

**“YANGI O‘ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR  
TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR  
BERUVCHI DSc.03/07.07.2025. FM/T.192.01 RAQAMLI ILMIY KENGASH**

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**FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI**

**O‘KTAMOV O‘KTAMJON XAYRULLA O‘G‘LI**

**GRAVITATSION OBYEKTLARNING QORONG‘U MATERIYA HALOSI  
VA MAGNIT MAYDONI MAVJUD BO‘LGAN MUHITDAGI O‘TA OG‘IR  
QORA TUYNUKLAR ATROFIDAGI DINAMIKASI**

**01.04.02 – Nazariy fizika**

**01.03.01 – Astronomiya**

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA  
DOKTORI (PhD) DISSERTATSIYASI  
AVTOREFERATI**

**Toshkent – 2025**

**Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertasiyasi  
avtoreferati mundarijasi**

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## KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

**Dissertatsiya mavzusining dolzarbligi va zaruriyati.** Hozirgi kunda LIGO-VIRGO va Event Horizon Telescope kabi ilg'or astronomik asboblarda yordamida qora tuynuklarning birlashishi va ikkilik neytron yulduzlardan kelib chiqadigan gravitatsion to'liqlarni to'g'ridan-to'g'ri aniqlash, shuningdek, M87 va Somon yo'li galaktikalari yadrosidagi o'ta og'ir qora tuynuklarning tasvirlarini olish mumkin bo'ldi. Ushbu ilg'or asboblarda yordamida gravitatsion to'liqlarni to'g'ridan-to'g'ri kuzatish va o'ta og'ir qora tuynukning batafsil tasvirini olish kuchli gravitatsion maydonlardagi astrofizik hodisalarni yoritadi. Ushbu zamonaviy kuzatishlar muqobil gravitatsiya nazariyalarida yangi yechimlar ishlab chiqish va parametrlarni aniq o'lchash hamda qorong'u materiyani mohiyatini tushunish bilan bog'liq muammolarni o'rganish uchun juda muhimdir. Qorong'u materiya va qora tuynuklar o'rtasidagi o'zaro ta'sirni tahlil qilish qorong'u materiyani asosiy tabiatini tushunishni yaxshilash uchun muhim ahamiyatga ega. Qorong'u materiya halolari va ularning qora tuynuk tizimlari bilan o'zaro ta'siri muhim bo'lib, astrofizik ma'lumotlar va simulyatsiyalar orqali qorong'u materiyani o'rganish va modellashtirish qimmatli ma'lumotlarni taqdim etishi mumkin. Shu maqsadda Einasto, Burkert va Dehnen kabi bir nechta analitik modellar qorong'u materiya halolaridagi qora tuynuk yechimlarini ifodalash uchun ishlab chiqilgan.

Respublikamizda relyativistik astrofizikasi sohasida fundamental tadqiqotlarga katta e'tibor qaratilmoqda. Fundamental tadqiqotlar va ilmiy izlanishlarning asosiy yo'nalishlari, shuningdek, ularning mamlakatimizda ilmfanni rivojlantirish uchun amaliy qo'llanilishi O'zbekiston Respublikasining 2022–2026 yillarga mo'ljallangan yanada rivojlanish strategiyasida batafsil bayon etilgan. So'nggi o'ttiz yil davomida Respublikada gravitatsion ixcham ob'ektlarning relyativistik astrofizikasi rivojlantirildi, qora tuynuklar, qurt teshiklari va singularliklarning energetik va optik xususiyatlari chuqur o'rganildi. Umumiy nisbiylik nazariyasining yangi effektlari magnitlangan neytron yulduzlarning relyativistik astrofizikasida kashf etildi, relyativistik magnetosferalar va plazma, shuningdek, magnitlashtirilgan relyativistik ob'ektlarni tushunishda yutuqlarga erishildi.

Ushbu dissertatsiya O'zbekiston Respublikasining 2017<sup>1</sup>-yil 7-fevraldagi UP-4947-sonli Prezident Farmoni, O'zbekiston Respublikasining yanada rivojlanish strategiyasi va 2018-yil 29-noyabrdagi O'zbekiston Hukumati tomonidan chiqarilgan "O'zbekistonda 2019–2021 yillarga mo'ljallangan asosiy tarkibiy islohotlar yo'nalishlari yo'l xaritasi" kabi davlat hujjatlarida belgilangan maqsadlarni amalga oshirishga qaratilgan.

**Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi.** Tadqiqot O'zbekiston Respublikasi fan va texnikaning

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<sup>1</sup> \*\*Eslatma\*\*: 2022-yil 1-yanvardagi O'zbekiston Respublikasi Prezidentining PF-60-sonli "2022-2026 yillarga mo'ljallangan Yangi O'zbekistonning rivojlanish strategiyasi to'g'risida"gi farmoni.

ustuvor yo‘nalishlariga muvofiq amalga oshirildi: II. “Quvvat, energiya va resurslarni tejash”.

**Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi.** Taqdimnoma Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan ilmiy loyihalar doirasida bajarilgan. FL-7923051796 ”Sferik simmetrik gravitasion maydonlarda skalyar maydonlar dinamikasi va kvant nurlatkichlarni modellashtirish”, F-FA-2021-432 “Kichik massali Rentgen qo‘shaloq tizimlaridan sun’iy yo‘ldosh orqali olingan ma’lumotlarni qayta ishlash va tahlil qilish”.

**Tadqiqotning maqsadi:** Qorong‘u modda va magnit maydoni mavjud muhitdagi o‘ta og‘ir qora tuynuklar (SMBH) atrofidagi zarrachalar dinamikasini o‘rganish, olingan natijalarni S2 yulduzining kuzatilgan pozitsiyasi bilan qiyosiy solishtirish va qorong‘u modda halosidagi yangi analitik Schwarzschild yechimiga o‘xshash qora tuynuk yechimlarini oluvchi nazariy formalizmlarni aniqlashtirishdan iborat.

**Tadqiqotning vazifalari:**

Dehnen tipidagi qorong‘u materiya halosida Schwarzschild yechimiga o‘xshash yangi analitik yechimlarini topish.

Olingan yechimning fazo-vaqt xususiyatlari va energiya shartlarini tahlil qilish.

LQGda (Loop Quantum Gravity) o‘z-o‘ziga dual qora tuynuk holati uchun maydon tenglamalarini topish va elektromagnit maydon to‘rt-vektorni aniqlash.

LQGda tashqi bir jinsli magnit maydonda joylashgan qora tuynuk yaqinidagi magnit dipollar va elektr zaryadlangan zarrachalar harakatini o‘rganish.

Tashqi bir jinsli magnit maydonga kiritilgan Schwarzschild qora tuynugi atrofida magnitlangan zarralarining bog‘langan va bog‘lanmagan harakatini tahlil qilish.

Sgr A\* atrofidagi magnit dipol momentiga ega S2 yulduzining kuzatilgan joylashuvi bilan olingan natijalarni qiyosiy tahlil qilish.

**Tadqiqot obyekti:** astrofizik ixcham ob'ektlar, zarrachalar dinamikasi, qorong‘u materiya halolari uchun analitik modellar, turli fazo-vaqtlarida bir xil magnit maydonlar, S2 yulduzi trayektoriyalari.

**Tadqiqot predmeti:** qora tuynuklarning kuzatuv xususiyatlari, LQGda magnit maydonga kiritilgan o‘z-o‘ziga dual qora tuynuk, o‘ta og‘ir qora tuynuk atrofidagi zarrachalar dinamikasi, zarrachalar harakatining differensial tenglamalarini yechish uchun analitik va raqamli usullar.

**Tadqiqot usullari:** nazariy fizika va astrofizika yondashuvlari, zamonaviy nazariy astrofizika va matematik fizika usullari, shuningdek, maydon va zarrachalar dinamikasi bilan bog‘liq differensial tenglamalarni yechish uchun analitik va raqamli texnikalar.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

Birinchi marta Dehnen tipidagi qorong‘u materiya halosida yadro modeli uchun Schwarzschild yechimiga o‘xshash analitik yechimi topildi.

Birinchi marta LQGda o'z-o'ziga dual qora tuynuk holati uchun maydon tenglamalari va elektromagnit maydon to'rt-vektor potentsiali yechimi olindi. LQGda o'z-o'ziga dual qora tuynuk yaqinidagi magnit dipollar va elektr zaryadlangan zarralar harakati o'rganildi.

Birinchi marta tashqi bir jinsli magnit maydonga kiritilgan Schwarzschild qora tuynugi atrofida aylanayotgan magnitlangan zarralarning bog'langan va bog'lanmagan harakati tahlil qilindi.

Birinchi marta S2 yulduzning nazariy joylashuvi uchun olingan natijalar yulduzining haqiqiy joylashuvi bilan taqqoslandi va S2 yulduzining magnit dipoli  $10^6 \text{ G}\cdot\text{sm}^3$  sifatida baholandi.

**Tadqiqotning amaliy natijalari** quyidagilarni o'z ichiga oladi:

Topilgan Dehnen tipidagi zichlik profiliga (1,4,2) ega qorong'u materiya halosi bilan o'ralgan statik qora tuynuk yechimining fazo-vaqt egri xususiyatlari va fizik jihatdan ma'noga egaligining asoslovchi energiya shartlari tekshirildi.

LQGda tashqi bir jinsli magnit maydonga kiritilgan o'z-o'ziga dual qora tuynuk fazo-vaqt geometriyasi atrofidagi muhitda magnit dipollar va elektr zaryadlangan zarralar harakati tahlil qilindi. Kvant tuzatish parametri P va magnit o'zaro ta'sir parametri  $\beta$  ning zarralar traektoriyasiga ta'siri baholandi. LQGda o'z-o'ziga dual qora tuynukning ekvatorial tekisligida magnetlangan va elektr zaryadlangan zarralar orbitasining mumkin bo'lgan turli holatlari tekshirildi.

Magnit maydonda joylashgan Schwarzschild qora tuynugi fazo vaqt geometriyasi atrofidagi magnitlangan zarralarning xususiy energiya va burchak momentiga asoslangan bog'langan va bog'lanmagan orbitalari tasniflandi. Sgr A\* atrofidagi S2 yulduzining kuzatilgan joylashuvlari va nazariy hisoblashlar magnit o'zaro ta'sir parametri  $\beta$  ning ikki xil qiymati uchun taqqoslandi va S2 yulduzining xususiy burchak momenti uchun uning minimal va maksimal qiymatlari topildi.

**Tadqiqot natijalarining ishonchliligi.** Dissertatsiyada standart matematik va nazariy fizika usullaridan foydalanish, shu jumladan yuqori samarali raqamli texnikalar va dasturiy ta'minotdan foydalanish orqali ta'minlanadi. Nazariy natijalarning cheklovchi holatlardagi mavjud ma'lumotlar, kuzatuv dalillari va boshqa tadqiqotchilarning topilmalari bilan mos kelishini ta'minlash uchun sinchkovlik bilan tekshiruv o'tkazildi. Bundan tashqari, xulosalar kuchli gravitatsiya rejimlaridagi gravitatsion ixcham obyektlar nazariyasining asosiy prinsiplariga mos keladi.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.**

Tadqiqot natijalarining ilmiy ahamiyati shundaki, taklif qilingan yangi analitik qorong'u materiya halosida joylashgan Schwarzschild yechimiga o'xshash qora tuynuk — qorong'u materiyaning tabiatini tushuntirishi mumkin.

Tadqiqot natijalarining amaliy ahamiyati shundaki, taklif qilingan model Sgr A\* atrofidagi muhitni tushuntirishi va S2 yulduzining magnit dipolini baholashga yordam berishi mumkin.

**Tadqiqot natijalarining qo'llanilishi:** Ixcham ob'ektlar dinamikasiga asoslanib, ularning xususiyatlariga oid ilmiy topilmalar xalqaro tadqiqotchilar tomonidan o'rganishlarda qo'llanilgan va yuqori ta'sirli xorijiy jurnallarda, jumladan (Journal of Cosmology and Astroparticle Physics, 2025(03), Volume 2025, Web-Sc, IF: 5.9; The European Physical Journal C, Volume 85, Article number 786, Web-Sc, IF: 4.2; The European Physical Journal C, Volume 85, Article number 494, Web-Sc, IF: 4.2; Universe, Web-Sc, IF: 2.6;)da nashr etilgan.

Mazkur ilmiy natijalar Vellore Institute of Technology instituti tomonidan qo'llab-quvvatlangan dasturlar doirasida qo'llanilgan (Dr. Pankaj Sheoran tomonidan taqdim etilgan rasmiy xat asosida).

**Tadqiqot natijalarining e'lon qilinganligi.** Tadqiqot mavzusi bo'yicha jami 5 ta ilmiy ish, jumladan xorijiy nufuzli jurnallarda 3 ta, O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish uchun tavsiya etgan ilmiy nashrlar ro'yxatida 2 ta ilmiy tezis chop etilgan.

**Dissertatsiyaning tuzilishi va hajmi.** Dissertatsiya kirish, 3 ta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat. Dissertatsiya hajmi 124 bet.

## DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismi mavzuning dolzarbligi va zarurligini, tadqiqotning respublika fan va texnologiyalarini rivojlantirishning ustuvor yo'nalishlariga mosligini, muammoning bilish darajasini, uning dissertatsiya tayyorlangan oliy ta'lim muassasasi tadqiqot rejalari bilan bog'liqligini, tadqiqotning maqsadi, vazifalari, obyekti, mavzu haqidagi qisqa ma'lumot, usullar, ilmiy yangilik, amaliy natija, ishonchlilik, ilmiy va amaliy ahamiyat, natijalarning amaliyotga joriy qilinishi, natijalarning tasdiqlanishi, nashr etilishi, shuningdek, dissertatsiyaning tuzilishi va hajmini ko'rsatadi.

Dissertatsiyaning **birinchi bobi** "Schwarzschild-like black holes in Dark matter halo" deb nomlanib Dehnen tipidagi zichlik profiliga ega bo'lgan qorong'u materiya halosi bilan o'ralgan statik qora tuynukni namoyish etuvchi yangi analitik Schwarzschild yechimiga o'xshash qora tuynuk yechimini olishga bag'ishlangan. Ushbu bobda yangi olingan qora tuynuk yechimining xususiyatlarini uning fazo-vaqt egri chizig'i xususiyatlari va energiya shartlarini o'rganish orqali tahlil qilinadi, bu qorong'u materiya halosining ushbu fundamental xususiyatlarga qanday ta'sir qilishi haqida tushuncha beradi. Ushbu yechim qora tuynuk-qorong'u materiya tizimlari o'zaro ta'sirining muqobil nuqtai nazarini ifodalashi mumkin, qorong'u materiya halolarining fundamental xususiyatlari haqida yangi tushunchalar beradi.

Dehnen tipidagi qorong'u materiya halosi zichlik taqsimoti:

$$\rho(r) = \rho_s \frac{\left[\left(\frac{r}{r_s}\right)^\alpha + 1\right]^{\frac{\gamma-\beta}{\alpha}}}{\left(\frac{r}{r_s}\right)^\gamma}, \quad (1)$$

bu yerda  $\rho_s$  haloning xarakterli zichligi (halo qulagan paytdagi koinot zichligi),  $r_s$  qorong'u materiya halosining xarakterli o'lchami, va  $(\alpha, \beta, \gamma)$

parametrlar zichlik profilining qiyaliklarini boshqaradi, bu bizning holatimizda (1,4,2) sifatida. [1] bizga haloning massa profilini quyidagicha topishga imkon beradi:

$$M_D = \frac{4\pi\rho_s r_s^3}{1 + \frac{r_s}{r}}. \quad (2)$$

Keyin sof qorong'u materiya halosi uchun chiziq elementi quyidagicha ifodalanishi mumkin:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2. \quad (3)$$

Shuningdek, qorong'u materiya ichida harakatlanayotgan sinov zarrasining urinma tezligi (2,3) tenglamalardan foydalanib quyidagicha yozilishi mumkin:

$$v_\phi = \frac{1}{r} \partial_r \left[ \log \sqrt{A(r)} \right] = \frac{M_D}{r}, \quad (4)$$

bu bizga  $A(r)$  ni quyidagicha topishga imkon beradi:

$$A(r) = \left( 1 + \frac{r_s}{r} \right)^{-8\pi r_s^2 \rho_s}. \quad (5)$$

Qorong'u materiya halosidagi qora tuynuk uchun Eynshteyn maydon tenglamasi quyidagicha ifodalanishi mumkin:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi [T_{\mu\nu}(D) + T_{\mu\nu}(BH)], \quad (6)$$

Bu yerda  $T_{\mu}^{\nu}(D) = \text{diag}[-\rho(r), P_r(r), P_t(r), P_t(r)]$  qorong'u materiya halosining energiya-moment tensori,  $T_{\mu\nu}$  qora tuynuk fazo-vaqti tufayli paydo bo'ladi. Keyinchalik, Eynshteyn maydon tenglamasini [6] yechib, Dehnen tipidagi qorong'u materiya halosidagi Schwarzschild qora tuynuk uchun chiziq elementini quyidagicha olishimiz mumkin:

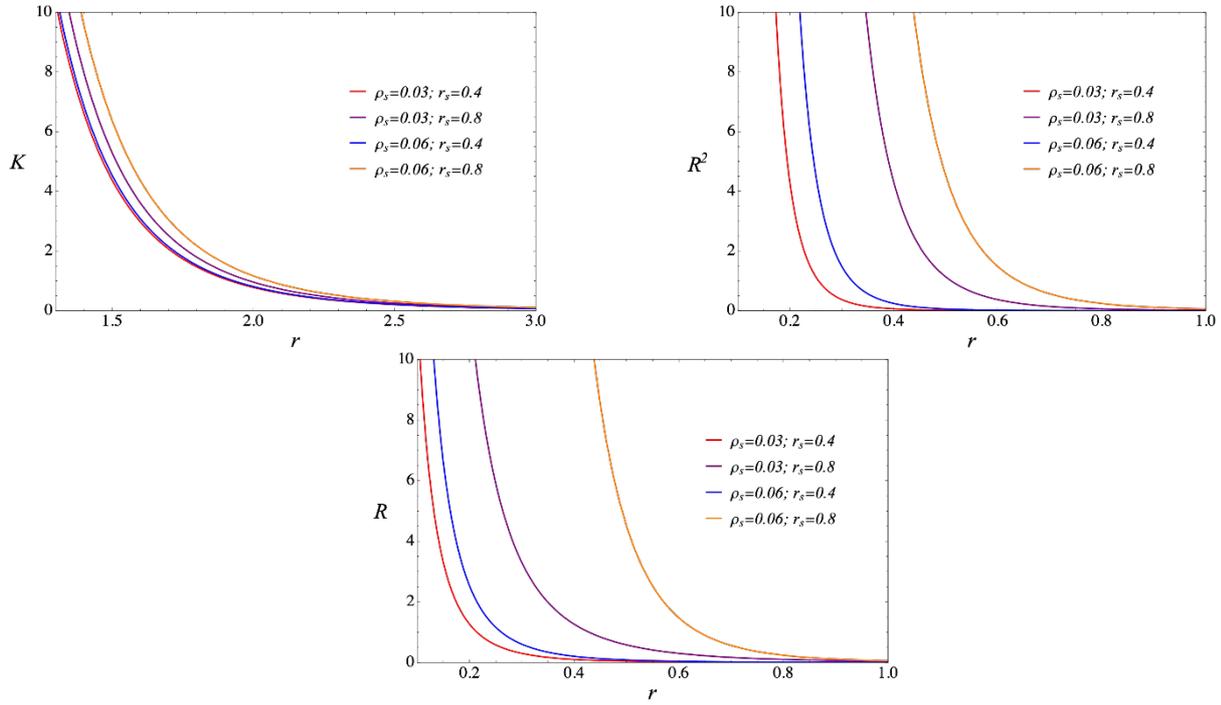
$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega^2, \quad (7)$$

bunda  $f(r)$ :

$$f(r) = 1 - \frac{2M}{r} - \frac{2M_D}{r_s} \log\left(1 + \frac{r_s}{r}\right) \left(1 + \frac{r_s}{r}\right). \quad (8)$$

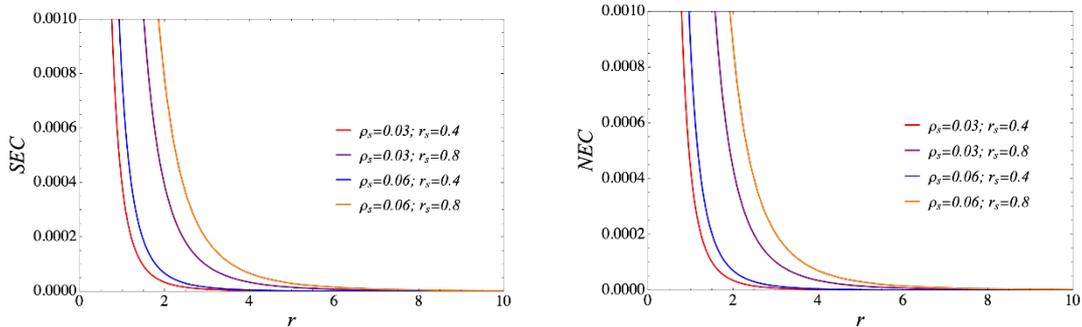
Ushbu qora tuynukning singularlik tuzilishini tekshirish uchun, fazo-vaqtning egri chiziq invariantlarini, masalan  $R$  Ricci skalyarini,  $R_{\mu\nu}R^{\mu\nu}$  Ricci

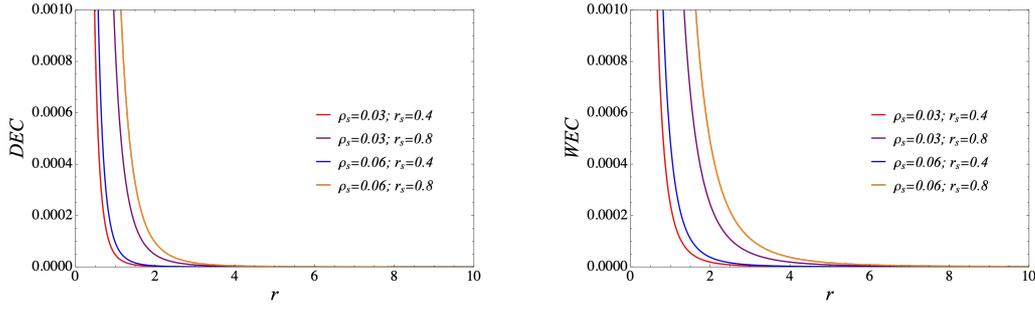
kvadratini,  $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$  va Kretshmann skalyari tahlil qilindi va ushbu invariantlarning radial bog'liqligini (Rasm 1)da tasvirlandi. Ushbu rasm (Rasm1)dan Dehnen tipidagi qorong'u materiyadagi qora tuynukning  $r=0$  da singular ekanligini payqash olish mumkin.



**Rasm. 1. Egri chiziq invariantlari.**

Umuman qabul qilingan holda, Eynshteyn tenglamalarining har qanday real yechimi uchta energiya shartiga, ya'ni zaif (WEC:  $(\rho(r) \geq 0, \rho(r) + P_i(r) \geq 0$  ( $i = r, \theta, \phi$ )), kuchli (SEC:  $\rho(r) + \sum_{i=1}^3 P_i \geq 0$ ) va dominant energiya (DEC:  $\rho(r) - |P_{\theta,\phi}| \geq 0$ ) shartlariga rioya qilishi kerak. Biz bu shartlarni vizual ravishda (Rasm 2.) da ko'rsatamiz.





**Rasm 2. NEC, WEC, DEC, SEC ning radial bog'liqligi  $\rho_s$  va  $r_s$  parametrlarining o'zgarishi bo'yicha.**

Dissertatsiyaning **ikkinchi bobi** "Dynamics of gravitational objects in the SMBH environment" deb nomlangan. Ushbu bobda LQG doirasidagi o'z-o'ziga dual qora tuynuk yaqinida, tashqi asymptotik bir xil magnit maydonga kiritilgan magnit dipollar va elektr zaryadlangan zarralarning harakati o'rganildi. Kvant tuzatish parametri va elektromagnit o'zaro ta'sirlarning zarra geodeziklariga ta'siri ko'rib chiqildi. Maydon tenglamalari chiqarildi va LQGdagi o'z-o'ziga dual qora tuynuk holati uchun elektromagnit to'rt vektorli potentsial aniqlandi. Magnit dipollar va elektr zaryadlangan zarralar uchun eng ichki barqaror doiraviy orbitlari (ISCO) batafsil o'rganildi, kvant tuzatish parametrining ISCO radiusiga sezilarli ta'sir qilishini va uning qisqarishiga olib kelishi ko'rsatildi. Bundan tashqari, magnit dipollarning ISCO radiusi magnit maydon o'zaro ta'siri tufayli elektr zaryadlangan zarralarnikidan katta ekanligi ko'rsatildi. Magnit dipollar va elektr zaryadlangan zarralar uchun ISCO parametrlarni (ya'ni  $r_{ISCO}$ ,  $l_{ISCO}$ ,  $\mathcal{E}_{ISCO}$ ,  $v_{ISCO}$ , va  $\Omega_{ISCO}$ ) batafsil qiymatlarni taqdim etgan holda o'rganildi. Bundan tashqari, kvant tuzatish parametri P natijasida yuzaga keladigan turli holatlarda zaryadlangan zarralarning traektoriyalari ko'rib chiqildi.

LQGdagi o'z-o'ziga dual qora tuynukning samarali metrikasi quyidagicha berilgan:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2[d\theta^2 + \sin^2\theta d\phi^2], \quad (9)$$

bu yerda:

$$A(r) = -g_{tt} = \frac{(r-r_+)(r-r_-)(r+r_*)}{r^4},$$

$$B(r) = \frac{1}{g_{rr}} = \frac{(r-r_+)(r-r_-)}{(r+r_*)^2},$$

bu yerda  $r_+ = 2M/(1+P)^2$  va  $r_- = 2MP^2/(1+P)^2$  qora tuynukning ikkita gorizontiga ishora qiladi, holbuki  $r_* = \sqrt{r_+r_-} = 2MP/(1+P)^2$  va ADM massa bilan  $M$ .

Keyin aylanmaydigan qora tuynuk uchun vektor potentsialning ansatzi quyidagicha ifodalanishi mumkin:

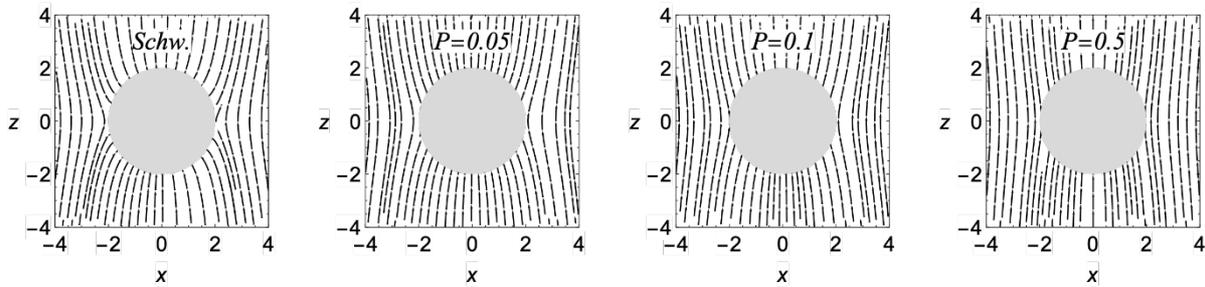
$$A^\mu = c_t(r, B_0)\xi_t^\phi + \left[ \frac{B_0}{2} + c_\phi(r, B_0) \right] \xi_\phi^\mu, \quad (10)$$

Bu yerda  $\xi_t^\mu = (1, 0, 0, 0)$  va  $\xi_\phi^\mu = (0, 0, 0, 1)$  vaqtga o'xshash va fazoga o'xshash Killing vektorlariga ishora qiladi. Aylanmaydigan vakuum qora tuynuki uchun Maxwell maydon tenglamalari bilan aniqlanishi mumkin  $F_{;v}^{\phi v} = 0$ , shuning uchun to'rt vektor potentsial uchun yakuniy javob:

$$A^\mu = \frac{B_0}{2} (\mathcal{B})^{\mathcal{A}} {}_2F_1(\mathcal{A}, \mathcal{A} + 3; 2\mathcal{A} + 3; \mathcal{B}) \xi_\phi^\mu, \quad (11)$$

yangi o'zgaruvchilar sifatida  $\mathcal{A} = \sqrt{\frac{1-2P}{1+2P}} - 1$  va  $\mathcal{B} = \frac{r(4P+1)}{2M(2P+1)}$  kiritildi.

Keyin Rasm 3 da LQGdagi o'z-o'ziga dual qora tuynuk yaqinidagi magnit maydon chiziqlarini chizildi.



**Rasm 3. LQGdagi o'z-o'ziga dual qora tuynuk yaqinidagi magnit maydon chiziqlarini kvant tuzatish parametri P ning turli kombinatsiyalari uchun ko'rsatadi.**

Endi biz tashqi magnit maydoniga joylashtirilgan LQG dagi qora tuynuk yaqinidagi magnitlangan zarralar uchun Hamilton-Yakobi tenglamasini quyidagicha yozishimiz mumkin:

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2 \left( 1 - \frac{\mathcal{U}}{m} \right), \quad (12)$$

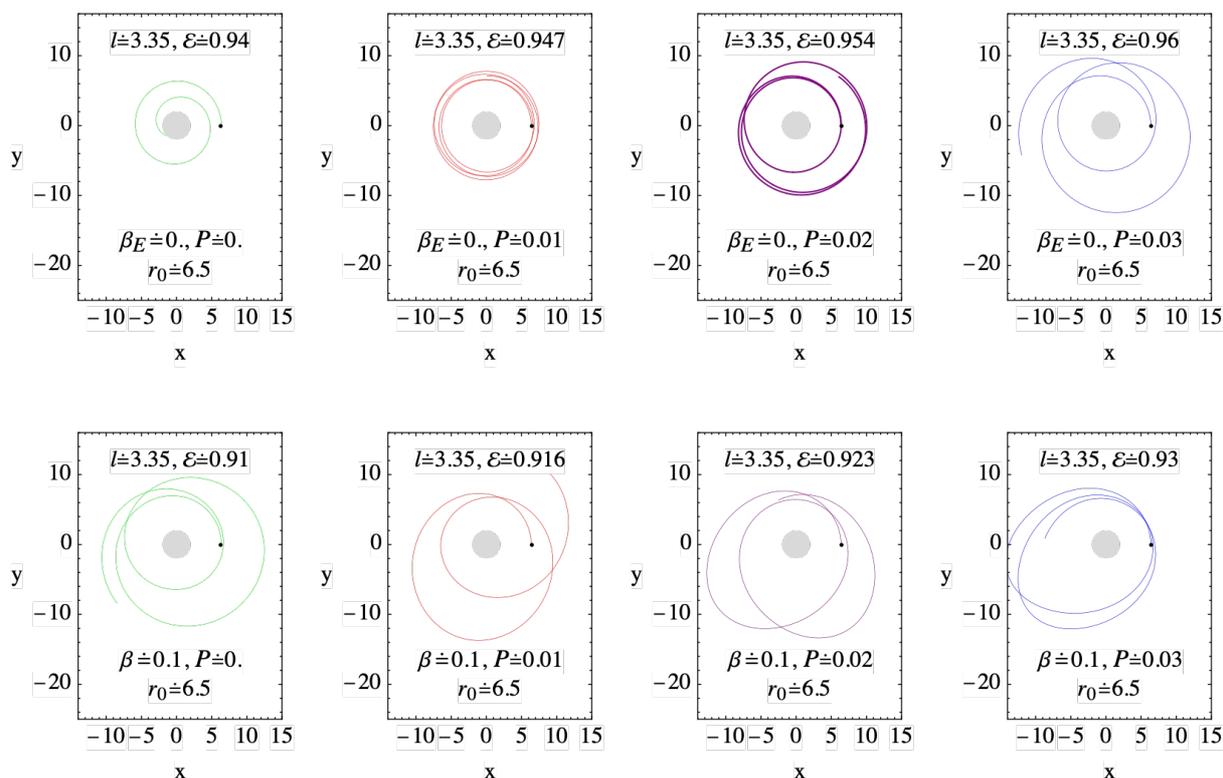
bu yerda  $\mathcal{U} = \mu_\alpha B^\alpha$  magnit o'zaro ta'sirni bildiradi. Keyin ekvatorial tekislikda Hamilton-Yakobi ta'sirini qo'llagan holda  $S = -Et + L\phi + S_r$ , magnitlangan zarralarning harakatini quyidagicha ifodalash mumkin:

$$\begin{aligned} \dot{t} &= -\frac{\mathcal{E}}{\left(1 + \frac{\mathcal{U}}{m}\right) g_{tt}}, \\ \dot{\phi} &= \frac{l}{\left(1 + \frac{\mathcal{U}}{m}\right) g_{\phi\phi}}, \\ \dot{r}^2 &= \frac{1}{g_{rr} g_{tt}} \left( \mathcal{E}^2 - A(r) \left[ 1 + \frac{l^2}{r^2} - \frac{\mathcal{U}}{m} \right] \right). \end{aligned} \quad (13)$$

Zaryadlangan zarrachalar uchun ham xuddi shu tahlilni amalga oshirishimiz mumkin:

$$g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} + qA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} + qA_\nu \right) = -m^2 \quad (14)$$

bu yerda  $q$  — elektr zaryadlangan zarrachaning zaryadi.



**Rasm 4. Tajribaviy zarrachaning zaryadi (yuqori panel) va magnet dipol momenti (pastki panel) bilan yo'nalishlari, tashqi bir jinsli magnet maydonga botirilgan o'z-o'ziga dual qora tuynuk atrofida.**

Unda, LQGdagi o'z-o'ziga dual qora tuynuk yaqinida tashqi bir jinsli magnet maydon ta'sirida elektr zaryadlangan zarrachaning harakat tenglamasi quyidagicha ifodalanadi:

$$\begin{aligned} \dot{t} &= -\frac{\varepsilon}{g_{tt}}, \\ \dot{\phi} &= \frac{l}{g_{\phi\phi}}, \\ \dot{r}^2 &= -\frac{1}{g_{rr}g_{tt}} \left[ \varepsilon^2 - A(r) \left( 1 + \left[ \frac{l}{r} + \frac{qA_\phi}{mr} \right]^2 \right) \right] \end{aligned} \quad (15)$$

Dissertatsiyaning **uchinchi bobi** “Orbits of particles with magnetic dipole moment around magnetized Schwarzschild black holes: Applications to S2 star orbit” deb nomlangan bo‘lib, tashqi asymptotik ravishda bir jinsli magnet maydonga kiritilgan Schwarzschild qora tuynugi atrofida magnitlangan zarrachalarning

bog'langan va bog'lanmagan harakatining keng qamrovli tahlilini o'z ichiga oladi, bu barcha mumkin bo'lgan bog'langan va bog'lanmagan orbita turlarini o'z ichiga oladi. Ayniqsa, planetar orbitalari uchun, biz o'z topilmalarimizni Sgr A\* atrofida magnit dipol momentiga ega S2 yulduzining kuzatilgan pozitsiyasi bilan solishtirish tahlilini o'tkazdik. Yulduzning magnit dipoli va tashqi magnit maydon o'rtasidagi magnit o'zaro ta'sir parametri, shuningdek, S2 yulduzining energiyasi va burchak momenti uchun maksimal va minimal qiymatlarni aniqlandi. Natijada, yulduzning magnit dipolini baholash amalga oshirildi. Bundan tashqari, biz gravitatsion Rutherford sochilishi kabi og'ishgan yo'nalishlarni o'rgandik. Orbital tenglamalar uchun yechimlarni olishda, biz elliptik integral va Jakobi elliptik funktsiyalarini qo'lladik, shuningdek tasvirlar hamda simulyatsiyalar tadqiqotni boyitadi.

Schwarzschild qora tuynugining chiziqli elementi quyidagicha ifodalanadi:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr + r^2d\Omega^2, \quad (16)$$

bu yerda

$$f(r) = 1 - \frac{2M}{r},$$

$$\text{va } d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2.$$

Waldning yondashuviga ko'ra, Maksvell tenglamasining yechimi bo'lgan to'rt-potensial quyidagicha ifodalanadi  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ , bu yerda  $\xi_\phi^\mu = (0,0,0,1)$  fazoviy Killing vektori sifatida ifodalanadi.

Tashqi bir jinsli magnit maydonga kiritilgan Schwarzschild qora tuynugi atrofida magnitlangan zarrachalarning orbitalari uchun Lagrangian quyidagicha ifodalanadi:

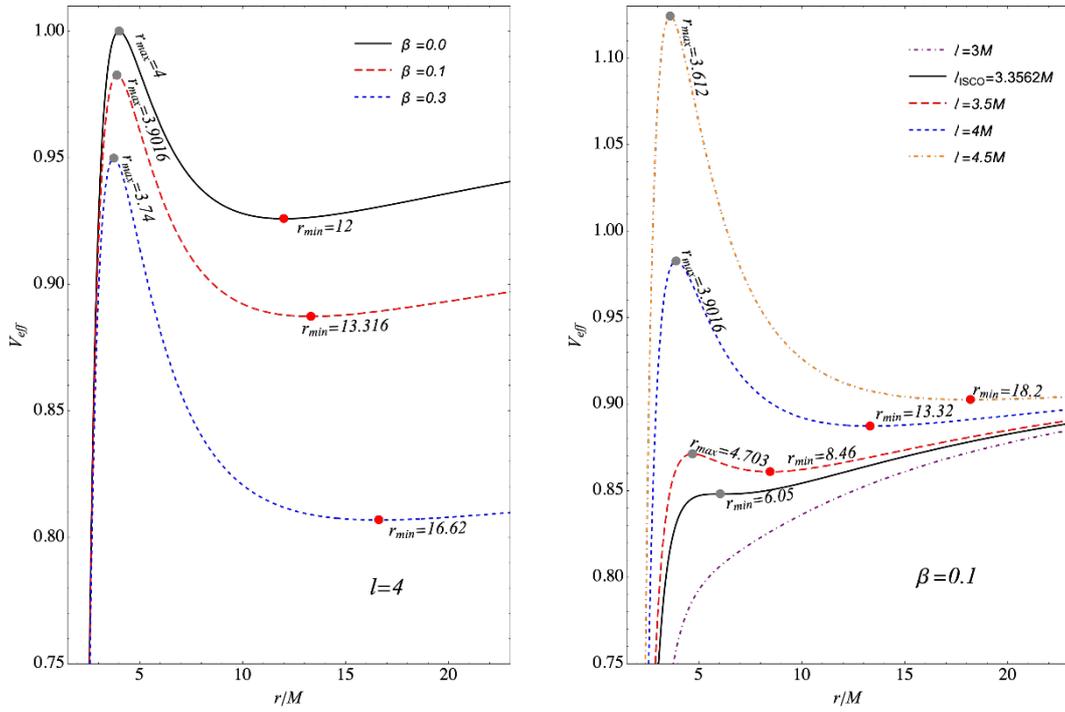
$$\mathcal{L} = \frac{1}{2}(m + \mathcal{U})g_{\mu\nu}u^\mu u^\nu - \frac{1}{2}\mathcal{U}, \quad (17)$$

bu yerda  $\mathcal{U} = \mu_\alpha B^\alpha$  magnit o'zaro ta'sir termini deb yuritiladi. Harakat tenglamalari esa quyidagi ko'rinishda yoziladi:

$$\begin{aligned} \dot{t} &= -\frac{\varepsilon}{[1+\beta\mathcal{F}]g_{tt}}, \\ \dot{\phi} &= \frac{l}{[1+\beta\mathcal{F}]g_{\phi\phi}}, \\ \dot{r}^2 &= \varepsilon^2 - V_{eff}, \end{aligned} \quad (18)$$

bu yerda

$$V_{eff}(r, l) = f(r) \left( [1 - \beta\mathcal{F}]^2 + \frac{l^2}{r^2} \right),$$



**Rasm 5. Effektiv potensialning radial profili magnit o‘zaro ta’sir parametri  $\beta$  va xususiy burchak momenti  $l$  ning belgilangan qiymatlari uchun tasvirlangan. Effektiv potensialdagi minimumlar ( $r_{min}$ ) barqaror aylana orbitalariga, maksimalar ( $r_{max}$ ) esa barqaror bo‘lmagan aylana orbitalariga mos keladi.**

Bu yerda yangi o‘zgaruvchilarni  $\beta = \mu B/m$  va  $\mathcal{F}(r, \theta) = \sqrt{f(r)} \sin \theta$  ko‘rinishida kiritamiz. Keying 5-rasmda turli holatlar uchun effektiv potensialning radial bog‘liqligini ko‘rsatdik.

Shuningdek, quyidagi (19)-tenglama yordamida zarrachaning harakatini ifodalaydigan differensial tenglamani olamiz:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4 [1 + \beta \sqrt{f(r)}]}{l^2} \left[ \mathcal{E} - f(r) \left( 1 + \frac{l^2}{r^2} - \beta \sqrt{f(r)} \right) \right], \quad (19)$$

bu yerda  $\beta = \frac{\mu B}{m}$  magnit o‘zaro ta’sir parametri,  $l$  va  $\mathcal{E}$  esa mos ravishda magnitlangan zarrachalarning xususiy burchak momenti va xususiy energiyasidir. Keyin, agar biz (19)-tenglamaga yangi o‘zgaruvchilarni kiritadigan bo‘lsak  $u = 1/r$ , magnitlangan zarrachalarning harakatini boshqaradigan quyidagi tenglamani olamiz:

$$\left(\frac{du}{d\phi}\right)^2 = (1 + \beta)(au^3 - bu^2 + cu + d), \quad (20)$$

bu yerda

$$a = 2M \left[ (2\beta + 1) - \frac{2\beta^2 M^2}{l^2} \right], \quad (21)$$

$$b = (\beta + 1) + \frac{2\beta(1 - 4\beta)M^2}{l^2},$$

$$c = \frac{M[2 - \beta(5\beta + 2\varepsilon^2 - 1)]}{l^2},$$

$$d = \frac{(\beta + 1)(\beta + \varepsilon^2 - 1)}{l^2}.$$

(20)-tenglama dan osonlik bilan shuni xulosa qilish mumkin: zarrachalar  $\varepsilon^2 < 1 - \beta$  uchun bog'langan orbitalar hosil qiladi,  $\varepsilon^2 > 1 - \beta$  uchun esa bog'lanmagan orbitalar hosil qiladi. Keyin, agar biz sinov zarrachalarining energiyasi  $\varepsilon^2 < 1 - \beta$  deb faraz qilsak va (20)-tenglamaning barcha ildizlari musbat bo'lsa (shart bilan  $u_1 < u_2 < u_3$ ), sinov zarrachalari planetar bog'langan orbitalar hosil qiladi. Shunday qilib, magnitlangan zarrachalar uchun harakatni  $u_1$  dan boshlanadigan yo'nalish quyidagicha bo'ladi:  $\phi = 0$

$$r(\phi) = \frac{1}{u_1 + (u_2 - u_1)sn^2(\theta_0, k_0)}, \quad (22)$$

bu yerda  $sn(\theta, k)$  — Jakobi elliptik sinus funktsiyasi, uning argumenti  $\theta_0 = \sqrt{a(1 + \beta)(u_3 - u_1)} \frac{\phi}{2}$  va moduli  $k_0 = \sqrt{\frac{u_2 - u_1}{u_3 - u_1}}$  bilan.

(22)- tenglamaning haqiqiy yechimlarini quyidagicha belgilash mumkin:

$$u_1 = \frac{1 - e}{\lambda}, \quad (23)$$

$$u_2 = \frac{1 + e}{\lambda},$$

$$u_3 = \frac{b}{(1 + \beta)a} - \frac{2}{\lambda}$$

bu yerda  $\lambda$  va  $e$  — mos ravishda latus rektum va eksantriklik. Shuningdek, Levin va Perez-Giz tasnifi yordamida:

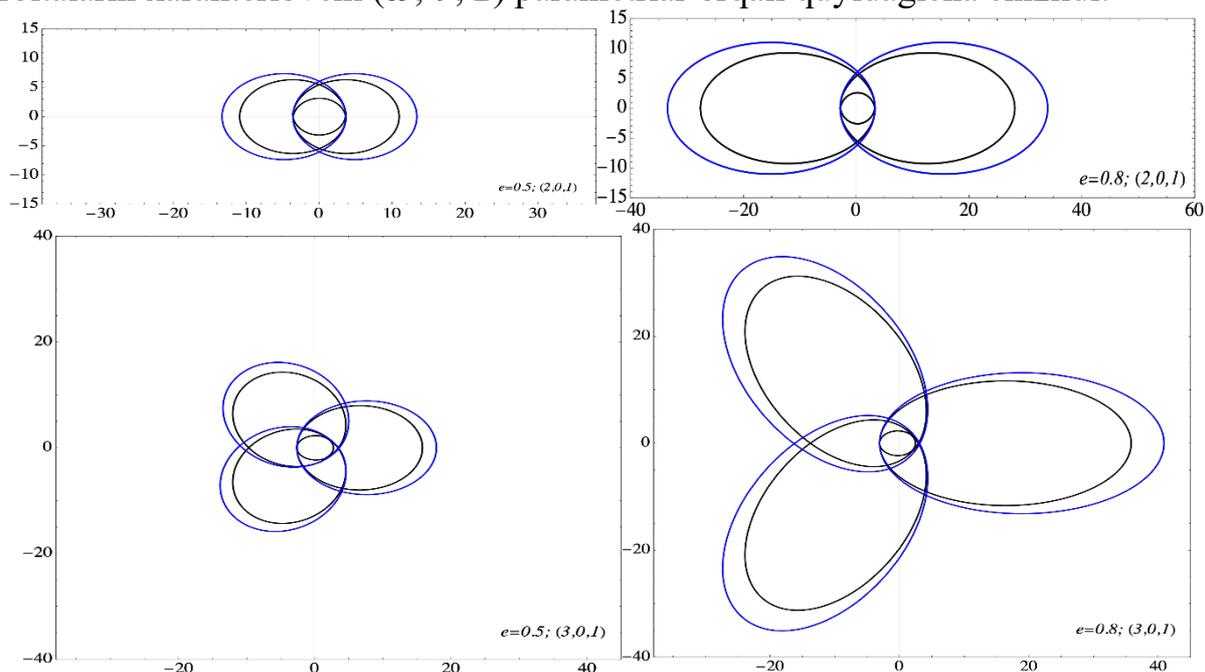
$$q + 1 = \omega + \frac{v}{z} + 1 = \frac{\Delta\phi}{2\pi}, \quad (24)$$

yuqridagi ifodadagi  $\omega$ ,  $v$  va  $z$  mos ravishda burilish, vertikal va zoom raqamlari sifatida ifodalanadi. Bu esa quyidagi tenglamani olish imkonini beradi:

$$q + 1 = \frac{2K(k_1)}{\pi\sqrt{b - \frac{a(1 + \beta)(3 - e)}{\lambda}}}. \quad (25)$$

Bu yerda  $K(k)$  — birinchi turdagi to'liq elliptik integral, moduli  $k_1 = \sqrt{\frac{2ea(1 + \beta)}{\lambda - a(1 + \beta)(3 - e)}}$  bilan. Endi (25)-tenglamani raqamli hisoblash orqali, planetar

orbitalarni vizual tarzda ko‘rsatish mumkin bo‘ladi. Turli qiymatlar uchun planetar orbitalarni xarakterlovchi ( $\omega$ ,  $v$ ,  $z$ ) parametrlar orqali quyidagicha chizildi:

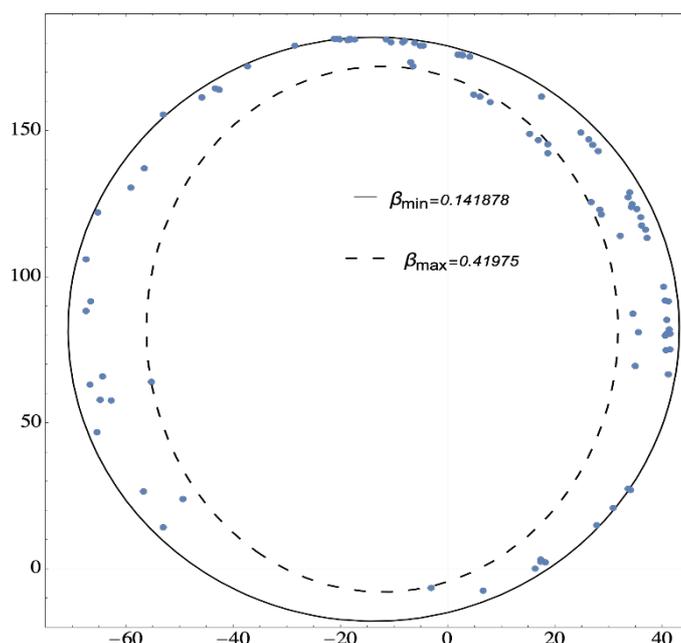


**Rasm 6. Turli eksantrikliklarga mos keluvchi orbitalar: ko‘k orbitalar  $\beta = 0.1$  ni, qora orbitalar esa  $\beta = 0.0$  ni ifodalaydi.**

Oddiy elliptik yo‘nalish uchun, S2 yulduzining orbitasi (1, 0, 0) bo‘lishi kerak, shuning uchun (25)- tenglama quyidagi ko‘rinishga ega bo‘ladi:

$$\frac{\pi}{2} = \frac{K(k_1)}{\sqrt{b - \frac{a(1+\beta)(3-e)}{\lambda}}}. \quad (26)$$

Natijada, Sgr A\* atrofidagi yulduzlar klasterida joylashgan S2 yulduzining joylashuvini kuzatish orqali, S2 yulduzining yo‘nalishini cheklashimiz va unga mos keluvchi magnit o‘zaro ta’sir terminini  $\beta$  topishimiz mumkin:



**Rasm 7. Sgr A\* atrofida aylanayotgan S2 yulduzining kuzatilgan yo‘nalishi.**

S2 yulduzining kuzatilgan ma'lumotlari yordamida biz magnit o'zaro ta'sir parametri  $\beta$  mumkin bo'lgan oraliqni oldik:

$$0.14249 \leq \beta \leq 0.419751.$$

## XULOSA

“Gravitatsion obyektning qorong'u materiya halosi va magnit maydoni mavjud bo'lgan muhitdagi o'ta og'ir qora tuynuklar atrofidagi dinamikasi” mavzusidagi dissertatsiya ishining natijalari asosida quyidagi xulosalar olindi:

1. Dehnen tipidagi qorong'u materiya halosi bilan o'ralgan Schwarzschildsimon qora tuynugi uchun yangi analitik yechim topildi. Yangi topilgan qora tuynuk yechimi uchun uning fazo-vaqt xususiyatlari va energiya shartlari qanoatlanishi ko'satildi. Ushbu yangi yechim qora tuynuk va qorong'u materiya sistemasining o'zaro ta'sirini ifodalaydigan va qorong'u materiya halolarining asosiy xususiyatlarini ochib beradigan yangi muqobil sifatida taqdim etildi.

2. LQG doirasidagi o'z-o'ziga dual va tashqi bir jinsli magnit maydonga joylashtirilgan qora tuynuk atrofida magnit dipollar va elektr zaryadlangan zarrachalar harakati trayektoriyasi aniqlandi. Kvant tuzatish parametri va elektromagnit o'zaro ta'sirlarning zarracha traektoriyalariga ta'siri aniqlandi. Maydon tenglamalari tahlil qilindi va LQGdagi o'z-o'ziga dual qora tuynuk holati uchun elektromagnit to'rt vektorli potentsiali topildi.

3. Magnit dipollar va elektr zaryadlangan zarrachalar uchun ISCO batafsil ko'rsatildi, bu esa kvant tuzatish parametri ISCO radiusiga sezilarli ta'sir ko'rsatib, uning kichrayishiga olib kelishi aniqlandi. Bundan tashqari, magnit maydon o'zaro ta'siri tufayli magnit dipollarning ISCO radiusi elektr jihatdan zaryadlangan zarralarnikidan kattaroq ekanligi ko'rsatiladi. ISCO parametrlari ( $r_{ISCO}$ ,  $l_{ISCO}$ ,  $E_{ISCO}$ ,  $v_{ISCO}$  va  $\Omega_{ISCO}$ ) magnit dipollar va elektr zaryadlangan zarrachalar uchun aniqlanib, batafsil qiymatlari keltirildi. Bundan tashqari, zaryadlangan zarrachalarning kvant tuzatish parametri P natijasida yuzaga keladigan turli traektoriyalari ko'rsatildi.

4. Schwarzschild qora tuynugi atrofida tashqi asimptotik bir jinsli magnit maydonda harakatlanayotgan magnitlangan zarralarning orbita jihatdan chegaralangan va chegaralanmagan harakati aniqlandi. Xususan, planetar orbitlari uchun biz topgan natijalarimizni Sgr A\* atrofida magnit dipol momentga ega S2 yulduzining kuzatilgan pozitsiyasi bilan taqqoslandi. Yulduzning magnit dipoli va tashqi magnit maydon o'rtasidagi magnit o'zaro ta'sir parametri uchun maksimal va minimal qiymatlari, shuningdek, S2 yulduzining energiyasi va burilish momenti topildi va yulduzning magnit dipoli momenti  $10^6 G \cdot cm^3$  ekanligi aniqlandi.

**SCIENTIFIC COUNCIL DSc. 03/07.07.2025. FM/T.192.01 ON AWARD  
OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES  
AT “NEW UZBEKISTAN” UNIVERSITY**

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**INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH**

**UKTAMOV UKTAMJON KHAYRULLA UGLI**

**DYNAMICS OF GRAVITATIONAL OBJECTS IN SMBH  
ENVIRONMENT WITH DARK MATTER HALO AND MAGNETIC  
FIELD**

**01.03.01 – Astronomy  
01.04.02 – Theoretical Physics**

**DOCTOR OF PHILOSOPHY IN PHYSICAL AND MATHEMATICAL  
SCIENCES (PhD)  
ABSTRACT OF THE DISSERTATION**

**Tashkent – 2025**

**The theme of dissertation of the doctor of philosophy (PhD) on technical sciences was registered by Supreme Attestation Commission at the Ministry of higher education, science and innovations of the Republic of Uzbekistan under B2024.2.PhD/FM1094.**

The doctoral (PhD) dissertation was carried out at Institute of fundamental and applied research. The abstract of the dissertation was posted in three (uzbek, english , russian(resume)) languages on the website of the Scientific Council ([www.ias.newuu.uz](http://www.ias.newuu.uz)) and on the information and education portal at "Ziyonet" ([www.ziyonet.uz](http://www.ziyonet.uz)).

<b>Scientific supervisors:</b>	<b>Ahmedov Bobomurat Juraevich</b> Doctor of physical and mathematical sciences, academician
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<b>Leading organization:</b>	<b>Al-Farabi Kazakh National University</b>

The defense of the dissertation will be held on “\_\_\_” \_\_\_\_\_ 2025 at \_\_\_ in the meeting of the Scientific Council No. DSc.03/07.07.2025.FM/T.192.01 at the Institute for Advanced Studies at “New Uzbekistan” university (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnahr Street 1, Institute for Advanced Studies at “New Uzbekistan” university, phone: +99871 202-41-11; e-mail: [info@newuu.uz](mailto:info@newuu.uz))

The dissertation can be looked through at the Information Resource Center of the Institute for Advanced Studies at “New Uzbekistan” university (registered under №\_). (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnahr Street 1, Institute for Advanced Studies at “New Uzbekistan” university, phone: +99871 202-41-11).

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## INTRODUCTION (Annotation of PhD dissertation)

**Relevance and necessity of the topic.** Nowadays, advanced astronomical instruments like the LIGO-VIRGO and Event Horizon Telescope enable direct detection of gravitational waves from merging black holes and binary neutron stars, as well as capturing images of the supermassive black hole at the core of the M87 and Milky Way galaxies. The direct observation of gravitational waves and the detailed imaging of a supermassive black holes using these advanced instruments shed light on astrophysical events in strong gravitational fields. These cutting-edge observations are vital for developing new solutions in alternative gravity theories and for exploring challenges tied to accurately measuring parameters and understanding the essence of dark matter. Analyzing the interaction between dark matter (DM) and black holes (BHs) is crucial for enhancing our understanding of DM's fundamental nature. Given the significance of DM halos and their interplay with BH systems, studying and modeling DM through astrophysical data and simulations can provide valuable insights. To this end, several analytical models, such as the Einasto, Burkert, and Dehnen models, have been developed to describe BH solutions within DM halos.

Our Republic places significant emphasis on fundamental research in the area of relativistic astrophysics of black holes. The primary areas of fundamental research and development, as well as their practical applications for advancing science in our nation, are detailed in the Strategy for the Further Development of the Republic of Uzbekistan for 2022–2026<sup>1</sup>. Over the last three decades, the Republic has advanced relativistic astrophysics of gravitational compact objects, thoroughly investigating the energy and optical characteristics of black holes, wormholes, and naked singularities. New effects of general relativity have been discovered in the relativistic astrophysics of magnetized neutron stars, and progress has been made in understanding relativistic magnetospheres and plasma, as well as magnetized relativistic objects.

This dissertation focuses on fulfilling objectives outlined in state regulatory documents, including Presidential Decree No. UP-4947 of February 7, 2017, on the Strategy for the Further Development of the Republic of Uzbekistan, and the "Roadmap of Key Structural Reform Directions in Uzbekistan for 2019–2021," issued by the Government of Uzbekistan on November 29, 2018.

**Conformity of the research to the main priorities of science and technology development of the republic.**

The research has been conducted in line with the priority areas of science and technology in the Republic of Uzbekistan, specifically under the category II: "Power, Energy, and Resource-Saving."

**The aim of the research** is to develop a theoretical formalism for obtaining new analytical Schwarzschild-like black hole solutions within the dark matter halo and for analyzing particle dynamics around supermassive black holes (SMBH) in an environment with dark matter and magnetic field, together with comparative

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<sup>1</sup> Decree No. PF-60 of the President of the Republic of Uzbekistan dated January 1, 2022 "On the Development Strategy of New Uzbekistan for 2022-2026".

analysis of the obtained results with the observed position of the S2 star around Sgr A\* SMBH.

**The tasks of the research:**

to find a novel analytical Schwarzschild-like BH in Dehnen-type DM halo.

to analyze spacetime curvature characteristics and energy conditions of the obtained solution.

to derive the field equations and determine the electromagnetic four-vector for the case of self dual black hole in LQG.

to study the motion of magnetic dipoles and electrically charged particles in the vicinity of a BH in LQG, in the vicinity of the Schwarzschild BH immersed in external uniform magnetic field.

to investigate the bound and unbound motion of magnetized particles orbiting a Schwarzschild black hole immersed in an external asymptotically uniform magnetic field.

to perform a comparative analysis of findings with the observed position of the S2 star carrying magnetic dipole moment around Sgr A\*.

**Connection of the topic of dissertation with the scientific works of scientific research organizations, where the dissertation was carried out.** The presentation was carried out within the framework of scientific projects funded by the Ministry of Innovative Development. FL-7923051796 “Modeling of Scalar Field Dynamics and Quantum Emitters in Spherically Symmetric Gravitational Fields”, F-FA-2021-432 “Processing and Analysis of Satellite Data from Low Mass X-ray Binary Systems”.

**The objects of the research** are astrophysical compact objects, particle dynamics, analytical models for DM halos, uniform magnetic fields in different spacetimes, S2 star trajectories.

**The subjects of the research** are observational properties of black holes, Selfdual Black hole immersed in magnetic field in LQG, particle dynamics around SMBH, analytical and numerical methods for solving differential equations of the motion of particles.

**The methods of the research** are the approaches involve theoretical physics and astrophysics, contemporary methods of theoretical astrophysics and mathematical physics, as well as analytical and numerical techniques for solving differential equations associated with field and particle dynamics.

**The scientific novelty of the research** is in the following:

For the first time, an analytical Schwarzschild-like black hole solution in a Dehnen-type DM halo for the core model has been derived.

For the first time, field equations and four-vector potential for the case of a self-dual black hole in LQG are derived. The motion of magnetic dipoles and electric charged particles in the vicinity of a self-dual black hole in LQG is learned.

For the first time, the bound and unbound motion of magnetized particles orbiting a Schwarzschild BH immersed in an external asymptotically uniform magnetic field is derived.

For the first time, results of the obtained analysis is compared with the real positions of the S2 star and magnetic dipole of the star is estimated as  $10^6 G \cdot cm^3$ .

Deflecting trajectories akin to gravitational Rutherford scattering is explored.

**Practical results** of the research are as follows:

Novel analytical Schwarzschild-like BH solution is derived, representing a static BH surrounded by a DM halo characterized by a Dehnen-type density profile (1,4,2). The properties of this BH is investigated by analyzing the spacetime curvature. The energy conditions, which are fundamental characteristics of the physical viability of spacetime, is examined.

A self-dual black hole immersed in an external asymptotically uniform magnetic field is considered. The motion of magnetic dipoles and electrically charged particles in the environment surrounding the spacetime geometry in LQG is studied. The combined effects of the quantum correction parameter  $P$  and the magnetic interaction parameter  $\beta$  on the ISCO radius is studied. We analyzed captured, bound and escape orbits of magnetized and electrically charged particles restricted from moving on the equatorial plane of the self-dual black hole in LQG for various possible cases.

The bound and unbound orbits of the magnetized particles as they approached a Schwarzschild BH in magnetic field are investigated. The effective potential experienced by approaching particles, classifying possible bound and unbound orbits based on their specific energy and angular momentum has been derived and analyzed. Comparisons between observed positions of the S2 star around  $Sgr A^*$  and theoretical predictions were made for two different values of  $\beta$ , calculating minimum and maximum values for this parameter and specific angular momentum of the S2 star.

**Reliability of the research results** is supported by the use of standard mathematical and theoretical physics methods in the dissertation, including highly effective numerical techniques and software. A meticulous verification was conducted to ensure the theoretical results aligned with existing data in limiting cases, observational evidence, and the findings of other researchers. Additionally, the conclusions are consistent with the core principles of the theory of gravitational compact objects in strong gravity regimes.

**Scientific and practical significance of the research results.** The scientific significance of the research results is that proposed a novel analytical Schwarzschild-like BH in DM halo can explain the nature of the dark matter.

The practical significance of the research results is that proposed model can explain environment of the  $Sgr A^*$  and help to estimate magnetic dipole of the S2 star.

**Application of the research results.** Drawing on the dynamics of the compact objects, the scientific findings related to their properties have been utilized in studies by international researchers and published in high-impact foreign journals, including (Journal of Cosmology and Astroparticle Physics, 2025(03), Volume 2025, Web-Sc, IF: 5.9; The European Physical Journal C, Volume 85, Article number 786, Web-Sc, IF: 4.2; The European Physical Journal C, Volume 85, Article number 494, Web-Sc, IF: 4.2; Universe, Web-Sc, IF: 2.6;).

These scientific results were used within the framework of programs supported by the Vellore Institute of Technology (based on an official letter provided

by Dr. Pankaj Sheoran).

**The publication of the research results.** 5 scientific works on the research topic have been published, including 3 in international refereed journals with high impact factors, among them 2 scientific abstracts in the list of scientific publications recommended by the Supreme Attestation Commission under the Ministry of Higher Education, Science, and Innovations of the Republic of Uzbekistan for publishing the main scientific results of dissertations.

**Volume and structure of the dissertation.** The dissertation consists of an introduction, three chapters, conclusion and a bibliography. The size of the dissertation is 122 pages.

## THE MAIN CONTENT OF DISSERTATION

**The introduction** of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

**The first chapter** entitled “Schwarzschild-like black holes in Dark matter halo”. In this chapter, we present a novel analytical Schwarzschild-like black hole (BH) solution that exhibits a static BH with a dark matter (DM) halo characterized by a Dehnen-type density profile. We study the properties of the newly derived BH solution by examining its spacetime curvature characteristics and energy conditions, providing insights into how the DM halo influences these fundamental characteristics. Additionally, we analyze the timelike geodesics of test particles in the obtained BH-DM spacetime, highlighting how the presence of the novel Dehnen-type DM halo alters the gravitational dynamics and modifies particle trajectories. This solution could represent an alternative perspective on the interaction of black hole-dark matter systems, providing new insights into the fundamental properties of DM halos.

Dehnen-type DM halo density distribution:

$$\rho(r) = \rho_s \frac{\left[\left(\frac{r}{r_s}\right)^\alpha + 1\right]^{\frac{\gamma-\beta}{\alpha}}}{\left(\frac{r}{r_s}\right)^\gamma}, \quad (1)$$

here,  $\rho_s$  is the characteristic density (the density of the universe at the moment when the halo collapsed) of the halo,  $r_s$  is the characteristic scale of the DM halo, and the parameters  $(\alpha, \beta, \gamma)$  control the slopes of the density profile which is in our case as (1,4,2). [1] enables us to find mass profile of the halo as:

$$M_D = \frac{4\pi\rho_s r_s^3}{1 + \frac{r_s}{r}}. \quad (2)$$

Then the line element for pure DM halo can be expressed as:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2. \quad (3)$$

Also, tangential velocity of a test particle moving within DM using Eqs. (2, 3) can be written as:

$$v_\phi = \frac{1}{r} \partial_r \left[ \log \sqrt{A(r)} \right] = \frac{M_D}{r}, \quad (4)$$

which enables us to find A(r) as:

$$A(r) = \left( 1 + \frac{r_s}{r} \right)^{-8\pi r_s^2 \rho_s}. \quad (5)$$

The Einstein field equation for BH in DM halo can be expressed as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi [T_{\mu\nu}(D) + T_{\mu\nu}(BH)], \quad (6)$$

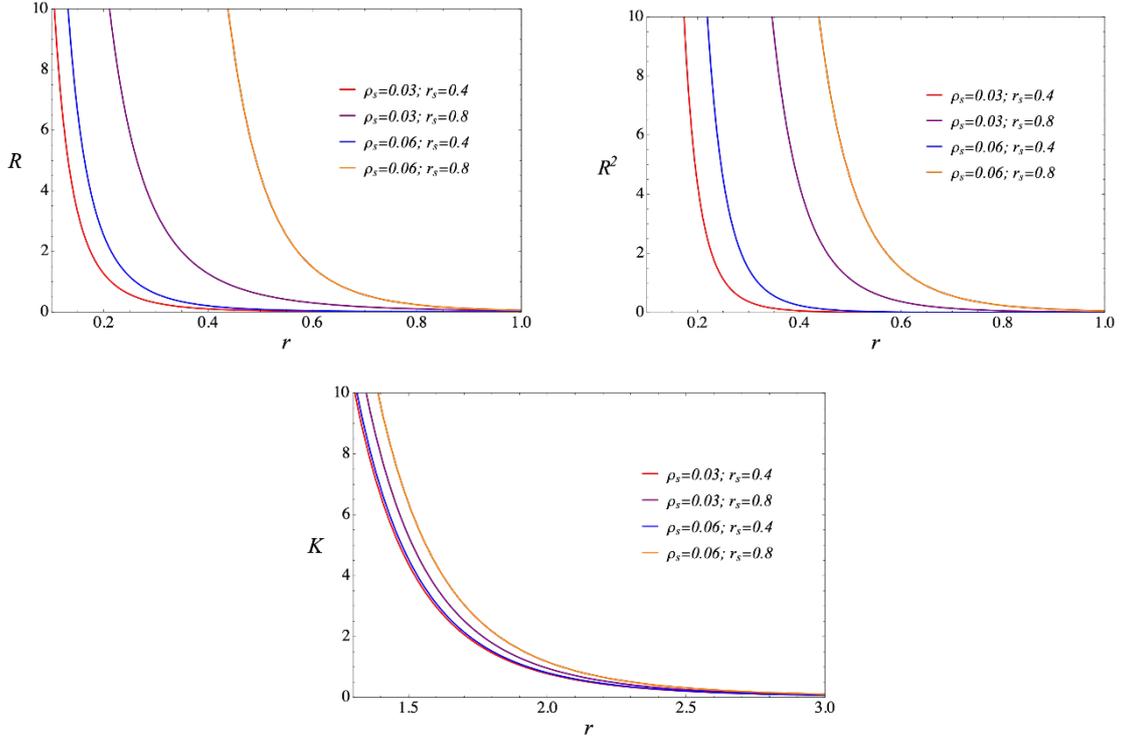
where  $T_\mu^\nu(D) = \text{diag}[-\rho(r), P_r(r), P_t(r), P_t(r)]$  is the energy-momentum tensor of the DM halo,  $T_{\mu\nu}$  appears due to BH space-time. Subsequently, solving Einstein field equation [6] we can obtain the line element for Schwarzschild BH in Dehnen type DM halo as:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega^2, \quad (7)$$

where f(r) is the lapse function:

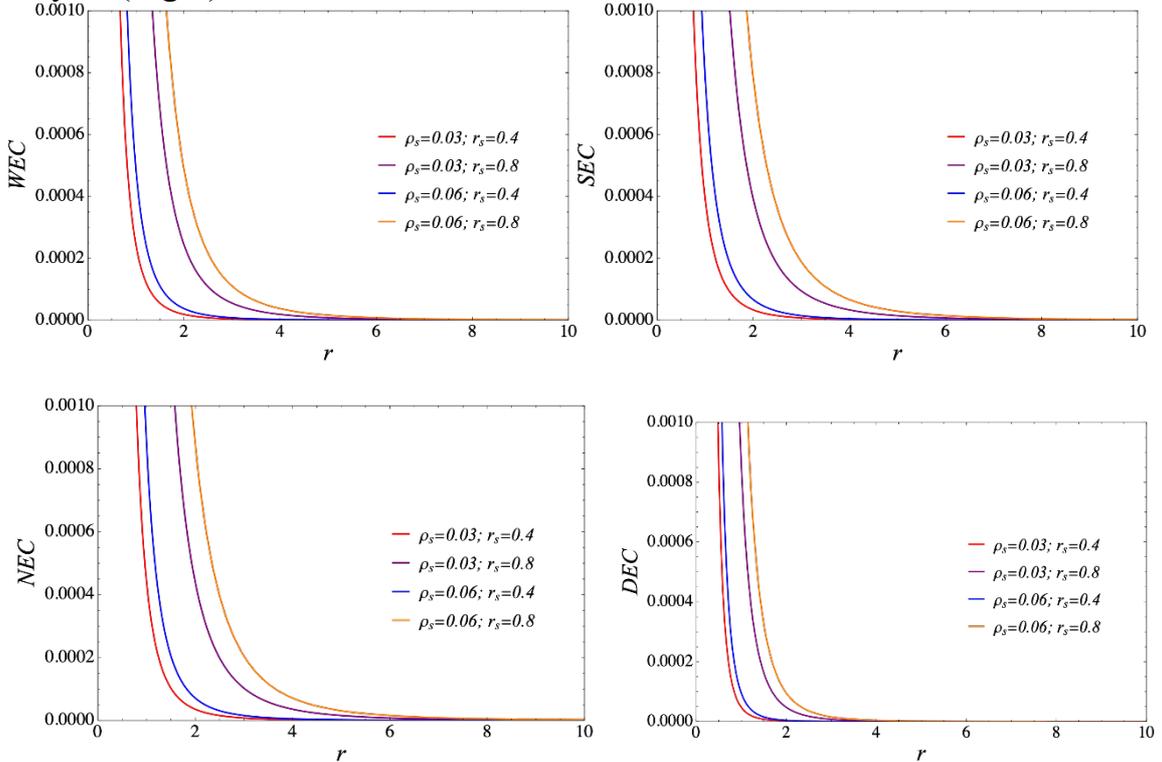
$$f(r) = 1 - \frac{2M}{r} - \frac{2M_D}{r_s} \log\left(1 + \frac{r_s}{r}\right)^{\left(1 + \frac{r_s}{r}\right)}. \quad (8)$$

To examine the singularity structure of this BH at  $r = 0$ , we analyse the curvature invariants of the spacetime such as Ricci scalar,  $R$ , Ricci square,  $R_{\mu\nu}R^{\mu\nu}$ , and Kretschmann scalars,  $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ , and plotted radial dependence of this invariants in (Fig.1). From this figure (Fig.1) one can recognize that BH in Dehnen type DM is singular at  $r=0$ .



**Figure 1. The curvature invariants.**

It is generally accepted that any realistic solution to the Einstein equations should obey the three energy conditions, namely the weak (WEC:  $\rho(r) \geq 0, \rho(r) + P_i(r) \geq 0$  with  $(i = r, \theta, \phi)$ ), the strong (SEC:  $\rho(r) + \sum_{i=1}^3 P_i \geq 0$ ), and the dominant energy (DEC:  $\rho(r) - |P_{\theta, \phi}| \geq 0$ ) conditions. We show this conditions visually in (Fig.2)



**Figure 2. The energy conditions.**

**The second chapter** of the dissertation is entitled as ‘‘Dynamics of gravitational objects in the SMBH environment.’’ In this chapter, we study the motion of magnetic dipoles and electrically charged particles in the vicinity of a self-dual black hole in Loop Quantum Gravity (LQG) immersed in an external asymptotically uniform magnetic field. We explore the effects of the quantum correction parameter and electromagnetic interactions on the particle geodesics. We derive the field equations and determine the electromagnetic four-vector potential for the case of a self-dual black hole in LQG. We investigate the innermost stable circular orbits (ISCOs) for both magnetic dipoles and electrically charged particles in detail, demonstrating that the quantum correction parameter significantly influences on the ISCO radius, causing it to shrink. Additionally, we show that the ISCO radius of magnetic dipoles is greater than that of electrically charged particles due to the magnetic field interaction. We investigate the ISCO parameters (i.e.,  $r_{ISCO}$ ,  $l_{ISCO}$ ,  $\mathcal{E}_{ISCO}$ ,  $v_{ISCO}$ , and  $\Omega_{ISCO}$ ) for magnetic dipoles and electrically charged particles, providing detailed values. Furthermore, we examine the trajectories of charged particles under various scenarios resulting from the quantum correction parameter  $P$ . Finally, analyzing the ISCO parameters that define the inner edge of the accretion disk, we explore the accretion disk around a self-dual black hole in LQG.

The effective metric of the self-dual black hole in LQG is given by:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2[d\theta^2 + \sin^2\theta d\phi^2], \quad (9)$$

here:

$$A(r) = -g_{tt} = \frac{(r-r_+)(r-r_-)(r+r_*)}{r^4},$$

$$B(r) = \frac{1}{g_{rr}} = \frac{(r-r_+)(r-r_-)}{(r+r_*)^2},$$

where  $r_+ = 2M/(1+P)^2$  and  $r_- = 2MP^2/(1+P)^2$  refers to the black hole's two horizons, while  $r_* = \sqrt{r_+r_-} = 2MP/(1+P)^2$  with ADM mass  $M$ .

Then vector potential's ansatz for non-rotating black hole can be expressed as:

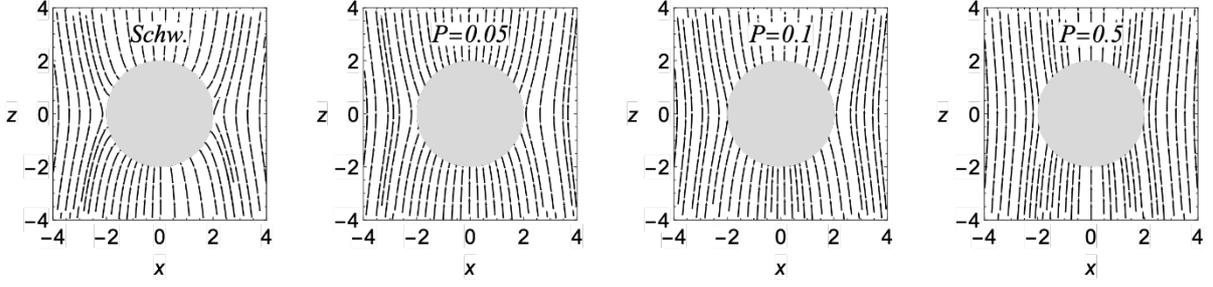
$$A^\mu = c_t(r, B_0)\xi_t^\mu + \left[\frac{B_0}{2} + c_\phi(r, B_0)\right]\xi_\phi^\mu, \quad (10)$$

here  $\xi_t^\mu = (1, 0, 0, 0)$  and  $\xi_\phi^\mu = (0, 0, 0, 1)$  refer to the time-like and space-like killing vectors. For a non-rotating vacuum black hole the Maxwell field equations can be defined by  $F_{;v}^{\phi v} = 0$  so final answer for four vector potential is:

$$A^\mu = \frac{B_0}{2}(\mathcal{B})^{\mathcal{A}} {}_2F_1(\mathcal{A}, \mathcal{A} + 3; 2\mathcal{A} + 3; \mathcal{B})\xi_\phi^\mu, \quad (11)$$

with new variables as  $\mathcal{A} = \sqrt{\frac{1-2P}{1+2P}} - 1$  and  $\mathcal{B} = \frac{r(4P+1)}{2M(2P+1)}$ .

Then in Fig. 3 we have plotted magnetic field lines in the vicinity of the self-dual black hole in LQG.



**Figure 3. The plot shows the magnetic field lines in the vicinity of the self-dual black hole in LQG for various combinations of the quantum correction parameter P.**

Now we can write Hamilton-Jacobi equation for magnetized particles in the vicinity of the BH in LQG immersed in external magnetic field:

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2 \left(1 - \frac{\mathcal{U}}{m}\right), \quad (12)$$

where  $\mathcal{U} = \mu_\alpha B^\alpha$  refers to the magnetic interaction. Then using Hamilton-Jacobi action at the equatorial plane  $S = -Et + L\phi + S_r$  the motion of the magnetized particles can be expressed as:

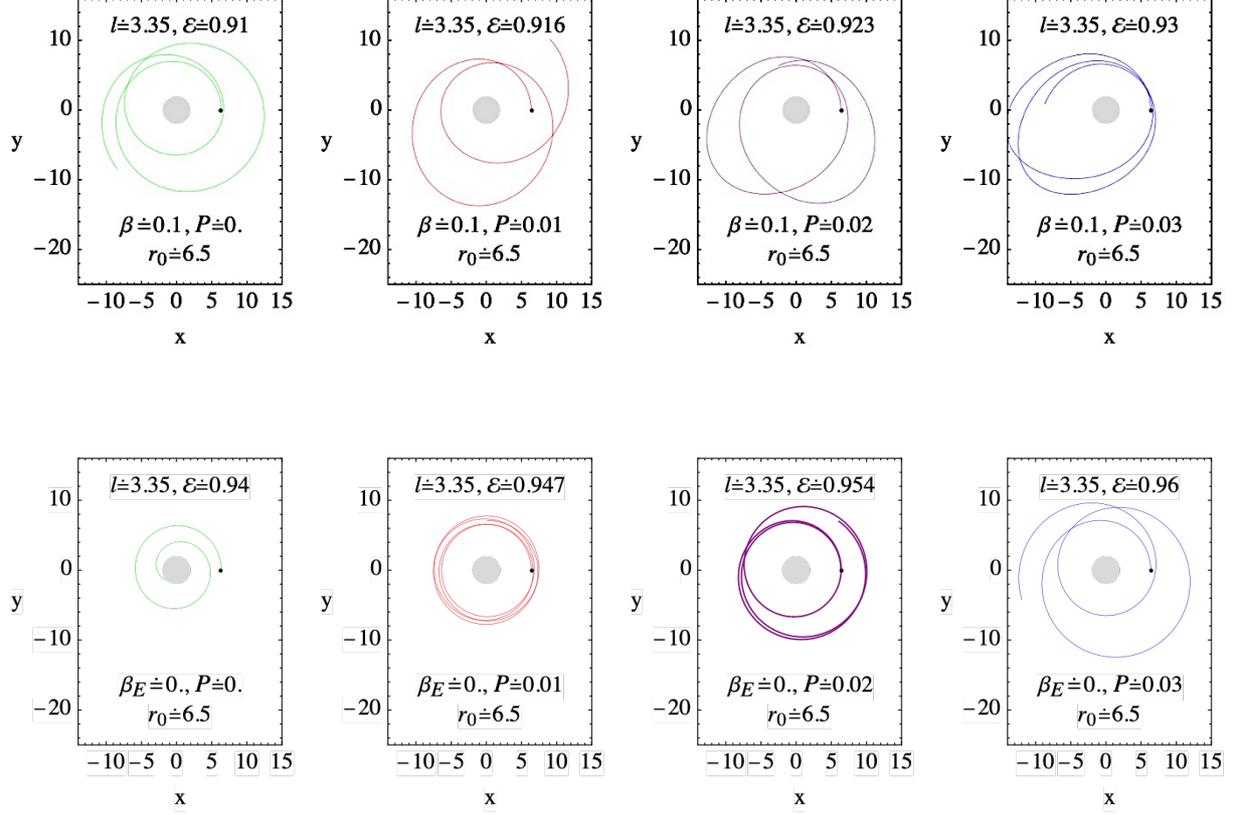
$$\begin{aligned} \dot{t} &= -\frac{\varepsilon}{(1+\frac{\mathcal{U}}{m})g_{tt}}, \\ \dot{\phi} &= \frac{l}{(1+\frac{\mathcal{U}}{m})g_{\phi\phi}}, \\ \dot{r}^2 &= \frac{1}{g_{rr}g_{tt}} \left( \varepsilon^2 - A(r) \left[ 1 + \frac{l^2}{r^2} - \frac{\mathcal{U}}{m} \right] \right). \end{aligned} \quad (13)$$

We can perform the same analysis for charged particles:

$$g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} + qA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} + qA_\nu \right) = -m^2, \quad (14)$$

where q is the charge of the electrically charged particle. Then, equation of the motion of the electrically charged particle in the vicinity of the self-dual black hole in LQG in external uniform magnetic field can be expressed as:

$$\begin{aligned} \dot{t} &= -\frac{\varepsilon}{g_{tt}}, \\ \dot{\phi} &= \frac{l}{g_{\phi\phi}}, \\ \dot{r}^2 &= -\frac{1}{g_{rr}g_{tt}} \left[ \varepsilon^2 - A(r) \left( 1 + \left[ \frac{l}{r} + \frac{qA_\phi}{mr} \right]^2 \right) \right]. \end{aligned} \quad (15)$$



**Figure 4. The trajectories of the test particle with charge (upper one) and magnetic dipole moment(bottom panel) in the vicinity of the self-dual black hole immersed in an external uniform magnetic.**

**The third chapter**, which is named as “Orbits of particles with magnetic dipole moment around magnetized Schwarzschild black holes: Applications to S2 star orbit”, includes a comprehensive analytical investigation of the bound and unbound motion of magnetized particles orbiting a Schwarzschild black hole immersed in an external asymptotically uniform magnetic field, which includes all conceivable types of bounded and unbounded orbits. In particular, for planetary orbits, we perform a comparative analysis of our findings with the observed position of the S2 star carrying magnetic dipole moment around Sagittarius A\* (Sgr A\*). We found maximum and minimum values for the parameter of magnetic interaction between the magnetic dipole of the star and the external magnetic field, as well as the energy and angular momentum of the S2 star. As a result, we obtain estimations of the magnetic dipole of the star in order of  $10^6 G \cdot cm^3$ . Additionally, we explore deflecting trajectories akin to gravitational Rutherford scattering. In obtaining the solutions for the orbital equations, we articulate the elliptic integrals and Jacobi elliptic functions, and illustrative figures and simulations augment our study. The line element of the Schwarzschild BH can be described as:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr + r^2d\Omega^2, \quad (17)$$

with the lapse function:

$$f(r) = 1 - \frac{2M}{r},$$

and  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ .

According to Wald's approach the four-potential, being solution of Maxwell's equation, can be expressed as  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ , where  $\xi_\phi^\mu = (0,0,0,1)$  represent a spacelike Killing vector. The Lagrangian for the magnetized particles orbiting around Schwarzschild BH immersed in external uniform magnetic field can be expressed as:

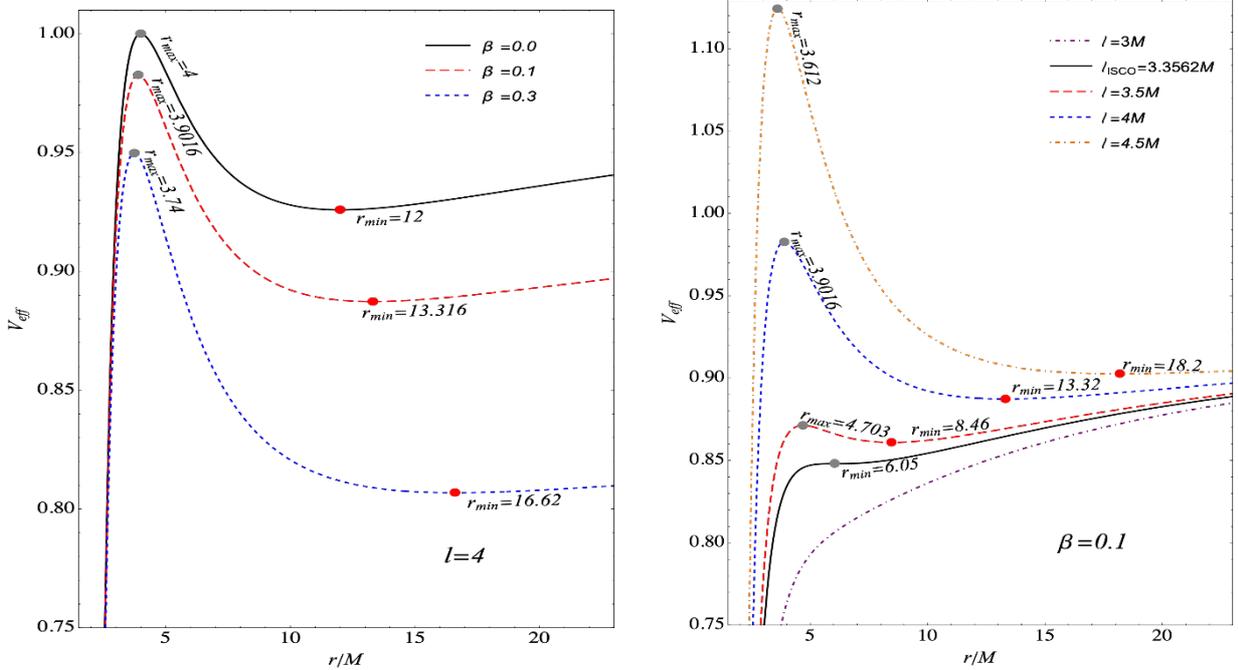
$$\mathcal{L} = \frac{1}{2}(m + \mathcal{U})g_{\mu\nu}u^\mu u^\nu - \frac{1}{2}\mathcal{U}, \quad (18)$$

where  $\mathcal{U} = \mu \frac{B^\alpha}{\alpha}$  is the magnetic interaction term. Then the equation of the motion is:

$$\begin{aligned} \dot{t} &= -\frac{\varepsilon}{[1+\beta\mathcal{F}]g_{tt}}, \\ \dot{\phi} &= \frac{l}{[1+\beta\mathcal{F}]g_{\phi\phi}}, \\ \dot{r}^2 &= \varepsilon^2 - V_{eff}, \end{aligned} \quad (19)$$

in which

$$V_{eff}(r, l) = f(r) \left( [1 - \beta\mathcal{F}]^2 + \frac{l^2}{r^2} \right),$$



**Fig.5.** The radial profile of the effective potential is depicted for fixed values of the magnetic coupling parameter  $\beta$ , and the specific angular momentum  $l$ . The minima ( $r_{min}$ ) in the effective potential corresponded to stable circular orbits, while the maxima ( $r_{max}$ ) correspond to unstable circular orbits.

here we have introduced new variables as  $\beta = \mu B/m$  and  $\mathcal{F}(r, \theta) = \sqrt{f(r)}\sin\theta$ . Then in Fig.5 we have shown radial dependence of the effective potential for different cases.

Also using Eqs.(19) we will get differential equation to govern particle motion:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4[1+\beta\sqrt{f(r)}]}{l^2} \left[ \mathcal{E} - f(r) \left( 1 + \frac{l^2}{r^2} - \beta\sqrt{f(r)} \right) \right], \quad (20)$$

here  $\beta = \frac{\mu B}{m}$  is the magnetic coupling parameter,  $l$  and  $\mathcal{E}$  are the specific angular momentum and specific energy of the magnetized particles, respectively. Then if we introduce new variable  $u = 1/r$  to the Eq.(20) we will have following equation to govern the motion of the magnetized particles:

$$\left(\frac{du}{d\phi}\right)^2 = (1 + \beta)(au^3 - bu^2 + cu + d), \quad (21)$$

where

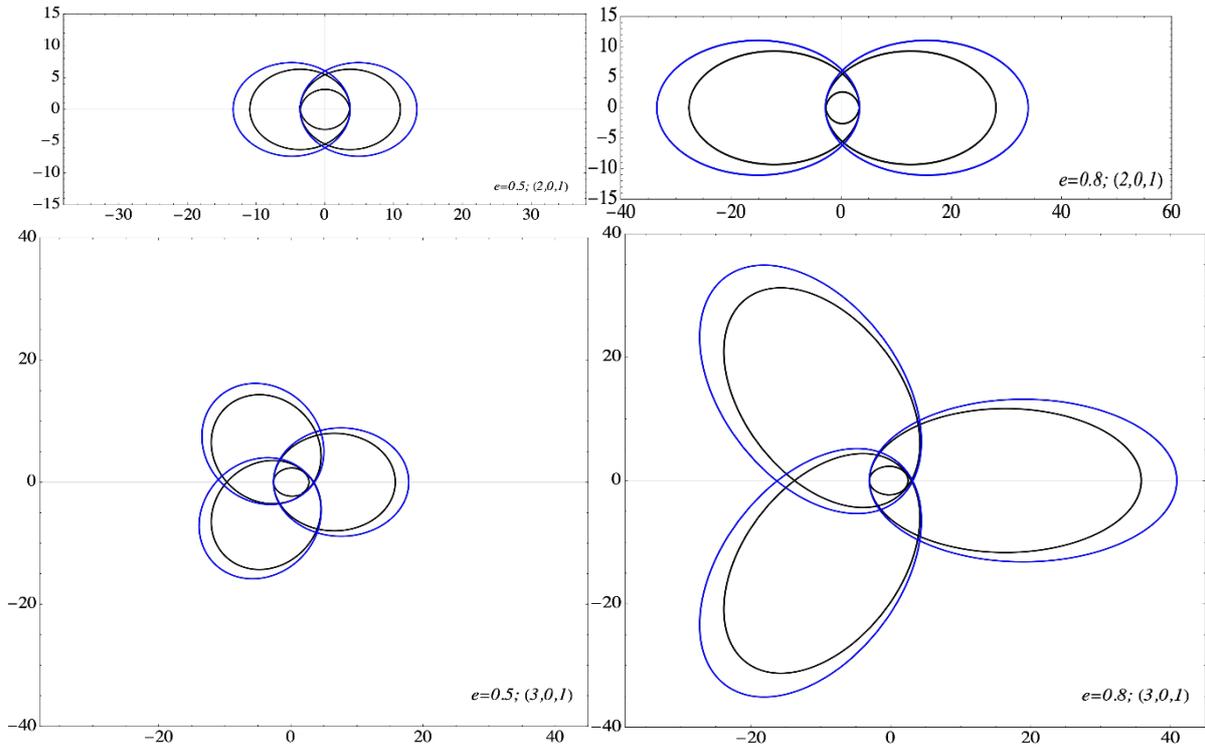
$$\begin{aligned} a &= 2M \left[ (2\beta + 1) - \frac{2\beta^2 M^2}{l^2} \right], \\ b &= (\beta + 1) + \frac{2\beta(1 - 4\beta)M^2}{l^2}, \\ c &= \frac{M[2 - \beta(5\beta + 2\mathcal{E}^2 - 1)]}{l^2}, \\ d &= \frac{(\beta + 1)(\beta + \mathcal{E}^2 - 1)}{l^2}. \end{aligned} \quad (22)$$

From equation (21) one can easily conclude particle make bound orbits for  $\mathcal{E}^2 < 1 - \beta$  and unbound orbits for  $\mathcal{E}^2 > 1 - \beta$ . Then if we assume test particles have energy  $\mathcal{E}^2 < 1 - \beta$  and all roots of the eq. (21) are positive (with the condition  $u_1 < u_2 < u_3$ ) test particles make planetary bound orbits. So, we will have trajectory for the magnetized particles starting their motion from  $u_1, \phi = 0$ :

$$r(\phi) = \frac{1}{u_1 + (u_2 - u_1)sn^2(\theta_0, k_0)}, \quad (23)$$

where  $sn(\theta, k)$  is the Jacobi elliptic sine function with the argument  $\theta_0 = \sqrt{a(1 + \beta)(u_3 - u_1)} \frac{\phi}{2}$  and the modulus  $k_0 = \sqrt{\frac{u_2 - u_1}{u_3 - u_1}}$ . The roots of the Eq. (23) can be labeled as:

$$\begin{aligned} u_1 &= \frac{1-e}{\lambda}, \\ u_2 &= \frac{1+e}{\lambda}, \\ u_3 &= \frac{b}{(1+\beta)a} - \frac{2}{\lambda} \end{aligned} \quad (24)$$



**Fig.6. Orbits corresponding to various eccentricities: blue orbits represent  $\beta = 0.1$ , while black orbits represent  $\beta = 0.0$ .**

where  $\lambda$  and  $e$  are the latus rectum and eccentricity, respectively. Also using Levin and Perez-Giz taxonomy:

$$q + 1 = \omega + \frac{v}{z} + 1 = \frac{\Delta\phi}{2\pi}, \quad (25)$$

where  $\omega$ ,  $v$ , and  $z$  represent the whirl, vertex, and zoom numbers, respectively.

Subsequently, we can find equation:

$$q + 1 = \frac{2K(k_1)}{\pi \sqrt{b - \frac{a(1+\beta)(3-e)}{\lambda}}}. \quad (26)$$

Here  $K(k)$  is the complete elliptic integral of the first kind with the modulus  $k_1 =$

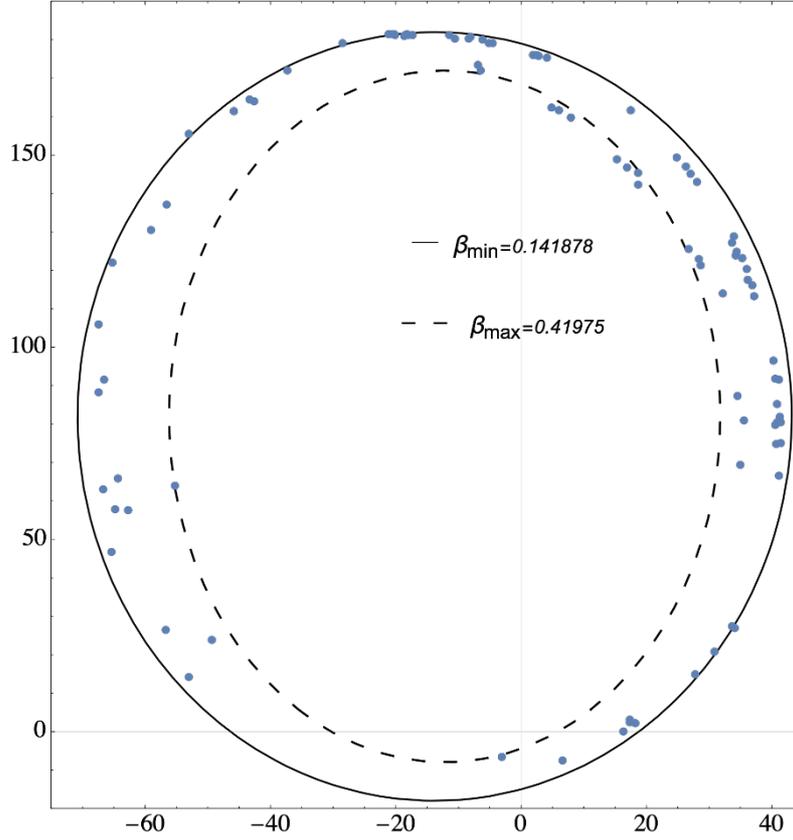
$$\sqrt{\frac{2ea(1+\beta)}{\lambda - a(1+\beta)(3-e)}}.$$

Now calculating Eq. (26) numerically, we can show planetary orbits visually. We have plotted planetary orbits for different values of the  $\omega$ ,  $v$ ,  $z$  in Fig.6

For a simple elliptic trajectory, the orbit of the S2 star should be  $(1, 0, 0)$ , so Eq.(26)

$$\frac{\pi}{2} = \frac{K(k_1)}{\sqrt{b - \frac{a(1+\beta)(3-e)}{\lambda}}} \quad (27)$$

Then using observations of phenomena of star S2, which is located in the star cluster close to Sgr A\*, we can constraint the trajectory of the S2 star and find corresponding magnetic interaction term  $\beta$ :



**Fig.7. Observed trajectory of the star S2 orbiting SgrA\*.**

Using observed data of the S2 star orbiting SgrA\* we obtained the possible range of the magnetic interaction parameter  $\beta$  as:

$$0.14249 \leq \beta \leq 0.419751.$$

## CONCLUSION

Based on the results of the dissertation titled "Dynamics of gravitational objects in the SMBH environment with a dark matter halo and magnetic field", the following conclusions were presented below:

1. An novel analytical Schwarzschild-like black hole solution with a Dehnen-type DM halo has been found. It has been shown that all energy conditions are fulfilled for newly derived BH solution. This solution could represent an alternative perspective on the interaction of black hole-dark matter systems, providing new insights into the fundamental properties of DM halos.

2. The motion of magnetic dipoles and electrically charged particles in the vicinity of a self-dual black hole in LQG immersed in an external asymptotically uniform magnetic field has been demonstrated by exploring the effects of the quantum correction parameter and electromagnetic interactions on the particle geodesics. The field equations have been derived, and the electromagnetic four-vector potential has been found for the self-dual black hole in LQG.

3. ISCOs for both magnetic dipoles and electrically charged particles have been found, demonstrating that the quantum correction parameter significantly influences

the ISCO radius, causing it to shrink. It has been shown that the ISCO radius of magnetic dipoles is greater than that of electrically charged particles due to the magnetic field interaction. The ISCO parameters (i.e.,  $r_{ISCO}$ ,  $l_{ISCO}$ ,  $\mathcal{E}_{ISCO}$ ,  $v_{ISCO}$ , and  $\Omega_{ISCO}$ ) have been found for magnetic dipoles and electrically charged particles, providing detailed values. The trajectories of charged particles have been demonstrated under various scenarios resulting from the quantum correction parameter  $P$ .

4. The orbitally bound and unbound motion of magnetized particles in vicinity of the Schwarzschild black hole immersed in an external asymptotically uniform magnetic field around has been found. In particular, the obtained results for planetary orbits have been compared with the observed position of the S2 star with a magnetic dipole moment around Sgr A\*SMBH. The maximum and minimum values of the magnetic interaction parameter between the star's magnetic dipole and the external magnetic field, as well as the energy and angular momentum of the S2 star have been determined, and it has been found that the star's magnetic dipole moment is of order  $10^6 G \cdot cm^3$ .

**НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025. FM/T.192.01  
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ  
ПЕРСПЕКТИВНЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА  
“НОВЫЙ УЗБЕКИСТАН”**

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**ИНСТИТУТ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ  
ИССЛЕДОВАНИЙ**

**УКТАМОВ УКТАМЖОН ХАЙРУЛЛА УҒЛИ**

**ДИНАМИКА ГРАВИТАЦИОННЫХ ОБЪЕКТОВ  
В ОКРЕСТНОСТИ СВЕРХМАССИВНЫХ ЧЕРНЫХ ДЫР  
С ГАЛО ТЕМНОЙ МАТЕРИЕЙ И МАГНИТНЫМ ПОЛЕМ**

**01.03.01 – Астрономия  
01.04.02 – Теоретическая физика**

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ  
ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

**Ташкент - 2025**

**Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования науки и инноваций Республики Узбекистан за номером B2024.2.PhD/FM1094.**

Диссертация выполнена в Институте фундаментальных и прикладных исследований.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета ([www.ias.newuu.uz](http://www.ias.newuu.uz)) и Информационно-образовательном портале «Ziyonet» ([www.ziyonet.uz](http://www.ziyonet.uz)).

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## **ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))**

**Актуальность и необходимость темы.** В настоящее время современные астрономические инструменты, такие как LIGO-VIRGO и телескоп Event Horizon, позволяют напрямую обнаруживать гравитационные волны от сливающихся черных дыр и двойных нейтронных звезд, а также получать изображения сверхмассивных черных дыр в центрах галактик M87 и Млечный путь. Непосредственное наблюдение за гравитационными волнами и детальное исследование изображения сверхмассивных черных дыр, полученные с помощью этих передовых инструментов проливают свет на астрофизические события в сильных гравитационных полях. Эти передовые наблюдения имеют решающее значение для разработки новых решений в альтернативных теориях гравитации и для исследования проблем, связанных с точным измерением параметров и пониманием сути темной материи. Анализ взаимодействия между темной материей (ТМ) и черными дырами (ЧД) важен для углубленного понимания основ ТМ. Учитывая значимость гало темной материи и их взаимодействие с системами черных дыр, исследование и моделирование ТМ через астрофизические данные и симуляции могут дать ценную информацию. В этой связи были разработаны несколько аналитических моделей, таких как модели Эйнато, Буркерта и Дехнена, для описания решений черных дыр в гало темной материи.

В нашей Республике придается большое значение фундаментальным исследованиям в области релятивистской астрофизики черных дыр. Основные направления фундаментальных исследований и разработок, а также их практическое применение для продвижения науки в нашей стране изложены в Стратегии дальнейшего развития Республики Узбекистан на 2022–2026<sup>1</sup> годы. За последние три десятилетия Республика сделала значительный вклад в релятивистскую астрофизику гравитационных компактных объектов, тщательно исследуя энергию и оптические характеристики черных дыр, кротовых нор и голых сингулярностей. Были открыты новые эффекты общей теории относительности в релятивистской астрофизике магнитных нейтронных звезд, а также сделан прогресс в понимании релятивистских магнитосфер и плазмы, а также намагниченных релятивистских объектов.

Данная диссертация направлена на выполнение задач, изложенных в государственных нормативных документах, включая Указ Президента № УР-4947 от 7 февраля 2017 года о Стратегии дальнейшего развития Республики Узбекистан и «Дорожную карту ключевых направлений структурных реформ в Узбекистане на 2019–2021 годы», изданную правительством Узбекистана 29 ноября 2018 года.

**Соответствие исследования основным приоритетам развития науки и технологий Республики.** Исследования диссертации проводились в соответствии с приоритетной областью научно-технологического развития

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<sup>1</sup> Указ Президента Республики Узбекистан № РР-60 от 1 января 2022 года «О Стратегии развития нового Узбекистана на 2022–2026 годы».

Республики Узбекистан, а именно с приоритетом П. «Энергия, энергоэффективность и ресурсосбережение».

**Целью исследования** является развитие теоретического формализма для получения новых аналитических решений для черных дыр типа Шварцшильда в гало темной материи и для анализа динамики частиц вокруг сверхмассивных черных дыр (СМЧД) с темной материей и магнитным полем, а также сравнительный анализ полученных результатов с наблюдаемым положением звезды S2 вокруг СМЧД Sgr A\*.

**Задачи исследования:**

Найти новое аналитическое решение Шварцшильдopodobной черной дыры в гало темной материи типа Дехнена.

Проанализировать характеристики кривизны пространства-времени и энергетические условия полученного решения.

Вывести уравнения поля и определить электромагнитный четырёхвектор для случая само-двойной черной дыры в LQG.

Изучить движение магнитных диполей и электрически заряженных частиц в окрестности черной дыры в LQG, вблизи черной дыры Шварцшильда, погруженной в внешнее однородное магнитное поле.

Исследовать связанные и несвязанные движения магнитных частиц, вращающихся вокруг черной дыры Шварцшильд, погруженной в внешнее асимптотически однородное магнитное поле.

Провести сравнительный анализ полученных результатов с наблюдаемым положением звезды S2 с предполагаемым магнитным дипольным моментом вокруг СМЧД Sgr A\*.

**Связь темы диссертации с научными работами научных исследовательских организаций, где проводилась диссертация.** Диссертация выполнена в рамках научных проектов, финансируемых Агентством инновационного развития, в частности, проекта F-FA-2021-510 «Исследования ядерного вещества нейтронных звезд в модифицированной гравитации», Ф-ФА-2021-432 «Анализ и обработка спутниковых данных рентгеновских двойных систем с малой массой».

**Объекты исследования** — астрофизические компактные объекты, динамика частиц, аналитические модели для гало темной материи, однородные магнитные поля в различных пространствах-времени, траектория звезды S2.

**Предмет исследования** — наблюдательные свойства черных дыр, модифицированная теория гравитации, само-двойная черная дыра, погруженная в магнитное поле в LQG, динамика частиц вокруг СМЧД, аналитические и численные методы решения дифференциальных уравнений поля и движения частиц.

**Методы исследования.** Включают подходы теоретической физики и астрофизики, современные методы теоретической астрофизики и математической физики, а также аналитические и численные методы решения дифференциальных уравнений, связанных с динамикой поля и частиц.

### **Научная новизна исследования заключается в следующем:**

Впервые получено аналитическое решение Шварцшильдоподобной черной дыры в гало темной материи типа Дехнена для модели ядра. Проанализированы характеристики кривизны пространства-времени и энергетические условия полученного пространства-времени.

Впервые выведены уравнения поля и четырёхвекторное потенциальное поле для случая само-двойной черной дыры в LQG. Изучено движение магнитных диполей и электрически заряженных частиц в окрестности само-двойной черной дыры в LQG.

Впервые исследовано движение частиц с магнитным дипольным моментом, вращающихся вокруг черной дыры Шварцшильда в магнитном поле.

Впервые результаты полученного анализа сравнены с реальными положениями звезды S2, и магнитный дипольный момент этой звезды оценен как  $10^6 G \cdot cm^3$ . Исследованы отклоненные траектории, аналогичные гравитационному рассеянию Резерфорда.

### **Практические результаты исследования:**

Получено новое аналитическое Шварцшильдоподобное решение для ЧД, представляющее статическую ЧД, окруженную гало темной материи с профилем плотности типа Дехнена (1,4,2). Свойства этой ЧД исследованы путем анализа кривизны пространства-времени. Рассмотрены энергетические условия, которые являются фундаментальными характеристиками физической состоятельности пространства-времени.

Рассматривается самодуальная черная дыра, погруженная во внешнее асимптотически однородное магнитное поле. Изучено движение магнитных диполей и электрически заряженных частиц в окружающей среде геометрии пространства-времени в рамках квантовой теории петель (LQG). Исследовано совместное влияние параметра квантовой коррекции  $P$  и параметра магнитного взаимодействия  $\beta$  на радиус внутренней стабильной круговой орбиты (ISCO). Проанализированы захваченные, связанные и уходящие орбиты магнитных и электрически заряженных частиц, ограниченных движением в экваториальной плоскости самодуальной черной дыры в LQG для различных возможных случаев.

Исследованы связанные и несвязанные орбиты магнитных частиц при их приближении к ЧД Шварцшильда в магнитном поле. Получен и проанализирован эффективный потенциал, испытываемый приближающимися частицами, классифицирующий возможные связанные и несвязанные орбиты на основе их удельной энергии и углового момента. Проведены сравнения между наблюдаемыми положениями звезды S2 вокруг СМЧД SgrA\* и теоретическими предсказаниями для двух различных значений параметра  $\beta$ , рассчитаны минимальные и максимальные значения этого параметра и удельного углового момента звезды S2.

**Надежность результатов исследования** подтверждается использованием стандартных математических и теоретических методов

физики в диссертации, включая эффективные численные методы и программное обеспечение. Также была проведена тщательная проверка теоретических результатов в ограниченных случаях, с наблюдательными данными и результатами других исследователей.

**Научная и практическая значимость результатов исследования** заключается в том, что предложенная модель черной дыры в гало темной материи может объяснить природу темной материи и служить для оценки магнитного диполя звезды S2 в окрестности СМЧД Sgr A\*.

**Применение результатов исследования.** Опираясь на динамику компактных объектов, научные результаты, связанные с их свойствами, были использованы в исследованиях международных ученых и опубликованы в высокорейтинговых зарубежных журналах, включая (Journal of Cosmology and Astroparticle Physics, 2025 , Volume 2025, Web-Sc, IF: 5.9; The European Physical Journal C, Volume 85, Article number 786, Web-Sc, IF: 4.2; The European Physical Journal C\*, Volume 85, Article number 494, Web-Sc, IF: 4.2; Universe, Web-Sc, IF: 2.6.)

Эти научные результаты были использованы в рамках программ, поддерживаемых Vellore Institute of Technology (на основании официального письма, предоставленного Dr. Pankaj Sheoran).

**Публикация результатов исследования.** Опубликовано 5 научных работ по теме исследования, включая 3 статьи в международных журналах с высокими импакт-факторами и 2 научных тезиса докладов в перечне научных изданий, рекомендованных Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан для публикации основных научных результатов диссертаций.

**Объем и структура диссертации.** Диссертация состоит из введения, трех глав, заключения и библиографии. Объем диссертации — 122 страниц.

## **ВЫВОДЫ**

На основе проведенного исследования по теме «**Динамика гравитационных объектов в окрестности сверхмассивных черных дыр с гало темной материей и магнитным полем**» для диссертации на соискание ученой степени доктора философии (PhD) были сделаны следующие выводы:

1. Найдено новое аналитическое Шварцшильдоподобное решение для черной дыры в гало темной материи типа Дехнена. Кроме того, вновь полученное решение для черной дыры анализируется путем изучения характеристик кривизны пространства-времени и энергетических условий. Это решение может представлять альтернативную перспективу на взаимодействие систем черная дыра–темная материя, предоставляя новые сведения о фундаментальных свойствах гало темной материи.

2. Анализ движения магнитных диполей и электрически заряженных частиц вблизи самодуальной черной дыры в LQG, погруженной во внешнее асимптотически однородное магнитное поле. Доказано влияние параметра квантовой поправки и электромагнитных взаимодействий на геодезические

траектории частиц и определен электромагнитный четырехвекторный потенциал для случая самодуальной черной дыры в LQG.

3. Подробный анализ ISCO для магнитных диполей и электрически заряженных частиц. Найдено, что параметр квантовой поправки существенно влияет на радиус ISCO и приводит к его уменьшению. Кроме того, показано, что радиус ISCO для магнитных диполей больше, чем для электрически заряженных частиц, из-за взаимодействия с магнитным полем. Проведен подробный анализ параметров ISCO (т.е.  $r_{ISCO}$ ,  $l_{ISCO}$ ,  $E_{ISCO}$ ,  $v_{ISCO}$  и  $\Omega_{ISCO}$ ) для магнитных диполей и электрически заряженных частиц. Кроме того, траектории заряженных частиц анализируются в различных сценариях, обусловленных параметром квантовой поправки  $P$ .

4. Анализ орбитально связанного и несвязанного движения магнитных частиц вблизи черной дыры Шварцшильда, погруженной во внешнее асимптотически однородное магнитное поле. В частности, полученные нами результаты для орбит были сравнены с наблюдаемой позицией звезды S2, обладающей магнитным дипольным моментом, вокруг СМЧД Sgr A\*. Определены максимальные и минимальные значения параметра магнитного взаимодействия между магнитным диполем звезды и внешним магнитным полем, а также энергия и угловой момент звезды S2. Показано, что звезда обладает магнитным дипольным моментом порядка  $10^6 G \cdot cm^3$ .

**E'LON QILINGAN ISHLAR RO'YXATI**  
**LIST OF PUBLISHED WORKS**  
**СПИСОК ОПУБЛИКОВАННЫХ РАБОТ**

**I bo'lim(I часть; I part)**

1. **U. Uktamov**, M. Alloqulov, S. Shaymatov, T. Zhu, and B. Ahmedov, "Particle dynamics and the accretion disk around a Self-dual Black Hole immersed in a magnetic field in Loop Quantum Gravity,"// **Physics of the Dark Universe**, - Elsevier (Netherlands) vol. 47, p. 101743, 2025. (No 1. Web of Science; **IF = 6.4**).

2. **U. Uktamov**, M. Fathi, J. Rayimbaev, and A. Abdujabbarov, "Orbits of particles with magnetic dipole moment around magnetized Schwarzschild black holes: Applications to the S2 star orbit", // **Physical Review D**, -New York (USA), vol. 110, no. 8, p. 084084, Oct. 2024. (No 1. Web of Science; **IF = 4.703**)

3. **U. Uktamov**, B. Narzilloev, and B. Ahmedov, "Perturbation of spacetime and electromagnetic field due to massive charged particle around electrically charged stringy black hole", // **European Physical Journal Plus**, – Springer (Germany), 2022. vol. 139, no. 10, p. 903, 2024. (No 1. Web of Science; **IF = 2.9**)

**II bo'lim (II часть; II part)**

4. **U.Uktamov**, "QUASIPERIODIC OSCILLATIONS AROUND A SCHWARZSCHILD BLACK HOLE SURROUNDED BY A DEHNEN TYPE DARK MATTER HALO", Fundamental and Applied Research in Physics, NUUZ, May 16-17, 2025, pgs.156-157.

5. **U.Uktamov**, "ON DESCRIPTION SHADOW OF THE ROTATING BARDEEN BLACK HOLE WITH STRING CLOUDS FIELD", FIZIKA FANINING RIVOJIDA ISTE'DODLI YOSHLARNING O'RNI (RIAK-XVI-2023), 28-29-aprel 2023-yil

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