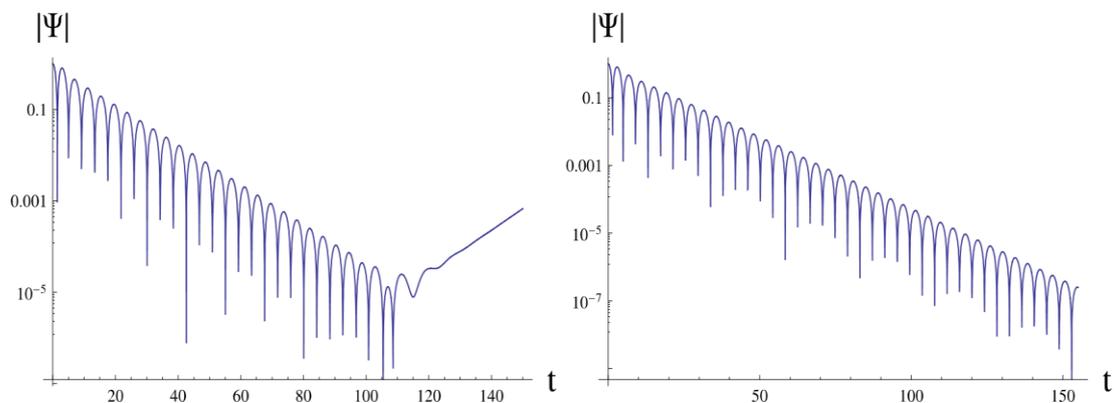
13-Rasm: Yarim-logarifmik grafiklarda m=3 (chap) va m=−3 (o'ng) uchun vaqt sohasidagi profil chiziqlari tasvirlangan. Bu yerda parametrlar quyidagicha olingan: ℓ=3, qB=0.1, a=0.3, μ=0.1, M=1.
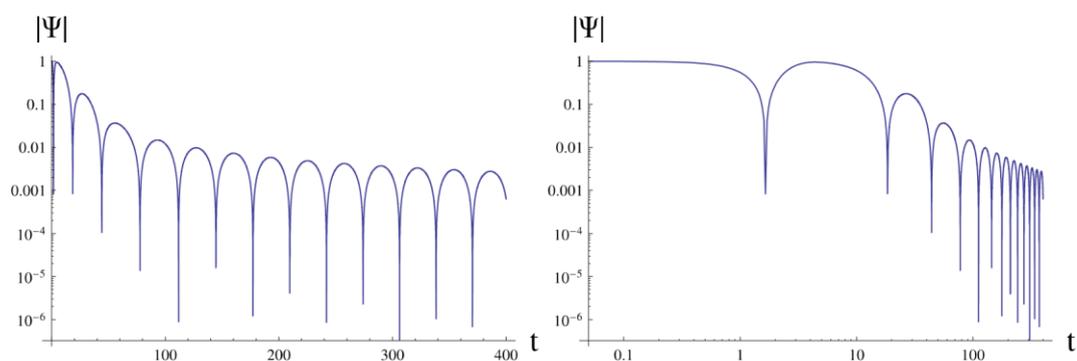
14-Rasm: Vaqt sohasi profillarining yarim logarifmik grafiklari (chapda) va logarifmik grafik (o'ngda); l = m = 0, qB = 0.1, a = 0.3, μ = 0.1, M = 1. Kvazinormal tebranishlarning qisqa davri tezda asimptotik dumlar bilan almashinadi, bu esa profildan chastotani yetarli aniqlik bilan ajratib olishni imkonsiz qiladi.
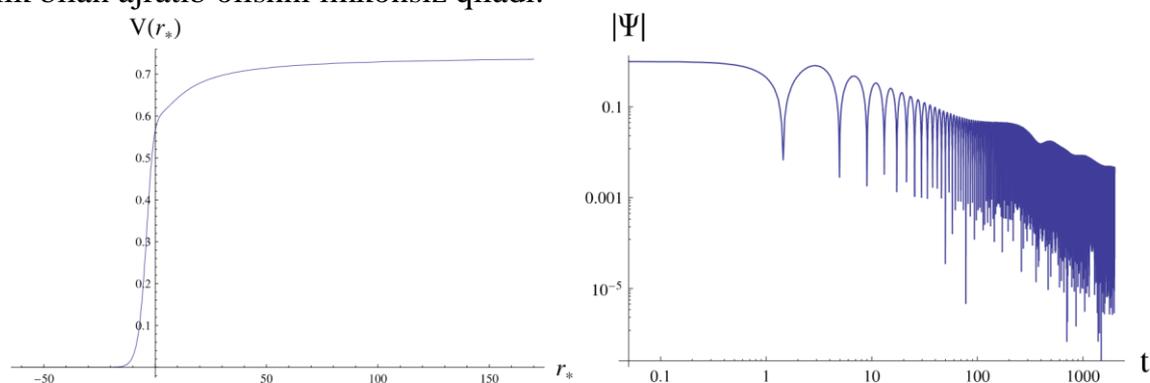
15-Rasm: Shuningdek, m=−2 uchun (ℓ=2, qB=0.61, a=0.3, μ=0, M=1) samarali potensial va logarifmik ko'rinishdagi vaqt sohasidagi profil grafigi tasvirlangan.

Garchi regular qora o'ralarning kvazinormal modalari ko'plab ishlarda keng qamrovli o'rganilgan bo'lsa-da, tashqi magnit maydon ishtirokidagi regular qora o'ralar uchun hech qanday keng qamrovli tadqiqotlar olib borilmagan. Ushbu ishda biz bu bo'shliqni to'ldirishga harakat qildik va magnit maydon zaryadlangan skalyar maydon spektrini sezilarli darajada o'zgartirishini namoyish etdik, bu esa o'z navbatida kvazi-rezonanslar deb ataluvchi uzoq yashovchi kvazinormal modalarning paydo bo'lishiga olib keladi.

Magnit maydon ishtirokida g'alayonlanishlar (perturbatsiyalar) evolyutsiyasining yana bir o'ziga xos jihati asimptotik dumlarning g'ayrioddiy harakatidir. Parametrlarning muayyan qiymatlarida bu dumlar darajali qobiqni namoyon etmaydi, aksincha, ossillyatsion (tebranuvchi) qobiqni namoyish etadi.

Xulosa

"Skalyar maydon mavjudligida kompakt ob'ektlar atrofida relyativistik astrofizik jarayonlar" mavzusida olib borilgan ilmiy tadqiqot asosida quyidagi ilmiy xulosalarga kelindi:

1. JNW yalang'och singulyarlik fazo-vaqtida skalyar maydonning itaruvchi ta'siri mavjudligi ko'rsatildi. Bu ta'sir ichki barqaror aylana orbitaning radiusini skalyar parametr n ga bog'liq holda nomonoton tarzda o'zgartiradi. Schwarzschild qora o'rasidan farqli ravishda, ma'lum kritik qiymatlarda IBAO butunlay yo'qoladi, bu esa singulyarlik atrofida barqaror orbitalarning buzilishini bildiradi.

2. JNW fazo vaqtida massiv zarralar uchun effektiv potensial, solishtirma energiya va burchak momentining aniq analitik ifodalari birinchi marta chiqarildi. Tadqiqotlar shuni ko'rsatdiki, skalyar bog'lanish parametri g_s ning qiymati musbat bo'lsa – IBAO kamayadi, manfiy bo'lsa – ortadi. Mazkur fazo vaqtda akretsiya disklarining nurlanish samaradorligi Schwarzschild qora o'rasiga nisbatan 12% gacha yuqori bo'lishi aniqlangan.

3. Ellis yumronqoziq inining bo'yin radiusi r_0 va skalyar bog'lanish parametri g_s barqaror orbitalarning radiusi orasidagi bog'lanish olindi. Radiatsion reaksiyani hisobga olganda, zarralar harakati qochish trayektoriyalarini namoyon etadi, bu esa qora o'ralardagi ushlab qolish xarakteridan keskin farq qiladi. Tebranish chastotalari hisoblab chiqildi va KDT larda sezilarli siljishlar aniqlangan.

4. Magnitlangan Ellis yumronqoziq inida zaryadlangan zarra dinamikasi o'rganildi. Magnit maydon ostidagi Ellis yumronqoziq inida zaryadlangan zarra Lorentz kuchi ta'sirida uning fundamental chastotalari bo'linib ketadi. Ma'lum magnit kuchlarida kvazi-rezonans rejimlar vujudga keladi. Nurlanish intensivligi skalyar va elektromagnit o'zaro ta'sirlarga bog'liqligi ko'rsatildi.

5. Markov zanjiri Monte-Carlo (MCMC) usuli birinchi bor JNW fazoviy-vaqti parametrlarini (M,g_s,n) KDT rengen nurlari ma'lumotlari (masalan, GRO J1655–40) asosida aniqlashda qo'llanildi. Eng yaxshi mos keluvchi qiymatlar n∽0.75, g_s ∽0.2 deb topildi, bu zaif skalyar maydon ustunligini ko'rsatadi.

6. Ellis yumronqoziq ini yechimidagi skalyar perturbatsiyalar uchun yechimlar aniq analitik ko'rinishda, ya'ni Heun funksiyasi orqali ifodalanishi ko'rsatildi. Bunda gravitatsion perturbatsiyalar esa Regge-Wheeler tipidagi tenglamaga bo'ysunadi va g'ayrioddiy tebranma quyruqlarga ega, bu holat qora o'ralarda kuzatiladigan quvvat qonuni(Power-Law)ga asoslangan so'nishdan farqlidir.

7. Tashqi magnit maydonga joylashtirilgan Schwarzschildga o'xshash qora o'ralar uchun kvazinormal rejimlar WKB yondashuvi va vaqt sohasidagi integratsiya usullari yordamida hisoblab chiqildi. Ular orasida uzoq yashovchi kvazi-rezonanslar va anomaliya ko'rsatuvchi kechki vaqt quyruqlari aniqlandi. Ushbu xususiyatlar kelajakdagi gravitatsiya to'lqinlari kuzatuvlari orqali, skalyar maydon mavjud bo'lgan fazo-vaqtlarni farqlash imkonini beradi.

SCIENTIFIC COUNCIL DSc. 03/07.07.2025.FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES AT "NEW UZBEKISTAN" UNIVERSITY

"NEW UZBEKISTAN" UNIVERSITY
ASTRONOMICAL INSTITUTE

DAVLATALIEV AKBARJON AKMALJON UGLI

RELATIVISTIC ASTROPHYSICAL PROCESSES AROUND COMPACT OBJECTS COUPLED WITH SCALAR FIELD

01.03.01 – Astronomy
01.04.02 – Theoretical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) IN PHYSICS AND MATHEMATICS

TASHKENT-2025

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The defense of the dissertation will be held on "___" _____ 2025 at ___ in the meeting of the Scientific Council No. DSc.03/07.07.2025.FM/T.192.01 at the Institute for Advanced Studies at "New Uzbekistan" University (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnahr Street 1, Institute for Advanced Studies at "New Uzbekistan" University, phone: +99871 202-41-11; e-mail: (info@newuu.uz)

The dissertation can be looked through at the Information Resource Center of the Institute for Advanced Studies at "New Uzbekistan" University (registered under №__). (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnahr Street 1, Institute for Advanced Studies at "New Uzbekistan" University, phone: +99871 202-41-11).

Abstract of dissertation sent out on « _____ » _____ 2025 year
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</div>

INTRODUCTION (Annotation of PhD dissertation)

Topicality and demand of the dissertation. With the increasing precision of astronomical observations such as gravitational wave detections (LIGO/Virgo) and highfrequency quasi-periodic oscillations (QPOs) from X-ray binaries—there is a growing demand for theoretical frameworks that extend beyond classical general relativity. This work provides essential insights into how scalar fields and external magnetic fields modify spacetime dynamics, offering testable predictions for future observations. The study of exotic compact objects (e.g. JNW naked singularities, Ellis wormholes) also aligns with ongoing efforts to probe the limits of Einstein theory and explore alternative gravity models. The dissertation addresses critical questions in modern relativistic astrophysics, particularly the interplay between compact objects (black holes, naked singularities, and wormholes) and scalar fields.

The dissertation's focus on scalar-tensor couplings and their astrophysical implications is highly relevant, given recent developments in modified gravity, dark matter, and dark energy research. Scalar fields are fundamental in theories like $f(R)$ gravity and string-inspired models, making this work pertinent to both theoretical and observational cosmology. Additionally, the analysis of quasi-resonant modes and QPOs contributes to the interpretation of data from next-generation telescopes (e.g., LISA, Athena) and X-ray observatories (e.g., NICER). The unusual late-time tails of perturbations in magnetized black holes, as explored in Chapter IV, could even shed light on anomalies in pulsar timing array (PTA) signals, such as those reported by NANOGrav.

By bridging particle dynamics, perturbation theory, and observational constraints, this dissertation advances the toolkit for studying compact objects in extreme environments. Its methodologies, such as the WKB approximation for quasinormal modes and MCMC analysis for QPOs, are transferable to other compact object scenarios, including neutron stars and exotic horizonless objects. The work also underscores the importance of interdisciplinary approaches, combining general relativity, quantum field theory, and data science to tackle open problems in astrophysics. As the community seeks to resolve tensions between general relativity and quantum mechanics, studies like this pave the way for new physics beyond the Standard Model. The topicality, demand, and actuality of the dissertation lie in its direct engagement with cutting-edge questions in relativistic astrophysics, its applicability to current and future observational campaigns, and its potential to influence theoretical paradigms in gravity and cosmology.

Furthermore, the dissertation aligns with the strategic scientific priorities in Uzbekistan and the broader international community. This dissertation work corresponds to the tasks of the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Decree of the President of the Republic of Uzbekistan No. PP-2789 "On Measures for Further Improvement of Academy of Sciences, Organization, Management, and Financing of Research Activities from 18.02.2017.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

Degree of study the problem. While the motion of test particles in black hole spacetimes has been extensively studied in general relativity (Subrahmanyan Chandrasekhar, Brandon Carter, James Bardeen, Valeri Frolov, Andrei Zelnikov, Pankaj S. Joshi, Remo Ruffini), the inclusion of scalar fields, particularly in the context of naked singularities (JNW spacetime) and wormholes (Ellis spacetime) – introduces novel complexities. This research work builds on foundational studies by Misner, Wheeler, and others but extends them by incorporating scalar couplings, radiation reaction effects, and magnetic fields, which have not been systematically explored in these spacetimes. This represents a significant advance beyond the classical treatments of geodesic motion.

Prior research on scalar fields in astrophysics has largely focused on cosmological applications or black hole perturbations (Claudia de Rham, Thomas P. Sotiriou, Vitor Cardoso, Kostas D. Kokkotas, Toby Crisford, Hideo Kodama), leaving gaps in understanding their role in naked singularities and wormholes. This dissertation fills these gaps by deriving exact analytical solutions (e.g., ISCO radii in JNW spacetime), numerical simulations of particle trajectories, and perturbation analyses (e.g., quasinormal modes in magnetized black holes). The study of quasiperiodic oscillations (QPOs) in scalarmodified spacetimes is particularly original, as it connects theoretical predictions with observational data from X-ray binaries, a link that has rarely been explored in the earlier literature. The inclusion of MCMC methods to constrain spacetime parameters further demonstrates a modern, data-driven approach to testing theoretical models.

Compared with similar studies, this work stands out by integrating scalar fields, electromagnetic interactions, and radiation reaction into a unified framework for multiple types of compact objects. However, certain limitations remain, such as the assumption of weak backreaction (neglecting particle effects on spacetime geometry) and the restriction to static, spherically symmetric solutions. Rotating compact objects (e.g., Kerr-like naked singularities) and nonlinear scalar couplings remain open challenges. Future research could extend this work by incorporating numerical relativity techniques or higher-dimensional theories (e.g., braneworld models), which would deepen the exploration of scalarfield dynamics in more realistic astrophysical scenarios. The dissertation provides a comprehensive but not exhaustive study of the problem, balancing analytical rigor with innovative applications while identifying clear pathways for future research.

Connection of the topic of the dissertation with the scientific research of the higher educational/research institutions, where the dissertation was carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development: F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity" (2021-2026).

The aim of the research of the dissertation is to develop a new theoretical model to study scalar field couplings and reveal new findings on particle dynamics near compact objects.

The tasks of the research:

– to develop a theoretical model coupling scalar fields to particle dynamics in JNW and Ellis spacetimes;
– to derive equations of motion for massive particles under scalar field influence;
– to analyze ISCO modifications due to scalar coupling parameters;
– to investigate radiation reaction effects on particle trajectories near naked singularities;
– to compute fundamental frequencies (orbital, radial, vertical) for QPO applications;
– to conduct MCMC analysis to constrain spacetime parameters using X-ray binary data;
– to explore charged particle dynamics in magnetized wormhole and black hole spacetimes.
– to study perturbation spectra (quasinormal modes, late-time tails) in scalar-modified
    geometries.

The objects of the research are the Janis-Newman-Winicour (JNW) naked singularity spacetime with scalar field coupling; the Ellis wormhole geometry interacting with external scalar fields; Schwarzschild-like regular black holes immersed in external magnetic fields; test particles (neutral and charged) in these modified spacetime geometries.

The subjects of the research are dynamics of massive particles under scalar field influence in exotic spacetimes; modification of orbital mechanics and fundamental frequencies due to scalar couplings;astrophysical implications of scalar-field modified compact objects (QPOs, ISCOs, etc.); perturbation spectra and stability properties of scalarcoupled compact objects.

The methods of the research are analytical modeling was employed to derive exact equations of motion and effective potentials for particles in scalar-coupled spacetimes (JNW, Ellis). Numerical simulations (Runge-Kutta integration) were used to solve geodesic equations and analyze particle trajectories under radiation reaction effects. Semianalytical WKB approximation and time-domain integration techniques were applied to compute quasinormal modes and perturbation spectra. Bayesian MCMC analysis was conducted to fit theoretical QPO frequencies to observational data from X-ray binaries.

The scientific novelty of the research is the follows:

– The research establishes the first complete theoretical model of particle dynamics in JNW naked singularity spacetime with scalar coupling, revealing unique ISCO behavior that distinguishes it from black holes.

- Novel findings demonstrate how scalar fields modify Ellis wormhole physics, including radiation-driven particle escape and characteristic QPO frequencies that differentiate exotic compact objects from black holes.
- The work pioneers MCMC-based constraints on scalar field parameters using X-ray binary data and identifies new spectral signatures for testing modified gravity theories in strong-field regimes.

Practical results of the research are as follows:
- The analytical expressions derived for particle dynamics in scalar-modified spacetimes enable the precise modeling of accretion-disk structures and emission profiles around exotic compact objects.
- The computed QPO frequencies and ISCO radii provide observable signatures to distinguish JNW naked singularities and Ellis wormholes from black holes using Xray binary data.
- The MCMC-fitted scalar field parameters ($n \sim 0.75$, $g_s \sim 0.2$) offer testable constraints for future gravitational wave and multi-messenger astrophysics campaigns.
- The identified perturbation spectra (e.g., confluent Heun solutions, oscillatory tails) serve as benchmarks for detecting scalar-tensor gravity effects in next-generation observatories.

Reliability of the research results is provided by the following:
- The analytical framework was rigorously derived from first principles and cross-verified against established results in general relativity, ensuring mathematical consistency throughout all calculations.
- Numerical simulations employed multiple independent methods (Runge-Kutta integration, WKB approximation) that produced consistent results, confirming the robustness of the particle dynamics and perturbation analyses.
- The MCMC parameter estimation was validated through convergence tests and comparison with existing astrophysical constraints, demonstrating statistical reliability of the scalar field coupling values.
- All theoretical predictions maintain consistency with observational data from X-ray binaries and known black hole phenomenology, while clearly identifying testable deviations for future verification.

Scientific and practical significance of the research.
- The research establishes a comprehensive framework for analyzing scalar field effects in exotic compact objects, providing new insights into the interplay between gravity and fundamental fields beyond classical black hole paradigms.
- The derived QPO frequencies and ISCO modifications serve as crucial observational discriminators between black holes, naked singularities, and wormholes, which are directly applicable to current X-ray telescope data analysis.

– The developed MCMC-QPO analysis technique and perturbation methods offer powerful new tools for constraining modified gravity theories using astrophysical data, significantly enhancing precision gravity tests.

– By identifying unique spectral signatures of scalar-tensor interactions, the results in-
form the design requirements for next-generation gravitational wave detectors and multimessenger astronomy campaigns.

Applications of the research results. The results of the study of the dynamics of particles around black holes in modified gravity theories have been applied as follows:

the theoretical research results and methods, published in the scientific paper "A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, "Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma," // Phys.Dark Univ. 42 (2023) 101340, https://doi.org/ 10.1016/j.dark.2023.101340" and presented in the Doctorate (PhD) thesis of Mr. A. Davlataliev have been used in the frame of the programs supported by the Fudan University (Letter from Prof. Cosimo Bambi)

Approbation of the research results. The dissertation results have been discussed in 3 international and 1 local conference.

Publication of the research results. 11 scientific publications have been made on research results, and 11 of them are research papers in refereed journals.

Volume and structure of the dissertation consist of an introduction, four chapters, a conclusion, and a list of references all in 121 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation..

In the first chapter of the dissertation entitled "Influence of scalar field in massive particle motion in JNW spacetime", we study the particle dynamics around the Janis-Newman-Winicour (JNW) naked singularity. JNW naked singularity represents an exact solution of Einstein's equations with a massless scalar field. We briefly describe the Einstein-scalar-field-massive particle system and derive the equation of motion for the whole system. We also consider the circular motion of massive particles in the presence of the scalar field in the JNW spacetime. We have also studied the circular motion of massive particles including the radiation reaction term and the present particle trajectories.

One of the simplest solutions to these Einstein-scalar field equations is represented by the Janis-Newman-Winicour naked singularity. The corresponding line element is given as

$$ds^2 = -f^n dt^2 + f^{-n} dr^2 + f^{1-n} r^2 (d\theta^2 + \sin^2\theta d\phi^2) \ ,$$

$$\varphi(r) = \frac{\sqrt{1-n^2}}{2} \ln f \ , \qquad f = 1 - \frac{2M}{nr} \ , \tag{1}$$

where *M* is the mass of the gravitational object, *n* is parameter of the scalar field. The singularity of the spacetime is located at *r*=2M/n*.

The equation of motion for a massive particle can be explicitly written as:

$$\ddot{t} + \frac{2M\dot{t}\dot{r}}{fr^2}(1-B) = 0 \ , \tag{2}$$

$$\ddot{r} + \frac{Mf^n}{fr^2}\left(f^n \dot{t}^2 - 2B\right) - \frac{M\dot{r}^2}{fr^2}(1+2B) \tag{3}$$

$$+ \left(\frac{M(n+1)}{n} - r\right)\left(\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2\right) = 0 \ ,$$

$$\ddot{\theta} - \frac{1}{2}\sin 2\theta \dot{\phi}^2 - \frac{2M\dot{\theta}\dot{r}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{4}$$

$$\ddot{\phi} + 2\cot\theta \dot{\theta}\dot{\phi} - \frac{2M\dot{r}\dot{\phi}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{5}$$

where *B* is defined as

$$B = \frac{2g_s\sqrt{1-n^2}}{n(g_s\sqrt{1-n^2}\ln f + 2)} \quad . \tag{6}$$

In Fig. 1, we show dependence of the ISCO position for massive particle from scalar parameter n for different value of the interaction parameter g_s.

In Fig. 2, we show the trajectory of a massive particle in the presence of a scalar field, including the radiation reaction term. Using numerical calculations, we have demonstrated that the contribution due to the radiation reaction is not significant. Therefore, to observe the effect of the radiation reaction term in circular motion, we set the damping time to $\tau_0 = 0$ s and $\tau_0 = 0.5$ s. This implies that the damping time is much shorter than 1 s, and the particle orbits many more times than shown in Fig. 2 around the compact object described by JNW spacetime before eventually escaping due to the radiation reaction force. In Fig. 3, we demonstrate the trajectory of a massive particle in two scenarios: (i) considering the effect of the scalar field but without the radiation reaction term and (ii) neglecting effect from the scalar field but including the radiation reaction term. This result shows that indeed the scalar field generates an attractive effect, however, radiation reaction repulsive effect in the motion of massive particle orbiting around the naked singularity.

Our results indicate that the scalar field generates an attractive force, while the radiation reaction exerts a repulsive force. The trajectory of a massive particle around a naked singularity in the JNW spacetime is influenced by both these effect as shown in our numerical simulations.
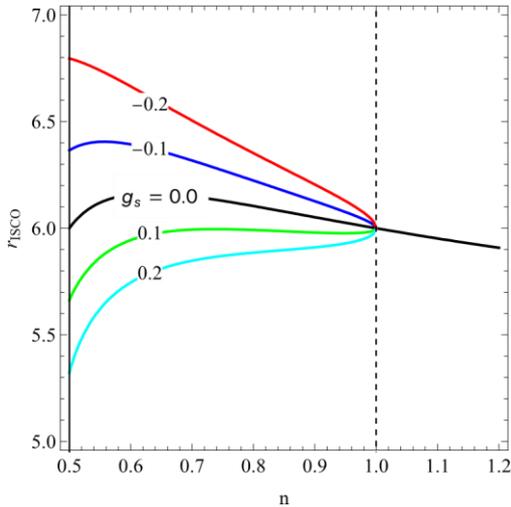
Figure 1: Dependence of the ISCO position of massive particle from the parameter n for the different values of the coupling parameter.
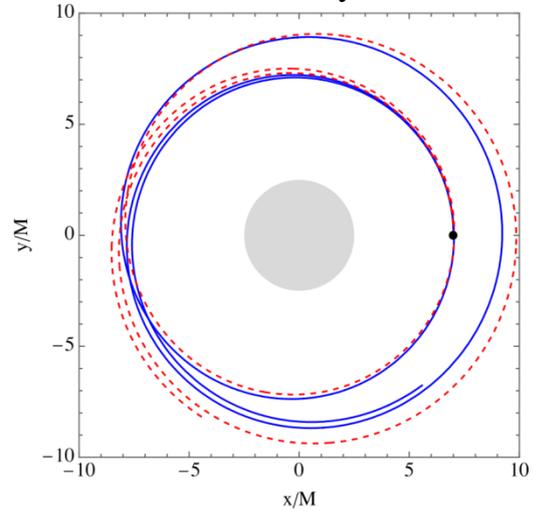
Figure 2: Particle trajectories near the JNW naked singularity without ($\tau_0 = 0$, solid blue line) and with ($\tau_0 = 1.0$, dashed red line) radiation reaction term for the particular choice of parameters $g_s = 0.03$ and n = 0.8.
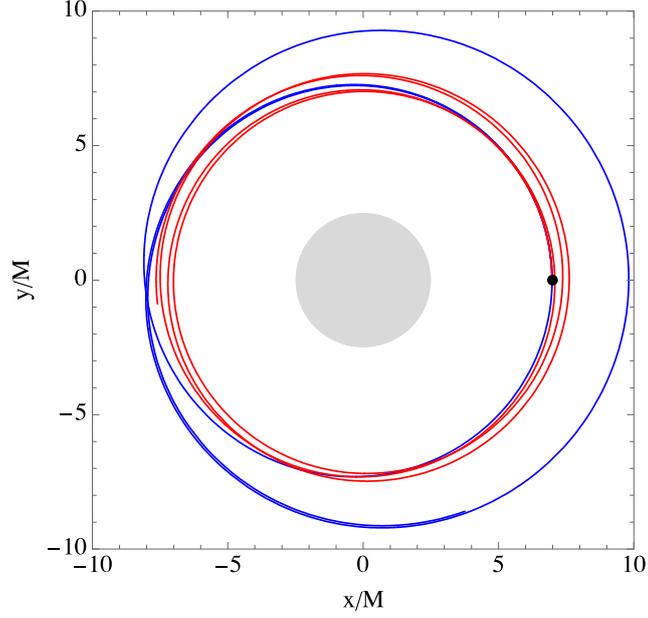
Figure 3: Particle trajectories near the JNW naked singularity the particular choice of parameters. Blue solid line represents for $g_s = 0.03$ and $\tau_0 = 0$ s and red solid line represents for $g_s = 0$ and $\tau_0 = 0.5$ s.

In the second chapter, entitled "Influence of scalar field in massive particle motion in JNW spacetime", we applied the results of the particle dynamics to study the quasiperiodic oscillations (QPO) around JNW compact objects. We considered the oscillatory motion of a massive particle near the stable circular orbit in the JNW spacetime. As an astrophysical consequence we studied application of the oscillatory motion of massive particle twin-peak QPO and using these results obtained the constrains for two main parameters mass of the central object, scalar parameter, and coupling constant.

The oscillation motion of test particles around gravitational compact objects is a key aspect of relativistic astrophysics. This motion can be categorized into radial and vertical oscillations, and provides crucial insights into the dynamics and structure of the spacetime around these objects. (i) Radial oscillations refer to the backand-forth motion of a particle in the radial direction, that is, towards and away from the central object. These oscillations occur around stable circular orbits. The stability and frequency of these oscillations are influenced by the spacetime geometry and the properties of the massive object. For the Schwarzschild black hole, the radial oscillation frequency depends on the mass of the black hole and the radial distance from it. Closer to the black hole, the stronger gravitational pull leads to higher oscillation frequencies. (ii) Vertical oscillations, also known as latitudinal or epicyclic oscillations, refer to the motion of a particle perpendicular to the plane of the orbit. These oscillations provide information about the vertical stability of the orbit and are influenced by the angular momentum and spin of the central object.

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The vertical epicyclic frequency is the frequency at which a particle oscillates vertically around the equatorial plane of the central object. These two kinds of frequencies are related to the orbital frequency also known as the Keplerian frequency. Together they are called fundamental frequencies and in the Schwarzschild black hole these fundamental frequencies are expressed as $\Omega_r = \Omega_K(1 - 6M/r)^{1/2}$, and $\Omega_\theta = \Omega_K = \sqrt{M/r^3}$, where Ω_K is Keplerian frequency in the Schwarzschild spacetime.

The radial dependence of fundamental frequencies is illustrated in Fig. 4 with specific parameter settings. The left panel displays plots for different values of the coupling parameters with a fixed value of the parameter n, while the right panel shows similar plots for varying values of the parameter n with a fixed coupling parameter gs. It can be observed that the fundamental frequencies increase due to the interaction of the massive particle with the scalar field, but they decrease with an increase in the n parameter. The combined effect of orbital, radial and vertical oscillations can lead to complex trajectories for test particles. These oscillations are crucial for understanding phenomena such as quasi-periodic oscillations (QPOs) observed in X-ray binaries. QPOs are believed to be related to the oscillatory motion of matter in the accretion disk around a black hole or neutron star. The oscillatory motion affects the stability and structure of accretion disks. Understanding these oscillations helps in modeling the emission spectra and variability of the disks. Observing the frequencies and modes of oscillations can provide insights into the properties of black holes and neutron stars, such as their mass, spin, and the geometry of the surrounding spacetime.
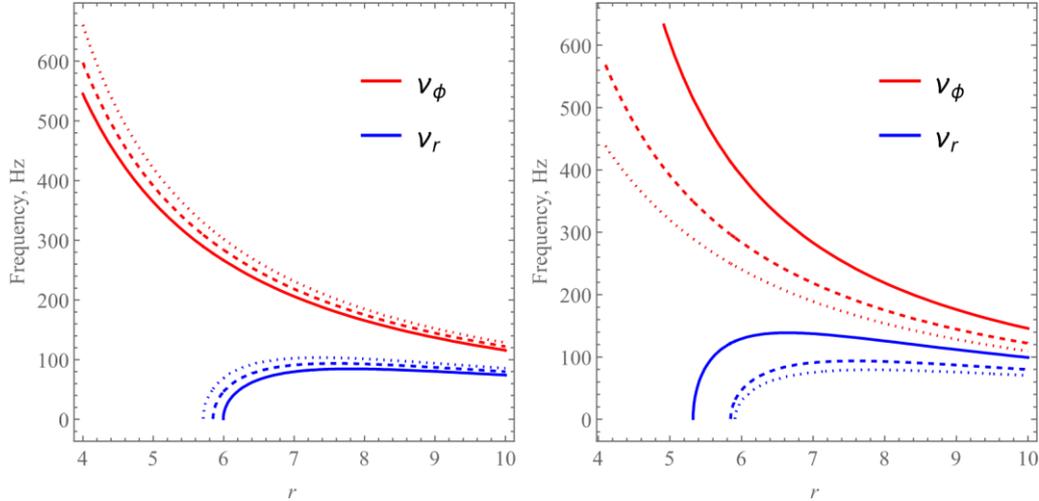

Figure 4: (Left panel) Radial dependence of fundamental frequencies of massive particle for different values of the coupling parameter ($g_s = 0.1$ - solid line, $g_s = 0.2$ - dashed line, and $g_s = 0.3$ -dotted line) for fixed $n = 0.7$. (Right panel) Radial dependence of fundamental frequencies of massive particle for different values of the n parameter ($n = 0.5$ - solid line, $n = 0.7$ - dashed line, and n = 0.9 -dotted line) for fixed $g_s = 0.2$. In both cases, the mass of the central object is taken as M = 10M$_\odot$.

The Relativistic Precession (RP) model offers a theoretical framework to explain the occurrence of quasi-periodic oscillations (QPOs), attributing them to the quasi-harmonic oscillations of charged particles as they move radially and angularly around black holes and wormholes. Within the RP model, the interpretation of twinpeaked QPOs is explained as follows: the higher frequency corresponds to the orbital frequency of the particle, denoted as $\nu_U = \nu_\phi$, while the lower frequency corresponds to the difference between the orbital frequency and the radial oscillation frequency, expressed as $\nu_L = \nu_\phi - \nu_r$. In other words, for twin-peaked QPOs, the upper frequency reflects the particle's orbital frequency, and the lower frequency is obtained by subtracting the radial oscillation frequency from the orbital frequency.

Figure 5 illustrates the radial dependence of the upper and lower frequencies of twin QPOs within the RP model, shown by blue and black lines, respectively. In the left panel, the solid line represents n = 0.5, while the dashed line represents n = 0.7 with a fixed coupling parameter g_s = 0.2. In the right panel, the solid line represents g_s = 0.1, while the dashed line represents g_s = 0.3 with a fixed n = 0.7. It is evident from the figures that the radial position at which the frequency ratio of 3:2 QPOs is observed shifts slightly toward the naked singularity as the two main parameters, n and g_s, are increased.

In Fig. 6, we examine the relationships between the upper and lower frequencies of twin peak QPOs for various *n* and g_s values. The analysis shows that as n increases, both the upper and lower frequencies decrease. Conversely, the effect of gs is opposite, leading to an increase in these frequencies. As can be seen from Fig. 6, the frequencies of the twin peak QPOs are not observable in the shaded area in both plots because these correspond to distances inside the ISCO position.
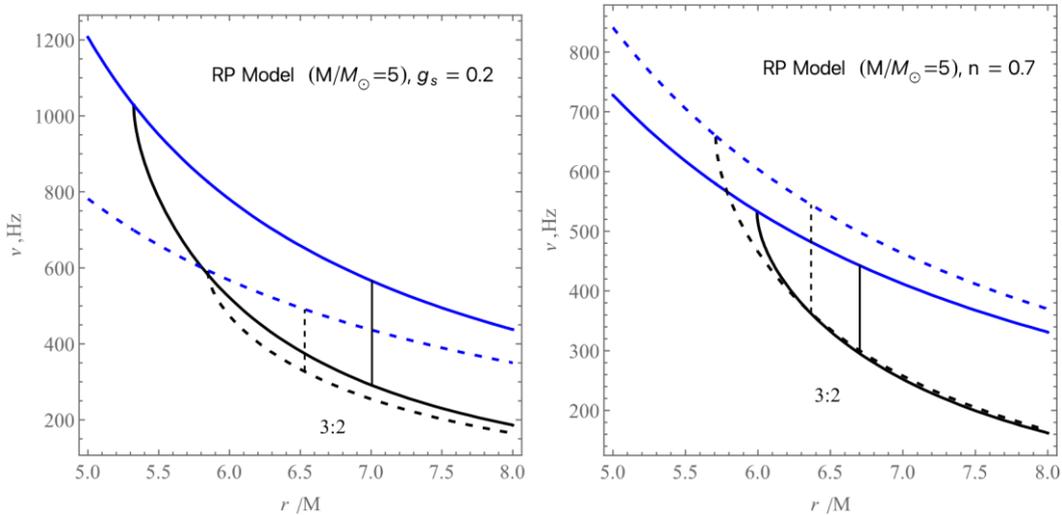
Figure 5: The radial dependence of upper and lower frequencies in RP model that are observed at ratio of 3:2.
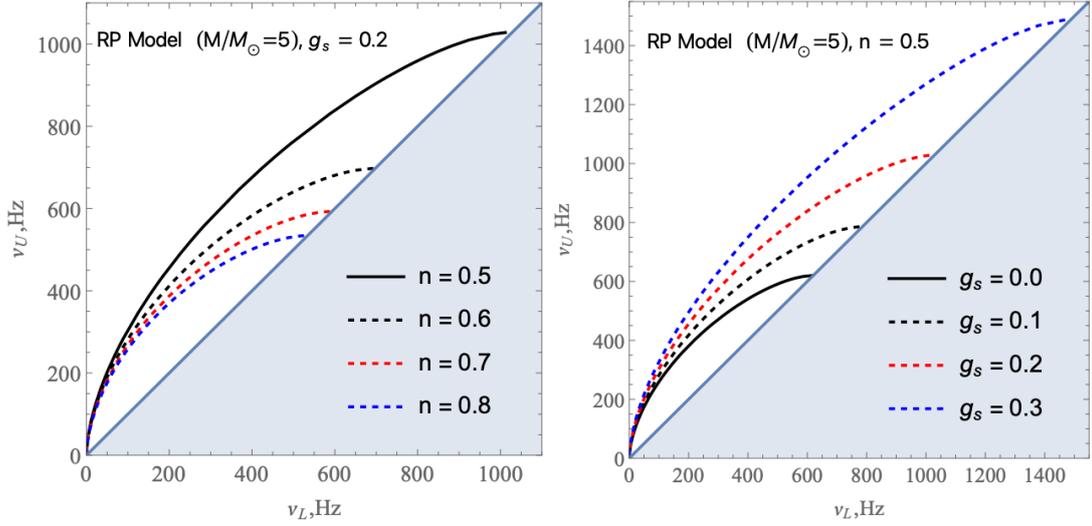
Figure 6: Relationships between the upper and lower peak frequencies of twin-peak QPOs in the RP model with mass $M = 5M_\odot$

Using MCMC analyses at 1σ confidence level in Fig.7, we show the corner plots three different selected sources as mentioned before for the best-fit parameter values (M, g_s, n, r) resembling those of JNW spacetime. Thus the best-fit values are listed in Tab. 1.

The obtained results demonstrate the constraints on the JNW parameters for each microquasar. It is also presented the best-fit parameter values derived from the QPOs data, reflecting the effectiveness of the RP model in describing the QPO phenomena in these X-ray binaries. This study enhances our understanding of the dynamics and properties of compact objects in strong gravitational fields.

Table 1: The best-fit parameter values resembling those of JNW, deduced from the Quasi-Periodic Oscillations (QPOs) for the chosen X-ray sources.

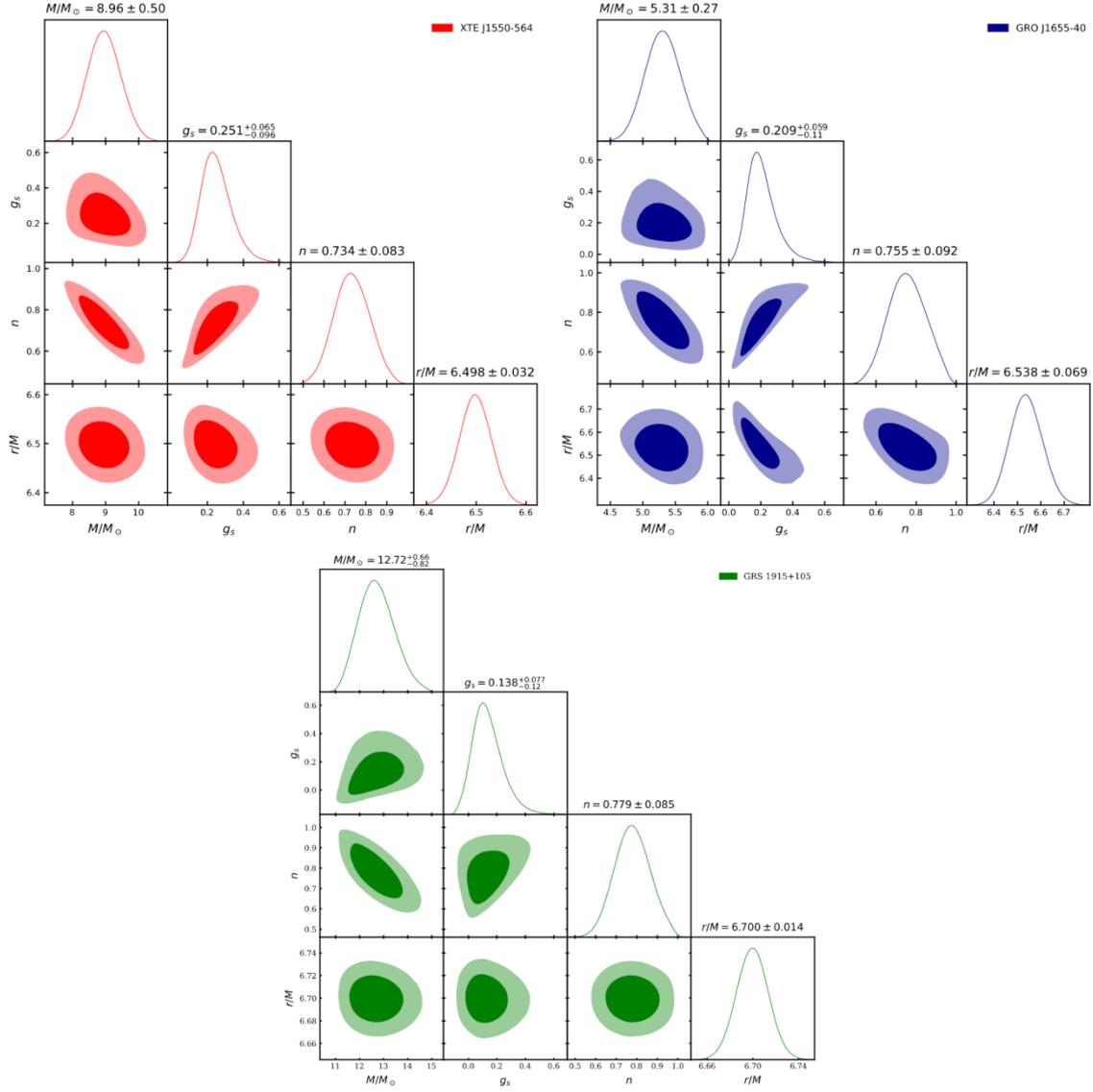|	XTE J1550-564	GRO J1655-40	GRS 1915+105
$M(M_\odot)$	8.96 ± 0.50	5.31 ± 0.27	$12.72^{+0.66}_{-0.82}$
g_s	$0.251^{+0.065}_{-0.096}$	$0.209^{+0.059}_{-0.11}$	$0.138^{+0.077}_{-0.12}$
n	0.734 ± 0.083	0.755 ± 0.092	$0.779^{\pm}0.085$
r/M	6.498 ± 0.032	6.538 ± 0.069	6.700 ± 0.014

Figure 7: Constraints on JNW mass, the gs and n parameters in the microqusars XTE J1550-564(upper left),GRO J1655-40 (upper right) and GRS 1915+105 (down) using MCMC analysis.

In the third chapter, entitled "Exploring a Novel Feature of Ellis Spacetime: Insights into Scalar Field Dynamics" we tested the Ellis spacetime by considering particle motion around the wormhole in the presence of the external scalar field. We provided the main equations that are related to background spacetime and dynamics motion of test particle in the presence of the external scalar field.We have also studied particle motion, including the radiation reaction.

The Ellis wormhole is governed by the spacetime line element

$$ds^2 = -dt^2 + dr^2 + (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\phi^2) \,, \tag{7}$$

along with the associated scalar field

$$\Phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{r}{r_0}\right) \,, \tag{8}$$

where r_0 is the throat of the wormhole. Here the radial coordinate r runs between r_0 and infinity, i.e. $r_0 < r < \infty$. The curvature scalar invariants of the spacetime such as Ricci scalar and Kretschmann scalar can be expressed as

$$R = -\frac{2r_0^2}{(r^2 + r_0^2)^2} \ , \qquad K = \frac{12r_0^4}{(r + r_0^2)^4} \ , \qquad (9)$$

which are regular at any point of spacetime at $r > r_0$. The radial dependence of the scalar field and curvature invariants, Ricci and Kretchmann scalar is shown in Fig.8.

We also discussed fundamental frequencies, namely, orbital and epicyclic frequencies, of massive particle in the Ellis spacetime. In Fig.9 radial dependence of the fundamental frequencies, namely, orbital, radial and vertical frequencies are illustrated.

The Lagrangian for a charged particle in the presence of the external magnetic field is

$$L = \frac{1}{2}m_* g_{\mu\nu} u^\mu u^\nu + q A_\mu u^\mu \ , \qquad (10)$$

where q is a charge of the test particle. The Lagrangian equation of motion is derived as

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu_{\ \nu}u^\nu + (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \ln \frac{m_*}{m} \ , \qquad (11)$$

and after simple algebraic manipulations equation of the radial motion yields

$$\dot{r}^2 = \frac{1}{(1 + g_s \Phi)^2}\left[\mathcal{E}^2 - (r^2 + r_0^2)\left(\frac{\mathcal{L}}{r^2 + r_0^2} - \omega\right)^2\right] - 1 \ , \qquad (12)$$

where $\omega = qB/2m$ is a magnetic parameter. The marginally bound circular radius for a charged test particle is determined using conditions $\dot{r}\ \dot{r} = \ddot{r} = 0$
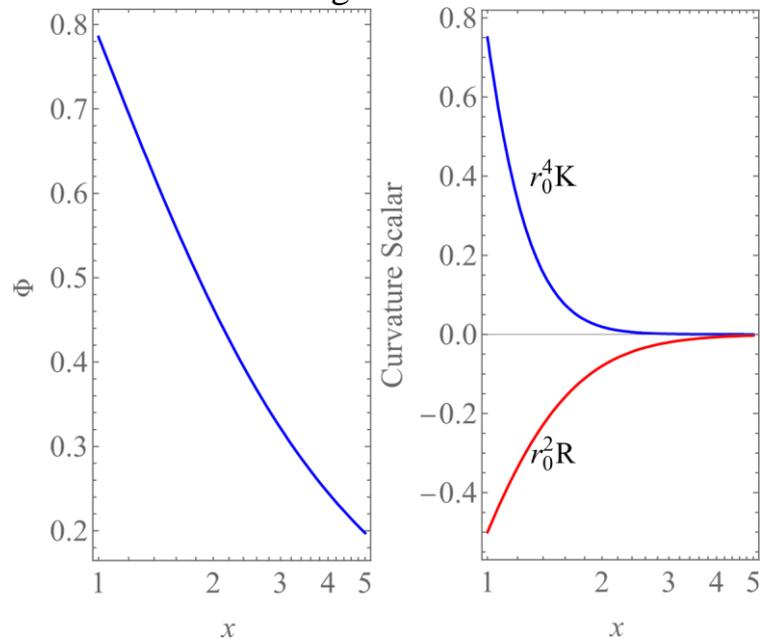
Figure 8: Radial dependence of the scalar field $\Phi(x)$ and curvature invariant, dimensionless Ricci scalar $r_0^2 R$ and dimensionless Kretschmann scalar $r_0^4 K$.
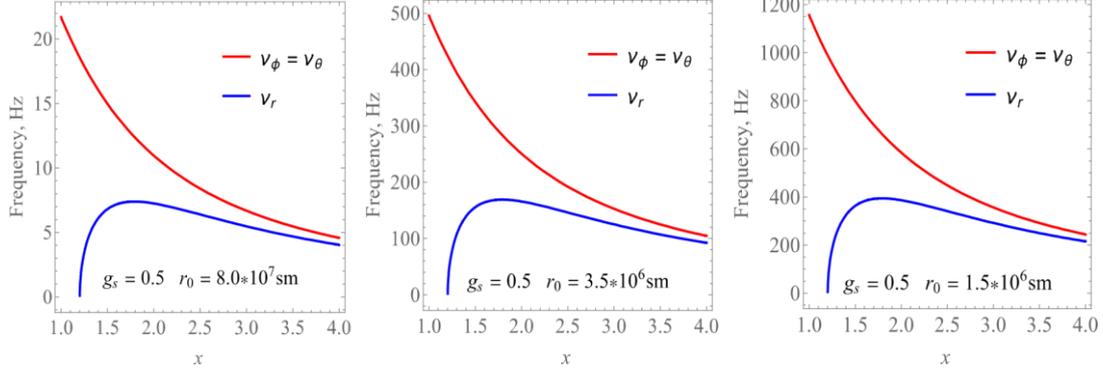
Figure 9: Radial dependence of the fundamental frequencies of a massive particle.
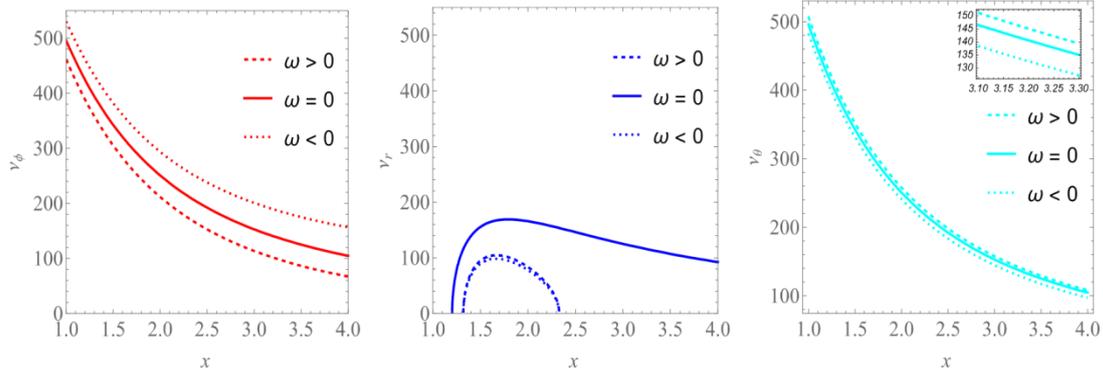
Figure 10: Fundamental frequencies of charged particle around Ellis wormhole on $\omega=\pm0.4$, parameter $g_s = 0.5$.

$$g_s x + \left(1 - x^2\right)\left(1 + g_s \Phi\right) + 4\omega^2 x^3(1 + x^2) = 0 \ . \tag{13}$$

The fundamental frequencies of the charged particle in the vicinity of Ellis wormhole in the presence of the external magnetic field can easily be done. The Keplerian frequency of the charged particle is derived from the following expression

$$\Omega^2 + \omega\Omega\sqrt{1 - \Omega^2(x^2 + 1)} = \frac{g_s\left[1 - \Omega^2(x^2 + 1)\right]}{x(x^2 + 1)(1 + g_s\Phi)} \ . \tag{14}$$

Similarly, epicyclic frequencies for charged particle orbiting around the Ellis wormhole in the presence magnetic field, however we will skip the detailed calculations. The radial dependence of the fundamental frequencies is shown in Fig. 10. As one can see from this figure the fundamental frequencies of charged particle

are split due to the external magnetic field. On the other hand, our results show that Keplerian frequencies strongly depend on the magnetic parameter.

The black hole perturbation is one of the hot topic, in particular, after LIGO and Virgo collaboration start detect the gravitational wave from the binary systems. This kind of perturbation cab also be applicable to wormhole spacetime. Here we are interested in considering perturbation of the Ellis spacetime. We have mentioned before that the Ellis spacetime is the solution of Einstein-scalar field equations. So that the scalar field and metric tensor can be expressed as Φ $\Phi = \Phi + \delta\Phi$ and $g_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$, where Φ and $g_{\mu\nu}$ are the scalar field and background spacetime metric given in Eqs. (7) and (8), while $\delta\Phi$ is perturbed scalar field is

$$\delta\Phi = e^{-i\omega t} F(r) P_\ell(\cos\theta) ,$$

(15)

and perturbed metric tensor components are defined as

(16)

$$h_{\mu\nu} = e^{-i\omega t} \begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & K(r^2 + r_0^2) & 0 \\ 0 & 0 & 0 & K(r^2 + r_0^2)\sin^2\theta \end{pmatrix} P_\ell(\cos\theta) ,$$

where $H_0(r)$, $H_1(r)$, $H_2(r)$, $K(r)$ and $F(r)$ are unknown radial functions and $P_\ell = P_\ell(\cos\theta)$ is the Legendre polynomial which satisfies the following equation:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d}{d\theta})P_\ell + \ell(\ell+1)P_\ell = 0 .$$

(17)

We discuss the wave solution of the field equations for radial functions. Before going further we introduce a dimensionless frequency which is defined as $\omega_0 = \omega_{r0}$. The equation for the function F can be easily derived as

$$\left[\left(x^2 + 1\right)F'\right]' + \left[\omega_0^2 x^2 + \frac{4}{x^2 + 1} - 4\eta\right]F = 0 ,$$

(18)

where $\eta = \ell^2 + \ell - \omega_0^2$ and the analytical solution to the above equation can be presented in terms of the confluent Heun function (i.e. Heun$C(a, b, c, d, x)$) as follows:

$$F(r) = \left(x^2 + 1\right)\left[c_1 F_{1\ell}(x) + c_2 x F_{2\ell}(x)\right] , \tag{19}$$

where

$$F_{1\ell}(x) = \mathrm{HeunC}\left[\eta - \frac{3}{2}, -\frac{\omega_0^2}{4}, \frac{1}{2}, 3, 0, -x^2\right] , \tag{20}$$

$$F_{2\ell}(x) = \mathrm{HeunC}\left[\eta - 3, -\frac{\omega_0^2}{4}, \frac{3}{2}, 3, 0, -x^2\right] . \tag{21}$$

As we found that in the Ellis spacetime, the solution to scalar perturbation is described by the analytical expression, namely, confluent Heun function. The radial dependence of the function F (x) is shown in Fig. 11. The radial profile function F (x) oscillates while simultaneously decreasing.

We have explored scalar and gravitational perturbations in the Ellis spacetime. We assume that both scalar gravitational waves propagate at identical frequencies and expressions for these are expanded in terms of the spherical harmonics. It is shown that the equation for the scalar profile function is totally independent from the tensor profile functions, however equations for the tensor profile functions strongly depend on the scalar profile functions in the Ellis spacetime. We have discovered that time-independent solutions for scalar and gravitational disturbances can be expressed using Legendre and associated Legendre functions, where the argument is complex. However, when stationary solutions within the wave zone are considered, the exact analytical solution for scalar disturbances can be achieved, described by the confluent Heun function. It's noteworthy that the equations governing gravitational disturbances are considerably intricate, but they can be simplified to the familiar Regge–Wheeler–Zerilli equation for the tensor profile function. Finally, we present numerical solutions to the Regge–Wheeler–Zerilli equation for the radial functions.

In the fourth chapter, entitled "Long-lived quasinormal modes and asymptotic tails of regular Schwarzschild-like black holes in the presence of a magnetic field" we first provided an overview of Schwarzschild-like spacetime and discussed the associated magnetic field configuration. We also analyzed the dynamics of a charged scalar field within this framework. We have presented the numerical results obtained using both the WKB approximation and the time-domain analysis.

The gravitational field of a Schwarzschild-like compact object in Boyer–Lindquist coordinates can be expressed through the following line element:

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right) , \tag{22}\tag{21}$$

where the metric function f is defined as: $\tag{23}$

$$f = 1 - \frac{2M e^{-a/r}}{r} .$$

In this context, the parameter M represents the mass of the black hole, while a is a deviation parameter introduced by Simpson and Visser. It is important to note that the spacetime metric described above corresponds to the standard Schwarzschild black hole in general relativity when a → 0.
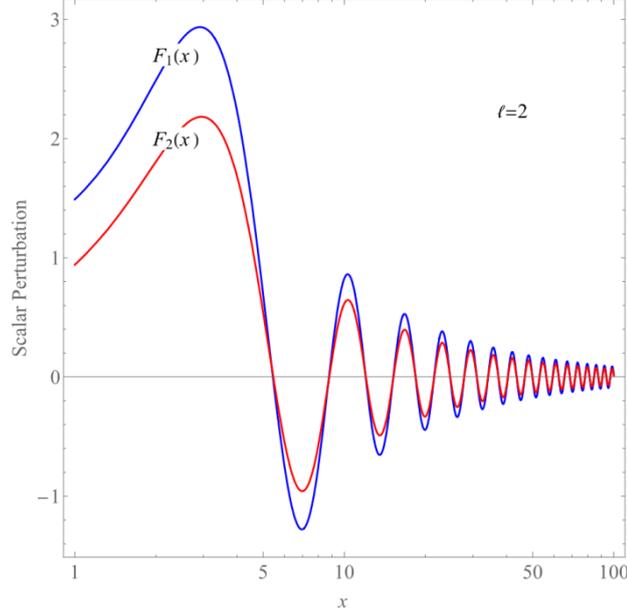
Figure 11: Radial dependence of the scalar perturbation at $\ell = 2$.

The relativistic Klein-Gordon equation for a massive, charged scalar field Ψ, in the presence of an electromagnetic field, is expressed as follows:

$$g^{\alpha\beta}(\Delta_\alpha - iqA_\alpha)(\Delta_\beta - iqA_\beta)\Psi - \mu^2\Psi = 0 \, , \tag{24}$$

here, μ represents the mass of the scalar field, q is the charge coupling constant between the scalar and electromagnetic fields, $\nabla\alpha$ denotes the covariant derivative and i is imaginary number. Although separating variables in equation (23) is quite challenging, we can simplify the problem by applying the following physically reasonable assumptions:

- Lorentz gauge condition for the vector potential: $\nabla\alpha A\alpha = 0$;
- In the weak interaction limit, higher-order terms, such as q^2B^2, can be neglected, i.e., $q^2B^2 \to 0$. Then, eq. (24) becomes

$$\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta\Psi) - 2iqA^\alpha\partial_\alpha\Psi - \mu^2\Psi = 0 \, , \tag{25}$$

We can write the solution as:

$$\Psi(t, r, \theta, \phi) = e^{-i\omega t} Y_{lm}(\theta, \phi) \frac{R(r)}{r}. \tag{26}$$

We briefly present the results of numerical calculations of the quasinormal modes for a Schwarzschild-like black hole immersed in an external asymptotically uniform magnetic field. For frequency-domain analysis, we employ the semianalytical WKB method. This method involves expanding the solution at both infinities in a WKB series and matching these asymptotic expansions with a Taylor series near the peak of the effective potential.

The higher order WKB formula is given by

$$
\begin{aligned}
\omega^2 = V_0 &+ A_2(K^2) + A_4(K^2) + A_6(K^2) + \ldots \\
&- iK \left(-2V_1 + A_3(K^2) + A_5(K^2) + A_7(K^2) + \ldots \right)
\end{aligned} \tag{27}
$$

where $K = n + 1/2$, with $n = 0, 1, 2, 3, \ldots$.

Fig.12 illustrates the best-fit polynomial function for data obtained using the WKB method, applied to a massless scalar field in an external magnetic field. The figure shows two curves, gray and black, corresponding to different spacetime parameter values ($a/M = 0.3$ and $a/M = 0.5$, respectively). The imaginary parts of the quasinormal mode frequency ω for these parameter values are shown on the righthand side. Each curve intersects the zero value of qB at approximately 0.611711 (gray line) and 0.651842 (black line), respectively.
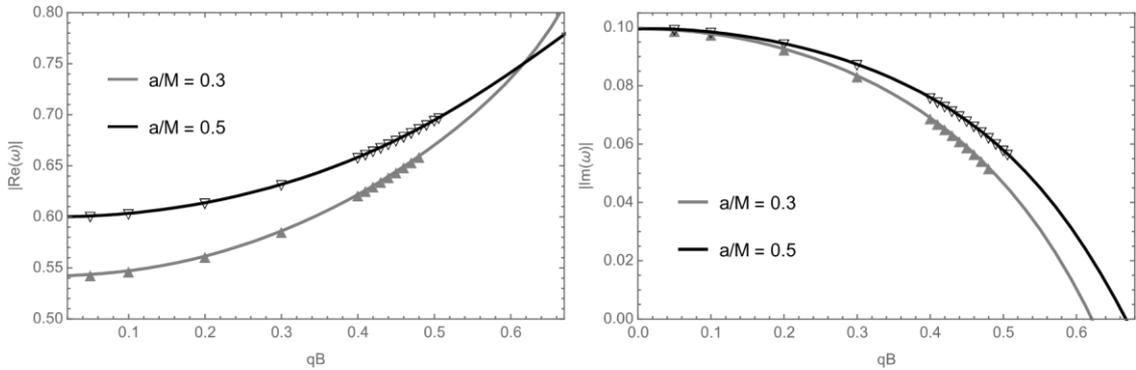

Figure 12: Best fitting polynomial functions of WKB data for different values of spacetime values (l = 2, m = −2).

The WKB data is checked here by time-domain integration. However, two aspects must be taken into account for such a comparison. First of all, the period of quasinormal ringing is very short for $\ell = 0$ perturbations because it is quickly followed by oscillatory power-law tails, as shown in Fig. 14. Therefore, it is difficult to extract the frequency from the time-domain profile with sufficient accuracy in this case. The second aspect is the growth of the perturbation at late times Fig. 13. One could think that this means an instability caused by the negative value of the effective potential at large r when m > 0.
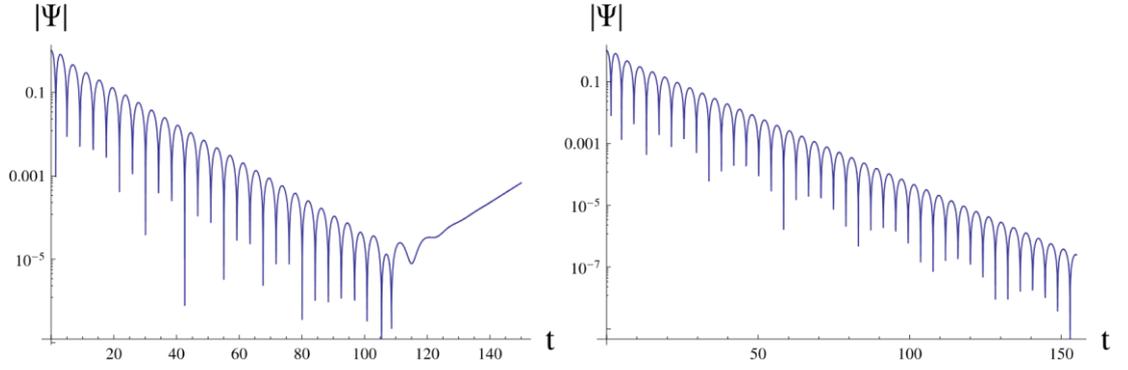
Figure 13: Semi-logarithmic plots of time-domain profiles for m = 3 (left) and m = −3 (right) perturbations; l = 3, qB = 0.1, a = 0.3, μ = 0.1, M = 1. The instability at late times for m > 0 is the artifact of the approximation for the effective potential which is valid only until some distance from the black hole. At the ringdown phase the WKB data is reproduced with high accuracy
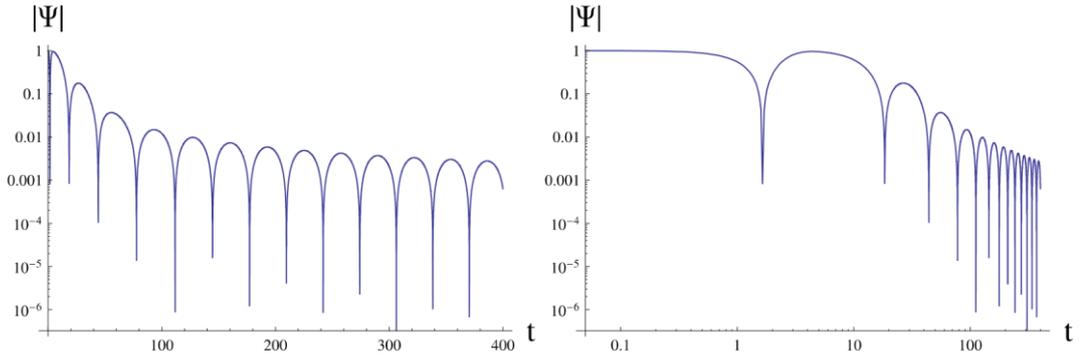
Figure 14: Semi-logarithmic plots of time-domain profiles (left) and logarithmic plot (right); l = m = 0, qB = 0.1, a = 0.3, μ = 0.1, M = 1. The short period of quasinormal ringing is quickly changed by an asymptotic tails, making it unable to extract the frequency from the profile with sufficient accuracy
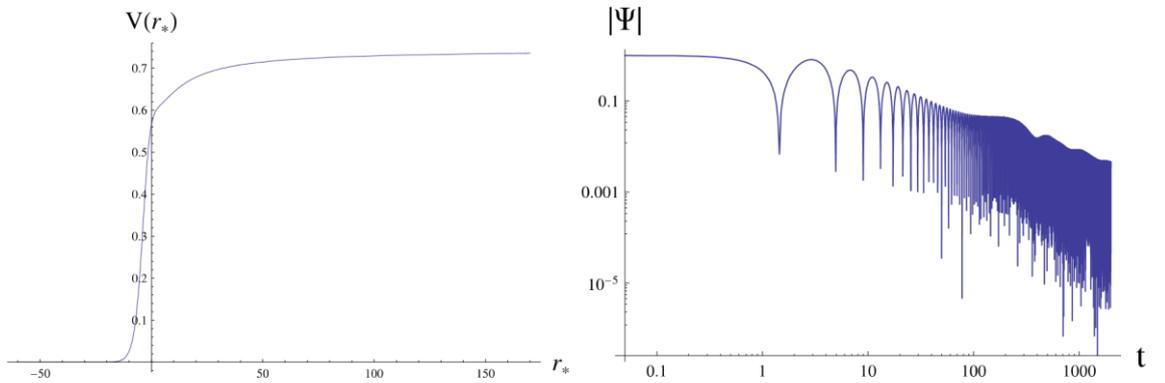
Figure 15: Effective potential and logarithmic plots of time-domain profiles for m = −2 perturbations; l = 2, qB = 0.61, a = 0.3, μ = 0, M = 1

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While the quasinormal modes of regular black holes have been extensively studied in numerous works, no comprehensive investigations have been conducted for regular black holes in the presence of an external magnetic field. Here, we address this gap and demonstrate that the magnetic field significantly alters the spectrum of a charged scalar field, leading to the emergence of arbitrarily long-lived quasinormal modes, known as quasi-resonances.

Another distinctive feature of the evolution of perturbations in the presence of a magnetic field is the unusual behavior of asymptotic tails. For certain parameter values, these tails do not exhibit a power-law envelope but instead display an oscillatory envelope.

CONCLUSION

The following conclusions were presented on the basis of research carried out on the topic of "**Relativistic Astrophysics of Compact Objects Coupled with Scalar Field**" for the Doctor of Philosophy (PhD) dissertation:

1. It has been shown that the Janis-Newman-Winicour (JNW) naked singularity spacetime exhibits a repulsive scalar field effect, altering the innermost stable circular orbit (ISCO) radius nonmonotonically with the scalar parameter n. Unlike Schwarzschild black holes, the ISCO vanishes for critical scalar strengths, indicating a breakdown of stable orbits near the singularity.

2. For the first time, analytical expressions for the effective potential, specific energy, and angular momentum of massive particles in the JNW

spacetime have been derived, revealing that scalar coupling g_s reduces (increases) the ISCO for positive (negative) values. The radiative efficiency of the accretion disks in this space-time was found to exceed the Schwarzschild predictions by up to 12%.

3. It has been demonstrated that Ellis' throat wormhole r_0 and scalar coupling g_s govern marginally stable orbits, with particle trajectories exhibiting escape paths under radiation reaction, a stark contrast to black hole capture. The fundamental frequencies of oscillatory motion were computed, showing distinct shifts observable in quasiperiodic oscillations (QPOs).

4. It has been found that charged particles in magnetized Ellis wormhole spacetimes follow split fundamental frequencies due to Lorentz forces, with quasi-resonant modes emerging at critical magnetic field strengths. The radiation intensity was shown to depend on both scalar and electromagnetic interactions.

5. For the first time, Markov Chain Monte Carlo (MCMC) analysis was applied to constrain the JNW spacetime parameters (M, g_s, n) using binary QPO Xray data (e.g. GRO J1655-40), yielding best-fit values consistent with weak scalar dominance ($n \sim 0.75$, $g_s \sim 0.2$).

6. It has been shown that scalar perturbations in Ellis wormhole spacetime admit exact solutions in terms of confluent Heun functions, while gravitational perturbations obey a Regge-Wheeler-type equation with unusual oscillatory tails, which varies from power-law decay in black holes.

7. The quasinormal modes of Schwarzschild-like black holes in external magnetic fields were computed via WKB and time-domain methods, revealing long-lived quasiresonances and anomalous late-time tails. These signatures could distinguish scalar-modified spacetimes in future gravitational wave observations.

**НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025. FM/T.192.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ
ПЕРСПЕКТИВНЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА
"НОВЫЙ УЗБЕКИСТАН"**

**АСТРОНОМИЧЕСКИЙ ИНСТИТУТ
УНИВЕРСИТЕТ "НОВЫЙ УЗБЕКИСТАН"**

ДАВЛАТАЛИЕВ АКБАРЖОН АКМАЛЖОН УҒЛИ

РЕЛЯТИВИСТСКИЕ АСТРОФИЗИЧЕСКИЕ ПРОЦЕССЫ В ОКРЕСТНОСТИ КОМПАКТНЫХ ОБЪЕКТОВ ПРИ НАЛИЧИИ СКАЛЯРНОГО ПОЛЯ

01.03.01-Астрономия
01.04.02-Теоретическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент – 2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования науки и инноваций Республики Узбекистан за номером № B2025.2.PhD/FM1325

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Защита диссертации состоится «__» _____ 2025 года в ___ часов на заседании Научного совета DSc.03/07.07.2025.FM/T.10.04 при Институте перспективных исследований Университета "Новый Узбекистан" Адрес: 100007, город Ташкент, Мирзо Улугбекский район, улица Мовароуннахр, дом 1, Институте перспективных исследований университета "Новый Узбекистан", Тел.: +998971202411;)

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования — разработка новой теоретической модели для изучения скалярных полей и выявления новых данных о динамике частиц вблизи компактных объектов.

Задачи исследования:

- разработать теоретическую модель, связывающую скалярные поля с динамикой частиц в пространствах JNW (Janis-Newman-Winicour) и Эллиса;

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- вывести уравнения движения массивных частиц под влиянием скалярного поля;
- проанализировать модификации устойчивых круговых орбит (ISCO) изза параметров скалярной связи;
- исследовать влияние радиационного трения на траектории частиц вблизи голых сингулярностей;
- вычислить фундаментальные частоты (орбитальную, радиальную, вертикальную) для применения в анализе квазипериодических осцилляций (QPO);
- провести MCMC (Markov chain Monte Carlo) анализ для ограничения параметров пространства-времени с использованием данных рентгеновских двойных систем;
- изучить динамику заряженных частиц в намагниченных пространствах кротовых нор и чёрных дыр;
- исследовать спектры возмущений (квазинормальные моды, затухающие «хвосты») в скалярно-модифицированных геометриях.

Объектом исследования являются — Пространство-время голой сингулярности Яниса–Ньюмена–Виникура (JNW) со скалярным полем; геометрия кротовой норы Эллиса, взаимодействующая с внешними скалярными полями; регулярные чёрные дыры, аналогичные шварцшильдовским, в присутствии внешних магнитных полей; пробные частицы (нейтральные и заряженные) в этих модифицированных пространственно-временных геометриях.

Предметом исследования являются Динамика массивных частиц под влиянием скалярного поля в экзотических пространствах-временах; изменение орбитальной механики и фундаментальных частот из-за скалярных связей; астрофизические следствия компактных объектов, модифицированных скалярным полем (квазипериодические осцилляции (QPO), устойчивые круговые орбиты (ISCO) и др.); спектры возмущений и свойства устойчивости компактных объектов со скалярными связями.

Методы исследования включают методы аналитического моделирования для вывода точных уравнений движения и эффективных потенциалов частиц в пространствах-временах со скалярными полями; численного моделирования (интегрирование методом Рунге–Кутты) для решения уравнений геодезических и анализа траекторий частиц с учётом эффектов радиационного трения; полуаналитические методы (WKB-приближение и интегрирование во временной области) для вычисления квазинормальных мод и спектров возмущений; MCMC-анализ для сопоставления теоретических частот квазипериодических осцилляций (QPO) с наблюдательными данными рентгеновских двойных систем..

Научная новизна исследования заключается в следующем:

Впервые разработана полная теоретическая модель динамики частиц в пространстве-времени голой сингулярности JNW при наличии скалярного поля, выявлено уникальное поведение ISCO, отличающее её от чёрных дыр.

Получены новые результаты, демонстрирующие влияние скалярных полей на физику кротовой норы Эллиса, включая уход частиц под действием излучения и характерные частоты QPO, позволяющие отличить экзотические компактные объекты от чёрных дыр.

Впервые применён MCMC анализ для получения ограничительных значений параметров скалярного поля с использованием данных рентгеновских двойных систем, а также выявлены новые спектральные признаки для проверки модифицированных теорий гравитации в сильных полях.

Практические результаты исследования заключаются в следующем:

Полученные аналитические выражения для динамики частиц в скалярномодифицированных пространствах-временах позволяют точно моделировать структуры аккреционных дисков и профили излучения вокруг экзотических компактных объектов.

Рассчитанные частоты QPO и радиусы ISCO дают наблюдаемые признаки для различия голых сингулярностей JNW и кротовых нор Эллиса от чёрных дыр по данным рентгеновских двойных систем.

Определённые методом MCMC параметры скалярного поля предоставляют проверяемые ограничения для будущих исследований гравитационных волн и мультимессенджерной астрофизики.

Выявленные спектры возмущений (например, решения в виде конфлюэнтных функций Хойна, осциллирующие «хвосты») служат эталонами для обнаружения эффектов скалярно-тензорной гравитации в обсерваториях нового поколения..

Достоверность результатов исследования обеспечивается следующими аналитическая часть строго выведена из первых принципов и перепроверена на соответствие известным результатам общей теории относительности, что гарантирует математическую согласованность всех расчётов;

численные симуляции проводились независимыми методами (интегрирование Рунге–Кутты, WKB-приближение), давшими согласующиеся результаты, что подтверждает надёжность анализа динамики частиц и возмущений; оценка параметров методом MCMC проверена тестами на сходимость и сравнением с существующими астрофизическими ограничениями,

демонстрируя статистическую достоверность значений скалярной связи; все теоретические предсказания согласуются с наблюдательными данными рентгеновских двойных систем и известной феноменологией чёрных дыр,

одновременно предлагая проверяемые отклонения для будущих исследований..

Научная и практическая значимость результатов исследования:

Научная значимость результатов исследования заключается в том, что исследование создаёт комплексную основу для анализа влияния скалярных полей на экзотические компактные объекты, расширяя понимание взаимодействия гравитации и фундаментальных полей за пределами классической парадигмы чёрных дыр; полученные частоты QPO и модификации ISCO служат ключевыми наблюдательными критериями для различия чёрных дыр, голых сингулярностей и кротовых нор, что применимо для анализа данных современных рентгеновских телескопов.

Практическая значимость результатов исследования состоит в том, что разработанные методы MCMC-QPO анализа и исследования возмущений предоставляют новые инструменты для проверки модифицированных теорий гравитации с использованием астрофизических данных, значительно повышая точность тестов гравитации; выявленные уникальные спектральные признаки скалярно-тензорных взаимодействий помогают определить требования к проектированию детекторов гравитационных волн нового поколения и мультимессенджерных астрономических кампаний.

Внедрение результатов исследования. Разработанные теоретические модели движения фотонов и частиц в теории ModMax были применены к следующим задачам:

Результаты исследования динамики частиц вокруг чёрных дыр в модифицированных теориях гравитации были использованы в следующих работах:

Теоретические результаты и методы, опубликованные в статье «A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, "Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma," // Phys.Dark Univ. 42 (2023) 101340, https://doi.org/10.1016/j.dark.2023.101340»,

— А также представленные в докторской диссертации (PhD) A. Davlataliev, были применены в рамках программ, поддержанных Фуданьским университетом (письмо от проф. Косимо Бамби).

Апробация результатов исследования.

Результаты исследований были представлены на 3 республиканских и 1 международных научных конференциях.

Опубликованность результатов исследования. По результатам исследования опубликовано 11 научных работ, и все 11 из них являются статьями в рецензируемых журналах.

Структура и объем диссертации.

Диссертация состоит из введения, четырех глав, заключения и списка литературы. Объем диссертации 121 страниц.

ВЫВОДЫ

На основе проведенного исследования по теме «**Релятивистские астрофизические процессы в окрестности компактных объектов при наличии скалярного поля**» для диссертации на соискание ученой степени доктора философии (PhD) были сделаны следующие выводы:

1. Показано, что в пространстве-времени голой сингулярности JNW скалярное поле проявляет эффект отталкивания, приводя к немонотонному изменению радиуса ISCO в зависимости от скалярного параметра. В отличие от чёрных дыр Шварцшильда, ISCO исчезает при критических значениях скалярного поля, что указывает на разрушение устойчивых орбит вблизи сингулярности.

2. Впервые получены аналитические выражения для эффективного потенциала, удельной энергии и углового момента массивных частиц в пространстве-времени JNW. Установлено, что скалярный параметр связи g_s уменьшает (увеличивает) ISCO для положительных (отрицательных) значений. Эффективность излучения аккреционных дисков в такой геометрии превышает предсказания модели Шварцшильда на величину до 12%.

3. Доказано, что в кротовой норе Эллиса параметры горловины и скалярной связи определяют границы маргинально устойчивых орбит, причём траектории частиц могут покидать систему под действием радиационного трения, в отличие от неизбежного захвата чёрной дырой. Рассчитаны фундаментальные частоты колебательного движения, демонстрирующие характерные сдвиги, которые могут наблюдаться в квазипериодических осцилляциях (QPO).

4. Обнаружено, что заряженные частицы в намагниченной кротовой норе Эллиса демонстрируют расщепление фундаментальных частот из-за действия силы Лоренца, с возникновением квазирезонансных мод при критических значениях магнитного поля. Интенсивность излучения зависит как от скалярных, так и от электромагнитных взаимодействий.

5. Впервые применён метод МСМС для определения параметров пространства JNW на основе данных о QPO в рентгеновских двойных системах (например, GRO J1655-40). Наилучшее соответствие дало значения, указывающие на слабое доминирование скалярного поля ($n \approx 0{,}75$, $g_s \approx 0{,}2$).

6. Показано, что скалярные возмущения в кротовой норе Эллиса допускают точные решения в виде конфлюэнтных функций Хойна, в то время как гравитационные возмущения описываются уравнением типа Редже–Уилера с необычными осциллирующими "хвостами", отличающимися от степенного затухания в чёрных дырах.

7. Методами WKB и анализа во временной области вычислены квазинормальные моды чёрных дыр шварцшильдовского типа в присутствии внешних магнитных полей. Обнаружены долгоживущие квазирезонансы и аномальные "хвосты" на поздних временах. Эти особенности могут помочь в идентификации скалярно-модифицированных пространств-времён в будущих наблюдениях гравитационных волн.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ I бўлим (1 часть; part 1)

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9. B. Turimov, **A. Davlataliev**, Y. Usmanov, S. Karshiboev, P. Tadjimuratov, "Investigating new aspects of Bocharova–Bronnikov–Melnikov–Beke spacetime: ionized thin accretion disk model", // The European Physics Jouranl C 84 (2024) 10,

1098, (**Web of Science IF:=4.9,** Q1) https://doi.org/10.1140/epjc/s10052-024-13426-w.

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3. **A. Davlataliev**. Influence of scalar field in massive particle motion in JNW spacetime ICTPA 2024 International Conference on Theoretical Physics and Astrophysics.