O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

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ACHILOV JAMOL

IDEAL SUYUQLIK VA DEFORMATSIYALANUVCHI MUHIT BILAN NOSTATSIONAR TA'SIRLASHUVCHI ANIZOTROP SILINDRIK QATLAMNING BO'YLAMA TEBRANISHLARI

5A440204 – Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi

Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi bo'yicha magistr darajasini olish uchun
MAGISTRLIK DISSERTATSIYASI

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M U N D A R I J A

KIRISH.

Mavzuning dolzarbligi. To'lqinlar tarqalish jarayonlarini o'rganish mexanikaning dolzarb masalalaridan biridir. Tutash muhitlar va muhandislik inshootlari elementlarida to'lqinlar tarqalish qonuniyatlarini tadqiq etish ana shu dolzarb masalalarning eng muhimlaridan hisoblanadi.

Doiraviy silindrik elastik qobiqlarining qatlam va qovushoq yoki ideal suyuqliklar bilan o'zaro nostatsionar ta'sirlashuvi haqidagi muammolar ham yuqoridagi aktual masalalar qatorida kiradi. Shu nuqtai-nazardan qaraganda dissertatsiya mavzusi doirasida tadqiq etilishi lozim bo'lgan suyuqlikni o'z ishida saqlovchi va deformatsiyalanuvchi muhit bilan ta'sirlashuvchi silindrik qatlamning bo'ylama-radial tebranishlari haqidagi masala ham deformatsiyalanuvchi qattiq jismlar mexanikasining dolzarb masalasidir.

Yuqorida aytilganlar bilan bir qatorda shuni ham ta'kidlash lozimki silindrik qatlamlarning suyuqliklar bilan o'zaro nostatsionar ta'sirlalashuvi haqidagi masalalar, juda ko'p hollarda faqat ideal suyuqlik uchungina yechilgan. Ayni paytda, suyuqlik qovushoq bo'lsa, bunday ishlarning soni juda kam. Qovushoq suyuqlik va silindrik qobiqlarning, umuman qattiq jismlarning, o'zaro nostatsionar ta'siri masalasini tadqiq qilish ko'lamining nisbatan torligi, asosan, qovishoq suyuqlik uchun Nave-Stoks tenglamasini yechishga duch kelinadigan, va shu vaqtdagacha hal bo'lmagan, matematik qiyinchiliklar bilan bog'liq. Bundan tashqari qatlamni o'rab turuvchi deformatsiyalanuvchi muhitning reologik xususiyatlaridan bog'liq holda masalaning ikkinchi tomoni ham juda murakkablashadi.

Xuddi ana shu munosabat bilan, endi qattiq jismlarning qovushoq suyuqliklar bilan o'zaro nostatsionar ta'sirlashuvi haqidagi masalalarni yangicha yo’nalishda tadqiq qilish imkoniyati paydo bo’ldi. Ikkinchi tomondan deformatsiyalanuvchi qattiq jismlarning, xususan silindrik qatlam va qobiqlarning, tutash muhitlar bilan o’zaro dinamik ta’sirlashuvi haqidagi masalalarni yechishda, qobiq ushun asosiy tenglamalar sifatida, taqribiy tebranish tenglamalardan foydalanadilar. Ana shunday tebranish tenglamalarni chiqarish ham bugungi kunda eng dolzarb bo’lib turibdi.

Ayni paytda, qatlam yoki qobiq deformatsiyalanuvchi muhitda joylashgan bo’lsa, bunday ishlarning soni juda kam. Silindrik qobiqning ichidagi suyuqlik qovushoq bo’lgan hol foydalanilgan adabiyotlar ro’yxatida ishlari zikr etilgan va olimlar jamoatchiligi tomonidan tan olingan olimlar Акилов Ж.А., Вестяк А.В., Горшков А.Г., Вольмир А. Тарлаковский Д.В. , Григолюк Э.И., Селезов И.Т. , Губенко В.С., Кузнецов В.Н., Гузь А.Н., Колтунов М.А., Кристенсен Р., Петрашень Г.И., Работнов Ю.А., Филиппов И.Г, Худойназаров Х.Х., Darmond R.P. , Rouleau W.T., Flugge W. va boshqalar tomonidan tahlil etilgan.

Qovushoq suyuqlik va silindrik qobiqlarning, umuman qattiq jismlarning, o’zaro nostatsionar ta’siri masalasini tadqiq qilish ko’lamining nisbatan torligi, asosan, qovishoq suyuqlik uchun Nave-Stoks tenglamasini yechishga duch kelinadigan, va shu vaqtdagacha hal bo’lmagan, matematik qiyinchiliklar bilan bog’liq. Xususan, shu vaqtgacha Nave-Stoks tenglamasining umumiy integrali topilmagan.

1980 yilda A.N. Guz tomonidan qovushoq suyuqlik uchun Nave-Stoks tenglamalari, to’rtta, bir-biriga bog’lik bo’lmagan, potensial funksiyalarga nisbatan tartibi ikkidan yuqori bo’lmagan differensial tenglamalarga keltirildi. Olingan tenglamalar qattiq jismlardagi to’lqin tarqalish tenglamalariga juda o’xshash bo’lib ularni yechish imkoniyati tug’ildi.

Dissertatsiyasining asosiy maqsadi, yuqorida sanab o’tilgan dolzarb masalalar mavzularidan kelib chiqqan holda, quyidagilardan iborat:

tashqi nostatsionar ta'sirlar ostidagi ideal suyuqlik bilan to'ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta'sirlashuvchi doiraviy silindrik anizotrop qatlamning (xususiy holda qobiqning) bo'ylama-radial tebranishlari umumiy tenglamalarini keltirib chiqarish, bu tenglamalarning xususiy va limitik hollarini tekshirish; olingan umumiy tenglamalardan amaliyotda qo'llash mumkin bo'lgan taqribiy tenglamalarni chiqarish hamda amaliy masalalar yechishda lozim bo'ladigan boshlang'ich va chegaraviy shartlarni topishdan iborat.

Olingan asosiy natijalarning ilmiy yangiligi quyidagilardan iborat:

- ichi ideal suyuqlik to'ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta'sirlashuvchi doiraviy silindrik anizotrop qatlamning bo'ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarildi;

- qatlam uchun olingan umumiy tenglamalarning xususiy va limitik hollarini tadqiq qilish, jumladan, ichiga ideal suyuqlik to'ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta'sirlashuvchi doiraviy silindrik anizotrop qobiqlarning bo'ylama-radial tebranishlari umumiy tenglamalari olindi;

-tebranish tenglamalar bilan bir qatorda anizotrop qatlam yoki anizotrop qobiqning istalgan kesimi nuqtalaridagi kuchlanganlik-deformatsiyalanganlik holatini talab etilgan aniqlikda hisoblashga imkon beruvchi algoritm ishlab chiqildi;

- muhandislik amaliyotida qo'llash mumkin bo'lgan taqribiy tenglamalar va amaliy masalalarning boshlang'ich va chegaraviy shartlari topildi.

Dissertatsiya himoyasiga quyidagi natijalar olib chiqilmoqda:

- ideal suyuqlik to'ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta'sirlashuvchi doiraviy silindrik anizotrop qatlamning bo'ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarildi;

- anizotrop qatlam uchun olingan umumiy tenglamalarning xususiy va limitik hollarini tadqiq qilish, jumladan, ichiga ideal suyuqlik to'ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta’sirlashuvchi doiraviy silindrik anizotrop qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalari olindi;

- anizotrop qatlam yoki qobiqning istalgan kesimi nuqtalaridagi kuchlanganlik-deformatsiyalanganlik holatini talab etilgan aniqlikda hisoblashga imkon beruvchi algoritm;

- amaliy masalalarning boshlang’ich va chegaraviy shartlari va muhandislik amaliyotida qo’llash mumkin bo’lgan, tartibi to’rtdan yuqori bo’lmagan taqribiy integro-differensial tenglamalar.

Tadqiqotlarning ilmiy ahamiyati ichida ideal suyuqlik to’ldirilgan doiraviy silindrik anizotrop qatlam va qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalarining yuqoridagi sistemalar tebranishlari haqidagi masalalarni yechishga qollash mumkinligi bo’lib, olingan natijalar tebranish jarayonidagi kuchlanganlik-deformatsiyalanganlik holatlarini aniqlashda ishlatilishi mumkinligidir.

Tadqiqotlarning amaliy ahamiyati olingan taqribiy tenglamalarning boshqa masalalarni, xususan yuqorida qaralgan doiraviy silindrik anizotrop qatlam, ideal suyuqlik va o’rab turuvchi deformatsiyalanuvchi muhitdan iborat sistemning majburiy tebranishlari haqidagi masalalarni ham yechishga qo’llash mumkin ekanligidir.

Dissertatsiya ishi 2 bobdan iborat.

Birinchi bobda doiraviy silindrik qatlam va qobiqlarning suyuqliklar deformatsiyalanuvchi jismlar bilan o’zaro nostatsionar ta’siriga bag’ishlangan ilmiy adabiyotlar sharhi, deformatsiyalanuvchi qattiq jism uchun potensial funksiyalarda ifodalangan to’lqin tenglamalari keltirilgan. Shuningdek siqiluvchi suyuqlik zarrachasi harakatning tenglamalari va ularning potensial funksialar orqali to’lqin tenglamalariga o’xshash ko’rinishdagi ifodalanishi keltirilgan.

Ushbu bobning *birinchi paragrafi* "Qovushoq suyuqlik va deformatsiyalanuvchi muhitning silindrik qatlam va qobiqlar bilan o'zaro nostatsionar ta'sirlashuvi. Masalaning zamonaviy holati" deb nomlangan va silindrik qatlam va qobiqlarning ideal va qovushoq suyuqliklar hamda deformatsiyalanuvchi muhitlar bilan o'zaro nostatsionar ta'sirlashuviga bag'ishlangan ilmiy tadqiqotlar va ularning natijalari bo'yicha ilmiy matbuotda e'lon qilingan maqolalarning ba'zilari haqida to'xtalib o'tilgan. Bunda ushbu tadqiqotlar yoki ular to'g'risidagi maqolalarni ikki yo'nalishga ajratilgan:

a) nostatsionar tebranishlar tenglamalarini chiqarish; b) ikki qattiq va suyuq muhitlarning o'zaro ta'sirlashuv masalalari.

Ikkinchi paragraph "Deformatsiyalanuvchi qattiq jism nuqtasida kuchlanish va deformatsiya orasidagi bog'lanishlar. Qattiq jismning muvozanat va harakat tenglamalari" deb ataladi. Ushbu paragraph doirasida Elastiklik nazariyasining dissertatsiya ishida kerak bo'ladigan va zarur bo'lganda foydalaniladigan asosiy tenglamalari va munosabatlarini qisqa ko'rinishda keltirilgan. Keltirilgan formulalar va tenglamalarning qanday yo'l bilan olinganligiga to'xtalib o'tirilmagan.

Uchinchi paragraf "Suyuqlik harakati uchun asosiy tenglamalar" nomi bilan yuritiladi. Bu paragrafda doiraviy silindrik elastik qobiqning ichidagi suyuqlikni umumiy holda qovushoq va siqiluvchi deb hisoblanadi va bundan tashqari suyuqlik zarrachalarining tebranishlarini ham kichik deb qabul qilinadi. U holda bunday suyuqlikni tavsiflash uchun Lagranj koordinatalarini qo'llash mumkin bo'lishi ko'rsatiladi va shuning uchun ham qaralayotgan suyuqlik uchun [7,8] ishlar natijalari asosida Nave-Stoksning vektor tenglamasi, uzviylik tenglamasi, suyuqlikdagi kuchlanish tenzorining komponentalarini aniqlash formulalari va holat tenglamasi keltirilgan.

To'rtinchi paragraf qovushoq-elastik jism nuqtasidagi kuchlanishlar va deformasiyalar orasidagi bog'lanishlarga bag'ishlangan. Boshqacha aytganda qovushoq-elastik jism nuqtasidagi σ_{ij} kuchlanishlar va ε_{ij} deformasiyalar orasidagi asosiy munosabatlarni topish uchun bu munosabatlarning chiziqli bo'lishi faraz qilinadi, qaysiki ko'chishlarning kichikligi haqidagi farazning qonuniyligini saqlab qolish uchun xizmat qiladi.

Ikkinchi bob ideal suyuqlik bilan to'ldirilgan va deformatsiyalanuvchi muhitda joylashgan doiraviy silindrik anizotrop qatlam va qobiqlarning bo'ylama-radial tebranish tenglamalarini keltirib chiqarish hamda tadqiq qilishga bag'ishlangan. Bu bob 5 ta paragrafdan iborat.

Bu bobning *birinchi paragrafi* ideal suyuqlik va deformatsiyalanuvchi muhit bilan ta'sirlashuvchi transversal-izotrop, qovushoq-elastik silindrik qatlamning bo'ylama-radial umumiy tenglamalarini keltirib chiqarishga bag'ishlangan. Bunda qatlamning tebranishlari uning yon sirtlarida ta'sir etuvchi dinamik zo'riqishlar ta'siri natijasida vujudga keladi deb hisoblanadi . Muhitlarning bo'linish chegaralarida ko'chishlar va kuchlanishlar uzluksiz deb faraz qilinadi.

Ikkinchi paragrafda yuqoridagi qaralayotgan gidroelastik sistemaning kuchlanganlik-deformatsiyalanganlik holatini aniqlashga imkon beruvchi qatlam nuqtalaridagi kuchlanish va ko'chishlarni hamda suyuqlik bosimini aniqlash formulalari chiqarilgan. Ushbu formulalar sistemaning kuchlanganlik-deformatsiyalanganlik holatini aniqlash bilan bir qatorda amaliy masalalar yechishda masalaning chegaraviy va boshlang'ich shartlarini to'g'ri qoyilishiga shart-sharoit yaratadilar.

Uchinchi paragrafda qovushoq-elastiklik xususiyati va o'rab turuvchi muhit hisobga olingan holda transversal-izotrop silindrik qatlamning bo'ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarilgan. Bunda ushbu bobning birinchi va ikkinchi paragraflarida olingan natijalardan xususiy hollarda kelib chiquvchi natijalardan foydalanilgan. Xususan bu, holda qatlam bilan ta'sirlashuvchi va uning ichiga to'ldirilgan deb hisoblangan ideal suyuqlik yo'q, yoki mavjud emas deb hisoblanadi. Ushbu sharoitda ideal suyuqlik zarrachalari tezliklari potensiali nolga teng bo’ladi va tebranish tenglamalari ancha soddalashadi.

Ushbu bobning *to’rtinchi paragrafida* ideal suyuqlik va deformatsiyalanuvchi muhit bilan ta’sirlashuvchi anizotrop silindrik qatlamning bo’ylama-radial umumiy tenglamalarining limitik va xususiy hollari qarab chiqilgan va tahlil qilingan. Xususan, ikkita limitik holat qaralgan, ya’ni:

a) qovushoq-elastik anizotrop doiraviy silindrik sterjen;

b) qovushoq-elastik transveresal-izotrop yupqa devorli doiraviy silindrik qobiq. Bundan tashqari qatlamni o’rab turuvchi muhit va uning ichidagi ideal suyuqliqlardan biri bor, ikkinchisi yo’q yoki aksincha ikkinchisi bor, birinchisi yo’q bo’lgan xususiy holatlar alohida-alohida tahlil qilingan.

Nihoyat *beshinchi paragrafda* ideal suyuqlik va deformatsiyalanuvchi muhit bilan ta’sirlashuvchi anizotrop silindrik qatlamning bo’ylama-radial tebranishlari taqribiy tenglamalari keltirib chiqarilgan. Bu tenglamalar umumiy tebranish tenglamalari tarkibidagi cheksiz yig’indilarda n=0 bo’lgan holda qatlamning oraliq sirti nuqtalarining U_z, U_r ko’chishlarining bosh qismlariga nisbatan chiqarilgan. Xuddi ana shu tenglamalar dissertatsiya ishida qaralgan gidroelastik sistemaning tebranishlari haqidagi masalalarni qo’yishda va yechishda ishlatilishi mumkin.

Dissertatsiya ishining oxirgi bo’limi *asosiy xulosalar* deb ataladi va bu bo’limda dissertatsiya ichida olingan eng muhim natijalar asosida chiqarilgan xulosalar keltirilgan.

Dissertatsiya ishi *foydalanilgan adabiyotlar* ro’yxati bilan yakunlanadi.

1 BOB

DOIRAVIY SILINDRIK QATLAMNING TEBRANISHLARI. ASOSIY TENGLAMA VA MUNOSABATLAR

1.1. Qovushoq suyuqlik va deformatsiyalanuvchi muhitning silindrik qatlam va qobiqlar bilan o'zaro nostatsionar ta'sirlashuvi. Masalaning zamonaviy holati.

Dinamik masalalarni yechishning turli usullari ichida elastik va qovushoq-elastik sterjen, plastina va qobiqlarning dinamikasini taqribiy va aniqlashtirilgan tenglamalar asosida o'rganish alohida o'rin tutadi. Bu tadqiqotchilarning, xuddi statik masalalardagi kabi, dinamik masalalarni yechishda ham masalaning soddalashtirilgan matematik modeliga o'tish uchun intilishlari bilan bog'iq.

Biz ushbu paragraf doirasida silindrik qobiqlarning ideal va qovushoq suyuqliklar bilan o'zaro nostatsionar ta'sirlashuviga bag'ishlangan ilmiy tadqiqotlar va ularning natijalari bo'yicha ilmiy matbuotda e'lon qilingan maqolalarning ba'zilari haqida to'xtalib o'tamiz. Bunda ushbu tadqiqotlar yoki ular to'g'risidagi maqolalarni ikki yo'nalishga ajratamiz: a) nostatsionar tebranishlar tenglamalarini chiqarish; b) ikki qattiq va suyuq muhitlarning o'zaro ta'sirlashuv masalalari.

Ma'lum bo'lgan klassik tenglamalar juda past chastotali jarayonlarnigina yaxshi tavsiflaydilar. Jarayonlar yuqori chastotali bo'lishlari bilan bu tenglamalar yaroqsiz bo'lib qoladilar. Shuning uchun ham tenglamalarni aniqlashtirishga urinishlar ko'p bo'lgan.

Bunday urinishlarda birinchilik L. Poxgammer, C. Kri, D.V.Reley va S.P.Timoshenkolarga tegishli [5]. Sterjenlarning klassik tebranish tenglamasini birinchi marta aniqlashtirish D.V.Releyga tegishli bo'lib, bunda u birinchi marta aylanish inersiyasini hisobga olgan. Keyinchalik D.V.Reley tenglamasi S.P.Timoshenko tomonidan ko'ndalang siljish deformatsiyasini hisobga olgan holda yanada aniqlashtirilgan. Klassik tenglamalarni aniqlashtirish masalasi bilan xuddi sterjenlar uchun bo'lgani kabi, plastina va qobiqlar uchun hozirgi kungacha davom etmoqda. Xususan bunday ish bilan L.G.Donnel, I.K.Nigul, V.I.Utesheva, E.B.Omenitskaya, K.Z.Galimov, I.Mirsky, H.Herrmann va boshqalar [5] shig'ullanishgan.

Mexanik sistemalarning tebranish tenglamalarini aniqlashtirish hozirgi kunda ham davom etmoqda. Bu chet el va vatanimiz olimlarining eng yangi ilmiy maqolalaridan ham ko'rinib turibdi. Masalan I.G.Filippov [22], X.Xudoynazarov [23], Sh.Markus [23] va boshqalarning [3,5] ishlari yuqoridagi fikrimizni tasdiqlaydi. Ko'rsatilgan ishlarning ko'pchiligida elastik yoki qovushoq-elastik mexanik sistemalarning taqribiy tebranish tenglamalari u yoki bu fizik yoki geometrik xarakterdagi gipotezalar asosida keltirib chiqarilgan. Ammo, ushbu gipotezalar tebranish tenglamalarni soddalashtirish bilan bir qatorda ba'zi xato xato va kamchiliklarga ham olib keladi.

Bunday xato va kamchiliklarni birinchilardan bo'lib V.V.Novojilov va R.M.Finkelshteynlar 1943 yilda [24] ko'rsatib o'tishgan va klassik nazariya yo'l qo'yadigan xato va kamchilklarni baholaganlar. Keyinchalik 1947 yilda X.M.Mushtari [24] tomonidan qobiqlarning klassik Kirxgoff-Lyav nazariyasiga tegishli tebranish taqribiy tenglamalarning qo'llanish sohasi aniqlandi va uning nisbatan tor ekanligi ko'rsatildi. 1961 yilga kelib professor V.M.Darevskiy Kirxgoff-Lyav nazariyasining turli variantlari turli xil xatolarga olib kelishini isbotladi [6]. Shuning uchun han umumiy holda, ya'ni ixtiyoriy qobiqlar va ixtiyoriy yuklamalar uchun elastiklik nazariyasining umumiy munosabatlarini sodda tenglik yoki munosabatlar bilan almashtirish masalasi doim ochiq holda qoladi.

Yuqorida nomlari zikr etilgan olimlarning ishlari va bu erda keltirilmagan ishlarning tahlili shuni ko'rsatadiki doiraviy silindrik elastik qobbiqlarning tebranisg nazariyasi hozirgi kungacha to'liq asoslanmagan. Shuning uchun ushbu nazariyani asoslashga bo'lgan urinishlar bugungi kungacha davom etib kelmoqda. Klassik tebranish nazariyasini aniqlashtirish ham nazariyani asoslashning xuddi o’zidir. Klassik nazariyalarni aniqlashtirish yoki tebranish nazariyasini asoslashda tebranish tenglamalarini keltiriib chiqarish asosiy rol o’ynaydi.

Tebranish tenglamalarini keltirib chiqarish usullari uchta asosiy yo’nalishlarga bo’linadi. Bu yo’nalishlardan birinchisiga dinamikada variatsion prinsiplarni ishlatishga asoslangan usullarni keltirish mumkin [5].

Ikkinchi yo’nalishga elastik ko’chishlar maydoni tuzuvchilarini qatorga, shu jumladan darajali qatorga, yoyishga asoslangan usullar kiradi. Ushbu metodning statika uchun variantini N.A.Kil’chevskiy [24] taklif etgan. Ushbu qator yoki darajali qatorga yoyish metodini dinamikaning keng sinflariga tadbiq etish birinchi bor P.S.Epstein tomonidan amalga oshirilgan. Bu avtorlar tadqiqotlaridan keyingi yillarda olib borilgan tadqiqotlar natijasidan ushbu metodlar sezilarli darajada rivojlantirildi.

Xuddi shu usul asosida V.Z.Vlasov [2,5] o’zining boshlang’ich funksiyalar usulini yaratdi va uni qobiqli sistemalar uchun muvaffaqiyat bilan qo’lladi. Elastik ko’chishlarni darajali qatorga yoyish metodini G.I.Petrashen [17], tekis deformatsiyalanuvchi tekislik (qatlam) haqidagi dinamik masala misolida, matematik jihatdan qat’iy asosladi.

Tebranish tenglamalarini keltirib chiqarish usullarining uchinchi yo’nalishiga, prof. I.G.Filippov va prof. X.X.Xudoynazarovlar [22,23,24,25,26,27] tomonidan sezilarli farajada rivojlantirilgan, elastiklik dinamik nazariyasining uch o’lchovli masalalarining aniq yechimlaridan foydalanish usuli kiradi. Professorlar I.G.Filippov va X.X.Xudoynazarov hamda ularning ko’p sonli o’quvchilari tomonidan jismlar va moddalarning har xil xususiyatlarini hisobga olgan holda sterjenlar, plastinalar va qobiqlarning tebranish tenglamalari keltirib chiqarilgan. Moddalarning har xil xususiyatlariga reologik, anizotropik, bir jinslimaslik, haroratga qarab o’zgarishlik va boshqalar kiradi. Bundan tashqari jismlar, ya’ni tadqiq etilayotgan mexanik sistemalarning xususiyatlariga ularning ko’ndalang kesimlari geometriyasining o'zgarishlari, bikrligining o'zgaruvchanligi va shunga o'xshash xususiyatlarini hisobga olish kiradi.

Xuddi ana shu usulni qo'llab deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi silindrik elastik va qovushoq-elastik qobiq va sterjenlarning tebranishlar nazariyasi prof. X.X.Xudoynazarov tomonidan yaratilgan [23]. Silindrik qobiq va sterjenlarning har xil maydonlar, xususan haroratlar va elektromagnit maydonlari bilan o'zaro nostatsionar ta'sirlashuvi haqidagi masalalar dots. F.Amirqulova ishlarida qaralgan.

Silindrik elastik va qovushoq-elastik qobiq va sterjenlarning ideal va qovushoq suyuqliklar bilan ta'sirlashuvi haqidagi masalalar tadqiqotiga juda ko'p avtorlarning ishlari qaratilgan.

Bunda ayniqsa mexanik sistemalarning ideal suyuqlik bilan ta'sirlashuvi masalalari juda keng va atroflicha tekshirilgan. Shu bilan bir qatorda, bunday sistemalarning qovushoq suyuqlik bilan ta'sirlashuvi haqidagi masalalar juda kam sonli tadqiqotchilar tomonidan tekshirilgan. Tadqiqotlar bu sohada kam o'tkazilishini sababi, qovushqoq suyuqlik harakat tenglamalarining yoki boshqacha aytganda Nave-Stoks tenglamasining echimi olinmaganligi, ya'ni bu tenglamalarni echish matematik jihatdan katta muammo ekanligidir.

O'tgan asrning 70-yillarida R.P.Darmond va W.T.Rouleau [31] lar tomonidan, qovushqoq siqilmaydigan suyuqliklar uchun Nave-Stoks tenglamasini to'rtta potensial funksiyalarga nisbatan tenglamalarga keltirirsh amalga oshiriladi. Nave-Stoks vektor tenglamasini eng umumiy holda ya'ni suyuqlik siqiluvchi va qovushoq bo'lgan holda potensial funksiyalarga nisbatan tenglamalar sistemasiga keltirish akademik A.N.Guz [7,8,9] tomonidan amalga oshirildi. Akademik A.N.Guzning ushbu ishlari asosida doiraviy silindrik elastik qobiqlarning qovushoq siqiluvchi suyuqliklar bilan o'zaro ta'sirlashuviga bag'ishlangan ishlar paydo bo'ldi. Xususan bunday ishlarga misol qilib tadqiqotchilardan B.Yalg'ashev [27,28,29] ishlarini ko'rsatish mumkin.

Mazkur dissertatsiya ishida prof. X.Xudoynazarov [23] tomonidan taklif etilgan nazariya asosida ideal suyuqlik bilan to'ldirilgan va deformatsiyalanuvchi mihitda joylashgan transversal-izotrop, doiraviy, silindrik qovushoq-elastik qobiqning bo'ylama-radial tebranishlari haqidagi masala tadqiq qilingan. Olingan natijalar asosida ba'zi ilmiy xulosalar chiqarilgan.

1.2. Deformatsiyalanuvchi qattiq jism nuqtasida kuchlanish va deformatsiya orasidagi bog'lanishlar. Qattiq jismning muvozanat va harakat tenglamalari.

Ushbu paragraph doirasida Elastiklik nazariyasining dissertatsiya ishida kerak bo'ladigan va zarur bo'lganda foydalaniladigan asosiy tenglamalari va munosabatlarini qisqa ko'rinishda keltiramiz. Keltirilgan formulalar va tenglamalarning qanday yo'l bilan olinganligiga to'xtalmaymiz.

Bir jinsli izotrop jism uchun umumlashgan Guk qonuni

$$\sigma_{ij} = \left[\lambda\delta_{ij}\delta_{k\ell} + \mu(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})\right]\varepsilon_{k\ell} \qquad (1.2.1)$$

ko'rinishda bo'ladi. Bu yerda $\theta = \delta_{k\ell}\varepsilon_{k\ell} = \varepsilon_{kk}$ - hajmiy deformatsiya;

$$\delta_{ik}\delta_{j\ell}\varepsilon_{k\ell} = \varepsilon_{ij}; \qquad \delta_{i\ell}\delta_{jk}\varepsilon_{k\ell} = \varepsilon_{ij},$$

ekanliklari uchun (1.2.1) formula

$$\sigma_{ij} = \lambda\delta_{ij}\theta + 2\mu\varepsilon_{ij} \qquad (1.2.2)$$

ko'rinishni oladi. Xuddi ana shu ifoda bir jinsli izotrop jism uchun Guk qonunini ifodalaydi va bu qonun oltita algebraik tenglamalaridan iborat

$$\begin{aligned} &\sigma_{11} = \lambda\theta + 2\mu\varepsilon_{11}; \qquad \sigma_{12} = 2\mu\varepsilon_{12};\\ &\sigma_{22} = \lambda\theta + 2\mu\varepsilon_{22}; \qquad \sigma_{23} = 2\mu\varepsilon_{23};\\ &\sigma_{33} = \lambda\theta + 2\mu\varepsilon_{33}; \qquad \sigma_{31} = 2\mu\varepsilon_{31}. \end{aligned} \qquad (1.2.3)$$

Ko'p hollarda Guk qonunining (1.2.3) ifodasi ε_{ij} larga nisbatan yechilgan holda ishlatiladi, ya'ni

$$
\begin{aligned}
&\varepsilon_{11} = \frac{1}{\mu(3\lambda+2\mu)}\left[(\lambda+\mu)\sigma_{11} - \frac{1}{2}\lambda(\sigma_{22}+\sigma_{33})\right];\\
&\varepsilon_{22} = \frac{1}{\mu(3\lambda+2\mu)}\left[(\lambda+\mu)\sigma_{22} - \frac{1}{2}\lambda(\sigma_{11}+\sigma_{33})\right];\\
&\varepsilon_{33} = \frac{1}{\mu(3\lambda+2\mu)}\left[(\lambda+\mu)\sigma_{33} - \frac{1}{2}\lambda(\sigma_{11}+\sigma_{22})\right];\\
&\varepsilon_{12} = \frac{1}{2\mu}\sigma_{12}; \qquad \varepsilon_{31} = \frac{1}{2\mu}\sigma_{31}; \qquad \varepsilon_{23} = \frac{1}{2\mu}\sigma_{23}.
\end{aligned}
\tag{1.2.4}
$$

Bu (1.2.3) tenglamalar bir jinslimas, lekin izotrop jism uchun ham o‘rinlidir.

Faraz qilaylik, deformatsiyalanuvchi qattiq jismning hajmi V va sirti S ga teng bo‘lsin hamda jismga uning nuqtalarining kichik ko‘chishlarini, va demak, kichik deformatsiyasini chaqiruvchi hajmiy va sirt kuchlari ta’sir etayotgan bo‘lsin. Jismga ta’sir qilayotgan hajmiy (massaviy) kuchlarni $\vec{f}$ bilan belgilaymiz. U holda bu jismning, muvozanat tenglamasi

$$\frac{\partial \sigma_{ij}}{\partial x_i} + \rho f_j = 0$$

yoki

$$\sigma_{ij,j} + \rho f_i = 0 \tag{1.2.5}$$

tenglamalar sistemasidan iborat bo‘ladi. (1.2.5) tengliklar *deformatsiyalangan jism muvozanatining uchta differensial tenglamasidan iboratdir.*

Ushbu tenglamalar tarkibidagi tengliklar izlanuvchi σ_{ij} funksiyalar bilan jism sirtiga qo‘yilgan sirt kuchlari orasidagi bog‘lanishlar *chegaraviy shartlar* deb ataladi va quyidagi ko’rinishga ega

$$
\begin{aligned}
&\sigma_{11}n_1 + \sigma_{12}n_2 + \sigma_{13}n_3 = F_1;\\
&\sigma_{21}n_1 + \sigma_{22}n_2 + \sigma_{23}n_3 = F_2;\\
&\sigma_{31}n_1 + \sigma_{32}n_2 + \sigma_{33}n_3 = F_3,
\end{aligned}
\tag{1.2.6}
$$

bu yerda n_j lar $\vec{n}$ birlik vektorining komponentalari, qaysiki uning yo‘naltiruvchi kosinuslariga teng: $\alpha_{ni} = n_i$.

Agar jism muvozanatda emas, balki harakatda bo‘lsa, (1.2.5) tenglamalarga Dalamber prinsipi asosida inersiya kuchlarini (inersiya kuchlari massaviy kuchlarga kiradi) qo‘shish kerak. U holda deformatsiyalanuvchi jismning harakat tenglamalarini quyidagicha yozish mumkin:

$$\sigma_{ij,j} + \rho(f_i - w_i) = 0, \qquad (1.2.7)$$

bu yerda w_i - vaqtning t paytida $M(x_k)$ nuqtada bo‘lgan jism zarrachasi tezlanishining x_i koordinat o‘qiga proyeksiyasi.

Agar jismning harakati davomida uning nuqtalarining u_i ko‘chishlari kichikligicha qolsalar, u holda

$$w_i = \frac{\partial^2 u_i}{\partial t^2}$$

va harakat tenglamasi

$$\sigma_{ij,j} + \rho f_i = \rho \frac{\partial^2 u_i}{\partial t^2} \qquad (1.2.8)$$

ko‘rinishni oladi.

Deformatsiyalanuvchi jism muvozanatda bo‘lsa, uning har bir nuqtasida (1.2.5) uchta defferensial tenglamalarni kuchlanish tenzorining oltita σ_{ij} komponentalari qanoatlantirishlari kerak. Jism sirtida σ_{ij} kuchlanishlar (1.2.6) chegaraviy shartlarni qanoatlantirishlari kerak.

Ko‘rinib turibdiki, (1.2.5) uch tenglama sistemasi olti noma’lumga nisbatan va u bir qiymatli yechimga ega bo‘lishi mumkin emas. Haqiqatan, bu tenglamalarning (1.2.6) chegaraviy shartlarni qanoatlantiruvchi ko‘plab yechimlarini topish mumkin. Ushbu yechimlarning har biri *statik jihatdan mumkin bo‘lgan kuchlanganlik holatini* ifodalaydi.

1.3. Suyuqlik harakati uchun asosiy tenglamalar

Doiraviy silindrik elastik qobiqning ichidagi suyuqlikni umumiy holda qovushoq va siqiluvchi deb hisoblaymiz. Bundan tashqari suyuqlik zarrachalarining tebranishlarini ham kichik deb qabul qilamiz. U holda bunday suyuqlikni tavsiflash uchun Lagranj koordinatalarini qo'llash mumkin bo'ladi, va demak mos ravishda, quyidagi munosabatlarga ega bo'lamiz [8]:

Nave-Stoksning vektor tenglamasi

$$\frac{\partial \vec{V}}{\partial t} - \nu' \Delta \vec{V} + \frac{1}{\rho_0'} gradP + \frac{1}{3} \nu' graddiv\vec{V} = 0, \qquad (r,\theta,z) \in V_2; \qquad (1.3.1)$$

uzviylik tenglamasi

$$\frac{1}{\rho_0'} \frac{\partial \rho'}{\partial t} + div\vec{V} = 0, \qquad (r,\theta,z) \in V_2; \qquad (1.3.2)$$

suyuqlikdagi kuchlanish tenzorining komponentalarini aniqlash formulalari

$$\begin{aligned} &P_{ij} = -P\delta_{ij} + \lambda' \delta_{ij} div\vec{V} + \mu' e_{ij}, \qquad (r,\theta,z) \in V_2; \\ &e_{ij} = \frac{1}{2}\left(V_{i,j} + V_{j,i}\right), \qquad (i, j = r,\theta,z); \end{aligned} \qquad (1.3.3.)$$

holat tenglamasi

$$\frac{\partial \rho}{\partial \rho'} = a_0^2 \quad , \qquad a_0 = const \qquad (1.3.4.)$$

Yuqorida qovushoq suyuqlik uchun keltirilgan asosiy munosabatlarda quyidagi belgilashlar qabul qilingan:

$\rho_{ij}(i, j = r, o, z)-$ suyuqlikda kuchlanish tenzori komponentalari;

ρ_o' $-$tinch holatdagi suyuqlik zichligi;

a_o $-$tinch holatdagi suyuqlikda tovush tarqalish tezligi; $\vec{V}-$suyuqlik zarrashasining tezlik vektori;

ρ' $-$suyuqlik zichligining qo'zg'olishi (harakatga kelgan, qo'zg'olgan suyuqlik zichligi;

$\rho-$suyuqlik bosimining qo'zg'olishi;

V_2 $-$fazoning suyuqlik egallagan qismining hajmi.

$V_i(i=r,o,z)$ – suyuqlik zarrashasi tezlik vektorining koordinat o'qlaridagi komponentalari;

λ' – qovushoqlik ikkinchi koeffitsiyenti, $\lambda' = -\frac{2}{3}\mu'$;

ν' – qovushoqlik kinematik koeffitsiyenti;

μ' – qovushoqlik koeffitsiyenti, $\mu' = \rho_0'\nu'$.

Keltirilgan (1.3.1)-(1.3.4) munosabatlar qobiq va suyuqliklar uchun alohida-alohida beriluvchi chegaraviy va boshlang'ich shartlar hamda ikki muhitni (qattiq va suyuq) ajratib turuvchi tebranuvchi sirtda beriladigan kinematik va dinamik shartlar bilan birgalikda doiraviy silindrik qobiq va qovushoq siqiluvchi suyuqlik uchun gidroelastik munosabatlarning yopiq (to'liq) sistemasini tashkil etadilar.

Endi (r,o,z) ortogonal koordinatalarninχg silindrik sistemasida keltirib chiqarilgan (1.3.1.), (1.3.3) va (1.3.4) tenglamalarning umumiy yechimlarini topish bilan shug'ullanamiz. Suyuqlik uchun G - skalyar va $\vec{\chi}$ vektor potensial-larni quyidagicha kiritamiz [7,8]:

$$\vec{V} = \frac{\partial}{\partial t}\left(gradG + rot\vec{\chi}\right); \qquad (r,\theta,z) \in V_2\,, \tag{1.3.5}$$

Bu yerda vektor $\vec{\chi}$ quyidagi tenglamani qanoatlantiradi.

$$\left(\frac{\partial}{\partial t} - \nu'\Delta\right)\vec{\chi} = 0\,, \qquad (r,\theta,z) \in V_2\,. \tag{1.3.6}$$

Oxirgi tenglama $\vec{\chi}$ vektorning χ_1, χ_2 va χ_3 komponentalariga nisbatan uchta skalyar tenglamalar sistemasiga ekvivalentdir. Suyuqlikning harakat tenglamasi, uzviylik tenglamasi va holat tenglamasi hammasi bo'lib uchta tenglamani tashkil etadilar. Lekin izlanuvchi noma'lumlar soni to'rtta, ya'ni – G,χ_1, χ_2, χ_3 lar. Shuning uchun (1.3.5) vektor tenglamada faqatgina rotorlari noldan farqli bo'lgan qo'shiluvchilargina qoldiramiz. Boshqacha aytganda $\vec{\chi}$ vektorini quyidagicha kiritamiz [9,10]

$$\vec{\chi} = \vec{e}_z\chi_1 + rot\left(\vec{e}_z\chi_2\right) \tag{1.3.7}$$

ya'uni (1.1.3.4) ga qo'yib

$$\vec{V}=\frac{\partial}{\partial t}\left(gradG+rot\left[\vec{e}_z\chi_1+rot\left(\vec{e}_z\chi_2\right)\right]\right);\quad (r,\theta,z)\in V_2, \qquad (1.3.8)$$

ga ega bo'lamizki, bunda χ_1 va χ_2 lar, (1.3.6) ga asosan

$$\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\chi_1=0 \quad \text{va} \quad \left(\frac{\partial}{\partial t}-\nu'\Delta\right)\chi_2=0;\quad (r,\theta,z)\in V_2. \qquad (1.3.9)$$

tenglamalarni qanoatlantiradilar.

Kiritilgan (1.3.8) taqdimotdan suyuqlik zarrachalari tezliklarining V_r,V_o,V_z komponentalari uchun ularni G,χ_1 va χ_2 skalyar potensiallar orqali ifodalovchi formulalarni keltirib chiqarish qiyin emas

$$\begin{aligned}
V_r&=\frac{\partial}{\partial t}\left[\frac{\partial G}{\partial r}+\frac{1}{r}\frac{\partial\chi_1}{\partial\theta}+\frac{\partial^2\chi_2}{\partial r\partial z}\right];\\
V_\theta&=\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial G}{\partial\theta}-\frac{\partial\chi_1}{\partial r}+\frac{1}{r}\frac{\partial^2\chi_2}{\partial\theta\partial z}\right]; \qquad (r,\theta,z)\in V_2, \qquad (1.3.10)\\
V_z&=\frac{\partial}{\partial t}\left[\frac{\partial G}{\partial z}-\frac{\partial^2\chi_2}{\partial r^2}-\frac{1}{r}\frac{\partial\chi_2}{\partial r}-\frac{1}{r^2}\frac{\partial^2\chi_2}{\partial\theta^2}\right];
\end{aligned}$$

Silindrik koordinatlari sistemasida suyuqlik zarrachalari tezliklari divergensiyasi quyidagi ko'rinishga ega

$$div\vec{V}=\frac{1}{r}\left[V_r+r\frac{\partial V_r}{\partial r}+\frac{\partial V_\theta}{\partial\theta}+r\frac{\partial V_z}{\partial z}\right]. \qquad (1.3.11)$$

(1.3.10) ifodani (1.3.11) formulaga qo'ysak

$$div\vec{V}=\Delta\frac{\partial G}{\partial t}\,,\qquad (r,\theta,z)\in V_2 \qquad (1.3.12)$$

ga ega bo'lamiz.

Suyuqlik uchun deformatsiya tezliklari tenzorining e_{ij}-komponentalari quyidagi ko'rinishlarga ega bo'ladilar

$$e_{rr}=\frac{\partial^3 G}{\partial r^2\partial t}+\frac{\partial^4\chi_2}{\partial r^2\partial z\partial t}\;;\qquad e_{\theta\theta}=\frac{1}{r}\frac{\partial}{\partial t}\left[\frac{\partial G}{\partial r}+\frac{\partial^2\chi_2}{\partial r\partial z}\right];$$

$$e_{zz}=\frac{\partial^3 G}{\partial z^2\partial t}-\frac{1}{r}\frac{\partial^3\chi_2}{\partial r\partial z\partial t}-\frac{\partial^4\chi_2}{\partial r^2\partial z\partial t};\quad e_{\theta z}=-\frac{\partial^3\chi_1}{\partial r\partial z\partial t};\qquad(1.3.13)$$

$$e_{rz}=2\frac{\partial^3 G}{\partial r\partial z\partial t}+\frac{\partial}{\partial r\partial t}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2\ ;\quad e_{r\theta}=\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial\chi_1}{\partial r}-\frac{\partial^2\chi_1}{\partial r^2}\right]$$

Xuddi shunday suyuqlik uchun kuchlanish tenzorining P_{ij}-komponentalari quyidagicha yoziladi

$$P_{\theta\theta}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial G}{\partial r}+\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}\right];$$

$$P_{zz}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial z^2}-\frac{1}{r}\frac{\partial^2\chi_2}{\partial r\partial z}-\frac{\partial^3\chi_2}{\partial r^2\partial z}\right];$$

$$P_{rr}=-\rho_0'\left(\frac{4}{3}\nu'\Delta_0-\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\lambda'\Delta_0\frac{\partial G}{\partial t}+\mu'\frac{\partial}{\partial t}\left[\frac{\partial^2 G}{\partial r^2}+\frac{\partial^3\chi_2}{\partial r^2\partial z}\right];\qquad(1.3.14)$$

$$P_{rz}=\mu'\frac{\partial}{\partial t}\left[2\frac{\partial^2 G}{\partial r\partial z}+\frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_2\right];$$

$$P_{r\theta}=\mu'\frac{\partial}{\partial t}\left(\frac{1}{r}-\frac{\partial}{\partial r}\right)\frac{\partial\chi_1}{\partial r}\ ;\qquad P_{z\theta}=-\mu'\frac{\partial^3\chi_1}{\partial r\partial z\partial t}$$

$$\Delta_0=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

Endi (1.3.12) ni (1.3.1), (1.3.2) va (1.3.3) munosabatlarga qo'yib

$$\frac{\partial\vec{V}}{\partial t}-\nu'\Delta\vec{V}+\frac{1}{\rho_0'}gradP-\frac{1}{3}\nu' grad\Delta\frac{\partial G}{\partial t}=0\,;\qquad(1.3.15)$$

$$\frac{1}{\rho_0'}\frac{\partial\rho'}{\partial t}+\Delta\frac{\partial G}{\partial t}=0\,;(r,\theta,z)\in V_2\qquad(1.3.16)$$

$$P_{ij}=P\delta_{ij}+\lambda\delta_{ij}\Delta\frac{\partial G}{\partial t}+\mu' e_{ij}\,,\qquad(1.3.17)$$

tenglamalarda ega bo'lamiz.

Ushbu olingan (1.3.15) tenglamadan quyidagini topamiz

$$\frac{1}{\rho_0'}gradP-\frac{1}{3}\nu' grad\Delta\frac{\partial G}{\partial t}=-\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\vec{V}$$

va bu yerdagi $\vec{V}$ tezlik vektorining o'rniga uning (1.3.5) ifodasini qo'yib, differensial (chiziqli) operatorlarning o'rinlarini almashtiramiz hamda (1.3.6) ga asosan quyidagi ifodaga ega bo'lamiz

$$grad\left(\frac{\rho}{\rho_0'}-\frac{\nu'}{3}\Delta\frac{\partial G}{\partial t}\right)=-grad\left(\frac{\partial}{\partial t}-\nu'\Delta\right)\frac{\partial G}{\partial t}$$

bu yerdan

$$\frac{\rho}{\rho_0'}-\frac{\nu'}{3}\Delta\frac{\partial G}{\partial t}=-\frac{\partial^2 G}{\partial t^2}+\nu'\Delta\frac{\partial G}{\partial t}$$

tenglamaga ega bo'lamiz.

Olingan tenglama suyuqlik bosimi qo'zg'olishini kiritilgan G – skalyar funksiya orqali ifodalash imkonini beradi:

$$P=\rho_0'\left[\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right]\frac{\partial G}{\partial t}\ , \qquad (r,\theta,z)\in V_2 . \qquad (1.3.18)$$

Endi (1.3.4)-holat tenglamasini quyidagicha almashtiramiz

$$\frac{\partial\rho}{\partial\rho'}=a_0^2 \;\Rightarrow\; \frac{\partial\rho}{\partial\rho'}\frac{\partial t}{\partial t}=a_0^2 \;\Rightarrow\; \frac{\partial\rho}{\partial t}\frac{\partial t}{\partial\rho'}=a_0^2 \;\Rightarrow\; \frac{\partial\rho}{\partial t}\frac{1}{\frac{\partial\rho'}{\partial t}}=a_0^2$$

bu yerda [7]

$$\frac{\partial\rho}{\partial t}=a_0^2\frac{\partial\rho'}{\partial t} \qquad \text{yoki} \qquad \frac{\partial\rho'}{\partial t}=\frac{1}{a_0^2}\frac{\partial\rho}{\partial t}.$$

Ushbu tenglamada P ning o'rniga uning (1.29) ifodasini qo'yamiz va natijada olamiz

$$\frac{\partial\rho'}{\partial t}=\frac{\rho_0'}{a_0^2}\left(\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right)\frac{\partial^2 G}{\partial t^2}, \quad (r,\theta,z)\in V_2 . \qquad (1.3.19)$$

Nihoyat (1.3.19) ifodani (1.3.16)-uzviylik tenglamasiga qo'yib quyidagini olamiz

$$\frac{1}{\rho_0'}\frac{\rho_0'}{a_0^2}\left(\frac{4}{3}\nu'\Delta-\frac{\partial}{\partial t}\right)\frac{\partial^2 G}{\partial t^2}+\Delta\frac{\partial G}{\partial t}=0$$

yoki

$$\frac{\partial}{\partial t}\left[\left(1+\frac{4\nu'}{3a_0^2}\right)\Delta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]G=0 \qquad (1.3.20)$$

shunday qilib qovushoq siqiluvchi suyuqlikning (1.3.1.), (1.3.2) va (1.3.4) harakat tenglamalari G, χ_1 va χ_2 potensiallarga nisbatan (1.3.9) va (1.3.20) to'lqin tenglamalariga keltiriladi va oxirgi tenglamalar yechimlari izlanadi. Bunda suyuqlik zarrachalari tezliklari vektorining komponentalari (1.3.10) formulalar yordamida, kuchlanish tenzori komponentalari (1.3.17) formulalar yordamida, suyuq-lik bosimi qo'zg'olishi (1.3.18) formula yordamida va nihoyat suyuqlik zichligining qo'zg'olishi – (1.3.19) formula yordamida ushbu G, χ_1 va χ_2 skalyar poten-siallar orqali ifodalanadi.

1.4. Qovushoq-elastik jism nuqtasidagi kuchlanishlar va deformasiyalar orasidagi bog'lanishlar

Endi qovushoq-elastik jism nuqtasidagi σ_{ij} kuchlanishlar va ε_{ij} deformasiyalar orasidagi asosiy munosabatlarni topish zarur. Bu munosabatlarning chiziqli bo'lishi faraz qilinadi, qaysiki ko'chishlarning kichikligi haqidagi farazga mos keladi.

Uhsbu munosabatlarni quyidagi ko'rinishda yozish mumkin [13]:

$$\sigma_{ij}(t) = G_{ijkl}(o)\varepsilon_{kl}(t) + \int_0^t \varepsilon_{kl}(t-s)\frac{dG_{ijkl}(s)}{ds}\, ds. \qquad (1.4.1)$$

Oxirgi (1.4.1.) munosabatlarni $G_{ijkl}(t)$ integrallash funksiyalarining differensiallari tarkibiga *t=0* bo'lganda kiradigan Dirak δ-funksiyani integrallash yordamida yanada soddaroq munosabatga kelish mumkin.

Relaksasiya va polzuchest funksiyalari. Endi (1.4.1) munosabatdagi integralni o'zgaruvchini $\tau = t-s$ qilib o'zgartirib va bo'laklab hisoblaymiz:

$$\sigma_{ij}(t) = G_{ijkl}(o)\varepsilon_{kl}(t) - \int_0^t \varepsilon_{kl}(\tau)\frac{dG_{ijkl}(t-\tau)}{d\tau}d\tau =$$

$$= G_{ijkl}(o)\varepsilon_{kl}(t) + \int_0^t \varepsilon_{kl}(\tau)\frac{dG_{ijkl}(t-\tau)}{d\tau}d\tau = G_{ijkl}(o)\varepsilon_{kl}(t) -$$

$$-\varepsilon_{kl}(\tau)G_{ijkl}(t-\tau)\Big|_0^t + \int_0^t \varepsilon_{kl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau = \int_0^t G_{ijkl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau.$$

Shunday qilib

$$\sigma_{ij}(t)=\int_0^t G_{ijkl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau. \qquad (1.4.2)$$

Ushbu kuchlanishlar va deformasiyalar orasidagi (1.4.2) bog'lanishlar qayishqoq-elastiklik nazariyasining umumiy va asosiy qonunlaridan biridir. Bu yerda $G_{ijkl}(t)$ integrallash funksiyalari mexanik xususiyatlarini aniqlaydilar va *relaksasiya funksiyalari* [12,13] deyiladi.

Yuqoridagi mulohazalar ko'rsatadiki kuchlanishlar va deformasiyalar orasidagi (1.4.2) bog'lanishlar xotiraga egalik haqidagi gipotezaga, silliqlik haqidagi farazlarga hamda matematik taqdim teoremasiga asoslangan. Bunda fizik intuisiya yoki biror-bir modelli tushunchalarga murojaat qilishga to'g'ri kelmaydi. Shu bilan birgalikda (1.4.2) munosabatlar juda oddiy fizik interpretasiyaga ega. Uni Bolsman superpozisiya prinsipining formulirovkasi deb qarash mumkin. Joriy kuchlanish deformasiya orttirmalari to'liq spektriga bo'lgan javoblar (otklich) superpozisiyasiga bog'liq.

Kuchlanishlar va deformasiyalar orasidagi bog'lanishlarning boshqa bir shaklini (1.4.2) munosabatlarda kuchlanishlar va deformasiyalarning rollarini almashtirib chiqarish mumkin. U holda

$$\varepsilon_{ij}(t)=\int_{-\infty}^{t} I_{ijkl}(t-\tau)\frac{d\sigma_{kl}}{d\tau}d\tau \qquad (1.4.3)$$

bu yerda

$$I_{ijkl}(t)=I_{jikl}(t)=I_{ijlk}(t); \qquad (1.4.4)$$

a) $-\infty<t<0$ da $I_{ijkl}(t)=0$

b) $0\le t<\infty$ da $I_{ijkl}(t)$ va $\dfrac{dI_{ijkl}(t)}{dt}$ lar uzluksiz.

Ushbu $I_{ijkl}(t)$ funksiyalar **polzuchest funksiyalari** [12,13] deyiladi. Xuddi relaksasiya funksiyalari kabi ular ham materialning mexanik xususiyatlarini xarakterlaydi.

Kuchlanishlar va deformasiyalar orasidagi qovushoq-elastik munosabatlarning izotrop shakli amaliy jihatdan muhim ahamiyatga ega. Avvalo relaksasiya funksiyalari tenzorini qaraymiz. To’rtinchi rang tenzorni izotrop tasvirlash eng umumiy holda quyidagicha bo’ladi:

$$G_{ijkl}(t)=\frac{1}{3}\left[G_2(t)-G_1(t)\right]\delta_{ij}\delta_{kl}+\frac{1}{2}G_1(t)\left(\delta_{ik}\delta_{jk}+\delta_{il}\delta_{jl}\right) \qquad (1.4.5)$$

Bu yerda $G_1(t)$ va $G_2(t)$ - bir- biriga bog’lanmagan, mustaqil relaksasiya funksiyalari; δ_{ij} - Kroneker simvoli.

Ma’lumki elastiklik nazariyasida σ_{ij} kuchlanish tenzori sharsimon va deviator qismlariga quyidagicha ajratiladi:

$$\sigma_{ij}=\frac{1}{3}\sigma_{kk}\delta_{ij}+s_{ij}, \qquad s_{ii}=0 \qquad (1.4.6)$$

bu yerda s_{ij} - lar σ_{ij} kuchlanish tenzorining deviator komponentalari,

$$\frac{1}{3}\sigma_{kk}\delta_{ij}=\frac{1}{3}\left(\sigma_{ij}+\sigma_{22}+\sigma_{33}\right)=\frac{1}{3}I_1\left(\sigma_{ij}\right) \qquad (1.4.7)$$

esa uning sharsimon qismi.

Xuddi shunga o’xshash, deformasiya tenzori ε_{ij} uchun quyidagi formulalarga egamiz:

$$\begin{aligned}&\varepsilon_{ij}=\frac{1}{3}\varepsilon_{kk}\delta_{ij}+e_{ij}; \qquad e_{ii}=0.\\&\frac{1}{3}\varepsilon_{kk}\delta_{ij}=\frac{1}{3}\left(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}\right)=\frac{1}{3}I_1\left(\varepsilon_{ij}\right).\end{aligned} \qquad (1.4.8)$$

(1.4.5), (1.4.6)-(1.4.8) formulalardan foydalanib o’tgan paragrafdagi (1.4.2) munosabatlarni quyidagicha yozish mumkin:

$$S_{ij}=\int_{-\infty}^{t}G_1(t-\tau)\frac{de_{ij}(\tau)}{d\tau}d\tau, \qquad (1.4.9)$$

$$\sigma_{kk}=\int_{-\infty}^{t}G_2(t-\tau)\frac{d\varepsilon_{kk}(\tau)}{d\tau}d\tau \qquad (1.4.10)$$

bu yerda

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}, \qquad S_{ii} = 0;$$
$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk}, \qquad e_{ii} = 0. \tag{1.4.11}$$

Ushbu (1.4.9) va (1.4.10) formulalarga o'xshash ravishda kuchlanishlar va deformasiyalar orasidagi polzuchest izotrop munosabatlarining integral shakli quyidagicha yoziladi:

$$e_{ij} = \int_{-\infty}^{t} I_1(t-\tau)\frac{dS_{ij}(\tau)}{d\tau}d\tau; \tag{1.4.12}$$

$$\varepsilon_{kk} = \int_{-\infty}^{t} I_2(t-\tau)\frac{d\sigma_{kk}(\tau)}{d\tau}d\tau, \tag{1.4.13}$$

bu yerda $I_1(t)$ va $I_2(t)$ lar ikkita mustaqil polzuchest izotrop funksiyalari.

Ko'rinib turibdiki $G_1(t)$ va $I_1(t)$ lar siljish holatiga mos keluvchi relaksasiya va polzuchest funksiyalari, $G_2(t)$ va $I_2(t)$ lar esa hajmiy kengayish holatiga to'g'ri keluvchi relaksasiya va polzuchest funksiyalaridir.

Tushunarliki (1.4.12) va (1.4.13) munosabatlar mos ravishda (1.4.9) va (1.4.10) munosabatlardan bog'liqdirlar. Bu yerdan, $I_\alpha(t)$ polzuchest va $G_\alpha(t)$ $(\alpha=1,2)$ relaksasiya funksiyalari o'zaro bog'langan bo'lishlari kelib chiqadi. Ana shu bog'lanishni topish uchun Laplas almashtirishini kiritamiz.

Laplas almashtirishi. $f(t)$ funksiyaning $\bar{f}(s)$ Laplas almashtirishi ushbu formula bilan aniqlanadi [19]:

$$L[f(t)] = \bar{f}(s) = \int_0^t f(t)e^{-st}dt \tag{1.4.14}$$

bu yerda s- almashtirish parametri. Bu parametr haqiqiy, umumiy holda esa kompleks son bo'lishi mumkin. Quyidagi shartlar ixtiyoriy $f(t)$ funksiyaning $\bar{f}(s)$ Laplas almashtirishi mavjud bo'lishi uchun zarur va yetarlidir:

1) $f(t)$ funksiya $t \geq 0$ bo'lganda har bir chekli intervalda bo'lak-bo'lak uzluksiz bo'lishi;

2) $t \to \infty$ bo'lganda $f(t)$ funksiyaning eksponenta tartibiga ega bo'lishi. Ushbu talab,shunday bir α o'zgarmasning mavjud bo'lishi kerakligi va uning uchun

$$\lim_{t\to\infty} e^{-\alpha t} f(t)=0$$

tenglik bajarilishi kerakligini anglatadi, ya'ni $t\to\infty$ bo'lganda $f(t)$ funksiya eksponentadan tez o'smasligi kerak.

Faraz qilaylik $f(t)$ funksiya va uning birinchi $(n-1)$ ta hosilalari uzluksiz bo'lsinlar. U holda $f(t)$ funksiyaning n- hosilasining Laplas almashtirishi quyidagicha formula bilan aniqlanadi:

$$L\left[\frac{d^n f(t)}{dt^n}\right]=S^n \bar{f}(s)-S^{n-1} f(0)-S^{n-2} f^{(1)}(0)-Sf^{(n-2)}(0)-f^{(n-1)}(0) \ , \qquad (1.4.15)$$

bu yerda

$$f^{(k)}(0)=\frac{d^k f(t)}{dt^k}\Big|_{t=0}$$

Ta'rif: Ikkita bo'lak-bo'lak uzluksiz $f(t)$ va $g(t)$ funksiyalarning tuguni (svertkasi) deb

$$\int_0^t f(t)g(t-\tau)d\tau$$

integralga aytiladi.

Ana shu tugun integralining Laplas almashtirishi

$$L\int_0^t f(t)g(t-\tau)d\tau=\bar{f}(s)\cdot\bar{g}(s) \qquad (1.4.16)$$

formula bilan aniqlanadi. Tugun integralining kommutativligi, ya'ni

$$\int_0^t f(t)g(t-\tau)d\tau=\int_0^t g(t)f(t-\tau)d\tau$$

Ekanligi

$$\bar{f}(s)\bar{g}(s)=\bar{g}(s)\bar{f}(s)$$

tenglik o'rinliligidan kelib chiqadi.

Faraz qilaylik $\bar{f}(s)$ Laplas almashtirishi kompleks tekislikning ajratilgan maxsus nuqtalaridan tashqari hamma joyida s kompleks o'zgaruvchining analitik funksiyasi bo'lsin. U holda $\bar{f}(s)$ almashtirishga teskari almashtirish

$$f(t)=L^{-1}\left[\bar{f}(s)\right]=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}\,\bar{f}(s)ds \tag{1.4.17}$$

formula bilan aniqlanadi. Bu yerda $i^2=-1$, ya’ni *i*- mavhum birlik; $\operatorname{Re}(s)=\gamma$ to’g’ri chiziq $\bar{f}(s)$ funksiyaning hamma maxsusliklaridan o’ng tomonda yotadi. Shunday qilib qaytarish formulasi *s* kompleks tekisligidagi mavhum o’qqa parallel bo’lgan chiziq bo’ylab integrallashni ko’zda tutadi. Bu integralni hisoblash odatda chegirmalar (vычitlar) nazariyasi yordamida amalga oshiriladi.

Endi (1.4.14), (1.4.15) va (1.4.16) formulalar asosida (1.4.9), (1.4.10) va (1.4.12), (1.4.13) ifodalarga Laplas almashtirishini qo’llaymiz.

$$\bar{S}_{ij}=s\bar{G}L_1\bar{e}_{ij};\qquad \bar{\sigma}_{kk}=s\bar{G}_2\bar{\varepsilon}_{kk} \tag{1.4.18}$$

$$\bar{e}_{ij}=s\bar{I}_1\bar{s}_{ij};\qquad \bar{\varepsilon}_{kk}=s\bar{I}_2\bar{\sigma}_{kk} \tag{1.4.19}$$

Ushbu tengliklardan:

$$\bar{e}_{ij}\frac{1}{s\bar{G}_1}\bar{s}_{ij}\quad\Rightarrow\quad\frac{1}{s\bar{G}_1}\bar{s}_{ij}=s\bar{I}_1\bar{s}_{ij}\quad\Rightarrow\quad\bar{I}_1=\frac{1}{s^2\bar{G}_1}$$

$$\bar{\varepsilon}_{kk}=\frac{1}{s\bar{G}_2}\bar{\sigma}_{kk}\;\Rightarrow\;\frac{1}{s\bar{G}_2}\bar{\sigma}_{kk}=s\bar{I}_2\bar{\sigma}_{kk}\;\Rightarrow\;\bar{I}_2=\frac{1}{s^2\bar{G}_2}.$$

bu ikkala tengliklarni umumlashtirib

$$\bar{I}_\alpha=\left(s^2\bar{G}_\alpha\right)^{-1},\qquad \alpha=1,2 \tag{1.4.20}$$

ga ega bo’lamiz.

Elastiklik nazariyasida modullar va beriluvchanliklar (podatlivost) orasidagi mos munosabatlar $\bar{I}_\alpha=\bar{G}_\alpha^{-1}$ ko’rinishga ega. Elastiklik nazariyasining ushbu natijasini qovushoq-elastiklik nazariyasi uchun intuitiv ravishda umumlashtirish $I_\alpha(t)=[G_\alpha(t)]^{-1}$ tenglikka olib kelgan bo’lardi. Oxirgi (1.4.20) tenglikdan ko’rinadiki bunday umumlashtirish xato bo’lgan bo’lardi.

Ikki $\varphi(t)$ va $\psi(t)$ funksiyalarning Stiltyes tuguni $\varphi*d\psi$ quyidagi

$$\varphi*d\psi=\int_{-\infty}^{t}\varphi(t-\tau)d\psi(\tau) \tag{1.4.21}$$

formula bilan aniqlanadi, bu yerda $t \to -\infty$ bo'lganda $\psi(t) \to 0$ va $\varphi(t)$ funksiya $0 \le t < \infty$ intervalda uzluksiz. Ushbu ta'rifdan foydalanib (1.4.1) bog'lanishni quyidagicha yozish mumkin

$$\sigma_{ij} = \varepsilon_{kl} * dG_{ijkl}. \tag{1.4.22}$$

Ikkinchi tomondan $t < 0$ bo'lganda $\varphi(t) = 0$ deb hisoblasak (1.4.21) shakl kommutativlik qonuniga bo'ysunishini ko'rsatish mumkin, ya'ni

$$\varphi * d\psi = \psi * d\varphi. \tag{1.4.23}$$

(1.4.23) tenglikni isbotlash uchun (1.4.21) ni bo'laklab integrallash va $t < 0$ bo'lganda $\varphi(t) = 0$ ekanligini hisobga olish yetarlidir.

Ushbu xossani hisobga olib kuchlanishlar va deformasiyalar orasidagi (1.4.6) bog'lanishni quyidagi ekvivalent ko'rinishda yozish mumkin

$$\sigma_{ij} = G_{ijkl} * d\varepsilon_{kl} \tag{1.4.24}$$

bu esa o'z navbatida (1.4.9) bog'lanishning simvolik taqdimotidir.

Kuchlanishlar va deformasiyalar orasidagi bog'lanishlarning izotrop shakli ham xuddi (1.4.24) kabi taqdim qilinishi mumkin, ya'ni

$$\begin{aligned} s_{ij} &= G_1 * de_{ij} = e_{ij} * dG_1; \\ \sigma_{kk} &= G_2 * d\varepsilon_{kk} = \varepsilon_{kk} * dG_2. \end{aligned} \tag{1.4.25}$$

bu yerda $t < 0$ bo'lganda $\varepsilon_{ij}(t) = 0$

Shunday tasvirlashning bundan keyingi qo'llanilishida Stiltyes tugunining assosiativlik va distributivlik xossalari foydali bo'ladi. Yuqorida $\varphi(t)$ va $\psi(t)$ funksiyalarga nisbatan qilingan farazlarda bu xossalar quyidagicha yoziladi.

$$\varphi * d(\psi * d\omega) = (\varphi * d\psi) * d\omega = \varphi * d\psi * d\omega, \tag{1.4.26}$$

va

$$\varphi * d(\psi + \omega) = \varphi * d\psi + \varphi * d\omega. \tag{1.4.27}$$

Kuchlanishlar va deformasiyalar orasidagi bog'lanishlarning biz yuqorida ko'rgan integral shakli yagona emas. Ular bir qancha ekanligini ta'kidlaymiz.

II Bob.

IDEAL SUYUQLIK VA DEFORMATSIYALANUVCHI MUHIT BILAN NOSTATSIONAR TA'SIRLASHUVCHI ANIZOTROP SILINDRIK QATLAMNING BO'YLAMA TEBRANISHLARI

Ushbu bob doirasida transversal-izotrop muhitda joylashgan va ichiga ideal suyuqlik to'ldirilgan transversal-izotrop, doiraviy silindrik qatlamning bo'ylama-radial tebranishlari o'rganilgan. Tebranishlar tenglamalari odatda klassik nazariyalarda qatlamning deformatsiylanish tabiatidan va kuchlanish tenzori ba'zi komponentalarining dinamik holatidan kelib chiqqan holda qo'llaniladigan gipoteza va farazlardan foydalanmasdan chiqariladi.

Bunda qatlam va o'rab turuvchi deformatsiyalanuvchi muhit materiallarining qovushoqlik xususiyatini tavsiflovchi qovushoq-elastiklik operatorlari yadrolari, umumiy holda ixtiyoriy deb hisoblanadi. Qatlam nuqtalarining tebranishlari jarayonini uyg'otuvchi tashqi zoriqishlar faqat [17] tadqiqot doirasidagina ixtiyoriy hisoblanadilar. Boshqacha aytganda tebranishlar natijasida tarqalayotgan to'lqinlar uzuliklari qatlamning ko'ndalang kesimi o'lchovlaridan kam bo'lishlari talab etiladi.

Birinchi marta sterjenning bo'ylama tebranishlari taqribiy tenglamalari juda sodda gipotezalardan foydalangan holda Y. Bernulli va D.Eylerlar tomonidan olingan. Keyinchalik, kochish va kuchlanishlarning izlanuvchi (noma'lum) kattaliklarini radial koordinataning darajalari bo'yicha darajali qatorga yoyish yo'li bilan Puasson doiraviy sterjenning bo'ylama tebranishlari tenglamasini keltirib chiqardi. Sterjenning tebranish tenglamalarida aylanish inersiyasini hisobga olish A. Reley tomonidan, ko'ndalang siljish deformatsiyasini hisobga olish esa S.P.Timoshenko tomonidan amalga oshirilgan[2,4,5,24].

Silindrik elastic qobiqning bo’ylama tebranishlari taqribiy tenglamalari, birinchi marta, Kirxgofning plastinalarga nisbatan ma’lum gipotezalari asosida A.Love [15] tomonidan taklif etilgan. Silindrik elastic qobiqning bo’ylama tebranishlari tenglamalarini ishlab chiqish o’tgan asrning o’rtalaridan boshlab juda tez rivojlanmoqda.

Silindrik qobiqning o’rta sirti nuqtalarining ko’chishlariga uchun elastiklik nazariyasi munosabtlari asosida simmetriya o’qiga nisbatan tebranishlari taqribiy tenglamalari [5,18,29] tadqiqotlarda ishlab chiqilgan.

Kirxgoff-Love gipotezalari asosida A.S.Volmir [3] silindrik qobiqning simmetriya o’qiga nisbatan simmetrik tebranishlari aniqlshtirilgan tenglamalarini taklif etgan. Elastik qobiqlarning asimptotik tenglamalari U.Nigul [2] ishlarida olingan va amaliy masalalarni yechishga qo’llanilgan. Chiziqli qovushoq-elastiklik nazariyasining uch o’lchovli masalasiga Laplas almashtirishi qo’llanilib va hosil bo’lgan differensial tenglamaning umumiy yechimidan silindrik qobiqning o’qqa nisbatan simmetrik tebranishlari taqribiy va aniqlashtirilgan tenglamalarini keltirib chiqarishga qo’llash [17,18,22-29] tadqiqotlarda amalga oshirilgan.

Ushbu bob doirasida anizotrop jism chiziqli qovushoq-elastiklik nazariyasining qovushoq-elastik muhitda joylashgan va ichida ideal suyuqlik saqlovchi qovushoq-elastik silindrik qatlam uchun uch o’lchovli masalasining umumiy yechimidan kelib chiqqan holda, bo’ylama-radial tebranishlari tanglamalari keltirib chiqarilgan. Bunda olingan tebranishlar natijaviy tenglamalari, asosiy izlanuvchi funksiyalar sifatida qatlamning biror “oraliq” sirti nuqtalari kochishlarining bosh qismlarini o’z tarkiblarida saqlaydilar. Ushbu “oraliq” sirt limitik holatlarda pirovardida qatlamning o’rta, ichki, tashqi Yoki istalgan boshqa sirtiga otishi mumkin.

Tebranishlar tenglamalari bilan bir qatorda qaralayotgan gidroelastik sistemaning ixtiyoriy kesimidagi kuchlanganli-deformatsiyalanganlik holatini vaqtning istalgan payti uchun aniqlashga imkon beradigan algoritm yaratilgan.

Bu algoritm qatlam, uni o'rab turuvchi muhit va suyuqlik nuqtalaridagi kuchlanishlar tenzorlari va ko'chish hamda tezlik vektorlari komponentalari uchun chiqarilgan formulalardan iborat bo'lib, qatlam kesimlaridagi mustahkamlik parametrlarini hisoblashga va amaliy masalalar yechishda chegaraviy shartlarni to'g'ri shakllantirishga imkoniyat yaratadi.

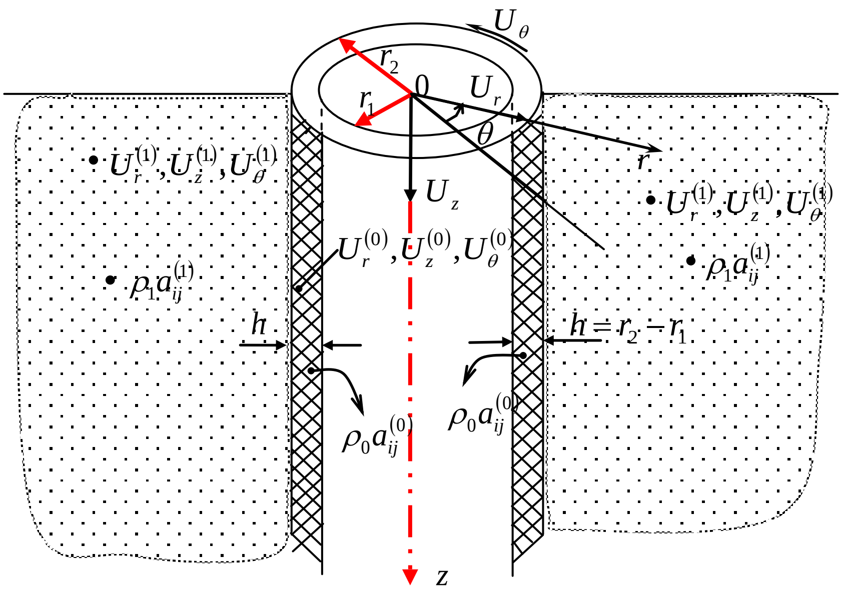
Silindrik qatlam, qobiq va doiraviy silindrik sterjanlar uchun ba'zi taqribiy va aniqlashtirilgan bo'ylama-radial tebranishlar tenglamalari keltirilgan. Tebranishlar tenglamalarining limitik va xususiy hollari hamda o'rab turuvchi muhitning reaksiyasi tahlil qilingan.

2.1. Ideal suyuqlik va deformatsiyalanuvchi muhit bilan ta'sirlashuvchi anizotrop silindrik qatlamning bo'ylama-
-radial tebranishlari umumiy tenglamalari

Deformatsiyalanuvchi qattiq jism mexanikasining juda kam o'rganilgan muammolari qatoriga tutash suyuq va qattiq muhitlar bilan ta'sirlashuvchi silindrik qatlam va qobiqlarning kuchlangan-deformatsiyalangan holatlarini aniqlash ham kiradi. Chunki bunday sistemalar fan va texnika sohalarida juda keng miqyosda qo'llaniladi. Jumladan, dinamik xususan seysmik va portlash yuklamalari ta'siridagi yer osti trubaprovodlari, tunnellar, tayyorlov va capital tog' ishlanmalari devorlari mustahkamlagichlari shunday konstruksiyalardir.

Quyida qovushoq-elastik muhitda joylashgan va ichida ideal suyuqlik saqlovchi qovushoq-elastik, anizotrop silindrik qatlam bo'ylama-radial tebranishlari haqidagi masala qaralgan. Bunday qatlam ko'ndalang kesimi doiraviy, yetarlicha uzunlikka ega yer osti inshootlari (tunnellar, tog' ishlanmalari mustahkamlagichlari, metro, truboprovodlar va boshqalar) ishini modellashtiradi. Masalaning hisob sxemasi va silindrik (r, θ,z) koordinatalar sistemasi o'qlarining yo'nalishlari quyida keltirilgan chizmada yaqqol aks ettirilgan.

Qatlamning tebranishlarini uning yon sirtlarida ta’sir qiluvchi tashqi dinamik kuchlar vujudga keltiradi deb hisoblanadi. Muhitlarni ajratib turuvchi sirtda, ya’ni muhitlar chegaralarida ko’chishlar va kuchlanishlarning uzluksizligi o’rinli deb hisoblanadi. Bo’ylama-radial tebranishlar holatita bu shartlar, ya’ni masalaning chegaraviy shartlari, quyidagicha bo’ladi



$$\sigma_{rr}^{(0)}(r_1,z,t) = -P^{(*)}, \quad \sigma_{rz}^{(0)}(r_1,z,t) = 0, \sigma_{rr}^{(0)}(r_2,z,t) = \sigma_{rr}^{(1)}(r_2,z,t) + P_r^{(2)}(z,t),$$

$$\sigma_{rz}^{(0)}(r_2,z,t) = 0, \quad \sigma_{rz}^{(1)}(r_2,z,t) + P_{rz}^{(2)}(z,t) = 0, \quad U_r^{(0)}(r_2,z,t) = U_r^{(1)}(r_2,z,t) + P_0(z,t) \tag{2.1.1}$$

$$U_r^{(0)} = \frac{\partial\varphi}{\partial r}, \qquad r = r_1, \ \ bu\ yerda \qquad P, P_r, P_{rz}, P_0 - tashqi\ ta'sir\ funksiyalari;$$

φ - suyuqlik zarrachalari tezliklari vektorining potensiali; $U_r^{(0)}$ - qatlam nuqtalarining radial ko’chishi. Bunda «0» bndeksi qatlamga, «1» indeksi esa o’rab turuvchi muhitga tegishli. Boshlang’ich shartlar nolga teng deb qabul qilingan.

Qatlamning bo’ylama-radial tebranishlari haqidagi qaralayotgan masala o’qqa nisbatan simmetrikdir. Shuning uchun ham ba’zi ko’chish va kuchlanishlar nolga teng bo’ladilar. Qolgan kuchlanish va ko’chishlar esa θ burchak koordinatasiga bog’liq bo’lmaydilar. Shuning uchun bu holda bo’ylama va radial ko’chishlar

$$U_r^{(m)} = U_r^{(m)}(r,z), \qquad U_z^{(m)} = U_z^{(m)}(r,z),$$

noldan farqli, buralma ko’chishlar esa nolga teng, ya’ni $U_\theta^{(m)} = 0$ (m=0,1).

Shuningdek qatlam (m=0) va o'rab turuvchi muhit (m=1) nuqtalaridagi kuchlanishlar tenzorlarining $\sigma_{ij}^{(m)}$-komponentalari ham burchak koordinatasiga bog'liq emas; Bu yerda $U_r^{(m)}$, $U_z^{(m)}$ - qatlam (m=0) va o'rab turuvchi muhit (m=1) nuqtalari ko'chish vektorlarining radial va bo'ylama tuzuvchilari.

Kuchlanish tenzorlarining noldan farqli komponentalarini $U_r^{(m)}$ *va* $U_z^{(m)}$ Ko'chishlar orqali ifodalaymiz. Transversal-izotrop jism uchun quyidagi munosabatlarga egamiz

$$\sigma_{rr}^{(m)} = G_{11}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial r}\right) + G_{12}^{(m)}\left(\frac{U_r^{(m)}}{r}\right) + G_{13}^{(m)}\left(\frac{\partial U_z^{(m)}}{\partial z}\right);$$

$$\sigma_{rz}^{(m)} = G_{44}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial z} + \frac{\partial U_z^{(m)}}{\partial r}\right);$$

$$\sigma_{\theta\theta}^{(m)} = G_{12}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial r}\right) + G_{11}^{(m)}\left(\frac{U_r^{(m)}}{r}\right) + G_{13}^{(m)}\left(\frac{\partial U_z^{(m)}}{\partial z}\right);$$

$$\sigma_{zz}^{(m)} = G_{13}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial r} + \frac{U_z^{(m)}}{r}\right) + G_{33}^{(m)}\left(\frac{\partial U_z^{(m)}}{\partial z}\right); \qquad (m = 0,1) \qquad (2.1.2)$$

bu yerda $G_{ij}^{(m)}$- quyidagi ko'rinishli qovushoq-elastiklik operatorlari

$$G_{ij}^{(m)}(\varsigma) = a_{ij}^{(m)}\left[\varsigma(t) - \int_0^t K_{ij}^{(m)}(t-\xi)\varsigma(\xi)d\xi\right],$$

$a_{ij}^{(m)}$-qatlam (m=0) va muhitning (m=1) elastiklik modullari.

Masalaning o'qa nisbatan simmetrikligini hisobga olsak qatlam, o'rab turuvchi muhit va suyuqlikning harakat tenglamalari [23] ushbu korinishda bo'ladilar:

$$\frac{\partial \sigma_{rr}^{(m)}}{\partial r} + \frac{\partial \sigma_{rz}^{(m)}}{\partial z} + \frac{\sigma_{rr}^{(m)} - \sigma_{\theta\theta}^{(m)}}{r} = \rho_m \frac{\partial^2 U_r^{(m)}}{\partial t^2};$$

$$\frac{\partial \sigma_{rz}^{(m)}}{\partial r} + \frac{\partial \sigma_{zz}^{(m)}}{\partial z} + \frac{\sigma_{rz}^{(m)}}{r} = \rho_m \frac{\partial^2 U_z^{(m)}}{\partial t^2}; \quad (\text{m=0,1}) \qquad (2.1.3)$$

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2};$$

Kuchlanishlarning (2.1.2) formulalar bo'yicha qiymatlarini (2.1.3) harakat tenglamalariga qo'yib, quyidagi tenglamalarga ega bo'lamiz

$$G_{11}^{(m)}\left(\frac{\partial^2 U_r^{(m)}}{\partial r^2}+\frac{1}{r}\frac{\partial U_r^{(m)}}{\partial r}-\frac{1}{r^2}U_r^{(m)}\right)+G_{44}^{(m)}\left(\frac{\partial^2 U_r^{(m)}}{\partial z^2}\right)+$$

$$-\left(G_{13}^{(m)}+G_{44}^{(m)}\right)\left(\frac{\partial^2 U_z^{(m)}}{\partial r\partial z}\right)=\rho_m\,\frac{\partial^2 U_r^{(m)}}{\partial t^2};$$

$$G_{44}^{(m)}\left(\frac{\partial^2 U_z^{(m)}}{\partial r^2}+\frac{1}{r}\frac{\partial U_z^{(m)}}{\partial r}\right)-G_{33}^{(m)}\left(\frac{\partial^2 U_z^{(m)}}{\partial z^2}\right)+\left(G_{13}^{(m)}+G_{44}^{(m)}\right)\times\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\frac{\partial U_z^{(m)}}{\partial z}=\rho_m\,\frac{\partial U_z^{(m)}}{\partial z^2}$$

$$\frac{\partial^2\varphi_0}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_0}{\partial r}+\frac{\partial^2\varphi_0}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\varphi_0}{\partial r^2};\qquad (m=0{,}1)$$

Ko'chishlar va tezlik;ar potensialini xuddi [17] kabi tasvirlaymiz

$$U_r^{(m)}=\int_0^\infty\left.\begin{matrix}\sin kz\\-\cos kz\end{matrix}\right\}dk\int_{(l)}U_r^{(m0)}e^{pt}dp;\qquad U_z^{(m)}=\int_0^\infty\left.\begin{matrix}\cos kz\\\sin kz\end{matrix}\right\}dk\int_{(l)}U_z^{(m0)}e^{pt}dp;$$

$$\varphi=\int_0^\infty\left.\begin{matrix}\sin kz\\-\cos kz\end{matrix}\right\}dk\int_{(l)}\varphi_0 e^{pt}dp,$$

Va kuchlanishlarning qiymatlarini harakat tenglamalariga qo'yib, ushbuni olamiz

$$G_{11}^{(m0)}\left(\frac{d^2U_r^{(m0)}}{dr^2}+\frac{1}{r}\frac{dU_r^{(m0)}}{dr}-\frac{1}{r^2}U_r^{(m0)}\right)-G_{44}^{(m0)}\left(k^2U_r^{(m0)}\right)-$$

$$-k\left(G_{13}^{(m0)}+G_{44}^{(m0)}\right)\left(\frac{dU_z^{(m0)}}{dr}\right)=\rho_m\;p^2U_r^{(m0)};$$

$$G_{44}^{(m0)}\left(\frac{d^2U_z^{(m0)}}{dr^2}+\frac{1}{r}\frac{dU_z^{(m0)}}{dr}\right)-G_{33}^{(m0)}\left(k^2U_z^{(m0)}\right)+k\left(G_{13}^{(m0)}+G_{44}^{(m0)}\right)\times\left(\frac{d}{dr}+\frac{1}{r}\right)U_r^{(m0)}=\rho_m\;p^2U_z^{(m0)}$$

$$\frac{d^2\varphi_0}{dr^2}+\frac{1}{r}\frac{d\varphi_0}{dr}-\alpha^2\varphi_0=0;\quad \alpha^2=k^2+p^2c^{-2},\qquad (2.1.4)$$

Belgilashlar kiritamiz

$$\Delta_0=\frac{d^2}{dt^2}+\frac{1}{r}\frac{d}{dt}-\frac{1}{r^2},\quad H_1^{(m0)}=\left(G_{13}^{(m0)}+G_{44}^{(m0)}\right)G_{11}^{(m0)^{-1}},\quad H_2^{(m0)}=\left(G_{13}^{(m0)}+G_{44}^{(m0)}\right)G_{44}^{(m0)^{-1}}$$

va (2.1.4) sistemaning ikkinchi tenglamasini r bo'yicha bir marta differensiallab, ega bo'lamiz:

$$\Delta_0 U_r^{(m0)} - \left[G_{11}^{(m0)} G_{44}^{(m0)} k^2 + \rho_m p^2 G_{11}^{(m0)^{-1}}\right] U_r^{(m0)} - kH_1^{(m0)} \frac{\partial U_r^{(m0)}}{\partial r} = 0;$$

$$\Delta_0 \frac{dU_z^{(m0)}}{dr} - \left[G_{33}^{(m0)} G_{44}^{(m0)^{-1}} k^2 + \rho_m p^2 G_{44}^{(m0)^{-1}}\right] \frac{dU_z^{(m0)}}{dr} - kH_2^{(m0)} \Delta_0 U_r^{(m0)} = 0; \qquad (2.1.5)$$

$$\frac{d^2\varphi_0}{dr^2} + \frac{1}{r}\frac{d\varphi_0}{dr} - \alpha^2 \varphi_0 = 0,$$

Birinchi tenglamadan

$$kH_1^{(m0)} \frac{dU_z^{(m0)}}{dr} = \left[\Delta_0 - \left(G_{11}^{(m0)^{-1}} G_{44}^{(m0)} k^2 + \rho_m p^2 G_{11}^{(m0)^{-1}}\right)\right] U_r^{(m0)}$$

ni topamiz va bu sistemani $U_r^{(m0)}$ funksiyaga nisbatab yechamiz

$$\Delta_0^2 U_r^{(m0)} - \left(\alpha_{1m}^2 + \alpha_{2m}^2\right) \Delta_0 U_r^{(m0)} + \alpha_{1m}^2 \alpha_{2m}^2 U_r^{(m0)} = 0;$$

$$\alpha_{1m}^2 + \alpha_{2m}^2 = \rho_m \left(G_{11}^{(m0)^{-1}} + G_{44}^{(m0)^{-1}}\right) p^2 + \left[G_{44}^{(m0)} G_{11}^{(m0)^{-1}} ++ G_{33}^{(m0)} G_{44}^{(m0)^{-1}} + G_{11}^{(m0)} G_{44}^{(m0)^{-1}} \left(G_{33}^{(m0)} + G_{44}^{(m0)}\right)^2\right] k^2;$$

$$\alpha_{1m}^2 \alpha_{2m}^2 = \rho_m^2 G_{11}^{(m0)^{-1}} G_{44}^{(m0)^{-1}} p^4 + \rho_m G_{11}^{(m0)^{-1}} \left(1 + G_{33}^{(m0)} G_{44}^{(m0)^{-1}}\right) p^2 k^2 + G_{11}^{(m0)^{-1}} G_{33}^{(m0)} k^4, \qquad (2.1.6)$$

(2.1.5) tenglamani quyidagi ko’rinishda yozish mumkin

$$\left(\Delta_0 - \alpha_{1m}^2\right)\left(\Delta_0 - \alpha_{2m}^2\right) U_r^{(m0)} = 0 \qquad (2.1.7)$$

Boggio T. [23] teoremasiga asosan (2.1.6) tenglamaning umumiy yechimi, quyidagi tenglamalar umumiy yechimlarining yig’indisiga teng

$$\frac{d^2 U_i^{(m0)}}{dr^2} + \frac{1}{r}\frac{dU_i^{(m0)}}{dr} - \left(\alpha_{im}^2 + \frac{1}{r^2}\right) U_i^{(m0)} = 0, \qquad (i = 1{,}2, \quad m = 0{,}1)$$

ya’ni

$$U_i^{(m0)} = U_1^{(m0)} + U_2^{(m0)}$$

Bu yerdan ko’rinadiki (2.1.7) tenglamaning qo’zg’olishlarning r=0 и r→∞ bo’lganda chegaralanganlik shartlarini qanoatlantiruvchi umumiy yechimi quyidagi ko’rinishga ega bo’lishi kerak:

Qatlam uchun, ya’ni m=0 bo’lganda

$$U_r^{(00)} = e_1 I_1(\alpha_{10} r) + e_2 K_1(\alpha_{10} r) + D_1 I_1(\alpha_{20} r) + D_2 K_1(\alpha_{20} r);$$

$$U_z^{(00)}=\frac{\alpha_{10}^2-\alpha_0^2}{k\alpha_{10}^2H_1^{(00)}}\left[e_1I_0(\alpha_{10}r)-e_2K_0(\alpha_{10}r)\right]+\frac{\alpha_{20}^2-\alpha_0^2}{k\alpha_{20}^2H_1^{(00)}}\left[D_1I_0(\alpha_{20}r)-D_2K_0(\alpha_{20}r)\right],$$

Muhit uchun, ya'ni m=1 bo'lganda (2.1.8)

$$U_r^{(10)}=D_3K_1(\alpha_{11}r)+D_4K_1(\alpha_{11}r);$$

$$U_z^{(10)}=-\frac{\alpha_{11}^2-\alpha_1^2}{k\alpha_{11}H_1^{(10)}}K_0(\alpha_{11}r)D_3-\frac{\alpha_{21}^2-\alpha_1^2}{k\alpha_{21}H_1^{(10)}}K_0(\alpha_{21}r)D_4;$$

$$\varphi_0=AI_0(\alpha r)\quad где\quad \alpha_m^2=\rho_m p^2G_{11}^{(m0)^{-1}}+G_{11}^{(m0)^{-1}}G_{44}^{(m0)}k^2\ \ (m=0{,}1)$$

Qatlamning $U_r^{(00)}$ *va* $U_z^{(00)}$ ko'chishlarini r-radial koordinataning darajalari bo'yicha darajali qatorlarga yoyib, quyidagi ifodalarga ega bo'lamiz

$$U_z^{(00)}=\sum_{n=0}^{\infty}\left[\frac{\alpha_{10}^2-\alpha_{10}^2}{k\alpha_{10}H_1^{(00)}}\alpha_{10}^{2n}e_{10}+\frac{\alpha_{20}^2-\alpha_{10}^2}{k\alpha_{20}H_1^{(00)}}\alpha_{20}^{2n}e_{20}\right]\frac{(r/2)^{2n}}{(n!)^2}+$$

$$\sum_{n=0}^{\infty}\eta_{3,n}\left[\frac{\alpha_{10}^2-\alpha_{10}^2}{k\alpha_{10}H_1^{(00)}}\alpha_{10}^{2n}e_2+\frac{\alpha_{20}^2-\alpha_{10}^2}{k\alpha_{20}H_1^{(00)}}\alpha_{20}^{2n}D_2\right]\frac{(r/2)^{2n}}{(n!)^2};$$

$$U_r^{(00)}=\frac{1}{r}\left(\frac{e_2}{\alpha_{10}}+\frac{D_2}{\alpha_{20}}\right)+\sum_{n=0}^{\infty}\left[\alpha_{10}^{2n+1}e_{10}+\alpha_{20}^{2n+1}e_{20}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!}+$$

$$\sum_{n=0}^{\infty}\eta_{4,n}(r)\left[\alpha_{10}^{2n+1}e_2+\alpha_{20}^{2n+1}D_2\right]\frac{(r/2)^{2n+1}}{n!(n+1)!},$$

$$\eta_{3,n}(r)=\ln\frac{r}{\xi}-\sum_{n=0}^{\infty}\frac{1}{k};\quad \eta_{4,n}(r)=\eta_{3,n}(r)-\frac{1}{2(n+1)},\tag{2.1.9}$$

va (2.1.9) almashtirilgan ko'chishlarning $r=\xi, n=0$ bo'lgandagi bosh qismlarini qaraymiz. Ushbu bosh qismlar (2.1.9) ifodalardagi cheksiz qatorlarning birinchi qo'shiluvchilariga teng bo'ladi

$$U_{r,0}^{(00)}=\alpha_{10}e_{10}+\alpha_{20}e_{20};\quad U_{r,1}^{(00)}=\xi^{-1}\left(\alpha_{10}^{-1}e_2+\alpha_{20}^{-1}D_2\right);$$

$$U_{z,0}^{(00)}=\frac{\alpha_{10}^2-\alpha_0^2}{k\alpha_{10}H_1^{(00)}}e_{10}+\frac{\alpha_{20}^2-\alpha_0^2}{k\alpha_{20}H_1^{(00)}}e_{20};\quad \xi U_{z,1}^{(0)}=\frac{\alpha_{10}^2-\alpha_0^2}{k\alpha_{10}H_1^{(00)}}e_2+\frac{\alpha_{20}^2-\alpha_0^2}{k\alpha_{20}H_1^{(00)}}D_2.$$

Bunda oraliq sirtning ξ - radiusi quyidagi formula bilan aniqlanadi [22,26]

$$\xi=\frac{r_1}{2}\left(\nu-\frac{r_1}{r_2}\right),\qquad 2+\frac{r_1}{r_2}\le\nu\le 2\frac{r_2}{r_1}+\frac{r_1}{r_2}$$

Ushbu sistemani yechib integrallash o'zgarmaslarini aniqlaymiz

$$e_{10}=\frac{1}{\alpha_0^2\Delta}\left[\frac{\alpha_{20}^2-\alpha_0^2}{k\alpha_{20}H_1^{(00)}}U_{r,0}^{(00)}-\alpha_{20}U_{z,0}^{(00)}\right];\qquad e_{20}=\frac{1}{\alpha_0^2\Delta}\left[\alpha_{10}U_{z,0}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{k\alpha_{10}H_1^{(00)}}U_{r,0}^{(00)}\right];$$

$$e_2=\frac{\xi}{\Delta}\left[\frac{\alpha_{20}^2-\alpha_0^2}{k\alpha_{20}H_1^{(00)}}U_{r,1}^{(00)}-\frac{1}{\alpha_{20}}U_{z,1}^{(00)}\right];\qquad D_2=\frac{\xi}{\Delta}\left[\frac{1}{\alpha_{10}}U_{z,1}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{k\alpha_{10}H_1^{(00)}}U_{r,1}^{(00)}\right],\tag{2.1.10}$$

bu yerda $\Delta=\dfrac{\alpha_{20}^2-\alpha_{10}^2}{k\alpha_{10}\alpha_{20}H_1^{(00)}}$.

Endi almashtirilgan $U_r^{(00)}$ va $U_z^{(00)}$ ko’chishlarni, keyinchalik esa masalaning chegaraviy shartlarini ham, ularning yuqorida kiritilgan $U_{z,0}^{(00)},U_{z,1}^{(00)},U_{r,0}^{(00)},U_{r,1}^{(00)}$ bosh qismlari orqali ifodalaymiz. Ko’chishlarning (2.1.9) ifodalari, ularga integrallash o’zgarmaslarining (2.1.10) qiymatlarini qo’ygandan keyin quyidagi ko’rinishga keladilar

$$U_z^{(00)}=\frac{1}{\alpha_0^2kH_1^{(00)}}\sum_{n=0}^{\infty}\left[kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2\left(P_n^{(00)}-\alpha_0^2\right)U_{z,0}^{(00)}-\left(\alpha_{10}^2-\alpha_0^2\right)\left(\alpha_{20}^2-\alpha_0^2\right)P_n^{(00)}U_{r,0}^{(00)}\right]\frac{(r/2)^{2n}}{(n!)^2}+$$

$$+\frac{1}{H_1^{(00)}}\sum_{n=0}^{\infty}\eta_{3,n}(r)\left[kH_1^{(00)}\left(P_{n+1}^{(00)}-\alpha_0^2P_n^{(00)}\right)U_{z,1}^{(00)}-\left(\alpha_{10}^2-\alpha_0^2\right)\left(\alpha_{20}^2-\alpha_0^2\right)P_n^{(00)}U_{r,1}^{(00)}\right]\frac{(r/2)^{2n}}{(n!)^2};$$

$$U_r^{(00)}=\frac{\xi}{r}U_{r,1}^{(00)}+\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\left[\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\right)U_{r,0}^{(00)}+kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2P_n^{(00)}U_{r,0}^{(00)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!}+$$

$$+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r)\left[\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2\right)U_{r,1}^{(00)}+kH_1^{(00)}P_{n+1}^{(00)}U_{z,1}^{(00)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!};\tag{2.1.11}$$

Bu yerda

$$P_n^{(00)}=\sum_{i=0}^{n-1}\alpha_{20}^{2(n-i-1)}\alpha_{10}^{2i};\quad P_0^{(00)}=0;\quad P_1^{(00)}=1;\quad P_2^{(00)}=\alpha_{10}^2+\alpha_{20}^2;\quad \alpha_{10}^2\alpha_{20}^2P_{-1}^{(00)}=0.\tag{2.1.12}$$

Ettita $e_j\,(j=1,2)$, A va $D_j\,(j=1,2,3,4)$ integrallash o’zgarmaslarini aniqlash uchun ettita (2.1.1) chegaraviy shartlarga egamiz, qaysiki ularga almashtirish qo’llanilgandan keyin ushbu ko’rinishda yoziladi

$$\left[\alpha_{10}H_1^{(00)}G_{11}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{\alpha_{10}}G_{13}^{(00)}\right]\left[e_1I_0(\alpha_{10}r_1)-e_2K_0(\alpha_{10}r_1)\right]+$$

$$\left[\alpha_{20}H_1^{(00)}G_{11}^{(00)}-\frac{\alpha_{20}^2-\alpha_0^2}{\alpha_{20}}G_{13}^{(00)}\right]\left[D_1I_0(\alpha_{20}r_1)-D_2K_0(\alpha_{20}r_1)\right]+$$

$$+\frac{H_1^{(00)}\left(G_{12}^{(00)}-G_{11}^{(00)}\right)}{r_1}\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)+D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right]=H_1^{(00)}\left[\rho_{ж}p\varphi_0\right];$$

$$\left(\alpha_{10}^2-\alpha_0^2+k^2H_1^{(00)}\right)\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)\right]+$$
$$+\left(\alpha_{20}^2-\alpha_0^2+k^2H_1^{(00)}\right)\left[D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right]=0;$$

$$\left(\alpha_{10}^2-\alpha_0^2+k^2H_1^{(00)}\right)\left[e_1I_1(\alpha_{10}r_2)+e_2K_1(\alpha_{10}r_2)\right]+$$
$$+\left(\alpha_{20}^2-\alpha_0^2+k^2H_1^{(00)}\right)\left[D_1I_1(\alpha_{20}r_2)+D_2K_1(\alpha_{20}r_2)\right]=0;$$

$$\left[\alpha_{10}H_1^{(00)}G_{11}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{\alpha_{10}}G_{13}^{(00)}\right]\left[e_1I_0(\alpha_{10}r_2)-e_2K_0(\alpha_{10}r_2)\right]+\left[\alpha_{20}H_1^{(00)}G_{11}^{(00)}-\frac{\alpha_{20}^2-\alpha_0^2}{\alpha_{20}}G_{13}^{(00)}\right]\times$$

$$\times\left[D_1I_0(\alpha_{20}r_2)-D_2K_0(\alpha_{20}r_2)\right]+\frac{H_1^{(00)}\left(G_{12}^{(00)}-G_{11}^{(00)}\right)}{r_2}\left[e_1I_1(\alpha_{10}r_2)+e_2K_1(\alpha_{10}r_2)+D_1I_1(\alpha_{20}r_2)-D_2K_1(\alpha_{20}r_2)\right]=$$
$$\frac{H_1^{(00)}\left(G_{13}^{(00)}-G_{11}^{(00)}\right)}{r_2}\left[D_3K_1(\alpha_{11}r_2)\right]+\left[\frac{\alpha_{11}^2-\alpha_1^2}{\alpha_{11}H_1^{(10)}}G_{13}^{(10)}H_1^{(00)}-\alpha_{11}H_1^{(00)}G_{11}^{(10)}\right]D_3K_0(\alpha_{11}r_2)+\frac{H_1^{(00)}\left(G_{12}^{(10)}-G_{11}^{(10)}\right)}{r_2}\times$$
$$\left[D_4K_1(\alpha_{21}r_2)\right]+\left[\frac{\alpha_{21}^2-\alpha_1^2}{\alpha_{21}H_1^{(10)}}G_{13}^{(10)}H_1^{(00)}-\alpha_{21}H_1^{(00)}G_{11}^{(10)}\right]D_4K_0(\alpha_{21}r_2)+H_1^{(00)}\left[P_r^{(20)}(k,p)\right]; \qquad (2.1.13)$$

$$\left[e_1I_1(\alpha_{10}r_2)+e_2K_1(\alpha_{10}r_2)+D_1I_1(\alpha_{20}r_2)+D_2K_1(\alpha_{20}r_2)\right]=$$
$$=D_3K_1(\alpha_{11}r_2)+D_4K_1(\alpha_{21}r_2)+P_r^{(20)}(k,p);$$

$$\left(\alpha_{11}^2-\alpha_1^2+k^2H_1^{(10)}\right)D_3K_1(\alpha_{11}r_2)+\left(\alpha_{21}^2-\alpha_1^2+k^2H_1^{(10)}\right)D_4K_1(\alpha_{21}r_2)+$$
$$+G_{44}^{(10)^{-1}}H_1^{(10)}\left[kP_r^{(20)}(k,p)\right]=0;$$

$$p\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)+D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right]=A\alpha I_1(\alpha r_1).$$

Oxirgi tenglamalardan A, D_3 va D_4 larni topamiz

$$A=\frac{p}{\alpha I_1(\alpha r_1)}C_1,\quad D_3=\frac{\left(\alpha_{21}^2-\alpha_1^2+k^2H_1^{(10)}\right)\left(C_2-P_0^{(0)}\right)+G_{44}^{(10)^{-1}}H_1^{(10)}\left(kP_{rz}^{(20)}\right)}{\left(\alpha_{21}^2-\alpha_{11}^2\right)K_1(\alpha_{11}r_2)},$$

$$D_4 = \frac{-\left(\alpha_{11}^2 - \alpha_1^2 + k^2 H_1^{(10)}\right)\left(C_2 - P_0^{(0)}\right) - G_{44}^{(10)^{-1}} H_1^{(10)}\left(kP_{rz}^{(20)}\right)}{\left(\alpha_{21}^2 - \alpha_{11}^2\right) K_1(\alpha_{11} r_2)},$$

bu yerda $\quad C_i = \left[e_1 I_1(\alpha_{10} r_i) + e_2 K_1(\alpha_{10} r_i) + D_1 I_1(\alpha_{20} r_i) + D_2 K_1(\alpha_{20} r_i)\right]$, (i=1,2)

Yuqoridagi (2.1.13) sistemaning birinchi va to'rtinchi tenglamalariga A, D_3 va D_4 o'zgarmaslarning topilgan qiymatlarini qo'yamiz

$$\left[\alpha_{10} H_1^{(00)} G_{11}^{(00)} - \frac{\alpha_{10}^2 - \alpha_0^2}{\alpha_{10}} G_{13}^{(00)}\right]\left[e_1 I_0(\alpha_{10} r_1) - e_2 K_0(\alpha_{10} r_1)\right] + \left[\alpha_{20} H_1^{(00)} G_{11}^{(00)} - \frac{\alpha_{20}^2 - \alpha_0^2}{\alpha_{20}} G_{13}^{(00)}\right] \times$$

$$\times \left[D_1 I_0(\alpha_{10} r_1) - D_2 K_0(\alpha_{10} r_1)\right] + \frac{H_1^{(00)}\left(G_{12}^{(0)} - G_{11}^{(00)}\right)}{r_1}\left[e_1 I_1(\alpha_{10} r_1) + e_2 K_1(\alpha_{10} r_1) + D_1 I_1(\alpha_{20} r_1) + D_2 K_1(\alpha_{10} r_1)\right] =$$

$$= H_1^{(00)} \frac{\rho_c p^2 I_0(\alpha r_1)}{\alpha I_1(\alpha r_1)} C_1, \qquad \frac{I_0(\alpha r_1)}{I_1(\alpha r_1)} \cong 1 \qquad (2.1.14)$$

$$\left[\alpha_{10} H_1^{(00)} G_{11}^{(00)} - \frac{\alpha_{10}^2 - \alpha_0^2}{\alpha_{10}} G_{13}^{(00)}\right]\left[e_1 I_0(\alpha_{10} r_2) - e_2 K_0(\alpha_{10} r_2)\right] + \left[\alpha_{20} H_1^{(00)} G_{11}^{(00)} - \frac{\alpha_{20}^2 - \alpha_0^2}{\alpha_{20}} G_{13}^{(00)}\right] *$$

$$\left[D_1 I_0(\alpha_{10} r_2) - D_2 K_0(\alpha_{10} r_2)\right] + \frac{H_1^{(00)}\left(G_{12}^{(0)} - G_{11}^{(00)}\right)}{r_2}\left[e_1 I_1(\alpha_{10} r_2) + e_2 K_1(\alpha_{10} r_2) + D_1 I_1(\alpha_{20} r_2) + D_2 K_1(\alpha_{10} r_2)\right] =$$

$$R_0^{(0)}\left[e_1 I_1(\alpha_{10} r_2) + e_2 K_1(\alpha_{10} r_2) + D_1 I_1(\alpha_{20} r_2) + D_2 K_1(\alpha_{10} r_2)\right] + R_1^{(0)}\left[P_0^{(0)}\right] + R_2^{(0)}\left[kP_{rz}^{(20)}\right] + H_1^{(00)}\left[P_r^{(20)}\right];$$

$$R_0^{(0)} = H_1^{(00)} \frac{\rho_c p^2}{\alpha}; \qquad R_1^{(0)} = \frac{H_1^{(00)}\left(G_{12}^{(00)} - G_{11}^{(00)}\right)}{r_2} +$$

$$+ \frac{G_{11}^{(10)} H_1^{(00)}\left(\alpha_1^4 + \alpha_{11}^2 \alpha_{21}^2 - \alpha_1^2(\alpha_{11}^2 + \alpha_{21}^2)\right) - k^2 H_1^{(10)}\left(\alpha_1^2 + \alpha_{11}\alpha_{21}\right) - \left(\alpha_1^2 + \alpha_{11}\alpha_{21} - kH_1^{(10)}\right)\alpha_{11}\alpha_{21} H_1^{(10)} G_{11}^{(10)} H_1^{(00)}}{\left(\alpha_{11} + \alpha_{21}\right)\alpha_{11}\alpha_{21} H_1^{(10)}}$$

$$R_2^{(0)} = \frac{G_{11}^{(00)} H_1^{(00)} G_{44}^{(10)^{-1}} \alpha_{11}\alpha_{21} - G_{13}^{(10)} H_1^{(00)} G_{44}^{(10)} H_1^{(10)}(\alpha_{11}\alpha_{21} - \alpha_1^2)}{\left(\alpha_{11} + \alpha_{21}\right)\alpha_{11}\alpha_{21} H_1^{(10)}}.$$

Endi (2.1.13) sistemaning birinchi ikkita tenglamasiga hamda (2.1.14) tenglamaga (2.1.10) ni qoyib, kiritilgan yangi $U_{z,0}^{(00)}, U_{z,1}^{(00)}, U_{r,0}^{(00)}, U_{r,1}^{(00)}$ funksiyalarga nisbatan to'rtta algebraic tenglamalar sistemasiga ega bo'lamiz

$$\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big\{k\alpha_{10}^2\alpha_{20}^2H_1^{(00)}\Big[\Big(H_1^{(00)}G_{11}^{(00)}-G_{13}^{(00)}\Big)P_n^{(00)}+\alpha_0^2G_{13}^{(00)}P_{n-1}^{(00)}\Big]U_{z,0}^{(00)}+\Big[H_1^{(00)}G_{11}^{(00)}\Big(\alpha_0^2P_{n+1}^{(00)}-P_{n+2}^{(00)}\Big)+$$

$$\Big(\alpha_{20}^2-\alpha_0^2\Big)P_n^{(00)}G_{13}^{(00)}\Big]U_{r,0}^{(00)}\Big\}\frac{(r_1/2)^{2n}}{(n!)^2}+\xi\sum_{m=0}^{\infty}\eta_{3,n}(r_1)\Big\{\Big[H_1^{(00)}G_{11}^{(00)}-kH_1^{(00)}G_{13}^{(00)}\Big(P_{n+1}^{(00)}-\alpha_0^2P_n^{(00)}\Big)\Big]U_{z,1}^{(00)}+$$

$$\Big[H_1^{(00)}G_{11}^{(00)}\Big(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)+\Big(\alpha_{10}^2-\alpha_0^2\Big)\Big(\alpha_{20}^2-\alpha_0^2\Big)P_n^{(00)}G_{13}^{(00)}\Big]U_{r,1}^{(00)}\Big\}\frac{(r_1/2)^{2n}}{(n!)^2}+\frac{H_1^{(00)}(G_{12}^{(00)}-G_{11}^{(00)})}{r_1}\times$$

$$\times\Bigg\{\frac{\xi}{r_1}U_{r,1}^{(00)}+\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big[\Big(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)U_{r,0}^{(00)}+kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2P_n^{(00)}U_{z,0}^{(00)}\Big]\frac{(r_1/2)^{2n+1}}{n!(n+1)^2}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_1)\times$$

$$\times\Big[\Big(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)U_{r,1}^{(00)}+kH_1^{(00)}P_{n+1}^{(00)}U_{z,1}^{(00)}\Big]\frac{(r_1/2)^{2n+1}}{n!(n+1)!}\Bigg\}=H_1^{(00)}R_0^{(0)}\Bigg\{\frac{\xi}{r_1}U_{r,1}^{(00)}+\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big[\Big(\alpha_0^2P_{n+1}^{(00)}-$$

$$-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)\Big]U_r^{(00)}+kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2P_n^{(00)}U_{z,0}^{(00)}\Big]\frac{(r_1/2)^{2n+1}}{n!(n+1)^2}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_1)\Big[\Big(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)U_{r,1}^{(00)}+$$

$$+kH_1^{(00)}P_{n+1}^{(00)}U_{z,1}^{(00)}\Big]\frac{(r_1/2)^{2n+1}}{n!(n+1)!}\Bigg\};$$

$$\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big\{\Big[\alpha_0^2P_{n+2}^{(00)}-\Big(\alpha_0^4+\alpha_{10}^2\alpha_{20}^2-\alpha_0^2k^2H_1^{(00)}\Big)P_{n+1}^{(00)}+\Big(\alpha_0^2-k^2H_1^{(00)}\Big)\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big]U_{r,0}^{(00)}+$$

$$kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2\Big[P_{n+1}^{(00)}+\Big(\alpha_0^2-k^2H_1^{(00)}\Big)P_n^{(00)}\Big]U_{z,0}^{(00)}\Big\}\frac{(r_1/2)^{2n+1}}{n!(n+1)!}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_1)\times$$

$$\Big\{\Big[\Big(\alpha_0^2P_{n+2}^{(00)}-\Big(\alpha_0^4+\alpha_{10}^2\alpha_{20}^2-\alpha_0^2k^2H_1^{(00)}\Big)P_{n+1}^{(00)}+\Big(\alpha_0^2-k^2H_1^{(00)}\Big)P_n^{(00)}\alpha_{10}^2\alpha_{20}^2\Big)U_{r,1}^{(00)}+\Big(\alpha_0^2-k^2H_1^{(00)}\Big)\times$$

$$\times\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big]U_{r,1}^{(00)}+kH_1^{(00)}\Big[P_{n+2}^{(00)}-\Big(\alpha_0^2-k^2H_1^{(00)}\Big)P_{n+1}^{(00)}\Big]U_{z,1}^{(00)}\Big\}\frac{(r_1/2)^{2n+1}}{n!(n+1)!}+\frac{\xi}{r_1}kH_1^{(00)}\Big(U_{z,1}^{(00)}+kU_{z,1}^{(00)}\Big)=0;$$

$$\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big\{k\alpha_{10}^2\alpha_{20}^2H_1^{(00)}\Big[\Big(H_1^{(00)}G_{11}^{(00)}-G_{13}^{(00)}\Big)P_n^{(00)}+\alpha_0^2G_{13}^{(00)}P_{n-1}^{(00)}\Big]U_{z,0}^{(00)}+\Big[H_1^{(00)}G_{11}^{(00)}\Big(\alpha_0^2P_{n+1}^{(00)}-P_{n+2}^{(00)}\Big)+$$

$$\left(\alpha_{20}^2-\alpha_0^2\right)P_n^{(00)}G_{13}^{(00)}\Big]U_{r,0}^{(00)}\Big\}\frac{(r_2/2)^{2n}}{(n!)^2}+\xi\sum_{m=0}^{\infty}\eta_{3,n}(r_2)\Big\{\Big[H_1^{(00)}G_{11}^{(00)}-kH_1^{(00)}G_{13}^{(00)}\left(P_{n+1}^{(00)}-\alpha_0^2P_n^{(00)}\right)\Big]U_{z,1}^{(00)}+$$

$$\Big[H_1^{(00)}G_{11}^{(00)}\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\right)+\left(\alpha_{10}^2-\alpha_0^2\right)\left(\alpha_{20}^2-\alpha_0^2\right)P_n^{(00)}G_{13}^{(00)}\Big]U_{r,1}^{(00)}\Big\}\frac{(r_2/2)^{2n}}{(n!)^2}+\frac{H_1^{(00)}\left(G_{12}^{(00)}-G_{11}^{(00)}\right)}{r_2}*$$

$$\left\{\frac{\xi}{r_2}U_{r,1}^{(00)}+\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big[\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\right)U_{r,0}^{(00)}+kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2P_n^{(00)}U_{z,0}^{(00)}\Big]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_2)*\right.$$

$$\left.\Big[\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\right)U_{r,1}^{(00)}+kH_1^{(00)}P_n^{(00)}U_{z,1}^{(00)}\Big]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}\right\}=R_1^{(0)}\left\{\frac{\xi}{r_2}U_{r,1}^{(00)}+\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big[\left(\alpha_0^2P_{n+1}^{(00)}-\right.\right.$$

$$-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big)\Big]U_{r,0}^{(00)}+kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2P_n^{(00)}U_{z,0}^{(00)}\Big]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_2)\Big[\left(\alpha_0^2P_{n+1}^{(00)}-\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\right)U_{r,1}^{(00)}+$$

$$\left.+kH_1^{(00)}P_{n+1}^{(00)}U_{z,1}^{(00)}\Big]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}\right\}+R_1^{(0)}\left[P_0^{(0)}\right]+R_2^{(0)}\left[kP_{rz}^{(20)}\right]+H_1^{(00)}\left[P_r^{(20)}\right];$$

$$\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\Big\{\Big[\alpha_0^2P_{n+2}^{(00)}-\left(\alpha_0^4+\alpha_{10}^2\alpha_{20}^2-\alpha_0^2k^2H_1^{(00)}\right)P_{n+1}^{(00)}+\left(\alpha_0^2-k^2H_1^{(00)}\right)\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big]U_{r,0}^{(00)}+$$

$$kH_1^{(00)}\alpha_{10}^2\alpha_{20}^2\left[P_{n+1}^{(00)}+\left(\alpha_0^2-k^2H_1^{(00)}\right)P_n^{(00)}\right]U_{z,0}^{(00)}\Big\}\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\xi\sum_{n=0}^{\infty}\eta_{4,n}(r_2)*$$

$$\Big\{\Big[\left(\alpha_0^2P_{n+2}^{(00)}-\left(\alpha_0^4+\alpha_{10}^2\alpha_{20}^2-\alpha_0^2k^2H_1^{(00)}\right)P_{n+1}^{(00)}+\right)U_{r,1}^{(00)}+\left(\alpha_0^2-k^2H_1^{(00)}\right)\alpha_{10}^2\alpha_{20}^2P_n^{(00)}\Big]U_{r,1}^{(00)}+$$

$$kH_1^{(00)}\left[P_{n+2}^{(00)}-\left(\alpha_0^2-k^2H_1^{(00)}\right)P_{n+1}^{(00)}\right]U_{z,1}^{(00)}\Big\}\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\frac{\xi}{r_2}kH_1^{(00)}\left(U_{z,1}^{(00)}+kU_{z,1}^{(00)}\right)=0;$$

Olingan tenglamalarni Fur'e va Laplas bo'yicha orqaga qaytarib, doiraviy silindrik qatlam oraliq sirtining $U_z^{(0)}$ *va* $U_r^{(0)}$ kochishlari bosh qismlariga nisbatan to'rtta integro-differensial tenglamalar sistemasiga ega bo'lamiz

$$a_{11}U_{z,0}^{(0)}+b_{11}U_{r,0}^{(0)}+\xi\left(c_{11}U_{z,1}^{(0)}+d_{11}U_{r,1}^{(0)}\right)=0;$$

$$a_{12}U_{z,0}^{(0)}+b_{12}U_{r,0}^{(0)}+\xi\left(c_{12}U_{z,1}^{(0)}+d_{12}U_{r,1}^{(0)}\right)=0;$$

(2.1.15)

$$\left(a_{21}-H_1^{(0)}Rq_{11}\right)U_{z,0}^{(0)}+\left(b_{21}-H_1^{(0)}Rq_{21}\right)U_{r,0}^{(0)}+\xi\left(c_{21}-H_1^{(0)}Rq_{31}\right)U_{z,1}^{(0)}+\xi\left(d_{21}-H_1^{(0)}Rq_{41}\right)U_{r,1}^{(0)}=0;$$

$$\left(b_{22}+R_1q_{12}\right)U_{r,0}^{(0)}-\left(a_{22}+R_1q_{22}\right)U_{z,0}^{(0)}-\xi\left(d_{22}+R_1q_{32}\right)U_{r,1}^{(0)}+\xi\left(c_{22}-R_1q_{42}\right)U_{z,1}^{(0)}=F,$$

bu yerda $\quad F = H_{10}^{(0)}\left[P_r^{(2)}\right] - R_1\left[P_0\right] - R_2\left[\frac{\partial P_{rz}^{(2)}}{\partial z}\right],$

va $a_{ji}, b_{ji}, c_{ji}, d_{ji} (j, i = 1,2)$ operatorlar quyidagi formulalar bilan aniqlanadilar

$$a_{1i} = \sum_{n=0}^{\infty} H_1^{(0)} \gamma_{20}^2 \frac{\partial}{\partial z}\left[P_{n+1}^{(0)} - \left(\gamma_{10} + H_1^{(0)} \frac{\partial^2}{\partial z^2}\right) P_n^{(0)}\right] \frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$b_{1i} = -\sum_{n=0}^{\infty}\left[P_{n+2}^{(0)} - \left(\gamma_{10}^2 + \gamma_{20}^2 + H_1^{(0)} \gamma_{10} \frac{\partial^2}{\partial z^2}\right) P_{n+1}^{(0)} + \left(\gamma_{10} + H_1^{(0)} \frac{\partial^2}{\partial z^2}\right) P_n^{(0)} \gamma_{20}^2\right] \frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$c_{1i} = -\sum_{n=0}^{\infty} \eta_{4,n}(r_i) \frac{\partial}{\partial z} \gamma_{10} H_1^{(0)}\left[P_{n+2}^{(0)} - \left(\gamma_{10} + H_1^{(0)} \frac{\partial^2}{\partial z^2}\right) P_{n+1}^{(0)}\right] \frac{(r_i/2)^{2n+1}}{n!(n+1)!} + \frac{1}{r_i} \gamma_{10} H_1^{(0)} \frac{\partial}{\partial z};$$

$$d_{1i} = -\sum_{n=0}^{\infty} \eta_{4,n}(r_i) \gamma_{10}\left[\gamma_{10} P_{n+2}^{(0)} - \left(\gamma_{10}^2 + \gamma_{20}^2 + H_1^{(0)} \gamma_{10} H_1^{(0)} \frac{\partial^2}{\partial z^2}\right) P_{n+1}^{(0)} + \left(\gamma_{10} + H_1^{(0)} \frac{\partial^2}{\partial z^2}\right) P_n^{(0)} \gamma_{20}^2\right] \times$$
$$\times \frac{(r_i/2)^{2n+1}}{n!(n+1)!} + \frac{1}{r_j} H_1^{(0)} \gamma_{10} \frac{\partial^2}{\partial z^2};$$

$$a_{1i} = \sum_{n=0}^{\infty} H_1^{(0)} \gamma_{20}^2 \frac{\partial}{\partial z}\left[\left(H_1^{(0)} G_{11}^{(0)} - G_{13}^{(0)}\right) P_n^{(0)} + \gamma_{10} G_{13}^{(0)} P_{n-1}^{(0)} + \frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)}\right] \frac{(r_i/2)^{2n}}{(n!)^2};$$

$$b_{2i} = \sum_{n=0}^{\infty}\left[H_1^{(0)} G_{11}^{(0)}\left(\gamma_{10} P_{n+1}^{(0)} - P_{n+2}^{(0)}\right) + G_{13}^{(0)}\left(\gamma_{10}^2 - \gamma_{10}\gamma_{30} + \gamma_{20}^2\right) P_n^{(0)} + \frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)}\right] \times$$
$$\times\left(\gamma_{10} P_{n+1}^{(0)} - P_{n+2}^{(0)}\right) \frac{(r_i/2)^{2n}}{(n!)^2};$$

$$c_{2i} = \sum_{n=0}^{\infty}\left[H_1^{(0)} G_{11}^{(0)}\left(\gamma_{10} P_{n+1}^{(0)} - P_{n+2}^{(0)}\right) + G_{13}^{(0)}\left(\gamma_{10}^2 - \gamma_{10}\gamma_{30} + \gamma_{20}^2\right) P_n^{(0)} +\right.$$
$$\left. + \frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)} \eta_{4,n}(r_i) \frac{\partial}{\partial z} P_{n+1}^{(0)} \frac{(r_i/2)^{2n}}{(n!)^2} + \frac{1}{r_i} \gamma_{10};\right.$$

$$d_{2i} = \sum_{n=0}^{\infty} \gamma_{10}\left[\eta_{3,n}(r_i)\left[H_1^{(0)} G_{11}^{(0)}\left(\gamma_{10} P_{n+1}^{(0)} - \gamma_{20}^2 P_{n+2}^{(0)}\right) + G_{13}^{(0)}\left(\gamma_{10}^2 - \gamma_{10}\gamma_{30} + \gamma_{20}^2\right) P_{n+1}^{(0)}\right] +\right.$$
$$\left. \frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)} \eta_{4,n}(r_i)\left(\gamma_{10} P_{n+1}^{(0)} - \gamma_{20}^2 P_{n+2}^{(0)}\right)\right] \frac{(r_i/2)^{2n}}{(n!)^2}; \qquad (i = 1,2)$$

$$q_{1i} = \sum_{n=0}^{\infty}\left(\gamma_{10} P_{n+1}^{(0)} - \gamma_{20}^2 P_{n+2}^{(0)}\right) \frac{(r_i/2)^{2n+1}}{n!(n+1)!}; \qquad q_{2i} = \sum_{n=0}^{\infty} \frac{\partial}{\partial z}\left(H_1^{(0)} \gamma_{20}^2 P_n^{(0)}\right) \frac{(r_i/2)^{2n+1}}{n!(n+1)!}; \quad (2.1.16)$$

$$q_{3i} = \frac{1}{r_i} \gamma_{10} + \sum_{n=0}^{\infty} \gamma_{10} \eta_{4,n}(r_i)\left(\gamma_{10} P_{n+1}^{(0)} - \gamma_{20}^2 P_{n+2}^{(0)}\right) \frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$q_{4i} = \sum_{n=0}^{\infty} \gamma_{10}\eta_{3,n}(r_i)\frac{\partial}{\partial z}\left(H_1^{(0)}\gamma_{20}^2 P_n^{(0)}\right)\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

Bu yerda $\gamma_{10}^n, \gamma_{20}^{2n}, \gamma_{30}^n, P_n^{(0)}$ operatorlar (z,t) o'zgaruvchilarda ushbularga teng

$$\gamma_{10}^n = G_{11}^{(0)^{-n}}\left[\rho_0\frac{\partial^2}{\partial t^2} - G_{44}^{(0)}\frac{\partial^2}{\partial z^2}\right]^n; \quad \gamma_{20}^{2n} = G_{11}^{(0)^{-n}}\left[\rho_0^2 G_{44}^{(0)^{-1}}\frac{\partial^4}{\partial t^4} - \rho_0\left(1 + G_{33}^{(0)} G_{44}^{(0)^{-1}}\right)\frac{\partial^4}{\partial z^2 \partial t^2} + G_{33}^{(0)}\frac{\partial^4}{\partial z^4}\right]^n;$$

$$\gamma_{30}^n = \left\{\rho_0\left(G_{11}^{(0)^{-1}} + G_{44}^{(0)^{-1}}\right)\frac{\partial^2}{\partial t^2} - \left[G_{44}^{(0)} G_{11}^{(0)^{-1}} + G_{44}^{(0)^{-1}} G_{33}^{(0)} + G_{44}^{(0)^{-1}} G_{11}^{(0)^{-1}}\left(G_{33}^{(0)} + G_{44}^{(0)}\right)^2\right]\frac{\partial^2}{\partial z^2}\right\}^n;$$

(2.1.17)

$$P_0^{(0)} \equiv 0; \quad P_1^{(0)} \equiv 1; \quad P_2^{(0)} \equiv \gamma_{30}; \quad \gamma_{20}^2 P_{-1} = -1; \quad P_3^{(0)} = \gamma_{30}^2 - \gamma_{20}^2;$$
$$P_4^{(0)} = \gamma_{30}\left(\gamma_{30}^2 - 2\gamma_{20}^2\right); \ldots$$

Keltirilgan (2.1.16) operatorlarning ko'rinishlaridan kelib chiqqan holda (2.1.15) tenglamalar sistemasining z koordinata va t vaqt bo'yicha hosilalar bo'yicha yuqori tartibli integro-differensial tenglamalar sistemasidan iborat ekanligini ko'rish qiyin emas. Bu tenglamalar anizotrop qatlam oraliq sirti nuqtalarining bo'ulama va radial kochishlari bosh qismlariga nisbatan keltirib chiqarildi.

2.2. Qatlam nuqtalaridagi kuchlanish va ko'chishlarni hamda suyuqlik bosimini aniqlash.

Deformatsiyalanuvchi qattiq jismlar mexaniksining asosiy masalasi tekshirilayotgan ob'ektning kuchlangan-deformatsiyalangan holatini aniqlashdan iborat. Shu nuqtai nazardan qaraganda tebranishlar tenglamalarini chiqarish bilan bir qatorda kuchlanishlar tenzorlari, ko'chish vektori va suyuqlik bosimini aniqlashga imkon beruvchi formulalarni chiqarish qo'yilgan masalani yechishning juda muhim bosqichlaridandir. Shu joyda ta'kidlash lozimki, qator olimlarning fikriga ko'ra (masalan U.K.Nigul, V.M.Darevskiy) Kirkhgoff-Love klassik nazariyasi qaralayotgan sistemaning, xususan silindrik qobiqning ham, tebranishlar tenglamalari hatto kvadraturada yechilganda ham, kuchlangan-deformatsiyalangan holatini bir qiymatli aniqlashga imkon bermaydi.

Shuning uchun ushbu paragraph kuchlanishlar tenzorlari, ko'chish va suyuqlik zarrachalari tezlik vektorlari komponentalari hamda suyuqlik bosimini aniqlashga imkon beruvchi formulalarni, tebranishlar tenglamalari yechimi bilan aniqlanuvchi, koordinata va vaqtning funksiyalari bo'lgan izlanuvchi kattaliklar (funksiyalar) orqali chiqarishga bag'ishlangan. Shu maqsadda qaralayotgan doiraviy silindrik transversal-izotrop qatlamning nuqtalarining $U_r^{(0)}$ va $U_z^{(0)}$ ko'chishlarini hamda qatlam nuqtalaridagi kuchlanish tenzorining noldan farqli komponentalarini $\sigma_{rr}^{(0)}, \sigma_{zz}^{(0)}, \sigma_{rz}^{(0)}$ va $\sigma_{\theta\theta}^{(0)}$ radial va bo'ylama ko'chishlar bosh qismlari $U_{r,0}^{(0)}, U_{r,1}^{(0)}, U_{z,0}^{(0)}$ va $U_{z,1}^{(0)}$ orqali ifodalaymiz.

Almashtirilgan $U_r^{(0)}$ va $U_z^{(0)}$ ko'chishlarning (2.1.11) formulalar bilan aniqlanuvchi ifodalarini Fur'e va Laplas bo'yicha orqaga qaytarib quyidagilarga ega bo'lamiz:

$$\begin{aligned} U_r^{(0)}(r,z,t) &= a_{r1}U_{r,0}^{(0)} + a_{r2}U_{z,0}^{(0)} - \xi\left(a_{r3}U_{r,1}^{(0)} + a_{r4}U_{z,1}^{(0)}\right); \\ U_z^{(0)}(r,z,t) &= a_{z1}U_{z,0}^{(0)} + a_{z2}U_{z,0}^{(0)} - \xi\left(a_{z3}U_{z,1}^{(0)} + a_{z4}U_{z,1}^{(0)}\right), \end{aligned} \tag{2.2.1}$$

bu yerda

$$\begin{aligned} a_{r1} &= \sum_{n=0}^{\infty}\left[P_{n+1}^{(0)} - \left(\gamma_{20}^2/\gamma_{10}\right)P_n^{(0)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!}; \\ a_{r2} &= \sum_{n=0}^{\infty}\left[kH_1^{(0)}\left(\gamma_{20}^2/\gamma_{10}\right)P_n^{(0)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!}; \\ a_{r3} &= \frac{1}{r} + \sum_{n=0}^{\infty}\eta_{4,n}(r)\left[\gamma_{10}P_{n+1}^{(0)} - \gamma_{20}^2P_n^{(0)}\right]\frac{(r/2)^{2n}}{(n!)^2}; \\ a_{r4} &= \sum_{n=0}^{\infty}\eta_{4,n}(r)\left[kH_1^{(0)}P_{n+1}^{(0)}\right]\frac{(r/2)^{2n}}{(n!)^2}; \end{aligned} \tag{2.2.2}$$

$$a_{z1}=\sum_{n=0}^{\infty}\left(\gamma_{20}^{2}/\gamma_{10}\right)\left[P_{n-1}^{(0)}\gamma_{10}^{2}P_{n}^{(0)}\right]\frac{(r/2)^{2n}}{(n!)^{2}};$$

$$a_{z2}=-\sum_{n=0}^{\infty}\left[\gamma_{20}^{2}-\gamma_{10}\gamma_{30}+\gamma_{10}kH_{1}^{(0)}\right]P_{n}^{(0)}\frac{(r/2)^{2n}}{(n!)^{2}};$$

$$a_{z3}=\sum_{n=0}^{\infty}\eta_{4,n}(r)\left[P_{n+1}^{(0)}-\gamma_{10}P_{n}^{(0)}\right]\frac{(r/2)^{2n}}{(n!)^{2}};$$

$$a_{z4}=-\frac{1}{kH_{1}^{(0)}}\sum_{n=0}^{\infty}\left[\gamma_{20}^{2}-\gamma_{10}\gamma_{30}+\gamma_{10}\right]P_{n}^{(0)}\frac{(r/2)^{2n}}{(n!)^{2}};$$

Kuchlanish tenzorining $\sigma_{ij}^{(0)}$ komponentalari uchun formulalarni chiqarish uchun ularni quyidagicha tasvirlaymiz

$$\left[\sigma_{rr}^{(0)},\sigma_{\theta\theta}^{(0)},\sigma_{zz}^{(0)}\right]=\int_{0}^{\infty}\left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\}dk\int_{(1)}\left[\sigma_{rr}^{(0)},\sigma_{\theta\theta}^{(0)},\sigma_{zz}^{(0)}\right]e^{ep}dp,$$

$$\sigma_{rz}^{(0)}=\int_{0}^{\infty}\left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\}dk\int_{(1)}\sigma_{rz}^{(00)}e^{pt}dp,$$

Bu formulalardan foydalanib almashtirilgan $\sigma_{rr}^{(0)}\sigma_{rz}^{(0)}\sigma_{rz}^{(0)}$ va $\sigma_{\theta\theta}^{(0)}$ kuchlanishlar uchun ularning Bessel va Makdonald modifitsirlangan funksiyalari orqali ifodalarini olamiz

$$\sigma_{rr}^{(00)}=\left[\alpha_{10}G_{11}^{(00)}-\frac{\alpha_{10}^{2}-\alpha_{0}^{2}}{\alpha_{10}H_{1}^{(00)}}G_{13}^{(00)}\right]\left[e_{1}I_{0}(\alpha_{10}r)-e_{2}K_{0}(\alpha_{10}r)\right]+\left[\alpha_{20}G_{11}^{(00)}-\frac{\alpha_{20}^{2}-\alpha_{0}^{2}}{\alpha_{20}H_{1}^{(00)}}G_{13}^{(00)}\right]\times$$

$$\times\left[D_{1}I_{0}(\alpha_{20}r)-D_{2}K_{0}(\alpha_{20}r)\right]+\frac{G_{12}^{(00)}-G_{11}^{(00)}}{r}\times$$

$$\times\left[e_{1}I_{1}(\alpha_{10}r)-e_{2}K_{1}(\alpha_{10}r)+D_{1}I_{0}(\alpha_{20}r)-D_{2}K_{0}(\alpha_{20}r)\right];$$

$$\sigma_{rz}^{(00)}=G_{44}^{(00)}\left\{\left(k+\frac{\alpha_{10}^{2}-\alpha_{0}^{2}}{kH_{1}^{(00)}}\right)\left[e_{1}I_{1}(\alpha_{10}r)-e_{2}K_{1}(\alpha_{10}r)\right]+\right.$$

$$+\left\{\left(k+\frac{\alpha_{10}^2-\alpha_0^2}{kH_1^{(00)}}\right)\left[D_1 I_0(\alpha_{20}r)+D_2K_0(\alpha_{20}r)\right]\right\};$$

$$\sigma_{zz}^{(00)}=\left[\alpha_{10}G_{13}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{\alpha_{10}H_1^{(00)}}G_{33}^{(00)}\right]\left[e_1I_0(\alpha_{10}r)-e_2K_0(\alpha_{10}r)\right]+$$

$$+\left[\alpha_{20}G_{13}^{(00)}-\frac{\alpha_{20}^2-\alpha_0^2}{\alpha_{20}H_1^{(00)}}G_{33}^{(00)}\right]\left[D_1I_0(\alpha_{20}r)-D_2K_0(\alpha_{20}r)\right];$$

(2.2.3)

$$\sigma_{\theta\theta}^{(00)}=\left[\alpha_{10}G_{12}^{(00)}-\frac{\alpha_{10}^2-\alpha_0^2}{\alpha_{10}H_1^{(00)}}G_{13}^{(00)}\right]\left[e_1I_0(\alpha_{10}r)-e_2K_0(\alpha_{10}r)\right]+$$

$$+\left[\alpha_{20}G_{12}^{(00)}-\frac{\alpha_{20}^2-\alpha_0^2}{\alpha_{20}H_1^{(00)}}G_{13}^{(00)}\right]\left[D_1I_0(\alpha_{20}r)-D_2K_0(\alpha_{20}r)\right]+$$

$$+\frac{G_{12}^{(00)}-G_{11}^{(00)}}{r}\left[e_1I_1(\alpha_{10}r)-e_2K_1(\alpha_{10}r)+D_1I_0(\alpha_{20}r)-D_2K_0(\alpha_{20}r)\right];$$

Hosil qilingan ifodalarning o'ng tomonlarini radial koordinataning darajalari bo'yicha qatorlarga yoyamiz va e_j и D_j lar o'rniga ularning (2.1.10) qiymatlarini qo'yib hamda olingan ifodalarni Fur'e va Laplas bo'yicha orqaga qaytarib ushbularga ega bo'lamiz:

$$\gamma_{10}\sigma_{rr}^{(0)}=H_1^{(0)-1}\left[a_2U_{z,0}^{(0)}+b_2U_{r,0}^{(0)}+\xi\left(c_2U_{z,1}^{(0)}+b_2U_{r,1}^{(0)}\right)\right];$$

$$\gamma_{10}\frac{\partial\sigma_{r я}^{(0)}}{\partial z}=H_1^{(0)-1}G_{44}^{(0)}\left[a_1U_{z,0}^{(0)}+b_1U_{r,0}^{(0)}+\xi\left(c_1U_{z,1}^{(0)}+b_1U_{r,1}^{(0)}\right)\right];$$

$$\gamma_{10}\sigma_{\theta\theta}^{(0)}=f_{\theta1}U_{z,0}^{(0)}+f_{\theta2}U_{r,0}^{(0)}+\xi\left(f_{\theta3}U_{z,1}^{(0)}+f_{\theta4}U_{r,1}^{(0)}\right);$$

(2.2.4)

$$\gamma_{10}\sigma_{zz}^{(0)}=f_{z1}U_{z,0}^{(0)}+f_{z2}U_{r,0}^{(0)}+\xi\left(f_{z3}U_{z,1}^{(0)}+f_{z4}U_{r,1}^{(0)}\right),$$

$$a_1=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[P_{n+1}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!};$$

$$b_1=-\sum_{n=0}^{\infty}\left[H_1^{(0)}P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\gamma_{20}^2\right]\frac{(r/2)^{2n+1}}{n!(n+1)!};$$

$$c_1=\sum_{n=0}^{\infty}\eta_{4,n}(r)H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z}\left[P_{n+2}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}\right]\frac{(r/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_i}\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z};$$

$$d_1=-\sum_{n=0}^{\infty}\eta_{4,n}(r)\gamma_{10}\left[\gamma_{10}P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\gamma_{20}^2\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\right]\times$$
$$\times\frac{(r/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_j}\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2};$$

$$a_2=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[\left(H_1^{(0)}G_{11}^{(0)}-G_{13}^{(0)}\right)P_{n+1}^{(0)}+\gamma_{10}G_{13}^{(0)}P_{n-1}^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}P_n^{(0)}\right]\frac{(r/2)^{2n}}{(n!)^2};$$

$$b_2=\sum_{n=0}^{\infty}\left[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{30}^2\right)P_n^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\right]\times$$
$$\times\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)\frac{(r/2)^{2n}}{(n!)^2};$$

$$c_2=\sum_{n=0}^{\infty}\gamma_{10}\left\{\eta_{3,n}(r)\frac{\partial}{\partial z}\left[H_1^{(0)}G_{11}^{(0)}-H_1^{(0)}G_{13}^{(0)}\left(P_{n+1}^{(0)}-\gamma_{10}P_n^{(0)}\right)\times\right.\right.$$

$$\left.\left.\times\frac{H_1^{(0)^2}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r)\frac{\partial}{\partial z}P_{n+1}^{(0)}\right]\right\}\frac{(r/2)^{2n}}{(n!)^2}+\frac{1}{r_i}\gamma_{10};$$

$$d_2=\sum_{n=0}^{\infty}\gamma_{10}\left\{\eta_{3,n}(r_i)\left[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)P_{n+1}^{(0)}\right]+\right\}$$

$$+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r)\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_n^{(0)}\right)\Bigg\}\frac{(r/2)^{2n}}{(n!)^2};$$

$$f_{\theta 1}=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\begin{bmatrix}\left(H_1^{(0)}G_{12}^{(0)}-G_{13}^{(0)}\right)P_n^{(0)}-\gamma_{10}G_{13}^{(0)}P_{n-1}^{(0)}+\\ +\dfrac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2(n+1)}P_n^{(0)}\end{bmatrix}\frac{(r/2)^{2n}}{(n!)^2};$$

$$f_{\theta 2}=-\sum_{n=0}^{\infty}\Bigg[H_1^{(0)}G_{12}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{30}^2\right)P_n^{(0)}+\Bigg];$$

$$+\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2(n+1)}\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)\Bigg]\frac{(r/2)^2}{(n!)^2};$$

(2.2.5)

$$f_{\theta 3}=\sum_{n=0}^{\infty}\gamma_{10}\begin{Bmatrix}\eta_{4,n}(r)\left[H_1^{(0)}G_{12}^{(0)}-G_{13}^{(0)}\left(P_{n+1}^{(0)}-\gamma_{10}P_n^{(0)}\right)\right]+\\ +\dfrac{H_1^{(0)}\left(G_{11}^{(0)}-G_{12}^{(0)}\right)}{2(n+1)}\eta_{4,n}(r)\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_n^{(0)}\right)\end{Bmatrix}\frac{(r/2)^{2n}}{(n!)^2}+\frac{1}{r_i}\gamma_{10};$$

$$f_{\theta 4}=\sum_{n=0}^{\infty}\gamma_{10}\begin{Bmatrix}\eta_{4,n}(r)\left[H_1^{(0)}G_{12}^{(0)}-\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{30}^2\right)P_n^{(0)}\right]+\\ +\dfrac{H_1^{(0)}\left(G_{11}^{(0)}-G_{12}^{(0)}\right)}{2(n+1)}\eta_{4,n}(r)\end{Bmatrix}\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)\Bigg\}\frac{(r/2)^{2n}}{(n!)^2};$$

$$f_{z1}=\sum_{n=0}^{\infty}\gamma_{20}^2\frac{\partial}{\partial z}\left[H_1^{(0)}G_{13}^{(0)}P_n^{(0)}+\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_n^{(0)}\right)G_{33}^{(0)}\right]\frac{(r/2)^2}{(n!)^2};$$

$$f_{z2}=\sum_{n=0}^{\infty}\left[\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_n^{(0)}\right)G_{13}^{(0)}-H_1^{(0)-1}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)G_{33}^{(0)}P_n^{(0)}\right]\frac{(r/2)^2}{(n!)^2};$$

$$f_{z3}=\sum_{n=0}^{\infty}\gamma_{10}\left\{\eta_{3,n}(r)\frac{\partial}{\partial z}\left[H_1^{(0)}G_{13}^{(0)}P_{n+1}^{(0)}+\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)G_{33}^{(0)}\right]\right\}\frac{(r/2)^2}{(n!)^2};$$

$$f_{z4}=\sum_{n=0}^{\infty}\left\{\eta_{3,n}(r)\left[\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_n^{(0)}\right)G_{13}^{(0)}-H_1^{(0)-1}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)G_{33}^{(0)}P_n^{(0)}\right]\right\}\frac{(r/2)^2}{(n!)^2};$$

Olingan (2.2.1) va (2.2.4) formulalar anizotrop qatlamning kuchlangan-deformatsiyalangan holatini aniqlashga, qatlamning bo’ylama-radial tebranishlari haqidagi masalalarning chegaraviy va boshlang’ich shartlaini shakllantirishga imkon beradi.

Qatlam nuqtalarining ko’chishlari (2.2.1) formulalar bilan, qatlam kesimlaridagi kuchlanishlar esa (2.2.2) formulalar bilan aniqlanishi mumkin. Bu holda $U_{r,0}^{(0)}, U_{r,1}^{(0)}, U_{z,0}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalar orniga ularning (2.1.15) tenglamalar sistemasini yechish natijasida topilgan qiymatlarini qo’ymoq zarur. Suyuqlikning faqat bosiminigina aniqlaymiz.

Suyuqlik tezlik potensialini (2.1.1) chegaraviy shartlarning oxirgi tenglamasidan topamiz va (2.1.8) yechimdan foydalanib, topamiz

$$\begin{aligned}&A\alpha I_1(\alpha r_1)=p\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)+D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right];\\&\alpha I_1(\alpha r_1)\varphi_0=pI_0(\alpha r_1)\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)+D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right];\end{aligned}\tag{2.2.6}$$

Ikkinchi tomondan P - bosim ifodasiga Laplas va Fur’e almashtirishlarini qo’llab suyuqlikning almashtirilgan bosimi uchun quyidagi formulaga ega bo’lamiz:

$$\overline{P}=-\rho_0p\varphi_0,\tag{2.2.7}$$

Endi (2.2.6) ni (2.2.7) ga qo’yamiz

$$\begin{aligned}&\alpha I_1(\alpha r_1)\overline{P}=-\rho_{ж}p^2I_0(\alpha r_1)\times\\&\times\left[e_1I_1(\alpha_{10}r_1)+e_2K_1(\alpha_{10}r_1)++D_1I_1(\alpha_{20}r_1)+D_2K_1(\alpha_{20}r_1)\right];\end{aligned}\tag{2.2.8}$$

Radial koordinataning darajalari bo'yicha (2.2.6) tenglamalar tarkibiga kiruvchi Bessel funksiyalani darajali qatorlarga yoyib, e_j, D_j operatorlar o'rniga ularning (2.1.10) formulalar bo'yicha qiymatlarini qoyib suyuqlikning almashtirilgan bosimi $U_{r,0}^{(0)}, U_{r,1}^{(0)}, U_{z,0}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalar orqali aniqlovchi formulani olamiz

$$\alpha I_1(\alpha r_1)\overline{P} = -\rho_{ж} p^2 I_0(\alpha r_1)\left[g_1 U_{r,0}^{(00)} + g_2 U_{z,0}^{(00)} + g_3 U_{r,1}^{(00)} + g_4 U_{z,1}^{(00)}\right]; \qquad (2.2.9)$$

Ushbuifodani Fur'e va Laplas bo'yicha teskari almashtirib, topamiz

$$P = -\rho_{ж}\frac{\partial}{\partial t}\left\{\frac{\xi}{r_1}U_{r,1}^{(0)} + \frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\left[\left(\alpha_0^2 P_{n+1}^{(0)} - \alpha_{10}^2\alpha_{20}^2 P_n^{(0)}\right)U_{r,0}^{(0)} + H_1^{(0)}\alpha_{10}^2\alpha_{10}^2 P_n^{(0)}\frac{\partial U_{z,0}^{(0)}}{\partial z}\right]\frac{(r_1/2)^{2n+1}}{!n(n+1)!} + \right.$$

$$\left. + \xi\frac{1}{\alpha_0^2}\sum_{n=0}^{\infty}\eta_{4,n}(r_1)\left[\left(\alpha_0^2 P_{n+1}^{(0)} - \alpha_{10}^2\alpha_{20}^2 P_n^{(0)}\right)U_{r,1}^{(0)} + H_1^{(0)} P_{n+1}^{(0)}\frac{\partial U_{z,1}^{(0)}}{\partial z}\right]\frac{(r_1/2)^{2n+1}}{!n(n+1)!}\right\}; \qquad (2.2.10)$$

yoki

$$\gamma_{10}P = -\rho_{ж}\frac{\partial}{\partial t}\left\{\frac{\xi}{r_1}\gamma_{10}U_{r,1}^{(0)} + \sum_{n=0}^{\infty}\left[\gamma_{10}U_{r,0}^{(0)}\right]\frac{(r_1/2)^{2n+1}}{!n(n+1)!} + \xi\sum_{n=0}^{\infty}\eta_{4,n}(r_1)\left[\gamma_{10}U_{r,1}^{(0)} + H_1^{(0)}\frac{\partial U_{z,1}^{(0)}}{\partial z}\right]\frac{(r_1/2)^{2n+1}}{!n(n+1)!}\right\}$$

Oxirgi (2.2.10) formula qatlamning suyuqlik egallagan bo'shliqlarida suyuqlik bosimini aniqlashga imkon beradi.

2.3. Qovushoq-elastiklik xususiyati va o'rab turuvchi muhit hisobga olingan holda transversal-izotrop silindrik qatlamning bo'ylama-radial tebranishlari umumiy tenglamalari

Faraz qilaylik qatlam ichida suyuqlik bo'lmasin ya'ni $\varphi \equiv 0$. Bu holda (2.1.15) tenglamalardan foydalanib doiraviy silindrik anizotrop qovushoq-elastik qatlam bo'ylama-radial tebranishlari tenglamalarini chiqarish mumkin. Bu holda (2.1.15) quyidagi ko'rinishga keladi

$$a_{1i}U_{z,0}^{(0)}+b_{1i}U_{r,0}^{(0)}+\xi\left(c_{1i}U_{z,1}^{(0)}+d_{1i}U_{r,1}^{(0)}\right)=0;\quad (i=1,2)$$

$$a_{21}U_{z,0}^{(0)}+b_{21}U_{r,0}^{(0)}+\xi\left(c_{21}U_{z,1}^{(0)}+d_{21}U_{r,1}^{(0)}\right)=0;\qquad (2.3.1)$$

$$\left(b_{22}+R_1q_{12}\right)U_{r,0}^{(0)}-\left(a_{22}+R_1q_{22}\right)U_{z,0}^{(0)}-\xi\left(d_{22}+R_1q_{32}\right)U_{r,1}^{(0)}+\xi\left(c_{22}-R_1q_{42}\right)U_{z,1}^{(0)}=F,$$

Bu esa doiraviy silindrik transversal-izotrop qatlamning qovushoq-elastiklik xususiyati va o'rab turuvchi muhit hisobga olingan bo'ylama-radial tebranishlari tenglamalaridir

Bu yerda a_{ji}, b_{ji}, c_{ji}, d_{ji} $(i,j=1,2)$ va q_{12}, q_{22}, q_{32}, q_{42} operatorlar quyidafi formulalar bilan ifodalanadilar

$$a_{1i}=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[P_{n+1}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$b_{1i}=-\sum_{n=0}^{\infty}\left[P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\gamma_{20}^2\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$c_{1i}=-\sum_{n=0}^{\infty}\eta_{4,n}(r_i)\gamma_{10}\left[P_{n+2}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_i}\gamma_{10}H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z};$$

$$d_{1i}=-\sum_{n=0}^{\infty}\eta_{4,n}(r_i)\gamma_{10}\left[\gamma_{10}P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+H_1^{(0)}\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\gamma_{20}^2\right]\times$$
$$\frac{(r_i/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_j}H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2};$$

$$a_{1i}=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[\left(H_1^{(0)}G_{11}^{(0)}-G_{13}^{(0)}\right)P_n^{(0)}+\gamma_{10}G_{13}^{(0)}P_{n-1}^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\right]\frac{(r_i/2)^{2n}}{(n!)^2};$$

$$b_{2i}=\sum_{n=0}^{\infty}\left[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)P_n^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\right]\times$$
$$\times\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)\frac{(r_i/2)^{2n}}{(n!)^2};$$

$$c_{2i} = \sum_{n=0}^{\infty}\Big[H_1^{(0)}G_{11}^{(0)}\big(\gamma_{10}P_{n+1}^{(0)} - P_{n+2}^{(0)}\big) + G_{13}^{(0)}\big(\gamma_{10}^2 - \gamma_{10}\gamma_{30} + \gamma_{20}^2\big)P_n^{(0)} +$$

$$\frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r_i)\frac{\partial}{\partial z}P_{n+1}^{(0)}\frac{(r_i/2)^{2n}}{(n!)^2} + \frac{1}{r_i}\gamma_{10};$$

$$d_{2i} = \sum_{n=0}^{\infty}\gamma_{10}\Big[\eta_{3,n}(r_i)\Big[H_1^{(0)}G_{11}^{(0)}\big(\gamma_{10}P_{n+1}^{(0)} - \gamma_{20}^2P_{n+2}^{(0)}\big) + G_{13}^{(0)}\big(\gamma_{10}^2 - \gamma_{10}\gamma_{30} + \gamma_{20}^2\big)P_{n+1}^{(0)}\Big] +$$

$$\frac{H_1^{(0)}(G_{12}^{(0)} - G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r_i)\big(\gamma_{10}P_{n+1}^{(0)} - \gamma_{20}^2P_{n+2}^{(0)}\big)\Big]\frac{(r_i/2)^{2n}}{(n!)^2}; \qquad (i = 1,2)$$

$$q_{12} = \sum_{n=0}^{\infty}\big(\gamma_{10}P_{n+1}^{(0)} - \gamma_{20}^2P_{n+2}^{(0)}\big)\frac{(r_2/2)^{2n+1}}{n!(n+1)!}; \qquad q_{22} = \sum_{n=0}^{\infty}\frac{\partial}{\partial z}\big(H_1^{(0)}\gamma_{20}^2P_n^{(0)}\big)\frac{(r_2/2)^{2n+1}}{n!(n+1)!}; \quad (2.3.2)$$

$$q_{32} = \frac{1}{r_2}\gamma_{10} + \sum_{n=0}^{\infty}\gamma_{10}\eta_{4,n}(r_2)\big(\gamma_{10}P_{n+1}^{(0)} - \gamma_{20}^2P_{n+2}^{(0)}\big)\frac{(r_2/2)^{2n+1}}{n!(n+1)!};$$

$$q_{42} = \sum_{n=0}^{\infty}\gamma_{10}\eta_{3,n}(r_2)\frac{\partial}{\partial z}\big(H_1^{(0)}\gamma_{20}^2P_n^{(0)}\big)\frac{(r_2/2)^{2n+1}}{n!(n+1)!};$$

$$\gamma_{10}^n = G_{11}^{(0)^{-n}}\left[\rho_0\frac{\partial^2}{\partial t^2} - G_{44}^{(0)}\frac{\partial^2}{\partial z^2}\right]^n; \qquad \gamma_{20}^{2n} = G_{11}^{(0)^{-n}}\left[\rho_0^2G_{44}^{(0)^{-1}}\frac{\partial^4}{\partial t^4} - \rho_0\big(1 + G_{33}^{(0)}G_{44}^{(0)^{-1}}\big)\frac{\partial^4}{\partial z^2\partial t^2} + G_{33}^{(0)}\frac{\partial^4}{\partial z^4}\right]^n;$$

$$\gamma_{30}^n = \left\{\rho_0\big(G_{11}^{(0)^{-1}} + G_{44}^{(0)^{-1}}\big)\frac{\partial^2}{\partial t^2} - \Big[G_{44}^{(0)}G_{11}^{(0)^{-1}} + G_{44}^{(0)^{-1}}G_{33}^{(0)} + G_{44}^{(0)^{-1}}G_{11}^{(0)^{-1}}\big(G_{33}^{(0)} + G_{44}^{(0)}\big)^2\Big]\frac{\partial^2}{\partial z^2}\right\}^n; \qquad (2.2.3)$$

Ushbu (2.3.1) tenglamalar (2.3.2) va (2.2.3) larni hisobga olganda doiraviy silindrik transversal-izotrop qatlamning qovushoq-elastiklik xususiyati va o’rab turuvchi muhit hisobga olingan bo’ylama-radial tebranishlari umumiy integro-differensial tenglamalaridir. Bu tenglamalar u yoki bu ko’rinishda amaliy masalalarni yechishga qo’llanilishi mumkin.

2.4. **Ideal suyuqlik va deformatsiyalanuvchi muhit bilanta’sirlashuvchi anizotrop silindrik qatlamning bo’ylama-radial umumiy tenglamalarining limitik va xususiy hollari**

Yuqorida, 2.1. paragrafda olingan doiraviy silindrik anizotrop qatlamning bo’ylama-radial tebranishlari tenglamalari (2.1.15) sistemasi, ta’kidlanganidek juda umumiydir. Ushbu paragraph doirasida bu tenglamalarning ba'zi xususiy va limitik hollari hamda tebranishlar taqribiy tenglamalarini qarab chiqamiz.

2.4.1. Asosiy (2.1.15) tenglamalar sistemasining limitik hollari

a) Anizotrop qovushoq-elastik sterjen.

Asosiy (2.1.15) tenglamalar sistemasida $\varphi = 0,\ r_1 = 0$ deb hisoblabdeformatsiyalanuvchi muhitda joylashgan, doiraviy ko'ndalang kesimli, qovushoq-elastik , anizotrop sterjen simmetriya o'qi nuqtalarining $U_z^{(0)}$ va $U_r^{(0)}$ ko'chishlari $U_{z,0}^{(0)}$ va $U_{r,0}^{(0)}$ bosh qismlariga nisbatan ikkita tenglama sistemasiga ega bo'lamiz

$$\begin{aligned} &a_{12}U_{z,0}^{(0)} + b_{12}U_{r,0}^{(0)} = 0, \\ &\left(b_{22} - R_1 q_{12}\right)U_{r,0}^{(0)} - \left(a_{22} + R_1 q_{22}\right)U_{z,0}^{(0)} = F; \end{aligned} \tag{2.4.1}$$

bu yerda a_{i2}, b_{i2} va q_{i2} $(i = 1,2)$ operatorlar (2.1.16) formulalar bilan aniqlanadilar.

b) Anizotrop qovushoq-elastik yupqa devorli silindrik qobiq.

Endi $r_2 = r_1(1+\varepsilon)$, bunda $\varepsilon > 0$ kichik parametr, deb hisoblab (2.1.15) tenglamalar sistemasidan doiraviy, anizotrop, qovushoq-elastik, yupqa devorli silindrik qobiqning bo'ylama-radial tebranishlari tenglamalarini olamiz. Bu holda $\ln\dfrac{r_i}{\xi} = 0,$ deb hisoblash zarur. Demak, $\eta_{3,n}(r)$ va $\eta_{4,n}(r)$ lar uchun quyidagi ifodakarni olamiz

$$\eta_{3,n}(r) = -\sum_{k=1}^{n}\frac{1}{k}; \qquad \eta_{4,n}(r) = -\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{2(n+1)}; \tag{2.4.2}$$

Bu qiymatlar (2.1.15) tenglamalar sistemasini hamda ko'chishlar va kuchlanishlar uchun formulalarni juda soddalashtiradi.

c) Asosiy (2.1.15) sistemadan foydalanib anizotrop qovushoq elastic qatlaning bo'ylama-radial tebranishlari tenglamalarini olish mumkin. Buning uchun можно $R_1 = 0$ va $\varphi = 0$ deb olish zarur. U holda (2.1.15) tenglamalar sistemasi quyidagi ko'rinishga keladi

$$a_{1i}U_{z,0}^{(0)}+b_{1i}U_{r,0}^{(0)}+\xi\left(c_{1i}U_{z,1}^{(0)}+d_{1i}U_{r,1}^{(0)}\right)=H_1^{(0)}G_{44}^{(0)^{-1}}\gamma_{10}\left[\frac{\partial}{\partial z}P_{rz}^{(i)}\right];\ (i=1,2)$$
$$a_{21}U_{z,0}^{(0)}+b_{21}U_{r,0}^{(0)}+\xi\left(c_{21}U_{z,1}^{(0)}+d_{21}U_{r,1}^{(0)}\right)=H_1^{(0)}\gamma_{10}\left[P_r^{(i)}\right];\qquad(2.4.3)$$

Bu esa doiraviy silindrik anizotrop qatlamning bo'ylama-radial tebranishlari tenglamasidir. Bu yerda $a_{ji}, b_{ji}, c_{ji}, d_{ji}, q_k, \gamma_{10}^n, \gamma_{20}^{2n}$ *va* γ_{30}^n (i,j=1,2) operatorlar quyidagicha aniqlanadi

$$a_{1i}=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[P_{n+1}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$b_{1i}=-\sum_{n=0}^{\infty}\left[P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\gamma_{20}^2\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$c_{1i}=-\sum_{n=0}^{\infty}\eta_{4,n}(r_i)\frac{\partial}{\partial z}\gamma_{10}H_1^{(0)}\left[P_{n+2}^{(0)}-\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}\right]\frac{(r_i/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_i}\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z};$$

$$d_{1i}=-\sum_{n=0}^{\infty}\eta_{4,n}(r_i)\gamma_{10}\left[\gamma_{10}P_{n+2}^{(0)}-\left(\gamma_{10}^2+\gamma_{20}^2+H_1^{(0)}\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_{n+1}^{(0)}+\left(\gamma_{10}+H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)P_n^{(0)}\gamma_{20}^2\right]\times$$
$$\frac{(r_i/2)^{2n+1}}{n!(n+1)!}+\frac{1}{r_j}H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2};$$

$$a_{1i}=\sum_{n=0}^{\infty}H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\left[\left(H_1^{(0)}G_{11}^{(0)}-G_{13}^{(0)}\right)P_n^{(0)}+\gamma_{10}G_{13}^{(0)}P_{n-1}^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\right]\frac{(r_i/2)^{2n}}{(n!)^2};$$

$$b_{2i}=\sum_{n=0}^{\infty}\left[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)P_n^{(0)}+\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\right]\times$$

$$\times\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)\frac{(r_i/2)^{2n}}{(n!)^2};\qquad(2.4.4)$$

$$c_{2i}=\sum_{n=0}^{\infty}\Big[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)P_n^{(0)}+$$

$$\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r_i)\frac{\partial}{\partial z}P_{n+1}^{(0)}\frac{(r_i/2)^{2n}}{(n!)^2}+\frac{1}{r_i}\gamma_{10};$$

$$d_{2i}=\sum_{n=0}^{\infty}\gamma_{10}\Big[\eta_{3,n}(r_i)\Big[H_1^{(0)}G_{11}^{(0)}\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)+G_{13}^{(0)}\left(\gamma_{10}^2-\gamma_{10}\gamma_{30}+\gamma_{20}^2\right)P_{n+1}^{(0)}\Big]+$$

$$\frac{H_1^{(0)}(G_{12}^{(0)}-G_{11}^{(0)})}{2(n+1)}\eta_{4,n}(r_i)\left(\gamma_{10}P_{n+1}^{(0)}-\gamma_{20}^2P_{n+2}^{(0)}\right)\Big]\frac{(r_i/2)^{2n}}{(n!)^2};\qquad(i=1,2)\tag{2.4.5}$$

$$\gamma_{10}^n=G_{11}^{(0)^{-n}}\left[\rho_0\frac{\partial^2}{\partial t^2}-G_{44}^{(0)}\frac{\partial^2}{\partial z^2}\right]^n;\qquad \gamma_{20}^{2n}=G_{11}^{(0)^{-n}}\left[\rho_0^2G_{44}^{(0)^{-1}}\frac{\partial^4}{\partial t^4}-\rho_0\left(1+G_{33}^{(0)}G_{44}^{(0)^{-1}}\right)\frac{\partial^4}{\partial z^2\partial t^2}+G_{33}^{(0)}\frac{\partial^4}{\partial z^4}\right]^n;$$

$$\gamma_{30}^n=\left\{\rho_0\left(G_{11}^{(0)^{-1}}+G_{44}^{(0)^{-1}}\right)\frac{\partial^2}{\partial t^2}-\left[G_{44}^{(0)}G_{11}^{(0)^{-1}}+G_{44}^{(0)^{-1}}G_{33}^{(0)}+G_{44}^{(0)^{-1}}G_{11}^{(0)^{-1}}\left(G_{33}^{(0)}+G_{44}^{(0)}\right)^2\right]\frac{\partial^2}{\partial z^2}\right\}^n;$$

2.5. Tebranishlar taqribiy tenglamalari (n=0)

Asosiy (2.1.15) tenglamalar sistemasi tashqi deformatsiyalanuvchi muhit va ichki siqiluvchi suyuqliq bilan o’zaro ta’sirlashuvchi doiraviy qovushoq-elastik transversal izotrop qatlamning bo’ylama-radial tebranishlari umumiy tenglamalaridir.Bu tenglamalar qatlamning radiusi ξ bo’lgan oraliq sirti nuqtalarining U_z, U_r ko’chishlari bosh qismlariga nisbatan chiqarilgan. Yuqoridagi (2.1.16) va (2.1.17) formulalar bilan aniqlanuvchi $a_{ji}, b_{ji}, c_{ji}, d_{ji}, q_k, \gamma_{10}^n, \gamma_{20}^{2n}$ *va* γ_{30}^n operatorlarning ko’rinishlaridan kelib chiqqan holda, (2.1.15) tenglamalar sistemasi hosilalar bo’yicha cheksiz yuqori tartibga ega. Shining uchun ham bu tenglamalarning chap tomonlaridagi hadlar sonini chegaralashga to’g’ri keladi.

Shu aytilganlardan kelib chiqan holda (2.1.15) tenglamalar sistemasida nolinchi yaqinlashish bilan chegaralanamiz va olamiz

$$\frac{r_1}{2}\left[H_1^{(0)}\gamma_{20}^2\right]\frac{\partial U_{z,0}^{(0)}}{\partial z}+\left[\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right]\frac{r_1}{2}U_{r,0}^{(0)}+\xi\left\{\left[\left(\ln\frac{r_1}{\xi}-\frac{1}{2}\right)H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z}\left(\gamma_{30}-\gamma_{10}-H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)\times\right.\right.$$

$$\left.\left.\frac{r_1}{2}+\frac{1}{r_1}H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z}\right]U_{z,1}^{(0)}+\left[\left(\ln\frac{r_1}{\xi}-\frac{1}{2}\right)\gamma_{10}\left(\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right)\right]\frac{r_1}{2}+\frac{1}{r_1}H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right]U_{r,1}^{(0)}\right\}=0;$$

$$\frac{r_2}{2}\left[H_1^{(0)}\gamma_{20}^2\right]\frac{\partial U_{z,0}^{(0)}}{\partial z}+\left[\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right]\frac{r_2}{2}U_{r,0}^{(0)}+\xi\left\{\left[\left(\ln\frac{r_2}{\xi}-\frac{1}{2}\right)H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z}\left(\gamma_{30}-\gamma_{10}-H_1^{(0)}\frac{\partial^2}{\partial z^2}\right)\times\right.\right.$$

$$\left.\left.\frac{r_2}{2}+\frac{1}{r_2}H_1^{(0)}\gamma_{10}\frac{\partial}{\partial z}\right]U_{z,1}^{(0)}+\left[\left(\ln\frac{r_2}{\xi}-\frac{1}{2}\right)\gamma_{10}\left(\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right)\right]\frac{r_2}{2}+\frac{1}{r_2}H_1^{(0)}\gamma_{10}\frac{\partial^2}{\partial z^2}\right]U_{r,1}^{(0)}\right\}=0;$$

$$-\left[H_1^{(0)}\left(G_{13}^{(0)}\gamma_{10}\frac{\partial}{\partial z}+Rq_{11}\right)\right]U_{z,0}^{(0)}+H_1^{(0)}\left(G_{11}^{(0)}(\gamma_{10}-\gamma_{30})+\frac{G_{12}^{(0)}-G_{11}^{(0)}}{2}\gamma_{10}-Rq_{21}\right)U_{r,0}^{(0)}+\xi\gamma_{10}H_1^{(0)}*$$

$$\left\{\left[\ln\frac{r_1}{\xi}\frac{\partial}{\partial z}\left(G_{11}^{(0)}-G_{13}^{(0)}\right)+\left(\ln\frac{r_1}{\xi}-\frac{1}{2}\right)\frac{\partial}{\partial z}\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}+\frac{1}{r_1}\gamma_{10}-Rq_{31}\right]U_{z,1}^{(0)}+\left[\ln\frac{r_1}{\xi}\left(G_{11}^{(0)}\gamma_{10}\right)+\right.\right.$$

$$\left.\left.+\left(\ln\frac{r_1}{\xi}-\frac{1}{2}\right)\frac{G_{12}^{(0)}-G_{11}^{(0)}}{2}\frac{1}{r_1}\gamma_{10}-Rq_{41}\right]U_{r,1}^{(0)}\right\}=0; \qquad (2.5.1)$$

$$\left\{\left[H_1^{(0)}G_{11}^{(0)}(\gamma_{10}-\gamma_{30})+\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\gamma_{10}+Rq_{22}\right]\right\}U_{r,0}^{(0)}+\left\{\left[-H_1^{(0)}G_{13}^{(0)}\gamma_{10}\frac{\partial}{\partial z}\right]-Rq_{12}\right\}U_{z,0}^{(0)}+$$

$$+\xi\left\{\left[\ln\frac{r_2}{\xi}\left(G_{11}^{(0)}H_1^{(0)}\gamma_{10}^2\right)+\left(\ln\frac{r_2}{\xi}-\frac{1}{2}\right)\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\gamma_{10}^2\right]-R_1q_{42}\right\}U_{r,1}^{(0)}+\xi\left\{\left[\ln\frac{r_2}{\xi}\frac{\partial}{\partial z}\left(G_{11}^{(0)}-G_{13}^{(0)}\right)H_1^{(0)}\gamma_{10}+\right.\right.$$

$$+\left(\ln\frac{r_2}{\xi}-\frac{1}{2}\right)\frac{\partial}{\partial z}\frac{H_1^{(0)^2}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\gamma_{10}+\frac{1}{r_2}-R_1q_{32}\Bigg\}U_{z,1}^{(0)}=F;$$

Bu yerda

$$F=H_1^{(0)}\left[P_r^{(2)}\right]-R_1\left[P_0\right]-R\left[\frac{\partial P_{rz}^{(2)}}{\partial z}\right]$$

$$q_{1i}=\gamma_{10}\frac{r_i}{2};\quad q_{2i}=0\quad q_{3i}=\frac{1}{r_i}\gamma_{10}+\gamma_{10}^2\eta_{4,n}(r_i)\frac{r_i}{2};\ \ q_{4,i}=\gamma_{10}\eta_{3,n}(r_i)\frac{\partial}{\partial z}\frac{r_i}{2};\ (i=1,2)$$

$$a_{1i}=\left[H_1^{(0)}\gamma_{20}^2\frac{\partial}{\partial z}\right]\frac{r_i}{2};\ b_{1i}=-\left[\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z^2}\right]\frac{r_i}{2};$$

$$c_{1i}=\left(\ln\frac{r_i}{\xi}-\frac{1}{2}\right)\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z}\left[\gamma_{30}-\gamma_{10}-H_1^{(0)}\frac{\partial}{\partial z^2}\right]\frac{r_i}{2}+\frac{1}{r_i}\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z};\qquad (2.5.2)$$

$$d_{1i}=\left(\ln\frac{r_i}{\xi}-\frac{1}{2}\right)\gamma_{10}\frac{\partial}{\partial z}\left[\gamma_{10}\gamma_{30}-\gamma_{10}^2-\gamma_{20}^2-\gamma_{10}H_1^{(0)}\frac{\partial}{\partial z^2}\right]\frac{r_i}{2}-\frac{1}{r_i}\gamma_{10}H_1^{(0)}\frac{\partial^2}{\partial z^2};$$

$$a_{2i}=H_1^{(0)}G_{13}^{(0)}\gamma_{10}\frac{\partial}{\partial z};\ b_{2i}=H_1^{(0)}G_{11}^{(0)}(\gamma_{10}-\gamma_{30})+\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\gamma_{10};$$

$$c_{2i}=\gamma_{10}\left\{\ln\frac{r_i}{\xi}\frac{\partial}{\partial z}H_1^{(0)}\left[G_{11}^{(0)}-G_{13}^{(0)}\right]+\left(\ln\frac{r_i}{\xi}-\frac{1}{2}\right)\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\frac{\partial}{\partial z}\right\};$$

$$d_{2i}=\gamma_{10}\left\{\ln\frac{r_i}{\xi}H_1^{(0)}G_{11}^{(0)}\gamma_{10}+\left(\ln\frac{r_i}{\xi}-\frac{1}{2}\right)\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\gamma_{10}+\frac{H_1^{(0)}\left(G_{12}^{(0)}-G_{11}^{(0)}\right)}{2}\frac{1}{r_i}\right\};$$

qaysiki

$$\gamma_{10}^n=G_{11}^{(0)^{-n}}\left[\rho_0\frac{\partial^2}{\partial t^2}-G_{44}^{(0)}\frac{\partial^2}{\partial z^2}\right]^n;$$

$$\gamma_{20}^{2n}=G_{11}^{(0)^{-n}}\left[\rho_0^2G_{44}^{(0)^{-1}}\frac{\partial^4}{\partial t^4}-\rho_0\left(1+G_{33}^{(0)}G_{44}^{(0)^{-1}}\right)\frac{\partial^4}{\partial z^2\partial t^2}+G_{33}^{(0)}\frac{\partial^4}{\partial z^4}\right]^n;$$

$$\gamma_{30}^n=\left\{\rho_0\left(G_{11}^{(0)^{-1}}+G_{44}^{(0)^{-1}}\right)\frac{\partial^2}{\partial t^2}-\left[G_{44}^{(0)}G_{11}^{(0)^{-1}}+G_{44}^{(0)^{-1}}G_{33}^{(0)}+G_{44}^{(0)^{-1}}G_{11}^{(0)^{-1}}\left(G_{33}^{(0)}+G_{44}^{(0)}\right)^2\right]\frac{\partial^2}{\partial z^2}\right\}.$$

Oxirgi tenglamalar sistemasi tashqi deformatsiyalanuvchi muhit va ichki siqiluvchi suyuqliq bilan o’zaro ta’sirlashuvchi doiraviy qovushoq-elastik transversal izotrop qatlamning bo’ylama-radial tebranishlari taqribiy tenglamalaridir. Bu tenglamalar o’zlarining tarkiblarida Aylanish inersiyasi, ko’ndalag siljish deformatsiyasi va qatlam va ta’sirlashuvchi muhitlar materiallarining boshqa xususiyatlarini hisobga oluvchi qo’shiluvchilarni saqlaydi.

ASOSIY XULOSALAR

Dissertatsiya ishi doirasida olingan natijalar quyidagi xulosalarni chiqarishga imkon beradi :

- tashqi deformatsiyalanuvchi muhit va ichki ideal suyuqliq bilan o’zaro ta’sirlashuvchi doiraviy qovushoq-elastik transversal-izotrop qatlamning bo’ylama-radial tebranishlari haqidagi masala shakllantirildi (qo’yildi);

- ichi ideal suyuqlik to’ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta’sirlashuvchi doiraviy silindrik transversal-izotrop qatlamning bo’ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarildi;

- qatlam uchun olingan umumiy tenglamalarning xususiy va limitik hollarini tadqiq qilish, jumladan, ichiga ideal suyuqlik to’ldirilgan va tashqi deformatsiyalanuvchi muhit bilan nostatsionar ta’sirlashuvchi doiraviy silindrik transversal-izotrop qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalari olindi, xususan natijalar quyidagi sistemalarni o’z ichiga oladi:

a) qovushoq-elastik anizotrop sterjen;

b) qovushoq-elastik silindrik anizotrop yupqa devorli qobiq;

c) elastic izotrop sterjen;

d) elastic anizotrop sterjen;

e) elastic izotrop va anizotrop silindrik yupqa devorli qobiq; va h.k.

-tebranish tenglamalar bilan bir qatorda anizotrop qatlam yoki transversal-izotrop qatlamning istalgan kesimi nuqtalaridagi kuchlanganlik-deformatsiyalanganlik holatini talab etilgan aniqlikda hisoblashga imkon beruvchi algoritm ishlab chiqildi;

- muhandislik amaliyotida qo’llash mumkin bo’lgan taqribiy tenglamalar va amaliy masalalarning boshlang’ich va chegaraviy shartlari topildi;

-ichida ideal suyuqlik to’ldirilgan doiraviy silindrik transversal-izotrop qatlam va qobiqlarning bo’ylama-radial tebranishlari umumiy tenglamalarining yuqoridagi sistemalar tebranishlari haqidagi masalalarni yechishga qollash mumkinligi, hamda olingan natijalar tebranish jarayonidagi kuchlanganlik-deformatsiyalanganlik holatlarini aniqlashda ishlatilishi mumkinligi ko’rsatib berildi;

- olingan taqribiy tenglamalarni boshqa masalalarni, xususan yuqorida qaralgan doiraviy silindrik anizotrop qatlam, ideal suyuqlik va o’rab turuvchi deformatsiyalanuvchi muhitdan iborat sistemning majburiy tebranishlari haqidagi masalalarni ham yechishga qo’llash mumkin.

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