V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

SHERALIYEVA SURAYYO ABDIKODIR QIZI

FILIFORM NILRADIKALLI YECHILUVCHAN LI VA LEYBNITS ALGEBRALARINING MARKAZIY HAMDA ABEL KENGAYTMALARI

01.01.06 – Algebra

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2025

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

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Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.2.PhD/FM1304 raqam bilan ro‘yxatga olingan.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Bugungi kunga kelib jahon miqyosida olib borilayotgan koʻplab ilmiy-amaliy tadqiqotlarning aksariyati algebraik tuzilmalarni oʻrganish, ularning oʻzaro bogʻlanishlari va umumiy xossalarini aniqlash masalalariga keltiriladi. Xususan, turli algebraik tizimlarning strukturasi, klassifikatsiyasi va ular orasidagi munosabatlarni tadqiq etish nazariy jihatdan katta ahamiyatga ega boʻlib, bu yoʻnalishlar matematikaning boshqa sohalari bilan ham uzviy bogʻlangan. Shu nuqtai nazardan, algebraik tuzilmalarni kengaytirish gʻoyalari mavjud nazariyalarni chuqurlashtirish, yangi algebraik modellarni yaratish hamda murakkab tizimlarning ichki tuzilishini yaxshiroq tushunish imkonini beradi. Bunday tadqiqotlar algebraik yondashuvlarning umumiy imkoniyatlarini kengaytirishga xizmat qiladi.

Algebralarning kengaytmalari va ularni tadqiq etish usullari, ayniqsa, yechiluvchan algebraik tizimlar doirasida, muhim ilmiy ahamiyat kasb etadi. Kengaytmalar yordamida algebraik tuzilmalarning yangi xossalari aniqlanadi, ularning tasniflash masalalariga yangi nuqtai nazardan yondashish imkoniyati paydo boʻladi. Shu bilan birga, bunday yondashuvlar algebraik tuzilmalar nazariyasining rivojlanishiga, ularning boshqa matematik sohalardagi qoʻllanilish imkoniyatlarini kengaytirishga xizmat qiladi. Shuningdek, ular algebraik tuzilmalarni chuqurroq oʻrganish va zamonaviy algebra nazariyasini boyitishda muhum oʻrin tutadi.

Soʻnggi yillarda mamlakatimizda ilmiy va amaliy qoʻllanilishiga ega boʻlgan fundamental fanlar - matematika, fizika, geologiya hamda biologiya fanlariga e'tibor kuchaymoqda. Jumladan, oxirgi yillarda algebraik obyektlar matematikaning turli sohalarida, ayniqsa geometriya, topologiya va fizikada oʻzining yangi tatbiqlarini topishda davom etmoqda. Bugungi kunga kelib turli xil koʻphilliklardagi nilpotent va yechiluvchan algebralarni, jumladan Li va Leybnits algebralarini tasniflash va kichik oʻlchamli noassotsiativ algebralarning algebraik hamda geometrik tasnifini aniqlash boʻyicha salmoqli natijalarga erishildi. "Algebra va funksional analiz" ustuvor yoʻnalishlari boʻyicha xalqaro standartlar darajasida tadqiqot olib borish matematika fanining asosiy vazifasi va faoliyat yoʻnalishi etib belgilandi.[1] Qaror ijrosini ta'minlashda ilmiy natijalarni tegishli ilm fanning turdosh sohasida qoʻllash uchun chekli oʻlchamli noassotsiativ algebralar nazariyasini ishlab chiqish, ularni tasniflash belgilangan vazifaning bajarilishini ta'minlashda muhim ahamiyatga ega.

Oʻzbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son "Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha harakatlar strategiyasi toʻgʻrisida"gi, 2017-yil 17-fevraldagi PQ-2789-son "Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari toʻgʻrisida"gi, 2017-yil 20-apreldagi PQ-2909-

[1] Oʻzbekiston Respublikasi Vazirlar mahkamasining 2017 yil 18 maydagi "Oʻzbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish toʻgʻrisida"gi 292-sonli qarori.

son “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”gi qarorlari, hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot O‘zbekiston Respublika fan va texnologiyalar rivojlanishining IV «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Li algebralari nazariyasiga XIX asr oxirida norveg matematigi Sofus Li tomonidan asos solingan bo‘lib, u inifinitesmal transformatsiyalarni o‘rganish maqsadida fanga kiritilgan. Keyinchalik V.Killing va E.Kartan tomonidan Li algebralarning tuzilishi, tasnifi hamda yechiluvchan va nilpotent algebralar sinflari bo ‘yicha fundamental natijalar olindi. Shu asosda Li algebralarning strukturaviy nazariyasi matematik fizika, geometriya va analiz sohalarining asosiy vositasiga aylandi. XX asrning o‘rtalaridan boshlab Li algebralarning nilpotent va yechiluvchan sinflarini tahlil qilish hamda ularning kengaytmalarini o‘rganish yo‘nalishida faol tadqiqotlar olib borila boshlandi. A.I.Malsev, N.Jakobson, I.Kaplansky, A.Kostrikin, M.Vern, D.Panyushev va boshqa olimlar tomonidan nilpotent va yechiluvchan algebralarning strukturaviy xossalarini, shuningdek, ularning markaziy kengaytmalari va deformatsiyalari o‘rganildi.

Shelbred va Sandlar tomonidan nilpotent Li algebralari uchun ishlab chiqilgan markaziy kengaytma usuli bir qator nilpotent algebralarning tasnifini olish imkonini berdi. Mazkur usul berilgan ko‘phillikdagi kichik o‘lchamli algebralardan foydalanib, ulardan katta o‘lchamli algebralarni tasniflash imkonini beradi. Markaziy kengaytma usuli yordamida nilpotent besh o‘lchamli Yordan algebralari, besh o‘lchamli kommutativ algebralar, olti o‘lchamli Malsev algebralari, olti o‘lchamli binar Li algebralari V.De-Graf, A.Xegazi, X.Abdelvaxab, A.Kalderon, A.Martin, I.Kaygorodov va boshqalarning ishlarida tasniflangan.

Yechiluvchan Li algebralar chekli o‘lchamli Li algebralar nazariyasida muhim rol o‘ynagani sababli, ularni tasniflash uchun turli metodlar ishlab chiqilgan. Shunday samarali usullardan biri Muborakjanov tomonidan taklif etilgan bo‘lib, u yechiluvchan Li algebralarni ularning nilradikallari va nilradikalning nil-erkli differensiallashlari orqali o‘rganish metodikasini ishlab chiqqan. Muborakjanovning natijalari asosida ma’lum nilradikallarga ega bo‘lgan - masalan, abel, filiform, kvazi-filiform va boshqa turdagi yechiluvchan Li algebralarning to‘liq tasnifi X.M.Ankochea Bermudes, R.Kampoamor-Stursberg, L.Garsiya Vergnoll, L.Snobl, D.Karashek J.S.Ndogmo, S.Trembley va P.Vinternitzlarning ilmiy ishlarida keltirilgan. Yechiluvchan Li algebralarning markaziy kengaytmasini aniqlash yordamida markazi nolga teng bo‘lgan algebralarni to‘liq olish imkoni yo‘qligi sababli Sand tomonidan yechiluvchan Li algebralar uchun markaziy kengaytma metodi umumlashtirilgan.

Jean-Louis Loday tomonidan kiritilgan Leybnits algebralari Li algebralarning tabiiy umumlashmasi sifatida algebraik strukturalar nazariyasiga yangi yo‘nalish olib kirdi. Leybnits algebralarda ko‘paytma amali antisimmetrik emasligi sababli, ularning kogomologiya va kengaytma nazariyasi Li algebralarnikidan sezilarli farq qiladi. Shu boisdan Leybnits algebralarning markaziy kengaytmalari, deformatsiyalari va izomorfizm sinflarini o‘rganish alohida ilmiy muammoga aylandi. Bu borada J.-L.Loday, T.Pirashvili, D.Barns, M.Ladra, J.Kasas, A.S.Djumadildayev, Sh. A. Ayupov, I. S. Raximov, B. A. Omirov, A. X. Xudoyberdiyev va boshqa ko‘plab olimlarning ishlari muhim o‘rin tutadi. Ular tomonidan Leybnits algebralarning strukturaviy invariantlarini, markaziy kengaytmalari, deformatsiyalari, kogomologiyasi va degeneratsiyalari tahlil qilingan. Shunungdek, Leybnits algebralari uchun Abel kengaytmalari ilk marotaba M. Kasas, E. Faro va A. M. Viyeteslarning ishlarida keltirilgan bo‘lsa, Abel bo‘lmagan kengaytmalarni aniqlash esa J. Liu, Y. Sheng va Q. Vanglar tomonidan tasniflangan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Ushbu dissertatsiya V.I. Romanovskiy nomidagi Matematika institutining ilmiy yo‘nalishlariga muvofiq ravishda amalga oshirilgan.

Tadqiqotning maqsadi nilradikallari filiform bo‘lgan chekli o‘lchamli yechiluvchan Li va Leybnits algebralarining kengaytmalari, xususan abel kengaytmalari orqali ularni klassifikatsiyalash va strukturaviy tahlil qilish uchun kompleks yondashuv ishlab chiqishdan iborat.

Tadqiqotning vazifalari:

nilpotent va yechiluvchan algebralarni klassifikatsiyalash jarayonida markaziy hamda abel kengaytmalar metodini qo‘llash va umumlashtirish;

yechiluvchan Li algebralarning bir o‘lchamli kengaytmalarini tavsiflash;

n-darajali Shrodinger algebraning markaziy kengaytmalari va infinitezimal deformatsiyalarini aniqlash;

filiform nilradikallarga ega kichik o‘lchamli yechiluvchi Leybnits algebralarning barcha o‘zaro izomorf bo‘lmagan abel kengaytmalarini aniqlash.

Tadqiqotning obyekti: nilpotent, yechiluvchan Li algebralari, *n*-darajali Shrodinger algebrasi, yechiluvchan Leybnits algebrasi.

Tadqiqotning premeti: *n*-darajali Shrodinger algebrasining markaziy kengaytmasi va infinitesimal deformatsiyalari, nilradikali filiform bo‘lgan yechiluvchan Li algebralari, nilradikali null-filfirom va tabiiy usulda gradiurlangan filiform bo‘lgan yechiluvchan Leybnits algebralari, besh o‘lchamli yechiluvchan Leybnits algebralari.

Tadqiqotning usullari. Dissertatsiyada kengaytmalar usuli, shuningdek, invariantlar nazariyasi usullari foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

n-darajali Shrodinger algebraning markaziy kengaytmalari va uning infinitezimal deformatsiyalari aniqlangan;

tabiiy usulda gradiurlangan filiform Li algebralarning barcha bir oʻlchamli markaziy kengaytmalari topilgan;

filiform nilradikalli yechiluvchan Li algebralarning bir oʻlchamli kengaytmalari aniqlangan va ularning toʻliq tuzilmalari tavsiflangan;

yechiluvchan Leybnits algebralarning Abel kengaytmalarini qurish usuli ishlab chiqilgan va besh oʻlchamli yechiluvchan Leybnits algebralarining Abel kengaytmalar aniqlangan.

Tadqiqotning amaliy natijalari. Olingan natijalar va dissertatsiyada qoʻllanilgan usullar oliy oʻquv yurtlari magistr talabalari va doktorantlari uchun maxsus kurslarda oʻqitilishi mumkin. Bundan tashqari, tadqiqot natijalari yechiluvchan Li va Leybnits algebralarning toʻliq klassifikatsiyasiga bevosita daxldor boʻlib, algebraik tizimlarning strukturaviy nazariyasiga sezilarli hissa qoʻshadi.

Tadqiqot natijalarining ishonchliligi. Matematik mulohazalarning qat’iyligi, algebralarning boshqa sinflaridagi ma’lum usullaridan, kogomologiyalar nazariyasi, algebralarining strukturaviy nazariyasidagi fundamental natijalardan foydalanilganligi bilan asoslanadi. Mathematica 13 dasturlash tilida yaratilgan maxsus dasturlar yordamida kichik oʻlchamli algebralarning differensiallashlari va avtomorfizmlariga doir natijalar tekshirilgan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Mazkur tadqiqot yechiluvchan Li va Leybnits algebralarining nazariy asoslarini chuqurlashtirishga xizmat qiladi. Ish davomida ishlab chiqilgan metodologiyalar va erishilgan natijalar algebra nazariyasiga sezilarli hissa qoʻshadi, hamda ushbu yoʻnalishdagi kelgusi tadqiqotlar uchun qat’iy nazariy asos yaratadi. Amaliy nuqati nazardan esa, ushbu tadqiqot natijalari matematik fizika, boshqaruv nazariyasi va kriptografiya kabi sohalarda qoʻllanish imkoniyatiga ega boʻlib, bu yerda oʻrganilgan algebraik tuzilmalar muhim ahamiyat kasb etadi.

Dissertatsiyadagi barcha natijalar va qoʻllanilgan usullardan oliy ta’lim muassasalarining magistratura va doktorantura talabalari uchun maxsus kurslarda foydalanish mumkin.

Tadqiqot natijalarining joriy qilinishi. Filiform nilradikalli yechiluvchan Li va Leybnits algebralarining markaziy hamda Abel kengaytmalari boʻyicha olingan natijalar asosida:

yechiluvchan Li va Leybnits algebralarning kengaytirish metodidan va filiform nilradikalli yechiluvchan Li algebralarning bir oʻlchamli kengaytmalaridan AP14870282 raqamli “Noassotsiativ algebralar turlari uchun Identifikatorlarni aniqlash” mavzusidagi xorijiy loyihada nonassotsiativ algebralarning boshqa sinflariga umumlashtirish maqsadida qoʻllanilgan (Astana IT Universitetining 2025 yil 5-noyabrdagi №1/1529-sonli ma’lumotnomasi, Qozogʻiston Respublikasi).

Ilmiy natijaning qo‘llanilishi binar va unar Leybnits algebralarining Abel kengaytmalarini aniqlash imkonini bergan;

kichik o‘lchamli yechiluvchan Li va Leybnits algebralarining markaziy va Abel kengaytmalari uchun ishlab chiqilgan usullari va ularning tasniflaridan F-FA-2021-423 raqamli “Operator algebralarining avtomorfizmlari, cheksiz o‘lchamli noassotsiativ algebralar va superalgebralarning klassifikatsiyasi” mavzusidagi fundamental loyihada noassotsiativ algebraik tuzilmalarni tasniflash uchun zamonaviy yondashuvlarni takomillashtirishda foydalanilgan (Matematika institutining 2025 yil 3-dekabrdagi №2/505-sonli ma’lumotnomasi). Ilmiy natijalarni qo‘llanilishi besh o‘lchamli nilpotent Leybnits algebralarining algebraik va geometrik tasniflarini olish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 5 ta xalqaro va 3 ta respublika ilmiy-amaliy anjumanlarda muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 14 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 2 tasi xorijiy, 4 tasi respublika jurnallarida, shuningdek 8 ta ma’ruza tezislari ilmiy konferensiya materiallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qism, to‘qqizta bo‘limga bo‘lingan uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 106 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiya kirish qism, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat.

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo‘nalishlariga mosligi ko‘rsatilgan. Shuningdek, bu qismda dissertatsiya mavzusi bo‘yicha muammoning o‘rganilganlik darajasi keltirilgan, tadqiqotning maqsadi, vazifalari, ob’ekti va premeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Li va Leybnits algebralarining asosiy tushunchalari**” nomli birinchi bobida Li va Leybnits algebralarining ta’riflari, ular uchun umumiy bo‘lgan asosiy tushunchalar va dissertatsiyaning keyingi boblarida foydalanilgan asosiy algebralarning strukturaviy tuzilishlari, Li algebralarining kogomologiyasi, hamda nilpotent Li algebralari uchun markaziy kengaytma va yechiluvchan Li algebralarning kengaytmalarini aniqlash metodlari keltirilgan. Shuningdek, kogomologik fazolariga oid asosiy tushunchalar va teoremalar, hamda Shrodinger algebrasining kogomologik fazolariga oid olingan natijalar bayon qilingan.

Birinchi bobning birinchi bo‘limida boshlang‘ich tushunchalar, Li va Leybnits algebralarining ta’riflari, kengaytmalarni aniqlash metodlari keltirilgan.

1-ta'rif: *F maydon ustida aniqlangan G algebraning ixtiyoriy x,y,z elementlari uchun quyidagi ayniyatlar bajarilsa,*

$$\left[x,\left[y,z\right]\right]+\left[y,\left[z,x\right]\right]+\left[z,\left[x,y\right]\right]=0,$$

$$\left[x,x\right]=0,$$

u holda G algebraga Li algebrasi deyiladi.

2-ta'rif. *F maydon ustida aniqlangan L algebraning ixtiyoriy x,y,z elementlari uchun quyidagi Leybnits ayniyati bajarilsa,*

$$\left[\left[x,y\right],z\right]=\left[x,\left[y,z\right]\right]+\left[\left[x,z\right],y\right],$$

L algebraga Leybnits algebrasi deyiladi..

Aytaylik, *G* Li (Leybnits) algebrasi berilgan bo'lsin. *G* algebra uchun xosilaviy va quyi markaziy qatorlarni mos ravishda quyidagicha aniqlaymiz.

$$G^{[1]}=\left[G,G\right],\ G^{[s+1]}=\left[G^{[s]},G^{[s]}\right],\quad s\geq 1,\ \ G^{1}=\left[G,G\right],\ G^{k+1}=\left[G^{k},G^{k}\right],\ k\geq 1.$$

3-ta'rif. *n-o'lchamli G algebra uchun $s\in\mathbb{N}$ (mos ravishda, $k\in\mathbb{N}$) natural son mavjud bo'lib, $G^{[s]}=0$ (mos ravishda, $G^{k}=0$) tenglik o'rinli bo'lsa, u holda G yechiluvchan (mos ravishda, nilpotent) algebra deyiladi.*

Li algebrasining maksimal nilpotent ideali algebraning nilradikali deb ataladi.

4-ta'rif. *Agar n-o'lchamli G algebra uchun* $\dim G^{i}=n-i$, $2\leq i\leq n$ *tenglik o'rinli bo'lsa, u holda G filiform Li* (Leybnits) *algebrasi deyiladi.*

5-ta'rif. *Agar n-o'lchamli L Leybnits algebrasi uchun* $\dim L^{i}=n+1-i$, $1\leq i\leq n+1$ *tenglik o'rinli bo'lsa, u holda L nol-filiform Leybnits algebrasi deyiladi.*

Nol-filiform Leybnits algebralari maksimal nilpotentlik indeksiga ega algebralar hisoblanadi. *n*-o'lchamli nol-filiform Leybnits algebrasi izomorfizm aniqligida yagona ekanligini Sh.A.Ayupov va B.A.Omirov ishlarida isbotlangan va uning ko'paytmasi quyidagicha ekanligi ko'rsatilgan:

$$NF_n:\quad [e_i,e_1]=e_{i+1},\quad 1\leq i\leq n-1,$$

bu yerda $\{e_1,e_2,\dots,e_n\}$ NF_n algebraning bazislari.

I.A.Karimjonov o'z PhD dissertatsiyasida nilradikali NF_n bo'lgan yechiluvchan Leybnits algebrasini aniqlab, $\{e_1,e_2,\dots,e_n,x\}$ bazisda uning ko'paytmasini quyidagicha bo'lishini isbotlagan:

$$R\left(NF_n\right):\begin{cases}[e_i,e_1]=e_{i+1},\quad 1\leq i\leq n-1,\ \ [x,e_1]=e_1,\\ [e_i,x]=-ie_i,\ \ 1\leq i\leq n.\end{cases}$$

6-ta'rif. Nilindeksi *s ga teng bo'lgan G algebra uchun quyidagi belgilashlarni kiritib olamiz:* $G_i=G^{i}/G^{i+1}$, $1\leq i\leq s-1$, $Gr(G)=G_1\oplus G_2\oplus\ldots\oplus G_{s-1}$. *Gr(G) vektor fazoda esa ko'paytmani quyidagicha aniqlaymiz:*

$$\left[x+G^{i+1},y+G^{j+1}\right]=\left[x,y\right]+G^{i+j+1},$$

bu yerda $x\in G^{i}\setminus G^{i+1}$, $y\in G^{j}\setminus G^{j+1}$. *U holda* $\left[G_i,G_j\right]\subseteq G_{i+j}$ *bo'lib, Gr(G) gradiurlangan algebra bo'ladi. Agar Gr(G) va G algebralar izomorf bo'lsa, u* *holda G algebrani tabiiy usulda gradiurlangan algebra deb ataladi.*

Fransuz matematiki M.Vern o'z ilmiy ishida ikki turdagi tabiiy usulda gradiurlangan Li algebralarini aniqlab, ularning umumiy ko'rinishi quyidagicha bo'lishini keltirib o'tgan:

$$n_{n,1}: \quad [e_i, e_1] = -[e_1, e_i] = e_{i+1}, \qquad 2 \le i \le n-1,$$

$$Q_{2n}: \quad \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, & 2 \le i \le 2n-2, \\ [e_i, e_{2n+1-i}] = -[e_{2n+1-i}, e_i] = (-1)^i e_{2n}, & 2 \le i \le n. \end{cases}$$

Shuni ta'kidalash joizki, ikkinchi tur faqat algebraning o'lchami juft bo'lgandagina mavjud bo'ladi.

A.Sh.Ayupov va B.A.Omirovlar esa o'z ishlarida tabiiy usulda gradiurlangan Li bo'lmagan Leybnits algebralarini tasniflab, quyidagi algebralarga ega bo'lishgan:

$$F_n^1 : [e_1, e_1] = e_3, \ [e_i, e_1] = e_{i+1}, \ 2 \le i \le n-1,$$

$$F_n^2 : [e_1, e_1] = e_3, \ [e_i, e_1] = e_{i+1}, \ 3 \le i \le n-1.$$

7-ta'rif. *(L,[-,-]) Leybnits algebrasining tasviri deb,* $\mathbb{V}$ *vektor fazo va quyidagi*

$$r_{[x,y]} = r_y \circ r_x - r_x \circ r_y, \quad l_{[x,y]} = r_y \circ l_x - l_x \circ r_y, \quad l_x \circ l_y = -l_x \circ r_y, \ \forall x, y \in L$$

shartlarni qanoatlantiruvchi $l : L \to gl(\mathbb{V})$ *va* $r : L \to gl(\mathbb{V})$ *chiziqli akslantirishlarga, ya'ni* $(\mathbb{V}, l, r)$ *uchlikka aytiladi.*

Xususan $r_x = -l_x$ bo'lsa, u holda $(\mathbb{V}, l, r)$ tasvir Li algebrasining tasviri bo'ladi.

G Li algebrasining *M* moduli uchun $C^0(G, M) = M$, $C^n(G, M) = Hom(G^{\otimes n}, M)$, $n > 0$ fazolarni qarab, $d^n : C^n(G, M) \to C^{n+1}(G, M)$ akslantirishni quyidagicha aniqlaymiz

$$d^n \psi(e_0, \ldots, e_n) = \sum_{i=0}^{n+1} (-1)^i \psi\left(e_0, \ldots, \hat{e}_i, \ldots, e_n\right) +$$
$$\sum_{1 \le i < j \le n} (-1)^{i+j} \psi\left([e_i, e_j], e_0, \ldots, \hat{e}_i, \ldots, \hat{e}_j, \ldots, e_n\right),$$

bu yerda $e_0, \ldots, e_n \in G$ va ushbu $\hat{}$ belgi – uning ostidagi element tashlab ketilishini bildiradi. d^n akslantirish uchun $d^{n+1} \circ d^n = 0$ xossa o'rinli ekanligi $d = \sum_{i \ge 0} d^i$ operator uchun $d \circ d = 0$ tenglik bajarilishini keltirib chiqaradi. Ushbu $H^n(G, M) = Z^n(G, M) / B^n(G, M)$ faktor fazo *G* algebraning *n*-tartibli kogomologik gruppasi deyiladi. Bu yerda $Z^n(G, M) = Ker(d^{n+1})$ va $B^n(G, M) = \mathrm{Im}(d^n)$ mos ravishda *n*-kotsikllari va *n*-kochegaralari deb ataladi.

Xususan, *n=2* bo'lgan xol uchun, agar *M* trivial *G*-modul bo'lsa, u holda 2-kogomologik gruppa Li algebrasining markaziy kengaytmasini xarakterlaydi. Bundan tashqari, agar *M=G* bo'lsa, u holda 2-kogomologik gruppa Li algebrasining infinitesimal deformatsiyasini tasniflaydi.

Agar R berilgan G algebraning ideali bo‘lsa, u holda biz har bir $C^n(R,M)$ da G-modul strukturasini tasniflashimiz mumkin. $n>0$ uchun G ning $C^n(R,M)$ dagi ta’sirini quyidagicha aniqlaymiz:

$$(v.\psi)(e_1,\ldots,e_n)=v.\psi(e_1,\ldots,e_n)-\sum_{i=1}^{n}\psi(e_1,\ldots,e_{i-1},[v,e_i],e_{i+1},\ldots,e_n),$$

bu yerda $v\in G,\ \psi\in C^n(R,M)$ va $e_1,\ldots,e_n\in R.$

Aytaylik, G nilpotent Li algebrasi va $\mathbb{V}$ vektor fazo bo‘lsin. Nilpotent Li algebralari uchun 2-kotsikl tushunchasi quyidagi ayniyatni qanoatlantiruvchi $\psi:G\times G\to\mathbb{V}$ bichiziqli akslantirish sifatida aniqlanadi:

$$\psi(x,[y,z])+\psi(y,[z,x])+\psi(z,[x,y])=0.$$

Barcha 2-kotsikllar fazosini $Z^2(G,\mathbb{V})$ kabi belgilaymiz.

Berilgan $f:G\to\mathbb{V}$ chiziqli almashtirish uchun $df:G\times G\to\mathbb{V}$ bichiziqli akslantirishni $df(x,y)=f([x,y])$ kabi belgilaymiz. Bunday df bichiziqli akslantirishlar 2-kochegaralar deyiladi va 2-kochegaralar fazosi $B^2(G,\mathbb{V})$ kabi belgilanadi. Ta’kidlash joizki, $B^2(G,\mathbb{V})\subseteq Z^2(G,\mathbb{V})$ va $H^2(G,\mathbb{V})=B^2(G,\mathbb{V})/Z^2(G,\mathbb{V})$ faktor fazo 2-kogomologik gruppa deyiladi.

$\psi\in Z^2(G,\mathbb{V})$ akslantirish yordamida $\overline{G}=G\oplus\mathbb{V}$ chiziqli fazoda quyidagi shaklda $[-,-]_\psi$ ko‘paytirish amalini aniqlaymiz:

$$[x+u,y+v]_\psi=[x,y]+\psi(x,y),\ \ \forall x,y\in G,\ u,v\in\mathbb{V}.$$

Yuqoridagi kabi aniqlangan $G_\psi=(\overline{G},[-,-]_\psi)$ algebra G algebraning $\mathbb{V}$ vektor fazo orqali aniqlangan markaziy kengaytmasi deb ataladi.

Aytaylik, G algebraning avtomorfizmlar gruppasi $Aut(G)$ berilgan bo‘lsin. Ushbu $Aut(G)$ gruppaning $Z^2(G,\mathbb{V})$ fazoga ta’sirini

$$\varphi.\psi(x,y)=\psi(\varphi(x),\varphi(x))$$

kabi aniqlaymiz, bu yerda $\varphi\in Aut(G)$ va $\psi\in Z^2(G,\mathbb{V})$. U holda, $B^2(G,\mathbb{V})$ qismfazo bu ta’sirga nisbatan invariant bo‘ladi. Demak, biz $Aut(G)$ gruppaning $H^2(G,\mathbb{V})$ fazoga ta’sirini ham qarashimiz mumkin.

$Ann(\psi)=\{x\in G:\psi(x,G)=0\}$ to‘plam ψ elementning annulyatori va $Ann(G)=\{x\in G:xG=0\}$ to‘plam esa G algebraning annulyatori deb ataladi. Qayd etish joizki, $Ann(G_\psi)=(Ann(\psi)\cap Ann(G))\oplus\mathbb{V}$ tenglik o‘rinli bo‘ladi.

Quyidagi natija A.Hegazi va H.Abdelvahab tomonidan Yordan algebralari uchun isbotlangan. Aslida bu natija ixtiyoriy noassotsiativ algebra uchun o‘rinlidir. Unga ko‘ra annulyatori noldan farqli har qanday algebra kichikroq o‘lchamli algebraning markaziy kengaytmasi orqali ifodalanadi..

1-tasdiq. *Aytaylik, G algebra n-o'lchamli muayyan tipdagi algebra va* $\dim\left(Ann\left(G\right)\right)=m\neq 0$ bo'lsin. *U holda, ayni tipdagi yagona (n-m)-o'lchamli G' algebra va* $Ann\left(\psi\right)\cap Ann\left(G'\right)=0$ *shartni qanoatlantiruvchi* $\psi\in Z^{2}\left(G',\mathbb{V}\right)$ *mavjud bo'lib,* $G\cong G'_{\psi}$, $G/Ann\left(G\right)\cong G'$ *munosabatlar o'rinli bo'ladi, bu yerda,* $\mathbb{V}$ *m-o'lchamli chiziqli fazo.*

Navbatdagi metod yechiluvchan Li algebralarga tatbiq etiluvchi kengaytirish usuli hisoblanadi. Mazkur metod T. Sand tomonidan olib borilgan tadqiqotlarda asoslab berilgan

Aytaylik, G yechiluvchan Li algebrasi va $\mathbb{V}$ vektor fazo bo'lsin. $\theta:G\to End(\mathbb{V})$ tasvir va quyidagi ayniyatni qanoatlantiruvchi anti-simmetrik bichiziqli akslantirishni olaylik:

$$\psi(x,[y,z])+\psi(z,[x,y])+\psi(y,[z,x])+\theta(x)\psi(y,z)+\theta(z)\psi(x,y)+\theta(y)\psi(z,x)=0,$$

bu yerda $x,y,z\in G.$

Yuqorida keltirilgan tenglikni qanoatlantiruvchi bichiziqli akslantirishlarni G ning θ ga mos 2-kotsilkli deyiladi va barcha 2-kotsikllar to'plami $Z^{2}(G,\theta,\mathbb{V})$ kabi belgilanadi. G ning θ ga mos 2-kochegaralarni $f:G\to\mathbb{V}$ chiziqli akslantirish va $\varphi\in Aut(G)$ uchun quyidagicha aniqlaymiz:

$$df\left(x,y\right)=f\left(\left[x,y\right]\right)+\theta\left(\varphi\left(y\right)\right)\left(f\left(x\right)\right)-\theta\left(\varphi\left(x\right)\right)\left(f\left(y\right)\right).$$

Barcha 2-kochegaralar to'plamini $B^{2}\left(G,\theta,\mathbb{V}\right)$ kabi belgilanadi va $Z^{2}\left(G,\theta,\mathbb{V}\right)$ ning qism fazosi bo'ladi. $Z^{2}\left(G,\theta,\mathbb{V}\right)$ ni $B^{2}\left(G,\theta,\mathbb{V}\right)$ bo'yicha faktor fazosi $H^{2}\left(G,\theta,\mathbb{V}\right)$ bilan belgilanib, u 2-kogomologik fazoni ifodalaydi

Vektor fazo $\overline{G}=G\oplus\mathbb{V}$ da berilgan $\psi\in Z^{2}\left(G,\theta,\mathbb{V}\right)$ uchun $G\left(\psi,\theta\right)$ Li strukturasini ixtiyoriy $u,v\in\mathbb{V},\ x,y\in G$ lar uchun quyidagicha aniqlaymiz:

$$\left[x+u,y+v\right]=\left[x,y\right]+\psi\left(x,y\right)+\theta\left(x\right)v-\theta\left(y\right)u.$$

$G\left(\psi,\theta\right)$ algebra G ning $\mathbb{V}$ orqali kengaytmasi bo'ladi.

G algebraning nilradikalini N va nilradikalning markazini *Z(G)* bilan belgilab olaylik. Biz nilradikali $\overline{N}$ ya'ni, N ning $\mathbb{V}$ orqali kengaytmasi bo'lgan $\overline{G}=G(\psi,\theta)$ algebrani o'rganamiz. Quyidagi belgilashlarni kiritamiz: $\psi^{0}=\psi|_{N\times N}$, $\theta^{0}=\theta|_{N}$, $Ann(\psi^{0})=\{x\in N\,|\,\psi^{0}(x,N)=0\}$.

T.Sund o'z ishida quyidagi tasdiqni yechiluvchan Li algebralar uchun isbotlagan.

2-tasdiq. *Bizga* $\overline{G}=G(\psi,\theta)$ *G ning* $\mathbb{V}$ *orqali kengaytmasi berilgan bo'lsin. U holda*

– $\overline{G}$ *ning nilradikali bo'lgan N berilgan N nilradikalning* $\mathbb{V}$ *orqali kengaytmasi bo'lishi uchun* $Ker(\theta)\supset N$ *bo'lishi zarur va yetarli.*

– $Ker(\theta)\supset N$ *bo'lsin. U holda* $Z(\overline{N})=\mathbb{V}$ *bo'lishi uchun* $Ann(\psi^{0})\cap Z(N)=0$ *bo'lishi zarur va yetarli.*

Bizga G algebraning ikkita $G(\psi_1,\theta_1)$ va $G(\psi_2,\theta_2)$ kengaytmalari berilgan bo‘lsin. Ular izomorf bo‘lishi uchun quyidagi tengliklar bajarilishi zarur va yetarli:

$$\theta_2\left(\alpha(x)\right)\left(\beta(a)\right)=\beta\left(\theta_1(x)(a)\right),$$

$$\psi_2\left(\alpha(x),\alpha(y)\right)-\theta_2\left(\alpha(y)\right)\left(f(x)\right)+\theta_2\left(\alpha(x)\right)\left(f(y)\right)=\beta\left(\psi_1(x,y)\right)+f\left([x,y]\right),$$

bu yerda $\alpha\in Aut(G),\ \beta\in Aut(\mathbb{V}),\ f\in Hom(G,\mathbb{V})$.

Yuqoridagi tengliklardan foydalangan holda $Aut(G)\times Aut(\mathbb{V})$ ni $\bigcup\limits_{\theta} Z^2(G,\theta,\mathbb{V})$ to‘plamga ta’sirini olamiz va bunda θ ga mos ψ 2-kotsikl ta’sir natijasida θ' ga mos ψ' 2-kotsiklga quyidagi shaklda o‘tadi:

$$\theta'(x)(a)=\beta\left(\theta\left(\alpha(x)\right)\left(\beta^{-1}(a)\right)\right),\quad \psi'(x,y)=\beta\left(\psi\left(\alpha(x),\alpha(y)\right)\right).$$

T.Sund o‘z ishida yechiluvchan Li algebrasining ikkita $G(\psi_1,\theta_1)$ va $G(\psi_2,\theta_2)$ kengaytmalari izomorf bo‘lishi uchun zarur va yetarli shartni quyidagicha ta’riflagan.

4-tasdiq. *Berilgan G yechiluvchan Li algebrasining ikkita $G(\psi_1,\theta_1)$ va $G(\psi_2,\theta_2)$ kengaytmalari izmorf bo‘lishi uchun $\psi_2-\beta\circ\psi_1\circ\alpha\in B^2(G,\theta_2,\mathbb{V})$ shartni qanoatlantiruvchi $\alpha\in Aut(G),\beta\in Aut(\mathbb{V})$ larning mavjud bo‘lishi hamda β operator θ_2 va $\theta_1\circ\alpha$ larni o‘zaro bog‘lovchi operator bo‘lishi zarur va yetarli.*

Birinchi bobning ikkinchi bo‘limida *n*-darajali Shrodinger algebrasining markaziy kengaytmasi va ikkinchi kogomologiyalar fazolari o‘rganilgan.

n-darajali Shrodinger algebra uch o‘lchamli sodda Li algebrasi va Geyzenberg algebralarining yarimto‘g‘ri ko‘paytmasidan iborat bo‘lib, $\{e,h,f,z,x_i,y_i,1\le i\le n\}$ bazisda ko‘paytmasi quyidagicha aniqlangan:

$$[h,e]=2e,\qquad [h,f]=-2f,\ [e,f]=h,\quad [h,x_i]=x_i,$$

$$[h,y_i]=-y_i,\quad [e,y_i]=x_i,\quad [f,x_i]=y_i,\ [x_i,y_i]=z.$$

n=1 bo‘lgan holda ikkinchi kohomologiyalar gruppasi Y.Zh.Vu, X.Q.Yue va L.Sh. Zhu larning ishlarida aniqlangan bo‘lib, $\dim(H^2(sch_1,\mathbb{C}))=0$ ekanligi isbotlangan. Shu sababli biz, $n\ge 2$ holni qaraganmiz.

1-teorema. *n-* darajali *Shrodinger algebra* sch_n *ning* $n\ge 2$ *uchun ikkinchi kohomologik fazosining o‘lchami* $\dfrac{n(n+1)}{2}-1$ *ga teng.*

2-teorema. $\dim(H^2(sch_2,sch_2))=1$ *va* $H^2(sch_n,sch_n)=0,\ n\ge 3.$

1-natija. *Quyidagicha aniqlangan* ψ *bichiziqli almashtirish* $H^2(sch_2,sch_2)$ *fazoning bazisi bo‘ladi:*

$$\psi(x_1,x_2)=2e,\ \psi(y_1,y_2)=-2f,\ \psi(x_1,y_2)=-h,\quad \psi(x_2,y_1)=h,$$

$$\psi(x_1,z)=3x_2,\ \psi(y_1,z)=3y_2,\quad \psi(x_2,z)=-3x_1,\ \psi(y_2,z)=-3y_1.$$

Dissertatsiyaning “**Yechiluvchan Li algebralarining kengaytmalari**” nomli ikkinchi bobi tabiiy usulda gradiurlangan filiform Li algebralarining markaziy kengaytmasi va nilradikali ushbu tabiiy usulda gradiurlangan filiform algebradan iborat bo‘lgan yechuluvchan Li algebralarining kengaytmalarini tasniflashga bag‘ishlangan.

Ma’lumki, L.Snobl va P.Vinternits o‘z ishlarida nilradikali $n_{n,1}$, J.M Ankochea Bermudes, R.Kampoamor-Shtursberg va L.Garsiya Veryollar esa nilradikali Q_{2n} bo‘lgan yechiluvchan Li algebralarini tasniflashgan.

Ikkinchi bobning birinchi bo‘limi, nilpotent $n_{n,1}$ *va* Q_{2n} Li algebralarining markaziy kengaytmalarini aniqlashga bag‘ishlangan.

Ushbu bo‘limning aosiy natijasi quyidagi teoramada keltirilgan.

3-teorema. $n_{n,1}$ *algebraning ixtiyoriy yoyilmaydigan markaziy kengaytmasi quyidagi o‘zaro izomorf bo‘lmagan algebralarning biriga izomorf bo‘ladi:*

$$n_{n+1,1},\ Q_{n+1},\ \text{and}\ \overline{G}_k\left(2\le k\le\left\lfloor\frac{n}{2}\right\rfloor\right):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n,\\ [e_i,e_{2k+1-i}]=(-1)^i e_{n+1}, & 2\le i\le n.\end{cases}$$

1-Izoh. Q_{n+1} *algebra faqat n toq bo‘lganda hosil bo‘ladi.*

Ikkinchi bobning ikkinchi bo‘limi filiform nilradikalli maksimal ko-o‘lchamga ega bo‘lgan yechiluvchan Li algebrasining kengaytmasini aniqlashga bag‘ishlangan. Bizga ma’lumki, bunday algebralar ikkita bo‘lib, ular quyidagi ko‘paytmalar bilan aniqlanadi:

$$s_{n,2}:\begin{cases}[e_i,e_1]=e_{i+1},\ 2\le i\le n-1,\ [e_1,x_1]=e_1,\\ [e_i,x_1]=(i-2)e_i,\ 3\le i\le n,\ [e_i,x_2]=e_i,\ 2\le i\le n,\end{cases}$$

$$\tau_{2n,2}:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\ [e_i,e_{2n+1-i}]=(-1)^i e_{2n},\ 2\le i\le n,\\ [e_i,x_1]=ie_i, & [e_i,x_2]=e_i,\quad 1\le i\le 2n-1,\\ [e_{2n},x_1]=(2n+1)e_{2n},\ [e_{2n},x_2]=2e_{2n}, & [x_2,e_{2n}]=-2e_{2n},\end{cases}$$

bu yerda $s_{n,2}$ nilradikali $n_{n,1}$ va $\tau_{2n,2}$ esa nildikali Q_{2n} bo‘lgan algebralar. Ma’lumki, Q_{2n} algebraning yoyilmaydigan markaziy kengaytmasi mavjud bo‘lmagani uchun, unga mos keluvchi yechiluvchan Li algebrasi ham yoyilmaydigan kengaytmaga ega emas. Shuning uchun, faqat $s_{n,2}$ algebraning kengaytmasini aniqlaymiz.

4-teorema. $\overline{G}$ *yechiluvchan Li algebrasi* $s_{n,2}$ *algebraning bir o‘lchamli yoyilmaydigan kengaytmasi bo‘lsin, u holda* $\overline{G}$ *algebra* $s_{n+1,2}$ *va* $\tau_{n+1,2}$ *algebralarning biriga izomorf bo‘ladi.*

Ikkinchi bobning uchinchi bo‘limida nilradikal $n_{n,1}$, Q_{2n} va ko-o‘lchami birga teng bo‘lgan algebralarning bir o‘lchamli kengaytmalari tasniflangan. Bu algebralarning ko‘paytmalari quyidagicha:

$$s^1_{n,1}(\beta):\begin{cases}[e_i,e_1]=e_{i+1},\ 2\le i\le n-1,\ [e_1,x]=e_1,\\ [e_i,x]=(i-2+\beta)e_i,\quad 2\le i\le n,\end{cases}\qquad s^2_{n,1}:\begin{cases}[e_i,e_1]=e_{i+1},\ 2\le i\le n-1,\\ [e_i,x]=e_i,\quad 2\le i\le n,\end{cases}$$

$$s^3_{n,1}: [e_i, e_1] = e_{i+1},\ 2 \le i \le n-1,\ \ [e_1, x] = e_1 + e_2,\ \ [e_i, x] = (i-1)e_i,\ \ \ 2 \le i \le n,$$

$$s^4_{n,1}(\alpha_3, \alpha_4, \dots, \alpha_{n-1}): [e_i, e_1] = e_{i+1},\ \ 2 \le i \le n-1,\ [e_i, x] = e_i + \sum_{l=i+2}^{n} \alpha_{l+1-i} e_l,,\ \ 2 \le i \le n,$$

$$\tau^1_{2n,1}(\alpha): \begin{cases} [e_i, e_1] = e_{i+1}, & 2 \le i \le 2n-2,\ [e_i, e_{2n+1-i}] = (-1)^i e_{2n},\ \ 2 \le i \le n, \\ [e_i, x] = (i-2+\alpha)e_i, & 2 \le i \le 2n-1,\ \ [e_1, x] = e_1, \\ [e_{2n}, x] = (2n+3+2\alpha)e_{2n}, & \end{cases}$$

$$\tau^2_{2n,1}: \begin{cases} [e_i, e_1] = e_{i+1}, \quad 2 \le i \le 2n-2,\ \ [e_i, e_{2n+1-i}] = (-1)^i e_{2n},\ \ 2 \le i \le n, \\ [e_i, x] = (i-n)e_i,\ \ 2 \le i \le 2n-1,\ \ [e_1, x] = e_1 + e_{2n},\ \ \ [e_{2n}, x] = e_{2n}, \end{cases}$$

$$\tau^3_{2n,1}(\alpha_4, \dots, \alpha_{2n-2}): \begin{cases} [e_i, e_1] = e_{i+1}, \quad 2 \le i \le 2n-2, [e_i, e_{2n+1-i}] = (-1)^i e_{2n}, 2 \le i \le n, \\ [e_{i+2}, x] = e_{i+2} + \sum\limits_{k=2}^{\left\lfloor \frac{2n+3-i}{2} \right\rfloor} \alpha_{2k} e_{2k+1+i},\ 0 \le i \le 2n-3,\ \ \ [e_{2n}, x] = 2e_{2n}. \end{cases}$$

Biz faqat nilradikali $n_{n,1}$ bo‘lgan yechiluvchan Li algebralarining kengaytmalarini aniqlaymiz.

Ushbu bo‘limda quyidagi natijalarni olganmiz.

5-teorema. *$\overline{G}$ yechiluvchan Li algebrasi $s^1_{n,1}(\beta)$ algebraning bir o‘lchamli yoyilmaydigan kengaytmasi bo‘lsin, u holda $\overline{G}$ algebra quyidagi algebralarning biriga izomorf bo‘ladi: $s^1_{n+1,1}(\beta)$, $\tau^1_{n+1,1}(\beta)$, $\tau^2_{n+1,1}$ va*

$$\overline{G}_k\left(2 \le k \le \left\lfloor \frac{n}{2} \right\rfloor\right): \begin{cases} [e_i, e_1] = e_{i+1}, & 2 \le i \le n,\ \ \ [e_1, x] = e_1, \\ [e_i, e_{2k+1-i}] = (-1)^i e_{n+1}, & 2 \le i \le k, \\ [e_i, x] = (n+i-2k)e_k,\ 2 \le i \le n+1. & \end{cases}$$

6-teorema. *$\overline{G}$ yechiluvchan Li algebrasi $s^2_{n,1}$ algebraning bir o‘lchamli yoyilmaydigan kengaytmasi bo‘lsin, u holda $\overline{G}$ algebra $s^2_{n+1,1}$, $s^4_{n+1,1}(0,0,\dots,1)$ va $\tau^3_{n+1,1}(0,0,\dots,0)$ algebralarning biriga izomorf bo‘ladi.*

7-teorema. *$s^3_{n,1}$ algebraning ixtiyoriy bir o‘lchamli yoyilmaydigan kengaytmasi $s^3_{n+1,1}$ algebraga izomorf bo‘ladi.*

8-teorema. *$s^4_{n,1}(\alpha_3, \alpha_4, \dots, \alpha_{n-1})$ algebraning ixtiyoriy bir o‘lchamli yoyilmaydigan kengaytmasi $s^4_{n+1,1}(\alpha_3, \alpha_4, \dots, \alpha_{n-1}, \alpha_n)$ va $\tau^3_{n+1,1}(\alpha_4, \alpha_6, \dots, \alpha_{n-1})$ algebralarga izomorf bo‘ladi.*

Dissertatsiyaning “**Yechiluvchan Leybnits algebralarining kengaytmalari**” nomli uchinchi bobida yechiluvchan Leybnits algebralarini Abel kengaytmalarini aniqlash metodini keltirib o‘tgan holda, ushbu metoddan foydalanib, nilradikali nol-filiform va tabiiy usulda gradiurlangan filiform algebralari bo‘lgan yechiluvchan Leybnits algebralarining Abel kengaytmalari tasniflangan. Shuningdek, trivial bo‘lmagan nilradikalga ega besh o‘lchamli yechiluvchan

Leybnits algebralarining bir o‘lchamli Abel kengaytmalari aniqlangan.

Uchinchi bobning birinchi bo‘limi, yechiluvchan Leybnits algebralari uchun Abel kengaytmalarini aniqlash metodiga bag‘ishlangan. J.Liu, Y.Sheng va Q.Vanglar o‘z ishlarida yechiluvchan chap Leybnits algebralari uchun Abel bo‘lmagan kengaytmalar metodini keltirib o‘tishgan. Biz ushbu ishdan ilhomlangan holda, yechiluvchan o‘ng Leybnits algebralari uchun Abel kengaytmalar metodini aniqladik.

Bizga L Leybnits algebrasi va $\mathbb{V}$ Abel Leybnits algebralari berilgan bo‘lsin. Aytaylik, $l: L \to gl(\mathbb{V})$, $r: L \to gl(\mathbb{V})$ chiziqli funksiyalar L ning $\mathbb{V}$ ga o‘ng va chap ta’sirlari va quyidagi ayniyatni qanoatlantiruvchi $\psi: L \times L \to \mathbb{V}$ bichiziqli akslantirish berilgan bo‘lsin:

$$\psi([x,y],z) - \psi(x,[y,z]) - \psi([x,z],y) - l_x\psi(y,z) - r_y\psi(x,z) + r_z\psi(x,y) = 0.$$

Yuqoridagi shartni qanoatlantiruvchi ψ bichiziqli akslantirishlar L ning (l,r) juftlikka mos 2-kotsikli deb ataladi. Barcha 2-kotsikllar to‘plami $Z^2(L,l,r)$ kabi belgilanadi. L ning (l,r) juftlikka mos 2-kochegaralarini quyidagicha aniqlaymiz:

$$df(x,y) = f([x,y]) - l_{\varphi(x)} f(y) - r_{\varphi(y)} f(x),$$

bu yerda $\varphi \in Aut(L)$ va $f \in Hom(L,\mathbb{V})$. Barcha 2-kochegaralar to‘plami $B^2(L,l,r)$ kabi belgilanadi.

$\psi \in Z^2(L,l,r)$ uchun biz $\overline{L} = L \oplus \mathbb{V}$ vektor fazoda $[-,-]_{(l,r,\psi)}$ ko‘paytmani quyidagicha aniqlaymiz:

$$[x+a, y+b]_{(l,r,\psi)} = [x,y]_L + \psi(x,y) + l_x b + r_y a, \ \ x,y \in L, \ a,b \in \mathbb{V}.$$

Ushbu ko‘paytma bilan birgalikda $\left(\overline{L},[-,-]_{l,r,\psi}\right)$ Leybnits algebrasi bo‘ladi va u L algebraning $\mathbb{V}$ orqali kengaytmasi deyiladi.

L yechiluvchan Leybnits algebrasi va N uning nilradikali, hamda $Z(N)$ nilradikalning markazi bo‘lsin. Aytaylik, $\overline{L} = L(\psi,l,r)$ bilan berilgan L algebraning ψ,l,r lar yordamida qurilgan kengaytmasini va $\overline{L}$ algebraning nilradikali esa $\overline{N}$ kabi belgilab olaylik. Bizga $\psi^0 = \psi|_{N\times N}$ va $l^0 = l|_N$, $r^0 = r|_N$ lar berilgan bo‘lsin. Quyidagi belgilashni kiritib olamiz: $G_{\psi^0} = \{X \in N : \psi^0(N,X) = \psi^0(X,N) = (0)\}$.

6-tasdiq. *$\overline{L}$ algebraning nilradikali $\overline{N}$ - N ning $\mathbb{V}$ orqali markaziy kengaytmasi bo‘lishi uchun $N \subseteq \ker l \cap \ker r$ bajarilishi zarur va yetarli. Shuningdek, agar $N \subseteq \ker l \cap \ker r$ o‘rinli bo‘lsa, u holda $Z(\overline{N}) = \mathbb{V}$ bo‘lishi uchun $G_{\psi^0} \cap Z(N) = \{0\}$ tenglik o‘rinli bo‘lishi zarur va yetarli.*

Navbatdagi tasdiqda, ikkita $\overline{L}_1 = L(\psi^1,l^1,r^1)$ va $\overline{L}_2 = L(\psi^2,l^2,r^2)$ kengaytmalar berilgan bo‘lsa ularning izomorf bo‘lish shartlarini keltirib o‘tamiz.

7-tasdiq. *Ikkita $\overline{L}_1 = L(\psi^1,l^1,r^1)$ va $\overline{L}_2 = L(\psi^2,l^2,r^2)$ Leybnits algebralari* *izomorf bo'lishi uchun ixtiyoriy* $x, y \in L, a \in \mathbb{V}$ *elementlar uchun quyidagi shartlarning qanoatlantiruvchi* $\varphi \in Aut(L),\ \phi \in Aut(\mathbb{V})$ *va* $f \in Hom(L, \mathbb{V})$ *lar mavjud bo'lishi zarur va yetarli:*

$$\psi^2(\varphi(x), \varphi(y)) = \varphi(\psi^1(x, y)) + f([x, y]) - l^2_{\varphi(x)} f(y) - r^2_{\varphi(y)},$$

$$l^2_{\varphi(x)} \phi(a) = \phi(l^1_x a), \quad r^2_{\varphi(y)} \phi(a) = \phi(r^1_y a).$$

Aytaylik, L va $\mathbb{V}$ algebralarning avtomorfizmlar gruppalari $Aut(L), Aut(\mathbb{V})$ berilgan bo'lsin. $\bigcup_{l,r} Z^2(L, l, r)$ ustidagi $Aut(L) \times Aut(\mathbb{V})$ ta'sirni quyidagicha aniqlaymiz, bu yerda (l, r) juftlikka mos ψ 2-kotsikl (l', r') juftlikka mos ψ' 2-kotsiklga o'tadi:

$$\psi'(x, y) = \phi(\psi(\varphi(x), \varphi(y))),\ l'_x(a) = \phi(l_{\varphi(x)} \phi^{-1}(a)),\ r'_x(a) = \phi(r_{\varphi(x)} \phi^{-1}(a)).$$

Bundan esa, quyidagicha xulosa qilishimiz mumkin, ya'ni $\overline{L}_1 = L(\psi^1, l^1, r^1)$ va $\overline{L}_2 = L(\psi^2, l^2, r^2)$ Leybnits algebralari izomorf bo'lishi uchun $\psi^2 - \phi \circ \psi^1 \circ \varphi \in B^2(L, l^2, r^2)$ bo'lishi zarur va yetarli. Shuningdek, quyidagi tasdiq o'rinli:

8-tasdiq. *Yechiluvchan Leybnits algebrasi L ning Abel algebrasi* $\mathbb{V}$ *orqali kengaytmalari bo'lgan* $\overline{L}_1 = L(\psi^1, l^1, r^1)$ *va* $\overline{L}_2 = L(\psi^2, l^2, r^2)$ *algebralar berilgan. U holda* $\overline{L}_1$ *va* $\overline{L}_2$ *Leybnits algebralari izomorf bo'lishi uchun* ψ^1 *va* ψ^2 *lar* $\bigcup_{l,r} H^2(L, l, r)$ *dagi* $Aut(L) \times Aut(\mathbb{V})$ *ning bir xil orbitasida yotishi zarur va yetarli.*

Uchinchi bobning ikkinchi bo'limi nol-filiform nilradikalga ega yechiluvchan Leybnits algebrasining kengaytmasini aniqlashga bag'ishlangan.

Ta'kidlab o'tganimizdek, I.A.Karimjonov o'z PhD dissertatsiyasida nilradikali nol-filiform bo'lgan yechiluvchan Leybnits algebrasini tasniflagan va $\{e_1, e_2, \dots, e_n, x\}$ bazisda uning ko'paytmasini quyidagicha bo'lishini isbotlagan:

$$R(NF_n): \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \le i \le n-1, \quad [x, e_1] = e_1, \\ [e_i, x] = -ie_i, & 1 \le i \le n. \end{cases}$$

Biz ushbu algebraning kengaytmasini aniqlab, quyidagi natijaga ega bo'ldik.

9-teorema. $\overline{R}(NF_n)$ *algebra* $R(NF_n)$ *Leybnits algebrasining bir o'lchamli Abel kengaytmasi bo'lsin, u holda* $\overline{R}(NF_n) \cong R(NF_{n+1})$.

Uchinchi bobning uchinchi bo'limida tabiiy usulda gradiurlangan filiform nilradikalga ega va ko-o'lchami maksimal bo'lgan yechiluvchan Leybnits algebrasining Abel kengaytmasi keltirilgan. I.A.Karimjonov o'z ishida bunday algebralarni izomorfizm aniqligida yagona ekanligini keltirib o'tgan. Biz faqat nilradikali F_n^1 bo'lgan yechiluvchan Leybnits algebrasining kengaytmasini qaraymiz. Bu algebraning umumiy ko‘rinishi quyidagicha:

$$R\left(F_n^1\right):\begin{cases}[e_1,x]=e_1, & [x,e_1]=-e_1, \quad [e_i,e_1]=e_{i+1}, \quad 2\le i\le n-1,\\ [e_i,x]=(i-1)e_i, & [e_i,y]=e_i, \qquad 2\le i\le n.\end{cases}$$

Bu bo‘limimizning asosiy natijasini keltirib o‘tamiz.

10-teorema. $\overline{R}\left(F_n^1\right)$ *algebra* $R\left(F_n^1\right)$ *Leybnits algebrasining* $\mathbb{V}=span\{e_{n+1}\}$ *Abel algebrasi yordamida kengaytmasi bo‘lsin. U holda* $\overline{R}\left(F_n^1\right)$ *quyidagi algebraga izomorf bo‘ladi:*

$$\overline{R}\left(F_n^1\right):\begin{cases}[e_1,x]=e_1, & [x,e_1]=-e_1, \quad [e_i,e_1]=e_{i+1}, \quad 2\le i\le n-1,\\ [e_i,x]=(i-1)e_i, & [e_i,y]=e_i, \qquad 2\le i\le n,\\ [e_{n+1},x]=ne_{n+1}, & [e_{n+1},y]=e_{n+1}.\end{cases}$$

To‘rtinchi bo‘limda, nilradikali trivial bo‘lmagan va ko-o‘lchami ikkiga teng bo‘lgan besh o‘lchamli yechiluvchan Leybnits algebrasining Abel kengaytmalarining strukturaviy ko‘rinishlari aniqlangan.

E.M.Kanete va A.X.Xudoyberdiyevlarning ishlarida to‘rt o‘lchamli Leybnits algebralari tasniflangan va barcha besh o‘lchamli Leybnits algebralari aniqlangan. Biz ushbu algebralar ichidan quyidagi trivial bo‘lmagan nilradikalga ega algebralarning Abel kengaytmalarini aniqlaymiz:

$$H:\begin{cases}[e_1,e_2]=e_3, \ [e_2,e_1]=-e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3,\\ [x_1,e_3]=-e_1, \ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2, \ [e_3,x_2]=e_3, \ [x_2,e_3]=-e_3,\end{cases}$$

$$L_1:[e_2,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3, \ [e_2,x_2]=e_2, \ [e_3,x_2]=e_3,$$

$$L_2:[e_2,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=2e_3, \ [e_2,x_2]=e_2,$$

$$L_3:[e_1,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=2e_3, \ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2.$$

Bu algebralarning Abel kengaytmalarini aniqlab, quyidagi natijalarga ega bo‘ldik:

11-teorema. $\overline{H}$ *algebra* H *Leybnits algebrasining* $\mathbb{V}=span\{e_4\}$ *Abel algebrasi yordamida kengaytmasi bo‘lsin. U holda* $\overline{H}$ *quyidagi algebraga izomorf bo‘ladi:*

$$\overline{H}:\begin{cases}[e_1,e_2]=e_3, \ [e_2,e_1]=-e_3, \ [e_1,e_3]=e_4, \ [e_3,e_1]=-e_4, \ [e_1,x_1]=e_1,\\ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3, \ [x_1,e_3]=-e_1, \ [e_4,x_1]=2e_4, \ [x_1,e_4]=-2e_4,\\ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2, \ [e_3,x_2]=e_3, \ [x_2,e_3]=-e_3,\\ [e_4,x_2]=e_4, \ [x_2,e_4]=-e_4.\end{cases}$$

12-teorema. $\overline{L}_1$ *algebra* L_1 *Leybnits algebrasining* $\mathbb{V}=span\{e_4\}$ *Abel algebrasi yordamida kengaytmasi bo‘lsin. U holda* $\overline{L}_1$ *quyidagi algebraga izomorf bo‘ladi:*

$$\overline{L}_1:\begin{cases}[e_2,e_1]=e_3,\ [e_3,e_1]=e_4,\ [e_1,x_1]=e_1, & [x_1,e_1]=-e_1,\ [e_3,x_1]=e_3,\\ [e_2,x_2]=e_2,\ [e_3,x_2]=e_3, [e_4,x_1]=2e_4,\ [e_4,x_2]=e_4. \end{cases}$$

13-teorema. *L_2 va L_3 algebralari uchun $G_{\psi^0}\cap Z(N)=\{0\}$ shartni qanoatlantiruvchi 2-kotsikl mavjud emas. Shuning uchun, bu algebralar yoyilmaydigan Abel kengaytmalarga ega emas.*

XULOSA

Ushbu dissertatsiyada kompleks sonlar maydoni ustida berilgan nilradikali tabiiy usulda gradiurlangan filiform Li algebrasi bo‘lgan yechiluvchan Li algebrasining kengaytmalari olingan va sodda ham yechiluvchan ham bo‘lmagan n-Shrodinger Li algebrasining markaziy kengaytmasi, infinitesimal deformatsiyalari tasniflangan. Bundan tashqari yechiluvchan Leybnits algebralarining Abel kengaytmalarini aniqlash metodi ishlab chiqilgan va ushbu metoddan foydalangan holda, nilradikali nol-filfirom va tabiiy usulda gradiurlangan filiform Leybnits algebralari bo‘lgan yechiluvchan Leybnits algebralarining Abel kengaytmalari aniqlangan. Shuningdek, kichik o‘lchamli nilradikali trivial bo‘lmagan besh o‘lchamli yechiluvchan Leybnits algebralarining ham Abel kengaytmakari tasniflangan.

Dissertatsiyaning asosiy natijalari quyidagilardan iborat:

1. *n*-darajali Shrodinger Li algebrasining markaziy kengaytmasi, infinitezimal deformatsiyalari tasniflangan;
2. Tabiiy usulda gradiurlangan filiform Li algebralarining markaziy kengaytmalari olingan;
3. Nilradikali filiform bo‘lgan yechiluvchan Li algebralarining kengaytmalari aniqlangan;
4. Yechiluvchan Leybnits algebrali ushun Abel kengaytmalar aniqlash metodi ishlab chiqilgan;
5. Nol-filiform nilradikalga ega yechiluvchan Leybnits algebralarining bir o‘lchamli Abel kengaytmalari tasniflangan;
6. Nilradikali tabiiy usulda gradiurlangan maksimal ko-o‘lchamga ega yechiluvchan Leybnits algebralarining Abel kengaytmalari olingan;
7. Besh o‘chamli yechiluvchan Leybnits algebralarining Abel kengaytmasi aniqlangan.

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INSTITUTE OF MATHEMATICS

SHERALIYEVA SURAYYO ABDIKODIR KIZI

CENTRAL AND ABELIAN EXTENSIONS OF SOLVABLE LIE AND LEIBNIZ ALGEBRAS WITH FILIFORM NILRADICAL

01.01.06 – algebra

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2025

The theme of the dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2025.2.PhD/FM1304.

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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of the dissertation. At present, a considerable part of scientific and applied research carried out worldwide is devoted to the study of algebraic structures, their interrelations, and their intrinsic properties. In particular, the investigation of the structure, classification, and mutual relationships of various algebraic systems is of fundamental theoretical importance and is closely interconnected with other areas of mathematics. From this standpoint, the study of extensions of algebraic structures constitutes an essential tool for deepening existing theories, constructing new algebraic models, and gaining a more precise understanding of the internal structure of complex algebraic systems. Such investigations contribute to the systematic development and generalization of algebraic methods.

The theory of algebraic extensions and the methods for their analysis are of special significance, especially in the context of solvable algebraic systems. Through the study of extensions, new structural properties of algebras can be revealed, and classification problems can be approached from a broader and more refined perspective. Furthermore, these methods foster the advancement of the theory of algebraic structures and extend their applicability to other branches of mathematics. Consequently, the investigation of algebraic extensions plays a crucial role in the in-depth study of algebraic systems and in the further enrichment of modern algebraic theory.

In recent years, increasing attention has been directed in our country toward the fundamental sciences-mathematics, physics, geology, and biology-which possess both scientific and practical significance. In particular, algebraic structures have continued to find new applications across various branches of mathematics, especially in geometry, topology, and physics. To date, substantial results have been achieved in the classification of nilpotent and solvable algebras of various dimensions, including Lie and Leibniz algebras, as well as in the algebraic and geometric classification of low-dimensional nonassociative algebras. Conducting research in "Algebra and Functional Analysis" at a level consistent with international standards has been designated as a primary objective and direction of activity for the mathematical sciences[1]. In fulfilling this mandate, the development of the theory of finite-dimensional nonassociative algebras and their classification plays a crucial role, as it enables the application of scientific results in related branches of the discipline and ensures the implementation of the tasks set forth in the aforementioned decision.

The subject and object of this dissertation align with the tasks outlined in the Decrees of the President of the Republic of Uzbekistan UP-4947 dated February 7, 2017 "On the strategy of action for the further development of the Republic of Uzbekistan", UP-2789 dated February 17, 2017 "On measures to further improvement of the activities of the Academy of Sciences, organization,

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[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292 dated May 17, 2017.

management, and financing of research activities, PP-4387 dated July 9, 2019 "On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics", UP-4708 dated May 7, 2020 "Quality of education in the field of mathematics", PP-60 dated January 28, 2022, "On the Development Strategy of New Uzbekistan for 2022-2026", and furthermore, regulations that encompass this research.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, Mathematics, Mechanics and Computer Science.

The degree of scrutiny of the problem. The theory of Lie algebras was founded in the late 19th century by the Norwegian mathematician M. Sophus Lie, who introduced it into mathematics for the purpose of studying infinitesimal transformations. Subsequently, fundamental results concerning the structure and classification of Lie algebras, as well as the classes of solvable and nilpotent algebras, were obtained by W. Killing and É. Cartan. On this basis, the structural theory of Lie algebras became a central tool in mathematical physics, geometry, and analysis. Beginning in the mid-20th century, intensive research commenced on the analysis of nilpotent and solvable Lie algebras and on the study of their extensions. Scholars such as A.I.Mal'cev, N.Jacobson, I.Kaplansky, A.Kostrikin, M.Vergne, D.Panyushev, and others investigated the structural properties of nilpotent and solvable algebras, as well as their central extensions and deformations.

The central extension method formulated by Skjelbred and Sund for the study of nilpotent Lie algebras has provided a powerful framework enabling the classification of numerous families of nilpotent algebraic structures. The essence of this approach lies in constructing higher-dimensional algebras from lower-dimensional ones of the same degree, thereby reducing classification problems to the analysis of central cocycles and second cohomology spaces. By employing this methodology, significant classification results have been obtained for various classes of nilpotent algebras. In particular, nilpotent five-dimensional Jordan algebras, five-dimensional commutative algebras, six-dimensional Malcev algebras, and six-dimensional binary Lie algebras have been systematically classified in the works of V.De Graaf, A.Hegazi, H.Abdelwahab, A.Calderón, A.Martín, I.Kaygorodov, and others.

Since solvable Lie algebras play a significant role in the theory of finite-dimensional Lie algebras, various methods have been developed for their classification. One of the effective approaches is the method proposed by Mubarakzhanov, who introduced a technique for studying solvable Lie algebras via their nilradicals and the nil-independent derivations of these nilradicals. Building on Mubarakzhanov's results, complete classifications of solvable Lie algebras with prescribed nilradicals-such as Abelian, filiform, quasi-filiform, and other types-have been established in the works of X. M. Ancochea Bermúdez, R. Campoamor-

Stursberg, L. García Vergnolle, L. Šnobl, D. Karasek, J. S. Ndogmo, S. Tremblay, and P. Winternitz. Because the determination of central extensions of solvable Lie algebras does not, in general, allow for a complete recovery of algebras with trivial center, Sund introduced a generalization of the central extension method specifically adapted to solvable Lie algebras.

Leibniz algebras, introduced by Jean-Louis Loday as a natural generalization of Lie algebras, opened a new direction in the theory of algebraic structures. Since the multiplication in Leibniz algebras is not antisymmetric, their cohomology and extension theories differ significantly from those of Lie algebras. Consequently, the study of central extensions, deformations, and isomorphism classes of Leibniz algebras has become an independent and important research problem. In this context, the works of J.-L.Loday, T.Pirashvili, D.Barnes, M.Ladra, J.Casas, A.S.Dzhumadildaev, Sh.A.Ayupov, I.S.Rakhimov, B.A.Omirov, A.Kh.Khudoyberdiyev, and many others have played a significant role. These studies analyzed the structural invariants of Leibniz algebras, their central extensions, deformations, cohomology, and degenerations. Moreover, abelian extensions for Leibniz algebras were first considered in the works of M.Casas, E.Faro, and A.M.Viyetes, while non-abelian extensions were classified by J. Liu, Y. Sheng, and Q. Wang.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This dissertation has been conducted in alignment with the research theme of the Institute of Mathematics named after V.I.Romanovskiy.

The aim of research work is to develop a comprehensive approach for the classification and structural analysis of finite-dimensional solvable Lie and Leibniz algebras whose nilradicals are filiform, through the study of their extensions and abelian extensions.

Research problems:

to apply and generalize the method of central and abelian extensions in the classification of nilpotent and solvable algebras;

to describe one-dimensional extensions of solvable Lie algebras;

to determine the central extensions and the infinitesimal deformations of the *n*-th Schrödinger algebra;

to determine all non-isomorphic abelian extensions of low-dimensional solvable Leibniz algebras with filiform nilradicals.

The research object: nilpotent Lie algebras, solvable Lie algebras, *n*-th Schrödinger algebra, solvable Leibniz algebras.

The research subject: the central extension and infinitesimal deformations of *n*-th Schrödinger algebra, solvable Lie algebra whose nilradical is filiform, solvable Leibniz algebra whose nilradical is null-filiform, and those naturally graded with a filiform nilradical; five-dimensional solvable Leibniz algebras.

Research methods: methods of extensions, as well as the methods of invariant theory, are applied in the dissertation.

Scientific novelty of the research work consists of the following:

the central extension and infinitesimal deformations of the *n*-th Schrödinger algebra are determined;

all one-dimensional central extensions of naturally graded filiform Lie algebras are found;

the one-dimensional extensions of solvable Lie algebras with filiform nilradicals are classified, and their complete structures are described;

method for constructing abelian extensions of solvable Leibniz algebras has been developed, and the abelian extensions of five-dimensional solvable Leibniz algebras have been determined.

Practical results of the research. The taken results and used methods in the dissertation can be taught as a graduate course for masters and doctoral students of higher education institutions. Furthermore, the results of the study are directly relevant to the full classification of solvable Lie and Leibniz algebras and contribute significantly to the structural theory of algebraic systems.

The reliability of the results of the study. The rigor of the mathematical arguments is ensured through the use of established methods from related classes of algebras, the application of cohomology theory, and reliance on fundamental results from the structural theory of algebras. In addition, specialized programs developed in the Mathematica 13 programming environment were employed to verify the results concerning derivations and automorphisms of low-dimensional algebras.

Scientific and practical significance of the research results. This research contributes to the deepening of the theoretical foundations of solvable Lie and Leibniz algebras. The methodologies developed and the results obtained throughout the work make a significant contribution to algebraic theory and provide a rigorous theoretical basis for future investigations in this field. From an applied perspective, the outcomes of this study have potential applications in areas such as mathematical physics, control theory, and cryptography, where the algebraic structures examined here play an important role.

All results and methods presented in the dissertation may also be utilized in specialized courses for master’s and doctoral students at institutions of higher education.

Implementation of the research results. Based on the results obtained on central and abelian extensions of solvable Lie and Leibniz algebras with filiform nilradicals:

the extension method for solvable Lie and Leibniz algebras, as well as one-dimensional extensions of solvable Lie algebras with filiform nilradicals, were applied in the international project AP14870282 entitled “Identification of identifiers for classes of non-associative algebras” with the aim of generalizing these results to other classes of non-associative algebras (certificate of Astana IT University No. 1/1529 dated November 5, 2025, Republic of Kazakhstan). The application of the obtained scientific results made it possible to determine abelian extensions of binary and unary Leibniz algebras;

the methods developed for central and abelian extensions of low-dimensional solvable Lie and Leibniz algebras, together with their classifications, were used in the fundamental project F-FA-2021-423 entitled “Automorphisms of operator algebras, classification of infinite-dimensional non-associative algebras and superalgebras” to improve modern approaches to the classification of non-associative algebraic structures (certificate of the Institute of Mathematics No. 2/505 dated December 3, 2025). The application of these scientific results enabled the derivation of algebraic and geometric classifications of five-dimensional nilpotent Leibniz algebras.

Approbation of the research results. The main results of the research have been discussed at 5 international and 3 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 14 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis. In addition, 2 of them were published in international journals, and 4 papers were published in national mathematical journals.

The structure and volume of the dissertation. The dissertation consists of the introduction, three chapters with conclusions, a general conclusion, and a bibliography. The total volume of the work is 106 pages.

THE MAIN CONTENT OF THE DISSERTATION

This dissertation is structured as follows: an introduction, three main chapters, a conclusion, and a bibliography.

The **Introduction** part of the dissertation includes the actuality and the demand of the research, the relevance of the research to the priority areas of science and technology. Moreover, in this part, the degree of scrutiny of the problem, the aim of the research work, the object and subject of the research, and the scientific novelty of the research work are presented. The theoretical and practical significance of the research is revealed, and implementation and approbation of the research results, publications of the research results, and the structure and volume of the dissertation are presented in this part.

The first chapter of the dissertation, entitled **“Basic concepts of Lie and Leibniz algebras”**, is devoted to the foundational definitions of Lie and Leibniz algebras, as well as the core concepts shared by both structures. The chapter also outlines the structural properties of the main algebraic objects employed throughout the subsequent chapters. Furthermore, it provides an overview of the cohomology theory of Lie algebras, methods for constructing central extensions of nilpotent Lie algebras, and techniques for determining extensions of solvable Lie algebras. In addition, the chapter presents essential definitions and theorems related to cohomology spaces, together with the results obtained on the cohomological structures of Schrödinger algebras.

The first section of Chapter one presents the preliminary concepts, the definitions of Lie and Leibniz algebras, as well as methods for determining their extensions.

Definition 1. *An algebra (G,[-,-]) over a field* $\mathbb{F}$ *is called a Lie algebra if for any* $x,y,z\in G$ *the following identities*

$$[x,[y,z]]+[y,[z,x]]+[z,[x,y]]=0,$$
$$[x,x]=0,$$

hold.

Definition 2. *A vector space with a bilinear multiplication (L,[-,-]) is called a Leibniz algebra if for any* $x,y,z\in L$ *the so-called Leibniz identity*

$$[[x,y],z]=[x,[y,z]]+[[x,z],y],$$

holds.

For an arbitrary Lie (Leibniz) algebra *G*, we define the *derived* and *central series* as follows:

$$G^{[1]}=[G,G],\ G^{[s+1]}=\left[G^{[s]},G^{[s]}\right],\quad s\geq 1,\quad G^{1}=[G,G],\ G^{k+1}=\left[G^{k},G^{k}\right],\ k\geq 1.$$

Definition 3. *Any n-dimensional Lie (Leibniz) algebra G is called solvable (nilpotent) if there exists* $s\in\mathbb{N}$ *(*$k\in\mathbb{N}$*) such that* $G^{[s]}=0$ *(*$G^{k}=0$*). Such a minimal number is called the index of solvability (nilpotency).*

The maximal nilpotent ideal of a Lie algebra is said to be the nilradical of the algebra.

Definition 4. *Any n-dimensional Lie (Leibniz) algebra G is said to be filiform if* $\dim G^{i}=n-i$*, for* $2\leq i\leq n$.

Definition 5. *Any n-dimensional Leibniz algebra* L *is said to be null-filiform if* $\dim L^{i}=n+1-i, 1\leq i\leq n+1$.

It is clear that null-filiform Leibniz algebras have a maximal index of nilpotency. It has been proven by Sh.A. Ayupov and B.A. Omirov that the *n*-dimensional null-filiform Leibniz algebra is unique up to isomorphism, and its multiplication is given as follows:

$$NF_n:[e_i,e_1]=e_{i+1},\quad 1\leq i\leq n-1,$$

where $\{e_1,e_2,\ldots,e_n\}$ is a basis of the algebra NF_n.

I.A. Karimjonov classified solvable Leibniz algebras with a nilradical and proved that their multiplication in a chosen basis has the following form:

$$R(NF_n):\begin{cases}[e_i,e_1]=e_{i+1},\quad 1\leq i\leq n-1,\ [x,e_1]=e_1,\\ [e_i,x]=-ie_i,\quad 1\leq i\leq n.\end{cases}$$

Definition 6. *Given a nilpotent Lie (Leibniz) algebra G with nilindex s put* $G_i=G^{i}/G^{i+1},\ 1\leq i\leq s-1$ *and* $Gr(G)=G_1\oplus G_2\oplus\ldots\oplus G_{s-1}$*. Define the product in the vector space Gr(G) as follows:*

$$\left[x+G^{i+1},y+G^{j+1}\right]=[x,y]+G^{i+j+1},$$

where $x\in G^{i}\setminus G^{i+1},\ y\in G^{j}\setminus G^{j+1}$*. Then* $\left[G_i,G_j\right]\subseteq G_{i+j}$ *and we obtain the graded algebra Gr(G). If Gr(G) and G are isomorphic, then we say that the algebra G is* *naturally graded.*

M. Vergne characterized two classes of naturally graded Lie algebras and described their general structure as follows:

$$n_{n,1}: \quad [e_i,e_1] = -[e_1,e_i] = e_{i+1}, \qquad 2 \le i \le n-1,$$

$$Q_{2n}: \quad \begin{cases} [e_i,e_1] = -[e_1,e_i] = e_{i+1}, & 2 \le i \le 2n-2, \\ [e_i,e_{2n+1-i}] = -[e_{2n+1-i},e_i] = (-1)^i e_{2n}, & 2 \le i \le n. \end{cases}$$

In fact, the second type will appear only in the case when the dimension of the algebra is even.

A.Sh. Ayupov and B.A. Omirov classified naturally graded non-Lie Leibniz algebras and obtained the following algebras:

$$F_n^1: [e_1,e_1] = e_3, \ [e_i,e_1] = e_{i+1}, \ 2 \le i \le n-1,$$

$$F_n^2: [e_1,e_1] = e_3, \ [e_i,e_1] = e_{i+1}, \ 3 \le i \le n-1.$$

In the subsequent chapters, we investigate the extensions of the solvable Lie and Leibniz algebras whose nilradicals are nontrivial, as previously outlined.

Definition 7. *A representation of a Leibniz algebra (L,[-,-]) is a triple* $(\mathbb{V},l,r)$, *where* $\mathbb{V}$ *is a vector space equipped with two linear maps* $l: L \to gl(\mathbb{V})$ *and* $r: L \to gl(\mathbb{V})$ *such that the following equalities hold:*

$$r_{[x,y]} = r_y \circ r_x - r_x \circ r_y, \quad l_{[x,y]} = r_y \circ l_x - l_x \circ r_y, \quad l_x \circ l_y = -l_x \circ r_y, \ \forall x,y \in L.$$

In particular, if $r_x = -l_x$, for all $x \in L$, then the representation $(\mathbb{V},l,r)$, reduces to a representation of a Lie algebra.

Now we consider the space of $C^0(G,M) = M$, $C^n(G,M) = Hom(G^{\otimes n},M)$, $n > 0$ structures for the module *M* over the Lie algebra *G*. Let $d^n: C^n(G,M) \to C^{n+1}(G,M)$ be a homomorphism defined as

$$d^n\psi(e_0,\dots,e_n) = \sum_{i=0}^{n+1}(-1)^i \psi(e_0,\dots,\hat{e}_i,\dots,e_n) +$$
$$\sum_{1\le i<j\le n}(-1)^{i+j}\psi([e_i,e_j],e_0,\dots,\hat{e}_i,\dots,\hat{e}_j,\dots,e_n),$$

where $e_0,\dots,e_n \in G$ and the sign $\hat{}$ indicates that the argument below it must be omitted. Set $Z^n(G,M) = Ker(d^{n+1})$ and $B^n(G,M) = \mathrm{Im}(d^n)$. The property $d^{n+1} \circ d^n = 0$ implies that the total differential $d = \sum_{i\ge 0} d^i$ satisfies $d \circ d = 0$. Therefore, the *n*-th cohomology space $H^n(G,M) = Z^n(G,M)/B^n(G,M)$ is well-defined.

We emphasize that for *n=2* if *M* is a trivial *G*-module, then the second cohomology group $H^2(G,M)$ characterizes the *central extensions* of the Lie algebra *G* In contrast, when *M*=*G* the group $H^2(G,G)$ classifies the *infinitesimal deformations* of *G*.

If *R* is an ideal of *G* then we define on each $C^n(R,M)$ the structure of an *G*-

module. For *n>0*, we define the action of G on $C^n(R,M)$ as follows:

$$(v.\psi)(e_1,\ldots,e_n) = v.\psi(e_1,\ldots,e_n) - \sum_{i=1}^{n}\psi(e_1,\ldots,e_{i-1},[v,e_i],e_{i+1},\ldots,e_n),$$

where $v\in G,\ \psi\in C^n(R,M)$ and $e_1,\ldots,e_n\in R$.

We now proceed to describe the method of constructing central extensions for nilpotent Lie algebras.

Let *(G,[-,-])* be a nilpotent Lie algebra over $\mathbb{C}$ and $\mathbb{V}$ be a vector space. The $\mathbb{C}$-linear space $Z^2(G,\mathbb{V})$is define as the set of all anti-symmetric bilinear maps $\psi: G\times G\to\mathbb{V}$ such that

$$\psi(x,[y,z])+\psi(y,[z,x])+\psi(z,[x,y])=0,\ \ x,y,z\in G.$$

These elements will be called 2-cocycles. The space of all 2-cocycles is denoted by $Z^2(G,\mathbb{V})$.

For a linear mapping $f: G\to\mathbb{V}$, if we define $df: G\times G\to\mathbb{V}$ by $df(x,y)=f([x,y])$, then $df\in Z^2(G,\mathbb{V})$. We define the set of 2-coboundaries as follows $B^2(G,\mathbb{V})=\{df \mid f\in Hom(G,\mathbb{V})\}$. Define the second cohomology space $H^2(G,\mathbb{V})$ as the quotient $B^2(G,\mathbb{V})/Z^2(G,\mathbb{V})$.

For $\psi\in Z^2(G,\mathbb{V})$ define on the linear space $\overline{G}=G\oplus\mathbb{V}$ the bilinear product $[-,-]_\psi$ by

$$[x+u,y+v]_\psi=[x,y]+\psi(x,y),\ \ \forall x,y\in G,\ u,v\in\mathbb{V}.$$

The algebra $G_\psi=\left(\overline{G},[-,-]_\psi\right)$ is called a *central extension* of G by $\mathbb{V}$. Let $Aut(G)$ be an automorphism group of G and let $\varphi\in Aut(G)$. For $\psi\in Z^2(G,\mathbb{V})$ define the action of the group $Aut(G)$ on $Z^2(G,\mathbb{V})$by

$$\varphi.\psi(x,y)=\psi(\varphi(x),\varphi(x)).$$

It is easy to verify that $B^2(G,\mathbb{V})$ is invariant under the action of $Aut(G)$. So, we have an induced action of $Aut(G)$ on $H^2(G,\mathbb{V})$.

Call the set $Ann(\psi)=\{x\in G:\psi(x,G)=0\}$ the *annihilator of* ψ. We recall that the *annihilator* of an algebra G is defined as the ideal $Ann(G)=\{x\in G: xG=0\}$. Observe that $Ann(G_\psi)=(Ann(\psi)\cap Ann(G))\oplus\mathbb{V}$.

The following result was proved for Jordan algebras by A.Hegazi and H.Abdelwahab. However, it can be applied to other types of non-associative algebras. It shows that every algebra with a non-zero annihilator is a central extension of a smaller-dimensional algebra.

Proposition 1. *Let G be an n-dimensional algebra of a certain type such that* $\dim(Ann(G))=m\neq 0$. *Then there exists, up to isomorphism, a unique* $(n-m)$-

dimensional algebra G' of the same type and a bilinear map $\psi \in Z^2(G',\mathbb{V})$ with $Ann(\psi) \cap Ann(G') = 0$, where $\mathbb{V}$ is a vector space of dimension m, such that $G \cong G'_\psi$ and $G / Ann(G) \cong G'$.

Let us fix a bases $e_1,\ldots,e_s$ of $\mathbb{V}$, and $\psi \in Z^2(G,\mathbb{V})$. Then ψ can be uniquely written as $\psi(x,y) = \sum_{i=1}^{s} \psi_i(x,y)e_i$, where $\psi_i \in Z^2(G,\mathbb{V})$. Moreover, $Ann(\psi) = Ann(\psi_1) \cap Ann(\psi_2) \cap \ldots \cap Ann(\psi_s)$. Furthermore, $\psi \in B^2(G,\mathbb{V})$ if and only if all $\psi_i \in B^2(G,\mathbb{V})$.

Proposition 2. *Let $\psi(x,y) = \sum_{i=1}^{s} \psi_i(x,y)e_i \in Z^2(G,\mathbb{V})$ and $Ann(\psi) \cap Ann(G) = 0$, then G_ψ has an annihilator component if and only if $[\psi_1],[\psi_1],\ldots,[\psi_s]$ are linearly dependent in $H^2(G,\mathbb{V})$.*

The next method we consider is the extension method for solvable Lie algebras, as presented by T. Sund in his works. Let G be a solvable Lie algebra and $\mathbb{V}$ be a vector space. Let $\theta: G \to gl(\mathbb{V})$ a representation, $\psi: G \times G \to \mathbb{V}$ an anti-symmetric bilinear mapping satisfying the condition

$$\psi(x,[y,z]) + \psi(z,[x,y]) + \psi(y,[z,x]) +$$
$$\theta(x)\psi(y,z) + \theta(z)\psi(x,y) + \theta(y)\psi(z,x) = 0,$$

where $x,y,z \in G$.

The bilinear mapping satisfying the previous condition is said to be a 2-cocycle on G with respect to θ. The set of all such 2-cocycles is denoted by $Z^2(G,\theta,\mathbb{V})$. The 2-coboundaries on G with respect to θ are defined as

$$df(x,y) = f([x,y]) + \theta(\varphi(y))(f(x)) - \theta(\varphi(x))(f(y))$$

for some linear map $f: G \to \mathbb{V}$ and $\varphi \in Aut(G)$. The set of all such 2-coboundaries is denoted by $B^2(G,\theta,\mathbb{V})$ and it is a subset of $Z^2(G,\theta,\mathbb{V})$. A factor space $Z^2(G,\theta,\mathbb{V})$. by $B^2(G,\theta,\mathbb{V})$is denoted as $H^2(G,\theta,\mathbb{V})$.

On the vector space $\overline{G} = G \oplus \mathbb{V}$ define Lie algebra structure $G(\psi,\theta)$ for the given $\psi \in Z^2(G,\theta,\mathbb{V})$ as follows:

$$[x+u, y+v] = [x,y] + \psi(x,y) + \theta(x)v - \theta(y)u,$$

for all $u,v \in \mathbb{V}$, $x,y \in G$. Note that the algebra $G(\psi,\theta)$ is an extension of G by $\mathbb{V}$.

Let us denote by N the nilradical of G and by $Z(G)$ the center of N. We study Lie algebras $\overline{G} = G(\psi,\theta)$ for which the nilradical $\overline{N}$ is central extension of N by $\mathbb{V}$. Denote $\psi^0 = \psi|_{N\times N}$, $\theta^0 = \theta|_N$, $Ann(\psi^0) = \{x \in N \mid \psi^0(x,N) = 0\}$.

The following proposition was established by T. Sund for solvable Lie algebras.

Proposition 3. *Let* $\overline{G} = G(\psi,\theta)$ *be an extension of G by* $\mathbb{V}$. *Then*

- The nilradical $\overline{N}$ of $\overline{G}$ is a central extension of N by $\mathbb{V}$ if and only if $Ker(\theta) \supset N$.
- Let $Ker(\theta) \supset N$. Then $Z(\overline{N}) = \mathbb{V}$ if and only if $Ann(\psi^0) \cap Z(N) = 0$.

Suppose we are given two $G(\psi_1,\theta_1)$ and $G(\psi_2,\theta_2)$ extensions of the algebra G. They are isomorphic if and only if the following equations hold:

$$\theta_2(\alpha(x))(\beta(a)) = \beta(\theta_1(x)(a)),$$

$$\psi_2(\alpha(x),\alpha(y)) - \theta_2(\alpha(y))(f(x)) + \theta_2(\alpha(x))(f(y)) = \beta(\psi_1(x,y)) + f([x,y]),$$

where $\alpha \in Aut(G)$, $\beta \in Aut(\mathbb{V})$, $f \in Hom(G,\mathbb{V})$.

Using the above equalities, we obtain an action $Aut(G) \times Aut(\mathbb{V})$ to the set $\bigcup_\theta Z^2(G,\theta,\mathbb{V})$ in which 2-cocycle ψ with respect to θ acts 2-cocycle ψ' with respect to θ' as follows:

$$\theta'(x)(a) = \beta(\theta(\alpha(x))(\beta^{-1}(a))),\quad \psi'(x,y) = \beta(\psi(\alpha(x),\alpha(y))).$$

T. Sund formulated the following necessary and sufficient condition for the isomorphism of two extensions $G(\psi_1,\theta_1)$ and $G(\psi_2,\theta_2)$ of a solvable Lie algebra.

Proposition 4. *Two extensions* $G(\psi_1,\theta_1)$ *and* $G(\psi_2,\theta_2)$ *are isomorphic if and only if there exist* $\alpha \in Aut(G)$, $\beta \in Aut(\mathbb{V})$ *such that* $\psi_2 - \beta \circ \psi_1 \circ \alpha \in B^2(G,\theta_2,\mathbb{V})$ *and* β *is an intertwining operator for representations* θ_2 *and* $\theta_1 \circ \alpha$.

The second section of Chapter one is devoted to the study of the central extensions and second cohomology spaces of *n*-th Schrödinger algebras.

The *n*-th Schrödinger algebra is defined as the semidirect product of the three-dimensional simple Lie algebra and the Heisenberg algebra. In the basis $\{e,h,f,z,x_i,y_i,1 \le i \le n\}$, equipped with the following non-trivial commutation relations:

$$[h,e] = 2e,\quad [h,f] = -2f,\ [e,f] = h,\quad [h,x_i] = x_i,$$

$$[h,y_i] = -y_i,\quad [e,y_i] = x_i,\quad [f,x_i] = y_i,\ [x_i,y_i] = z.$$

In the case *n=1*, the second cohomology group was computed by Y. Zh. Wu, X. Q. Yue, and L. Sh. Zhu, and shown to be trivial, i.e., $\dim(H^2(sch_1,\mathbb{C})) = 0$. Hence, we focus on the case $n \ge 2$.

Theorem 1. *For the n-th Schrödinger algebra* sch_n, *the dimensional of 2-cohomology space is equal to* $\frac{n(n+1)}{2} - 1$ *for all* $n \ge 2$.

Theorem 2. $\dim(H^2(sch_2,sch_2)) = 1$ *and* $H^2(sch_n,sch_n) = 0,\ n \ge 3$.

Corollary 1. *The alternating bilinear map* ψ, *defined as below, forms a basis* *on*

$$\psi(x_1,x_2)=2e,\ \psi(y_1,y_2)=-2f,\ \psi(x_1,y_2)=-h,\ \ \psi(x_2,y_1)=h,$$
$$\psi(x_1,z)=3x_2,\ \psi(y_1,z)=3y_2,\quad \psi(x_2,z)=-3x_1,\ \ \psi(y_2,z)=-3y_1.$$

The second chapter of the dissertation, entitled "**Extensions of solvable Lie algebras**" is devoted to the classification of central extensions of naturally graded filiform Lie algebras and to the classification of extensions of solvable Lie algebras whose nilradical is a naturally graded filiform Lie algebra.

As is well known, L. Šnobl and P. Winternitz classified solvable Lie algebras whose nilradical is $n_{n,1}$, whereas J. M. Ancochea Bermúdez, R. Campoamor-Stursberg, and L. García Vergnolle studied those with nilradical Q_{2n}.

In the first section of the second chapter, we determine the central extensions of the nilpotent Lie algebras $n_{n,1}$ and Q_{2n}.

Now, we present the main result of this section.

Theorem 3. *An arbitrary non-split central extension of the algebra* $n_{n,1}$ *is isomorphic to one of the following pairwise non-isomorphic algebras:*

$$n_{n+1,1},\ Q_{n+1},\ \text{and}\ \overline{G}_k\left(2\le k\le\left\lfloor\frac{n}{2}\right\rfloor\right):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n,\\ [e_i,e_{2k+1-i}]=(-1)^i e_{n+1}, & 2\le i\le n.\end{cases}$$

Remark 1. *The algebra* Q_{n+1} *appears only in case of n being odd.*

In the second section of the second chapter, we give an algebraic of extensions of solvable Lie algebras with a filiform nilradical of maximal dimension. It is known that there are exactly two such algebras, which are defined by the following multiplication rules:

$$s_{n,2}:\begin{cases}[e_i,e_1]=e_{i+1},\ \ 2\le i\le n-1,\ \ [e_1,x_1]=e_1,\\ [e_i,x_1]=(i-2)e_i,\ \ 3\le i\le n,\ [e_i,x_2]=e_i,\ \ 2\le i\le n,\end{cases}$$

$$\tau_{2n,2}:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\ \ [e_i,e_{2n+1-i}]=(-1)^i e_{2n},\ \ 2\le i\le n,\\ [e_i,x_1]=ie_i, & [e_i,x_2]=e_i,\quad 1\le i\le 2n-1,\\ [e_{2n},x_1]=(2n+1)e_{2n}, & [e_{2n},x_2]=2e_{2n},\ \ [x_2,e_{2n}]=-2e_{2n},\end{cases}$$

where, $s_{n,2}$ is the algebra with nilradical $n_{n,1}$, while $\tau_{2n,2}$ is the algebra whose nilradical is Q_{2n}. Since the algebra Q_{2n} does not admit a non-trivial central extension, the corresponding solvable Lie algebra also does not possess a non-trivial extension of this type. Therefore, we focus solely on determining the extensions of the algebra $s_{n,2}$.

Theorem 4. *Let* $\overline{G}$ *be a one-dimensional non-split extension of the solvable Lie algebra* $s_{n,2}$ *then* $\overline{G}$ *is isomorphic to one of the algebras* $s_{n+1,2}$ *and* $\tau_{n+1,2}$.

In the third section of the second chapter, we intend to obtain full classification of one-dimensional extensions of algebras whose nilradicals are $n_{n,1}$, and Q_{2n} with codimension one. The multiplication rules for these algebras are given as follows:

$$s_{n,1}^1(\beta):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n-1, \quad [e_1,x]=e_1,\\ [e_i,x]=(i-2+\beta)e_i, & 2\le i\le n,\end{cases} \quad s_{n,1}^2:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n-1,\\ [e_i,x]=e_i, & 2\le i\le n,\end{cases}$$

$$s_{n,1}^3:[e_i,e_1]=e_{i+1},\ 2\le i\le n-1,\quad [e_1,x]=e_1+e_2,\quad [e_i,x]=(i-1)e_i,\quad 2\le i\le n,$$

$$s_{n,1}^4(\alpha_3,\alpha_4,\dots,\alpha_{n-1}):[e_i,e_1]=e_{i+1},\quad 2\le i\le n-1,\ [e_i,x]=e_i+\sum_{l=i+2}^{n}\alpha_{l+1-i}e_l,,\quad 2\le i\le n,$$

$$\tau_{2n,1}^1(\alpha):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\ [e_i,e_{2n+1-i}]=(-1)^i e_{2n},\quad 2\le i\le n,\\ [e_i,x]=(i-2+\alpha)e_i, & 2\le i\le 2n-1,\ [e_1,x]=e_1,\\ [e_{2n},x]=(2n+3+2\alpha)e_{2n}, \end{cases}$$

$$\tau_{2n,1}^2:\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\quad [e_i,e_{2n+1-i}]=(-1)^i e_{2n},\quad 2\le i\le n,\\ [e_i,x]=(i-n)e_i, & 2\le i\le 2n-1,\quad [e_1,x]=e_1+e_{2n},\quad [e_{2n},x]=e_{2n},\end{cases}$$

$$\tau_{2n,1}^3(\alpha_4,\dots,\alpha_{2n-2}):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le 2n-2,\ [e_i,e_{2n+1-i}]=(-1)^i e_{2n},\ 2\le i\le n,\\ [e_{i+2},x]=e_{i+2}+\sum_{k=2}^{\left\lfloor\frac{2n+3-i}{2}\right\rfloor}\alpha_{2k}e_{2k+1+i}, & 0\le i\le 2n-3,\quad [e_{2n},x]=2e_{2n}.\end{cases}$$

We consider only the extensions of solvable Lie algebras whose nilradical is $n_{n,1}$.

Here is the main results of this section.

Theorem 5. *Let $\overline{G}$ be a one-dimensional extension of the solvable Lie algebra $s_{n,1}^1(\beta)$ then $\overline{G}$ is isomorphic to one of the following algebras $s_{n,1}^1(\beta)$, $\tau_{n+1,1}^1(\beta)$, $\tau_{n+1,1}^2$ and*

$$\overline{G}_k\left(2\le k\le\left\lfloor\frac{n}{2}\right\rfloor\right):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\le i\le n,\quad [e_1,x]=e_1,\\ [e_i,e_{2k+1-i}]=(-1)^i e_{n+1}, & 2\le i\le k,\\ [e_i,x]=(n+i-2k)e_k, & 2\le i\le n+1.\end{cases}$$

Theorem 6. *Let $\overline{G}$ be a one-dimensional extension of the solvable Lie algebra $s_{n,1}^2$ then $\overline{G}$ is isomorphic to the following algebras $s_{n+1,1}^2$, $s_{n+1,1}^4(0,0,\dots,1)$, $\tau_{n+1,1}^3(0,0,\dots,0)$.*

Theorem 7. *Any one-dimensional non-split extension of the solvable Lie algebra $s_{n,1}^3$ is isomorphic to the algebra $s_{n+1,1}^3$.*

Theorem 8. *One-dimensional non-split extensions of the solvable Lie algebra $s_{n,1}^4(\alpha_3,\alpha_4,\dots,\alpha_{n-1})$ are the algebras $s_{n+1,1}^4(\alpha_3,\alpha_4,\dots,\alpha_{n-1},\alpha_n)$ and $\tau_{n+1,1}^3(\alpha_4,\alpha_6,\dots,\alpha_{n-1})$.*

The third chapter of the dissertation,. entitled **“Extensions of solvable Leibniz algebras”**, rigorously develops a method for computing abelian extensions of solvable Leibniz algebras. Employing this approach, a comprehensive classification of abelian extensions has been achieved for solvable Leibniz algebras whose nilradicals are either null-filiform or naturally graded filiform. Additionally, the study identifies and characterizes one-dimensional abelian extensions of five-dimensional solvable Leibniz algebras possessing a non-trivial nilradical structure.

The first section of chapter three is devoted to the method of determining abelian extensions for solvable Leibniz algebras. In their work, J. Liu, Y. Sheng, and Q. Wang introduced a method for constructing non-abelian extensions of solvable left Leibniz algebras. Inspired by their approach, we develop a method for determining abelian extensions of solvable right Leibniz algebras.

Let L, $\mathbb{V}$ be Leibniz algebras, and let $l: L \to gl(\mathbb{V})$, $r: L \to gl(\mathbb{V})$ are representations $\psi: L \times L \to \mathbb{V}$ a bilinear map satisfying

$$\psi([x,y],z) - \psi(x,[y,z]) - \psi([x,z],y) - l_x\psi(y,z) - r_y\psi(x,z) + r_z\psi(x,y) = 0.$$

The bilinear maps ψ satisfying above we call a 2-cocycles on L with respect to the pair (l,r) and the set of all such 2-cocycles denote by $Z^2(L,l,r)$. The 2-coboundary on L with respect to the pair (l,r) is defined as follows

$$df(x,y) = f([x,y]) - l_{\varphi(x)} f(y) - r_{\varphi(y)} f(x),$$

where $\varphi \in Aut(L)$ and $f \in Hom(L,\mathbb{V})$. The set of such bilinear maps is denoted by $B^2(L,l,r)$.

For $\psi \in Z^2(L,l,r)$, define on the linear space $\overline{L} = L \oplus \mathbb{V}$ the bilinear product $[-,-]_{(l,r,\psi)}$ by

$$[x+a, y+b]_{(l,r,\psi)} = [x,y]_L + \psi(x,y) + l_x b + r_y a, \ \ x,y \in L, \ a,b \in \mathbb{V}.$$

Together with this product, $\left(\overline{L}, [-,-]_{l,r,\psi}\right)$ becomes a Leibniz algebra and is called an extension of the algebra L by $\mathbb{V}$.

Let L be a solvable Leibniz algebra, N its nilradical, and $Z(N)$ the center of N. Suppose that the Leibniz algebra $\overline{L} = L(\psi,l,r)$ is the extension of the given algebra L generated by ψ, l, r, and denote by $\overline{N}$ a nilradical of the algebra $\overline{L}$. Let $\psi^0 = \psi|_{N\times N}$ and $l^0 = l|_N$, $r^0 = r|_N$. Denote

$$G_{\psi^0} = \left\{X \in N : \psi^0(N,X) = \psi^0(X,N) = (0)\right\}.$$

Proposition 6. *The nilradical $\overline{N}$ of $\overline{L}$ is a central extension of N by $\mathbb{V}$ if and only if $N \subseteq \ker l \cap \ker r$. Moreover, if $N \subseteq \ker l \cap \ker r$, then $Z\left(\overline{N}\right) = \mathbb{V}$ if and only if $G_{\psi^0} \cap Z(N) = \{0\}$.*

In the following proposition, we present the conditions under which two given extensions, $\overline{L}_1 = L(\psi^1, l^1, r^1)$ and $\overline{L}_2 = L(\psi^2, l^2, r^2)$, are isomorphic.

Proposition 7. *Two Leibniz algebras* $\overline{L}_1 = L(\psi^1, l^1, r^1)$ *and* $\overline{L}_2 = L(\psi^2, l^2, r^2)$ *are isomorphic if and only if there exists* $\varphi \in Aut(L)$, $\phi \in Aut(\mathbb{V})$ *and* $f \in Hom(L, \mathbb{V})$ *such that*

$$\psi^2(\varphi(x), \varphi(y)) = \varphi(\psi^1(x,y)) + f([x,y]) - l^2_{\varphi(x)} f(y) - r^2_{\varphi(y)},$$

$$l^2_{\varphi(x)}\phi(a) = \phi(l^1_x a), \quad r^2_{\varphi(y)}\phi(a) = \phi(r^1_y a).$$

for any $x, y \in L, a \in \mathbb{V}$.

Let $Aut(L)$, $Aut(\mathbb{V})$ *be an automorphism groups of* L and $\mathbb{V}$, respectively. we define an action $Aut(L) \times Aut(\mathbb{V})$ on $\bigcup_{l,r} Z^2(L, l, r)$ under which a 2-cocycle ψ with respect to the pair (l, r) is transformed into a 2-cocycle ψ' with respect to (l', r') as follows:

$$\psi'(x,y) = \phi(\psi(\varphi(x), \varphi(y))), \; l'_x(a) = \phi(l_{\varphi(x)}\phi^{-1}(a)), \; r'_x(a) = \phi(r_{\varphi(x)}\phi^{-1}(a)).$$

Therefore, we conclude that two Leibniz algebras $\overline{L}_1 = L(\psi^1, l^1, r^1)$ and $\overline{L}_2 = L(\psi^2, l^2, r^2)$ are isomorphic if and only if $\psi^2 - \phi \circ \psi^1 \circ \varphi \in B^2(L, l^2, r^2)$ and ϕ is an intertwining operator for l^2 and $l^1 \circ \varphi$ (respectively, r^2 and $r^1 \circ \varphi$). Thus, we obtain the following proposition.

Proposition 8. *Let* $\overline{L}_1 = L(\psi^1, l^1, r^1)$ *and* $\overline{L}_2 = L(\psi^2, l^2, r^2)$ *be extensions of the solvable Leibniz algebra L by the abelian algebra* $\mathbb{V}$. *Then the Leibniz algebras* $\overline{L}_1$ *and* $\overline{L}_2$ *are isomorphic if and only if* ψ^1 *and* ψ^2 *are in the same* $Aut(L) \times Aut(\mathbb{V})$ *orbit in* $\bigcup_{l,r} H^2(L, l, r)$.

In the second section of the third chapter, we determine the extension of solvable Leibniz algebras whose nilradical is of null-filiform type. As previously noted, I.A.Karimjonov identified solvable Leibniz algebras with a null-filiform nilradical and proved that, with respect to the basis $\{e_1, e_2, \ldots, e_n, x\}$, their multiplication structure is given as follows:

$$R(NF_n): \begin{cases} [e_i, e_1] = e_{i+1}, \quad 1 \le i \le n-1, \quad [x, e_1] = e_1, \\ [e_i, x] = -ie_i, \quad 1 \le i \le n. \end{cases}$$

Throught the analysis of the extension of this algebra, we obtained the following result:

Theorem 9. *Let* $\overline{R}(NF_n)$ *be a one-dimensional abelian extension of the solvable Leibniz algebra* $R(NF_n)$, *then* $\overline{R}(NF_n) \cong R(NF_{n+1})$.

In the third section of Chapter three, the abelian extension of a solvable

Leibniz algebra with a naturally graded filiform nilradical and maximal codimension is presented. I.A. Karimjonov has shown that such algebras are unique up to isomorphism. In this section, we consider only the extension of a solvable Leibniz algebra whose nilradical is of this type. The general form of this algebra is given as follows:

$$R(F_n^1):\begin{cases}[e_1,x]=e_1, & [x,e_1]=-e_1, \quad [e_i,e_1]=e_{i+1}, \quad 2\le i\le n-1,\\ [e_i,x]=(i-1)e_i, & [e_i,y]=e_i, \qquad 2\le i\le n.\end{cases}$$

Theorem 10. *Let* $\overline{R}(F_n^1)$ *be an extension of the solvable Leibniz algebra* $R(F_n^1)$ *by the abelian algebra* $\mathbb{V}=span\{e_{n+1}\}$ *Then* $\overline{R}(F_n^1)$ *is isomorphic to the following algebra:*

$$\overline{R}(F_n^1):\begin{cases}[e_1,x]=e_1, & [x,e_1]=-e_1, \quad [e_i,e_1]=e_{i+1}, \quad 2\le i\le n-1,\\ [e_i,x]=(i-1)e_i, & [e_i,y]=e_i, \qquad 2\le i\le n,\\ [e_{n+1},x]=ne_{n+1}, & [e_{n+1},y]=e_{n+1}.\end{cases}$$

In the fourth section, the structural forms of abelian extensions of five-dimensional solvable Leibniz algebras with a non-trivial nilradical of codimension two are determined.

In the works of E.M. Cañete and A.Kh. Khudoyberdiyev, four-dimensional Leibniz algebras have been classified, and all five-dimensional Leibniz algebras have been identified. From among these algebras, we determine the Abelian extensions of those that possess a non-trivial nilradical, specifically the following cases:

$$H:\begin{cases}[e_1,e_2]=e_3, \ [e_2,e_1]=-e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3,\\ [x_1,e_3]=-e_1, \ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2, \ [e_3,x_2]=e_3, \ [x_2,e_3]=-e_3,\end{cases}$$

$$L_1:[e_2,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3, \ [e_2,x_2]=e_2, \ [e_3,x_2]=e_3,$$

$$L_2:[e_2,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=2e_3, \ [e_2,x_2]=e_2,$$

$$L_3:[e_1,e_1]=e_3, \ [e_1,x_1]=e_1, \ [x_1,e_1]=-e_1, \ [e_3,x_1]=2e_3, \ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2.$$

By determining the abelian extensions of these algebras, we obtained the following results:

Theorem 11. *Let* $\overline{H}$ *be an extension of the solvable Leibniz algebra H by the abelian algebra* $\mathbb{V}=span\{e_4\}$ *Then* $\overline{H}$ *is isomorphic to the following algebra:*

$$\overline{H}:\begin{cases}[e_1,e_2]=e_3, \ [e_2,e_1]=-e_3, \ [e_1,e_3]=e_4, \ [e_3,e_1]=-e_4, \ [e_1,x_1]=e_1,\\ [x_1,e_1]=-e_1, \ [e_3,x_1]=e_3, \ [x_1,e_3]=-e_1, \ [e_4,x_1]=2e_4, \ [x_1,e_4]=-2e_4,\\ [e_2,x_2]=e_2, \ [x_2,e_2]=-e_2, [e_3,x_2]=e_3, \ [x_2,e_3]=-e_3, \ [e_4,x_2]=e_4,\\ [x_2,e_4]=-e_4.\end{cases}$$

Theorem 12. *Let* $\overline{L}_1$ *be an extension of the solvable Leibniz algebra* L_1 *by the abelian algebra* $\mathbb{V}=span\{e_4\}$ *Then* $\overline{L}_1$ *is isomorphic to the following algebra:*

$$\overline{L}_1:\begin{cases}[e_2,e_1]=e_3,\ [e_3,e_1]=e_4,\ [e_1,x_1]=e_1,\quad [x_1,e_1]=-e_1,\ [e_3,x_1]=e_3,\\ [e_2,x_2]=e_2,\ [e_3,x_2]=e_3,\ [e_4,x_1]=2e_4,\ [e_4,x_2]=e_4.\end{cases}$$

Theorem 13. In the case of the algebras L_2 and L_3, no 2-cocycle exists that satisfies the condition $G_{\psi^0}\cap Z(N)=\{0\}$. Consequently, these algebras do not possess any non-split abelian extensions.

CONCLUSION

In this dissertation, the extensions of solvable Lie algebras with a naturally graded filiform Lie algebra as their nilradical, defined over the field of complex numbers, are obtained. Moreover, the central extensions and infinitesimal deformations of the non-simple and non - solvable *n*-Schrödinger Lie algebras are classified. In addition, a method for determining abelian extensions of solvable Leibniz algebras is developed. Using this method, the abelian extensions of solvable Leibniz algebras with null-filiform and naturally graded filiform nilradicals are explicitly described. Furthermore, the abelian extensions of five-dimensional solvable Leibniz algebras with non-trivial nilradicals of small dimension are also classified.

Main results of the dissertation are the following:

1. the central extensions and infinitesimal deformations of the *n*-Schrödinger Lie algebras have been classified;

2. the central extensions of naturally graded filiform Lie algebras have been obtained;

3. the extensions of solvable Lie algebras with filiform nilradicals have been determined;

4. a method for determining abelian extensions of solvable Leibniz algebras has been developed;

5. one-dimensional abelian extensions of solvable Leibniz algebras with null - filiform nilradicals have been classified;

6. abelian extensions of solvable Leibniz algebras with naturally graded filiform nilradicals and maximal codimension have been obtained.

7. abelian extensions of five-dimensional solvable Leibniz algebras have been determined.

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ЦЕНТРАЛЬНЫЕ И АБЕЛЕВЫ РАСШИРЕНИЯ РАЗРЕШИМЫХ АЛГЕБР ЛИ И ЛЕЙБНИЦА С ФИЛИФОРМНЫМ НИЛЬРАДИКАЛОМ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования: разработка комплексного подхода к классификации и структурному анализу конечномерных разрешимых алгебр Ли и Лейбница с филиформным нильрадикалом посредством изучения их центральных и абелевых расширений.

Научная новизна исследования:

определены центральные расширения и бесконечно малые деформации n-й алгебры Шрёдингера;

получено полное описание всех одномерных центральных расширений естественно градуированных филиформных алгебр Ли;

классифицированы одномерные расширения разрешимых алгебр Ли с филиформным нильрадикалом и описаны их структуры;

разработан метод построения абелевых расширений разрешимых алгебр Лейбница и найдены абелевы расширения пяти-мерных разрешимых алгебр Лейбница.

Внедрение результатов исследования. На основе полученных результатов по центральным и абелевым расширениям разрешимых алгебр Ли и Лейбница с филиформным нильрадикалом:

метод расширения разрешимых алгебр Ли и Лейбница, а также одномерные расширения разрешимых алгебр Ли с филиформным нильрадикалом были использованы в зарубежном проекте AP14870282 «Определение идентификаторов для классов неассоциативных алгебр» с целью обобщения на другие классы неассоциативных алгебр (справка Astana IT University №1/1529 от 5 ноября 2025 года, Республика Казахстан). Применение полученных научных результатов позволило определить абелевы расширения бинарных и унарных алгебр Лейбница;

разработанные методы и классификации центральных и абелевых расширений маломерных разрешимых алгебр Ли и Лейбница были использованы в фундаментальном проекте F-FA-2021-423 «Автоморфизмы операторных алгебр, классификация бесконечномерных неассоциативных алгебр и суперbegалгебр» для совершенствования современных подходов к классификации неассоциативных алгебраических структур (справка Института математики №2/505 от 3 декабря 2025 года). Применение научных результатов позволило получить алгебраическую и геометрическую классификации пятимерных нильпотентных алгебр Лейбница.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав с выводами по каждой главе, общего заключения и списка литературы. Общий объём работы 106 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
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I bo‘lim (part 1; часть 1)

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II bo‘lim (part 2; часть 2)

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