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UMAROV ELMUROD TO’LQINOVICH

# Binomial qonun bo’yicha taqsimlangan tarmoqlanish jarayoni

5A130102-Ehtimollar nazariyasi va matematik statistika

Magistr
akademik darajasini olish uchun yozilgan
dissertatsiya

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## *Mundarija*

# KIRISH

<u>*Dissertatsiya mavzusining asoslanishi va uning dolzarbligi*</u>: Ushbu magistrlik dissertatsiya “Binomial qonun bo’yicha taqsimlangan tarmoqlanish jarayoni” deb nomlangan . Tarmoqlanish jarayoni juda ko’p sohalarida keng qo’llaniladi: jumladan demografiya, biologiya, meditsina, fizika, ommaviy xizmat ko’rsatish nazariyasida va xokazolarda. Shu o’rinda Prezidentimiz Islom Abdug’aniyevich Karimov ma’ruzalarida quyida fikrlarni aytib o’tdilar.

[1]”Hisobot yilida ta’lim sohasini rivojlantirish va isloh etish masalasi doimiy ravishda e’tiborimiz markazida bo’ldi.

Ta’lim-tarbiya sohasining yaxlit, uzluksiz tizimini shakllantirish va mustahkamlash, jumladan, umumiy o’rta ta’limdan boshlab o’rta maxsus, kasb-hunar hamda oliy ta’limgacha bo’lgan barcha bosqichlarda yuksak bilimli va malakali kasb tayyorgarligiga ega bo’lgan avlodni tarbiyalash jarayonini takomillashtirish ishlari izchil davom ettirildi.

2013 yilning 1 yanvaridan boshlab biz uchun yangi bo’lgan oliy o’quv yurtidan keyingi ta’lim, doktorlik ilmiy ishlarini tayyorlash va himoya qilish, ilmiy daraja hamda ilmiy unvonlar berish tizimi joriy etilmoqda.

Ilmiy kengashlar asosan nafaqat yuqori malakali kadrlar tayyorlash maskani, ayni vaqtda ilmiy tadqiqotlar olib boriladigan markazga aylanishi lozim bo’lgan yetakchi oliy o’quv yurtlari qoshida tashkil etiladi.

Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasining tashkiliy tuzilishi va uning nizomi tubdan qayta ko’rib chiqildi.

2013 yilning 1 yanvaridan boshlab mamlakatimizda oliy o’quv yurtlari faoliyatini baholashning reyting tizimi joriy etilmoqda. Reyting tuzish vazifasi

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[1] I.A.Karimov 2012-yilda mamlakatimizni ijtimoiy-iqtisodiy rivojlantirish yakunlari hamda 2013-yilga mo’ljallangan iqtisodiy dasturning eng muhim ustuvor yo’nalishlariga bag’ishlangan O’zbekiston Respublikasi vazirlar Mahkamasining majlisidagi ma’ruzasi 2013-yil 18-yanvar

Vazirlar Mahkamasi huzuridagi Davlat test markazi zimmasiga yuklanadi. Ushbu markaz reyting baholarini tuzishning hozircha vaqtinchalik tasdiqlangan metodikasi asosida Hukumatga mamlakatimizda oliy ta'limning qanday rivojlanayotgani to'g'risida har yili ta'liliy ma'lumotlar taqdim etadi.

Oliy o'quv yurtlari faoliyati baholanadigan mezonlarni shakllantirishda oliy o'quv yurtlaridagi o'qitish sifati va ilmiy salohiyat darajasi indeksiga asosiy e'tibor qaratiladi va ularning har biri natijasiga ko'ra, 35 foizdan eng yuqori ballar qo'yiladi. Shuningdek, talaba va bitiruvchilarning ish beruvchilar o'rtasida so'rov o'tkazish natijasida aniqlanadigan malaka indeksiga alohida ahamiyat beriladi va bu ko'rsatkich 20 foiz bilan baholanadi. Qolgan 10 foiz boshqa ko'rsatkichlar bo'yicha beriladi.

Reyting tizimini joriy etishning ma'nosi va ahamiyati faqat har bir oliy o'quv yurtining mamlakatimiz oliy o'quv yurtlari orasida qanday o'rinni egallab turgani haqida xolis ma'lumotga ega bo'lishdan iborat emas.

Eng asosiysi, shu asnoda oliy o'quv yurtlari o'rtasida sog'lom raqobat va musobaqa muhitini shakllantirish, shuningdek, ishimizdagi e'tibordan chetda qolib kelayotgan jihatlar va rezervlarni baholash, yuqori malakali kadrlar tayyorlash darajasi hamda sifatini yanada oshirish bo'yicha aniq takliflarni ishlab chiqish imkoniyati paydo bo'ladi.

Ta'lim sohasidagi ishlarimizni sarhisob qilar ekanmiz, Fransiyadagi dunyoning eng yaxshi beshta biznes maktabi qatoriga kiradigan "Inssad" xalqaro biznes maktabining 2012 yilgi "Innovatsiyalarning global indeksi" ma'ruzasida bayon etilgan ma'lumotlarni keltirish o'rinli, deb o'ylayman. Ma'ruza Jahon intellektual mulk tashkiloti bilan hamkorlikda tayyorlangan.

Ushbu ma'ruzada dunyoning 141 mamlakatidagi innovatsion rivojlanish kompleks tarzda tahlil qilingan. Tahlilning asosiy tarkibiy qismlaridan biri inson kapitalini rivojlantirish darajasi bo'lib, mazkur ko'rsatkich bo'yicha bizning mamlakatimiz 35-o'rinni egallagan. Ta'lim tizimini rivojlantirish darajasi bo'yicha esa O'zbekiston – shunga e'tibor beringlar – dunyoning 141 mamlakati orasida ikkinchi o'rinni band etgan.

O'ylaymanki, bu o'rinda ortiqcha izohga hojat yo'q".

*Tadqiqot ob'yekti va predmetining belgilanishi:* Ilmiy tadqiqot ob'yektini tarmoqlanish jarayoni uchun limit teoremalarga oid ilmiy manbalar, tarmoqlanish jarayonini o'rganishga bag'ishlangan yurtimizdagi va xorijdagi tadqiqotlar tashkil qiladi.

Tarmoqlanish jarayoni uchun limit teoremalar masalasining talqin etilishining tamoyillarini o'rganish va ilmiy-nazariy tahlil qilishdan iborat.

*Tadqiqot maqsadi va vazifalari:* Magistrlik ishining maqsadi ehtimollar nazariyasining asosiy yo'nalishlaridan biri bo'lgan tarmoqlanish jarayonidagi binomial qonun bo'yicha taqsimlanishni o'rganish va kelajakda bu sohada ilmiy izlanishlar olib borishdan iborat.

*Tadqiqotning asosiy masalalari va farazlari:* Tasodifiy sondan boshlangan tarmoqlanish jarayoni uchun limit teoremalar o'rganish.

*Mavzu bo'yicha qisqacha adabiyotlar tahlili* Men dissertatsiyani yozishda asosan quyidagi adabiyotlardan foydalandim: Xarris T.E, Теория ветвящихся случайных процессов. М: "Мир", 1966, Севастьянов Б.А. "Ветвящиеся процессы " М., Наука, 1971. Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoniga oid yetarli ma'lumot oldim.

*Tadqiqotda qo'llanilgan uslublarning qisqacha tavsifi:* Ushbu dissertatsiyaning ilmiy–nazariy tomonlarini ishlab chiqishda tarmoqlanish jarayoni soxasida ish olib borishgan matematik olimlarimizdan B.A.Sevastyanov, A.M.Zubkov, V.A.Vatutin, Sh.K.Farmonov, I.S.Badalboev va boshqa yirik olimlar, xozirda esa O’zbekistonlik ustozlarimiz nazariy tadqiqotlariga tayanildi.

*Tadqiqot natijalarining nazariy va amaliy ahamiyati:* Tarmoqlanish jarayoni keng tadbiqga ega, ya’ni demografiya, biologiya, meditsina, fizika, ommaviy xizmat ko’rsatish nazariyasida va hokazolarda bu metoddan unumli foydalaniladi.

*Tadqiqotning ilmiy yangiligi:* Ishda $\mu_n$ Puasson taqsimoti bo’yicha , $\nu_n$ binomial taqsimoti bo’yicha taqsimlangan bo’lsa $F_n(x)$ ni aniqlash kabi masalalar ko’rilgan.

*Dissertatsiya tarkibining qisqacha tavsifi:* Magistrlik dissertatsiya kirish, asosiy qism (3 ta bob), xulosa, foydalanilgan adabiyotlar ro’yxati, internet materiallaridan iborat. Hajmi 110 betdan tashkil topgan.

Davlatimiz rahbari Islom Karimov tashabbusi bilan 2014-yil mamlakatimizda “Sog‘lom bola yili” deb e’lon qilinishi jamoatchiligimiz tomonidan keng qo‘llab-quvvatlandi. Joriy yilning 19-fevral kuni O‘zbekiston Respublikasi Prezidentining “Sog‘lom bola yili” Davlat dasturi to‘g‘risida”gi qarori qabul qilindi. Shuni ishonch bilan aytish mumkinki, bu mustaqilligimizning ilk kunlaridan jismonan sog‘lom va ma’nan yetuk barkamol avlodni tarbiyalash ustuvor vazifa etib belgilangan ijtimoiy yo‘naltirilgan davlat siyosatining mantiqiy davomi bo‘ldi. “Ona va bola”, “Yoshlar ”, “Barkamol avlod ”, “Oila” va boshqa nomlar bilan atalgan yillarda amalga oshirilgan ishlar xalqimizning ezgu orzusi bo‘lgan sog‘lom bola tarbiyalashdek olijanob maqsadga hamohangdir.

Hech shubhasiz, har birimiz farzandlarimizni sog‘lom va har tomonlama barkamol qilib tarbiyalash, ularning baxt-saodati, yorug‘ kelajagini ko‘rishni niyat qilamiz.

O‘tgan davrda keng miqyosli va teran mazmunli ulkan ishlar, mamlakatimiz va jamiyatimiz taraqqiyoti uchun g‘oyat muhim ahamiyat kasb etuvchi vazifalarni bajarishga yo‘naltirilgan qator umummilliy dasturlar, birinchi navbatda, “Sog‘lom ona – sog‘lom bola” dasturi amalga oshirildi. Farzandlarimiz va xalqimiz baxti uchun, kelajagimiz uchun qilinayotgan bu ezgu ishlar izchillik bilan davom etib, tobora kengayib borayotgani, yuksak samaralar berayotgani quvonarlidir.

Kadrlar tayyorlash milliy dasturi va Maktab ta’limini rivojlantirish umummilliy davlat dasturining amalga oshirilishi samaralari mamlakatimizda barkamol avlodni tarbiyalashga qaratilayotan ulkan e’tiborning yaqqol tasdig‘idir.

“Sog‘lom bola yili” Davlat dasturida ko‘zda tutilgan chora-tadbirlar, avvalo shu muhim masalalarni hal qilishga qaratilgan. Joriy yilda Davlat dasturini amalga oshirish jarayonida bu ishlarning barchasi yanada yuqori darajaga ko‘tariladi. Dasturda avvalo ham jismoniy, ham ma’naviy jihatdan sog‘lom, mustaqil fikrlay oladigan, yuksak intellektual salohiyatli, chuqur bilimli va zamonaviy dunyoqarashga ega, mamlakat taqdiri va kelajagi uchun mas’uliyatni o‘z zimmasiga olishga qodir barkamol avlodni shakllantirish, davlat va jamiyatning barcha kuch va imkoniyatlarini ana shu maqsadlarga safarbar etishga doir keng miqyosli chora-tadbirlarni amalga oshirish ko‘zda tutilgan.

Dastur yetti bo‘lim va 125 banddan iborat bo‘lib, unda bolalar tug‘ilishi, ta’lim-tarbiyasi, oilada sog‘lom muhitni, uning iqtisodiy va ma’naviy-ahloqiy asoslarini mustahkamlash, ijtimoiy soha rivojiga ajratilayotgan mablag‘lar samaradorligini oshirish bilan bog‘liq barcha masalalar aks etgan.

# __I bob. Galton-Vatson tarmoqlanish jarayoni

18-asrda o'rtalarida Galton va Vatson familiyalarini yo'qolib ketishi masalalarini yechish jarayonida ehtimollar nazariyasi tadbiqlaridan hisoblangan tarmoqlanuvchi jarayon uchun model tuzdilar , keyinchalik 1927-yilda Xolden 1930-yil Stefenson , 1938- yilda Kolmogorov, 1931-yilda Lodkov kabilar bu fanni boshlang'ich masalalarini yechdilar . Ko'pgina fanlarda bu jarayon o'z aksini topdi. Demografik jarayonda , genetika masalalarida , ximiyada , fizikada keng tadbiq qilinmoqda.

Hozirgi davrda Rossiyada , AQSH da , Yaponiyada , O'zbekistonda bu jarayon o'rganilmoqda va tadbiq qilinmoqda . O'zbekistonda Shokir Farmonov rahbarligida o'rganilmoqda.

## *1.1. Tarmoqlanish jarayoni ta'rifi*

### *Bir xilli tarmoqlanuvchi jarayon ta'rifi*

Faraz qilaylik $\mu_0,\mu_1,,,,,\mu_n$, musbat, butun qiymatlarni qabul qiladigan tasodifiy miqdorlar quyidagi shartlarni qanoatlantirsin:

$$P\ (\mu_0 = 1) = 1, \qquad P(\mu_1 = k) = P_k\ , \qquad \sum_{k=0}^{\infty} P_k = 1,$$

(1.1.1)

$\{\xi_ø\}$ tasodifiy miqdorlar o'zaro bog'liqsiz va har biri $\mu_1$ day taqsimlangan bo'lib,

$$P(\mu_{n+1} = k/\ \ \mu_n = l) = P(\xi_1 + \xi_2 +,,,+\xi_L = k),$$

$$(\ P(\mu_{n+1} = 0/\mu_n = 0) = 1) \qquad (1.1.2)$$

bajarilsin.

**1-TA'RIF**. (1.1.1) va (1.1.2) shartni qanoatlantiruvchi $\{\mu_n\}$ tasodifiy miqdorlar ketma-ketligi bir jinsli bir xilli nuqtaviy vaqtli tarmoqlanish jarayoni deyiladi.

Bu yerda $\mu_n$ n-chi avloddagi zarrachalar sonini bildiradi. Bunday jarayonni Gal'ton-Vatson jarayoni deb yuritamiz. (1.1.2) ifoda ixtiyoriy avloddagi har bir zarracha o'zaro bog'liqsiz ko'payishini ko'rsatadi.

Tarmoqlanish jarayonini o'rganishdagi qulay usullardan biri hosil qiluvchi funksiyadir. Quyidagi hosil qiluvchi funksiyani kiritamiz:

$$F(x) = \sum_{k=0}^{\infty} p_k x^k, \qquad |x| \le 1. \ \sum P_k = 1$$

Agar $F(x)$ bu funksiyaning $x = 1$ nuqtadagi birinchi, ikkinchi va uchinchi xosilalari mavjud bo'lsa, bularni mos ravishda

$$À = F^1(1), \ \ B = F^1(1), \quad D = F^{111}(1)$$

kabi belgilaymiz.

Ishonch xosil qilish mumkinki

$$F_n(x) = \sum_{k=0}^{\infty} P(\mu_n = k)x^k = F(F_{n-1}(x)) = F_{n-1}(F(x)),$$

(1.1.3)

$n = 1,2,,,,$ $\quad F(x) = F_1(x).$ (1.1.3) dan foydalanib $\mu_n$ ni matematik kutilmasini $M\mu_n = A^n$ ga tengligini topamiz.

Dispersiya esa

$$D\mu_n = \begin{cases} \dfrac{(B+A-A^2)A^n(A^n-1)}{A^2-A}, & A \ne 1, \\ n(B+A-A^2), & A = 1 \end{cases}$$

(1.1.4)

**2-TA'RIF:** Agar ehtimollik bilan $\{\mu_n\}$ ni chekli sondagi hadlaridan tashqari hadlari 0 ga teng bo'lsa, u holda jarayon tugaydi deyiladi.

Xususan,

$$P(\mu_n = 0) = 1$$

bajarilsa jarayon n-nchi avlodda tugaydi.

$\{\mu_n\}$ jarayoni tugash ehtimolligini $q$ bilan belgilaymiz,

u holda

$$q = \lim_{n\to\infty} P(\mu_n = 0) = \lim_{n\to\infty} F_n(0)$$

**1-TEOREMA** Agar $A \le 1$ bo'lsa $\{\mu_n\}$ jarayoni tugash ehtimoli birga teng bo'ladi. $A > 1$ da esa $\{\mu_n\}$ jarayoni tugash ehtimolligi

$$F(x) = x \qquad (1.1.5)$$

tenglama manfiy bo'lmagan va birdan kichik bo'lgan yagona q yechimiga tengdir.

### *Ko'p xilli tarmoqlanuvchi jarayon*

**3-TA'RIF**. Aytaylik $T = (T_1, T_2, ..., T)$ $K$ o'lchovli vektor bo'lib, uning komponentlari manfiy bo'lmagan butun qiymatlardan iborat bo'lsin. $e_i \, (1 \le i \le k)$ bilan esa $i$-nchi komponentasi 1, qolgan komponentlari 0 dan iborat vektorni belgilaymiz.

**4-TA'RIF.** Ko'p xolatli Galt'ton-Vatson tarmoqlanish jarayoni deb vaqt bo'yicha bir jinsli T-xolatdagi Markov

$$\mu(0),\ \mu(1),\ \mu(2), ....\mu(n), ... \qquad (1.1.6)$$

jarayoniga aytiladi, bu yerda $\mu(0)$ doim aniq butun vektor deb qabul qilinadi,

$\mu(l) = (\mu_1(l),\ \mu_2(l), ....\mu_k(l)),\quad l = 0,1,2....,$ da esa $\mu_i(l)$ $l$-nchi avloddagi $i$ nchi $(1 \le i \le k)$ xildagi zarrachalar sonini ifodalaydi deb faraz qilamiz. $\mu_i^{\ j}(l),\ i, j = \overline{1,k},$ $j$-nchi xildan boshlangan $l$-nchi avloddagi $i$-nchi xildagi zarrachalar sonini belgilaydi.

Hosil qiluvchi funksiyani quyidagicha kiritamiz:

$$F_n^{\ j}(s) = F_n^{\ j}(s_1, s_2, ....s_k) = \sum_{\substack{m_j = 0 \\ i = \overline{1,k}}} P\left(\mu_1^{\ j}(n) = m_1, , , , \mu_k^{\ j}(n) = m_k\right) \cdot$$

$$s_1^{m_1} \cdot s_2^{m_2} \cdots s_k^{m_k},\ 1 \le j \le k,\quad \left|s_j\right| \le 1,\quad 1 \le i \le k$$

$$F_n(s) = \left(F_n{}'(s), F_n^{\ 2}(s), ....F_n^{\ k}(s)\right), \qquad F_1(s) = F(s),$$

$F_n(s)$ vektor ixtiyoriy $n_1,\ n_2 \ge 0$ uchun $F_0(s) = s$ shartni qanoatlantiruvchi

$$F_{n_1 + n_2}(s) = F_{n_1}\left(F'_{n_2}(s),\ F_{n_2}^2(s), ....F_{n_k}^k(s)\right) \qquad (1.1.7)$$

tenglamani qanoatlantiradi. Quyidagi belgilashlarni kiritamiz:

$$A_j^j(n) = M\mu_j^j(n), \;\; A_j^j(1) = a_j^j$$

$$B_l^j = M(\mu_j^j(1)\;\mu_l^j(1)),\; i \neq l,\; B_{l,l}^j = M(\mu_l^j(1)(\mu_l^j(1)-1)) \quad P(\mu_1^j(n) = m_1, ....,$$

$$\mu_k^j(n) = m_k) = P_{m_1,m_2..,m_k}^{j\;(n)}, \qquad n \geq 0,$$

u holda (1.1.10) ga ko'ra

$$\frac{\partial F^j(1,1,....1)}{\partial s_j} = A_j^j(1), \;\; i, j = 1,2,....,k.$$

$$A_j^j(n) = \sum_{l=1}^{k} A_n^j(n-1)\,A_j^l.$$

Ma'lumki, agar $\|A_i^j\|$ matritsa yoyilmaydigan bo'lsa, u holda, musbat oddiy xos son $\rho$ mavjud bo'lib bu son $\|A_i^j\|$ matritsani boshqa xos sonlardan absolyut qiymat jihatdan oshmaydi va $\rho$ ga mos o'ng $u = (u_1, u_2, ...u_k)$ chap xos vektorlar musbat va

$$A_1^j u_1 + A_2^j u_2 +,,,, + A_k^j u_k = \rho u_j$$

$$v_1 A_i^1 + v_2 A_i^2 +,,,, + v_k A_i^k = \rho v_i \qquad (1.1.8)$$

$$u_1 V_1 + u_2 v_2 + . \;.\; . + u_k v_k = 1$$

tenglikni qanoatlantiradi. Bunday tarmoqlanish jarayoniga yoyilmaydigan tarmoqlanish jarayoni deyiladi. Biz yoyilmaydigan davriy bo'lmagan ko'p xilli Gal'ton-Vatson jarayonini qaraymiz, bu shartlar buzilgan hollarni ham topish mumkin.

Atrey va Ney $A_i^j$(n) uchun $n \to \infty$da quyidagi asimtotik formulani keltirib chiqaradi;

$$A_i^j(n) = \rho^n u_j v_i + r_{j,i}(n), \qquad (1.1.9)$$

bu yerda $r_{j,;}(n) < c\rho^n$.

**5-TA'RIF**. Agar $\rho < 1$ bo'lsa kritikgacha $\rho = 1$ kritikdan keyingi ko'p xilli Gal'ton-Vatson jarayoni deyiladi. $F_n^j(0,0)$ $j$-xildan boshlangan jarayonni $n$-nchi davrda tugash ehtimolini bildiradi.

**6-TA'RIF.** $q^j$ bilan $j-$nchi tipdan boshlangan zarrachani tugash ehtimolini belgilaymiz.

Ma’lumki

$$\lim_{n\to\infty} F_n^j(0,0,...,0) = q^j, \qquad j = 1,2,....k \qquad (1.1.10)$$

va $q^j$

$$s_j = F_1^j(s_1, s_2, ...s_k)$$

tenglamaning yechimidan iboratdir, shu bilan birga

$$\lim_{n\to\infty} F_n(s_1,..,s_k) = q = (q_1, q_2, ...q_k), \qquad (1.1.11)$$

**7-TA’RIF**. $t\downarrow 0$ da

$$P^{j(t)}_{m_1,m_2,..m_k} = \delta^{l_j}_{m_1,m_2,..m_k} + P^{j\quad t+0(t)}_{m_1,m_2,..m_k}, \qquad \sum_{m_1,m_2,..m_k} P^j_{m_1,m_2} = 0$$

deb qabul qilamiz, bu yerda

$$e_j = (\delta_j^1, \delta_j^2, ..., \delta_j^k), \qquad \delta_i^j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases},$$

$$\delta_m^\alpha = (\delta_m^{\alpha_1}, \delta_m^{\alpha_2}, .., \delta_m^{\alpha_k}), \qquad m = (m_1, m_2, .., m_k),$$

$$\delta_m^{\alpha_i} = \begin{cases} 1, & \alpha_i = m \\ 0, & \alpha_i \neq m. \end{cases}$$

Bunday kiritilgan jarayonga $k$ xilli uzluksiz vaqtli tarmoqlanish jarayoni deb ataladi.

Quyidagi hosil qiluvchi funksiya kiritamiz:

$$\mathrm{f}^j(s) = \sum_m P_m^j s^m = \sum_{m_1=0,...,m_k=0} P_m^j, ..., S_k^{m_k}$$

$$\mathrm{f}(s) = (\mathrm{f}'(s), ..., \mathrm{f}^k(s))$$

u holda (1.1.11) dan $t\downarrow 0$ da $s(|s| \leq 1)$ bo’yicha tekis

$$F(t,s) = s + tf(s) + 0(t), \qquad (1.1.12)$$

bu yerda

$$F^j(t,s) = \sum_m P^j_{m_1,m_2,..m_k}(t) s_1^{m_1} s_2^{m_2} ... s_k^{m_k}, \qquad F(t,s) = (F'(t,s), ..., F^k(t,s))$$

Uzluksiz xol uchun

$$q^j = \lim_{t\to\infty} F_t^j(0,0,...0)$$

tenglamaning yechimini qanoatlantiradi.

**2-TEOREMA**. $F(t,s)$ $|s| < 1$ da

$$\frac{\partial F(t,s)}{\partial t} = f(F(t,s)), \quad F(0,s) = s \tag{1.1.13}$$

differentsial tenglamalarni qanoatlantiradi, xususan

$$\frac{\partial F(t,s)}{\partial t} = \sum_{j=1}^{k} f^{j}(s)\frac{\partial F(t,s)}{\partial s^{j}},\ F(0,s) = s. \tag{1.1.14}$$

Quyidagi belgilashni kiritamiz:

$$a_i^j = \frac{\partial f^{j}(s)}{\partial s_i}\Bigg|_{s=1}, \qquad 1 = (1,1,...,1),\ b_{i,l}^{j} = \frac{\partial^2 f^{j}(s)}{\partial s_i \partial s_l}, \qquad \overline{A}_i^j(t) = \frac{\partial F^{j}(t,s)}{\partial s_i}\Bigg|_{s=1} \tag{1.1.15}$$

Bundan va

$$F(t_1 + t_2, s) = F(t_1, F'(t_2,s),...,F^{k}(t_2,s)) \tag{1.1.16}$$

ifodadan

$$\overline{A}_i^j(t_1 + t_2) = \sum_{l=1}^{k} \overline{A}_l^j(t_1)\,\overline{A}_i^l(t_2) \tag{1.1.17}$$

ni xosil qilamiz.

**3-TEOREMA**. Agar $a_i^j$ lar chekli bo’lsa u holda $\overline{A}_i^j(t)$ lar ham chekli va

$$\frac{d\overline{A}_i^j(t)}{dt} = \sum_{l=1}^{k} a_l^j \overline{A}_i^j(t), \qquad \overline{A}_i^j(0) = \delta_i^j \tag{1.1.18}$$

tenglamalarni qanoatlantiradi.

Xuddi ko’p xilli Galton-Vatson jarayonidagidek $\left\| a_i^j \right\|$ yoyilmaydigan matritsa uchun

$$\sum_{l=1}^{k} a_l^j u^l = \overline{\rho} u_j, \qquad \sum v_j a_i^j = \overline{\rho} v_j, \tag{1.1.19}$$

o’rinli, bu yerda $\overline{\rho}$ $\left\| a_i^j \right\|$ matritsani oddiy soni va (1.1.19) bu hol uchun ham o’rinli:

$$\overline{A}_i^j(t) = u^{j} v_i e^{t\overline{\rho}} + r_{j,i}(t),\ \ r_{j,i}(t) < ce^{ts}, \tag{1.1.20}$$

**8-TA’RIF**. Agar $\overline{\rho} < 0$ bo’lsa kritikgacha $\overline{\rho} = 0$ kritik, $\overline{\rho} > 0$ kritikdan keyingi ko’p xilli uzluksiz vaqtli jarayon deyiladi.

$$Q_n = 1 - P(\mu_n = 0).$$

ifoda G-V jarayonini davom etish ehtimolligini ifodalaydi.

Ikki xilli Gal’ton-Vatson jarayoni uchun chegaraviy masala

$r_{x,y}^{l}, r_{x}^{l}$ tasodifiy miqdorlarni quyidagicha kiritamiz:

$$\tau_{x,y}^{l}=\begin{cases} i, & агар \ \max\limits_{1\le n\le i}\mu_1^l(n)<x,\ \max\limits_{1\le n\le i}\mu_2^l(n)<y \\ & лекин\,(\mu_1^{\,l}(i+1)\ge x)\cup(\mu_2^{\,l}(i+1)\ge y) \\ +\infty & агар \max\limits_{1\le n\le i}\mu_1^{\,l}(n)<x,\ \max\limits_{1\le n\le i}\mu_2^{\,l}(n)<y, \end{cases} \quad (1.97) \quad (1.1.21)$$

$$\tau^{l}{}_{x}=\begin{cases} i, & агар \max\limits_{1\le n\le i}(u_1\mu_1^l(n)+u_2\mu_2^l(n)<x), \\ & лекин\, u_1\mu_1^l(i+1)+u_2\mu_2^l(i+1)\ge x \\ +\infty & агар \max\limits_{n}(u_1\mu_1^l(n)+u_2\mu_2^l(n))<x \end{cases} \qquad (1.1.22)$$

$$\tau_{r}^{l}=\begin{cases} i, & агар \max\limits_{1\le n\le i}\left|\mu^l(n)\right|<r \\ & лекин\left|\mu^l(i+1)\right|\ge r \\ +\infty & агар \max\limits_{n}\left|\mu^l(n)\right|<r \end{cases} \qquad (1.1.23)$$

$$\left|\mu^l(n)\right|=\sqrt{(\mu_1^l(n))^2+(\mu_2^l(n))^2}, \qquad l=1,2$$

Agar $\rho>1$ bo’lsa, u holda $1-q$ ehtimollik bilan GAL’TON –VATSON jarayoni ixtiyoriy belgilangan sathdan chiqib ketadi.

$\tau_{x,y}^{l}, \tau_{x}^{l}, \tau_{r}^{l}$ tasodifiy miqdorlar bilan bog’langan masalalarni chegaraviy masala deb nomlaymiz. Bu tasodifiy miqdorlarning taqsimotini aniqlash hozircha hal etilmagan. Shuning uchun $\tau_{x,y}^{l}, \tau_{x}^{l}, \tau_{r}^{l}$ tasodifiy miqdorlarning $x, y, z\to+\infty$ dagi holatni o’rganamiz. Bu masalani bir xilli GAL’TON-VATSON jarayoni uchun Nagaev A.V. va Badalov I.S lar o’rganganlar.

## *1.2. Kritikgacha tarmoqlanish jarayoni*

Kritikkacha tarmoqlanish jarayoniga doir asosiy teoremalar

**4-TEOREMA**: Agar A<1, B$<+\infty$ bo'lsa u holda

$$Q_n = kA^n\,(1+0\,(1)) \qquad (1.2.1)$$

Bu yerda k μ, ning taqsimotiga bog'liq o'zgarmas son.

5-TEOREMA: Agar $A<1, M\mu^{1+\delta}<+\infty$, δ>0 bo'lsa u holda $n\to\infty$ da (1.1.24) o'rinli

**6-TEOREMA** $A<1$ da (1.2.1) ifodaning o'rinli bo'lishi uchun

$$-\int_0^1 \frac{1-Ax-F(1-x)}{x^2}\,dx<+\infty \qquad (1.2.2)$$

shartni bajarilishi zarur va yetarlidir. Ushbu tenglikni ko'rsatish mumkin:

$$\frac{1-Ax-F(1-x)}{x^2} = -\sum_{k=0}^{\infty} r_k x^k \; r_k = \sum_{i\ge k}\sum_{j=1} P_j$$

Xususan, $B<+\infty$ da (1.2.2) bajariladi.

**7-TEOREMA** $A<1$ da (1.2.1) ifodaning bajarilishi uchun $M\mu_1 \ln\mu_1<+\infty$ bo'lishi zarur va yetarlidir.

**8-TEOREMA** Agar $A<1$ bo'lsa u holda

$$\lim_{n\to\infty} P(\mu_n k/\mu_n>0)=P_k^*, \qquad \sum_{k=1}^{\infty} P_k^*=1$$

va $P_k^*$ ni xosil qiluvchi funksiyasi

$$F^*(x)=\sum_{k=1}^{\infty} P_k^* x^k$$

quyidagi funksional tenglamani qanoatlantiradi:

$$1-F^*(F(x)) = A(1-F^*(x)) \qquad (1.2.3)$$

**9-TEOREMA**. $F^*(x)$ ning taqsimoti matematik kutilmaga ega bo'lishi uchun M $(\mu_1\log\mu_1)$ ning mavjud bo'lishi zarur va yetarli, shu bilan birga

$$\sum_{m=1}^{\infty} mp_m^* = k^{-1}$$

Bu yerda k (1.2.1) formula bilan izoxlangan.

**9A-TEOREMA** .Agar $A<1$, $B<+\infty$ bo'lsa

$$\lim_{n\to+\infty} P\left\{\frac{\mu_0+\mu_1+\mu_2+...+\mu_n - n(F*(1))'}{\sqrt{n\mathrm{H}}} \le x/\mu_n > 0\right\} = \Phi(x)$$

bu yerda

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}dt, \qquad H = D\left(\sum_{i=0}^{n}\mu_n\right)$$

**10-TEOREMA.** Faraz qilaylik $A<1,\ B<+\infty$, $n\to\infty$ da $m\to+\infty,\ \sigma_n^{\ 2}\to+\infty, \sigma_1 = 0(\sigma_n^{\ 2})$, u holda

$$P(r_{n\nu} < x) = G\left(\frac{x}{\delta}\right) * \varPhi\left(\frac{x}{\sqrt{1+\delta^2}}\right) + 0(1),$$

bu yerda $*$-kompozitsiya belgisi, $\delta = \dfrac{A^n\sigma}{\sigma_n}$ teorema isboti uchun quyidagi lemmani keltiramiz.

LEMMA. Yuqoridagi bajarilganda

$$\Psi_n(s) = \theta(\delta s)e^{-\frac{s^2}{2}(1+\delta^2)} + 0(1)$$

tenglik o'rinlidir.

Lemmaning iboti. Quyidagicha ifodalaymiz:

$$\Psi_n(s) = \sum_{k=1}^{\infty} q_k e^{\frac{i(k-m)A^n s}{\sigma_n}}\left[e^{-\frac{isA^n}{\sigma_n}} F_n(e\frac{i\delta}{\sigma_n})\right]^k .$$

Ma'lumki

$$\overline{\Psi}_n(s) = \sum_{k=1}^{\infty} q_k e^{\frac{i(k-m)A^n s}{\sigma_n}}\left[e^{-\frac{isA^n}{\sigma_n}} F_n(e\frac{i\delta}{\sigma_n})\right]^m = = \theta(\delta r)\left[e^{\frac{-isA^n}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}})\right]^m .$$

U holda

$$\left|\Psi_n(s) - \overline{\Psi}_n(s)\right| \le \sum_{k=1}^{\infty} q_k \left| e^{\frac{i(k-m)A^n S}{\sigma_n}}\left\{\left[e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right]\left[e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right]^m\right\}\right. =$$

$$= \sum_{|k-1|>\sigma_n^2} q_k \left| e^{\frac{i(k-m)A^n s}{\sigma_n}} \left\{ \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right] - \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right]^m \right\} \right| + \sum_{|k-1|\le\sigma_n^2} q_k \left| e^{\frac{(k-m)A^n S}{\sigma_n}} \right.$$

$$\left\{ \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right] - \left[ e^{\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right)\right] \right\}^m = T_1 + T_2, \qquad (1.2.3)$$

$n \to \infty$ da

$$M\left(\frac{\nu - m}{\sigma_n^2}\right) \to 0, \; M\left(\frac{\nu - m}{\sigma^2}\right)^2 = \frac{\sigma^2}{\sigma_n^4} \to 0.$$

Shuning uchun $n \to \infty$ da ehtimollik bilan $\frac{\nu - m}{\sigma_n^2} \to 0$.

Natijada

$$T_1 \le 2 \sum_{|k-m|>\sigma_n^2} q_k = 2P\left\{\left|\frac{\nu - m}{\sigma_n^2}\right| > 1\right\} = 0(1), \qquad (1.2.4)$$

$F_n(0)$ funksiyani Teylor qatoriga yoyamiz:

$$F_n\left(e^{\frac{it}{\sigma_n}}\right) = 1 + \frac{isA^n}{\sigma_n} - \frac{s^2 M(\mu_n)^2}{2\sigma_n^2} + T(n) \qquad (1.2.5)$$

bu yerda

$$T(n) = -\frac{s^2}{2\sigma_n^2} F_n''\left(e^{\frac{is}{\sigma_n}}\right) + \frac{s^2}{2\sigma_n^2} M(\mu_n)^2, \qquad |\overline{S}| < |S|$$

Ikkinchi tomondan $N$ natural son uchun

$$|T(n)| \ge \frac{S^2}{2\sigma_n^2} \left| \sum_{k=0}^{\infty} k^2 P\left(\mu_n^{(1)} = k\right) e^{\frac{is}{\sigma_n}} - M(\mu_n^{(1)})^2 \right| \le \frac{S^2}{2\sigma_n^2} \sum_{k=0}^{\infty} k^2 P\left(\mu_n^{(1)} = k\right) \left| e^{\frac{is}{\sigma_n}} - 1 \right| \le$$

$$\le \frac{s^2}{2\sigma_n^2} \left[ \sum_{k\le N} \frac{k^3 |\overline{s}|}{\sigma_n} P(\mu_n^{(1)} = k) + 2\sum_{k>N} k^2 P\left(\mu_n^{(1)} = k\right)\right] = \frac{T^2}{2\sigma_n^2} (T_1(n) + T_2(n)). \qquad (1.2.6)$$

$T_1(n)$ va $T_2(n)$ larni baholaymiz.

$$T_1(n) \le \frac{N|S|}{\sigma_n} \sum_{k\le N} K^2 P(\mu_n^{(1)} = k) < c \frac{N|S|}{\sigma_n} A^n.$$

Natijada $N$ ni tanlash hisobiga

$$T_1(n) = 0(A^n) \qquad (1.2.7)$$

tenglikka ega bo'lamiz.

Agar

$$\sum_{k=1}^{\infty} k^2 P(\mu_n^{(1)} = k) = 0(A^n)$$

munosabatni hisobga olsak

$$T_2(n) = \sum_{K>N} 2K^2 P(\mu_n^{(1)} = k) = 0(A^n) \qquad (1.2.8)$$

tenglik bajariladi. U holda (1.2.6)-(1.2.8) larga asosan

$$T(n) = 0\left(\frac{S^2 A^n}{\sigma_n^2}\right). \qquad (1.2.9)$$

Demak (1.2.5) va (1.2.9) lardan

$$F_n\left(e^{\frac{is}{\sigma_n}}\right) = 1 + \frac{isA^n}{\sigma_n} - \frac{S^2 M(\mu_n^{(1)})^2}{2\sigma_n^2} + 0\left(\frac{S^2 A^n}{\sigma_n^2}\right)$$

yoki

$$e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) = 1 - \frac{D\mu_n}{2\sigma_n^2} S^2 + 0\left(\frac{S^2 A^n}{\sigma_n^2}\right). \qquad (1.2.10)$$

Robbins ishidan foydalanib $T_2$ ni baholaymiz:

$$T_2 \le \sum_{|k-m|\le\sigma_n^2} q_k \left| \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^k - \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^m \right| =$$

$$\le \sum_{m-\sigma_n^2 \le k \le m} q_k \left| \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^k - \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^m \right| +$$

$$+ \sum_{m\le k\le \sigma^2{}_n+m} q_k \left| \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^k - \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^m \right| \le$$

$$\le \max_{m-\sigma_n^2 \le k < m} \left| \left[ e^{-\frac{isA^n}{\sigma_n}} F_n\left(e^{\frac{is}{\sigma_n}}\right) \right]^{m-k} - 1 \right| + \max_{m\le k\le\sigma_n^2+m} \left| \left[ e^{-\frac{isA^n}{\sigma_n}} F\left(e^{\frac{is}{\sigma_n}}\right) \right]^{m-k} - 1 \right| = T_{2,1} + T_{2,2}$$

$n \to \infty$ da (1.2.10) dan

$$\left[e-\frac{isA^n}{\sigma_n}F_n\left(e^{\frac{iT}{\sigma_n}}\right)\right]^{\frac{\sigma_n^2}{D\mu_n}}=e^{-\frac{s^2}{2}}+0(1)$$

demak

$$T_{2,1}\le\max_{m-\sigma_n^2\le k\le m}\left|e^{-\frac{S^2D\mu_n(m-k)}{\sigma_n^2}}-1\right|+0(1)\le\frac{S^2}{2}D\mu_n^{(1)}e^{-\frac{S^2}{2}D\mu_n^{(1)}}+0(1).$$

Xuddi shunday

$$T_{2,2}\le\frac{S^2}{2}D\mu_n e^{\frac{S^2}{2}d\mu_b}+0(1)\,.$$

Oxirgi ikkita tengsizlikdan

$$T_2\le\frac{S^2}{2}D\mu_n e^{\frac{S^2}{2}D\mu_n}+0(1)$$

$\lim\limits_{n\to\infty}D\mu_n=0$ ligini hisobga olsak, oxirgi tengsizlik va (1.2.5), (1.2.4) lardan

$$\Psi_n(s)=\overline{\Psi}(s)+0(1) \qquad (1.2.11)$$

kelib chiqadi.

(1.2.10) yoyilmadan foydalanib

$$\overline{\Psi}_n(s)=\theta(\delta s)e^{\ln\left[1-\frac{D\mu_nS^2}{2\sigma_n^2}+0\left(\frac{S^2A^n}{\sigma_n^2}\right)\right]}=\theta(\delta s)\exp\left(-\frac{s^2}{2}-\frac{mD\mu_n}{\sigma_n^2}\right)+0(1)=\theta(\delta s)\,.$$

$$\exp\left(-\frac{s^2}{2}\left(1-\frac{A^{2n}}{\sigma_n^2}\sigma^2\right)\right)+0(1)=\theta(\delta s)e^{-\frac{S^2}{2}\left(1-\delta^2\right)}+0(1)$$

ni hosil qilamiz.

(1.2.11) va oxirgi tengsizlikdan lemmaning isboti kelib chiqadi.

11-teorema isboti. Essen teoremasini lemmaga qo’llab, keltirib chiqariladi.

**11-TEOREMA**. $A<1, D<+\infty$ bajarilsin, u holda

$$\sup_x\left|S_n(x)-\Phi(x)\right|\le\frac{c}{\sqrt{c_nA^n}}\,\beta(n),$$

bu yerda $\beta(n)=\overline{\beta}_n\sqrt{A^n}:\overline{\sigma}^3n$

Eslatma. Ishonch hosil qilish mumkinki

$$\beta(n) \le \frac{\left(6B^2 + 10B + 2D + 4A^2\right)\sqrt{A}}{\left(A - A^3\right)(1 - A)\sqrt{\left(B - A^2 + A^3\right)}}$$

11-teoremaning isbotida ham Essen teoremasidan foydalaniladi.

## *Ko'p xilli kritikgacha bo'lgan tarmoqlanuvchi jarayon uchun limit teoremalar*

**12-TEOREMA**. Agar $\rho < 1$ bo'lsa, u holda $s$ ni har bir komponentlari bo'yicha monoton o'smaydigan $K(s)$ haqiqiy funksiya mavjudki $n \to +\infty$ da

$$\frac{VR_n(s)}{\rho^n} \downarrow k(s) \ge 0, \qquad 0 \le s \le 1 \qquad (1.2.12)$$

$$\lim_{n\to\infty} \frac{R_n(s)}{\rho^n} = k(s)u, \qquad 0 \le s \le 1 \qquad (1.2.13)$$

Bu yerda $R_n(s) = 1 - F_n(s) = (R'_n(s), R_n^2(s), ..., R^k{}_n(s))$.

**13-TEOREMA**. $n \to +\infty$ da jarayonning davom etish ehtimolligi

$$R_n^j(0) = \theta_n^j = K(0)u_i\rho^n(1 + 0(1))$$

(1.2.14)

ga teng. Xususan, agar boshlanishida $\alpha = (\alpha_1, \alpha_2, ..., \alpha_k) = \mu(0)$ bo'lsa, u holda $n \to +\infty$ da

$$P\left(\mu(n) > 0 / \mu(0) = \alpha\right) = k(0)\sum_{i=1}^{k} u_i\alpha_i\rho^n\left(1 + 0(1)\right) \qquad (1.2.15)$$

**14-TEOREMA**. $t \to +\infty$ da

$$P(\mu(n) = \alpha / \mu(n) \ne 0, \ \ \mu(0) = \beta \ne 0) \to P_\alpha * \qquad (1.2.16)$$

va $P_\alpha *$ ni hosil qiluvchi funksiyasi

$$F*(s) = \sum_\alpha P_\alpha^* s^\alpha$$

quyidagi tenglamani qanoatlantiradi:

$$1 - F*(F(s)) = \rho(1 - F*(s)), \qquad (1.2.17)$$

ya'ni $P_\alpha *$ boshlang'ich shartga bog'liq emas.

**15-TEOREMA**. $\{P_\alpha^*\}$ taqsimot faqat va faqat $k(0)$ musbat bo'lgandagina matematik kutilmaga ega bo'ladi:

$$A_i^* = F_i^*(1) = \left.\frac{\partial F^*(s)}{\partial s_i}\right|_{s=1} = \frac{v_j}{k(0)}$$

**16-TEOREMA**. $K(0) > 0$ bo'lishi uchun $M\mu_i^j \log \mu_i^j(1) < +\infty$ bajarilishi zarur va yetarlidir. Shu bilan birga (1.2.17) yagona yechimga egadir.

**17-TEOREMA**. Agar $\rho < 1, B_{k,i}^j < +\infty,\ c_n^j \sim d_j\ \rho^{-n},\ d_j > 0,\ i,k,j = 1,2$

$$\lim_{n\to\infty} P(\mu_1(n) = \alpha_1, \qquad \mu_2(n) = \alpha_2 / \mu^j(0) = (c_n^1, c_n^2)) = q_{\alpha_1,\alpha_2}$$

bo'lsa, u holda $q_{\alpha_1,\alpha_2}$ ehtimollikning hosil qiluvchi funksiyasi

$$\sum_{\alpha_1,\alpha_2}^{\infty} q_{\alpha_1,\alpha_2} S_1^{\alpha_1} S_2^{\alpha_2} = \exp(k(0)\cdot < d, u > (F^*(S_1,S_2) - 1)),$$

bu yerda $u = (u_1, u_2),\quad d = (d_1, d_2),\ k(s),\ F^*(S_1,S_2)$ lar esa $P(r^l_{\underset{x,y}{=}} + \infty, T^l = +\infty) = 0$ va

$$\Delta_2(n) \le \sup_x \left| P_n(x) - \int_0^{+\infty} \frac{1}{\sqrt{z}} \varphi_\phi\left(\frac{x}{\sqrt{z}}\right) dk_n(x) \right| +$$

$$\sup_x \left| \int_0^{+\infty} \frac{1}{\sqrt{z}} \varphi_\Phi\left(\frac{x}{\sqrt{z}}\right) dk_n(z) - \int_0^{+\infty} \frac{1}{\sqrt{z}} \varphi_\phi\left(\frac{x}{\sqrt{z}}\right) dk(z) \right| = J_3 + J_4$$

larda keltirilgan.

Isbot. Ma'lumki, tarmoqlanish jarayoni ta'rifidan

$$P(\mu_1(n) = \alpha_1,\ \mu_2(n) = \alpha_2 / \mu(0) = (c_n^1, c_n^2)) =$$

$$P(\mu_1^1(n) = \alpha_1,\ \mu_2^1(n) = \alpha_2) *^{C_n^1} (P(\mu_1^2(n) = \alpha_1, \mu_2^l(n) = \alpha_2)) *^{C_n^1} \qquad (1.2.18)$$

Bu ehtimollikning xosil qiluvchi funksiyasi

$$(F_n^1(S_1,S_2))^{C_n^1} \cdot (F_n^2(S_1,S_2))^{C_n^1} \qquad (1.2.19)$$

bo'ladi. Agar $n \to \infty$ da

$$(F_n^j(S_1,S_2) - F_n^j(0{,}0)) : (1 - F_n^j(0{,}0)) \to F^*(S_1,S_2)(1 + 0(1))$$

ni hisobga olsak, u holda

$$P(\tau_{x,y}^l = +\infty, T^l < +\infty) = P(\tau_{x,y}^l = +\infty, 1 \le T^l \le N) + P(\tau^l_{\underset{x,y}{=}} +\infty, N < T^l < +\infty) =$$

$$= P(\tau_{x,y}^l = +\infty, 1 \le T^l \le N) + \theta_7 E, \qquad 0 \le \theta_7 \le 1,$$

ni

$$F_n^j(S_1,S_2)=1+k(0)u_j\rho^n(1-F^*(S_1,S_2))(1+0(1)$$

Agar $\left|S_j\right|\leq 1$ligini e'tiborga olsak

$$\ln F_n^j(S_1,S_2)=-k(0)u_j\rho^n(1-F^*(S_1,S_2))(1+0(1))$$

hosil bo'ladi.

Bu ifodadan

$$\lim_{n\to\infty} c_n^j \ln k_n^j(S_1,S_2)=k(0)\ \ u_j d_j(1-F^*(S_1,S_2))(1+0(1)) \qquad (1.2.20)$$

Natijada (1.2.18)-(1.2.20) dan 17-teorema isboti kelib chiqadi.

### *1.3. Kritik tarmoqlanish jarayoni*

Kritik tarmoqlanish jarayoniga doir asosiy teoremalar

**18-TEOREMA**. Agar $À = 1, F''(1) < +\infty$ ,bo'lsa

$$Q_n = \frac{2}{Bn}(1 + 0(1).$$

**19-TEOREMA.** Agar $À = 1, \quad F^r(1) < +\infty$, r ≥3 bo'lsa,

$$Q_n = \frac{2}{Bn} + \sum_{J=2}^{r-1} n^{-j} \sum_{k=0}^{i} A_{jk} \ln^k n + 0\,(n^{1-r} \ln n) \qquad (1.3.1)$$

Bu yerda $À_{jk}$ $\mu_1$ ning taqsimotiga bog'liq miqdor.

Xususan, $F(1-x) = 1 - x = x^{i+\alpha} L(x), \quad 0 < \alpha < 1,$ bo'lsa

$$Q_n = n^{-\frac{1}{\alpha}} L_1(n) \qquad n \to +\infty \qquad (1.3.2)$$

Bu yerda $L(x), L_1(n)$ lar sekin o'zgaruvchili funksiyalar.

***Kritik tarmoqlanuvchi jarayon uchun limit teoremalar***

**20-TEOREMA**. Agar $À = 1, 0 < B < +\infty$ shartlar bajarilsa u holda

$$\lim_{n\to\infty} P\left(\frac{2\mu_n}{Bn} \le x / \mu_0 = 1, \mu_n > 0\right) = 1 - e^{-x} \qquad (1.3.3)$$

munosabat o'rinli bo'ladi.

**20[a]-TEOREMA**. Agar $A = 1, \quad B < +\infty$ va $n \to \infty$ da $m \to \infty, \ \sigma \to \infty, \ \sigma = 0(\sigma_n{}^2)$ bajarilsa, u holda

$P\left(\frac{z_{ny} - m}{\sqrt{mnB + \sigma^2}} < x\right) = G\left(\frac{x}{\delta_1}\right) * \Phi\left(\frac{x}{\sqrt{1-\delta_1{}^2}}\right) + 0(1)$ bajariladi, bu yerda $\delta_1 = \frac{\sigma}{\sigma_m}$,

***Ko'p xilli kritik tarmoqlanuvchi jarayon uchun limit teoremalar***

**21-TOEREMA**. Agar $\rho = 1, \ B_{i,l}^j < +\infty, \quad i, j, l = \overline{1,k}$ shartlari bajarilsa, $n \to \infty$ da $0 \le s \le 1$ da tekis

$$R_n^j(s) = \frac{u^j \sum_{l=1}^{k} V_l(1-s_l)}{1+\frac{Bn}{2}\sum_{m=1}^{k} v_m(1-s_m)} + 0(\frac{1}{n})$$

(1.3.4)

bajariladi, bu yerda

$$B = \sum_{i,m,l=1}^{k} v_i B_{lm}^i u_l u_m > 0.$$

Xususan

$$\theta_n^j \sim \frac{2u_j}{Bn}.$$

Agar jarayon $\mu(0) = \alpha$ dan boshlansa $n \to \infty$ da

$$P(\mu(n) \neq 0 / \mu(0) = \alpha) = \frac{2\sum_{i=1}^{k} \alpha_i u_i}{Bn}(1+0(1)). \qquad (1.3.5)$$

**22-TOEREMA**. Faraz qilaylik $\rho = 1,\ B_{i,l}^j < +\infty$ bajarilsin, u holda $\xi^l(n) = (\xi^l(n),...,\xi^l{}_k(n))$ vektor $\xi^l(n) \neq 0$ sharti ositida $n \to \infty$ da taqsimot bo'yicha $\xi = (\xi_1, \xi_2,...,\xi_k)$ tasodifiy vektorga intiladi va 1 ehtimollik bilan $\xi_1 = \xi_2 = ... = \xi_{k,}$

$$P(\xi_i \leq x) = 1 - e^{-x}, \quad x \geq 0,\ i = \overline{1,k} \qquad (1.3.6)$$

ya'ni $l$ boshlang'ich holatga bog'liq emas, bu yerda

$$\xi_j^{lj}(n) = \frac{1\mu_j^l(n)v_j^{-1}}{2Bn}$$

(1.3.7)

**23-TEOREMA.** Agar $\rho = 1, 0 < B_{i,k}^j < +\infty,\ C_n^j \sim d_j n,\ (d_j > 0),$

bo'lsa

$$\lim_{n\to\infty} P\left(\frac{2\mu_i(n)}{nB} < x, \quad \frac{2\mu_2(n)}{nB} < y / \mu(0) = (c_n^1, c_1^2)\right) = G(x,y)$$

bu yerda $G(x,y)$ Laplas almashtirishi

$$\int_0^{+\infty}\int_0^{+\infty} e^{-\lambda, x-\lambda_2 y} dG(x,y) = e - \frac{2(<u,d>)(<\theta\lambda>)}{B(1+<v\lambda>)},$$

ga teng; $\lambda = (\lambda_1 \lambda_2)$

$$B=\sum_{k,l,m=1}^{\lambda} v_k B_{lm}^k u_m u_l$$

Isbot: (1.2.18) ga ko'ra

$$M\left(\exp\left(-\frac{2\lambda_1\mu_1(n)}{nB}-\frac{2\lambda_2\mu_2(n)}{nB}\right)\Big/\mu(0)=(C_n^1,C_n^2)\right)=$$

$$=\left(F'_n\left(e^{-\frac{2\lambda_1}{nB}},e^{-\frac{2\lambda}{nB}}\right)\right)^{C_n^1}\left(F_n^2\left(e^{-\frac{2\lambda_1}{nB}},e^{-\frac{2\lambda}{nB}}\right)\right)^{C_n^2}$$

(1.3.8)

U holda

$$\left(M\left(\exp\left(\frac{i(\xi^l-u_l)<v_1\tau>}{\sqrt{<k_1,D\xi>}}\right)\right)\right)^{k_l}=\exp\left(-\frac{(<\tau,v>)^2k_lD\xi^l}{2<k,D\xi>}\right)+0(1)$$

dan

$$F_n^j\left(e^{-\frac{2\lambda_1}{nB}},e^{-\frac{2\lambda}{nB}}\right)=1-\frac{2u_j(<v\lambda>)}{nB(1+<v\lambda>)}(1+0(1))\ .$$

ifodani hosil qilamiz.

Bu tenglikdan

$$\lim_{n\to+\infty} C_n^j\ln F_n^j\left(e^{-\frac{2\lambda_1}{nB}},e^{-\frac{2\lambda}{nB}}\right)=\frac{-2u_jd_j(<v\lambda>)}{B(1+<v\lambda>)}$$

(1.3.9)

Natijada(1.3.8),

$$F_n^j(S_1,S_2)=1+\sum_{k=1}^{2}A_i^j(n)(S_k-1)+\frac{1}{2}\sum_{k,l=1}^{\lambda}B_{k,l}^j(n)(S_l-1)(S_k-1)+0(B_{k,l}^j|S_k-1|\cdot|S_l-1|)$$

(1.3.10)

ga ega bo'lamiz, bu yerda

$$M\left(\mu_i^j(n)\left(\mu_i^j(n)-1\right)\right)=B_i^j(n)$$

lardan teorema isboti kelib chiqadi.

**23ᵃ-TEOREMA**. Agar $\rho=1, B_{i,k}^j<\infty, \lim_{n\to\infty} C_n^j=+\infty,\ j,k,l=1,2,$ $C_n^1\sim C_n^2, b_n\sim\rho^n\sqrt{<C_nA>}$ bo'lsa

$$\lim_{n\to\infty} P\left\{\frac{\mu_1(n)-\rho^n v_1(<uc_n>)}{bn}<x_1 \ \frac{\mu_2(n)-\rho^n v_2(<uc_n>)}{bn}<x_2 \Big/ \mu(0)=(C_n^1,C_n^2)\right\}=\theta(x_1,x_2)$$

va

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{u<tx>}d\theta(x_1,x_2)=e^{-\frac{(<t_1v>)^2}{2}},$$

bajariladi, bu yerda $A=(A^1,A^2),\quad A^j=D\xi^j$

$$C_n=(C_n^1,C_n^2),\ x=(x_1,x_2),\qquad t=(t_1,t_2)$$

ISBOT: (1.2.19) ga ko'ra

$$\varphi_n(t_1,t_2)=M\Big(\exp\Big(\big(i\mu_1(n)-\rho^n v_1<uc_n>\big)t_1+(\mu_2/n)-\rho^n v_2<uc_n>\big)t_2:b\Big)/\mu(0)=$$

$$=(C_n^1,C_n^2))=\exp\left(-\frac{\rho^n(v_1t_1(<uc_n>)+v_2t_2(<uc_n>))}{bn}\right)\times$$

$$\times\left(F_n^1\left(e^{\frac{it_1}{bn}},e^{\frac{it_2}{bn}}\right)\right)^{C_n^1}\cdot\left(F_n^2\left(e^{\frac{it_1}{bn}},e^{\frac{it_2}{bn}}\right)\right)^{C_n^2} \tag{1.3.11}$$

Ifodani hosil qilamiz.

$F_n^j(S_1,S_2)$ ni (1,1) nuqta atrofida Teylor qatoriga yoyib

(1.3.10) dan isbotlash mumkinki $\rho>1$da

$$B_i^j(n)=\frac{\sum_{\theta_i m=1}^{2} d_y^j B_{im}u_iu_mv_kv_e}{\begin{vmatrix}\rho^2-a_1^1 & a_2^1\\ a_1^2 & \rho^2-a_2^2\end{vmatrix}}\rho^{2n}(1+0(1)) \tag{1.3.12}$$

Bu formuladagi $d_y^j,\ j,y=1{,}2$ lar Agar $B_{i,l}^j<+\infty,\qquad i,j,l=\overline{1,k}$ bo'lsa $\xi^j$ ni dispersiyasi

$$D\xi^j=\frac{\sum_{\gamma,i,l=1}^{k} d_\gamma^j u_iu_l}{\begin{vmatrix}\rho^2-a_1^1 & a_2^1\ldots & a_k^1\\ a_1^2 & \rho^2-a_2^2\ldots & a_k^2\\ a_1^k & a_2^k\ldots & \rho^2-a_k^k\end{vmatrix}}$$

bo'ladi, bu yerda $d_\gamma^j$ maxrajdagi diterminantdagi $\rho^2\delta_\gamma^j-a_\gamma^j$ elementning algebraik to'ldiruvchisi.

(1.1.9) va (1.3.12) ko'ra, $S_i=e^{\frac{iti}{bn}},\qquad l=1{,}2$ da

$$F_n^j\left(e^{\frac{it_1}{bn}},e^{\frac{it_2}{bn}}\right)=1+i\,\frac{\rho^n u_j}{bn}(<vt>)-\frac{\rho^{2n}}{2b_n^2}\sum_{i,k=1}^{2}(A^j-u_j^2)\times v_k v_i t_i t_k+0\left(\frac{\rho^{2n}}{bn^2}\right)$$

Natijada

$$\ln F_n^j\left(e^{\frac{it_1}{bn}},e^{\frac{it_2}{bn}}\right)=i\rho^{n\frac{u_j}{bn}}(<vt>)-\frac{\rho^{2n}A^j}{2bn^2}(<vt>)^2+0\left(\frac{\rho^2 n}{bn^2}\right).$$

U holda (1.3.11) va oxirgi tenglikdan $|t_j|<T$ tengsizlikni qanoatlantiruvchi ixtiyoriy $T>0$ uchun

$$\ln\varphi_n(t_1,t_2)=-\frac{1}{2}\,\frac{\rho^{2n}}{bn^2}(<C_nA>)(<vt>)^2+0\left(\frac{\rho^{2n}(C_n^1+C_n^2)}{bn^2}\right)$$

ifodaga ega bo'lamiz.

Oxirgi tenglikdan 23[a]-teoremaning isboti kelib chiqadi.

**24-TEOREMA**. Agar $\text{À}=1,\ \text{ñ}<+\infty$ bo'lsa (1.3.3) ning yaqinlashish tezligi $0\left(\frac{\ln^2 n}{n}\right)$ kabi bo'ladi.

### *1.4.Kritikdan keyingi tarmoqlanish jarayoni*

Kritikdan keyingi tarmoqlanish jarayoniga doir asosiy teoremalar

**25-TEOREMA** Agar $A > 1$ da $q > 0$ bo'lsa,

$$F_n(0) = q - d[F(q)]^n + 0([F(q)]^{2n}), \qquad (1.4.1)$$

bu yerda d- musbat o'zgarmas son.

**26- TEOREMA.** Agar $A > 1,\ B < \infty$ bo'lsa u holda $W_n = \frac{\mu_n}{A^n}$ tasodifiy miqdor o'rta kvadratik va bir ehtimollik bilan $n \to \infty$ da **W** tasodifiy miqdorga intiladi, shu bilan birga

$$MW = 1, \qquad DW = \frac{D\mu_1}{A^2 - A} \qquad (1.4.2)$$

bo'ladi. Bundan tashqari $P(W > 0) > 0$ bajarilishi uchun $M(W \ln W) < \infty$ bo'lishi zarur va yetarlidir.

**27-TEOREMA** f(s) xarakteristik funksiya

$$f(AS) = F(0, f(s)) \qquad (1.4.3)$$

tenglamani qanoatlantiradi.

**28-TEOREMA** . $D\mu_1 > 0$ shart bajarilsa, u holda

$$P(W \le x / \ w > 0) = \frac{k(x) - k(0)}{1 - k(0)} = \frac{k(x) - q}{1 - q} \qquad (1.4.4)$$

taqsimot funksiya absalyut uzluksiz va $D(W/W > 0) > 0$, bu yerda

$$K(z) = P(W \le x).$$

**29-TEOREMA** .Agar $B < +\infty$ bajarilsa, u holda bir ehtimollik bilan quyidagi tenglik o'rinli bo'ladi:

$$\lim_{n\to\infty} \frac{\mu_0 + \mu_1 +,,+ \mu_n}{A^n} = \frac{Aw}{A-1}. \qquad (1.4.5)$$

Teorema : Agar $A = 1$ va $f_1^{11}(1) = b < +\infty$ bo'lsa $1 - P_{n_0} = \frac{2}{bn}(1 + o(1))$

Isbot: $q_n(x) = 1 - f_n(x),$ $f(x)$ ni $x = 1$ nuqta atrofida Teylor qatoriga yozamiz.

$$f(x) = 1 + (x-1) + \frac{b}{2}(x-1)^2 + o(x-1)^2$$

$$1 - f(x) = (1-x) - \frac{b}{2}(1-x)^2 + o(x-1)^2$$

$$\frac{1-f(x)}{1-x}=1-\frac{b}{2}(1-x)+o(1-x)\ , \qquad x=f_k(x)$$

$$\frac{1-f_{k+1}(x)}{1-f_k(x)}=1-\frac{b}{2}(1-f_k(x))+o(1-f_k(x))$$

$$\frac{q_{k+1}(x)}{q_k(x)}=1-\frac{b}{2}q_k(x)+o(q(x)),$$
$$0\le q_k(x)\le q_k(0)\to 0$$

$$\frac{1}{q_{k+1}(x)}-\frac{1}{q_k(x)}=\frac{b}{2}\frac{q_k(x)}{q_{k+1}(x)}+o(1)$$

Xususan $\frac{1}{q_{k+1}(0)}-\frac{1}{q_k(0)}=\frac{b}{2}-\frac{q_k(0)}{q_{k+1}(0)}+o(1),$

$q_0(0)=1;$ $\qquad f_0(x)=x$ $\qquad$ dan

$$\frac{1}{q_n(0)}-\frac{1}{q_0(0)}=n\frac{b}{2} \qquad \frac{1}{n}\sum_{k=o}^{n-1}\left[\frac{q_k(0)}{q_{k+1}(0)}+\alpha_k\right], \qquad \alpha_k\to 0$$

$\sum_{k=0}^{n-1}(\frac{q_k(0)}{q_{k+1}(0)}+\alpha_k)$ ni hisoblash uchun Chezar o'rtachasini keltiramiz.

$\frac{1}{n}\sum_{1}^{n}(\alpha_n-\alpha)\approx\sum_{1}^{n}\frac{\alpha_n}{n}$ yaqinlashuvchi
($\alpha_n\to\alpha$ da)

Demak $\frac{1}{q_n(0)}-1=\frac{nb}{2}\frac{1}{n}\sum_{0}^{n-1}\left[\frac{q_k(0)}{q_{k+1}(0)}+\alpha_k\right],$ $\quad \frac{1}{n}\sum_{0}^{n-i}\frac{q_k(0)}{q_{k+1}(0)}\sim 1\Rightarrow q_n(0)=\frac{2}{nb}(1+o(1))$

Agar $\tau=$ bilan jarayoni yashash davrini belgilasak $P(\tau=k)=P_{k+1,0}-P_{k,0}$,

Unda
$$M\tau=\sum_{k+1}^{\infty}k\rho(\tau=k-1)=\sum_{1}^{\infty}k(\frac{2}{kb}-\frac{2}{(k-1)b})\approx\sum_{k=1}^{\infty}\frac{2}{b_k}\sim+\infty$$

Kritik tarmoqlanish jarayoni uchun limit teorema

Terema : Agar $A>1$ $\ b<+\infty$ bo'lsa $P\{\mu_n q_n<x/\mu_n>0\}=\begin{cases}1-e^{-x} & x>0\\ 0, & x<0\end{cases}$

Isbot: $\frac{1}{n}\sum_{0}^{n}\frac{q_k(x)}{q_{k-1}(x)}=\frac{1}{n}(\frac{1}{q_n(x)}-\frac{1}{q_0(x)})=\frac{1}{n}(\frac{1}{q_n(x)}-\frac{1}{1-x})=\frac{b}{2}(1+o(1))$

$x=e^{itq_n}$ da

$$\frac{\frac{2}{bn}}{q_n(e^{itq_n})}-\frac{\frac{2}{bn}}{1-e^{itq_n}}=1+o(1)$$

$$\frac{2}{b_n}\sim q_n(0)=1-P_{n,0} \qquad \alpha\frac{e^{\alpha x}-1}{\alpha x}\to\alpha$$

$$e^{\alpha x}=1+\alpha x+o(\alpha)$$

$\mu_n q_n < x/\mu_n > 0$ ni hosil qiluvchi funksiyasi

$$F_n(\alpha)=\frac{f_n(x)-f_n(0)}{1-f_n(0)}=1-\frac{1-f_n(x)}{1-f_n(0)};$$

$$\frac{1-f_n(e^{itq_n})}{1-\rho_{n,0}}\to\frac{1}{1-\frac{1}{it}}$$

$$R_n(x)\sim 1-\frac{1}{1-\frac{1}{it}}=1-\frac{it}{it-1}=\frac{it-1-it}{it-1}=\frac{1}{1-it}$$

Taxlil $\quad \rho(\varepsilon<\mu_n q_n<\frac{1}{\varepsilon}/\mu_n>0)=e^{-\varepsilon}-e^{-\frac{1}{\varepsilon}}\to 1$

Demak $\quad \varepsilon\frac{nb}{2}<\mu_n<\frac{nb}{2e}$

3- terema: $f_1^{III}(1)<\infty$ bo'lsa

$$\rho(\frac{2\mu_n}{b_n}\le x/\mu_0=1, \quad \mu_n>x)=1-e^{-x}+0(\frac{\ln^2 n}{n});$$

Uzluksiz jarayon uchun ham uchala teoremani isbotlash mumkin.

### *Kritikdan keyingi Gal'ton-Vatson jarayoni uchun integral teoremalarda yaqinlashish tezligi*

Quyidagi belgilashni kiritamiz:

$$\sigma^2=M(w-1)^2=\frac{B-A^2+A}{A^2-A}, \quad \beta_3=M(w-1)^3, \quad \eta_n=\frac{A^{\frac{n}{2}}(W-W_n)}{\sigma},$$

$$\varphi_1(t) = Me^{-\frac{t^2}{2}w}, \qquad \psi(t) = Me^{it(w-1)},$$

$$\varphi_{1,n} = Me^{ith_n}, \qquad G(x) = \int_0^{+\infty} \Phi\left(\frac{x}{\sqrt{y}}\right) dp(w-y).$$

Ma'lumki

$$\lim_{n\to\infty} P(\eta_n < x) = G(x), \quad \varphi_1(t) = \int_{-\infty}^{+\infty} e^{itx} dG(x)$$

Shu bilan birga

$$A^{\frac{n}{2}}(W - W_n) = A^{-\frac{n}{2}} \sum_{k=1}^{\mu_n} \left(W^{(k)} - 1\right),$$

bu yerda $\{w^{(k)}\}$ o'zaro bog'liq bo'lmagan, W bilan bir hil taqsimlangan tasodifiy miqdorlardir.

**30-TEOREMA.** Agar $A > 1,\ D < +\infty$ bajarilsa, u holda shunday $c_1$ va $c_2$ absolyut o'zgarmas sonlar mavjudki

$$\sup_X \left|P(\eta_n < x) - G(x)\right| \le c_1 \frac{\beta_3}{\sigma^3 \sqrt{A^n}} + c_2 \frac{\max(1,\sigma)}{\sqrt[6]{A^n}}$$

tengsizlik bajariladi.

Teoremani isbotlash uchun quyidagi lemmani keltiramiz:

LEMMA. Agar $A > 1,\ D < \infty$ bajarilsa

$$|t| \le \frac{\sigma^3}{5\beta_3} \quad \sqrt{A^n}$$

uchun

$$\left|\varphi_{1,n}(t) - \varphi_1(t)\right| \le \frac{7\beta_3 |t|^3}{6\sigma^3 \sqrt{A^n}} M\left(W_n e^{-\frac{t^2}{2}W_n}\right) + \frac{t^2}{2} M|W_n - W| e^{-\min\left((W_n, W)\frac{t^2}{2}\right)}$$

munosabat o'rinli.

Lemmaning isboti. Lemma shartiga ko'ra

$$\left|\psi\left(\frac{t}{\sigma\sqrt{A^n}}\right)\right| \ge 1 - \frac{t^2}{2A^n} - \frac{|t|^3 \beta_3^{\,3}}{6\sigma^3 A^{3n:2}} \ge \frac{24}{25}$$

(1.4.6)

Demak, $\psi(\cdot)$ ni logarifmlash mumkin:

$$\ln\psi\left(\frac{t}{o\sqrt{A^n}}\right)=-\frac{t^2}{2A^n}+\frac{t^3\tilde{\beta}_3}{6\sigma^3 A^{3n:2}},$$

(1.4.7)

bu yerda

$$\tilde{\beta}_3=\left|\frac{d^3}{dz^3}\ln\psi(z)\right|_{z=\frac{\theta t}{o\sqrt{A^n}}},\quad |\theta|\le 1.$$

Natijada,(1.4.6) dan foydalanib

$$\left|\tilde{\beta}_3\right|=\left|\frac{d^3}{dz^3}\ln\psi(z)\right| \qquad (1.4.8)$$

$$z=\frac{\theta t}{\sigma\sqrt{A^n}}\le 7\beta_3$$

ni hosil qilamiz.

Agar

$$\varphi_{1,n}(t)=\int_0^{+\infty}\left(\psi\left(\frac{t}{o\sqrt{A^n}}\right)\right)^{A^n y}dp(W_n<y)$$

tenglikni hisobga olsak

$$\left|\varphi_{1,n}(t)-\varphi_1(t)\right|\le\left|\int_0^{+\infty}\left(\psi\left(\frac{t}{\sigma\sqrt{A^n}}\right)\right)^{A^n y}dp(W_n<y)-\int_0^{+\infty}e^{-\frac{t^2}{2}y}dp(W_n<y)\right|+\left|\int_0^{+\infty}e^{-\frac{t^2}{2}y}dp(W_n<y)-\right.$$

$$\left.-\int_0^{+\infty}e^{-\frac{t^2}{2}y}dp(W<y)\right|=J_1+J_2.$$

(1.4.9)

Ushbu

$$\left|e^x-1\right|\le|x|e^{|x|}$$

tengsizlik va (1.4.7), (1.4.8) larni hisobga olgan holda

$$J_1\le\int_0^{+\infty}e^{-\frac{t^2}{2}y}\left|e^{\frac{\overline{\beta}_3 t^3}{6\sigma^3A^{n:2}}y}-1\right|dp(w_n<y)\le\frac{7\beta_3|t|^3}{6\sigma^3\sqrt{A^n}}\int_0^{+\infty}ye^{-\frac{t^2}{2}y}\,e^{\frac{7\beta_3|t|^3}{6\sigma\sqrt{A^n}}y}\,dp(W_n<y)$$

ifodaga ega bo’lamiz.

Agar $t$ ni lemma shartini qanoatlantiradigan holda tanlasak

$$\frac{t^2}{2}-\frac{7\beta_3|t|^3}{6\sigma\sqrt{A^n}}\geq\frac{t^2}{4}$$

tengsizlik hosil bo'ladi.

U holda

$$J_1\leq\frac{7\beta_3|t|^3}{6\sigma\sqrt{A^n}}\int_0^{+\infty}ye^{-\frac{t^2}{4}y}dp(W_n<y)=\frac{7\beta_3|t|^3}{6\sigma\sqrt{A^n}}M(W_ne^{-\frac{t^2}{4}W_n}). \qquad (1.4.10)$$

Ushbu

$$\left|e^{-\alpha}-e^{-\beta}\right|\leq|\alpha-\beta|e^{-\min(\alpha,\beta)}$$

tengsizlikga ko'ra

$$J_2=\left|Me^{-\frac{t^2}{2}W_n}-Me^{-\frac{t^2}{2}W}\right|\leq\int_0^{+\infty}\int_0^{+\infty}\left|e^{-\frac{t^2}{2}y}-e^{-\frac{t^2}{2}z}\right|\cdot dp(W_n<y,W<z)\leq$$

$$\leq\frac{t^2}{2}\int_0^{+\infty}\int_0^{+\infty}|y-z|e^{-\min(y,z)\frac{t^2}{2}z}dp(\;W_n<yw<z)=\frac{t^2}{2}M(|W_n-W|e^{-\min(W_n,W)\frac{t^2}{2}}). \quad (1.4.11)$$

(1.4.10)-(1.4.11) lardan lemmaning isboti kelib chiqadi.

30-teoremaning isboti. Quyidagi belgilashni kiritamiz:

$$T=\sqrt[6]{A^n},\qquad\qquad E_n=\int_{-T}^{T}\frac{\varphi_{1,n}(t)-\varphi_1(t)}{|t|}dt,$$

Lemmaga ko'ra

$$E_n\leq\frac{7\beta_3}{6\sigma^3\sqrt{A^n}}\int_{-T}^{T}t^2M(W_ne^{-\frac{t^2}{2}W_n})dt+$$

$$+\frac{1}{2}\int_{-T}^{T}|t|M\left(|W_n-W|e^{-\min(W_n,W)\frac{t^2}{2}}\right)dt=J_1+J_2. \qquad (1.4.12)$$

Ishonch hosil qilish mumkinki

$$J_1=\frac{7\beta_3}{3\sigma^3\sqrt{A^n}}\int_0^T t^2\int_0^{+\infty}ye^{-\frac{t^2}{2}y}dp(W_n<y)\,dt=$$

$$=\frac{7\beta_3}{3\sigma^3\sqrt{A^n}}\int_0^T\int_0^{+\infty}t^2ye^{-\frac{t^2}{2}y}dp(W_n<y)dt\leq\frac{c\beta_3}{\sigma^3\sqrt{A^n}}T=c\frac{\beta_3}{\sigma^3\sqrt[3]{A^n}}, \qquad (1.4.13)$$

Koshi-Bunyakovskiy tengsizligini

$$M\left(\left|W_n - W\right| e^{-\min(W_n, W)\frac{t^2}{2}}\right) \le M\left(\left|W_n - W\right|\right)$$

ga qo'llab

$$J_2 \le \sqrt{M(W_n - W)^2}\, T^2$$

tengsizlikka ega bo'lamiz.

Agar ([84]) $M(W_n - W)^2 = \dfrac{\sigma^2}{A^n}$ ni hisobga olsak

$$J_2 \le \frac{\sigma}{\sqrt[6]{A^n}}, \qquad (1.4.14)$$

Natijada, (1.4.12)-(1.4.13) larni e'tiborga olgan holda Essen teoremasidan teoremaning isboti kelib chiqadi.

Agar $A > 1, D < +\infty$ bajarilsa

$$\sup_x \left| P\left(\frac{A^n (W - W_n)}{\sigma\sqrt{\mu_n}} < x / \mu_n > 0\right) - \phi(x)\right| \le c_3 \frac{\beta_3 \sqrt{M(\mu_n^{-1} / \mu_n > 0)}}{\sigma^3}$$

tengsizlik o'rinlidir. Bu yerda $C_3$-absolyut o'zgarmas son. Quyidagi teorema bu natijani ma'lum darajada yaxshilaydi:

**31-TEOREMA.** Agar $A > 1, D < +\infty$ bo'lsa

$$\left| P\left(\frac{A^n (W - W_n)}{\sigma\sqrt{\mu_n}} < x / \mu_n > 0\right) - \Phi(x)\right| \le c_4 \frac{\beta_3 \sqrt{M(\mu_n^{-1} / \mu_n > 0)}}{\sigma^3 (1 + |x|^3)}$$

bo'ladi. bu yerda $C_4$-absolyut o'zgarmas son.

Isbot. Quyidagi munosabatlarni to'g'ri bo'lishini ko'rish qiyin emas

$$P\left(\frac{A^n (W - W_n)}{\sigma\sqrt{\mu_n}} < x / \mu_n > 0\right) = \frac{1}{P(\mu_n > 0)} \sum_{k=1}^{\infty} P(\mu_n = k)$$

$$P\left(\lim_{r\to\infty} \frac{(\mu_n^{(1)} + \mu_r^{(2)} + \ldots + \mu_r^{(\mu_n)}) : A^r - \mu_n}{\sigma\sqrt{\mu_n}} < x / \mu_n = k\right) =$$

$$= \frac{1}{P(\mu_n > 0)} \sum_{k=1}^{\infty} P(\mu_n = k)\; P\left(\frac{W^{(1)} + W^{(2)} + \ldots W^{(k)} - k}{\sigma\sqrt{k}} < x\right).$$

Bu tengsizlikdan foydalanib

$$\left| P\left( \frac{A^n (W - W_n)}{\sigma\sqrt{\mu_n}} < x / \mu_n > 0 \right) - \Phi(x) \right| =$$

$$\left| \sum_{k=1}^{\infty} \frac{P(\mu_n = k)}{P(\mu_n > 0)} \left( P\left( \frac{W^{(1)} - 1) + ..... + (W)}{} \right) - \phi(x) \right) \right| \qquad (1.4.15)$$

ni hosil qilamiz. Markaziy limit teoremaning notekis yaqinlashishi haqidagi teoremaga ko'ra

$$\left| P\left( \frac{(W^{(1)} - 1) + ,,, + (W^{(k)} - 1)}{\sigma\sqrt{k}} < x \right) - \Phi(x) \right| \leq \frac{C_4 \beta_3}{\sigma^3 \sqrt{k} (1 + |2|^3)} \qquad (1.4.16)$$

Ikkinchi tomondan

$$\sum_{k=1}^{\infty} \frac{P(\mu_n = k)}{P(\mu_n > 0)\sqrt{k}} = M\left( \mu_n^{-\frac{1}{2}} / \mu_n > 0 \right) \leq \sqrt{M(\mu_n^{-1} / \mu_n > 0)}$$

Oxirgi tengsizlikni hisobga olib (1.4.15), (1.4.16) dan 31-teoremaning isbotiga ega bo'lamiz.

**32-TEOREMA.** Agar $A > 1, D < +\infty$ bo'lsa

$$\sup_x |S_n(x) - \Phi(x)| \leq \frac{c}{\sqrt{c_n}} \cdot \frac{\overline{B}_n}{\overline{\sigma}_n^2}$$

tengsizlik o'rinli bo'ladi.

Bu yerda

$$\overline{\sigma}_n^2 = Mr^2 = M\left( \frac{\mu_n - A^n}{b_n^2} \right)^2, \quad \overline{\beta}_n^3 = M|r_n|^3, \ \overline{\varphi}_n(\tau) = Me^{iTr_n},$$

$$\overline{b}_n = \sqrt{c_n} \left[ A^n (A^n - 1)(B - A^n + A)(A^2 - A)^{-1} \right]^{\frac{1}{2}}.$$

Eslatma.

$$\frac{\overline{\beta}_n}{\overline{\sigma}_n^2} \leq \frac{(6AB^2 + 10A^3 B + 2A^2 D + 4A^5) A^2}{(A^3 - A)(A - 1)\sqrt{(B - A^2 + A)^3}}.$$

Tengsizlikning bajarilishiga ishonch hosil qilish qiyin emas.

32-teoremani isbotlash uchun quyidagi lemma kerak bo'ladi.

LEMMA. Agar $A > 1, D < +\infty$ bo'lsa, u holda

$$|\delta| \leq \frac{\sqrt{C_n}}{5} \cdot \frac{\overline{\sigma}_n^3}{\overline{\beta}_n^3}$$

uchun

$$\left|\overline{\varphi}_n^{\,C_n}(\tau)-e^{-\frac{\tau^2}{2}}\right|\le\frac{7}{6}\cdot\frac{\overline{\beta}_n^{\,3}|\tau|^3}{\overline{\sigma}_n^{\,3}\sqrt{C_n}}e^{-\frac{\tau^2}{4}}.$$

Isbot. Bevosita tekshirib ko'rish mumkinki

$$M\left(\mu_n-A^n\right)^2=A^n\left(A^n-1\right)\left(B-A^2+A\right)\left(A^2-A\right)^{-1},\qquad(1.4.17)$$

$$M\left(\mu_n-A^n\right)^3=\frac{\alpha A^{3n}-\beta A^{2n}+\gamma A^n}{(A^3-A)(A^2-A)},\qquad(1.4.18)$$

bu yerda

$$\alpha=3B(B-A^2+A)+(A^2-A)(D+2A^3-2A)$$

$$\beta=3B(AB-2A^3+2A+B)+3(A^3-A)(A^2-A)$$

$$\gamma=(A^3-A-D)(A^2-A)+3B(Ab-A^3+A)$$

(1.4.17) ga ko'ra

$$\overline{\sigma}_n^{\,2}=\frac{1}{c_n}.\qquad(1.4.19)$$

Oxirgi tenglikni hisobga olsak

$$\left|\overline{\varphi}(\tau)\right|\ge1-\frac{r^2}{2}\overline{\sigma_n}^2-\frac{|\tau|^3}{6}\overline{\beta_n}^3=1-\frac{\overline{\sigma}_n^{\,2}\tau^2}{2}\left[1+\frac{|\tau|\overline{\beta_n}^3}{3\sqrt{C_n}\,\overline{\sigma_n}^3}\right]\ge$$

$$\ge1-\frac{\overline{\sigma}_n^{\,2}\tau^2}{2}\ge1-\frac{\overline{\sigma}_n^{\,2}\tau^2}{2}\left[1+\frac{1}{3\cdot5}\right]=1-\frac{8\tau^2}{15c_n}.$$

Ikkinchi tomondan

$$\tau^2\le\frac{c_n}{25}\cdot\frac{\left(\overline{\sigma}_n^{\,3}\right)^2}{(\overline{\beta}_n^{\,3})^2}\le\frac{c_n}{25}.$$

Demak

$$\left|\overline{\varphi}_n(\tau)\right|\ge\frac{24}{25}.$$

(1.4.19) ni hisobga olib

$$C_n\ln\overline{\varphi}_n(\tau)=-\frac{r^2}{2}+\frac{C_n}{6}\tau^3\left[\frac{d^3}{dz^3}\ln\overline{\varphi}_n(z)\right]_{z=z\theta}\quad|\theta|\le1\qquad(1.4.20)$$

ifodani hosil qilamiz.

Lyapunov tengsizlikni ko’llab

$$\left|\frac{d^3 \ln \overline{\varphi}_n(z)}{dz^3}\right|_{z=\theta\tau} \le \frac{\overline{\beta_n}^3 + 3\overline{\sigma_n}^2 + 2(M|r_n|^3)}{|\overline{\varphi}_n(\tau)|^3} \le \frac{6\beta_n^3}{\left(\frac{24}{25}\right)^3} \le 7\overline{\beta}_n^3. \qquad (1.4.21)$$

Agar $|e^x - 1| \le |x| e^{|x|}$ tengsizlikni hisobga olsak (1.4.20), (1.4.21) ga ko’ra

$$\left|\overline{\varphi}_n^{cn}(\tau) - e^{-\frac{\tau^2}{2}}\right| \le \overline{\beta}_n^3 \exp\left(-\frac{\tau^2}{2} + \frac{7c_n|\tau|^3 \overline{\beta}_n^3}{6}\right) \qquad (1.4.22)$$

hosil qilamiz.

Lemmaning isboti (1.4.21),(1.4.22) lardan kelib chiqadi.

32-teoremaning isboti. Ishonch hosil qilish mumkinki,

$$M\left(e^{\frac{\mu_n - A^n c_n}{b_n} i\tau} \Big/ z_0 = c_n\right) = \overline{\varphi}_n^{c_n}(\tau).$$

Natijada, oxirgi tenglamaning hisobga olib, Essen teoremasini lemmaga qo’llab 32-teoremaning to’g’riligiga ishonch hosil qilamiz.

**33-TEOREMA.** Agar $\sup_x k'(x) \le k < +\infty \;\; A >!, D\mu_1 < \infty$ bo’lsa

$$\sup_x |k_n(x) - k(x)| \le c\left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}},$$

bu yerda $k_n(x) = P(W_n \le x),$ $c$-absolyut o’zgarmas son.

(1.4.3) ga ko’ra

$$\varphi(at) = F(\varphi(t))$$

yoki

$$\varphi(t) = F_n\left(\varphi\left(\frac{t}{A^n}\right)\right).$$

Xuddi shunday

$$\varphi_n(t) = F_n\left(e^{\frac{it}{A^n}}\right).$$

O’rta qiymat haqidagi teoremaga asosan, oxirgi ikkita tenglikka ko’ra

$$|\varphi_n(t) - \varphi(t)| = \left|F_n\left(e^{\frac{it}{A^n}}\right) - F_n\left(\varphi\left(\frac{t}{A^n}\right)\right)\right| = \left|F'_n\left(e^{i\theta_1 t}\right)\right| \left|e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right| \le$$

$$\le \left|F'_n\left(e^{i\theta_1 t}\right)\right| \cdot \left|e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right| \le A^n \left|e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right|, \quad |\theta_1| \le 1. \qquad (1.4.23)$$

Ma'lumki

$$e^{\frac{it}{A^n}} = 1 + \frac{it}{A^n} + \theta_2 \frac{t^2}{2A^{2n}}, |\theta_2| \le 1,$$

$$\varphi\left(\frac{t}{A^n}\right) = 1 + \frac{it}{A^n} + \theta_3 \frac{D\mu_1 + A^2 - A}{A^2 - A} \cdot \frac{t^2}{2A^{2n}}, \quad |\theta_3| \le 1.$$

U holda bu tengliklardan (1.4.23) ni quyidagicha yozamiz:

$$|\varphi_n(t) - \varphi(t)| \le \frac{D\mu_1 + A^2 - A}{A^2 - A} \cdot \frac{t^2}{A^n}. \qquad (1.4.24)$$

U holda (1.4.24) ga ko'ra Berri-Esseen teoremasini qo'llab ushbu tengsizlikni hosil qilamiz:

$$\sup_x |k_n(x) - k(x)| \le c \int_{|t| < \sqrt[3]{A^n}} |\varphi_n(t) - \varphi(t)| \cdot \frac{dt}{|t|} +$$

$$+ c \frac{\sup_x k'(x)}{\sqrt[3]{A^n}} \le c \left( \frac{D\mu_1 + A^2 - A}{A^2 - A} A^{-n} \int_{|t| < \sqrt[3]{A^n}} |t| dt + \frac{k}{\sqrt[3]{A^n}} \right).$$

Oxirgi ifodadan 33-teorema isboti kelib chiqadi.

**34-TEOREMA.** Aytaylik

$h = 0, \quad M|\xi_1|^3 = \beta_3 < +\infty, \; A > 1, \; D\mu_1 < +\infty, \; k < +\infty.$ u holda $n \ge 1$ uchun

$$\Delta_1(x) = \sup_x |T_n(x) - G(x)| \le c \left[ \frac{\beta_3}{H^3} \left(M\mu_1^{-1}\right)^{\frac{n}{2}} + \left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}} \right],$$

bu yerda $T_n(x) = P\left(\frac{S\mu_n}{H\sqrt{A^n}} < x\right), \; G(x) = \int_0^{+\infty} \phi\left(\frac{x}{\sqrt{z}}\right) dk(z)$,

Teoremani isbotlash uchun quyidagi lemmaning isbotsiz keltiramiz.

LEMMA. Agar $\beta_3 < +\infty$ va $A > 1$ bajarilsa u holda

$|t| \le T_n = \frac{H^2 \sqrt{A^n}}{5\beta_3}$ uchun

$$\left| Me^{it\frac{S\mu_n}{H\sqrt{A^n}}} - Me^{-\frac{t^2}{2}W_n} \right| \le \frac{7\beta_3 |t|^3}{6H^2\sqrt{A^n}} M\left(W_n e^{-\frac{t^2}{2}W_n}\right),$$

bu yerda $\beta_3 = M|\xi_1|^3 < +\infty$

34- teoremaninng isboti. $\Delta_1(x)$ ni quyidagi yig'indilarga ajratamiz:

$$\Delta_1(x) \le \sup_x |T_n(x) - G_n(x)| + \sup_x |G_n(x) - G(x)| = J_1 + J_2 \quad . \tag{1.4.25}$$

Bu yerda

$$G_n(x) = \int_0^{+\infty} \Phi\left(\frac{x}{\sqrt{z}}\right) dk_n(x)$$

Berri-Essen teoremasiga ko'ra, lemmadan foydalangan holda

$$J \le c \int_{|t|<T_n} \left| Me^{it\frac{S\mu_n}{\sigma\sqrt{A^n}}} - Me^{-\frac{t^2}{2}W_n} \right| \frac{dt}{|t|} + \frac{c}{T_n} M(W_n^{-\frac{1}{2}}) \tag{1.4.26}$$

ni hosil qilamiz.

Bevosita ishonch hosil qilish mumkinki

$$\int_{|t|\le T_n} t^2 M\left(W_n e^{-\frac{t^2}{2}W_n}\right) dt \le 2\int_0^{T_n} t^2 \int_0^{+\infty} ye^{-\frac{t^2}{2}y} dG_n(y) dt \le CM\left(\frac{1}{\sqrt{W_n}}\right) \tag{1.4.27}$$

u holda (1.4.26), (1.4.27) dan foydalanib

$$J_1 \le \frac{C\beta_3}{H^3} M\left(\frac{1}{\sqrt{W_n}}\right) \tag{1.4.28}$$

Agar $F(0) = 0$ tenglikni e'tiborga olsak

$$M\left(\frac{1}{\sqrt{\mu_n}}\right) \le \left(M\left(\frac{1}{\mu_n}\right)\right)^{\frac{1}{2}} \le \left(M\left(\frac{1}{\mu_1}\right)\right)^{\frac{n}{2}} \tag{1.4.29}$$

ni hosil qilamiz.

$J_2$ ni baxolash uchun 33-teoremani qo'llab

$$J_2 = \sup_x |G_n(x) - G(x)| = \sup_x \left| \int_0^{+\infty} [k_n(x) - k(x)] d\Phi\left(\frac{x}{\sqrt{z}}\right) \right| \le c\left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}}$$

ega bo'lamiz.

Natijada (1.4.25), (1.4.28)-(1.4.29) lardan 34-teorema isboti kelib chiqadi.

**35-TEOREMA**. Agar $h = 0,\ \beta_3 < +\infty, A > 1,\ D\mu_1 < +\infty,\ k < +\infty\ AP(\mu_1 = 1) < 1,$ bo'lsa

$$\Delta_2(n) = \sup_x \left| P_n(x) - \int_0^{+\infty} \frac{1}{\sqrt{z}} \varphi_\Phi\left(\frac{x}{\sqrt{z}}\right) dk(z) \right| \le$$

$$c\left[\frac{\beta_3}{H^3}\sqrt{MW_n^{-1}}\left(M\mu_1^{-1}\right)^{\frac{n}{2}}+\left(k+\frac{D\mu_1+A^2-A}{A^2-A}+\max\left(MW_n^{-1},MW^{-1}\right)\right)A^{-\frac{n}{6}}\right]$$

bu yerda $\varphi_\phi(x)$-standart normal taqsimotning zichlik funksiyasi, $P_n(x)$ $T_n(x)$ ni zichlik funksiyasi

35-teorema isboti. Xuddi (1.4.25) dagidek

$$\Delta_2(n)\le \sup_x\left|P_n(x)-\int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\phi\left(\frac{x}{\sqrt{z}}\right)dk_n(x)\right|+$$

$$\sup_x\left|\int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\Phi\left(\frac{x}{\sqrt{z}}\right)dk_n(z)-\int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\phi\left(\frac{x}{\sqrt{z}}\right)dk(z)\right|=J_3+J_4 \qquad (1.4.30)$$

Quyidagi belgilashni kiritamiz:

$P_{nx}(x)$ $\dfrac{\xi_1+\xi_2+...+\xi_k}{H\sqrt{k}}$ ni zichlik funksiyasi bo'lsin.

U holda

$$J_3=\sup_x\left|\sum_{k=1}^{\infty}P_n(k)\sqrt{\frac{A^n}{k}}\;P_{nk}\left(x\sqrt{\frac{A^n}{k}}\right)-\int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\phi\left(\frac{x}{\sqrt{z}}\right)dk_n(z)\right|\le \sup_x\sum_{k=1}^{\infty}P_n(n)\sqrt{\frac{A^n}{k}}.$$

$$\cdot\left|P_{nk}\left(x\sqrt{\frac{A^n}{k}}\right)-\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2kA^{-n}}}\right| \qquad (1.4.31)$$

Ikkinchi tomondan

$$\sup_x\left|P_{nk}\left(x\sqrt{\frac{A^n}{k}}\right)-\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2kA^{-n}}}\right|\le\frac{c\beta_3}{\sigma^3\sqrt{k}}$$

u holda (1.4.31) dan

$$J_3\le\frac{3\beta_3}{H^3}M\frac{1}{\sqrt{W_n}\sqrt{\mu_n}}\le\frac{c\beta_3}{H^3}\sqrt{MW_n^{-1}}\;\sqrt{M\mu_n^{-1}}\,. \qquad (1.4.32)$$

$J_4$ ni baholaymiz:

$$J_4\le\sup_x\left|\int_{|t|\le A^{\frac{n}{6}}}e^{-itx}\left[\int_0^{+\infty}e^{-\frac{t^2}{2}z}dk_n(z)-\int_0^{+\infty}e^{-\frac{t^2}{2}z}dk(z)\right]dt\right|+\sup_x\left|\int_{|t|>A^{\frac{n}{6}}}e^{-tx}\int_0^{+\infty}e^{-\frac{t^2z}{2}}dk_n(z)dt\right|$$

$$+\sup_x\left|\int_{|t|>A^{\frac{n}{6}}}e^{-itx}\int_0^{+\infty}e^{-\frac{t^2z}{2}}dk(z)dt\right|=J_5+J_6+J_7\,. \qquad (1.4.33)$$

$J_5$ da ichki integralni bo'laklab integrallab

$$J_5 \le \frac{1}{2} \int\limits_{|t|\le A^{\frac{n}{6}}} t^2 \int\limits_0^{+\infty} e^{-\frac{t^2 z}{2}} |K_n(z) - K(z)| dzdt$$

ni hosil qilamiz.

33-teoremani qo'llab

$$J_5 \le c\left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}} \int\limits_{|t|\le A^{\frac{n}{6}}} t^2 \int\limits_0^{+\infty} e^{-\frac{t^2 z}{2}} dzdt \le c\left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}} \cdot A^{\frac{n}{6}}.$$

(1.4.34)

Ushbu tengsizliklarning to'g'riligiga bevosita ishonch hosil qilish mumkin:

$$J_6 \le \frac{C}{A^{\frac{n}{6}}} M\left(\frac{1}{W_n}\right), \; J_7 \le \frac{c}{A^{\frac{n}{6}}} M\left(\frac{1}{W}\right)$$

(1.4.35)

Natijada (1.4.33)-(1.4.35) lardan

$$J_4 \le c\left[k + \frac{D\mu_1 + A^2 - A}{A^2 - A} + \max\left(M\left(W_n^{-1}\right), M\left(W^{-1}\right)\right)\right] A^{-\frac{n}{6}}.$$

(1.4.36)

Agar (1.4.30), (1.4.33), (1.4.36) hisobga oladigan bo'lsak teoremaning isboti kelib chiqadi.

**36-TEOREMA.** Faraz qilaylik $h = 0,\; x \to \infty$ da

$$\int\limits_{-x}^{x} x^2 dF(x) \to H^2,\; A > 1,\; D\mu_1 < +\infty, \qquad \sup_x k'(x) = k,$$

$MW^{-1} < +\infty$ va $\xi_i$ ning $\bar{f}(t) = Me^{-i\xi_i t}$ xarakteristik funksiyasi absolyut integrallanuvchi, $B_n^{\;2} \sim L(B_n)A^n$ bo'lsin u holda

$$\lim_{n\to\infty} \sup_x \left| P_n(x) - \int\limits_0^{+\infty} \frac{1}{\sqrt{z}} \varphi_\Phi\left(\frac{x}{\sqrt{z}}\right) dk(z) = \right| = 0,$$

bu yerda $P_n(x)$ $S\mu_1 : B_n$ ning zichlik funksiyasi, $B_n$ normallovchi son, $L(x)$-sekin o'zgaruvchi funksiya.

Isbot. Teorema shartlari asosida $E > 0,\ B > 0$ sonlar uchun

$$2\pi\left|P_n(x) - \int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\phi\left(\frac{x}{\sqrt{z}}\right)dk(z)\right| = \left|\int_{-\infty}^{+\infty}e^{-itx}\left[\Psi_n(t) - \int_0^{+\infty}e^{-\frac{t^2}{2}z}dk(z)\right]dt\right| \leq$$

$$\leq \left|\int_{|t|\leq B}e^{-itx}\left[\Psi_n(t) - \int_0^{+\infty}e^{-\frac{t^2}{2}z}dk(z)\right]dt\right| + \left|\int_{B<|t|<EB_n}e^{-itx}\Psi_n(t)dt\right| + \left|\int_{|t|\geq EB_n}e^{-itx}\Psi_n(x)dt\right| +$$

$$\left|\int_{|t|>B}e^{-itx}\int_0^{+\infty}e^{-\frac{T^2z}{2}}dk(z)dt\right| = J_1 + J_2 + J_3 + J_4\,,$$

(1.4.36.a)

bu yerda $\Psi_n(t)$ $\frac{S\mu_n}{B_n}$ ning xarakteristik funksiyasi.

Teorema shartidan foydalanib

$$\lim_{n\to\infty}\left[\bar{\mathrm{f}}\left(\frac{t}{B_n}\right)\right]^{A^n} = e^{-\frac{t^2}{2}}\,,$$

u holda ([2])

$$\lim_{n\to\infty}\Psi_n(t) = \lim_{n\to\infty}\int_0^{+\infty}\left[\bar{f}\left(\frac{t}{B_n}\right)\right]^{A^nz}dk_n(z) = \int_0^{+\infty}e^{-\frac{t^2z}{2}}dG(z),$$

yoki

$$\underset{|<B|}{Sup}\left|\Psi_n(t) - \int_0^{+\infty}e^{-\frac{t^2}{2}z}dk(z)\right| = 0(1)\,.$$

Buni e'tiborga olsak

$$J_1 \leq 2\underset{|t|\leq B}{Sup}\left|\Psi_n(t) - \int_0^{+\infty}e^{-\frac{t^2}{2}z}dk(z)\right| = 0(1)\,.$$

(1.4.36.b)

Ixtiyoriy $E > 0$ uchun $|t| \leq E$ da shunday $C > 0$ son topiladiki

$$\left|\bar{\mathrm{f}}(t)\right| \leq e^{-ct^2}\,.$$

Bundan foydalanib

$$J_2 = \left|\int_{B\leq|t|\leq EB_n}e^{-itx}\int_0^{+\infty}\left[\bar{\mathrm{f}}\left(\frac{t}{B_n}\right)\right]^{A^nz}dK_n(x)dt\right| \leq B_n\int_0^{+\infty}\int_{B\cdot B_n^{-1}\leq|t|\leq E}e^{-CA^nt^2z}dtdK_n(z) \leq \frac{cB_n}{A^n}\int_0^{+\infty}\frac{1}{z}$$

$$\int_{B\cdot B_n^{-1}\le|t|\le E} \frac{dt}{t^2} dk_n(x) \le \frac{cB_n}{BA^n}^2 MW_n^{-1}$$

Agar $B_n^{\,2} \sim L(B_n)A^n$ va $MW_n^{-1} < +\infty$ ekanini e'tiborga olsak $B \to +\infty$ da

$$J_2 \to 0 .$$

$|t| \ge E$ da esa shunday $c > 0$ topiladiki

$$\left|\bar{\mathrm{f}}(t)\right| \le e^{-c} .$$

Natijada

$$J_3 = B_n \int_0^{+\infty} \int_{|t|\ge E} \left|\bar{\mathrm{f}}(t)\right|^{A^n z} dt\, dk_n(z) \le \frac{CB_n}{A^n} \int_0^{+\infty} \frac{dk_n}{z} = c\frac{B_n}{A^n} MW_n^{-1} .$$

Bundan $n \to \infty$ da

$$J_3 \to 0$$

(1.4.36.v)

$J_4$ da integral ostidagi funksiyani $z$ bo'yicha bo'laklab integrallab, topamiz:

$$J_4 \le \left| \int_{|t|>B} e^{-itx} \left( \int_0^1 \frac{K'(z)}{t^{2\delta} z^{\delta}} dz + \int_1^{+\infty} \frac{2}{zt^2} dk(z)dt \right) \right| \le$$

$$\le \left| \int_{|t|\ge B} e^{-itx} \left( \frac{2}{t^2} + \frac{\underset{x}{Sup}\, K'(z)}{(1-\delta)t^{2\delta}} \right) dt \right| \le \frac{4}{B} + \frac{A}{B^{2\delta-1}}, \qquad \frac{1}{2} < \delta < 1$$

Bundan $B \to \infty$

$$J_4 \to 0 .$$

(1.4.36.d)

(1.4.36.a)- (1.4.36.d) larni hisobga olsak 36-teoremaning isboti kelib chiqadi.

***Ko'p xilli kritikdan keyingi tarmoqlanuvchi jarayon uchun limit teorema***

**37-TEOREMA.** Agar $\rho > 1$ bajarilsa u holda $\mu^j(n) : \rho^n = w^j(n)$ bir ehtimollik bilan $n \to \infty$ da $W_j$ tasodifiy vektorga intiladi va shu bilan birga $W^j \ne 0$ da $W^j$ ni yo'nalishi $v$ bilan bir xil yo'nalgan bo'ladi, ya'ni

$$w^j = \xi^j v = (\xi^j v_1, \xi^j v_2, ..., \xi^j v_k) = (w_1^j, w_2^j, .., w_k^j).$$

**38-TEOREMA.** $M\xi^j = u_j$ bajarilishi uchun

$$M(\mu_i(1) \log \mu_j(1) / \mu(0) = l_j) < +\infty$$

bajarilishi zarur va yetarlidir.

$P(\xi^j < x / \xi^j > 0)$ taqsimot funksiya absolyut uzluksiz.

Agar $B_{i,l}^j < +\infty, \quad i, j, l = \overline{1,k}$ bo'lsa $\xi^j$ ni dispersiyasi

$$D\xi^j = \frac{\sum_{\gamma,i,l=1}^{k} d_\gamma^j u_i u_l}{\begin{vmatrix} \rho^2 - a_1^1 & a_2^1 & \dots & a_k^1 \\ a_1^2 & \rho^2 - a_2^2 & \dots & a_k^2 \\ a_1^k & a_2^k & \dots & \rho^2 - a_k^k \end{vmatrix}}$$

(1.4.37)

bo'ladi, bu yerda $d_\gamma^j$ maxrajdagi determinantdagi $\rho^2 \delta_\gamma^j - a_\gamma^j$ elementning algebraik to'ldiruvchisi.

***Ikki xilli kritikdan keyingi jarayon hosil qiluvchi funksiyasining ayrim xossalari***

Quyidagi belgilashni kiritamiz.

$$m_j^l = \left.\frac{\partial F_1^j(s_1 s_2)}{\partial s_j}\right|_{\substack{S_1 = q_1 \\ S_2 = q_2}}, l, j = 1,2.$$

Ma'lumki ([89]) $\|m_j^l\|$ matritsaning oddiy xos soni $\rho_q (0 \le \rho_q < 1)$ mavjud bo'lib, ikkinchi xos soni absolyut qiymat jihatidan $\rho_q$ dan kichikdir.

Agar $q_i > 0, \ i = 1,2$ bo'lsa, u holda $\rho_q$ ga mos o'ng $u^* = (u_1^*, u_2^*)$ va chap $v^* = (v_1^*, v_2^*)$ xos vektorlarini musbat qilib tanlash mumkin va ularning elementlari

$$m_1^l u_1^* + m_2^l u_2^* = \rho u_l^*$$

$$v_1^* m_2^1 + v_2^* m_2^2 = \rho v_l^*, \qquad l = 1,2$$

(1.4.38)

tenglikni qanoatlantiradi.

**39-TEOREMA**. Agar $\rho > 1$ va $q_j > 0, \ j = 1,2$ bo'lsa, u holda $0 \le s_1 \le q_1, \ 0 \le s_2 \le q_2$ tengsizlikni qanoatlantiruvchi $s_1, s_2$ uchun

$$\lim_{n\to\infty} \frac{F_n(q_1,q_2)-F_n(s_1,s_2)}{v*(F_n(q_1,q_2)-F_n(s_1,s_2))} = u*$$

(1.4.39)

bajariladi.

Isboti. $F_1(s_1,s_2)$ ni $(q_1,q_2)$ nuqta atrofida Teylor qatoriga yoyib

$$F_1(s_1,s_2) = F_1(q_1,q_2)(s_1,s_2) + (M - E(s_1,s_2))(s-q)$$

(1.4.40)

ni hosil qilamiz, bu yerda $M = \|m_j^l\|$, $E(s_1,s_2)$ matritsa $0 \le s_j \le s_j^1 \le q,\ j=1,2$ qanoatlantiruvchi $s_1$ va $s_2$ uchun $0 \le E(s_1^1,s_2^1) \le E(s_1,s_2) \le M$ ni qanoatlantiradi va $\lim_{s\uparrow q} E(s_1,s_2) = 0$ $E_n(s_1,s_2) = E(F'_n(s_1,s_2), F_n^{\ 2}(s_1,s_2))$ belgilash kiritib (1.4.40) dan

$$F_n(q) - F_n(s) = (M - E_n(s))(M - E_{n-1}(s))....(M - E_1(s))(q-s)$$

(1.4.41)

ifodaga ega bo'lamiz. Oxirgi tenglikni $\rho_q^{\ k}$ ga bo'lib va

$$c = \frac{M}{\rho_q}, \qquad M_n(s) = \frac{E_q(s)}{\rho_q},$$

$B_n(s) = (c - M_n(s))(c - M_{n-1}(s))\ldots(c - M_1(s))$ belgilashlarni hisobga olib

$$\frac{F_n(q)-F_n(s)}{v*(F_n(q)-F_n(s))} = \frac{B_n(s)(q-s)}{v*B_n(s)(q-s)}$$

(1.4.42)

munosabatni hosil qilamiz.

Ma'lumki

$$\lim_{n\to\infty} \frac{B_n(s)(q-s)}{v*B_n(s)(q-s)} = u$$

(1.4.43)

(1.4.42) va (1.4.43) lardan teoremaning isboti kelib chiqadi.

**40-TEOREMA**. 39-teorema sharti ostida $S_1, S_2$ bo'yicha monoton o'smaydigan haqiqiy $K(s_1,s_2)$ funksiya mavjudki,

$$\lim_{n\to\infty} \frac{v*(F_n(q)-F_n(s))}{\rho_q^n} \downarrow K(s)$$

(1.4.44)

va

$$\lim_{n\to\infty} \frac{F_n(q)-F_n(s)}{\rho_q^n} = F(s)u*$$

(1.4.45)

Isboti. Quyidagi belgilashni kiritamiz:

$$\Delta_n(s) = \frac{v*(F_n(q)-F_n(s))}{\rho_q^n}, \quad 0 \le s_i \le q_i, \ \ i = 1{,}2$$

Agar (1.4.38) va (1.4.40) larni hisobga olsak $\Delta_{n+1}(s) \le \Delta_n(s)$ tengsizlikka ega bo'lamiz.

Bundan $\Delta_n(s)$ ni monoton kamayuvchiligi kelib chiqadi.

39-teoremani (1.4.45) ga qo'llasak

$$\frac{F_n(q)-F_n(s)}{\rho_q{}^n} = \frac{F_n(q)-F_n(s)}{v*(F_n(q)-F_n(s))} \cdot \frac{v*(F_n(q)-F_n(s))}{\rho_q^n} \to K(s)u*$$

Teorema isboti kelib chiqadi.

**41-TEOREMA**. Agar $a_l^l > 1,\ B_{\gamma\rho}^l < +\infty,\ e, j, v = 1{,}2$ bo'lsa, $x \to +\infty$ va $xv_2 < yv_1$ da

$$\sup_{-\infty<m<+\infty} \left| P\left(\tau_{x,y}^l = \left[\frac{\ln xv_1^{-1}}{\ln \rho}\right] + m\right) - (1-q_l)S^l(m, x, v_1^{-1}) \right| = 0(1),$$

(1.4.46)

$y \to +\infty$ da $yv_1 \le xv_2$ da

$$\sup_{-\infty<m<+\infty} \left| P\left(\tau_{x,y}^l = \left[\frac{\ln xv_2^{-1}}{\ln \rho}\right] + m\right) - (1-q_l)S^l(m, y, v_2^{-1}) \right| = 0(1),$$

(1.4.47)

$x, y \to +\infty$ da

$$P(\tau_{x,y}^l = +\infty) = q_l + 0(1)$$

(1.4.48)

Bu Teoremani isbotlash uchun quyidagi lemma kerak bo'ladi.

LEMMA. Agar $a_l^l > 1,\ B_{\gamma,j}^l < +\infty,\ \ l, j, v = 1{,}2$ bo'lsa, u holda ixtiyoriy $E > 0$ uchun $x, y \to +\infty$ da $i$ bo'yicha tekis

$$\left|P(\tau_{x,y}^{l} < i) - P((\mu_1^{l}(i) \geq x) \cup (\mu_2^{\,l}(i) \geq y))\right| < E$$

(1.4.49)

bajariladi.

Lemmaning isboti. Quyidagi belgilashni kiritamiz

$$R = R(x,y) = 0(\min(\log_{\rho} x, \log_{\rho} y))$$

(1.4.50)

Faraz qilaylik $1 \leq i \leq R$

Ishonch hosil qilish mumkinki

$$P(\tau_{x,y}^{l} < i) = P((\max_{1\leq n\leq i} \mu_1^{l}(n) \geq x) \cup (\max_{1\leq n\leq i} \mu_2^{l}(n) \geq y)) \leq P(\max_{1\leq n\leq i} \mu_1^{l}(n) \geq x) + P(\max_{1\leq n\leq i} \mu_2^{l}(n) \geq y) \leq$$

$$\leq \sum_{n=1}^{i} P(\mu_1^{l}(n) \geq x) + \sum_{n=1}^{i} P(\mu_2^{l}(n) \geq y)$$

(1.4.51)

bo'ladi.

Oxirgi tengsizlikni o'ng tomoniga Chebishev tengsizligini qo'llab (1.1.9) ga asosan

$$P\left(\tau_{x,y}^{l} < i\right) \leq c \frac{\rho^{i}}{\min(x,y)}$$

(1.4.52)

hosil qilamiz, bu yerda $ci, x, y$ ga bog'liq emas o'zgarmas son.

Xuddi shunday usul bilan ushbu

$$P((\mu_1^{l}(i) \geq x) \cup (\mu_2^{l}(i) \geq y)) \leq \frac{c\rho^{i}}{\min(x,y)}$$

(1.4.53)

tengsizlikni o'rinliligini ko'rsatish mumkin.

(1.4.52),(1.4.53) dan ixtiyoriy $E > 0$ uchun $R$ ni tanlash hisobiga

$$P(\tau_{x,y}^{l} < i) \leq \frac{E}{2}, P((\mu_1^{l}(i) \geq x) \cup (\mu_2^{l}(i) > y)) \leq \frac{E}{2}$$

(1.4.54)

Endi, faraz qilaylik $i \geq R+1$

Quyidagi belgilashlarni kiritamiz:

$$D_k = \bigcup_{n=R+1}^{i} (\mu_k^l(n+1) \le \mu_k^l(n), \mu_1^l(i) + \mu_2^l(i) > 0),\ l,k = 1,2$$

$$\overline{P}_{l_1,l_2}(n) = P(\mu_1^l(n) = l_1, \mu_2{}^l(n) = l_2 / \mu_1{}^l(n) > 0,\ \mu_2^l(n) > 0),\ \ l = 1,2$$

$$P_{l_1,l2}^l(n) = P(\mu_1^l(n) = l_1, \mu_2^l(n) = l_2)$$

Ma'lumki

$$P(D_k) \le \sum_{n=R+1}^{i} P(\mu_k^l(n+1) \le \mu_k^l(n), \mu_1^l(n) + \mu_2^l(n) > 0)$$

(1.4.55)

Quyidagi xodisani

$$(\mu_k^l(n+1) \le \mu_k^l(n),\ \mu_1^l(n) + \mu_2^l(n) > 0) = (\mu_k^l(n+1) \le \mu_1^l(n) = 0,\ \mu_2^l(n) = 0) +$$

$$(\mu_k^l(n+1) = \mu_k^l(n),\ \mu_1^l(n) > 0,\ \mu_2^l(n) = 0) + (\mu_k^l(n+1) =$$

$$\mu_k^l(n), \mu_1^l(n) > 0,\ \mu_2^l(n) > 0) = L_{1,k} \cup L_{2,k} \cup L_{3,k} \qquad (1.4.56)$$

ko'rinishda ifodalaymiz.

$K = 1$ teng holni qaraymiz $k = 2$ bo'lgan hol xuddi shunday baxolanadi. $L_{3,1}$ hodisani ehtimolligini hisoblaymiz:

$$P(L_{3,1}) \le P(\mu_1{}^l(n) > 0, \mu_2{}^l(n) > 0) \sum_{\substack{e_1 \ge 1 \\ e_2 \ge 1}} \overline{P}^l_{l_1,l_2}(n) P(S_{1,l}^1 + S_{1,l_2}^2 \le l_1) \qquad (1.4.57)$$

bu yerda

$$S_{1,l}^1 = \xi_{1,1}^1 + \xi_{1,2}^1 + \ldots \xi_{1,l_1}^1, S_{1,l_2}^2 = \xi_{1,1}^2 + \xi_{1,2}^2 + \ldots \xi_{1,l_2}^2,$$

$\xi_{1,j}^1$ va $\xi_{2,j}^2$ o'zaro bog'liqsiz va mos ravishda $\mu_1{}^1(1), \mu_1{}^2(1)$ lar bilan bir xil taqsimlangan. $a_1^1 > 1$ va Chebeshev tengsizligini hisobga olib

$$P\left(S_{1,l_1}^1 + S_{1,l_2}^2 \le l_1\right) \le P\left(\left|\frac{S_{1,l_1}^1 + S_{1,l_2}^2}{l_1 + l_2} - \frac{a_1^1 l_1 + a_1^2 l_2}{l_1 + l_2}\right| > \frac{a_1^1 l_1 + a_1{}^2 l_2}{l_1 + l_2} - \frac{l_1}{l_1 + l_2}\right) \le$$

$$\le \frac{\max(D\xi_{1,1}^1, D\xi_{1,1}^2)(l_1 + l_2)}{\min((a_1^1 - 1)^2 (a_1^2)^2)(l_1 + l_2)^2} \le \frac{c}{l_1 + l_2}$$

(1.4.58)

Natijada $L_{3,1}$ hodisaning ehtimolligi uchun

$$P(L_{3,1}) \le c \sum_{l_1,l_2 \ge 1} P_{l_1,l_2}^l(n) \frac{1}{l_1 + l_2}$$

tengsizlikni hosil qilamiz.

Bu tengsizlikka o'rta arifmetik va so'ngra Koshi-Bunyakovskiy tengsizligini qo'llab

$$P(L_{3,1}) \le c\sqrt{\sum_{l_1,l_2\ge 1}\frac{P_{l_1 l_2}(n)}{l_1\cdot l_2}}$$

(1.4.59)

ga ega bo'lamiz.

$\alpha_n = \left[\frac{n}{2}\right],\ \beta_n = n-\alpha_n$ belgilashlarni kiritib ildiz ostida ifodani baholaymiz:

$$T = \sum_{l_1,l_2\ge 1}\frac{P^l_{l_1,l_2}(n)}{l_1\cdot l_2} = \int_0^1\int_0^1\left(\sum_{l_1,l_2\ge 1}P^l_{l_1,l_2}(n)S_1^{\,l_1-1}S_2^{\,l_2-1}\right)ds_1 ds_2 = T_1+T_2+T_3+T_4+T_5+T_6+T_7.$$

(1.4.60)

Bu yerdagi $T_i, i=\overline{1,7}$ qo'shiluvchilar integralni $0\le S_1\le 1,\ 0\le S_2\le 1$ chegara- sini quyidagicha ajratilgan integrallardir:

$T_1:\quad (0\le s_1\le q_1,\ 0\le s_2\le q_2),$

$T_2:\quad (\exp(-\rho^{-\beta_n}) < s_1\le 1,\ \ 0<s_2\le \exp(-\rho^{-\beta_n})),$

$T_3:\quad (0<S_1<\exp(-\rho^{-\beta_n}), \exp(-\rho^{-\beta_n})<s_2\le 1),$

$T_4:\quad (0<s_1\le q_1, q_2<s_2<\exp(-\rho^{-\beta_n})),$

$T_5:\quad (q_1<s_1<\exp(-\rho^{-\beta_n}),\ 0<s_2<q_2),$

$T_6:\quad (q_1\le s_1<\exp(-\rho^{-\beta_n}), q_2\le s_2<\exp(-\rho^{-\beta_n})),$

$T_7:\quad (\exp(-\rho^{-\beta_n})\le s_1\le 1,\ \exp(-\rho^{-\beta_n}\le s_2\le 1)).$

Bevosita ishonch hosil qilish mumkinki:

$$T_2 = 0\left(\rho^{-\frac{n}{2}}\right),\ T_3 = 0\left(\rho^{-\frac{n}{2}}\right),\ T_7 = 0\left(\rho^{-n}\right)$$

(1.4.61)

$F_n^l(s_1,s_2)$ monotonligi hisobga olib

$$T_1 \le 3(F_n^l(q_1,q_2) - F_n^l(0,0)) = 0(\rho_q^{\ n}).$$

(1.4.62)

Ma'lumki $n\to\infty$ da $b_1, b_2, \lambda(0<b_1,\ b_2<1,\ \ 0<\lambda<1)$

uchun

$$F_n^{\,l}(s_1 s_2) \to q_l,\ \left|\frac{\partial F_n^{\,l}(s_1,s_2)}{\partial s_j}\right|_{\substack{S_1 \le b_1 \\ S_2 \le b_2}} \le c\lambda^n$$

(1.4.63)

o’rinlidir.

oldingilardan dan foydalanib

$$F_n^{\,l}(e^{-S_1\rho^{-n}}, e^{-S_2\rho^{-n}}) \to V^l(s_1,s_2)$$

(1.4.64)

ni hosil qilamiz, bu yerda $V^l(s_1,s_2)W^l$ ni Laplas almashtirishi.

Agar $S^l(x) = p(\xi^l < x/\xi^l > 0)$ ni absolyut uzluksizligini hisobga olsak, u holda $s_1 > \alpha_1 > 0, \quad s_2 > \alpha_2 > 0$ lar uchun

$$\left|V^l(s_1,s_2)\right| < 1 \qquad (1.4.65)$$

Endi $T_6$ ni baxolaymiz:

$$T_6 = \int\limits_{q_1}^{\exp(-\rho^{-\beta_n})} \int\limits_{q_2}^{\exp(-\rho^{-\beta_n})} \left( \sum_{l_1,l_2 \ge 1} P_{l_1,l_2}^{l}(n)\, s_1^{\,l_1-1} s_2^{\,l_2-1} \right) ds_1 ds_2 \le c\left(F_n^l(e^{-\rho^{-\beta_n}}, e^{-\rho^{-\beta_n}}) - F_n^{\,l}(0,0)\right) =$$

$$c\left(F_{n-\beta_n}^l\left(F_{\beta_n}^1\left(e^{-\rho^{-\beta_n}}, e^{-\rho^{-\beta_n}}\right), F_{\beta_n}^2\left(e^{-\rho^{-\beta_n}}, e^{-\rho^{-\beta_n}}\right)\right) - F_n^{\,l}(0,0)\right).$$

Agar (1.4.64) ni hisobga olsak, u holda oxirgi ifodani quyidagicha yozish mumkin:

$$T_6 \le c\left(F_{n-\beta_n}^l\left(V'(1,1) + E'_{\beta_n},\ V'(1,1) + E^2{}_{\beta_n}\right) - F_n^l(0,0)\right)$$

bu yerda

$$\lim_{n\to\infty} E_{\beta_n}^j = 0, \qquad j = 1,2$$

Oxirgi ifodaga (1.4.63), (1.4.65) larni qo’llagandan so’ng o’rta qiymat haqidaga teoremaga asosan

$$T_6 \le c\left(\left(F_{n-\beta_n}^l\left(V'(1,1) + E\beta_n', V^2(1,1) + E_{\beta_n}^2\right) - q_l\right) + (q_l - F_n^{\,l}(0,0)\right) = 0(\max(\lambda^{\frac{n}{2}}, \rho_q^n))$$

(1.4.66)

Shunday usulda $T_4$ va $T_5$ lar baholanadi:

$$T_4 = 0\left(\max\left(\lambda^{\frac{n}{2}}, \rho_q{}^n\right)\right),\ T_5 = 0(\max(\lambda^{\frac{n}{2}}, \rho_q{}^n))$$

(1.4.67)

(1.4.60),(1.4.62),(1.4.66),(1.4.67) larni yig'ib:

$$T = 0(\rho_1{}^n)$$

(1.4.68)

ga ega bo'lamiz, bu yerda $1 > \rho_1 > \max(\lambda^{\frac{1}{2}}, \rho^{-\frac{1}{2}}, \rho_q)$

Natijada (1.4.59),(1.4.60) lardan

$$P(L_{3,1}) = 0(\rho_1^{\frac{n}{2}})$$

(1.4.69)

Endi $L_{2,1}$ hodisani ehtimolligini baholaymiz:

$$P(L_{2,1}) = P(\mu_1^l(n) > 0,\ \mu_2{}^l(n) = 0)\cdot\sum_{l_1=1}^{\infty}(P(\mu_1^l(n) = l_1, \mu_2^l(n) = 0 / \mu_1^l(n) > 0, \mu_2^l(n) = 0)\cdot$$

$$P(\mu_1^l(n+1) \le \mu_1^l(n) / \mu_1^l(n) = l_1,\ \mu_2^l(n) = 0)) = P(\mu_1^l(n) > 0, \mu_2^l(n) = 0)\cdot$$

$$\sum_{l_1=1}^{\infty}\overline{\overline{P}}^{l}_{l_1,0}(n)P(s^1_{1,l_1} \le l_1)$$

(1.4.70)

bu yerda

$$\overline{\overline{P}}_{l_1,0}(n) = P(\mu_1^l(n) = l_1, \mu_2^l(n) = 0 / \mu_1^l(n) > 0,\ \mu_2^l(n) = 0)$$

$a_1^1 > 1$ni hisobga olib, Chebeshev tengsizligini qo'llash orqali ushbu tengsizlikka ega bo'lamiz:

$$P(S^1_{1,l_1} \le l_1) \le P\left(\left|\frac{s^1_{1,l_1}}{l_1} - a_1^1\right| > a_1^1 - 1\right) \le \frac{l_1 D\xi^1_{1,1}}{l_1{}^2(a_1^1-1)^2} \le \frac{c}{l_1}$$

(1.4.71)

Agar (1.4.70),(1.4.71) larni yig'sak

$$P(L_{2,1}) \le cP\left(\mu_1^l(n) > 0, \mu_2^l(n) = 0\right)\sum_{l_1=1}^{\infty}\frac{\overline{\overline{P}}^{l}_{l_1,0}(n)}{l_1}$$

hosil bo'ladi.

Oxirgi tengsizlikni quyidagi yig'indi ko'rinishida ifodalaymiz:

$$P(L_{2,1}) \le c\int_0^1 \left( \sum_{l=1}^{\infty} \overline{\overline{P}}_{l_1,0}(n) s_1^{l_1-1} \right) ds_1 = \int^{q_1} \left( \sum_{l_1=1}^{\infty} \overline{\overline{P}}_{l_1,0}(n) s_1^{l_1-1} \right) ds_1 + \int_{q_1}^{\exp(-\rho^{\beta_n})} \left( \sum_{l_1=1}^{\infty} \overline{\overline{P}}_{l_1,0}(n) s_1^{l_1-1} \right) ds_1 +$$

$$\int_{\exp(-\rho^{-\beta_n})}^{1} \left( \sum_{l_1=1}^{\infty} \overline{\overline{P}}_{l_1,0}(n) s_1^{l_1-1} \right) ds_1 = T_1^1 + T_2^1 + T_3^1$$

(1.4.72)

Agar $F_n^{\,l}(s_1,s_2)$ ni $s$ bo'yicha, monoton o'suvchiligini hisobga olsak

$$T_1^1 \le c\left(F_n^{\,l}(q,0) - F_n^{\,l}(0,0)\right) \le c\left(F_n^{\,l}(q_1,q_2) - F_n^{\,l}(0,0)\right)$$

tengsizlikka ega bo'lamiz. Bu tengsizlikka (1.3.69) ni qo'llab

$$T_1^{'} = 0(\rho_q^{\,n})$$

(1.4.73)

baxoni olamiz

$F_n^{\,l}(s_1,s_2)$ ni monotonligidan foydalanib

$$T_2^1 \le c\left(F_n^{\,l}\left(e^{-\rho^{-\beta_n}},0\right) - F_n^{\,l}(0,0)\right) \le c\left(F_n^{\,l}(e^{-\rho^{-\beta_n}}, e^{-\rho^{-\beta_n}}) - F_n^{\,l}(0,0)\right)$$

tengsizlikka ega bo'lamiz.

U holda xuddi $T_6$ ni baxolagandek $T_2^{'}$ ni ham baholash mumkin:

$$T_2' = 0\left( \max\left( \lambda^{\frac{n}{2}}, \rho_q^{\,n} \right)\right)$$

(1.4.74)

Ishonch hosil qilish mumkinki

$$T_3' = 0(\rho^{-\frac{n}{2}})$$

(1.4.75)

Demak (1.4.72)-(1.4.75) ga ko'ra

$$P(L_{2,1}) = 0(\rho_1^{\,n})$$

(1.4.76)

Endi $L_{1,1}$ hodisaning ehtimolligini baholaymiz:

$$P(L_{1,1}) = P(\mu_1^l(n0 = 0,\ \mu_2^l(n) > 0)$$

$$\sum_{l_2=1}^{\infty}\left(P(\mu_1^l(n)=0),\ \mu_2^l(n)=l_2/\mu_1^l(n)=0,\ \mu_2^l(n)>0\right)\cdot P(\mu_1^l(n+1)\le\mu_1^l(n)/\mu_1^l(n)=0,\ \mu_2^l(n)=l_2)=$$

$$\sum_{l_2=1}^{\infty}P_{0,l_2}^l(n)P(S_{1,l_2}^2\le 0)=\sum_{l_2=1}^{\infty}P_{0,l_2}^l(n)P\left(\xi_1^2+\xi_{1,2}^2+\ldots+\xi_{1,l_2}^2=0\right)=\sum_{l_2=1}^{\infty}P_{0,l_2}^l(n)\left(P\left(\xi_{1,1}^2=0\right)\right)^{l_2}.$$

Agar $P(\xi_{1,1}^2=0)<1$ligini hisobga olsak, u holda shunday $c$ o'zgarmas son topiladiki

$$P(L_{1,1})=c\sum_{l_2=1}^{\infty}\frac{P_{0,l_2}^l(n)}{l_2}$$

Natijada xuddi $P(L_{2,1})$ ni baxolagandek

$$P(L_{1,1})=0(\rho_1{}^n) \tag{1.4.77}$$

ifodaga ega bo'lamiz.

Agar (1.4.55), (1.4.56), (1.4.59), (1.4.76), (1.4.77) larni e'tiborga olsak

$$P(D_1)=\sum_{n=R+1}^{i}0(\rho_1^{\frac{n}{2}})=0(\rho_1^{\frac{R}{2}})$$

Xuddi shu kabi

$$P(D_2)=0(\rho_1^{\frac{R}{2}})$$

u holda $x, y$ larni ixtiyoricha katta qilib tanlab

$$P(D_1)\le\frac{E}{3},\qquad\qquad P(D_2)\le\frac{E}{3} \tag{1.4.78}$$

larni hosil qilamiz.

Endi $\tau_{x,y}^l$ tasodifiy miqdorni $i$ dan kichik bo'lish ehtimolini quyidagicha ifodalaymiz:

$$P(\tau_{x,y}^l<i)=P\left(\left(\max_{1\le n\le i}\mu_1^l(n)\ge x\right)\cup\left(\max_{1\le n\le i}\mu_2^l(n)>y\right)\right)=P\left(\max_{1\le n\le i}\mu_1^l(n)\ge x\right)+P\left(\max_{1\le n\le i}\mu_2^l(n)\ge y\right)-$$
$$-P\left(\max_{1\le n\le i}\mu_1^l(n)\ge x,\ \max_{1\le n\le i}\mu_2^l(n)\ge y.\right) \tag{1.4.79}$$

Bu tenglikni birinchi qo'shiluvchisini baholaymiz:

$$P\left(\max_{1\le n\le i}\mu_1^l(n)\ge x\right)=P\left(\left(\max_{1\le n\le R}\mu_1^l(n)\ge x\right)\cup\left(\max_{R+1\le n\le i}\mu_1^l(n)\ge x\right)\right)=P\left(\max_{1\le n\le R}\mu_1^l(n)\ge x\right)+P\left(\max_{R+1\le n\le i}\mu_1^l(n)\ge x\right)-$$
$$-P\left(\max_{1\le n\le R}\mu_1^l(n)\ge x,\ \max_{R+1\le n\le i}\mu_1^l(n)\ge x\right)=P\left(\max_{R+1\le n\le i}\mu_1^l(n)\ge x\right)+\theta P(\max_{1\le n\le R}\mu_1^l(n)\ge x),\qquad 0\le\theta\le 1$$

Agar (1.4.51)-(1.4.54) larni hisobga olsak

$$\left|P(\max_{1\le n\le i}\mu_1^l(n)\ge x)-P(\max_{R+1\le n\le i}\mu_1^l(n)\ge x)\right|<\frac{E}{3}$$

(1.4.80)

U holda tarmoqlanish jarayon ta’rifiga ko’ra ixtiyoriy $m>0$ son uchun

$$(\mu_1^l(i)+\mu_2^l(i)=0)\subset(\mu_1^l(i+m)+\mu_2^l(i+m)=0)$$

(1.4.81)

Demak,

$$(\max_{R+1\le n\le i}\mu_1^l(n)\ge x,\ \mu_1^l(i)+\mu_2^l(i)=0)\subset(\max_{R+1\le n\le i}\mu_1^l(n)\ge x,\ \ \mu_1^l(i+\bar{\tau}_x^{\ -l})+\mu_2^l(i+\bar{\tau}_x^{\ l})=0),$$

(1.4.82)

bu yerda $\bar{\tau}_x^{\ l}$ bilan birinchi marta $\mu_1^l(n)$ ni $x$ satxga erishish holatini belgilaymz.

$\tau_x^{\ l}$ Markov zanjarini xossasiga bo’ysinishi hisobga olsak

$$P(\max_{R+1\le n\le i}\mu_1^l(n)\ge x,\mu_1^l(i)+\mu_2^l(i)=0)\le P(R+1\le\bar{r}_x^{\ l}\le i,\ (\bar{r}_x^{\ l})\ge x,\ \mu_1^l(i+\bar{r}_x^{\ l})+\mu_2^l(i+\bar{r}_x)=0)\le$$

$$\le\sum_{R=[x]}^{\infty}P(\mu_1^l(\bar{r}_x)=R)\,P(\mu_1^l(i+\bar{r}_x^{\ l})+\mu_2^l(i+\bar{r}_x^{\ l})=0/\mu_1^{\ l}(r_x^{-l})=R)=$$

$$\sum_{R=[x]}^{\infty}P(\mu_1^l(\bar{r}_x)=R)\,P(\mu_1^l(i)+\mu_2^l(i)=0/\mu_1^l(0)=R)\le\sum_{R=[x]}^{\infty}P(\mu_1^l(\bar{r}_x)=R)\,q_l^R\le q_l^{[x]}\qquad(1.4.83)$$

$D_1$ hodisani to’ldirmasi

$$\overline{D}_1=\left(\bigcap_{n=R+1}^{i}\left(\mu_1^l(n+1)>\mu_1^l(n),\ \mu_1^l(i)+\mu_2^l(i)>0\right)\right)\cup\left(\mu_1^l(i)+\mu_2^l(i)=0\right)$$

ligini hisobga olib (1.4.83) ga ko’ra

$$P(\max_{R+1\le n\le i}\mu_1^l(n)\ge x)=P(\overline{D}_1,\ \max_{R+1\le n\le i}\mu_1^l(n)\ge x,)+P(D_1,\ \max_{R+1\le n\le i}\mu_1^l(n)\ge x)=$$

$$P(\mu_1^l(i)\ge x)+\theta_1P(D_1)+q_l^{[x]},\qquad 0\le\theta_1\le 1\qquad(1.4.84)$$

Natijada (1.4.78), (1.4.80),(1.4.84) lardan

$$\left|P(\max_{1\le n\le i}\mu_1^l(n)\ge x)-P(\mu_1^l(i)\ge x)\right|<E\qquad(1.4.85)$$

Shu usulda

$$\left|P(\max_{1\le n\le i}\mu_2^l(n)\ge y)-P(\mu_2^l(i)\ge y)\right|<E\qquad(1.4.86)$$

ligini ko’rsatish mumkin.

Ma’lumki, ixtiyoriy $A_1, A_2, B_1, B_2$ hodisalar uchun

$$P((A_1 \cup A_2) \cap (B_1, B_2)) = P((A_1 \cap B_1) \cup (A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2) =$$

$$P(A_1 \cap B_1) + P(A_2 \cap B_2) - 5\theta_2 P(A_1) - \theta_3 P(B_1), \qquad 0 \le \theta_2, \quad \theta \le 1$$

(1.4.87)

u holda

$$P(\max_{1\le n\le i} \mu_1^l(n) \ge x, \max_{1\le n\le i} \mu_2^l(n) \ge y) = P((\max_{1\le n\le R} \mu_1^l(n) \ge x) \cup (\max_{R+1\le n\le i} \mu_1^l(n) \ge x),$$

$$(\max_{1\le n\le R} \mu_2^l(n) \ge y) \cup (\max_{R+1\le n\le i} \mu_2^l(n) \ge y)) = P(\max_{R+1\le n\le i} \mu_1^l(n) \ge x, \max_{R+1\le n\le i} \mu_2^l(n) \ge y) +$$

$$P(\max_{1\le n\le R} \mu_1^l(n) \ge x, \max_{1\le n\le i} \mu_2^l(n) \ge y) - 5\theta P(\max_{1\le n\le R} \mu_1^l(n) \ge x) - \theta_3 P(\max_{1\le n\le R} \mu_2^l(n) \ge y).$$

Oxirgi ifodaga (1.4.51)-(1.4.54) ni qo’llab

$$\left| P(\max_{1\le n\le i} \mu_1^l(n) \ge x, \max_{1\le n\le i} \mu_2^l(n) \ge y) - P(\max_{R+1\le n\le i} \mu_1^l(n) \ge x, \max_{R+1\le n\le i} \mu_2^l(n) \ge y) \right| \le \frac{E}{3} \qquad (1.4.88)$$

ga ega bo’lamiz.

Ixtiyoriy $B$ hodisa uchun

$$P(B) = P(B \cap \overline{D}_1 \cap \overline{D}_2) + P(B \cap \overline{D}_1 \cap D_2) + P(B \cap D_1) =$$

$$= P(B \cap \overline{D}_1 \cap \overline{D}_2) + \theta_4 P(D_2) + \theta_5 P(D_1), \qquad 0 \le \theta_4, \quad \theta_5 \le 1$$

Shuning uchun (1.4.83) va (1.4.87) larga ko’ra

$$P(\max_{R+1\le n\le i} \mu_1^l(n) \ge x, \max_{R+1\le n\le i} \mu_2^l(n) \ge y) = P(\max_{R+1\le n\le i} \mu_1^l(n) \ge x, \max_{R+1\le n\le i} \mu_2^l(n) \ge y, \overline{D}_1, \overline{D}_2) + \theta_4 P(D_2) +$$

$$+\theta_5 P(\overline{D}_1) = P(\mu_1^l(i) \ge x, \mu_2^l(i) \ge y) + 3\theta_6 P(\max_{R+1\le n\le i} \mu_1^l(n) \ge x, \max_{R+1\le n\le i} \mu_2^l(n) \ge y, \mu_1^l(i) + \mu_2^l(i) = 0) +$$

$$+\theta_4 P(D_2) + \theta_4 P(D_2) + \theta_5 P(D_1) \le P(\mu_1^l(i) \ge x, \mu_2^l(i) \ge y) + 3q_l^{[x]} +$$

$$\theta_4 P(D_2) + \theta_5 P(D_1), \qquad 0 \le \theta_6 \le 1$$

(1.4.89)

ni hosil qilamiz.

Natijada yetarlicha katta $x$ va $y$ lar uchun (1.4.79), (1.4.88), (1.4.89) ifodalarni hisobga olib

$$\left| P(\max_{1\le n\le i} \mu_1^l(n) \ge x, \max_{1\le n\le i} \mu_2^l(n) \ge y) - P(\mu_1^l(i) \ge x, \mu_2^l(i) \ge y) \right| < E$$

(1.4.90)

ga ega bo’lamiz.

Shundan qilib, (1.4.79), (1.4.85), (1.4.86),(1.4.90) lardan, $i \geq R$ uchun xam lemmaning isboti kelib chiqdi.

41-teoremaning isboti. Umumiylikka halal keltirmaslik uchun $xv_2 < yv_1$ deb faraz qilaylik.

Lemma isbotidan ko'rinadiki $i \geq R$ hol asosiy bo'lib, $i \leq R$ holi o'zidan o'zi ko'rinadi.

Quyidagi belgilashni kiritamiz;

$$i = \left[\frac{\ln xv_1^{-1}}{\ln \rho}\right] + m, \qquad -\infty < m < +\infty$$

(1.4.91)

Ishonch hosil qilish mumkinki

$$P\big((\mu_1^l(i) \geq x) \cup (\mu_2^l(i) \geq y)\big) = P(\mu_1^l(i) \geq x) \cup (\mu_2^l(i) \geq y), \mu_1^l(i) + \mu_2^l(i) > 0) = P(\mu_1^l(i) + \mu_2^l(i) > 0)$$

$$P((\mu_1^l(i) \geq x) \cup (\mu_2^l(i) \geq y / \mu_1^l(i) + \mu_2^l(i) > 0) = P(\mu_1^l(i) + \mu_2^l(i) > 0) - P(\mu_1^l(i) + \mu_2^l(i) > 0)$$

$$P(\mu_1^l(i) < x,\ \mu_2^l(i) < y / \mu_1^l(i) + \mu_2^l(i) > 0) = P(\mu_1^l(i) + \mu_2^l(i) > 0) - P(\mu_1^l(i) + \mu_2^l(i) > 0) \cdot$$

$$P(\mu_1^l(i)\rho^{-i}v_1^{-1} < x\rho^{-i}v_1^{-1}, \mu_2^l(i)\rho^{-i}v_2^{-1} < y\rho^{-i}v_2^{-1} / \mu_1^l(i) + \mu_2^l(i) > 0)$$

(1.4.92)

(1.4.92) ga ko'ra

$$\rho^{-i} = xv_1^{-1}\rho^{-m+w(xv_1^{-1})}$$

va

$$\lim_{i \to +\infty} P(\mu_1^l(i) + \mu_2^l(i) > 0) = 1 - q_l$$

ligini hisobga olsak (1.4.91),(1.4.92) lardan yetarlicha katta $x, y$ lar uchun

$$\left| P\left( \mu_1^l\left(\left[\frac{\ln xv_1^{-1}}{\ln \rho}\right] + m\right) \cup \left( \mu_2^l\left(\left[\frac{\ln xv_1^{-1}}{\ln \rho}\right] + m\right) \geq y\right)\right) - (1 - q_l) \cdot (1 - s^l(\rho^{-m+w(xv_1^{-1})})) \right| < E$$

(1.4.93)

tengsizlikka ega bo'lamiz.

Ma'lumki

$$P(\tau_{x,y}^l = i) = P(\tau_{x,y}^l < i + 1) - P(\tau_{x,y}^l < i),$$

(1.4.94)

U holda (1.4.94) dan, lemmaga ko'ra, 41-teorema birinchi qismning isboti kelib chiqadi.

41-teoremaning 2 - qismini isboti ham xuddi shu usulda olib boriladi.

Endi 41-teorema uchinchi qismining isbotini keltiramiz. Faraz qiliaylik boshlanishda bir dona $l$ tipli zarracha bo'lsin va uni yashash davri $T^l$ bo'lsin, ya'ni

$$T^l = \begin{cases} n, & агар \mu_1^l(n) + \mu_2^l(n) > \ \ лекин\ \mu_1^l((n+1) + \mu_2^l(n+1) = 0 \\ +\infty & агар \min_n (\mu_1^l(n) + \mu_2^l(n) > 0), \end{cases}$$

$l = 1,2$

Agar ixtiyoriy $A, B$ xodisalar uchun

$$P(AB) = P(A) - P(A\overline{B})$$

ligini hisobga olsak, u holda

$$P(T^l = n) = F_{n+1}^l(0,0) - F_n^l(0,0)$$

Bundan

$$\sum_{n=1}^{\infty} P(T^l = n) = q_l,\ P(T^l = +\infty) = 1 - q_l$$

Osonlik bilan ishonch hosil qilish mumkinki

$$P(\tau_{x,y}^l = +\infty) = P(\tau_{x,y}^l = +\infty, T^l = +\infty) + P(\tau_{x,y}^l = +\infty, T^l < \infty)$$

$$P(\tau_{x,y}^l = +\infty, T^l = +\infty) \le P(\mu_1^l(N) < x, \mu_2^l(N) < y, \mu_1^l(N) + \mu_2^l(N) > 0) =$$

$$= P\left(\frac{\mu_1^l(N)}{\rho^N v_1} < \frac{x}{\rho^N v_1},\ \frac{\mu_2^l(N)}{\rho^N v_2} < \frac{y}{\rho^N v_2}, \mu_1^l(N) + \mu_2^l(N) > 0\right)$$

(1.4.95)

Natijada oxirgi tengsizlikdan

$$P(r_{x,y}^l = +\infty, T^l = +\infty) = 0$$

(1.4.96)

ni hosil qilamiz.

Yetarlicha katta $N$ lar uchun

$$P(N < T^l < +\infty) = \sum_{n=N+1}^{\infty} P(T^l = n) < E$$

Bundan

$$P(\tau^l_{x,y} = +\infty, T^l < +\infty) = P(\tau^l_{x,y} = +\infty, 1 \le T^l \le N) + P(\tau^l_{\overline{\overline{x,y}}} = +\infty, N < T^l < +\infty) =$$

$$= P(\tau^l_{x,y} = +\infty, 1 \le T^l \le N) + \theta_7 E, \quad 0 \le \theta_7 \le 1,$$

(1.4.97)

$$P(\tau^l_{\overline{\overline{x,y}}} = +\infty, 1 \le T^l \le N) = P(1 \le T^l \le N) - P(\tau^l_{x,y} = +\infty, \quad 1 \le T^l \le N)$$

(1.4.98)

Ikkinchi tomondan

$$P(\tau^l_{\overline{\overline{x,y}}} = +\infty, 1 \le T^l \le N) \le P(\tau^l_{x,y} < \infty, \mu^l_1(N+1) + \mu^l_2(N+1) = 0) =$$

$$= P((\max_n \mu^l_1(n) \ge x) \cup (\max_n \mu^l_2(n) \ge y), \ \mu^l_1(N+1) + \mu^l_2(N+1) = 0) =$$

$$= P((\max_{n \le N} \mu^l_1(n) \ge x) \cup (\max_{n \le N} \mu^l_2(n) \ge y)) \le \sum_{n=1}^{N} P(\mu^l_1(n) \ge x) + \sum_{n=1}^{N} P(\mu^l_2(n) \ge y)$$

U holda (1.4.51),(1.4.52) ga ko'ra, yetarlicha katta $x$ uchun, oxirgi tengsizlikdan

$$P(\tau^l_{x,y} < +\infty, 1 \le T^l \le N) < E$$

(1.4.99)

Shu bilan birga yetarlicha katta $N$ uchun

$$\left| P(1 \le T^l \le N) - q_l \right| = \sum_{n=N+1}^{\infty} P(T^l = n) < E$$

(1.4.100)

Shunday qilib (1.4.95)-(1.4.100) larga asosan va $E$ ni ixtiyoriyligini hisobga olib 41-teorema uchinchi qismining to'g'riligiga ishonch hosil qilamiz. Quyidagi ikkala teoremani isboti 41-teoremaning isboti kabi bo'lganligi uchun ularni isbotsiz keltiramiz:

**42-TEOREMA**. Agar $\rho > 1, \ B^l_{k,j} < +\infty, \ l,k,j = 1,2$

bo'lsa u holda $x \to \infty$ da

$$\sup_{-\infty < m < +\infty} \left| P\left( r^l_x s = \left[ \frac{\ln x}{\ln \rho} \right] + m \right) - (1 - q_l) s^l(m,x) \right| = 0(1),$$

$$P(r^l_x = +\infty) = q_l + 0(1)$$

bo'ladi.

**43-TEOREMA**. Agar $a_l^l > 1,\ B_{k,j}^l < +\infty, \quad l,k,j = 1,2$ bo'lsa, u holda $r \to \infty$ shart bajarilganda

$$\sup_{-\infty<m<+\infty}\left| P\left( \tau_r^l = \left[ \frac{\ln \frac{r}{\sqrt{v_1^2 + v_2^2}}}{\ln \rho} \right] + m \right) - (1-q_l) S^l \left( m, \frac{r}{\sqrt{v_1^2 + v_2^2}} \right) \right| = 0(1),$$

$$P(\tau_r^l = +\infty) = q_l + 0(1)$$

## Ikki xilli kritikdan keyingi Galton Vatson jarayoni uchun integral teoremalar

Teoremalarni keltirishdan oldin bir nechta lemmalarni isbotlaymiz.

1-LEMMA. Agar $\rho > 1,\ B_{j,k}^l < +\infty, \quad l,j,k = 1,2$ bajarilsa u holda ixtiyoriy $\tau_1, \tau_2$ uchun $n \to \infty$ da

$$\Omega_n^l(r_1, r_2) = M(\exp(i\rho^{\frac{n}{2}}((W_1^l - v_1 < W^l(n), u >)\tau_1 + (W_2^l - v_2 < W^l(n), u >)\tau_2))) =$$

$$= M\left( \exp\left( -\frac{\xi^l < v_1 D\xi > (< v_1, \tau >)^2}{2} \right) \right) + 0(1)$$

bajariladi, bu yerda va bundan keyin $< x, y >$ bilan $x, y$ ni skalyar ko'paytmasini belgilaymiz, ya'ni $x = (x_1, x_2),\ y = (y_1, y_2)$ bo'lsa $< x, y >= x_1 y_1 + x_2 y_2$

Isbot. Ikki Gal'ton-Vatson jarayonining ta'rifidan foydalanib

$$P\left(\mu_j^l(r) = m / \mu^l(n) = (k_1, k_2)\right) = (P(\mu_{j*}^l(r-n) = m))^{*k_1} * (P(\mu_2^l(r-n) = m))^{*k_2} \tag{1.4.101}$$

ni hosil qilamiz, bu yerda taqsimot yig'masini bilidiradi.

U holda (1.4.101) dan

$$\Omega_n^l(\tau_1, \tau^2) = \lim_{r\to} M(\exp p\left( i\rho^{\frac{n}{2}}((W_1^l(r) - v_1 < W^l(n), u > \right)\tau_1 + (W_2^l(r) - v_2 < W^l(n), u >)\tau_2))) =$$

$$= \lim_{r\to\infty} \sum_{k_1,k_2=0}^{\infty} P_{k_1,k_2}^l(n) M(\exp(i\rho^{\frac{n}{2}}((W_1^l(r) - v_1 < W^l(n), u >)\tau + (W_2^l(r) - v_2 < W^l(n), u > \tau_2)))$$

$$/ \mu_1^l(n) = k_1, \mu_2^l(n) = k_2) = \sum_{k_1,k_2=0}^{\infty} P_{k_1,k_2}^l(n)(M(\exp(i\rho^{-\frac{n}{2}}(\xi^1 - u_i) < v\tau >)))^{k_1}$$

$$(M(\exp(i\rho^{-\frac{n}{2}}(\xi^2-u_2)<v,\tau>)))^{k_2}=M((\left((M\exp(i\rho^{-\frac{n}{2}}(\xi^1-u_1)<v,\tau>))^{\rho^n}\right)^{\frac{\mu_1^l(n)}{\rho^n}}\cdot$$

$$\cdot\left((M\exp(i\rho^{-\frac{n}{2}}(\xi^2-u_2)<v,\tau>))^{\rho^n}\right)^{\frac{\mu_2^l(n)}{\rho^n}}$$

(1.4.102)

tenglikka ega bo'lamiz, bu yerda $\tau=(\tau_1,\tau_2)$ yetarlicha katta $n$ lar uchun

$$M(\exp(i\rho^{-\frac{n}{2}}(\xi^l-u_l)<v_1,\tau))=1-\rho^{-n}(<v,\tau>)^2\cdot\frac{D\xi^l}{2}+0(\rho^{-n}(\tau_1{}^2+\tau_2{}^2)),\ e=1,2$$

(1.4.103)

Ixtiyoriy $T_0>0$ uchun (1.3.103) dan $|\tau_l|<T_0$ lardan

$$\lim_{n\to\infty}\ln(M(\exp(ip^{-\frac{n}{2}}(\xi^l-u_l)<\mathrm{v}_1,\tau>))^{\rho^n}=-\frac{(<v_1\tau>)^2}{2}D\xi^2$$

(1.4.104)

Natijada 1-lemmaning isboti (1.4.102) va (1.4.104) lardan kelib chiqadi.

2-LEMMA: Agar $\rho>1, B_{k,j}^l<\infty,\quad l,j,k=1,2$ bo'lsa, u holda $\tau_1$ uchun $n\to\infty$ da

$$\Omega_n^l(\tau_1)=M\left(\exp\left((\xi^l-<u,w_1^l(n)>)i\tau\rho^{\frac{n}{2}}\right)\right)=M(\exp\left(-\frac{\xi^l<v,D\xi>}{2}\tau_1^2)\right)+0(1)$$

Isbot. Jarayon ta'rifiga ko'ra $\tau>n$ uchun

$$\Omega_n^l(\tau_1)=\lim_{r\to+\infty}M\left(\exp(\xi_j^l(r)-<u_1w^l(n)>\right)i\tau_1=M(((Me^{i\tau_1\rho^{-\frac{n}{2}}}(\xi^1-u_1)^{\rho^n(v_1)})^{\frac{\mu_1^l(n)}{v_1\rho^n}}$$

$$((Me^{i\tau_1\rho^{-\frac{n}{2}}}(\xi^2-u_2))^{\rho^n v_2})^{\frac{\mu_2^l(n)}{v_2\rho^n}}$$

ifodani hosil qilamiz, bu yerda $\xi_j^l(r)=\dfrac{\mu_j^l(r)}{v_j\rho^r},\quad e,j=1,2$

Bu tenglikdan 2-lemma isboti kelib chiqadi.

3-LEMMA. Agar $\rho>1,\quad D_{k,j}^l<+\infty,\ j,k=1,2$ bo'lsa u holda ixtiyoriy $\tau_1,\tau_2>0$ sonlar uchun $n\to\infty$ da

$$\overline{\Omega}_n^l(\tau_1,\tau_2)=M\left(\exp\left(i\rho^n\left(\frac{(W_1^l-v_1<W^l(n),u>)\tau_1+(W_2^l-v_2<W^l(n),u>)\tau_2}{\sqrt{<\mu^l(n),D\xi>}}\Big/E\right)\right)\right)=$$

$$\exp\left(-\frac{(<\tau,v>)^2}{2}\right)+0(1),$$

bu yerda $\xi=(\xi^1,\xi^2)$, $E=(\mu_1^l(n)>0,\ \mu_2^l(n)>0)$.

Isbot. (1.4.101)ga ko'ra

$$\overline{\Omega}_n^l(\tau_1,\tau_2)=\sum_{k_1,k_2=1}^{\infty}\frac{P_{k_1,k_2}^l(n)}{P(E)}\cdot\left(M\left(\exp\left(\frac{i(\xi'-u_1)<v_1\tau>}{\sqrt{<k_1D\xi>}}\right)\right)\right)^{k_1}\cdot\left(M\left(\exp\left(\frac{i(\xi^2-u_2)<v_1\tau>}{\sqrt{<k_1D\xi>}}\right)\right)\right)^{k_2},$$

bu yerda $K=(k_1,k_2)$

O'ng tomondagi yig'indini to'rtta qismga ajratamiz:

$$\overline{\Omega}_n^l(\tau_1,\tau_2)=T_1+T_2+T_3+T_4, \tag{1.4.105}$$

bu yerda $\tau_1$ da yig'indi chegarasi $1\le k_1,\ k_2\le[\rho^{\frac{n}{2}}]$,

$T_2$ da $1\le k_1<\left[\rho^{\frac{n}{2}}\right],\left[\rho^{\frac{n}{2}}\right]\le k_2<\infty$, $T_3$ da $\left[\rho^{\frac{n}{2}}\right]\le k_1<\infty$,

$1\le k_2<\left[\rho^{\frac{n}{2}}\right]$, $T_4$ da $k_1\ge\left[\rho^{\frac{n}{2}}\right]$, $k_2\ge\left[\rho^{\frac{n}{2}}\right]$

Ishonch hosil qilish mumkinki

$$|T_1|\le P\left(\mu_1^l(n)<[\rho^{\frac{n}{2}}],\mu_2^l(n)<\left[\rho^{\frac{n}{2}}\right]\Big/E\right)\le P\left(\frac{\mu_1^l(n)}{\rho^n v_1}<\frac{1}{v_1\rho^{\frac{n}{2}}}\Big/E\right),$$

$$|T_2|\le P\left(\mu_1^l(n)\ge\left[\rho^{\frac{n}{2}}\right],\mu_2^l(n)\ge\left[\rho^{\frac{n}{2}}\right]\Big/E\right)\le P\left(\frac{\mu_1^l(n)}{\rho^n v_1}<\frac{1}{v_1\rho^{\frac{n}{2}}}\Big/E\right),$$

$$|T_2|\le P\left(\mu_1^l(n)\ge\left[\rho^{\frac{n}{2}}\right],\mu_2^l(n)\ge\left[\rho^{\frac{n}{2}}\right]\Big/E\right)\le P\left(\frac{\mu_1^l(n)}{\rho^n v_1}<\frac{1}{v_1\rho^{\frac{n}{2}}}\Big/E\right).$$

Agar $P\left(\xi_j^l(n)<x/\xi_j^l(n)x\right)$ ning limiti $P\left(\xi^l<x/\xi>0\right)$ ning absolyut uzluksizligini hisobga olsa, u holda $n\to\infty$ da

$$T_1=0(1),\qquad T_2=0(1),\qquad T_3=0(1) \tag{1.4.106}$$

$k_1,k_2>\rho^{\frac{n}{2}}$ bo'lgan holda

$$M\left(\exp\left(\frac{i(\xi^l - u_l) < v_1\tau >}{\sqrt{< k_1 D\xi >}}\right)\right) = 1 - \frac{(< \tau, v >)^2 D\xi^l}{1 < k_1, D\xi >} + 0\left(\frac{\tau_1{}^2 + \tau_2{}^2}{< k_1 D\xi >}\right), \qquad l = 1,2$$

Bundan

$$\left(M\left(\exp\left(\frac{i(\xi^l - u_l) < v_1\tau >}{\sqrt{< k_1, D\xi >}}\right)\right)\right)^{k_l} = \exp\left(-\frac{(< \tau, v >)^2 k_l D\xi^l}{2 < k, D\xi >}\right) + 0(1)$$

(1.4.107)

Ishonch hosil qilish mumkinki

$$\sum_{k_1, k_2 > \left[\rho^{\frac{n}{2}}\right]} P^l_{k_1,k_2}(n) = P(E) + 0(1)$$

(1.4.108)

Natijada 3-lemma isboti(1.4.105)-(1.4.108) lardan kelib chiqadi.

4-LEMMA. Agar $\rho > 1, D^l_{i,j,k} < +\infty, l, k, j = 1,2$ bo'lsa u holda

$|\tau_1| \le \frac{1}{4h_{k_1,k_2}}, \; h_{k_1,k_2} = \frac{< k_1\beta >}{(< k_1 D\xi >)^{3/2}}$ uchun

$$\left|\left(\overline{\Omega}^1_{k_1,k_2}(\tau_1)\right)^{k_1}\left(\overline{\Omega}^2_{k_1,k_2}(\tau_1)\right)^{k_2} - e^{-\frac{\tau_1{}^2}{2}}\right| \le 16 h_{k_1,k_2}|\tau_1|^3 e^{-\frac{\tau_1{}^2}{3}},$$

bu yerda $\overline{\beta} = M\left|\xi^j - u_j\right|^3, \; j = 1,2, \; D^l_{i,j,v} = M\mu^l_i(1)\mu^l_j(1)\mu^l_v(1),$

$$\overline{\Omega}^1_{k_1,k_2}(\tau_1) = M\left(\exp\frac{i\tau_1(\xi^l - u_l)}{\sqrt{< k_1 D\xi >}}\right), \qquad l = 1,2$$

5-LEMMA. Agar $\rho > 1, \; D^l_{i,j,y} < \infty$ bo'lsa, u holda

$$|\tau_1| \le \frac{\min(3\min(D\xi^i) < D\xi, v > \min\limits_i (D\xi^i)^{3/2} v_j^{3/4}}{14\max\limits_{1 \le j \le k} \overline{\rho}_j}$$

(1.4.109)

uchun

$$\left|\Omega^l_n\left(\frac{\tau_1}{\sqrt{< v, D\xi >}}\right) - Me^{-\frac{\xi^l\tau_1{}^2}{2}}\right| \le \frac{7|\tau_1|^3 \max\overline{\beta}_i}{6(< v, D\xi >)^{3/2}}\left[M\left(\left(W^l_1(n) + W^l_2(n)\right)\right)\times\right.$$

$$\times\exp\left(-\frac{\tau_1{}^2}{4}\cdot\frac{W^l_1(n) + W^l_2(n)}{< v_1 D\xi >}\min_{1 \le j \le 2} D\xi^j + \frac{\tau_1{}^2}{2}M\left(\left(\frac{< D\xi_1 W^l(n) >}{< v_1 D\xi >} - \xi\right)\right)\times\right.$$

$$\times \exp\left(-\frac{\tau_1^{\ 2}}{2}\min\left(\frac{D\xi_1 W^l(n)>}{<v_1, D\xi>}, \xi^l\right)\right)\Bigg]$$

ISBOT:

$$\Omega_n^l\left(\frac{\tau_1}{\sqrt{<vD\xi>}}\right) = \int_0^{+\infty}\int_0^{+\infty}\left(\Psi^1\left(\frac{\tau_1}{\sqrt{<vD\xi>\rho^n}}\right)\right)^{v_1 y_1 \rho^n}\left(\Psi^2\left(\frac{\tau_1}{\sqrt{<vD\xi>\rho^n}}\right)\right)^{v_2 y_2 \rho^n} \times$$

$$\times dp\left(\frac{w_1^i(n)}{v_1 < y_1}, \frac{w_2^j(n)}{v_2} < y_2\right)$$

(1.4.110)

ni hosil qilamiz, bu yerda

$$\Psi^l(\tau_1) = Me^{i\tau_1(\xi^l - u_l)}, \quad l = 1{,}2$$

$\Psi^j(\tau_1)$ ни $\frac{\tau_1}{\sqrt{<vD\xi>\rho hn}}$ darajalari bo'yicha qatorga yoyib

$$\left|\Psi^l\left(\frac{\tau_1}{\sqrt{<vD>\rho^n}}\right)\right| \geq 1 - \frac{\tau_1^{\ 2}}{2}\cdot\frac{D\xi_l}{<vD\xi>\rho^n} - \frac{|\tau_1|^3}{6}\cdot\frac{\overline{\beta}_2}{(<vD\xi>)^{3/2}\cdot\rho^{3n/2}} =$$

$$= 1 - \frac{\tau_1^{\ 2}}{2\rho^n}\left(\frac{D\xi_l}{<vD\xi>} + \frac{|\tau_1|}{3}\cdot\frac{\overline{\beta}_2}{(<vD\xi>)^{3/2}\sqrt{\rho^n}}\right)$$

(1.4.111)

5-lemma shartini hisobga olsak

$$|\tau_1| \leq \frac{\min\limits_{1\leq i\leq 2}(D\xi_i)^{3/2}}{14\max\limits_{1\leq i\leq 2}}\cdot v_l^{\ 3/4}\sqrt{\rho^n}$$

(1.4.112)

Shu bilan birga

$$\frac{D\xi_l}{<vD\xi>} \leq \frac{1}{v_l}$$

(1.4.113)

U holda (1.4.112)(1.4.113) ga ko'ra

$$\frac{D\xi_l}{<vD\xi>} + \frac{|\tau_1|}{3}\cdot\frac{\overline{\beta}_2}{(<vD\xi>)^{3/2}\sqrt{\rho^n}} \leq \frac{16}{15}\cdot\frac{l}{v_j^{\ 3/2}}$$

(1.4.114)

Ma'lumki dispersiya va uchinchi absolyut momet to'g'risidagi tengsizlikka ko'ra

$$\min_{1\le j\le 2}\left(D\xi_j\right)^{3/2} \le \max_{1\le j\le 2} \overline{\beta}_j$$

U holda oxirgi tengsizlikdan (1.4.111)(1.4.114) larga asosan

$$\left|\Psi^l\left(\frac{\tau_1}{\sqrt{<vD\xi>\rho^n}}\right)\right| \ge \frac{24}{25}$$

ni hosil qilamiz. Natijada  ni logarifmlash mumkinlagi kelib chiqadi:

$$\ln\Psi^l\left(\frac{\tau_1}{\sqrt{<vD\xi>\rho^n}}\right) = -\frac{\tau_1}{2\rho^n}\cdot\frac{D\xi_l}{<vD\xi>} + \frac{\tau_1^3}{6}\cdot\frac{\overline{\beta}_l}{(<vD\xi>)^{3/2}\rho^{\frac{3n}{2}}},$$

(1.4.115)

bu yerda

$$\left|\overline{\beta}_l\right| \le \left|\frac{d^3}{dz^3}\ln\Psi^l(z)\right|_{t=\frac{\theta\tau_1}{\sqrt{<vD\xi>\rho^n}}} \le 7\beta_j,\ j=1,2$$

(1.4.116)

(1.4.110) va (1.4.115) lardan

$$\tau_1 = \left|\Omega_n^l\left(\frac{\tau_1}{\sqrt{<vD\xi>\rho^n}}\right) - \int_0^{+\infty}\int_0^{+\infty} e^{-\frac{\tau_1^2}{2}\cdot\frac{y_1v_1D\xi_1+y_2v_2D\xi_2}{<v_1D\xi>}} \times dp\left(\frac{w_1^l(n)}{v_1}<y_1, \frac{W_2^l(n)}{v_2}<y_2\right) \le\right|$$

$$\int_0^{+\infty}\int_0^{+\infty} e^{-\frac{\tau_1^2}{2},\frac{y_1v_1D\xi_1+y_2v_2D\xi_2}{<v_1D\xi>}} \times\left|\exp\left(\frac{\tau_1^3(v_1\overline{\beta}_1y_1+v_2\overline{\beta}_2y_2)}{6(<vD\xi>)^{3/2}\sqrt{\rho^n}}\right)-1\right|dp\left(\frac{w_1^l(n)}{v_1}<y_1, \frac{w_2^l(n)}{v_2}y_2\right)$$

tengsizlikka ega bo'lamiz.

5-lemma shartidan $\tau_1$ ni tanlashga ko'ra

$$\frac{\min_i D\xi i}{<v,D\xi>} - \frac{7|\tau_1|\min_n\beta_i}{3\sqrt{(<v,D\xi>)^3\rho^n}} \ge \frac{\min_i D\xi i}{2<v,D\xi>},$$

(1.4.117)

natijada

$$\left|e^x-1\right| \le |x|e^{|x|}$$

ni hisobga olib (1.4.116),(1.4.117) lardan

$$T_1 \le \frac{7|\tau_1|^3}{6\sqrt{(<v_1 D\xi>)^3}\rho^n} \max_i \beta_i M(W_1^l(n)+W_2^l(n)) \times \exp\left(-\frac{\tau_1^2 (W_1^l(n)+W_2^l(n))}{4<v_1 D\xi>} \min_i v_1 D\xi\right)$$

(1.4.118)

Ushbu $\left|e^{-\alpha}-e^{-\beta}\right| \le |\alpha-\beta| \exp(-\min(\alpha,\beta))$ ga ko'ra

$$T_2 = \left|\int_0^{+\infty}\int_0^{+\infty} \exp\left(-\frac{\tau^2}{2}\cdot\frac{y_1 v_1 D\xi_1 + y_2 v_2 D\xi_2}{<v_1 D\xi>}\right) dp\left(\frac{W_1^l(n)}{v_1}<y_1, \frac{W_2^l(n)}{v_2}<y_2\right) - \int_0^{+\infty} e^{-\frac{\tau_1^2}{2}y_3} dp(\xi^l<y_3)\right| \le$$

$$\le \frac{\tau_1^2}{2} M\left(\left|\frac{<D\xi, W^l(n)>}{<v_1 D\xi>} - \xi^l\right|\right) \times \exp\left(-\frac{\tau_1^2}{2}\min\left(\frac{<D\xi W^l(n)>}{<v, D\xi>}, \xi^l\right)\right)$$

(1.4.119)

Agar

$$\left|\Omega_n^l\left(\frac{\tau_1}{\sqrt{<v, D\xi>}}\right) - Me^{-\frac{\xi^2\tau_1^2}{2}}\right| \le T_1 + T_2$$

ligini hisobga olsak (1.4.118),(1.4.119) lardan 5-lemmaning isboti kelib chiqadi.

**44-TEOREMA**. Agar $\rho>1, \quad D_{j,k,v}^l<+\infty, \quad e, j, k = 1,2$

bo'lsa

$$\left|P\left(\frac{\rho^n(\xi^l - <W^l(n), u>)}{\sqrt{<\mu^l(n)D\xi>}} < x/E\right) - \Phi(x)\right| \le \frac{c}{\rho^n} M\left(\frac{<w^l(n), \beta>}{(<W^l(n), D\xi>)^{3/2}} \Big/ E\right), \quad \beta = (\bar\beta_1, \bar\beta_2)$$

bo'ladi.

Isbot. (1.4.101) ga asosan

$$M\left(\exp\left(\frac{i\tau_1(\xi^l - <W^l(n), u>)}{\sqrt{<\mu^l(n)D\xi>}} \Big/ E\right)\right) = \sum_{k_1,k_2=1}^{\infty} \frac{P_{k_1,k_2}^l(n)}{P(E)} \cdot \left(\overline{\Omega}_{k_1,k_2}^1(\tau_1)\right)^{k_1} \left(\overline{\Omega}_{k_1,k_2}^2(\tau_1)\right)^{k_2}, \quad e=1,2$$

yoki

$$\left|M\left(\left(\exp\left(\frac{i\tau_1(\xi^l - <W^l(n), u>)}{\sqrt{<\mu^l(n)D\xi>}}\right)\right) \Big/ E\right) - e^{-\frac{\tau_1^2}{2}}\right| \le \sum_{k_1,k_2=1}^{\infty} \frac{P_{k_1,k_2}^l(n)}{P(E)} \left|\left(\overline{\Omega}_{k_1,k_2}^1(\tau_1)\right)^{k_1} \left(\overline{\Omega}_{k_1,k_2}^2(\tau_1)\right)^{k_2} - e^{-\frac{\tau_1^2}{2}}\right|$$

So'ngi munosabatda 4-lemmani qo'llab teoremani to'g'riligiga ishonch hosil qilamiz.

**47-TEOREMA**. Agar $\rho>1, \quad D_{j,k,v}^l<+\infty, \quad e, j, k = 1,2$

bo'lsa

$$\sup_x \left| P\left\{ \frac{\sqrt{\rho^n}(\xi^l - \langle u, W^l(n)\rangle)}{\sqrt{\langle u_1 D\xi\rangle}} < x \right\} - \int_0^{+\infty} \Phi\left(\frac{x}{\sqrt{y}}\right) dp(\xi^l < y) \right| \le c \frac{\max \beta_j}{\sqrt{\langle v_1 D\xi\rangle \min_j D\xi^j \sqrt[3]{\rho^n}}} +$$

$$+ \max\left( \sqrt{M\left(\frac{\langle D\xi, W^l(n)\rangle}{\langle v_1 D\xi\rangle} - \xi^l\right)^2 \frac{\rho^n}{1+\rho_1^{\,n}}}, 1 \right) \times \max\left( \sqrt[6]{\left(\frac{\rho 1}{\rho}\right)^n}, \rho^{-\frac{n}{6}} \right),$$

bu yerda $\rho_1 = |\overline{\rho}| < \rho$

Isbot. Avval $\rho_1 \le 1$ xolni karaylik. Quyidagi belgilarni kiritamiz:

$$\Delta_n^l = \sup_x \left| P\left\{ \frac{\sqrt{\rho^n}(\xi^l - \langle W^l(n)\rangle)}{\sqrt{\langle u_1 D\xi\rangle}} < x \right\} - \int_0^{+\infty} \Phi\left(\frac{x}{\sqrt{y}}\right) dp(\xi^l < y) \right|,$$

$$E_n^l = \int_{-\rho^{\frac{n}{6}}}^{\rho^{\frac{n}{6}}} \frac{\left| \Omega_n^l\left(\frac{\tau_1}{\sqrt{\langle v_1 D\xi\rangle}}\right) - Me^{-\frac{\xi^l \tau_1^2}{2}} \right|}{|\tau_1|} d\tau_1$$

U holda, 5- lemmaga ko'ra

$$R_1^l = \frac{7\max \beta_i}{6(\langle v_1 D\xi\rangle)^{3/2}\sqrt{\rho^n}} \int_{-\rho^{\frac{n}{2}}}^{\rho^{\frac{n}{2}}} \tau_1^{\,2} M((W_1^l(n) + W_2^l(n)))e - \frac{\tau^2}{4} \cdot \frac{W_1^l(n) + W_2^l(n)}{\langle v, D\xi\rangle} \min_i D\xi^i \, d\tau \le$$

$$\le c \frac{\max_i \beta_i}{\sqrt{\rho^n} \langle v_1 D\xi\rangle} \rho^{\frac{n}{6}} = c \frac{\max_i \beta_i}{\langle v_1 D\xi\rangle \min_i D\xi \sqrt[3]{\rho^n}},$$

(1.4.120)

$$R_2^l = \frac{1}{2} \int_{-\rho^{\frac{n}{6}}}^{\rho^{\frac{n}{6}}} |\tau_1| M\left( \left| \frac{\langle D\xi_1 W^l(n)\rangle}{\langle v_1 D\xi\rangle} - \xi^l \right| \times \exp\left( -\frac{\tau_1^{\,2}}{2} \min\left( \frac{\langle D\xi, W^l(n)\rangle}{\langle v, D\xi\rangle}, \xi^l \right) \right) \right) d\tau_1 \le$$

$$\le \sqrt{M\left(\frac{\langle D\xi, W^l(n)\rangle}{\langle v, D\xi\rangle} - \xi^l\right)^2} \rho^{-\frac{2n}{6}}$$

(1.4.121)

bu yerda

$$E_n^l \le R_1^l + R_2^l$$

(1.4.122)

Endi $\rho_1 > 1$ bo'lsin, u holda

$$R_1^l \le C \frac{\max\limits_i \beta i}{\sqrt{< v_1 D\xi >} \min D\xi^i \sqrt[3]{\rho^n} \sqrt[6]{\rho_1{}^n}},$$

(1.4.123)

$$R_2^l \le \sqrt{M\left(\frac{< D\xi_1 W^l(n) >}{< v_1 D\xi) >} - \xi^l\right) \sqrt[6]{\left(\rho^{-1} \cdot \rho_1\right)^n}}$$

(1.4.124)

Natijada Essen teoremasini qo'llab (1.4.120)-(1.4.124) lardan teoremaning to'g'riligiga ishonch hosil qilamiz.

Quyidagi teoremani isbati 44,45 teorema isbotidek bo'lgani uchun, isbotsiz keltiramiz:

**46-TEOREMA**. Agar $\rho > 1, \quad D_{j,k,v}^l < +\infty, \quad e, j, k = 1,2$

bajarilsa

$$a) \sup_{x_1, x_2} \left| P\left\{ \frac{\sqrt{\rho^n}\,(W_i^l - v_i < u, W^l(n) >)}{v^l \sqrt{< v_1 D\xi >}} < x_i, \qquad i = 1,2 \right\} - \int_0^{+\infty} \Phi\left( \frac{\min\limits_{1\le i\le 2}}{\sqrt{y}} \right) dp\left(\xi^l < y\right) \right| \le$$

$$\le c \left| \frac{\max\limits_i \beta i}{\sqrt{< v_1 D\xi >} \min\limits_i D\xi^i \sqrt[3]{\rho^n}} + \max\left( M\left( \frac{< D\xi, W^l(n) >}{v_1 D\xi} \right)^2 \cdot \frac{\rho^n}{1 + \rho_1{}^n}, 1 \right) \max \sqrt[6]{\left(\frac{\rho_1}{\rho}\right)}, \rho^{-\frac{6}{n}}. \right.$$

$$б) \sup_{x_1, x_2} \left| P\left( \frac{\rho^n (W_i^l - v_i < u_1 W^l(n)) >}{v_2^l \sqrt{< \mu^l(n) D\xi >}} < x_i, \qquad i = 1,2 / E \right) - \Phi\left( \min_{1\le i\le 2} x_i \right) \right| \le$$

$$\frac{c}{\rho^n} M\left( \frac{< \beta_i W^l(n) >}{(D\xi, W^l(n) >)^{3/2}} \Big/ E \right)$$

Bunday natijalarni bir xilli jarayonlar uchun o'rganilgan.

44-46 teoremalar deyarli o'zgarishsiz ikki xilli uzluksiz vaqtli tarmoqlanish jarayoni uchun xam o'rinlidir.

**47-TEOREMA**. Agar $\rho > 1,\ B_{k,l}^j < +\infty,\ \lim\limits_{n\to\infty} \ln^j, = +\infty$

$C_n^1 \sim c_n^2$ bo'lsa, u holda

$$\lim_{n\to\infty} P\left\{ \frac{\mu_i(n) v_j{}^{-1} - \rho^n (< u c_n >)}{bn} < x / \mu(0) = (C_n^1, C_n^2) \right\} = \Phi(x)$$

ISBOT: Tarmoqlanish jarayoni xossasiga ko'ra

$$P\left(\mu_i(n)=\alpha_1 / \mu(0)=(C_n^1, C_n^2)\right)=P\left(\mu_j^1(n)=\alpha_1\right)*^{C_n^1} \times P(\mu_j^2(n)=\alpha_1)*^{C_n^2}$$

U holda bu tenglikdan

$$M\left(e^{\frac{i\tau/\mu_j(n)v_j^{-1}-\rho^n(<uC_n>)}{bn}} \Big/ \mu(0)=(C_n^1, C_n^2)\right)=e^{\frac{-i\rho^n(<uC_n>)\tau}{bn}}\left[M\left(e^{\frac{i\tau\mu_i(n)}{v_j bn}}\right)\right]^{C_n^1}\cdot\left[M\left(e\frac{i\tau\mu_j{}^2(n)}{v_j bn}\right)\right]^{C_n^2}$$

Ifodaga ega bo'lamiz.

Agar (1.4112)

$$B_i^j(n)=\frac{\sum_{\theta_i m=1}^{2} d_y^j B_{im} u_i u_m v_k v_e}{\begin{vmatrix} \rho^2-a_1^1 & a_2^1 \\ a_1^2 & \rho^2-a_2^2 \end{vmatrix}}\rho^{2n}(1+0(1))$$

(1.4.124.a)

ni hisobga olsak

$$M\left(\exp\left(\left(i\tau\mu_j^k(n)\right): v_j bn\right)\right)=1+i\tau\frac{uk}{bn}\rho^n-\frac{\tau^2 A^k\rho^{2n}}{2b_n{}^2}+(\rho^{2n}: b_n{}^2)$$

- Natijada oxirgi tenglikdan 47-teoremani to'g'riligiga ishonch hosil qilamiz.

**48-TEOREMA**. Agar $\rho>1, D_{i,j,v}^l<+\infty, C_n^1 \sim C_n^2$ bo'lsa

$$\sup_x\left|P\left\{\frac{\mu_j v_j{}^{-1}-\rho^n<u_1 C_n>}{bj(n)}<x \Big/ \mu(0)=\left(C_n^1, C_n^2\right)\right\}-\Phi(x)\right| \leq$$

$$C\frac{1}{\sqrt{\min_i C_n^i}}\cdot\frac{\max_{ij}\beta_3^{i,j}(n)}{\min_{i,j}\sigma_{i,j}{}^3(n)}, \qquad \frac{\max_{i,j}\beta^{i,j}(n)}{\min_{i,j}\sigma_{i,j}{}^3(n)}\leq C_1,$$

bu yerda $C_1$ $n$ ga bog'liq bo'lmagan o'zgarmas son

$$b_j(n)=\sqrt{M\left(\mu_j^1(n)v_j{}^{-1}-u_1\rho^n\right)^2+M\left(\mu_j^2(n)v_j{}^{-1}-u_2\rho^n\right)^2},$$

$$\beta_3^{i,j}(n)=M\left|\left(\mu_j^i(n)v_j{}^{-1}-u_j\rho^n\right): b_j(n)\right|^3,$$

$$\sigma^2{}_{i,j}(n)=M\left(\left(\mu_i^j(n)v_j{}^{-1}-u_i\rho^n\right): b_j(n)\right)^2$$

Teoremaning isboti

$$P(\mu_1(n)=\alpha_1,\ \mu_2(n)=\alpha_2 / \mu(0)=(c_n^1, c_n^2))=$$

$$P(\mu_1^1(n)=\alpha_1,\ \mu_2^1(n)=\alpha_2)*^{C_n^1}(P(\mu_1^2(n)=\alpha_1,\mu_2^l(n)=\alpha_2))*^{C_n^1}$$

(1.4.124.b)

Bu ehtimollikning hosil qiluvchi funksiyasi

$$(F_n^1(S_1,S_2))^{C_n^1}\cdot(F_n^2(S_1,S_2))^{C_n^1}$$

(1.4.124.v)

lardan foydalangan holda teorema isboti olib boriladi.

**Ikki xilli kritikdan keyingi tarmoqlanish jarayoni**

**uchun lokal teorema**

Faraz qilaylik $q_j=0,\ j=1{,}2$ bo'lsin, agar $q_i\neq 0$ bo'lsa u holda shakl almashtirish bilan $q_j=0$ holga keltirish mumkin.

Quyidagi belgilar kiritamiz:

$$m_i^j=\left.\frac{\partial F_i^j(S_1,S_2)}{\partial S_i}\right|_{\substack{S_1=0\\S_2=0}}\qquad A=\|m_i^j\|,$$

$S_q-A$ matritsaning perron ildizi bo'lsin

$$\varphi_n^j(t_1,t_2)=Me^{i(<t,W^j(n)>)},t=(t_1,t_2)$$

$$\varphi^j(t_1,t_2)=Me^{i(<tW^j>)}.$$

Ma'lumki $\varphi^j(t_1,t_2)$ quyidagi tenglamani qanoatlantiradi:

$$\varphi^j(t_1,\rho^n,t_2\rho^n)=F_n^j(\varphi'(t_1,t_2),\varphi^2(t_1,t_2)),\ n\geq 1$$

(1.4.125)

**49-TEOREMA**: Agar $\rho>1,\ B_{k,l}^j<\infty,\ j,k,l=1{,}2$ bajarilsa u holda $n$ ga bog'liq bo'lmagan shunday musbat $c$ soni topiladiki

$$\left|\rho^{2n}P_{r_1,r_2}^j(n)-w^j(\rho^{-n}r_1,\rho^{-n}r_2)\right|\leq C\left\{\left(\rho^{-n}r_1\right)^{-1}\left(\rho^{-n}r_2\right)^{-n}\rho_1{}^n+\rho_2{}^n\right\},$$

bu yerda $w^j(x,y)W^j$ ning zichlik funksiyasi $\rho_1$ va $\rho_2$ lar shunday sonlarki $0<\rho,\ \rho_2<1,\ \ P_{r_1,r_2}^j(n)=P\left(\mu_i^j(n)=r_i,i=1{,}2\right)$

Teoremani isbotlash uchun bir nechta lemmalarni keltiramiz.

1-LEMMA: Agar $\rho>1,\ \ \mathrm{B}_{k,l}^j<\infty,\ j,k,l=1{,}2$ bo'lsa

$1^0.\ \lim_{n\to\infty} \rho^{\delta_0 n}\varphi^j(\rho^n t_1, \rho^n t_2) = k(\varphi^1(t_1,t_2), \varphi^2(t_1,t_2)), \qquad t_1^2 + t_2^2 \neq 0, \quad \delta_0 = -\log_\rho \rho_q$

(1.4.126)

$2^0.\ \sup_{\substack{-\infty<t_1<+\infty \\ -\infty<t_2<+\infty}} \left(\sqrt{t_1^2 + t_2^2}\right)^{\delta_0} \left|\varphi^j(t_1,t_2)\right| < +\infty,$

(1.4.127)

$3^0.\ \lim_{n\to\infty} \frac{\rho^{(1+\delta_0)^n \partial}\varphi^j(\rho^{n_t}, \rho^{n_t})}{\partial t_l} = \sum_{y=1}^{2} \frac{\partial k(\varphi_1(t_1,t_2), \varphi^2(t_1,t_2))}{\partial \varphi^y(t_1,t_2)} \times$

$\frac{\partial \varphi^y(t_1,t_2)}{\partial t_1},\ t_1^2 + t_2^2 \neq 0,\ j,l = 1,2$

(1.4.128)

$4^0.\ \sup_{\substack{-\infty<t_1<+\infty \\ -\infty<t_2<+\infty}} \left(\sqrt{t_1^2 + t_2^2}\right)^{1+\delta_0} \left|\frac{\partial \varphi^j(t_1,t_2)}{\partial t_l}\right| < +\infty,$

(1.4.129)

$5^0.\ \sup_{\substack{-\infty<t_1<+\infty \\ -\infty<t_2<+\infty}} \left(\sqrt{t_1^2 + t_2^2}\right)^{2+\delta_0} \left|\frac{\partial^2 \varphi^j(t_1,t_2)}{\partial t_l}\right| < +\infty, \qquad l, y = 1,2$

(1.4.130)

Lemmadagi - $K(S_1,S_2)$ funksiya (1.4.111) da aniqlangan.

Lemmaning isboti: (1.4.125) ga ko'ra

$$\rho^{\delta_0 n}\varphi^j(\rho^n t_1 \neq \rho^n t_2) = F_n^j\left(\varphi^1(t_1,t_2), \varphi^2(t_1,t_2)\right)\big/\rho^{-\delta_0 n} = F_n^j\left(\varphi^1(t_1,t_2), \varphi^2(t_1,t_2)\right)\big/\rho_q^{\,n}$$

(1.4.124.a) ga ko'ra

$$\frac{F_n(S_1,S_2)}{\rho_q^{\,n}} \uparrow K(S_1,S_2)u* \qquad (1.4.131)$$

u holda (1.4.130), (1.4.131) dan 1-lemmaning $1^0$-qismi kelib chiqadi.

$W^j$ taqsimot funksiyasining absolyut uzluksizligini hisobga olsak

$$\beta_j = \sup_{1\le\sqrt{t_1^2+t_2^2}\le\rho} \left|\varphi^j(t_1,t_2)\right| < 1, \qquad j = 1,2$$

(1.4.132)

bo'ladi. Ikkinchi tomondan (1.4.125) va (1.4.131) ga ko'ra

$$\left(\rho^n\sqrt{t_1^2 + t_2^2}\right)^{\delta_0}\left|\varphi^j\left(\rho^n t_1, \rho^n t_2\right)\right| \le \left(\sqrt{t_1^2 + t_2^2}\right)^{\delta_0}$$

$$F_n^j\left(\left|\varphi^1(t_1,t_2)\right|,\left|\varphi^2(t_1,t_2)\right|\right)\Big/\rho_q^n \le \left(\sqrt{t_1^{\ 2}+t_2^{\ 2}}\right)^{\delta_0} k\left(\left|\varphi^1(t_1,t_2)\right|,\left|\varphi^2(t_1,t_2)\right|\right)u_j^*$$

(1.4.133)

Natijada (1.4.132) va (1.4.133) ga ko'ra

$$\underset{1\le\sqrt{t_1^{\ 2}+t_2^{\ 2}}\le\rho}{Sup}\left(\rho^n\sqrt{t_1,t_2}\right)^{\delta_0}\left|\varphi^j\left(\rho^n t_1,\rho^n t_2\right)\right|\le\rho^{\delta_0}k(\beta_1,\beta_2)<\infty$$

Bunda

$$\underset{\rho^n\le\sqrt{t_1,t_2}\le\rho^{n+1}}{Sup}\left(\sqrt{t_1,t_2}\right)^{\delta_0}\left|\varphi^j(t_1,t_2)\right|<\rho^{\delta_0}k(\beta_1,\beta_2),$$

yoki

$$\underset{1\le\sqrt{t_1,t_2}}{Sup}\left(\sqrt{t_1,t_2}\right)^{\delta_0}\left|\varphi^j(t_1,t_2)\right|<+\infty$$

(1.4.134)

lekin

$$\underset{\sqrt{t_1,t_2}<1}{Sup}\left(\sqrt{t_1,t_2}\right)^{\delta_0}\left|\varphi^j(t_1,t_2)\right|<+\infty$$

ligi doim to'g'ri va demak oxirgi tengsizlikni (1.4.125) bilan birlashtirsak 1-lemmaning $2^0$ qismi kelib chiqadi.

(1.4.116) tenglama $t_l$ bo'yicha diferensiallab topamiz:

$$\frac{\rho^n\partial\varphi^j(\rho^n t_1,\rho^n t_2)}{\partial t_l}=\sum_{y=1}^{2}\frac{\partial F_n^j(\varphi^1(t_1,t_2),\varphi^2(t_1,t_2))}{\partial\varphi^y(t_1,t_2)}\cdot\frac{\partial\varphi^y(t_1,t_2)}{\partial t_l},\qquad l=1,2$$

Bundan

$$\rho^{n(1+\delta_0)}\cdot\frac{\partial\varphi^j(\rho^n t_1,\rho^n t_2)}{\partial t_l}=\sum_{y=1}^{2}\frac{\partial F_n^j(\varphi^1(t_1,t_2),\varphi^2(t_1,t_2))}{\rho_q^n\partial\varphi^y(t_1,t_2)}\cdot\frac{\partial\varphi^{(n)}(t_1,t_2)}{\partial t_l},\qquad l=1,2$$

(1.4.135)

Natijada oxirgi tenglikdan (1.4.134) ni hisobga olsak lemmaning $3^0$ qismi kelib chiqadi.

(1.4.134) dan

$$\frac{\partial^2 F_n(S_1,S_2)}{\rho_q^n\cdot\partial S_i\partial Slj}\uparrow\frac{\partial^2 k(S_1,S_2)}{\partial S_i\partial S_j}u*,\qquad i,j=\overline{1,2}$$

(1.4.136)

ifodaga ega bo'lamiz.

(1.4.135) va (1.4.136) dan

$$\left(\rho^n\sqrt{t_1^2+t_2^2}\right)^{1+\delta_0}\left|\frac{\partial\varphi^j(\rho^n t_1,\rho^n t_2)}{\partial t_l}\right|\le\left(\sqrt{t_1^2+t_2^2}\right)^{1+\delta_0}\cdot$$

$$\cdot\sum_{y=1}^{2}\frac{\partial k\left(\left|\varphi^1(t_1+t^2)\right|,\left|\varphi^2(t^2+t^2)\right|\right)}{\partial\varphi^y(t_1,t_2)}\left|\frac{\partial\varphi^y(t_1,t_2)}{\partial t_l}\right|$$

(1.4.137)

Ixtiyoriy $t_1$ va $t_2$ uchun

$$\left|\frac{\partial\varphi^y(t_1,t_2)}{\partial t_1}\right|\le MW_l^y=v_l M\xi^y=v_l u_y,\qquad y,l=1,2$$

ligini hisobga olsak (1.4.132) va (1.4.137) lardan

$$\sup_{1\le\sqrt{t_1^2+t_2^2}\le\rho}\left(\rho^n\sqrt{t_1^2+t_2^2}\right)^{1+\delta_0}\left|\frac{\partial\varphi^j(\rho^n t_1,\rho^n t_2)}{\partial t_l}\right|\le\rho^{1+\delta_0}\sum_{y=1}^{2}\frac{\partial k(\beta_1,\beta_2)}{\partial S_y}\cdot u_y v_l<+\infty$$

yoki

$$\sup_{1\le\sqrt{t_1^2+t_2^2}}\left(\sqrt{t_1^2+t_2^2}\right)^{1+\delta_0}\left|\frac{\partial\varphi^j(t_1,t_2)}{\partial t_l}\right|<+\infty$$

(1.4.138)

lekin

$$\sup_{\sqrt{t_1^2+t_2^2}<1}\left(\sqrt{t_1^2+t_2^2}\right)^{1+\delta_0}\left|\frac{\partial\varphi^j(t_1,t_2)}{\partial t_l}\right|<+\infty$$

ekanligini hisobga olsak (1.4.128) dan lemmaning $4^0$ qismi hosil bo’ladi shu usulni qo’llab lemmaning 5 qismini ham isbotlash mumkin.

2-LEMMA. Agar $B_{k,l}^j<+\infty$ bo’lsa u holda shunday o’zgarmas $C$ son mavjud bo’ladiki barcha $T_1>0, T_2>0$ lar uchun

$$\left|W^j(x_1,x_2)-\frac{1}{(2\pi)^2}\int_{-T_1}^{T_1}\int_{-T_2}^{T_2}e^{-i(<t,x>)}\varphi^j(t_1,t_2)dt_1,dt_2\right|\le C\frac{T_1^{-\delta_0}+x_1T_1T_2^{-\delta_0}}{x_1\cdot x_2},$$

bu yerda $x=(x_1,x_2)$

ISBOT: Ma’lumki

$$\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_l\partial t_y}=-\int_0^{+\infty}\int_0^{+\infty}x_l x_y e^{i(<t,x>)}W^j(x_1,x_2)dx_1,dx_2$$

(1.4.139)

Ikkinchi tomondan 1-lemmaning $5^0$ qismiga ko’ra

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1\partial t_2}\right|dt_1dt_2<+\infty,$$

Demak, (1.4.139) dan aylantirish formulasiga asosan

$$x_1x_2W^j(x_1,x_2)=-\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-i(<t,x>)}\frac{\partial\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2$$

ga ega bo'lamiz. Bu ifodaning o'ng tomonini bo'laklarga bo'lamiz.

$$x_1x_2W^j(x_1,x_2)=\frac{1}{(2\pi_i)^2}\int_{-T_1}^{T_1}\int_{-T2}^{T_2}e^{-i(<t,x>)}\cdot\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2+\frac{1}{(2\pi i)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-i(<t,x>)}\cdot\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2+$$

$$+\frac{1}{(2\pi i)^2}\int_{T_1}^{+\infty}\int_{-\infty}^{+\infty}e^{-i(<t,x>)}\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2+\frac{1}{(2\pi i)^2}\int_{T_1}^{T_1}\int_{-\infty}^{T_2}e^{-i(<t,x>)}\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2+$$

$$+\frac{1}{(2\pi i)^2}\int_{T_2}^{T_1}\int_{T_2}^{+\infty}e^{-i(<t,x>)}\frac{\partial^2\varphi^j(t_1,t_2)}{\partial t_1,\partial t_2}dt_1dt_2=T_1+T_2+T_3+T_4+T_5$$

(1.4.140)

$T_1$ ni bo'laklab integrallab

$$T_1=\frac{1}{(2\pi i)^2}\int_{-T_2}^{T_2}e^{i(<t,x>)}\frac{\partial\varphi^j(t_1,t_2)}{\partial t_2}\Bigg|_{-T_1}^{T_2}dt_2+\frac{x_ii}{(2\pi i)^2}\int_{-T_1}^{T_1}\int_{-Tl2}^{T_2}e^{-i(<t,x>)}\frac{\partial\varphi^j(t_1,t_2)}{\partial t_2}dt_1dt_2=T_{1,1}+T_{1,2}$$

(1.4.141)

ni hosil qilamiz.

Agar birinchi qo'shiluvchiga birinchi lemmaning $4^0$qismini qo'llasak

$$\left|T_{1,1}\right|\le\int_{-T_2}^{T_2}\frac{dt_2}{\left(\sqrt{T_1^{\,2}+T_2^{\,2}}\right)^{1+\delta_0}}\le C_2T_1^{-\delta_0}$$

(1.4.142)

Xuddi shu kabi $T_{1,2}$ ni ham bo'laklab integrallaymiz

$$T_{1,2}=\frac{x,i}{(2\pi)^2}\int_{-T_1}^{T_1w}e^{-i(<t,x>)}\varphi^j(t_1,t_2)\Big|_{-T_2}^{T_2}+\frac{x_1x_2i^2}{(2\pi i)^2}\int_{-T_1}^{T_1}\int_{-T_2}^{T_2}e^{-i(<t,x>)}\varphi^j(t_1,t_2)dt_1dt_2=T_{1,2}{}^{(1)}+T_{1,2}{}^{(2)}$$

(1.4.143)

1-lemmaning $2^0$qismiga asosan

$$\left|T_{1,2}{}^{(1)}\right|\le x_1w\int_{-T_1}^{T}\left|\varphi^j(t_1T_2)-\varphi^j(t_1-T_2)\right|dt_1\le 2x_1\int_{-T_1}^{T_1}\frac{dt_1}{\left(\sqrt{T_2^2+T_1^{\,2}}\right)^{\delta_0}}\le C_3x_1T_1T_2^{-\delta_0}$$

1-lemmaning $5^0$ qismiga asosan

$$|T_2| \le \int_{-\infty}^{-T_1}\int_{-\infty}^{\infty} \frac{dt_1 dt_2}{\left(\sqrt{t_1^2 + t_2^2}\right)^{2+\delta_0}} \le cT_1^{-\delta_0}$$

(1.4.144)

$$|T_3| \le cT_1^{-\delta_0}, \ |T_4| \le cT_1^{-\delta_0}, \ |T_5| \le cT_1^{-\delta_0}$$

(1.4.145)

2-lemmaning isboti (1.4.140)-(1.4.145) lardan kelib chiqadi.

3-LEMMA: Agar $B_{k,l}^j < +\infty$ bo'lsa,

$$\left|\varphi^j(t_1,t_2) - \varphi^j(t_1,t_2)\right| \le \max(t_1^2, t_2^2)\sum_{l=1}^{2} M(W_l^j - W_l^j(n))^2 + \max\left(|t_1|,|t_2|\sum_{l=1}^{2}\sqrt{M\left(W_l^j - W^j(n)\right)^2}\right)$$

bo'ladi.

ISBOT: Ishonch hosil qilish mumkinki

$$\varphi_n^j(t_1+t_2) - \varphi^j(t_1,t_2) = M\left(e^{i(<t,w^j(n)>)} - e^{i(<t,w^j(n)>)}\right) =$$

$$M\left(e^{i(<t,w^j(n)>)}\left[1 + i(<t,(W^j - W^j(n))>) - e^{i(<t,w^j(n)>)}\right]\right) + M(it_1 e^{i(<t,w^j(n)>)}\left(W_1^j - W_1^j(n)\right) +$$

$$it_2 e^{i(<t,w^j(n)>)}(W_2^j - W_2^j(n))$$ 3

(1.4.146)

quyidagi

$$\left|e^{in} - 1 - ix\right| \le \frac{x2}{2}$$

tengsizlikdan foydalanib

$$\left|1 + i(<t,(W^j - w^j(n))>) - e^{-i(<t,(W^j - W^j(n))>)}\right| \le \frac{(<t, W^j - W^j(n)>)^2}{2}$$

(1.4.147)

tengsizlikni hosil qilamiz.

Koshi Bunyakovskiy tengsizligidan

$$|M|\left(it_1 e^{i(<t,W^j(n)>)}\left(W_e^j - W_e^j(n)\right)\right) \le |t_l|\sqrt{M(W_e^j - W_e^j(n))^2}, \qquad j,e = 1,2$$

(1.4.148)

ga ega bo'lamiz, u holda (1.4.146) –(1.4.148) lardan 3-lemmaning isboti kelib chiqadi.

Teoremaning isboti: Aylantirish formulasiga asosan

$$\rho^{2n}P_n^j(r_1,r_2)-W^j(\rho^{-n}r_1,\rho^{-n}r_2)=\frac{\rho^{2n}}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}F_n^j(e^{it_1},e^{-t_2})e^{-i(<r,t>)}dt_1dt_2-W^j\left(\rho^{-n}(r_1,r_2)\right)=$$

$$\left[\frac{\rho^{2n}}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}F_n^j(e^{it_1},e^{-t_2})e^{-i(<r,t>)}dt_1dt_2-\frac{1}{(2\pi)^2}\int_{-\rho(1-\alpha)\frac{\delta n}{2}}^{\rho(1-\alpha)\frac{\delta n}{2}}\int_{-\rho(1-\alpha)\frac{\delta n}{2}}^{\rho(1-\alpha)\frac{\delta n}{2}}e^{-i(<r,t>)}\rho^{-n}\varphi^j(t_1,t_2)dt_1dt_2\right]+$$

$$+\left[W^j\left(\rho^{-n}r_1,\rho^{-n}r_2\right)-\frac{1}{(2\pi)^2}\int_{-\rho(1-\alpha)\frac{\delta n}{2}}^{\rho(1-\alpha)\frac{\delta n}{2}}\int_{-\rho(1-\alpha)n}^{\rho(1-\alpha)n}e^{-i(<r,t>)\rho^{-n}}\varphi^j(t_1t_2)dt_1dt_2\right]=T_1+T_2$$

(1.4.149)

bu yerda $\max\left(0,\frac{\delta_0-2}{\delta_0}\right)<\alpha<1, \qquad r=(r_1,r_2)$

Quyidagi belgilashni kiritamiz:

$$x_1=r_1\rho^{-n},\qquad x_2=r_2\rho^{-n},\qquad T_1=\rho^{(1-\alpha)\frac{\delta_0 n}{2}},\ \ T_2=\rho^{(1-\alpha)n}$$

2-lemmani qo'llab

$$|T_2|\le C\left[(\rho^{-n}r_1)^{-1}(\rho^{-n}r_2)\rho^{-(1-\alpha)\frac{\delta_0^2 n}{2}}+(\rho^{-n}r_2)^{-1}\rho^{-(1-\alpha)\frac{\delta_0 n}{2}}\right]$$

(1.4.150)

baxoga ega bo'lamiz.

$T_1$ ni ikkita yig'indiga ajratamiz:

$$T_1=\left[\frac{\rho^{2n}}{(2\pi)^2}\int_{-\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}^{\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}\int_{-\rho^{-n\alpha}}^{\rho^{-n\alpha}}F_n^j(e^{it_1},e^{it_2})e^{-i(<t,r>)}dt_1dt_2-\frac{1}{(2\pi)^2}\int_{-\rho^{(1-\alpha)\frac{\delta_0 n}{2}}}^{\rho^{(1-\alpha)\frac{\delta_0 n}{2}}}\int_{-\rho^{(1-\alpha)n}}^{\rho^{(1-\alpha)n}}e^{-i(<t,r>)}\varphi^j(t_1,t_2)dt_1dt_2\right]+$$

$$+\frac{\left(\rho^{-n}r_1\right)^{-1}\left(\rho^{-n}r_2\right)^{-1}}{(2\pi)^2}\left[\int_{-\pi}^{((1-\alpha)\frac{\delta_0}{2}-1)n}\int_{-\pi}^{\pi}r_1,r_2F_n^j(e^{-t_1},e^{it_2})e^{-i(<t,r>)}dt_1dt_2+\int_{\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}^{\pi}\int_{-\pi}^{\pi}r_1r_2F_n^j\left(e^{it_1},e^{it_2}\right)\cdot\right.$$

$$\cdot e^{-i(<t,r>)}dt_1dt_2+\int_{-\rho^{((1-\alpha)\frac{\delta_0}{2}-1)}}^{\rho^{((1-\alpha)\frac{\delta_0}{2}-1)}}\int_{\pi}^{\rho^{-\alpha n}}r_1,r_2F_n^j(e^{-t_1},e^{it_2})e^{-i(<t,r>)}dt_1dt_2+$$

$$\left. \int_{-\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}^{\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}} \int_{\rho^{-\alpha n}}^{\pi} r_1, r_2 F_n^j (e^{-t_1}, e^{it_2}) e^{-i(<t,r>)} dt_1 dt_2 \right] = T_{1,1} + T_{1,2}$$

(1.4.151)

Birinchi integralda almashtirish bajarib

$$|T_{1,1}| \le \int_{-\rho^{(1-\alpha)\frac{\delta_0 n}{2}}}^{\rho^{(1-\alpha)\frac{\delta_0 n}{2}}} \int_{-\rho^{(1-\alpha)n}}^{(1-\alpha)n} \left| F_n^j (e^{it_1 \rho^{-n}}, e^{it_2 \rho^{-n}}) - \varphi^j (t_1, t_2) \right| dt_1 dt_2$$

tengsizlikka ega bo'lamiz.

Ko'rsatish mumkinki

$$M\left(W_e^j(n) - W_e^j\right)^2 \le C\left(\rho^{-n} + \rho^{-n} |\overline{\rho}|^n\right),$$

(1.4.152)

bu yerda $\overline{\rho}\|A_i^j\|$ matritsaning ikkinchi xarakteristik sonidir.

3-lemmaga asosan (1.4.152) ni qo'llab,

$$|T_{1,1}| \le C\left(\rho^{\left(\frac{3\delta_0}{2}(1-\alpha)+\frac{1}{2}-\alpha\right)n} + \rho^{\left(\frac{\delta_0}{2}(1-\alpha)+\frac{5}{2}-3\alpha\right)n}\right)\left(1+|\overline{\rho}|^{\frac{n}{2}}\right),$$

$$\alpha > \max\left(\frac{3\delta_0+1}{3\delta_0+2}, \ \frac{3\delta_0+1+\log_\rho|\overline{\rho}|}{3\delta_0+2}, \ \frac{\delta_0+5}{\delta_0+6}, \ \frac{\delta_0+5+\log_\rho|\overline{\rho}|}{\delta_0+6}, \ \frac{\delta_0-2}{\delta_0}\right)$$

(1.4.153)

ga ega bo'lamiz. $T_{1,2}$ ifodada bo'laklab integrallab, soddalashtirgandan so'ng

$$\left(\rho^{-n} r_1\right)\left(\rho^{-n} r_2\right)|T_{1,2}| \le C\left|F_n^j\left(e^{i\rho-\alpha n}, e^{((1-\alpha)\frac{\delta_0}{2}-1)}\right)\right| + C\int_{-\pi}^{\pi}\left|\frac{\partial F_n^j\left(e^{it_1}, e^{i\rho(1-\alpha)\frac{\delta_0}{2}-1)}\right)}{\partial t_1}\right| dt_1 +$$

$$+C \int_{-\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}^{\pi} \int_{-\pi}^{\pi} \left|\frac{\partial^2 F_n^j (e^{-t_1}, e^{it_2})}{dt_1 dt_2}\right| dt_1 dt_2 = T_{1,2}^{(1)} + T_{1,2}^{(2)} + T_{1,2}^{(3)}$$

(1.4.154)

bo'ladi.

(1.1.7) dan

$$F_n^j\left(e^{i\rho^{-\alpha n}}, e^{((1-\alpha)\frac{\delta_0}{2}-1)n}\right) = F_{n-[\alpha n]}^j\left(F_{[\alpha n]}^1\left(e^{i\rho_{t_1}{}^{-\alpha n}}, e^{\rho^{((1-\alpha)\frac{\delta_0}{2}-1)n}}\right)\right), F_{[\alpha n]}^\alpha\left(e^{i\rho^{-\alpha n}}, e^{i\rho_{t_2}{}^{((1-\alpha)\frac{\delta_0}{2}-1)n}}\right)$$

(1.4.155)

Ikkinchi tomondan

$$F_{[\alpha n]}^\alpha\left(e^{i\rho^{-\alpha n}}, e^{((1-\alpha)\frac{\delta_0}{2}-1)n}\right) = \varphi_{[\alpha n]}^j\left(\rho^{-\alpha n+[\alpha n]}, \rho^{((1-\alpha)\frac{\delta_0}{2}-1)n+[\alpha n]}\right)$$

va $n \to \infty$

$$\varphi_{[\alpha n]}^j(t_1, t_2) \to \varphi^j(t_1, t_2)$$

Xarakteristik funksiyasi xossasiga asosan $t_1{}^2 + t_2{}^2 \neq 0$ uchun

$$\left|\varphi^j(t_1, t_2)\right| < 1$$

U holda yetarligicha katta $n$ lar uchun

$$\underset{\substack{1 \le t_1 \le \rho \\ -\infty < t_2 < +\infty}}{Sup} \left|\varphi_{[\alpha n]}^j(t_1, t_2)\right| < 1$$

Buni hisobga olib (1.2.12) va (1.4.155) lar aososida

$$T_{1,2}{}^{(1)} \le C\rho_q{}^{n(1-\alpha)}$$

(1.4.156)

(1.1.7) dan

$$\frac{\partial F_n^j\left(e^{it_1}, e^{it_2}\right)}{\partial t_e} = \sum_{y=1}^{2} \frac{\partial F_{n-[\beta n]}^j\left(F_{[\beta n]}^1, F_{[\beta n]}^2\right)}{\partial F_{[\beta n]}^y} \cdot \frac{\partial F_{[\beta n]}^y}{\partial t_e}$$

(1.4.157)

bu yerda

$$0 < \beta < \frac{\delta_0}{2 + \delta_0}$$

Ma'lumki

$$\left|\frac{\partial F^{j}_{[\beta n]}\left(e^{it_1}, e^{it_2}\right)}{\partial t_e}\right| \le C\rho^{\beta n}, \qquad e, j = 1,2$$

(1.4.158)

Kasten va Stigum ishida

$$\left|\frac{\partial F^{j}_{[\beta n]}(S_1, S_2)}{\partial S_y}\right| \le C\lambda^{n}(S_1, S_2), \quad 0 < |\lambda(S_1, S_2)| < 1,$$

(1.4.159)

ekanligi ko'rsatilgan, bu yerda

$$\lim_{\substack{S_1 \to 0 \\ S_2 \to 0}} \lambda(S_1, S_2) = \rho_q$$

Natijada (1.4.159) dan foydalanib

$$\left|\frac{\partial F^{j}_{n-[\beta n]}\left(F^{1}_{[\beta n]}, F^{2}_{[\beta n]}\right)}{\partial F^{y}_{[\beta n]}}\right| \le C\lambda^{n-\beta n}$$

(1.4.160)

bahoni olamiz.

(1.4.157),(1.4.158), (1.4.160) lardan

$$T_{1,2}{}^{(2)} \le C\left(\rho^{-n} + \rho^{\beta n}\lambda^{n-\beta n}\right) \le C\left(\rho^{-n} + \rho^{\beta n}\rho_q{}^{n(1-\beta)}\right)$$

(1.4.161)

ni ishonch hosil qilamiz.

Ishonch hosil qilish mumkinki

$$\frac{\partial^2 F^{j}_{n}\left(e^{it_1}, e^{it_2}\right)}{\partial t_e \partial t_y} = \sum_{y_1, y_2 = 1}^{2} \frac{\partial^2 F^{j}_{n-[\beta n]}\left(F^{1}_{[\beta n]}, F^{2}_{[]\beta n}\right)}{\partial F^{y_1}_{[\beta n]}\, \partial F^{y_2}_{[\beta n]}} \cdot \frac{\partial F^{y_1}_{[\beta n]}\, \partial F^{y_2}_{[\beta n]}}{\partial F_e\, \partial F_y} + \sum_{y_1 = 1}^{2} \frac{\partial F^{j}_{n-[\beta n]}\left(F^{1}_{[\beta n]}, F^{2}_{[\beta n]}\right)}{\partial F^{y}_{[\beta n]}} \cdot$$

$$\cdot \frac{\partial^2 F^{y}_{[\beta n]}}{\partial t_e dt_y}, \qquad S, l, y = 1,2$$

(1.4.162)

Ixtiyoriy $n$ uchun

$$\left|\frac{\partial^2 F^j_{[\beta n]}\left(e^{it_1}, e^{it_2}\right)}{\partial t_e \partial t_y}\right| \le C\rho^{2\beta n}$$

(1.4.163)

(1.4.159) bahoni olish usulini qo'llab

$$\left|\frac{\partial^2 F^j_{n-[\beta n]}\left(S_1, S_2\right)}{\partial S_1, \partial S_2}\right| \le C\lambda^{n-(1-\beta)}, \qquad |S_j| < 1$$

(1.4.164)

ga ega bo'lamiz.

Natijada yetarlicha katta $n$ lar uchun (1.4.162)-(1.4.164) ni hisobga olgan holda

$$T_{1,2}^{(3)} \le C\left(\rho^{-n} + \rho^{2\beta n}\rho_q^{\,n-\beta n}\right)$$

(1.4.165)

Teoremaning to'g'riligiga ishonch hosil qilish uchun (1.4.149)-(1.4.151), (1.4.153), (1.4.154), (1.4.157),(1.4.161),(1.4.165) larni yig'amiz:

$$\left|\rho^{2n}P_n^j(r_1, r_2) - W^j\left(\rho^{-n}r_1, \rho^{-n}r_2\right)\right| \le C\Bigg\{\left(\rho^{-n}r_1\right)^{-1}\left(\rho^{-n}r_2\right)^{-1}\Bigg[\rho^{(\alpha-1)\frac{\delta_0}{2}} + \rho_q^{\,n(1-\alpha)} + \rho^{-n} + \rho^{2\beta n}\rho_q^{\,n(1-\beta)} +$$

$$\rho^{(\alpha-1)\frac{\delta_0}{2}}\Bigg] + \left(\rho^{\left(\frac{3\delta_0}{2}(1-\alpha)+\frac{1}{2}-\alpha\right)n}\right) + \left(\rho^{\left(\frac{\delta_0}{2}(1-\alpha)+\frac{5}{2}-3\alpha\right)n}\right)\left(1+|\overline{\rho}|^{\frac{3}{2}}\right)\Bigg\},$$

(1.4.166)

va

$$\rho_1 = \max\left(\rho^{(1-\alpha)\frac{\delta_0}{2}}, \rho_q^{\,1-\alpha}, \rho^{-1}, \rho^{2\beta}\rho_q^{\,1-\beta}, \rho^{(\alpha-1)\frac{\delta_0}{2}}\right),$$

$$\rho_2 = \max\left( \rho^{\frac{3\delta_0}{2}(1-\alpha)+\frac{1}{2}-\alpha}, \rho^{\frac{\delta_0}{2}(1-\alpha)+\frac{5}{2}-3\alpha}, \rho^{\frac{3\delta_0}{2}(1-\alpha)+\frac{1}{2}-\alpha}\left|\overline{\rho}\right|^{\frac{1}{2}}, \rho^{\frac{\delta_0}{2}(1-\alpha)+\frac{5}{2}-3\alpha}\left|\overline{\rho}\right|^{\frac{1}{2}} \right)$$

Belgilashni kiritib (1.4.166) dan teorema isbotlanganligiga ishonch hosil qilamiz.

*I bob bo'yicha xulosalar*

Dissertatsiyaning 1-bobi « Galton Vatson tarmoqlanish jarayoni » deb nomlangan va kritikgacha tarmoqlanish jarayoni, kritik tarmoqlanish jarayoni , kritikdan keyingi tarmoqlanish jarayoni va bu jarayonlarga oid limit teoremalar keltirilgan.

## II bob. Binomial qonun bo'yicha taqsimlangan tarmoqlanish Jarayoni

### 2.1. *Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoni uchun limit teorema*

$\mu_n$ Galton Vatson tarmoqlanish jarayoni bo'lsin.

Agar $p(\mu_1 = 0) = q_1, \quad p(\mu_1 = 1) = 1 - q_1 = p_1$ bo'lsa , u holda $\mu_n$ quyidagi xususiyatlarga ega bo'ladi.

$$\mu_n = \mu_n^{\ 1} + \mu_n^{\ 2} + \ldots + \mu_n^{\ \mu_{n-1}}$$

$$\text{p}(\mu_n = 0) = q_n \qquad \text{p}(\mu_{n-1} = 1) = p_n = 1 - q_n$$

Agar $n$ momentda $c_n$ zarrachaga ega bo'lsa va har bir zarracha $i = \overline{1, c_n}$ bog'langan.Bu yerda $\mu_n^{\ 1} + \mu_n^{\ 2} + \ldots + \mu_n^{\ c_n}$ quyidagi ehtimolliklarga ega .

$p(\mu_n^i = 0) = q_i, \quad p(\mu_n^i = 1) = p_i$

$S_{c_n} = \mu_n^1 + \mu_n^2 + \ldots + \mu_n^{c_n}$ u holda $S_{c_n}$ binomial taqsimotga ega bo'ladi.

$M\mu_n^i = p_i < 1 \qquad S_{c_n}$ Puasson taqsimotiga ega

$$\lambda_{c_n} = \sum_{i=1}^{c_n} p_i, \quad \lambda_{c_n}(r) = \sum_{i=1}^{c_n} p_i^r, \quad r \ge 2 \quad \theta_{c_n} = \frac{\lambda_{c_n}(r)}{\lambda_{c_n}}$$

$$F_{c_n}(x) = p(s_{c_n} \le x), \quad \pi(m, \lambda_{c_n}) = \frac{\lambda_{c_n}^m}{m!} e^{-\lambda_{c_n}}, \quad m = 0,1,\ldots$$

$$\Pi(x, \lambda_{c_n}) = \sum_{m=0}^{[x]} \pi(m, \lambda_{c_n}), \quad \pi(m, \lambda_{c_n}) = \pi(m, \lambda_{c_n}) - \pi(m-1, \lambda_{c_n})$$

$$\Delta^{k+1}\pi(m, \lambda_{c_n}) = \Delta^k \pi(m, \lambda_{c_n}) - \Delta^k \pi(m-1, \lambda_{c_n}), \quad k = 1,2,\ldots,$$

$$\Delta^{1\pi}(m, \lambda_{c_n}) = \Delta\pi(m, \lambda_{c_n})$$

Teorema. $c_n \to \infty$ va ixtiyoriy $\nu = 1,2,\ldots$

$$F_{c_n}(x) = \Pi(x, \lambda_{c_n}) + \sum_{k=1}^{\nu} \left\{ a_{2k} \Delta^{2k-1} \pi(x, \lambda_{c_n}) + a_{2k+1} \Delta^{2k} \pi(x, \lambda_{c_n}) \right\} + R_\nu(x)$$

Bu yerda $a_2 = -\frac{\sum_{j=1}^{c_n} p_j^2}{2}, \quad a_3 = -\frac{\sum_{j=1}^{c_n} p_j^3}{3}, \quad a_r = -\frac{1}{r}(\lambda_{c_n}(r) + \sum_{k=2}^{r-2} a_k \lambda_{c_n}(r-k)), \quad r \ge 4$ va

$$|R_\nu(x)| \le \frac{\overline{c_1}}{1 - \sqrt{\theta_{c_n}}} \theta_{c_n}^{\nu+1}, \quad \overline{c_1} = \frac{1 + \sqrt{\frac{\pi}{2}}}{2}$$

$\nu = 1 va \quad r = 4$

$$F_{c_n}(x) = \Pi(x, \lambda_{c_n}) + a_2 \Delta\pi(x, \lambda_{c_n}) + a_3 \Delta^2 \pi(x, \lambda_{c_n}) + R_n(x)$$

$$a_4 = -\frac{1}{4}(\lambda_{c_n}(4) + a_2 \lambda_{c_n}(2)),$$

$$|R_1(x)| \le \frac{\overline{c_1}}{1 - \sqrt{\theta_{c_n}}} \theta_{c_n}^2, \quad \overline{c_1} = \frac{1 + \sqrt{\frac{\pi}{2}}}{2}$$

$\{X_j\}, - j = 1,2,3,...,$ $p_j$ *va* $q_j = 1 - q_j$ ehtimolliklari 0 va1 qiymatlarni qabul qiluvchi bog'liqsiz tasodifiy miqdor bo'lsin .

$S_n = X_1 + .... + X_n$

$S_n$ tasodifiy miqdor binomial taqsimot deyiladi.Puasson qonunidagi tasodifiy yig'indi uchun bu taqsimotni asimptotik yoyilmasini topaylik.

$$\lambda = \sum_{j=1}^{n} p_j, \quad \lambda_r = \sum_{j=1}^{n} p_j^r \quad (r \ge 2).\theta = \frac{\lambda_2}{\lambda}$$

$$F_n(x) = P\{S_n \le x\} \quad \pi(m; a) = a^m e^{-a} / m! \quad (m = 0,1,...), \pi(m; a) = 0 \quad (m = -1,-2,-3,...);$$

$\Pi(x, a) = \sum_{m=0}^{[x]} \pi(m; a)$ -Puasson taqsimot funksiyasi

Agar $X$ - qandaydir tasodifiy miqdor bo'lsin.

$$F(x; X) = P\{X \le x\}, \quad \varphi(t; X) = \mathrm{E}\exp(itX).$$

Agar X-butun sonli nomanfiy tasodifiy miqdor bo'lsa $f(z; X) = Ez^X$

$$\Delta\pi(m; a) = \pi(m; a) - \pi(m - 1; a),$$

$$\Delta^{k+1}\pi(m; a) = \Delta^k \pi(m; a) - \Delta^k \pi(m - 1; a), \quad k = 1,2,...$$

Agar $h(m)$ – qandaydir butun argumentli funksiya bo'lsin.

Teorema 1: Ixtiyoriy $\nu = 0,1,..$ da

$$F_n(x) = \ddot{I}\ (x; \lambda) + \sum_{k=1}^{\nu} \{a_{2k} \Delta^{2k-1} \pi(x; \lambda) + a_{2k+1} \Delta^{2k} \pi(x; \lambda)\} + R_\nu(x)$$

$$\_\_ \quad a_2 = -\frac{\lambda_2}{2}, \quad a_3 = -\frac{\lambda_3}{3}, \quad a_r = -\frac{1}{r}(\lambda_r + \sum_{k=2}^{r} a_k \lambda_{r-k}) \ (r \ge 4)$$

$$|R_\nu(x)| \le \frac{c_1}{1 - \sqrt{\theta}} \theta^{\nu+1} \quad (c_1 = \frac{1 + \sqrt{\frac{\pi}{2}}}{2})$$

Teorema lemma1 orqali isbotlanadi.

K- butun noman'fiy qiymatlarni qabul qiluvchi tasodiy miqdorlar sinfi bo'lsin.

Lemma 1: ([1]) Agar $X \in K$ bo'lsa

$$F(m; X) = a_0(X)\ddot{I}\ (m; \lambda(X)) + \sum_{k=1}^{\infty} a_k(X) \Delta^{k-1} \pi(m; \lambda(X)), \quad (1)$$

Bu yerda

$$a_k(X)=\frac{(-1)^k}{2\pi i}e^{\lambda(X)}\oint_{\tilde{A}} e^{-u} f\left(\frac{u}{\lambda(X)};X\right)\frac{\lambda^k(X)}{(u-\lambda(X))^{k+1}}du \qquad (2)$$

k=0,1,2,..., $\lambda$ $\lambda(X)=EX,$ $\Gamma-u=\lambda(X)$ *markaziy nuqtaning atrofi*

1) Haqiqiy yig'indi

$X\in K$ tasodifiy miqdorning funksiyasini kiritamiz

$$\tilde{F}^{(l)}(m;X)=a_0(X)\Pi(m;\lambda(X))+\sum_{k=1}^{l}a_k(X)\Delta^{k-1}\pi(m;\lambda(X)),$$

$\tilde{F}^{(l)}(x;X)$ Fur'e Stiltesa darajali funksiyasi

$$\tilde{\varphi}^{(l)}(t;X)=\sum_{m=0}^{\infty}e^{itm}\left\{\tilde{F}^{(l)}(m;X)-\tilde{F}^{(l)}(m-1;X)\right\}$$

Lemma 2: Teng kuchli tenglama

$$\varphi(t;X)=\exp\left\{-\lambda(X)(1-e^{it})\right\}\sum_{k=0}^{\infty}a_k(X)(1-e^{it})^k$$

$$\tilde{\varphi}^{(l)}(t;X)=\exp\left\{-\lambda(X)(1-e^{it})\right\}\sum_{k=0}^{l}a_k(X)(1-e^{it})^k$$

Lemma 3:Koeffisiyentlari $\{a_k(S_n)\}, k=0,1,2,...,$ rekkurent formula orqali aniqlanadi $a_0(S_n)=1, a_1(S_n)=0$

$$a_k(S_n)=-\frac{1}{k}\left(\lambda_k+\sum_{j=2}^{k-2}a_j(S_n)\lambda_{k-j}\right) \qquad (k=2,3,...)$$

Isboti. $S_n$ tasodifiy miqdorning xarakteristik funksiyasi

$$\varphi(t;S_n)=\prod_{j=1}^{n}\varphi(t;X_j)=\prod_{j=1}^{n}(1-p_jz)=\psi(z) \qquad (z=1-e^{it})$$

$$\psi(z)=\exp\left\{-\lambda z-\frac{\lambda_2}{2}z^2-\frac{\lambda_3}{3}z^3-...\right\}=e^{-\lambda z}g(z),$$

Bu yerda g(z)= $ó÷ç\left\{-\frac{\lambda_2}{2}z^2-\frac{\lambda_3}{3}z^3-...\right\}=\exp(\delta(z))$ .

$g(z)=b_0+b_1z+b_2z^2...$ bo'lsin

$$b_k=\frac{1}{k!}\frac{d^k}{dz^k}\exp(\delta(z))\Big|_{z=0}=\frac{1}{k!}\sum_{j=0}^{k-1}C_{k-1}^j\frac{d^j}{dz^j}\exp(\delta(z))\Big|_{z=0}\frac{d^{k-1-j}}{dz^{k-1-j}}\delta'(z)\Big|_{z=0}=-\frac{1}{k}\sum_{j=0}^{k-2}b_j\lambda_{k-j}.$$

$$g(z)=e^{\lambda z}\psi(z)=\sum_{k=0}^{\infty}a_k(S_n)z^k$$

$$g(z)=a_0(S_n)+a_1(S_n)z+a_2(S_n)z^2+...$$

$|z|<2.$ , $a_k(S_n)=b_k \qquad (k=0,1,2,...);$

## 2.2 *Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoni uchun limit teorema*

*Tarmoqlanish jarayonida barcha davrdagi zarrachalar yig`indisi uchun limit teoremalar*

Faraz qilaylik $\{\mu_n\}_1^{\infty}$ Galton-Vatson jarayonini tashkil qilsin. Ma'lumki, bunday jarayon uchun quyidagi xususiyat o`rinlidir:

$$\mu_n = \mu_{n-1}^{(1)} + \mu_{n-1}^{(2)} + \ldots + \mu_{n-1}^{(\mu_1)} = \mu_1^{(1)} + \mu_1^{(2)} + \ldots + \mu_1^{(\mu_{n-1})} \qquad (1)$$

bu yerda $\mu_{n-1}^{(j)}$, $j=\overline{1,\mu_1}$ lar bog`liqsiz $\mu_{n-1}$ bilan $\mu_1^{(j)}$, $j=\overline{1,\mu_{n-1}}$ lar ham bog`liqsiz $\mu_1$ bilan bir xil taqsimlangan tasodifiy miqdorlardir $\rho(\mu_0=1)=1$, [1].

Ma'lumki tarmoqlanish jarayoni biologiyada, kimyoda, aholishunoslikda, genetikada, fizikada va boshqa soxalarda keng qo`llaniladi. Xususan, kiyoda uran moddasini parchalanishida, radiaktiv moddalarni tarqalishi jarayonlarni matematik modelini tuzishda ishlatiladi.

Quyidagi belgilashlarni kiritamiz:

$$f_1(S) = \sum_{k=0}^{\infty} P(\mu_1 = k) S^k = \sum_{k=0}^{\infty} P_k S^k, \qquad f_1'(1) = a, \ \sigma^2 = D\mu_1$$

(1) ga asosan $f_n(S) = MS^{\mu_n}$ uchun

$$f_{n-1}(f_1(S)) = f_1(f_{n-1}(S)) = f_n(S) \qquad (2)$$

o`rinli. (2) dan $S$ =1 nuqta atrofida birinchi va ikkinchi tartibli hosilalarini hisoblab quyidagilarni hosil qilamiz

$$f_n(S) = M\mu_n$$

$$D\mu_n = \begin{cases} \dfrac{\sigma^2 a^n (a^{n-1}-1)}{a^2 - a}, & m \neq 1 \\ n\sigma^2, & m = 1 \end{cases}$$

Masalani qo`yilishi:

$$\bar{\mu} = \mu_0 + \mu_1 + \ldots + \mu_n + \ldots, \qquad \bar{\mu}_n = \mu_0 + \mu_1 + \ldots + \mu_n + \ldots, \qquad (3)$$

larni o`rganamiz. Buning uchun birinchi navbatda $M\,\bar{\mu}_n$ va $M\mu$ larni hisoblaymiz, ya'ni

$$M\,\bar{\mu}_n = \sum_{k=0}^{n} a^k = 1 + \frac{(1-a^n)}{1-a}, \qquad \lim_{n\to\infty} \bar{\mu}_n = \begin{cases} 1 + \dfrac{a}{1-a}, & a < 1 \\ +\infty, & a \geq 1 \end{cases}$$

Ma'lumki, $a$ > 1 jarayon cheksiz davom etadi. $a \leq 1$ da jarayon ertami, kechmi tugaydi. Agar $P_k \neq 0$ ni qanoatlantiruvchi $k$ -1 $d$ ga bo`linsa, $a$ > 1, $P_0$ >0, $k \to 0$ larda

$$P(\mu = k) = \frac{d}{\sqrt{2\pi f^n(1)}} \frac{1}{\sqrt{k^3}} + O\left(\frac{1}{\sqrt{k^5}}\right)$$

bajariladi ([2])

Peyks [3] $a < 1$ hol uchun, $n \to \infty$ da quyidagi munosabat o`rinliligini ko`rsatgan

$$P\left(\frac{\mu_0 + \mu_1 + ... + \mu_n - nB'(1)}{\sqrt{Hn}} < x, \ | \ \mu_n > 0\right) \to \phi(x),$$

bu yerda $\phi(x)$ standart normal qonun, $H = D\overline{\mu}$ , $\lim\limits_{n\to\infty} M\left(\frac{S^{\mu_n}}{\mu_n} > 0\right) = B(S)$.

Ma'lumki, Duon O.V. Viskov, Boyd [3] lar

$$P(\mu_0 + \mu_1 + ... + \mu_n + ... = j / \mu_0 = 2) = \frac{1}{j} P(\xi_1 + \xi_2 + ... + \xi_j + ... = j - 1) \qquad (4)$$

ligini ko`rsatganlar, bu yerda $\xi_j$ lar bog`liqsiz va $\mu_1$ bilan bir xil bog`liqsiz taqsimlangan.

Ehtimollar xossasidan

$$\begin{gathered} P(\xi_1 + \xi_2 + ... + \xi_j + ... < j) - P(\xi_1 + \xi_2 + ... + \xi_j + ... < j - 1) = \\ = P(\xi_1 + \xi_2 + ... + \xi_j + ... = j - 1) \end{gathered} \qquad (5)$$

Bularni quyidagicha ham mumkin.

$$P\left(\frac{\sqrt{\xi_1 + \xi_2 + ... + \xi_j - aj}}{\sqrt{j D\mu_1}} < \frac{j - aj}{\sqrt{j D\mu_1}}\right) - P\left(\frac{\sqrt{\xi_1 + \xi_2 + ... + \xi_j - aj}}{\sqrt{j D\mu_1}} < \frac{j - 1 - aj}{\sqrt{j D\mu_1}}\right) \qquad (6)$$

Quyidagi teorema o`rinli:

**Teorema**. Agar

$$|\,\tau\,| < T_j = \frac{\sqrt{j}\,(D\mu_1)^{\frac{3}{2}}}{\beta_3}, \qquad \beta_3 = M(\mu - a)^3, \quad j \geq 2$$

bo`lsa, u holda

$$P(\mu_0 + \mu_1 + ... + \mu_n + ... = j \ | \mu_0 = 2) = \frac{1}{j}\left(\phi\left(\frac{j - aj}{\sqrt{j D\mu_1}}\right) - \phi\left(\frac{j - 1 - aj}{\sqrt{j D\mu_1}}\right)\right) +$$

$$= \left(\frac{\beta_3}{6\sqrt{D\mu_1}} + \frac{\sqrt{2}}{3\sqrt{\pi}}\right) \int_0^{T_j} \frac{\tau^2}{\sqrt{j}} e^{\frac{j-1}{3j}\tau^2} d\tau$$

o`rinli bo`ladi.

Teoremani isbotlash uchun quyidagi lemmani keltiramiz.

$$\left| \left( f_1\left(e^{\frac{i\tau}{\sqrt{j D\mu_1}}}\right) e^{-\frac{i\tau k}{\sqrt{j D\xi_1}}} \right)^j - e^{-\frac{\tau}{2}} \right| = \left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}} + \frac{\sqrt{2}}{3\sqrt{\pi}}\right) \int_0^{T_j} \frac{|\tau|^3}{\sqrt{j}} e^{-\frac{j-1}{3j}\tau^2} d\tau$$

Ma'lumki, agar $G(x)$ taqsimot funksiya uchun $\alpha_k$ tartibli moment mavjud bo`lsa, u holda unga mos $g(t)$ xarakteristik funksiyani quyidagicha yozish mumkin.

$$g(t) = \sum_{j=0}^{k-1} \frac{\alpha\, j}{j\,!} (j\,t)^j + \gamma(t) \frac{\overline{\beta}_k}{k!} t^k ,$$

bu yerda $\overline{\beta}_k = M\,|\,\xi - \mathrm{M}\xi|^k$, $\gamma(t)$ kompleks o`zgaruvchili funksiya bo`lib $|\,\gamma(t < 1)|$.

Oxirgi tenglikdan foydalanib

$$f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau k}{\sqrt{jD\xi_1}}}=1-\frac{\tau^2}{2j}+\gamma(\tau)\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^2 j^3}} \qquad (7)$$

ni hosil qilamiz.

Normal qonunning xarakteristik funksiyasini

$$e^{-\frac{\tau^2}{2}}=\left(1-\frac{\tau^2}{2j}+\gamma(t)\frac{\beta_3(\phi)}{3!\,j^{\frac{3}{2}}}\tau^3\right)^j \qquad (8)$$

Ko`rinishda ifodalaymiz, bu yerda

$$\beta_3(\phi)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}|x|^3e^{-\frac{x^2}{2}}dx=\frac{2\sqrt{2}}{\sqrt{\pi}} \qquad (9)$$

Ixtiyoriy ikkita xarakteristik funksiya uchun

$$\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right)^j-\left(e^{-\frac{\tau^2}{2}}\right)^j=\sum_{l=0}^{j-1}\left[\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right)^{j-l-1}e^{-\frac{\tau^2}{2}l}\right]$$

$$\left[f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}-e^{-\frac{\tau^2}{2j}}\right].$$

(7) – (9) dan foydalanib topamiz

$$f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}-e^{-\frac{\tau^2}{2j}}=J(\tau)\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^3 j^3}}+\gamma(t)\frac{2\sqrt{2}\,\tau^3}{6\sqrt{\pi}} \qquad (11)$$

yoki

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}-e^{-\frac{\tau^2}{2j}}\right|\le\left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\frac{|\tau|^3}{\sqrt{j^3}} \qquad (12)$$

(7) dan

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right|\le1-\frac{\tau^2}{2j}+\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^3 j^3}}\le1-\frac{\tau^2}{2j}\left(\frac{\beta_3|\tau|}{3\sqrt{(D\mu_1)^3 j^3}}\right)\le1-\frac{\tau^2}{3j} \qquad (13)$$

ma’lumki, $0\le\alpha\le1$ da $1-\alpha\le e^{-\alpha}$.

U holda (13) dan

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right|\le e^{-\frac{\tau^2}{3j}} \qquad (14)$$

Shu bilan birga

$$e^{-\frac{\tau^2}{2k}}\le e^{-\frac{\tau^2}{3k}}$$

u holda $|\tau|\le T_j$ bo`lgan holda (14), (15) dan

$$\left|\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right)^{j-l-1}-e^{-\frac{\tau^2}{2k}j}\right|\le e^{-\frac{(j-1)\tau^2}{3j}}d\tau \qquad (16)$$

Natijada (10), (12), (16) lardan

$$\left|\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\xi_1}}}\right)^{j-l-1}-e^{-\frac{\tau^2}{2k}j}\right|\le\left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\frac{\tau^2}{\sqrt{j}}e^{-\frac{(j-1)\tau^2}{3j}}d\tau$$

Lemma isbotlandi.

Teorema isboti. (4) va (6) va Berdi-Esseyen tengsizligi va lemmaga ko`ra

$$P(\mu_0+\mu_1+...+\mu_n+...=j\ |\mu_0=2)=\frac{1}{j}\left[P\left(\frac{\sqrt{\xi_1+\xi_2+...+\xi_j-aj}}{\sqrt{jD\mu_1}}<\frac{j-aj}{\sqrt{jD\mu_1}}\right)-\right.$$

$$\left.-P\left(\frac{\sqrt{\xi_1+\xi_2+...+\xi_j-aj}}{\sqrt{jD\mu_1}}<\frac{j-1-j-a}{\sqrt{jD\mu_1}}\right)\right]=\frac{1}{j}$$

$$\left[\Phi\left(\frac{j-ja}{\sqrt{jD\mu_1}}\right)-\Phi\left(\frac{j-1-j-a}{\sqrt{jD\mu_1}}\right)+\left(\frac{\beta_3}{3!\sqrt{jD\mu_1}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\int_0^{T_j}\frac{\tau^2}{\sqrt{j}}e^{-\frac{(j-1)}{3j}\tau^2}d\tau\right]$$

Teorema isbotlandi.

## *2.3. Tasodifiy sondan boshlangan tarmoqlanish jarayoni yig'indisi uchun limit teorema*

*Tasodifiy sondan boshlangan tarmoqlanish jarayoni*

Tasodifiy sondan boshlangan tarmoqlanish jarayoni

$$Z_{n,\nu}=\mu_n^{(1)}+\mu_n^{(2)}+...+\mu_n^{(\nu)}$$

$\mu_n^{(i)}$ bog'liqsiz $\mu_n$ bilan bir xil taqsimlangan,

$$q_k=p(\nu=k),\qquad M\nu=m,\ \ D\nu=\sigma^2\qquad \theta(s)=Mc^{\frac{i(\nu-m)s}{\sigma_1}}$$

$$r_{n,\nu}=\frac{Z_{n,\nu}-MZ_{n,\nu}}{\sqrt{DZ_{n,\nu}}}=\frac{\mu_n^{(1)}+\mu_n^{(2)}+...+\mu_n^{(\nu)}-mA^n}{\sqrt{DZ_{n,\nu}}},$$

$$\psi_n(s)=Me^{isZ_{n,\nu}};\qquad g(f_n(\Omega))=Mx^{Z_{n,\nu}}\qquad G(x)=P(\frac{\nu-m}{\sigma_1}<x)$$

$$DZ_{n,\nu}=\sigma_n^2=\begin{cases}m\dfrac{(B+A-A^2)(A^n-1)}{A^2-A}+A^{2n}\sigma^2, A\neq 1\\ mnB+\sigma^2, \qquad A=1\end{cases}$$

$$MZ_{n,\nu}=mA^n\qquad\qquad \psi_n(s)=\sum_{k=1}^{\infty}q_k e^{-\frac{ismA^n}{\sigma^n}}[F_n\left(e^{\frac{is}{\sigma_n}}\right)]^k$$

1-Teorema: $A<1,\quad B<+\infty\quad n\to\infty$ da $m\to\infty,\quad \tau_n^2\to+\infty,$

$\tau_1=0(\tau_n^2)$ u holda

$$P(r_{n,\nu} < x) = G(\frac{x}{\delta}) \cdot F(\frac{x}{\sqrt{1+\delta^2}}) + 0(1), \qquad \delta = \frac{A^n \tau}{\tau_n}$$

Lemma: 1-teorema shartlari bajarilganda

$$\psi_n(s) = (\delta^s) e^{-\frac{i(k-m)A^n S}{\sigma_n}} [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{i\delta}{\sigma_n}})]^n$$

$$\overline{\psi}_n(s) = \sum_{k=1}^{\infty} q_k e \frac{i(k-m)A^n S}{\sigma_n} [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{i\delta}{\sigma_n}})]^m = \theta(\delta(s)[e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{i\delta}{\sigma_n}})]^m)$$

U holda

$$\left|\psi_n(s) - \overline{\psi_n}(s)\right| \le \sum_{k=1}^{\infty} q_k \left| e \frac{i(k-m)A^n S}{\sigma_n} \left\{ [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}}) - [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{i\delta}{\sigma_n}})]^m \right\} \right| =$$

$$= \sum_{|k-1|>\sigma^2}^{\infty} q_k \left| e \frac{i(k-m)A^n S}{\sigma_n} \left\{ [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}})] - [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}})]^m \right\} + \sum_{|k-1|<\sigma_n^2}^{\infty} q_k \right| e \frac{(k-m)A^n S}{\sigma_n} \left\{ [e^{-\frac{isA^n}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}} F_n(e^{\frac{is}{\sigma_n}})\right.$$

$$= T_1 + T_2$$

Teorema shartiga ko'ra

$$M\left(\frac{\nu m}{\sigma_n^2}\right) \to 0, \qquad M\left(\frac{\nu - m}{\sigma^2}\right)^2 = \frac{\sigma_1^2}{\sigma_n^2} \Rightarrow 0$$

$$\frac{\gamma - m}{\sigma_n^2} \to 0 \qquad\qquad \sigma_1 \le 2 \sum_{|k-m|>\sigma_n^2} q_k = 2P\left\{\left|\frac{\nu - m}{\sigma_n^2}\right| \ge\right\} = 0 \qquad (1)$$

$F_n(0)$ ni yozamiz

$$F_n\left(e^{\frac{it}{\sigma_n}}\right) = 1 + \frac{iSA^n}{\sigma^n} - \frac{S^2 M\mu_n^2}{2\sigma_n^2} + T(n)$$

$$T(n) = -\frac{S^2}{2\sigma_n^2} F_n^{II}(e^{\frac{it}{\sigma_n}}) + \frac{\overline{S}^2}{2\sigma_n^2} M\mu_k^2, \quad \left|\overline{S}\right| < |S|$$

$$\left|T(n) \ge \frac{S^2}{2\sigma_n^2} \left|\sum_{k=0}^{\infty} k^2 P(\mu^{(1)} = k) e^{\frac{is}{\sigma_n}} - M\left(\mu_n^{(1)}\right)^2 \le \frac{S^2}{2\sigma_n^2} \sum_{k=0}^{\infty} k^2 P\left(\mu_n^{(1)} = k\right)\right| e^{\frac{is}{\sigma_n}} - 1\right| \le$$

$$\le \frac{S^2}{2\sigma_n^2}\left[\sum_{k<N} \frac{k^3|\overline{S}|}{\sigma_n} P\left(\mu_n^1 = k\right) + 2\sum k^2 P(\mu^{(1)} = k)\right] = \frac{\sigma^2}{2\sigma_n^2}\left(T_1(n) + T_2(n)\right)$$

(1)

$$T_1(n) \le \frac{N(s)}{\sigma_n} \sum_{k \le N} k^2 P(\mu_n^{(1)} = k) \le c \frac{M|s|}{\sigma_n} A^n \Rightarrow T_1(n) = 0(A^n) \qquad (2)$$

Agar $\sum k^2 P(\mu_n^{(1)} = k) = \theta(A^n)$ ligini hisobga olsak

$$\sigma_2(n)=\sum_{k>N}^{\infty}2k^2P(\mu_n^{(1)}=k)=0(A^n) \qquad (3)$$

(1)-(3) dan

$$T(n)=0(\frac{S^2A^n}{\sigma_n^2})$$

Demak $$F_n(e^{\frac{is}{\sigma_n}})=1+\frac{iSA^n}{\sigma_n}-\frac{S^2N(\mu_n^{(1)})^2}{2\sigma_n^2}+0\left(\frac{s^2A^n}{\sigma_n^2}\right)$$

yoki $$e^{-\frac{isA^n}{\sigma_n}}F_n\left(e^{\frac{is}{\sigma_n}}\right)=1-\frac{D\mu_n}{2\sigma_n^2}S^2+O\left(\frac{S^2A^n}{\sigma_n^2}\right) \quad (4)$$

xuddi shunday Robbins ishidan foydalanib

$$\sigma_2=\tfrac{S^2}{2}D\mu_n e^{-\frac{S^2}{2}D\mu_n}+O(1)$$

Demak

$$\psi_n(s)=\overline{\psi}(s)+O(1)$$

(4) dan

$$\overline{\psi}_n(s)=\theta(\delta s)e^{\ln\left[1-\frac{mD\mu_nS^2}{2\sigma_n^2}+O\left(\frac{S^2A^n}{\sigma_n^2}\right)\right]}=\theta(\delta s)\exp\left(-\frac{s^2}{2}-\frac{mD\mu_n}{\sigma_n^2}\right)+O(i)=$$

$$=\theta(\delta s)\exp\left(-\frac{S^2}{2}(1-\frac{A^{2n}\sigma^2}{\sigma_n^2})\right)+\theta(\delta s)e^{-\frac{s^2(1-\delta^2)}{2}}+O(1)=\theta(\delta s)e^{-\frac{s^2}{2}(1-\delta^2)}+O(1)$$

2-teorema: $A=1,\quad \beta<\infty$ va $n\to\infty$ da $m\to\infty,\quad \sigma\to\infty \quad \sigma=0(\sigma_n^2)$

u holda

$$P\left(\frac{Z_{n,\nu}-m}{\sqrt{mn\beta+\sigma^2}}<x\right)=\sigma\left(\frac{x}{\delta_1}\right)*\Phi\left(\frac{x}{\sqrt{1-\delta_1^2}}\right)+O(1)$$

$$\delta_1=\frac{\sigma}{\delta_m}$$

Uzluksiz bo'lgan holda $a<0,\quad b<+\infty$ va $t\to\infty$ da

$$P\left(\frac{z_{t\gamma}-me^{at}}{\sigma_t}<x\right)\to\Phi(x),\quad \sigma=o(\sigma_t^2),\quad \sigma_t^2\to\infty,\quad m\to\infty$$

*Tarmoqlanish jarayoni va tasodifiy qo'shiluvchilarga oid limit teoremalar*

Faraz qilaylik $\{\xi_i\}$ bog'liqsiz va bir xil taqsimlangan tasodifiy miqdor. $\mu_n$-Galton –Vatson jarayonini tashkil qilsin u holda

$S\mu_n = \xi_1 + \xi_2 + ... + \xi_{\mu_n}$ $\quad M\xi_1 = h \quad D\xi_1 = H \quad \mu_n$ va $\xi_i$ larga bog'liqmas

Bu sxemaga tarmoqlanish jarayoni va tasodifiy qo'shiluvchilar deyiladi.

1-teorema: (Heydi teoremasi) $A > 1, \quad q = 0$

$$M\mu_1^2 < \infty \qquad M\xi_i^2 < \infty$$

$$\sup_x (P\left(\frac{S\mu_n}{H\sqrt{A^n}} < x\right) - \int_0^\infty \Phi\left(\frac{x}{\sqrt{z}}\right) dx(z)) \to 0$$

$$K(z) = P(w < z) \qquad T_n(x)$$

$$P_n(x); \qquad P\left(\frac{S\mu_n}{H\sqrt{A^n}} < x\right) = T_n(x)$$

2. $W_n \to W$ yaqinlashish tezligi

Teorema: $M\mu_n^2 < \infty \qquad \sup_x K(x) \le k < \infty \quad A < 1, \quad M\mu_1^3 < \infty$

$$\sup_x |K_n(x) - K(x)| \le c\left(k + \frac{D\mu_1 + A^2 - A}{A^2 - A}\right) A^{-\frac{n}{3}}$$

$$K_n(x) > P(w_n < x) \qquad \varphi(at) = F(\varphi(t))$$

$$\varphi(t) = Me^{itW} \qquad \varphi(t) = F_n\left(\varphi\left(\frac{t}{A^n}\right)\right)$$

$$\varphi_n(t) = Me^{itW_n} = F_n\left(e^{\frac{it}{A_n}}\right)$$

O'rta qiymat haqidagi teoremaga ko'ra

$$|\varphi_n(t) - \varphi(t)| = \left| F_n\left(e^{\frac{it}{A_n}}\right) - F_n\left(\varphi\left(\frac{t}{A^n}\right)\right)\right| = \left| F_n\left(e^{iat}\right)\left| e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right|\right| \le$$

$$\le \left|F_n\left(e^{iat}\right)\right| \left| e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right| \le A^n \left| e^{\frac{it}{A^n}} - \varphi\left(\frac{t}{A^n}\right)\right|$$

$$\varphi\left(\frac{t}{A^n}\right) = 1 + \frac{it}{A^n} + \theta_3 \frac{D\mu_1 + A^2 - A}{A^2 - A} \frac{t^2}{2A^{2n}}, \qquad (\theta_3) < 1$$

$|\varphi_n(t) - \varphi(t)| \le \dfrac{D\mu_1 + A^2 - A}{A^2 - A} \dfrac{t^2}{A^n}$, u holda

Berri Essen teoramasini qo'llaymiz

$$\sup_x\left|K_n(x)-K(x)\right|\le c\int_{|t|<\sqrt[3]{A^n}}\left|\varphi_n(t)-\varphi(t)\right|\frac{dt}{t}+\frac{\sup K^1(x)}{\sqrt[3]{A^n}}\le c\frac{D\mu_1+A^2-A}{A^2-A}A^{-n}$$

$$\left|\int_{|t|<\sqrt[3]{A^n}}|t|dt+\frac{K}{\sqrt[3]{A^n}}\right|$$

3. $h=0,\quad M|\xi|^3=\beta_3<+\infty,\quad A>1,\qquad D\mu_1<\infty,\quad k<+\infty$

$$\sup_{x<0}\left|T_n(x)-G(x)\right|\le c\left[\frac{\beta_3}{H^3}M\left(\mu^{-1}\right)^{\frac{n}{2}}+\left(k+\frac{D\mu_1+A^2-A}{A^2-A}\right)A^{-\frac{n}{3}}\right],$$

$$T_n(x)=P\left(\frac{S\mu_n}{H\sqrt{A^n}}<x\right)\qquad G(x)=\int_0^{+\infty}\varphi\left(\frac{x}{\sqrt{z}}dk(z)\right)$$

$\Phi(z)*K(z)$

$$T_n(x)=P\left(\frac{S\mu_n}{H\sqrt{\mu_n}}<x\right)$$

Lemma: $\beta_3<\infty,\quad A>1\qquad |t|<T_n=\dfrac{H^2\sqrt{A^n}}{5\beta_3}$

$$\left|Me^{it}\frac{S\mu_n}{H\sqrt{A^n}}-Me^{-\frac{t^2}{2}W_n}\right|\le\frac{7\beta_3|t|^3}{6H^3\sqrt{A^n}}M\left(W_ne^{-\frac{t^2}{2}W_n}\right)$$

4-teorema: Agar $h=0,\quad \beta_3<+\infty,\quad D\mu_1<+\infty,\quad k<+\infty$

$$AP(\mu_1=1)<1$$

$$\sup_x\left|P_n(x)-\int_0^{+\infty}\frac{1}{\sqrt{z}}\varphi_\phi\left|\frac{x}{\sqrt{z}}\right|dk(z)\right|\le\left[\frac{\beta_3}{H_3}\left(M\mu_1^{-1}\right)^{\frac{n}{2}}+\left(k+\frac{D\mu_1+A^2-A}{A^2-A}+\max\left(MW_n^{-1},MW^{-1}\right)\right)A^{-\frac{n}{6}}\right]$$

Aytaylik $\xi_1,\ \xi_2,\ ...,\xi_n,...$ o'zaro bog'liqsiz bir xil taqsimlangan tasodifiy miqdor, $\xi_1$ning taqsimot funksiyasi

$$T(x), M\xi_1=h,\ D\xi_1=H^2\ \ va\,\mu_1,\mu_2,....\mu_n$$

ketma-ketlik Gal'ton-Vatson jarayoni $\{\xi_i\}$ bilan o'zaro bog'liqsiz bo'lsin.

Quyidagi belgilashni kiritamiz:

$$S_{\mu_n}=\xi_1+\xi_2+,,,,+\xi_{\mu_n},\qquad n=1,2....$$

$S_{\mu_n}$ ni o'rganish tiklash nazariyasi uchun muhimdir.

Umuman, $A>1, q=0$ va $n\to\infty$ da

$$\sup_x\left|P\left(\frac{S_{\mu_n}}{H\sqrt{A^n}}<x\right)-\int_0^{+\infty}\Phi\left(\frac{x}{\sqrt{z}}\right)dk(z)\right|\to0$$

(1)

ligi isbotlangan, bu yerda $k(z) = P(w < z)$

$T_n(x)$ va $P_n(x)$ bilan, mos ravishda $\frac{S_{\mu_n}}{H\sqrt{h^2}}$ ni taqsimot va zichlik funksiyalarining ifodalaymiz.

### *Puasson qonun bilan taqsimlangan tarmoqlanish jarayoni uchun limit teorema*

Faraz qilaylik $\mu_1$ va $\gamma_n$ lar bog‘liqsiz va $\lambda_1$, $\lambda_2$ parametr bo‘yicha Puasson qonuni bo‘yicha taqsimlangan tasodifiy miqdorlar bo‘lib, $\{\mu_n\}$, $n = 1,2...,k,...$ Galton-Vatson jarayonini tashkil qilgan tasodifiy miqdor bo‘lsin [1], $\gamma_n$ lar bog‘liqsiz bir xil taqsimlangan miqdor.

Quyidagi yig‘indini tuzamiz.

$$Z_{n,\gamma_n} = \mu_n^{(1)} + \mu_n^{(2)} + ... + \mu_n^{(\gamma_n)},$$

bu yerda $\mu_n^i$ lar bog‘liqsiz va $\mu_n$ bilan bir xil taqsimlangan bo‘lsin, $i = 1, \bar{\gamma}_n$, $n = 1,2,...,...$

Quyidagi belgilashlarni kiritamiz:

$$\omega_k(n) = P(\gamma_n = k),\ \theta(\tau) = M\exp\left(\frac{i(\gamma_n - \lambda_2)\tau}{\sqrt{\lambda_2}}\right),\ F_n(x) = MS^{\mu_n},$$

$$\eta_{n,\gamma_n} = \frac{Z_{n,\gamma_n} - MZ_{n,\gamma_n}}{\sqrt{DZ_{n,\gamma_n}}} \qquad G(x) = P\left(\frac{\gamma_n - \lambda_2}{\sqrt{\lambda_2}} < x\right). \tag{1}$$

$\mu_1$ va $\gamma_n$ lar $\lambda_1$ va $\lambda_2$ parametrli Puasson qonuni bo‘yicha taqsimlanganligidan $M\mu_1 = \lambda_1$, $M\gamma_1 = \lambda_2$, $D\mu_1 = \lambda_1$, $D\gamma_1 = \lambda_2$, $MZ_{n,\gamma_n} = \lambda_2\lambda_1^n$

$$\sigma_n^2 = DZ_{n,y} = \begin{cases} \frac{\lambda_1^{n+1}\left(\lambda_1^{n+1} - 1\right)}{\lambda_1^2 - \lambda_1} \cdot \lambda_2 & ,\lambda_1 \neq 1 \\ \lambda_2 n & \lambda_1 = 1 \end{cases}$$

Tarmoqlanish jarayoni ta’rifiga ko‘ra

$$\psi_n(\tau) = MC^{i\tau h_{n,\gamma_n}} = \sum_{k=1}^{\infty} \omega_k(n) e^{-\frac{i\tau_1\lambda_2\lambda_1^n}{\sigma_n}} \left[F_n\left(\exp\left(\frac{i\tau}{\sigma_n}\right)\right)\right]^k = \sum_{k=1}^{\infty} \omega_k \exp\left(\frac{i(k-\lambda_2)\lambda_1^n\tau}{\sigma_n}\right)\left[F_n\left(\exp\left(-\frac{i\tau\lambda_1^n}{\sigma_n}\right)\right)\right]^k \tag{2}$$

$\eta_{n,\gamma_n}$ uchun quyidagi teorema o‘rinli.

**Teorema:** Agar $\lambda_1 < 1$ bajarilsa, u holda $n \to \infty$ da $\sqrt[4]{\lambda_2} \cdot \lambda_1^n \cdot \ln\lambda_2 \to 0$

$$P\left(\eta_{n,\gamma_n} < x\right) = G\left(\frac{\sqrt{2}x}{\sigma}\right) * \Phi\left(\frac{x}{\sqrt{1-\sigma^2}}\right) + C_1\left(\sqrt[4]{\lambda_2}\lambda_1^{n+1} : \sigma_n^{\ 3} + \frac{\lambda_2}{\sigma_n^4}\ln\lambda_2 + \sqrt[4]{\lambda_2}\lambda_1^{\ n}\ln\lambda_2\right) + o(1)$$

o‘rinli, $c_1$ - o‘zgarmas son, * kompozitsiya belgisi, $\sigma = \frac{\sqrt{\lambda_1^{\ n}(1-\lambda_1)}}{1-\lambda_1^n}$

Teoremani isboti uchun quyidagi lemmani keltiramiz:

**Lemma.** Teorema sharti bajarilganda va $|\tau| < \sqrt[8]{\lambda_2}$ da

$$\psi_n(\tau) = \theta\left(\frac{\sigma\tau}{\sqrt{2}}\right) e^{-\frac{\tau^2(1-\sigma^2)}{2}}\left(1 + \frac{/\tau/^3}{6\sigma_n^3}\ \lambda_2 \cdot \lambda_1^{\ n}\right) + \frac{\lambda_2}{\sigma_n^{\ 4}} + c_1\tau^2\lambda_1^{\ n} e^{-\tau^2\lambda_1^n}\left(1 + o(1)\right)$$

o‘rinli.

**Lemma isboti.** Quyidagi xarakteristik funksiyani kiritamiz

$$\bar{\psi}_n(\tau) = \sum_{k-1}^{\infty} \omega_k \exp\frac{i(k-\lambda_2)\lambda_1^{\ n}\tau}{\sigma_n}\left[\exp\left(-\frac{i\tau\lambda_1^n}{\sigma_n}\right) F_n\left(\exp\left(\frac{i\tau}{\sigma_n}\right)\right)\right]^{\lambda_2} = \theta\left(\frac{\sigma\tau}{\sqrt{2}}\right)\left[\exp\left(-\frac{i\tau\lambda_1^{\ n}}{\sigma_n}\right)\left[F_n\left(e^{\frac{i\tau}{\theta_n}}\right)\right]\right]^{\lambda_2}$$

(3)

(2) va (3) dan

$$\left|\psi_n(\tau) - \bar{\psi}_n(\tau)\right| = \sum_{k=1}^{\infty}\omega_k\left|\exp\left(\frac{i(k-\lambda_2)\lambda_1^{\ n}\tau}{\sigma_n}\right)\left[e^{-\frac{i\tau\lambda_1^n}{\sigma_n}} \cdot F_n\left(e^{-\frac{i\tau}{\sigma_n}}\right)\right]^k - \right.$$

$$\left. -\left[e^{\frac{i\tau\lambda_1^n}{\sigma_n}} F_n\left(e^{\frac{i\tau}{\sigma_n}}\right)\right]^{\lambda_2}\right| = \sum_{|k-\lambda_2| > \sigma_n^2}(\square) + \sum_{|k-\lambda_2| \le \sigma_n^2}(\square) = J_1 + J_2 \qquad (4)$$

Teorema shartiga ko‘ra $n \to \infty$ da

$$M\frac{\gamma_n - \lambda_2}{\sigma_n^{\ 2}} = 0, \quad M\left(\frac{\gamma_n - \lambda_2}{\sigma_n^{\ 2}}\right)^2 = \frac{\lambda_2}{\sigma_n^{\ 4}} \to 0$$

Demak $n \to \infty$ da ehtimollik bo‘yicha $\frac{\gamma_n - \lambda_2}{\sigma_n^{\ 2}} \to 0$ va

$$J_1 \le 2\sum_{|k-\lambda_2| > \sigma_n^2}\omega_k \le 2\sum_{k=1}^{\infty}\frac{(k-\lambda_2)^2}{\sigma_n^{\ 4}}\omega_k = \frac{\lambda_2}{\sigma_n^{\ 4}} \qquad (5)$$

$J_2$ ni baholash uchun yig‘indini ikkiga bo‘lamiz:

$$J_2 \le \sum_{|k-\lambda_2|\le\sigma_n^2} \left| \left[ \exp\left(-\frac{i\tau\lambda_1^n}{\sigma_n}\right) F_n\left(e^{\frac{i\tau}{\sigma_n}}\right)\right]^k - \left[\exp\left(-\frac{i\tau\lambda_1^n}{\sigma_n}\right) F_n\left(e^{\frac{i\tau}{\sigma_n}}\right)\right]^{\lambda_2} \right| \le \sum_{\lambda_2-\sigma_n^2\le k\le\lambda_2} + \sum_{\lambda_2<k<\sigma_n^2} \le$$

$$\le \max_{\lambda_2-\sigma_n^2\le k\le\lambda_2} \left| \left[ e^{-\frac{i\tau\lambda_1^n}{\sigma_n}} F_n\left(e^{\frac{i\tau}{\sigma_n}}\right)\right]^{\lambda_2-k_2} -1 \right| + \max_{\lambda_2\le k\le\sigma_n+\lambda_2} \left| \left[ e^{-\frac{i\tau\lambda_1^n}{\sigma_n}} F_n\left(e^{\frac{i\tau}{\sigma_n}}\right)\right]^{k-\lambda_2} -1 \right| = T_{2,1}+T_{2,2}$$

(6)

$F_n(,)$ ning harakteristik funksiyani Teylor qatoriga yoyib

$$F_n\left(e^{\frac{i\tau}{\sigma_n}}\right) = 1+\frac{i\tau\lambda_1^n}{\sigma_n} - \frac{\tau^2 M\left(\mu_n^1\right)^2}{2\sigma_n^2} + \frac{(i\overline{\tau})^3 M\left(\mu_n^1\right)^3}{6\sigma_n^3} + o(1), \qquad |\overline{\tau}|<|\tau|,$$

ni hosil qilamiz.

Bundan

$$\mathrm{e}^{-\frac{\mathrm{i}\tau\lambda_1^{\mathrm{n}}}{\sigma_n}} F_n(e^{\frac{i\tau}{\sigma_n}}) = e^{\frac{-\tau^2 D\mu_n}{2\sigma_n^2} + \frac{(-i\tau)3M(\mu_n-\lambda_1^n)^3}{6\sigma_n^3}} \tag{7}$$

Oxirgi tenglik va $\left|e^x-1\right| \le |x| e^{|x|}$ ga ko‘ra

$$J_{2,1} \le \max_{\lambda_2-\sigma_n^2\le k\le\lambda_2} \left| [e^{\frac{-\tau^2 D\mu_n}{2\sigma_n^2} + \frac{(-i\tau)^3 M(\mu_n'-\lambda_1^n)^3}{6\sigma_n^3}}]-1 \right|^{\lambda_2-k} \le \left| \frac{-\tau^2 D\mu_n}{2\sigma_n^2} + \frac{(i\tau)3M(\mu_n'-\lambda_1^n)^3}{6\sigma_n^3} \right| \cdot$$

$$(\lambda^2-k)\cdot e^{-\frac{\tau^2}{2\sigma_n^2}D\mu_n + \frac{|\tau|\beta_3(n)}{6\sigma_n^3})\sigma_n^2} \le \left( \frac{\tau^2\sigma_n^2 D\mu_n}{2\sigma_n^2} + \frac{|\tau|^3 \beta^3(n)\cdot\sigma_n^2}{6\sigma_n^3} \right) e^{-\frac{\tau^2}{2}D\mu_n + \frac{|\tau|^3}{6\sigma_n}\beta_3(n)} =$$

$$= \left( \frac{\tau^2 D\mu_n}{2} + \frac{|\tau|^3 \beta_3(n)}{6\sigma_n} \right) e^{-\frac{\tau^2 D\mu_n}{2} + \frac{|\tau|^3 \beta_3(n)}{6\sigma_n}} \tag{8}$$

Bu yerda $\beta_3(n) = M\left|\mu_n - \lambda_1^n\right|^3$.

Xuddi shunday

$$J_{2,2} \le \left( \frac{\tau^2 D\mu_n}{2} + \frac{|\tau|^3 \beta_3(n)}{6\sigma_n} \right) e^{-\frac{\tau^2 D\mu_n}{2} + \frac{|\tau|^3 \beta_3(n)}{6\sigma_n}}, \tag{9}$$

(6),(8), (9) dan

$$J_2 \le c_1 \tau^2 D\mu_n e^{-\tau^2\beta_3(n)} \tag{10}$$

(4),(5), (6), (10) dan

$$\left|\varphi_n(\tau) - \overline{\varphi_n}(\tau)\right| \le \left( \frac{2\lambda_2}{\sigma_n^4} + \tau^2 D\mu_n e^{-\tau^2 D\mu_n} \right)\left(1+o(1)\right) \tag{11}$$

Bundan

$$\psi_n(\tau)=\overline{\psi_n}(\tau)+\left(\frac{\lambda_2}{\sigma_n^4}+\tau^2 D\mu_n e^{-\tau^2 D\mu_n}\right)\left(1+o(1)\right) \qquad (12)$$

(7) ni e'tibor olgan holda

$$\left[e^{\frac{-i\tau\lambda_1}{\sigma_n}}F_n(e^{\frac{i\tau}{\sigma_n}})\right]^{\lambda_2}=e^{[-\frac{\tau^2}{2\sigma_n^2}D\mu_n+\frac{(i\tau)^3 M(\mu_n-\lambda_1{}^n)^3}{6\sigma_n^3}]\lambda_2}=e^{\frac{-\tau^2}{2\sigma_n^2}D\mu_n\lambda_2}\left|1+\frac{(i\tau)^3 M(\mu_n-\lambda_1{}^n)^3}{6\sigma_n^3}\lambda_2\right|=$$

$$=e^{\frac{-\tau^2}{2}(1-\sigma^2)}(1+\frac{|\tau|^3\lambda_2\lambda_1^n}{6\sigma_n^3}) \qquad (13)$$

yoki uchdan

$$\overline{\psi_n}(\tau)=\theta\left(\frac{\sigma i}{\sqrt{2}}\right)e^{-\frac{\tau^2}{2}(1-\sigma^2)}\left(1+\frac{|\tau|^3}{6\sigma_n^3}\lambda_2\lambda_1^n\right)\left(1+o(1)\right)$$

Demak (3), (2),(4), (13) dan lemma isboti kelib chiqadi.
Lemmaga Essen tengsizligini qo‘llab teorema isbotini keltirib chiqaramiz.

*II bob bo'yicha xulosalar*

Dissertatsiyaning 2-bobi " Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoni " deb nomlangan va bobda Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoni uchun va tasodifiy sondan boshlangan tarmoqlanish jarayoni yig'indisi uchun limit teoremalar keltirilgan.

## *III bob. 3.1.Tasodifiy sondan boshlangan tarmoqlanish jarayoni uchun limit teoremalar*

Faraz qilaylik $\{\xi_i\}^{\infty}$ o'zaro bog'liqsiz bir xil taqsimlangan tasodifiy miqdorlar va bularga bog'liq bo'lmagan $\{\mu_n\}^{\infty}$ Galton Vatson jarayoni bo'lsin.

Tarmoqlanish jarayonidan boshlangan tasodifiy miqdorlar yig'indisi ko'pgina jarayonlarga tadbiq qilinishi mumkin, xususan, kimyoga, xizmat ko'rsatish nazariyasiga, biologiyaga (zoologiya, entomologiya, genetika), yadro fizikasiga, aholishunoslik jarayonlariga tadbiq qilinishi mumkin.

Xususan, yadro zanjir reaksiyasida quyidagi holatda ko'rish mumkin: Bunda zarracha deganda neytronlarni qaraymiz. Bu zarracha boshqa zarrachalar bilan to'qnashib, yangi o'ziga o'xshagan zarrachaga aylanadi, har bir to'qnashish davrini nomerlab chiqish mumkin, agar $p(\mu_1 = k) = p_k$ $\left(k = \overline{0,\infty}\right)$ bilan bitta neytronni boshqa zarralar bilan to'qnashganda k zarrachaga aylanishini va $p_0$ bilan esa jarayonni tugash ehtimolligini ifodalasak, u holda keyingi to'qnashishlar shunday ( $\sum_{k=0}^{\infty} p_k = 1$ ) xarakterga ega bo'lsa, bu jarayon Galton - Vatson jarayonidir.

Xuddi shunday gen va mutatsiyani ham ma'lum shartlar ostida diskret tarmoqlanish jarayonini hosil qilishni ko'rish mumkin: $G_1$ Mendel qonuniga asosan, har bir to'qnashuvga ikki juft genlar mavjud bo'lib oddiy sxemaga bu juftlik A yoki a ko'rinishida deb faraz qilishi mumkin.

Natijada argonizmda AA, Aa, aa genotik hosil bo'lishi mumkin. Umumiy holda xar bir genotik $A_{1,}A_2,...,A_w$ holatda bo'ladi, xususan bu holda $C_w^{\lambda}$ genotik hosil bo'ladi.

Yangi organizmda ikki turli xil jinslarni to'qnashish natijasida yangi hosil bo'ladigan individum o'zini xususiyatini ikkita turli xil jinsdan oladi, masalan birinchi jins $A_1a_1$ genotipda ikkinchi jins $A_2a_2$ genotipda bo'lsin, u holda yangi individum $A_3a_3$ genotipda bo'lib $A_3$ *yo* $A_1$ *yo* $A_2$ dan xuddi shunday $a_3$ *yo* $a_1$ *yo* $a_2$ dan oladi. Quyidagi belgilanishni kiritamiz

$$S_{\mu_n} = \xi_1 + \xi_2 + ... + \xi_{\mu_n}, \; M\xi_1 = a, \; D\xi_1 = \sigma^\lambda, \; M\mu_1 = A, \; D\mu_1 = \sigma_1^\lambda$$

Agar $\mu_n$ Puasson qonuni bo'yicha taqsimlangan bo'lsa , u xolda $S_{\mu_n}$ sug'urta ta'rifi uchun optimal baxoni topishga imkon berdi .

Quyida teorema o'rinli .

Teorema. Agar $\{\xi_i\}_{i=1}^{\infty}$ lar bog'liqsiz va $\mu_n$ ga bog'liq bo'lmasa, $\mu_n$ $A^n$ atrofda jamlangan bo'lsa. u xolda $D\mu_1 = \sigma_1 < \infty$ , $M\xi_i^4 < \infty$, $\rho(\mu_n = 0) = 0$ bajarilganda

$$F_n(x) = \Phi\left( \frac{x}{A^n} \sqrt{A^n b^2 + a^2\left(\sigma_1^2 A^n (A^n - 1)\right) : A(A-1)} + aA^n - \mu_n a \right) +$$

$$+ c_1 \frac{M\left|\xi_1 - m\xi_1\right|^3}{\sqrt{\mu_n}(D\xi_1)^{\frac{3}{2}}} + c_1 \frac{M\left(\xi_1 - m\xi\right)^4}{\mu_n (D\xi_1)^3}$$

Bajariladi , bu yerda Φ(x)standart normal taqsimot , $c_i, i = 1.2$ absalyut o'zgarmas sonlar.

Isbot. Ma'lumki

$$F_n(x) = P\left( \frac{\xi_1 + \xi_2 + ... + \xi_{\mu_n} - \mu S_{\mu_n}}{\sqrt{DS_{\mu_n}}} < \frac{x}{A^n} \right) = P\left( \frac{\xi_1 + \xi_2 + ... + \xi_{\mu_n} - aA^n}{\sqrt{A^n b^2 + a^2\left(\sigma_1^2 A^n (A^n - 1)\right) : A(A-1)}} < \frac{x}{A^n} \right) =$$

$$= \sum_{k=1}^{\infty} p(\mu_n = k) p\left[ \xi_1 + \xi_2 + ... + \xi_k < \frac{x}{A^n} \sqrt{b^2 A^2 + a^2 (\tau_1^2 A^n (A^n - 1)) : A(A-1)} + aA^n \right] =$$

$$= \sum_{k=1}^{\infty} p(\mu_n = k) p\left( \frac{\xi_1 + \xi_2 + ... + \xi_k - ka}{\sqrt{k}\sqrt{\tau^2}} < \left[ \frac{x\sqrt{b^2 A^2 + a^2 (\tau_1^2 A^n (A^n - 1)) : A(A-1)}}{A^n} + aA^n - ka \right] : \sqrt{k\tau^2} \right) =$$

$$= \sum_{k=1}^{\infty} p(\mu_n = k)\phi(\left[ \frac{x}{A^n} \sqrt{A^n\, b^2 + \left[a^2\tau_1^2 A^n (A^n - 1)\right] : A(A-1) + aA^n - kA^n} \right] : \sqrt{k\tau^2}) : \sqrt{k\tau} + C_1 \frac{M|\xi_1 - M\xi_1|^3}{\sqrt{k\tau^3}} +$$

$$+ C_2 \frac{M(\xi_1 - M\xi_1)^4}{k\tau^4} = \phi(\frac{x}{A^n} \sqrt{A^n b^2 + \left[a^2\tau_1^2 A^n (A^n - 1)\right] : A(A-1)})$$

## *3.2.Разложение распределения нормального закона для сумм докритического ветвящихся процессов*

Пусть $\{\mu_{ð}\}_1^{\infty}$ образует процесс Гальтона- Ватсона. Рассмотрим распределение величин $\mu = \mu_0 + \mu_1 + \cdots + \mu_n + \cdots,$

$$\mu_0 + \mu_1 + \ldots + \mu_n = \overline{\mu_n}.$$

Обозначим через $F(s)$ и $F_n(s)$ производящие функции $\mu$ и $\overline{\mu_n}$ соответственно.

Предположим, что $\mu$ конечна с вероятности единица, тогда F(1)=1 (см[1]).

Хокинс и Улам (см[2]) показали, что

$$F(s) = sf[F(s)], \quad f(s) = MS^{\mu_1} \tag{1}$$

Пейкс (см[2]) доказал, что если $A = M\mu_1 < 1, f''(1) < \infty$, то при $n \to \infty$

$$P\left\{ \frac{\overline{\mu_n} - nB'(1)}{\sqrt{nH}} < x / \mu_n > 0 \right\} \to \phi(x),$$

где $M\left(S^{\mu_n} / \mu_n > 0\right) = B(s),$ $H = D\mu$ ,

$\phi(x)$ -стандартный нормальный закон.

Дуасс, В. В .Висков, Бойд (см[2]) получили, что для процесса Гальтона-Ватсона

$$P\{\mu = j / \mu_0 = k\} = k \cdot j^{-1} \quad P\{\mu_{1,1} + \mu_{1,2} + \ldots \mu_{1,j} = j - k\}, \tag{2}$$

где $\mu_{1,e}$, $e = \overline{1, j}$ независимы и одинаково распределены как $\mu_1$.

Нормируя и центрируя имеем

$$\frac{\mu_{1,1} - A}{\delta\sqrt{j}} + \frac{\mu_{1,2} - A}{\delta\sqrt{j}} + \cdots + \frac{\mu_{1,n} - A}{\delta\sqrt{j}} = \frac{j - k - jA}{\delta\sqrt{j}}, \tag{3}$$

где $\delta^2 = D\mu_1$.

Пусть четвертый момент $\xi_k$ с нулевым средним и единичной дисперсией ,а также характеристической функцией f(t) такой, что (см[3])

$$\int_{-\infty}^{+\infty} |f(t)|^{\gamma} dt < \infty$$

для некоторого $\gamma > 0$,кроме того $\mu(t) \geq e^{-\frac{t^2}{2}}$ и число T>0 такое, что $|f(t)| \leq \mu(t)$ для всех $|t| \leq T$.

Тогда для любого $n \geq \max(2, \gamma)$ и всех x для плотности $\xi_1, \xi_2, ... \xi_n$ $P_n(x)$ справедливо разложение

$$P_n(x) = \varphi(x) + \frac{a_3}{6\sqrt{n}}(x^3 - 3x)\varphi(x) + R, \tag{4}$$

где $\varphi(x)$ плотность стандартного нормального закона, $a_3 = \mu\xi_k^3$, а для величины R остаточной части разложения плотности справедлива оценка

$$|R| \leq \left(\frac{\beta_4}{4!} + \frac{1}{8}\right)\frac{B_{4,n-1}}{n} + \frac{1}{2}\left(\frac{a_3}{6}\right)^2 \frac{B_{6,n}}{n} + \frac{|a_3| \cdot B_{5,n-1}}{12\sqrt{n^3}} + \frac{1}{2} \cdot \frac{|a_3|}{6} \cdot \left(\frac{\beta_4}{4!} + \frac{1}{8}\right) \cdot \frac{B_{7,n-1}}{\sqrt{n^3}} +$$

$$+ \frac{1}{4}\left(\frac{a_3}{6}\right)^2 \cdot \frac{B_{8,n-1}}{n^2} + \frac{|a_3|}{3\pi\sqrt{n}}\left(\frac{T^2 n}{2} + 1\right)e^{-\frac{T^2 n}{2}} + \frac{\sqrt{n}}{\pi}\alpha^{n-\gamma}(T)\int_T^{\infty} |f(t)|^{\gamma} dt - \frac{1}{\pi T\sqrt{n}} \cdot e^{-\frac{T^2 n}{2}},$$

где $\beta_4$ четвертый момент $\xi_i$, кроме того

$$B_{k,n-1} = \frac{1}{2\pi}\int_{-T\sqrt{n}}^{T\sqrt{n}} |t|^k \mu^{n-1}\left(\frac{1}{\sqrt{n}}\right)dt, \qquad k=4,5,7,8$$

$$B_{6,n} = \frac{1}{2\pi}\int_{-T\sqrt{n}}^{T\sqrt{n}} t^6 \mu^n\left(\frac{t}{\sqrt{n}}\right)dt, \alpha(t) = \max\{|f(t)| : t \geq T\} < 1 .$$

Используя (3) равенство(2) представим в виде

$$P(\mu_0 + \mu_1 + ... + \mu_n + ... = j / \mu_0 = k) = kj^{-1} p(\mu_{1,1} + \mu_{1,2} + ... + \mu_{1,j} = j - k) =$$

$$= kj^{-1} P\left(\frac{\mu_{1,1} - A}{\delta\sqrt{j}} + \frac{\mu_{1,2} - A}{\delta\sqrt{j}} + ... + \frac{\mu_{1,j} - A}{\delta\sqrt{j}} = \frac{j - k - jA}{\delta\sqrt{j}}\right).$$

Применив (4) к последнему равенству имеем следующее утверждение:

**Теорема.** Пусть A<1, M $\mu_1^4 < \infty$ а также

$$\int_{-\infty}^{+\infty} \left| M\left(e^{\frac{it(\mu_1 - A)}{\delta}}\right)\right|^{\gamma} dt < \infty, \quad \gamma > 0$$

и $|M(e^{\frac{it(\mu_1-A)}{\delta}})| \leq e^{-\frac{t^2}{2}},$ при |t|<T, T>0.

Тогда для любого ***j***≥***max*** (2 ,$\gamma$ ) и всех $\chi = \frac{j - k - jA}{\delta\sqrt{j}}$ для плотности $\overline{P_j}(x)$ величины $\left\{\frac{\mu_{1,e} - A}{\delta}\right\}_1^j$ справидливо разложение типа (4), т-е

$$\overline{P_j}(x) = \left( \varphi(x) + \frac{M\left(\frac{\mu_1 - A}{\sigma}\right)^3}{6\sqrt{j}} \left(x^3 - 3x\right)\varphi(x) + R_1 \right) k \cdot j^{-1},$$

где

$$|R|_1 \le \left(\frac{\overline{\beta_4}}{4!} + \frac{1}{8}\right)\frac{\overline{B}_{4,J-1}}{J} + \frac{1}{2}\left(\frac{\overline{a}_3}{6}\right)^2 \frac{\overline{B}_{6,j}}{j} + \frac{|\overline{a}_3|\overline{B}_{5,j-1}}{12\sqrt{j^3}} + \frac{1}{2}\cdot\frac{|\overline{a_3}|}{6}\left(\frac{\overline{\beta_4}}{4!} + \frac{1}{8}\right)\cdot\frac{\overline{B}_{7,j-1}}{\sqrt{j^3}} +$$

$$\frac{1}{4}\cdot\left(\frac{\overline{a_3}}{6}\right)^2 \cdot \frac{\overline{B}_{8,j-1}}{j^2} + \frac{|\overline{a_3}|}{3\pi\sqrt{j}}\left(\frac{T^2 j}{2} + 1\right)C^{-\frac{T^2 j}{2}} + \frac{\sqrt{j}}{\pi}\overline{\alpha}^{\,j-\gamma}(T)\int_T^{+\infty}\left|M\left(e^{\frac{it(\mu_1 - A)}{\delta}}\right)\right|^{\gamma} dt - \frac{1}{\pi T\sqrt{j}}\cdot e^{-\frac{T^2 j}{2}},$$

где $\overline{\beta_4} = M\left(\frac{\mu_1 - A}{\delta}\right)^4$, $\overline{a_3} = M\left(\frac{\mu_1 - A}{\delta}\right)^3$, $\overline{B_k}_{,\,j-1} = \frac{1}{2\pi}\int_{-T\sqrt{j}}^{T\sqrt{j}} |t|^k e^{-\frac{(j-1)t^2}{2j}} dt,$

k=4,5,7,8, $\overline{B}_{6,j} = \frac{1}{2\pi}\int_{-T\sqrt{j}}^{T\sqrt{j}} t^6 e^{-\frac{t^2}{2}} dt,$ $\overline{\alpha}(T) = \max\left\{\left|M\cdot e\frac{it(\mu_1 - A)}{\sigma}\right| : t \ge T\right\} < 1.$

Доказательство теоремы проводится аналогично как утверждению формулы (4) .

Для сравнения сумма четвертых моментов $\mu$ можно использовать функциональное уравнение (1).

## *3.3.Tarmoqlanish jarayonida barcha davrdagi zarrachalar yig'indisi uchun limit teoremalar*

Faraz qilaylik $\{\mu_n\}_1^\infty$ Galton-Vatson jarayonini tashkil qilsin. Ma'lumki, bunday jarayon uchun quyidagi xususiyat o'rinlidir

$$\mu_n = \mu_{n-1}^{(1)} + \mu_{n-1}^{(2)} + \cdots + \mu_{n-1}^{(\mu_1)} = \mu_1^{(1)} + \mu_1^{(2)} + \cdots + \mu_1^{(\mu_{n-1})} \qquad (1)$$

bu yerda $\mu_{n-1}^{(j)}, j = \overline{1,\mu_1}$ lar bog'liqsiz $\mu_{n-1}$ bilan, $\mu_1^{(j)}, j = \overline{1,\mu_{n-1}}$ lar ham bog'liqsiz $\mu_1$ bilan bir xil taqsimlangan tasodifiy miqdorlardir $\rho(\mu_0 = 1) = 1,$ [1].

Ma'lumki t.j. biologiyada, kimyoda, aholishunoslikda, genetikada, fizikada va boshqa sohalarda keng qo'llaniladi. Xususan kimyoda uran moddasini parchalanishida, radiaktiv moddalarni tarqalishi jarayonlarni matematik modelini tuzishda ishlatiladi.

Quyidagi belgilashlarni kiritamiz:

$$f_1(S)=\sum_{k=0}^{\infty}P(\mu_1=k)S^k=\sum_{k=0}^{\infty}P_kS^k,\ f_1'(1)=a,\ \sigma^2=D\mu_1.$$

(1) ga asosan $f_n(S)=MS^{\mu_n}$ uchun

$$f_{n-1}(f_1(S))=f_1(f_{n-1}(S))=f_n(S) \quad (2)$$

o'rinli. (2) dan $S=1$ nuqta atrofida birinchi va ikkinchi tartibli hosilalarini hisoblab quyidagilarni hosil qilamiz

$$f_n(S)=M\mu_n$$

$$D\mu_n=\begin{cases}\dfrac{\sigma^2a^n(a^n-1)}{a^2-a}, & m\neq 1\\ n\sigma^2, & m=1\end{cases}$$

.

Masalani qo'yilishi:

$$\overline{\mu}=\mu_0+\mu_1+\cdots+\mu_n+\cdots,\qquad \overline{\mu_n}=\mu_0+\mu_1+\cdots+\mu_n+\cdots, \quad (3)$$

larni o'rganamiz. Buning uchun birinchi navbatda $M\overline{\mu_n}$ va $M\mu$ larni hisoblaymiz, ya'ni

$$M\overline{\mu_n}=\sum_{k=0}^{n}a^k=1+\frac{(1-a^n)}{1-a},\qquad \lim_{n\to\infty}\overline{\mu_n}=\begin{cases}1+\dfrac{a}{1-a}, & a<1\\ +\infty, & a\geq 1\end{cases}$$

.

Ma'lumki, $a>1$ jarayon cheksiz davom etadi. $a\leq 1$ da jarayon ertami, kechmi tugaydi.

Agar $P_k\neq 0$ ni qanoatlantiruvchi k-1 $d$ ga bo'linsa, $a>1, P_0>0, k\to 0$ larda

$$P(\mu = k) = \frac{d}{\sqrt{2\pi f''(1)}}\frac{1}{\sqrt{k^3}} + \mathrm{O}\left(\frac{1}{\sqrt{k^5}}\right)$$

bajariladi ([2] )

Peyks [3] $a<1$ hol uchun, $f'''(1)<+\infty$ va $n\to\infty$ da quyidagi munosabat o'rinliligini ko'rsatgan

$$P\left(\frac{\mu_0+\mu_1+\cdots+\mu_n-nB'(1)}{\sqrt{Hn}}<x\Big/\mu_n>0\right)\to\phi(x),$$

bu yerda $\phi(x)$ standart normal qonun, $H=D\overline{\mu},\ \lim\limits_{n\to\infty}M\left(S^{\mu_n}/\mu_n>0\right)=B(S).$

Ma'lumki, Duon, O. V. Viskov, Boyd [3] lar

$$P(\mu_0+\mu_1+\cdots+\mu_n+\cdots=j/\mu_0=2)=\frac{1}{j}P\left(\xi_1+\xi_2+\cdots+\xi_j+\cdots=j-1\right) \qquad (4)$$

ligini ko'rsatganlar, bu yerda $\xi_j$ lar bog'liqsiz va $\mu_1$ bilan bir xil bog'liqsiz taqsimlangan.

Ehtimollar xossasidan

$$P\left(\xi_1+\xi_2+\cdots+\xi_j+\cdots<j\right)-P\left(\xi_1+\xi_2+\cdots+\xi_j+\cdots<j-1\right)=$$
$$=P\left(\xi_1+\xi_2+\cdots+\xi_j+\cdots=j-1\right). \qquad (5)$$

Bularni quyidagicha ham yozish mumkin.

$$P\left(\frac{\sqrt{\xi_1+\xi_2+\cdots+\xi_j-aj}}{\sqrt{jD\mu_1}}<\frac{j-aj}{\sqrt{jD\mu_1}}\right)-P\left(\frac{\sqrt{\xi_1+\xi_2+\cdots+\xi_j-aj}}{\sqrt{jD\mu_1}}<\frac{j-1-aj}{\sqrt{jD\mu_1}}\right). \qquad (6)$$

Quyidagi teorema o'rinli:

Teorema. Agar

$$|\tau|<T_j=\frac{\sqrt{j}\,(D\mu_1)^{\frac{3}{2}}}{\beta_3},\ \beta_3=M(\mu_1-a)^3,\ j\ge 2$$

bo'lsa, u holda

$$P(\mu_0+\mu_1+\cdots+\mu_n+\cdots=j/\mu_0=2)=\frac{1}{j}\Phi\left(\frac{j-ja}{\sqrt{jD\mu_1}}\right)-\Phi\left(\frac{j-1-ja}{\sqrt{jD\mu_1}}\right)+$$

$$+\quad\left(\frac{\beta_3}{6\sqrt{D\mu_1}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\int_0^{T_j}\frac{\tau^2}{\sqrt{j}}e^{\frac{j-1}{3j}\tau^2}d\tau$$

o'rinli bo'ladi.

Teoremani isbotlash uchun quyidagi lemmani keltiramiz.

Lemma.

$$\left|\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau k}{\sqrt{jD\xi_1}}}\right)^j-e^{-\frac{\tau}{2}}\right|\leq=\left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\int_0^{T_j}\frac{|\tau|^3}{\sqrt{j}}e^{-\frac{j-1}{3j}\tau^2}d\tau\ .$$

Ma'lumki, agar G(x) taqsimot funksiya uchun $\alpha_k$ tartibli moment mavjud bo'lsa, u holda unga mos $g(t)$ xarakteristik funksiyani quyidagicha yozish mumkin.

$$g(t)=\sum_{j=0}^{k-1}\frac{\alpha j}{j!}(jt)^j+\gamma(t)\frac{\overline{\beta_k}}{k!}t^k,$$

Bu yerda $\overline{\beta_k}=M|\xi-M\xi|^k$, $\gamma(t)$ kompleks o'zgaruvchili funksiya bo'lib $|\gamma(t)<1|$.

Oxirgi tenglikdan foydalanib

$$f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau k}{\sqrt{jD\xi_1}}}=1-\frac{\tau^2}{2j}+\gamma(\tau)\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^2 j^3}} \qquad (7)$$

ni hosil qilamiz.

Normal qonunning xarakteristik funksiyasini

$$e^{-\frac{\tau^2}{2}}=\left(1-\frac{\tau^2}{2j}+\gamma(t)\frac{\beta_3(\phi)}{3!\,j^{\frac{3}{2}}}\tau^3\right)^j \qquad (8)$$

ko'rinishda ifodalaymiz, bu yerda

$$\beta_3(\phi)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}|x|^3e^{-\frac{x^2}{2}}dx=\frac{2\sqrt{2}}{\sqrt{\pi}} \qquad (9)$$

Ixtiyoriy ikkita xarakteristik funksiya uchun

$$\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right)^j-\left(e^{-\frac{\tau^2}{2}}\right)^j=\sum_{l=0}^{j-1}\left[\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right)^{j-l-1}e^{-\frac{\tau^2}{2}l}\right].$$

$$\left[f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}-e^{-\frac{\tau^2}{2j}}\right].$$

(7) - (9) dan foydalanib topamiz

$$f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}-e^{-\frac{\tau^2}{2j}}=J(\tau)\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^3 j^3}}+\gamma(t)\frac{2\sqrt{2}\tau^3}{6\sqrt{\pi}} \qquad (11)$$

yoki

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}-e^{-\frac{\tau^2}{2j}}\right|\le\left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\frac{|\tau|^3}{\sqrt{j^3}} \qquad (12)$$

(7) dan

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right|\le 1-\frac{\tau^2}{2j}+\frac{\beta_3\tau^3}{6\sqrt{(D\mu_1)^3 j^3}}\le 1-\frac{\tau^2}{2j}\left(\frac{\beta_3|\tau|}{3\sqrt{(D\mu_1)^3 j^3}}\right)\le 1-\frac{\tau^2}{3j} \qquad (13)$$

ma’lumki, $0\le\alpha\le 1$ da $1-\alpha\le e^{-\alpha}$.

U holda (13) dan

$$\left|f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right|\le e^{-\frac{\tau^2}{3j}} \qquad (14)$$

Shu bilan birga

$$e^{-\frac{\tau^2}{2k}}\le e^{-\frac{\tau^2}{3k}}$$

u holda $|\tau|\le T_j$ bo’lgan holda (14), (15) dan

$$\left|\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right)^{j-l-1}-e^{-\frac{\tau^2}{2k}j}\right|\le e^{-\frac{(j-1)\tau^2}{3j}}d\tau \qquad (16)$$

Natijada (10), (12), (16) lardan.

$$\left|\left(f_1\left(e^{\frac{i\tau}{\sqrt{jD\mu_1}}}\right)e^{-\frac{i\tau a}{\sqrt{jD\mu_1}}}\right)^j - e^{-\frac{\tau^2}{2}}\right| \le \left(\frac{\beta_3}{3!\sqrt{(D\mu_1)^3}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\frac{\tau^3}{\sqrt{j}}e^{-\frac{(j-1)\tau^2}{3j}}d\tau$$

Lemma isbotlandi.

Teorema isboti. (4)va(6) va Berdi-Esseyen tengsizligi va lemmaga ko'ra

$$P(\mu_0+\mu_1+\cdots+\mu_n+\cdots=j/\mu_0=2)=\frac{1}{j}\left[P\left(\frac{\sqrt{\xi_1+\xi_2+\cdots+\xi_j-aj}}{\sqrt{jD\mu_1}}<\frac{j-aj}{\sqrt{jD\mu_1}}\right)-\right.$$

$$\left.-P\left(\frac{\xi_1+\xi_2+...+\mu_j-ja}{\sqrt{jD\mu_1}}<\frac{j-1-j-a}{\sqrt{jD\mu_1}}\right)\right]=\frac{1}{j}$$

$$\left[\Phi\left(\frac{j-ja}{\sqrt{jD\mu_1}}\right)-\Phi\left(\frac{j-1-ja}{\sqrt{jD\mu_1}}\right)+\left(\frac{\beta_3}{3!\sqrt{jD\mu_1}}+\frac{\sqrt{2}}{3\sqrt{\pi}}\right)\int_0^{T_j}\frac{\tau^2}{\sqrt{j}}e^{-\frac{(j-1)}{3j}\tau^2}d\tau\right]$$

Teorema isbotlandi.

## *III bob bo'yicha xulosalar*

Dissertatsiyaning 3-bobida Tasodifiy sondan boshlangan tarmoqlanish jarayoni uchun limit teoremalar va Tarmoqlanish jarayonida barcha davrdagi zarrachalar yig'indisi uchun limit teoremalar keltirilgan.

# XULOSA

Magistrlik dissertatsiyasi kirish, asosiy qism, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan.

Magistrlik ishida ehtimollar nazariyasining asosiy yo'nalishlaridan biri bo'lgan tarmoqlanish jarayoni uchun limit teoremalarni o'rganish va ayrim natijalar olishdan iborat ishlar bajarildi , kelajakda bu sohada ilmiy izlanishlar olib boramiz degan umiddaman.

Tarmoqlanish jarayoni uchun limit teoremalar keyingi vaqtda matematikada va hayotda eng dolzarb mavzuligi, hamda bu sohani hayotga tatbiqi va tatbiqiy ahamiyati ko'p bo'lganligi uchun bu sohani o'rganish muhim ahamiyatga ega. Bu sohani kelajakda genetika soxasi bilan bog'lab yana davom ettirish mumkin.

Magistrlik dissertatsiyasining kirish qismida Prezidentimiz Islom Abdug'aniyevich Karimovning ma'ruzalaridan, allomalarimizning fan sohasiga qo'shgan xissalari qisqacha bayon qilingan.

Asosiy qismning 1-bobida « Tarmoqlanish jarayonlari» haqida to'liq ma'lumotlar berilgan.

2-bobda «Binomial qonun bo'yicha taqsimlangan tarmoqlanish jarayoni» bayon etilgan.

3-bobda Tarmoqlanish jarayoniga oid chop etilgan maqolalar(olingan natijalar) keltirilgan.

Ushbu dissertatsiyaning ilmiy–nazariy tomonlarini ishlab chiqishda tarmoqlanish jarayoni soxasida ish olib borishgan matematik olimlarimizdan A.H.Колмогоров, T.Xarris, B.A.Sevastyanov, A.M.Zubkov, V.A.Vatutin, Atrey, Ney, hozirda esa o'zbekistonlik ustozlarimiz Sh.K.Farmonov, I.S.Badalboev va boshqa yirik olimlar nazariy tadqiqotlariga tayanildi.

Dissertatsiyada asosan tasodifiy sondan boshlangan tarmoqlanish jarayoni uchun limit teoremalar, tarmoqlanish jarayonida barcha davrdagi zarrachalar yig'indisi uchun limit teoremalar keltirilgan.

Aslida $Z_{n,\nu_n}$ ni o'rganish muhim masalalardan biri hisoblanadi, chunki $Z_{n,\nu_n}$ ni taqsimotini $n \to \infty$ da aniq ko'rinishini doim aniqlab bo'lmaydi , Hamon shunday ekan $Z_{n,\nu_n}$ ni ayrim xususiy hollarda taqsimoti o'rganildi, lekin bu ishni davom ettirish mumkin va bu bir nechta magistrlik dissertatsiyasini tashkil qilish mumkin va uni tadbiqini ham ishlab chiqish lozim .meni kelajakdagi maqsadim barcha hollarni ko'rib chiqish va tadbiqlar bo'yicha ham ish qilishdir.

**"Biz tayanchimiz va suyanchimiz, g'ururimiz va iftixorimiz bo'lmish bolalarimizga, farzandlarimizga ishonch bilan, hurmat-e'tibor bilan qarashni kelajagimizga bo'lgan ishonch, millatimizga, xalqimizga bo'lgan hurmat-ehtirom ifodasi, deb bilamiz."** *I.A. Karimov*

"Sog'lom bola yili" Davlat dasturi buning yana bir yorqin tasdig'idir.

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