SAMARQAND DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/30.12.2019.FM.02.01 RAQAMLI ILMIY KENGASH

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

IKROMOVA DILDORA ISROILOVNA

SODDA MAXSUSLIKLAR BILAN BOG‘LANGAN EKSTREMAL MASALALAR

01.01.01–Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Samarqand – 2025

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Contents of dissertation abstract of the doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda olib borilayotgan koʻplab ilmiy va amaliy tadqiqotlarda, xususan, matematik fizika va spektral analizning koʻpgina muammolarini yechishda katta va kichik parametrlarga ega boʻlgan tebranuvchan integrallarni baholash masalalarini tadqiq qilishga alohida ahamiyat berilmoqda. Hozirgi kunda rivojlangan mamlakatlarda qat'iy giperbolik tenglamalar uchun Koshi masalasining yechimlariga dastlabki (a-prior) baholar olish masalasi tebranuvchan yadroli oʻrama operatorlarining $L^p(\mathbb{R}^n)$ kabi Lebeg fazolarida chegaralanganlik muammosi bilan bogʻliq. Matematika va nazariy fizikada, shuningdek, Furye analizi va garmonik analizda koʻplab modellar differensial tenglamalar va ulardanda umumiyroq boʻlgan psevdo-differensial tenglamalar yordamida ifodalanadi. Shuning uchun garmonik analizga oid tadqiqotlarni rivojlantirish matematika sohasida muhim ahamiyatga ega boʻlmoqda.

Jahonda garmonik analiz va uning matematik fizika hamda Furye optikaga tatbiqlari bilan bogʻliq ilmiy tadqiqotlar keng miqyosda olib borilmoqda. Ma'lumki, differensial tenglamalarning silliq funksiyalar aniqligidagi taqsimotlar fazosidan boʻlgan umumiy yechimini topish qiyin muammo hisoblanadi. Shubhasiz, yechimning maxsusligi unga silliq funksiyani qoʻshish bilan oʻzgarmaydi. Shuning uchun koʻp hollarda differensial tenglamalarning silliq funksiyalar aniqligidagi taqsimot yechimlarini tuzishga oid tadqiqotlar muhim oʻrin tutmoqda. Bu borada amaliyotda umumiy taqsimot ma'nosidagi yechimdan koʻra kuchsiz yechimlar keng qoʻllaniladi. Integrallanuvchi funksiyalar fazosiga asoslangan differensial tenglamalar yechimlarini tahlil qilish uchun koʻplab tadqiqotlar olib borilmoqda. Masalan, qat'iy giperbolik tenglamalar, shuningdek, diskret tenglamalar yechimlari koʻplab ishlarda oʻrganilgan. Boshlangʻich shartlarga bogʻliq boʻlgan va dispersiv baholar deb ataluvchi yechimlar uchun a-priori (dastlabki) baholar matematik fizika sohasida koʻplab tatbiqlarga ega boʻlgani bois, bunday tadqiqotlarni rivojlantirish dolzarb vazifalardan biri hisoblanmoqda.

Respublikamizda amaliy va fundamental fanlar orqali ham nazariy, ham amaliy ahamiyatga ega boʻlgan tadqiqotlar boʻyicha keng qamrovli izlanishlar olib borilmoqda. Furye analizi va uning operatorlarining spektral yoyilmalari hamda matematik fizika, amaliy matematika sohalaridagi tatbiqlari ustida muhim ilmiy tadqiqotlarga katta e'tibor qaratilmoqda. "Algebra va uning tatbiqlari, differensial tenglamalar va ularning tatbiqlari, chiziqsiz tizimlar, dinamik tizimlar va ularning tatbiqlari, matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi"[1] fanlarning ustuvor yoʻnalishlari boʻyicha xalqaro standartlar darajasida tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yoʻnalishlari etib belgilangan. Ushbu vazifani amalga oshirilishini ta'minlash uchun tebranuvchan integrallarning nazariy tushunishni shakllantirish

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[1] Oʻzbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi PQ-4387-son "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek, Oʻzbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida" gi qarori.

juda muhimdir, sirt o‘lchovlari Furye almashtirishi, shunindek, tebranuvchan yadroli o‘rama bilan aniqlangan Furyening integral operatorlari uchun baholar olish muhim ilmiy ahamiyatga ega hisoblanadi.

O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-sonli “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari, 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi va 2023-yil 11-sentyabrdagi PF-158-son “O‘zbekiston-2030” strategiyasi to‘g‘risidagi Farmonlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Bizga ma’lumki qat’iy giperbolik tenglamalar uchun Koshi masalasining yechimi o‘rama operator ko‘rinishida beriladi. O‘rama operatorning yadrosi sirtda mujassamlashgan o‘lchovlar yoki silliq gipersirtlarda mujassamlashgan zaryadlar Furye almashtirishi bilan berilgan tebranuvchan integral orqali aniqlanadi. Agar gipersirtning balandligi ikkidan qat’iy kichik bo‘lsa, shu gipersirt bilan bog‘liq bo‘lgan faza funksiyasi, faqat Arnoldning sodda maxsusliklariga egadir.

Boshqa tomondan, sirtlarda mujassamlangan zaryadlar Furye almashtirishining integrallanish ko‘rsatkichining aniq qiymatini topish masalasi matematikaning ayrim sohalari uchun muhimdir. I.M.Vinogradov, Xya-Lo-Gen, A.A.Karatsuba va boshqalar tomonidan ishlab chiqilgan trigonometrik yig‘indilarni baholash usuli uchun bu masalani yechish hal qiluvchi ahamiyatga ega. Integrallanuvchilik muammosini o‘rganish uchun yana bir turtki uning biror daraja bilan yig‘iluvchi funksiyalar Furye almashtirishini o‘lchovi nolga teng bo‘lgan to‘plamlarga cheklash (toraytirish) bilan bog‘liqligidir. Furye almashtirishini silliq sirtlarga cheklash bo‘yicha ba’zi natijalar ma’lum. Ushbu natijalar bilan batafsilroq tanishish uchun Steinning klassik monografiyasiga murojaat qilish maqsadga muvofiqdir.

Biroq, bizning bu masalaga qiziqishimiz matematik fizikaning boshqa muammosidan kelib chiqadi. Aniqroq aytganda, bizni giperbolik tenglamalarning xarakteristik sirtlari bilan bog‘liq bo‘lgan o‘rama operatorlari yoki Furyening integral operatorlari uchun baholar bilan bog‘liq muammolar qiziqtiradi. Bundan tashqari, funksiyalarni manfiy daraja bilan integrallash masalasi umumlashgan Fridrixs modelining spektri haqidagi muammo bilan bog‘liq. Bu mulohazalar matematik fizika va garmonik analiz kabi zamonaviy sohalarda muhim ahamiyatga ega va bu yuqorida muhokama qilingan masalalarning ahamiyatini va dolzarbligini tushuntiradi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim yoki ilmiy tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiya tadqiqoti Samarqand davlat universitetida OT-F4-69- "Garmonik analiz, darajali geometriya va ularning matematik fizika masalalarini yechishga tadbiqlari" (2017-2021) mavzusidagi ilmiy tadqiqot loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi. Sirtlarda mujassamlashgan o'lchovlar Furye almashtirishini cheksizdagi xarakterini tadqiq qilish va qat'iy giperbolik tenglamalarga qo'yilgan Koshi masalasi yechimlari uchun a-prior baholar olish. Ya'ni bu tenglamalar bilan bog'langan o'rama operatori chegaralangan bo'ladigan kritik ko'rsatkich qiymatini topish. Bundan tashqari, sirt o'lchovlarining Furye almashtirishining biror daraja bilan integrallanishi masalasini ko'rib chiqishdan iborat.

Tadqiqotning vazifalari:

E_7 va E_8 tipidagi maxsusliklarga ega bo'lgan funksiyalar grafiklari sifatida berilgan gipersirtlarda mujassamlashgan o'lchovlarning Furye almashtirishining baholarini aniqlash;

D tipidagi maxsusliklarga ega bo'lgan sirtlar bilan bog'langan o'lchovlar orqali aniqlangan Randol tipidagi maksimal funksiyalarning integrallanish ko'rsatkichini topish va uni aniqligini isbotlash;

E_7 va E_8 tipidagi maxsusliklarga ega bo'lgan funksiyalar grafigi shaklida berilgan sirtlarda mujassamlashgan o'lchovlar bilan bog'liq Rendol tipidagi maksimal funksiyalarning integrallanish ko'rsatkichini topish va integrallanish ko'rsatkichining aniqligini isbotlash;

Qat'iy giperbolik tenglamalar uchun qo'yilgan Koshi masalasining yechimi bilan bog'langan o'rama operatorlari chegaralangan bo'ladigan kritik ko'rsatkichni aniqlash.

Tadqiqotning ob'yekti. Xarakteristik gipersirt, gipersirtlarda mujassamlashgan o'lchovlar, sirt o'lchovlarining Furye almashtirishi, o'rama operatorlaridan iborat.

Tadqiqotning predmeti. O'lchov Furye almashtirishining cheksizdagi xarakterini o'rganish va tebranuvchan integrallar bilan bog'liq bo'lgan Rendol maksimal funksiyasining xarakterini, shuningdek, qat'iy giperbolik tenglamalar uchun Koshi masalasining yechimlari bilan bog'langan o'rama operatori uchun a-prior baholarni tadqiq qilishdan iborat.

Tadqiqotning usullari. Tadqiqotda matematik analiz, kompleks analiz, funktsional analiz, matematik fizika, tebranuvchan integrallar nazariyasi, differensiallanuvchi funksiyalarning maxsusliklari nazariyasi, darajali geometriya usullari, shu jumladan Nyuton ko'pyoqliklari usullari, differensial geometriya usullari qo'llaniladi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat**:**

E_7 va E_8 tipidagi maxsusliklarga ega boʻlgan funksiyalar grafiklari sifatida berilgan gipersirtlarda mujassamlashgan oʻlchovlarning Furye almashtirishining yangi baholari topilgan va bu baholarning aniqligi koʻrsatilgan;

D tipidagi maxsusliklarga ega boʻlgan sirtlar bilan bogʻlangan oʻlchovlar orqali aniqlangan Rendol tipidagi maksimal funksiyalarning integrallanish koʻrsatkichi topilgan va uni aniqligi hamda bahoning D tipidagi maxsuslik karraligiga bogʻliq emasligi isbotlangan;

E_7 va E_8 tipidagi maxsusliklarga ega boʻlgan sirtlarda mujassamlashgan oʻlchovlar bilan bogʻliq Rendol tipidagi maksimal funksiyalarning integrallanish koʻrsatkichi topilgan va integrallanish koʻrsatkichining aniqligi isbotlangan;

qat’iy giperbolik tenglamalar uchun qoʻyilgan Koshi masalasining yechimi bilan bogʻlangan oʻrama operatorlari chegaralangan boʻladigan kritik koʻrsatkichining aniq qiymati topilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

tadqiqot fundamental xarakterga ega. Lekin sirtlarda mujassamlashgan oʻlchovlar Furye almashtirishining xarakterini topish masalasi sonlar nazariyasidagi gomotetik sohalarda butun sonlar sonini topish masalasida, hisoblash matematikasida kubatur formulalarda qoldiq hadni baholash masalalarida qoʻllanilgan;

integral operatorlarining chegaralanganligi haqidagi natijalar yordamida matematik fizika tenglamalarining psevdo-differensial operatorlar bilan bogʻliq umumlashgan toʻlqin tenglamalari yechimlari baholangan.

Tadqiqot ishonchliligi. Matematik analiz, matematik fizika, funksional analiz, kompleks analiz, differensial geometriya hamda qat’iy matematik mantiq usullarni qoʻllanilishi bilan asoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, ulardan matematik fizikaning differensial tenglamalari, xususan, umumlashgan toʻlqin tenglamasi, qat’iy giperbolik tenglamalar uchun Koshi masalasi va diskret Shryodinger, diskret Klayn-Gordon tenglamalari yechimlarini baholashda foydalanilishi mumkinligi bilan izohlanadi.

Tadqiqot ishining amaliy ahamiyati fizik jarayonlarni tavsiflovchi modellar bilan bogʻlangan matematik fizika tenglamalarini sonli yechishda nazariy asos sifatida qoʻllanilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Sodda maxsusliklar bilan bogʻlangan ekstremal masalalarga oid natijalar asosida:

gipersirtlardagi oʻlchovlar Furye almashtirishlarining aniq baholariga oid natijalar Qozogʻiston respublikasi Xoja Ahmad Yassaviy nomidagi xalqaro qozoq-turk universitetining № AP09259074 sonli “Kasr tartibli differensial tenglamalar yechimlarini qurish usullari, boshlangʻich va chegaraviy masalalarning yechilish usullari” mavzusidagi loyihada Giperbolik tenglamalar uchun Koshi masalasini yechishda qoʻllanilgan (Qozogʻiston respublikasi Xoja Ahmad Yassaviy nomidagi xalqaro qozoq-turk universitetining 2024 yil 7 noyabr, 03/3108 raqamli ma’lumotnomasi). Olingan natijalar yuqori tartibli qat’iy giperbolik tenglamalar uchun Koshi masalasi yechimi silliqlik ko‘rsatkichining aniq bahosini olish imkonini bergan;

yadrosi tebranuvchan integral operatorlarning chegaralanganligiga oid natijalar O‘zbekiston Respublikasi Innovatsion rivojlanish vazirligining "Ikkinchi va yuqori tartibli aralash tipdagi tenglamalar uchun to‘g‘ri va teskari masalalarni o‘rganish" fundamental tadqiqot OT F4–88, 2017–2020 loyihasida yuqori tartibli giperbolik tenglama uchun chegaraviy masalalarni yechishda qo‘llanilgan (V.I.Romanovskiy nomidagi matematika instituti 2024 yil 6 noyabrdagi, 2/402 raqamli ma’lumotnoma). Olingan natijalar Koshi masalasining boshlang‘ich shartlaridan foydalangan holda yuqori tartibli giperbolik tenglamalarning yechimi uchun aniq baho olish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqot natijalari 7 ta ilmiy amaliy anjumanlarda, jumladan 2 ta xalqaro va 5 ta respublika ilmiy amaliy anjumanlarida muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 14 ta ilmiy ish chop etilgan, shulardan O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 7 ta maqola, jumladan 4 tasi Scopus va Web of Science bazasiga kiruvchi xorijiy jurnallarda va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning Hajmi 115 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqotning maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning **“Zarur ta’riflar va ma’lum natijalar”** deb nomlanuvchi birinchi bobida biz kerakli ta’riflar va tadqiqotimiz bilan bog‘liq ba’zi ma’lum natijalarni keltiramiz. Shuningdek, biz dissertasiyada foydalanilgan belgilar bilan tanishamiz. Bundan tashqari, sodda Arnol’d maxsusliklari haqidagi ba’zi natijalarning sodda isbotlarini keltiramiz. Ayniqsa, biz biror nuqtada E_7 yoki E_8 tipidagi maxsusliklarga ega bo‘lgan funksiyalardan normal shakli haqidagi tasdiqning elementar isbotini keltiramiz. Shuningdek, E_7, E_8 tipidagi maxsusliklarga ega bo‘lgan funksiyalarning chiziqli, maxsus nuqtani o‘zgartirmaydigan, almashtirishlarga nisbatan normal shaklini beramiz. Shuni ta’kidlash joizki biz foydalanadigan asosiy vositalardan biri Nyuton ko‘pyoqliklari va muofiqlashgan koordinatalar sistemasidir. Biz silliq funksiyaning, markazi maxsus nuqtada bo‘lgan, darajali qatoriga mos Nyuton ko‘pyoqligi bilan bog‘liq tushunchalarni kiritamiz va silliq funksiyaning balandligi va chiziqli balandligini aniqlaymiz.

Dissertatsiyaning “**Sodda maxsusliklar bilan bog‘liq sirt o‘lchovlarining Furye almashtirishining integrallanuvchiligi**” deb nomlanuvchi ikkinchi bobi sirt o‘lchovlari Furye almashtirishlari bilan bog‘langan Rendol maksimal funksiyalarining baholari va ularning sirt o‘lchovlari Furye almashtirishlarining biror daraja bilan integrallanishi masalasiga tadbiqlariga bag‘ishlangan.

II bobda biz sodda Arnold maxsusliklariga ega bo‘lgan silliq funksiyalar grafiklari bilan $\mathbb{R}^3$ da berilgan S silliq sirtlarida mujassamlashgan o‘lchovlar Furye almashtirishining baholarni muhokama qilamiz. Ushbu sirtlarning bosh egriliklari ma’lum bir nuqtada nolga aylanadi deb faraz qilamiz. Shuni ta’kidlash joizki, Evklid fazosidagi har qanday silliq gipersirt o‘zining har bir nuqtasining yetarli kichik atrofida (ya’ni lokal ma’noda) biror silliq funksiyaning grafigi sifatida berilishi mumkin. Bundan tashqari, agar $n \leq 6$ bo‘lsa, har qanday umumiy holatdagi silliq gipersirt faqat sodda Arnold maxsusligiga ega silliq funksiya grafigi sifatida ifodalanadi. Biroq, mos keladigan faza funksiyasi oddiy maxsusliklarning universal (bunga versal deb yuritiladi) qo‘zg‘alishi ya’ni deformatsiyasidir. Biz sodda maxsusliklarga ega bo‘lgan funksiyalarning chiziqli qo‘zg‘alishlarini (deformatsiyasini) ko‘rib chiqamiz. Arnold terminologiyasi ma’nosida bu qo‘zg‘alishlar universal emas (ya’ni versal emas). Shunday qilib, mos keladigan gipersirt umumiy holatda emas. Bunday holda, tegishli tebranuvchan integrallarning xarakaterini o‘rganish muammosi ancha nozik muammodir.

Shubhasiz, umumiy holda fazasi maxsusliklarga ega bo‘lgan ixtiyoriy funksiya bo‘lgan tebranuvchan integrallarni o‘rganish geometrik tahlilning murakkab ochiq masalasidir. Biz o‘zimizni balandligi ikkidan kichik bo‘lgan funksiyalar bilan cheklaymiz. Shunday qilib, mos keladigan funksiya faqat sodda, ya’ni A, D, E tipidagi maxsusliklarga ega bo‘ladi. Tegishli sirtning ikkala bosh egriliklari D va E tipidagi maxsusliklar holida koordinatalar boshida nolga aylanadi.

II bob natijalarining qat’iy matematik bayoniga o‘tamiz. Aytaylik, $S \subset \mathbb{R}^3$ silliq gipersirt bo‘lsin va $\varphi \in C_0^\infty(S)$ shu gipersirtda aniqlangan tashuvchisi kompakt cheksiz silliq funksiya bo‘lsin. $d\mu(X) := \varphi(X)dS$ tenglik bilan aniqlangan zaryadni qaraymiz, bunda dS, S gipersirtdagi indusirlangan Lebeg o‘lchovidir. Xususan, agar φ manfiy bo‘lmagan funksiya bo‘lsa, biz Borel sirt o‘lchoviga ega bo‘lamiz.

$d\mu$ zaryadning Furye almashtirishi quyidagi integral bilan aniqlanadi:

$$\widehat{d\mu}(\xi) = \int_S e^{iX\xi} d\mu(X),$$

bu $d\mu$ zaryad bilan aniqlangan taqsimotning Furye almashtirishiga mos keladi (bu tasdiq V. P. Palamodov monografiyasidan ko‘rinib turibdi), bunda $X\xi$, X va ξ vektorlarning skalyar ko‘paytmasi.

Biz $\widehat{d\mu}(\xi)$ ga mos keladigan Randolning maksimal funksiyasini aniqlaymiz:

$$M(\omega) = \sup_{r>0} r|\widehat{d\mu}\,(r\omega)|.$$

Bu yerda $\omega \in S^2$ (S^2 - esa markazi $\mathbb{R}^3$ fazoning koordinatalar boshida bo‘lgan birlik sfera). Agar S silliq gipersirt bo‘lsa, deyarli barcha $\omega \in S^2$ uchun $M(\omega)$ - chekli va - Borel ma’nosida o‘lchovli funksiya bo‘lishini ko‘rsatish mumkin.

E’tirof etamizki, agar S qat’iy qavariq gipersirt (yoki umumiyroq Gauss egriligi biror nuqtada ham nolga teng bo‘lmagan gipersirt) bo‘lsa, biz $\omega \in S^2$ yo‘nalishi bo‘yicha quyidagi tekis bahoga ega bo‘lamiz:

$$\left|\widehat{d\mu}(r\omega)\right| \leq Cr^{-1}.$$

Shuning uchun bu holda M chegaralangan funksiya bo‘ladi.

Ushbu bobda biz quyidagi masalani ko‘rib chiqamiz: Quyidagi to‘plamning infimumi p_S ni toping:

$$p_S = inf\ \{p \in [1, \infty)\ har\ qanday\ \varphi \in C_0^\infty(S)\ uchun\ \widehat{d\mu} \in L^p(\mathbb{R}^3)\},$$

bu yerda $L^p(\mathbb{R}^3)$ $(1 \leq p < \infty)$ p-daraja bilan $\mathbb{R}^3$ to‘plamda integrallanuvchi funksiyalar fazosi, albatta, agar $p = \infty$, bo‘lsa biz muhim chegaralangan (ya’ni muhim qiymatlar to‘plami chegaralangan) funksiyalar fazosiga ega bo‘lamiz.

p_S soniga S gipersirt ustida mujassamlashgan zaryad $d\mu$ (ishora almashinuvchi o‘lchov) Furye almashtirishining aniq integrallanish ko‘rsatkichi deb ataladi.

Umuman olganda, $p_S = \infty$ holati yuz berishi mumkin. Masalan, gipertekislikda mujassamlashgan trivial bo‘lmagan istalgan o‘lchovning Furye almashtirishi p ning har qanday chekli qiymati uchun integrallanmaydi.

Biroq, agar gipersirt E. M. Stein tomonidan kiritilgan "egrilik" sharti deb ataladigan shartni qanoatlantirsa, u holda p_S chekli son bo‘ladi, ammo bu sonning aniq qiymatini topish muammosi juda qiyin masala hisoblanadi. Shunga o‘xshash masalalar bilan I.M. Vinogradov, Xua-Lo-Gen, G. I. Arxipov, A.A. Karatsuba, V.N. Chubarikov, G. Mokenhaupt va bir qancha mashhur matematiklar shug'ullangan.

Bundan keyin, biror tayinlangan nuqtaning yetarlicha kichik atrofida, aytaylik, koordinatalar boshining yetarli kichik atrofida, gipersirt $S, x_3 = \phi(x_1, x_2)$ silliq funksiya grafigi sifatida berilgan bo'lib, bu funksiya $\phi(0) = 0$ va $\nabla\phi(0) = 0$ ya'ni $\partial_1\phi(0) = \partial_2\phi(0) = 0$ shartlarni qanoatlantirsin va ϕ koordinatalar boshida $D_{k+1}^{\pm}(3 \le k \le \infty)$ yoki $E_k(6 \le k \le 8)$ kabi sodda maxsusliklar qatoriga kiruvchi maxsusliklarga ega bo'lsin.

Ushbu II bobning asosiy natijalari quyidagi teoremada jam bo'lgandir.

2.1-teorema. *Aytaylik, $S \subset \mathbb{R}^3$, $\phi(x_1, x_2)$ funksiyasining grafigi sifatida aniqlangan silliq gipersirt bo'lsin, ya'ni,*

$$S := \{x \in \mathbb{R}^3 : x_3 = \phi(x_1, x_2)\},$$

bu yerda ϕ - quyidagi shartlarni qanoatlantiradigan silliq funksiya:
i) ixtiyoriy $\alpha_1 + \alpha_2 \le 2$ shartni qanoatlantiruvchi $(\alpha_1\alpha_2) \in \mathbb{Z}_+^2$, multiindeks uchun $\partial_1^{\alpha_1}\partial_2^{\alpha_2}\phi(0,0) = 0$ tenglik tenglik o'rinli;
ii) ϕ funksiya nol nuqtada karraligi ko'pi bilan 7 bo'lgan maxsuslikka ega.
U holda nol nuqtaning shunday U atrofi mavjud bo'lib, istalgan $\varphi \in C_0^\infty(U)$ zichlik funksiyasi uchun $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3) := \bigcap_{p>3} L^p(\mathbb{R}^3)$ munosabat o'rinli bo'ladi.

Izoh. *Shuni ta'kidlash kerakki, agar*

$$D^2\phi(0,0) := \begin{pmatrix} \partial_1^2\phi(0,0) & \partial_1\partial_2\phi(0,0) \\ \partial_2\partial_1\phi(0,0) & \partial_2^2\phi(0,0) \end{pmatrix} \neq 0$$

ya'ni, agar sirtning bosh egriliklaridan kamida bittasi $\mathbb{R}^3$ fazo koordinatalar boshida nolga aylanmasa, u holda, umuman olganda, 2.1- teoremaning tasdig'i o'rinli bo'lmaydi.

Masalan, aytaylik, $\phi(x_1, x_2) = x_1^2 + x_2^5$ bo'lsin; bu funksiya uchun kritik nuqtaning karraligi 4 ga teng. Shunda biz buni osongina ko'rsata olamizki quyidagi munosabat $\widehat{d\mu} \notin L^{22/7}(\mathbb{R}^3)$ o'rinli bo'ladi. Bu ikkala bosh egrilikning nolga aylanish shartining mohiyatini va muhimligini tushuntiradi.

Boshqa tomondan, agar kritik nuqtaning karraligi 7 dan katta bo'lsa, u holda 2.1- teoremaning tasdig'i bajarilmaydi, buni biz E_8 tipidagi maxsuslikka ega bo'lgan funksiya misolida ko'rishimiz mumkin. Aniqroq qilib aytganda, agar ϕ funksiya E_8 maxsuslikning normal shakliga ega bo'lsa, ya'ni $\phi(x_1, x_2) = x_1^3 + x_2^5$ bo'lsa, u holda $m(\phi) = 8$ va $\widehat{d\mu} \notin L^{22/7}(\mathbb{R}^3)$, ya'ni 2.1-teoremaning tasdig'i bajarilmaydi.

Shunday qilib, bizda $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$ munosabatning haqiqiyligini kafolatlaydigan kritik nuqtalarning karraligi uchun aniq baho olindi. Asosiy teoremadagi integrallanish ko'rsatkichining aniqligi quyidagi tasdiqdan kelib chiqadi.

2.4-tasdiq. *Aytaylik, $S \subset \mathbb{R}^3$ koordinatalar boshini saqlovchi C^2-silliq gipersirt bo'lsin, $d\mu$ o'lchov esa quyidagi tenglik bilan aniqlangan bo'lsin:*

$$\widehat{d\mu}(\xi) := \int_S e^{ix\xi}\varphi(x)dS,$$

va φ yetarlicha kichik tashuvchiga ega bo'lib, nomanfiy uzluksiz funksiya bo'lib va $\varphi(0) > 0$ shart bajarilsin. U holda ixtiyoriy $1 \leq p \leq 3$ son uchun $\widehat{d\mu} \notin L^p(\mathbb{R}^3)$ munosabat o'rinli bo'ladi.

Quyidagi teorema II bobning 2.4-paragrafida isbotlangan.

2.6- teorema. *Aytaylik, $S \subset \mathbb{R}^3$ silliq gipersirt quyidagi grafik sifatida berilgan bo'lsin:*

$$S = \{x \in \mathbb{R}^3 : x_3 = \phi(x_1, x_2)\}$$

bundagi silliq funksiya quyidagi shartlarni qanoatlantiradi:

(i) $\phi(0,0) = 0, \nabla\phi(0,0) = 0$*;*

(ii) ϕ funksiya $\mathbb{R}^2$ fazo koordinatalar boshida $D_{k+1}(3 \leq k < \infty)$ tipidagi maxsuslikka ega.

U holda nol nuqtaning shunday $U \subset \mathbb{R}^3$ atrofi mavjud bo'lib, ixtiyoriy $\varphi \in C_0^\infty(U)$ zichlik funksiya uchun, $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$ munosabat o'rinli bo'ladi.

Izoh. *Shunisi qiziqki, D tipdagi maxsuslik sinfining o'ziga xosligi shundan iboratki, o'lchov Furye almashtirishining aniq L^p-bahosi kritik nuqtaning karraligiga (ya'ni, k ga) bog'liq emas. Biroq, bu hodisa boshqa maxsusliklar hollarida kuzatilmaydi. Bundan tashqari, shunga o'xshash natija $R -$ sharti ostida D_∞ tipidagi cheksiz silliq funksiya maxsusliklari uchun o'rinli bo'ladi. Bunday natijalar D_∞ ga ega funksiya grafigi sifatida berilgan S analitik sirtlar uchun N.A. Soleevaning maqolasida isbotlangan. Shunday qilib, 2.6- teorema yuqorida zikr etilgan maqola natijalarini silliq D_{k+1} $(k < \infty)$ maxsusliklar uchun va $R -$ sharti ostida D_∞ silliq sirtlar holi uchun kengaytiradi.*

2.5-paragrafda quyidagi teoremani isbotlaymiz.

2.14-teorema. *Aytaylik, $S \subset \mathbb{R}^3$quyidagi shartlarni qanoatlantiruvchi $x_3 = \phi(x_1, x_2)$ silliq funksiya grafigi sifatida berilgan silliq gipersirt bo'lsin:*

(i) $\phi(0,0) = 0, \nabla\phi(0,0) = 0$*;*

(ii) ϕ funksiya $\mathbb{R}^2$ fazo koordinatalar boshida E_6 yoki E_7 tipidagi maxsusliklarga ega.

U holda nol nuqtaning shunday U atrofi mavjud bo'lib, istalgan $\varphi \in C_0^\infty(U)$ zichlik funksiya uchun $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$ tegishlilik munosabati o'rinli bo'ladi.

Asosiy 2.1-teoremaning isboti 2.6- teorema va 2.14- teoremaga asoslanadi. Dissertatsiya ishining **"Furye integral operatorlarining kritik ko'rsatkichi uchun baholar"** deb nomlangan uchinchi bobi qat'iy giperbolik tenglamalar uchun Koshi masalasini yechimlari bilan bog'liq o'rama operatorini o'rganishga bag'ishlangan.

III bobda asosan o'rama operatori uchun $L^p(\mathbb{R}^3) \to L^{p'}(\mathbb{R}^3)$ chegaralanganlik masaslasini qaraymiz, bunda $1 \leq p \leq 2$ va p' qo'shma daraja, ya'ni $1/_p + 1/_{p'} = 1$ tenglikni qanoatlantiruvchi son. Bu o'rama operatorining yadrosi quyidagi munosabatdan aniqlanadi:

$$M_k = F^{-1} e^{i\varphi(\xi)} a_k(\xi) F$$

bu yerda $F -$ Furye almashtirishi operatori, $\varphi \in C^{\infty}(\mathbb{R}^n \setminus \{0\})$ koordinata boshidan tashqarida silliq, darajasi 1 bo‘lgan bir jinsli funksiya, $a_k \in C^{\infty}(\mathbb{R}^n)$ funksiya esa katta ξ lar uchun darajasi $-k$ bo‘lgan bir jinsli funksiya.
Biz M. Sugimotoning maqolasida isbotlangan natijalarni umumlshtiramiz.
Bu baholar

$$\Sigma = \{\xi \in \mathbb{R}^{\nu} \setminus \{0\}: \varphi(\xi) = 1\}$$

gipersirtda mujassamlashgan o‘lchov Furye almashtirishining asimptotik xarakterini tadqig‘iga asoslanadi.
Shuni ta’kidlaymizki, Furye almashtirishining xarakteri Σ gipersirtning geometrik xossalariga bog‘liq.

Endi biz M_k lokallashtirilgan o‘rama operator uchun baholarni qaraymiz.

Faraz qilaylik, a_k amplituda funksiyasi $(0,0,1)$ nuqtaning yetarlicha kichik konussimon atrofida mujassamlashgan va $\Sigma \subset \mathbb{R}^3$ gipersirtning bu atrofdagi qismi silliq hamda $\phi(0,0) = 0$ va $\nabla\phi(0,0) = 0$ shartlarini qanoatlantiradigan $x_3 = 1 + \phi(x_1, x_2)$ funksiya grafigi sifatida berilgan.

Bu bobdagi asosiy natijalarimizni bayon qilish uchun bizga quyidagi tasdiq kerak bo‘ladi.

3.1-tasdiq. *Faraz qilaylik ϕ, $\mathbb{R}^2$ koordinatalar boshining biror atrofida aniqlangan silliq funksiya bo‘lib, quyidagi shartlarni qanoatlantirsin: $\partial_2^2\phi(0,0) \neq 0$ hamda $(0,2)$ dan farqli har qanday $|\gamma| \leq 2$ shartni qanoatlantirivchi γ uchun $\partial^{\gamma}\phi(0,0) = 0$ bo‘lsin. U holda ϕ funksiya koordinatalar boshining yetarlicha kichik atrofida quyidagi shaklda yozilishi mumkin:*

$$\phi(x_1, x_2) = b(x_1, x_2)\big(x_2 - \psi(x_1)\big)^2 + b_0(x_1),$$

bu yerda b, b_0 va ψ silliq funksiyalar hamda $b(0,0) \neq 0$. Agar ψ (mos ravishda b_0) tekis funksiya bo‘lmasa, u holda ψ (mos ravishda b_0) funksiya $\psi(x_1) = x_1^m\omega(x_1)$, $m \geq 2$ shaklida $\omega(0) \neq 0$ ni qanoatlantiradigan silliq ω funksiya, (mos ravishda $b_0(x_1) = x_1^n\beta(x_1)$, $\beta(0) \neq 0, n \geq 2$ ni qanoatlantiruvchi silliq funksiya bilan yozilishi mumkin.

Bundan tashqari, agar Σ, C^{∞} gipersirt hamda b_0 tekis funksiya bo‘lsa, u holda $b_0 \equiv 0$ deb faraz qilamiz. Bu shart Mueller tomonidan kiritilgan "$R-$ condition" deb ataladigan shartga mos keladi. Albatta, agar ϕ haqiqiy analitik funksiya bo‘lsa, $R -$sharti avtomatik ravishda bajariladi.

Ushbu III bobda biz III bobning asosiy natijalari bo‘lgan quyidagi teoremani isbotlaymiz.

3.3-teorema. *Aytaylik, $\Sigma \subset \mathbb{R}^3 \setminus \{0\}$ sirt $v = (0,0,1) \in \Sigma$ nuqtada kamida bitta nolmas bosh egrilikka ega silliq sirt va $1 \leq p \leq 2$ tayinlangan son, shuningdek (m, n) 3.1-tasdiqda aniqlangan juftlik bo‘lsin.*
U holda quyidagi tasdiqlar o‘rinli bo‘ladi:
(i) Agar $2m \geq n$ (hamda $2 \leq n \leq \infty$) bo‘lsa, u holda $k_p(v) = \left(5 - \frac{2}{n}\right)\left(\frac{1}{p} - \frac{1}{2}\right)$;
(ii) Agar Σ $R -$shartni qanoatlantiruvchi silliq gipersirt va $m \geq 3$ hamda $2m < n \leq \infty$ bo‘lsa u holda

$$k_p(v) = max\left\{\left(5-\frac{1}{m}\right)\left(\frac{1}{p}-\frac{1}{2}\right), \left(6-\frac{2(m+1)}{n}\right)\left(\frac{1}{p}-\frac{1}{2}\right)-\frac{1}{2}+\frac{m}{n}\right\}$$

tenglik o'rinli bo'ladi.

Endi D va E tipidagi maxsusliklar uchun shunga o'xshash natijalarni ko'rib chiqamiz. Shunday qilib, D tipidagi maxsuslikdan farqli $h(\phi) < 2$, bo'lgan ixtiyoriy funksiyalar uchun aniq natijalarga erishamiz.

3.16-teorema. *Agar ϕ funksiya $E_k(6 \le k \le 8)$ tipidagi maxsusliklarga ega bo'lsa, u holda quyidagi munosabat*

$$k_p(v) = \left(6-\frac{2}{h(\phi)}\right)\left(\frac{1}{p}-\frac{1}{2}\right)$$

o'rinli bo'ladi.

Shunga o'xshash natija M. Sugumoto tomonidan qaralmagan chunki (0,0,1) nuqtada ikkala bosh egrilik ham nolga aylanadi.

XULOSA

Mazkur dissertatsiya ishi sirt oʻlchovlarining Furye almashtirishining xarakterini oʻrganishga va olingan natijalarni tebranuvchan yadroli oʻrama operatorlari uchun chegaralanganlik muammosiga qoʻllashga bagʻishlangan. Xususan, qat'iy giperbolik tenglamalar uchun Koshi masalasining yechimlari silliq funksiyalar aniqligida bunday oʻrama operatorlari shaklida yozilishi mumkin. Dissertatsiyaning asosiy mazmuni jami kirish, uchta bob 16 ta paragrafdan iborat:

Tadqiqotning asosiy natijalari quyidagilardan iborat:

E_7 va E_8 tipidagi maxsuslikka ega boʻlgan funksiya grafigi sifatida berilgan silliq gipersirtlarda mujassamlashgan zaryadlarning (oʻlchovlarning) Furye almashtirishining baholari olingan, olingan baholarning aniqligi isbotlangan;

tegishli faza funksiyasining normal shakliga bogʻliq holda gipersirtlar sinflari oʻrganilgan integrallanish koʻrsatkichining maksimalligini ta’minlaydigan funksiya kritik nuqtasining maksimal karraliligi topilgan;

olingan baholar sodda maxsusliklar bilan bogʻliq boʻlgan gipersirtlarga mujassamlashgan oʻlchovlar Furye almashtirishini biror daraja bilan integrallanishi muammosini yechishga qoʻllanilgan va integrallanish darajasi D tipidagi maxsuslik karraliligiga bogʻliq emasligi isbotlangan;

qat’iy giperbolik tenglamalar uchun Koshi masalasini yechimi bilan bogʻliq boʻlgan oʻrama operatori chegaralangan boʻladigan darajalar toʻplamning infimumi topilgan. Sugumoto tipidagiga oʻxshash baholar olingan va olingan bahoning aniqligi isbotlangan;

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SAMARQAND STATE UNIVERSITY

IKROMOVA DILDORA ISROILOVNA

EXTREMAL PROBLEMS RELATED TO SIMPLE SINGULARITIES

01.01.01 – Mathematical analysis

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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme dissertation. In many scientific and applied research conducted in the world, in particular, in solving most problems of mathematical physics and spectral analysis, special importance is attached to the study of the evaluation of oscillatory integrals with large and small parameters. Nowadays, in developed countries, the problem of obtaining a-prior estimates of solutions to the Cauchy problem for strictly hyperbolic equations is linked to the boundedness issue of convolution operators with an oscillatory kernel in Lebesgue spaces such as $L^p(\mathbb{R}^n)$. In mathematics and theoretical physics, as well as in Fourier analysis and harmonic analysis, many models are described using differential equations and more general pseudo-differential equations.

In practice, we usually deal with solutions to differential equations in the sense of distributions. For example, in physics, experts are mostly interested in solutions with discontinuous points. The set of these points is called to be singularities of the solutions. The study of these singularities is the subject of harmonic analysis. Therefore, the development of research in harmonic analysis is becoming an important area of focus in mathematics.

In the world, finding a general solution to a differential equation in the space of distributions is a difficult problem. Surely, the singularities of the solution will not change if a smooth function is added to it. Therefore, in many cases, authors give priority to research on the construction of solutions to differential equations, up to smooth functions, and have a wide range of investigations. In this regard, weak solutions than the general distribution solution are often used in practice.The study of solutions to differential equations based on the space of integrable functions has been discussed in many articles. For example, the solution to the strictly hyperbolic equations, as well as discrete equations, have been studied in many papers. A-priori estimates for solutions, which depend on initial conditions and are known as dispersive estimates, have numerous applications in the field of mathematical physics and developing investigation is becoming actual.

In our Republic, conducting research at the level of international standards in the priority areas of[2] "Algebra and its applications, differential equations and their applications, non-linear systems, dynamic systems and mathematical modeling of their applications, stochastic analysis, medical-biological informatics, computational mathematics" is identified as one of the main tasks of the scientific researches. To ensure the successful implementation of the decision, it is crucial to develop a theoretical understanding of oscillatory integrals in relation to the Fourier transform of measures on surfaces, as well as estimates for Fourier integral operators defined by convolution with oscillatory kernels.

The subject and object of our dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February

[2] The Cabinet of Ministers of the Republic of Uzbekistan, dated May 18, 2017, issued Decision No. 292 "On the organization of the activities of scientific research institutions created under the Academy of Sciences of the Republic of Uzbekistan".

7, 2017, PF-4947, "On the strategy of action for the further development of the Republic of Uzbekistan", PQ-4387 dated July 9, 2019 "On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan" and PQ-4708 of May 7, 2020 "On measures to improve the quality of education and research in the field of mathematics", Decrees No. PF-60, dated January 28, 2022, "on the development strategy for new Uzbekistan for 2022-2026" and No. PF-158, dated September 11, 2023, "on Uzbekistan-2030 strategy", this dissertation serves to implement the objectives outlined in regulatory legal acts as well as in other regulations related to this activity.

Dependence of research to priority directions of development of science and technologies of the republic. This study was performed in accordance with the Republic of Uzbekistan IV priority areas of science and technology. "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem. As we know, the solution to the Cauchy problem for strictly hyperbolic equations can be expressed in terms of a convolution operator, the kernel of the convolution operator is given by an oscillatory integral, which is determined by the Fourier transform of measures or charges concentrated on smooth hypersurfaces. If the height of the hypersurface is strictly less than two, the phase function associated with this hypersurface has only simple Arnold singularities.

On the other hand, finding the exact value of the integrability index for the Fourier transform of charges concentrated on surfaces is an important problem in certain areas of mathematics. The solution to this problem is crucial for the method of trigonometric summation developed by I.M.Vinogradov, Xya-Lo-Gen, A.A.Karatsuba and so on. Another motivation for studying the problem of integrability is its connection with the restriction of the Fourier transform of integrability with some degree functions to subsets with measure zero. Some results on the restriction of the Fourier transform to smooth surfaces are known. For a more detailed discussion of these results, please see Stein's classic monograph.

However, our interest in this problem comes from another problem in mathematical physics. More specifically, we are interested in the problems related to estimates for convolution operators or Fourier integral operators, which are associated with characteristic surfaces of hyperbolic equations. Additionally, the issue of integrating functions with negative degrees is related to the spectral problem of the generalized Friedrichs model. These arguments are important in modern fields such as mathematical physics and harmonic analysis, and this explains the significance and actuality of the issue discussed above.

The connection of the topic of the dissertation with the research work of the higher educational institution, in which the dissertation is carried out. The dissertation work is done in accordance with the planned theme of scientific research "Harmonic analysis, power geometry and its applications to problems of mathematical physics" (OT-F4-69, Samarkand State University, 2017-2021).

The aim of the research is Investigation of the behavior of the Fourier transform at infinity for measures concentrated on surfaces and obtaining a priori estimates for solutions to the Cauchy problem for strictly hyperbolic equations. That is, to find the value of the critical exponent at which the convolution operator associated with these equations will become bounded. In addition, it consists considering the integrability with a certain degree of Fourier transform of surface-carried measures.

Research tasks: are

Determine the estimates of the Fourier transform of measures concentrated on hypersurfaces, defined as graphs of functions with singularities of types E_7 and E_8;

Finding the integrability index of Randol-type maximal functions, determined using measures related to surfaces with special singularities of type D, and proving the sharpness of the integrability index;

Finding the integrability index of Randol-type maximal functions related to measures concentrated on surfaces with singularities of type E_7 and E_8, and proving the sharpness of the integrability index;

Determining the critical exponent at which the convolution operators associated with solutions to the Cauchy problem for strictly hyperbolic equations become bounded.

The research objects are characteristic hypersurface, measures supported on hypersurfaces, Fourier transform of surface-carried measures, convolution operators.

The research subjects are investigation of the behavior of Fourier transforms of measures and the Randol maximal function associated to oscillatory integrals, as well as a priori estimates for the convolution operator given by the solutions to the Cauchy problem for strictly hyperbolic equations.

Research methods. The research uses the methods of mathematical analysis, complex analysis, functional analysis, mathematical physics, theory of the oscillatory integrals, theory of singularities of differentiable maps, methods of power geometry, including Newton polyhedrons methods, methods of differential geometry.

The scientific novelty of the research is as follows:

new estimates for the Fourier transform of measures concentrated on hypersurfaces given as graphs of functions with singularities of types E_7 and E_8 have been obtained, and the sharpness of these estimates has been demonstrated;

the integrability exponent of Randol-type maximal functions defined via measures associated with surfaces possessing D-type singularities has been found, and it has been proved that both the sharpness of this exponent and the estimate itself do not depend on the multiplicity of the D-type singularity;

the integrability exponent of Randol-type maximal functions associated with measures concentrated on surfaces having singularities of types E_7 and E_8 has been determined, and the sharpness of this exponent has been proved;

for the convolution operators related to the Cauchy problem for strictly hyperbolic equations, the exact value of the critical exponent at which these operators become bounded is found.

Practical results of the research:

It consists of: the research is fundamental in nature, but the problem of the behavior of the Fourier transform of surface-carried measures is applied in the theory of numbers, in problems of finding the number of integer points in homothetic domains, and in computational mathematics, in problems of estimating the residual term in cubature formulas;

Using the results on the boundedness of integral operators, the solutions to generalized wave equations in mathematical physics related to pseudodifferential operators were studied.

The reliability of the results of the research based on using the methods of mathematical analysis, mathematical physics, functional analysis, complex analysis, differential geometry, as well as the rigor of mathematical reasoning.

The scientific and practical significance of the research results. The scientific value of the results of the study lies in the fact that they can be used in investigation of solutions to differential equations of mathematical physics. In particular, it is explained by the fact that it can be used in generalized wave equation, Cauchy problem for strictly hyperbolic equations and discrete Schrödinger equation, discrete Klein-Gordon equations. The practical significance of the dissertation work is explained by the fact that the scientific results obtained in this work can serve as a theoretical basis of numerical solutions of equations of mathematical physics, which describe models for physical process.

Implementation of the research results. Based on the results of the extreme problems related to simple singularities:

the results are used in the project “methods for constructing solutions of fractional-order differential equations, methods for solving elementary and boundary problems” No. AP09259074 of the international Kazakh-Turkish university named after Khoja Ahmad Yassavi of the Republic of Kazakhstan, (2024-7 November, 2024, No. 03/3108 of the international Kazakh-Turkish university named after Khoja Ahmad Yassavi of the Republic of Kazakhstan), it was used in solving of the Cauchy problem for hyperbolic equations. The results obtained made it possible to obtain an exact estimate of the smoothness of the solution of the Cauchy problem for high-order strictly hyperbolic equations;

results on the boundedness of integral operators with oscillating kernels used in solving boundary value problem for higher order hyperbolic equation in fundamental research OT F4–88, 2017-2020 of the Ministry of innovative development of the Republic of Uzbekistan "Study of correct and inverse problems for second and high order mixed type equations" (V.I.Romanovsky Institute of Mathematics, reference No. 2/402 dated November 6, 2024). The application of these scientific results made it possible to obtain an exact estimate for solving high-order hyperbolic equations using the initial data of the Cauchy problem.

Approbation of the research results. The results of the research were discussed at 7 scientific practical conferences, including 2 International and 5 Republican scientific practical conferences.

Publications of the research results. On the topic of the dissertation 14 scientific works have been published, 7 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 4 of the papers were published in international journals of mathematics indexed by Scopus and Web of Science, while 3 other papers were published in national mathematical journals.

The volume and structure of the dissertation. The dissertation work consists of an introduction, three chapters, conclusion and bibliography. The volume of the thesis is 115 pages.

MAIN CONTENT OF THE DISSERTATION

In the introduction is given the actuality and relevance of the thesis topics, determined the appropriate research priority areas of science and technology of the Republic, presented a review of international research on the theme of the dissertation and the degree of scrutiny of the problem, formulated goals and objectives, identified the object and subject of study, scientific novelty and practical results of the research are stated, revealed the theoretical and practical importance of the obtained results, information on the implementation of the research results about the published works and the structure of the dissertation are given.

In the chapter I named "**Necessary definitions and known results**" we present the necessary definitions and some well-known results related to our research. We will also give the list of notations used in the thesis. We also provide some elementary ways to prove results on simple Arnol'd's singularities. In particular, we provide an elementary proof for the statement regarding the normal form of functions that have singularities of type E_7 or E_8 at some point. We also give the normal forms of functions with singularities of type E_7 and E_8, with respect to linear transformations that preserve the singular point. It is worth noting that one of the main tools we use is the Newton polyhedron and an adapted coordinate system. We introduce concepts related to the Newton polyhedron associated with a power series of a smooth function, whose center of expansion is located at a singular point. We also determine the height and linear height of this smooth function.

In chapter II, named as "Integrability of the Fourier transform of surface-carried measures related to simple singularities", is devoted to the question of the estimates of the Randol maximal functions connected to Fourier transform of surface-carried measures and application of this estimates to problem on integrability with some degree of Fourier transform of corresponding measures.

In chapter II, we discuss estimates for the Fourier transform of measures concentrated on smooth surfaces S in $\mathbb{R}^3$, which are given by the graphs of smooth functions having simple Arnol'd's singularities. We assume that the principal curvatures of these surfaces vanish at a certain point. Note that any smooth hypersurface in Euclidean space locally can be given as the graph of a smooth function. Moreover, if $n \leq 6$ then any generic smooth hyper-surface can be given as the graph of smooth function having only simple Arnol'd's singularities. However, the corresponding phase function is the universal perturbation of simple singularities. We consider, the linear perturbation of the functions having simple singularities. The perturbation is not universal (i.e. is not unifolding) in the sense of Arnold terminology. So, the corresponding hyper-surface is not generic. In this case problem, on investigation of the behavior of the corresponding oscillatory integrals, is more subtle.

Surely, investigation of the arbitrary function having singularities is the complicated open problem of the geometric analysis. We restrict ourselves with functions having height less than two. Then the corresponding function has only simple singularities i.e. singularities of type A, D, E. Both principal curvatures of the corresponding surface vanish for the case of D and E type singularities.

We present a rigorous mathematical description of Chapter II. Let $S \subset \mathbb{R}^3$ be a smooth hypersurface, and let $\varphi \in C_0^\infty(S)$ be a smooth function with compact support. Consider the charge $d\mu(X) := \varphi(X)dS$, where dS is the induced Lebesgue measure on the hypersurface S. In particular, if φ is a nonnegative function, then we are dealing with a Borel surface-carried measure.

The Fourier transform of the charge $d\mu$ is defined by the following integral:

$$\widehat{d\mu}(\xi) := \int_S \ e^{iX\xi} d\mu(X),$$

which corresponds to the Fourier transform of the distribution determined by the charge $d\mu$ (as can be seen from V. P. Palamodov's monograph), where $X\xi$ is the inner product of the vectors X and ξ.

We define a Randol's maximal function corresponding to $\widehat{d\mu}$ by

$$M(\omega) = \sup_{r>0} r\left|\widehat{d\mu}(r\omega)\right|,$$

where $\omega \in S^2$ (S^2 is the unite sphere in $\mathbb{R}^3$ centered at the origin). It can be shown that if S is a smooth hypersurface then for a.e. $\omega \in S^2$ $M(\omega)$ is finite and M is the Borel's measurable function.

Note that if S is the strictly convex hypersurface (a hypersurface with non-zero Gaussian curvature) then we have the following uniform with respect to the direction $\omega \in S^n$ bound:

$$\left|\widehat{d\mu}(r\omega)\right| \leq Cr^{-1}.$$

Therefore in this case M is a bounded function.

In this chapter, we consider the following problem: *Find the infimum p_S of the following set:*

$$\left\{p \in [1, \infty): for\ any\ \ \varphi \in C_0^\infty(S),\ \widehat{d\mu} \in L^p(\mathbb{R}^3)\right\},$$

where $L^p(\mathbb{R}^{n+1})$-is the space of pth-integrable functions, $p(1 \leq p < \infty)$ of course, if $p = \infty$, then we are dealing with the space of essentially bounded functions.

The number p_S is called to be an exact integrability exponent of the Fourier transform of the charge (signed measure) $d\mu$, concentrated on the hypersurface S.

Generally speaking, the case $p_S = \infty$ is possible. For example, the Fourier transform of a nonzero measure concentrated on a hyperplane is not integrable for any finite value of p.

However, if the hypersurface satisfies the so-called "curvature" condition introduced by E. M. Stein, then p_S is a finite number, although the problem of determining the exact value of this number is very difficult. With similar issues, I.M. Vinogradov, Hua-Lo-Gen, G. I. Arkhipov, A. A. Karatsuba, V. N. Chubarikov, and G. Mokenhaupt, as well as several other famous mathematicians, were involved.

Further, suppose that, in a sufficiently small neighborhood of a fixed point, say, the origin, the hypersurface S is given as the graph of a smooth function $x_3 = \phi(x_1, x_2)$, satisfying the conditions $\phi(0) = 0$ and $\nabla\phi(0) = \partial_1\phi(0) = \partial_2\phi(0)$, and ϕ has singularities of type $D^{\pm}_{k+1}(3 \le k \le \infty)$ and $E_k(6 \le k \le 8)$. Such singularities are included in the list of simple singularities.

The main results of this chapter II is the following Theorem.

Theorem 2.1. *Let $S \subset \mathbb{R}^3$ be a smooth hypersurface defined as a graph of the function $\phi(x_1, x_2)$, i.e.,*

$$S = \{x \in \mathbb{R}^3 \colon x_3 = \phi(x_1, x_2)\},$$

where ϕ is a smooth function satisfying the following conditions:

i) for an arbitrary multiindex $(\alpha_1, \alpha_2) \in \mathbb{Z}^2_+$, with the condition $\alpha_1 + \alpha_2 \le 2$, the equality $\partial_1^{\alpha_1}\partial_2^{\alpha_2}\phi(0,0) = 0$ holds;

ii) ϕ has a singularity of multiplicity at most 7 at zero.

Then there exists a neighborhood of zero U such that for any density $\varphi \in C_0^\infty(U)$, the inclusion $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3) \coloneqq \bigcap_{p>3} L^p(\mathbb{R}^3)$ holds.

Remark. *It should be noted that if*

$$D^2\phi(0,0) \coloneqq \begin{pmatrix} \partial_1^2\phi(0,0) & \partial_1\partial_2\phi(0,0) \\ \partial_2\partial_1\phi(0,0) & \partial_2^2\phi(0,0) \end{pmatrix} \ne 0,$$

i.e., if at least one of the principal curvatures of the surface does not vanish at the origin $\mathbb{R}^3$, then, generally speaking, the statement of Theorem 2.1. does not hold. For example, let $\phi(x_1, x_2) = x_1^2 + x_2^5$; for this function, the multiplicity of the critical point is equal to 4. Then we can easily show that $\widehat{d\mu} \notin L^{22/7}(\mathbb{R}^3)$. This explains the essentiality of the condition that both principal curvatures vanish at the origin.

On the other hand, if the critical point multiplicity is greater than 7, then the statement of Theorem 2.1. does not hold, as we can see from the example of a function with a singularity of type E_8. More precisely, if $\phi(x_1, x_2) = x_1^3 + x_2^5$, then $m(\phi) = 8$, and $\widehat{d\mu} \notin L^{22/7}(\mathbb{R}^3)$. Thus, we have an exact estimate for the critical point multiplicity, which guarantees the validity of the inclusion $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$.

The sharpness of the main theorem follows from the following Proposition.

Proposition 2.4. *Let $S \subset \mathbb{R}^3$ be a C^2-smooth hypersurface containing the origin, let $d\mu$ be a measure defined by the equality*

$$\widehat{d\mu}(\xi) \coloneqq \int_S e^{-ix\xi}\varphi(x)dS,$$

and let φ be a nonnegative continuous function concentrated in a sufficiently small neighborhood of zero such that $\varphi(0) > 0$. Then $\widehat{d\mu} \notin L^p(\mathbb{R}^3)$ for any $1 \le p \le 3$.

The following Theorem is proved in the section 2.4 of the chapter II.

Theorem 2.6. *Let a smooth hypersurface $S \subset \mathbb{R}^3$ be defined as the graph*

$$S = \{x \in \mathbb{R}^3 | x_3 = \phi(x_1, x_2)\}$$

of a smooth function satisfying the following conditions:

(i) $\phi(0,0) = 0, \nabla\phi(0,0) = 0$;

(ii) ϕ has a singularity of type $D_{k+1} (3 \le k < \infty)$ at the origin $\mathbb{R}^2$.

Then there exists a neighborhood of zero $U \subset \mathbb{R}^3$ such that for any density $\varphi \in C_0^\infty(U)$, the inclusion $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$ holds.

Remark. *It is interesting to note that, in the case of singularities of class D, the exact L^p-estimate for the Fourier transform of the measure does not depend on the multiplicity (i.e., on k) of the critical point. However, this phenomenon is not observed in other singularities. Moreover, analogical result holds true for the D_∞ type singularities under the R −condition. Such kind of results for the case when the analytic surface S given as the graph of function having D_∞ was proved in the paper by N.A. Soleeva. Thus Theorem 2.6 extends results of the paper to the smooth surfaces for D_{n+1} for $n < \infty$ and for the case D_∞ case under the R −condition.*

We prove the following Theorem in section 2.5.

Theorem 2.14. *Let $S \subset \mathbb{R}^3$ be a smooth hypersurface defined as the graph of a smooth function $x_3 = \phi(x_1, x_2)$ satisfying the following conditions:*

(i) $\phi(0,0) = 0, \nabla\phi(0,0) = 0$;

(ii) ϕ has a singularity of type E_6 or E_7 at the origin $\mathbb{R}^2$.

Then there exists a neighborhood of zero U such that for any density $\varphi \in C_0^\infty(U)$ the inclusion $\widehat{d\mu} \in L^{3+0}(\mathbb{R}^3)$ holds.

The proof of the main theorem is based on the theorems 2.6 and theorem 2.14.

In the third chapter of the dissertation called **"Estimates for the critical exponent of Fourier integral operators**" is devoted to study the convolution operator related to solutions of the Cauchy problem for strictly hyperbolic equations.

In chapter III, we will mainly focus on the $L^p(\mathbb{R}^3) \to L^{p'}(\mathbb{R}^3)$ boundedness issue for the convolution operator. Here, p and p' are conjugate exponents, which means they satisfy the equation 1/p+1/p'=1. The kernel of this convolution operator is determined from the following relation:

$$M_k = F^{-1} e^{i\varphi(\xi)} a_k(\xi) F$$

where F is the Fourier transform operator, $\varphi \in C^\infty(\mathbb{R}^\nu \backslash \{0\})$ is homogeneous of order one function, $a_k \in C^\infty(\mathbb{R}^\nu_\xi)$ is a homogeneous function of order $-k$ for large ξ. We generalize the results presented in M. Sugimoto's paper.

These estimates are based on the study of the asymptotic behavior of the Fourier transform of a measure concentrated on a hypersurface:

$$\Sigma = \{\xi \in \mathbb{R}^\nu \setminus \{0\} : \varphi(\xi) = 1\}.$$

Note that the behavior of the Fourier transform depends on the geometric properties of the Σ-hypersurface.

Now, we consider the estimates for the localized convolution operator M_k. Assume the amplitude function a_k is supported in a sufficiently small conic neighborhood of the point $(0,0,1)$ and the hypersurface $\Sigma \subset \mathbb{R}^3$ is given as the graph of a smooth function $x_3 = 1 + \phi(x_1, x_2)$ satisfying the conditions $\phi(0,0) = 0$ and $\nabla\phi(0,0) = 0$.

In order, to formulate our results of this chapter we need the following Proposition.

Proposition 3.1. *Assume that ϕ is a smooth function defined in a neighborhood of the origin of $\mathbb{R}^2$ satisfying the conditions: $\partial_2^2\phi(0,0) \neq 0$ and also $\partial^\gamma\phi(0,0) = 0$ for any $|\gamma| \leq 2$ with $\gamma \neq (0,2)$.*

Then, ϕ can be written in the following form on a sufficiently small neighborhood of the origin:

$$\phi(x_1, x_2) = b(x_1, x_2)\big(x_2 - \psi(x_1)\big)^2 + b_0(x_1),$$

where b, b_0 and ψ are smooth functions with $b(0,0) \neq 0$. The function ψ (resp. b_0) can be written as $\psi(x_1) = x_1^m\omega(x_1)$ with a smooth function ω satisfying $\omega(0) \neq 0$, $m \geq 2$ (resp. $b_0(x_1) = x_1^n\beta(x_1)$, with a smooth function β satisfying $\beta(0) \neq 0, n \geq 2$ unless ψ (resp. b_0) is a flat function.

Further, we assume that if Σ is a C^∞ hypersurface and b_0 is a flat function then $b_0 \equiv 0$. This condition agrees with so-called "$R-$condition" introduced in the monograph by Detlef Müller. Surely, if ϕ is a real analytic function then the $R-$ condition is automatically fulfilled.

In this chapter III we prove the following statement, which is the main results of the chapter III.

Theorem 3.3. *Let $\Sigma \subset \mathbb{R}^3 \setminus \{0\}$ be a smooth surface having at least one non-vanishing principal curvature at the point $v = (0,0,1) \in \Sigma$ and $1 \leq p \leq 2$ be a fixed number and also (m, n) be the pair defined by the Proposition 1. Then the following statements hold:*

(i) If $2m \geq n$ (with $2 \leq n \leq \infty$) then $k_p(v) = (5 - \frac{2}{n})(\frac{1}{p} - \frac{1}{2})$;

(ii) If Σ is a smooth hypersurface satisfying the $R-$condition and $m \geq 3$ and also $2m < n \leq \infty$ then

$$k_p(v) = max\left\{\left(5 - \frac{1}{m}\right)\left(\frac{1}{p} - \frac{1}{2}\right), \left(6 - \frac{2(m+1)}{n}\right)\left(\frac{1}{p} - \frac{1}{2}\right) - \frac{1}{2} + \frac{m}{n}\right\}.$$

Then we consider analogical results for D and E type singularities. Thus, obtain the sharp results for the case when $h(\phi) < 2$, but the D type singularities.

Theorem 3.16. *If the function ϕ has $E_k(6 \leq k \leq 8)$ type singularities then the relation*

$$k_p(v) = \left(6 - \frac{2}{h(\phi)}\right)\left(\frac{1}{p} - \frac{1}{2}\right)$$

holds.

A similar result is not taken into account by M. Sugimoto, because at the point (0,0,1), both principal curvatures are zero.

CONCLUSION

The dissertation work is devoted to study behavior of the Fourier transform of surface-carried measures and applications of the obtained results to boundedness problem for convolution operators with oscillatory kernel. In particular, solutions, up to smooth functions, to the Cauchy problem for strictly hyperbolic equations can be written as such convolution operators. The main content of the dissertation is an introduction, three chapters consist of 16 sections.

The main results of the research are as follows:

Estimates for the Fourier transforms of measures concentrated on smooth hypersurfaces, given as graphs of functions with singularities such as E_7 and E_8, have been obtained, and the sharpness of these estimates has been proved;

Classes of hypersurfaces are studied based on the normal form of the corresponding phase function. The maximum multiplicity of the critical point of this function is found, which provides the maximum integration index;

The obtained estimates were used to solve the problem of integrability with a certain degree of Fourier transform of measures supported on a hypersurface associated with simple singularities. It was proved that the degree of integrability does not depend on the multiplicity of singularities of type D.

The smallest element of the set of exponents has been identified, for which the convolution operator is bounded. This operator is relevant to the solution of the Cauchy problem for hyperbolic equations. As a result, estimates similar to those proposed by Sugimoto have been derived, and their accuracy has been demonstrated.

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ИКРОМОВА ДИЛДОРА ИСРОИЛОВНА

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С диссертацией можно ознакомиться в Информационно-ресурсном центре Самаркандского государственного университета (зарегистрирована за № 94). (Адрес: 140104, г. Самарканд, Университетский бульвар, 15. Тел.: (+99866) 231-06-32, факс: (99866) 235-19-38).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является изучение поведения преобразования Фурье поверхностных мер на бесконечности и получить априорные оценки для решений задачи Коши для строго гиперболических уравнений. Мы также рассматриваем интегрируемость преобразования Фурье поверхностных мер с некоторой степени. Эти поверхности заданы графиком функции, имеющей простые особенности Арнольда.

Объекты исследования: характеристическая гиперповерхность, меры, сосредоточенные на гиперповерхностях, преобразование Фурье поверхностных мер, операторы свертки, строго гиперболические уравнения.

Научная новизна исследования заключается в следующем:

получены новые оценки для преобразований Фурье мер, сосредоточенных на гиперповерхностях, представленных в виде графиков функций с такими особенностями, как E_7 и E_8, и доказана точность этих оценок;

найден показатель интегрируемости максимальных функций типа Рендола, определённых через меры, связанные с поверхностями, обладающими особенностями типа D, и доказано, что как точность показателя, так и сама оценка не зависят от кратности особенности типа D;

определен показатель интегрируемости максимальных функций типа Рендола, связанных с мерами, сосредоточенными на поверхностях с особенностями типов E_7 и E_8, и доказана точность этого показателя;

найдено точное значение критического показателя, при котором свёрточные операторы, связанные с решением задачи Коши для строго гиперболических уравнений, являются ограниченными.

Внедрение результатов исследования. На основе результатов экстремальные задачи связанные с простыми особенностями:

результаты использованы в проекте «Методы построения решений дробно-дифференциальных уравнений, методы решения элементарных и граничных задач» № AP09259074 Международного казахско-турецкого университета имени Ходжи Ахмеда Яссави Республики Казахстан (2024 г. – 7 ноября 2024 г., справка № 03/3108 Международного казахско-турецкого университета имени Ходжи Ахмеда Яссави Республики Казахстан), при решении задачи Коши для гиперболических уравнений. Полученные результаты позволили получить точную оценку гладкости решения задачи Коши для строго гиперболических уравнений высокого порядка;

результаты исследований ограниченности интегральных операторов с осциллирующими ядрами, используемые при решении краевой задачи для гиперболического уравнения высокого порядка, получены в рамках фундаментальной научно-исследовательской работы ОТ Ф4–88, 2017–2020 гг. Министерства инновационного развития Республики Узбекистан «Исследование корректных и обратных задач для уравнений смешанного типа второго и высокого порядков» (Институт математики им. В.И.Романовского

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Объем и структура диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации составляет 115 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
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Bosmaxona tasdiqnomasi

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"Times" garniturasi. Ofset bosma usuli.
Shartli b.t. 2,25. Adadi 60 nusxa. Buyurtma 17/3.

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Manzil: Samarqand viloyati, Samarqand tumani, Xishrav MFY.