O‘ZBEKISTON MILLIY UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/30.12.2019.FM.01.02 RAQAMLI ILMIY KENGASH

ANDIJON DAVLAT UNIVERSITETI

ATABAYEV ODILJON XUSNIDDIN O‘G‘LI

NOCHIZIQLI MANBA VA YUTILISHGA EGA NOCHIZIQLI DIFFUZIYA JARAYONLARINI MATEMATIK MODELLASHTIRISH

05.01.07-Matematik modellashtirish. Sonli usullar va dasturlar majmui
(fizika-matematika fanlari)

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2025

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
DISSERTATSIYASI AVTOREFERATI**

Toshkent – 2025

Falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.4.PhD/FM1214 raqam bilan ro'yxatga olingan.

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Ilmiy rahbar:	**Aripov Mersaid** fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Sadullayeva Shaxlo Azimbayevna** fizika-matematika fanlari doktori, professor
	Xaydarov Abdugappar fizika-matematika fanlari nomzodi, dotsent
Yetakchi tashkilot:	**Urganch davlat universiteti**

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Dissertatsiya bilan O'zbekiston Milliy universitetining Axborot resurs markazida tanishish mumkin (___ raqami bilan ro'yxatga olingan). Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi 4-uy. Tel.: (+99871) 246-02-24.

Dissertatsiya avtoreferati 2025-yil "___" _____ kuni tarqatildi.
(2025-yil "___" ______ dagi _____ raqamli reyestr bayonnomasi).

B.F.Abduraximov
Ilmiy darajalar beruvchi ilmiy kengash raisi o'rinbosari, f.-m.f.d., professor

Z.R.Raxmonov
Ilmiy darajalar beruvchi ilmiy kengash ilmiy kotibi, f.-m.f.d. professor

A.S.Matyakubov
Ilmiy darajalar beruvchi ilmiy kengash qoshidagi ilmiy seminar raisi, f.-m.f.d., dotsent

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda neft-gaz va ishlab chiqarish sohalarida fizik, biologik, mexanik, ekologik, seysmologik kabi murakkab nochiziqli jarayonlarni matematik modellashtirish, ularni sonli-analitik tadqiq etish usullarini takomillashtirishga alohida ahamiyat berilmoqda. Bu borada xususiy hosilali differensial tenglamalar va ularning sistemalari yordamida tasvirlanuvchi murakkab nochiziqli matematik modellashtirish masalalari va ularni sonli tadqiq etish usullari hamda katta hajmli hisoblashlarni amalga oshirish uchun samarali algoritmlar yaratish muhim ahamiyat kasb etadi. Shunga ko'ra tabiiy jarayonlarni matematik jihatdan tavsiflash va nochiziqli matematik modellarni amaliyotga joriy etish uchun samarali sonli yechish usullarni yaratish va takomillashtirish, nochiziqli diffuziya jarayonlarining matematik modellarini qurish, nochiziqli matematik modellar ustida murakkab hisoblashlarni amalga oshirish algoritmlari va dasturiy ta'minotini yaratishga alohida e'tibor qaratilmoqda.

Jahonda biologik populyatsiya, reaksiya-diffuziya va termoyadro kabi jarayonlarning matematik modellarini tadqiq etishda sonli yechish usullarini ishlab chiqish va qo'llashga qaratilgan ilmiy-tadqiqotlar olib borilmoqda. Bunda, energetika, tibbiyot hamda neft va gaz sohalarida amaliy tatbiqqa ega bo'lgan reaksiya-diffuziya, issiqlik o'tkazuvchanlik, filtratsiya, biologik populyatsiya kabi jarayonlarni sonli modellashtirish bo'yicha tadqiqotlar ustivor hisoblanmoqda. Shunga ko'ra, nochiziqli matematik modellarni o'rganishda, yechimlarni sonli hisoblash va vizuallashtirish uchun algoritm va dasturiy ta'minot ishlab chiqish va natijalarni amaliyotga tatbiq etish maqsadli ilmiy tadiqotlardan hisoblanadi.

Mamlakatimizda fundamental fanlarni rivojlantirish bilan bir qatorda olingan ilmiy natijalarni amaliyotga tatbiq etishga, ayniqsa amaliy ahamiyati katta bo'lgan yangi innovatsion texnologiyalarni yaratish, nochiziqli masalalarni matematik modellashtirishda, ularni sonli-analitik yechish usullarini takomillashtirishga alohida e'tibor qaratilmoqda. Xususan, chiziqsiz matematik modellarning sifat xossalarini o'rganish usullarini ishlab chiqish, yechimlarning vaqt bo'yicha aniq baholarini topish, chiziqsiz effektlarni aniqlash, tejamkor sonli sxemalar ishlab chiqish; chiziqsiz jarayonlarning matematik modellarini o'rganishga yordam beruvchi amaliy dasturlar majmuini yaratish bo'yicha salmoqli natijalarga erishildi. "Funksional analiz, algebra, differensial tenglamalar, matematik fizika, matematik modellashtirish, hisoblash matematikasi va diskret matematika, ehtimollar nazariyasi va matematik statistika"[1] kabi ustuvor yo'nalishlar bo'yicha xalqaro standartlar darajasidagi ilmiy izlanishlar olib borish O'zR FA V.I.Romanovskiy nomidagi Matematika instituti faoliyatining asosiy vazifalaridan biri hisoblanadi. Qaror ijrosini ta'minlashda nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi parabolik tipdagi tenglamalar va sistemalar yechimlarining asimptotikalarini aniqlash, baholarini olish, sifat xossalarini tadqiq

[1] O'zbekiston Respublikasi Prezidentining 2020-yil 7-maydagi "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi PQ-4708-son qarori.

etish usullarini ishlab chiqish, sonli yechish dasturlarini yaratish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2022-yil 28-yanvardagi PF-60-son “2022-2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida” gi Farmoni, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta‘lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi Qarori, 2019-yil 8-oktabrdagi PF-5847-son “O‘zbekiston Respublikasi oliy ta‘lim tizimini 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”gi Farmoni, 2019-yil 27-apreldagi PQ-3682-son “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi Qarori va 2021-yil 1-apreldagi PF-6198-son “Ilmiy va innovatsion faoliyatni rivojlantirish bo‘yicha davlat boshqaruvi tizimini takomillashtirish to‘g‘risida”gi Farmonlari, hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining asosiy ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlantirishning IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Jahonda nochiziqli tenglamalar va tenglamalar sistemalari yechimlari sifat xossalarini o‘rganish hamda sonli yechimlarini tahlil qilish bo‘yicha bir qancha tadqiqotlar olib borilgan. Mazkur tadqiqotlar natijalari asosida turli sohalardagi nochiziqli jarayonlarni ifoda etuvchi matematik modellar, xususan nochiziqli manba va yutilishga ega nochiziqli diffuziya tenglamalari uchun yangi effektlar olingan. Keyinchalik, A.A.Samarskiy, V.A.Galaktionov, J.L.Vazquez, S.P.Kurdyumov, A.P.Mixaylovlarning ishlarida yechimning asimptotik xossalari o‘rganilgan. Shuningdek, A.Friedman, J.Mcleod, M.Escobedo, M.A.Herrero, H.A.Levine, K.Deng, P. Souplet, N.Bedjaoui, Zh. Li, W. Du, H. Li, Ch.Mu boshqalar global yechim mavjudlik shartlarini aniqladilar. Ilmiy tadqiqotlarning asosiy e’tibori global yechimning mavjudligi, chekli vaqtda chegaralanmagan yechim xossalarini o‘rganishga qaratilgan edi. Ulardan tashqari Wiegner M., Winkler M., Gage E., Angenent S., Jin Ch., Yin J., Yunzhu Gao, Qiu Meng, Yingjia Guo, Zhi-wen Duan, Li Zhou tomonidan yechimlarning asimptotikasi, uning chegaralanmaganligi, chekli tezlikda tarqalish effekti va issiqlik tarqalishining fazoviy lokallashishi, manba yoki yutilishga ega nochiziqli muhitlarda ta’sirlanish jarayonining chekli vaqtda mavjud bo‘lishi kabilar aniqlandi.

O‘zbekistonda turli jarayonlarning matematik modellarini tasvirlovchi parabolik turdagi nochiziqli masalalar bilan B. M. Xujayarov, N. Muhitdinov, A. S. Rasulov, M. Aripov, J. Toxirov, N. Ravshanov, Sh. Sadullayeva, A. Xaydarov, A.Xasanov, B.Babajanov, A. Matyakubov, F.Kabiljanova, Z. Raxmonov va ularning shogirdlari shug‘ullanishgan. Ularning asosiy ishlari divergent va nodivergent tenglama va tenglamalar sistemasi bilan ifodalanadigan nochiziqli diffuziya masalasi yechimlari xossalarini sonli o‘rganishga bag‘ishlanadi hamda bu metodlarni filtratsiya, diffuziya, issiqlik o‘tkazuvchanlik jarayonlarini modellashtirishga bag‘ishlangan. Ular tomonidan avtomodel tahlil asosida tabiatshunoslikning turli sohalarida uchraydigan jarayonlarni ifodalovchi nochiziqli masalalar yechimlarining sifat xossalari tadqiq etilgan va sonli natijalar olib tahlillar keltirilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universiteti ilmiy tadqiqot ishlari rejasiga muvofiq, OT-F4-30 “Ikki marta nochiziqli kross sistemaning konvektiv ko‘chish, o‘zgaruvchan zichlik, manba yoki yutish ta’siridagi sifat xossalarini tadqiq qilish” ilmiy tadqiqot loyihasi va AL-9224104601 “Nochiziqli, divergent va nodivergent ko‘rinishdagi parabolik tenglama va sistemalar bilan tasvirlanuvchi jarayonlarni matematik modellashtirish” mavzusidagi (amaliy) ilmiy tadqiqot loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega bo‘lgan muhitlarda parabolik tipdagi tenglamalar va ularning sistemalari bilan tavsiflanuvchi nochiziqli matematik modellarning sifat xossalarini sonli hamda analitik tadqiq etishdan iborat.

Tadqiqotning vazifalari:

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli model yechimlarining vaqt bo‘yicha globallik hamda global bo‘lmaslik shartlarini topish;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya sistemalari yechimlarining sonli hisoblashlarda muhim bo‘lgan quyi va yuqori baholarini olish;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli masalalarni sonli hisoblash uchun zarur bo‘lgan boshlang‘ich yaqinlashishni topish;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarning matematik modellari sifat xossalarini o‘rganish uchun sonli hisoblash sxemalarini qurish;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya masalasini yechish va ularni vizuallashtirish uchun dasturiy vositalar majmuini ishlab chiqish.

Tadqiqotning obyekti o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega parabolik tenglama va ularning sistemalari bilan tavsiflanuvchi diffuziyaning nochiziqli jarayonlari olingan.

Tadqiqotning predmeti o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlari matematik modellari sifat xossalarini tadqiq qilish, sonli yechish sxemalarini qurish, dasturiy majmualarini yaratishdan iborat.

Tadqiqotning usullari. Ushbu dissertatsiya ishida nochiziqli ajratish algoritmi, avtomodel va taqribiy avtomodel usullari, etalon tenglamalar usuli, yechimlarni taqqoslash teoremalari, ayirmali sxemalar, iteratsiya va haydash usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli model yechimlarining vaqt bo‘yicha globallik hamda global bo‘lmaslik shartlari topilgan;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya sistemalari yechimlarining sonli hisoblashlarda muhim bo‘lgan quyi va yuqori baholari olingan;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli masalalarni sonli hisoblash uchun zarur bo‘lgan boshlang‘ich yaqinlashishlar topilgan;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarining matematik modellari sifat xossalarini o‘rganish uchun sonli hisoblash sxemalari qurilgan;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya masalasini yechish va ularni vizuallashtirish uchun dasturiy vositalar majmui ishlab chiqilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya masalalari yechimlari uchun quyi va yuqori baholar olingan;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya masalalar uchun sonli yechimlar qurilgan va dasturlar majmui yaratilgan.

Tadqiqot natijalarining ishonchliligi. Dissertatsiya ishida olingan tasdiqlar solishtirish teoremalari va maksimum prinsipi asosida qat’iy isbotlangan hamda hisoblash eksperimenti natijalari bilan tasdiqlangan, olingan natijalarning saqlanish qonunlariga muvofiqligi bilan asoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati avtomodel yechimni qurish usullari, asimptotik formulalar, sonli yechish sxemalari va algoritmlaridan viruslarning tarqalishi, gaz va suyuqliklar filtratsiyasi, diffuziya, issiqlik o‘tkazuvchanlik jarayonlari modellarini sonli va analitik yechishda qo‘llanilishi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati iteratsion jarayonlar qurilganligi, sonli hisoblash sxemasi va dasturiy ta’minot yaratilganligi, ulardan nochiziqli issiqlik o‘tkazuvchanlik, gaz va suyuqliklar filtratsiyasi, diffuziya, viruslar tarqalishi, biologik populyatsiya masalalari uchun samarali hisoblash tajribalarini o‘tkazishga imkon berishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. O‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlari masalalarini matematik va sonli modellashtirish bo‘yicha olingan ilmiy natijalar asosida:

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya sistemalari yechimlarining quyi va yuqori baholaridan OT-F4-88 “Ikkinchi va yuqori tartibdagi aralash tipdagi tenglamalar uchun to‘g‘ri va teskari masalalarni tadqiq etish” mavzusidagi fundamental grant loyihasida vertikal yarim yo‘lakda singulyar koeffitsientli va spectral parametrli buziluvchan tenglama uchun Dirixle masalasini yechishda foydalanilgan. (V.I.Romanovskiy nomidagi Matematika institutining 2024-yil 5-dekabrdagi 02/437-sonli ma'lumotnomasi). Ilmiy natijalarni qo'llash vertikal yarim yo'lakda singulyar koeffitsientli va spektral parametrli buziluvchan tenglama uchun Dirixle masalasining aniq yechimini topish imkonini bergan.

o'zgarmas yoki o'zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli masalalarni sonli hisoblash uchun zarur bo'lgan boshlang'ich yaqinlashishni topish ilmiy natijasidan OT-F4-04 "Spektral usulni matritsaviy nochiziqli evolyusion tenglamalarni yechishga tadbiqlari, Yurak-qon tomir tizimining biomexanikasi" mavzusidagi fundamental grant loyihasida yuklangan hadli Kaup sistemasiga qo'yilgan Koshi masalasini davriy funksiyalar sinfida yechishda foydalanilgan. (Urganch davlat universitetining 2024-yil 6-dekabrdagi 04-235/2-sonli ma'lumotnomasi). Ilmiy natijalardan foydalanish yuklangan hadli Kaup sistemasi uchun qo'yilgan Koshi masalasining davriy yechimlarining aniq ko'rinishlarini aniqlash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot ishi natijalari 11 ta ilmiy-amaliy anjumanlarda, jumladan, 7 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Tadqiqot mavzusi bo'yicha jami 19 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 1 tasi xorijiy (Scopus, Q2) va 5 tasi respublika jurnallarida nashr etilgan. Shuningdek, EHM uchun yaratilgan dasturning rasmiy ro'yxatdan o'tkazilganligi to'g'risida 2 ta mualliflik guvohnomasi olingan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa, foydalanilgan adabiyotlar ro'yxati va ilovalardan iborat. Dissertatsiyaning hajmi 96 betdan tashkil topgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi ko'rsatilgan, mavzu bo'yicha xorijiy ilmiy tadqiqotlar sharhi, muammoning o'rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning **"Nochiziqli manba va yutilishga ega nochiziqli parabolik tenglama yechimlarining xossalari"** deb nomlangan birinchi bobida nochiziqli manba va yutilishga ega nochiziqli diffuziya modeli yechimlarining global mavjudligi, chekli vaqt ichida chegaralanmagan bo'lishi va asimptotik harakati kabi sifat xossalari ahamiyati muhokama qilingan.

1.1-paragrafda populyatsiya dinamikasi, nochiziqli muhitda issiqlik o‘tkazuvchanlik va reaksiya-diffuziya jarayonlari nochiziqli manba va yutilishga ega ikki karra nochiziqli buziluvchan parabolik tenglama bilan modellashtirilishi mumkin. Xususan, ushbu

$$\frac{\partial u}{\partial t} = \nabla\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + u^{q_1} - u^{q_2}, \quad x \in \Omega, \quad t > 0, \tag{1}$$

$$u(t,x) = 0, \quad x \in \partial\Omega, \quad t > 0, \tag{2}$$

$$u(0,x) = u_0(x), \quad x \in \Omega, \tag{3}$$

ikki karra nochiziqli masala reaksiya-diffuziya jarayonlarini ifodalaydi, bu yerda $n,\ m,k > 1,\ p > 2,\ q_1 > 1,\ q_2 \geq 1,\ q_1 \neq q_2$ va $\Omega \subset R^N$ esa silliq chegara $\partial\Omega$ bilan chegaralangan sohadir. Yuqoridagi masaladagi boshlang‘ich shart $u_0(x)$ berilgan notrivial, nomanfiy, chegaralangan va mos ravishda silliq funksiyadir.

1.2-paragrafda suyuqlik dinamikasi, biologik populyatsiya, iqtisodiy matematika sohalaridagi nochiziqli modellashtirish masalalariga oid tadqiq etilgan ishlar va olingan natijalar sharhi bayon qilingan.

I bobning 3-paragrafida natijalarni kelgusida taqdim etish uchun zarur bo‘lgan ba’zi yordamchi tasdiqlar va ta’riflar keltirilgan.

1-ta’rif. Faraz qilaylik, $T > 0$, $Q_T = \Omega \times (0,T)$, $E = \{u \in L^{2q_1}(Q_T) \cup L^{2q_2}(Q_T); u_t, \nabla u \in L^2(Q_T)\}$ va $E_0 = \{u \in E; u = 0 on \partial\Omega\}$ bo‘lsin. Agar ixtiyoriy nomanfiy $\varphi \in E_0$ funksiya uchun

$$\iint_{Q_T} u_t \varphi dxdt + \iint_{Q_T} \left|\nabla u^k\right|^{p-2} \nabla u^m \nabla \varphi dxdt \geq (\leq) \iint_{Q_T} u^{q_1}\varphi - u^{q_2}\varphi dxdt$$

$$u(x,t) \geq (\leq) 0 \ on\ \partial\Omega \times (0,T)\ and\ u(x,0) \geq (\leq) u_0(x)\ a.e.\ in\ \Omega.$$

o‘rinli bo‘lsa, u holda nomanfiy $u(t,x) \in E$ funksiya (1) masalaning Q_T sohadagi umulashgan yuqori (quyi) yechimi deyiladi.

2-ta’rif. Agar $u(t,x)$ (1)-(3) masalaning ham umulashgan yuqori, ham umulashgan quyi yechimi bo‘lsa, u holda u umumlashgan yechim deyiladi.

3-ta’rif. Agar $u(t,x)$ har bir $T < \infty$ da (1)-(3) ning Q_T sohadagi umulashgan yechimi bo‘lsa, u holda u global yechim deyiladi.

4-ta’rif. Agar (1) tenglama $u(t,x) > 0$ va $\left|\nabla u^k\right| > 0$ da parabolik tipga tegishli bo‘lib, $u(t,x) = 0$ yoki $\left|\nabla u^k\right| = 0$ da oddiy differensial tenglamani ifodalasa, u holda (1) tenglama “buziluvchan” deyiladi.

5-ta’rif. Agar shunday $T < \infty$ topilsaki,

$$\lim_{t \to T^-} \left\|u(t)\right\|_{L^\infty(\Omega)} = \infty$$

shart bajarilsa, u holda (1) ko‘rinishdagi nochiziqli parabolik tenglamaning yechimi $u(t,x)$chegaralanmagan deyiladi.

1-teorema. *Faraz qilaylik, $u_1(t,x)$ va $u_2(t,x)$ quyidagi masalaning mos ravishda quyi va yuqori yechimlari bo‘lsin*

$$\frac{\partial u}{\partial t} = \nabla\cdot\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + u^{q_1} - u^{q_2}, \quad x\in\Omega, t>0,$$

$$u(t,x)=0,\ x\in\partial\Omega,\ t>0$$

$$u(0,x)=u_0(x),\ x\in\Omega$$

Agar, Ω *sohada* $u_1(0,x)\le u_2(0,x)$ *shart bajarilsa, u holda barcha* $t>0$ *va* $x\in\Omega$ *da* $u_1(t,x)\le u_2(t,x)$ *o'rinli bo'ladi.*

Dissertatsiyaning **"Nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini matematik modellashtirish"** deb nomlanuvchi ikkinchi bobi o'zgarmas yoki o'zgaruvchan zichlikli nochiziqli manba va yutilishga ega nochiziqli diffuziya masalasi yechimi baholarini olish va sonli yechishga bag'ishlangan.

II bobning 1-paragrafida (1)-(3) masala yechim sifat xossalarini o'rganishda $\left(m+k(p-2),q_1,q_2\right)$ parametrlar oraliqlari quyidagi 3 qismga ajratilgan:

(i) $q_1<\max\{m+k(p-2),q_2\}$;

(ii) $q_1=\max\{m+k(p-2),q_2\}$;

(iii) $q_1>\max\{m+k(p-2),q_2\}$.

2-teorema. *Agar* $q_1<\max\{m+k(p-2),q_2\}$ *bo'lsa, u holda (1)-(3) masalaning barcha yechimlari chegaralangan bo'ladi.*

3-teorema. *Agar* $q_1=m+k(p-2)$, $q_1>q_2$ *va* $\lambda_1\ge 1$ *bo'lsa, u holda (1)-(3) masalaning barcha yechimlari global bo'ladi.*

4-teorema. *Agar quyidagi shartlardan*

- $q_1=m+k(p-2)>q_2$ va $\lambda_1<1$
- $q_1>\max\{m+k(p-2),q_2\}$

biri bajarilsa, u holda (1)-(3) masala global va noglobal yechimlarga ega.

5-teorema. *Agar* $q_1>\max\{m+k(p-2),q_2\}$ *shart bajarilganda (1)-(3) masalaning yechimi* $u(t,x)$ *chekli* T *vaqt ichida chegaralanmagan bo'lsa, u holda shunday musbat* c *son mavjudki, quyidagi ifoda o'rinli bo'ladi:*

$$\max_{\Omega} u(t,x)\ge c(T-t)^{-\frac{1}{q_1-1}} \quad agar \quad t\to T.$$

6-teorema. *Agar* $q_1>m+k(p-2)\ge q_2$ *shart bajarilganda (1)-(3) masalaning yechimi* $u(t,x)$ *chekli* T *vaqt ichida chegaralanmagan bo'lsa, u holda shunday musbat* C *son mavjudki, quyidagi baho o'rinli*

$$\max_{\Omega} u(t,x)\le C(T-t)^{-\frac{1}{q_1-1}} \quad agar \quad t\to T.$$

II bobning 2-paragrafida ushbu o'zgaruvchan zichlikli nochiziqli manba va yutilishga ega ikki karra nochiziqli parabolik tenglama qaralgan

$$\frac{\partial u}{\partial t} = \nabla\left(|x|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + u^{q_1} - u^{q_2},\ x\in\Omega,\ t>0, \quad (4)$$

$$u(t,x)=0,\ x\in\partial\Omega,\ t>0, \quad (5)$$

$$u(0,x)=u_0(x),\ x\in\Omega, \tag{6}$$

bu yerda $n,\ m,k>1,\ p>2,\ q_1>1,\ q_2\geq 1,\ q_1\neq q_2$ va $\Omega\subset R^N$ esa silliq chegara $\partial\Omega$ bilan chegaralangan sohadir. Bu yerda $u_0(x)$ berilgan notrivial, nomanfiy, chegaralangan va mos ravishda silliq funksiyadir.

7-teorema. *Agar* $n<p$ *va* $q_1<\max\{m+k(p-2),q_2\}$ *bo'lsa, barcha nomanfiy yechimlari global bo'ladi.*

8-teorema. *Agar* $n<p$ *va* $q_1=m+k(p-2)>q_2$ *bo'lsa, u holda (4)-(6) masalaning global va chegaralanmagan yechimlari mavjud.*

9-teorema. *Agar* $n<p$ *va* $q_1>\max\{m+k(p-2),q_2\}$ *bo'lsa, u holda (4)-(6) masalaning yetarlicha katta boshlang'ich shartlardagi har qanday nomanfiy yechimi, chekli vaqtda chegaralanmagan bo'ladi.*

2.3-paragraf esa o'zgarmas va o'zgaruvchan zichlikli nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini matematik modellashtirishga bag'ishlangan.

1 o'lchovli hol. Dastlab nochiziqli diffuziya masalasi 1 o'lchovli hol $x\in[0,L]$ va $t>0$ da qaralgan bo'lib berilgan masala quyidagi ko'rinishga ega bo'ladi

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)+u^{q_1}-u^{q_2},\quad x\in(0,L),\quad t>0, \tag{7}$$

$$u(t,0)=0,\quad u(t,L)=0,\quad t\geq 0, \tag{8}$$

$$u(0,x)=u_0(x),\quad x\in[0,L], \tag{9}$$

Masalani sonli yechish uchun sonli sxema va algoritmlar ishlab chiqilgan. Hisoblash uchun x va t bo'yicha to'r qurilib

$$\bar{\omega}_h=\{x_i=ih,\ h>0,\ i=0,1,...,n,\ hn=L\},\ \bar{\omega}_\tau=\{t_j=j\tau,\ \tau>0,\ j=0,1,...,m,\ \tau m=T\}$$

,

approksimatsiya qilingan va quyidagi masala bilan almashtirilgan

$$\begin{cases}\dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h^2}\left[P_{i+1}\left(y^{j+1}\right)\left(y_{i+1}^{j+1}-y_i^{j+1}\right)-P_i\left(y^{j+1}\right)\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right]+f_i\left(y^{j+1}\right),\\ i=1,2,...,n-1;\ j=0,1,...,m-1\\ y_i^0=u_o(x_i),\ i=0,1,...,n\\ y_0^j=0,\quad j=1,2,...,m\\ y_n^j=0,\quad j=1,2,...,m\end{cases}$$

Bu yerda P_{i+1} va P_i diffuziya koeffitsientini ifodalovchi nochiziqli hadlardir. Bu hadlar mos ravishda quyidagi formulalar yordamida hisoblanadi

$$P_{i+1}\left(y^{j+1}\right)=\frac{1}{2}\left[\left(y_i^{j+1}\right)^{m-1}\left|\frac{\left(y_{i+1}^{j+1}\right)^k-\left(y_i^{j+1}\right)^k}{h}\right|^{p-2}-\left(y_{i+1}^{j+1}\right)^{m-1}\left|\frac{\left(y_i^{j+1}\right)^k-\left(y_{i-1}^{j+1}\right)^k}{h}\right|^{p-2}\right]$$

$$P_i\left(y^{j+1}\right)=\frac{1}{2}\left[\left(y_{i-1}^{j+1}\right)^{m-1}\left|\frac{\left(y_i^{j+1}\right)^k-\left(y_{i-1}^{j+1}\right)^k}{h}\right|^{p-2}-\left(y_i^{j+1}\right)^{m-1}\left|\frac{\left(y_{i-1}^{j+1}\right)^k-\left(y_{i-2}^{j+1}\right)^k}{h}\right|^{p-2}\right]$$

Nochiziqli hadlarni chiziqlashtirish quyidagi usullarda amalga oshirildi (qarang 1-jadval)

1-jadval.

Nochiziqli hadlarni chiziqlilashtirish

#	Nochiziqli had	Chiziqlashtirish	Nochiziqli had	Chiziqlashtirish
1	u^{q_1} - manba	Nyuton	u^{q_2} - yutilish	Pikar
2	u^{q_1} - manba	Nyuton	u^{q_2} - yutilish	Nyuton
3	u^{q_1} - manba	Pikar	u^{q_2} - yutilish	Pikar
4	u^{q_1} - manba	Pikar	u^{q_2} - yutilish	Nyuton

Mazkur jadvaldagi 1-holat, ya’ni manba va yutilish hadidagi nochiziqliklar mos ravishda Nyuton va Pikar metodlari yordamida chiziqli holatga keltiriladi

$$f_i\left(y^{j+1}\right)=q_1\left(y_i^j\right)^{q_1-1}\left(y_i^{j+1}-y_i^j\right)+\left(y_i^j\right)^{q_1}-\left(y_i^j\right)^{q_2}$$

So‘ngra quyidagiga ega bo‘linadi

$$\frac{y_i^{j+1,s+1}-y_i^j}{\tau}=\frac{1}{h^2}\left[P_{i+1}(y^{j+1,s})(y_{i+1}^{j+1,s}-y_i^{j+1,s})-P_i(y^{j+1,s})(y_i^{j+1,s}-y_{i-1}^{j+1,s})\right]+f_i(u^{j+1,s}),$$

bunda iteratsion jarayon $\max_i\left|y_i^{j+1,s+1}-y_i^{j+1,s}\right|<\varepsilon$ shart bajarilgunicha davom etadi.

Izoh. *Barcha hisoblashalarda aniqlik $\varepsilon=10^{-3}$ sifatida olingan.*

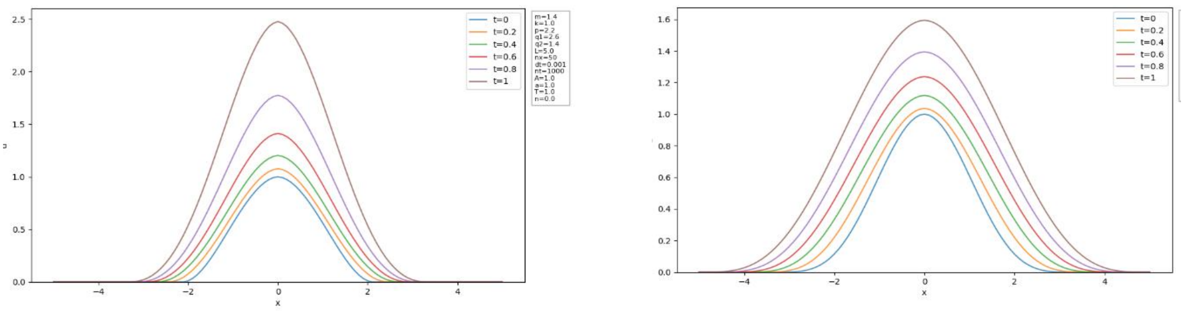
Yechimning tegishli dastlabki yaqinlashishlarini topish juda muhimdir. Sonli hisoblashlarda boshlang‘ich yaqinlashish sifatida quyidagicha olingan.

$$u_0(x)=\left(T+(q_1-1)t\right)^{-\frac{1}{q_1-1}}A\left(a-b\xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{m-1+k(p-2)}},\ \xi=r\tau^{-\frac{1}{p}},$$

$$\tau(t)=\begin{cases}\dfrac{\left(T+(q_1-1)t\right)^{-\frac{m+k(p-2)-q_1}{q_1-1}}}{q_1-\left(m+k(p-2)\right)} & \text{if } m+k(p-2)\neq q_1\\[2ex] \dfrac{1}{q_1-1}\ln\left(T+(q_1-1)t\right) & \text{if } m+k(p-2)=q_1\end{cases},$$

$$b=\frac{m-1+k(p-2)}{p}\left(\frac{1}{A^{m-1+k(p-2)}k^{p-2}p}\right)^{\frac{1}{p-1}},\quad a>0$$

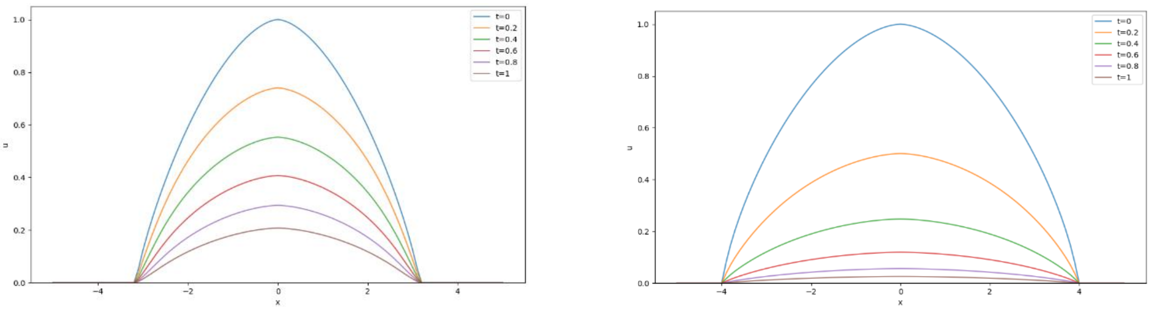
Parametrlarning turli qiymatlarida (7)-(9) masala uchun olingan sonli natijalar. Turli parametr qiymatlari nochiziqli diffuziya va manba hadlari tarqalish tezligiga qanday ta’sir etayotganini ko‘rish mumkin.



m=1.4, k=1.0, p=2.2, q_1=2.6, q_2=1.4 m=1.1, k=1.1, p=2.1, q_1=2.1, q_2=1.1

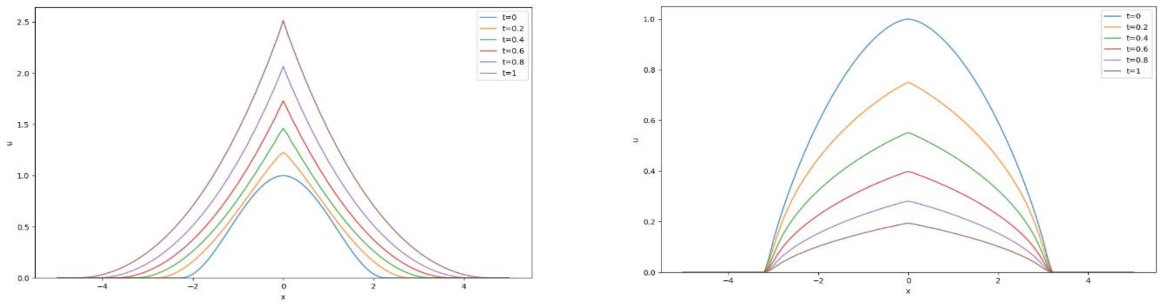
1-rasm. 1 o‘lchovli holatda (1)-(3) masalaning sonli yechimi.

2-rasmda o‘zgarmas zichlikli holatda jarayon kechishining natijalari tasvirlangan bo‘lsa, 3-rasmda o‘zgaruvchan zichlikli holatni ifodalaydi. 4-rasmda jarayon 2 o‘lchovli holatdagi olingan sonli natijalarni vizuallashtiradi.



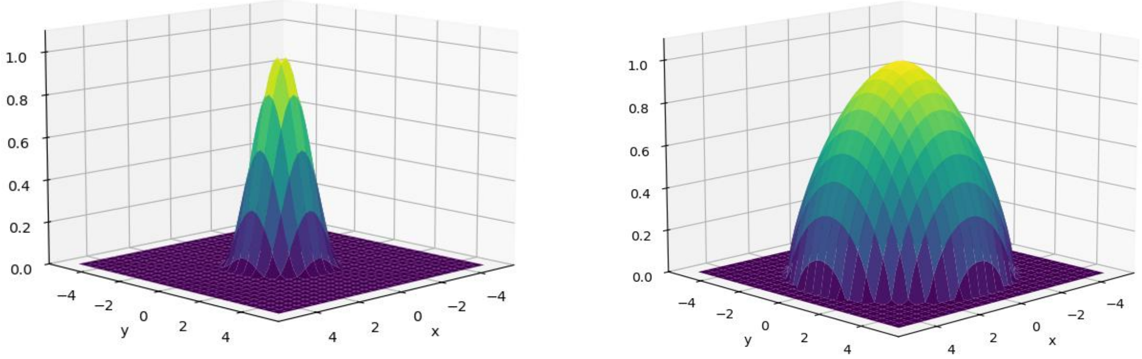
m=1.3, k=3.0, p=2.5, q_1=2.2, q_2=3.4 m=1.5, k=3.5, p=2.4, q_1=2.1, q_2=5.4

2-rasm. 1 o‘lchovli holatda (1)-(3) masalaning sonli yechimi.



n=1, m=1.4, k=1.1, p=2.1, q_1=2.3, q_2=1.1 n=1, m=1.3, k=3.0, p=2.5, q_1=2.2, q_2=3.5

3-rasm. 1 o‘lchovli holatda (4)-(6) masalaning sonli yechimi.



m=1.4, k=1.0, p=2.2, q_1=2.6, q_2=1.4 m=1.5, k=3.5, p=2.4, q_1=2.1, q_2=5.4

4-rasm. 2 o‘lchovli holatda (1)-(3) masalaning sonli yechimi.

Hisoblash natijalari tanlangan metod va boshlang‘ich yaqinlashish yaxshi tanlanganini bildirib, 3 ta iteratsiyadan so‘ng yaqinlashish sodir bo‘ldi.

Dissertatsiyaning III bobi **“Nochiziqli manba va yutilishga ega nochiziqli parabolik sistemalar bilan ifodalangan nochiziqli diffuziya jarayonlarini matematik modellashtirish”**ga bag‘ishlangan. Unda o‘zgarmas yoki o‘zgaruvchan zichlikli nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi parabolik sistemalar qaralgan.

3.1 paragrafda ushbu

$$\begin{aligned}
&\frac{\partial u}{\partial t}=\nabla\left(u^{m_1-1}\left|\nabla u^{k_1}\right|^{p_1-2}\nabla u\right)+v^{q_1}-\alpha_1 u^{r_1}, \quad x\in\Omega, t>0,\\
&\frac{\partial v}{\partial t}=\nabla\left(v^{m_2-1}\left|\nabla v^{k_2}\right|^{p_2-2}\nabla v\right)+u^{q_2}-\alpha_2 v^{r_2}, \quad x\in\Omega, t>0,\\
&u(x,t)=v(x,t)=0, \quad x\in\partial\Omega, t>0,\\
&u(x,0)=u_0(x), \quad v(x,0)=v_0(x), \quad x\in\Omega.
\end{aligned} \tag{10}$$

sistema o‘rganilgan bo‘lib, u biologik talqinda ikki tur o‘rtasidagi o‘zaro ta’sirni ifodalaydi, bu yerda $p_i\ge 2$, $k_i,m_i\ge 1$, $q_i,r_i,\alpha_i\ge 0$ $(i=1,2)$ va $\Omega\subset R^N$ esa silliq chegara $\partial\Omega$ bilan chegaralangan sohadir. Bu yerda boshlang‘ich qiymatlar quyidagi shartlarni qanoatlantiradi $u_0(x),v_0(x)\in C^{2+\zeta}(\bar{\Omega})$, $0<\zeta<1$, $u_0(x),v_0(x)\ge 0$, $u_0(x),v_0(x)\not\equiv 0$.

(10) masalaga o‘xshash ikki karra nochiziqli parabolik tenglamalar turli sohalarda, jumladan biologiyada uchraydi. Ular g‘ovak muhitda no-Nyuton oqimlarning kontsentratsiyasi yoki yonuvchi moddalarning harorat taqsimoti kabi hodisalarni tavsiflaydi.

Ushbu masala uchun quyidagi $\mu_1=\max\{m_1+k_1(p_1-2),r_1\}$, $\mu_2=\max\{m_2+k_2(p_2-2),r_2\}$ belgilashlar kiritilgan.

10-teorema. *Agar $q_1q_2<\mu_1\mu_2$ bo‘lsa, u holda (10) masalaning barcha nomanfiy yechimlari global bo‘ladi.*

11-teorema. *Faraz qilaylik, $q_1q_2=\mu_1\mu_2$ bo‘lsin. U holda quyidagi xulosalar o‘rinli:*

1. Agar $r_1>m_1+k_1(p_1-2)$; $r_2>m_2+k_2(p_2-2)$ ya’ni $q_1q_2=r_1r_2$ bo‘lsa, u holda (10) masala yetarlicha kichik boshlang‘ich qiymatlarda global yechimga va yetarlicha katta α_1, α_2 ning qiymatlarida chegaralanmagan yechimga ega bo‘ladi.

2. Agar $r_1<m_1+k_1(p_1-2)$ va $r_2<m_2+k_2(p_2-2)$ bo‘lsa, ya’ni $q_1q_2=(m_1+k_1(p_1-2))(m_2+k_2(p_2-2))$, u holda (10) masala yetarlicha kichik boshlang‘ich qiymatlarda global yechimga ega.

3. Agar $r_1 < m_1 + k_1(p_1 - 2)$ *va* $r_2 > m_2 + k_2(p_2 - 2)$ *bo'lsa, ya'ni* $q_1q_2 = (m_1 + k_1(p_1 - 2))r_2$*, u holda (10) masala yetarlicha kichik boshlang'ich qiymatlarda chegaralanmagan yechimga ega.*

4. Agar $r_1 > m_1 + k_1(p_1 - 2)$ *va* $r_2 < m_2 + k_2(p_2 - 2)$ *bo'lsa, ya'ni* $q_1q_2 = r_1(m_2 + k_2(p_2 - 2))$*, u holda (10) masala yetarlicha katta boshlang'ich qiymatlarda chegaralanmagan yechimga ega.*

III bobning 2-paragrafida ushbu

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \nabla\left(|x|^{n_1} u^{m_1-1}\left|\nabla u^{k_1}\right|^{p_1-2}\nabla u\right) + v^{q_1} - \alpha_1 u^{r_1}, \quad x\in\Omega,\ t>0,\\
&\frac{\partial v}{\partial t} = \nabla\left(|x|^{n_2} v^{m_2-1}\left|\nabla v^{k_2}\right|^{p_2-2}\nabla v\right) + u^{q_2} - \alpha_2 v^{r_2}, \quad x\in\Omega,\ t>0,\\
&u(x,t) = v(x,t) = 0, \quad x\in\partial\Omega,\ t>0,\\
&u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x\in\Omega.
\end{aligned} \tag{11}$$

o'zgaruvchan zichlikli nochiziqli manba va yutilishga ega diffuziya tenglamalar sistema o'rganilgan, bu yerda $p_i \ge 2$, $k_i, m_i \ge 1$, $n_i, q_i, r_i, \alpha_i \ge 0$ ($i = 1,2$) va $\Omega \subset R^N$ esa silliq chegara $\partial\Omega$ bilan chegaralangan sohadir. Yuqoridagi masaladagi boshlang'ich qiymatlar quyidagi shartlarni qanoatlantiradi $u_0(x), v_0(x) \in C^{2+\zeta}(\bar{\Omega})$, $0 < \zeta < 1$, $u_0(x), v_0(x) \ge 0$, $u_0(x), v_0(x) \not\equiv 0$.

12-teorema. *Faraz qilaylik* $n_1n_2 < p_1p_2$ *bo'lsin. Agar* $q_1q_2 < \mu_1\mu_2$ *bo'lsa, u holda (11) masalaning barcha nomanfiy yechimlari global bo'ladi.*

13-teorema. *Faraz qilaylik,* $n_1n_2 < p_1p_2$ *bo'lsin. Agar* $q_1q_2 > \mu_1\mu_2$ *bo'lsa, u holda (11) masalaning manfiy bo'lmagan yechimlari yetarlicha katta boshlang'ich qiymatlarda chegaralanmagan hamda yetarlicha kichik boshlang'ich qiymatlarda global bo'ladi.*

3-bobning 3-paragrafi (10) masalani sonli yechish va natijalarni vizuallashtirishga bag'ishlangan.

2-o'lchovli holda nochiziqli diffuziya masalasi $\Omega = [0,b_1]\times[0,b_2]$ sohada $t>0$ da qaralgan. Bunda (10) masala uchun quyidagi ifodaga ega bo'lamiz

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \left(\frac{\partial}{\partial x_1}\left(u^{m_1-1}\left|\frac{\partial u^{k_1}}{\partial x_1}\right|^{p_1-2}\frac{\partial u}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(u^{m_1-1}\left|\frac{\partial u^{k_1}}{\partial x_2}\right|^{p_1-2}\frac{\partial u}{\partial x_2}\right)\right) + v^{q_1} - u^{r_1},\\
&\frac{\partial v}{\partial t} = \left(\frac{\partial}{\partial x_1}\left(v^{m_2-1}\left|\frac{\partial v^{k_2}}{\partial x_1}\right|^{p_2-2}\frac{\partial v}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(v^{m_2-1}\left|\frac{\partial v^{k_2}}{\partial x_2}\right|^{p_2-2}\frac{\partial v}{\partial x_2}\right)\right) + u^{q_2} - v^{r_2},
\end{aligned} \tag{12}$$

$$u(x_1,x_2,t)=0,\ v(x_1,x_2,t)=0,\ x\in\partial\Omega \tag{13}$$

$$u(0,x)=u_0(x)\geq 0,\quad v(0,x)=v_0(x)\geq 0,\ 0\leq x_i\leq b_i,\ i=1,2 \tag{14}$$

x_1 va x_2 uchun $h_1=\dfrac{b_1}{n_1}$ va $h_2=\dfrac{b_2}{n_2}$ qadamlar bilan quyidagi to‘rni

$$\bar{\omega}_h=\left\{x_{ij}=\left(x_1^i,x_2^j\right),\ x_1^i=ih_1,\ x_2^j=jh_2,\ i,j=0,1,...,n_\alpha,\ \alpha=1,2\right\},$$

vaqt bo‘yicha esa $\tau=\dfrac{T}{m}$ qadam bilan quyidagini hosil qilamiz

$$\bar{\omega}_\tau=\left\{t_l=l\tau,\ \tau>0,\ l=0,1,...,m,\ \tau m=T\right\},\ T>0$$

(12) masalani sonli yechish uchun Pismen Rekfordning o‘zgaruvchan yo‘nalishlar metodidan foydalanilgan:

$$\begin{cases}\dfrac{y_{i,j}^{l+1/2}-y_{i,j}^{l}}{0.5\cdot\tau}=\Lambda_1 y^{l+1/2}+\Lambda_2 y^{l}+\left(Y^{l}\right)^{q_1}-\left(y^{l}\right)^{r_1}\\ \dfrac{y_{i,j}^{l+1}-y_{i,j}^{l+1/2}}{0.5\cdot\tau}=\Lambda_1 y^{l+1/2}+\Lambda_2 y^{l+1}+\left(Y^{l+1}\right)^{q_1}-\left(y^{l+1}\right)^{r_1}\end{cases} \tag{15}$$

$$\begin{cases}\dfrac{Y_{i,j}^{l+1/2}-Y_{i,j}^{l}}{0.5\cdot\tau}=\Lambda_1 Y^{l+1/2}+\Lambda_2 Y^{l}+\left(y^{l}\right)^{q_2}-\left(Y^{l}\right)^{r_2}\\ \dfrac{Y_{i,j}^{l+1}-Y_{i,j}^{l+1/2}}{0.5\cdot\tau}=\Lambda_1 Y^{l+1/2}+\Lambda_2 Y^{l+1}+\left(y^{l+1}\right)^{q_2}-\left(Y^{l+1}\right)^{r_2}\end{cases} \tag{16}$$

bu yerda

$$\Lambda_1 y^{l+1/2}=\frac{1}{h_1^2}\left[P_{i+1,j}\left(y^{l+1/2}\right)\left(y_{i+1,j}^{l+1/2}-y_{i,j}^{l+1/2}\right)-P_{i,j}\left(y^{l+1/2}\right)\left(y_{i,j}^{l+1/2}-y_{i-1,j}^{l+1/2}\right)\right]$$

$$\Lambda_2 y^{l}=\frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l}\right)\left(y_{i,j+1}^{l}-y_{i,j}^{l}\right)-Q_{i,j}\left(y^{l}\right)\left(y_{i,j}^{l}-y_{i,j-1}^{l}\right)\right]$$

$$\Lambda_2 y^{l+1}=\frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l+1}\right)\left(y_{i,j+1}^{l+1}-y_{i,j}^{l+1}\right)-Q_{i,j}\left(y^{l+1}\right)\left(y_{i,j}^{l+1}-y_{i,j-1}^{l+1}\right)\right]$$

$i=1,2,...n_1-1$; $j=1,2,...n_2-1$; $f(y)=v^{q_1}-u^{r_1}$

$$\Lambda_1 Y^{l+1/2}=\frac{|x|^{n_1}}{h_1^2}\left[Q_{i+1,j}\left(Y^{l+1/2}\right)\left(Y_{i+1,j}^{l+1/2}-Y_{i,j}^{l+1/2}\right)-Q_{i,j}\left(Y^{l+1/2}\right)\left(Y_{i,j}^{l+1/2}-Y_{i-1,j}^{l+1/2}\right)\right]$$

$$\Lambda_2 Y^{l}=\frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l}\right)\left(y_{i,j+1}^{l}-y_{i,j}^{l}\right)-Q_{i,j}\left(y^{l}\right)\left(y_{i,j}^{l}-y_{i,j-1}^{l}\right)\right]$$

$$\Lambda_2 y^{l+1}=\frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l+1}\right)\left(y_{i,j+1}^{l+1}-y_{i,j}^{l+1}\right)-Q_{i,j}\left(y^{l+1}\right)\left(y_{i,j}^{l+1}-y_{i,j-1}^{l+1}\right)\right]$$

$i=1,2,...n_1-1$; $j=1,2,...n_2-1$;

$P_{i,j}\left(y^{l}\right)$ va $Q_{i,j}\left(y^{l}\right)$ lar esa quyidagicha approksimatsiya qilingan

$$P_{i+1,j}\left(y^{l+1/2}\right)=\frac{1}{2}\left[\left(y_{i,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i+1,j}^{l+1/2}\right)^{k}-\left(y_{i,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}+\left(y_{i+1,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1/2}\right)^{k}-\left(y_{i-1,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}\right]$$

$$P_{i,j}\left(y^{l+1/2}\right)=\frac{1}{2}\left[\left(y_{i-1,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1/2}\right)^{k}-\left(y_{i-1,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}+\left(y_{i,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i-1,j}^{l+1/2}\right)^{k}-\left(y_{i-2,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}\right]$$

$$Q_{i,j+1}\left(y^{l}\right)=\frac{1}{2}\left[\left(y_{i,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j+1}^{l}\right)^{k}-\left(y_{i,j}^{l}\right)^{k}}{h_2}\right|^{p-2}+\left(y_{i+1,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l}\right)^{k}-\left(y_{i-1,j}^{l}\right)^{k}}{h_2}\right|^{p-2}\right]$$

$$Q_{i,j}\left(y^{l}\right)=\frac{1}{2}\left[\left(y_{i,j-1}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l}\right)^{k}-\left(y_{i,j-1}^{l}\right)^{k}}{h_2}\right|^{p-2}+\left(y_{i,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j-1}^{l}\right)^{k}-\left(y_{i,j-2}^{l}\right)^{k}}{h_2}\right|^{p-2}\right]$$

$$Q_{i,j}\left(y^{l+1}\right)=\frac{1}{2}\left[\left(y_{i,j-1}^{l+1}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1}\right)^{k}-\left(y_{i,j-1}^{l+1}\right)^{k}}{h_2}\right|^{p-2}+\left(y_{i,j}^{l+1}\right)^{m-1}\left|\frac{\left(y_{i,j-1}^{l+1}\right)^{k}-\left(y_{i,j-2}^{l+1}\right)^{k}}{h_2}\right|^{p-2}\right]$$

Izoh. *Barcha hisob-kitoblarda iteratsiya aniqligi* $\varepsilon=10^{-3}$ *deb qaralgan.*

Sonli yechishda yechimning tegishli boshlang‘ich yaqinlashishini topish juda muhim ahamiyatga ega. Sonli hisoblashlarda boshlang‘ich yaqinlashish sifatida quyidagicha olingan

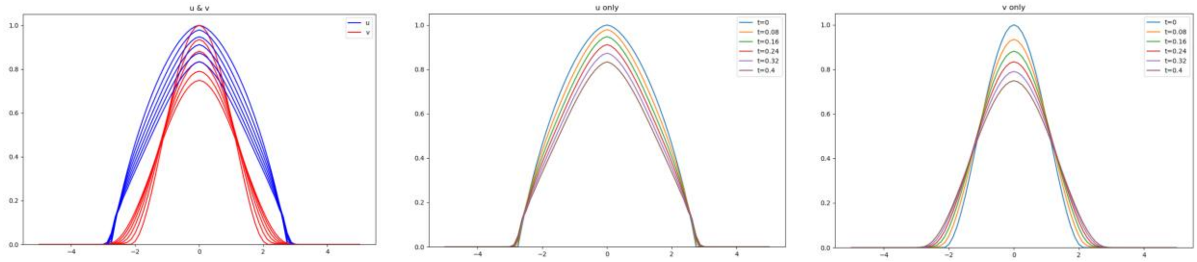
$$u_0(x)=T^{-\gamma_1}\left(A^{\frac{p_1}{p_1-1}}-\xi_1^{\frac{p_1}{p_1-1}}\right)_+^{\frac{p_1-1}{m_1-1+k_1(p_1-2)}},\quad v_0(x)=T^{-\gamma_2}\left(A^{\frac{p_2}{p_2-1}}-\xi_2^{\frac{p_2}{p_2-1}}\right)^{\frac{p_2-1}{m_2-1+k_2(p_2-2)}}$$

$$\xi_1=|x|/T^{\beta_1},\qquad \xi_2=|x|/T^{\beta_2},\qquad \gamma_1=\frac{q_1+1}{q_1q_2-1},\qquad \gamma_2=\frac{q_2+1}{q_1q_2-1},$$

$$\beta_1=\frac{1-\gamma_1\left(m_1+k_1\left(p_1-2\right)-1\right)}{p_1}>0,\ \beta_2=\frac{1-\gamma_2\left(m_2+k_2\left(p_2-2\right)-1\right)}{p_2}>0$$

Parametrlarning turli qiymatlarida olingan natijalar quyidagicha. Xususan, 5-rasmda global yechim erkin chegaraning tarqalish chekli tezligi ifodalangan.

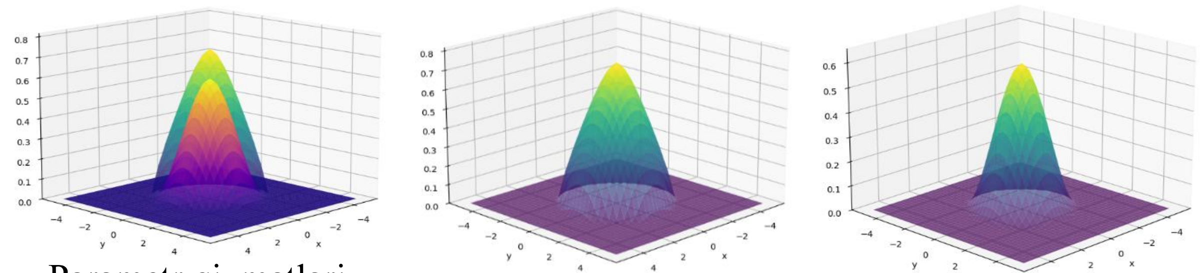
u & v | u | v



Parametr qiymatlari keyingi yacheykalarda | $m_1=1.9$, $k_1=1.7$, $p_1=2.5$, $q_1=2.2$, $r_1=1.0$ | $m_2=1.4$, $k_2=1.2$, $p_2=2.1$, $q_2=2.5$, $r_2=1.0$

5-rasm. 1 o‘lchovli holatda (10) masalaning sonli yechimi.

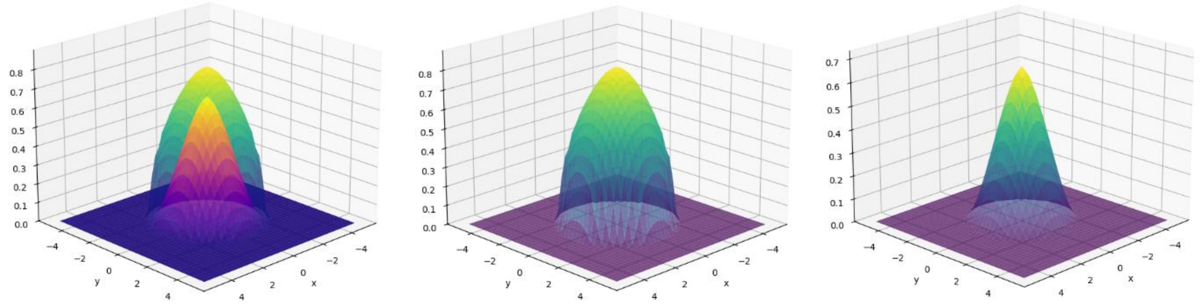
u & v | u | v



Parametr qiymatlari keyingi yacheykalarda | $m_1=1.9, k_1=1.7, p_1=2.5, q_1=2.2, r_1=1.0$ | $m_2=1.4, k_2=1.2, p_2=2.1, q_2=2.5, r_2=1.0$

6-rasm. 2 o‘lchovli holatda (10) masalaning sonli yechimi.

u & v | u | v



Parametr qiymatlari keyingi yacheykalarda | $m_1=2.5, k_1=2.5, p_1=2.1, q_1=2.1, r_1=3.0$ | $m_2=1.2, k_2=1.2, p_2=2.5, q_2=2.9, r_2=1.0$

7-rasm. 2 o‘lchovli holatda (10) masalaning sonli yechimi.

3.4 paragrafda Nonlinear Diffusion Modeling nomli dasturlar majmuining tavsifi va undan foydalanish bo‘yicha ko‘rsatmalar keltirilgan. Dasturlar majmui nochiziqli diffuziya jarayonlarini modellashtirishga mo‘ljallangan. Dasturlar majmui Python dasturlash tilidan foydalanib, Spyder IDE dasturlash muhitida yozilgan, unda matplotlib, numpy kutubxonalari yodamida vizuallashtirishni amalga oshirilgan. Hisoblash nuqtai nazaridan, nochiziqli diffuziya masalalarini yechish va sonli natijalarni olish qiyin. Bunday masalalarni hal qilish barqaror va hisoblash jihatdan samarali sxemalarni talab qiladi.

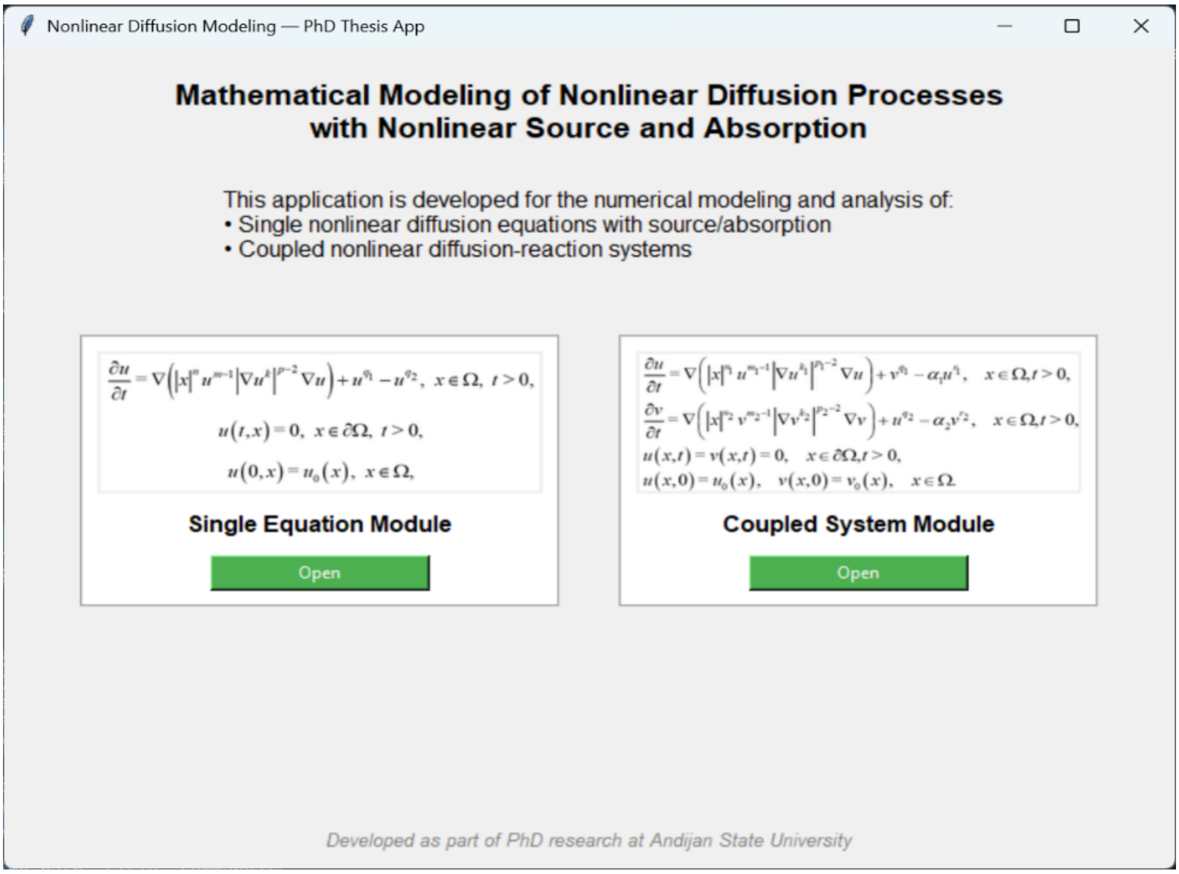
Dasturlar majmui Python 3.x da modulli arxitekturadan foydalangan holda maxsus ishlab chiqilgan bo‘lib quyidagi qismlardan iborat:

• Asosiy interfeys (main.py) - Tkinter-ga asoslangan foydalanuvchi interfeysi, bitta tenglama va sistema modullariga kirish imkoniyatlarini taqdim etadi.

• Tenglama (equation.py) – dinamik vizuallashtirish animatisya imkoniyatlariga ega Pikar / Nyuton chiziqlashtirish usullarini qo‘llab-quvvatlovchi manba / yutilishga ega bir va ikki o‘lchovli nochiziqli diffuziya tenglamalarini yechish imkoniyatini taqdim etadi.

• Sistema (system.py) – 2D / 3D vizuallashtirish imkoniyatiga ega nochiziqli reaksiya-diffuziya sistema yechimlarini simulyatsiya qilish imkoniyatini taqdim etadi.

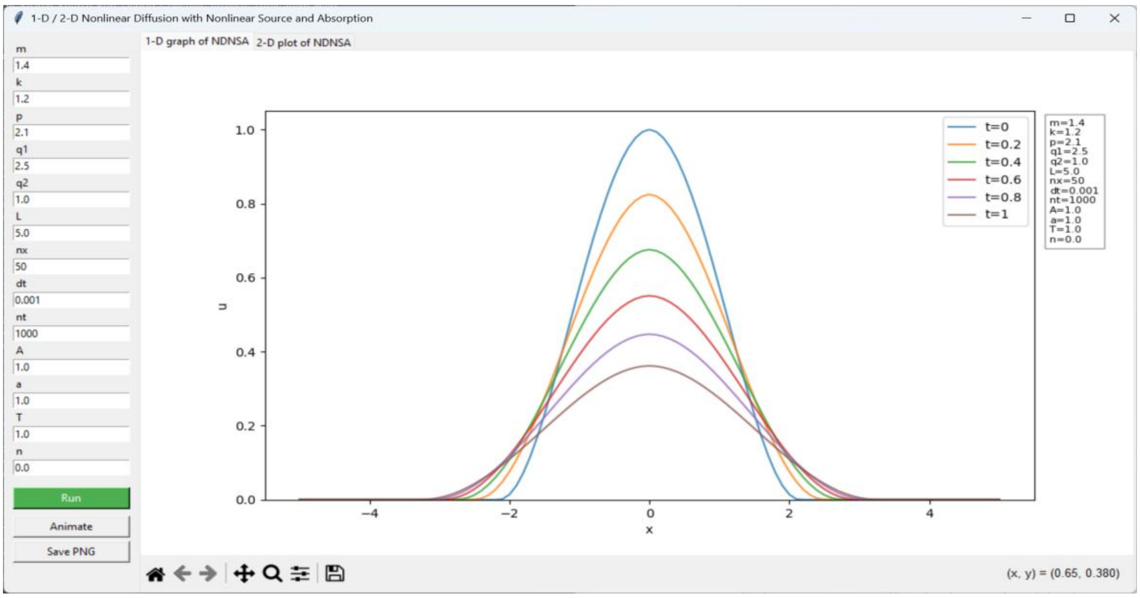
• Yordamchi dasturlar – ma’lumotlarni eksport qilish, xatolarni tahlil qilish va natijalarni aks ettirishga bag‘ishlangan.



8-rasm. Dasturining asosiy oynasiinterfeysi (modul tanlash oynasi).

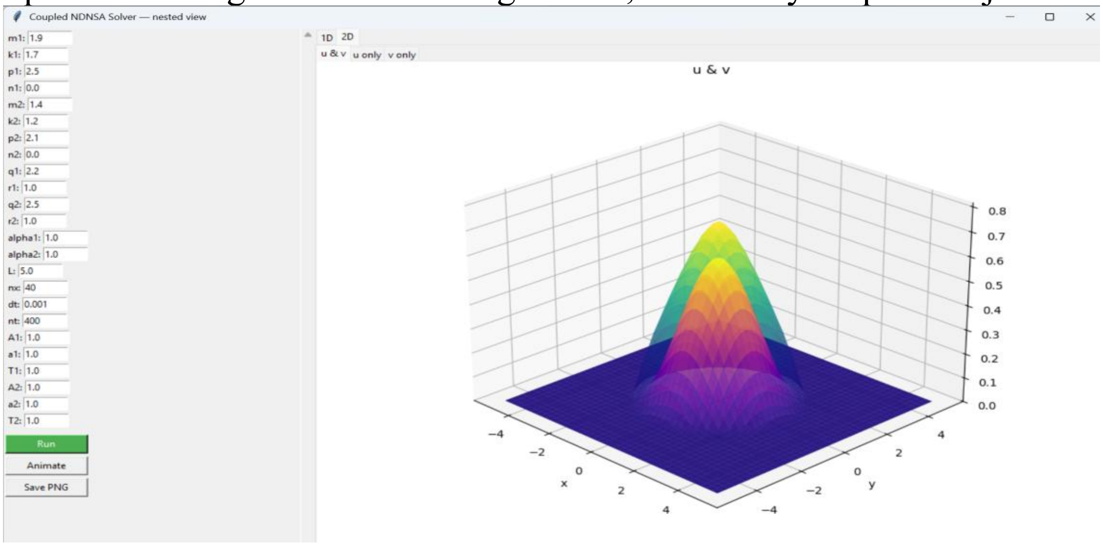
Ilovaning asosiy oynasi (8-rasmga qarang) Tkinter yordamida ishlab chiqilgan yagona kirish nuqtasi hisoblanadi. Foydalanuvchilar bitta tenglama va sistema modullariga o‘tishga imkon beradi.

Tenglama moduli (9-rasmga qarang) manba yoki yutilishga ega nochiziqli diffuziya jarayonlarini simulyatsiya qilishga mo‘ljallangan. Foydalanuvchilar fazo, vaqt va model parametrlarini kiritishi, so‘ng dastur vaqt o‘tishi bilan yechim profillarini hisoblashi va vizuallashtirishi nazarda tutilgan. Qolaversa, 1D va 2D yorliqlar yordamida diskretizatsiyalarni tanlashga imkoniyati taqdim etilgan.



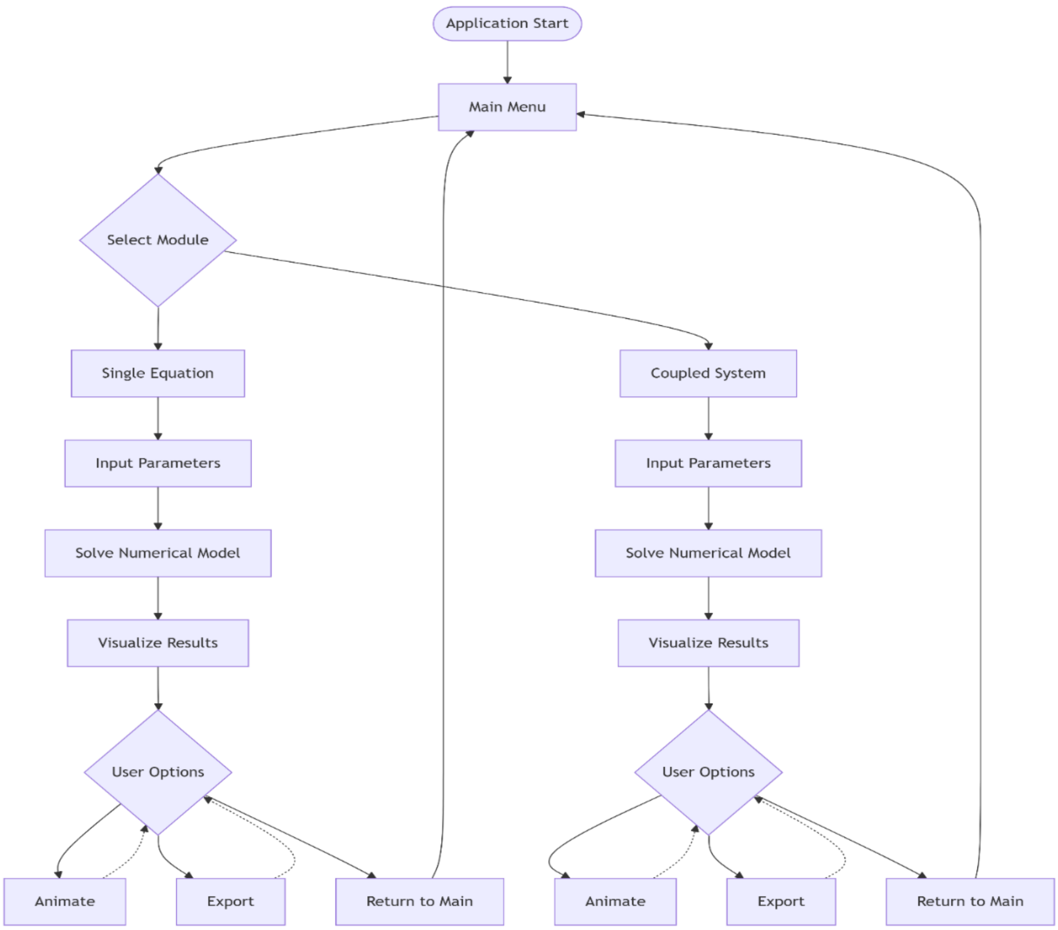
9-rasm. Dasturning tenglama moduli.

Diffuziya jarayonlarini ifodalovchi tenglamalar sistemasini yechishga mo‘ljallangan (10-rasmga qarang) dasturiy ta’minot tenglamani yechish moduli imkoniyatlarini kengaytiradi. U tizim parametrlarini, boshlang‘ich va chegara shartlarini qabul qiladi. Bitta tenglama holatida bo‘lgani kabi, 1D va 2D yorliqlari mavjud.



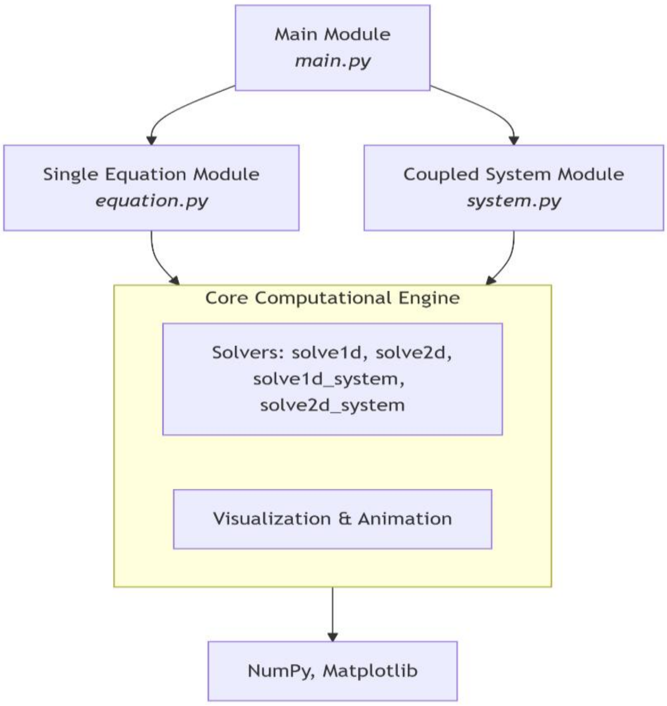
10-rasm. Dasturning sistema moduli.

Ilovaning umumiy ish oqimi 11-rasmda tasvirlangan. Jarayon interfeys orqali parametrlarni aniqlash bilan boshlanadi, so‘ngra qayta ishlash va tekshirish jarayonlari amalga oshiriladi. Raqamli modul chekli ayirmali sxema yordamida hisoblashlarni amalga oshiradi, shundan so‘ng natijalar vizuallashtiriladi va eksport qilish tartiblariga o‘tkaziladi.



11-rasm. Hisoblash dasturining sxemasi.

Dasturning yuqori darajadagi arxitekturasi 12-rasmda tasvirlangan.



12-rasm. Dasturning arxitekturasi.

XULOSA

Ushbu dissertatsiya ishida nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi masalani modellashtirish, sonli yechimlarni olish va tahlil qilish vazifalari bajarildi. Bunda nazariy tahlil, sonli usullar va dasturlar majmui yaratildi:

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli model yechimlarining vaqt bo‘yicha globallik hamda global bo‘lmaslik shartlari topildi;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya sistemalari yechimlarining sonli hisoblashlarda muhim bo‘lgan quyi va yuqori baholari olindi;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarini ifodalovchi nochiziqli masalalarni sonli hisoblash uchun zarur bo‘lgan boshlang‘ich yaqinlashishlar topildi;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya jarayonlarining matematik modellari sifat xossalarini o‘rganish uchun sonli hisoblash sxemalari qurildi;

o‘zgarmas yoki o‘zgaruvchan zichlikli, nochiziqli manba va yutilishga ega nochiziqli diffuziya masalasini yechish va vizuallashtirish uchun dasturiy vositalar majmui ishlab chiqildi.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREESDSc.03/30.12.2019.FM.01.02
NATIONAL UNIVERSITY OF UZBEKISTAN

ANDIJAN STATE UNIVERSITY

ATABAEV ODILJON KHUSNIDDIN UGLI

MATHEMATICAL MODELING OF NONLINEAR DIFFUSION PROCESSES WITH NONLINEAR SOURCE AND ABSORPTION

05.01.07-Mathematical simulation. Numerical methods and Software
(Physical-Mathematical sciences)

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

Tashkent – 2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan under number B2024.4.PhD/FM1214.

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Scientific supervisor:	**Aripov Mersaid** Doctor of physical and mathematical sciences, professor
Official opponents:	**Sadullaeva Shakhlo Azimbaevna** Doctor of physical and mathematical sciences, professor
	Khaydarov Abdugappar Candidate of physical and mathematical sciences, docent
Leading organization:	**Urgench State University**

Defense will take place "___" ________ 2025 at ____ at the meeting of Scientific Council DSc.03/30.12.2019.FM.01.02 at National University of Uzbekistan. (Address: 4 University str. Almazar district, Tashkent, 100174, Uzbekistan. Tel.: (+99871) 227-12-24, Fax: (+99871) 246-53-21, 246-02-24, e-mail: nauka@nuu.uz).

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Abstract of dissertation sent out on "___" _________ 2025 year.
(Mailing report No__________ on "___" ___________2025 year).

B.F.Abdurakhimov
Co-chairman of Scientific Council on award of scientific degrees, D.Ph.-M.S., professor

Z.R.Rakhmonov
Scientific secretary of Scientific Council on award of scientific degrees, D.Ph.-M.S., professor

A.S.Matyakubov
Chairman of Scientific Seminar under Scientific Council on award of scientific degrees, D.Ph.-M.S., docent

INTRODUCTION (abstract of PhD thesis)

Relevance and necessity of the dissertation topic. In the world, special attention is paid to the mathematical modeling of complex nonlinear processes, such as physical, biological, mechanical, ecological, seismological, and other processes, and to the improvement of their numerical-analytical research methods in the oil and gas and production sectors. In this regard, the development of complex nonlinear mathematical modeling problems described using partial differential equations and their systems, methods for their numerical research, and the creation of effective algorithms for performing large-scale calculations are of great importance. Accordingly, special attention is paid to the creation and improvement of effective numerical solution methods for the mathematical description of natural processes and the implementation of nonlinear mathematical models in practice, the construction of mathematical models of nonlinear diffusion processes, the creation of algorithms and software for performing complex calculations on nonlinear mathematical models.

In the world, scientific research is being conducted aimed at the development and application of numerical solution methods in the study of mathematical models of processes such as biological population, reaction-diffusion, and thermonuclear fusion. In this regard, research on numerical modeling of processes such as reaction-diffusion, heat transfer, filtration, biological population, which have practical applications in the energy, medicine, and oil and gas sectors, is considered a priority. Accordingly, in the study of nonlinear mathematical models, the development of algorithms and software for numerical calculation and visualization of solutions and the application of results to practice are considered targeted scientific research.

In our country, along with the development of fundamental sciences, special attention is paid to the application of the obtained scientific results to practice, especially the creation of new innovative technologies of great practical importance, the improvement of methods for numerical-analytical solution of nonlinear problems in mathematical modeling. In particular, the development of methods for studying the qualitative properties of nonlinear mathematical models, finding accurate estimates of solutions over time, determining nonlinear effects, and developing economical numerical schemes; Significant results have been achieved in creating a set of practical programs that help study mathematical models of nonlinear processes. Conducting scientific research at the level of international standards in priority areas such as "Functional analysis, algebra, differential equations, mathematical physics, mathematical modeling, computational mathematics and discrete mathematics, probability theory and mathematical statistics"[1]. is one of the main tasks of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan. In ensuring the implementation of the decision, it is important to determine the asymptotics of solutions of equations and systems of parabolic type representing nonlinear diffusion processes with a nonlinear source and absorption,

[1] Resolution of the President of the Republic of Uzbekistan No. DP-4708 dated May 7, 2020 “On measures to improve the quality of education and develop scientific research in the field of mathematics”

to obtain estimates, to develop methods for studying qualitative properties, and to create numerical solution programs.

This dissertation research to a certain extent serves to solve the problems outlined in the Decree of the President of the Republic of Uzbekistan DP-No. 60 dated January 28, 2022 "On the development strategy of the New Uzbekistan for 2022-2026", RP - No. 4708 dated May 7 2020 “On measures to improve the quality of education and develop scientific research in the field of mathematics”, DP- No. 5847 dated October 8, 2019 “On approval of the Concept for the development of the higher education system of the Republic of Uzbekistan until 2030”, RP - No. 3682 dated April 27, 2018 "On measures to further improve the system of practical implementation of innovative ideas, technologies and projects", RP - No. 6198 dated April 1, 2021 "On Improving the System of State Administration for the Development of Scientific and Innovative Activities" as well as in other regulatory legal acts related to this area of activity.

The connection of the study with the priority areas of development of science and technology of the republic. This dissertation work is carried out in accordance with the priority directions of development of science and technology of the Republic of Uzbekistan IV. “Mathematics, Mechanics and Computer Science”.

The level of study of the problem. In the world, a few studies have been conducted on the qualitative properties of solutions of nonlinear equations and systems of equations and to analyze their numerical solutions. Based on the results of these studies, new effects were obtained for mathematical models expressing nonlinear processes in various fields, particularly nonlinear diffusion equations with a nonlinear source and absorption. Later, the asymptotic properties of the solution were studied in the works of A.A. Samarsky, V.A. Galaktionov, J.L. Vazquez, S.P. Kurdyumov, A.P. Mikhailov. Also, A. Friedman, J. McLeod, M. Escobedo, M.A. Herrero, H.A. Levine, K. Deng, P. Souplet, N. Bedjaoui, Zh. Li, W. Du, H. Li, Ch. Mu and others determined the conditions for the existence of a global solution. The focus of scientific research was on the existence of a global solution, the study of the properties of an unbounded solution in finite time. In addition, Wiegner M., Winkler M., Gage E., Angenent S., Jin Ch., Yin J., Yunzhu Gao, Qiu Meng, Yingjia Guo, Zhi-wen Duan, Li Zhou determined the asymptotics of solutions, its unboundedness, the effect of diffusion at finite speeds and spatial localization of heat dissipation, the existence of the process of influence in nonlinear environments with a source or absorption in finite time, etc.

B. M. Khujayarov, N. Muhitdinov, A. S. Rasulov, M. Aripov, J. Tokhirov, N. Ravshanov, Sh. Sadullayeva, A. Khaydarov, A.Khasanov, B.Babajanov, A. Matyakubov, F.Kabiljanova, Z. Rakhmonov and their students were engaged in nonlinear problems of the parabolic type, describing mathematical models of various processes in Uzbekistan. Their main works are devoted to the numerical study of the properties of solutions to nonlinear diffusion problems expressed by divergent and non-divergent equations and systems of equations, and the application of these methods to model filtration, diffusion, and heat transfer processes. They studied the qualitative properties of solutions to nonlinear problems representing processes occurring in various fields of natural science based on self-similar analysis and presented numerical results and analyses.

The connection of the dissertation with scientific research projects. The dissertation research was carried out as part of a research in accordance with the plan of research work of the National University of Uzbekistan named after Mirzo Ulugbek with the scientific research projects OT-F4-30 “Study of qualitative properties under the influence of convective migration, variable density, source or absorption of a double-nonlinear cross-system” and the topic AL-9224104601 “Mathematical modeling of processes described by parabolic equations and systems of nonlinear, divergent and nondivergent forms”.

The purpose of the dissertation work is to conduct numerical and analytical investigations of the qualitative properties of nonlinear mathematical models described by parabolic-type equations and their systems in media with constant or variable density, nonlinear sources, and absorption.

The tasks of the research:

to determine the conditions for global existence and non-existence in time of the solutions of nonlinear diffusion models with constant or variable density, nonlinear source and absorption;

to obtain lower and upper bounds that are important for numerical simulation of nonlinear diffusion systems with constant or variable density, nonlinear source, and absorption;

to determine effective initial approximation required for the numerical solution of nonlinear diffusion problems with constant or variable density, nonlinear source and absorption;

to apply and analyze numerical calculation schemes for studying the qualitative behavior of mathematical models of nonlinear diffusion processes with constant or variable density, nonlinear source and absorption;

to develop a problem-oriented software package for solving and visualizing nonlinear diffusion problems with constant or variable density, nonlinear source, and absorption.

The object of the research is nonlinear diffusion processes described by parabolic equations and systems with constant and variable densities, nonlinear sources and absorptions.

The subject of the research is to study the qualitative properties of mathematical models of nonlinear diffusion processes with constant or variable density, nonlinear source and absorption, as well as the development of numerical solution schemes, and create software packages.

Research methods. This dissertation work uses a nonlinear separation algorithm, self-similar and approximate self-similar methods, the method of standard equations, solution comparison theorems, differential schemes, iteration and sweep methods.

The scientific novelty of the research is as follows:

The conditions ensuring global existence and non-existence in time of the solutions of nonlinear diffusion models with constant or variable density, nonlinear source and absorption have been determined;

important qualities for numerical simulation that is lower and upper bounds of the solutions of nonlinear diffusion systems with constant or variable density, nonlinear source and absorption have been obtained;

effective initial approximations required for the numerical solution of nonlinear diffusion problems with constant or variable density, nonlinear source and absorption have been determined;

numerical calculation schemes have been applied and analyzed to study the qualitative behavior of nonlinear diffusion model with constant or variable density, nonlinear source and absorption;

a problem-oriented software package has been developed for solving and visualizing nonlinear diffusion problems with constant or variable density, nonlinear source and absorption.

The practical results of the research are as follows:

Lower and upper bounds were obtained that ensure the accuracy of numerical simulations of nonlinear diffusion processes with constant or variable density, nonlinear source and absorption;

Numerical methods were implemented, and a software package was developed to perform computational experiments and visualize solutions of nonlinear diffusion problems with constant or variable density, nonlinear source, and absorption.

Reliability of research results. The assertions obtained in the dissertation are rigorously proven based on comparison theorems and the maximum principle and are validated by computational experiments demonstrating the consistency of the obtained results with the conservation laws.

Scientific and practical significance of the research results. The scientific significance of the research lies in the development and the application of methods for constructing self-similar solutions, deriving asymptotic formulas, and designing numerical schemes and algorithms for analytical and numerical studies of models describing virus propagation, gas and liquid filtration, diffusion, and heat transfer processes.

The practical significance of the research is determined by implementation of iterative processes, the creation of a numerical computation scheme, and the development of software package that enables effective computational experiments that can be applied to nonlinear heat transfer, gas and liquid filtration, diffusion, virus propagation, and biological population problems.

Implementation of research results. The scientific results obtained on the mathematical and numerical modeling of nonlinear diffusion processes with constant or variable density, nonlinear source and absorption have been implemented in practice in the following areas:

the lower and upper bounds of the solutions of nonlinear diffusion systems with constant or variable density, nonlinear source and absorption were used in the fundamental grant project OT-F4-88 “Investigation of direct and inverse problems for mixed-type equations of the second and higher order” to solve the Dirichlet problem for a perturbative equation with singular coefficients and spectral parameters in a vertical half-pass. (Reference of the V.I. Romanovsky Institute of Mathematics dated December 5, 2024 No. 02/437). As a result, an exact solution of the Dirichlet problem for a perturbative equation with singular coefficients and spectral parameters in a vertical half-stripe was found.

the scientific result of determining the initial approximation necessary for the numerical calculation of nonlinear problems representing nonlinear diffusion processes with constant or variable density, nonlinear source and absorption was used to solve the Cauchy problem posed for the Kaup system in the class of periodic functions in the fundamental grant project OT-F4-04 “Applications of the spectral method to solving matrix nonlinear evolution equations, Biomechanics of the cardiovascular system” (Reference of Urgench State University dated December 6, 2024 No. 04-235/2). As a result, the imposed solution made it possible to determine the exact forms of periodic solutions of the Cauchy problem posed for the Kaup system.

Approval of research results. The results of this research work were discussed at 11 scientific and practical conferences, including 7 international and 4 republican scientific and practical conferences.

Publication of research results. A total of 19 scientific works were published on the topic of the research, of which 6 articles were published in scientific publications recommended for publication of the main scientific results of thesis of Doctor of Philosophy of the Higher Attestation Commission of the Republic of Uzbekistan, including 1 foreign (**Scopus, Q2**) and 5 in republican journals. Also, 2 author's certificates were obtained on the official registration of the program created for the computer.

Structure and scope of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion, a list of used literature and appendices. The volume of the dissertation consists of 96 pages.

THE MAIN CONTENT OF THE DISSERTATION

The introduction is based on the relevance and necessity of the thesis, the compliance of the research with the priority directions of the development of science and technology of the republic, a review of foreign scientific research on the topic, the degree of study of the problem is given, the purpose, tasks, object and subject of the research are described, the scientific novelty and practical results of the research are outlined, the theoretical and practical significance of the results obtained is revealed, Data on the introduction, published works and the structure of the dissertation are presented.

The first chapter of the thesis, entitled **"Properties of solutions of nonlinear parabolic equations with nonlinear source and absorption",** discusses the global availability of solutions of the nonlinear diffusion model with nonlinear source and absorption, the importance of qualitative properties such as unbounded within a finite time and asymptotic motion.

In paragraph 1.1, population dynamics, heat conductivity in nonlinear media, and reaction-diffusion processes can be modeled by a double nonlinear disruptive parabolic equation with nonlinear source and absorption. In particular, this

$$\frac{\partial u}{\partial t} = \nabla\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + u^{q_1} - u^{q_2}, \quad x \in \Omega, \quad t > 0, \tag{1}$$

$$u(t,x) = 0, \quad x \in \partial\Omega, \quad t > 0, \tag{2}$$

$$u(0,x) = u_0(x), \quad x \in \Omega, \tag{3}$$

represents a double nonlinear matter reaction-diffusion processes, where n, $m,k>1$, $p>2$, $q_1>1$, $q_2\geq 1$, $q_1\neq q_2$ and $\Omega\subset R^N$ with $\partial\Omega$ The initial condition $u_0(x)$ is a notrivial, non-negative, bounded, and correspondingly smooth function.

Paragraph 1.2 describes the research work and the results obtained on nonlinear modeling in the fields of fluid dynamics, biological population, economic mathematics.

Paragraph 3 of Chapter I In this section the general framework of solutions for the nonlinear parabolic problem (1)-(3) is discussed. Key definitions, notations and auxiliary results have been introduced that will be used throughout the work.

Definition 1. Let $T>0$, $Q_T=\Omega\times(0,T)$, $E=\{u\in L^{2q_1}(Q_T)\cup L^{2q_2}(Q_T);u_t,\nabla u\in L^2(Q_T)\}$ and $E_0=\{u\in E;\ u=0\ on\ \partial\Omega\}$. If for any nonnegative function $\varphi\in E_0$ following

$$\iint_{Q_T}u_t\varphi dxdt+\iint_{Q_T}\left|\nabla u^k\right|^{p-2}\nabla u^m\nabla\varphi dxdt\geq(\leq)\iint_{Q_T}u^{q_1}\varphi-u^{q_2}\varphi dxdt$$

$$u(x,t)\geq(\leq)0\ on\ \partial\Omega\times(0,T)\ and\ u(x,0)\geq(\leq)u_0(x)\ a.e.\ in\ \Omega.$$

statement holds, then nonnegative function $u(t,x)\in E$ called the *weak upper (lower)* solution of problem (1)-(3) in Q_T.

Definition 2. If the function $u(t,x)$ is both weak upper and weak lower solution of problem (1)-(3), then it is called *weak* solution.

Definition 3. If function $u(t,x)$ is the weak solution of (1)-(3) in Q_T for all $T<\infty$, then it is called *global* solution.

Definition 4. The problem (1) is called "degenerate" if it presents the parabolic equation in the case when $u(t,x)>0$, $|\nabla u|>0$, and it becomes ordinary differential equation for $u(t,x)=0$ or $|\nabla u|=0$.

The fact that the solution of the partial derivative differential equation is not bounded in a finite time shows the property of singularity at some point within this field.

Definition 5. If there exists such $T<\infty$, and following

$$\lim_{t\to T^-}\|u(t)\|_{L^\infty(\Omega)}=\infty$$

condition holds, then the solution of problem (1)-(3) called blow-up.

Theorem 1. *Let $u_1(t,x)$ and $u_2(t,x)$ are lower and upper solutions of following problem*

$$\frac{\partial u}{\partial t}=\nabla\cdot\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right)+u^{q_1}-u^{q_2},\quad x\in\Omega,t>0,$$

$$u(t,x)=0,\ x\in\partial\Omega,\ t>0$$

$$u(0,x)=u_0(x),\ x\in\Omega$$

If the condition $u_1(0,x)\leq u_2(0,x)$ holds in Ω, then for all $t>0$ and $x\in\Omega$ $u_1(t,x)\leq u_2(t,x)$ holds.

Chapter II of the dissertation called **"Mathematical modeling of nonlinear diffusion processes with nonlinear source and absorption"** and there discussed solution estimates and numerical solution of nonlinear diffusion problem with nonlinear source and absorption with variable density.

The following theorems play a fundamental role in mathematical modeling, numerical simulation and software implementation of problem (1)-(3). In paragraph 2.1 the parameter region $\left(m+k\left(p-2\right),q_1,q_2\right)$ divided into 3 classes in order to study the qualitative properties of solution of problem (1)-(3):

(i) $q_1 < \max\{m+k\left(p-2\right),q_2\}$;

(ii) $q_1 = \max\{m+k\left(p-2\right),q_2\}$;

(iii) $q_1 > \max\{m+k\left(p-2\right),q_2\}$.

Theorem 2. *If* $q_1 < \max\{m+k\left(p-2\right),q_2\}$, *then all solutions of problem (1)-(3) are bounded.*

Theorem 3. *If* $q_1 = m+k\left(p-2\right)$, $q_1 > q_2$ *and* $\lambda_1 \ge 1$, *then all solutions of problem (1)-(3) are global.*

Theorem 4. *The problem (1)-(3) has both global and nonglobal solutions under the following conditions:*

- $q_1 = m+k\left(p-2\right) > q_2$ va $\lambda_1 < 1$
- $q_1 > \max\{m+k\left(p-2\right),q_2\}$.

Theorem 5. *Suppose* $q_1 > \max\{m+k\left(p-2\right),q_2\}$. *If the solution* $u\left(t,x\right)$ *of problem (1)-(3) blows-up in finite time* T, *then there exists a positive constant* c *such that:*

$$\max_\Omega u\left(t,x\right) \ge c\left(T-t\right)^{-\frac{1}{q_1-1}} \quad as \quad t\to T.$$

Theorem 6. *Suppose* $q_1 > m+k\left(p-2\right) \ge q_2$. *If the solution* $u\left(t,x\right)$ *of problem (1)-(3) blows-up in finite time* T, *then there exists a positive constant* C *such that*

$$\max_\Omega u\left(t,x\right) \le C\left(T-t\right)^{-\frac{1}{q_1-1}} \quad as \quad t\to T.$$

Paragraph 2.2 of the work is dedicated to the study of doubly nonlinear parabolic equation with nonlinear source and absorption terms and variable density

$$\frac{\partial u}{\partial t} = \nabla\left(\left|x\right|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + u^{q_1} - u^{q_2},\ x\in\Omega,\ t>0, \tag{4}$$

$$u\left(t,x\right)=0,\ x\in\partial\Omega,\ t>0, \tag{5}$$

$$u\left(0,x\right)=u_0\left(x\right),\ x\in\Omega, \tag{6}$$

where n, $m,k>1$, $p>2$, $q_1>1$, $q_2\ge 1$, $q_1\ne q_2$ and $\Omega\subset R^N$ is a bounded domain with smooth boudary $\partial\Omega$, Here $u_0\left(x\right)$ is a nontrivial, nonnegative, bounded, and appropriately smooth function.

Theorem 7. *If* $n<p$ *and* $q_1 < \max\left\{m+k\left(p-2\right),q_2\right\}$, *then all non-negative solutions of problem (4)-(6) are global.*

Theorem 8. *If* $n<p$ *and* $q_1=m+k(p-2)>q_2$, *then there exists both global and blow-up solutions of problem (4)-(6).*

Theorem 9. *If* $n<p$ *and* $q_1>\max\{m+k(p-2),q_2\}$, *then any non-negative solution of (4)-(6) with large initial data blows-up in finite time.*

Third paragraph of Chapter 2 dedicated to mathematical modeling of doubly nonlinear parabolic problem with nonlinear source and absorption terms and variable density.

1 dimensional case. First, the nonlinear diffusion problem considered in $x\in[0,L]$ and $t>0$, i.e. one-dimensional case, that is

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)+u^{q_1}-u^{q_2},\quad x\in(0,L),\quad t>0, \tag{7}$$

$$u(t,0)=0,\quad u(t,L)=0,\quad t\geq 0, \tag{8}$$

$$u(0,x)=u_0(x),\quad x\in[0,L]. \tag{9}$$

Special numerical scheme and algorithms were developed for numerical solution of the problem. Following spatial and time grids were constructed with respect to x and t as

$$\bar{\omega}_h=\{x_i=ih,\, h>0,\, i=0,1,...,n,\, hn=L\},\ \bar{\omega}_\tau=\{t_j=j\tau,\, \tau>0,\, j=0,1,...,m,\, \tau m=T\}$$

,

and approximated as following

$$\begin{cases}\dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h^2}\left[P_{i+1}\left(y^{j+1}\right)\left(y_{i+1}^{j+1}-y_i^{j+1}\right)-P_i\left(y^{j+1}\right)\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right]+f_i\left(y^{j+1}\right),\\ i=1,2,...,n-1;\ j=0,1,...,m-1\\ y_i^0=u_o(x_i),\ i=0,1,...,n\\ y_0^j=0,\ \ j=1,2,...,m\\ y_n^j=0,\ \ j=1,2,...,m\end{cases}$$

Here P_{i+1} and P_i nonlinear terms representing diffusion coefficients. These terms are calculated as following

$$P_{i+1}\left(y^{j+1}\right)=\frac{1}{2}\left[\left(y_i^{j+1}\right)^{m-1}\left|\frac{\left(y_{i+1}^{j+1}\right)^k-\left(y_i^{j+1}\right)^k}{h}\right|^{p-2}-\left(y_{i+1}^{j+1}\right)^{m-1}\left|\frac{\left(y_i^{j+1}\right)^k-\left(y_{i-1}^{j+1}\right)^k}{h}\right|^{p-2}\right]$$

$$P_i\left(y^{j+1}\right)=\frac{1}{2}\left[\left(y_{i-1}^{j+1}\right)^{m-1}\left|\frac{\left(y_i^{j+1}\right)^k-\left(y_{i-1}^{j+1}\right)^k}{h}\right|^{p-2}-\left(y_i^{j+1}\right)^{m-1}\left|\frac{\left(y_{i-1}^{j+1}\right)^k-\left(y_{i-2}^{j+1}\right)^k}{h}\right|^{p-2}\right]$$

Linearization of nonlinear terms made up with different methods (see Table 1)

Table 1.

Choosen linearization methods for nonlinear terms

#	Nonlinear term	Linearization	Nonlinear term	Linearization
1	u^{q_1} - source	Newton	u^{q_2} - absorption	Picard
2	u^{q_1} - source	Newton	u^{q_2} - absorption	Newton
3	u^{q_1} - source	Picard	u^{q_2} - absorption	Picard
4	u^{q_1} - source	Picard	u^{q_2} - absorption	Newton

Now proceed with 1st choice of linearization, that is apply Newton's method for source term and Picard's iteration method for absorption term, respectively

$$f_i\left(y^{j+1}\right)=q_1\left(y_i^j\right)^{q_1-1}\left(y_i^{j+1}-y_i^j\right)+\left(y_i^j\right)^{q_1}-\left(y_i^j\right)^{q_2}.$$

This will lead to the following

$$\frac{y_i^{j+1,s+1}-y_i^j}{\tau}=\frac{1}{h^2}\left[P_{i+1}(y^{j+1,s})(y_{i+1}^{j+1,s}-y_i^{j+1,s})-P_i(y^{j+1,s})(y_i^{j+1,s}-y_{i-1}^{j+1,s})\right]+f_i(u^{j+1,s}),$$

here iteration process is continued until $\max_i\left|y_i^{j+1,s+1}-y_i^{j+1,s}\right|<\varepsilon$.

Remark. *In all calculations iteration accuracy is set to* $\varepsilon=10^{-3}$.

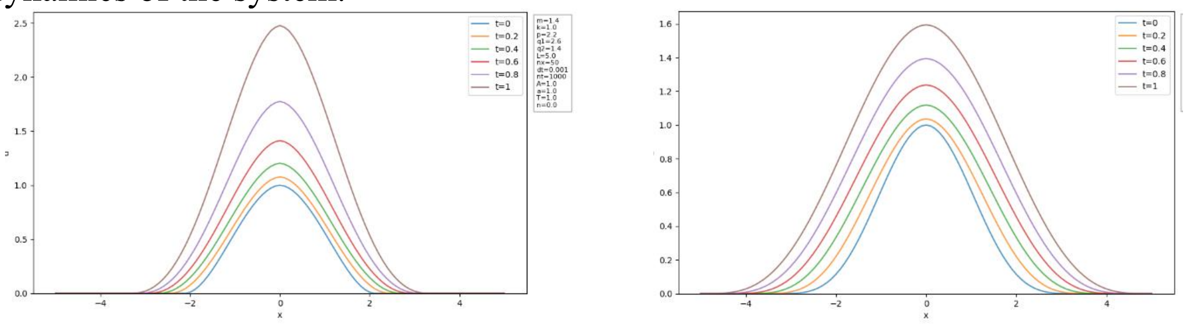
It is crucial to find an appropriate initial approximation of the solution. Therefore, the initial data calculated using the function

$$u_0(x)=\left(T+(q_1-1)t\right)^{-\frac{1}{q_1-1}}A\left(a-b\xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{m-1+k(p-2)}},\ \xi=r\tau^{-\frac{1}{p}},$$

$$\tau(t)=\begin{cases}\dfrac{\left(T+(q_1-1)t\right)^{-\frac{m+k(p-2)-q_1}{q_1-1}}}{q_1-\left(m+k(p-2)\right)} & \text{if } m+k(p-2)\neq q_1\\[2ex] \dfrac{1}{q_1-1}\ln\left(T+(q_1-1)t\right) & \text{if } m+k(p-2)=q_1\end{cases},$$

$$b=\frac{m-1+k(p-2)}{p}\left(\frac{1}{A^{m-1+k(p-2)}k^{p-2}p}\right)^{\frac{1}{p-1}},\quad a>0$$

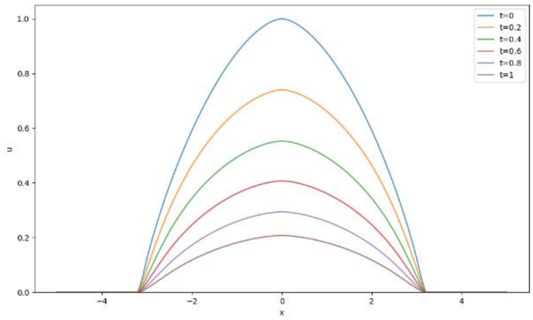
Numerical results of the problem (1)-(3) for various parameter sets. Different parameter values affect the propagation rate and profile shape, demonstrating how nonlinear diffusion and interaction intensity influence the dynamics of the system.



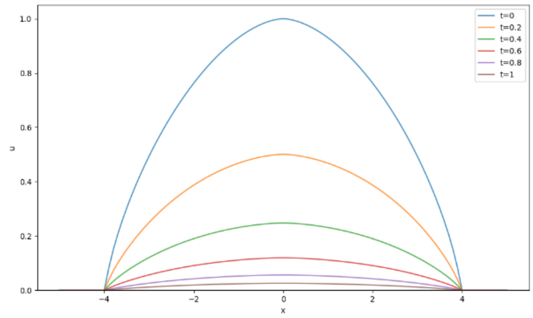
m=1.4, k=1.0, p=2.2, q_1=2.6, q_2=1.4 m=1.1, k=1.1, p=2.1, q_1=2.1, q_2=1.1

Figure 1. Numerical solution of problem (1)-(3) in 1D case.

While in Fig. 22 one can observe how process goes with constant density on problem (1)-(3), in Fig 3 the process shown with variable density both in one dimensional cases. Fig 4 shows the results in two dimensional cases.

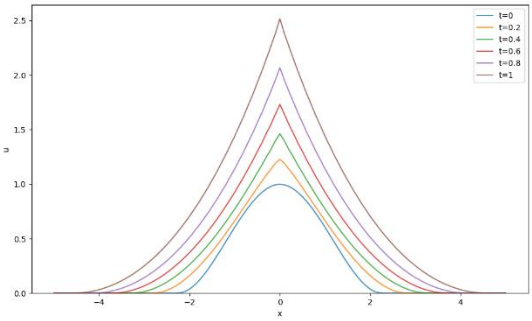


m=1.3, k=3.0, p=2.5, q_1=2.2, q_2=3.4

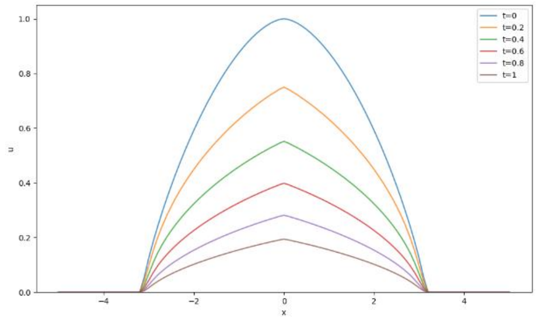


m=1.5, k=3.5, p=2.4, q_1=2.1, q_2=5.4

Figure 2. Numerical solution of problem (1)-(3) in 1D case.

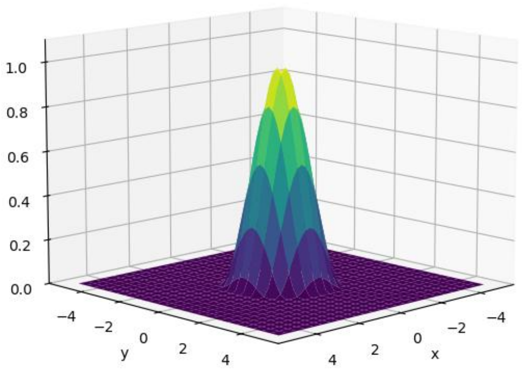


n=1, m=1.4, k=1.1, p=2.1, q_1=2.3, q_2=1.1

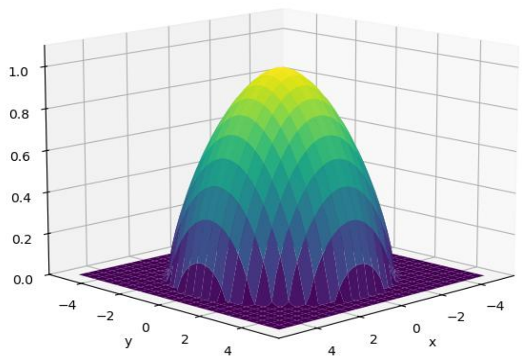


n=1, m=1.3, k=3.0, p=2.5, q_1=2.2, q_2=3.5

Figure 3. Numerical solution of problem (4)-(6) in 1D case.



m=1.4, k=1.0, p=2.2, q_1=2.6, q_2=1.4



m=1.5, k=3.5, p=2.4, q_1=2.1, q_2=5.4

Figure 4. Numerical solution of problem (1)-(3) in 2D case.

The calculation results showed that the chosen method and initial approximation were well chosen, and after 3 iterations, the approximation occurred.

Third chapter of the dissertation work is dedicated to **"Mathematical modeling of nonlinear diffusion processes with nonlinear source and absorption"**. The chapter considers in brief parabolic systems in the form of nonlinear diffusion problem with nonlinear source and absorption terms both in constant and variable density cases.

Particularly **Paragraph 3.1** dealt with model that show synergistic interactions between two species, where the presence of one species promotes the growth of the other and inhibits its own growth as following

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \nabla\left(u^{m_1-1}\left|\nabla u^{k_1}\right|^{p_1-2} \nabla u \right) + v^{q_1} - \alpha_1 u^{r_1}, \quad x \in \Omega, t > 0,\\
&\frac{\partial v}{\partial t} = \nabla\left(v^{m_2-1}\left|\nabla v^{k_2}\right|^{p_2-2} \nabla v \right) + u^{q_2} - \alpha_2 v^{r_2}, \quad x \in \Omega, t > 0,\\
&u(x,t) = v(x,t) = 0, \quad x \in \partial\Omega, t > 0,\\
&u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in \Omega.
\end{aligned} \tag{10}$$

where $p_i \ge 2$, $k_i, m_i \ge 1$, $q_i, r_i, \alpha_i \ge 0$ $(i = 1,2)$ and $\Omega \subset R^N$ is a bounded domain with smooth boundary $\partial\Omega$. Here initial data has following property $u_0(x), v_0(x) \in C^{2+\zeta}(\bar{\Omega})$, $0 < \zeta < 1$, $u_0(x), v_0(x) \ge 0$, $u_0(x), v_0(x) \not\equiv 0$.

Additionally, doubly nonlinear parabolic problems similar to (10) arise in various fields including biology. They describe phenomena such as the concentration of non-Newtonian flows in porous media or the temperature distribution of flammable substances.

Denote here $\mu_1 = \max\{m_1 + k_1(p_1 - 2), r_1\}$, $\mu_2 = \max\{m_2 + k_2(p_2 - 2), r_2\}$.

Theorem 10. *If $q_1 q_2 < \mu_1 \mu_2$, then all nonnegative solutions of problem (10) are global.*

Theorem 11. *Let $q_1 q_2 = \mu_1 \mu_2$. Then following conclusions can be made:*

1. If $r_1 > m_1 + k_1(p_1 - 2)$, $r_2 > m_2 + k_2(p_2 - 2)$, that is $q_1 q_2 = r_1 r_2$, then problem (10) is globally solvable for small initial data and blow-up in finite time solutions for sufficiently large α_1, α_2.

2. If $r_1 < m_1 + k_1(p_1 - 2)$, $r_2 < m_2 + k_2(p_2 - 2)$, that is $q_1 q_2 = (m_1 + k_1(p_1 - 2))(m_2 + k_2(p_2 - 2))$, then problem (10) is globally solvable for small initial data.

3. If $r_1 < m_1 + k_1(p_1 - 2)$, $r_2 > m_2 + k_2(p_2 - 2)$, that is $q_1 q_2 = (m_1 + k_1(p_1 - 2)) r_2$, then problem (10) is globally solvable for small initial data.

4. If $r_1 > m_1 + k_1(p_1 - 2)$, $r_2 < m_2 + k_2(p_2 - 2)$, that is $q_1 q_2 = r_1(m_2 + k_2(p_2 - 2))$, then there is a blow-up finite time solution of problem (10) for large initial values.

Paragraph 3.2 is devoted to the study of system with variable density

$$\begin{aligned}
&\frac{\partial u}{\partial t}=\nabla\left(|x|^{n_1}u^{m_1-1}\left|\nabla u^{k_1}\right|^{p_1-2}\nabla u\right)+v^{q_1}-\alpha_1 u^{r_1},\ x\in\Omega,\ t>0,\\
&\frac{\partial v}{\partial t}=\nabla\left(|x|^{n_2}v^{m_2-1}\left|\nabla v^{k_2}\right|^{p_2-2}\nabla v\right)+u^{q_2}-\alpha_2 v^{r_2},\ x\in\Omega,\ t>0,\\
&u(x,t)=v(x,t)=0,\ x\in\partial\Omega,\ t>0,\\
&u(x,0)=u_0(x),\ v(x,0)=v_0(x),\ x\in\Omega
\end{aligned}\tag{11}$$

here $p_i\ge 2$, $k_{i,}m_i\ge 1$, $n_i,q_i,r_i,\alpha_i\ge 0$ ($i=1,2$) and $\Omega\subset R^N$ is bounded domain with smooth boundary $\partial\Omega$. The initial data $u_0(x),v_0(x)\in C^{2+\zeta}(\bar{\Omega})$, $0<\zeta<1$, $u_0(x),v_0(x)\ge 0$, $u_0(x),v_0(x)\not\equiv 0$.

Theorem 12. *Let $n_1n_2<p_1p_2$. If $q_1q_2<\mu_1\mu_2$, then all nonnegative solutions of problem (11) are gloabl.*

Theorem 13. *Suppose $n_1n_2<p_1p_2$. If $q_1q_2>\mu_1\mu_2$, then nonnegative solutions of problem (11) blow up in finite time for sufficiently large initial data and exists globally for small initial values.*

Paragraph 3.3 is dedicated for numerical solution of problem (10).

In this case we numerically solve the nonlinear diffusion problem in two spatial dimensions considering the problem posed over the domain $\Omega=[0,b_1]\times[0,b_2]$ and $t>0$. Then for the problem (10) we have following two-dimensional nonlinear reaction-diffusion problem with nonlinear absorption

$$\begin{aligned}
&\frac{\partial u}{\partial t}=\left(\frac{\partial}{\partial x_1}\left(u^{m_1-1}\left|\frac{\partial u^{k_1}}{\partial x_1}\right|^{p_1-2}\frac{\partial u}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(u^{m_1-1}\left|\frac{\partial u^{k_1}}{\partial x_2}\right|^{p_1-2}\frac{\partial u}{\partial x_2}\right)\right)+v^{q_1}-u^{r_1},\\
&\frac{\partial v}{\partial t}=\left(\frac{\partial}{\partial x_1}\left(v^{m_2-1}\left|\frac{\partial v^{k_2}}{\partial x_1}\right|^{p_2-2}\frac{\partial v}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(v^{m_2-1}\left|\frac{\partial v^{k_2}}{\partial x_2}\right|^{p_2-2}\frac{\partial v}{\partial x_2}\right)\right)+u^{q_2}-v^{r_2},
\end{aligned}\tag{12}$$

$$u(x_1,x_2,t)=0,\quad v(x_1,x_2,t)=0,\quad x\in\partial\Omega \tag{13}$$

$$u(0,x)=u_0(x)\ge 0,\quad v(0,x)=v_0(x)\ge 0,\ 0\le x_i\le b_i,\ i=1,2 \tag{14}$$

We construct uniform spatial grid of x_1 and x_2 with steps $h_1=\dfrac{b_1}{n_1}$ and $h_2=\dfrac{b_2}{n_2}$

$$\bar{\omega}_h=\left\{x_{ij}=\left(x_1^i,x_2^j\right),\ x_1^i=ih_1,\ x_2^j=jh_2,\ i,j=0,1,...,n_\alpha,\ \alpha=1,2\right\}$$

and temporal grid with step $\tau=\dfrac{T}{m}$

$$\bar{\omega}_\tau=\{t_l=l\tau,\ \tau>0,\ l=0,1,...,m,\ \tau m=T\},\ T>0$$

For the numerical solution of (12) we use method of alternating directions by Peacemen and Rachford:

$$\begin{cases} \dfrac{y_{i,j}^{l+1/2} - y_{i,j}^{l}}{0.5\cdot\tau} = \Lambda_1 y^{l+1/2} + \Lambda_2 y^{l} + \left(Y^{l}\right)^{q_1} - \left(y^{l}\right)^{r_1} \\ \dfrac{y_{i,j}^{l+1} - y_{i,j}^{l+1/2}}{0.5\cdot\tau} = \Lambda_1 y^{l+1/2} + \Lambda_2 y^{l+1} + \left(Y^{l+1}\right)^{q_1} - \left(y^{l+1}\right)^{r_1} \end{cases} \quad (15)$$

$$\begin{cases} \dfrac{Y_{i,j}^{l+1/2} - Y_{i,j}^{l}}{0.5\cdot\tau} = \Lambda_1 Y^{l+1/2} + \Lambda_2 Y^{l} + \left(y^{l}\right)^{q_2} - \left(Y^{l}\right)^{r_2} \\ \dfrac{Y_{i,j}^{l+1} - Y_{i,j}^{l+1/2}}{0.5\cdot\tau} = \Lambda_1 Y^{l+1/2} + \Lambda_2 Y^{l+1} + \left(y^{l+1}\right)^{q_2} - \left(Y^{l+1}\right)^{r_2} \end{cases} \quad (16)$$

where

$$\Lambda_1 y^{l+1/2} = \frac{1}{h_1^2}\left[P_{i+1,j}\left(y^{l+1/2}\right)\left(y_{i+1,j}^{l+1/2} - y_{i,j}^{l+1/2}\right) - P_{i,j}\left(y^{l+1/2}\right)\left(y_{i,j}^{l+1/2} - y_{i-1,j}^{l+1/2}\right)\right]$$

$$\Lambda_2 y^{l} = \frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l}\right)\left(y_{i,j+1}^{l} - y_{i,j}^{l}\right) - Q_{i,j}\left(y^{l}\right)\left(y_{i,j}^{l} - y_{i,j-1}^{l}\right)\right]$$

$$\Lambda_2 y^{l+1} = \frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l+1}\right)\left(y_{i,j+1}^{l+1} - y_{i,j}^{l+1}\right) - Q_{i,j}\left(y^{l+1}\right)\left(y_{i,j}^{l+1} - y_{i,j-1}^{l+1}\right)\right]$$

$i = 1,2,\dots n_1 - 1$; $j = 1,2,\dots n_2 - 1$; $f\left(y\right) = v^{q_1} - u^{r_1}$

$$\Lambda_1 Y^{l+1/2} = \frac{|x|^{n_1}}{h_1^2}\left[Q_{i+1,j}\left(Y^{l+1/2}\right)\left(Y_{i+1,j}^{l+1/2} - Y_{i,j}^{l+1/2}\right) - Q_{i,j}\left(Y^{l+1/2}\right)\left(Y_{i,j}^{l+1/2} - Y_{i-1,j}^{l+1/2}\right)\right]$$

$$\Lambda_2 Y^{l} = \frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l}\right)\left(y_{i,j+1}^{l} - y_{i,j}^{l}\right) - Q_{i,j}\left(y^{l}\right)\left(y_{i,j}^{l} - y_{i,j-1}^{l}\right)\right]$$

$$\Lambda_2 y^{l+1} = \frac{1}{h_2^2}\left[Q_{i,j+1}\left(y^{l+1}\right)\left(y_{i,j+1}^{l+1} - y_{i,j}^{l+1}\right) - Q_{i,j}\left(y^{l+1}\right)\left(y_{i,j}^{l+1} - y_{i,j-1}^{l+1}\right)\right]$$

$i = 1,2,\dots n_1 - 1$; $j = 1,2,\dots n_2 - 1$;

For the nonlinear part of $P_{i,j}\left(y^{l}\right)$ and $Q_{i,j}\left(y^{l}\right)$ we used following approximation

$$P_{i+1,j}\left(y^{l+1/2}\right) = \frac{1}{2}\left[\left(y_{i,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i+1,j}^{l+1/2}\right)^{k} - \left(y_{i,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2} + \left(y_{i+1,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1/2}\right)^{k} - \left(y_{i-1,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}\right]$$

$$P_{i,j}\left(y^{l+1/2}\right) = \frac{1}{2}\left[\left(y_{i-1,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1/2}\right)^{k} - \left(y_{i-1,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2} + \left(y_{i,j}^{l+1/2}\right)^{m-1}\left|\frac{\left(y_{i-1,j}^{l+1/2}\right)^{k} - \left(y_{i-2,j}^{l+1/2}\right)^{k}}{h_1}\right|^{p-2}\right]$$

$$Q_{i,j+1}\left(y^{l}\right) = \frac{1}{2}\left[\left(y_{i,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j+1}^{l}\right)^{k} - \left(y_{i,j}^{l}\right)^{k}}{h_2}\right|^{p-2} + \left(y_{i+1,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l}\right)^{k} - \left(y_{i-1,j}^{l}\right)^{k}}{h_2}\right|^{p-2}\right]$$

$$Q_{i,j}\left(y^{l}\right)=\frac{1}{2}\left[\left(y_{i,j-1}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l}\right)^{k}-\left(y_{i,j-1}^{l}\right)^{k}}{h_2}\right|^{p-2}+\left(y_{i,j}^{l}\right)^{m-1}\left|\frac{\left(y_{i,j-1}^{l}\right)^{k}-\left(y_{i,j-2}^{l}\right)^{k}}{h_2}\right|^{p-2}\right]$$

$$Q_{i,j}\left(y^{l+1}\right)=\frac{1}{2}\left[\left(y_{i,j-1}^{l+1}\right)^{m-1}\left|\frac{\left(y_{i,j}^{l+1}\right)^{k}-\left(y_{i,j-1}^{l+1}\right)^{k}}{h_2}\right|^{p-2}+\left(y_{i,j}^{l+1}\right)^{m-1}\left|\frac{\left(y_{i,j-1}^{l+1}\right)^{k}-\left(y_{i,j-2}^{l+1}\right)^{k}}{h_2}\right|^{p-2}\right]$$

Remark. *In all our calculations iteration accuracy is set to* $\varepsilon=10^{-3}$.

It is critical to find the appropriate initial approximation of the solution in the numerical solution. In finite calculations, the initial approximation is chosen as following

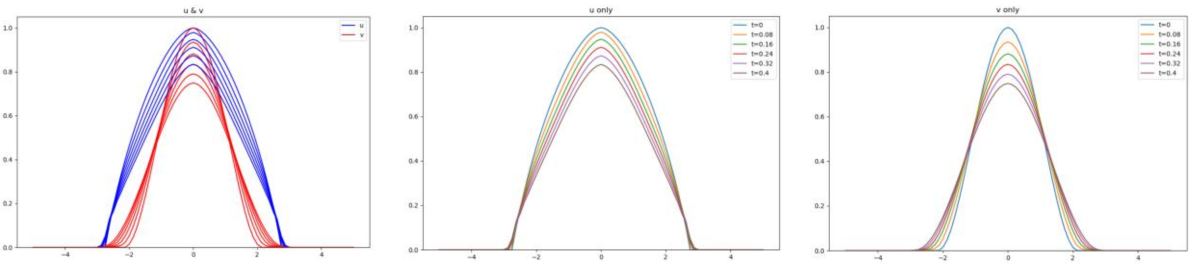
$$u_0(x)=T^{-\gamma_1}\left(A^{\frac{p_1}{p_1-1}}-\xi_1^{\frac{p_1}{p_1-1}}\right)_+^{\frac{p_1-1}{m_1-1+k_1(p_1-2)}},\quad v_0(x)=T^{-\gamma_2}\left(A^{\frac{p_2}{p_2-1}}-\xi_2^{\frac{p_2}{p_2-1}}\right)^{\frac{p_2-1}{m_2-1+k_2(p_2-2)}}$$

$$\xi_1=|x|/T^{\beta_1},\qquad \xi_2=|x|/T^{\beta_2},\qquad \gamma_1=\frac{q_1+1}{q_1q_2-1},\qquad \gamma_2=\frac{q_2+1}{q_1q_2-1},$$

$$\beta_1=\frac{1-\gamma_1\left(m_1+k_1\left(p_1-2\right)-1\right)}{p_1}>0,\ \beta_2=\frac{1-\gamma_2\left(m_2+k_2\left(p_2-2\right)-1\right)}{p_2}>0$$

Numerical results obtained for different parameter values as below. Particulary, Figure 5 represents global solution free boundary distribution speed.

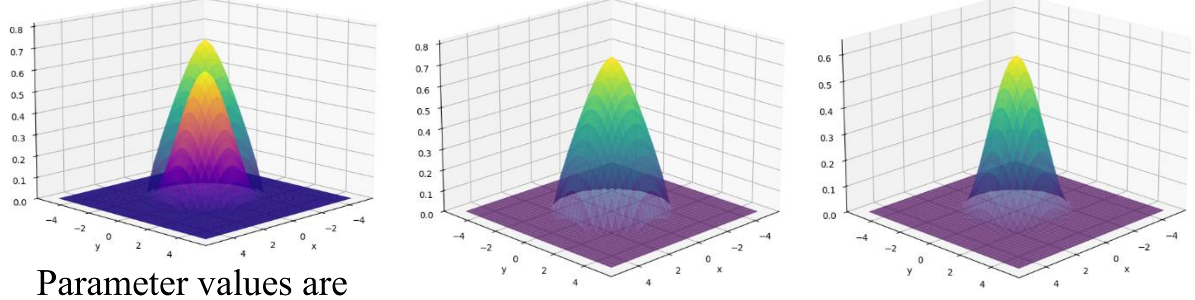
u & v | u | v



Parameter values are same as next cells | $m_1=1.9$, $k_1=1.7$, $p_1=2.5$, $q_1=2.2$, $r_1=1.0$ | $m_2=1.4$, $k_2=1.2$, $p_2=2.1$, $q_2=2.5$, $r_2=1.0$

Figure 5. Numerical solution of problem (10) in 1D case.

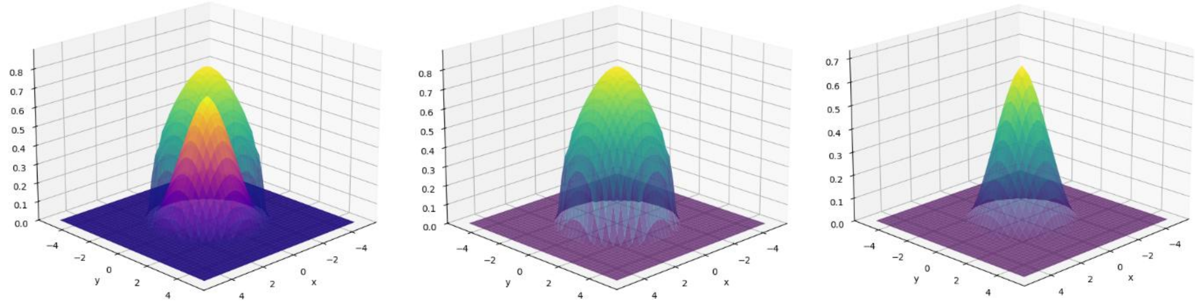
u & v | u | v



Parameter values are same as next cells | $m_1=1.9$, $k_1=1.7$, $p_1=2.5$, $q_1=2.2$, $r_1=1.0$ | $m_2=1.4$, $k_2=1.2$, $p_2=2.1$, $q_2=2.5$, $r_2=1.0$

Figure 6. Numerical solution of problem (10) in 2D case.

u & v | u | v



Parameter values are same as next cells

$m_1=2.5$, $k_1=2.5$, $p_1=2.1$, $q_1=2.1$, $r_1=3.0$

$m_2=1.2$, $k_2=1.2$, $p_2=2.5$, $q_2=2.9$, $r_2=1.0$

Figure 7. Numerical solution of problem (10) in 2D case.

Paragraph 3.4 gives the explanation and instruction manual of software Nonlinear Diffusion Modeling. The software is intended for the modeling of nonlinear diffusion processes. It is written in the Spyder IDE programming environment using the Python programming language.

A dedicated software package was developed in Python 3.x using a modular architecture:

• Main interface (main.py) – Tkinter-based GUI serving as a launcher with access to the single-equation and system modules.

• Equation solver (equation.py) – solves one- and two-dimensional nonlinear diffusion equations with source/absorption, supporting explicit schemes and Picard/Newton linearization, with dynamic visualization.

• System solver (system.py) – simulates coupled diffusion–reaction systems, using splitting and sweep algorithms for tridiagonal problems, with 2D/3D visualization (surface plots, contours, animations).

• Auxiliary utilities – handle data export, error analysis, and advanced plotting.

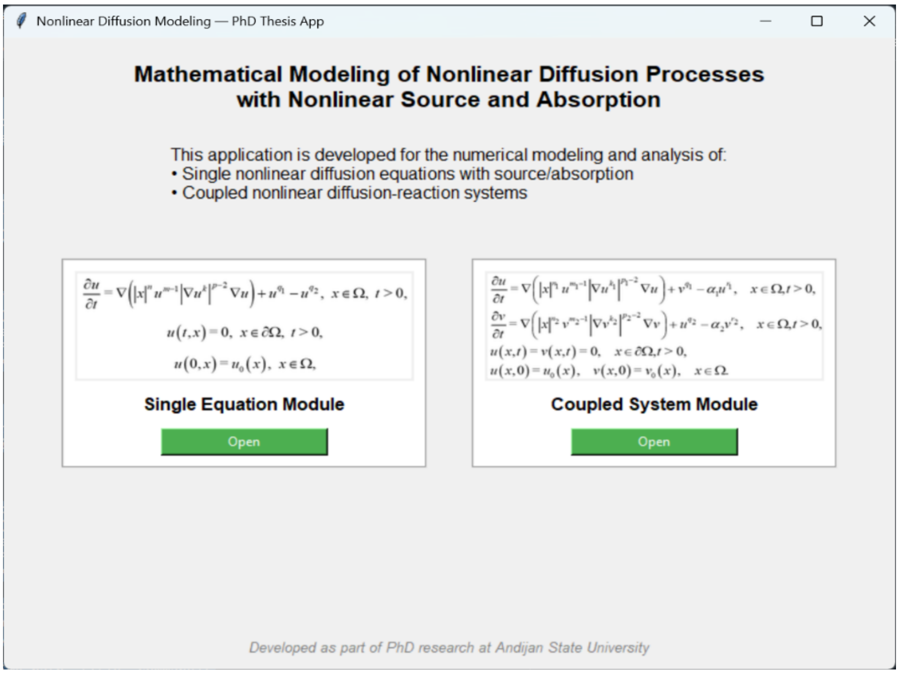


Figure 8. Main interface of the nonlinear diffusion modeling application (module selection window).

The main window of the application (Fig. 8) provides a unified entry point, developed using Tkinter. Users can switch between the single-equation and coupled-system modules. The design emphasizes clarity and accessibility, enabling direct access to modeling tasks without programming effort.

The single-equation module (Fig. 9) simulates nonlinear diffusion with source or absorption. Users define spatial, temporal, and model parameters, after which the solver computes and visualizes solution profiles over time. Two sub-tabs allow selection between 1D and 2D discretization, supporting geometry-specific visualization.

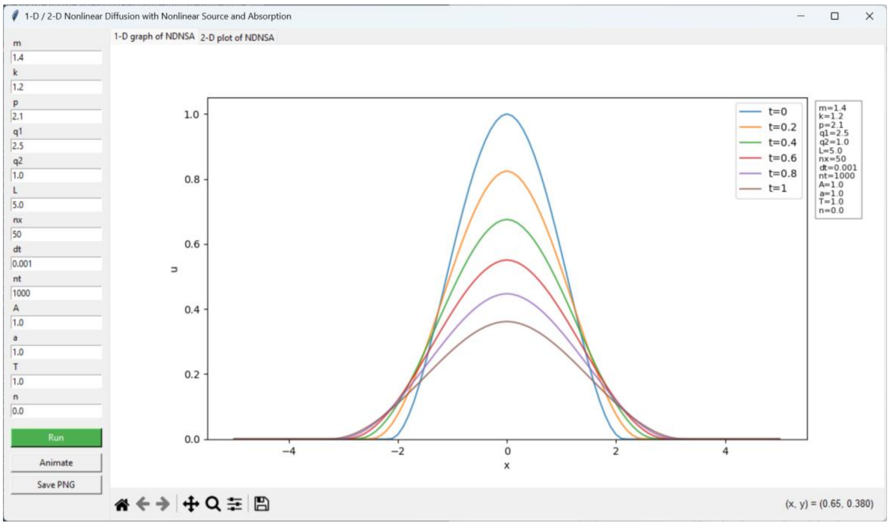


Figure. 9. Interface of the single nonlinear diffusion equation module with parameter input and computation options.

The coupled-system module (Fig. 10) extends the software to multi-component diffusion–reaction systems. It accepts system parameters, initial and boundary conditions, and visualizes the dynamics of each component. As in the single-equation case, both 1D and 2D sub-tabs are available.

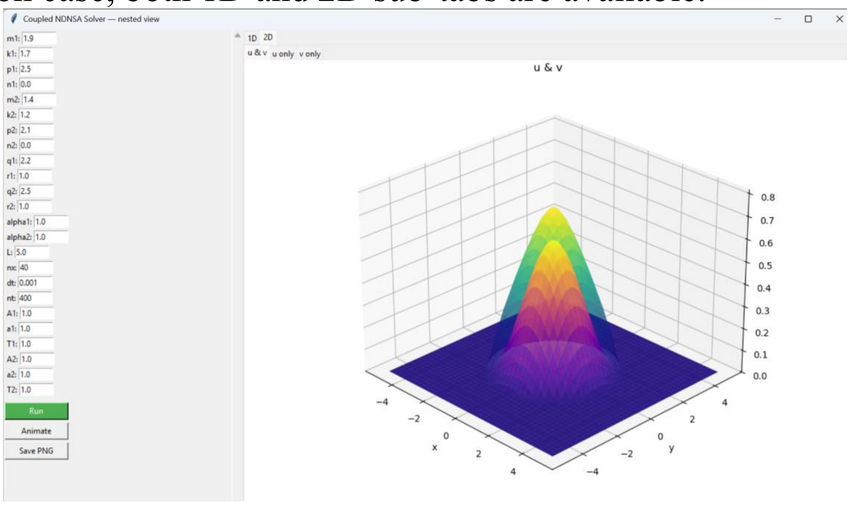


Figure. 10. Interface of the coupled nonlinear diffusion–reaction system module, supporting multi-equation simulations.

The general workflow of the application is illustrated in Fig. 11. The process begins with parameter specification via the GUI, followed by preprocessing and validation. The numerical module executes the finite-difference scheme, after which results are passed to visualization and export routines.

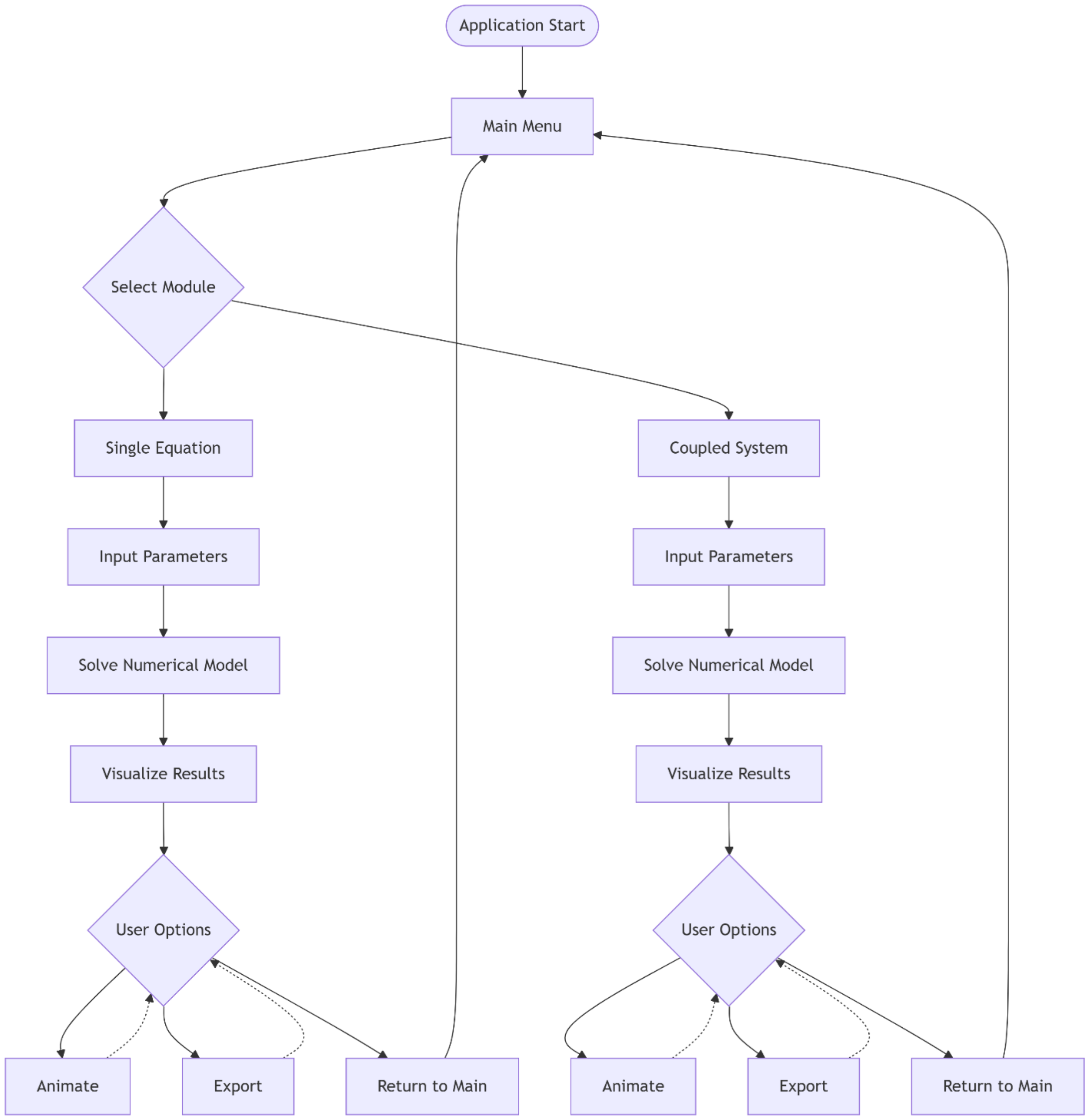


Figure. 11. Flowchart of the computational program, from user input to visualization and export.

The High-level software architecture of the application is summarized in Fig. 12.

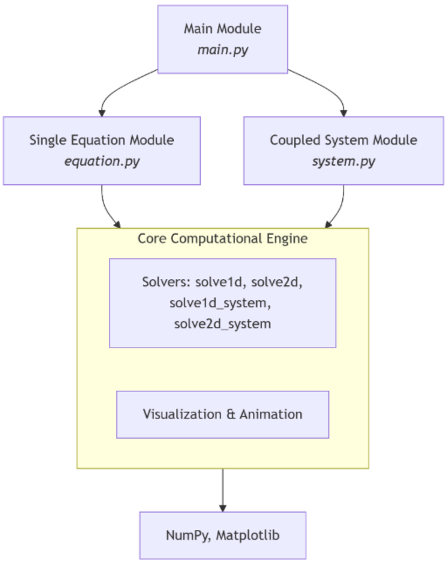


Figure. 12. High-level software architecture of the application.

In summary, the developed software translates nonlinear diffusion models into an interactive and extensible computational platform. The modular design ensures flexibility for incorporating new methods, while the graphical interface facilitates accessibility for users without programming expertise. Together, the algorithmic framework and its implementation provide the necessary tools for numerical experiments supporting the theoretical analysis of nonlinear diffusion problems.

CONCLUSION

The dissertation successfully addressed key tasks related to the modeling, analysis, and numerical solutions of nonlinear diffusion problem with nonlinear source and absorption terms that describe biological population, virus propagation and other processes. These tasks addressed through a combination of theoretical analysis, numerical methods, and software development, as summarized below:

the conditions ensuring global existence and non-existence in time of the solutions of nonlinear diffusion models with constant or variable density, nonlinear source and absorption have been determined;

useful qualitative behavior as lower and upper bounds of the solutions for numerical simulation of nonlinear diffusion systems with constant or variable density, nonlinear source and absorption have been obtained

effective initial approximations required for the numerical solution of nonlinear diffusion problems with constant or variable density, nonlinear source and absorption have been determined;

numerical calculation schemes have been applied and analyzed to study the qualitative behaviour of nonlinear diffusion model with constant or variable density, nonlinear source and absorption;

a problem-oriented software package has been developed for solving and visualizing nonlinear diffusion problems with constant or variable density, nonlinear source and absorption.

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ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
НАЦИОНАЛЬНОМ УНИВЕРСИТЕТЕ УЗБЕКИСТАНА

АНДИЖАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

АТАБАЕВ ОДИЛЖОН ХУСНИДДИН УГЛИ

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕЛИНЕЙНЫХ ПРОЦЕССОВ ДИФФУЗИИ С НЕЛИНЕЙНЫМ ИСТОЧНИКОМ И СТОКОМ

05.01.07 – Математическое моделирование. Численные методы и комплексы программ (физико-математические науки)

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент – 2025

Тема диссертации доктора философии (Doctor of Philosphy) по физико-математическим наукам зарегистрировано в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инновации Республики Узбекистан за B2024.4.PhD/FM1214.

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Научнқй руководитель:	**Арипов Мерсаид** доктор физико-математических наук, профессор
Официальные оппоненты:	**Садуллаева Шахло Азимбаевна** доктор физико-математических наук, профессор **Хайдаров Абдугаппар** кандидат физико-математических наук, доцент
Ведущая организация:	**Ургенчский государственный университет**

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С диссертацией можно ознакомиться в Информационно-ресурсном центре Национального университета Узбекистана (зарегистрировано за №___). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 4. Тел.: (+99871) 246-02-24).

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Б.Ф.Абдурахимов
Заместитель председателя Научного совета по присуждению ученых степеней, д.ф.-м.н., профессор

З.Р.Рахмонов
Ученый секретарь Научного совета по присуждению ученых степеней, д.ф.-м.н., профессор

А.С.Матякубов
Председатель научного семинара при Научном совете по присуждению ученых степеней, д.ф.-м.н., доцент

ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Цель исследования численно и аналитически изучить качественные свойства нелинейных математических моделей, характеризующихся уравнениями параболического типа и их системами в средах с постоянной или переменной плотностью, нелинейными источниками и стоками.

Объектом исследования являются параболические уравнения с постоянной или переменной плотностью, нелинейными источником и стоком, а также нелинейные процессы диффузии, характеризуемые их системами.

Предметом исследования является изучение качественные свойства математических моделей нелинейных диффузионных процессов с постоянной или переменной плотностью, нелинейным источником и поглощением, построение схем численного решения, создание пакетов программ.

Научная новизна исследования заключается в следующем:

найдены условия глобального существования и не существования во времени решений нелинейных моделей, представляющих собой нелинейные диффузионные процессы с постоянной или переменной плотностью, нелинейным источником и поглощением;

получены нижние и верхние оценки решений нелинейных диффузионных систем с постоянной или переменной плотностью, нелинейным источником и поглощением;

найдены начальные приближения, необходимые для численного расчета нелинейных задач, представляющих нелинейные диффузионные процессы с постоянной или переменной плотностью, нелинейным источником и стоком;

построены математические модели нелинейных диффузионных процессов с постоянной или переменной плотностью, нелинейным источником и поглощением, схемы численного расчета для исследования качественных свойств;

разработан комплекс программных средств для решения задачи нелинейной диффузии с постоянной или переменной плотностью, нелинейным источником и поглощением, а полученные решения визуализированы.

Внедрение результатов исследований. Научные результаты, полученные по математическому и численному моделированию задач нелинейных диффузионных процессов с постоянной или переменной плотностью, нелинейным источником и поглощением, реализуются на практике по следующим направлениям:

нижние и верхние границы решений нелинейных систем диффузии с постоянной или переменной плотностью, нелинейным источником и поглощением были использованы при решении задачи Дирихле для возмущенного уравнения с сингулярными коэффициентами и спектральными параметрами в вертикальном полуполосе в фундаментальном грантовом проекте ОТ-Ф4-88 «Исследование прямых и обратных задач для уравнений смешанного типа второго и высшего порядка». (Справка Института математики им. В.И. Романовского РАН № 02/437 от 5 декабря 2024 г.). В результате найдено точное решение задачи Дирихле для возмущенного уравнения с сингулярными коэффициентами и спектральными параметрами в вертикальной полуполосе.

научный результат по нахождению начального приближения, необходимого для численного расчета нелинейных задач, представляющих собой нелинейные диффузионные процессы с постоянной или переменной плотностью, нелинейным источником и поглощением, был использован для решения задачи Коши, поставленной в системе Каупа в классе периодических функций в фундаментальном грантовом проекте ОТ-Ф4-04 «Применение спектрального метода к решению матричных нелинейных эволюционных уравнений, Биомеханика сердечно-сосудистой системы» (Справка Ургенчского государственного университета от 06.12.2024 № 04-235/2). В результате навязанное решение позволило определить точные формы периодических решений задачи Коши, поставленной для системы Каупа.

Публикация результатов исследования. Всего по теме исследования опубликовано 19 научных работ, из них 6 статей в научных изданиях, рекомендованных ВАК РУз для публикации научных результатов, в том числе 1 зарубежная (Scopus, Q2) и 5 отечественных журналах. Также получены 2 авторских свидетельства об официальной регистрации программы, созданной для ЭВМ.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения, списка использованной литературы и приложений. Объем диссертации составляет 96 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo‘lim (I часть; part I)

1. Aripov, M.M., O.X. Atabaev, and A.M. AL-Marashi. “On the Behavior of Solutions of a Doubly Nonlinear Degenerate Parabolic System with Nonlinear Sources and Absorptions with Variable Densities.” Bulletin of the Karaganda University-Mathematics 2025, 117, no. 1: 12–23. https://doi.org/10.31489/2025m1/12-23. (№3, Scopus, IF=0.7)

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