

**V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI  
HUZURIDAGI ILMIY DARAJALAR BERUVCHI  
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

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**NAMANGAN DAVLAT UNIVERSITETI**

**ABDUKAXOROVA ZULXUMOR TUXTASINOVNA**

**BA'ZI MODELLAR UCHUN KUCHSIZ DAVRIY UMUMLASHGAN  
 $p$ -ADIK GIBBS O'LCHOVLARI**

**01.01.01 – Matematik analiz**

**FIZIKA-MATEMATIKA FANLARI  
bo'yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI**

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**Abdukaxorova Zulxumor Tuxtasinovna**

Ba'zi modellar uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari .....3

**Abdukahorova Zulkhumor Tukhtasinovna**

Weakly periodic  $p$ -adic generalized Gibbs measures for some models .....21

**Абдукахорова Зулхумор Тухтасиновна**

Слабо периодические  $p$ -адические обобщённые меры Гиббса для некоторых моделей.....37

**E'lon qilingan ilmiy ishlar ro'yxati**

Список опубликованных работ

List of published works.....41

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## KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

**Dissertatsiya mavzusining dolzarbligi va zarurati.** Jahon miqyosida amalga oshirilayotgan fizik va biologik jarayonlarning termodinamik xususiyatlarini o'rganishga qaratilgan turli ilmiy va amaliy tadqiqotlar aksariyat hollarda statistik mexanika modellariga olib kelinadi. Mazkur modellar tabiiy ravishda Arximed va noarximed modellarga ajraladi. Noarximed muhitda sodir bo'layotgan jarayonlarning modellarini tadqiq qilishda  $p$ -adik Gibbs o'lchovlari asosiy vosita hisoblanadi va faza almashishlar nazariyasida markaziy o'rin tutadi.  $p$ -Adik Gibbs o'lchovlari nafaqat statistik fizikaning nazariy rivojlanishi uchun, balki kimyo, materialshunoslik, biologiya va axborot texnologiyalari sohalaridagi amaliy muammolarni hal qilish uchun ham muhim ahamiyatga ega. Biroq, statistik mexanikaning klassik modellari uchun ham  $p$ -adik Gibbs o'lchovlari to'plamining to'liq tavsifi ochiq va murakkab muammo bo'lib qolmoqda, bu esa bunday o'lchovlarni boyitish, tasniflash va topilgan o'lchovlar xossalarini tahlil qilishni dolzarb tadqiqot yo'nalishiga aylantiradi.

Hozirgi kunda dunyoda panjarali sistemalarda aniqlangan Gamiltonian uchun  $p$ -adik Gibbs o'lchovlari mavjudligini aniqlash va bunday o'lchovlar to'plamini to'liq tavsiflash dolzarb vazifalardan biridir. Statistik fizikaning faza almashishlar nazariyasida berilgan fizik model uchun kamida ikkita umumlashgan  $p$ -adik Gibbs o'lchovini mavjudligi muhim ahamiyatga ega. Shu munosabat bilan tashqi maydonli va tashqi maydonsiz Gamiltonianlarga mos keluvchi translyatsion-invariant, davriy va kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining mavjudligini aniqlash hamda bunday o'lchovlar to'plamini to'liq tavsiflash va topilgan o'lchovlarning chegaralanganligini tekshirish orqali qaralayotgan modellar uchun faza almashinuvi mavjudligini aniqlash maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda rivojlanishning asosiy tayanchlaridan biri hisoblangan fundamental fanlarning ilmiy va amaliy qo'llanilishiga alohida e'tibor qaratilmoqda. Xususan, so'nggi yillarda Keli daraxtidagi haqiqiy va  $p$ -adik sonlar maydonida statistik mexanikaning klassik modellari uchun translyatsion-invariant, davriy va kuchsiz davriy Gibbs o'lchovlarining mavjudligini aniqlashda, shuningdek, bunday o'lchovlar to'plamini boyitishda salmoqli natijalarga erishildi. Respublikamizda algebra va uning tatbiqlari, differensial tenglamalar va uning tatbiqlari, chiziqsiz tizimlar, dinamik tizimlar va ularning tatbiqlarini matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi yo'nalishlari fundamental ilmiy tadqiqotlarning ustuvor yo'nalishlari etib belgilandi<sup>1</sup>. Qarorning ijrosini ta'minlash maqsadida, statistik mexanikaning klassik modellari uchun faza

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<sup>1</sup> O'zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab-quvvatlash shuningdek, O'zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risida"gi № PQ-4387-son qarori.

almashishlar nazariyasini rivojlantirish va bu ilmiy natijalarni fanning turdosh sohalarida qo'llash muhim ahamiyatga ega.

O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha harakatlar strategiyasi to'g'risida"gi va 2022-yil 28-yanvardagi PF-60-son "2022-2026-yillarga mo'ljallangan Yangi O'zbekistonning Taraqqiyot strategiyasi to'g'risida"gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab-quvvatlash, shuningdek O'zbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risida"gi va 2020-yil 7-maydagi PQ-4708-son "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

**Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo'nalishlariga bog'liqligi.** Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yo'nalishi doirasida bajarilgan.

**Muammoning o'rganilganlik darajasi.** Amerikalik olim J.U.Gibbs doimiy haroratga ega va tashqi muhit bilan issiqlik muvozanatida bo'lgan tizimlar uchun muhim ahamiyat kasb etuvchi Gibbs taqsimoti tushunchasini kiritgan. Haqiqiy qiymatli limit Gibbs o'lchovlarining umumiy xususiyatlari R.L.Dobrushin, O.Lenford va D.Ryuel ishlarida asoslab berilgan. Ushbu o'lchovlarning zamonaviy nazariy asoslari R.Bakster, H.O.Georgi, V.A.Malyshev, R.A.Minlos, K.Preston, D.Ryuel, Ya.G.Sinai, G.Gallavotti, F.Bonetto, J.Gentile, Jin Zin-Justin, N.N. G'anixo'jayev, U.A.Rozikov va F.M.Muxamedovlarning ilmiy ishlarida keng qamrovli tarzda ishlab chiqilgan. Bundan tashqari, R.L. Dobrushin haqiqiy qiymatli limit Gibbs o'lchovlarining mavjudligi haqida muhim teoremani isbotlagan. Statistik mexanikaning klassik modellari uchun Gibbs o'lchovlari muammosi R.L.Dobrushin, G.R.Braytveit, P.Uinkler, Yu.M.Suxov, J.Martin, N.N.G'anixo'jayev, U.A.Rozikov, F.M.Muxamedov, D.Gandolfo, M.M.Rahmatullayev, R.M.Xakimov, G.I.Botirov, O.N.Hakimov, E.Normatov, Sh.Shoyusupov, F.Haydarov, M.A.Rasulova, A.M.Tuxtaboyev, J.Dehqonov, O.Sh.Qarshiboyev va boshqa tadqiqotchilarning ishlarida chuqur o'rganilgan. Haqiqiy qiymatli Gibbs o'lchovlari nazariyasida o'lchovlarni davom ettirishga oid Kolmogorov teoremasi alohida ahamiyatga ega. Uning noarximed ehtimollik taqsimotlariga mos keluvchi muqobili N.N.G'anixo'jayev, F.M.Muxamedov va U.A.Rozikov hamda F.M.Muxamedovlar tomonidan isbotlangan. Shuningdek, muvofiqlik shartini qanoatlantiruvchi noarximed o'lchovlar uchun limit o'lchovning mavjudligi A.Yu.Xrennikov va S.Ludkovskiy tomonidan ko'rsatib berilgan. Ushbu natijalar panjarali tizimlarda turli  $p$ -adik modellar uchun Gibbs taqsimotlarini tadqiq etishda muhim nazariy asos bo'lib xizmat qilgan. N.N.G'anixo'jayev, F.M.Muxamedov va U.A.Rozikovlar o'z tadqiqotlarida  $\mathbb{Z}$

panjaradagi Izing modeli uchun  $p$ -adik Gibbs taqsimotining yagonaligini isbotlagan. Shuningdek, R.Gandolfo, U.A.Rozikov va J.Ruislarning ishlari natijasida  $k$ -tartibli Keli daraxtida qattiq sfera modeli uchun  $p$ -adik limit Gibbs taqsimoti mavjud bo'lishining shartlari aniqlangan.

Panjaradagi Izing modeli uchun  $p$ -adik Gibbs taqsimotining yagonaligi N.N.G'anixo'jayev, F.M.Muxamedov va U.A.Rozikov ishlarida isbotlangan. N.N.G'anixo'jayev va H.Akinning ishlarida esa Izing-Vannimenus modeli uchun  $p$ -adik Gibbs o'lchovining yagona emasligi ko'rsatilgan. O.N.Hakimov ikkinchi tartibli Keli daraxtidagi Vannimenus modeli uchun translyatsion-invariant va davriy  $p$ -adik kvazi Gibbs o'lchovlarini o'rgangan. Ushbu ishlar F.M.Muxamedov, M.K.Saburov va O.N.Hakimovning ishlarida Izing-Vannimenus modeli uchun davom ettirilgan. Bir jinsli Izing modeli uchun yagona  $p$ -adik Gibbs o'lchovining mavjudligi M.Hamrayev, F.M.Muxamedov va U.A.Rozikov ishlarida isbotlangan. Izing modeli uchun umumlashgan  $p$ -adik Gibbs o'lchovi tushunchasi O.N.Hakimov tomonidan kiritildi va bu model uchun translyatsion-invariant va  $G_2^{(2)}$ -davriy umumlashgan  $p$ -adik Gibbs o'lchovlari ham o'rganilgan. Bundan tashqari, Izing modeli uchun faza almashishi mavjudligi ko'rsatilgan. Umumlashgan  $p$ -adik Gibbs o'lchovlariga mos dinamik sistemaning xaotik tabiatga egaligi F.M.Muxamedov, O.N.Hakimov, H.Akin va M.Dogan tomonidan tadqiq qilingan.

2008-yilda ilk bora U.A.Rozikov, M.M.Rahmatullaevlar tomonidan haqiqiy sonlar maydonida kuchsiz davriy Gibbs o'lchovlari tushunchasi kiritilgan va Izing modeli uchun bunday o'lchovlarning mavjudligi tadqiq qilingan. Xususan, Keli daraxtining gruppaviy tasviridagi indeksleri ikki va to'rtga teng bo'lgan normal qism gruppalariga mos keluvchi kuchsiz davriy Gibbs o'lchovlari haqiqiy sonlar maydoni ustida aniq tavsiflangan. Shuni ta'kidlash joizki, yuqorida qayd etilgan ko'plab tadqiqotlar mavjudligiga qaramay, Keli daraxtidagi har qanday model uchun barcha limit Gibbs o'lchovlarining to'liq tasnifi hali ham olinmagan. Bundan tashqari, kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari hozirgacha to'liq o'rganilmagan.

**Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi.** Dissertatsiya tadqiqoti Namangan davlat universitetining ilmiy-tadqiqot ishlari rejasining "Fundamental tadqiqotlar" tarmog'i doirasida bajarilgan.

**Tadqiqot maqsadi** Keli daraxtida tashqi maydonli va tashqi maydonsiz Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining mavjudligini isbotlash va ularning chegaralanganlikka tekshirish orqali berilgan modellarda faza almashinuvi mavjudligini ko'rsatish iborat.

**Tadqiqotning vazifalari:**

Ikkinchi tartibli Keli daraxtida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining mavjudligini isbotlash;

Ikkinchi va uchinchi tartibli Keli daraxtlaridagi Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovlarini to'plamini tavsiflash va topilgan o'lovlarining chegaralanganligini tekshirish orqali qaralayotgan modellar uchun faza almashinuvi masalasini hal qilish;

Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun ikki indeksli normal qism gruppaga mos keluvchi kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovlarini aniqlash va bu model uchun faza almashinuvi mavjudligini ko'rsatish;

Uchinchi tartibli Keli daraxtida bir jinsli tashqi maydonli Izing modeli uchun translyatsion-invariant umumlashgan  $p$ -adik Gibbs o'lovlarini to'plamini to'liq tavsiflash va faza almashish masalasini tadqiq qilish.

**Tadqiqot obyekti:** Keli daraxtidagi tashqi maydonli va tashqi maydonsiz  $p$ -adik Izing modellari.

**Tadqiqot predmeti.** Gruppalar va graflar nazariyasi, Gibbs o'lovlarini nazariyasi, algebra va sonlar nazariyasi,  $p$ -adik analiz, chiziqli bo'lmagan Markov jarayonlari.

**Tadqiqot usullari.** Tadqiqot ishida  $p$ -adik analiz, sonlar nazariyasi, funksional analiz, gruppalar nazariyasi, o'lovlar nazariyasi, chiziqli algebra usullaridan foydalanilgan.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

Ikkinchi tartibli Keli daraxtida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovining mavjudligi isbotlangan;

Ikkinchi va uchinchi tartibli Keli daraxtlarida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovlarini qurilgan. Qurilgan o'lovlar chegaralanganlikka tekshirilgan;

Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun indeksli ikkiga teng bo'lgan normal qism gruppaga mos keluvchi kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovlarini qurilgan va bu model uchun  $p > 2$  bo'lganda faza almashishi mavjudligi isbotlangan;

Uchinchi tartibli Keli daraxtidagi bir jinsli tashqi maydonli Izing modeli uchun translyatsion-invariant umumlashgan  $p$ -adik Gibbs o'lovlarini aniqlangan va qaralayotgan model uchun  $p > 3$  bo'lganda faza almashishi mavjudligi isbotlangan.

**Tadqiqotning amaliy natijalari** quyidagilardan iborat:

$p$ -Adik analiz va sonlar nazariyasi metodlaridan atrof-muhit bilan issiqlik muvozanatida bo'lgan noarximed fizik sistemalar uchun Gibbs o'lovlarini qurishda va faza almashishlari mavjudligini tekshirishda foydalanish mumkin.

Shuningdek, kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovlarini to'plamida turli faza almashishlarning mavjudligini ta'minlaydigan parametr qiymatlarining aniq ifodasi xizmat ko'rsatish nazariyasi masalalarini yechishda qo'llanishi mumkin.

**Tadqiqot natijalarining ishonchliligi.** Sonlar nazariyasi,  $p$ -adik analiz, noarximed funksional analiz, noarximed Markov tasodifiy maydonlar nazariyasi,  $p$ -adik ehtimollar nazariyasi, Gibbs va noarximed o'lchovlar nazariyasi usullaridan foydalanilgan. Olingan natijalar qat'iy matematik mulohazalarga asoslanib isbotlangan.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.** Tadqiqot natijalarining ilmiy ahamiyati  $p$ -adik statistik mexanika va umumlashgan Gibbs o'lchovlari nazariyasi bo'yicha mavjud natijalarni boyitishi, xususan, kuchsiz davriy o'lchovlar orqali faza almashish hodisalarining xilma-xilligini aniqlashda yangi yondashuvni taklif qilishi bilan belgilanadi.

Tadqiqot natijalarining amaliy ahamiyati  $p$ -adik modellar yordamida murakkab tizimlarning (biologik, fizik va axborot tizimlari) xatti-harakatini modellashtirish hamda ularga oid masalalarning aniq yechimlarini izlash imkoniyatini berishi bilan izohlanadi.

**Tadqiqot natijalarining joriy qilinishi.** Izing modeli uchun umumlashgan  $p$ -adik Gibbs o'lchovlari bo'yicha olingan natijalar asosida:

ikkinchi va uchinchi tartibli Keli daraxtlarida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari tavsifidan va topilgan o'lchovlarning chegaralanganlikka tekshirish usullaridan №374824-2022 raqamli "Faza almashish muammolari va kritik hodisalar. Ular tenglamalarining matematik nuqtayi nazari, tez almashishlar va asimptotikalar" mavzusidagi xorijiy loyihada Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarini tavsiflashga doir nazariy yondashuvlar faza almashishlari va kritik hodisalarni aniqlash usullarini takomillashtirishda foydalanilgan. (O'sh davlat universitetining 2025-yil 15-oktyabrdagi №1400-sonli ma'lumotnomasi, Qirg'iziston). Ilmiy natijaning qo'llanishi faza almashishlarini va kritik haroratlarni aniqlash imkonini bergan;

Keli daraxtlaridagi Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari to'plamining to'liq tasnifi G0003247 raqamli "Tarmoq modellarining renormalizatsiyalangan gruppalari bilan bog'liq xaotik va aralash  $p$ -adik dinamik tizimlar" mavzusidagi xorijiy loyihada Izing modeli uchun umumlashgan  $p$ -adik Gibbs o'lchovlarini va fazaviy o'tishlarni tadqiq etishda foydalanilgan (Birlashgan Arab Amirliklari universitetining 2025-yil 23-oktyabrdagi ma'lumotnomasi, BAA). Ilmiy natijaning qo'llanishi  $p$ -adik dinamik sistemalar nazariyasiga yangi tushunchalar kiritish va fizik modellarning termodinamik xususiyatlarini o'rganish uchun amaliy asos yaratish imkonini bergan;

ikkinchi tartibli Keli daraxtida berilgan  $p$ -adik Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining mavjudligini tekshirish usullaridan F-FA-2021-425 raqamli "Panjarali sistemalarda gradient ehtimollik o'lchovlari" mavzusidagi fundamental loyihada panjarali sistemalarda faza almashishlari mavjudligini aniqlashda foydalanilgan (Matematika institutining 2025-yil 27-oktyabrdagi №2/416-sonli ma'lumotnomasi). Ilmiy natijaning

qo'llanishi panjarali sistemalarda yaqin qo'shni ta'sirga ega modellar uchun davriy bo'lmagan Gibbs o'lchovlarini tavsiflash imkonini bergan.

**Tadqiqot natijalarining aprobatsiyasi.** Mazkur tadqiqot natijalari 12 ta ilmiy-amaliy anjumanlarda, jumladan, 8 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

**Tadqiqot natijalarining e'lon qilinganligi.** Dissertatsiya tadqiqoti mavzusi bo'yicha jami 18 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy Attestatsiya Komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta, jumladan, 4 tasi xorijiy va 2 tasi respublika jurnallarida nashr etilgan.

**Dissertatsiyaning tuzilishi va hajmi.** Dissertatsiya kirish qismi, uchta bob, o'nta paragraf, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 114 betni tashkil etgan.

## DISSERTATSIYANING ASOSIY MAZMUNI

**Kirish** qismida tadqiqot mavzusining dolzarbligi va tanlanish asoslari bayon etilgan, uning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga muvofiqligi ko'rsatilgan, mavzuga oid xorijiy va mahalliy ilmiy adabiyotlarning tahlili keltirilgan, muammoning o'rganilganlik darajasi ochib berilgan. Shuningdek, tadqiqotning maqsad va vazifalari, obykti va predmeti aniqlangan, olingan natijalarning ilmiy yangiligi, nazariy hamda amaliy ahamiyati yoritilgan. Bundan tashqari, tadqiqot natijalarining tatbiq etilishi, nashr etilgan ilmiy maqolalar va dissertatsiyaning tuzilishi to'g'risida ma'lumotlar berilgan.

Dissertatsiyaning "***p*-Adik analiz va *p*-adik o'lchovlar haqida**" deb nomlangan birinchi bobi olingan natijalarni tavsiflash uchun ishlatiladigan asosiy tushunchalar va ta'riflar, shuningdek, tadqiqot davomida qo'llanilgan fundamental teoremlarni taqdim etadi. Bundan tashqari, ushbu bobda Keli daraxtidagi Izing modeli uchun *p*-adik Gibbs o'lchovining yagonaligi, umumlashgan *p*-adik Gibbs o'lchovlari tushunchasi va ikkinchi tartibli Keli daraxtidagi Izing modelida bunday o'lchovlar uchun olingan natijalar muhokama qilinadi. Ushbu bob keyingi boblarda taqdim etilgan asosiy ilmiy natijalarni tushuntirish va asoslash uchun nazariy hamda metodologik poydevor bo'lib xizmat qiladi.

$\mathbb{Q}$  ratsional sonlar maydoni bo'lsin. Ma'lumki, biror fiksirlangan *p* tub son uchun ixtiyoriy noldan farqli *x* ratsional sonni  $x = p^r \frac{m}{n}$  ko'rinishda tasvirlash mumkin. Bu yerda  $m, r \in \mathbb{Z}$ ,  $n \in \mathbb{N}$ , *m* va *n* sonlari *p* tub songa bo'linmaydi. Bu holda *x* ratsional sonni *p*-adik normasi quyidagicha aniqlanadi:

$$|x|_p = p^{-r}.$$

Shuningdek,  $x=0$  bo'lganda esa uning *p*-adik normasi quyidagicha aniqlanadi:

$$|0|_p = 0.$$

Ushbu norma noarximed normasidir, ya'ni barcha  $x, y \in \mathbb{Q}$  uchun u kuchli uchburchak tengsizligini qanoatlantiradi:

$$|x + y|_p \leq \max\{|x|_p, |y|_p\}.$$

$\mathbb{Q}$  ratsional sonlar maydonining  $p$ -adik norma bo'yicha to'ldirmasiga  $p$ -adik sonlar maydoni deyiladi va bu maydon  $\mathbb{Q}_p$  kabi belgilanadi. Har qanday nolga teng bo'lmagan  $x$   $p$ -adik sonni yagona kanonik ko'rinishda yozish mumkin:

$$x = p^{\gamma(x)}(x_0 + x_1 p + x_2 p^2 + \dots),$$

bu yerda  $\gamma(x), x_j \in \mathbb{Z}$  va  $x_0 > 0, 0 \leq x_j \leq p - 1$ . Bu holda,  $|x|_p = p^{-\gamma(x)}$ .

Quyida biz tadqiqotimiz uchun muhim bo'lgan to'plamlarni keltiramiz:  $a \in \mathbb{Q}_p$  va  $r > 0$  sonlar berilgan bo'lsin.  $a$  markazli  $r$  radiusli ochiq shar :

$$B(a, r) = \{x \in \mathbb{Q}_p : |x - a|_p < r\},$$

$a$  markazli  $r$  radiusli yopiq shar:

$$\overline{B(a, r)} = \{x \in \mathbb{Q}_p : |x - a|_p \leq r\},$$

$a$  markazli  $r$  radiusli sfera:

$$S(a, r) = \{x \in \mathbb{Q}_p : |x - a|_p = r\},$$

$p$ -adik butun sonlar:

$$\mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\},$$

$p$ -adik birlar:

$$\mathbb{Z}_p^* = \{x \in \mathbb{Q}_p : |x|_p = 1\}$$

kabi aniqlanadi.

$p$ -adik Gibbs o'lchovlar nazariyasida muhim ahamiyatga ega bo'lgan quyidagi funktsiyani keltiramiz.

$p$ -adik eksponensial funktsiya quyidagi qator ko'rinishida aniqlanadi:

$$\exp_p(x) = \sum_{n=1}^{\infty} \frac{x^n}{n!},$$

hamda u  $B(0, p^{-1/(p-1)})$  sharda yaqinlashadi.

$p$ -adik eksponensial funktsiyaning qiymatlari to'plami

$$\mathcal{E}_p = \{x \in \mathbb{Q}_p : |x - 1|_p < p^{-1/(p-1)}\}.$$

Faraz qilaylik,  $(X, \mathfrak{B})$ -o'lchovli fazo, bunda  $\mathfrak{B}$  oila  $X$  to'plamning qism to'plamlaridan tashkil topgan algebra va  $\mathbb{K}$  noarximed maydon bo'lsin. Agar  $\mu: \mathfrak{B} \rightarrow \mathbb{K}$  funktsiya chekli additiv bo'lsa, ya'ni juft-jufti bilan o'zaro kesishmaydigan  $A_1, A_2, \dots, A_n \in \mathfrak{B}$  uchun  $A = \bigcup_{k=1}^n A_k \Rightarrow \mu(A) = \sum_{k=1}^n \mu(A_k)$  o'rinli bo'lsa, u holda  $\mu$  to'plam funktsiyasiga *noarximed o'lchov* deyiladi. Agar  $\mu(X) = 1$  bo'lsa, bunday o'lchov *noarximed ehtimollik o'lchovi* deyiladi.

Agar  $\mu$  noarximed o'lchovi chegaralangan, ya'ni  $\sup\{\mu(A)|_p : A \in \mathfrak{B}\} < \infty$  bo'lsa, u holda  $\mu$  chegaralangan noarximed o'lchov deyiladi.

$\Gamma^k = (V, L)$   $k$ -tartibli Keli daraxti bo'lsin. Bunda  $V$  daraxtning uchlari to'plami,  $L$  uning qirralari to'plami. Agar  $x, y \in V$  uchlar uchun ularni tutashtruvchi  $l \in L$  qirra mavjud bo'lsa, u holda  $x$  va  $y$  uchlar yaqin qo'shnilar deyiladi va  $l = \langle x, y \rangle$  kabi belgilanadi. Keli daraxtida  $x, y \in V$  uchlar orasidagi yo'l deb  $\langle x, x_1 \rangle, \dots, \langle x_{d-1}, y \rangle$  yaqin qo'shnilar ketma-ketligiga aytiladi.  $x$  va  $y$  ni tutashtruvchi eng qisqa yo'ldagi qirralar soni  $d(x, y)$  bilan aniqlanadi.

Fiksirlangan  $x^0 \in V$  uchun quyidagicha belgilashlar kiritilgan:

$$W_n = \{x \in V : d(x, x^0) = n\}; \quad V_n = \bigcup_{j=0}^n W_j; \quad S(x) = \{y \in W_{n+1} : d(x, y) = 1\}, \quad x \in W_n;$$

$$L_n = \{l = \langle x, y \rangle \in L : x, y \in V_n\}; \quad S_1(x) = \{y \in V : d(x, y) = 1\}; \quad \{x_\downarrow\} = S_1(x) \setminus S(x).$$

$G_k$ -barpo etuvchilari mos ravishda  $a_1, a_2, \dots, a_{k+1}$  bo'lgan  $k+1$  ta ikkinchi tartibli siklik gruppalarining ozod ko'paytmasidan iborat gruppaga bo'lsin.

Quyidagi tasdiq N.N.G'anixo'jayevning ishlaridan ma'lum.

**1-tasdiq.**  $k$ -tartibli Keli daraxtining  $V$  uchlar to'plami bilan  $G_k$  gruppaga o'rtasida o'zaro bir qiymatli moslik mavjud bo'ladi.

$\Phi \subset \mathbb{Z}$  chekli to'plam berilgan bo'lsin.  $\Lambda \subset V$  to'plamda aniqlangan  $x \in \Lambda \rightarrow \sigma_\Lambda(x) \in \Phi$  funksiya konfiguratsiya deyiladi.  $\Lambda$  to'plamda aniqlangan barcha konfiguratsiyalar to'plami  $\Omega_\Lambda = \Phi^\Lambda$  kabi belgilanadi. Xususan,  $\Omega = \Omega_V$ .

Berilgan  $\sigma \in \Omega_\Lambda$  va  $\varphi \in \Omega_{V \setminus \Lambda}$  konfiguratsiyalar uchun quyidagi aniqlangan:

$$(\sigma \vee \varphi)(x) = \begin{cases} \sigma(x), & \text{agar } x \in \Lambda, \\ \varphi(x), & \text{agar } x \in V \setminus \Lambda. \end{cases}$$

$\sigma \in \Omega$  konfiguratsiyaning energiyasi ushbu

$$H(\sigma) = \sum_{\substack{\Lambda \subset V: \\ \text{diam}(\Lambda) \leq r}} I(\sigma_\Lambda),$$

*Gamiltonian* orqali beriladi, bu yerda  $r \in \mathbb{N}$ ,  $\text{diam}(\Lambda) = \max_{x, y \in \Lambda} d(x, y)$  va

$I : \Omega_\Lambda \rightarrow \mathbb{Q}_p$  berilgan potensialni ifodalaydi.

$\mathfrak{B}$  oila  $\Omega$  to'plamning silindrik qism to'plamlaridan tuzilgan algebra bo'lsin.

**1-ta'rif.**  $(\Omega, \mathfrak{B})$  da aniqlangan  $\mu$   $p$ -adik ehtimollik o'lchovi berilgan bo'lsin. Faraz qilaylik, ixtiyoriy  $\Lambda \subset V$  uchun

$$\mu(\sigma | \varphi) = \frac{\exp_p \{H(\sigma \vee \varphi)\}}{Z_{\Lambda, \varphi}}, \quad \forall \sigma \in \Omega_\Lambda, \quad \forall \varphi \in \Omega_{V \setminus \Lambda},$$

o'rinli bo'lsin. Bu yerda  $Z_{\Lambda, \varphi}$  normallovchi o'zgarma bo'lib, u quyidagicha aniqlanadi:

$$Z_{\Lambda, \varphi} = \sum_{\omega \in \Omega_{\Lambda}} \exp_p \{H(\omega \vee \varphi)\}.$$

Bunday o'lovga  $p$ -adik Gibbs o'lovchi deyiladi.

Birinchi bob uchunchi paragrafda Izing modeli uchun  $p$ -adik Gibbs o'lovchining yagonaligi, umumlashgan  $p$ -adik Gibbs o'lovchi tushunchasi va 2-tartibli Keli daraxtida Izing modeli uchun olingan natijalar keltirilgan.

$\Phi = \{-1, 1\}$  bo'lsin.  $\Omega_{V_n}$  da Izing modelining Gamiltoniani quyidagicha aniqlanadi:

$$H_n(\sigma_n) = J \sum_{\langle x, y \rangle \in L_n} \sigma_n(x) \sigma_n(y), \quad (1)$$

bunda  $0 < |J|_p < p^{-1/(p-1)}$  o'zgarmasdir.

$h: x \rightarrow h_x, \forall x \in V, h_x \in \mathbb{Q}_p$  funksiya bo'lsin.  $\Omega_{V_n}$  dagi quyidagicha aniqlangan  $\mu_h^{(n)}$   $p$ -adik ehtimollik o'lovchini qaraylik:

$$\mu_h^{(n)}(\sigma_n) = \frac{1}{Z_n^{(h)}} \exp_p \{H_n(\sigma_n)\} \prod_{x \in W_n} h_x^{\sigma_n(x)}, \quad n = 1, 2, \dots, \quad (2)$$

bu yerda  $Z_n^{(h)}$  normallovchi bo'luvchi bo'lib, u

$$Z_n^{(h)} = \sum_{\sigma_n(x) \in \Omega_{V_n}} \exp_p \{H_n(\sigma_n)\} \prod_{x \in W_n} h_x^{\sigma_n(x)},$$

formula bilan aniqlanadi.

Agar (2) formula orqali aniqlangan  $p$ -adik ehtimollik o'lovchilari uchun ixtiyoriy  $n \in \mathbb{N}$  va  $\sigma_{n-1} \in \Phi^{V_{n-1}}$  da

$$\sum_{\omega_n \in \Omega_{W_n}} \mu_h^{(n)}(\sigma_{n-1} \vee \omega_n) = \mu_h^{(n-1)}(\sigma_{n-1}), \quad (3)$$

tenglik o'rinli bo'lsa, u holda bu o'lovlar muvofiqlik shartini qanoatlantiradi, deyiladi. Agar (2) formula orqali aniqlangan  $p$ -adik ehtimollik o'lovchi (3) shartni qanoatlantirsa, u holda N.N.G'anixo'jayev, F.M.Muxamedov, U.A.Rozikovlar tomonidan isbotlangan Kolmogorovning o'lovni davom ettirish haqidagi teoremasining  $p$ -adik muqobiliga ko'ra, ixtiyoriy  $n \in \mathbb{N}$  va  $\sigma_n \in \Omega_{V_n}$  uchun  $\mu_h(\{\sigma \in \Omega: \sigma|_{V_n} \equiv \sigma_n\}) = \mu_h^{(n)}(\sigma_n)$  shartni qanoatlantiruvchi  $\Omega$  da yagona  $p$ -adik limit Gibbs o'lovchi mavjud bo'ladi.

F.M.Muxamedov ishlarida  $p$ -adik Gibbs o'lovchilari uchun faza almashish tushunchasi keltirilgan. Agar berilgan model uchun kamida ikkita  $\mu, \nu$   $p$ -adik Gibbs o'lovchilari mavjud bo'lib, ulardan biri chegaralangan, boshqasi esa chegaralanmagan bo'lsa, u holda berilgan model uchun faza almashish mavjud deyiladi. Bundan tashqari, agar ikkita turli  $h$  va  $s$  funksiyalar mavjud bo'lib, ularga mos  $\mu_h, \mu_s$   $p$ -adik Gibbs o'lovchilarining har ikkisi yoki chegaralangan, yoki chegaralanmagan bo'lsa, u holda kvazi faza almashishi ro'y beradi deyiladi.

M.Hamrayev, U.A.Rozikov hamda F.M.Muxamedovlarning ishlarida Keli daraxtidagi bir jinsli Izing model uchun  $p$ -adik Gibbs o'lovchilarining yagonalik

masalasi tadqiq qilingan. Ular bu model uchun faza almashinuvi mavjud emasligini, ya'ni yagona  $p$ -adik Gibbs o'lchovi mavjud ekanligini isbotlaganlar.

2014-yilda O.N.Hakimov Izing modeli uchun umumlashgan  $p$ -adik Gibbs o'lchovlari tushunchasini kiritdi. Agar qandaydir  $h$  funksiyasi uchun  $\mu_h^{(n)}$  o'lchovlari (3) muvofiqlik shartini qanoatlantirsa, u holda yagona  $p$ -adik ehtimollik o'lchovi mavjud bo'lib, uni  $h$  ga bog'liqligi sababli  $\mu_h$  bilan belgilangan. Bunday  $\mu_h$  o'lchov  $p$ -adik Izing modeliga mos umumlashgan  $p$ -adik Gibbs o'lchovi deyilgan.

Agar  $\forall x \in V$  uchun  $h_x \in \mathcal{E}_p$  bo'lsa, bunday  $h = \{h_x, x \in V\}$  miqdorlarga mos kelgan umumlashgan  $p$ -adik Gibbs o'lchovi  $p$ -adik Gibbs o'lchovi bo'ladi.

Ma'lumki, Keli daraxtida Izing modeli uchun (2) ko'rinishda aniqlangan  $\{\mu_h^{(n)}\}_{n \geq 1}$  ehtimollik o'lchovlari ketma-ketligi (3) muvofiqlik shartini qanoatlantirishi uchun ixtiyoriy  $x \in V \setminus \{x^0\}$  da  $h = \{h_x, x \in V\}$  miqdorlar to'plami ushbu

$$h_x^2 = \prod_{y \in S(x)} \frac{\theta h_y^2 + 1}{h_y^2 + \theta}, \quad (4)$$

tenglikni qanoatlantirishi zarur va yetarli, bu yerda  $\theta = \exp_p \{2J\}$ .

Shuningdek, O.N.Hakimov ikkinchi tartibli Keli daraxtida Izing modeli uchun translyatsion-invariant va davriy umumlashgan  $p$ -adik Gibbs o'lchovlarni topdi. Topilgan o'lchovlarni chegaralanganlikka tekshirib, qaralayotgan model uchun faza almashishi masalasini hal qildi.

Dissertatsiyaning ikkinchi bobi "**Keli daraxtida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari**" deb nomlanadi. "*Keli daraxtidagi kuchsiz davriy Gibbs o'lchovlari*" deb nomlangan 2.1-paragrafda asosiy tushunchalar, ma'lum natijalar keltirilgan. "*Keli daraxtida kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovining mavjudligi*" deb nomlangan 2.2-paragrafda Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining mavjudligi isbotlandi. "*Ikkinchi va uchinchi tartibli Keli daraxtlarida kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari*" deb nomlangan 2.3-paragrafda 2- va 3-tartibli Keli daraxtlaridagi Izing modeli uchun bunday o'lchovlar qurilgan va o'rganilgan. Bu o'lchovlarning chegaralanganlik va faza almashinuvi masalalari "*Kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining chegaralanganligi va faza almashish muammosi*" deb nomlangan 2.4-paragrafda hal qilingan.

Ilk bora 2008-yilda U.A.Rozikov va M.M.Rahmatullayevlar tomonidan translyatsion-invariant va davriy o'lchovlar tushunchalaridan ko'ra umumiyroq bo'lgan kuchsiz davriy Gibbs o'lchovi tushunchasi kiritildi. Quyida bu tushunchalar keltirilgan.

$G_k^* - G_k$  gruppning chekli indeksli normal qism gruppasi va  $G_k / G_k^* = \{H_0, H_1, \dots, H_{r-1}\}$  - chekli ( $r \geq 1$  indeksli) faktor gruppasi bo'lsin.

**2-ta'rif.** Barcha  $x \in H_i$  lar uchun  $h_x = h_i$  tenglik o'rinli bo'ladigan  $h = \{h_x, x \in G_k\}$  miqdorlar to'plami  $G_k^*$ -kuchsiz davriy deb ataladi.  $G_k$ -davriy miqdorlar translyatsion-invariant deb ataladi.

**3-ta'rif.** Ixtiyoriy  $x \in H_i$  va  $x_{\downarrow} \in H_j$  lar uchun  $h_x = h_{i,j}$  o'rinli bo'ladigan  $h = \{h_x, x \in G_k\}$  miqdorlar to'plami  $G_k^*$ -davriy deb ataladi.

**4-ta'rif.**  $G_k^*$ -(kuchsiz) davriy  $h$  ga mos  $\mu$  umumlashgan  $p$ -adik Gibbs o'lchovi  $G_k^*$ -(kuchsiz) davriy umumlashgan  $p$ -adik Gibbs o'lchovi deb ataladi.  $G_k$ -davriy o'lchov translyatsion-invariant o'lchov deyiladi.

Ta'kidlash lozimki,  $G_k$  gruppning ixtiyoriy ikki indeksli normal bo'luvchisi

$$H_A = \{x \in G_k : \sum_{i \in A} \omega_x(a_i) - \text{juftson}\}$$

ko'rinishida bo'lib,  $\emptyset \neq A \subseteq N_k = \{1, 2, 3, \dots, k+1\}$  va  $\omega_x(a_i) - x \in G_k$  so'zdagi  $a_i$  harflar sonidir.  $H_A - G_k$  ning normal qism gruppasi. Ko'rish mumkinki, kuchsiz davriy Gibbs o'lchovlari  $G_k$  ning normal qism gruppasining tanlanishiga bog'liq.  $|A| = k+1$  bo'lgan holda, ya'ni  $A = N_k$  bo'lsa, kuchsiz davriylik tushunchasi odatdagi davriylik tushunchasi bilan ustma-ust tushadi. Shuning uchun biz  $A \neq N_k$  bo'lgan  $A \subset N_k$  holni qaraymiz. Shunda  $H_A$ -kuchsiz davriy  $h_x$  lar to'plami quyidagi ko'rinishda bo'ladi:

$$h_x = \begin{cases} h_{00}, & \text{agar } x \in H_0, x_{\downarrow} \in H_0 \text{ bo'lsa,} \\ h_{01}, & \text{agar } x \in H_0, x_{\downarrow} \in H_1 \text{ bo'lsa,} \\ h_{10}, & \text{agar } x \in H_1, x_{\downarrow} \in H_0 \text{ bo'lsa,} \\ h_{11}, & \text{agar } x \in H_1, x_{\downarrow} \in H_1 \text{ bo'lsa,} \end{cases}$$

bu yerda  $H_0 := H_A$  va  $H_1 := G_k \setminus H_A$ .

Haqiqiy sonlar maydoni ustida ikki indeksli va to'rt indeksli normal qism gruppalariga mos kuchsiz davriy Gibbs o'lchovlari U.A.Rozikov, M.M.Rahmatullayev va J.D.Dehqonovlarning tadqiqotlarida yetarlicha o'rganilgan. Bundan tashqari, D.Gandolfo, J.Ruiz va S.Shlosmanlar tomonidan  $k$ -tartibli Keli daraxtida Gibbs o'lchovlarining kuchsiz davriy Gibbs o'lchovlaridan farqli yangi sinfi qurilgan.

Ikkinchi bob birinchi paragrafda  $p$ -adik sonlar maydoni ustida 2-tartibli Keli daraxtida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari mavjudligi isbotlanib, quyidagi natija olindi:

**1-teorema.** Agar  $p \equiv 1 \pmod{4}$  bo'lsa, u holda 2-tartibli Keli daraxtida Izing modeli uchun kamida bitta  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovi mavjud bo'ladi.

1-teoremada  $p$ -adik sonlar maydonida kamida bitta kuchsiz davriy umumlashgan Gibbs o'lchovi mavjudligi ko'rsatilgan. Ushbu natijaga asoslanib,

ikkinchi tartibli Keli daraxtida Izing modeli uchun barcha kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlarining tavsifi quyidagi teoremda keltirilgan.

**2-teorema.** Aytaylik,  $|A|=1$  bo'lsin. U holda ikkinchi tartibli Keli daraxtida Izing modeli uchun oltita turli  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari mavjud bo'ladi.

Ikkinchi tartibli Keli daraxtida Izing modeli uchun olib borilgan tadqiqotlar uchinchi tartibli Keli daraxtiga kengaytirilib,

$$I_1 = \{(h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = h_{11} = h_{01} = h_{10}\},$$

$$I_2 = \{(h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = h_{11}, h_{01} = h_{10}\},$$

$$I_3 = \left\{ (h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = \frac{1}{h_{11}}, h_{01} = \frac{1}{h_{10}} \right\}$$

invariant to'plamlar ustida quyidagi natijalar olindi:

**3-teorema.** Aytaylik,  $k=3$ ,  $|A|=1$  va  $\mathcal{M}_1(p) - I_2$  invariant to'plamdagi va  $\mathcal{M}_2(p) - I_3$  invariant to'plamdagi (4) ning  $H_A$ -kuchsiz davriy yechimlariga mos  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari soni bo'lsin. U holda quyidagi tasdiqlar o'rinli:

$$\mathcal{M}_1(p) = \begin{cases} 6, & \text{agar } p \equiv 1 \pmod{12}, \\ 4, & \text{agar } p \equiv 5 \pmod{12}, \\ 1, & \text{boshqa hollarda.} \end{cases} \quad \mathcal{M}_2(p) = \begin{cases} 6, & \text{agar } p \equiv 1 \pmod{12}, \\ 3, & \text{agar } p \equiv 5 \pmod{12}, \\ 1, & \text{boshqa hollarda.} \end{cases}$$

Qaralayotgan model uchun qanday faza almashish yuz berishini aniqlashda o'lchovning chegaralanganligini tekshirish muhim hisoblanadi. Shu boisdan, 2.4-paragrafda topilgan o'lchovlar chegaralanganlikka tekshirildi.

**4-teorema.** Aytaylik,  $p \equiv 1 \pmod{4}$  va  $|A|=1$  bo'lsin. U holda ikkinchi tartibli Keli daraxtida Izing modeli uchun oltita chegaralanmagan  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari mavjud bo'ladi.

3-tartibli Keli daraxtida (4) ning  $I_2$  invariant to'plamdagi kuchsiz davriy yechimlari orasida ixtiyoriy  $p$  tub sonda mavjud bo'ladigan  $h_x^2 = h_0^{(1)}$ ,  $\forall x \in V$  yechimlarga mos o'lchovni  $\mu_h^1$  bilan belgilangan, shu invariant to'plamda quyidagi natija olingan:

**5-teorema.** Aytaylik,  $k=3$ ,  $|A|=1$  va  $\mathcal{h} - (4)$  ning  $I_2$  invariant to'plamdagi  $H_A$ -kuchsiz davriy yechimi bo'lsin. U holda Keli daraxtida Izing modeli uchun quyidagi tasdiqlar o'rinli:

1.  $p=2$  bo'lganda faqat  $\mu_h^1$   $H_A$ -kuchsiz davriy (translyatsion-invariant bo'lmagan) umumlashgan  $p$ -adik Gibbs o'lchovi chegaralanmagan bo'ladi.

2.  $p \neq 2$  bo'lganda faqat  $\mu_h^1$   $H_A$ -kuchsiz davriy (translyatsion-invariant bo'lmagan) umumlashgan  $p$ -adik Gibbs o'lchovi chegaralangan bo'ladi.

**6-teorema.** Aytaylik,  $k=3$ ,  $|A|=1$  va  $h - (4)$  ning  $I_3 \setminus I_2$  to'plamdagi  $H_A$ -kuchsiz davriy yechimi bo'lsin. Agar quyidagi shartlardan biri bajarilsa, u holda barcha  $H_A$ -kuchsiz davriy (translyatsion-invariant bo'lmagan) umumlashgan

$p$ -adik Gibbs o'lchovlari chegaralanmagan bo'ladi:

$$1) p \equiv 1 \pmod{12}.$$

$$2) p \equiv 5 \pmod{12}.$$

**1-natija.** Aytaylik,  $p \equiv 5 \pmod{12}$  bo'lsin. U holda uchinchi tartibli Keli daraxtida  $p$ -adik Izing modeli uchun faza almashinuvi ro'y beradi.

**1-izoh.** Agar  $p \equiv 1 \pmod{12}$  bo'lsa, u holda qaralayotgan model uchun faza almashinuvi mavjud bo'ladi. Shuni ta'kidlash kerakki, bu shart M.M.Rahmatullayev, O.N.Hakimov va A.M.To'xtaboyevlarning ishlaridagi shart bilan mos keladi.

Dissertatsiyaning uchinchi bobi "**Tashqi maydonli Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lchovlari**" deb nomlangan. "*Keli daraxtida tashqi maydonli Izing modeli*" deb nomlangan 3.1-paragrafda tashqi maydonli  $p$ -adik Izing modeli uchun ma'lum faktlar va olingan natijalar keltirilgan. Tashqi maydonli  $p$ -adik Izing modelining Gamiltoniani

$$H_n(\sigma_n) = J \sum_{\langle x,y \rangle \in L_n} \sigma_n(x)\sigma_n(y) + \alpha \sum_{x \in V_n} \sigma_n(x) \quad (5)$$

ko'rinishida bo'lib, bu yerda  $J, \alpha \in B(0, p^{-1/(p-1)}) \setminus \{0\}$  o'zgarmaslardir. Ta'kidlash kerakki, agar  $\alpha = 0$  bo'lsa, u holda (5) Gamiltonian tashqi maydonsiz  $p$ -adik Izing modelining Gamiltonianini ifodalaydi. Shuning uchun  $\alpha \neq 0$  bo'lgan hol o'rganilgan.

Ma'lumki, Keli daraxtida tashqi maydonli  $p$ -adik Izing modeli uchun (2) ko'rinishda aniqlangan  $\{\mu_h^{(n)}(\sigma_n)\}_{n \geq 1}$  o'lchovlar ketma-ketligi (3) muvofiqlik shartini qanoatlantirishi uchun ixtiyoriy  $x \in V \setminus \{x^0\}$  da

$$\hat{h}_x = \eta^{k+1} \prod_{y \in S(x)} \frac{\theta \hat{h}_y + 1}{\hat{h}_y + \theta} \quad (6)$$

tenglikni qanoatlantirishi zarur va yetarli, bu yerda  $\theta = \exp_p\{2J\}$ ,  $\eta = \exp_p\left(\frac{2\alpha}{k+1}\right)$ ,  $\hat{h}_x = \eta \exp_p\{2h_x\}$ .

Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun translyatsion-invariant va  $G_2^{(2)}$ -davriy Gibbs o'lchovlari F.M.Muxamedov, M.M.Rahmatullayev, A.M.To'xtaboyev va R.M.Mamajanovlar tomonidan o'rganilgan.

“Tashqi maydonli Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o‘lchovlari” deb nomlangan 3.2-paragrafda aynan shu model uchun  $|A|=1$  va  $|A|=2$  bo‘lgan hollarda  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o‘lchovlari tadqiq qilingan.  $|A|=1$  bo‘lganda (6) ni invariant to‘plamda yechib, quyidagi natijalar olingan:

**7-teorema.** Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun kamida bitta  $H_A$ -kuchsiz davriy (translyatsion-invariant bo‘lmagan) umumlashgan  $p$ -adik Gibbs o‘lchovi mavjud bo‘ladi.

Bu o‘lchov chegaralanganlikka tekshirilgan hamda quyidagi natija olindi:

**8-teorema.** Faraz qilaylik,  $\hat{h}_x$  – (6) tenglamaning  $H_A$ -kuchsiz davriy yechimi bo‘lsin. U holda ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun kamida bitta chegaralanmagan  $H_A$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o‘lchovi mavjud bo‘ladi.

**9-teorema.** Faraz qilaylik,  $|A|=1$  bo‘lsin. U holda ikkinchi tartibli Keli daraxtida  $p$ -adik Izing modeli uchun quyidagi mulohazalar o‘rinli:

- 1) agar  $p=2$ . bo‘lsa, u holda kvazi faza almashinuvi yuz beradi;
- 2) istalgan  $p$  toq tub son uchun esa, faza almashinuvi sodir bo‘ladi.

$|A|=2$  bo‘lganda  $G_k$  ning ikki indeksli normal qism gruppasi

$$H_{A_2} = \{x \in G_k : (\omega_x(a_i) + \omega_x(a_j)) \equiv 0 \pmod{2}\}, i, j \in \{1, 2, 3\}, i \neq j$$

ko‘rinishida bo‘ladi. Agar  $|A|=1$  va  $|A|=2$  bo‘lganda (6) tenglamaning yechimlariga mos bo‘lgan o‘lchovlar mos ravishda  $H_{A_1}$ - va  $H_{A_2}$ -kuchsiz davriy o‘lchovlar deb belgilansa, u holda (6) ning kuchsiz davriy yechimlari quyidagi natijani olishni ta‘minladi:

**10-teorema.** Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun kamida bitta  $H_{A_1}$ -kuchsiz davriy va kamida bitta  $H_{A_2}$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o‘lchovi mavjud bo‘ladi.

Bu o‘lchovlarni chegaralanganlikka tekshirish, quyidagi natijani berdi:

**11-teorema.** Faraz qilaylik,  $p \geq 3$  bo‘lsin. U holda ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun kamida bitta chegaralanmagan  $H_{A_1}$ -kuchsiz davriy va kamida bitta chegaralanmagan  $H_{A_2}$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o‘lchovi mavjud bo‘ladi.

**12-teorema.** Har qanday  $p$  toq tub sonda, ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun faza almashinuvi sodir bo‘ladi.

Uchinchi bobning uchinchi paragrafi “3-tartibli Keli daraxtida bir jinsli tashqi maydonli Izing modeli uchun translyatsion-invariant umumlashgan  $p$ -adik Gibbs o‘lchovlari” deb nomlangan bo‘lib,  $\mathbb{Q}_p$  maydonda (6) ning translyatsion-invariant yechimlari mavjud bo‘lishi uchun  $p$  tub songa shartlar

topilgan. Topilgan o'lovchlarni chegaralanganlikka tekshirish bilan quyidagi natija olindi:

**13-teorema.** Faraz qilaylik,  $p > 3$  bo'lsin. U holda uchinchi tartibli Keli daraxtida bir jinsli tashqi maydonli Izing modeli uchun quyidagi tasdiqlar o'rinli:

1) agar  $p \equiv 1 \pmod{6}$  bo'lsa, u holda bitta chegaralangan va uchta chegaralanmagan translyatsion-invariant umumlashgan  $p$ -adik Gibbs o'lovchlari mavjud bo'ladi;

2) agar  $p \not\equiv 1 \pmod{6}$  bo'lsa, u holda bitta chegaralangan va bitta chegaralanmagan translyatsion-invariant umumlashgan  $p$ -adik Gibbs o'lovchlari mavjud bo'ladi.

**14-teorema.** Aytaylik,  $p > 3$  bo'lsin. U holda bir jinsli tashqi maydonli Izing modelida faza almashinuvi mavjud bo'ladi.

## XULOSA

Dissertatsiya ikkinchi va uchinchi tartibli Keli daraxtlarida tashqi maydonli va tashqi maydonsiz Izing modellariga mos kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovchlari to'plamini tavsiflashga hamda bu modellarga mos  $p$ -adik faza almashish turlarini tekshirishga bag'ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1.  $p$ -adik sonlar maydoni ustida 2-tartibli Keli daraxtida Izing modeli uchun kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovchlari mavjudligi isbotlangan.

2. Agar  $p \equiv 1 \pmod{4}$  bo'lsa, 2-tartibli Keli daraxtida Izing modeli uchun to'rtta kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovchlari mavjudligi ko'rsatildi. Bu o'lovchlar chegaralanganlikka tekshirilgan.

3. Uchinchi tartibli Keli daraxtida Izing modeli kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovchlari topilgan.

4. Agar  $p \equiv 5 \pmod{12}$  bo'lsa, u holda uchinchi tartibli Keli daraxtida Izing modeli uchun faza almashishi mavjudligi isbotlangan.

5. Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun invariant to'plam ustida har qanday tub son uchun kamida bitta chegaralanmagan  $H_{A_1}$ - va kamida bitta chegaralanmagan  $H_{A_2}$ -kuchsiz davriy umumlashgan  $p$ -adik Gibbs o'lovchlari mavjudligi isbotlangan.

6. Ikkinchi tartibli Keli daraxtida tashqi maydonli Izing modeli uchun  $p = 2$  bo'lganda kvazi faza almashishi ro'y berishi hamda har qanday  $p$  toq tub son uchun faza almashishi yuz berishi ko'rsatilgan.

7. Uchinchi tartibli Keli daraxtida bir jinsli tashqi maydonli Izing modeli uchun barcha translyatsion-invariant umumlashgan  $p$ -adik Gibbs o'lovchlari topilgan.

8.  $p > 3$  bo'lganda, uchinchi tartibli Keli daraxtida bir jinsli tashqi maydonli Izing modeli uchun faza almashishi mavjud ekanligi isbotlangan.

**SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES  
DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED  
AFTER V.I. ROMANOVSKY**

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**NAMANGAN STATE UNIVERSITY**

**ABDUKAHOROVA ZULKHUMOR TUKHTASINOVNA**

**WEAKLY PERIODIC  $p$ -ADIC GENERALIZED GIBBS MEASURES FOR  
SOME MODELS**

**01.01.01-Mathematical analysis**

**ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON  
PHYSICAL AND MATHEMATICAL SCIENCES**

**Namangan – 2025**

**The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number № B2023.3.PhD/FM903**

Dissertation has been prepared at Namangan State University.

The abstract of the dissertation is posted in three languages (Uzbek, English, Russian (resume)) on the website of Scientific Council (<https://kengash.mathinst.uz>) and the “ZiyoNet” information and educational portal (<http://www.ziynet.uz>).

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Defense will take place on “ 16 ” December 2025 at 16:00 at the meeting of Scientific Council DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I.Romanovskiy. (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40, e-mail: [uzbmath@umail.uz](mailto:uzbmath@umail.uz), Website: [www.mathinst.uz](http://www.mathinst.uz)).

Dissertation is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovskiy (registered for No. 217). (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40).

Abstract of dissertation sent out on “ 02 ” December 2025 year.  
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**INTRODUCTION (abstract of PhD thesis)**

**Actuality and demand of the theme of dissertation.** Various scientific and applied studies conducted worldwide to investigate the thermodynamic properties of physical and biological processes often lead to models of statistical mechanics. These models are naturally divided into Archimedean and non-Archimedean types. In the study of models describing processes occurring in a non-Archimedean setting,  $p$ -adic Gibbs measures serve as a fundamental tool and play a central role in the theory of phase transitions.  $p$ -Adic Gibbs measures are important not only for the theoretical development of statistical physics but also for solving practical problems in fields such as chemistry, materials science, biology, and information technology. However, even for classical models of statistical mechanics, a complete description of the set of  $p$ -adic Gibbs measures remains an open and complex problem, which makes the construction, classification, and analysis of the properties of such measures an important and topical area of research.

Nowadays in the world, one of the actual problems is to determine the existence of  $p$ -adic Gibbs measures for a given Hamiltonian on lattice systems and to provide a complete description of the set of such measures. In the theory of phase transitions in statistical physics, the existence of at least two generalized  $p$ -adic Gibbs measures for a given physical model is of significant importance. In this regard, determining the existence of translation-invariant, periodic, and weakly periodic generalized  $p$ -adic Gibbs measures corresponding to Hamiltonians with and without an external field, as well as providing a full description of the set of such measures and verifying the boundedness of the obtained measures in order to establish the occurrence of a phase transition for the considered models, constitute one of the essential directions of current scientific research.

In our country, special attention is paid to the scientific and practical applications of the fundamental sciences, which are considered one of the main pillars of development. In particular, in recent years, substantial progress has been made in determining the existence of translation-invariant, periodic, and weakly periodic Gibbs measures for classical models of statistical mechanics on the Cayley tree, both over the fields of real and  $p$ -adic numbers, as well as in enriching the set of such measures. In the Republic, the following fields have been identified as priority directions for fundamental scientific research: algebra and its applications, differential equations and their applications, mathematical modeling of nonlinear and dynamical systems and their applications, stochastic analysis, medical-biological informatics, and computational mathematics<sup>1</sup>. To ensure the implementation of this Resolution, the development of the theory of phase transitions for classical models of statistical mechanics, and the application of the obtained theoretical results in related scientific fields, are of significant importance.

The subject and object of the research of this dissertation are in line with the tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the Strategy of Actions for the Further Development of the Republic of Uzbekistan”, UP-60 of January 28, 2022 “On the

Development Strategy of New Uzbekistan for 2022–2026”, UP-2789 dated February 17, 2017 “On measures to further improve the activities of the Academy of Sciences, organization, management and financing of research activities”, PP-4387 of July 9, 2019 “On measures to further develop mathematical education and science, and radically improve the activities of the V.I. Romanovskiy Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan”, and PP-4708 dated May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics”, as well as other normative-legal documents related to the development of basic sciences.

**Connection of research to priority directions of development of science and technologies of the Republic.** This study was conducted in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

**The degree of scrutiny of the problem.** The American scientist J. Willard Gibbs introduced the concept of the Gibbs distribution, which plays a crucial role for systems with constant temperature in thermal equilibrium with their environment. The general properties of real-valued limiting Gibbs measures were established in the studies of R.L.Dobrushin, O.Lanford, and D.Ruelle. The modern theoretical foundations of these measures were extensively developed in the scientific works of R.Baxter, H.O.Georgii, V.A.Malyshev, R.A.Minlos, K.Preston, D.Ruelle, Ya.G.Sinai, G.Gallavotti, F.Bonetto, J.Gentile, Jin Zin-Justin, N.N.Ganikhodjaev, U.A.Rozikov, and F.M.Mukhamedov. Moreover, R.L.Dobrushin proved an important theorem on the existence of real-valued limiting Gibbs measures. The problem of Gibbs measures for classical models of statistical mechanics has been thoroughly investigated in the works of R.L.Dobrushin, G.R.Brightwell, P.Winkler, Yu.M.Suhov, J.Martin, N.N.Ganikhodjaev, U.A.Rozikov, F.M.Mukhamedov, D.Gandolfo, M.M.Rakhmatullaev, R.M.Khakimov, G.I.Botirov, O.N.Khakimov, E.Normatov, Sh.Shoyusupov, F.Haydarov, M.A.Rasulova, A.M.Tukhtabaev, B.U.Abraev, O.Sh.Karshibayev, J.Dehqonov, and other researchers. For the Ising model on the lattice, the uniqueness of the  $p$ -adic Gibbs distribution was demonstrated by N.N.Ganikhodjaev, F.M.Mukhamedov, and U.A.Rozikov. The work of N.N.Ganikhodjaev and H.Akin demonstrated the non-uniqueness of the  $p$ -adic Gibbs measure for the Ising-Vannimenus model. O.N.Khakimov examined translation-invariant and periodic  $p$ -adic quasi Gibbs measures on the Cayley tree of order two for the Vannimenus model. These works were continued for the Ising-Vannimenus model by F.M.Mukhamedov, M.K.Saburov and O.N.Khakimov.

The existence of a unique  $p$ -adic Gibbs measure for the homogeneous Ising model was established by M.Khamraev, F.M.Mukhamedov and U.A.Rozikov. O.N.Khakimov introduced the notion of  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree. He investigated translation-invariant and  $G_2^{(2)}$ -periodic  $p$ -adic generalized Gibbs measures for this model. A boundedness criterion for translation-invariant and periodic  $p$ -adic generalized Gibbs measures was

obtained, and the existence of a phase transition for the homogeneous Ising model was proved. The chaotic behavior of the dynamical system associated with  $p$ -adic generalized Gibbs measures has been investigated by F.M. Mukhamedov, O.N. Hakimov, H. Akin, and M. Dogan.

In 2008, over the field of real numbers, the notion of a weakly periodic Gibbs measure was introduced. The existence of such measures for the Ising model was investigated by U.A.Rozikov and M.M.Rakhmatullaev. In particular, on  $\mathbb{R}$ , weakly periodic Gibbs measures corresponding to normal subgroups of indexes 2 and 4 in the group representation of the Cayley tree were explicitly described. It should be noted that, despite the existence of numerous studies mentioned above, a complete classification of all limiting Gibbs measures for any model on the Cayley tree has not yet been obtained. Moreover, weakly periodic  $p$ -adic generalized Gibbs measures have not been fully investigated so far.

**Connection of the theme of the dissertation with the research works of scientific research Institute, where the dissertation is carried out.** Dissertation research was carried out within the “Fundamental research” branch of the research plan of Namangan State University.

**The aim of this research** is to establish the existence of weakly periodic generalized  $p$ -adic Gibbs measures for the Ising model on the Cayley tree with and without an external field, and, by examining their boundedness, to demonstrate the occurrence of a phase transition in the given models.

**Research problems are as follows:**

To prove the existence of  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree of order two;

To describe the set of  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on Cayley trees of orders two and three, to verify the boundedness of the obtained measures, and to address the problem of phase transition for the considered models;

For the cases  $|A|=1$  and  $|A|=2$  to determine the weakly periodic  $p$ -adic generalized Gibbs measures corresponding to the normal subgroup of index 2 to the Ising model with an external field on the Cayley tree of order 2, and to demonstrate the existence of phase transition for this model;

To provide a complete characterization of translation-invariant  $p$ -adic generalized Gibbs measures for the Ising model with a homogeneous external field on the Cayley tree of order three, and to investigate the problem of phase transition for this model.

**The research object:**  $p$ -adic Ising models without an external field and with an external field on the Cayley tree.

**The research subject:** Theory of groups and graphs, Gibbs measures theory, Algebra and number theory,  $p$ -adic analysis, non-linear Markov processes.

**Research methods:** In the research the methods of  $p$ -adic analysis, number theory, functional analysis, group theory, measures theory, theory of linear algebra are

used.

**The scientific novelty of the research** consists of the followings:

the existence of  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measure for the Ising model on the Cayley tree of order two has been established;

the  $H_A$ -weakly periodic generalized  $p$ -adic Gibbs measures for the Ising model on Cayley trees of orders two and three have been investigated. The obtained measures were examined for boundedness, and the existence of phase transitions was demonstrated;

the weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model with an external field on the Cayley tree of order two, corresponding to the normal subgroup  $H_A$  of index two, for the cases  $|A|=1$  and  $|A|=2$  have been determined, and it has been proven that a phase transition exists when  $p > 2$ ;

for the Ising model with a homogeneous external field on the Cayley tree of order three, the translation-invariant  $p$ -adic generalized Gibbs measures have been identified, and it has been established that a phase transition occurs when  $p > 3$ .

**Practical results of the research** consist of the following:

The obtained results and the methods used in the dissertation can be taught as a course for graduate students of higher education and after the higher education. Also, the exact expression of parameter values that ensure the existence of the different phase transitions in a set of weakly periodic  $p$ -adic generalized Gibbs measures can be used to solve service theory problems.

**The reliability of the results of the study.** The results have been obtained by using the methods of  $p$ -adic analysis, non-Archimedean functional analysis, non-Archimedean Markov theory of random fields,  $p$ -adic probability theory, number theory, Gibbs and non-Archimedean measure theory. The obtained results are mathematically strongly proved.

**Scientific and practical significance of research results.** The scientific importance of the results of the research work is explained by the fact that various  $p$ -adic models of statistical mechanics can be used in the developing of Gibbs measures theory.

The practical significance of the research determines by the changing of state of physical systems is investigated and it gives opportunity to solve some problems of combinatorics and telecommunications.

**Implementation of the research results.** Based on the results obtained for the generalized  $p$ -adic Gibbs measures for the Ising model:

Theoretical approaches derived from the description of weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley trees of orders two and three, and the methods for checking the boundedness of these measures, were used in the foreign project No. 374824-2022, titled “*Phase transition problems and critical phenomena. Mathematical aspects of their equations, fast transitions and asymptotics*” to improve methods for determining phase transitions and critical phenomena. (Osh State University Certificate No. 1400 dated 15.10.2025, Kyrgyzstan).

The application of this scientific result enabled the determination of phase transitions and critical temperatures;

The complete description of  $H_A$ -weakly periodic generalized  $p$ -adic Gibbs measures for the Ising model on Cayley trees has been utilized within the international research grant project “*Chaotic and mixed  $p$ -adic dynamical systems associated with renormalized groups of lattice models*” G0003247, United Arab Emirates University, 2025 (United Arab Emirates University Certificate, dated October 23, 2025). These results provided a foundation for introducing new insights into the theory of  $p$ -adic dynamical systems and furnish a practical framework for investigating the thermodynamic properties of physical models;

The methods used to establish the existence of weakly periodic  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model on the Cayley tree of order two were applied in the fundamental research project No. F-FA-2021-425, titled “*Gradient Probability Measures on Lattice Systems,*” to determine the occurrence of a phase transition in lattice systems (Institute of Mathematics Certificate No. 2/416 dated 27.10. 2025). The application of this scientific result made it possible to characterize non-periodic Gibbs measures for models with nearest-neighbor interactions on lattice systems.

**Approbation of the research results.** The main results of the research have been discussed in 8 international and 4 national scientific conferences.

**Publications of the research results.** On the topic of the dissertation 6 research papers have been published in the scientific journals, all of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, including 4 in foreign and 2 in Republic journals.

**The structure and volume of the dissertation.** The dissertation consists of an introduction, three chapters, conclusion and bibliography. The volume of the thesis is 114 pages.

## THE MAIN CONTENT OF THE DISSERTATION

**In the introduction** besides the motivation of research theme and correspondence to the priority research areas of science and technology of the Republic, we present the degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we reduce the theoretical and practical importance of the obtained results, and give information on the implementation of the research results, the published works and the structure of dissertation.

The first chapter of the dissertation, entitled “**On  $p$ -adic analysis and  $p$ -adic measures**”, presents the basic concepts and definitions used to present the results obtained in this work, together with the fundamental theorems applied during the research. Furthermore, this chapter is devoted to the uniqueness problem of the  $p$ -adic Gibbs measure for the Ising model on the Cayley tree, the notion of  $p$ -adic generalized Gibbs measures, and the results obtained for these measures on the Cayley tree of order two. This chapter serves as the theoretical and methodological foundation

for explaining and substantiating the main scientific results presented in the subsequent chapters.

Let  $\mathbb{Q}$  be the field of rational numbers. For a fixed prime number  $p$ , every rational number  $x \neq 0$  can be represented in the form  $x = p^r \frac{m}{n}$  where,  $m \in \mathbb{Z}$ ,  $n \in \mathbb{N}$ , and  $m, n$  are not divisible by  $p$ . The  $p$ -adic norm of  $x \in \mathbb{Q}$  is defined as follows

$$|x|_p = p^{-r}.$$

Additionally, the  $p$ -adic norm of zero is defined by

$$|0|_p = 0.$$

This norm is non-Archimedean; that is, it satisfies the strong triangle inequality

$$|x + y|_p \leq \max\{|x|_p, |y|_p\}$$

for all  $x, y \in \mathbb{Q}$ .

The completion of  $\mathbb{Q}$  with respect to the  $p$ -adic norm yields the field of  $p$ -adic numbers, denoted by  $\mathbb{Q}_p$ . Every nonzero element  $x \in \mathbb{Q}_p$  has a unique canonical representation of the form:

$$x = p^{\gamma(x)}(x_0 + x_1 p + x_2 p^2 + \dots),$$

where  $\gamma(x), x_j \in \mathbb{Z}$  with  $x_0 > 0, 0 \leq x_j \leq p - 1$ . In this case,  $|x|_p = p^{-\gamma(x)}$ .

Below, we present the sets that are important in our research.

Let  $a \in \mathbb{Q}_p$ , and  $r > 0, r \in \mathbb{R}$ . The open ball of radius  $r$  centred at  $a$  is the set

$$B(a, r) = \{x \in \mathbb{Q}_p : |x - a|_p < r\},$$

the closed ball of radius  $r$  centred at  $a$  is the set

$$\overline{B(a, r)} = \{x \in \mathbb{Q}_p : |x - a|_p \leq r\},$$

the sphere of radius  $r$  centred at  $a$  is the set

$$S(a, r) = \{x \in \mathbb{Q}_p : |x - a|_p = r\},$$

the set of  $p$ -adic integers is defined as

$$\mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\},$$

the set of  $p$ -adic units is

$$\mathbb{Z}_p^* = \{x \in \mathbb{Q}_p : |x|_p = 1\}.$$

We now introduce a function that is important in the theory of  $p$ -adic Gibbs measures.

$p$ -adic exponential is defined by

$$\exp_p(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

which converges for  $x \in B(0, p^{-1/(p-1)})$ .

The range of the  $p$ -adic exponential function is given as follows:

$$\mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x - 1|_p < p^{-1/(p-1)}\right\}.$$

Let  $(X, \mathfrak{B})$  be a measurable space, where  $\mathfrak{B}$  is an algebra of subsets  $\mathfrak{X}$  and let

$\mathbb{K}$  be a non-Archimedean field. A function  $\mu: \mathfrak{B} \rightarrow \mathbb{K}$  is said to be a non-Archimedean measure if for any  $A = \bigcup_{k=1}^n A_k$ ,  $A_k \in \mathfrak{B}$ ,  $k = \overline{1, n}$  such that  $A_i \cap A_j = \emptyset$ ,  $i \neq j$ , the following holds:

$$\mu\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n \mu(A_i).$$

A non-Archimedean measure is called *probability measure* if  $\mu(X) = 1$ . A non-Archimedean measure  $\mu$  is called a bounded if  $\sup\{|\mu(A)|_p : A \in \mathfrak{B}\} < \infty$ .

Let  $\Gamma^k = (V, L)$  be a Cayley tree of order  $k \geq 1$ .  $V$  is the set of vertices,  $L$  is the set of edges. Two vertices  $x$  and  $y$  are called nearest neighbors, if there exist an edge  $l \in L$  connecting them and denote by  $l = \langle x, y \rangle$ . For  $x, y \in V$ , a *path* connecting them is a sequence of edges  $\langle x, x_1 \rangle, \dots, \langle x_{d-1}, y \rangle$ . The distance  $d(x, y)$  between  $x$  and  $y$  is defined as the number of edges in the shortest path connecting them.

For a fixed  $x^0 \in V$ , we consider the following sets:

$$W_n = \{x \in V : d(x, x^0) = n\}, \quad V_n = \bigcup_{j=0}^n W_j, \quad S(x) = \{y \in W_{n+1} : d(x, y) = 1\}, \quad x \in W_n$$

$$L_n = \{l = \langle x, y \rangle \in L \mid x, y \in V_n\}; \quad S_1(x) = \{y \in V : d(x, y) = 1\}; \quad \{x_\downarrow\} = S_1(x) \setminus S(x).$$

Let  $G_k$  be a free product of  $k + 1$  cyclic groups of the second order with generators  $a_1, a_2, \dots, a_{k+1}$ , respectively.

The following proposition was proved by N.N.Ganikhodjaev.

**Proposition 1.** There exists a one-to-one correspondence between the set of vertices  $V$  of the Cayley tree  $\Gamma^k = (V, L)$  and the group  $G_k$ .

Let  $\Phi \subset \mathbb{Z}$  be a finite set. For  $\Lambda \subset V$ , a configuration  $\sigma_\Lambda$  on  $\Lambda$  is defined as a function

$$x \in \Lambda \rightarrow \sigma_\Lambda(x) \in \Phi.$$

The set all of configurations on  $\Lambda$  is denoted by  $\Omega_\Lambda = \Phi^\Lambda$ , in particular,  $\Omega = \Omega_V$ .

For given configurations  $\sigma \in \Omega_\Lambda$  and  $\varphi \in \Omega_{V \setminus \Lambda}$  we define a configuration on  $\Omega$  as follows:

$$(\sigma \vee \varphi)(x) = \begin{cases} \sigma(x), & \text{if } x \in \Lambda, \\ \varphi(x), & \text{if } x \in V \setminus \Lambda. \end{cases}$$

The energy of a configuration  $\sigma \in \Omega$  is given by the formal Hamiltonian

$$H(\sigma) = \sum_{\substack{\Lambda \subset V: \\ \text{diam}(\Lambda) \leq r}} I(\sigma_\Lambda),$$

where  $r \in \mathbb{N}$ ,  $\text{diam}(\Lambda) = \max_{x, y \in \Lambda} d(x, y)$  and  $I: \Omega_\Lambda \rightarrow \mathbb{Q}_p$  is a given potential.

Let  $\mathfrak{B}$  be algebra of cylindrical subsets of  $\Omega$ .

**Definition 1.** Let  $\mu$  be a  $p$ -adic probability measures on  $(\Omega, \mathfrak{B})$ . Assume that for all  $\Lambda \subset V$  and  $\sigma \in \Omega_\Lambda, \varphi \in \Omega_{V \setminus \Lambda}$  the following equality holds

$$\mu(\sigma | \varphi) = \frac{\exp_p \{H(\sigma \vee \varphi)\}}{Z_{\Lambda, \varphi}},$$

where  $Z_{\Lambda, \varphi}$  is the normalizing constant, i.e.,

$$Z_{\Lambda, \varphi} = \sum_{\omega \in \Omega_{\Lambda}} \exp_p \{H(\omega \vee \varphi)\}.$$

Such a measure is called a *p*-adic Gibbs measure.

In §1.3, the uniqueness of the *p*-adic Gibbs measure for the Ising model, the concept of the *p*-adic generalized Gibbs measure, and the results obtained for the Ising model on the Cayley tree of order two are presented.

Let  $\Phi = \{-1, 1\}$ . The formal Hamiltonian of *p*-adic Ising model is

$$H_n(\sigma_n) = J \sum_{\langle x, y \rangle \in L_n} \sigma_n(x) \sigma_n(y)$$

where  $0 < |J|_p < p^{-1/(p-1)}$  is a constant.

Assume that  $h: x \rightarrow h_x, \forall x \in V, h_x \in \mathbb{Q}_p$  is a mapping and consider the *p*-adic probability measures  $\mu_h^{(n)}(\sigma_n)$  on  $\Omega_{V_n}$  defined by

$$\mu_h^{(n)}(\sigma_n) = \frac{1}{Z_n^{(h)}} \exp_p \{H_n(\sigma_n)\} \prod_{x \in W_n} h_x^{\varphi^{(n)}(x)} \quad n = 1, 2, \dots, \quad (1)$$

here,  $Z_n^{(h)}$  is the corresponding normalizing factor

$$Z_n^{(h)} = \sum_{\sigma_n(x) \in \Omega_{V_n}} \exp_p \{H_n(\sigma_n)\} \prod_{x \in W_n} h_x^{\varphi^{(n)}(x)}.$$

We say that *p*-adic probability measures (1) are compatible if all  $n \in \mathbb{N}$  and  $\sigma_{n-1} \in \Phi^{V_{n-1}}$ :

$$\sum_{\omega_n \in \Omega_{W_n}} \mu_h^{(n)}(\sigma_{n-1} \vee \omega_n) = \mu_h^{(n-1)}(\sigma_{n-1}). \quad (2)$$

We note that if *p*-adic probability measures (1) satisfies the compatibility condition (2), then according to non-Archimedean analogue of the Kolmogorov extension theorem, which was proved by N.N.Ganikhodjaev, F.M.Mukhamedov, U.A.Rozikov, there exists a unique *p*-adic measure  $\mu_h$  on  $\Omega$  such that

$$\mu_h(\{\sigma \in \Omega: \sigma|_{V_n} \equiv \sigma_n\}) = \mu_h^{(n)}(\sigma_n),$$

for all  $\sigma_n \in \Omega_{V_n}, n \in \mathbb{N}$ .

In the works of F.M.Mukhamedov, the concept of phase transitions for *p*-adic Gibbs measures is presented. If there are at least two distinct *p*-adic Gibbs measures  $\mu$  and  $\nu$  such that one is bounded and the other is unbounded, then one says that a *phase transition* occurs. Moreover, it is said a *quasi phase transition* occurs if there are two different functions  $s$  and  $h$  such that there exist the corresponding measures  $\mu_s$  and  $\mu_h$  they are either both bounded or both unbounded.

In the works of M.Khamrayev, U.A.Rozikov, and F.M.Mukhamedov, the uniqueness of *p*-adic Gibbs measures for the homogeneous Ising model on the Cayley

tree was investigated. They proved that no phase transition occurs for this model, that is, a unique  $p$ -adic Gibbs measure exists.

In 2014, O.N. Khakimov introduced the concept of  $p$ -adic generalized Gibbs measures for the Ising model.

If for some function  $h$  the measures  $\mu_h^{(n)}$  satisfy the compatibility condition (2), then there is a unique  $p$ -adic probability measure, which we denote by  $\mu_h$ , since it depends on  $h$ . Such a measure  $\mu_h$  is said to be a  $p$ -adic generalized Gibbs measure corresponding to the  $p$ -adic Ising model.

If  $h_x \in \mathcal{E}_p$  for any  $x \in V \setminus x_0$  then from the  $p$ -adic generalized Gibbs measure we obtain the usual  $p$ -adic Gibbs measure.

It is known from the work of O.N.Khakimov that, the sequence of  $p$ -adic probability distributions  $\{\mu_h^{(n)}\}_{n \geq 1}$  defined by formula (2), is consistent if and only if, for any  $x \in V \setminus \{x^0\}$ , the following relation holds:

$$h_x^2 = \prod_{y \in S(x)} \frac{\theta h_y^2 + 1}{h_y^2 + \theta}$$

where  $\theta = \exp_p\{2J\}$ .

In addition, in the case of the Ising model on the Cayley tree of order two, the author established the translation-invariant and periodic  $p$ -adic generalized Gibbs measures, examined the boundedness of these measures, and solved the problem of a phase transition for this model.

The second chapter is titled “**Weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree**”. § 2.1, titled “*Weakly periodic Gibbs measures on the Cayley tree*”, introduces the main concepts and some known results. The existence of these measures is investigated in §2.2, titled “*The existence of weakly periodic  $p$ -adic generalized Gibbs measures on the Cayley tree*”. In §2.3, titled “*Weakly periodic  $p$ -adic generalized Gibbs measures on the Cayley trees of orders two and three*”, such measures are constructed for the Ising model on Cayley trees of orders two and three. Their boundedness and the problem of phase transition are then studied in §2.4, titled “*Boundedness of weakly periodic  $p$ -adic generalized Gibbs measures and the problem of a phase transition*”.

For the first time, in 2008, U.A.Rozikov and M.M.Rakhmatullaev introduced the notion of a weakly periodic Gibbs measure, which is more general than the concepts of translation-invariant and periodic measures. Below we present these concepts.

Let  $G_k^*$  be a normal subgroup with index  $r \geq 1$  of the group  $G_k$  and  $G_k / G_k^* = \{H_0, H_1, \dots, H_{r-1}\}$  be a finite quotient group.

**Definition 2.** A set  $h = \{h_x, x \in G_k\}$  of quantities is called  $G_k^*$ -periodic if  $h_x = h_i$ , for all  $x \in H_i$ . A  $G_k^*$ -periodic function is called a translation-invariant.

**Definition 3.** A set of quantities  $h = \{h_x, x \in G_k\}$  is called  $G_k^*$ -weakly periodic if  $h_x = h_{x_j}$ , for any  $x \in H_i$  and  $x_j \in H_j$ .

**Definition 4.** A  $p$ -adic generalized Gibbs measure  $\mu_h$  is said to be  $G_k^*$ -(weakly) periodic if it corresponds to a  $G_k^*$ -(weakly) periodic  $h$ . A  $G_k^*$ -periodic measure is called a translation-invariant measure.

Note that any normal divisor of index two of the group  $G_k$  has the following form

$$H_A = \{x \in G_k : \sum_{i \in A} \omega_x(a_i) \text{ is an even number}\},$$

where  $\emptyset \neq A \subseteq N_k = \{1, 2, 3, \dots, k+1\}$ , and  $\omega_x(a_i)$  is the number of letters  $a_i$  in a word  $x \in G_k$ .  $H_A$  is a normal subgroup of the  $G_k$ . It can be checked that the weakly periodic Gibbs measure depends on choosing the normal subgroup of  $G_k$ . Note that in case  $|A| = k+1$ , i.e.,  $A = N_k$  a notion of weakly periodicity coincides with the usual periodicity (where  $|A|$  is the number of elements of the set  $A$ ). Therefore, we consider  $A \subset N_k$  such that  $A \neq N_k$ . Then  $H_A$ -weakly periodic collection  $h_x$  has the following form

$$h_x = \begin{cases} h_{00}, & \text{if } x \in H_0, x_j \in H_0, \\ h_{01}, & \text{if } x \in H_0, x_j \in H_1, \\ h_{10}, & \text{if } x \in H_1, x_j \in H_0, \\ h_{11}, & \text{if } x \in H_1, x_j \in H_1, \end{cases}$$

where  $H_0 := H_A$  and  $H_1 := G_k \setminus H_A$ .

Weakly periodic Gibbs measures corresponding to two- or four-indexed normal subgroups were thoroughly investigated in the field of real numbers by U.A.Rozikov, M.M.Rakhmatullaev, and J.D.Dekhkonov. Furthermore, a new class of Gibbs measures, similar to weakly periodic ones, was constructed on the Cayley tree of order  $k$  by D.Gandolfo, J.Ruiz, and S.Shlosman.

In §2.1, the existence of weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree of order two in the  $p$ -adic setting was established, which leads to the following result:

**Theorem 1.** If  $p \equiv 1 \pmod{4}$  then there exists at least one weakly periodic (non-periodic)  $p$ -adic generalized Gibbs measure for the Ising model on the Cayley tree of order two.

Theorem 1 establishes the existence of at least one weakly periodic generalized Gibbs measure in the field of  $p$ -adic numbers. Motivated by this result, we undertake the description of all weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree of order two.

**Theorem 2.** Let  $k = 2$ ,  $|A| = 1$ . There exist six distinct  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measures for the Ising model on the

Cayley tree.

The analysis carried out for the Ising model on the Cayley tree of order two was extended to the case of the Cayley tree of order three, and the following results were obtained on the invariant sets

$$\begin{aligned} I_1 &= \left\{ (h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = h_{11} = h_{01} = h_{10} \right\}, \\ I_2 &= \left\{ (h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = h_{11}, h_{01} = h_{10} \right\}, \\ I_3 &= \left\{ (h_{00}, h_{01}, h_{10}, h_{11}) \in \mathbb{Q}_p^4 : h_{00} = \frac{1}{h_{11}}, h_{01} = \frac{1}{h_{10}} \right\}. \end{aligned}$$

**Theorem 3.** Let  $k = 3$ ,  $|A| = 1$ ,  $\mathcal{M}_1(p)$  and  $\mathcal{M}_2(p)$  be the number of  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measures corresponding to the  $H_A$ -weakly periodic solutions of (3) on the invariant set  $I_2$  and  $I_3$ , respectively. Then the following assertion holds:

$$\mathcal{M}_1(p) = \begin{cases} 6, & \text{if } p \equiv 1 \pmod{12}, \\ 4, & \text{if } p \equiv 5 \pmod{12}, \\ 1, & \text{otherwise.} \end{cases} \quad \mathcal{M}_2(p) = \begin{cases} 6, & \text{if } p \equiv 1 \pmod{12}, \\ 3, & \text{if } p \equiv 5 \pmod{12}, \\ 1, & \text{otherwise.} \end{cases}$$

In determining the occurrence of phase transition for the considered model, it is important to verify the boundedness of measures. Therefore, in §2.4, the following results are presented by generalizing those obtained for the Cayley tree of order two.

**Theorem 4.** Let  $p \equiv 1 \pmod{4}$ ,  $|A| = 1$ . Then there exist six  $H_A$ -weakly periodic (non-periodic) unbounded  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model on the Cayley tree of order two.

On the Cayley tree of order three, among the weakly periodic solutions of (3) on the invariant set  $I_2$ , we denoted by  $\mu_h^1$  the measure corresponding to the solution  $h_x^2 = h_0^{(1)}$ ,  $\forall x \in V$ , which exists for any prime number  $p$ . Now we present the following result obtained on this invariant set:

**Theorem 5.** Let  $k = 3$ ,  $|A| = 1$  and  $h$  be an  $H_A$ -weakly periodic solution of (3) on the invariant set  $I_2$ . Then for the Ising model on the Cayley tree the following assertions hold:

1. Only  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measure  $\mu_h^1$  is unbounded if  $p = 2$ .
2. Only  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measure  $\mu_h^1$  is bounded if  $p \neq 2$ .

By analyzing the obtained measures on  $I_3$ , we obtained the following theorem:

**Theorem 6.** Let  $k = 3$ ,  $|A| = 1$  and  $h$  be an  $H_A$ -weakly periodic solution of (3) on  $I_3 \setminus I_2$ . Then all  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic

generalized Gibbs measures are unbounded if one of the following conditions hold:

- 1)  $p \equiv 1 \pmod{12}$ .
- 2)  $p \equiv 5 \pmod{12}$ .

**Corollary 1.** Let  $p \equiv 5 \pmod{12}$ . Then there exists a phase transition for the  $p$ -adic Ising model on the Cayley tree of the third-order.

**Remark 1.** If  $p \equiv 1 \pmod{12}$ , then there exists a phase transition. Note that this condition coincides with the condition in M.M.Rakhmatullaev, O.N.Khakimov and A.M.Tuxtabaev's work.

**Chapter 3** is entitled “**Weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model with an external field**”. §3.1, titled “*The Ising model with an external field on the Cayley tree*”, presents well-known facts together with the obtained results for the  $p$ -adic Ising model with an external field.

A Hamiltonian in  $\Omega_{V_n}$  (i.e.,  $H_n : \Omega_{V_n} \rightarrow \mathbb{Q}_p$ ) of the  $p$ -adic Ising model with an external field is defined by

$$H_n(\sigma_n) = J \sum_{\langle x,y \rangle \in L_n} \sigma_n(x)\sigma_n(y) + \alpha \sum_{x \in V_n} \sigma_n(x), \quad (4)$$

where  $J, \alpha \in B(0, p^{-1/(p-1)}) \setminus \{0\}$  are constants.

If  $\alpha = 0$ , then equation (4) reduces to the Hamiltonian of the  $p$ -adic Ising model without an external field. Therefore, we consider the case  $\alpha \neq 0$ .

It is known from the work of F.M.Muxamedov, M.M.Rakhmatullaev, A.M.Tuxtabayev, and R.M.Mamadjanov that for the  $p$ -adic Ising model with an external field on the Cayley tree, the sequence of  $p$ -adic probability measures

$\{\mu_h^{(n)}\}_{n \geq 1}$  defined by formula (2), is consistent if and only if, for any  $x \in V \setminus \{x^0\}$ , the following relation holds:

$$\hat{h}_x = \eta^{k+1} \prod_{y \in S(x)} \frac{\theta \hat{h}_y + 1}{\hat{h}_y + \theta} \quad (5)$$

where  $\theta = \exp_p\{2J\}$ ,  $\eta = \exp_p\left(\frac{2\alpha}{k+1}\right)$ ,  $\hat{h}_x = \eta \exp_p\{2h_x\}$ .

For the Ising model with an external field on the second-order Cayley tree, translation-invariant and periodic  $p$ -adic generalized Gibbs measures were studied by F.M.Mukhamedov, M.M.Rakhmatullaev, A.M.Tukhtabayev, and R.M.Mamadjanov.

In §3.2, titled “*Weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model with an external field*”,  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measures for the considered model are studied in the cases  $|A|=1$  and  $|A|=2$ . For  $|A|=1$ , the following results were obtained:

**Theorem 7.** There exists at least one  $H_A$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measure for the Ising model with an external field on the Cayley tree of order two.

We examined the boundedness of this measure and obtained the following

theorem:

**Theorem 8.** Let  $\hat{h}_x$  be an  $H_A$ -weakly periodic solution of (5). Then corresponding  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measure  $\mu_{\hat{h}_x}$  is unbounded for the  $p$ -adic Ising model with external field on the Cayley tree of order two .

**Theorem 9.** Let  $k = 2$  and  $|A|=1$ . For the  $p$ -adic Ising model with an external field on the Cayley tree of order two the following statements hold:

1. if  $p = 2$  then a quasi phase transition occurs;
2. for any odd prime  $p$  a phase transition occurs.

In the case  $|A|=2$ , the normal subgroup of index two in the group  $G_k$  is given by:

$$H_{A_2} = \left\{ x \in G_k : \left( \omega_x(a_i) + \omega_x(a_j) \right) \equiv 0 \pmod{2} \right\}, \quad i, j \in \{1, 2, 3\}, i \neq j,$$

that is, the total number of letters  $a_i$  and  $a_j$  in the word  $x$  is even. If we denote the corresponding measures obtained for  $|A|=1$  and  $|A|=2$  by  $H_{A_1}$ - and  $H_{A_2}$ -weakly periodic measures, respectively, then the weakly periodic solutions of (5) led us to the following result:

**Theorem 10.** For the Ising model with an external field, there exist at least one  $H_{A_1}$ -weakly periodic (non-translation-invariant) and at least one  $H_{A_2}$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measure on the Cayley tree of order two.

Checking the boundedness of these measures yields the following result:

**Theorem 11.** Let  $p \neq 3$ . For the Ising model with an external field on the Cayley tree of order two, there exists at least one unbounded  $H_{A_1}$ -weakly periodic (non-translation-invariant) and at least one unbounded  $H_{A_2}$ -weakly periodic (non-translation-invariant)  $p$ -adic generalized Gibbs measure.

**Theorem 12.** For any odd number  $p$ , a phase transition occurs in the Ising model with an external field on the Cayley tree of order two.

§3.3, titled “*Translation-invariant generalized  $p$ -adic Gibbs measures for the Ising model with a homogeneous external field on the Cayley tree of order three*”, is devoted to finding the conditions on the prime number  $p$  under which translation-invariant solutions of (5) exist. Checking the boundedness of the obtained measures gave us the following result:

**Theorem 13.** Let  $p > 3$ . Then, for the Ising model with a homogeneous external field on the Cayley tree of order three, the following statements hold:

1. If  $p \equiv 1 \pmod{6}$ , then there exist one bounded and three unbounded translation-invariant  $p$ -adic generalized Gibbs measure.
2. If  $p \not\equiv 1 \pmod{6}$ , then there exist one bounded and one unbounded translation-

invariant  $p$ -adic generalized Gibbs measures.

**Theorem 14.** If  $p > 3$  then there exists a phase transition for the Ising model with a homogeneous external field.

### Conclusion

This dissertation is devoted to the characterization of the set of weakly periodic generalized  $p$ -adic Gibbs measures for the Ising model with and without an external field on second and third-order Cayley trees. Furthermore, it examines the types of  $p$ -adic phase transitions corresponding to these models.

The results obtained in this dissertation can be summarized as follows:

1. The existence of weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the second-order Cayley tree in the  $p$ -adic case was proved.

2. It was shown that for  $p \equiv 1 \pmod{4}$ , there are four weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the second-order Cayley tree. The boundedness of these measures was also examined.

3. Weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the third-order Cayley tree were found. In particular, it was proved that if  $p \equiv 1 \pmod{12}$ , the number of such measures is at most twelve, whereas if  $p \equiv 5 \pmod{12}$ , there are at most seven, and at least two in other cases.

4. It was proved that for  $p \equiv 5 \pmod{12}$ , a phase transition exists for the Ising model on the third-order Cayley tree.

5. For the Ising model with an external field on the Cayley tree of order two, it is proved that for any prime number  $p$ , there exists at least one unbounded  $H_{A_1}$ -weakly periodic and at least one unbounded  $H_{A_2}$ -weakly periodic generalized  $p$ -adic Gibbs measure.

6. It was shown that a quasi phase transition occurs for the Ising model with an external field on the second-order Cayley tree when  $p = 2$ , and that a phase transition occurs for any odd prime number  $p$ .

7. All translation-invariant  $p$ -adic generalized Gibbs measures were found for the Ising model with a homogeneous external field on the third-order Cayley tree.

8. It was proved that a phase transition exists for the Ising model with a homogeneous external field on the third-order Cayley tree when  $p > 3$ .

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УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ  
В.И.РОМАНОВСКОГО**

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**НАМАНГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ**

**АБДУКАХОРОВА ЗУЛХУМОР ТУХТАСИНОВНА**

**СЛАБО ПЕРИОДИЧЕСКИЕ  $p$ -АДИЧЕСКИЕ ОБОБЩЕННЫЕ МЕРЫ  
ГИББСА ДЛЯ НЕКОТОРЫХ МОДЕЛЕЙ**

**01.01.01 – Математический анализ**

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-  
МАТЕМАТИЧЕСКИМ НАУКАМ**

**Наманган – 2025**

**Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образования, Науки и Инноваций Республики Узбекистан за № В2023.3.PhD/FM903.**

Диссертация выполнена в Наманганском государственном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (<https://kengash.mathinst.uz>) и на Информационно-образовательном портале «Ziyonet» (<http://www.ziyonet.uz>).

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Защита диссертации состоится 16 декабря 2025 года в 16:00 часов на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И. Романовского. (Адрес: 102174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871) 207-91-40, e-mail: [uzbmath@umail.uz](mailto:uzbmath@umail.uz), Website: [www.mathinst.uz](http://www.mathinst.uz))

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И. Романовского (зарегистрирована за № 217). (Адрес: 102174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871) 207-91-40).

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## **ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))**

**Целью исследования** является установление существования слабопериодических обобщённых  $p$ -адических гиббсовских мер для модели Изинга на дереве Кэли как при наличии внешнего поля, так и при его отсутствии, а также, проверяя их ограниченность, показать существование фазового перехода в данных моделях.

**Объектом исследования** являются  $p$ -адические модели Изинга без внешнего поля и с внешним полем на дереве Кэли.

**Научная новизна исследования** состоит в следующем:

доказано существование слабопериодической обобщённой  $p$ -адической гиббсовской меры для модели Изинга на дереве Кэли второго порядка;

на деревьях Кэли второго и третьего порядков для модели Изинга построены слабопериодические обобщённые  $p$ -адические гиббсовские меры и исследована их ограниченность;

на дереве Кэли второго порядка для модели Изинга во внешнем поле построены слабопериодические обобщённые  $p$ -адические гиббсовские меры, соответствующие нормальной подгруппе индекса два, и доказано существование фазового перехода при  $p > 2$  для данной модели;

для однородной модели Изинга во внешнем поле на дереве Кэли третьего порядка найдены трансляционно-инвариантные обобщённые  $p$ -адические гиббсовские меры, и доказано существование фазового перехода при  $p > 3$  для рассматриваемой модели.

**Внедрение результатов исследования.** На основе полученных результатов для обобщённых  $p$ -адических мер Гиббса для модели Изинга:

теоретические подходы, полученные при описании слабо периодических обобщённых  $p$ -адических мер Гиббса для модели Изинга на деревьях Кэли второго и третьего порядка, а также методы проверки ограниченности этих мер, были использованы в зарубежном проекте № 374824-2022 под названием “Задачи фазовых переходов и критические явления. Математические аспекты их уравнений, быстрые переходы и асимптотика” для совершенствования методов определения фазовых переходов и критических явлений (Справка Ошского государственного университета № 1400 от 15.10.2025, Кыргызстан). Применение данного научного результата позволило определить фазовые переходы и критические температуры;

полное описание  $H_A$ -слабо периодических обобщённых  $p$ -адических мер Гиббса для модели Изинга на деревьях Кэли было использовано в рамках международного исследовательского грантового проекта «Хаотические и смешанные  $p$ -адические динамические системы, связанные с ренормализационными группами решёточных моделей» (UAEU, грант № G0003247, Университет Объединённых Арабских Эмиратов, 2025 г.) (сертификат, выданный Университетом ОАЭ, от 23 октября 2025 г.). Эти результаты послужили основой для введения новых понятий в теорию  $p$ -

адических динамических систем и исследования термодинамических свойств физических моделей;

методы, использованные для установления существования слабо периодических  $p$ -адических обобщённых мер Гиббса для  $p$ -адической модели Изинга на дереве Кэли второго порядка, были применены в фундаментальном исследовательском проекте № F-FA-2021-425 «Градиентные вероятностные меры на решёточных системах» для определения наличия фазового перехода в решёточных системах (Справка Института математики № 2/416 от 27.10.2025). Применение данного научного результата позволило охарактеризовать непериодические меры Гиббса для моделей с взаимодействием ближайших соседей на решёточных системах.

**Структура и объем диссертации.** Диссертация состоит из введения, трех глав, разбитых на десять параграфов, заключения и списка использованной литературы. Объем диссертации составляет 114 страниц.

**E'LON QILINGAN ILMIY ISHLAR RO'YXATI**  
**LIST OF PUBLISHED WORKS**  
**СПИСОК ОПУБЛИКОВАННЫХ РАБОТ**

**I bo'lim (part I; часть I)**

1. Rahmatullaev M.M., Abdukaxorova Z.T.  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measure for the  $p$ -adic Ising model on a Cayley tree of order two. //  $p$ -Adic Numbers, Ultrametric Analysis and Applications. – 2024, Vol. 16, No. 3, pp. 233–263. (3. Scopus. IF=0.41).

2. Rahmatullaev M.M., Abdukaxorova Z.T.  $H_A$ - weakly periodic  $p$ -adic generalized Gibbs measure for the Ising model on a Cayley tree. // Lobachevskii Journal of Mathematics – 2024, Vol. 45, No. 1, pp. 500–512. (3. Scopus. IF=0.44).

3. Rahmatullaev M.M., Abdukaxorova Z.T. On  $H_A$ - weakly periodic  $p$ -adic generalized Gibbs measure for the  $p$ -adic Ising model with an external field on a Cayley tree.// Reports On Mathematical Physics – 2025. Vol. 95. № 1.– pp. 93-109. (3. Scopus. IF=0.35).

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**II bo'lim (part II; часть II)**

7. Rahmatullaev M.M., Abdukaxorova Z.T. On weakly periodic  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model on the Cayley tree of order three. VII International Scientific Conference "Non-local boundary value problems and related problems of mathematical biology, informatics and physics", Nalchik, December 4–8, 2023, pp. 365-366.

8. Rahmatullaev M. M., Abdukaxorova Z. T. Phase transition for the  $p$ -adic Ising model with a homogeneous external field on the Cayley tree. International Scientific Conference "Mathematical Analysis and Dynamical Systems". Tashkent, May 20-21, 2025 – pp.113-114.

9. Raxmatullayev M.M., Abdukaxorova Z.T., Tukhtabaev A. M. Weakly periodic  $p$ -adic generalized Gibbs measures for the Ising model on the Cayley tree of order two. Modern Problems of Mathematics and Mechanics: of the International Conference, Baku, April 26-28, 2023, pp. 334-337.

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11. Rahmatullaev M.M., Abdukaxorova Z.T. The weakly periodic  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model on the Cayley tree. "Actual problems of mathematical modeling and information technology" International scientific and practical conference, Nukus, May 2-3, 2023, p. 36-38.

12. Rahmatullaev M.M., Abdukaxorova Z.T. The existence of  $H_A$ -weakly periodic  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model on the Cayley tree of order two. "Actual Problems of Applied Mathematics and Information Technologies", International Conference, Samarkand, September 25-26, 2023, pp. 221.

13. Abdukaxorova Z.T. Translation-invariant  $p$ -adic Generalized Gibbs Measures for the Ising Model with a Homogenous External Field on the Cayley Tree. "Actual Problems of Algebra, Analysis, Topology and Computational Mathematics", International Scientific Conference, Tashkent, May 30-31, 2025, p. 79-80.

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18. Abdukaxorova Z.T. Weakly periodic  $p$ -adic generalized Gibbs measures for the  $p$ -adic Ising model with external field on the Cayley tree of order two. In: "Olima ayollar - millat kelajagining tarbiyachisi" Respublika ilmiy-amaliy anjumani, Toshkent, 2024-yil 13-fevral, pp. 6-12.

Avtoreferat “O‘zbekiston matematika jurnali” tahririyatida 2025-yil 24-noyabrda tahrirdan o‘tkazilib, o‘zbek, ingliz va rus tillaridagi matnlar o‘zaro muvofiqlashtirilgan.

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