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OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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Qo‘lyozma huquqida

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“Adamar ma’nosidagi karrali maxsus operatorlar va ularning ba’zi bir tadbiqlari“ mavzusidagi ma’ruzasi

5A 130102- “Ehtimollar nazariyasi va matematik statistika” kafedrasi
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MAGISTRLIK DISSERTATSIYASI

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“Matematik fizika va funksional analiz” kafedrasi mudiri **dots. A.M.Xalxo‘jayev**

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Mundarija

Kirish

Mavzuning dolzarbligi va ta'rixi: Matematik fizikaning ko'pgina chegaraviy masalalari, jumladan, yoqlama integro – differensial tenglamalar va boshqa shunga o'xshash masalalarning yechimlari Adamar ma'nosidagi integral tenglamalarga keltiriladi. Shu sababli Adamar ma'nosidagi maxsus integral tenglamalarni o'rganish muhim ahamiyat kasb etadi.

Hozirgacha Adamar ma'nosidagi maxsus integral tenglamalar boshqa Fredgolm integral tenglamalari, singulyar integral tenglamalarga nisbatan kam o'rganilgan. Adamar ma'nosidagi karrali maxsus integrallar va ularning tadbiqlari deyarli hozirgacha kam o'rganilgan. Shuning uchun magistrlik dissertatsiya mavzusi dolzarb masalalardan biri bo'lib hisoblanadi.

Adamar ma'nosidagi integrallar nazariyasi va ularning har xil tatbiqlari bilan ko'pgina ko'zga ko'ringan olimlar ilmiy izlanishlar olib borishgan. Jumladan, N.P. Vekua [2], J. Adamar [1], Klaus Viner [3], S.P. Raslambekov [5], N.S. Gabbasov [6], Z. Prisdorf [12], A. G'oziyev [7], Yu.U. Kim [16], K.D. Sakalyuk [15] va boshqalar.

Masalaning qo'yilishi: Ma'lumki, N.P. Vekua [2] ushbu

$$\varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\varphi(\tau)}{(b-\tau)^{1+\mu}} d\tau = f(t) \tag{1}$$

ko'rinishdagi integral tenglamani qaragan va katma – ket yaqinlashish usuli yordamida uning yechimining mavjudligi va yagonaligini isbotlagan.

Mazkur dissertatsiya ishida N.P. Vekua [2] ning olgan natijalarini ko'p o'zgaruvchili funksiyalar uchun ham o'rinli bo'ladimi, degan masalani yechishga qaratilgan.

Ishning ilmiy tadqiqot metodlari: Dissertatsiya ishida asosan ketma – ket yaqinlashish usuli, qisib akslantirish prinsipi, takroriy yadrolar, rezolventa usullaridan foydalaniladi.

Ishning yangilik darajasi: Dissertatsiya ishi ilmiy xarakterga ega bo'lib, magistrni kelgusida ilmiy ishlar bilan shug'ullanishga o'rgatish maqsad qilib qo'yilgan.

Ishning nazariy va amaliy ahamiyati: Magistrlik dissertatsiya mavzusi ilmiy xarakterga ega bo'lib, dissertatsiya ishida olinishi ko'zda tutilayotgan natijalardan matematik fizikaning chegaraviy masalalarini yechishda, yoqlama integro – differensial tenglamalarni yechishda va shunga o'xshash izlanishlarda, bakalavrlarning dasturidagi o'rganishi ko'zda tutilgan integral tenglamalarni yechishda, maxsus kurslar va talabalar tanlovi kurslarida foydalanish mumkin.

Ishning asosiy maqsadi va vazifalari: Magistrlik ishida Adamar ma'nosidagi maxsus operatorlarning ba'zi muhum xossalarini o'rganib, o'rganilgan xossalarni chiziqli va chiziqli bo'lmagan Adamar ma'nosidagi maxsus integral tenglamalarni yechishga qo'llash maqsad qilib qo'yilgan. Magistrning vazifasi Adamar ma'nosidagi karrali maxsus operatorlar va ularning ba'zi bir muhim xossalarini mustaqil o'rganish va o'rganilgan xossalarni chiziqli maxsus integral tenglamalarni yechishga tadbiq qilishdan iborat.

Mavzuning ishlab chiqarishga bog'liqligi (i. ch. korxonasi, muassasa bilan hamkorlik): Magistrlik ishidan maxsuslik darajasi birdan katta bo'lgan uzoqlashuvchi maxsus integrallarning chekli qismini ajratib olishda integral tenglamalar kursida integral tenglamalar yadrosining maxsusligi birdan katta bo'lgan hollarda integral tenglamalarni yechishga qo'llash mumkin.

Ishning tuzilishi: Dissertatsiya kirish va uch bobdan iborat bo'lib, kirish qismida Adamar ma'nosidagi maxsus integral tenglamalar nazariyasining qisqacha rivojlanish tarixi, birinchi bobda asosan, Adamar ma'nosidagi integral tushunchasi va uning asosiy xossalari, ikki karrali Adamar ma'nosidagi integrallarning o'rinlarini almashtirish haqidagi teorema, bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli yordamida isbot qilingan.

Ikkinchi bobi ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglamani yechishga bag'ishlangan bo'lib, u ikki paragrafdan iborat bo'lib, uning birinchi paragrafida avvalo ikki karrali Adamar ma'nosidagi integral tushunchasi kiritilib, so'ngra, maxsuslik darajasi $p>1$ bo'lganda ikki o'zgaruvchili funksiya uchun Adamar ma'nosidagi integral tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli yordamida isbotlangan.

Ikkinchi paragrafda maxsuslik darajasi $p>1$ bo'lganda ikki o'zgaruvchili Adamar ma'nosidagi integral tenglamani qisib akslantirish usuli yordamida yechimning mavjudligi va yagonaligi isbotlangan.

Ishning oxirgi uchunchi bobida bir o'zgaruvchili va ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi integral tenglama iteratsiyalangan yadro va rezolventa yordamida yechib ko'rsatilgan. Dissertatsiya xulosa bilan birga 63 betdan iborat.

Olingan natijalarning qisqacha bayoni: N.P. Vekua [2] ning bir o'zgaruvchili funksiyalar uchun olingan natijalarini ikki o'zgaruvchili funksiyalar uchun umumlashtirish, Adamar ma'nosidagi chiziqli integral tenglamalari yechimlarining mavjudligi va yagonaligi haqidagi yangi teoremalarni isbot qilishdan iborat.

Bu ishda ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimlarining mavjudligi va yagonaligi haqidadgi teoremalar ketma – ket yaqinlashish usuli, qisib akslantirish prinsipi, iteratsiyalangan yadro va rezolventa usuli yordamida isbotlanib, yangi natijalarga erishildi. Bu teoremalar birinchi bobdagi teoremalar va yechish usullaridan foydalanib isbotlandi.

1-teorema. Agar $f(t,\tau)$ funksiya (Δ_1) da uzluksiz va uzluksiz 2-martagacha aralash xususiy hosilalarga ega bo`lib, $K(x,s,t,\tau)$ funksiya esa $(\Delta)$ da uzluksiz va to'rt martagacha uzluksiz va uzluksiz aralash xususiy hosilalarga ega bo`lib,

$$|\lambda| < \frac{1}{B} \tag{2}$$

bo`lsa,

$$B = 2\left(\frac{1}{\mu_1(b-a)^{\mu_1}\mu_2(d-c)^{\mu_2}} + \frac{(b-a)^{1-\mu_1}}{\mu_2(d-c)^{\mu_2}(1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{(1-\mu_2)\mu_1(b-a)^{\mu_1}} + \right.$$
$$\left. + \frac{(d-c)^{1-\mu_2}(b-a)^{1-\mu_1}}{(1-\mu_2)(1-\mu_1)}\right)$$

u holda

$$\varphi(x,y) - \lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau = f(x,y) \tag{3}$$

integral tenglama absolyut va tekis yaqinlashuvchi

$$\varphi(x,y) = \varphi_0(x,y) + \lambda\varphi_1(x,y) + \lambda^2\varphi_2(x,y) + \dots + \lambda^n\varphi_n(x,y) + .. \tag{4}$$

qatordan iborat bo'lgan yagona yechimga ega bo'ladi.

2 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada uzluksiz va to'rt martagacha uzluksiz aralash xususiy hosilalarga ega bo'lsa, u holda A operator M fazoni o'zini o'ziga ko'chiradi va chegaralangan bo'ladi.

Ushbu

$$\varphi(x,y) - \lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}d\tau dt = g(x,y) \tag{5}$$

chiziqli ikki karrali Adamar ma'nosidagi integral tenglamani qaraymiz, bunda $K(x,y,t,\tau)$ va $g(x,y)$ funksiyalar mos ravishda D va D_1 sohalarda aniqlangan, λ esa parametr.

3 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada to'rt martagacha uzluksiz aralash xususiy hosilalarga, $g(x,y)$ funksiya esa D_1 sohada ikki martagacha uzluksiz aralash xususiy hosilalaga ega bo'lsa, u holda

$$\varphi(x,y)-\lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}d\tau dt = g(x,y) \qquad (6)$$

integral tenglama λ parametrning

$$|\lambda| < \frac{1}{\frac{NE}{\mu_1\mu_2}} = \frac{\mu_1\mu_2}{NE} \qquad (7)$$

tengsizlikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi.

4 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada $4p$ martagacha uzluksiz aralash hosilalarga, $g(x,y)$ funksiya esa D_1 sohada $2p$ martagacha uzluksiz aralash xususiy hosilalarga ega bo'lsa, u holda

$$\varphi(x,y)-\lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}}d\tau dt = g(x,y) \qquad (8)$$

integral tenglama λ parametrning

$$|\lambda| < \frac{1}{N_1E_1}$$

tengsizlikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi.

I Bob. Adamar ma'nosidagi bir o'zgaruvchili chiziqli integral tenglama.

1-§. Adamar ma'nosidagi integral tushunchasi va uning xossalari.

1.1.1. Ba'zi ta'rif va tasdiqlar.

Ushbu

$$I(b) = \int_a^b \frac{A(t)}{(b-t)^{\frac{1}{2}}} dt \tag{1.1.1}$$

integral berilgan bo'lib, bunda $A(t)$ $[a,b]$ oraliqda integrallanuvchi (yoki Lipshis shartini qanoatlantiruvchi) funksiya deb faraz qilinadi. Agar biz bu integralni b parameter bo'yicha formal ravishda integrallasak, u holda quyidagi ifodaga ega bo'lamiz:

$$-\frac{1}{2}\int_a^b \frac{A(t)}{(b-t)^{\frac{3}{2}}} dt + \left[\frac{A(t)}{(b-t)^{\frac{1}{2}}}\right]_{t=b}$$

qaysiki bu ifoda ma'noga ega emas, chunki birinchi qo'shiluvchidagi integral ostidagi funksiya $\frac{3}{2}$ darajadagi maxsuslikka ega, ikkinchi qo'shiluvchi ham ma'noga ega emas. Lekin (1.1.1) integralning b parameter bo'yicha hosilaga ega bo'lishi uchun ifoda ma'noga ega. Endi quyidagi ifodaning limitini qaraymiz:

$$\lim_{t \to b}\left[\int_b^\tau \frac{A(t)}{(b-t)^{\frac{3}{2}}} dt + \frac{B(\tau)}{\sqrt{b-\tau}}\right] \tag{1.1.2}$$

bunda $B(\tau)$ - ixtiyoriy differensiallanuvchi (yoki Lipshis shartini qanoatlantiruvchi) hamda

$$B(b) = -2A(a) \tag{1.1.3}$$

shartni qanoatlantiruvchi funksiya. Agar $B(\tau)$ - ixtiyoriy funksiya yuqoridagi shartlarni qanoatlantirsa, u holda bo'laklab integrallash yo'li bilan ko'rsatish mumkin.

(1.1.2) ifodaning limiti $B(\tau)$ funksiyaning ko'rinishga bog'liq bo'lmasdan mavjud bo'ladi. (1.1.2) limitga uzoqlashuvchi

$$\int_a^b \frac{A(t)}{(b-t)^{\frac{3}{2}}}dt$$

integralning “chekli qismi” deyiladi va uni Adamar ma’nosida

$$\Gamma\int_a^b \frac{A(t)}{(b-t)^{\frac{3}{2}}}dt$$

kabi belgilanadi. Demak

$$\Gamma\int_a^b \frac{A(t)}{(b-t)^{\frac{3}{2}}}dt = \lim_{\tau\to b}\left[\int_a^\tau \frac{A(t)}{(b-t)^{\frac{3}{2}}}dt - \frac{B(\tau)}{\sqrt{b-\tau}}\right] \qquad (1.1.4)$$

bunda $B(\tau)$ ni (1.1.4) limit mavjud bo’ladigan qilib tanlanadi. Xususiy holda $A(\tau)$ va $B(\tau)$ lar hosilaga ega bo’lsa (yoki Lipshis shartini qanoatlantirsa) (1.1.4) limit $B(\tau)$ ni tanlashga bog’liq bo’lmagan holda har doim mavjud bo’ladi. Endi (1.1.1) integralning b parameter bo’yicha hosilasini Adamar ma’nosidagi integral orqali ushbu

$$I'(b) = -\frac{1}{2}\Gamma\int_a^b \frac{A(t)}{(b-t)^{\frac{3}{2}}}dt$$

ko’rinishda yozish mumkin. Maxsuslik darajasi 1 dan katta bo’lgan integrallar uchun ham bu ta’rifni yozish mumkin, ya’ni

$$\int_a^b \frac{A(t)}{(b-t)^{p+\mu}}dt \quad 0<\mu<1$$

uzoqlashuvchi integralning “chekli qismi” ni

$$\Gamma\int_a^b \frac{A(t)}{(b-t)^{p+\mu}}dt = \lim_{\tau\to b}\left[\int_a^\tau \frac{A(t)}{(b-t)^{p+\mu}}dt - \frac{\psi(\tau)}{(b-\tau)^{p+\mu-1}}\right] \qquad (1.1.5)$$

ko’rinishda aniqlash mumkin, bunda $\psi(\tau)$ (1.1.5) limit mavjud bo’ladigan qilib tanlanadigan ixtiyoriy differensiallanuvchi funksiya. Xususiy holda, $A(\tau)$ va $\psi(\tau)$ funksiyalar p marta differensiallanuvchi bo’lsa, bu limit $\psi(\tau)$ ni tanlashga bog’liq bo’lmagan holda har doim mavjud bo’ladi.

$$\int_a^b \frac{A(t)}{(b-t)^k}dt\,,\ k\in N$$

uzoqlashuvchi integralning chekli qismi ushbu

$$\int_a^b \frac{A(t)}{(b-t)^k}dt = \lim_{\varepsilon \to 0}\left\{ \int_a^{b-\varepsilon} \frac{A(t)}{(b-\tau)^k}dt - q^*, \ln\varepsilon - \varepsilon^{-(k-1)}\left(q_0 + q_1\varepsilon + q_2\varepsilon^2 + \ldots + q_{k-2}\varepsilon^{k-2}\right)\right\} \quad (1.1.6)$$

bunda $\varepsilon > 0, q^*, q_0, \ldots, q_{k-2}$ koeffitsientlar shunday tanlanadiki, natijada (1.1.6) limit mavjud bo'lsin.

1.1.2. Adamar ma'nosidagi integralning xossalari

Adamar ma'nosidagi integralning ta'rifidan uning Riman integralining xossalariga o'xshash bir qancha xossalari kelib chiqadi:

1^0. $\forall \tilde{n} \in (a,b)$ uchun $\Gamma\int_a^b \frac{A(t)}{(b-t)^{1+\mu}}dt = \Gamma\int_a^c \frac{A(t)}{(b-t)^{1+\mu}}dt + \Gamma\int_c^b \frac{A(t)}{(b-t)^{1+\mu}}dt$, $0 < \mu < 1$.

2^0. $\Gamma\int_a^b \frac{\tilde{n}A(t)}{(b-t)^{1+\mu}}dt = \tilde{n}\,\Gamma\int_a^b \frac{A(t)}{(b-t)^{1+\mu}}dt, \ c \in R$.

3^0. Agar $A_1(t)$ va $A_2(t)$ funksiyalarning har biri $[a,b]$ da integrallanuvchi (Adamar ma'nosida) bo'lsa, $A_1(t) \pm A_2(t)$ funksiyalar ham (Adamar ma'nosida) integrallanuvchi bo'ladi, ya'ni

$$\Gamma\int_a^b \frac{A_1(t) \pm A_2(t)}{(b-t)^{1+\mu}}dt = \Gamma\int_a^b \frac{A_1(t)}{(b-t)^{1+\mu}}dt + \Gamma\int_c^b \frac{A_2(t)}{(b-t)^{1+\mu}}dt,$$

tenglik o'rinli bo'ladi.

4^0. Agar Adamar integrali ostidagi funksiya oddiy Riman ma'nosida integrallanuvchi bo'lsa, u holda Adamar integralining qiymati, Riman integrali qiymatiga teng bo'ladi.

5^0. Agar $A(t)$ funksiya analitik funksiya bo'lsa, u holda Adamar integralini b parameter bo'yicha differensiallash mumkin, bu operatsiyani bajarishda mos hadlarning b nuqtadagi qiymatini tashlab yuborish mumkin.

6^0. $A(t)$ funksiyaning ishorasiga qarab, uning Adamar integralining ishorasi to'g'risida biror narsa aytish mumkin emas.

Misol. $\Gamma\int_a^b \frac{1}{(b-t)^{\frac{3}{2}}}dt = -\frac{2}{\sqrt{b-a}}$

Bundan keyin har doim $\Gamma\int_a^b \frac{A(t)}{(b-t)^{p+\mu}}dt = \int_a^b \frac{A(t)}{(b-t)^{p+\mu}}dt$ deb yozishga kelishib olamiz.

Takroriy integrallarda integrallarning o'rinlarini almashtirishda quyidagi Fubin teoremasi muhim rol o'ynaydi.

1.1.1 – teorema. Agar $f(x,y)$ funksiya $\Omega = \Omega_1 \times \Omega_2, (\Omega_i = [a_i, b_i], i = 1,2)$ da aniqlangan va o'lchovli bo'lsa hamda

$$\int_{\Omega_1} dx \int_{\Omega_2} f(x,y)dy, \int_{\Omega_2} dy \int_{\Omega_1} f(x,y)dx, \iint_{\Omega_1\times\Omega_2} f(x,y)dxdy .$$

integrallarning birortasi absolyut yaqinlashuvchi bo'lsa, ular bir – biriga teng bo'ladi.

1.1.3. Qisib akslantirish prinsipi.

$[a,b]$ oraliqda uzluksiz va uzluksiz hosilaga ega bo'lgan funksiyalar sinfini M bilan belgilaymiz. $f \in M$ funksiyaning normasi deb, $\|f\|_M = \max\left\{\max_{a\le x\le b}|f(x)|\right.$ va $\left.\max_{a\le x\le b}|f'(x)|\right\}$. Bunday aniqlangan norma quyidagi aksiomalarni qanoatlantiradi:

1) $\|f\| = 0$ faqat va faqat $f \equiv 0$ bo'lsa;

2) $\|cf\| = |c|\|f\|$ c – haqiqiy o'zgarmas son;

3) $\|f_1 + f_2\| \le \|f_1\| + \|f_2\|$.

$M \subset C^1[a,b]$. M fazo metrik fazo bo'ladi: $f_1, f_2 \in M$. Bu nuqtalar orasidagi masofani

$$\rho(f_1, f_2) = \|f_1 - f_2\|$$

kiritamiz. Bunday aniqlangan fazo metrik fazoning hamma aksiomalarini qanoatlantiradi.

$\{f_n\} \subset M$ ketma – ketlik norma bo'yicha chegaralangan deyiladi, agarda $\{\|f_n\|\}$ - sonli to'plam chegaralangan bo'lsa.

Ravshanki, $\{f_n\}$ ketma – ketlik norma bo'yicha chegaralangan bo'lsa, u holda:

$$\{f_n(x)\} \text{ va } \{f_n^{'}(x)\}$$

bir vaqtda chegaralangan bo'ladi.

1.1.1 – tasdiq. Agar $\{f_n\} \subset M$ berilgan bo'lib,

$$\lim_{n\to\infty}\|f - f_n\| = 0\,, \ f \in M\ .$$

bo'lsa, u holda tekis $f_n(x) \underset{n\to\infty}{\rightarrow} f(x)$ va $f_n^{'}(x) \underset{n\to\infty}{\rightarrow} f^{'}(x)$ bo'ladi va aksincha $n \to \infty$ da tekis $f_n(x) \to f(x)$, $f_n^{'}(x) \to f^{'}(x)$ bo'lsa, $n \to \infty$ da $\|f - f_n\| \to 0$ bo'ladi.

1.1.1 – ta'rif. $\{f_n\} \subset M$ ketma – ketlik $f \in M$ ga norma bo'yicha yaqinlashuvchi deyiladi, agarda $\lim\limits_{n\to\infty}\|f - f_n\| = 0$ bo'lsa.

1.1.2 – tasdiq. $\{f_n\} \subset M$ ketma – ketlik norma bo'yicha yaqinlashuvchi bo'lishi uchun $\forall \varepsilon > 0$ uchun $\exists N(\varepsilon)$, $n > N$ va $m > N$ lar uchun

$$\|f_n - f_m\| < \varepsilon$$

bajarilishi zarur va yetarli.

M fazo kiritilgan norma bo'yicha to'liq fazo bo'ladi, ya'ni $\{f_n\} \subset M$ fundamental ketma – ketlik norma bo'yicha yaqinlashuvchi bo'lsa.

M - to'liq metrik fazo berilgan bo'lsin.

1.1.2 – teorema. A – to'liq M fazoni o'zini – o'ziga ko'chiruvchi operator bo'lsin. Agar $\forall x, y \in M$ uchun

$$\rho(Ax, Ay) \le \alpha\rho(x, y) \tag{1.1.7}$$

tengsizlik bajarilsa (bunda $\alpha < 1$, x va y larga bog'liq bo'lmagan o'zgarmas son), u holda $\exists x_0$ shunday x_0 nuqta mavjud bo'lib, $Ax_0 = x_0$ tenglik o'rinli bo'ladi. x_0 nuqtaga A operatorning qo'zg'almas nuqtasi deyiladi.

Isbot. $\forall x_0 \in M$ nuqtani olib, uni belgilaymiz. So'ngra $x_1 = Ax_0, x_2 = Ax_1 = A^2x_0,...,\ x_n = Ax_{n-1} = A^nx_0,...$ elementlarni tuzamiz va $\{x_n\}$ ketma – ketlikning M fundamental ketma – ketlik ekanligini ko'rsatamiz. Buning uchun

$$\rho(x_1, x_2) = \rho(Ax_0, Ax_1) \le \alpha\rho(x_0, x_1) = \alpha\rho(x_0, Ax_0),$$

$$\rho(x_2, x_3) = \rho A(Ax_1, Ax_2) \le \alpha\rho(x_1, x_2) = \alpha\rho(Ax_0, Ax_1) = \alpha^2\rho(x_0, Ax_0),$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\rho(x_n, x_{n+1}) \le \alpha^n\rho(x_0, Ax_0),...$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\rho(x_n, x_{n+p}) \le \rho(x_n, x_{n+1}) + \rho(x_{n+1}, x_{n+2}) + ... + \rho(x_{n+p-1}, x_{n+p}) \le$$

$$\le \left(\alpha^n + \alpha^{n-1} + ... + \alpha^{n+p-1}\right)\rho(x_0, Ax_0) = \frac{\alpha^n + \alpha^{n+p}}{1-\alpha}\rho(x_0, Ax_0)$$

Shartga ko'ra $\alpha < 1$ bo'lganligi uchun

$$\rho(x_n, x_{n+p}) < \frac{\alpha^n}{1-\alpha}\rho(x_0, Ax_0) \qquad (1.1.8)$$

bundan $p > 0$ ligini e'tiborga olib $n \to \infty$ da limitga o'tsak, $\rho(x_n, x_{n+p}) \underset{n\to\infty}{\longrightarrow} 0$ ekanligi kelib chiqadi. Demak, $\{x_n\}$ ketma – ketlik fundamental ketma – ketlik ekan. M fazo to'liq bo'lganligi uchun $\exists x \in M$ bo'lib, u $\{x_n\}$ ketma – ketlikning limiti mavjud bo'ladi, ya'ni

$$\lim_{n\to\infty} x_n = x$$

Endi $Ax = x$ ekanligini isbotlaymiz.

Haqiqatan ham

$\rho(x, Ax) \le \rho(x, Ax_n) + \rho(x_n, Ax_n) = \rho(x, x_n) + \rho(Ax_{n-1}, Ax) \le \rho(x, x_n) + \alpha\rho(x_{n-1}, x), \forall\varepsilon > 0$ va istalgancha katta n lar uchun

$$\rho(x, x_n) < \frac{\varepsilon}{2}, \rho(x, x_{n-1}) < \frac{\varepsilon}{2}$$

tengsizliklar o'rinli bo'ladi. Demak,

$$\rho(x, Ax) < \varepsilon$$

tengsizlik bajariladi. $\varepsilon > 0$ ning ixtiyoriyligidan $\rho(x, Ax) = 0$, bundan $Ax = x$ eganligi kelib chiqadi, ya'ni x nuqta A operator uchun qo'zg'almas nuqta bo'lar ekan. Bu nuqtaning yagonaligini isbotlaymiz. Faraz qilaylik A operatorning qo'zg'almas nuqtasi ikkita bo'lsin, ya'ni $Ax = x$ va $Ay = y$ bo'lsin.

U holda

$$\rho(x, y) = \rho(Ax, Ay) \leq \alpha\rho(x, y), \quad \rho(x, y) = 0$$

bo'lganligi uchun keyingi tengsizlikdan $\alpha \leq 1$ ekanligi kelib chiqadi, bunday bo'lishi mumkin emas. Demak, $\rho(x, y) = 0$, bundan $x = y$ ekanligi kelib chiqadi. (1.1.7) tenglikda $n \to \infty$ da limitga o'tsak, natijada n - yaqinlashish qo'yilgan xatoni hisoblash uchun kerak bo'ladigan (1.1.8) tengsizlikni hosil qilamiz.

1.1.1-eslatma. x qo'zg'almas nuqtaga yaqinlashuvchi $\{x_n\}$ ketma – ketlikni tuzishda $\forall x_0 \in M$ nuqtani olish mumkin. x_0 nuqtani tanlash faqat $\{x_n\}$ ketma – ketlikning x limitiga intilish tezligiga ta'sir qilish mumkin.

1.1.1 Misol. $K(t, s)$ - haqiqiy funksiya $[a \leq t, s \leq b]$ kvadratda aniqlangan va o'lchovli funksiya bo'lib,

$$\int_a^b \int_a^b K^2(t, s)\,dt\,ds < \infty \tag{1.1.9}$$

shartni qanoatlantirsin, $f(t) \in L_2[a, b]$ bo'lsin. Shu shartlarda

$$x(t) = f(t) + \lambda \int_a^b K(t, s)x(s)\,ds \tag{1.1.10}$$

integral tenglama λ parametrning kichik qiymatida yagona $x(t) \in L_2[a, b]$ yechimga ega bo'ladi.

Yechish. $Ax = f(t) + \lambda \int_a^b K(t, s)x(s)\,ds$ operatorning har bir $x(t) \in L_2[a, b]$ funksiyani $L_2[a, b]$ fazoga ko'chirishni ko'rsatamiz:

$f(t) \in L_2[a, b]$ bo'lishi uchun,

$$g(t) = \int_a^b K(t, s)x(s)\,ds$$

funksiyaning $L_2[a,b]$ ga qarashli ekanligini ko'rsatish yetarli . (1.1.9) shartdan Fubin teoremasiga asosan $K^2(t,s)$ funksiya s bo'yicha $[a,b]$ da deyarli hamma $t\in[a,b]$ nuqtalar uchun integrallanuvchi bo'ladi. Bundan deyarli hamma $t\in[a,b]$ nuqtalar uchun

$$g(t)=\int_a^b K(t,s)x(s)ds$$

integral mavjud. U holda Bunyakovskiy tengsizligiga asosan

$$g^2(t)=\left(\int_a^b K(t,s)x(s)ds\right)^2\le\int_a^b K^2(t,s)ds\int_a^b x^2(s)ds$$

tengsizlik o'rinli bo'ladi.

$$\int_a^b x^2(s)ds$$

integral mavjud bo'lganligi uchun, u o'zgarmas sonni ifodalaydi. Fubin teoremasiga va (1.1.9) shartga asosan

$$\int_a^b K^2(t,s)ds$$

integral $t\in[a,b]$ bo'yicha integrallanuvchi bo'lgani uchun $g^2(t)$ ham $t\in[a,b]$ bo'yicha integrallanuvchi bo'ladi va

$$g^2(t)\le\int_a^b K^2(t,s)ds\int_a^b x^2(s)ds$$

tengsizlik o'rinli bo'ladi. Demak, $Ax\in L_2[a,b]$ ekan. Endi $\rho(Ax,Ay)$ ni baholaymiz, bunda $x,y\in L_2[a,b]$.

$$\rho(Ax,Ay)=\left\{\int_a^b\left(\lambda\int_a^b K(t,s)x(s)ds-\lambda\int_a^b K(t,s)y(s)ds\right)^2dt\right\}^{\frac{1}{2}}=|\lambda|\left\{\int_a^b\left(\int_a^b K(t,s)[x(s)-y(s)]ds\right)^2\right\}^{\frac{1}{2}}\le$$

$$\le|\lambda|\left(\int_a^b\int_a^b K^2(t,s)dtds\right)^{\frac{1}{2}}\left(\int_a^b[x(s)-y(s)]^2ds\right)^{\frac{1}{2}}=|\lambda|\left(\int_a^b\int_a^b K^2(t,s)dtds\right)^{\frac{1}{2}}\rho(x,y)$$

tengsizlik o'rinli bo'ladi.

Agarda

$$|\lambda| < \frac{1}{\left(\int_a^b\int_a^b K^2(t,s)dtds\right)^{\frac{1}{2}}} \tag{1.1.11}$$

shart bajarilsa A operator qisuvchi bo'ladi. Demak, qisish prinsipiga asosan (1.1.10) integral tenglama $L_2[a,b]$ da yagona yechimga ega bo'ladi va bu yechimni ketma – ket yaqinlashtirish usuli bilan topiladi.

1.2 –§. Adamar ma'nosidagi chiziqli integral tenglamani ketma-ket yaqinlashish usuli bilan yechish.

1. Ushbu

$$\varphi(x) - \lambda\int_a^b \frac{K(x,t)\varphi(t)}{(b-t)^{1+\mu}}dt = f(x) \tag{1.2.1}$$

chiziqli integral tenglamani qaraymiz, bunda $K(x,t):\{(x,t): a \le x,t \le b\}$ da berilgan uzluksiz va 2 martagacha uzluksiz xususiy hosilalarga ega, $f(x):\{a \le t \le b\}$ uzluksiz va uzluksiz xususiy hosilalarga ega, $0 < \mu < 1$, λ - esa parametr, $\varphi(x)$ - izlauvchi funksiya. (1.2.1) dagi integralni Adamar ma'nosida tushuniladi.

(1.2.1) tenglamani ketma – ket yaqinlashish usuli bilan yechish uchun avvalo quyidagi teoremani isbotlaymiz.

1.2.1- teorema. Agar $A(t,\tau)$ va $\psi(\tau)$ funksiyalar $[a,b]$ da p – martagacha uzluksiz hosilalarga ega bo'lsa va $\psi(\tau)$ funksiya ushbu

$$\psi(\tau) = \sum_{i=0}^{\infty}\psi_i(\tau)$$

tekis yaqinlashuvchi qator shaklida tasvirlangan bo'lsa, hamda

$$\psi^{(k)}(\tau) = \sum_{i=0}^{\infty}\psi^{(k)}{}_i(\tau) \qquad (k = 1,2,\ldots, p)$$

qator tekis yaqinlashuvchi bo'lsa, u holda

$$\int_a^b \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau = \sum_{i=0}^{\infty}\int_a^b \frac{A(t,\tau)\psi_i(\tau)}{(b-\tau)^{p+\mu}}d\tau \qquad (1.2.2)$$

tenglik o'rinli bo'ladi.

Isbot. (1.2.2) tenglikning chap tomoniga Adamar integralining ta'rifini qo'llaymiz:

$$\int_a^b \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau = \lim_{\tau\to b}\left[\int_a^{\tau} \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau + \frac{\mathrm{B}_0(t,\tau)+(b-\tau)\mathrm{B}_1(t,\tau)+\ldots+(b-\tau)^{p-1}\mathrm{B}_{p-1}(t,\tau)}{(b-\tau)^{p-1+\mu}}\right]$$

$\left[\int_a^{\tau} \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau + \frac{\mathrm{B}_0(t,\tau)+(b-\tau)\mathrm{B}_1(t,\tau)+\ldots+(b-\tau)^{p-1}\mathrm{B}_{p-1}(t,\tau)}{(b-\tau)^{p-1+\mu}}\right]$ qavs ichidagi $\int_a^b \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau$ integralni p – marta bo'laklab integrallaymiz:

$$\begin{aligned}\int_a^b \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}}d\tau &= -\frac{A(t,a)\psi(a)}{a_1.b_1} + \frac{[A(t,a)\psi(a)]'_a}{a_1a_2.b_2} - \frac{[A(t,a)\psi(a)]''_a}{a_1a_2a_3.b_3} + \ldots + \\ &+ \frac{(-1)^{p-1}[A(t,a)\psi(a)]^{(p-1)}_a}{a_1a_2\ldots a_pb_p} + \frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{[A(t,\tau)\psi(\tau)]^p_\tau}{(b-\tau)^{\mu}}d\tau\end{aligned} \qquad (1.2.3)$$

bunda,

$$a_k = p+\mu-k,\ \ b_k = (b-a)^{p+\mu-k} \quad (k=1,2,\ldots,\ p).$$

Xuddi shunday

$$\begin{aligned}\int_a^b \frac{A(t,\tau)\psi_i(\tau)}{(b-\tau)^{p+\mu}}d\tau &= -\frac{A(t,a)\psi_i(a)}{a_1.b_1} + \frac{[A(t,a)\psi_i(a)]'_a}{a_1a_2.b_2} - \frac{[A(t,a)\psi_i(a)]''_a}{a_1a_2a_3.b_3} + \ldots + \\ &+ \frac{(-1)^{p-1}[A(t,a)\psi_i(a)]^{(p-1)}_a}{a_1a_2\ldots a_pb_p} + \frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{[A(t,\tau)\psi_i(\tau)]^p_\tau}{(b-\tau)^{\mu}}d\tau\end{aligned} \qquad (1.2.4)$$

(1.2.4) tenglikning chap va o'ng tomonini hadma – had qo'shish natijasida

$$\begin{aligned}\sum_{i=0}^{\infty}\int_a^b \frac{A(t,\tau)\psi_i(\tau)}{(b-\tau)^{p+\mu}}d\tau &= -\frac{A(t,a)\psi(a)}{a_1.b_1} + \frac{[A(t,a)\psi(a)]'_a}{a_1a_2.b_2} - \frac{[A(t,a)\psi(a)]''_a}{a_1a_2a_3.b_3} + \ldots + \\ &+ \frac{(-1)^{p-1}[A(t,a)\psi(a)]^{(p-1)}_a}{a_1a_2\ldots a_p.b_p} + \frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{[A(t,\tau)\psi(\tau)]^p_\tau}{(b-\tau)^{\mu}}d\tau\end{aligned}$$

Keyingi tenglikni (1.2.3) bilan solishtirish natijasida teoremaning isbot bo'lganiga, ya'ni (1.2.2) ning o'rinli ekanligiga ishonch hosil qilamiz.

Bu teorema qatorlarni hadma – had integrallash (integral, Adamar ma'nosidagi integral bo'lganda) mumkinligining yetarli shartini ifodalaydi. Bu esa, Adamar ma'nosidagi integral tenglamaga ketma – ket yaqinlashish usulini qo'llash mumkinligiga imkon tug'diradi.

2. Ushbu

$$\varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\varphi(\tau)}{(b-\tau)^{1+\mu}} d\tau = f(t) \tag{1.2.5}$$

tenglamani qaraymiz, bunda $0 < \mu < 1$, t va τ lar $[a \le t \le b, a \le \tau \le b]$ yopiq kvadratda o'zgaradi.

$K(t,\tau)$ va $f(t)$ funksiyalar mos ravishda $(D) = [a,b;a,b]$ va $(D_1) = [a,b]$ sohalarda aniqlangan bo'lib, $K(t,\tau)$ funksiya (D) da uzluksiz va ikkinchi tartibgacha uzluksiz aralash xususiy hosilalarga ega, $f(t)$ funksiya esa, (D_1) da uzluksiz va birinchi tartibli uzluksiz hosilaga ega, $\varphi(t)$ - izlanuvchi funksiya, λ - parametr.

(1.2.5) tenglamadagi integral Adamar ma'nosida tushuniladi.

(1.2.5) tenlamaning yechimini ushbu

$$\varphi(t) = \sum_{n=0}^{\infty} \lambda^k \varphi_k(t) \tag{1.2.6}$$

qator shaklida izlaymiz:

$\varphi(t)$ ning bu qiymatini (1.2.5) ga olib borib qo'yib, uni formal ravishda hadma – had integrallab, λ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib topamiz:

$$\varphi_0(t) = f(t);$$

$$\varphi_1(t) = \int_a^b \frac{K(t,\tau) f(\tau)}{(b-\tau)^{1+\mu}} d\tau \; ;$$

$$\varphi_2(t) = \int_a^b \frac{K(t,\tau)\varphi_1(\tau)}{(b-\tau)^{1+\mu}} d\tau \; ;$$

. . . .

$$\varphi_n(t) = \int_a^b \frac{K(t,\tau)\varphi_{n-1}(\tau)}{(b-\tau)^{1+\mu}} d\tau \; ;$$

....

Agar

$$\varphi(t)=\sum_{k=0}^{\infty}\lambda^k\varphi_k(t),\ \sum_{k=0}^{\infty}\lambda^k\varphi'_k(t)$$

qatorlar tekis yaqinlashuvchi bo'lsa, u holda 1.2.1 – teoremaga asosan, qatorlarni hadma – had integrallash mumkin bo'ladi.

$f(t)$ funksiya va uning hosilasining absolyut qiymatlarining eng kattasini, ya'ni $N=\max\{\max|f(t)|\max|f'(t)|\}$ bilan, $K(t,\tau)$ funksiya maksimum qiymati, hamda 2 – tartibgacha hosilalari maksimum qiymatlarining eng kattasini, ya'ni

$$M=\max\{\max|K(t,\tau)|\max|K'_\tau(t,\tau)|\max|K''_{\tau^2}(t,\tau)|\}$$

bilan belgilaymiz. Natijada,

$$|\varphi_0(t)|=|f(t)|\le N\ ,$$

$\varphi_1(t)$ ni baholash uchun chekli ayirmalar formulasidan, ya'ni

$$K(t,\tau)f(\tau)=K(t,b)f(b)-(b-\tau)\left[K'_\tau(t,b+\theta h)f(b+\theta h)+K(t,b+\theta h)f'(b+\theta h)\right],\qquad(1.2.7)$$

bunda $h=\tau-b,\ o<\theta<1$ formulasidan foydalanib topamiz:

$\varphi_1(t)$ ning ifodasiga (1.2.7) ni qo'yamiz, natijada

$$\varphi_1(t)=K(t,b)f(b)\int_a^b\frac{d\tau}{(b-\tau)^{1+\mu}}-\int_a^b\frac{K'_\tau(t,b+\theta h)f(b+\theta h)+K(t,b+\theta h)f'(b+\theta h)}{(b-\tau)^{\mu}}d\tau$$

ega bo'lamiz.

$$\int_a^b\frac{d\tau}{(b-\tau)^{1+\mu}}=-\frac{1}{\mu(b-a)^{\mu}}$$

ni e'tiborga olgan holda,

$$|\varphi_1(t)|\le\frac{MN}{\mu(b-a)^{\mu}}+\frac{2MN\ (b-a)^{1-\mu}}{1-\mu}=NB\ ,$$
$$B=\frac{M}{\mu(b-a)^{\mu}}+\frac{2MN\ (b-a)^{1-\mu}}{1-\mu}\qquad(1.2.8)$$

topamiz.

Ravshanki, $\varphi_1^{'}(t)=\int\limits_a^b \frac{K_t^{'}(t,\tau)f(\tau)}{(b-\tau)^{1+\mu}}d\tau$.

Xuddi yuqoridagi $\varphi_1(t)$ ni baholagan singari, $\varphi_1^{'}(t)$ uchun ham

$$\left|\varphi_1^{'}(t)\right|\leq NB \tag{1.2.9}$$

tenglikni hosil qilamiz.

Endi $\varphi_2(t)$ ni baholaymiz: buning uchun ushbu

$$K(t,\tau)\varphi_1(\tau)=K(t,b)\varphi_1(b)-(b-\tau)\left[K_\tau^{'}(t,b+\theta_1 h)\varphi_1(b+\theta_1 h)+K(t,b+\theta_1 h)\varphi_1^{'}(b+\theta_1 h)\right],$$

chekli ayirmalar haqidagi formuladan foydalangan holda, keyingi ifodani $\varphi_2(t)$ ning ifodasiga olib borib qo'yamiz. Natijada,

$$\varphi_2(t)=K(t,b)\varphi_1(b)\int\limits_a^b\frac{d\tau}{(b-\tau)^{1+\mu}}-\int\limits_a^b\frac{K_\tau^{'}(t,b+\theta_1 h)\varphi_1(b+\theta_1 h)+K(t,b+\theta_1 h)\varphi_1^{'}(b+\theta_1 h)}{(b-\tau)^{\mu}}d\tau$$

Bundan, $\varphi_1(t)$ va $\varphi_1^{'}(t)$ larning baholaridan, ya'ni (1.2.8) va (1.2.9) lardan foydalanib topamiz:

$$\left|\varphi_2(t)\right|\leq NB^2;\ \left|\varphi_2^{'}(t)\right|\leq NB^2. \tag{1.2.10}$$

Xuddi shunday yo'l bilan

$$\left|\varphi_3(t)\right|\leq NB^3;\ \left|\varphi_3^{'}(t)\right|\leq NB^3. \tag{1.2.11}$$

va hakozo induksiya usuli yordamida

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left|\varphi_i(t)\right|\leq NB^i;\ \left|\varphi_i^{'}(t)\right|\leq NB^i \qquad (i=1,2,3,\ldots.) \tag{1.2.12}$$

Shunday qilib, (1.2.6) qator uchun majorant bo'lgan ushbu

$$N+N|\lambda|B+N|\lambda|^2B^2+\ldots \tag{1.2.13}$$

sonli qatorga ega bo'lamiz. Bu qator

$$|\lambda|<\frac{1}{B}=\frac{(b-a)^{\mu}}{\frac{M}{\mu}+\frac{2M}{1-\mu}-(b-a)} \tag{1.2.14}$$

shartni qanoatlantirganda yaqinlashuvchi bo'ladi.

Xuddi shunday (1.2.6) qatorning hosilasidan tuzilgan

$$\sum_{k=0}^{\infty} \lambda^k \varphi'_k(t) \qquad (1.2.6')$$

qator ham (1.2.14) shart bajarilganda tekis yaqinlashuvchi bo'lishini ko'rsatish mumkin.

Shunday qilib, agarda (1.2.14) shartni qanoatlantirganda (1.2.5) tenglama yechimga ega bo'ladi, yechim esa, (1.2.6) qator shaklda ifoda qilinadi.

Yechimning yagonaligi.

Agar λ (1.2.14) shartni qanoatlantirsa, u holda, yuqorida ketma – ket yaqinlashish usuli bilan topilgan yechim yagona bo'ladi.

Isbot. Faraz qilaylik (1.2.5) tenglama ikkita $\varphi(t)$ va $\varphi^*(t)$ $(\varphi(t) \neq \varphi^*(t))$ yechimlarga ega bo'lsin. U holda, $\psi(t) = \varphi(t) - \varphi^*(t)$ ayirma ushbu

$$\psi(t) = \lambda \int_a^b \frac{K(t,\tau)\psi(\tau)}{(b-\tau)^{1+\mu}} d\tau \qquad (1.2.15)$$

bir jinsli tenglamani qanoatlantiradi.

$$N_1 = \{\max \left|\max |\psi(t)|, \max |\psi'(t)|\right|\}$$

deb belgilaymiz.

Agar (1.2.15) ning chap tomoni uchun yuqoridagiday baholash usullarini qo'llasak, natijada

$$|\psi(t)| \leq N_1 C|\lambda|,$$

Bunda $C = B$ yoki $C = E$ bahoga ega bo'lamiz.

Ravshanki, $\left|\psi^{(k)}(t)\right| \leq N_1 C|\lambda| \quad (k = 1,2,..., p)$ tengsizlik ham o'rinli bo'ladi. Bu tengsizliklardan

$$\psi(t) \leq N_1 C^2 |\lambda|^2;$$

$$\psi(t) \leq N_1 C^n |\lambda|^n \quad (n = 1,2,...).$$

$|\tilde{N}\lambda| < 1$ ekanligini e'tiborga olgan holda

$$\psi(t) \leq \lim_{n\to\infty} N_1 C^n |\lambda|^n = 0 \, .$$

Shunday qilib, agar λ (1.2.14) tengsizlikni qanoatlantirganda (1.2.5) tenglama yagona yechimga ega bo’ladi, ya’ni quyidagi teorema o’rinli bo’ladi.

Shunday qilib quyidagi teoremaga kelamiz.

1.2.2 – teorema. Agar $K(t,\tau)$ va $f(t)$ funksiyalar mos ravishda (D) va (D_1) sohalarda aniqlangan uzluksiz bo’lib, $K(t,\tau)$ funksiya (D) da uzluksiz 2 martagacha xususiy hosilalarga, $f(t)$ funksiya esa (D_1) uzluksiz $f'(t)$ hosilaga ega bo’lsa, hamda

$$|\lambda| < \frac{1}{B}$$

shartni qanoatlantirganda yagona yechimga ega bo’ladi, bu yechim ketma – ket yaqinlashish usuli bilan topiladi.

3. Ushbu

$$\varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\varphi(\tau)}{(b-\tau)^{p+\mu}} d\tau = f(t) \qquad (1.3.16)$$

integral tenglamani qaraylik, bunda p – butun musbat son, $0 < \mu < 1$. $K(t,\tau)$, $f(t)$ funksiyalar mos ravishda (D) va (D_1) sohalarda aniqlangan bo’lib, $K(t,\tau)$ funksiya 2p – martagacha uzluksiz hosilalarga, $f(t)$ funksiya esa p martagacha uzluksiz hosilaga ega deb faraz qilinadi.

(1.2.16) tenglamaning yechimini ushbu

$$\varphi(t) = \sum_{k=0}^{\infty} \lambda^k \varphi_k(t) \qquad (1.2.17)$$

qator shaklida izlaymiz.

(1.2.17) ni (1.2.16) ga qo’yib, formal ravishda qatorni hadma – had integrallab, λ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib topamiz:

$$\varphi_0(t) = f(t);$$

$$\varphi_1(t)=\int_a^b \frac{K(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau\ ;$$

$$\varphi_2(t)=\int_a^b \frac{K(t,\tau)\varphi_1(\tau)}{(b-\tau)^{p+\mu}}d\tau\ ;$$

....

$$\varphi_n(t)=\int_a^b \frac{K(t,\tau)\varphi_{n-1}(\tau)}{(b-\tau)^{p+\mu}}d\tau\ ;$$

....

Qatorni formal ravishda hadma – had integrallash to'g'ri bo'ladi, agarda (1.2.17) qator va uning hosilasidan tuzilgan

$$\sum_{k=0}^{\infty}\lambda^k\varphi'_k(t) \qquad (1.2.17')$$

qator tekis yaqinlashuvchi bo'lsa.

Agar

$$N=\max\left\{\max|f(t)|,\max|f'(t)|,\max|f''(t)|,\ldots,\max|f^{(p)}(t)|\right\},$$

$$M=\max\left\{\max|K(t,\tau)|,\max|K'_\tau(t,\tau)|,\ldots,\max|K^{(2p)}(t,\tau)|\right\}$$

belgilasak, u holda

$$|\varphi_0(t)|=|f(t)|\le N\ ;$$

$\varphi_1(t)$ ni baholash uchun $f(\tau)K(t,\tau)$ ko'paytmani quyidagi formula bo'yicha yoyamiz:

$$K(t,\tau)f(\tau)=K(t,b)f(b)-(b-\tau)[K(t,b)f(b)]'_b+\frac{(b-\tau)^2}{2!}[K(t,b)f(b)]''_b-\ldots+$$
$$+\frac{(-1)^p(b-\tau)^p}{p!}[K(t,b+\theta h)f(b+\theta h)]_b^{(p)}$$

Keyingi ifodani $\varphi_1(t)$ ning ifodasidagi integral ostiga olib borib qo'yamiz, natijada

$$\varphi_1(t) = K(t,b)f(b)\int_a^b \frac{d\tau}{(b-\tau)^{p+\mu}} - [K(t,b)f(b)]_b^{'} \cdot \int_a^b \frac{d\tau}{(b-\tau)^{p+\mu-1}} + \frac{[K(t,b)f(b)]_b^{''}}{2!}\int_a^b \frac{d\tau}{(b-\tau)^{p-2+\mu}} -$$
$$- \dots + \frac{(-1)^p}{p!}\int_a^b \frac{[K(t,b+\theta_1 h)f(b+\theta_1 h)]_\tau^{(p)}}{(b-\tau)^{\mu}} d\tau;$$
$$|\varphi_1(t)| \le \frac{MN}{(p+\mu-1)(b-a)^{p+\mu-1}} + \frac{2MN}{(p+\mu-2)(b-a)^{p+\mu-2}} + \frac{2^2 MN}{2!(p+\mu-3)(b-a)^{p+\mu-3}} + \dots + \tag{1.2.18}$$
$$+ \frac{2^p MN\,(b-a)^{1-\mu}}{p!(1-\mu)} = ND$$

bunda

$$D = \frac{M}{(p+\mu-1)(b-a)^{p+\mu-1}} + \frac{2M}{(p+\mu-2)(b-a)^{p+\mu-2}} + \dots + \frac{2^p M\,(b-a)^{1-\mu}}{p!(1-\mu)}$$

Ravshanki, $K(t,\tau)$, $f(\tau)$ funksiyalarga qo'yilgan shartlarda, $\varphi_1(t)$ funksiyaning hosilalari uchun (1.2.17) baho o'rinli bo'ladi:

$$\left|\varphi_1^{(k)}(t)\right| \le ND \quad (k = 1,2,\dots p) \tag{1.2.19}$$

Endi $\varphi_2(t)$ ni baholaymiz:

$K(t,\tau)\varphi_1(\tau)$ ni quyidagi formula bo'yicha yoyamiz:

$$K(t,\tau)\varphi_1(\tau) = K(t,b)\varphi_1(b) - (b-\tau)[K(t,b)\varphi_1(b)]_b^{'} + \dots + \frac{(-1)^p}{p!}(b-\tau)^p [K(t,b+\theta_1 h)\varphi_1(b+\theta_1 h)]_\tau^{(p)}$$

Keyingi ifodani $\varphi_2(t)$ ning ifodasidagi integral ostiga qo'yish natijasida

$$\varphi_2(t) = K(t,b)\varphi_1(b)\cdot\int_a^b \frac{d\tau}{(b-\tau)^{p+\mu}} - [K(t,b)\varphi_1(b)]_b^{'}\int_a^b \frac{d\tau}{(b-a)^{p+\mu-1}}d\tau + \frac{[K(t,b)\varphi_1(b)]_b^{''}}{2!}\int_a^b \frac{d\tau}{(b-a)^{p+\mu-2}} - \dots +$$
$$+ \frac{(-1)^p}{p!}\int_a^b \frac{[K(t,b+\theta_1 h)\varphi_1(b+\theta_1 h)]_\tau^{(p)}}{(b-\tau)^{\mu}} d\tau$$

ega bo'lamiz.

Bundan, (1.2.18) va (1.2.19) larni e'tiborga olgan holda,

$$|\varphi_2(t)| \le ND^2, \quad |\varphi_2^{'}(t)| \le ND^2$$

bahoni olamiz.

Xuddi shunday, induksiya usuli yordamida

$$|\varphi_i(t)| \le ND^i, \quad |\varphi_i^{'}(t)| \le ND^i \qquad (i = 1,2,\dots, p) \tag{1.2.19'}$$

Shunday qilib, (1.2.17) qatorga mos kelgan majorant qator parametr λ ushbu

$$|\lambda| < \frac{1}{\frac{M}{(p+\mu-1)(b-a)^{p+\mu-1}} + \frac{2M}{(p+\mu-2)(b-a)^{p+\mu-2}} + \frac{2^2 M}{2(p+\mu-3)(b-a)^{p+\mu-3}} + \ldots + \frac{2^p M (b-a)^{1-\mu}}{p!(1-\mu)}}$$

(1.2.20)

shartni qanoatlantirsa yaqinlashuvchi bo'ladi.

Shunday qilib, agar λ parametr (1.2.20) shartni qanoatlantirsa, u holda (1.2.17) qator tekis yaqinlashuvchi bo'ladi.

Xuddi shunday (1.2.17) qatorning p – tartibgacha hosilasidan tuzilgan qator ham, λ parametr (1.2.20) shartni qanoatlantirsa tekis yaqinlashuvchi bo'ladi.

$\varphi_k(t)$ $(k = 0,1,2,\ldots)$ funksiyalarni baholashning boshqa usulini keltiramiz.

(1.2.17) formuladan foydalanib, $\varphi_1(t)$ ni quyidagicha tasvirlaymiz:

$$\varphi_1(t) = \int_a^b \frac{K(t,\tau) f(\tau)}{(b-\tau)^{p+\mu}} d\tau = -\frac{K(t,a) f(a)}{a_1 b_1} + \frac{[K(t,a) f(a)]_a'}{a_1 a_2 b_2} - \frac{[K(t,a) f(a)]_a''}{a_1 a_2 a_3 b_3} + \ldots +$$

$$+ \frac{(-1)^p [K(t,a) f(a)]_a^{(p-1)}}{a_1 a_2 \ldots a_p b_p} + \frac{(-1)^p}{a_1 a_2 \ldots a_p} \int_a^b \frac{[K(t,\tau) f(\tau)]_\tau^{(p)}}{(b-\tau)^\mu} d\tau$$

bunda

$$a_k = p + \mu - k, \; b_k = (b-a)^{p+\mu-k} \quad (k = 1,2,\ldots, p).$$

Bundan,

$$|\varphi_1(t)| \le \frac{MN}{a_1 b_1} + \frac{2MN}{a_1 a_2 b_2} + \frac{2^2 MN}{a_1 a_2 a_3 b_3} + \ldots + \frac{2^{p-1} MN}{a_1 a_2 \ldots a_p b_p} + \frac{2^p MN}{a_1 a_2 \ldots a_p} \cdot \frac{(b-a)^{1-\mu}}{1-\mu},$$

$|\varphi_1(t)| \le NE$, bunda

$$E = \frac{M}{a_1 b_1} + \frac{2M}{a_1 a_2 b_2} + \frac{2^2 M}{a_1 a_2 a_3 b_3} + \ldots + \frac{2^{p-1} M}{a_1 a_2 \ldots a_p b_p} + \frac{2^p M}{a_1 a_2 \ldots a_p} \cdot \frac{(b-a)^{1-\mu}}{1-\mu}$$

Ravshanki, $\varphi_1(t)$ funksiyaning p – tartibgacha hosilasi uchun ham xuddi shu bahoni olish mumkin. (2) formuladan foydalanib $\varphi_2(t)$ ni quyidagicha tasvirlaymiz:

$$\varphi_2(t) = \int_a^b \frac{K(t,\tau)\varphi_1(\tau)}{(b-\tau)^{p+\mu}} d\tau = -\frac{K(t,a)\varphi_1(a)}{a_1 b_1} + \frac{[K(t,a)\varphi_1(a)]_a'}{a_1 a_2 b_2} - \ldots +$$

$$+ \frac{(-1)^p [K(t,a)\varphi_1(a)]_a^{(p-1)}}{a_1 a_2 \ldots a_p b_p} + \frac{(-1)^p}{a_1 a_2 \ldots a_p} \int_a^b \frac{[K(t,\tau)\varphi_1(\tau)]_\tau^{(p)}}{(b-\tau)^\mu} d\tau$$

Bundan, $|\varphi_2(t)| \le NE^2$

Induksiya usuli yordamida

$$|\varphi_k(t)| \le NE^k \tag{1.2.21}$$

ekanligini topamiz.

(1.2.17) funksional qator uchun majorant qator tuzsak, bu sonli qator

$$|\lambda| < \frac{1}{E},$$

$$E = \frac{1}{\dfrac{M}{a_1 b_1} + \dfrac{2M}{a_1 a_2 b_2} + \ldots + \dfrac{2^{p-1}M}{a_1 a_2 \ldots a_2} \cdot \dfrac{(b-a)^{1-\mu}}{1-\mu}} \tag{1.2.22}$$

shartni qanoatlantirganda yaqinlashuvchi bo'ladi.

(1.2.22) shart (1.2.17) funksional qatorning yaqinlashish sharti bo'lib hisoblanadi.

Xuddi shunday (1.2.17) funksional qator hosilasidan tuzilgan qator uchun ham (1.2.22) shart tekis yaqinlashish sharti bo'lib hisoblanadi.

(1.2.20) va (1.2.22) shartlarni solishtirish natijasida, $p = 1$ bo'lganda (1.2.20) shartning aniqlik darajasi (1.2.22) shartga nisbatan yaxshi (aniqroq) ekanligiga ishonch hosil qilamiz.

Agar $p > 1$ bo'lganda, (1.2.20) va (1.2.22) shartlarning aniq darajasi μ va (a,b) oraliqning uzunligiga bog'liq.

(a,b) oraliqning uzunligi kichik, μ esa istalgancha birga yaqin bo'lganda (1.2.22) shart yaxshi natija beradi;

Masalan, $p = 2$, $b - a = \frac{1}{4}$, $\mu = \frac{9}{10}$ bo'lganda (1.2.22) baho qulay. Agar $p = 2$, $b - a = 1$ bo'lganda $\forall \mu \in (0,1)$ uchun (1.2.20) baho qulay bo'ladi.

Yechimning yagonaligi.

Agar λ (1.2.20) yoki (1.2.22) shartni qanoatlantirsa, u holda, yuqorida ketma – ket yaqinlashish usuli bilan topilgan yechim yagona bo'ladi.

Isbot. Faraz qilaylik (1.2.16) tenglama ikkita $\varphi(t)$ va $\varphi^*(t)$ $(\varphi(t) \neq \varphi^*(t))$ yechimlarga ega bo'lsin. U holda, $\psi(t) = \varphi(t) - \varphi^*(t)$ ayirma ushbu

$$\psi(t) = \lambda \int_a^b \frac{K(t,\tau)\psi(\tau)}{(b-\tau)^{p+\mu}} \qquad (1.2.23)$$

bir jinsli tenglamani qanoatlantiradi.

$$N_1 = \left\{\max \left|\max |\psi(t)|, \max |\psi'(t)|, \ldots, \max |\psi^{(p)}(t)|\right|\right\}$$

deb belgilaymiz.

Agar (1.2.23) ning chap tomoni uchun yuqoridagiday baholash usullarining birini qo'llasak, natijada

$$|\psi(t)| \leq N_1 C |\lambda|,$$

Bunda $C = B$ yoki $C = E$ bahoga ega bo'lamiz.

Ravshanki, $|\psi^{(k)}(t)| \leq N_1 C|\lambda|$ $(k = 1, 2, \ldots, p)$ tengsizlik ham o'rinli bo'ladi. Bu tengsizliklardan

$$\psi(t) \leq N_1 C^2 |\lambda|^2;$$

$$\psi(t) \leq N_1 C^n |\lambda|^n \quad (n = 1, 2, \ldots).$$

$|\tilde{N}\lambda| < 1$ ekanligini e'tiborga olgan holda

$$\psi(t) \leq \lim_{n \to \infty} N_1 C^n |\lambda|^n = 0.$$

Shunday qilib, agar λ (1.2.20) yoki (1.2.22) tengsizlikni qanoatlantirganda (1.2.16) tenglama yagona yechimga ega bo'ladi, ya'ni quyidagi teorema o'rinli bo'ladi.

Shunday qilib, quyidagi teorema isbot bo'ladi.

1.2.3 – teorema. Agar $K(t,\tau)$ va $f(\tau)$ funksiyalar mos ravishda (D) va (D_1) sohalarda aniqlangan bo'lib, ular mos ravishda 2p va p marta uzluksiz hosilalarga ega bo'lsa, hamda λ (1.2.20) yoki (1.2.22) shartlarning birini qanoatlantirsa, u holda (1.2.16) Adamar ma'nosida chiziqli tenglama yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli bilan topiladi.

1.3- §. Adamar ma'nosidagi integrallarda integrallarning o'rinlarini almashtirish formulasi.

1.3.1 – teorema. Agar $K(t,\tau)$ va $f(\tau)$ funksiyalar mos ravishda (D) va (D_1) sohalarda aniqlangan bo'lib, $K(t,\tau)$ funksiya 2 – tartibgacha uzluksiz aralash hosilalarga, $f(\tau)$ funksiya esa birinchi tartibli uzluksiz hosilaga ega bo'lsa, u holda

$$\int_a^b \frac{K(t,s)ds}{(b-s)^{1+\mu}} \int_a^b \frac{K(s,\tau)f(\tau)}{(b-s)^{1+\mu}} d\tau = \int_a^b \frac{f(\tau)}{(b-\tau)^{1+\mu}} d\tau \int_a^b \frac{K(t,s)K(s,t)}{(b-s)^{1+\mu}} ds \tag{1.3.1}$$

tenglik o'rinli bo'ladi.

Odatda, bu formula Adamar integrallarining o'rinlarini almashtirish formulasi deb yuritiladi.

Isbot. Agar $A(t,a)$ va $\psi(\tau)$ funksiyalar τ bo'yicha birinchi tartibli hosilalarga ega bo'lsa, u holda

$$\int_a^b \frac{A(t,\tau)\psi(\tau)}{(b-\tau)^{1+\mu}} d\tau = -\frac{A(t,a)\psi(a)}{\mu(b-a)^{\mu}} - \frac{1}{\mu}\int_a^b \frac{A_\tau^{'}(t,\tau)\psi(\tau) + A(t,\tau)\psi^{'}(\tau)}{(b-a)^{\mu}} d\tau \tag{1.3.2}$$

tenglik o'rinli bo'ladi. Bu tenglikning o'rinli ekanligiga Adamar ma'nosidagi integralning ta'rifini eslash yetarli.

(1.3.2) formulani (1.3.1) tenglikning chap tomonidagi ichki integralga qo'llash natijasida:

$$\int_a^b \frac{A(s,\tau)f(\tau)}{(b-\tau)^{1+\mu}} d\tau = -\frac{K(s,a)f(a)}{\mu(b-a)^{\mu}} - \frac{1}{\mu}\int_a^b \frac{A_\tau^{'}(s,\tau)f(\tau) + A(s,\tau)f^{'}(\tau)}{(b-a)^{\mu}} d\tau \tag{1.3.3}$$

ega bo'lamiz. (1.3.3) ni (1.3.1) ning chap tomoniga olib borib qo'yamiz:

$$\int_a^b \frac{K(t,s)ds}{(b-s)^{1+\mu}} \int_a^b \frac{K(s,\tau)f(\tau)}{(b-\tau)^{1+\mu}} d\tau = \int_a^b \frac{K(t,s)\left[-\frac{K(s,a)f(a)}{\mu(b-a)^{\mu}} - \frac{1}{\mu}\int_a^b \frac{K_\tau^{'}(s,\tau)f(\tau) + K(s,\tau)f^{'}(\tau)}{(b-\tau)^{\mu}} d\tau\right]}{(b-s)^{1+\mu}} ds$$

Bu tenglikning o'ng tomonidagi tashqi integralga (1.3.2) formulani qo'llaymiz:

$$\int_a^b \frac{K(t,s)\left[-\frac{K(s,a)f(a)}{\mu(b-a)^{\mu}}-\frac{1}{\mu}\int_a^b \frac{K'_{\tau}(s,t)f(\tau)+K(s,t)f'(\tau)}{(b-\tau)^{\mu}}d\tau\right]}{(b-s)^{1+\mu}}ds =$$

$$= -\frac{K(t,a)\left[-\frac{K(a,a)f(a)}{\mu(b-a)^{\mu}}-\frac{1}{\mu}\int_a^b \frac{K'_{\tau}(a,\tau)f(\tau)+K(a,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau\right]}{\mu(b-a)^{\mu}}-$$

$$-\frac{1}{\mu}\int_a^b \left\{K'_s(t,s)\left[-\frac{K(s,a)f(a)}{\mu(b-a)^{\mu}}-\frac{1}{\mu}\int_a^b \frac{K'_{\tau}(s,\tau)f(\tau)+K(s,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau\right]-\right.$$

$$\left.-K(t,s)\left[\frac{K'_s(s,a)f(a)}{\mu(b-a)^{\mu}}+\frac{1}{\mu}\int_a^b \frac{K''_{\tau s}(s,\tau)f(\tau)+K'_s(s,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau\right]\frac{ds}{(b-s)^{\mu}}\right.$$

Shunday qilib, oxirgi natija quyidagicha bo’ladi:

$$\begin{aligned}
&\int_a^b \frac{K(t,s)ds}{(b-s)^{1+\mu}}\int_a^b \frac{K(s,\tau)f(\tau)}{(b-\tau)^{1+\mu}}d\tau = \frac{K(t,a)K(a,a)f(a)}{\mu^2(b-a)^{2\mu}}+\\
&+\frac{K(t,a)}{\mu^2(b-a)^{\mu}}\int_a^b \frac{K'_{\tau}(a,\tau)f(\tau)+K(a,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau+\frac{f(a)}{\mu^2(b-a)^{\mu}}\int_a^b \frac{K'_s(t,s)K(s,a)}{(b-s)^{\mu}}ds+\\
&+\frac{f(a)}{\mu^2(b-a)^{\mu}}\int_a^b \frac{K'_s(s,a)K(t,s)}{(b-s)^{\mu}}ds+\frac{1}{\mu^2}\int_a^b \frac{K'_s(t,s)ds}{(b-s)^{\mu}}\int_a^b \frac{K'_{\tau}(s,\tau)f(\tau)}{(b-\tau)^{\mu}}d\tau+\\
&+\frac{1}{\mu^2}\int_a^b \frac{K'_s(t,s)ds}{(b-s)^{\mu}}\int_a^b \frac{K(s,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau+\frac{1}{\mu^2}\int_a^b \frac{K(t,s)ds}{(b-s)^{\mu}}\int_a^b \frac{K'_{\tau s}(s,\tau)f(\tau)}{(b-\tau)^{\mu}}d\tau+\\
&+\frac{1}{\mu^2}\int_a^b \frac{K(t,s)ds}{(b-s)^{\mu}}\int_a^b \frac{K'_s(s,\tau)f'(\tau)}{(b-\tau)^{\mu}}d\tau
\end{aligned} \tag{1.3.4}$$

Endi (1.3.1) tenglikning o’ng tomonidagi ichki integralga (1.3.2) ni qo’llash natijasida:

$$\int_a^b \frac{K(t,s)K(s,\tau)}{(b-s)^{1+\mu}}ds = -\frac{K(t,a)K(a,\tau)}{\mu(b-a)^{\mu}}-\frac{1}{\mu}\int_a^b \frac{K'_s(t,s)K(s,\tau)+K(t,s)K'_s(s,\tau)}{(b-s)^{\mu}}ds\ ,$$

$$\int_a^b \frac{f(\tau)}{(b-\tau)^{1+\mu}} d\tau \int_a^b \frac{K(t,s)K(s,\tau)}{(b-s)^{1+\mu}} ds =$$

$$= \int_a^b \frac{f(\tau)\left[-\dfrac{K(t,a)K(a,\tau)}{\mu(b-a)^{\mu}} - \dfrac{1}{\mu}\displaystyle\int_a^b \dfrac{K_s^{'}(t,s)K(s,\tau)+K(t,s)K_s^{'}(s,\tau)}{(b-s)^{\mu}}ds\right]}{(b-s)^{1+\mu}} d\tau =$$

$$= \frac{f(a)\left[-\dfrac{K(t,a)K(a,a)}{\mu(b-a)^{\mu}} - \dfrac{1}{\mu}\displaystyle\int_a^b \dfrac{K_s^{'}(t,s)K(s,a)+K_s^{'}(s,a)K(t,s)}{(b-s)^{\mu}}ds\right]}{\mu(b-a)^{\mu}} -$$

$$-\frac{1}{\mu}\int_a^b \left\{\left[-\frac{K(t,a)K_\tau^{'}(a,\tau)}{\mu(b-a)^{\mu}} - \frac{1}{\mu}\int_a^b \frac{K_s^{'}(t,s)K_\tau^{'}(s,t)+K_{s\tau}^{''}(s,\tau)K(t,s)}{(b-s)^{\mu}}ds\right] f(\tau) +\right.$$

$$\left. + f^{'}(\tau)\left[-\frac{K(t,a)K(a,\tau)}{\mu(b-a)^{\mu}} - \frac{1}{\mu}\int_a^b \frac{K_s^{'}(t,s)K(s,\tau)+K_s^{'}(s,\tau)K(t,s)}{(b-s)^{\mu}}d\tau\right]\right\} \frac{d\tau}{(b-\tau^{\mu})}$$

Shunday qilib, oxirgi natija quyidagicha bo'ladi:

$$\begin{aligned}
&\int_a^b \frac{f(\tau)d\tau}{(b-\tau)^{1+\mu}} \int_a^b \frac{K(t,s)K(s,\tau)}{(b-s)^{1+\mu}} ds = \frac{K(t,a)K(a,a)f(a)}{\mu^2(b-a)^{2\mu}} + \\
&+ \frac{K(t,a)}{\mu^2(b-a)^{\mu}} \int_a^b \frac{K_\tau^{'}(a,\tau)f(\tau)+K(a,\tau)f^{'}(\tau)}{(b-\tau)^{\mu}} d\tau + \frac{f(a)}{\mu^2(b-a)^{\mu}} \int_a^b \frac{K_s^{'}(t,s)K(s,a)}{(b-s)^{\mu}} ds + \\
&+ \frac{f(a)}{\mu^2(b-a)^{\mu}} \int_a^b \frac{K_s^{'}(s,a)K(t,s)}{(b-s)^{\mu}} ds + \frac{1}{\mu^2} \int_a^b \frac{f(\tau)d\tau}{(b-\tau)^{\mu}} \int_a^b \frac{K_s^{'}(t,s)K_\tau^{'}(s,\tau)}{(b-s)^{\mu}} ds + \\
&+ \frac{1}{\mu^2} \int_a^b \frac{f(\tau)d\tau}{(b-\tau)^{\mu}} \int_a^b \frac{K_{s\tau}^{'}(s,\tau)K(t,s)}{(b-s)^{\mu}} ds + \frac{1}{\mu^2} \int_a^b \frac{f^{'}(\tau)d\tau}{(b-\tau)^{\mu}} \int_a^b \frac{K_s^{'}(t,s)K(s,\tau)}{(b-s)^{\mu}} ds + \\
&+ \frac{1}{\mu^2} \int_a^b \frac{f^{'}(\tau)d\tau}{(b-\tau)^{\mu}} \int_a^b \frac{K_s^{'}(s,\tau)K(t,s)}{(b-s)^{\mu}} ds
\end{aligned} \tag{1.3.5}$$

Agar (1.3.4) va (1.3.5) tengliklarni solishtirsak, u holda (1.3.1) formulaning to'g'riligiga ishonch hosil qilamiz. Bu teoremani e'tiborga olib quyidagi teoremani isbot qilish qiyin emas.

1.3.2 – teorema. Agar $K(t,\tau)$ va $f(\tau)$ funksiyalar mos ravishda D va D_1 sohalarda aniqlangan bo'lib, $K(t,\tau)$ funksiya $2p$ tartibgacha hamma uzluksiz hosilalarga, $f(\tau)$ funksiya esa uzluksiz p – tartibgacha hosilaga ega bo'lsa, u holda

$$\int_a^b \frac{K(t,s)ds}{(b-s)^{p+\mu}} \int_a^b \frac{K(s,\tau)f(\tau)}{(b-\tau)^{p+\mu}} d\tau = \int_a^b \frac{f(\tau)}{(b-\tau)^{p+\mu}} d\tau \int_a^b \frac{K(t,s)K(s,\tau)}{(b-s)^{p+\mu}} ds \tag{1.3.6}$$

tenglik o'rinli bo'ladi.

Odatda bu formulani ham ikki karrali Adamar integrallarida integrallarning o'rinlarini almashtirish formulasi deb yuritiladi.

Bu teorema Adamar integrallarida ham iteratsiyalangan yadro va rezolventa tushunchalarini kiritish imkonini beradi.

1.4 - §. Qisib akslantirish prinsipi yordamida yechish Adamar ma'nosidagi chiziqli integral tenglamani yechish.

Ushbu

$$\varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\varphi(\tau)}{(b-\tau)^{p+\mu}} d\tau = f(t) \tag{1.4.1}$$

Adamar ma'nosidagi chiziqli integral tenglama yechimining mavjudligi va yagonaligini isbotlash uchun qo'llaymiz.

Bu yerda $K(t,\tau)$ - integral tenglama yadrosi, $\varphi(t)$ - berilgan funksiya, $f(t)$ - izlanayotgan (noma'lum) funksiya, λ esa haqiqiy parameter.

Ko'rsatamizki, qisib akslantirish prinsipini λ parametrning yetarlicha kichik qiymatlarida qo'llash mumkin.

Faraz qilamiz, $K(t,\tau) - [a,b] \times [a,b]$ kvadratda uzluksiz funksiya bo'lsin. Shunday ekan, musbat M son bo'lib, barcha $t,\tau \in [a,b]$ uchun $|K(t,\tau)| \le M$ tengsizlik bajarildi. To'la C[a,b] fazoni o'zini – o'ziga

$$g(t) = \varphi(t) - \lambda \int_a^b \frac{K(t,\tau) f(\tau)}{(b-a)^{p+\mu}} d\tau \tag{1.4.2}$$

formula asosida akslantiruvchi $g = Af$ akslantirish berilgan bo'lsin.

U holda

$$\rho(g_1, g_2) = \max_{a \le t \le b} |g_1(t) - g_2(t)| \le |\lambda| M (b-a) \cdot \max_{a \le t \le b} |f_1(t) - f_2(t)|$$

yoki

$$\rho(Af_1, Af_2) \le |\lambda| M (b-a) \cdot \rho(f_1, f_2)$$

Shunday ekan

$$|\lambda| < \frac{1}{M\,(b-a)} \qquad (1.4.3)$$

bo'lganda A qisib akslantirish bo'ladi.

Qisib akslantirishlar prinsipiga asoslanib xulosa qilamizki, (1.4.3) shartni qanoatlantiruvchi ixtiyoriy λ da (1.4.1) Adamar ma'nosidagi integral tenglamasi yagona uzluksiz yechimga ega.

Bu yechimga intiluvchi ketma – ket yaqinlashishlar $f_0, f_1, \dots, f_n, \dots$

$$f_n(t) = \varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\, f_{n-1}(\tau)}{(b-a)^{p+\mu}} d\tau$$

ko'rinishga ega, bu yerda f_0 sifatida uzluksiz funksiyani olish mumkin.

I Bob bo'yicha xulosa

Adamar ma'nosidagi maxsus integralga keltirilib, ularning asosiy xossalari o'rganildi. Bu o'rganilgan xossalar yordamida bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimi mavjudligi va yagonaligi haqidagi 1.2.2 va 1.2.3 – teoremalar isbotlangan. Ta'kidlash mumkinki, 1.2.2 – teorema integral tenglamaning maxsuslik darajasi $1+\mu \quad (0<\mu<1)$ bo'lganda N.P. Vekua [2] ishida isbotlangan. Shu natija asosida tenglamaning maxsuslik darajasi $p+\mu$ (p – musbat butun son) bo'lgan ishida 1.2.3 – teorema batafsil ketma – ket yaqinlashish usuli yordamida yechimining mavjudligi va yagonaligi isbot qilindi.

Bir o'zgaruvchili funksiya uchun Adamar ma'nosidagi integral tenglamaning maxsuslik darajasi $p+\mu \quad (0<\mu<1)$ bo'lganda uning yechimining mavjudligi va yagonaligi qisib akslantirish usuli yordamida ko'rsatilgan.

II Bob. Adamar ma’nosidagi ikki o’zgaruvchili chiziqli integral tenglama.

2.1-§. Maxsuslik darajasi $p > 1$ bo’lganda ikki o’zgaruvchili Adamar ma’nosidagi integral tenglamani ketma – ket yaqinlashish usuli bilan yechish.

Ushbu

$$\varphi(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} dtd\tau = f(x,y) \tag{2.1.1}$$

Adamar ma’nosidagi integral tenglamani qaraymiz, bunda $K(x,y,t,\tau)$ $\Delta = \{a \le x, y \le b; c \le t, \tau \le d\}$ sohada aniqlangan uzluksiz va to’rt martagacha uzluksiz aralash hosilalarga ega bo’lgan funksiya, $f(x,y)$ esa, $\Delta_1 = \{c \le t, \tau \le d\}$ sohada uzluksiz va ikki martagacha uzluksiz aralash hosilalarga ega bo’lgan funksiya, $0 < \mu_1 < 1, 0 < \mu_2 < 1$, $\varphi(x,y)$ - izlanuvchi funksiya, a, b, c, d, λ - lar o’zgarmas haqiqiy sonlar $(\lambda \ne 0)$.

(2.1.1) integral tenglamaning yechimini ushbu

$$\varphi(x,y) = \varphi_0(x,y) + \lambda\varphi_1(x,y) + \lambda^2\varphi_2(x,y) + \dots + \lambda^n\varphi_n(x,y) + \dots \tag{2.1.2}$$

qator ko’rinishda izlaymiz. (2.1.2) qatorni (2.1.1) tenglamaning yechimi deb faraz qilamiz va uni berilgan tenglamaga qo’yib, λ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib $\varphi_0(x,y), \varphi_1(x,y), \dots, \varphi_n(x,y), \dots$ larning ifodasini birin ketin aniqlaymiz:

$$\varphi_0(x,y) = f(x,y);$$

$$\varphi_1(x,y) = \int_a^b \int_c^d \frac{K(x,y,t,\tau) f(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} dtd\tau;$$

$$\varphi_2(x,y) = \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi_1(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} dtd\tau;$$

....

$$\varphi_n(x,y) = \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi_{n-1}(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} dtd\tau;$$

....

Shu ifodalar yordamida (2.1.2) qatorni quyidagi ko'rinishda yozib olamiz:

$$\varphi(x,y)= f(x,y)+\lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)f(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau+\lambda^2\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi_1(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau+$$
$$+\ldots+\lambda^n\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi_{n-1}(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau+\ldots \qquad (2.1.3)$$

Yuqorida 2 - § da isbot qilingan 1.2.2 – teoremaga asosan,

$$\varphi(x,y)=\sum_{k=0}^{\infty}\lambda^k\varphi_k(x,y),$$

agarda yuqoridagi qator va uning hamma aralash xususiy hosilalaridan tuzilgan qatorlar tekis va absolyut yaqinlashuvchi bo'lsa, qatorlarni hadma – had integrallash mumkin bo'ladi va (2.1.2) qator (2.1.1) integral tenglamaning yehimi bo'ladi.

Endi bu qatorlarning tekis yaqinlashuvchi ekanligini isbotlaymiz. Buning uchun $f(x,y)$ funksiya va uning ikkinchi tartibgacha hamma aralash xususiy hosilalari maksimumlarining maksimumi N bilan, $K(x,y,t,\tau)$ va uning hamma to'rt martagacha aralash xususiy hosilalari maksimumlarining maksimumi E bilan belgilab, quyida tengsizliklarni hosil qilamiz:

$$|\varphi_0(x,y)|=|f(x,y)|\le N, \qquad (2.1.4)$$

$\varphi_1(x,y)$ ni baholash uchun quyidagi chekli ayirmalar haqidagi formulalardan foydalanamiz:

$$K(x,y,t,\tau)f(t,\tau)=K(x,y,d,\tau)f(d,\tau)-$$
$$-(d-t)\left[K_t'(x,y,d+\theta_1h_1,\tau)f(d+\theta_1h_1,\tau)+K(x,y,d+\theta_1h_1,\tau)f_t'(d+\theta_1h_1,\tau)\right] \qquad (*)$$

$$K(x,y,d,\tau)f(d,\tau)=K(x,y,d,b)f(d,b)-$$
$$-(b-\tau)\left[K_\tau'(x,y,d,b+\theta_2h_2)f(d,b)+K(x,y,d,b+\theta_2h_2)f_\tau'(d,b+\theta_2h_2)\right] \qquad (**)$$

bunda $h_1=d-t, h_2=b-\tau, 0<\theta_1,\theta_2<1$.

(*) ni $\varphi_1(x,y)$ ning ifodasidagi integral ostiga olib borib qo'yish natijasida

$$\varphi_1(x,y)=\int_a^b\int_c^d \frac{\{K(x,y,d,\tau)f(d,\tau)-}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}$$

$$\frac{-(d-t)\left[K_t^{'}(x,y,d+\theta_1 h_1,\tau)f(d+\theta_1 h_1,\tau)+K(x,y,d,\tau)f_t^{'}(d+\theta_1 h_1,\tau)\right]\}}{}dtd\tau=$$

$$=\int_c^d \frac{1}{(d-t)^{1+\mu_2}}dt\int_a^b \frac{K(x,y,d,\tau)f(d,\tau)}{(b-\tau)^{1+\mu_1}}d\tau- \tag{2.1.5}$$

$$-\int_c^d \frac{dt}{(d-t)^{\mu_2}}\left[\int_a^b \frac{K_t^{'}(x,y,d+\theta_1 h_1,\tau)f(d+\theta_1 h_1,\tau)}{(b-\tau)^{1+\mu_1}}d\tau-\right.$$

$$\left.-\int_a^b \frac{K(x,y,d,\tau)f_t^{'}(d+\theta_1 h_1,\tau)}{(b-\tau)^{1+\mu_1}}d\tau\right]$$

keyingi (2.1.5) tenglamaning o'ng tomonidagi ichki integralning suratidagi funksiyalarga τ bo'yicha chekli ayirmalar haqidagi (**) formulani qo'llaymiz: bunda, $\int_c^d \frac{dt}{(d-t)^{1+\mu_2}}=-\frac{1}{\mu_2(d-c)^{\mu_2}},\int_a^b \frac{d\tau}{(b-\tau)^{1+\mu_1}}=-\frac{1}{\mu_1(b-a)^{\mu_1}}$ ni e'tiborga olgan holda,

$$\int_a^b \frac{K(x,y,d,\tau)f(d,\tau)}{(b-\tau)^{1+\mu_1}}d\tau=K(x,y,d,b)f(d,b)\cdot\int_a^b \frac{d\tau}{(b-\tau)^{1+\mu_1}}-$$

$$-\int_a^b \frac{\left[K_\tau^{'}(x,y,d,b+\theta_2 h_2)f(d,b)+K(x,y,d,b+\theta_2 h_2)f_\tau^{'}(d,b+\theta_2 h_2)\right]}{(b-\tau)^{\mu_1}}d\tau=$$

$$=-\frac{1}{\mu_1(b-a)^{\mu_1}}\cdot K(x,y,d,b)f(d,b)+ \tag{2.1.6}$$

$$+\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_\tau^{'}(x,y,d,b+\theta_2 h_2)f(d,b)+K(x,y,d,b+\theta_2 h_2)f_\tau^{'}(d,b+\theta_2 h_2)\right]$$

$$\int_a^b \frac{K_t^{'}(x,y,d+\theta_1 h_1,\tau)f(d+\theta_1 h_1,\tau)}{(b-\tau)^{1+\mu_1}}d\tau=K_t^{'}(x,y,d+\theta_1 h_1,b)f(d+\theta_1 h_1,b)\int_a^b \frac{d\tau}{(b-\tau)^{1+\mu_1}}-$$

$$-\int_a^b \frac{K_{t\tau}^{''}(x,y,d+\theta_1 h_1,b+\theta_2 h_2)f(d+\theta_1 h_1,b)+K_t^{'}(x,y,d+\theta_1 h_1,b)f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)}{(b-\tau)^{\mu_1}}d\tau=$$

$$=-\frac{1}{\mu_1(b-a)^{\mu_1}}\cdot K_t^{'}(x,y,d+\theta_1 h_1,b)f(d+\theta_1 h_1,b)- \tag{2.1.7}$$

$$-\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_{t\tau}^{''}(x,y,d+\theta_1 h_1,b+\theta_2 h_2)f(d+\theta_1 h_1,b)+\right.$$

$$\left.+K_t^{'}(x,y,d+\theta_1 h_1,b)f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)\right];$$

$$\int_a^b \frac{K(x,y,d,\tau) f_t^{'}(d+\theta_1 h_1,\tau)}{(b-\tau)^{1+\mu_1}} d\tau = K(x,y,d,b) f_t^{'}(d+\theta_1 h_1,b) \int_a^b \frac{d\tau}{(b-\tau)^{1+\mu_1}} -$$

$$-\int_a^b \frac{K_t^{'}(x,y,d,b+\theta_2 h_2) f_t^{'}(d+\theta_1 h_1,b) + K(x,y,d,b) f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)}{(b-\tau)^{\mu_1}} d\tau =$$

$$= -\frac{1}{\mu_1 (b-a)^{\mu_1}} \cdot K(x,y,d,b) f_t^{'}(d+\theta_1 h_1,b) - \tag{2.1.8}$$

$$-\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_t^{'}(x,y,d,b+\theta_2 h_2) f_t^{'}(d+\theta_1 h_1,b) + \right.$$

$$\left. + K_t^{'}(x,y,d,b) f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)\right]$$

(2.1.6), (2.1.7), (2.1.8) larni e'tiborga olib, (2.1.5) dan

$$\varphi_1(x,y) = \frac{1}{\mu_1 (b-a)^{\mu_1}} \cdot \frac{1}{\mu_2 (d-c)^{\mu_2}} K(x,y,d,b) f(d,b) -$$

$$-\frac{1}{\mu_2 (d-c)^{\mu_2}} \cdot \frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_\tau^{'}(x,y,d,b+\theta_2 h_2) f(d,b) + \right.$$

$$\left. + K(x,y,d,b+\theta_2 h_2) f_\tau^{'}(d,b+\theta_2 h_2)\right] +$$

$$+\frac{(d-c)^{1-\mu_2}}{1-\mu_2} \cdot \frac{1}{\mu_1 (b-a)^{\mu_1}} \cdot K_t^{'}(x,y,d+\theta_1 h_1,b) f(d+\theta_1 h_1,b) +$$

$$+\frac{(d-c)^{1-\mu_2}}{1-\mu_2} \cdot \frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_{t\tau}^{''}(x,y,d+\theta_1 h_1,b+\theta_2 h_2) f_t^{'}(d+\theta_1 h_1,b) + \right.$$

$$\left. + K_t^{'}(x,y,d+\theta_1 h_1,b) f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)\right] -$$

$$-\frac{(d-c)^{1-\mu_2}}{1-\mu_2} \cdot \frac{(b-a)^{1-\mu_1}}{\mu_1 (b-a)^{\mu_1}} \cdot K(x,y,d,b) f_t^{'}(d+\theta_1 h_1,b) -$$

$$-\frac{(d-c)^{1-\mu_2}}{1-\mu_2} \cdot \frac{(b-a)^{1-\mu_1}}{1-\mu_1}\left[K_\tau^{'}(x,y,d,b+\theta_2 h_2) f_t^{'}(d+\theta_1 h_1,b) + \right.$$

$$\left. + K(x,y,d,b) f_{t\tau}^{''}(d+\theta_1 h_1,b+\theta_2 h_2)\right] \tag{2.1.9}$$

ega bo'lamiz.

(2.1.9) dan

$$\left|\varphi_1(x,y)\right| \le \frac{NE}{\mu_1(b-a)^{\mu_1}\mu_2(d-c)^{\mu_2}} + \frac{1}{\mu_2(d-c)^{\mu_2}}\cdot\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\cdot 2NE +$$

$$+\frac{(d-c)^{1-\mu_2}}{1-\mu_2}\cdot\frac{1}{\mu_1(b-a)^{\mu_1}}\cdot EN + \frac{(d-c)^{1-\mu_2}}{1-\mu_2}\cdot\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\cdot 2EN +$$

$$+\frac{(d-c)^{1-\mu_2}}{1-\mu_2}\frac{1}{\mu_1(b-a)^{\mu_1}}EN + \frac{(d-c)^{1-\mu_2}}{1-\mu_2}\frac{(b-a)^{1-\mu_1}}{1-\mu_1}2EN =$$

$$= NE\left[\frac{1}{\mu_1(b-a)^{\mu_1}\mu_2(d-c)^{\mu_2}} + \frac{2(b-a)^{1-\mu_1}}{\mu_2(d-c)^{\mu_2}(1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{1-\mu_2}\cdot\frac{1}{\mu_1(b-a)^{\mu_1}} +\right.$$

$$\left.+\frac{2(d-c)^{1-\mu_2}}{1-\mu_2}\frac{(b-a)^{1-\mu_1}}{1-\mu_1} + \frac{(d-c)^{1-\mu_2}}{1-\mu_2}\cdot\frac{1}{\mu_1(b-a)^{\mu_1}} + \frac{2(d-c)^{1-\mu_2}}{1-\mu_2}\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\cdot\right] =$$

$$= 2NE\left(\frac{1}{\mu_1(b-a)^{\mu_1}\mu_2(d-c)^{\mu_2}} + \frac{(b-a)^{1-\mu_1}}{\mu_2(d-c)^{\mu_2}(1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{1-\mu_2}\frac{1}{\mu_1(b-a)^{\mu_1}} +\right.$$

$$\left.+\frac{(d-c)^{1-\mu_2}}{1-\mu_2}\frac{(b-a)^{1-\mu_1}}{1-\mu_1}\right) = NE\cdot B,$$

bunda,

$$B = 2\left(\frac{1}{\mu_1(b-a)^{\mu_1}\mu_2(d-c)^{\mu_2}} + \frac{(b-a)^{1-\mu_1}}{\mu_2(d-c)^{\mu_2}(1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{(1-\mu_2)\mu_1(b-a)^{\mu_1}} +\right.$$
$$\left.+\frac{(d-c)^{1-\mu_2}(b-a)^{1-\mu_1}}{(1-\mu_2)(1-\mu_1)}\right) \qquad (2.1.10)$$

Shunday qilib $\left|\varphi_1(x,y)\right| \le BEN$.

Ravshanki, $$\left(\varphi_1^{'}(x,y)\right)_x^{'} = \int_a^b\int_c^d \frac{K_x^{'}(x,y,t,\tau)f(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau\,,$$

$$\left(\varphi_1^{'}(x,y)\right)_y^{'} = \int_a^b\int_c^d \frac{K_y^{'}(x,y,t,\tau)f(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau$$

Bu yerdan, yuqoridagi $\varphi_1(x,y)$ baholashni etiborga olgan holda

$$\left|\left(\varphi_1^{'}(x,y)\right)_x^{'}\right| = BEN\,,\left|\left(\varphi_1^{'}(x,y)\right)_y^{'}\right| = BEN \qquad (2.1.11)$$

tengsizlikni o`rinli ekanligiga ishonch hosil qilish qiyin emas.

Endi $\varphi_2(x,y)$ ni baholaymiz.

Chekli ayirmalar haqidagi formuladan foydalanib,

$$K(x,y,t,\tau)\varphi_1(t,\tau) = K(x,y,d,\tau)\varphi_1(b,\tau) -$$
$$-(d-t)\left[K_t^{'}(x,y,d+\theta_1h_1)\varphi_1(d+\theta_1h_1,\tau) + K(x,y,d+\theta_1h_1,\tau)\varphi_t^{'}(d+\theta_1h_1,\tau)\right], \qquad (\alpha)$$

$$K(x,y,d,\tau)\varphi_1(d,\tau)=K(x,y,d,b)\varphi_1(d,b)-$$
$$-(b-\tau)\left[K_\tau^{'}(x,y,d,b+\theta_2h_2)\varphi_1(d,b)+K(x,y,d,b+\theta_2h_2)\varphi_\tau^{'}(d,b+\theta_2h_2)\right] \qquad (\beta)$$

ni hosil qilamiz.

(α) ni $\varphi_2(x,y)$ ning ifodasidagi integral ostiga olib borib qo`yish natijasida

$$\varphi_2(x,y)=\int_c^d\frac{dt}{(d-t)^{1+\mu_2}}\cdot\int_a^b\frac{K(x,y,d,\tau)\varphi_1(d,\tau)}{(b-\tau)^{1+\mu_1}}-$$
$$-\int_c^d\frac{dt}{(d-t)^{\mu_2}}\left[\int_a^b\frac{K_t^{'}(x,y,d+\theta_1h_1,\tau)\varphi_1(d+\theta_1h_1,\tau)}{(b-\tau)^{1+\mu_1}}d\tau-\int_a^b\frac{K(x,y,d,\tau)\kappa_t^{'}(d+\theta_1h_1,\tau)}{(b-\tau)^{1+\mu_1}}d\tau\right] \qquad (2.1.5^{'})$$

ni hosil qilamiz.

$\varphi_1(x,y),(\varphi_1^{'}(x,y))_x^{'},(\varphi_1^{'}(x,y))_y^{'}$ funksiyalarning (2.1.10) va (2.1.11) dagi baholarni e'tiborga olgan holda

$$|\varphi_2(x,y)|\le ENB^2,\left|(\varphi_2^{'}(x,y))_x^{'}\right|\le ENB^2,\left|(\varphi_2^{'}(x,y))_y^{'}\right|\le ENB^2$$

tengsizliklarga ega bo`lamiz.

Xuddi shunday yo`l bilan, induksiya usuli yordamida

$$|\varphi_n(x,y)|\le ENB^n,\left|(\varphi_n^{'}(x,y))_x^{'}\right|\le ENB^n,\left|(\varphi_n^{'}(x,y))_y^{'}\right|\le ENB^n$$

tengsizliklarga ega bo`lamiz.

Shunday qilib, (2.1.2) qatorga mos kelgan majorant sonli qatorning ko`rinishi

$$EN+EN|\lambda|B+EN|\lambda|^2B^2+...+EN|\lambda|^nB^n+... \qquad (2.1.12)$$

shaklda bo`ladi.(2.1.12) qator λ parameter

$$|\lambda|<\frac{1}{B} \qquad (2.1.13)$$

shartni qanoatlantirganda yaqinlashuvchi bo`ladi.

Demak, (2.1.12) qator (2.1.13) shart bajarilganda tekis yaqinlashuvchi bo'ladi.

Xuddi shunday (2.1.2) qatorning hosilalaridan tashkil topgan qatorlar ham, ya'ni

$$\sum_{k=0}^{\infty}(\varphi_k(x,y))_x^{'},\sum_{k=0}^{\infty}(\varphi_k(x,y))_y^{'}$$

qatorlar ham (2.1.13) shart bajarilganda yaqinlashuvchi bo`ladi.

Biz hozircha (2.1.2) qator (2.1.1) integral tenglamaning yechimi ekanligini isbotlagandik. Endi shu yechim yagona ekanligini ko`rsatamiz. Buning uchun teskarisini, ya’ni (2.1.1) integral tenglamaning yana bitta $V(x,y)$ yechimga ega bo`lsin deb faraz qilamiz. U holda

$$V(x,y)-\lambda\int_a^b\int_c^d\frac{K(x,s,t,\tau)V(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau = f(x,y)$$

Buni (2.1.1) dan ayiramiz;

$$\varphi(x,y)-V(x,y)=\lambda\int_a^b\int_c^d\frac{K(x,y,t,\tau)[\varphi(t,v-V(t,\tau))]}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau\ ,$$

$\varphi(x,y-V(x,y))=\alpha(x,y)$ deb belgilasak, u holda

$$\alpha(x,y)=\lambda\int_a^b\int_c^d\frac{K(x,y,t,\tau)\alpha(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}dtd\tau \qquad (2.1.14)$$

$\alpha(x,y)$ funksiya va uning ikki martagacha aralash hosilalarinig maksimumlarining maksimumini N_1 deb belgilaymiz.

(2.1.14) ning chap tomonini baholash uchun yuqorida qo’llanilgan usulni qo’llasak, u holda

$$|\alpha(x,y)|\le N_1EB|\lambda|$$

ekanligini ko’ramiz. Bu bahodan foydalanib

$$|\alpha(x,y)|\le N_1E|\lambda|^2B^2\ ,$$

…………………………...

$$|\alpha(x,y)|\le N_1E|\lambda|^nB^n\ ,\ n-1,2,...$$

…………………………

Demak, $|\alpha(x,y)|\le\lim\limits_{n\to\infty}N_1E|\lambda|^nB^n=0$, chunki $|\lambda B|<1$.

Shunday qilib quyidagi teorema isbot qilindi.

2.1.1-teorema. Agar $f(t,\tau)$ funksiya (Δ_1) da uzluksiz va uzluksiz 2-martagacha aralash xususiy hosilalarga ega bo`lib, $K(x,s,t,\tau)$ funksiya esa $(\Delta)$ da uzluksiz va to'rt martagacha uzluksiz va uzluksiz aralash xususiy hosilalarga ega bo`lib,

$$|\lambda| < \frac{1}{B} \qquad (2.1.13)$$

bo`lsa,

$$B = 2\left(\frac{1}{\mu_1 (b-a)^{\mu_1} \mu_2 (d-c)^{\mu_2}} + \frac{(b-a)^{1-\mu_1}}{\mu_2 (d-c)^{\mu_2} (1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{(1-\mu_2)\mu_1 (b-a)^{\mu_1}} + \right.$$
$$\left. + \frac{(d-c)^{1-\mu_2}(b-a)^{1-\mu_1}}{(1-\mu_2)(1-\mu_1)} \right)$$

u holda (2.1.1) integral tenglama absolyut va tekis yaqinlashuvchi (2.1.2) qatordan iborat bo'lgan yagona yechimga ega bo'ladi.

Ushbu

$$\varphi(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} dt d\tau = f(x,y) \qquad (2.1.15)$$

bunda p – butun musbat son, $0 < \mu_1, \mu_2 < 1$, $f(x,y)$ funksiya (Δ_1) sohada 2p martagacha uzluksiz aralash xususiy hosilalarga, $K(x,y,t,\tau)$ funksiya esa, (Δ) sohada 4p martagacha uzluksiz aralash xususiy hosilalarga ega bo'lsin deb faraz qilinadi.

(2.1.15) integral tenglamaning yechimini ushbu

$$\varphi(x,y) = \sum_{k=0}^{\infty} \lambda^k \varphi_k(x,y) \qquad (2.1.16)$$

qator shaklida izlaymiz. $\varphi(x,y)$ ning (2.1.16) dagi qiymatini (2.1.15) ga olib borib qo'yib, formal ravishda qatorni hadma – had integrallaymiz. λ ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib

$$\varphi_0(x,y) = f(x,y)$$

$$\varphi_1(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau) f(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} dt d\tau \ ,$$

$$\varphi_2(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi_1(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} dt d\tau \ ,$$

……………………………………

$$\varphi_n(x,y)-\lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi_{n-1}(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}}dtd\tau ,$$

…………………………………....

ketma – ketlikni hosil qilamiz.

Yuqoridagi qatorni formal ravishda hadma– had integrallash to'g'ri bo'lishi uchun (2.1.16) qator va uning 2p martagacha hosilasidan tashkil topgan qatorlar tekis yaqinlashuvchi bo'lishi kerak.

Buni isbotlash uchun $f(x,y)$ funksiyaning absolyut qiymati va uning 2p martagacha aralash xususiy hosilalari absolyut qiymatlari maksimumini N bilan, $K(x,y,t,\tau)$ funksiyaning absolyut qiymati va uning 4p martagacha aralash xususiy hosilalari absolyut qiymatlari maksimumini E bilan belgilaymiz.

U holda,

$$|\varphi_0(x,y)|=|f(x,y)|\le N ,$$

$\varphi_1(x,y)$ ni baholaymiz.

$\varphi_1(x,y)$ funksiyani baholash uchun $K(x,y,t,\tau)f(t,\tau)$ funksiyani t bo'yicha quyidagi formulaga yoyamiz:

$$K(x,y,t,\tau)f(t,\tau)=K(x,y,d,\tau)f(d,\tau)-(d-t)[K(x,y,d,\tau)f(d,\tau)]_d^{'}+$$
$$+\frac{(d-t)^2}{2!}[K(x,y,d,\tau)f(d,\tau)]_d^{''}-...+\frac{(-1)^p(d-t)^p}{p!}[K(x,y,d+\theta_1h_1,\tau)f(d+\theta_1h_1,\tau)]_d^{(p)} \quad 2.1.17)$$

(2.1.17) ifodani $\varphi_1(x,y)$ ning ifodasidagi integral ostiga olib borib qo'yamiz:

$$\varphi_1(x,y)=\int_a^b\int_c^d \frac{1}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}}\Big[K(x,y,d,\tau)f(d,\tau)-(d-t)[K(x,y,d,\tau)f(d,\tau)]_d^{'}+$$

$$+\frac{(d-t)^2}{2!}[K(x,y,d,\tau)f(d,\tau)]_d^{''}-...+\frac{(-1)^p(d-t)^{p-1}}{(p-1)!}[K(x,y,d,\tau)f(d,\tau)]_d^{(p-1)}\Big]+$$

$$+\frac{(-1)^p(d-t)^p}{p!}[K(x,y,d+\theta_1h_1,\tau)f(d+\theta_1h_1,\tau)]_t^{(p)}\Big]dtd\tau=$$

$$=\int_a^b\frac{1}{(b-\tau)^{p+\mu_1}}\Big\{K(x,y,d,\tau)f(d,\tau)\int_c^d\frac{dt}{(d-t)^{p+\mu_2}}-[K(x,y,d,\tau)f(d,\tau)]_d^{'}\int_c^d\frac{dt}{(d-t)^{p+\mu_2-1}}+$$

$$+\frac{[K(x,y,d,\tau)f(d,\tau)]_d^{''}}{2!}\int_c^d\frac{dt}{(d-t)^{p+\mu_2-2}}-...+$$

$$+\frac{(-1)^p}{(p-1)!}[K(x,y,d,\tau)f(d,\tau)]_d^{(p-1)}\int_c^d\frac{dt}{(d-t)^{1+\mu_2}}+$$

$$+\frac{(-1)^p}{p!}\int_c^d\frac{[K(x,y,d+\theta_1h_1,\tau)f(d+\theta_1h_1,\tau)]_t^{(p)}}{(d-t)^{\mu}}dt\Big\}d\tau \tag{2.1.18}$$

$$\varphi_1(x,y)=\int_a^b\int_c^d\frac{K(x,y,t,\tau)f(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}}dtd\tau \tag{2.1.19}$$

$\varphi_1(x,y)$ funksiyani baholash uchun uning ifodasidagi ichki integralga Adamar ma’nosidagi integralning ta’rifini qo’llaymiz:

$$\int_a^b\frac{K(x,y,t,\tau)f(t,\tau)}{(d-t)^{p+\mu_2}}dt=-\frac{K(x,y,c,\tau)f(c,\tau)}{c_1d_1}+\frac{[K(x,y,c,\tau)f(c,\tau)]_c^{'}}{c_1c_2d_2}-$$

$$-\frac{[K(x,y,c,\tau)f(c,\tau)]_c^{''}}{c_1c_2c_3d_3}+...+\frac{(-1)^p[K(x,y,c,\tau)f(c,\tau)]_c^{(p-1)}}{c_1c_2...c_pd_p}+ \tag{2.1.20}$$

$$+\frac{(-1)^p}{c_1c_2...c_p}\int_c^d\frac{[K(x,y,t,\tau)f(t,\tau)]_t^{(p)}}{(d-t)^{\mu_2}}dt$$

bunda

$$c_k=p+\mu_2-k\,,\quad d_k=(d-c)^{p+\mu_2-k}\qquad (k=1,2,...,\ p)$$

(2.1.19) ni (2.1.20) ga olib borib qo’yamiz:

$$\varphi_1(x,y)=\int_a^b\int_c^d\frac{K(x,y,t,\tau)f(t,\tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}}dtd\tau=-\frac{1}{c_1d_1}\int_a^b\frac{K(x,y,c,\tau)f(c,\tau)}{(b-\tau)^{p+\mu_1}}d\tau+$$

$$+\frac{1}{c_1c_2d_2}\int_a^b\frac{[K(x,y,c,\tau)f(c,\tau)]_c^{'}}{(b-\tau)^{p+\mu_1}}d\tau-\frac{1}{c_1c_2c_3d_3}\int_a^b\frac{[K(x,y,c,\tau)f(c,\tau)]_c^{''}}{(b-\tau)^{p+\mu_1}}d\tau+...+ \tag{2.1.21}$$

$$+\frac{(-1)^p}{c_1c_2...c_pd_p}\int_a^b\frac{[K(x,y,c,\tau)f(c,\tau)]_c^{(p-1)}}{(b-\tau)^{p+\mu_1}}d\tau+\frac{(-1)^p}{c_1c_2...c_p}\int_a^b\int_c^d\frac{[K(x,y,t,\tau)f(t,\tau)]_t^{(p)}}{(d-t)^{\mu_2}(b-\tau)^{p+\mu_1}}dtd\tau$$

(2.1.21) ning o'ng tomonidagi τ bo'yicha olingan integrallarning har biriga Adamar integralining ta'rifini qo'llaymiz:

$$\int_a^b \frac{K(x,y,c,\tau)f(c,\tau)}{(b-\tau)^{p+\mu_1}}d\tau = -\frac{K(x,y,c,a)f(c,a)}{a_1b_1} + \frac{[K(x,y,c,\tau)f(c,a)]_a^{'}}{a_1a_2b_2} -$$
$$-\frac{[K(x,y,c,\tau)f(c,a)]_a^{''}}{a_1a_2a_3b_3} + \ldots + \frac{(-1)^p[K(x,y,c,\tau)f(c,a)]_a^{(p-1)}}{a_1a_2\ldots a_pb_p} + \qquad (\alpha_0)$$
$$+\frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{[K(x,y,c,\tau)f(c,\tau)]_\tau^{(p)}}{(b-\tau)^{\mu_1}}d\tau$$

$$\int_a^b \frac{[K(x,y,c,\tau)f(c,\tau)]_c^{'}}{(b-\tau)^{p+\mu_1}}d\tau = -\frac{(K(x,y,c,a)f(c,a))_c^{'}}{a_1b_1} + \frac{\left[(K(x,y,c,a)f(c,a))_c^{'}\right]_a^{'}}{a_1a_2b_2} -$$
$$-\frac{\left[(K(x,y,c,a)f(c,a))_c^{'}\right]_a^{''}}{a_1a_2a_3b_3} + \ldots + \frac{(-1)^p\left[(K(x,y,c,a)f(c,a))_c^{'}\right]_a^{(p-1)}}{a_1a_2\ldots a_pb_p} + \qquad (\alpha_1)$$
$$+\frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{\left[(K(x,y,c,\tau)f(c,\tau))_c^{'}\right]_\tau^{(p)}}{(b-\tau)^{\mu_1}}d\tau$$

$$\int_a^b \frac{[K(x,y,c,\tau)f(c,\tau)]_c^{''}}{(b-\tau)^{p+\mu_1}}d\tau = -\frac{(K(x,y,c,a)f(c,a))_c^{''}}{a_1b_1} + \frac{\left[(K(x,y,c,a)f(c,a))_c^{''}\right]_a^{'}}{a_1a_2b_2} -$$
$$-\frac{\left[(K(x,y,c,a)f(c,a))_c^{''}\right]_a^{''}}{a_1a_2a_3b_3} + \ldots + \frac{(-1)^p\left[(K(x,y,c,a)f(c,a))_c^{''}\right]_a^{(p-1)}}{a_1a_2\ldots a_pb_p} + \qquad (\alpha_2)$$
$$+\frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{\left[(K(x,y,c,\tau)f(c,\tau))_c^{'}\right]_\tau^{(p)}}{(b-\tau)^{\mu_1}}d\tau$$

....

$$\int_c^d \frac{dt}{(d-t)^{\mu_2}}\int_a^b \frac{((x,y,t,\tau)f(t,\tau))_t^{(p)}}{(b-\tau)^{p+\mu_1}}d\tau = \int_c^d \frac{dt}{(d-t)^{\mu_2}}\left\{-\frac{(K(x,y,t,a)f(t,a))_t^{(p)}}{a_1b_1} + \right.$$
$$+\frac{\left[(K(x,y,t,a)f(t,a))_t^{(p)}\right]_a^{'}}{a_1a_2b_2} - -\frac{\left[(K(x,y,t,a)f(t,a))_t^{(p)}\right]_a^{''}}{a_1a_2a_3b_3} + \ldots +$$
$$+\frac{(-1)^p\left[(K(x,y,t,a)f(t,a))_t^{(p)}\right]_a^{(p-1)}}{a_1a_2\ldots a_pb_p} + \frac{(-1)^p}{a_1a_2\ldots a_p}\int_a^b \frac{\left[(K(x,y,t,\tau)f(t,\tau))_t^{(p)}\right]_\tau^{(p)}}{(b-\tau)^{\mu_1}}d\tau$$

2.2-§. Maxsuslik darajasi $p > 1$ bo'lganda ikki o'zgaruvchili Adamar ma'nosidagi integral tenglamani qisib akslantirish prinsipi yordamida yechish.

Ushbu

$$A\varphi = \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} d\tau dt \qquad (2.2.1)$$

integral operatorni qaraymiz, bunda $K(x,y,t,\tau)$ funksiya $D = \{[a \le x, y \le b; c \le t, \tau \le d]\}$ sohada aniqlangan bo'lib, to'rt martagacha uzluksiz aralash xususiy hosilalarga ega. $D_1 = \{[c \le t, \tau \le d]\}$ sohada uzluksiz va ikki martagacha uzluksiz aralash xususiy hosilalarga ega bo'lgan $f(x,y)$ funksiyalar sinfini M bilan belgilaymiz. M to'plamdagi funksiyalarning normasini

$$\|f\|_M = \max\left\{\max\left|f(t,\tau)\right|, \max\left|f_t^{'}(t,\tau)\right|, \max\left|f_\tau^{'}(t,\tau)\right|, \max\left|f_{\tau t}^{''}(t,\tau)\right|,\right.$$
$$\left.\max\left|f_{t\tau}^{''}(t,\tau)\right|, \max\left|f_{\tau\tau}^{''}(t,\tau)\right|, \max\left|f_{tt}^{''}(t,\tau)\right|\right\}$$

deb kiritamiz. Bunday kiritilgan norma ushbu aksiomalarni qanoatlantiradi:

1) $\|f\| = 0$ faqat va faqat $f = 0$ bo'lsa;

2) $\|cf\| = |c|\|f\|$ c – o'zgarmas son;

3) $\|f_1 + f_2\| \le \|f_1\| + \|f_2\|$

Agar M to'plamdagi f_1 va f_2 elementlar orasidagi masofani

$$\rho(f_1, f_2) = \|f_1 - f_2\|$$

deb kiritsak, bu masofa metrik fazoning hamma aksiomalarini qanoatlantiradi. M fazo shu metrika bo'yicha to'liq bo'ladi. $\varphi(t,\tau) \in M$ bo'lsin. $A\varphi = f(x,y) \in M$ dan ekanligini ko'rsatamiz.

$$A\varphi = \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} d\tau dt$$

Adamar integralining chekli qismini ta'rif bo'yicha ajratamiz. Avvalo ta'rifni ichki integral uchun qo'llaymiz:

$$g(x,y)=\int_a^b \frac{\left[-\frac{K(x,y,c,\tau)\varphi(c,\tau)}{\mu_2(d-c)^{\mu_2}}-\frac{1}{\mu_2}\int_c^d \frac{K_t'(x,y,t,\tau)\varphi(t,\tau)+K(x,y,t,\tau)\varphi_t'(t,\tau)}{(d-t)^{\mu_2}}\right]}{(b-\tau)^{1+\mu_1}}d\tau= \tag{2.2.2}$$

$$=\int_a^b \frac{\psi(x,y,\tau)}{(b-\tau)^{1+\mu_1}}d\tau$$

$$\psi(x,y,\tau)=-\frac{K(x,y,c,\tau)\varphi(c,\tau)}{\mu_2(d-c)^{\mu_2}}-\frac{1}{\mu_2}\int_c^d \frac{K_t'(x,y,t,\tau)\varphi(t,\tau)+K(x,y,t,\tau)\varphi_t'(t,\tau)}{(d-t)^{\mu_2}}$$

Bu integralga yana Adamar integralining ta’rifini qo’llaymiz:

$$g(x,y)=-\frac{\psi(x,y,a)}{\mu_1(b-a)^{\mu_1}}-\frac{1}{\mu_1}\int_a^b \frac{\psi_\tau'(x,y,t)}{(b-a)^{\mu_1}}dt \tag{2.2.3}$$

(2.2.2) ni e’tiborga olgan holda, (3.1.3) ni quyidagicha yozamiz:

$$\begin{aligned}
g(x,y)=&\frac{K(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}}+\frac{1}{\mu_1\mu_2}\int_c^d \frac{K_t'(x,y,t,a)\varphi(t,a)+K(x,y,t,a)\varphi_t'(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b \frac{K_\tau'(x,y,c,\tau)\varphi(c,\tau)+K(x,y,c,\tau)\varphi_\tau'(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{\tau t}''(x,y,t,\tau)\varphi(t,\tau)+K_\tau'(x,y,t,\tau)\varphi_t'(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_t'(x,y,t,\tau)\varphi_\tau'(t,\tau)+K_\tau'(x,y,t,\tau)\varphi_{\tau t}''(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned} \tag{2.2.4}$$

Yuqorida $K(x,y,t,\tau)$ va $\varphi(t,\tau)$ funksiyalarga qo’yilgan shartlarda $f(x,y)$ funksiya ikki martagacha uzluksiz aralash xususiy hosilalarga ega. (2.2.4) dan bu xususiy hosilalarni topamiz:

$$\begin{aligned}
g_x'(x,y)=&\frac{K_x'(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}}+\frac{1}{\mu_1\mu_2}\int_c^d \frac{K_{tx}''(x,y,t,a)\varphi(t,a)+K_x'(x,y,t,a)\varphi_t'(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b \frac{K_{\tau x}''(x,y,c,\tau)\varphi(c,\tau)+K_x'(x,y,c,\tau)\varphi_\tau'(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{\tau tx}'''(x,y,t,\tau)\varphi(t,\tau)+K_{\tau x}''(x,y,t,\tau)\varphi_t'(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau+\\
&+\frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{tx}''(x,y,t,\tau)\varphi_\tau'(t,\tau)+K_x'(x,y,t,\tau)\varphi_{\tau t}''(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned} \tag{2.2.5}$$

$$
\begin{aligned}
g_y^{'}(x,y) = {} & \frac{K_y^{'}(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}} + \frac{1}{\mu_1\mu_2}\int_c^d \frac{K_{ty}^{''}(x,y,t,a)\varphi(t,a) + K_y^{'}(x,y,t,a)\varphi_t^{'}(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b \frac{K_{\tau y}^{''}(x,y,c,\tau)\varphi(c,\tau) + K_y^{'}(x,y,c,\tau)\varphi_\tau^{'}(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{\tau ty}^{'''}(x,y,t,\tau)\varphi(t,\tau) + K_{\tau y}^{''}(x,y,t,\tau)\varphi_t^{'}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{ty}^{''}(x,y,t,\tau)\varphi_\tau^{'}(t,\tau) + K_y^{'}(x,y,t,\tau)\varphi_{\tau t}^{''}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned}
\tag{2.2.6}
$$

Xuddi shunday (2.2.5) va (2.2.6) lardan

$$
\begin{aligned}
g_{xy}^{''}(x,y) = {} & \frac{K_{xy}^{''}(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}} + \frac{1}{\mu_1\mu_2}\int_c^d \frac{K_{txy}^{'''}(x,y,t,a)\varphi(t,a) + K_{xy}^{''}(x,y,t,a)\varphi_t^{'}(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b \frac{K_{txy}^{'''}(x,y,c,\tau)\varphi(c,\tau) + K_{xy}^{''}(x,y,c,\tau)\varphi_t^{'}(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{\tau txy}^{''''}(x,y,t,\tau)\varphi(t,\tau) + K_{\tau xy}^{'''}(x,y,t,\tau)\varphi_t^{'}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{txy}^{'''}(x,y,t,\tau)\varphi_\tau^{'}(t,\tau) + K_{xy}^{''}(x,y,t,\tau)\varphi_{\tau t}^{''}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned}
\tag{2.2.7^1}
$$

$$
\begin{aligned}
g_{yx}^{''}(x,y) = {} & \frac{K_{yx}^{''}(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}} + \frac{1}{\mu_1\mu_2}\int_c^d \frac{K_{tyx}^{'''}(x,y,t,a)\varphi(t,a) + K_{yx}^{''}(x,y,t,a)\varphi_t^{'}(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b \frac{K_{tyx}^{'''}(x,y,c,\tau)\varphi(c,\tau) + K_{yx}^{''}(x,y,c,\tau)\varphi_t^{'}(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{\tau tyx}^{''''}(x,y,t,\tau)\varphi(t,\tau) + K_{\tau yx}^{'''}(x,y,t,\tau)\varphi_t^{'}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau \; + \\
& + \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K_{tyx}^{'''}(x,y,t,\tau)\varphi_\tau^{'}(t,\tau) + K_{yx}^{''}(x,y,t,\tau)\varphi_{\tau t}^{''}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned}
\tag{2.2.7^2}
$$

$$
\begin{aligned}
g''_{xx}(x,y) &= \frac{K''_{xx}(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}} + \\
&+ \frac{1}{\mu_1\mu_2}\int_c^d \frac{K'''_{txx}(x,y,t,a)\varphi(t,a)+K''_{xx}(x,y,t,a)\varphi'_t(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b \frac{K'''_{txx}(x,y,c,\tau)\varphi(c,\tau)+K''_{xx}(x,y,c,\tau)\varphi'_t(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K''''_{\tau txx}(x,y,t,\tau)\varphi(t,\tau)+K'''_{\tau xx}(x,y,t,\tau)\varphi'_t(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K'''_{txx}(x,y,t,\tau)\varphi'_\tau(t,\tau)+K''_{xx}(x,y,t,\tau)\varphi''_{\tau t}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned}
\tag{2.2.7^3}
$$

$$
\begin{aligned}
g''_{yy}(x,y) &= \frac{K''_{yy}(x,y,c,a)\varphi(c,a)}{\mu_1\mu_2(b-a)^{\mu_1}(d-c)^{\mu_2}} + \frac{1}{\mu_1\mu_2}\int_c^d \frac{K'''_{tyy}(x,y,t,a)\varphi(t,a)+K''_{yy}(x,y,t,a)\varphi'_t(t,a)}{(b-a)^{\mu_1}(d-t)^{\mu_2}}dt + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b \frac{K'''_{tyy}(x,y,c,\tau)\varphi(c,\tau)+K''_{yy}(x,y,c,\tau)\varphi'_t(c,\tau)}{(b-\tau)^{\mu_1}(d-c)^{\mu_2}}d\tau + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K''''_{\tau tyy}(x,y,t,\tau)\varphi(t,\tau)+K'''_{\tau yy}(x,y,t,\tau)\varphi'_t(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau + \\
&+ \frac{1}{\mu_1\mu_2}\int_a^b\int_c^d \frac{K'''_{tyy}(x,y,t,\tau)\varphi'_\tau(t,\tau)+K''_{yy}(x,y,t,\tau)\varphi''_{\tau t}(t,\tau)}{(b-\tau)^{\mu_1}(d-t)^{\mu_2}}d\tau
\end{aligned}
\tag{2.2.7^4}
$$

ni topamiz. $K(x,y,t,\tau)$ va uning hamma to'rt martagacha xususiy hosilalarining maksimumlarining maksimumini N deb belgilaymiz. (2.2.4) – (2.2.7) lardan

$$
\begin{aligned}
&\max|g(x,y)| \le \frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.\\
&\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\| = \frac{NE}{\mu_1\mu_2}\|\varphi\|;\\
&\max|g'_x(x,y)| \le \frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.\\
&\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\| = \frac{NE}{\mu_1\mu_2}\|\varphi\|;\\
&\max|g'_y(x,y)| \le \frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.\\
&\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\| = \frac{NE}{\mu_1\mu_2}\|\varphi\|
\end{aligned}
\tag{2.2.8}
$$

Xuddi shunday

$$\max\left|g''_{xy}(x,y)\right|\le\frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.$$
$$\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\|=\frac{NE}{\mu_1\mu_2}\|\varphi\|; \qquad (2.2.9^1)$$

$$\max\left|g''_{yx}(x,y)\right|\le\frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.$$
$$\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\|=\frac{NE}{\mu_1\mu_2}\|\varphi\| \qquad (2.2.9^2)$$

$$\max\left|g''_{xx}(x,y)\right|\le\frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.$$
$$\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\|=\frac{NE}{\mu_1\mu_2}\|\varphi\|; \qquad (2.2.9^3)$$

$$\max\left|g''_{yy}(x,y)\right|\le\frac{N}{\mu_1\mu_2}\left[1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\right.$$
$$\left.+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}}\right]\|\varphi\|=\frac{NE}{\mu_1\mu_2}\|\varphi\| \qquad (2.2.9^4)$$

bunda

$$E=1+\frac{2(d-c)^{1-\mu_2}}{(1-\mu_2)(b-a)^{\mu_1}}+\frac{2(b-a)^{1-\mu_1}}{(1-\mu_1)(d-c)^{\mu_2}}+\frac{4(b-a)^{1-\mu_1}(d-c)^{1-\mu_2}}{(1-\mu_1)(1-\mu_2)(b-a)^{\mu_1}(d-c)^{\mu_2}} \qquad (2.2.10)$$

(2.2.10), (2.2.4) - ($2.2.7^1$),($2.2.7^2$),($2.2.7^3$),($2.2.7^4$) lardan A operator chegaralangan ekanligini ko’rish qiyin emas, ya’ni

$$\|A\varphi\|_M\le\frac{NE}{\mu_1\mu_2}\|\varphi\|_M \qquad (2.2.11)$$

Shunday qilib quyidagi teorema isbotlandi.

2.2.1 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada uzluksiz va to'rt martagacha uzluksiz aralash xususiy hosilalarga ega bo'lsa, u holda A operator M fazoni o'zini o'ziga ko'chiradi va chegaralangan bo'ladi.

Ushbu

$$\varphi(x,y)-\lambda\int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}d\tau dt = g(x,y) \qquad (2.2.12)$$

chiziqli ikki karrali Adamar ma'nosidagi integral tenglamani qaraymiz, bunda $K(x,y,t,\tau)$ va $g(x,y)$ funksiyalar mos ravishda D va D_1 sohalarda aniqlangan, λ esa parametr.

2.2.2 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada to'rt martagacha uzluksiz aralash xususiy hosilalarga, $g(x,y)$ funksiya esa D_1 sohada ikki martagacha uzluksiz aralash xususiy hosilalaga ega bo'lsa, u holda (2.2.12) integral tenglama λ parametrning

$$|\lambda| < \frac{1}{\frac{NE}{\mu_1\mu_2}} = \frac{\mu_1\mu_2}{NE} \qquad (2.2.13)$$

tengsizlikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi.

Isbot. $B\varphi = \int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}d\tau dt + g(x,y)$ operatorni qaraymiz. Bu operatorni M fazoning har bir $\varphi(t,\tau)\in M$ elementini, yana shu fazoning o'ziga ko'chirishini ko'rsatamiz. Shartga ko'ra $g(x,y)\in M$ bo'lganligi uchun

$$B_0\varphi = \int_a^b\int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}}d\tau dt$$

operatorning har bir $\varphi(t,\tau)\in M$ funksiyani yana M fazoga ko'chirishini, ya'ni $f(x,y)=B_0\varphi\in M$ ekanligini ko'rsatish yetarli. Teoremaning shartlari bajarilganda 2.2.1 teorema o'rinli bo'ladi.

Shuning uchun $\varphi(t,\tau)\in M$ dan bo'lganda B_0 operator M fazoni o'zini o'ziga ko'chiradi va chegaralangan bo'ladi, ya'ni $B_0\varphi\in M$. Demak, $\varphi(t,\tau)\in M$ bo'lar ekan. Endi B operatorning qisuvchi operator ekanligini ko’rsatamiz. Buning uchun $\rho(B\varphi_1, B\varphi_2)$ ni baholaymiz. (2.2.11) ga asosan

$$\rho(B\varphi_1, B\varphi_2) = |\lambda| \|B\varphi_1 - B\varphi_2\| = |\lambda| \|B(\varphi_1 - \varphi_2)\| \leq \frac{NE}{\mu_1 \mu_2} \|\varphi_1 - \varphi_2\| \qquad (2.2.14)$$

bunda

$$|\lambda| \frac{NE}{\mu_1 \mu_2} \|\varphi_1 - \varphi_2\|$$

bo’lganligi uchun B operator qisuvchi operator bo’lar ekan. Demak, qisib akslantirish prinsipiga asosan (2.2.12) integral tenglama M fazoda yagona yechimga ega bo’lar ekan. Bu yechim ketma – ket yaqinlashish usuli yordamida topiladi.

2.2.3 - teorema. Agar $K(x, y, t, \tau)$ funksiya D sohada $4p$ martagacha uzluksiz aralash hosilalarga, $g(x, y)$ funksiya esa D_1 sohada $2p$ martagacha uzluksiz aralash xususiy hosilalarga ega bo’lsa, u holda

$$\varphi(x, y) - \lambda \int_a^b \int_c^d \frac{K(x, y, t, \tau)\varphi(t, \tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} d\tau dt = g(x, y) \qquad (2.2.15)$$

integral tenglama λ parametrning

$$|\lambda| < \frac{1}{N_1 E_1}$$

tenglikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo’ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi, bunda N_1 va E_1 lar 2.2.2 – teoremadagi N va E lar kabi, $K(x, y, t, \tau)$, $g(x, y)$ funksiyalar va ularning hosilalari orqali ifoda qilinadigan o’zgarmas sonlar.

Isbot. $B\varphi = \int_a^b \int_c^d \frac{K(x, y, t, \tau)\varphi(t, \tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} d\tau dt + g(x, y)$ operatorni qaraymiz. Bu operatorni M fazoning har bir $\varphi(t, \tau) \in M$ elementini, yana shu fazoning o’ziga ko’chirishini ko’rsatamiz. Shartga ko’ra $g(x, y) \in M$ bo’lganligi uchun

$$B_0\varphi = \int_a^b \int_c^d \frac{K(x, y, t, \tau)\varphi(t, \tau)}{(b-\tau)^{p+\mu_1}(d-t)^{p+\mu_2}} d\tau dt$$

operatorning har bir $\varphi(t,\tau)\in M$ funksiyani yana M fazoga ko'chirishini, ya'ni $f(x,y)=B_0\varphi\in M$ ekanligini ko'rsatish yetarli. Teoremaning shartlari bajarilganda 2.2.1 teorema o'rinli bo'ladi.

Shuning uchun $\varphi(t,\tau)\in M$ dan bo'lganda B_0 operator M fazoni o'zini o'ziga ko'chiradi va chegaralangan bo'ladi, ya'ni $B_0\varphi\in M$. Demak, $\varphi(t,\tau)\in M$ bo'lar ekan. Endi B operatorning qisuvchi operator ekanligini ko'rsatamiz. Buning uchun $\rho(B\varphi_1,B\varphi_2)$ ni baholaymiz. (2.2.11) ga asosan

$$\rho(B\varphi_1,B\varphi_2)=|\lambda|\|B\varphi_1-B\varphi_2\|=|\lambda|\|B(\varphi_1-\varphi_2)\|\leq\frac{NE}{(p+\mu_1-1)(p+\mu_2-1)}\|\varphi_1-\varphi_2\| \quad (2.2.16)$$

bunda

$$\rho(B\varphi_1-B\varphi_2)\leq|\lambda|\frac{NE}{(p+\mu_1-1)(p+\mu_2-1)}\|\varphi_1-\varphi_2\|$$

bo'lganligi uchun B operator qisuvchi operator bo'lar ekan. Demak, qisip akslantirish prinsipiga asosan (2.2.15) integral tenglama M fazoda yagona yechimga ega bo'lar ekan. Bu yechim ketma – ket yaqinlashish usuli yordamida topiladi.

II Bob bo'yicha xulosa

Ko'p o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi karrali maxsus integral tenglamaning maxsuslik darajasi $1+\mu$ $(0<\mu<1)$ bo'lganda, hamda maxsuslik darajasi $p+\mu$ (p – musbat butun son) bo'lgan hollarda tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli yordamida isbotlanagan (2.1.1 va 2.1.2 - teoremalar).

Ko'p o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi karrali maxsus integral tenglamaning maxsuslik darajasi $p>1$ bo'lgan hol uchun uning yechimining mavjudligi va yagonaligi qisib akslantirish usuli yordamida isbotlangan.

III Bob. Adamar ma'nosidagi chiziqli integral tenglamani iteratsiyalangan yadro va rezolventa yordamida yechish.

3.1-§. Bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi integral tenglamani iteratsiyalangan yadro va rezolventa yordamida yechish.

Biz yuqorida ko'rdikki ushbu

$$\varphi(t) - \lambda \int_a^b \frac{K(t,\tau)\varphi(\tau)}{(b-\tau)^{p+\mu}} d\tau = f(t) \qquad (3.1.1)$$

integral tenglamaning yechimi qator shaklida, ya'ni

$$\varphi(t) = \sum_{k=0}^{\infty} \lambda^k \varphi_k(t) \qquad (3.1.2)$$

ko'rinishda izlangan edi, bunda

$$\varphi_0(t) = f(t);$$

$$\varphi_k(t) = \int_a^b \frac{K(t,\tau)\varphi_{n-1}(\tau)}{(b-\tau)^{p+\mu}} d\tau \quad (k = 1,2,\dots)$$

Bundan,

$$\varphi_1(t) = \int_a^b \frac{K(t,\tau) f(\tau)}{(b-\tau)^{p+\mu}} d\tau ;$$

$$\varphi_2(t) = \int_a^b \frac{K(t,s)\varphi_1(s)}{(b-s)^{p+\mu}} ds = \int_a^b \frac{K(t,s)ds}{(b-s)^{p+\mu}} \int_a^b \frac{K(s,\tau) f(\tau)}{(b-\tau)^{p+\mu}} d\tau$$

Bunga [1] (1.3.1) formulani qo'llash natijasida

$$\varphi_2(t) = \int_a^b K^{(2)}(t,\tau) f(\tau) d\tau ,$$

bunda

$$K^{(2)}(t,\tau) = \int_a^b \frac{K(t,s)K(s,\tau)}{(b-s)^{p+\mu}} ds ,$$

$$\varphi_3(t)=\int_a^b \frac{K(t,s)\varphi_2(s)}{(b-s)^{p+\mu}}ds=\int_a^b \frac{K(t,s)ds}{(b-s)^{p+\mu}}\int_a^b \frac{K^{(2)}(s,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau=$$

$$=\int_a^b \frac{f(\tau)d\tau}{(b-\tau)^{p+\mu}}\int_a^b \frac{K(t,s)K^{(2)}(s,\tau)}{(b-\tau)^{p+\mu}}ds=\int_a^b \frac{K^{(2)}(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau$$

Xuddi shunday

$$\varphi_n(t)=\int_a^b \frac{K^{(n)}(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau ,$$

bunda

$$K^{(n)}(t,\tau)=\int_a^b \frac{K(t,s)K^{n-1}(s,\tau)}{(b-s)^{p+\mu}}ds$$

…………………………..

$K^{(2)}(t,\tau), K^{(3)}(t,\tau),...$ – yadrolarga iteratsiyalangan yadro deyiladi.

Integrallarning o’rinlarini almashtirish formulasidan foydalanib, quyidagi tengliklarni isbotlash mumkin:

$$K^{(n)}(t,\tau)=\int_a^b \frac{K^{\grave{o}-1}(t,s)K(s,\tau)}{(b-s)^{p+\mu}}ds , \tag{3.1.3}$$

$$K^{(i+j)}(t,\tau)=\int_a^b \frac{K^{i}(t,s)K^{j}(s,\tau)}{(b-s)^{p+\mu}}ds , \tag{3.1.4}$$

bunda i va j butun musbat sonlar.

Iteratsiyalangan yadrolar yordamida ushbu

$$\Gamma(t,\tau,\lambda)=K(t,\tau)+\lambda K^{(2)}(t,\tau)+...+\lambda^{n-1}K^{(n)}(t,\tau)+... \tag{3.1.5}$$

qatorni tuzamiz.

Agar λ (1.2.20) yoki (1.2.22) shartlarni qanoatlantirganda (1.2.17) qator singari (3.1.5) va uning p – martagacha hosilasidan tuzilgan qatorlarning tekis yaqinlashuvchi ekanligini isbotlash mumkin. (3.1.5) qator bilan ifoda qilingan $\Gamma(t,\tau,\lambda)$ funksiyaga rezolventa deyiladi. (3.1.2) qatorni quyidagicha yozib olamiz.

$$\varphi(t)=f(t)+\lambda\int_a^b \frac{K(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau+\lambda^2\int_a^b \frac{K^{(2)}(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau+...+\lambda^n\int_a^b \frac{K^{(n)}(t,\tau)f(\tau)}{(b-\tau)^{p+\mu}}d\tau+... ,$$

$$\varphi(t)=f(t)+\lambda\int_a^b \frac{K(t,\tau)+\lambda K^{(2)}(t,\tau)+\ldots+\lambda^{n-1}K^{(n)}(t,\tau)+\ldots}{(b-\tau)^{p+\mu}}f(\tau)d\tau \quad ,$$

$$\varphi(t)=f(t)+\lambda\int_a^b \frac{\Gamma(t,\tau,\lambda)}{(b-\tau)^{p+\mu}}d\tau \tag{3.1.6}$$

(3.1.6) formula (3.1.1) Adamar ma'nosidagi integral tenglamaning yechimini ifodalaydi. (Oshkor shaklda topilgan yechimi)

3.2-§. Ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi integral tenglamani iteratsiyalangan yadro va rezolventa yordamida yechish.

2. Ushbu

$$\varphi(x,y)-\lambda\int_a^b\int_c^d \frac{K(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2 = f(x,y) \ , \tag{3.2.1}$$

integral tenglamaning yechimini qator shaklida, ya'ni

$$\varphi(x,y)=\sum_{k=0}^{\infty}\lambda^k\varphi_k(x,y) \ , \tag{3.2.2}$$

ko'rinishda izlaymiz, bunda

$$\varphi_0(x,y)=f(x,y);$$

…………………………………………………….

$$\varphi_k(x,y)=\int_a^b\int_c^d \frac{K(x,y,t_1,t_2)\varphi_{n-1}(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2 \ , \quad (k=1,2,\ldots).$$

Bundan,

$$\varphi_1(x,y)=\int_a^b\int_c^d \frac{K(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2 \ ;$$

$$\varphi_2(x,y)=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)\varphi_1(s_1,s_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2=$$
$$=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2\int_a^b\int_c^d \frac{K(s_1,s_2,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2=$$
$$=\int_a^b\int_c^d \frac{f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)K(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2=$$
$$=\int_a^b\int_c^d \frac{K^{(2)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2$$

Bunga [1] (1.3.1) formulani qo'llash natijasida

$$\varphi_2(x,y)=\int_a^b\int_c^d \frac{K^{(2)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2\,,$$

bunda

$$K^{(2)}(x,y,t_1,t_2)=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)K(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2\,,$$

$$\varphi_3(x,y)=\int_a^b \frac{K(x,y,s_1,s_2)\varphi_2(s_1,s_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2=$$
$$=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2\int_a^b\int_c^d \frac{K^{(2)}(s_1,s_2,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2=$$
$$=\int_a^b\int_c^d \frac{f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)K^{(2)}(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2=\,,$$
$$=\int_a^b\int_c^d \frac{K^{(3)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2$$

bunda

$$K^{(3)}(x,y,t_1,t_2)=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)K^{(2)}(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2\,.$$

Xuddi shunday

$$\varphi_n(t)=\int_a^b \frac{K^{(n)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2\,,$$

bunda

$$K^{(n)}(x,y,t_1,t_2)=\int_a^b\int_c^d \frac{K(x,y,s_1,s_2)K^{(n-1)}(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2$$

…………………………..

$K^{(2)}(x,y,t_1,t_2), K^{(3)}(x,y,t_1,t_2),...$ – yadrolarga iteratsiyalangan yadro deyiladi.

Integrallarning o'rinlarini almashtirish formulasidan foydalanib, quyidagi tengliklarni isbotlash mumkin:

$$K^{(n)}(x,y,t_1,t_2)=\int_a^b \frac{K^{m-1}(x,y,s_1,s_2)K(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2, \qquad (3.2.3)$$

$$K^{(i+j)}(x,y,t_1,t_2)=\int_a^b \frac{K^{i}(x,y,s_1,s_2)K^{j}(s_1,s_2,t_1,t_2)}{(b-s_2)^{p+\mu_1}(d-s_1)^{p+\mu_2}}ds_1ds_2, \qquad (3.2.4)$$

bunda i va j butun musbat sonlar.

Iteratsiyalangan yadrolar yordamida ushbu

$$\Gamma(x,y,t_1,t_2,\lambda)=K(x,y,t_1,t_2)+\lambda K^{(2)}(x,y,t_1,t_2)+...+\lambda^{n-1}K^{(n)}(x,y,t_1,t_2)+..., \qquad (3.2.5)$$

qatorni tuzamiz.

Agar λ (1.2.20) yoki (1.2.22) shartlarni qanoatlantirganda (1.2.17) qator singari (3.2.5) va uning p – martagacha hosilasidan tuzilgan qatorlarning tekis yaqinlashuvchi ekanligini isbotlash mumkin. (3.2.5) qator bilan ifoda qilingan $\Gamma(x,y,t,\tau,\lambda)$ funksiyaga rezolventa deyiladi. (3.2.2) qatorni quyidagicha yozib olamiz.

$$\varphi(x,y)=f(x,y)+\lambda\int_a^b\int_c^d \frac{K(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2+$$

$$+\lambda^2\int_a^b\int_c^d \frac{K^{(2)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2+...+\lambda^n\int_a^b\int_c^d \frac{K^{(n)}(x,y,t_1,t_2)f(t_1,t_2)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2+...,$$

$$\varphi(x,y)=f(x,y)+\lambda\int_a^b \frac{K(x,y,t_1,t_2)+\lambda K^{(2)}(x,y,t_1,t_2)+...+\lambda^{n-1}K^{(n)}(x,y,t_1,t_2)+...}{(b-t_2)^{p+\mu_1}(d-t)^{p+\mu_2}}f(t_1,t_2)dt_1dt_2,$$

$$\varphi(x,y)=f(x,y)+\lambda\int_a^b \frac{\Gamma(x,y,t_1,t_2,\lambda)}{(b-t_2)^{p+\mu_1}(d-t_1)^{p+\mu_2}}dt_1dt_2. \qquad (3.2.6)$$

(3.2.6) formula (3.2.1) Adamar ma'nosidagi integral tenglamaning yechimini ifodalaydi. (Oshkor shaklda topilgan yechimi)

Shunday qilib, osonlik bilan ko’rish mumkinki, Adamar ma’nosidagi integral tenglamalar uchun ham Fredgolm integral tenglamalar nazariyasidagi hamma formulalar kabi formulalar o’rinli bo’lar ekan.

III Bob boyicha xulosa

Mazkur bobda bir o’zgaruvchili funksiyalar uchun Adamar ma’nosidagi chiziqli integral tenglamani iteratsiyalangan yadro va rezolventa yordamida yechib ko’rsatilgan.

Olingan natijalardan foydalanib, ikki o’zgaruvchili funksiyalar uchun Adamar ma’nosidagi karrali maxsus chiziqli integral tenglamalar uchun iteratsiyalangan yadro va rezolventa tushunchalari kiritilib, ular yordamida qaralayotgan tenglama yechimi topildi.

Xulosa

1. Mazkur, magistrlik dissertatsiya ishi bo'yicha tadqiqot natijalari bo'yicha xulosalar va ularning asoslanishi:

1. Adamar ma'nosidagi maxsus integralga keltirilib, ularning asosiy xossalari o'rganildi. Bu o'rganilgan xossalar yordamida bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimi mavjudligi va yagonaligi haqidagi 1.2.2 va 1.2.3 – teoremalar isbotlangan. Ta'kidlash mumkinki, 1.2.2 – teorema integral tenglamaning maxsuslik darajasi $1+\mu \quad (0<\mu<1)$ bo'lganda N.P. Vekua [2] ishida isbotlangan. Shu natija asosida tenglamaning maxsuslik darajasi $p+\mu$ (p – musbat butun son) bo'lgan ishida 1.2.3 – teorema batafsil ketma – ket yaqinlashish usuli yordamida yechimining mavjudligi va yagonaligi isbot qilindi.

2. Bir o'zgaruvchili funksiya uchun Adamar ma'nosidagi chiziqli integral tenglamaning yechimi iteratsiyalangan yadro va rezolventa yordamida olingan.

3. Bir o'zgaruvchili funksiya uchun Adamar ma'nosidagi integral tenglamaning maxsuslik darajasi $p+\mu \quad (0<\mu<1)$ bo'lganda uning yechimining mavjudligi va yagonaligi qisib akslantirish usuli yordamida ko'rsatigan.

4. Ko'p o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi karrali maxsus integral tenglamaning maxsuslik darajasi $1+\mu \quad (0<\mu<1)$ bo'lganda, hamda maxsuslik darajasi $p+\mu$ (p – musbat butun son) bo'lgan hollarda tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli yordamida isbotlanagan (2.1.1 va 2.1.2 - teoremalar).

5. Ko'p o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi karrali maxsus integral tenglamaning maxsuslik darajasi $p>1$ bo'lgan hol uchun uning yechimining mavjudligi va yagonaligi qisib akslantirish usuli yordamida isbotlangan.

6. Qaralayotgan Adamar ma'nosidagi karrali maxsus integral tenglama yechimi iteratsiyalangan yadrolar va rezolventa yordamida ham olingan.

7. Mazkur magistrlik dissertatsiya ishining ilmiylik darajasini quyidagicha ifodalash mumkin:

a) Adamar ma'nosidagi bir o'zgaruvchili maxsus integral tushunchasi ko'p o'zgaruvchili (ikki o'zgaruvchili) funksiyalar uchun umumlashtirilgan va uning asosiy xossalari o'rganildi.

b) N.P. Vekuaning [2] ishida keltirilgan va isbotlangan bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi integral tenglamaning maxsuslik darajasi $1+\mu \quad (0<\mu<1)$ bo'lgan holda isbot qilingan teoremani, maxsuslik darajasi $p+\mu$ (butun musbat son) hol uchun batafsil isbot qilindi.

v) Maxsuslik darajasi $p+\mu_1$ va $p+\mu_2$ bo'lgan Adamar ma'nosidagi karrali maxsus integral tenglamalar uchun umumlashtirilgan.

c) Adamar ma'nosidagi karrali maxsus integral tenglamalar uchun iteratsiyalangan yadrolar va rezolventa tushunchalari kiritilgan va ular yordamida qaralayotgan tenglama yechimi topildi.

2. Erishilgan asosiy natijalar: N.P. Vekua [2] ning bir o'zgaruvchili funksiyalar uchun olingan natijalarini ikki o'zgaruvchili funksiyalar uchun umumlashtirish, Adamar ma'nosidagi chiziqli integral tenglamalari yechimlarining mavjudligi va yagonaligi haqidagi yangi teoremalarni isbot qilishdan iborat.

Bu ishda ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimlarining mavjudligi va yagonaligi haqidadgi teoremalar ketma – ket yaqinlashish usuli, qisib akslantirish prinsipi, iteratsiyalangan yadro va rezolventa usuli yordamida isbotlanib, yangi natijalarga erishildi. Bu teoremalar birinchi bobdagi teoremalar va yechish usullaridan foydalanib isbotlandi.

1-teorema. Agar $f(t,\tau)$ funksiya (Δ_1) da uzluksiz va uzluksiz 2-martagacha aralash xususiy hosilalarga ega bo`lib, $K(x,s,t,\tau)$ funksiya esa $(\Delta)$ da uzluksiz va to'rt martagacha uzluksiz va uzluksiz aralash xususiy hosilalarga ega bo`lib,

$$|\lambda| < \frac{1}{B} \tag{2}$$

bo`lsa,

$$B = 2\left(\frac{1}{\mu_1 (b-a)^{\mu_1} \mu_2 (d-c)^{\mu_2}} + \frac{(b-a)^{1-\mu_1}}{\mu_2 (d-c)^{\mu_2} (1-\mu_1)} + \frac{(d-c)^{1-\mu_2}}{(1-\mu_2)\mu_1 (b-a)^{\mu_1}} + \right.$$

$$\left. + \frac{(d-c)^{1-\mu_2} (b-a)^{1-\mu_1}}{(1-\mu_2)(1-\mu_1)} \right)$$

u holda

$$\varphi(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} dt d\tau = f(x,y) \tag{3}$$

integral tenglama absolyut va tekis yaqinlashuvchi

$$\varphi(x,y) = \varphi_0(x,y) + \lambda\varphi_1(x,y) + \lambda^2 \varphi_2(x,y) + \ldots + \lambda^n \varphi_n(x,y) + .. \tag{4}$$

qatordan iborat bo’lgan yagona yechimga ega bo’ladi.

2 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada uzluksiz va to’rt martagacha uzluksiz aralash xususiy hosilalarga ega bo’lsa, u holda A operator M fazoni o’zini o’ziga ko’chiradi va chegaralangan bo’ladi.

Ushbu

$$\varphi(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} d\tau dt = g(x,y) \tag{5}$$

chiziqli ikki karrali Adamar ma’nosidagi integral tenglamani qaraymiz, bunda $K(x,y,t,\tau)$ va $g(x,y)$ funksiyalar mos ravishda D va D_1 sohalarda aniqlangan, λ esa parametr.

3 - teorema. Agar $K(x,y,t,\tau)$ funksiya D sohada to’rt martagacha uzluksiz aralash xususiy hosilalarga, $g(x,y)$ funksiya esa D_1 sohada ikki martagacha uzluksiz aralash xususiy hosilalaga ega bo’lsa, u holda

$$\varphi(x,y) - \lambda \int_a^b \int_c^d \frac{K(x,y,t,\tau)\varphi(t,\tau)}{(b-\tau)^{1+\mu_1}(d-t)^{1+\mu_2}} d\tau dt = g(x,y) \tag{6}$$

integral tenglama λ parametrning

$$|\lambda| < \frac{1}{\frac{NE}{\mu_1\mu_2}} = \frac{\mu_1\mu_2}{NE} \tag{7}$$

tengsizlikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi.

4 - teorema. Agar $K(x, y, t, \tau)$ funksiya D sohada $4p$ martagacha uzluksiz aralash hosilalarga, $g(x, y)$ funksiya esa D_1 sohada $2p$ martagacha uzluksiz aralash xususiy hosilalarga ega bo'lsa, u holda

$$\varphi(x, y) - \lambda \int_a^b \int_c^d \frac{K(x, y, t, \tau)\varphi(t, \tau)}{(b - \tau)^{p+\mu_1}(d - t)^{p+\mu_2}} d\tau dt = g(x, y) \tag{8}$$

integral tenglama λ parametrning

$$|\lambda| < \frac{1}{N_1 E_1}$$

tengsizlikni qanoatlantiruvchi qiymatida M fazoda yagona yechimga ega bo'ladi va bu yechim ketma – ket yaqinlashish usuli orqali topiladi.

3. Magistr shaxsan erishgan yutuqlari: Mazkur dissertatsiya ishida N.P. Vekua [2] ning olgan natijalarini ko'p o'zgaruvchili funksiyalar uchun ham o'rinli bo'ladimi, degan masalani yechishga qaratilgan. N.P. Vekua [2] Adamar ma'nosidagi integral tushunchasi va uning asosiy xossalari, ikki karrali Adamar ma'nosidagi integrallarning o'rinlarini almashtirish haqidagi teorema, bir o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli yordamida isbot qilgan. Bundan tashqari integral tenglamaning rezolventa yordamidagi yechimi ham yechib ko'rsatilgan.

Magistrlik dissertatsiya ishida erishgan yutuqlar shundan iboratki ikki o'zgaruvchili funksiyalar uchun Adamar ma'nosidagi chiziqli integral tenglamani yechishga bag'ishlangan bo'lib, ikki karrali Adamar ma'nosidagi integral tushunchasi kiritilib, so'ngra, maxsusulik darajasi $1 + \mu_1$ va $1 + \mu_2$ bo'lganda, maxsuslik darajasi p>1 bo'lganda ikki o'zgaruvchili funksiya uchun Adamar ma'nosidagi integral tenglama yechimining mavjudligi va yagonaligi ketma – ket yaqinlashish usuli, qisib akslantirish usuli, iteratsiyalangan yadro va rezolventa yordamida yechib ko'rsatilgan.

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