FARG‘ONA DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
PhD.03/30.12.2019.FM.05.04 RAQAMLI ILMIY KENGASH

NAMANGAN DAVLAT UNIVERSITETI

UMARALIYEVA NARGIZA TASHKINBAYEVNA

BOSHQARUV FUNKSIYALARI TURLI CHEGARALANISHLI DINAMIK O‘YINLAR VA ULARNING ANIMATSION MODELLARI

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI

Farg‘ona – 2025

UDK: 517.977.5:519.837.2

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

**FARG‘ONA DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
PhD.03/30.12.2019.FM.05.04 RAQAMLI ILMIY KENGASH**

NAMANGAN DAVLAT UNIVERSITETI

UMARALIYEVA NARGIZA TASHKINBAYEVNA

BOSHQARUV FUNKSIYALARI TURLI CHEGARALANISHLI DINAMIK O‘YINLAR VA ULARNING ANIMATSION MODELLARI

01.01.02 – Differensial tenglamalar va matematik fizika

**FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

Farg‘ona – 2025

Fizika-matematika fanlari bo'yicha falsafa doktori (Doctor of Philosophy) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2022.4.PhD/FM194 raqam bilan ro'yxatga olingan.

Dissertatsiya Namangan davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus (резюме)) Ilmiy kengash veb-saytida (www.fdu.uz) va « ZiyoNET» ta'lim axborot tarmog'ida (www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar:	**Samatov Baxrom Tadjiaxmatovich** fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Mamadaliyev Numanjon** fizika-matematika fanlari doktori, professor
	Ibaydullayev To'lanboy Tursunboyevich fizika-matematika fanlari nomzodi, professor
Yetakchi tashkilot:	**Termiz davlat universiteti**

Dissertatsiya himoyasi Farg'ona davlat universiteti huzuridagi PhD.03/30.12.2019.FM.05.04 raqamli Ilmiy kengashning 2025 yil «01» 11 soat 10.00 dagi majlisida bo'lib o'tadi. (Manzil: 150100, Farg'ona shahar, Murabbiylar ko'chasi, 19-uy. Tel.: (+99873) 244-44-02, faks: (+99873) 244-44-93, e-mail: fardu_info@umail.uz).

Dissertatsiya bilan Farg'ona davlat universitetining Axborot-resurs markazida tanishish mumkin (590 raqam bilan ro'yxatga olingan). (Manzil: 150100, Farg'ona shahar, Murabbiylar ko'chasi, 19 uy. Tel.: (+99873) 244-44-94).

Dissertatsiya avtoreferati 2025-yil «20» 10 kuni tarqatildi.
(2025-yil «17» 10 dagi 10 raqamli reestr bayonnomasi).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI FARG'ONA DAVLAT ... O'ZBEKISTON

Sh.T.Karimov
Ilmiy darajalar beruvchi ilmiy kengash raisi o'rinbosari , f.-m.f.d., dotsent

I.U.Xaydarov
Ilmiy darajalar beruvchi ilmiy kengash ilmiy kotibi, f.-m.f.n., dotsent

Yu.P.Apakov
Ilmiy darajalar beruvchi ilmiy kengash qoshidagi ilmiy seminar raisi o'rinbosari, f.-m.f.d., professor

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda yuz berayotgan ilm-fan va texnologiyalarning jadal rivojlanishi qarama-qarshi maqsadli dinamik tizimlar uchun matematik jihatdan modellarni qurish va ularni amaliy jarayonlarga samarali qo‘llash yetakchi o‘rinlardan birini egallamoqda. Dunyo miqyosida bunday tizimlarni tahlil qilish uchun eng samarali vositalardan biri bu — dinamik o‘yinlar nazariyasi bo‘lib, uning yordamida qurilgan modellarni boshqaruv nazariyasi, iqtisodiyot, robototexnika, mudofaa va sun’iy intellekt kabi sohalar orqali amaliyotga joriy etishni taqozo etadi. Shu jihatdan, dinamik o‘yinlar nazariyasining asosiy obyekti hisoblangan qarama-qarshi boshqaruvli jarayonlarni tadqiq qilish, bu bo‘yicha boshqaruvlari turli chegaralanishlarga ega bo‘lgan diskret va differensial o‘yin masalalarini o‘rganish, matematik modellarini ishlab chiqish va ularni algoritmik shaklda yechish asosida vizual animatsiyalash orqali dinamik harakatlarni joriy vaqtda ko‘rish va tahlil qilishda animatsion modellardan samarali foydalanish muhim ahamiyatga ega hisoblanadi.

Jahonda ilmiy-amaliy tadqiqotlar natijasida yuzaga keladigan ko‘plab amaliy va nazariy muammolarni hal qilishda vaqt, resurs, xarajat, sifat va imkoniyatlarni optimallashtirish masalalariga yo‘naltirilgan ilmiy-tadqiqot ishlari olib borilmoqda. Bu borada, dinamik o‘yinlarda boshqaruv funksiyalari geometrik, yig‘indi va Langenhop tipidagi turli chegaralanishli qarama-qarshi maqsadli jarayonlarni matematik tadqiq qilib, amalda qo‘llanadigan barqaror modellar tuzishga alohida e’tibor qaratilmoqda. Bunday modellarni ishlab chiqish nazariy natijalarni real amaliy jarayonlarda qo‘llash imkoniyatini oshirmoqda. Mazkur yondashuv murakkab tizimlarda boshqaruv samaradorligini ta’minlash va strategik qarorlar qabul qilishda muhim ahamiyat kasb etmoqda.

Respublikamizda fundamental fanlarning ilmiy va amaliy tatbiqlariga ega bo‘lgan dolzarb yo‘nalishlariga e’tibor berilgan holda zamonaviy innovatsion texnologiyalarni rivojlantirish yuzasidan keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Xususan, “algebra va uning tatbiqlari, differensial tenglamalar va uning tatbiqlari, chiziqsiz tizimlar, dinamik sistemalar va ularning tatbiqlarini matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi” kabi ustuvor yo‘nalishlar bo‘yicha muhim vazifalar belgilab berilgan[1]. Ushbu vazifalarni amalga oshirishda, jumladan, differensial tenglamalar, dinamik sistemalar va dinamik o‘yinlar nazariyalarini takomillashtirish maqsadida diskret va differensial o‘yinlarda boshqaruvlari turli chegaralanishlarga ega bo‘lgan quvish-qochish masalalari uchun optimal strategiyalarni qurish hamda ularning animatsion modellarini yaratish muhim ahamiyat kasb etmoqda.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-sonli “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi

[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi qarori.

to‘g‘risida”gi farmoni, 2017-yil 17-fevraldagi PF-2789-sonli “Fanlar akademiyasi faoliyatini tashkil etish, boshqarish va moliyalashtirishni takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2017-yil 20-apreldagi PF-2909-sonli “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2022-yil 28-yanvardagi PF-60-sonli “2022–2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi farmonlari, 2018 yil 27-apreldagi PQ-3682-son “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi PQ-4387-sonli “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-sonli “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari, hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustivor yo‘nalishlariga bog‘liqligi. Dissertatsiya tadqiqoti Namangan davlat universitetining ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial tenglamalar va uning turdosh matematik sohalarining dolzarb muammolari” dasturi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Differensial o‘yinlar nazariyasining asoslari 1940-yillarning oxirlarida R. Isaacs tomonidan RAND korporatsiyasida harbiy quvish-qochish (P-E) masalalari uchun matematik modellar ishlab chiqilishi bilan boshlangan. R. Isaacs o‘z nazariyasida klassik variatsion hisobning Gamilton–Yakobi usullarini qo‘llagan va ularni boshqaruv funksiyalari bilan belgilangan harakatlarga tadbiq qilgan. Bu g‘oyalar keyinchalik L.D. Berkovitz, W.H. Fleming, A. Friedman kabi olimlar tomonidan yanada rivojlantirilib, o‘yinlar nazariyasi va variatsion hisobning yangi usullari kiritilgan. Differensial o‘yinlardagi masalalar, axborot ustunligi qaysi tomonlama berilishiga qarab, quvish yoki qochish holatlariga bo‘linadi. Bu yondashuv masalalarining tasnifi L.S. Pontryagin tomonidan taklif qilingan va bu yo‘nalish A.A. Chikrii, N.L. Grigorenko, N.N. Petrov, M.S. Nikolskiy, N.Yu. Satimov, A.A. Azamov va boshqa olimlar tomonidan rivojlantirilgan.

Differensial o‘yinlardan tashqari, pozitsion strategiyali diskret o‘yinlar ham keng o‘rganilgan. Diskret pozitsion o‘yinlarda strategiyalar faqat joriy holatga bog‘liq bo‘lib, bu optimal strategiyalarni qurish va tahlil qilish jarayonini soddalashtiradi. Ushbu sohada R. Isaacs, L.S. Pontryagin, N.N. Krasovskiy, A.I. Subbotin, A.A. Azamov, L.A. Petrosyan va A.A. Chikrii tomonidan muhim hissalar qo‘shilgan. O‘zbekistondagi ushbu yo‘nalishdagi ilmiy maktab N.Yu. Satimov rahbarligida shakllandi. U 35 yildan ortiq vaqt davomida “Optimal boshqaruv va differensial o‘yinlar” ilmiy seminariga rahbarlik qilgan. Hozirda ushbu seminar A.A. Azamov boshchiligida olib borilmoqda. Ilmiy maktab vakillari tomonidan muhim natijalarga erishilgan. A.Z. Fazilov, B.B. Rixsiev, A.A. Xamdamov, G.I. Ibragimov, M.Sh. Mamatov, B.T. Samatov, A.Sh. Kuchkarov va N.A. Mamadaliev tadqiqotlari asosida differensial va diskret modellar doirasida quvish–qochish masalalariga samarali yechimlar topilgan.

L.A. Petrosyan taklif qilgan parallel yaqinlashish strategiyasi (П-strategiya) yordamida guruhli quvish masalalari va R. Isaacs tomonidan kiritilgan “Hayot chizig‘i” masalasiga samarali yechimlar taklif etgan. A.A. Azamov ushbu o‘yinning ko‘p quvlovchi va bitta qochuvchidan iborat holatini ko‘p qiymatli akslantirishlar orqali analitik yechgan. G.I. Ibragimov yig‘indi chegaralanishga ega chiziqli diskret quvish o‘yinlarini o‘rganib, bosqichma-bosqich strategiyalar va kafolatli yechim shartlarini aniqlagan. B.T. Samatov boshqaruv funksiyalariga integral, chiziqli, nostatsionar, Gronuoll, Langenhop tipidagi va aralash chegaralanishlar qo‘yilgan differensial o‘yinlar uchun П-strategiyani qo‘llab quvish–qochish masalalarini hal qilgan. N.A. Mamadaliyev boshqaruv funksiyalari integral cheklovlarga bo‘ysunadigan, kechikish argumentlariga ega tizimlar bilan tavsiflangan differensial o‘yinlardagi quvish va qochish masalalarini tadqiq qilgan hamda muhim ilmiy natijalarga erishgan.

Dissertatsiya mavzusining olib borilayotgan oliy ta’lim muassasasidagi ilmiy tadqiqot ishlari bilan bog‘liqligi. Ushbu tadqiqot Namangan davlat universitetining ilmiy tadqiqot ishlari rejasidagi “Fundamental tadqiqotlar” yo‘nalishi doirasida amalga oshirilgan.

Tadqiqotning maqsadi boshqaruv funksiyalariga qo‘yilgan turli xil chegaralanishlar, jumladan, geometrik, yig‘indi va Langenhop tipidagi chegaralanishlarga ega dinamik quvish-qochish masalalarini hal qilishdan iborat.

Tadqiqotning vazifalari:

o‘yinchilarning boshqaruv funksiyalari geometrik chegaralanishga ega bo‘lgan, ko‘p quvlovchi va bitta qochuvchili diskret o‘yinda quvish masalasi yechilishini kafolatlovchi yetarli shartlarni aniqlash hamda o‘yinning animatsion modelini qurish;

o‘yinchilarning boshqaruv funksiyalari yig‘indi chegaralanishga ega bo‘lgan, sodda harakatli diskret o‘yinda quvish masalasi yechilishini kafolatlovchi yetarli shartlarni aniqlash hamda o‘yinning animatsion modelini qurish;

o‘yinchilarning boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo‘lgan differensial o‘yinda qochish masalasi yechilishini kafolatlovchi yetarli shartlarni aniqlash;

o‘yinchilarning boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo‘lgan diskret o‘yinda quvish masalasi yechilishini kafolatlovchi yetarli shartlarni aniqlash hamda o‘yinning animatsion modelini qurish.

Tadqiqotning obyekti – boshqaruv funksiyalari turli chegaralanishli dinamik o‘yinlardir.

Tadqiqotning predmeti – boshqaruv funksiyalari turli chegaralanishli geometrik, integral va Langenhop tipidagi dinamik o‘yinlarning differensial va diskret quvish–qochish masalalari, ularni ifodalovchi differensial tenglamalar, tengsizliklar hamda ularning yechimlarini qurish strategiyalaridir.

Tadqiqotning usullari. Tadqiqotda diskret va differensial o‘yinlardagi quvish masalalari uchun П-strategiyadan, qochish masalalari uchun esa kechikkan boshqaruvdan foydalanilgan bo‘lib, bu jarayonda matematik analiz, dinamik o‘yinlar nazariyasi, differensial tenglamalar, funksional analiz hamda optimal boshqaruv nazariyasining asosiy tushuncha va teoremalariga tayanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

boshqaruv funksiyalari geometrik chegaralanishga ega bo‘lgan, ko‘p quvlovchi va bitta qochuvchili diskret o‘yinda quvish masalasini yechish uchun quvlovchilar tomonidan qo‘llaniladigan parallel yaqinlashish usuli (П-strategiya) asosida kafolatlovchi yetarli shartlar topilib, o‘yinning 2D animatsion modeli ishlab chiqilgan;

boshqaruv funksiyalari yig‘indi chegaralanishga ega bo‘lgan, sodda harakatli diskret o‘yinda quvish masalasi yechilishi uchun tutishni kafolatlovchi yetarli shartlar quvlovchi tomonidan qo‘llaniladigan parallel yaqinlashish usuli (П-strategiya)dan foydalanib topilgan;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo‘lgan differensial o‘yinda qochish masalasi yechilishi uchun qochuvchining kechikkan boshqaruvi orqali aniqlangan yetarli shartlarda o‘yinchilar orasidagi masofaning quyi chegarasi topilgan;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo‘lgan diskret o‘yinda quvish masalasining П-strategiya yordamida topilgan yechimi asosida o‘yinning 2D animatsion modeli ishlab chiqilgan.

Tadqiqotning amaliy natijasi quyidagilardan iborat:

optimal boshqaruv va dinamik o‘yinlar nazariyasi asoslarining amaliy tatbiqlariga bevosita aloqadorligi nuqtayi nazaridan differensial va rekurrent tenglamalar orqali ifodalanuvchi ziddiyatli boshqaruv masalalari hal qilingan. Olingan nazariy natijalar asosida obyektlarning dinamik harakatlarini vizual tarzda kuzatish imkonini beruvchi animatsion modellar ishlab chiqilgan bo‘lib, robototexnika, xavfsizlik tizimlari va avtomatlashtirilgan boshqaruv tizimlariga oid masalalar ushbu modellar yordamida tahlil qilingan.

Tadqiqot natijalarining ishonchliligi. Matematik optimal boshqaruv nazariyasi asoslari, oddiy differensial tenglamalar nazariyasi, funksional va qavariq tahlil, matematik tahlil, dinamik o‘yinlar nazariyasining quvish-qochish muammolariga oid teorema va lemmalardan foydalanilgan. Tadqiqot ishida bayon qilingan natijalar matematikadagi qat’iy yondashuvlar asosida isbotlanganligi bilan izohlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Dissertatsiya ishining ilmiy ahamiyati boshqaruv funksiyalariga geometrik, yig‘indi va Langenhop tipidagi chegaralanishlar qo‘yilgan holda diskret va differensial o‘yinlarda quvish–qochish masalalarining optimal yechimlarini ta’minlaydigan strategiyalarning qurilishi va ularning amaliyotga tatbiq etilishi bilan belgilanadi. Keltirilgan natijalar boshqaruv va dinamik o‘yinlar nazariyalaridagi klassik metodlarni takomillashtirish, shuningdek ularning amaliyotdagi tadbiqlari bilan bog‘liq muammolarning yechimlarini aniqlashda qo‘llanilishi bilan ilmiy ahamiyat kasb etadi.

Tadqiqotning amaliy ahamiyati esa quvish–qochish masalalaridagi diskret harakatlarni vizuallashtiruvchi 2D animatsion modellarni qurish va amalga tatbiq etish orqali ifodalanadi. Ushbu modellar nazariy natijalarni vizualizatsiya qilish orqali tasdiqlash imkonini beradi hamda robototexnika, xavfsizlik tizimlari va avtomatlashtirilgan boshqaruv tizimlarida qo‘llash uchun asos bo‘lib xizmat qilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Boshqaruv funksiyalari turli chegaralanishli dinamik o‘yinlar va ularning animatsion modellari bo‘yicha olingan natijalar asosida:

boshqaruv funksiyalariga geometrik chegara qo‘yilgan ko‘p quvlovchilardan va bir qochuvchidan iborat diskret o‘yin masalasining animatsion modeli uchun yaratilgan kompyuter dasturidan ФЗ-201905171 raqamli “G‘ovak muhitlarda suyuqlik va gazlarni anomal filtrlash jarayonini tadqiq etish uchun gidrodinamik modellar va samarali algoritmlar yaratish” mavzusidagi amaliy loyihada suvli qatlamlar bilan chegaralangan gaz konlarini kompleks tadqiq qilishda flyuidlar filtratsiya jarayonining kompyuter modelini ishlab chiqishda foydalanilgan (O‘zbekiston Respublikasi Fanlar Akademiyasi V.I. Romanovskiy nomidagi Matematika institutining 2025-yil 25-iyundagi 2/272-sonli ma’lumotnomasi). Natijada, koordinatalar bo‘yicha ajralish va differensial haydash usullariga asoslangan g‘ovak muhitdagi filtratsiya masalalarini hal qilish va ularning sonli algoritmlarini yaratish imkonini bergan;

boshqaruv funksiyalari geometrik va Langenhop tipidagi chegaralanishli dinamik o‘yin masalalarini yechishda optimal yaqinlashish strategiyalari (П-strategiyalar) asosida masalalar yechimlarini kafolatlovchi shartlardan hamda masalalarni tutish jarayonini vizual tarzda ifodalovchi yaratilgan animatsion modellaridan OT-F4-33 raqamli “Differensial tenglamalar bilan tavsiflanuvchi ziddiyatli boshqaruv uchun yangi usullarni ishlab chiqish va ularning sonli tadbiqi” mavzusidagi fundamental loyihada boshqaruv funksiyalari turli chegaralanishli dinamik o‘yin masalalarini hal etishda foydalanilgan (O‘zbekiston Milliy universitetining 2025-yil 18-iyundagi 04/11-7621-sonli ma’lumotnomasi). Natijada, Langenhop va geometrik chegaralarga ega diskret tenglamalar bilan ifodalangan ziddiyatli holat jarayonlarini boshqarish masalalari uchun yangi usullarni ishlab chiqish, ularni sonli hal etish va natijalarni amaliy jihatdan to‘g‘riligini tasdiqlash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqot davomida olingan natijalar 16 ta ilmiy-amaliy anjumanda muhokama qilingan bo‘lib, ulardan 10 tasi xalqaro, 6 tasi esa respublika miqyosidagi konferensiyalardir.

Tadqiqot natijalarining e’lon qilinganligi. Mazkur tadqiqot davomida jami 26 ta ilmiy ish nashr etilgan. Ulardan 7 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan fan doktori ilmiy darajasini olish uchun tavsiya etilgan ilmiy jurnallarda chop etilgan. Xususan, ushbu ishlarning 2 tasi xalqaro ilmiy jurnallarda, 5 tasi respublika miqyosidagi ilmiy jurnallarda nashr qilingan, 16 tasi esa ilmiy anjuman tezislari shaklida taqdim etilgan.

Shuningdek, ushbu tadqiqot doirasida yaratilgan dasturiy mahsulotlar O‘zbekiston Respublikasi Adliya vazirligi tomonidan “EHM uchun dastur” sifatida davlat ro‘yxatidan o‘tkazilib, ularga 3 ta guvohnoma berilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa, foydalanilgan adabiyotlar ro‘yxati va ilovalardan iborat. Dissertatsiyaning umumiy hajmi 109 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlarning tahlili berilgan, muammoning o‘rganilganlik darajasi yoritilgan, tadqiqotning maqsad va vazifalari, obyekti va predmeti ko‘rsatilgan, tadqiqot natijalarining ilmiy yangiligi ochib berilgan, olingan natijalarning nazariy va amaliy ahamiyati ko‘rsatilgan, tadqiqot natijalarining tatbiqi, shuningdek, nashr etilgan ilmiy ishlar va dissertatsiyaning tuzilishi haqida ma’lumotlar keltirilgan.

Birinchi bob **“Boshqaruvlari geometrik chegaralanishli diskret o‘yinlar va ularning animatsion modellari”** deb nomlanadi va boshqaruv funksiyalari geometrik chegaralanishga ega ikkita diskret quvish masalalari o‘rganilgan.

$\mathbb{R}^d$ fazoda ($d \geq 2$), ikkita obyekt — quvuvchi P va qochuvchi E — qadamma-qadam harakatlari quyidagi rekurrent tenglamalar bilan ifodalanadi:

$$P\colon\ x_n = x_{n-1} + u, \quad E\colon\ y_n = y_{n-1} + v, \quad n = 1,2,\dots . \tag{1}$$

Bu yerda x_n va y_n mos ravishda P va E o‘yinchilarining n-qadamdagi holatlarini, x_0 va y_0 esa ularning boshlang‘ich holatlarini ifodalaydi va $x_0 \neq y_0$. u va v mos ravishda P va E obyektlarning boshqaruv parametrlari bo‘lib, ular $\mathbb{R}^d$ da aniqlangan o‘zgarmas vektorlar ketma-ketligi $\{u_n\}$ va $\{v_n\}$ elementlari sifatida tanlanadi hamda quyidagi geometrik chegaralanishga (qisqacha G-chegaralanish) bo‘ysunadi:

$$u_n \in S_\alpha \coloneqq \{u \in \mathbb{R}^d\colon \|u\| \leq \alpha\},\ v_n \in S_\beta \coloneqq \{v \in \mathbb{R}^d\colon \|v\| \leq \beta\}, \tag{2}$$

bu yerda $\alpha > 0, \beta \geq 0$, S_α va S_β esa $\mathbb{R}^d$ da markazi koordinata boshida joylashgan Yevklid sharlaridir.

Boshqaruvlar ketma-ketlilari (2) ni rekurrent tenglamalar (1) ga qo‘yib, quyidagi ko‘rinishdagi yechimni, ya’ni $\mathbb{R}^d$ fazodagi P va E ob’yektlarning izlarini hosil qilamiz:

$$P\colon\ x_n = x_0 + \sum_{i=1}^{n} u_i\,, \quad E\colon\ y_n = y_0 + \sum_{i=1}^{n} v_i\,, \quad n = 1,2,\dots . \tag{3}$$

Quvlovchining maqsadi — biror $N \in \mathbb{N}$ da $x_N = y_N$ tenglikka erishish (Discrete Pursuit Problem (qisqacha DPP) — diskret quvish masalasi), qochuvchining maqsadi esa barcha $n \in \mathbb{N}$ uchun $x_n \neq y_n$ tengsizlikni saqlab qolishdir (Discrete Evasion Problem (qisqacha DEP) — diskret qochish masalasi).

Yangi o‘zgaruvchi $z_n = x_n - y_n$ kiritamiz. Shunda rekurrent tenglamalar (1) quyidagi ko‘rinishga ega bo‘ladi

$$z_n = z_{n-1} + u - v, \tag{4}$$

yechimilar (3) esa quyidagicha yoziladi:

$$z_n = z_0 + \sum_{i=1}^{n} (u_i - v_i)\,, \qquad z_0 = x_0 - y_0.$$

Qisqalik uchun, boshqaruv funksiyalari (2) geometrik chegaralanishlarga ega bo'lgan va harakatlari (1) yoki (4) rekurrent tenglamalar orqali ifodalanadigan diskret quvish–qochish masalasini *G-o'yin* (G-chegaralanishli o'yin) deb ataymiz.

1 - ta'rif. Quyidagi boshqaruvlar ketma-ketligini

$$\boldsymbol{u}_n = \begin{cases} \boldsymbol{u}_n^A, & \text{agar } |z_{n-1}| > \mu(v_n), \\ \boldsymbol{u}_n^F, & \text{agar } |z_{n-1}| \le \mu(v_n), \end{cases} \tag{5}$$

G-o'yin uchun DPPning Π_G*-strategiyasi* deb ataymiz, bu yerda

$$\begin{gathered} \boldsymbol{u}_n^A = v_n - \mu(v_n)\xi_0, \\ \boldsymbol{u}_n^F = v_n - z_{n-1}, \\ \mu(v_n) = \langle v_n, \xi_0 \rangle + \sqrt{\langle v_n, \xi_0 \rangle^2 + \alpha^2 - |v_n|^2}, \qquad \xi_0 = z_0/|z_0|, \end{gathered} \tag{6}$$

bu yerda $\langle v_n, \xi_0 \rangle - v_n$ va ξ_0 vektorlarining $\mathbb{R}^d$dagi skalyar ko'paytmasi $n = 1,2,\dots$. Kelgusi o'rinlarda $\boldsymbol{u}_n^A$ ketma-ketligi *yaqinlashish strategiyasi,* $\boldsymbol{u}_n^F$ ketma-ketligi *yakuniy (yoki tugallovchi) strategiya* deb ataymiz.

1-lemma. Agar $\alpha \ge \beta$ bo'lsa, unda har bir juftlik $(v_n, z_{n-1}) \in S_\beta \times \mathbb{R}^d$ uchun quyidagilar o'rinli:

a) $\boldsymbol{u}_n$ ketma-ketligi aniqlangan bo'ladi;

b) har qanday $v_n \in S_\beta$ uchun $\alpha - \beta \le \mu(v_n) \le \alpha + \beta$ tengsizliklar bajariladi.

1-teorema. G-o'yin uchun DPPda $\alpha > \beta$ bo'lsin va quvlovchi Π_G-strategiyani (5) qo'llasin. Unda:

1) Agar $i = 1,2,\dots,n$ qadamlar uchun $|z_{i-1}| > \mu(v_i)$ tengsizlik bajarilsa, unda barcha $i = 1,2,\dots,n$ qadamlar uchun $z_i \ne 0$ bo'ladi;

2) Agar n-qadamda $|z_n| \le \mu(v_{n+1})$ tengsizlik bajarilsa, $n+1$-qadamda qochuvchi tutib olinadi, ya'ni $z_{n+1} = 0$.

2-lemma. Agar qandaydir n-qadamda $0 < |\mathrm{z_n}| \le \alpha - \beta$ tengsizlik bajarilsa, u holda quvlovchi yakuniy strategiya (6)ni qo'llagan holda quvishni $n+1$-qadamda yakunlaydi.

2-teorema. G-o'yin uchun DPPda $\alpha > \beta$ bo'lsin. Unda quvlovchi Π_G-strategiya (5)ni qo'llab, quvishni

$$N(z_0) = \left[\frac{|z_0|}{\alpha - \beta}\right] + 1$$

qadamdan kechikmay yakunlaydi. Bu yerda $[\cdot]$ belgi, qavs ichidagi sonning butun qismini bildiradi.

1.2-paragrafda biz Pshenichniy masalasining[*] diskret analogini — o'rganamiz, ya'ni $\mathbb{R}^d$ fazoda bir nechta quvlovchi P^1, P^2,..., P^m tomonidan bitta qochuvchini E diskret quvush masalasini o'rganamiz. O'yinchilar harakati qadamma-qadam harakatni ifodalovchi quyidagi rekurrent tenglamalar bilan beriladi:

$$P^i\colon\ x_n^i = x_{n-1}^i + u^i,\ \ i \in I_m = \{1,2,3,\dots,m\};\ E\colon\ y_n = y_{n-1} + v, \tag{7}$$

[*] B.N. Pshenichnii. Simple pursuit by several objects. Cybernetics and System Analysis, 1976.

bu yerda $n = 1,2, \ldots$. Boshlang‘ich holatlar farqli deb qaraladi, ya’ni barcha $i \in I_m$ uchun $x_0^i \neq y_0$. u^i va v mos ravishda quvlovchilar P^i va qochuvchi E ning boshqaruv parametrlari bo‘lib, ular $\mathbb{R}^d$ da aniqlangan o‘zgarmas vektorlar ketma-ketligi $\{u_n^i\}_{n\in N}$ va $\{v_n\}_{n\in N}$ elementlari sifatida tanlanadi va quyidagi geometrik chegaralanishlarni (qisqacha G-chegaralanish) qanoatlantiradi:

$$|u_n^i| \leq 1, \;\; i \in I_m; \;\;\; |v_n| \leq 1. \tag{8}$$

Demak, barcha o‘yinchilarning boshqaruv imkoniyatlari teng deb qaraladi.

Quvlovchilarning maqsadi — qandaydir chekli N qadamda hech bo‘lmaganda biror $j \in I_m$ uchun $x_N^j = y_N$ tenglikka erishish (Group Discrete Pursuit Problem, (qisqacha GDPP)— guruhli diskret quvish masalasi), qochuvchining maqsadi esa barcha qadamlar davomida barcha $i \in I_m$ uchun $x_n^i \neq y_n$ tengsizlikni saqlab qolishdir (Group Discrete Evasion Problem (qisqacha GDEP) — guruhli diskret qochish masalasi).

1.1-paragrafdagi kabi, bu yerda ham diskret quvish maslasini soddalashtirish uchun yangi o‘zgaruvchilar kiritiladi $z_n^i = x_n^i - y_n$, $i \in I_m$ va $z_0^i = x_0^i - y_0$. bunda (7)-tenglamalardan quyidagi rekurrent tenglamalar hosil bo‘ladi

$$z_n^i = z_{n-1}^i + u_n^i - v_n, \; i \in I_m. \tag{9}$$

Ushbu (9) tenglamalarning yechimi quyidagi ko‘rinishga ega

$$z_n^i = z_0^i + \sum_{k=1}^{n} (u_k^i - v_k)\,, i \in I_m.$$

Qisqalik uchun, boshqaruv funksiyalari (8) geometrik chegaralanishlarga ega bo‘lgan va harakatlari (7) yoki (9) rekurrent tenglamalar bilan ifodalanadigan guruhli diskret quvish–qochish masalasini *G-o‘yin* (G-chegaralanishli o‘yin) deb ataymiz.

2-ta’rif. Quyidagi boshqaruvlar ketma-ketligini

$$\boldsymbol{u}_n^i = \begin{cases} \boldsymbol{u}_n^{A_i}, & \text{agar } |z_{n-1}^i| > \mu_i(v_n), \\ \boldsymbol{u}_n^{F_i}, & \text{agar } |z_{n-1}^i| \leq \mu_i(v_n), \end{cases} \tag{10}$$

G-o‘yin uchun GDPPda i-quvlovchining Π_G^i*-strategiya*si deb ataymiz, bu yerda

$$\boldsymbol{u}_n^{A_i} = v_n - \mu_i(v_n)\xi_0^i, \qquad \boldsymbol{u}_n^{F_i} = v_n - z_{n-1}^i,$$

$$\mu_i(v_n) = \langle v_n, \xi_0^i \rangle + \sqrt{\langle v_n, \xi_0^i \rangle^2 + 1 - |v_n|^2}, \quad \xi_0^i = \frac{z_0^i}{|z_0^i|}, \quad n = 1,2,\ldots, \quad i \in I_m.$$

3-teorema. G-o‘yin uchun GDPPda barcha quvlovchilar Π_G^i-strategiyani (10) qo‘llasin. Unda:

1) Agar $k = 1,2,\ldots,n$ qadamalarda barcha $i \in I_m$ uchun $|z_{k-1}^i| > \mu_i(v_k)$ bajarilsa, u holda $k = 1,2,\ldots,n$ qadamlarda barcha $i \in I_m$ uchun $z_k^i \neq 0$ bo‘ladi;

2) Agar qandaydir $j \in I_m$ uchun n -qadamda $|z_n^j| \leq \mu_j(v_{n+1})$ bajarilsa, u holda $n+1$ - qadamda P^j quvlovchi quvishni yakunlaydi, ya’ni $z_{n+1}^j = 0$.

4-teorema. *(Pshenichniy teoremasining diskret analogi)* Agar G-o'yin uchun GDPPda boshlang'ich holatlar $z_0^i,\ i \in I_m$ uchun quyidagi shart bajarilsa

$$0 \in int\, conv\{\xi_0^1, \xi_0^2, \ldots, \xi_0^m\},$$

u holda Π_G^i-strategiyalar (10) yordamida quvlovchilar quvishni

$$N(z_0^1, z_0^2, \ldots, z_0^m) = \left[\sum_{i=1}^{m} \left|z_0^i\right| / \gamma\right] + 1,$$

qadamdan kechikmay yakunlaydi, bu yerda

$$\gamma = \frac{2\delta}{1+2\delta} \text{ va } \delta = \min_{|p|=1} \max_{i \in I_m} \langle p, \xi_0^i \rangle > 0.$$

Ikkinchi bob "**Boshqaruvlari yig'indi chegaralanishli diskret o'yin va uning animatsion modeli**" deb nomlanib, oddiy harakatli diskret o'yinning yig'indi chegaralanishli holatini tadqiq etishga bag'ishlangan.

Bunda *P* va *E* obyektlarning harakati rekurrent tenglamalar (1) orqali ifodalanadi. Quvuvchi va qochuvchining boshqaruv ketma-ketliklari $\{\boldsymbol{u}_i\}_{i=1}^{\infty}$, $\boldsymbol{u}_i \in \mathbb{R}^d$ va $\{\boldsymbol{v}_i\}_{i=1}^{\infty}$, $\boldsymbol{v}_i \in \mathbb{R}^d$ mos ravishda quyidagi yig'indi chegaralanishlarni (qisqacha S-chegaralanish) qanoatlantiradi:

$$\| \boldsymbol{u} \|_{\ell_2} = \sqrt{\sum_{i=1}^{\infty} |\boldsymbol{u}_i|^2} \le \rho_0, \quad \| \boldsymbol{v} \|_{\ell_2} = \sqrt{\sum_{i=1}^{\infty} |\boldsymbol{v}_i|^2} \le \sigma_0, \tag{11}$$

bu yerda $\rho_0 > 0$ va $\sigma_0 \ge 0$.

Qisqalik uchun, o'yinchilarning boshqaruv funksiyalari (11) yig'indi chegaralanishlarga ega bo'lgan va harakatlari (1) yoki (4) rekurrent tenglamalar orqali ifodalanadigan diskret quvish–qochish masalasini *S-o'yin* (S-chegaralanishli o'yin) deb ataymiz.

3-ta'rif. Quyidagi boshqaruvlar ketma-ketligini

$$\boldsymbol{U}_n^* = \begin{cases} \boldsymbol{u}_n^*, & \text{agar} \quad |\boldsymbol{z}_{n-1}| > \eta(v_n), \\ \boldsymbol{u}_n^{**}, & \text{agar} \quad |\boldsymbol{z}_{n-1}| \le \eta(v_n), \end{cases} \tag{12}$$

S-o'yin uchun DPPning Π_S-*strategiya*si deb ataymiz, bu yerda

$$\boldsymbol{u}_n^* = v_n - \eta(v_n)\xi_0, \quad \boldsymbol{u}_n^{**} = v_n - z_{n-1},$$
$$\eta(v_n) = max\{0, \delta_0^2/|z_0| + 2\langle v_n, \xi_0 \rangle\},$$

$\xi_0 = \frac{z_0}{|z_0|}$, $z_0 = x_0 - y_0$, $\delta_0^2 = \rho_0^2 - \sigma_0^2$ va $\langle v_n, \xi_0 \rangle - v_n$ va ξ_0 vektorlarining $\mathbb{R}^d$dagi skalyar ko'paytmasi $n = 1,2,\ldots$. Kelgusi o'rinlarda $\boldsymbol{u}_n^*$ ketma-ketligi *yaqinlashish strategiyasi,* $\boldsymbol{u}_n^{**}$ ketma-ketligi *yakuniy (yoki tugallovchi) strategiya* deb ataymiz.

3-lemma. Agar qandaydir n-qadamda $|z_{n-1}| > \eta(v_n)$ tengsizlik o'rinli bo'lsa, u holda n-qadamda $|\boldsymbol{u}_n^*|^2 = |v_n|^2 + \frac{\delta_0^2}{|z_0|}\eta(v_n)$ tenglik bajariladi.

4-ta'rif. S-o'yinda quyidagi ketma-ketliklar

$$\rho_{n-1}^2 = \rho_0^2 - \sum_{i=1}^{n-1} |\boldsymbol{U}_i^*|^2, \quad \sigma_{n-1}^2 = \sigma_0^2 - \sum_{i=1}^{n-1} |\boldsymbol{v}_i|^2,$$

mos ravishda P va E obyektlarning joriy resurslari, quyidagi ketma-ketlik esa

$$\delta_{n-1}^2 = \rho_{n-1}^2 - \sigma_{n-1}^2 = \delta_0^2 - \sum_{i=1}^{n-1} (|\mathbf{U}_i^*|^2 - |\mathbf{v}_i|^2),$$

har ikki obyektlarning joriy resurslari farqi deb ataymiz.

5-teorema S-o'yin uchun DPPda $\rho_0 > \sigma_0$ bo'lsin va quvlovchi Π_S-strategiyani (12) qo'llasin. Unda:

1) Agar $k = 1,2, \dots, n$ qadamlar uchun $|z_{k-1}| > \eta(v_k)$ tengsizlik bajarilsa, u holda barcha $k = 1,2, \dots, n$ qadamlar uchun $z_k \neq 0$ va $z_k = z_0 h_k$, $\delta_k^2 = \delta_0^2 h_k$, $0 < h_k \leq 1$ bajariladi, bu yerda

$$h_k = 1 - \frac{1}{|z_0|} \sum_{i=1}^{k} \eta\,(v_i), \quad k = 1,2, \dots, n.$$

2) Agar n-qadamda $|z_{n-1}| \leq \eta(v_n)$ tengsizlik bajarilsa, u holda n-qadamda qochuvchi tutiladi, ya'ni $z_n = 0$.

6-teorema. S-o'yin uchun DPPda $\rho_0 > \sigma_0$ bo'lsin. Unda quvlovchi Π_S-strategiyani (12) qo'llab, tutishni

$$N(z_0) = \left[\left(\frac{|z_0|}{\rho_0 - \sigma_0} \right)^2 \right] + 1$$

qadamdan kechikmay amalga oshiradi.

Uchinchi bob "**Boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo'lgan dinamik o'yinlar va ularning animatsion modellari**" deb nomlanadi. Bobda o'yinchilar boshqaruv funksiyalariga qo'yilgan Langenhop tipidagi chegaralanishga ega bo'lgan differensial va diskret quvish–qochish o'yinlari o'rganiladi.

Dastlab $\mathbb{R}^d$ fazoda P va E obyektlarning harakati quyidagi differensial tenglamalar bilan ifodalanadigan qochish masalasi ko'rib chiqilgan.:

$$P: \dot{x} = u, \ x(0) = x_0; \quad E: \dot{y} = v, \ y(0) = y_0, \tag{13}$$

bu yerda $x, y, u, v \in \mathbb{R}^d, d \geq 2$ va $x_0 \neq y_0$ — boshlang'ich holatlar. Boshqaruvlar $u(t)$ va $v(t)$ mos ravishda P va E obyektlarning tezlik vektorlari bo'lib, ular $[0, +\infty)$ oraliqda aniqlangan o'lchanuvchi funksiyalar bo'lishi talab etiladi: $u(\cdot): [0, +\infty) \to \mathbb{R}^d$ va $v(\cdot): [0, +\infty) \to \mathbb{R}^d$.

Langenhop tipidagi chegaralanish (qisqacha, *La*-chegaralanish):

$$|u(t)|^2 \leq \rho^2 - 2k \int_0^t |u(s)|^2 ds, |v(t)|^2 \leq \sigma^2 - 2k \int_0^t |v(s)|^2 ds, t \geq 0, \tag{14}$$

bu yerda $k > 0$, ushbu sinflarni mos ravishda U_{La} va V_{La} bilan belgilaymiz.

Geometrik chegaralanish (qisqacha, *G*-chegaralanish):

$$|u(t)| \le \rho e^{-kt}, \quad |v(t)| \le \sigma e^{-kt}, \qquad t \ge 0,$$

bunday sinflarni mos ravishda U_G, V_G bilan belgilaymiz.

Integral chegaralanish (qisqacha, *I*-chegaralanish):

$$\int_0^t |u(s)|^2 ds \le \frac{\rho^2}{2k}(1 - e^{-2kt}), \int_0^t |v(s)|^2 ds \le \frac{\sigma^2}{2k}(1 - e^{-2kt}), \qquad t \ge 0,$$

bunday sinflarni mos ravishda U_I ,V_I bilan belgilaymiz.

La-, *G*- va *I*-chegaralanishlarni qanoatlantiradigan boshqaruv funksiyalari sinflari orasidagi aniqlangan munosabatga asoslanib, P va E obyektlar uchun uchta ruxsat etilgan boshqaruv sinfi kiritamiz. Agar U (mos ravishda V) U_{La}, U_G, U_I (mos ravishda V_{La}, V_G, V_I) sinflaridan biri bo‘lsa, u holda $u(\cdot) \in U$ va $v(\cdot) \in V$ uchun o‘yinchilarning harakatlari quyidagicha beriladi:

$$P: x(t) = x_0 + \int_0^t u(s)\, ds, \qquad E: y(t) = y_0 + \int_0^t v(s)\, ds, \;\; t \ge 0.$$

Quvlovchining maqsadi — qandaydir $t^* > 0$ da $x(t^*) = y(t^*)$ tenglikni amalga oshirish, qochuvchining maqsadi esa har qanday $t \ge 0$ uchun $x(t) \ne y(t)$ tengsizlikka erishish.

Soddalik uchun, (13) tenglamalar bilan harakatlanadigan hamda boshqaruv funksiyalari *La*-chegaralanishli (14) quvish–qochish masalasini *La*-o‘yin (*La*-chegaralanishli o‘yin) deb ataymiz.

4-lemma. $U_G \subset U_{La} \subset U_I, \quad V_G \subset V_{La} \subset V_I.$

5-ta’rif. *La*-o‘yinda $\sigma \ge \rho$ bo‘lsin. Unda qochuvchi uchun E_{La}-strategiya deb quyidagi funksiyaga aytamiz

$$\boldsymbol{v}_{La}(t, u_\varepsilon(t)) = \begin{cases} 0, & \text{agar } 0 \le t < \varepsilon, \\ -\sqrt{|u(t-\varepsilon)|^2 + \theta e^{-2k(t-\varepsilon)}}\xi_0, & \text{agar } t \ge \varepsilon, \end{cases} \tag{15}$$

bu yerda

$$u_\varepsilon(t) = \begin{cases} 0, \text{ agar } 0 \le t < \varepsilon, \\ u(t-\varepsilon), \text{ agar } t \ge \varepsilon, \end{cases} u_\varepsilon(t) \in \mathbb{R}^n, \theta = \sigma^2 - \rho^2, \xi_0 = z_0/|z_0|.$$

6-ta’rif. E_{La}-strategiya (15) qochuvchi uchun yutuqli deb ataymiz, agar har qanday $u(\cdot) \in U_{La}$ uchun:

a) Koshy masalasi

$$\dot{z} = u(t) - \boldsymbol{v}_{La}\big(t, u_\varepsilon(t)\big), \qquad z(0) = z_0,$$

yechimida barcha $t \ge 0$ uchun $z(t) \ne 0$ bo‘lsa, ya’ni quvish hech qachon yakunlanmasa;

b) $[0, t]$ vaqt oralig‘ida barcha $t \ge 0$ uchun $\mathbf{v}_{La}(t, u_\varepsilon(\cdot)) \in V_{\mathrm{La}}$ bajarilsa.

5-lemma. Agar $u(\cdot) \in U_I$ va $k > 0$ bo‘lsa, u holda quvlovchining trayektoriyasi uchun barcha $t \geq 0$ da $x(t) \in S_{\nu(t)}(x_0)$, bu yerda $\nu(t) = \rho\sqrt{\frac{t}{2k}}$.

7-teorema. Agar *La*-o‘yinda $\rho \leq \sigma, k > 0$ va $0 < \varepsilon \leq \frac{2k|z_0|^2}{\rho^2}$ bo‘lsa, u holda E_{La}-strategiya (15) qochuvchi uchun yutuqli bo‘ladi va o‘yinchilarning orasidagi masofasi uchun quyidagi munosabat o‘rinli bo‘ladi

$$|z(t)| > \begin{cases} 0, \text{ agar } 0 \leq t < \varepsilon, \\ \sqrt{\Phi_E(t-\varepsilon) + \Psi_E(t-\varepsilon)} - \sqrt{\Phi_E(t-\varepsilon)}, \text{agar } t \geq \varepsilon, \end{cases}$$

bu yerda $\Phi_E(t) = t\rho^2(1 - e^{-2kt})/(2k), \Psi_E(t) = (\sigma^2 - \rho^2)(1 - e^{-kt})^2/k^2$.

3.2-paragrafda biz Langenhop–Geometrik chegaralanishli differensial quvish masalasini o‘rganamiz. Bunda P va E obyektlarining harakati (13)–harakat tenglamalari bilan ifodalanadi. Qochuvchining boshqaruv funksiyasi $v(\cdot): [0, +\infty) \to \mathbb{R}^d$ quyidagi geometrik chegaralanishni (*G*-chegaralanish) qanoatlantiradi

$$|v(t)| \leq \beta, \; t \geq 0, \beta \geq 0. \tag{16}$$

Qisqalik uchun, (13) tenglamalar orqali harakatlanadigan hamda quvlovchi boshqaruv funksiyasi Langenhop tipidagi chegaralanishli (14), qochuvchining boshqaruv funksiyasi esa geometrik chegaralanishli (16) quvish-qochish masalasini *LaG-o‘yin* (*LaG* chegaralanishli o‘yin) deb ataymiz.

7-ta’rif. *LaG*-o‘yinda

$$\boldsymbol{u}_{LaG}(t, v) = v - \lambda_{LaG}(t, v)\xi_0 \tag{17}$$

funksiya quvuvchining Π_{LaG}-strategiyasi deb ataymiz, bu strategiya $[0, \mathrm{T}_{LaG}]$ vaqt oralig‘ida qo‘llaniladi. Bu yerda:

$$\mathrm{T}_{LaG} = \frac{1}{k} ln\frac{\rho}{\beta}, \; \rho > \beta > 0, \tag{18}$$

$$\lambda_{LaG}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \rho^2 e^{-2kt} - |v|^2}, \; \xi_0 = z_0/|z_0|, \tag{19}$$

va $\langle v, \xi_0 \rangle - v$ va ξ_0 vektorlarining $\mathbb{R}^d$dagi skalyar ko‘paytmasi. T_{LaG} kafolatlangan tutish vaqti deb ataymiz.

1–tasdiq. Barcha $(t, v) \in [0, \mathrm{T}_{LaG}] \times S_\beta$ uchun $\lambda_{LaG}(t, v)$ (19) funksiyasi aniqlangan, uzluksiz va quyidagicha chegaralangan

$$\rho e^{-kt} - \beta \leq \lambda_{LaG}(t, v) \leq \rho e^{-kt} + \beta.$$

2–tasdiq. Barcha $(t, v) \in [0, \mathrm{T}_{LaG}] \times S_\beta$ uchun $|\boldsymbol{u}_{LaG}(t, v)| = \rho e^{-kt}$ tenglik o‘rinli.

6-lemma. O‘yinchilar harakati quyidagi sistema bilan boshqarilayotgan bo‘lsin

$$\dot{z} = \boldsymbol{u}_{\boldsymbol{LaG}}(t, v(t)) - v(t), \;\; z(0) = z_0,$$

hamda quvlovchi Π_{LaG}-strategiyani (17) qo'llasin. Unda barcha $t \in [0,\ \mathrm{T}_{LaG}]$ uchun quyidagi skalyar funksiya $\Lambda_{LaG}\big(t, v(\cdot)\big) = 1 - \frac{1}{|z_0|}\int_0^t \lambda_{LaG}\big(s, v(s)\big)ds$ quyidagi bahoni qanoatlantiradi:

$$\Upsilon_{LaG}(t) = 1 - \frac{1}{|z_0|}\Big(\frac{\rho}{k}(1 - e^{-kt}) + \beta t\Big) \le \Lambda_{LaG}\big(t, v(\cdot)\big) \le$$
$$\le 1 - \frac{1}{|z_0|}\Big(\frac{\rho}{k}(1 - e^{-kt}) - \beta t\Big) = \Lambda_{LaG}(t).$$

7-lemma. Agar $\rho > \beta$ va $|z_0| \le \frac{1}{k}\left(\rho - \beta\left(1 + ln\frac{\rho}{\beta}\right)\right)$ shartlar bajarilsa, u holda $\Lambda_{LaG}(t) = 0$ tenglama $[0,\ \mathrm{T}_{LaG}]$ oraliqda yagona musbat ildizga ega bo'ladi.

8-teorema. Agar 7-lemma shartlari bajarilsa, u holda quvlovchi Π_{LaG}-strategiyani (17) qo'llab, T_{LaG} dan (18) kechikmay qochuvchini tutadi.

3.3-paragrafda La-chegaralanishli diskret quvish masalasini o'rganilgan. Diskret o'yinda quvlovchining boshqaruvlariga quyidagi chegaraviy shartlarni qanoatlantiruvchi to'plamlar aniqlanadi:

Geometrik chegaralanish (qisqacha, G-chegaralanish)

$$\mathbf{U}_G^n = \left\{\{u_i\}_{i=1}^n : |u_i|^2 \le (1-k)^{i-1}\rho_0^2, i = 1,2,\dots,n\right\};$$

Langenhop tipidagi chegaralanish (qisqacha, La-chegaralanish)

$$\mathbf{U}_{La}^n = \{\ \{u_i\}_{i=1}^n :\ \ |u_1|^2 \le \rho_0^2, |u_i|^2 \le \rho_0^2 - k(|u_1|^2 + |u_2|^2 + \dots + |u_{i-1}|^2),\ \ i = 2,3,\dots,n\}; \quad (20)$$

Yig'indi chegaralanish (qisqacha, S-chegaralanish)

$$\mathbf{U}_S^n = \left\{\{u_i\}_{i=1}^n : \sum_{i=1}^n |u_i|^2 \le \frac{\rho_0^2}{k}[1 - (1-k)^n]\right\}.$$

(Shu tarzda, ρ_0 o'rniga σ_0 qo'yib, $\mathbf{V}_G^n, \mathbf{V}_L^n, \mathbf{V}_S^n$ to'plamlar ham aniqlanadi.) La-, G- va S-chegaralanishlarni qanoatlantiradigan boshqaruv funksiyalar to'plamlari orasidagi aniqlangan munosabatga asoslanib, P va E obyektlar uchun uchta ruxsat etilgan boshqaruvni kiritamiz.

Soddalik uchun, (1) yoki (4) rekkurent tenglamalar bilan harakatlanadigan va parametrlari $0 < k < 1$, $\rho_0 > 0$, $\sigma_0 \ge 0$ bo'lgan, boshqaruvlari La-chegaralanishli (20) diskret quvish–qochish masalasini La-o'yin (La-chegaralanishli o'yin) deb ataymiz.

8-lemma. Agar $0 < k < 1$ bo'lsa, u holda barcha $n = 1,2,\dots$ uchun $\mathbf{U}_G^n \subset \mathbf{U}_L^n \subset \mathbf{U}_S^n$, $\mathbf{V}_G^n \subset \mathbf{V}_L^n \subset \mathbf{V}_S^n$ munosabatlar bajariladi.

8-ta'rif. Quyidagi ketma-ketlik

$$\boldsymbol{u}_n = \begin{cases} \boldsymbol{u}_n^*, & \text{agar } |z_{n-1}| > \ell(v_n), \\ \boldsymbol{u}_n^{**}, & \text{agar } |z_{n-1}| \le \ell(v_n), \end{cases} \quad (21)$$

La-o'yin uchun DPPdagi Π_{La}-strategiya deb ataymiz. Bu yerda:

$$\boldsymbol{u}_n^* = v_n - \ell(v_n)\xi_0, \qquad \boldsymbol{u}_n^{**} = v_n - z_{n-1},$$
$$\ell(v_n) = \langle v_n, \xi_0\rangle + \sqrt{\langle v_n, \xi_0\rangle^2 + (1-k)^{n-1}\delta_0},$$
$$\xi_0 = \frac{z_0}{|z_0|}, \qquad z_0 = x_0 - y_0, \qquad \delta_0 = \rho_0^2 - \sigma_0^2,$$

va $\langle v_n, \xi_0\rangle - v_n$ va ξ_0 vektorlarining $\mathbb{R}^d$dagi skalyar ko'paytmasi $n = 1,2,\dots$. Kelgusi o'rinlarda $\boldsymbol{u}_n^*$ ketma-ketligi *yaqinlashish strategiyasi,* $\boldsymbol{u}_n^{**}$ ketma-ketligi *yakuniy (*yoki *tugallovchi) strategiya* deb ataymiz.

9-lemma. Agar qandaydir n qadamda $|z_{n-1}| > \ell(v_n)$ tengsizlik bajarilsa, u holda shu n -qadam uchun quyidagi tenglik o'rinli bo'ladi

$$|\boldsymbol{u}_n^*|^2 = |v_n|^2 + (1-k)^{n-1}\delta_0.$$

9-ta'rif. La-o'yin uchun DPPda quyidagi ketma-ketliklar

$$\rho_{n-1}^2 = \rho_0^2 - k\sum_{i=1}^{n-1} |\boldsymbol{u}_i|^2, \quad \sigma_{n-1}^2 = \sigma_0^2 - k\sum_{i=1}^{n-1} |v_i|^2, \qquad n = 2,3,\dots,$$

mos ravishda *P* va *E* obyektlarning joriy resurslari, quyidagi ketma-ketlik esa

$$\delta_{n-1} = \rho_{n-1}^2 - \sigma_{n-1}^2 = \delta_0 - k\sum_{i=1}^{n-1} (|\boldsymbol{u}_i|^2 - |v_i|^2), \;\; n = 2,3,\dots,$$

har ikki obyektlarning joriy resurslari farqi deb ataymiz.

10-lemma. Agar $|z_{n-1}| > \ell(v_n)$ tengsizlik qandaydir $n-$ qadamdan oldin bajarilsa, u holda $\delta_{j-1} = (1-k)^{j-1}\delta_0$, $j = 1,2,\dots,n$ tenglik shu $n-$qadamdan oldin bajaradi.

9-teorema. La-o'yin uchun DPPda $\rho > \sigma, 0 < k < 1$ bo'lsin va quvlovchi Π_{La}-strategiyani (21) qo'llasin. Unda:

1) Agar $j = 1,2,\dots,n$ qadamlar uchun $|z_{j-1}| > \ell(v_j)$ tengsizlik bajarilsa, u holda barcha $j = 1,2,\dots,n$ qadamlar uchun
$z_j \neq 0$ va $z_j = z_0 h_j$, $h_j > 0$, $\delta_{j-1} = (1-k)^{j-1}\delta_0$ o'rinli bo'ladi, bu yerda

$$h_j = 1 - \frac{1}{|z_0|}\sum_{i=1}^{j} \ell(v_i), j = 1,2,\dots,n;$$

2) Agar n-qadamda $|z_{n-1}| \le \ell(v_n)$ tengsizlik bajarilsa, u holda n-qadamda qochuvchi tutib olinadi, ya'ni $z_n = 0$.

1-faraz. La-o'yin uchun DPPda $\rho > \sigma, 0 < k < 1$ va quyidagi shartni qanoatlantiruvchi eng kichik natural son $n = N_{La}(z_0)$ mavjud bo'lsin,

$$|z_0| \le \sqrt{p_n^2 + \delta_0 q_n^2} - p_n$$

bu yerda $p_n^2 = \frac{\sigma_0^2 n}{k}[1-(1-k)^n]$ va $q_n = \frac{1-(1-k)^{\frac{n}{2}}}{1-\sqrt{1-k}}$. U holda $n = N_{La}(z_0)$ soni quvishning yakunlanishini kafolatlovchi qadam deb ataymiz.

10-teorema. La-o'yin uchun DPPda 1-faraz bajarilsin. Unda quvlovchi Π_{La}-strategiyani (21) qo'llab tutishni $N_{La}(z_0)$-qadamdan kechikmay yakunlaydi.

1-izoh. Agar $k \to 0$ bo'lsa, u holda $p_n^2 \to \sigma_0^2 n^2, q_n^2 \to n^2$. Yensen tengsizligining** asosiy teoremasidan $|z_0| \leq n(\rho_0 - \sigma_0)$ munosabat hosil bo'ladi. Demak, $k = 0$ holatida quvishning kafolatlangan tutish qadami quyidagicha

$$N_{La}(z_0) = \begin{cases} \left[\frac{|z_0|}{\rho - \sigma}\right] + 1, & \text{agar } \left\{\frac{|z_0|}{\rho - \sigma}\right\} > 0, \\ \left[\frac{|z_0|}{\rho - \sigma}\right], & \text{agar } \left\{\frac{|z_0|}{\rho - \sigma}\right\} = 0, \end{cases}$$

bu yerda $[a]$ — a sonining butun qismi, $\{a\}$ — uning kasr qismi.

** Khardi G., Littlvud Dzh., Polya G. Neravenstva [Inequalities] Moscow: IL, 1948. (in Russian)

XULOSA

Dissertatsiyada boshqaruv funksiyalari turli chegaralanishli dinamik o'yinlarning quvish-qochish masalalari uchun quyidagi tadqiqotlar bajarildi.

1. Boshqaruv funksiyalari geometrik chegaralanishga ega bo'lgan, ko'p quvlovchi va bitta qochuvchili diskret o'yinda quvish masalasini yechish uchun quvlovchilar tomonidan qo'llaniladigan parallel yaqinlashish usuli (П-strategiya) asosida kafolatlovchi yetarli shartlar topilib, o'yinning 2D animatsion modeli ishlab chiqilgan.
2. Boshqaruv funksiyalari yig'indi chegaralanishga ega bo'lgan, sodda harakatli diskret o'yinda quvish masalasi yechilishi uchun tutishni kafolatlovchi yetarli shartlar quvlovchi tomonidan qo'llaniladigan parallel yaqinlashish usuli (П-strategiya)dan foydalanib topilgan;
3. Boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo'lgan differensial o'yinda qochish masalasi yechilishi uchun qochuvchining kechikkan boshqaruvi orqali aniqlangan yetarli shartlarda o'yinchilar orasidagi masofaning quyi chegarasi topilgan;
4. Boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega bo'lgan diskret o'yinda quvish masalasining П-strategiya yordamida topilgan yechimi asosida o'yinning 2D animatsion modeli ishlab chiqilgan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES
PhD.03/30.12.2019.FM.05.04 FERGANA STATE UNIVERSITY

NAMANGAN STATE UNIVERSITY

UMARALIYEVA NARGIZA TASHKINBAYEVNA

DYNAMIC GAMES WITH VARIOUS CONSTRAINTS ON CONTROL FUNCTIONS AND THEIR ANIMATION MODELS

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

Fergana – 2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Higher Attestation Commission under the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan under number B2022.4.PhD/FM194.

Dissertation has been prepared at Namangan state university.

The abstract of the dissertation is posted in three languages (Uzbek, English, Russian (resume)) on the website (www.fdu.uz) and the "ZiyoNET" information and educational portal (www.ziyonet.uz).

Scientific supervisors:	**Samatov Bahrom Tadjiahmatovich** Doctor of Physical and Mathematical Sciences, Professor
Official opponents:	**Mamadaliev Numanjon** Doctor of Physical and Mathematical Sciences, Professor
	Ibaydullaev Tulanboy Tursunbaevich Candidate of Physical and Mathematical Sciences, Professor
Leading organization:	**Termez State University**

Defense will take place "1" 11 2025 at 1000 at the meeting of Scientific Council number PhD.03/30.12.2019.FM.05.04 at Fergana State University (Address: Murabbiylar str. 19, Fergana, Uzbekistan, 150100, Phone: (+99873) 244-44-02, fax: (+99873) 244-44-93, e-mail: fardu_info@umail.uz).

Dissertation is possible to review in Information-resource centre at Fergana State University (is registered № 590). (Address: Murabbiylar str. 19, Fergana, Uzbekistan, 150100, Phone: (+99873) 244-44-94).

Abstract of dissertation sent out on "20" 10 2025 year.
(Mailing report №. 10 on "17" 10 2025 year).

Sh.T.Karimov
Deputy chairman of Scientific Council on award of scientific degrees, D.Ph.M.S., Dotsent

I.U.Khaydarov
Scientific Secretary of Scientific Council on award of scientific degrees, C.Ph.M.S., Dotsent

Yu.P.Apakov
Deputy chairman of the Scientific Seminar under Scientific Council on award of scientific degrees, D.Ph.M.S., Professor

INTRODUCTION (abstract of the PhD thesis)

Actuality and demand of the theme of dissertation. The rapid development of science and technology around the world is leading the way in constructing mathematical models for dynamic systems with opposing goals and their effective application to practical processes. One of the most effective tools for analyzing such systems worldwide is dynamic game theory, which requires the implementation of models built with its help in practice through areas such as control theory, economics, robotics, defense, and artificial intelligence. In this regard, the study of conflict-controlled processes, which are the main object of dynamic game theory, the study of discrete and differential game problems with different control constraints, the development of mathematical models, and the effective use of animation models in visual animation based on their algorithmic solution, is of great importance in viewing and analyzing dynamic actions in real time.

Worldwide, research is being carried out to optimize time, resources, costs, quality, and capabilities in solving numerous theoretical and applied problems arising from scientific developments. In this context, special attention is given to the mathematical study of conflicting goal processes in dynamic games with various constraints on control functions—geometric, integral, and Langenhop types—and to the construction of stable models suitable for practical use. The development of such models expands the applicability of theoretical results to real processes. This approach plays an important role in enhancing control efficiency in complex systems and in supporting strategic decision-making.

In our republic, paying attention to the current directions of fundamental sciences with scientific and practical applications, comprehensive measures are being taken to develop modern innovative technologies, and certain results are being achieved. In particular, important tasks[1] in have been set in such priority areas as "algebra and its applications, differential equations and their applications, nonlinear systems, mathematical modeling of dynamic systems and their applications, stochastic analysis, medical and biological informatics, computational mathematics". In implementing these tasks, in particular, in order to improve the theories of differential equations, dynamic systems and dynamic games, it is important to build optimal strategies for pursuit–evasion problems with different control constraints in discrete and differential games and create their animated models.

In accordance with the Decree of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan," the Decree UP-2789 of February 17, 2017 "On Measures to Improve the Organization, Management, and Financing of the Activities of the Academy of Sciences," the Decree UP-2909 of April 20, 2017 "On Measures for the Further Development of the Higher Education System," the

[1] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 № PQ-4387 "On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan".

Decree UP-60 of January 28, 2022 “On the Development Strategy of New Uzbekistan for 2022–2026,” the Resolution PP-3682 of April 27, 2018 “On Measures to Further Improve the System of Introducing Innovative Ideas, Technologies, and Projects into Practice,” the Resolution PP-4387 of July 9, 2019 “On State Support for the Further Development of Mathematical Education and Sciences, as well as on the Radical Improvement of the Activities of the V.I. Romanovskiy Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan,” and the Resolution PP-4708 of May 7, 2020 “On Measures to Improve the Quality of Education and Develop Scientific Research in the Field of Mathematics,” as well as other relevant normative and legal documents, this dissertation research serves, to a certain extent, in the implementation of the tasks defined therein.

Connection of research to priority directions of development of science and technologies of the Republic. This dissertation research was carried out within the framework of the program “Topical Problems of Differential Equations and Related Mathematical Fields” in accordance with the research plan of Namangan State University.

The degree of scrutiny of the problem. The foundation of differential game theory was laid by R. Isaacs, who developed mathematical models for military pursuit-evasion (P-E) problems at the RAND Corporation in the late 1940s. His work applied Hamilton–Jacobi methods from classical variational calculus to dynamic situations governed by control functions. These ideas were later developed by researchers such as L.D. Berkovitz, W.H. Fleming, and A. Friedman, who introduced additional tools from game theory and calculus of variations. Problems in differential games are classified into pursuit or evasion cases depending on which side holds the information advantage. This classification approach was proposed by L.S. Pontryagin and further developed by researchers such as A.A. Chikrii, N.L. Grigorenko, N.N. Petrov, M.S. Nikolskiy, N.Yu. Satimov, A.A. Azamov, and others.

In addition to differential games, discrete games with positional strategies have also been extensively studied. In discrete positional games, strategies depend only on the current state, which simplifies the process of constructing and analyzing optimal strategies. Significant contributions to this area have been made by R. Isaacs, L.S. Pontryagin, N.N. Krasovskii, A.I. Subbotin, A.A. Azamov, L.A. Petrosyan, and A.A. Chikrii. The scientific school of Uzbekistan in this direction was formed under the leadership of N.Yu. Satimov, who led the scientific seminar “Optimal control and differential games” for more than 35 years. Today it is headed by A.A. Azamov. Significant results have been achieved by representatives of the scientific school. Based on the research of A.Z. Fazilov, B.B. Rikhsiev, A.A. Khamdamov, G.I. Ibragimov, M.Sh. Mamatov, B.T. Samatov, A.Sh. Kuchkarov, and N.A. Mamadaliev, effective solutions to the pursuit-evasion problems were found within both differential and discrete models.

Using the parallel approach strategy (Π-strategy) proposed by L.A. Petrosyan, solutions were developed for group pursuit problems, as well as for the “Life Line” problem introduced by R. Isaacs. A.A. Azamov was the first to analytically solve the “Life-Line” game involving multiple pursuers and a single evader by employing a support function based on multivalued reflection. G.I. Ibragimov studied linear discrete pursuit games with integral constraints. He proposed step-by-step strategies and established sufficient conditions for guaranteed capture. B.T. Samatov developed adapted versions of the Π-strategy for simple-motion differential games with control functions subject to integral, linear, non-stationary, Gronwall-type, and Langenhop-type constraints, including mixed forms. N.A. Mamadaliyev conducted research on pursuit and evasion problems in differential games described by systems with delay arguments, where control functions are subject to integral constraints, and obtained significant scientific results.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This research was performed within the "Fundamental Research" network of the scientific research work plan of Namangan State University.

The aim of the research is to solve dynamic pursuit–evasion problems with various constraints on control functions, including geometric, summary, and Langenhop-type constraints.

Problems of the research:

to determine sufficient conditions that guarantee the solvability of the pursuit problem in a discrete game with multiple pursuers and one evader, where the players' control functions are subject to geometric constraints and to develop an animation model of the game;

to determine sufficient conditions that ensure the solvability of the pursuit problem in a simple-motion discrete game under summary constraints on the players' control functions and to develop an animation model of the game;

to identify sufficient conditions that guarantee the solvability of the evasion problem in a differential game where the players' control functions are subject to Langenhop-type constraints;

to determine sufficient conditions that guarantee the solvability of the pursuit problem in a discrete game under Langenhop-type constraints on the players' control functions and to develop an animation model of the game.

The object of the research is dynamic games with various constraints on control functions.

The subject of the research is the differential and discrete pursuit–evasion problems of dynamic games with geometric, summary, and Langenhop-type constraints on control functions, the differential equations and inequalities describing them, as well as strategies for constructing their solutions.

Methods of the research. It includes solving pursuit problems in discrete and differential games using Π-strategy and evasion problems using control with delay. Also, basic theorems and concepts of mathematical analysis, dynamic game theory, differential equations, functional analysis, and optimal control theory are used.

Scientific novelty of the research is composed of the following:

for a discrete game with multiple pursuers and one evader, where the control functions are subject to geometric constraints, sufficient conditions guaranteeing pursuit have been established based on the parallel approach method (Π-strategy) applied by the pursuers, and a 2D animation model of the game has been developed;

for a discrete game with simple motion in which the control functions are subject to summary constraints, sufficient conditions guaranteeing capture have been obtained using the parallel approach method (Π-strategy) applied by the pursuer;

for differential game with Langenhop-type constraints on the control functions, in the evasion problem, sufficient conditions have been determined under which, using the evader's delayed control, the lower bound of the distance between the players is found;

based on the solution of the pursuit problem for a discrete game with Langenhop-type constraints on the control functions, obtained using the Π-strategy, a 2D animation model of the game has been developed.

Practical results of the research. The practical significance of the research lies in the direct applicability of the results presented in the dissertation, as well as the methods used to obtain them, to conflict control problems described by differential and recurrent equations. Based on the obtained theoretical results, animation models have been developed to enable visual observation of the dynamic behavior of objects. These models have been employed to analyze problems related to robotics, security systems, and automated control systems.

The reliability of the results of the research. The foundations of mathematical optimal control theory, ordinary differential equations theory, functional and convex analysis, mathematical analysis, and theorems and lemmas related to pursuit-evasion problems of dynamic game theory are applied. The results presented in the research work are explained by the fact that they are proven based on rigorous approaches in mathematics.

The scientific and practical significance of the research results. The scientific significance of the dissertation work lies in the construction and implementation of strategies that provide optimal solutions to pursuit and evasion problems in discrete and differential games with various constraints on the control functions, namely geometric, summary and Langenhop-type constraints. The results presented are explained by the fact that they can be used to improve classical methods in control and dynamic game theory and identify solutions to problems related to their practical applications.

The practical significance of the research is expressed through the development and implementation of 2D animation models that visualize discrete motions in pursuit problems. These models enable the verification of theoretical results through visualization and serve as a foundation for applications in robotics, security systems, and automated control systems.

Implementation of the research result. Based on the results obtained for dynamic games with various constraints on control functions and their animation models:

the animation model developed for the discrete game with multiple pursuers and one evader under geometric constraints on control functions was utilized in the applied project No. FZ-201905171 titled "Development of hydrodynamic models and efficient algorithms for studying anomalous filtration processes of fluids and gases in porous media." It was used to create a computer model of fluid filtration for the comprehensive study of gas fields bounded by water-bearing layers (according to Reference No. 2/272 issued by the V.I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan on June 25, 2025). As a result, it became possible to solve filtration problems in porous media based on coordinate separation and differential driving methods, as well as to develop their numerical algorithms;

in solving dynamic game problems with geometric and Langenhop-type constraints on control functions, the guaranteed conditions based on optimal approach strategies (Π-strategies), as well as the developed animation models visualizing the capture process, were utilized within the framework of the fundamental project OT-F4-33 titled "Development of new methods for conflict control described by differential equations and their numerical implementation" (based on Certificate No. 04/11-7621 issued by the National University of Uzbekistan on June 18, 2025). As a result, it became possible to develop new methods for solving control problems of conflicting processes described by discrete equations with geometric and Langenhop-type constraints, to solve them numerically, and to practically verify the correctness of the obtained results.

Approbation of the research results. The results of the research were discussed at 16 scientific and practical conferences, including 10 international and 6 Republican scientific and practical conferences.

Publications of the research results. During the course of this research, a total of 26 scientific works have been published. Among them, 7 are listed in the official registry of scientific journals recommended by the Higher Attestation Commission of the Republic of Uzbekistan for defending dissertations to obtain the degree of Doctor of Philosophy. Specifically, 2 of these works were published in international journals, 5 in national scientific journals, and 16 were presented as conference abstracts.

Also, three Certificates of State Registration were granted by the Ministry of Justice of the Republic of Uzbekistan in accordance with the Law of the Republic of Uzbekistan "On Legal Protection of Computer Programs and Databases," recognizing software developed as part of this research.

The structure and volume of the dissertation. The dissertation is composed of the introduction, three chapters, conclusion, bibliography and appendices. The total volume of the dissertation is 109 pages.

THE MAIN CONTENT OF THE DISSERTATION

The introduction presents the motivation for the research topic and its correspondence to the priority research areas of science and technology in the Republic. A review of international research on the topic of the dissertation and the extent to which the problem has been studied is provided. The goals and objectives of the research are formulated, and the object and subject of the study are identified. The scientific novelty and practical results of the research are stated, along with the theoretical and practical significance of the obtained results. Information on the implementation of the research results, published works, and the structure of the dissertation is also included.

The first chapter is titled "**Discrete games with geometric constraints on controls and their animation models**" is devoted to the study of two discrete pursuit problems with geometric constraints on control functions.

In the space $\mathbb{R}^d$, where $d \geq 2$, two objects — P (the pursuer) and E (the evader) — perform stepwise movements according to the recurrence equations:

$$P:\ x_n = x_{n-1} + u, \quad E:\ y_n = y_{n-1} + v, \tag{1}$$

where x_n and y_n denote the positions of P and E at step n ($n = 1,2,\dots$) with initial positions x_0 and y_0 respectively, such that $x_0 \neq y_0$. The controls u and v are the control parameters of objects P and E, respectively, selected as sequences of constant vectors $\{u_n\}$ and $\{v_n\}$in $\mathbb{R}^d$, subject to the following geometric constraints (briefly, G-constraints):

$$u_n \in S_\alpha \coloneqq \{u \in \mathbb{R}^d: \|u\| \leq \alpha\},\ v_n \in S_\beta \coloneqq \{v \in \mathbb{R}^d: \|v\| \leq \beta\}, \tag{2}$$

where $\alpha > 0, \beta \geq 0$, and S_α, S_β are Euclidean balls centered at the origin in $\mathbb{R}^d$.

By substituting the control sequences (2) into the recurrence equations (1), we obtain the following solutions in the form of sequences, namely, the traces of objects P and E in the space $\mathbb{R}^d$:

$$P:\ x_n = x_0 + \sum_{i=1}^{n} u_i\,, \qquad E:\ y_n = y_0 + \sum_{i=1}^{n} v_i\,, \qquad n = 1,2,\dots\,. \tag{3}$$

The goal of the pursuer is to achieve $x_N = y_N$ for some finite $N \in \mathbb{N}$ (Discrete Pursuit Problem, DPP), while the evader aims to maintain $x_n \neq y_n$ for all $n \in \mathbb{N}$ (Discrete Evasion Problem, DEP).

We introduce a new variable $z_n = x_n - y_n$. Then the recurrence equations (1) take the form

$$z_n = z_{n-1} + u - v, \tag{4}$$

using (3) the solution is

$$z_n = z_0 + \sum_{i=1}^{n} (u_i - v_i)\,, \qquad z_0 = x_0 - y_0.$$

For simplicity, we call the discrete pursuit–evasion game, in which the control functions satisfy the geometric constraints (2) and the motions are described by the recurrence equations (1) or (4), the *G-game* (G-constraints game).

Definition 1. We call the sequence

$$\boldsymbol{u}_n = \begin{cases} \boldsymbol{u}_n^A, & if\ \ |z_{n-1}| > \mu(v_n), \\ \boldsymbol{u}_n^F, & if\ |z_{n-1}| \leq \mu(v_n), \end{cases} \tag{5}$$

the *Π_G-strategy* in the DPP for the *G*-game, where

$$\begin{gathered} \boldsymbol{u}_n^A = v_n - \mu(v_n)\xi_0, \\ \boldsymbol{u}_n^F = v_n - z_{n-1}, \\ \mu(v_n) = \langle v_n, \xi_0\rangle + \sqrt{\langle v_n, \xi_0\rangle^2 + \alpha^2 - |v_n|^2},\ \ \xi_0 = z_0/|z_0|, \end{gathered} \tag{6}$$

and $\langle v_n, \xi_0 \rangle$ means the inner product of the vectors v_n and ξ_0 in $\mathbb{R}^d$, $n = 1,2,\dots$. Further, we call sequence $\boldsymbol{u}_n^A$ *the approach strategy*, and sequence $\boldsymbol{u}_n^F$ *the final* (or *finishing) strategy*.

Lemma 1. *If $\alpha \geq \beta$, then for each pair $(v_n, z_{n-1}) \in S_\beta \times \mathbb{R}^d$:*
a) the sequence $\boldsymbol{u}_n$ is defined;
b) the inequalities $\alpha - \beta \leq \mu(v_n) \leq \alpha + \beta$ hold for any $v_n \in S_\beta$.

Theorem 1. Let $\alpha > \beta$ and the pursuer implements the Π_G-strategy (5) in the DPP for the *G*-game. Then:
1) If, for steps $i = 1,2,\dots,n$, the inequality $|z_{i-1}| > \mu(v_i)$ holds, then $z_i \neq 0$ for all steps $i = 1,2,\dots,n$;
2) If, at step n, the inequality $|z_n| \leq \mu(v_{n+1})$ holds, then at step $n+1$ the evader is captured, i.e., $z_{n+1} = 0$.

Lemma 2. If at some step n the inequality $0 < |z_n| \leq \alpha - \beta$ is satisfied, then the pursuer, using the final strategy (6), completes the pursuit at step $n+1$.

Theorem 2. Let $\alpha > \beta$ be satisfied in the DPP for the G-game. Then the pursuer, implementing the Π_G-strategy (5), completes the pursuit no later than in step

$$N(z_0) = \left[\frac{|z_0|}{\alpha-\beta}\right] + 1,$$

where the sign $[\cdot]$ denotes the integer part of the number inside this sign.

In Section 1.2, we study a group discrete pursuit problem — a discrete analogue of the Pshenichnii problem* — in which multiple pursuers P^1, P^2,..., P^m chase an evader E in $\mathbb{R}^d$. The motion of all players follows recurrence equations that model simple step-by-step dynamics.

$$P^i\colon x_n^i = x_{n-1}^i + u^i,\ \ i \in I_m = \{1,2,3,\dots,m\},\quad E\colon y_n = y_{n-1} + v, \tag{7}$$

where $n = 1,2,\dots$. The initial positions are assumed to be distinct $x_0^i \neq y_0$, for all $i \in I_m$. The controls u^i and v are the control parameters of the pursuers P^i and the evader E, respectively, selected as sequences of constant vectors $\{u_n^i\}_{n\in N}$ and $\{v_n\}_{n\in N}$ in $\mathbb{R}^d$, subject to the following geometric constraints (briefly, G-constraints):

$$|u_n^i| \leq 1,\ \ i \in I_m,\ \ |v_n| \leq 1. \tag{8}$$

Thus, all players are assumed to have equal control capabilities.

The pursuers aim to ensure $x_N^j = y_N$ for some $j \in I_m$ at a finite step *N* (Group Discrete Pursuit Problem, GDPP), while the evader seeks to avoid this by maintaining $x_n^i \neq y_n$ for all $i \in I_m$ at every step (Group Discrete Evader Problem, GDEP).

Similar to Section 1.1, here we simplify the solution of the discrete pursuit problem by introducing new variables: $z_n^i = x_n^i - y_n$, $i \in I_m$, and $z_0^i = x_0^i - y_0$. Then from (7) we obtain the recurrence equations

$$z_n^i = z_{n-1}^i + u_n^i - v_n, \qquad i \in I_m. \tag{9}$$

with solutions of the recurrence equations (9) given by

* B.N. Pshenichnii. Simple pursuit by several objects. Cybernetics and System Analysis, 1976.

$$z_n^i = z_0^i + \sum_{k=1}^{n}(u_k^i - v_k), \qquad i \in I_m.$$

For simplicity, we call the group discrete pursuit–evasion game, in which the control functions satisfy the geometric constraints (8) and the motions are described by the recurrence equations (7) or (9), a *G-game* (G-constraints game).

Definition 2.

$$\boldsymbol{u}_n^i = \begin{cases} \boldsymbol{u}_n^{A_i}, & if\ |z_{n-1}^i| > \mu_i(v_n), \\ \boldsymbol{u}_n^{F_i}, & if\ |z_{n-1}^i| \le \mu_i(v_n), \end{cases} \tag{10}$$

we call the Π_G^i*-strategy* of the i-pursuer in the GDPP for the G-game, where

$$\boldsymbol{u}_n^{A_i} = v_n - \mu_i(v_n)\xi_0^i, \qquad \boldsymbol{u}_n^{F_i} = v_n - z_{n-1}^i,$$

$$\mu_i(v_n) = \langle v_n, \xi_0^i \rangle + \sqrt{\langle v_n, \xi_0^i \rangle^2 + 1 - |v_n|^2}, \qquad \xi_0^i = \frac{z_0^i}{|z_0^i|}, \qquad n = 1,2,\ldots, \qquad i \in I_m.$$

Theorem 3. Let in the GDPP for the G-game the pursuers implement Π_G^i-strategy (10). Then:

1) If, for steps $k = 1,2,\ldots,n$ the inequality $|z_{k-1}^i| > \mu_i(v_k)$ holds for all $i \in I_m$, then $z_k^i \neq 0$ for all $k = 1,2,\ldots,n$ and all $i \in I_m$;

2) If, at step n, the inequality $|z_n^j| \le \mu_j(v_{n+1})$ holds for some $j \in I_m$, then at step $n+1$ the pursuer P^j completes the pursuit, i.e. $z_{n+1}^j = 0$.

Theorem 4. (Discrete analogue of Pshenichii's Theorem) If in the GDPP for the G-game for the initial states $z_0^i,\ i \in I_m$, the condition

$$0 \in int\ conv\{\xi_0^1, \xi_0^2, \ldots, \xi_0^m\}$$

is satisfied, then with the help of Π_G^i-strategies (10) the pursuers complete the pursuit no later than step

$$N(z_0^1, z_0^2, \ldots, z_0^m) = \left[\sum_{i=1}^{m} |z_0^i| / \gamma\right] + 1,$$

where

$$\gamma = \frac{2\delta}{1+2\delta} \text{ and } \delta = \min_{|p|=1} \max_{i \in I_m} \langle p, \xi_0^i \rangle > 0.$$

The second chapter is titled "**Discrete game with summary constraints on controls and its animation model.**" is devoted to the study of the discrete game with simple motion with summary constraints.

Let the motions of the objects *P* and *E* be described by the recurrence equations (1). The control sequences of the objects *P* and *E* $\{\boldsymbol{u}_i\}_{i=1}^{\infty}$, $\boldsymbol{u}_i \in \mathbb{R}^d$ and $\{\boldsymbol{v}_i\}_{i=1}^{\infty}$, $\boldsymbol{v}_i \in \mathbb{R}^d$ are assumed to satisfy the following summary constraints (briefly, S-constraints):

$$\| \boldsymbol{u} \|_{\ell_2} = \sqrt{\sum_{i=1}^{\infty} |\boldsymbol{u}_i|^2} \le \rho_0, \ \| \boldsymbol{v} \|_{\ell_2} = \sqrt{\sum_{i=1}^{\infty} |\boldsymbol{v}_i|^2} \le \sigma_0, \tag{11}$$

where $\rho_0 > 0$ and $\sigma_0 \ge 0$.

For simplicity, we call the discrete pursuit–evasion game, in which the players' control functions satisfy the summary constraints (11) and their motions are governed by the recurrence equations (1) or (4), the *S-game* (S-constraints game).

Definition 3. We call the sequence

$$\boldsymbol{U}_n^* = \begin{cases} \boldsymbol{u}_n^*, & if \quad |z_{n-1}| > \eta(v_n), \\ \boldsymbol{u}_n^{**}, & if \quad |z_{n-1}| \le \eta(v_n), \end{cases} \tag{12}$$

the Π_S*-strategy* in the DPP for the S-game, where

$$\boldsymbol{u}_n^* = v_n - \eta(v_n)\xi_0, \qquad \boldsymbol{u}_n^{**} = v_n - z_{n-1},$$
$$\eta(v_n) = max\{0, \delta_0^2/|z_0| + 2\langle v_n, \xi_0\rangle\},$$
$$\xi_0 = \frac{z_0}{|z_0|}, \;\; z_0 = x_0 - y_0, \;\; \delta_0^2 = \rho_0^2 - \sigma_0^2,$$

and $\langle v_n, \ \xi_0\rangle$ means the inner product of the vectors v_n and ξ_0 in $\mathbb{R}^d$, $n = 1,2, \ldots$. Further, we will call sequence $\boldsymbol{u}_n^*$ *the approach strategy*, and sequence $\boldsymbol{u}_n^{**}$ *the final* (or *finishing) strategy*.

Lemma 3. If at some step n the inequality $|\boldsymbol{z}_{n-1}| > \eta(\boldsymbol{v}_n)$ is satisfied, then at step n the equality $|\boldsymbol{u}_n^*|^2 = |\boldsymbol{v}_n|^2 + \frac{\delta_0^2}{|z_0|}\eta(\boldsymbol{v}_n)$, holds.

Definition 4. In S-game we call sequences

$$\rho_{n-1}^2 = \rho_0^2 - \sum_{i=1}^{n-1} |\boldsymbol{U}_i^*|^2, \quad \sigma_{n-1}^2 = \sigma_0^2 - \sum_{i=1}^{n-1} |\boldsymbol{v}_i|^2,$$

the current resources of the objects P and E, respectively, and sequence

$$\delta_{n-1}^2 = \rho_{n-1}^2 - \sigma_{n-1}^2 = \delta_0^2 - \sum_{i=1}^{n-1} (|\mathbf{U}_i^*|^2 - |\mathbf{v}_i|^2),$$

the difference in the current resource of the players.

Theorem 5. Let $\rho_0 > \sigma_0$ in the DPP for the S-game and the pursuer implements the Π_S-strategy (12). Then:

1) If, for steps $k = 1,2, \ldots, n$ the inequality $|\mathrm{z}_{k-1}| > \eta(\mathrm{v}_k)$ holds, then for all steps $k = 1,2, \ldots, n$ we have $\mathrm{z}_k \neq 0$ and $\mathrm{z}_k = \mathrm{z}_0 h_k$, $\delta_k^2 = \delta_0^2 h_k$, $0 < h_k \le 1$, where

$$h_k = 1 - \frac{1}{|z_0|}\sum_{i=1}^{k} \eta\,(v_i), \quad k = 1,2, \ldots, n;$$

1) If, at step n, the inequality $|z_{n-1}| \le \eta(v_n)$ holds, then at step n, the evader is captured, i.e. $z_n = 0$.

Theorem 6. Let $\rho_0 > \sigma_0$ in the DPP for the S-game. Then the pursuer, implementing Π_S -strategy (12), completes the pursuit no later than in step

$$N(z_0) = \left[\left(\frac{|z_0|}{\rho_0 - \sigma_0}\right)^2\right] + 1.$$

The third chapter is titled "**Dynamic games with Langenhop-type constraints on controls and their animation models.**" It is devoted to the study of differential and discrete pursuit–evasion games with Langenhop-type constraints on the players' control functions.

Initially, the evasion problem is considered in the space $\mathbb{R}^d$, where the motion of the pursuer and the evader is described by the differential equations

$$P: \dot{x} = u,\ x(0) = x_0; \qquad E: \dot{y} = v,\ y(0) = y_0, \tag{13}$$

where $x, y, u, v \in \mathbb{R}^d, d \geq 2; x_0$ and y_0 are initial positions of the objects. It is assumed that $x_0 \neq y_0$; u and v are the velocity vectors, which serve as parameters of the equations. Here u and v must be a measurable functions $u(\cdot): [0, +\infty) \to \mathbb{R}^d$ and $v(\cdot): [0, +\infty) \to \mathbb{R}^d$, respectively.

Langenhop type constraint (briefly, *La* -constraint):

$$|u(t)|^2 \leq \rho^2 - 2k \int_0^t |u(s)|^2 ds, |v(t)|^2 \leq \sigma^2 - 2k \int_0^t |v(s)|^2 ds, \qquad t \geq 0, \tag{14}$$

where $k > 0$ and we denote such classes by U_{La} and V_{La} .

Geometric constraint (briefly, *G*-constraint):

$$|u(t)| \leq \rho e^{-kt},\ |v(t)| \leq \sigma e^{-kt},\ \ t \geq 0,$$

and we denote such classes by U_G and V_G.

Integral constraint (briefly, *I*-constraint) of form:

$$\int_0^t |u(s)|^2 ds \leq \frac{\rho^2}{2k}(1 - e^{-2kt}), \qquad \int_0^t |v(s)|^2 ds \leq \frac{\sigma^2}{2k}(1 - e^{-2kt}), \qquad t \geq 0,$$

and we denote such classes by U_I and V_I.

Based on the identified relationship among the classes of control functions satisfying *La*-, *G*-, and *I*-constraints, we introduce three classes of admissible controls for the pursuer and the evader.

If U (resp. V) is one of the classes U_{La}, U_G, U_I (resp. V_{La}, V_G, V_I), then for $u(\cdot) \in U$ and $v(\cdot) \in V$, the motions of the players are given by

$$P: x(t) = x_0 + \int_0^t u(s)\, ds, \qquad E: y(t) = y_0 + \int_0^t v(s)\, ds,\ \ t \geq 0.$$

The goal of the pursuer is to achieve $x(t^*) = y(t^*)$ for some $t^* > 0$, while the evader aims to ensure that $x(t) \neq y(t)$ for all $t \geq 0$.

For simplicity, we call the pursuit–evasion problem governed by the equations (13), where the control functions are subject to *La*-constraints (14), the *La*-game (*La*-constraints game).

Lemma 4. $U_G \subset U_{La} \subset U_I, \quad V_G \subset V_{La} \subset V_I$.

Definition 5. Let $\sigma \geq \rho$ in the La-game. Then by the E_{La}-*strategy* of the evader we mean the function

$$\boldsymbol{v}_{La}(t, u_\varepsilon(t)) = \begin{cases} 0,\ if\ 0 \leq t < \varepsilon, \\ -\sqrt{|u(t-\varepsilon)|^2 + \theta e^{-2k(t-\varepsilon)}}\xi_0,\ if\ t \geq \varepsilon, \end{cases} \tag{15}$$

where

$$u_\varepsilon(t) = \begin{cases} 0, \text{if } 0 \leq t < \varepsilon, \\ u(t-\varepsilon), \text{if } t \geq \varepsilon, \end{cases} u_\varepsilon(t) \in \mathbb{R}^n, \theta = \sigma^2 - \rho^2, \xi_0 = z_0/|z_0|.$$

Definition 6. We call E_{La}-strategy (15) winning for the evader, if for every $u(\cdot) \in U_{La}$:

a) the solution $z(t)$ to the Cauchy problem

$$\dot{z} = u(t) - \boldsymbol{v}_{La}\big(t, u_\varepsilon(t)\big), z(0) = z_0$$
is nonzero for all $t \geq 0$, i.e. $z(t) \neq 0$ for all $t \geq 0$;
b) the condition $\mathbf{v}_{La}(t, u_\varepsilon(\cdot)) \in V_{\mathrm{La}}$ holds on the time interval $[0, t]$ for all $t \geq 0$.

Lemma 5. If $u(\cdot) \in U_I$ and $k > 0$, then the trajectory of the pursuer satisfies $x(t) \in S_{\nu(t)}(x_0)$ for all $t \geq 0$, where $\nu(t) = \rho\sqrt{\frac{t}{2k}}$.

Theorem 7. If in the *La*-game $\rho \leq \sigma$, $k > 0$ and $0 < \varepsilon \leq \frac{2k|z_0|^2}{\rho^2}$, then E_{La}-strategy (15) is winning for E and the following estimate for the distance between the players holds for all $t \geq 0$:

$$|z(t)| > \begin{cases} 0, \text{ if } 0 \leq t < \varepsilon, \\ \sqrt{\Phi_E(t-\varepsilon) + \Psi_E(t-\varepsilon)} - \sqrt{\Phi_E(t-\varepsilon)}, \text{ if } t \geq \varepsilon, \end{cases}$$

where $\Phi_E(t) = t\rho^2(1 - e^{-2kt})/(2k), \Psi_E(t) = (\sigma^2 - \rho^2)(1 - e^{-kt})^2/k^2$.

In Section 3.2, we study a differential pursuit problem with Langenhop–Geometric constraints. In this case, the motions of the objects *P* and *E* are described by the motion equations (13). The control function of the evader $v(\cdot): [0, +\infty) \to \mathbb{R}^d$ satisfies the following geometric constraint (briefly, *G*- constraint)

$$|v(t)| \leq \beta, \quad t \geq 0, \beta \geq 0. \tag{16}$$

For brevity, we call the pursuit–evasion game governed by the motion equations (13), where the pursuer's control is subject to the Langenhop-type constraint (14) and the evader's control satisfies the geometric constraint (16), the *LaG-game* (*LaG* constraints game).

Definition 7. In the *LaG-game*, we call the function

$$\boldsymbol{u}_{LaG}(t, v) = v - \lambda_{LaG}(t, v)\xi_0, \tag{17}$$

the Π_{LaG}-strategy of the pursuer on the time interval $[0,\ \mathrm{T}_{LaG}]$, where

$$\mathrm{T}_{LaG} = \frac{1}{k} ln\frac{\rho}{\beta}, \text{ for } \rho > \beta > 0, \tag{18}$$

$$\lambda_{LaG}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \rho^2 e^{-2kt} - |v|^2}, \ \xi_0 = z_0/|z_0|, \tag{19}$$

and $\langle v, \xi_0 \rangle$ indicates the inner product of the vectors v and ξ_0 in $\mathbb{R}^d$. The time T_{LaG} is referred to as the guaranteed time of capture.

Proposition 1. For all $(t, v) \in [0,\ \mathrm{T}_{LaG}] \times S_\beta$, the function $\lambda_{LaG}(t, v)$ (19) is defined, continuous and bounded as

$$\rho e^{-kt} - \beta \leq \lambda_{LaG}(t, v) \leq \rho e^{-kt} + \beta.$$

Proposition 2. For all $(t, v) \in [0,\ \mathrm{T}_{LaG}] \times S_\beta$, the following identity holds

$$|\boldsymbol{u}_{LaG}(t, v)| = \rho e^{-kt}.$$

Lemma 6. Let the motion of the players be governed by the system

$$\dot{z} = \boldsymbol{u}_{\boldsymbol{LaG}}(t, v(t)) - v(t), \quad z(0) = z_0,$$

and the pursuer applies Π_{LaG}-strategy (17). Then for all $t \in [0,\ \mathrm{T}_{LaG}]$, the scalar function

$$\Lambda_{LaG}\big(t, v(\cdot)\big) = 1 - \frac{1}{|z_0|}\int_0^t \lambda_{LaG}\big(s, v(s)\big)ds$$

satisfies the estimate

$$\Upsilon_{LaG}(t) = 1 - \frac{1}{|z_0|}\Big(\frac{\rho}{k}(1-e^{-kt}) + \beta t\Big) \le \Lambda_{LaG}\big(t, v(\cdot)\big) \le$$
$$\le 1 - \frac{1}{|z_0|}\Big(\frac{\rho}{k}(1-e^{-kt}) - \beta t\Big) = \Lambda_{LaG}(t).$$

Lemma 7. Let $\rho > \beta$ and $|z_0| \le \frac{1}{k}\left(\rho - \beta\left(1 + ln\frac{\rho}{\beta}\right)\right)$ be satisfied. Then the equation $\Lambda_{LaG}(t) = 0$ has a unique positive root on the interval $[0,\ \mathrm{T}_{LaG}]$.

Theorem 8. If the conditions of Lemma 7 are satisfied, then the pursuer, applying the Π_{LaG}-strategy (17), captures the evader no later than time T_{LaG} (18).

In Section 3.3. The discrete pursuit problem with La-constraints is studied. In the discrete game, the following sets of admissible controls for the pursuer are defined:

Geometric constraint (briefly, *G*-constraint)

$$\mathbf{U}_G^n = \{\{u_i\}_{i=1}^n : |u_i|^2 \le (1-k)^{i-1}\rho_0^2, i = 1,2,\dots,n\};$$

Langenhop-type constraint (briefly, La-constraint)

$$\mathbf{U}_{La}^n = \{\ \{u_i\}_{i=1}^n:\quad |u_1|^2 \le \rho_0^2, |u_i|^2 \le \rho_0^2 - k(|u_1|^2 + |u_2|^2 + \dots + |u_{i-1}|^2),\quad i = 2,3,\dots,n\}; \tag{20}$$

Summary constraint (briefly, S-constraint)

$$\mathbf{U}_S^n = \left\{\{u_i\}_{i=1}^n : \sum_{i=1}^n |u_i|^2 \le \frac{\rho_0^2}{k}[1-(1-k)^n]\right\}.$$

(Analogously, by replacing ρ_0 with σ_0, the sets $\mathbf{V}_G^n, \mathbf{V}_L^n, \mathbf{V}_S^n$ are also defined.) Based on the established relations between the sets of admissible control functions under *La*-, *G*-, and *S*-constraints, we introduce three admissible controls for both the pursuer and the evader.

For simplicity, we call the discrete pursuit–evasion game governed by recurrence equations (1) or (4) under the *La*-constraint (20) with parameters $0 < k < 1, \rho_0 > 0, \sigma_0 \ge 0$ the *La*-game (*La*-constraint game).

Lemma 8. If $0 < k < 1$ then $\mathbf{U}_G^n \subset \mathbf{U}_{La}^n \subset \mathbf{U}_S^n$, $\mathbf{V}_G^n \subset \mathbf{V}_L^n \subset \mathbf{V}_S^n$ are satisfied for all $n = 1,2,\dots$.

Definition 8. We call the sequence

$$\boldsymbol{u}_n = \begin{cases} \boldsymbol{u}_n^*, & \text{if } |z_{n-1}| > \ell(v_n), \\ \boldsymbol{u}_n^{**}, & \text{if } |z_{n-1}| \le \ell(v_n), \end{cases} \tag{21}$$

the *Π_{La}-strategy* in the DPP for the *La*-game, where

$$\boldsymbol{u}_n^* = v_n - \ell(v_n)\xi_0,\ \boldsymbol{u}_n^{**} = v_n - z_{n-1},$$
$$\ell(v_n) = \langle v_n, \xi_0\rangle + \sqrt{\langle v_n, \xi_0\rangle^2 + (1-k)^{n-1}\delta_0},$$
$$\xi_0 = \frac{z_0}{|z_0|}, z_0 = x_0 - y_0, \delta_0 = \rho_0^2 - \sigma_0^2,$$

and $\langle v_n, \xi_0\rangle$ means the inner product of the vectors v_n and ξ_0 in $\mathbb{R}^d, n = 1,2,\dots$. Further, we will call sequence $\boldsymbol{u}_n^*$ *the approach strategy*, and sequence $\boldsymbol{u}_n^{**}$ *the final* (or *finishing*) *strategy*.

Lemma 9. If at some step n the inequality $|z_{n-1}| > \ell(v_n)$ is satisfied, then at step n the equality $|\boldsymbol{u}_n^*|^2 = |v_n|^2 + (1-k)^{n-1}\delta_0$ hold.

Definition 9. In the DPP for the *La*-game we call sequences:

$$\rho_{n-1}^2 = \rho_0^2 - k\sum_{i=1}^{n-1} |\boldsymbol{u}_i|^2, \qquad \sigma_{n-1}^2 = \sigma_0^2 - k\sum_{i=1}^{n-1} |v_i|^2, n = 2,3,\ldots,$$

the current resources of objects P and E, respectively, and sequence

$$\delta_{n-1} = \rho_{n-1}^2 - \sigma_{n-1}^2 = \delta_0 - k\sum_{i=1}^{n-1} (|\boldsymbol{u}_i|^2 - |v_i|^2), n = 2,3,\ldots,$$

the difference in the current resource of the players.

Lemma 10. If inequality $|z_{n-1}| > \ell(v_n)$ is satisfied before some step n, then equality $\delta_{j-1} = (1-k)^{j-1}\delta_0,\ j = 1,2,\ldots,n$ is true before this step n.

Theorem 9. Let in the DPP for the *La*-game $\rho > \sigma, 0 < k < 1$ and the pursuer implements the Π_{La}-strategy (21). Then:

1) If for steps $j = 1,2,\ldots,n$ inequality $|z_{j-1}| > \ell(v_j)$ holds, then for all steps $j = 1,2,\ldots,n$ we have $z_j \neq 0$ and $z_j = z_0 h_j,\ h_j > 0,\ \delta_{j-1} = (1-k)^{j-1}\delta_0$ holds, where

$$h_j = 1 - \frac{1}{|z_0|}\sum_{i=1}^{j} \ell(v_i), j = 1,2,\ldots,n;$$

2) If inequality $|z_{n-1}| \leq \ell(v_n)$ is satisfied in step n, then in step n the evader is captured, i.e. $z_n = 0$.

Assumption 1. Let in the DPP for the *La*-game $\rho > \sigma, 0 < k < 1$ and there exists the smallest natural number $n = N_{La}(z_0)$ for which the inequality

$$|z_0| \leq \sqrt{p_n^2 + \delta_0 q_n^2} - p_n$$

is satisfied, where

$$p_n^2 = \frac{\sigma_0^2 n}{k}[1-(1-k)^n] \text{ and } q_n = \frac{1-(1-k)^{\frac{n}{2}}}{1-\sqrt{1-k}}.$$

Then we will call the number $N_{La}(z_0)$ the step that guarantees the completion of the pursuit.

Theorem 10. Let Assumption 1 be satisfied in the DPP for the *La*-game. Then the pursuer implementing Π_{La}-strategy (21), completes the pursuit no later than at step $N_{La}(z_0)$.

Remark 1. If $k \to 0$, then $p_n^2 \to \sigma_0^2 n^2, q_n^2 \to n^2$. Therefore, from main theorem, Jensen's inequality** we have $|z_0| \leq n(\rho_0 - \sigma_0)$. Thus, the guaranteed step of completion of the pursuit in the case $k = 0$ is equal to

$$N_{La}(z_0) = \begin{cases} \left[\dfrac{|z_0|}{\rho-\sigma}\right] + 1, & \text{if } \left\{\dfrac{|z_0|}{\rho-\sigma}\right\} > 0, \\ \left[\dfrac{|z_0|}{\rho-\sigma}\right], & \text{if } \left\{\dfrac{|z_0|}{\rho-\sigma}\right\} = 0, \end{cases}$$

**Khardi G., Littlvud Dzh., Polya G. Neravenstva [Inequalities] Moscow: IL, 1948. (in Russian) where $[a]$ is the integer part of the number a, and $\{a\}$ is the fractional part of the number a.

CONCLUSIONS

This dissertation investigated pursuit and evasion problems in dynamic games under various types of constraints on control functions.

1. For a discrete game with multiple pursuers and one evader, where the control functions are subject to geometric constraints, sufficient conditions guaranteeing pursuit have been established based on the parallel approach method (Π-strategy) applied by the pursuers, and a 2D animation model of the game has been developed;
2. For a discrete game with simple motion in which the control functions are subject to summary constraints, sufficient conditions guaranteeing capture have been obtained using the parallel approach method (Π-strategy) applied by the pursuer;
3. For differential game with Langenhop-type constraints on the control functions, in the evasion problem, sufficient conditions have been determined under which, using the evader's delayed control, the lower bound of the distance between the players is found;
4. Based on the solution of the pursuit problem for a discrete game with Langenhop-type constraints on the control functions, obtained using the Π-strategy, a 2D animation model of the game has been developed.

НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.05.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ФЕРГАНСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ

НАМАНГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

УМАРАЛИЕВА НАРГИЗА ТАШКИНБАЕВНА

ДИНАМИЧЕСКИЕ ИГРЫ С РАЗЛИЧНЫМИ ОГРАНИЧЕНИЯМИ НА УПРАВЛЯЮЩИЕ ФУНКЦИИ И ИХ АНИМАЦИОННЫЕ МОДЕЛИ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ
диссертации доктора философии (PhD)
по ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Фергана – 2025

Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан за №B2022.4.PhD/FM194.

Диссертация выполнена в Наманганском государственном университете.

Автореферат диссертации на трёх языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.fdu.uz) и на Информационно-образовательном портале «ZiyoNET» (www.ziyonet.uz).

Научные руководители:	**Саматов Бахром Таджиахматович** доктор физико-математических наук, профессор
Официальные оппоненты:	**Мамадалиев Нуманжан** доктор физико-математических наук, профессор
	Ибайдуллаев Туланбой Турсунбоевич кандидат физико-математических наук, профессор
Ведущая организация:	**Термезский государственный университет**

Защита диссертации состоится « 1 » 11 2025 года в 10 00 часов на заседании Научного совета PhD.03/30.12.2019.FM.05.04 при Ферганском государственном университете. (Адрес: 150100, г. Фергана, ул. Мураббийлар, 19. Тел.: (+99873) 244-44-02, факс: (+99873) 244-44-93, e-mail: fardu_info@umail.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Ферганского государственного университета (зарегистрирована за № 590). (Адрес: 150100, г. Фергана, ул. Мураббийлар, 19. Тел.: (+99873) 244-44-94).

Автореферат диссертации разослан « 20 » 10 2025 года.
(протокол рассылки № 10 от « 17 » 10 2025 года).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI FARG'ONA DAVLAT UNIVERSITETI

Ш.Т.Каримов
Заместитель председателя научного совета по присуждению ученых степеней, д.ф.-м.н., доцент

И.У.Хайдаров
Ученый секретарь научного совета по присуждению ученых степеней, к.ф.-м.н., доцент

Ю.П.Апаков
Заместитель председателя научного семинара при научном совете по присуждению ученых степеней, д.ф.-м.н., профессор

Целью исследования является решение динамических задач преследования и убегания при различных типах ограничений на управляющие функции, таких как геометрические, суммарные и ограничения типа Лангенхопа.

Объектом исследования являются динамические игры с различными ограничениями на управляющие функции.

Научная новизна исследования заключается в следующем:

для дискретной игры с несколькими преследователями и одним убегающим, в которой функции управления имеют геометрические ограничения, найдены достаточные условия гарантированного преследования на основе метода параллельного сближения (П-стратегии), применяемого преследователями, и разработана 2D анимационная модель игры;

для дискретной игры с простым движением, где функции управления имеют суммарные ограничения, найдены достаточные условия, гарантирующие поимку, на основе применения преследователем метода параллельного сближения (П-стратегии);

в дифференциальной игре с ограничениями типа Лангенхопа на управляющие функции задача уклонения решена с использованием запаздывающего управления убегающего, при этом в найденных достаточных условиях определена нижняя граница расстояния между игроками;

на основе решения задачи преследования для дискретной игры с ограничениями Лангенхопа на функции управления, найденного с применением П-стратегии, разработана 2D анимационная модель игры.

Внедрение результатов исследования. На основе полученных результатов по динамическим играм с различными ограничениями на функции управления и их анимационным моделям:

Анимационная модель, разработанная для дискретной игры с несколькими преследователями и одним убегающим при геометрических ограничениях на управляющие функции, была использована в прикладном проекте № FZ-201905171 «Разработка гидродинамических моделей и эффективных алгоритмов для исследования аномальных процессов фильтрации жидкостей и газов в пористых средах». Она применялась для создания компьютерной модели фильтрации жидкости при комплексном исследовании газовых месторождений, ограниченных водоносными пластами (согласно справке № 2/272 Института математики им. В.И. Романовского Академии наук Республики Узбекистан от 25 июня 2025 года). В результате стало возможным решение задач фильтрации в пористой среде, основанных на методах разделения по координатам и дифференциального управления, а также разработка их численных алгоритмов;

При решении задач динамических игр с геометрическими и ограничениями типа Лангенхопа на управляющие функции были использованы гарантированные условия, основанные на оптимальных стратегиях сближения (П-стратегиях), а также разработанные анимационные модели, визуализирующие процесс захвата, в рамках фундаментального проекта OT-F4-33 «Разработка новых методов конфликтного управления, описываемого дифференциальными уравнениями, и их численная реализация» (согласно справке № 04/11-7621 Национального университета Узбекистана от 18 июня 2025 года). В результате стало возможным разработать новые методы для решения задач управления конфликтными процессами, описанными дискретными уравнениями с ограничениями типа Лангенхопа и геометрическими ограничениями, численно их решать и практически подтверждать правильность полученных результатов.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo'lim (part I; часть I)

1. Umaraliyeva N.T., Soyibboyev U.B., Uralova S.I. Evasion problem for the second order differential game // Scientific Bulletin of Namangan State University. – 2019. – No. 4. – P. 8–13. (01.00.00; No. 14).
2. Samatov B.T., Umaraliyeva N.T., Uralova S.I. Differential games with the Langenhop-type constraints on controls // Lobachevskii Journal of Mathematics. – 2021. – Vol. 42, Issue 12. – P. 2942–2951. (3. Scopus, IF: 0.435).
3. Samatov B.T., Umaraliyeva N.T., Soyibboyev U.B., Uralova S.I. The pursuit problem in a differential game with Langenhop–geometric constraints on controls // Scientific Bulletin of Namangan State University. – 2022. – No. 11. – P. 20–25. (01.00.00; No. 14).
4. Samatov B.T., Umaraliyeva N.T. The Π-strategy and its animation model in plane // AIP Conference Proceedings. – 2024. – Vol. 3244. – Article 020026. (3. Scopus. IF: 0,41)
5. Умаралиева Н.Т. Алгоритм П-Стратегии в дискретных играх преследования с суммарными ограничениями на плоскости // Научный вестник Наманганского государственного университета. – 2025. – №2. – С. 9–14. (01.00.00; № 14).
6. Umaraliyeva N.T. Discrete game with summary constraints on controls and its animation model // Scientific Bulletin. Physical and Mathematical Research. – Andijan, 2025. – Vol. 7, Issue 2. – P. 45–55. (01.00.00; No. 13).
7. Samatov B.T., Umaraliyeva N.T. Discrete game with Langenhop-type constraints on controls and its animation model // Bulletin of the Institute of Mathematics. – 2025. – Vol. 8, No. 4. – P. 104–120. (01.00.00; No. 17).

II bo'lim (part II; часть II)

8. Inomiddinov S.N., Umaraliyeva N.T., Umarov E.T. The Pursuit Problem with Gronwall Constraint for the Evader // International Conference "Control, optimization and dynamic systems". – Andijan, Uzbekistan, October 17–19, 2019. – P. 30–31.
9. Samatov B.T., Inomiddinov S.N., Umaraliyeva N.T., Uralova S.I. Differential games of the second order with integral constraints // International scientific conference "Nonclassical equations of mathematical physics and their applications" (Uzbekistan–Russia). – Tashkent, October 24–26, 2019. – P. 180–181.
10. Soyibboyev U.B., Umaraliyeva N.T., Uralova S.I. Tezlanishli harakat uchun qochish masalasi // "Matematika va informatikaning zamonaviy muammolari" mavzusidagi Respublika ilmiy anjumani to'plami. – Farg'ona, 22–23-may, 2019-yil. – B. 48–49.

11. Samatov B.T., Inomiddinov S.N., Umaraliyeva N.T. The second order differential games for integral constraints // Inverse and Ill-Posed Problems, The International Conference. – Samarkand, Uzbekistan, October 2–4, 2019. – P. 28–29.
12. Inomiddinov S.N., Umaraliyeva N.T., Uralova S.I. The relation between Gronwall, geometric and integral constraints on control parameters of players // "Matematika, fizika va axborot texnologiyalarining dolzarb muammolari" mavzusidagi Respublika ilmiy anjumani to'plami. – Buxoro, 15-aprel, 2020-yil. – B. 163-164.
13. Samatov B.T., Umaraliyeva N.T., Inomiddinov S.N., Uralova S.I. Differential games of the second order with integral constraints // Scientific Bulletin of Namangan State University. – 2020. – No. 3. – P. 7–14.
14. Samatov B.T., Umaraliyeva N.T., Inomiddinov S.N., Jamalov O.O. Qarama-qarshi maqsadli dinamik ob'yektlarning to'qnashish masalasi uchun animatsion model // O'zbekiston Respublikasi Adliya Vazirligi huzuridagi intellektual mulk agentligi. – EHM uchun dastur. – 2021. – № DGU 11150.
15. Soyibboyev U.B., Umaraliyeva N.T., Zoxidova Z.S. On pursuit problem in the third order differential game with geometric constraints // Scientific Bulletin of Namangan State University. – 2022. – Special Issue. – P. 33–38.
16. Samatov B.T., Uralova S.I., Umaraliyeva N.T. Linear evasion differential game with Langenhop constraints // "Yosh matematiklarning yangi teoremalari-2022" mavzusidagi Respublika ilmiy anjumani to'plami. – Namangan, 13–14-may, 2022-yil. – B. 190–191.
17. Samatov B.T., Uralova S.I., Umaraliyeva N.T. Linear differential games with La-constraints on controls // Международная научная конференция "Неклассические уравнения математической физики и их приложения". – Ташкент, 6–8 октября 2022 г. – С. 50–51.
18. Uralova S.I., Umaraliyeva N.T. On some pursuit-evasion problems with La-constraints // "Modern problems of analysis" materials of the republican scientific conference. – Karshi, Uzbekistan, June 2–3, 2023. – P. 200–202.
19. Samatov B.T., Umaraliyeva N.T., Xolmirzayev X.E., Soyibboyev U.B. Boshqaruvi geometrik chegaralanishli ko'p quvlovchili tutish masalasining animatsion modeli // O'zbekiston Respublikasi Adliya Vazirligi huzuridagi intellektual mulk agentligi. – EHM uchun dastur. – 2023. – № DGU 27302.
20. Samatov B.T., Umaraliyeva N.T., Xolmirzayev X.E., Abdurahmanov A.Sh. Geometrik chegaralanishli quvish-qochish masalasiga animatsion model // O'zbekiston Respublikasi Adliya Vazirligi huzuridagi intellektual mulk agentligi. – EHM uchun dastur. – 2023. – № DGU 27303.
21. Samatov B.T., Umaraliyeva N.T., Abduraxmanov A.Sh. Boshqaruvlari integral chegaralanishli quvish diskret o'yinida П-strategiyaning tadbiqi // O'zbekiston Respublikasi Toshkent shahar ilg'or tadqiqotlar ilmiy markazi xalqaro ilmiy forum. – Toshkent, 13-yanvar, 2023-yil. – B. 808–810.
22. Umaraliyeva N.T. П-Strategy in a Discrete Pursuit Games // "Fizikaning zamonaviy muammolari va rivojlanish istiqbollari" I xalqaro ilmiy-amaliy konferensiya. – Namangan, October 22–23, 2024. – P. 935–936.

23. Samatov B.T., Umaraliyeva N.T. Π-Strategy in a Discrete Game with Multiple Pursuers // “Fizikaning zamonaviy muammolari va rivojlanish istiqbollari” I xalqaro ilmiy-amaliy konferensiya. – Namangan, October 22–23, 2024. – P. 937–938.
24. Samatov B.T., Umaraliyeva N.T. A Discrete Pursuit Problem with Total Constraints on Controls // X All-Russian Scientific and Practical Conference “Modern Problems of Physical and Mathematical Sciences Topics”. – Oryol, Russia, November 29–30, 2024. – P. 210–213.
25. Samatov B.T., Umaraliyeva N.T. A Discrete Pursuit Problem with Langenhop type Constraints on Controls // VII International Conference “Topological Methods in Dynamics and Related Topics”. – Nizhny Novgorod, Russia, December 13–15, 2024. – P. 83–84.
26. Samatov B.T., Umaraliyeva N.T. Π-Strategy Algorithm in Discrete Pursuit Game with Total Constraints // Proceedings of the All-Russian Conference with International Participation “Control Theory and Mathematical Modeling”, dedicated to the memory of Professors N.V. Azbelev and E.L. Tonkov. – Izhevsk, Russia, June 16–20, 2025. – Part 2. – P. 21–25.

Avtoreferat Farg‘ona davlat universiteti «FarDU. Ilmiy xabarlar – Научный вестник. ФерГУ» ilmiy – metodik jurnal tahririyatida tahrirdan o‘tkazilib, o‘zbek, rus va ingliz tillaridagi matnlar o‘zaro muvofiqlashtirildi.

Bosishga ruxsat etildi: 17.10.2025-yil
Bichimi 60x84 $^{1}/_{16}$, «Times New Roman» garniturasi.
Shartli bosma tabog‘i 2,75 Adadi: 100.

Namangan shahar Hamroh ko‘chasi 71^{A} uy.
“Yashin sanoat” bosmaxonasida chop etildi.