“YANGI O‘ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/07.07.2025.FM/T.192.01 RAQAMLI ILMIY KENGASH

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

“YANGI O‘ZBEKISTON” UNIVERSITETI

RAXIMOVA GULZODAXON ISROILJON QIZI

MODIFIKATSIYALANGAN GRAVITATSIYA NAZARIYALARI DOIRASIDA KOMPAKT OBYEKTLAR ATROFIDA TERMODINAMIK VA ENERGETIK JARAYONLAR

01.03.01 - Astronomiya
01.04.02 - Nazariy fizika

FIZIKA VA MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI

AVTOREFERATI

Toshkent-2026

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertasiyasi avtoreferati mundarijasi

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta‘lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.3.PhD/FM1360 raqami bilan ro‘yxatga olingan.

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Dissertatsiya avtoreferati 2026-yil “___”_____kuni tarqatildi.

(2026-yil “___”_____dagi___ raqamli reestr bayonnomasi)

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KIRISH

Dissertatsiya mavzusining dolzarbligi

Mavzu umumiy nisbiylik nazariyasidagi (UNN) fundamental masalalarni, masalan, singulyarliklar va kvant mexanikasi bilan nomutanosiblikni hal qilishga qaratilgan muqobil gravitatsiya nazariyalariga qiziqish ortib borayotgani tufayli nihoyatda dolzarbdir. Astrofizik kuzatuvlardagi so‘nggi yutuqlar, shu jumladan gravitatsion to‘lqinlarni aniqlash va Hodisalar gorizonti teleskopi (Event Horizon Telescope (EHT)) tomonidan qora tuynuklarning tasvirlanishi, ushbu nazariyalarni sinash uchun misli ko‘rilmagan imkoniyatlarni taqdim etdi.

Mazkur turdagi tadqiqotlarga bo‘lgan ehtiyoj koinotning qorong‘u tarkibiy qismlari — qorong‘u modda va qorong‘u energiyaning tabiati umumiy nisbiylik nazariyasi doirasida hanuz to‘liq izohlanmaganligi bilan bog‘liqdir. Modifikatsiyalangan gravitatsiya nazariyalari ushbu muammolarga potensial yechimlar taklif etib, mazkur dissertatsiya ishining dolzarbligini zamonaviy astrofizika va kosmologiya nuqtai nazaridan asoslaydi. Dissertatsiyada ekstremal fizik sharoitlarda gravitatsiyaning fundamental xususiyatlarini tadqiq etish uchun muhim bo‘lgan zarrachalar dinamikasi (jumladan, zarrachalarning tezlanish jarayonlari) hamda tizimning termodinamik xossalari batafsil o‘rganilgan. Xususan, qora tuynuklarning yuqori energiyali zarracha tezlatkichlari sifatida namoyon bo‘lishini ko‘rsatadigan Bañados–Silk–West (BSW) mexanizmi yuqori energiyali astrofizika va kosmik nurlar hosil bo‘lish jarayonlarini tushunishda muhim nazariy ahamiyatga ega.

Qolaversa, dissertatsiya O‘zbekiston va xalqaro hamjamiyatning strategik ilmiy ustuvorliklariga mos keladi. Ushbu dissertatsiya ishi quyidagi davlat normativ hujjatlarining vazifalariga muvofiqdir: O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi PF-4947-sonli farmoni, O‘zbekiston Respublikasi Prezidentining 2017-yil 18-fevraldagi “Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789-sonli Qarori.

Tadqiqotning O‘zbekiston Respublikasi fan va texnologiyalarni rivojlantirishning ustuvor yonalishlariga dolzarbligi Dissertatsiya ishi O‘zbekiston Respublikasi fan va texnologiyalarni rivojlantirishning ustuvor yo‘nalishlariga muvofiq amalga oshirilgan: II. “Energetika, energiya va resurslarni tejash”.

Muammoning o‘rganilganlik darajasi

Modifikatsiyalangan gravitatsiya nazariyalarida kompakt obyektlar atrofidagi termodinamik va energetik jarayonlarni tushunish muammosi keng o‘rganilgan bo‘lsada, jiddiy bo‘shliqlar saqlanib qolmoqda. Klassik ishlar qora tuynuk soyalari va geodezik harakatlar uchun asos yaratdi (Bardeen, Luminet, Event Horizon Telescope

Collaboration, Abdujabbarov, Ahmedov, Bambi), keyingi tadqiqotlar esa qora tuynuklarni zarracha tezlatgichlari sifatida ishlatish kontseptsiyasini kiritdi (Banados, Silk, West, Zaslavskii, Wei, Liu, Atamurotov). Eynshteyn-Gauss-Bonnet (EGB) va skalyar-tenzor modellari kabi modifikatsiyalangan gravitatsiya nazariyalari gorizont tuzilishi, foton sferalari va zarrachalar dinamikasi kabi masalalar doirasida keng o‘rganilgan. Biroq mavjud tadqiqotlarning aksariyati alohida jihatlarga, jumladan qora tuynuk soyasi yoki termodinamik xossalarga (Bekenstein, Hawking, Glavan, Lin, Ghosh, Ahmedov, Jamil) qaratilgan bo‘lib, ushbu hodisalarni yagona va izchil nazariy doira doirasida birlashtirish masalasi yetarlicha yoritilmagan. Bundan tashqari, BSW mexanizmi Kerr va Kerr-Newman fazo-vaqtlarida yaxshi o‘rganilgan bo‘lsa-da, uning modifikatsiyalangan gravitatsiya nazariyalaridagi, xususan, qorong‘u materiya yoki chiziqli bo‘lmagan elektrodinamika kabi qo‘shimcha maydonlarga ega bo‘lgan sharoitlardagi qo‘llanilishi yetarlicha tadqiq qilinmagan.

M87* va Sgr A* tasvirlarini olish kabi kuzatuv astronomiyasidagi so‘nggi yutuqlar umumiy nisbiylik nazariyasidan chetga chiqishlarni hisobga oluvchi kompleks modellarga bo‘lgan ehtiyojni kuchaytirdi. 4D EGB gravitatsiya nazariyasi, Simpson-Visser metrikalari va o’ta oquvchan qorong‘u moddasi (PFDM) muhitlaridagi aylanuvchi qora tuynuklarga oid tadqiqotlar paydo bo‘ldi (Simpson, Visser, Kiselev, Abdujabbarov, Atamurotov, Ghosh), ammo kuzatuv ma‘lumotlari bilan tizimli taqqoslashlar hali ham cheklangan. Termodinamik xususiyatlar, shu jumladan Xoking nurlanishi va faza o‘tishlari soddalashtirilgan stsenariylarda tahlil qilingan, ammo termal fluktuatsiyalar va yuqori tartibli tuzatishlarning ta‘siri to‘liq tushunilmagan. Dissertatsiya ushbu bo'shliqlarni modifikatsiyalangan gravitatsiya nazariyasida zarrachalar dinamikasi va termodinamikasini birlashtirish orqali bartaraf etadi, kompakt obyektlarni yanada yaxlit tushunishni ta‘minlaydi. Ish ham analitik, ham raqamli usullardan foydalangan holda nazariy bashoratlarni kuzatilishi mumkin bo‘lgan hodisalar bilan bog‘lashni maqsad qilgan va shu bilan muqobil grvitatsiya nazariyalarini davom etayotgan takomillashtirishga hissa qo‘shadi.

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy talim ilmiy tadqiqot muassasalarining ilmiy ishlari bilan bog‘liqligi. Dissertatsiya O‘zbekiston Respublikasi Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan quyidagi ilmiy loyiha doirasida bajarilgan: F-FA-2021-510 “Modifikatsiyalangan gravitatsiya nazariyasida neytron yulduzlarining yadroviy materiyasini o‘rganish” (2021-2026-yillar).

Tadqiqotning maqsadi modifikatsiyalangan gravitatsiya nazariyalarida aylanayotgan qora tuynuklar atrofida kechadigan astrofizik jarayonlarni kompleks tahlil qilishdan iborat.

Tadqiqot vazifalari:

- modifikatsiyalangan gravitatsiya nazariyalarida (Eynshteyn–Gauss–Bonnet, Simpson–Visser va PFDM modellari) aylanayotgan va zaryadlangan qora tuynuklarning fazo-vaqt tuzilishini tahlil qilish, jumladan gorizont konfiguratsiyalari, ekstremallik shartlari va ergosfera xususiyatlarini o‘rganish hamda ushbu xususiyatlarning qo‘shimcha gravitatsion parametrlar bilan qanday bog‘liqligini aniqlash;
- modifikatsiyalangan gravitatsiyada aylanayotgan qora tuynuklar atrofida sinov zarralarining harakat tenglamalarini effektiv potensial yondashuvi yordamida keltirib chiqarish va tadqiq etish;
- ekstremal va ekstremal bo‘lmagan rejimlarda qora tuynuklar yaqinida zarrachalarning tezlanish mexanizmini o‘rganish;
- modifikatsiyalangan gravitatsiya fonida sinov zarralari uchun eng ichki barqaror aylana orbitlari (ISCO) va effektiv potensiallarni tadqiq etish;
- modifikatsiyalangan gravitatsiyada aylanayotgan qora tuynuklarning termodinamik kattaliklarini, xususan Xoking harorati, entropiya va Gibbs erkin energiyasini keltirib chiqarish;
- termal fluktuatsiyalar va yuqori tartibli entropiya tuzatmalari ta‘sirini hisobga olgan holda, termodinamik barqarorlik va fazaviy holat xarakteristikalarini tadqiq etish.

Tadqiqot obyektlari modifikatsiyalangan gravitatsiya nazariyalaridagi aylanuvchi qora tuynuklar, zaryadlangan qora tuynuklar, gorizont va ergosfera tuzilmalari, shuningdek, zarracha dinamikasi va termodinamik tizimlardir.

Tadqiqot predmetlari zarrachalarning tezlanish mexanizmlari, effektiv potensiallar va barqarorlik, termodinamik xususiyatlar, qorong‘u modda.

Tadqiqot metodlari hisoblash matematikasi metodlari, nazariy astrofizika metodlari, matematik fizikaning zamonaviy metodlari, maydon va zarracha harakati uchun differensial tenglamalarni hisoblashning analitik va raqamli usullaridan iborat. Modifikatsiyalangan gravitatsiya nazariyalarida qora tuynuk yechimlarini keltirib chiqarish hamda metrik komponentlar tahlili orqali gorizont va ergosfera tuzilmalarini hisoblash uchun metrik tahlil va analitik hisoblash metodi qo‘llanilgan.

Zaryadlangan qorong‘u modda bilan modifikatsiyalangan qora tuynuklar uchun chiziqli bo‘lmagan maydon tenglamalarini raqamli yechish maqsadida raqamli modellashtirish va hisoblash simulyatsiyalari metodi qo‘llanilgan. Xoking harorati, entropiya hamda Gibbs erkin energiyasini baholashda sirt gravitatsiyasi va Evklid harakati usullariga asoslangan barqarorlik tahlili qo‘llanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

- Zarrachalar dinamikasi va termodinamikasining birinchi yagona tadqiqoti bizga zarrachalarning tezlanishini va termodinamik barqarorlikni modifikatsiyalangan gravitatsiya nazariyalarida (masalan, 4D EGB, Simpson-Visser) yagona tizim doirasida birlashtirish imkonini berdi, shu bilan birga avval faqat alohida o'rganilgan o'zaro bog'liqliklarni ochib berdi.
- PFDM modeli doirasida qora tuynuklar uchun kuzatiladigan belgilar (jumladan, eng ichki barqaror aylana orbitaning — ISCO — siljishlari) aniqlanib, ular umumiy nisbiylik nazariyasidan tajribada tekshirilishi mumkin bo‘lgan og‘ishlarni ko‘rsatadi.
- Nazariy natijalar termal fluktuatsiyalarni va yuqori tartibli entropiya tuzatishlarini modifikatsiyalangan gravitatsiya nazariyasi qora tuynuklariga kiritadi, shu bilan Xoking nurlanishi va faza otishlariga kvant/statistik effektlarning ta‘sirini tushunishdagi boshliqlarni bartaraf etadi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

- Zarrachalarning tezlanish jarayonlari bo‘yicha olingan natijalar modifikatsiyalangan fazo-vaqtlarida gorizont yaqinidagi zarrachalar to‘qnashuvi uchun energiya chegaralarini miqdoriy jihatdan aniqlaydi, bu esa yuqori energiyali astrofizika (masalan, kosmik nurlar, AGN jetlari) uchun amaliy ahamiyatga ega.
- Egzotik kompakt obyektlar uchun barqarorlik mezonlarini tahlil qilish modifikatsiyalangan gravitatsiyadagi qora tuynuklarning barqaror bo‘lishi mumkin bo‘lgan parametr oralig‘ini aniqlab beradi, bu esa nazariy modellar va sonli nisbiylik simulyatsiyalarini shakllantirish va boshqarishda asos bo‘lib xizmat qiladi.
- Natijalar akkretsiya diskidagi materiyaning spini va qora tuynukning fundamental parametrlari o‘rtasida to‘g‘ridan-to‘g‘ri, miqdoriy bogliqlikni taminlaydi, bu esa standart modeldan tashqaridagi “sochli” qora tuynuklarning mavjudligini sinab ko‘rish uchun astrofizik kuzatuvlardan foydalanishning yangi usulini taklif etadi.

Tadqiqot natijalarining ishonchliligi matematik fizika, hisoblash matematikasi va relyativistik astrofizikaning zamonaviy, sinovdan o‘tgan usullarini qo‘llash orqali taminlanadi. Natijalar qatiy ravishda umumiy nisbiylik nazariyasi va nazariy fizikaning matematik apparati doirasida olindi. Hisoblashning zamonaviy raqamli va analitik usullari ham qo‘llanilgan, natijalar mavjud kuzatuv ma‘lumotlari va boshqa mualliflarning natijalari bilan taqqoslangan. Dissertatsiyaning tuzilmaviy xulosalari kompakt obyektlar astrofizikasining asosiy qoidalariga mos keladi. Barcha olingan yechimlar modifikatsiyalangan gravitatsiya nazariyalarining (masalan, 4D EGB va

PFDM) matematik asoslariga qat‘iy rioya qiladi. Hisob-kitoblar geodezikalar uchun Gamilton-Yakobi formalizmi kabi o‘rnatilgan usullar yordamida o‘zaro tekshiriladi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati quyidagilardan iborat:

- Dissertatsiya natijalari zarrachalar dinamikasi hamda 4D Eynshteyn-Gauss-Bonnet va Simpson-Visser fazo-vaqtlaridagi kompakt obyektlar termodinamikasini yagona, o‘zaro mos formalizmga birlashtiradi va ular oldingi tadqiqotlarda yetarlicha o‘rganilmagan hodisalar o‘rtasidagi bog‘liqliklarni ochib beradi.
- Dissertatsiyadagi tadqiqot BSW zarracha tezlanish mexanizmini modifikatsiyalangan gravitatsiyadagi aylanuvchi qora tuynuklarga kengaytirib, qo‘shimcha maydonlar (skalyar va PFDM) mavjud bo‘lgan fazo-vaqtlar uchun kritik impuls momenti shartlari va to‘qnashuv energiyasi chegaralarini hosil qiladi.
- Yuqori tartibli kvant tuzatmalari asosidagi termodinamik tahlil modifikatsiyalangan gravitatsiya nazariyasidagi qora tuynuklar uchun logarifmik tuzatmalar va termal fluktuatsiyalarni o‘z ichiga olgan keng qamrovli termodinamik tavsifni ishlab chiqadi hamda ularning yarim-klassik yaqinlashuvlardan tashqaridagi barqarorligi va fazaviy o‘tish xususiyatlaridagi noaniqliklarni bartaraf etadi.

Tadqiqot natijalarini amaliyotga joriy etish

Modifikatsiyalangan gravitatsiya nazariyalari doirasida kompakt obyektlar atrofida termodinamik va energetik jarayonlarni o‘rganish natijalari quyidagicha qo‘llanilgan: “G. Rakhimova, F. Atamurotov, F. Javed, A. Abdujabbarov, G. Mustafa, Thermodynamical analysis of charged rotating black hole surrounded by perfect fluid dark matter, Nuclear Physics B, V. 996, 116363 (2023)”, ilmiy maqolasida e‘lon qilingan va G. Raximovaning fan doktori (PhD) dissertatsiyasida taqdim etilgan nazariy tadqiqot natijalari va usullari Fudan universiteti tomonidan qo‘llab-quvvatlanadigan dasturlar doirasida foydalanilgan (Prof. Cosimo Bambi xati).

Tadqiqot natijalarining aprobatsiyasi

Dissertatsiya natijalari 4 ta respublika konferensiyasida muhokama qilingan.

Tadqiqot natijalarining nashr etilishi

Dissertatsiya mavzusi bo‘yicha jami 12 ta ilmiy ish chop etilgan bo‘lib, shulardan 6 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan PhD dissertatsiyalarining asosiy ilmiy natijalarini e‘lon qilish uchun tavsiya etilgan xalqaro ilmiy jurnallarda nashr etilgan.

Dissertatsiyaning hajmi va tuzilishi

Dissertatsiya kirish, uchta bob, xulosa va adabiyotlar ro‘yxatidan iborat bolib, umumiy hajmi 108 sahifani tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismi mavzuning dolzarbligi va zarurligi, tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustuvor yo‘nalishlariga muvofiqligi, muammoning o‘rganilganlik darajasi, uning dissertatsiya bajarilgan oliy ta‘lim muassasasining ilmiy-tadqiqot rejalari bilan bog‘liqligi, tadqiqotning maqsadi, vazifalari, obyekti, predmeti, metodlari, ilmiy yangiligi, olingan natijalarning amaliy natijasi, ishonchliligi, ilmiy va amaliy ahamiyati, natijalarning amaliyotga joriy etilishi, aprobatsiyasi, nashr etilishi, shuningdek, dissertatsiyaning tuzilishi va hajmini ko‘rsatadi.

Birinchi bob, “4D Eynshteyn-Gauss-Bonnet gravitatsiya nazariyasida aylanuvchi zaryadlangan qora tuynuk yaqinida zarrachalarning tezlanishi” deb nomlangan bolib, u 4D Eynshteyn-Gauss-Bonnet gravitatsiya nazariyasi doirasida zaryadlangan aylanuvchi qora tuynukdagi gorizont tuzilishi va ergosferani o‘rganishga bag‘ishlangan. Bu nazariya massa (M) va aylanish parametri (a) dan tashqari zaryad (Q) va Gauss-Bonnet parametri (β) tufayli qo'shimcha parametrlarni kiritadi. Qizig‘i shundaki, parametrning har bir qiymati $Q(\beta)$ uchun kritik GB parametri $\beta = \beta_E$ ($Q = Q_E$) mavjud bo‘lib, u degenerativ gorizontga ega ekstremal qora tuynukka mos keladi. $\beta < \beta_E (Q < Q_E)$ bo‘lganda, u ikkita gorizontga ega ekstremal bo‘lmagan qora tuynukni, $\beta > \beta_E (Q > Q_E)$ bo‘lganda esa qora tuynuk umuman mavjud emasligini tasvirlaydi. Ekstremal qiymat $\beta_E (Q_E)$ ga GB parametri β va ergosfera ham ta‘sir qiladi. Biz, shuningdek, ushbu qora tuynuk gorizonti yaqinida teng massali ikkita zarrachaning to‘qnashuvini o‘rganamiz va $\beta(Q)$ parametrining ta‘sirini aniq ko‘rsatamiz. Fazo-vaqtda zarrachalar harakatini boshqaruvchi eng ichki barqaror aylana orbitalar (ISCO) va effektiv potensial turli parametr qiymatlari uchun tahlil qilindi. Massalar markazi energiyasi (E_{CM}) aylanish parametriga a hamda β va Q parametrlariga bog‘liq. Biz ekstremal va ekstremal bo‘lmagan holatlar uchun gorizont yaqinida to‘qnashayotgan ikkita zarrachaning E_{CM} energiyasini tekshiramiz. Ko‘rsatilishicha, ekstremal holatlarda, to‘qnashayotgan zarrachalardan biri kritik impuls momentiga ega bo‘lsa, E_{CM} ixtiyoriy ravishda yuqori bo‘lishi mumkin, bu esa 4D Eynshteyn-Gauss-Bonnet gravitatsiya nazariyasida zaryadlangan aylanuvchi qora tuynukning zarracha tezlatgichi sifatida ishlashi mumkinligini ko‘rsatadi. Qora tuynuk yechimining murakkabligiga qaramay, massa, Xoking harorati va entropiya kabi qora tuynuklarning termodinamik miqdorlari uchun gorizont radiusi orqali aniq ifoda keltirib chiqarilgan. Gauss-Bonnet parametri va elektr zaryadining ta‘siri tufayli ushbu miqdorlar Kerr yechimidan sezilarli chetlanishlarni ko‘rsatadi. 4D Eynshteyn-Gauss-Bonnet gravitatsiya nazariyasida aylanuvchi zaryadlangan qora tuynukning gravitatsion maydonini quyidagi metrika bilan Boyer-Lindquist koordinatalaridan foydalangan holda tasvirlash mumkin:

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}(adt - (r^2 + a^2)d\phi)^2, \quad (1)$$

metrik funksiyalar quyidagicha aniqlanadi:

$$\rho^2 = r^2 + a^2 \cos^2\theta\,, \tag{2}$$

$$\Delta = r^2 + a^2 + \frac{r^4}{2\beta}\left(1 - \sqrt{1 + 4\beta\left(\frac{2M}{r^3} - \frac{Q^2}{r^4}\right)}\right). \tag{3}$$

Biz $\theta = \pi/2$ ekvatorial tekislikda m massaga ega vaqtsimon zarrachaning harakatini ko‘rib chiqamiz, bu yerda qutb tezligi $\dot{\theta}$ nolga teng bo‘ladi. Aylanuvchi zaryadlangan qora tuynukning fazo-vaqtida zarrachaning umumlashtirilgan impulslari quyidagi shaklda ifodalanadi:

$$P_t = g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\,, \tag{4}$$

$$P_\phi = g_{\phi\phi}\dot{\phi} + g_{t\phi}\dot{t}, \tag{5}$$

bu yerda P_t va P_ϕ harakat doimiyliklari bo'lib, ular mos ravishda zarrachaning E energiyasiga va L impuls momentiga mos keladi. Soddalik uchun zarrachaning tinch massa qiymatini $m = 1$ deb olamiz. Nuqtali belgi xos vaqtga nisbatan differensiallashni bildiradi. Biz $u_\mu u^\mu = -1$ normallashtirish sharti bilan birgalikda (4)-(5)-tenglamalardan massiv zarracha harakat tenglamalarini hisoblab chiqdik, ular quyida keltirilgan:

$$u_t = \frac{1}{r^2}\left[\frac{(a^2 + r^2)}{\Delta}T + a(L - aE)\right], \tag{6}$$

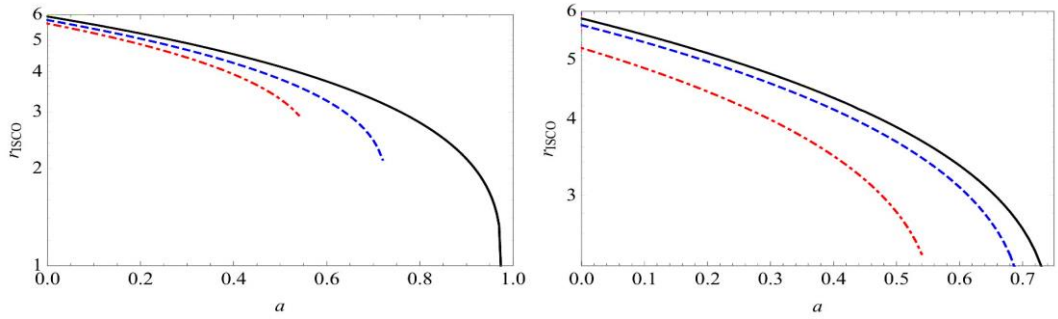
$$u_\phi = \frac{1}{r^2}\left[\frac{a}{\Delta}T + (L - aE)\right], \tag{7}$$

$$u_r = \pm\frac{\sqrt{T^2 - \Delta(r^2 + (L - aE)^2)}}{r^2}, \tag{8}$$

bu yerda $T = E(a^2 + r^2) - aL$ hamda (8)-tenglamadagi “+” va “-” belgilari mos ravishda chiqib ketayotgan va kirib kelayotgan geodeziklarga ishora qiladi.

Zarrachalar harakatini baholash uchun biz ushbu qora tuynukning xususiyatlari va effektiv potensialini tushunishimiz kerak. Zaryadlangan aylanuvchi qora tuynuklar uchun cheksizlikda tinch holatda turgan sinov zarrachasi uchun effektiv potensial quyidagi formula yordamida hisoblanishi mumkin:

$$V_{\text{eff}} = -\frac{1}{2g_{rr}}\left(\frac{E^2 g_{\phi\phi} + 2ELg_{t\phi} + L^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}\right), \tag{9}$$



Rasm 1: Ushbu rasm eng ichki barqaror aylana orbitalarning (ISCO) spin parametrlari bo‘yicha β va Q ning turli qiymatlari uchun o‘zgarishini ko‘rsatadi. Chap tomondagi grafik belgilangan $Q = 0.2$ va turli β qiymatlari uchun: $\beta = 0$ (qora), $\beta = 0.2$ (ko‘k) va $\beta = 0.4$ (qizil). Xuddi shunday, o‘ng tomondagi rasm belgilangan $\beta = 0.2$ va turli Q qiymatlari uchun: $Q = 0$ (qora), $Q = 0.3$ (ko‘k) va $Q = 0.6$ (qizil).

(9)-tenglamadan effektiv potensialning aniq shaklini osongina olish mumkin:

$$V_{\text{eff}} = -\frac{(aL - (a^2 + r^2)E)^2 - \Delta(r^2 + (L - aE)^2)}{2r^4}. \qquad (10)$$

$\dot{r} = 0;\quad \partial_r V_{\text{eff}} = 0;\quad \partial_r^2 V_{\text{eff}} = 0$ bir vaqtlik tenglamalar yechimi zarrachaning eng ichki barqaror aylana orbitasini r_{ISCO} aniqlaydi. Bu yerda r_{ISCO} uchun analitik yechim topish oson emas, lekin 1- rasmda biz r_{ISCO} qiymatini β va Q ning turli qiymatlari uchun raqamli hisobladik. Bundan tashqari, ushbu rasmlardan ko‘rinib turibdiki, agar boshqa parametrlar o‘zgarmas bo‘lib qolsa, β va Q parametrning ortishi eng ichki barqaror aylana orbitasi radiusining kamayishiga olib keladi.

Zarraning impulsi qiymati egri fazo-vaqtda uning geodezik harakatini tahlil qilishda muhim ahamiyatga ega. Statsionar va aksial-simmetrik qora tuynuk fazo-vaqtida sinov zarrachasining harakati V_{eff} effektiv radial potensial orqali tavsiflanishi mumkin. Shuning uchun, fizik jihatdan ruxsat etilgan harakat quyidagi shartni qanoatlantirishi lozim: $V_{\text{eff}} \geq 0$.

Impuls momenti uchun chegaraviy qiymatlar cheksizlikdan tashlab yuborilgan zarra burilish nuqtasiga duch kelmasdan qora tuynuk gorizontiga yetib borishi mumkin bo‘lgan impuls momentlari oralig‘ini aniqlaydi. Ushbu qiymatlar tushuvchi va tushmaydigan trayektoriyalar orasidagi chegarani belgilovchi beqaror aylana orbitlarning mavjudligi bilan aniqlanadi.

1-Jadval va 2-Jadval da keltirilgan impuls momentining chegaraviy qiymatlari L_{min} va L_{max} quyidagi aylana orbit shartlarini qo‘llash orqali topiladi:

$$V_{\text{eff}} = 0 \text{ and } \frac{dV_{\text{eff}}}{dr} = 0. \tag{11}$$

Ushbu tenglamalarning yechimlari beqaror aylana harakatiga mos keladi. Mos impuls momentlari ruxsat etilgan interval chegaralarini belgilaydi: $L_{min} < L < L_{max}$.

Agar impuls momenti ushbu intervaldan tashqarida bo‘lsa, effektiv potensial gorizontdan tashqarida burilish nuqtasini hosil qiladi va natijada zarra qora tuynukka yetib bora olmaydi. Shuning uchun, chegaraviy impuls momentlari gorizontga yetib borishni ta‘minlovchi global kinematik shartni ifodalaydi.

Kritik impuls momenti hodisalar gorizontining bevosita yaqinida zarracha harakatining o‘zini tutishi bilan belgilanadi. U radial tezlikning aynan gorizontda nolga tenglashish shartidan kelib chiqib aniqlanadi:

$$V_{\text{eff}}(r_h) = 0. \tag{12}$$

Shu sababli, (8)-tenglamadan foydalanib, impuls momentining kritik qiymatini aniqlash mumkin. Ya‘ni, $r \to r_h^E$ da quyidagi ifoda hosil bo‘ladi:

$$L_{cr} = \frac{(a^2 + (r_H^E)^2)E}{a}. \tag{13}$$

Ushbu nozik sozlangan trayektoriya BSW mexanizmida asosiy ro‘l o’ynaydi, chunki u gorizontga ixtiyoriy darajada yaqin sohada sodir bo‘ladigan zarrachalar to‘qnashuvlarida massa-markazi energiyasining cheksiz o‘sishiga olib keladi.

Chegaraviy va kritik impuls momentlari bir xil $V_{\text{eff}} = 0$ shartidan kelib chiqsa-da, ularning fizik mazmuni tubdan farqlidir. Chegaraviy impuls momentlari gorizontdan tashqaridagi radiuslarda aylanma orbita shartlari orqali aniqlanadi va cheksizlikdan kelayotgan zarra harakatiga global cheklov qo‘yadi. Aksincha, kritik impuls momenti faqat gorizontga oid lokal shart bilan aniqlanadi va o‘zi mustaqil ravishda zarraning gorizontga yetib borishini kafolatlamaydi.

BSW effektining fizik jihatdan amalga oshishi uchun kritik impuls momenti ruxsat etilgan interval ichida bo‘lishi zarur: $L_{cr} \in [L_{min}, L_{max}]$.

Agar L_{cr} maksimal ruxsat etilgan qiymatdan katta bo‘lsa, $L = L_{cr}$ bo‘lgan zarra gorizontdan tashqarida burilish nuqtasiga duch keladi va gorizontga yetib bora olmaydi, hatto gorizont shartini qanoatlantirgan taqdirda ham. Bunday holatda massa-markazi energiyasining formal cheksizligi kinematik jihatdan taqiqlanadi.

Impuls momentining chegaraviy qiymatlari, shuningdek qora tuynukning mos spin parametri, β, Q va hodisa gorizonti radiusi r_h ekstremal va ekstremal bo‘lmagan holatlar uchun mos ravishda 1- va 2- jadvallarda keltirilgan.

1-Jadval: Turli ekstremal holatlar uchun impuls momentining chegaraviy qiymatlari

β	Q	a_E	r_H^E	L_{min}	L_{max}	L_{cr}
0.1	0.1	0.852804	1.14881	-4.71391	2.35980	2.40035
0.1	0.2	0.838440	1.14423	-4.69208	2.36291	2.40001
0.1	0.3	0.813929	1.13654	-4.65479	2.36996	2.40094
0.3	0.3	0.622437	1.18055	-4.49305	2.83298	2.86155
0.4	0.4	0.501169	1.15498	-4.35397	3.00904	3.16288
0.5	0.5	0.358248	1.10509	-4.17220	3.19564	3.76714

Biz 4D Eynshteyn-Gauss-Bonnet gravitatsiya nazariyasida zaryadlangan aylanuvchi qora tuynuklarning zarracha harakati, BSW mexanizmi va termodinamik xususiyatlarini o‘rgandik va olingan bir qancha natijalarni ta‘kidlab o‘tamiz.

Parametrlarning kichik qiymatlari uchun ushbu qora tuynuk ikkita gorizontga ega bo‘ladi va ushbu parametrlar ortishi bilan bu gorizontlar bir-biriga yaqinlashadi.

2-Jadval: Turli ekstremal bo‘lmagan holatlar uchun impuls momentining chegaraviy qiymatlari.

β	Q	a	r_H^+	r_H^-	L_{min}	L_{max}	L_{cr}
0.1	0.3	0.4	1.79992	0.40315	-4.32006	3.45952	8.49932
0.1	0.4	0.4	1.75454	0.42731	-4.28909	3.40803	8.09605
0.2	0.3	0.4	1.72661	0.51472	-4.31206	3.42612	7.85292
0.2	0.4	0.4	1.67569	0.54524	-4.28079	3.37120	7.41985
0.3	0.3	0.4	1.64295	0.61796	-4.30389	3.38812	7.14819
0.3	0.4	0.4	1.58352	0.65778	-4.27231	3.32845	6.66883

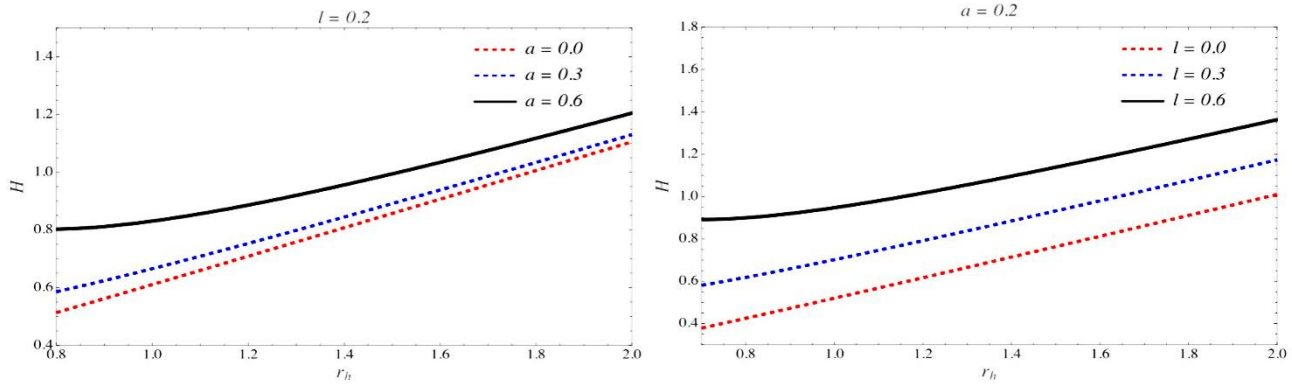
Oxir-oqibat, ushbu parametrlarning o‘ziga xos qiymatlari (ular ekstremal qiymatlar deb ataladi) uchun gorizontlar to‘qnashadi va bitta gorizont hamda ekstremal qora tuynuk

deb ataladigan qora tuynuk paydo bo‘ladi. Ekstremal qiymatlardan kattaroq qiymatlar uchun qora tuynuk mavjud emas.

BSW shuni ko‘rsatdiki, ekstremal Kerr qora tuynuklari holatida to‘qnashayotgan ikki zarra uchun massa-markaz energiyasi E_{CM} ixtiyoriy darajada katta qiymatlarga yetishi mumkin. BSW tahlilini 4D Eynshteyn–Gauss–Bonnet gravitatsiyasidagi zaryadlangan va aylanayotgan qora tuynuklarga kengaytirish natijasida esa quyidagi xulosaga kelinadi: 4D Eynshteyn–Gauss–Bonnet gravitatsiyasidagi ekstremal zaryadlangan aylanayotgan qora tuynuk gorizontiga yaqin sohada sodir bo‘ladigan to‘qnashuvlarda E_{CM} ning ixtiyoriy darajada oshishiga erishib bo‘lmaydi. Buning sababi kritik impuls momentining qiymati ruxsat etilgan maksimal impuls momenti qiymatidan oshib ketishidir. Shuningdek, ekstremal bo‘lmagan qora tuynuk holatida ham E_{CM} ning cheksiz qiymatiga erishish mumkin emas.

Ushbu qora tuynukning Xoking harorati keltirib chiqarilgan va uning chegaraviy holatlari muhokama qilingan. Qora tuynuk yechimining murakkabligiga qaramay, massa, Xoking harorati va entropiya kabi parametrlarni o‘z ichiga olgan qora tuynuklarning termodinamik xususiyatlari uchun gorizont radiusi bilan aniq formulalar olingan. Ushbu miqdorlar Gauss-Bonnet parametrlari va elektr zaryadining ta‘siri tufayli Kerr yechimidan sezilarli chetlanishlarni namoyish etadi.

Ikkinchi bob “Aylanuvchi Simpson-Visser qora tuynuk atrofida zarrachalar tezlanishi va termodinamikasi” deb nomlanib, unda gorizont va ergosfera tuzilishini o‘rganish uchun aylanuvchi Simpson-Visser qora tuynugi fonida vaqtsimon geodezik harakat ko‘rib chiqilgan. Eng ichki barqaror aylana orbitalar (ISCO) va zarrachaning fazo-vaqtdagi harakatini boshqaruvchi effektiv potensial SV qora tuynugidagi parametrlarning turli qiymatlari uchun o‘rganilgan. Ikki to‘qnashayotgan zarrachaning massalar markazi energiyasi ham ekstremal, ham ekstremal bo‘lmagan holatlar uchun tekshirilgan. Bundan tashqari, SV qora tuynugining termodinamik xususiyatlari ham batafsil o‘rganilgan.



Rasm2: Grafik entalpiyaning gorizont radiusi bo‘yicha o‘zgarishini ko‘rsatadi.

Aylanuvchi Simpson-Visser kompakt obyektining gravitatsion maydonini Boyer-Lindquist koordinatalarida quyidagi interval orqali ifodalash mumkin:

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}(adt - (r^2 + a^2)d\phi)^2, \quad (14)$$

metrik funksiyalar quyidagicha aniqlanadi:

$$\rho^2 = r^2 + a^2\cos^2\theta, \quad (15)$$

$$\Delta = r^2 + a^2 - 2Mre^{-\frac{l}{r}}, \quad (16)$$

bu yerda a va l mos ravishda SV qora tuynukning spini va chetlanish parametrlari deb atalishi mumkin. Ushbu metrika oldin Schvarzschild metrikasida qilinganidek, $M \to M(r) = Me^{-l/r}$ modifikatsiyasi bajarilgan Kerr metrikasidagi regulyarizatsiya usulidan olingan. Bu yerda l parametrini Kerr yechimidan chetlanishni miqdoriy baholovchi kattalik sifatida ko‘rish mumkin. Ushbu qora tuynuk atrofidagi fazo-vaqt odatdagi aylanuvchi obyekt atrofidagidek harakatlanadi va $l \to 0$ limiti Kerr qora tuynugiga mos keladi. Biz ushbu qora tuynukning Simpson-Visser qora tuynugidagi termodinamikasini tahlil qilamiz. Metrikaning umumiy shaklini quyidagicha yozish mumkin:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi. \quad (17)$$

Sinov zarrachasining hodisa gorizonti nuqtasida, impuls momenti nolga teng bo‘lganda, burchak tezligi aylanuvchi Kerrga o‘xshash qora tuynuk uchun $\Omega_H = -g_{t\phi}/g_{\phi\phi}$ formula yordamida hisoblanishi mumkin:

$$\Omega_H = \frac{a}{r_h^2 + a^2}. \quad (18)$$

Biz keyingi hisob-kitoblarimiz uchun (18) ifodasidan foydalanamiz. Endi biz SV qora tuynugining termodinamik miqdorlarini hisoblashga o‘tamiz. Qora tuynuk massasi (M, bu entalpiyaga H teng) (16) tenglamani nolga tenglashtirish orqali aniqlanadi. Natijada quyidagini olish mumkin:

$$H = \frac{(a^2 + r_h^2)e^{\frac{l}{r_h}}}{2r_h}. \quad (19)$$

2-rasmda H funksiyasi sifatida entalpiyani namoyish etadi. Ko‘rish mumkinki, a va l parametrlarining ortishi belgilangan r_h uchun kattaroq massaga (entalpiyaga) mos keladi.

Ushbu bob doirasida ISCO radiusi ham SV fazo-vaqtida uning l va a parametrlari bilan muhokama qilingan. Ushbu parametrlarning ortishi ISCO radiusini kichiklashtiradi.

Massiv zarracha orbitalaridan foydalanib, biz BSW effekti orqali ekstremal va ekstremal bo‘lmagan holatlar uchun qora tuynukdan energiya olishni tekshirdik. Massalar markazi energiyasi ekstremal SV qora tuynugi uchun ixtiyoriy ravishda yuqori bo‘lishi mumkin, bu esa ekstremal bo‘lmagan SV qora tuynugi uchun to‘g'ri emas. Termodinamika xususiyatlari turli holatlar uchun o‘rganilgan: entalpiya, Xoking harorati, entropiya va Gibbs erkin energiyasi. Aniqroq aytganda, qora tuynukning l va a parametrlarining ortishi Gibbs erkin energiyasining ham ortishiga olib keladi. Ushbu turdagi qora tuynukning harorati uchun buning aksi to‘g‘ri keladi.

Uchinchi bob “O‘ta oquvchan qorong‘u moddasi bilan o‘ralgan zaryadlangan aylanuvchi qora tuynukning termodinamik tahlili” deb nomlanib, u o‘ta oquvchan qorong‘u moddasi bilan o‘ralgan zaryadlangan aylanuvchi qora tuynuklar atrofidagi termodinamika va zarrachalar tezlanishini o‘rganishga bag‘ishlangan. Biz o‘ta oquvchan qorong‘u moddasi doirasida qora tuynuk gorizontlarini, ergosferani va eng ichki barqaror aylana orbitalarni muhokama qilamiz. O‘ta oquvchan qorong‘u moddasi Xoking harorati, Gibbs erkin energiyasi va termodinamikaga oid boshqa fizik miqdorlarga ta‘sir qilishi kuzatilgan. Termodinamikaning ba‘zi turli jihatlari jalb qilingan parametrlarning turli qiymatlari doirasida muhokama qilinadi. Bundan tashqari, biz ko‘rib chiqilayotgan qora tuynuk yechimlarining termal fluktuatsiyalarini muhokama qilamiz, ular harorat fluktuatsiyalari tufayli qora tuynuk atrofida zarrachalarning tasodifiy harakatiga ishora qiladi. Xususan, termal fluktuatsiyalar qora tuynukning entropiyasi va Xoking nurlanishiga ta‘sir ko‘rsatib, fizik kattaliklarning o‘zgarish xususiyatlarini tushunishda muhim ahamiyat kasb etadi. O‘ta oquvchan qorong‘u moddasiga ega zaryadlangan aylanuvchi qora tuynuklarning tuzatilgan termodinamik miqdorlari o‘ta oquvchan qorong‘u moddasisiz zaryadlangan aylanuvchi qora tuynuknikidan kichikroq ekanligi aniqlangan. Biz adabiyotlarda keltirilgan taxminlar va oldingi qora tuynuk yechimlarining termodinamikasi bizning natijalarimizga mos kelishini ta‘kidlaymiz. O‘ta oquvchan qorong‘u moddasidagi zaryadlangan aylanuvchi qora tuynuk quyidagi interval orqali tasvirlanadi:

$$ds^2 = -\frac{1}{\rho^2}(\Delta - a^2\sin^2\theta)dt^2 - \frac{2a\sin^2\theta}{\rho^2}\left[2Mr - Q^2 - \gamma r\ln\left(\frac{r}{\gamma}\right)\right]dtd\phi + \rho^2 d\theta^2 + \sin^2\theta\left[r^2 + a^2 + \frac{a^2\sin^2\theta}{\rho^2}\left(2Mr - Q^2 - \gamma\, r\ln\left(\frac{r}{\gamma}\right)\right)\right]d\phi^2 + \frac{\rho^2}{\Delta}dr^2, \qquad (20)$$

bu yerda Δ va ρ quyidagicha aniqlanadi:

$$\Delta = r^2 + a^2 - 2Mr + Q^2 + \gamma r \ln\left(\frac{r}{\gamma}\right),$$
$$\rho^2 = r^2 + a^2 \cos^2\theta\,, \qquad (21)$$

bu yerda M - qora tuynukning massasi, a - aylanish parametri, Q - qora tuynukning zaryadi va γ - o‘ta oquvchan qorong‘u moddasi (PFDM) parametri.

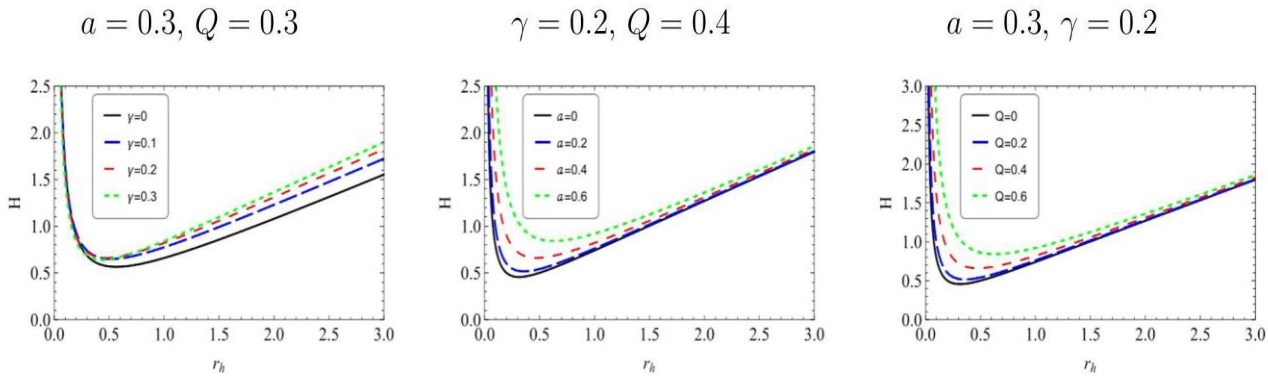
Biz o‘ta oquvchan qorong‘u moddasidagi zaryadlangan aylanuvchi qora tuynukning termodinamikasini muhokama qilamiz. Berilgan metrika uchun qora tuynukning hodisa gorizonti quyidagi shart orqali olinadi “$\Delta = 0$”:

$$r_h^2 + a^2 - 2Mr + Q^2 + \gamma r_h \ln\frac{r_h}{\gamma} = 0. \qquad (22)$$

Shuni osongina payqash mumkinki, u ikkita gorizontga ega: ichki gorizont r_- va tashqi gorizont r_+, ular PFDM parametri γ ga bog‘liq. Qora tuynukning hodisa gorizonti uchun (22) shartdan foydalanib, biz qora tuynuk massasini M (u entalpiya H ga teng) keltirib chiqarishimiz mumkin:

$$H = \frac{1}{2}\left[r_h + \frac{(a^2 + Q^2)}{r_h} + \gamma \ln\frac{r_h}{\gamma}\right]. \qquad (22)$$

Entalpiya munosabatining (22) grafik tahlili 3- rasmda berilgan. Grafiklardan shuni aniq ko‘rish mumkinki, entalpiya r_h ning kichik qiymatlari uchun kamayadi va γ ning ortishi bunga ta‘sir qilmaydi. Biroq, γ parametri ortganda u $r_h = 0.5$ dan boshlab o‘sishni boshlaydi. Ayni paytda, spin parametri a va zaryad Q ortganda, entalpiya qora tuynuk yaqinida farq qiladi va qora tuynukdan uzoqlashgan sari parametr o‘zgarishlaridan qat‘iy nazar ko‘tariladi.



Rasm 3: Entalpiyaning turli parametrlar uchun gorizont radiusi bo‘yicha o‘zgarishi.

Hozirgi vaqtda qora tuynuklarning xususiyatlarini o‘rganish uchun bir qancha kuzatuv parametrlari mavjud, ammo faqat qora tuynuklarning zaryad yoki aylanish parametrlari muhokama qilinadigan aniq ma‘lumotlar yo‘q. Bu yerda biz o‘ta oquvchan qorong'u

moddasi mavjudligida zaryadlangan aylanuvchi qora tuynukni nazariy tahlil qilishga harakat qildik va quyidagilarni o‘rgandik.

Dastlab, ushbu qora tuynukning gorizontlari va ergosfera sohasi chegaralari muhokama qilindi. Shuni ta‘kidlash joizki, biz gorizontlarning ikkala radiusi o‘zaro ustma-ust tushadigan aylanish parametrining ma‘lum bir kritik $a = a_E$ qiymati uchun ekstremal qora tuynuk holatini hisobladik. O‘ta oquvchan qorong‘u moddasi (PFDM) parametrlarining gorizontlar va ergosfera sohasiga ta‘siri ekstremal va ekstremal bo‘lmagan qora tuynuklar misolida o‘rganildi. Ushbu sohalar shaklining o‘zgarishi grafik ko‘rinishda tahlil qilindi. Tadqiqot natijasida PFDM ning γ parametri ham gorizont, ham ergosfera chegaralarini qisqartirishi va ularning deyarli yagona sohaga birlashishiga olib kelishi aniqlandi. ISCO radiusining barcha parametrlar (γ, Q va a) doirasida kamayishi kuzatilgan.

Biz ushbu qora tuynukning PFDM dagi termodinamik xarakteristikalarini o‘rgandik va γ ning ortishi entalpiya H va Gibbs erkin energiyasi G ning ortishiga olib kelishini aniqladik. Aksincha, qora tuynuk harorati γ ning o‘sishi bilan kamayadi. Ishda PFDM parametri γ ning ISCO radiusiga ta'siri ham muhokama qilingan, xususan, γ ortishi bilan ISCO radiusi kamayadi. Biz qora tuynuk gorizonti yaqinida ikkita bir xil massiv zarrachaning to‘qnashuvini o‘rgandik va ekstremal hamda ekstremal bo‘lmagan holatlar uchun massa markazi energiyasining o‘zgarish qonuniyatlarini o‘rgandik.

Ikkinchi tartibli, oddiy logarifmik va yuqori tartibli tuzatish hadlarining PFDM bilan va PFDM siz zaryadlangan aylanuvchi qora tuynukning Gelmgols erkin energiyasi, ichki energiya, entalpiya va Gibbs erkin energiyasiga ta‘siri kuzatilgan. Shuni ta‘kidlash joizki, solishtirma issiqlik ma‘lum o‘lchamdagi qora tuynuklar uchun barqaror tuzilmani, ammo sezilarli darajada kattaroq qora tuynuklar uchun beqaror dizaynni ko‘rsatadi.

Xulosalar

Falsafa doktori (PhD) dissertatsiyasi uchun “Modifikatsiyalangan gravitatsiya nazariyalarida kompakt obyektlar atrofidagi termodinamik va energetik jarayonlar” mavzusida o‘tkazilgan tadqiqotlar asosida quyidagi xulosalar taqdim etildi:

- Ilk bor, modifikatsiyalangan gravitatsiya fazo-vaqtlarida (4D Eynshteyn–Gauss–Bonne, Simpson–Visser va o‘ta oquvchan qorong‘u modda modellari) gorizont tuzilishi, ergosfera geometriyasi va ekstremallik shartlarining qo‘shimcha gravitatsion parametrlarga tizimli ravishda bog‘liqligi ko‘rsatib berildi — bu esa “qora tuynuk sochlari” va kuzatiluvchi fazo-vaqt xususiyatlari o‘rtasidagi bevosita bog‘liqlikni ochib beradi.

- Ilk bor, ushbu nazariy modellarda eng ichki barqaror aylana orbita (ISCO) parametrga bog‘liq siljishlarga ega ekanligi aniqlanib, akkretsion disk kuzatuvlari asosida modifikatsiyalangan gravitatsiya nazariyalarini tekshirish uchun yangi diagnostik vosita taqdim etildi.

- Ilk bor, modifikatsiyalangan gravitatsiyada BSW zarrachalarni tezlatish mexanizmining sezilarli darajada kuchayishi mumkinligi isbotlandi hamda qorong‘u modda yoki chiziqli bo‘lmagan elektromagnit maydonlar bilan o‘ralgan qora tuynuklar uchun kritik impuls momentining aniq shartlari olindi.

- Ilk bor, qora tuynuk entropiyasiga kiritilgan logarifmik va yuqori tartibli kvant tuzatmalari, ayniqsa o‘ta oquvchan qorong‘u modda mavjudligida yangi termodinamik fazoviy o‘tishlarga va o‘zgargan barqarorlik mezonlariga olib kelishi aniqlandi.

SCIENTIFIC COUNCIL DSc.03/07.07.2025.FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES AT "NEW UZBEKISTAN" UNIVERSITY

TASHKENT STATE TECHNICAL UNIVERSITY AFTER ISLAM KARIMOV
"NEW UZBEKISTAN" UNIVERSITY

RAXIMOVA GULZODAXON ISROILJON QIZI

THERMODYNAMIC AND ENERGETIC PROCESSES AROUND COMPACT OBJECTS IN MODIFIED THEORIES OF GRAVITY

01.03.01 - Astronomy
01.04.02 - Theoretical Physics

DOCTOR OF PHILOSOPHY IN PHYSICAL AND MATHEMATICAL SCIENCES (PhD)

ABSTRACT OF DISSERTATION

Tashkent-2026

The theme of dissertation of the Doctor of Philosophy (PhD) on physical and mathematical sciences is registered by Supreme Attestation Commission of Higher

Education, Science and Innovations of the Republic of Uzbekistan under B2025.3.PhD/FM1360.

The dissertation has been carried out at Tashkent state technical university after Islam Karimov and "New Uzbekistan" university.

The abstract of the dissertation was posted in three (Uzbek, English, Russian (resume)) languages onthe website of the Scientific Council (www.ias.ifar.uz) and on the information and education portal at "Ziyonet" (www.ziyonet.uz).

Scientific supervisors:	**Abdujabbarov Ahmadjon Adiljanovich** Doctor of physical and mathematical sciences, professor
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Leading organization:	**Al-Farabi Kazakh National University**

The defense of the dissertation will be held on "___"_____ 2026 at ___ in the meeting of the Scientific Council No. DSc.03/07.07.2025.FM/T.192.01 at the Institute for Advanced Studies, "New Uzbekistan" University (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnaxr Street 1, Institute for Advanced Studies, phone: +99871 202-41-11; e-mail: info@newuu.uz

The dissertation can be looked through at the Information Resource Center of the Institute for Advanced Studies under "New Uzbekistan" university (registered under №__). (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnaxr Street 1,Institute for Advanced Studies at "New Uzbekistan" university, phone: +99871 202-41-11).

The abstract of the dissertation was distributed on "___"_____, 2026.

(Registry record №___ dated "___"_____, 2026).

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B.M. Narzilloev

Chairman of the Scientific Seminar of the Scientific Council on Award of Scientific Degrees, DSc**.**

INTRODUCTION

Topicality and demand of the theme of the dissertation.

The theme is highly topical due to the growing interest in alternative theories of gravity, which aim to resolve fundamental issues in General Relativity (GR), such as singularities and incompatibility with quantum mechanics. Recent advancements in astrophysical observations, including gravitational wave detections and black hole imaging by the Event Horizon Telescope, have provided unprecedented opportunities to test these theories.

The demand for such research stems from the need to understand the dark components of the universe, dark matter and dark energy, which remain unexplained within standard GR. Modified gravity theories offer potential solutions, making this dissertation relevant to contemporary astrophysics and cosmology. The work investigates key phenomena like particle acceleration, and thermodynamic properties, which are crucial for probing the nature of gravity in extreme environments. For instance, the Banados-Silk-West (BSW) mechanism, which suggests black holes can act as particle accelerators, has implications for highenergy astrophysics and cosmic-ray production.

Moreover, the dissertation aligns with the strategic scientific priorities in Uzbekistan and the broader international community. This dissertation work corresponds to the tasks of the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 “On the Strategy of Actions for the Further Development of the Republic of Uzbekistan” dated February 07, 2017, Decree of the President of the Republic of Uzbekistan No. PP-2789 “On Measures for Further Improvement of Academy of Sciences, Organization, Management, and Financing of Research Activities” from 18.02.2017.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. Dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. “Power, energy and resource-saving”.

|Degree of study of the problem

The problem of understanding thermodynamic and energetic processes around compact objects in modified theories of gravity has been extensively studied, yet significant gaps remain. Classical works laid the foundation for black hole shadows and geodesic motion (Bardeen, Luminet, Event Horizon Telescope Collaboration, Abdujabbarov, Ahmedov, Bambi), while later studies introduced the concept of black holes as particle accelerators (Banados, Silk, West, Zaslavskii, Wei, Liu, Atamurotov). Modified gravity theories, such as Einstein-Gauss-Bonnet (EGB) and scalar-tensor models, have been explored in contexts like horizon structure, photon spheres, and particle dynamics. However, most existing research focuses on isolated aspects, such as shadows or thermodynamics (Bekenstein, Hawking, Glavan, Lin, Ghosh, Ahmedov,

Jamil), without integrating these phenomena into a cohesive framework. Additionally, while the BSW mechanism is well studied in Kerr and Kerr-Newman spacetimes, its implications in modified gravity, particularly with additional fields like dark matter or nonlinear electrodynamics, remain underexplored.

Recent advancements in observational astronomy, such as the imaging of M87* and SgrA*, have intensified the need for comprehensive models that account for deviations from GR. Studies on rotating black holes in 4D EGB gravity, SimpsonVisser metrics, and perfect fluid dark matter (PFDM) environments have emerged (Simpson, Visser, Kiselev, Abdujabbarov, Atamurotov, Ghosh), yet systematic comparisons with observational data are still limited. Thermodynamic properties, including Hawking radiation and phase transitions, have been analyzed in simplified scenarios, but the effects of thermal fluctuations and higher-order corrections are not fully understood. The dissertation addresses these gaps by unifying particle dynamics and thermodynamics in modified gravity, providing a more holistic understanding of compact objects. By leveraging both analytical and numerical methods, the work aims to bridge theoretical predictions with observable phenomena, contributing to the ongoing refinement of alternative gravity theories.

Connection of the topic of the dissertation with the scientific research of the higher educational/research institutions, where the dissertation was carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development: F-FA-2021-510 “Investigations of nuclear matter of neutron stars in modified gravity” (2021-2026).

The aim of the research is the comprehensive analysis of astrophysical processes around rotating black holes in modified theories of gravity.

The tasks of the research:

- to analyze the spacetime structure of rotating and charged black holes in modified gravity theories (Einstein-Gauss-Bonnet, Simpson-Visser, and PFDM models), including horizon configurations, extremality conditions, and ergosphere properties, and to determine how these features depend on additional gravitational parameters;
- to derive and investigate the equations of motion for test particles of rotating black holes in modified gravity, using the effective potential approach;
- to study particle acceleration mechanism near black holes in both extremal and non-extremal regimes;
- to examine the innermost stable circular orbits (ISCOs) and effective potentials for test particles in modified gravity backgrounds;

- to derive the thermodynamic quantities of rotating black holes in modified gravity, such as Hawking temperature, entropy and Gibbs free energy;
- to examine thermodynamic stability and phase behavior, including the effects of thermal fluctuations and higher-order entropy corrections.

The objects of the research are rotating black holes in modified gravity, charged black holes, horizon and ergosphere structures, particle dynamics and thermodynamic systems.

The subjects of the research are particle acceleration mechanisms, effective potentials and stability, thermodynamic properties, dark matter.

The methods of the research are methods of computational mathematics, methods of theoretical astrophysics, modern methods of mathematical physics, analytical and numerical methods of calculating differential equations for field and particle motion. The method of metric analysis and analytical calculations has been used for the derivation of black hole solutions in modified gravity and the computation of horizon and ergosphere structures via metric component analysis. Method of numerical simulations and computational modeling has been used to solve nonlinear field equations numerically for charged/dark matter-modified black holes. Stability analysis has been used for evaluating Hawking temperature, entropy, and Gibbs free energy via surface gravity and Euclidean action methods.

The scientific novelty of the research is the follows:

- The first unified study of dynamics of particles and thermodynamics allowed us to combine particle acceleration and thermodynamic stability in modified gravity (e.g., 4D EGB, Simpson-Visser) within a single framework, revealing interdependencies previously studied only in isolation.
- Obtained derivations of observable signatures (e.g. ISCO shifts) for black holes in PFDM providing testable deviations from General Relativity.
- The theoretical results incorporate thermal fluctuations and higher-order entropy corrections into modified gravity black holes, addressing gaps in understanding quantum/statistical effects on Hawking radiation and phase transitions.

Practical results of the investigation are as follows:

- The results on particle acceleration processes quantify energy limits for near horizon particle collisions in modified spacetimes, with implications for high energy astrophysics (e.g., cosmic rays, AGN jets).
- Analysis of stability criteria for exotic compact objects identify parameter ranges where black holes in modified gravity remain stable, guiding theoretical models and numerical relativity simulations.

- the results provide a direct, quantitative link between the spin of matter in an accretion disk and the fundamental parameters of a black hole, offering a new method to use astrophysical observations to test for the existence of “hairy” black holes beyond the standard model.

Reliability of the research results provided by applying modern proven methods of mathematical physics, computational mathematics, and relativistic astrophysics. The results were obtained strictly within the mathematical apparatus of general relativity and theoretical physics. Modern numerical and analytical methods of calculation are also used, and the results are compared with available observational data and the results of other authors. The structured conclusions of the thesis correspond to the basic rules of astrophysics of compact objects. All derived solutions strictly adhere to the mathematical frameworks of modified gravity theories (e.g., 4D EGB and PFDM). Calculations are cross-verified using established methods like Hamilton-Jacobi formalism for geodesics.

The scientific and practical significance of the research results are follows:

- The results of the dissertations integrate particle dynamics, and thermodynamics of compact objects in 4D Einstein-Gauss-Bonnet, Simpson-Visser spacetimes into a single self-consistent formalism, revealing previously unexplored interconnections between these phenomena.

- The study in the dissertation extends the Banados-Silk-West particle acceleration mechanism to rotating black holes in modified gravity, deriving critical angular momentum conditions and collision energy limits for spacetimes with additional fields (scalar and PFDM).

- The analysis of thermodynamics with higher-order quantum corrections develops a comprehensive thermodynamic description incorporating logarithmic corrections and thermal fluctuations for modified gravity black holes, resolving ambiguities in their stability and phase transition behavior beyond semiclassical approximations.

Application of the research results

The results of study of thermodynamic and energetic processes around compact objects in modified theories of gravity have been applied as follows: the theoretical research results and methods, published in the scientific paper “G. Rakhimova, F. Atamurotov, F. Javed, A. Abdujabbarov, G. Mustafa, Thermodynamical analysis of charged rotating black hole surrounded by perfect fluid dark matter, Nuclear Physics B, V. 996, 116363 (2023)”, and presented in the Doctorate (PhD) thesis of Ms. G.Raximova have been used in the frame of the programs supported by the Fudan University (Letter from Prof. Cosimo Bambi).

Approbation of the research results
The dissertation results have been discussed in 4 republic conferences.

Publication of the research results
On the theme of the dissertation, 12 scientific works were published, including 6 scientific papers in international scientific journals recommended by the Supreme Attestation Comission of the Republic of Uzbekistan for publishing basic scientific results of PhD dissertations.

Volume and structure of the dissertation
The dissertation is presented on 108 pages and consists of an introduction, three chapters, conclusion, and a bibliography.

THE MAIN CONTENT OF DISSERTATION

The introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

The first chapter, entitled "Particle Acceleration Near a Rotating Charged Black Hole in 4D Einstein-Gauss-Bonnet Gravity" is devoted to study the horizon structure and ergosphere in a charged rotating black hole within 4D Einstein-Gauss-Bonnet gravity, which introduces additional parameters (Q) because of the charge and Gauss-Bonnet parameter (β), besides the mass (M) and rotation parameter (a). Interestingly, for each value of the parameter $Q(\beta)$, there is a critical GB parameter $\beta = \beta_E(Q = Q_E)$ that corresponds to an extremal black hole with degenerate horizons. For $\beta < \beta_E(Q < Q_E)$, it describes a non-extremal black hole with two horizons, and for $\beta > \beta_E(Q > Q_E)$, no black hole exists. The extremal value $\beta_E(Q_E)$ is also affected by the GB parameter α and the ergosphere. We also study the collision of two equal-mass particles near the horizon of this black hole and explicitly show the effect of the parameter $\beta(Q)$. The innermost stable circular orbits (ISCO) and the effective potential, which governs the motion of particles in spacetime, have been analyzed for different parameter values. The centre-of-mass energy (E_{CM}) depends on the rotation parameter a and the parameters β and Q. We investigate the E_{CM} of two colliding particles near the horizon for both extremal and non-extremal cases. It is shown that in extremal cases, when one of the colliding particles has a critical angular momentum, the E_{CM} can be

arbitrarily high, suggesting that the charged rotating in 4D Einstein-Gauss-Bonnet gravity can function as a particle accelerator. Despite the complexity of the BH solution, an exact expression for the thermodynamic quantities of black holes, such as the mass, Hawking temperature, and entropy, is derived in terms of the horizon radius. These quantities show significant deviations from the Kerr solution because of the influence of the Gauss-Bonnet parameter and electric charge.

In 4D Einstein-Gauss-Bonnet gravity, the gravitational field of a rotating charged black hole can be described using Boyer-Lindquist coordinates, resulting in the following metric:

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}(adt - (r^2 + a^2)d\phi)^2, \quad (1)$$

with the metric functions defined as:

$$\rho^2 = r^2 + a^2\cos^2\theta\,, \quad (2)$$

$$\Delta = r^2 + a^2 + \frac{r^4}{2\beta}\left(1 - \sqrt{1 + 4\beta\left(\frac{2M}{r^3} - \frac{Q^2}{r^4}\right)}\right). \quad (3)$$

We consider the motion of a time-like particle with a rest mass m in the equatorial plane $\theta = \pi/2$ where the polar velocity $\dot{\theta}$ becomes zero. The generalized momenta of the particle in the spacetime of a charged rotating BH are expressed in the form:

$$P_t = g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\,, \quad (4)$$

$$P_\phi = g_{\phi\phi}\dot{\phi} + g_{t\phi}\dot{t}\,, \quad (5)$$

where P_t and P_ϕ are the constants of motion, corresponding to the particle's energy E and the angular momentum L, respectively. For simplicity, we set the rest-mass of the particle $m = 1$. The overdot denotes differentiation with respect to the proper time. We calculated the equations of motion of a massive particle from Eqs. (4)-(5) along with the normalization condition $u_\mu u^\mu = -1$, given as below:

$$u_t = \frac{1}{r^2}\left[\frac{(a^2 + r^2)}{\Delta}T + a(L - aE)\right], \quad (6)$$

$$u_\phi = \frac{1}{r^2}\left[\frac{a}{\Delta}T + (L - aE)\right], \quad (7)$$

$$u_r = \pm\frac{\sqrt{T^2 - \Delta(r^2 + (L - aE)^2)}}{r^2}\,, \quad (8)$$

where $T = E(a^2 + r^2) - aL$ and + and - signs in Eq. (8) refer to the outgoing and incoming geodesics, respectively.

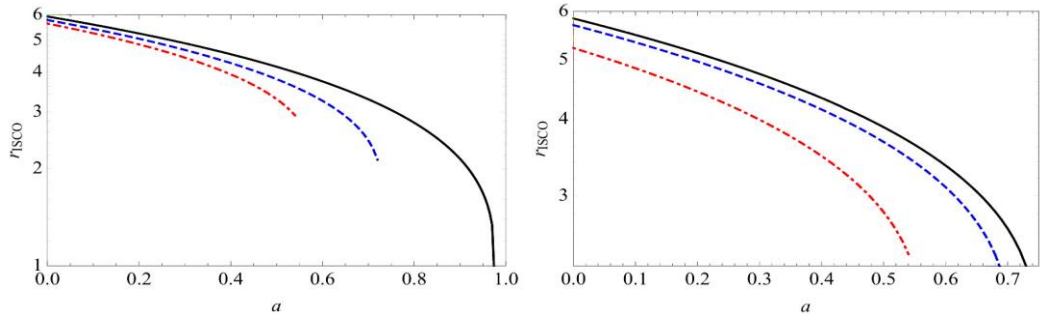

Figure 1: The figure shows the variation of the innermost stable circular orbits with respect to spin parameters for the different values of β and Q. The plot on the left-hand side is for the fixed $Q = 0.2$ and different β, namely $\beta = 0$ (black), $\beta = 0.2$ (blue) and $\beta = 0.4$ (red). Similarly, the figure on the right-hand side is for the fixed $\beta = 0.2$ and different Q, i.e. $Q = 0$ (black), $Q = 0.3$ (blue) and $Q = 0.6$ (red).

To evaluate the motion of particles, we have to understand the properties and effective potential of this BH. An effective potential for a test particle which is at rest at infinity for charged rotating BHs can be calculated with the following formula:

$$V_{\text{eff}} = -\frac{1}{2g_{rr}}\left(\frac{E^2 g_{\phi\phi} + 2EL g_{t\phi} + L^2 g_{tt}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}\right), \tag{9}$$

from Eq. (9), one can get an explicit form of effective potential easily:

$$V_{\text{eff}} = -\frac{(aL - (a^2 + r^2)E)^2 - \Delta(r^2 + (L - aE)^2)}{2r^4}. \tag{10}$$

The solution to the simultaneous equations $\dot{r} = 0;\ \ \partial_r V_{\text{eff}} = 0;\ \ \partial_r^2 V_{\text{eff}} = 0$ defines the innermost stable circular orbit r_{ISCO} of the particle. It isn't easy to find an analytic solution for r_{ISCO} here, but in Fig.1, we calculated the r_{ISCO} numerically for the different values of β and Q. Furthermore, it is evident from these figures that an increase in any parameter leads to a decrease in the radius of the innermost stable circular orbit, assuming the other parameters remain constant.

The magnitude of the particle's momentum is significant for analyzing its geodesics in curved spacetime. The motion of a test particle in a stationary and axisymmetric black hole spacetime can be described in terms of an effective radial potential V_{eff} . So, physically admissible motion requires: $V_{\text{eff}} \geq 0$.

The limiting values of the angular momentum define the range of angular momenta for which a particle released from infinity can reach the black hole horizon without encountering a turning point. These values are determined by the existence of unstable circular orbits, which act as separatrices between plunging and non-plunging trajectories.

The limiting angular momenta L_{min} and L_{max} which is depicted in tables 1 and 2 are obtained by imposing the circular-orbit conditions:

$$V_{\text{eff}} = 0 \text{ and } \frac{dV_{\text{eff}}}{dr} = 0\,. \tag{11}$$

Solutions of these equations correspond to unstable circular motion. The associated angular momenta define the boundaries of the allowed interval: $L_{min} < L < L_{max}$.

For angular momenta outside this interval, the effective potential develops a turning point outside the horizon, preventing the particle from reaching the black hole. Therefore, the limiting angular momenta represent a global kinematical condition ensuring horizon accessibility.

The critical angular momentum is defined by the behavior of the particle motion in the immediate vicinity of the event horizon. It is obtained from the condition that the radial velocity vanishes exactly at the horizon:

$$V_{\text{eff}}(r_h) = 0. \tag{12}$$

Thus, using Eq.(8), one may determine the critical value of the angular momentum. This means that when $r \to r_h^E$ we can get:

$$L_{cr} = \frac{(a^2 + (r_H^E)^2)E}{a}. \tag{13}$$

This fine-tuned trajectory plays a central role in the Banãdos–Silk–West (BSW) mechanism, as it leads to an unbounded center-of-mass energy in particle collisions occurring arbitrarily close to the horizon.

Although both the limiting and critical angular momenta are obtained from conditions involving $V_{\text{eff}} = 0$, they have fundamentally different physical meanings. The limiting angular momenta are determined by circular-orbit conditions at radii outside the horizon and impose a global constraint on particle motion from infinity. In contrast, the

critical angular momentum is determined by a local horizon condition and does not by itself guarantee that the particle can reach the horizon.

For the BSW effect to be physically realizable, the critical angular momentum must lie within the allowed range: $L_{cr} \in [L_{min}, L_{max}]$.

If L_{cr} exceeds the maximum allowed angular momentum, a particle with $L = L_{cr}$ encounters a turning point outside the horizon and cannot reach it, despite satisfying the horizon condition. In such cases, the formal divergence of the center-of-mass energy is kinematically forbidden.

The angular momentum's limiting values, along with the BH's corresponding spin, β, Q and radius of the event horizon for the extremal case and non-extremal case, are presented in Tables 1 and 2.

Table 1: The limiting values of angular momentum for different extremal cases.

β	Q	a_E	r_H^E	L_{min}	L_{max}	L_{cr}
0.1	0.1	0.852804	1.14881	-4.71391	2.35980	2.40035
0.1	0.2	0.838440	1.14423	-4.69208	2.36291	2.40001
0.1	0.3	0.813929	1.13654	-4.65479	2.36996	2.40094
0.3	0.3	0.622437	1.18055	-4.49305	2.83298	2.86155
0.4	0.4	0.501169	1.15498	-4.35397	3.00904	3.16288
0.5	0.5	0.358248	1.10509	-4.17220	3.19564	3.76714

Table 2: The limiting values of angular momentum for different non-extremal cases.

β	Q	a	r_H^+	r_H^-	L_{min}	L_{max}	L_{cr}
0.1	0.3	0.4	1.79992	0.40315	-4.32006	3.45952	8.49932
0.1	0.4	0.4	1.75454	0.42731	-4.28909	3.40803	8.09605
0.2	0.3	0.4	1.72661	0.51472	-4.31206	3.42612	7.85292
0.2	0.4	0.4	1.67569	0.54524	-4.28079	3.37120	7.41985
0.3	0.3	0.4	1.64295	0.61796	-4.30389	3.38812	7.14819
0.3	0.4	0.4	1.58352	0.65778	-4.27231	3.32845	6.66883

We have investigated the particle motion, BSW mechanism and thermodynamic properties of charged rotating black holes in 4D Einstein-Gauss-Bonnet gravity are explored, and we highlight several results obtained.

For smaller values of parameters, this black hole has two horizons, and with the increase of these parameters, these horizons become closer. Eventually, for the specific values (these are called extreme values) of these parameters, horizons collide and there will be one horizon and a black hole called extremal black hole. For the greater values than extremal ones, there is no black hole.

BSW demonstrated that the E_{CM} of two colliding particles can reach arbitrarily high levels in the case of extremal Kerr black holes. By extending the BSW analysis to the charged rotating black hole in 4D Einstein-Gauss-Bonnet gravity, it is impossible to achieve arbitrary high E_{CM} when the collision occurs near the horizon of an extremal charged rotating black holes in 4D Einstein-Gauss-Bonnet gravity due to the exceeding the values of the critical angular momenta than maximum allowed value of angular momentum. Also, it is impossible to achieve infinite amount of E_{CM} in non-extremal BH case.

Hawking temperature of this black hole is derived and limiting cases of it are discussed. Despite the intricacy of the black hole solution, a precise formulation for the thermodynamic properties of black holes, including parameters like mass, Hawking temperature, and entropy, is obtained with the horizon radius. These quantities exhibit notable deviations from the Kerr solution due to the influence of GB parameters and electric charge.

In the second chapter, entitled "Particle Acceleration and Thermodynamics of the rotating Simpson-Visser black hole" we have considered a time-like geodesic in the background of rotating Simpson-Visser black hole (BH) to examine structure of the horizon and ergosphere. The innermost stable circular orbits (ISCO) and the effective potential, which controls the particle's motion in spacetime, have been studied for different values of parameters in SV BH. The center-of-mass (CM) energy of two colliding particles near the horizon has been investigated for both extremal and non-extremal cases. Furthermore, thermodynamic properties of the SV black hole have also been investigated in detail.

The gravitational field of a rotating Simpson-Visser compact object in Boyer-Lindquist coordinates can be expressed through the following line element:

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}(adt - (r^2 + a^2)d\phi)^2, \quad (14)$$

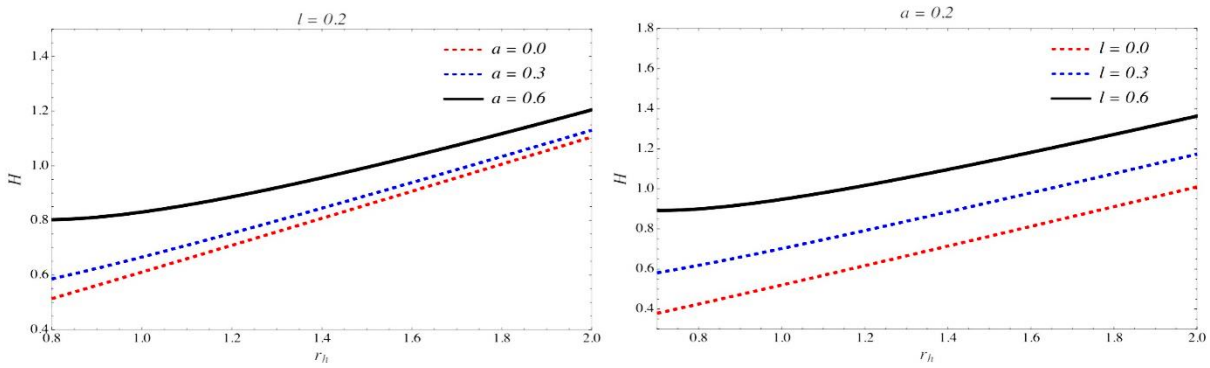

Figure 2: Plot shows the variation of enthalpy with respect to horizon radius for different values of a and l.

with the metric functions defined as:

$$\rho^2 = r^2 + a^2 \cos^2\theta\,, \tag{15}$$

$$\Delta = r^2 + a^2 - 2Mre^{-l/r}, \tag{16}$$

where a and l can be referred to as spin and deviation parameters of the SVBH, respectively. This metric is derived from the regularization procedure in the Kerr metric like did in the Schwardchild metric before, where $M \to M(r) = Me^{-l/r}$ modification is performed. Here, l can be viewed as quantifying the deviation from Kerr. The spacetime around this black hole behaves as one around typical rotating and the limit $l \to 0$ corresponds to Kerr BH.

We analyze the thermodynamics of this BH in SV BH. The general form of the metric can be written as:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi. \tag{17}$$

The angular velocity of a test particle at the event horizon, with vanishing angular momentum, can be calculated using the formula $\Omega_H = -g_{t\phi}/g_{\phi\phi}$ for the rotating Kerr-like BH:

$$\Omega_H = \frac{a}{r_h^2 + a^2}. \tag{18}$$

We will use the expression (18) for our further calculations. Now we turn to calculate the thermodynamical quantities of SV BH. The BH mass (M, which is equal to enthalpy H) can be derived by making Eq. (16) equals to zero. As a result one may obtain:

$$H = \frac{(a^2 + r_h^2)e^{\frac{l}{r_h}}}{2r_h}. \tag{19}$$

Figure 3 demonstrates the enthalpy as a function of r_h. One may see that an increase in the parameters a and l corresponds to a greater mass (enthalpy) for fixed r_h.

Within this chapter the radius of ISCO is also discussed in SV spacetime with its parameters l and a. An increase in these parameters makes the ISCO radius smaller.

Using massive particle orbits, we checked energy extraction from BH for the extremal and non-extremal cases through the BSW effect. The center of mass energy can be arbitrarily high for the extremal SV BH, which is not true for the non-extremal SV BH.

The properties of thermodynamics are studied for different cases: enthalpy, hawking temperature, entropy, and Gibbs free energy. To be more precise, an increase of BH's parameters l and a also causes the Gibbs free energy to increase. The opposite is true for the temperature of this type of BH.

The third chapter, entitled "Thermodynamical Analysis of Charged Rotating Black Hole Surrounded by Perfect Fluid Dark Matter" is devoted to studying thermodynamics and particle acceleration around charged rotating BHs surrounded by perfect fluid dark matter. We discuss BH horizons, ergosphere and inner stable circular orbits in the framework of perfect fluid dark matter. It is observed that perfect fluid dark matter effects Hawking's temperature, Gibbs free free energy, and other physical quantities related to thermodynamics. Some different aspects of thermodynamics are discussed within the scope of different values of the involved parameters. Further, we discuss the thermal fluctuations of considered BH solutions, which refer to the random motion of particles around a BH due to temperature fluctuations. Specifically, thermal fluctuations are affecting the entropy and Hawking radiation of a BH, which are crucial for understanding the behavior of physical quantities. It is found that corrected thermodynamical quantities of charged rotating perfect fluid dark matter BHs are smaller than the charged rotating BH without perfect fluid dark matter. We emphasize that the assumptions made in the literature and the thermodynamics of the previous BH solutions are well consistent with our results. The charged rotating BH in perfect fluid dark matter is described by line element:

$$ds^2 = -\frac{1}{\rho^2}(\Delta - a^2\sin^2\theta)dt^2 - \frac{2a\sin^2\theta}{\rho^2}\left[2Mr - Q^2 - \gamma r\ln\left(\frac{r}{\gamma}\right)\right]dtd\phi + \rho^2 d\theta^2 + \sin^2\theta\left[r^2 + a^2 + \frac{a^2\sin^2\theta}{\rho^2}\left(2Mr - Q^2 - \gamma\ln\left(\frac{r}{\gamma}\right)\right)\right]d\phi^2 + \frac{\rho^2}{\Delta}dr^2, \quad (20)$$

here Δ and ρ are defined as:

$$\Delta = r^2 + a^2 - 2Mr + Q^2 + \gamma r\ln\left(\frac{r}{\gamma}\right),$$
$$\rho^2 = r^2 + a^2\cos^2\theta, \quad (21)$$

where M is the mass of the BH, a is the rotational parameter, Q is the charge of the BH and γ is the PFDM parameter.

We discuss the thermodynamics of the charged rotating BH in PFDM. The event horizon of BH for the given metric is obtained by condition "$\Delta = 0$":

$$r_h^2 + a^2 - 2Mr + Q^2 + \gamma r_h \ln\frac{r_h}{\gamma} = 0. \tag{22}$$

It can be easily noticed that it has two horizons: the inner one $r_$and the outer one r_+, which depend on the PFDM parameter γ. Using the condition for the BH event horizon (22) we can derive the BH mass M, which is equivalent to the enthalpy H :

$$H = \frac{1}{2}\left[r_h + \frac{(a^2 + Q^2)}{r_h} + \gamma \ln\frac{r_h}{\gamma}\right]. \tag{23}$$

The graphical analysis of the enthalpy relation (23) is given in Fig. (3). It is clear from the plots that enthalpy decreases in the beginning and the rise of γ does not matter for it. However, it starts to increase from $r_h = 0.5$ when γ parameter increases. Whereas, when the spin parameter a and charge Q rise the enthalpy differs close to the BH and goes up becoming indifferent to parameter changes far away from the BH.

There are several observational parameters to study properties of BH but there is no exact data in which only charge or rotational parameters of BHs are discussed. Here, we have tried to theoretically analyze the charged rotating BH in the presence of PFDM and investigated the following:

First, we have discussed the borders for horizons and ergosphere region of this BH. It is necessary to mention that we have calculated an extremal BH for a

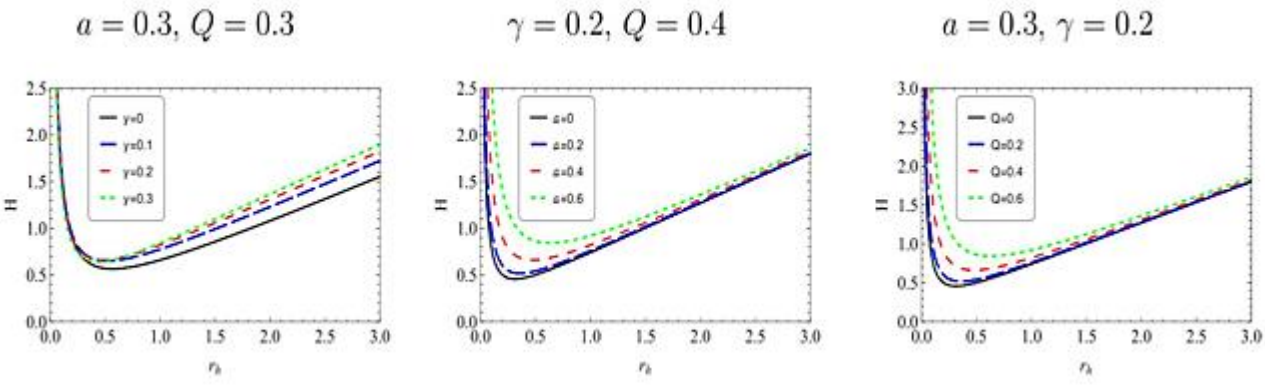

Figure 3: Enthalpy variation with horizon radius for different parameters

certain critical value of the rotational parameter $a = a_E$, where the two radii of the horizons have coincided. We have studied the influence of PFDM parameters on

horizons and ergosphere region for two cases: an extremal and non-extremal BH. We take the behavior of the shape of these regions graphically. We find that the PFDM parameter γ shortens the border for both the horizon and ergosphere regions, almost merging into one region. The ISCO radius has been observed to decrease within the scope of all parameters (γ, Q and a).

We have studied the thermodynamic characteristics of this BH in PFDM and have discovered that the increase in γ results in an increase in the enthalpy H and the Gibbs free energy G. In contrast, the temperature of the BH decreases with growth γ. The effect of the PFDM parameter γ on the ISCO radius has also been discussed in the work, namely, with increasing γ the ISCO radius decreases. We have studied the collision of two identical massive particles near the horizon of the BH and studied the behavior of the CM energy for an extremal and non-extremal case.

The effect of second-order, simple logarithmic, and higher-order correction terms on the Helmholtz free energy, internal energy, enthalpy, and Gibbs free energy of the charged rotating BH with and without PFDM have been observed. It is worth noting that the specific heat indicates a stable structure for BHs of a certain size but an unstable design for BHs that are significantly larger.

Conclusions

The following conclusions were presented based on research carried out on the topic of "Thermodynamic and Energetic Processes Around Compact Objects in Modified Theories of Gravity" for the Doctor of Philosophy (PhD) dissertation:

- For the first time, it has been demonstrated that the horizon structure, ergosphere geometry and extremality conditions in modified gravity spacetimes (4D Einstein–Gauss–Bonnet, Simpson–Visser, and perfect fluid dark matter models) depend systematically on additional gravitational parameters—revealing a direct link between "black hole hair" and observable spacetime features.
- For the first time, it has been shown that the innermost stable circular orbit (ISCO) undergoes parameter-dependent shifts in these frameworks, providing a new diagnostic tool to test modified gravity using accretion disk observations.
- For the first time, it has been derived that the Banados–Silk–West (BSW) particle acceleration mechanism can be significantly enhanced in modified gravity, with explicit critical angular momentum conditions obtained for black holes surrounded by dark matter or nonlinear electromagnetic fields.
- For the first time, it has been found that logarithmic and higher-order quantum corrections to black hole entropy led to new thermodynamic phase transitions and modified stability criteria, especially in the presence of perfect fluid dark matter.

НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025.FM/T.192.01 ПО ПРИСУЖДЕНИЮ УЧЁНОЙ СТЕПЕНИ ПРИ ИНСТИТУТЕ ПЕРЕДОВЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА “НОВЫЙ УЗБЕКИСТАН”

ТАШКЕНТСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ ИСЛАМА КАРИМОВА

УНИВЕРСИТЕТ “НОВЫЙ УЗБЕКИСТАН”

РАХИМОВА ГУЛЗОДАХОН ИСРОИЛЖОН КИЗИ

ТЕРМОДИНАМИЧЕСКИЕ И ЭНЕРГЕТИЧЕСКИЕ ПРОЦЕССЫ ВОКРУГ КОМПАКТНЫХ ОБЪЕКТОВ В МОДИФИЦИРОВАННЫХ ТЕОРИЯХ ГРАВИТАЦИИ

01.03.01 — Астрономия
01.04.02 — Теоретическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ

НА СОИСКАНИЕ СТЕПЕНИ ДОКТОРА ФИЛОСОФИИ (PHD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент — 2026

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей Аттестационной Комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан под номером B2025.3.PhD/FM1360.

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ВВЕДЕНИЕ

Актуальность и востребованность темы диссертации.

Тема весьма актуальна в связи с растущим интересом к альтернативным теориям гравитации, которые направлены на решение фундаментальных вопросов общей теории относительности (ОТО), таких как сингулярности и несовместимость с квантовой механикой. Недавние достижения в астрофизических наблюдениях, включая обнаружение гравитационных волн и получение изображения черной дыры с помощью телескопа Event Horizon, предоставили беспрецедентные возможности для проверки этих теорий.

Спрос на такие исследования обусловлен необходимостью понимания темной компоненты Вселенной, таких как, темная материя и темная энергия, которые остаются необъяснимыми в рамках стандартной ОТО. Модифицированные теории гравитации предлагают потенциальные решения, что делает эту диссертацию актуальной для современной астрофизики и космологии. В работе исследуются такие ключевые явления, как ускорение частиц и термодинамические свойства, которые имеют решающее значение для изучения природы гравитации в экстремальных условиях. Например, механизм Банадоса-Силка-Уэста (BSW), который предполагает, что черные дыры могут действовать как ускорители частиц, имеет значение для астрофизики высоких энергий и образование космических лучей.

Более того, диссертация соответствует стратегическим научным приоритетам Узбекистана и более широкому международному сообществу. Данная диссертационная работа соответствует задачам следующих государственных нормативных документов: Указ Президента Республики Узбекистан № УП-4947 “О стратегии действий по дальнейшему развитию Республики Узбекистан” от 07 февраля 2017 года, Указ Президента Республики Узбекистан № ПП-2789 “О мерах по дальнейшему совершенствованию деятельности академии наук, организации, управления и финансирование научно-исследовательской деятельности” с 18.02.2017.

Актуальность исследования для приоритетных направлений развития науки и технологий Республики Узбекистан. Диссертационное исследование выполнено в соответствии с приоритетными направлениями развития науки и технологий в Республике Узбекистан: II. “Энергетика, энерго- и ресурсосбережение”.

Степень изученности проблемы

Проблема понимания термодинамических и энергетических процессов вокруг компактных объектов в модифицированных теориях гравитации была широко изучена, однако остаются значительные пробелы. Классические работы

заложили основу для изучения теней черных дыр и геодезического движения (Bardeen, Luminet, сотрудничество с телескопом Event Horizon Telescope, Абдужаббаров, Ахмедов, Bambi), в то время как более поздние исследования ввели концепцию черных дыр как ускорителей частиц (Banados, Silk, West, Zaslavskii, Wei, Liu, Атамуратов). Модифицированные теории гравитации, такие как Эйнштейна-Гаусса-Бонне (EGB) и скалярно-тензорные модели, были исследованы в таких контекстах, как структура горизонта, фотонные сферы и динамика частиц. Однако большинство существующих исследований сосредоточено на изолированных аспектах, таких как термодинамика (Bekenstein, Hawking, Glavan, Lin, Ghosh, Ahmedov, Jamil), без интеграции этих явлений в единую структуру. Кроме того, в то время как механизм BSW хорошо изучен в пространстве-времени Керра и Керра-Ньюмана, его значение в модифицированной гравитации, особенно в таких дополнительных областях, как темная материя или нелинейная электродинамика, остается недостаточно изученным.

Недавние достижения в наблюдательной астрономии, такие как получение изображений M87 $*$ и SgrA*, усилили потребность во всеобъемлющих моделях, учитывающих отклонения от ОТО. Появились исследования вращающихся черных дыр в условиях 4D-гравитации EGB, метрики Симпсона-Виссера и идеальной текучей темной материи (PFDM) (Simpson, Visser, Kiselev, Abdujabbarov, Atamurotov, Ghosh), однако систематические сравнения с данными наблюдений по-прежнему ограничены. Термодинамические свойства, включая излучение Хокинга и фазовые переходы, были проанализированы в упрощенных сценариях, но влияние тепловых флуктуаций и поправок более высокого порядка до конца не изучено. Диссертация устраняет эти пробелы, объединяя динамику частиц и термодинамику в модифицированной гравитации, обеспечивая более целостное понимание компактных объектов. Используя как аналитические, так и численные методы, работа направлена на то, чтобы связать теоретические предсказания с наблюдаемыми явлениями, способствуя постоянному совершенствованию альтернативных теорий гравитации.

Связь темы диссертации с научными исследованиями высших учебных заведений/научно-исследовательских учреждений, в которых выполнялась диссертация. Диссертация выполнена в рамках научных проектов, финансируемых Министерством инновационного развития: F-FA-2021-510 “Исследования ядерного вещества нейтронных звезд в условиях измененной гравитации” (2021-2026).

Целью исследования является комплексный анализ астрофизических процессов вокруг вращающихся черных дыр в модифицированных теориях гравитации.

Задачи диссертации:

- проанализировать структуру пространства-времени вращающихся и заряженных черных дыр в модифицированных теориях гравитации (модели Эйнштейна-Гаусса-Бонне, Симпсона-Виссера и PFDM), включая конфигурации горизонта, условия экстремальности и свойства эргосферы, и определить, как эти характеристики зависят от дополнительных гравитационных параметров;
- вывести и исследовать уравнения движения для пробных частиц вращающихся черных дыр в модифицированной гравитации, используя подход эффективного потенциала;
- изучить механизм ускорения частиц вблизи черных дыр как в экстремальном, так и в неэкстремальном режимах;
- рассмотреть внутренние устойчивые круговые орбиты (ISCO) и эффективные потенциалы для пробных частиц на фоне модифицированной гравитации;
- вывести термодинамические величины вращающихся черных дыр в модифицированной гравитации, такие как температура Хокинга, энтропия и свободная энергия Гиббса;
- изучить термодинамическую стабильность и фазовое поведение, включая эффекты тепловых флуктуаций и поправок энтропии высшего порядка.

Объектами исследования являются вращающиеся черные дыры с модифицированной гравитацией, заряженные черные дыры, структуры горизонта и эргосферы, динамика частиц и термодинамические системы.

Предметами исследования являются механизмы ускорения частиц, эффективные потенциалы и стабильность, термодинамические свойства, эффекты темной материи.

Методами исследования являются методы вычислительной математики, методы теоретической астрофизики, современные методы математической физики, аналитические и численные методы расчета дифференциальных уравнений для поля и движения частиц. Метод метрического анализа и аналитических расчетов был использован для получения решений для черных дыр в условиях модифицированной гравитации и расчета структур горизонта и эргосферы с помощью анализа метрических компонентов. Для численного решения нелинейных уравнений поля для черных дыр, модифицированных заряженной/темной материей, были использованы методы численного моделирования. Анализ стабильности был использован для оценки температуры Хокинга, энтропии и свободной энергии Гиббса с помощью поверхностной гравитации и евклидовых методов воздействия.

Научная новизна исследования заключается в следующем:

- Первое объединенное исследование динамики частиц и термодинамики позволило нам объединить ускорение частиц и термодинамическую стабильность в условиях модифицированной гравитации (например, 4D EGB, Симпсона-Виссера) в рамках единой концепции, выявив взаимосвязи, ранее изучавшиеся только изолированно.
- Получены выводы о наблюдаемых сигнатурах (например, сдвигах ISCO) для черных дыр в PFDM, обеспечивающие проверяемые отклонения от общей теории относительности.
- Теоретические результаты учитывают тепловые флуктуации и поправки на энтропию более высокого порядка в модифицированных гравитационных черных дырах, устраняя пробелы в понимании квантовых/статистических эффектов излучения Хокинга и фазовых переходов.

Практические результаты исследования заключаются в следующем:

- Результаты, касающиеся процессов ускорения частиц, количественно определяют энергетические пределы для столкновений частиц вблизи горизонта в измененном пространстве-времени, что имеет значение для астрофизики высоких энергий (например, космических лучей, струй AGN).
- Анализ критериев стабильности экзотических компактных объектов позволяет определить диапазоны параметров, в которых черные дыры в условиях измененной гравитации остаются стабильными, что является основой для теоретических моделей и численного моделирования теории относительности.
- Полученные результаты обеспечивают прямую количественную связь между вращением вещества в аккреционном диске и фундаментальными параметрами черной дыры, предлагая новый метод использования астрофизических наблюдений для проверки существования “волосатых” черных дыр за пределами стандартной модели.

Достоверность результатов исследования обеспечивается применением современных проверенных методов математической физики, вычислительной математики и релятивистской астрофизики. Результаты были получены строго в рамках математического аппарата общей теории относительности и теоретической физики. Также используются современные численные и аналитические методы расчета, а результаты сравниваются с имеющимися данными наблюдений и результатами других авторов. Структурированные выводы диссертации соответствуют основным правилам астрофизики компактных объектов. Все полученные решения строго соответствуют

математическим рамкам модифицированных теорий гравитации (например, 4D EGB и PFDM). Расчеты проверяются с использованием общепринятых методов, таких как формализм Гамильтона-Якоби для геодезических.

Научная и практическая значимость результатов исследования заключаются в следующем:

- Результаты диссертаций объединяют динамику частиц и термодинамику компактных объектов в трехмерных системах Эйнштейна-Гаусса-Бонне, Симпсона-Виссера в единый самосогласованный формализм, выявляя ранее неисследованные взаимосвязи между этими явлениями.
- Исследование, представленное в диссертации, распространяет механизм ускорения частиц Банадоса-Силка-Веста на вращающиеся черные дыры в условиях измененной гравитации, выводя критические условия углового момента и пределы энергии столкновения для пространственновременных систем с дополнительными полями (скалярными и PFDM).
- Анализ термодинамики с квантовыми поправками более высокого порядка позволяет разработать всеобъемлющее термодинамическое описание, включающее логарифмические поправки и тепловые флуктуации для черных дыр с измененной гравитацией, разрешающее неоднозначности в их стабильности и поведении при фазовых переходах за пределами полуклассических приближений.

Применение результатов исследования
Результаты изучения термодинамических и энергетических процесс воуруг компактных объектов в модифицированных теориях гравитации были применены следующим образом: Результаты и методы теоретического исследования, опубликованные в научной статье “G. Rakhimova, F. Atamurotov, F. Javed, A. Abdujabbarov, G. Mustafa, Thermodynamical analysis of charged rotating black hole surrounded by perfect fluid dark matter, Nuclear Physics B, V. 996, 116363 (2023)”, и представленные в докторской диссертации Г. Рахимовой были использованы в в рамках программ, поддерживаемых Университетом Фудань (Письмо от Проф. Cosimo Bambi)

Апробация результатов исследования
Результаты диссертации были обсуждены на 4 республиканских конференциях.

Публикация результатов исследования
По теме диссертации опубликовано 12 научных работ, в том числе 6 научных статей в международных научных журналах, рекомендованных Высшей

Аттестационной Комиссией Республики Узбекистан для публикации основных научных результатов PhD-диссертаций.

Объем и структура диссертации
Диссертация изложена на 108 страницах и состоит из введения, трёх глав, заключения и списка литературы.

Выводы

По результатам исследований, проведенных по теме “Термодинамические и энергетические процессы вокруг компактных объектов в модифицированных теориях гравитации” для диссертации на соискание ученой степени доктора философии (PhD), были представлены следующие выводы:

- Впервые продемонстрировано, что структура горизонта, геометрия эргосферы и условия экстремальности в пространстве-времени модифицированной гравитации (4D гравитация Эйнштейна–Гаусса–Бонне, модели Симпсона–Виссера и совершенной жидкой тёмной материи) систематически зависят от дополнительных гравитационных параметров, что выявляет прямую связь между “волосами” чёрной дыры и наблюдаемыми свойствами пространства-времени.
- Впервые показано, что радиус внутренней устойчивой круговой орбиты (ISCO) испытывает параметрически обусловленные смещения в этих теориях, что предоставляет новый диагностический инструмент для проверки модифицированной гравитации на основе наблюдений аккреционных дисков.
- Впервые получено, что механизм ускорения частиц Банадоса–Силка–Уэста (BSW) может быть существенно усилен в теориях модифицированной гравитации; при этом явно выведены условия на критический угловой момент для чёрных дыр, окружённых тёмной материей или нелинейными электромагнитными полями.
- Впервые установлено, что логарифмические и более высокие квантовые поправки к энтропии чёрных дыр приводят к возникновению новых термодинамических фазовых переходов и изменённых критериев устойчивости, особенно в присутствии совершенной жидкой тёмной материи.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo‘lim (part I; I часть)

1. Rakhimova G., Atamurotov F., Javed F., Abdujabbarov A., Mustafa G. Thermodynamical analysis of charged rotating black hole surrounded by perfect fluid dark matter // Nuclear physics B 2023, Volume 996, article id.116363 (2023) (№1 Web of Science, IF: 2.8).
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3. Nosirov A., Atamurotov F., Rakhimova G., Abdujabbarov A., Ghosh S.G Particle acceleration near a rotating charged black hole in 4D Einstein-Gauss-Bonnet gravity // Nuclear Physics Section B, 2024, Volume 1005, article id. 116583 (№1 Web of Science, IF: 2.8)
4. Rakhimova G., Abdulkhamidov F., Atamurotov F., Abdujabbarov A., Mustafa G. Spinning particle motion around charged black hole from T-duality // Chinese Journal of Physics, 2025, Vol. 94, p.66-78. (№1 Web of Science, IF: 4.6)
5. Rakhimova G., Atamurotov F., Juraeva N., Abdujabbarov A., Mustafa G., Spinning particle motion around charged decoupled hairy black hole // Physics of the Dark Universe, 2025 Volume. 47, article id. 101721 (№1 Web of Science, IF: 6.4)
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II bo‘lim (II часть; II part)

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2. Rakhimova G., Ro‘zmetov Q., Abdujabbarov A., Thermodynamics of charged rotating black hole surrounded by perfect fluid dark matter// “Fizika fanining rivojida iste‘dodli yoshlarning o‘rni” Respublika ilmiy-amaliy konferensiya (RIAK-XVI-2023) materiallari to‘plami: O‘zMU, 28-29-aprel, 2024-yil. Toshkent.57-60 betlar.
3. Rakhimova G., Abduvokhidov A., Davletova P., Thermodynamics of the rotating Simpson–Visser black hole// “Fizika fanining rivojida iste‘dodli yoshlarning o‘rni” Respublika ilmiy-amaliy konferensiya (RIAK-XVII-2024) materiallari to‘plami: O‘zMU,5-6-aprel,2024-yil. Toshkent.113-115 betlar.
4. Rakhimova G., Ibrokhimov T., Khayitboyev F., Atamurotov F., Particle acceleration of the rotating Simpson–Visser black hole // “Oliy harbiy ta‘lim muassasalarida tabiiy va texnik fanlarni o‘qitishda zamonaviy yondashuvlar va innovatsiyalar” mavzusida Respublika ilmiy-uslubiy anjuman. Toshkent. 2024-yil 29-mart.109-111-betlar.