O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

Qo’lyozma huquqida

539.3

BEGJANOV AMIRBEK

AYLANISH INERSIYASI VA KO’NDALANG SILJISH DEFORMATSIYASINI HISOBGA OLGANDA BALKANING BO’YLAMA MAJBURIY TEBRANISHLARI.

5A140302 – Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi

Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi bo’yicha magistr darajasini olish uchun

MAGISTRLIK DISSERTATSIYASI

Ish ko’rib chiqildi va himoyaga
ruxsat berildi.
«Mexanika» кafedrasi mudiri
Dotsent Sh.Berdiyev ____________
Mexanika-matematika fakul’teti dekani
dots. A.Soleev _____________
«____»_____________ 2014y.

Ilmiy rahbar:
prof.X.Xudoynazarov ___________
Ilmiy maslahatchi:
Sh.Burqutboyev________

Samarqand – 2014

M U N D A R I J A

KIRISH

Hozirgi zamon texnikasining juda tez sur'atlar bilan rivojlanishi deformatsiyalanuvchi qattiq jismlar mexanikasi oldiga yangidan-yangi, vaqt o'tgan sari tobora murakkablashib borayotgan masalalarni qo'ymoqda. Shu paytgacha ishlatib kelingan an'anaviy materiallar yuqori bosimli va yuqori haroratli o'ta murakkab sharoitlarda ishlatilmoqda, yangi-yangi materiallar har xil yuqori haroratlarga chidamli qotishmalar, kompozit materiallar, o'ta mustahkam va yaroqli modulli tolalar amaliyotda qo'llanilmoqda.

Bunday o'zgarishlar jismning elastik modeli bilan bir qatorda, deformatsiyalanuvchi qattiq jismning boshqa, mukammaliroq modellarini ham yaratishga, muhandislik qurilmalari hisobida ishlab chiqilganiga ancha bo'lgan bo'lsada shu vaqtgacha foydalanilmagan usullardan, xususan plastiklik, qovushoq-elastiklik, polzuchest nazariyalari usullaridan foydalanishga olib kelmoqda. Bu masalalar ayniqsa jism nuqtalarida yuzaga keladigan kuchlanishlar statistik va ehtimollar nazariyalari usullaridan foydalanishga to'g'ri kelganida yaqqol namoyon bo'ladi.

Keyingi bir necha o'n yillik davomida deformatsiyalanuvchi qattiq jismlar mexanikasining "yemirilish mexanikasi", "kompozit materiallar mexanikasi", "Nanomexanika" va shu kabi qator yangi yo'nalishlar paydo bo'lishini ham ana shu yangi talablar taqozo qildi. Ushbu yo'nalishlar rivoji ham, eng avvalo elastik jism modeliga tayanadi.

Dissertatsiya ishida tadqiqot predmeti muhandislik qurilmalari elementlarining tarkibiy qismi bo'lgan doiraviy balkalarda, ularga berilgan tashqi ta'sir natijasida vujudga keladigan tebranishlarni o'rganish. Tebranishlar nostatsionar harakterga ega bo'lgan hollarda bunday elementlarda paydo bo'ladigan nostatsionar to'lqinlar tarqalish jarayonlarini, ularning o'zlariga xos xususiyatlarini hisobga olgan holda o'rganish va tadqiq qilishdan iborat. Nostatsionar tebranishlarni o'rganishda ularning xususiy chastotalarini topish, xususiy amplitudalarini aniqlash va tebranish shakllarini topish masalalarini qo'yish ularni hal qilish va ilmiy xulosalar chiqarish.

Erkin tebranishlarning topilgan fizik-mexanik xarakteristikalaridan nostatsionar tebranishlar biror vaqt davomida ta'sir etuvchi tashqi dinamik ta'sir natijasida uyg'otilgan hollar uchun tadbiq etish, ulardan foydalana bilish ham dissertatsiya ishining predmetini tashkil etadi.

Dissertatsiya ishda tadqiqot ob'ektietib, yuqorida aytilganlardan kelib chiqqan holda, ko'ndalang kesimi doiraviy bo'lgan, uzunligi chekli, elastikmaterialdan yasalgan balkadan iboratdir.Bunda, balkaning bo'ylama tebranishlarida vujudga keladigan kuchlanganlik-deformatsiyalanganlik holatini balka materialiningmurakkab xususiyatlarini hisobga olgan holda aniqlash.

Mavzuning dolzarbligi Doiraviybalka va sterjenlar juda ko'p va xilma-xil muhandislik qurilmalarining tarkibiy qismlarini tashkil etadilar. Bundan tashqari bunday balkalar ko'plab mashina va mexanizmlarning elementlari hamdir. Shunday holda bu balkalar turli xil dinamik tashqi ta'sirlar ostida ishlaydilar va ularning kesmlarida turli xil yuklanishlar vujudga keladi. Balkalarvasterjenlardagi ana shunday yuklanishlarni aniqlash masalasi deformatsiyalanuvchi qattiq jismlar mexanikasining dolzarb masalalaridan hisoblanadi ana shunday yuklar ta'siri ostidagi balkalar kesmlaridagi kuchlanganlik-deformatsiyalanganlik holatlarini aniqlash.Ammo, balkalar nuqtalaridagi, tashqi dinamik ta'sirlar natijasida vujudga keladigan kuchlanganlik-deformatsiyalanganlik holatlarini aniqlash analitik usullar bilan topish hamma vaqt ham mumkin bo'lavermaydi [1]. Bunday holda masalani yechish uchun sonli usullardan foydalanishga to'g'ri keladi. Shu nuqtai nazardan qaraganda dissertatsiya ishida qo'yilgan masala zamonaviy dolzarb masalalar qatoriga kiradi.

Bundan tashqari masalalarni yechish uchun ularning matematik modelini yaratishda asosiy rol o'ynovchi tebranish tenglamalarini [1] ishning natijalaridan foydalanib keltirib chiqaramiz. Mana shu aytib o'tilgan hisobga olingan holda dissertatsiya ishining maqsad va vazifalari belgilangan.

Ishning maqsad va vazifalari Mazkur magistrlik dissertatsiya ishining asosiy maqsadi elastik balkalarning ko'ndalang kesimi doiraviy bo'lgan hol uchun nostatsionar tebranishlarini tadqiq qilish. Bunda tadqiqotni klassik va aniqlashtirilgan tebranish tenglamalari asosida olib borish va tenglamalarni analitik usullar yordamida yechish talab etiladi. Ana shulardan kelib chiqqan holda dissertatsiya ishining asosiy vazifalari qilib quyidagilar belgilangan:

1. Elastiklik nazariyasi asosiy munosabatlarini o'rganish;
2. Ko'ndalang kesimi doiraviy balkalarning bo'ylama-radial tebranishlari umumiy tenglamalarini keltirib chiqarish va undan xususiy hollarda klassik va aniqlashtirilgan tenglamalarni keltirish;
3. Differensial tenglamalarni yechishning analitik usullarini, xususan, o'zgarmaslarni variatsiyalash, operatsion hisob usullaridan Laplas almashtirishlarini o'rganish va amaliy masalalar yechishga tadbiq etish;
4. Amaliy masalalar yechish;
5. Olingan natijalar asosida ilmiy xulosalar chiqarish.

Muammoning ishlab chiqilish darajasi. Doiraviy elastik sterjenlarda bo'ylama to'lqinlar tarqalishini tadqiq etish masalasi bilan juda ko'p olimlar ancha qadim zamonlardan buyon (Bernulli, Eyler) shug'illanishadi. Ammo, fan va texnikaning hozirgi zamon taraqqiyot darajasi moddalarning yangi-yangi xususiyatlarini, jumladan ularning reologik, anizotropik temperaturaviy va h.k. xususiyatlarini hisobga olgan holda tadqiqotlar o'tkazishni talab etmoqda. Bundan tashqari balkalardagi bo'ylama to'lqinlar tarqalishi masalasi aniqlashtirilgan tenglamalar asosida nisbatan kam miqdordagi ilmiy ishlarda tadqiq etilgan.

Tadqiqotning ilmiy ahamiyati. Doiraviy balkalarvasterjenlarning bo'ylama tebranishlari aksariyat hollarda analitik yechimlar asosida tadqiq qilingan. Balkalarvasterjenlarning bo'ylama tebranishlari haqidagi masalalarni tadqiq etish masalasi keyingi vaqtlarda katta ilmiy va amaliy ahamiyatga ega bo'lmoqda. Shu nuqtai nazardan qaraganda dissertatsiya ishida qaralgan va yechilishi uchun analitik usullar tadbiq etilgan masalalarning ilmiy ahamiyati juda katta.

Tadqiqotning amaliy ahamiyati. Hozirgi zamon texnikasi, qurilish, yer osti va yer usti inshoatlari, aviatsiya, kemasozlik va boshqa juda ko'plab sohalarda ko'ndalang kesimi doiraviy bo'lgan balkalar muhandislik qurilmalarining asosiy elementlaridan biri sifatida ishlatiladi. Ekspluatatsiya jarayonida bunday balkalar intensiv va impulsiv dinamik yuklar ta'siri ostida bo'ladilar va juda ko'p hollarda ularning dinamik chidamlilik darajasini eksperimentdan emas, balki hisoblashlar yordamida aniqlashga to'g'ri keladi.

Yuqorida aytilganlar doiraviy elastik balkalarvasterjenlarning nostatsionar tebranishlarini tadqiq qilish, ularning tebranish chastotasi, amplitudasi, shakli va boshqa xarakteristikalarini aniqlash muhim amaliy ahamiyatga ega ekanligini ko'rsatadi. Dissertatsiya ishida qaralgan balkalarning bo'ylama tebranishlari haqida masalalar tadqiqotlari ham shunday tadqiqotlar jumlasiga kiradi va muhim amaliy ahamiyatga ega bo'lgan masalalardir.

Dissertatsiya ishining tuzilishi. Magistrlik dissertatsiya ishi kirish, ikkita bob, xulosa hamda foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib 70kompyuter sahifasida bayon qilingan. Ana shu hajm doirasiga 5ta rasmlar ham kiradi.

Dissertatsiya asosiy bo'limlarining qisqacha mazmuni. Dissertatsiya ishining kirish qismida ishning predmeti, va dolzarbligi tavsiflashga atroflicha to'xtalib o'tilgan. Ishning predmeti ishonarli dalillar asosida ko'rsatilgan va shu asosda uning ob'yekti konkretlashtirilgan.

Doiraviy elastik balkalar va sterjenlar bo'ylama tebranishlarini klassik va aniqlashtirilgan tenglamalar asosida tadqiq qilish zamonaviy mexanikaning o'ta dolzarb mavzularidan ekanligi isbotlangan. Dissertatsiya ishining maqsadi va bu maqsadni amalga oshirish uchun bajarilishi kerak bo'lgan vazifalar belgilab berilgan. Shunday vazifalardan beshtasi alohida ajratilgan va sanab o'tilgan. Ularning har biriga xarakteristika berilgan.

Dissertatsiya ishida hal qilinishi kerak bo'lgan muammo tahlil qilingan, uning shu vaqtgacha bo'lgan ishlab chiqilish darajasi aniqlangan. Olingan natijalarning ilmiy va amaliy ahamiyatlari sodda qilib tushintirib berilgan. Ilmiy ahamiyat amaliy ahamiyatdan ajratib ko’rsatilgan.Dissertatsiya ishining tuzilishi, qisqa bir necha satrlarda lo’nda qilib bayon qilingan.

Dissertatsiya ishining birinchi bobi “Doiraviy elastik balkaning nostatsionar bo’ylama tebranish tenglamalari”deb ataladi va o’z ichiga to’rtta paragrafni olgan.Dissertatsiya ishining ushbu bobi avvalo silindrik qatlamlar, qobiqlar va sterjenlar tipidagi konstruksiya elementlari hamda elastik va qovushoq-elastik jismlarning tutash muhit bilan o’zaro nostatsionar ta’sirlashuvi haqidagi ilmiy adabiyotlar sharhiga bag’ishlangan. Shuningdek, ushbu bob doirasida, deformatsiyalanuvchi qattiq jism uchun potensial funksiyalarda ifodalangan to’lqin tenglamalari keltirilgan.Doiraviy silindrik elastik sterjenlarning buralma nostatsionar tebranishlarining umumiy tenglamalari keltirib chiqarilgan. Olingan tenglamalar koordinata va vaqt bo’yicha cheksiz katta tartibga ega va undan xususiy hollarda taqribiy klassik va aniqlashtirilgan tenglamalarni olish mumkinligidan kelib chiqqan holda umumiy tenglamalardan, xususiy holda, klassik ya’ni ikkinchi tartibli tebranish tenglamasi hamda turli aniqlashtirilgan tenglamalar keltirib chiqarilgan.

Birinchi paragrafda doiraviy ko’ndalang kesimga ega bo’lgan silindrik qobiq va sterjenlarning nostatsionar (buralma, bo’ylama va ko’ndalang) tebranishlariga doir o’tkazilgan ilmiy tadqiqot ishlariga qisqacha sharh berilgan. Bunda asosiy e’tibor doiraviy yoki doiraviy bo’lmagan balkalar vasterjenlarga qaratilgan.Shunday masalalar sinfiga qaralayotgan muhandislik konstruksiyasini o’rab turgan yoki uning ichida joylashgan tutash muhitda tarqaluvchi akustik va nostatsionar to’lqinlar ta’siri ostidagi muhandislik konstruksiyalari elementlarning hisobi ham kiradi. Bunday masalalar umuman olganda tashqi dinamik yuklar ixtiyoriy bo’lgan holning hususiy holidir.Amalda tashqi yuklar faqat akustik yoki nostatsionar to’lqinlar ta’siridangina iborat bo’lmaydi.

Ikkinchi paragraf elastik jismning dinamik deformatsiyalangan holati va uni aniqlovchi munosabatlar deb nomlangan. Ma’lumki elastiklik nazariyasi kursidaelastik jism uchun asosiy tenglamalar va munosabatlar chiqarilgan. Bu tenglamalar va munosabatlar yopiq sistemani tashkil etadi va jismga ta’sir etgan tashqi kuchlarni hisobga olgan holda uning kuchlangan-deformatsialangan holatini aniqlashga imkon beradi. Bu tenglamalar odatda uch turga bo‘linadir: *geometrik, statik (dinamik) va fizik tenglamalar.* Paragraf doirasida dissertatsiya ishida zarur bo’ladigan tenglamalar va munosabatlarni avvalo tenzor yozuvi ko’rinishida hamda ularning silindrik koordinatalar sistemasidagi ko’rinishlarini keltirilgan.

Uchinchi paragraf“Doiraviy elastik sterjenning bo’ylama tebranishlari haqidagi masalaning qoyilishi”deb ataladi. Bir jinsli va izotrop, radiusi r_0 bo’lgan doiraviy silindrik elastik sterjenni silindrik koordinatalar sistemasidaqaraymiz. Tebranish tenglamalarini chiqarishda sterjenni uch o’lchovli jism deb hisoblay-miz.Ushbu dissertatsiya doirasida balkaning bo’ylama tebranishlari tadqiq etiladi.Balkaning yoki sterjenning bo’ylama tebranishlari uning o’qiga nisbatan simmetrik tebranish-lardan iboratdir.Shuning uchun sterjen nuqtalarining $\vec{U}(r,\theta,z,t)$ ko’chish vektori-ning faqat ikkita tuzuvchisi tadqiq qilingan.

To’rtinchi paragraf“Doiraviy elastik sterjenning bo’ylama tebranishlari tenglamalarini keltirib chiqarish umumiy usuli” deb ataladi. Tebranishlar tenglamalarini keltirib chiqarishning elastiklik dinamik nazariyasiga asoslangan usullari uchta asosiy yo’nalishlarga bo’linadilar. Dinamikada variatsion prinsiplardan foydalanishga asoslangan usullarni bu yo’nalishlarning birinchisini tashkil etadi deb hisoblash mumkin.Ikkinchi asosiy yo’nalishga elastik ko’chishlar komponentalarini qatorlarga (shu jumladan darajali qatorlarga) yoyishga asoslangan usullarni kiritish mumkin. Bu usulning statik masalalar uchun ishlab chiqilgan umumiy sxemasi birinchi marta N.A. Kilchevskiy [73] tomonidan taklif etilgan.

Uchinchi asosiy yo’nailishga elastiklik dinamik nazariyasining uch o’lchovli masalalarining aniq yechimlaridan foydalanish usuli kiradi. Ushbu dissertatsiya doirasida keltirib chiqariladigan tenglamalarning barchasi ana shu usul asosida olingan. Tenglamalardan tashqari, ushbu usul yordamida qaralayotgan sistema (bu holda doiraviy brus) kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatlarini bir qiymatli aniqlashga imkon beruvchi algoritm ham ishlab chiqilgan.

Dissertatsiya ishining ikkinchi bobi doiraviy elastik balka va sterjenlarning bo'ylama tebranishlarini uning aniqlashtirilgan tenglamasi asosida tatqiq etishga bag'ishlangan va "doiraviy elastik balkaning majburiy bo'ylama tebranishlarini aylanish inersiyasi va ko'ndalang siljish deformatsiyasini hisobga olib tadqiq etish" debnomlangan.Ko'ndalang kesimi doiraviysilindrik qobiqning bo'ylama tebranish umumiy tenglamalaridan, qobiqning ichki radiusi nolga teng bo'lgan xususiy holi uchun doiraviy qayishoq-elastik sterjenning umumiy tenglamalari keltirib chiqarilgan. Olingan umumiy tebranish tenglamalaridan xususiy hollarda klassik va aniqlashtirilgan tenglamalar keltirib chiqarilgan.Unda doiraviy elastik balkaning bo'ylama tebranishlari klassik va aniqlashtirilgan tenglamalari keltirib chiqarilgan. Shuningdek aniqlashtirilgan tenglamalarning turli xil ko'rinishlari muhokama etilgan. Aniqlashtirilgan tenglamalarning har bir hadi qanday mexanik ma'noga ega ekanligi ochib berilgan.Olingan tenglamalarning boshqa avtorlar tomonidan olingan shunday tenglamalar bilan taqqoslash natijalari bayon etilgan.uyuqlik bilan ta'sirlashuvchi doiraviy silindrik sterjenning bo'ylama tebranishlarihaqidagi masalavauzunligi bir tomonlama chegaralangan va bir uchi qistirib mahkamlangan doiraviy balkaning bo'ylama tebranishlarida uning kuchlangan-deformatsiyalangan holatini aniqlash haqidagi masala yechilgan.

Ushbu bobning *birinchi paragrafi* konkret amaliy masalalarni yechish uchun qo'llaniladigan tenglamalar qarab chiqilgan va"Doiraviy elastik balkaning bo'ylama tebranishlari klassik va aniqlashtirilgan tenglamalari"deb nomlangan.
Olingan tenglamalarning boshqa avtorlar tomonidan olingan shunday tenglamalar bilan taqqoslash natijalari bayon etilgan.Shuningdek aniqlashtirilgan tenglamalarning turli xil ko'rinishlari muhokama etilgan. Aniqlashtirilgan tenglamalarning har bir hadi qanday mexanik ma'noga ega ekanligi ochib berilgan.

Bobning *ikkinchi paragrafida* doiraviy sterjenning uchiga qo'yilgan zarba ta'sirida uyg'otilgan bo'ylama deformatsiyalanish jarayoni haqidagi masala, sterjen bilan ta'sirlashuvchi suyuqlikni hisobga olgan holda, yechilgan. Olingan sonli natijalar, ko'chishlar va kuchlanishlarning vaqtga va koordinataga bog'liq grafiklari ko'rinishida berilgan. Natijalar asosida xulosalar chiqarilgan.

Bobning *uchinchi paragrafida*tashqi yuklanish bo'lmagan holda, ya'ni doiraviy silindrik sterjenning sirti tashqi yuklardan xoli bo'lgan holda va Puasson koeffitsiyenti o'zgarmas bo'lganda, uzunligi bir tomonlama chegaralangan va bir uchi qistirib mahkamlangan doiraviy balkaning bo'ylama tebranishlarida uning kuchlangan-deformatsiyalangan holatini aniqlash haqidagi masala yechiladi.Bu yerda ham ko'chishlar (sterjenning uchidan har xil uzoqlikda yotuvchi kesimlarida) va kuchlanishlar grafiklari olingan sonli natijalar asosida qurilgan . Mumkin bo'lgan va yoki sodir bo'lishi mumkin bo'lgan ba'zi xususiy hollar qarab chiqilgan. Ilmiy xulosalar olingan.

I BOB

DOIRAVIY ELASTIK BALKA VA STERJENLARNING NOSTATSIONAR BO'YLAMA TEBRANISH TENGLAMALARI.

Disertatsiya ishining ushbu bobi avvalo silindrik qatlamlar, qobiqlar va sterjenlar tipidagi konstruksiya elementlari hamda elastik va qovushoq-elastik jismlarning tutash muhit bilan o'zaro nostatsionar ta'sirlashuvi haqidagi ilmiy adabiyotlar sharhiga bag'ishlangan. Shuningdek, ushbu bob doirasida, deformatsiyalanuvchi qattiq jism uchun potensial funksiyalarda ifodalangan to'lqin tenglamalari keltirilgan.Doiraviy silindrik elastik sterjenlarning buralma nostatsionar tebranishlarining umumiy tenglamalari keltirib chiqarilgan. Olingan tenglamalar koordinata va vaqt bo'yicha cheksiz katta tartibga ega va undan xususiy hollarda taqribiy klassik va aniqlashtirilgan tenglamalarni olish mumkinligidan kelib chiqqan holda umumiy tenglamalardan, xususiy holda, klassik ya'ni ikkinchi tartibli tebranish tenglamasi hamda turli aniqlashtirilgan tenglamalar keltirib chiqarilgan.

§1.1. Doiraviy elastik sterjen va balkalarning nostatsionar tebranishlari bo'yicha ilmiy tadqiqotlar.

Hozirgi zamon fani va texnikasi, ularning turli sohalarining rivoji, muhandislik amaliyoti ehtiyojlari hamda yangi texnologiyalar deformatsyalanuvchi qattiq jismlar mexanikasi oldiga yangidan-yangi nazariy va amaliy masalarni qo'ymoqda. Hususiy holda, bunday masalalar qatoriga silindrik qatlam, qobiq va sterjenlar tipidagi konstruktiv elementlarning nostatsionar tebranishlari haqidagi masalalar ham kiradi [1,2,5,9].

Shunday masalalar sinfiga qaralayotgan muhandislik konstruksiyasini o'rab turgan yoki uning ichida joylashgan tutash muhitda tarqaluvchi akustik va nostatsionar to'lqinlar ta'siri ostidagi muhandislik konstruksiyalari elementlarning hisobi ham kiradi. Bunday masalalar umuman olganda tashqi dinamik yuklar ixtiyoriy bo'lgan holning hususiy holidir.Amalda tashqi yuklar faqat akustik yoki nostatsionar to'lqinlar ta'siridangina iborat bo'lmaydi.

Moddalarning nostatsionar xulqining (o'zlarini nostatsionar tutishlarining) mukammallashgan modellarini, ularga har xil muhitlar ta'sirini, shuningdek moddalarning turli fizik-mexanik hususiyatlarini hisobga olgan holda yaratish ham yuqoridagi masalalar sinfiga tegishli muammolardir.

Sanab o'tilgan muammolarni hal qilishga juda ko'p sonli ilmiy ishlar bag'ishlangan va ochiq matbuotda chop etilgan [9]. Bunday ilmiy ishlar hozirgi kunda ham bizning Vatanimiz olimlari, shuningdek yaqin va uzoq chet el olimlari tomonidan bajarilmoqda.

Dinamikaning nostatsionar masalalarini tadqiq qilishning turli nazariy va eksperimental usullari, shuningdek deformatsiyalanuvchi qattiq jismlarning tutash muhit bilan o'zaro ta'sirlashuvi bilan bog'langan ko'plab masalalar, hamda voqea va hodisalarning effektiv matematik modellarini yaratish bilan bog'liq fundamental muammolar K.Sh.Bobomurodov, V.Q.Qobulov, T.Bo'riyev, A. S.Volmir,

S. M. Beloserkovskiy, A. N.Guz, V.D.Kubenko, A.N. Babayev, Sh. Mamatqulov, M. Mirsaidov, X. A. Rahmatullin, T. R. Rashidov, I. G. Fillipov, S.Markus, X.Xudoynazarov, T. Sh.Shirinqulovlarning ilmiy maqolalari va monografiyalarida yoritilgan.

Deformatsiyalanuvchi qattiq jismlarning suyuqlik yoki tuproqli muhit bilan o'zaro nostatsionar ta'sirlashuvi haqidagi muammoni tadqiq etishga bag'ishlangan fundamental tadqiqotlar A.N. Guz, I.G. Petrashen, I.G. Filippov, X.Xudoynazarov va boshqa olimlar ishlarida uchraydi.

Deformatsiyalanuvchi qattiq jismlarning tebranishlari haqidagi masalalarni tadqiq etishda, xususan, tutash muhit bilan o'zaro ta'sirlashuvchi silindrik sterjenlar tebranishlarini tadqiq etishda, sterjen uchun hal qiluvchi tenglamalar sifatida asosan tebranish taqribiy tenglamalari qabul qilinadi.Yuqorida sanab o'tilgan avtorlar ishlarning aksariyat ko'pchiligida turli xil gipotezalarga asoslanib chiqarilgan taqribiy tenglamalar ishlatilgan.Bunday gipotezalar xususan, sterjen uchun Krixgoff gipotezalari, harakat tenglamalari strukturalarini sezilarli darajada soddalashtirishga olib keladi. S.P.Timoshenko tipidagi aniqlashtirilgan tebranish tenglamalarini keltirib chiqarishda ham fizik va geometrik harakterdagi turli hil gipoteza va farazlardan foydalaniladi.Shuning uchun ham ishlatishga qabul qilingan tebranish nazariyasiga bog'liq ravishda olib borilayotgan tadqiqotlarning turli xil yo'nalishlari paydo bo'ldi.

Tebranish tenglamalarini keltirib chiqarishga hamda deformatsiyalanuvchi qattiq jismlar tebranishlari uchun aniqlashtirilgan nazariyalarini ishlab chiqishda, xususan silindrik qatlam qobiqlar va sterjinlarning tebranish nazariyalarini ishlab chiqishga bag'ishlangan ilmiy tadqiqotlar tahlili hamda bu muammolarning turli yo'nalishlari batafsil tahlili [8,13,14] ishlarda keltirilgan.

Muhandislik konstruksiyalari elementlarining tebranishlar nazariyalari hamda taqribiy tenglamalarini keltirib chiqarish masalalari prof. I. G. Flippov va uning o'quvchilari [10,11,13,15] tomonidan ishlab chiqilgan. Bu ishlarda qovushoq-elastiklik nazariyasining masalalarini uch o'lchovli asosda qo'yish yo'li bilan qovushoq-elastik plastina va doiraviy sterjenlarning bo'ylama va ko'ndalang tebranish tenglamalari keltirib chiqarilgan. Shuningdek bunday tenglamalar o'rab turuvchi muhit va ishqalanish kuchini hisobga olgan holda ham chiqarilgan. Bunda plastina va sterjenlarning anizotropik xossalari va harorat ta'siri ham (bog'liq nazariya) hisobga olingan. Olingan umumiy tenglamalar asosida klassik (Krihgoff-Love) va aniqlashtirilgan S.P.Timoshenko tipida hamda boshqa koordinata va vaqt o'zgaruvchilariga nisbatan yanada yuqoriroq hosilalarni o'z tarkibida saqlovchi yanada yuqori tartibli tenglamalar chiqarilgan.

Ishlab chiqarilgan umumiy va aniqlashtirilgan tenglamalar asosida amaliy ahamiyatga ega masalalar yechilgan. Bu masalalar qobiqlar, plastinalar va sterjenlarning erkin va majburiy tebranishlarini tadqiq etishga bag'ishlangan. Dinamik impulsli tashqi yuklar ta'siri ostidagi chiziqli qovushoq-elastik va terma-qovushoq-elastik muhit dinamikasini har xil analitik usullar bilan tadqiq etish natijalari keltirilgan. Bunday natijalar to'lqin maydoni asosiy parametrlarini fizik fazoning turli nuqtalarida vaqtning istalgan payti uchun aniqlashga imkon beradi.

Ushbu yo'nalishda [13] monografiyaini alohida ko'rsatib o'tish lozim. Chunki bu ish mazkur magistrlik disertatsiyasida qaralgan masalalarga to'g'ridan-to'g'ri aloqador va unda silindrik qatlam, qobiq, sterjenlarning tebranishlari nazariyalari yoritilgan. Avvalo shuni ta'kidlash lozimki, ushbu monografiyada ishlab chiqilgan usul prof. I.G.Filippov tomonidan plastina va sterjenlar uchun ishlab chiqilgan edi.Professor X.Xudoynazarov tomonidan ushbu usul silindrik qatlam va qobiqlar uchun rivojlantirildi. Bunday usul nafaqat sof qatlam va qobiqlar uchun balka ularni o'rab turuvchi deformatsiyalanuvchi qattiq va suyuq muhitlarni, xususan ideal suyuqlikni hisobga olgan holda ishlab chiqilgan.

Ko'rsatilgan ishda birinchi marta qatlamning tebranishlari haqidagi ma'lumotlarni (informatsiyani) o'zida saqlovchi va tashuvchi asosiy sirt sifatida qatlamning, klassik nazariyalardagi kabi o'rta sirti emas, balki oraliq sirti kiritilgan Ushbu oraliq sirt, kiritilgan χ parametrning ma'lum bir qiymatlarida qatlamning ichki va tashqi, yoki o'rta sirtiga o'tishi mumkin. Bunda χ parametrning qiymatlari spektri uzluksiz bo'lib yuqoridan va pastdan chegaralangan. Bu yerda olingan tenglamalar, klassik yoki S.P.Timoshenko tipidagi tenglamalardan farqli ravishda aylanish inertsiyasi va ko'ndalang siljish deformatsiyasi effektlarini avtomatik tarzda hisobga oladi. Shuni takidlash lozimki, asosiy sirt sifatida silindrik qatlamning o'rta sirti qabul qilinishi zarur degan davodan voz kechish, bundan oldingi ma'lum bo'lgan nazariyalarga nisbatan ancha katta aniqlikda kinematiik va dinamik kontakt shartlarini ifodalashga imkon beradi. Ma'lumki, bunday kontakt shartlari silindrik qatlam, qobiq va sterjenlarning deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvi masalalarida paydo bo'ladi.

Ko'ndalang kesimi doiraviy, uzunligi cheksiz qovushoq-elastik silindrik qatlam sirtlariga vaqtning t=0 paytida tashqi nostatsionar zo'riqishlar ta'sir etadi va bu zo'riqishlar qatlamning buralma, bo'ylama-radial va ko'ndalang tebranishlarini qo'zg'atadilar deb hisoblanadi [5,14,24]. Silindrik qatlamning harakat tenglamalari sifatida qovushoq-elastiklik nazariyasining uch o'lchovli harakat tenglamalarining

Φ- bo'ylama va $\vec{\Psi}$- ko'ndalang to'lqinlar potensiallalri orqali ifodalangan ko'rinishlari foydalanilgan:

$$L(\Phi) = \rho\frac{\partial^2\Phi}{\partial t^2};\ M(\vec{\Psi}) = \rho\frac{\partial^2\vec{\Psi}}{\partial t^2},$$

bu yerda $\vec{\Psi}$ vektorning komponentalari $\vec{\Psi}(\Psi_1,\Psi_2,\Psi_3)$- vektor maydoninig solinoidligi shartini qanoatlantirishlari kerak; L,M- qovushoq-elastiklik operatorlari. Qatlam nuqtalarining $U_r,\ U_z, U_\theta$ –ko'chishlari Φ va Ψ_j potensiallar orqali quyidagicha ifodalanadilar:

$$U_r = \frac{\partial}{\partial r}\left(\Phi + \frac{\partial}{\partial z}\Psi_2\right) + \frac{1}{r}\frac{\partial\Phi_1}{\partial\theta};\ U_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta} - \frac{\partial\Psi_2}{\partial r} + \frac{1}{r}\frac{\partial^2\Psi_2}{\partial z\partial\theta};$$

$$U_z = \frac{\partial\Phi}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Psi_2.$$

Qatlam nuqtalaridagi σ_{ij} kuchlaishlari ham Φ va Ψ_j potensiallar orqali xuddi yuqoridagi kabi ifodalanadilar.

Umumiy holda qovushoq-elastik silindrik qatlamning tebranishlari uning radiuslari $r = r_i\ (i = 1,2)$ bo'lgan sirtlariga ta'sir etuvchi zo'riqishlarning ta'sir etishlari natijasida qo'zg'atiladi deb hisoblanadi. Boshqacha qilib aytganda masalaning chegaraviy shartlari quyidagicha ko'rinishga ega bo'ladi:

$$\sigma_{r\theta} = f_{r\theta}^{(i)}(z,t);\ \sigma_{rr} = f_{rr}^{(i)}(z,t);\ \sigma_{rz} = f_{rz}^{(i)}(z,t);$$
$$\text{bunda}\quad r = r_i,\ i = 1,2.$$

Bulardan tashqari qobiq va uni o'rab turuvchi muhit orasidagi kontakt sirtida ko'chishlar tengligi sharti bajarilishi kerak. Potensiallar uchun boshlang'ich shartlar nolga teng deb hisoblanadi, ya'ni vaqtning t<0 bo'lgan qiymatlari uchun silindrik qatlam tinch holatda bo'lgan deb hisoblanadi.

Ishning davomida [8,14] ishlardagi kabi tashqi zo'riqishlar

$$f_{kj}^{(i)}(z,t) = \int_0^\infty \left.\begin{matrix}\sin qz\\ -\cos qz\end{matrix}\right\} dq \int_{(l)} f_{kj}^{(io)} e^{pt}dp,\ (i=1,2;\ k, j = r,\theta,z)$$

ko'rinishda tasvirlangan. Bu yerda $f_{kj}^{(o)}(p,q)$ $(o=1,2;)$ $(k, j = r,\theta,z)$ funksiyalar $f_{kj}^{(o)}(z,t)$ zo'riqish funksiyalarining tasviri bo'lib,

$$|q| \le q_0; \ |\sin p| \le w_0$$

sohadan tashqarida hisobga olmaslik darajasida kichik deb hisoblanadi; (l) p - tekislikdagi mavhum o'qning $(-i\omega_0, i\omega_0)$ qismiga o'ngdan yondosh ochiq kontur

Qo'yilgan masalani yechish uchun Φ va Ψ_j potensiallar ham yuqoridagi ko'rinishda tasvirlanadi[12]. Potensiallarning aytilgan tasvirlari harakat tenglamalariga qo'yilib, Besselning oddiy differentsial tenglamalari keltirib chiqarilgan.Olingan Bessel tenglamalarining aniq yechimlarini topish esa qiyinchilik tug'dirmaydi. Keyinchalik qatlam nuqtalarining buralma bo'ylama va ko'ndalang ko'chishlarining bosh qismlarini yangi izlanuvchi funksiyalar sifatida kiritib, hamda p va q parametrlar bo'yicha teskari almashtirishlarni bajarib, silindrik qatlamning yangi kiritilgan fungsiyalarga nisbatan tebranish tenglamalari olingan.

Ta'kidlash lozimki, olingan tenglamalar strukturalarida [10,16,21,23] bo'ylama va radial koordinatalar hamda *t* vaqt bo'yicha cheksiz hosilalarga ega hadlar qatnashadi. Tabiiyki bu narsa olingan tenglamalarni amaliy masalalar yechish uchun ishlatishga imkon bermaydi.Shuning uchun tenglamalar tarkibiga kiruvchi darajali cheksiz qatorning yaqinlashish sohasini aniqlash asosida qatorlarning chekli sondagi hadlarini olish bilan chegaralanish mumkinligi haqida xulosa chiqarilgan.

Tenglamalar tarkibiga kiruvchi darajali cheksiz qatorlar nolinchi, birinchi va hokazo (n=0,1 va hokazo) yaqinlashishlar bilan chegaralanilgan va natijada tebranishlar taqribiy tenglamalari keltirib chiqarilgan. Bu tenglamalar taqribiy bo'lishlariga qaramasdan klassik va S.P.Timoshenko [24] tipidagi tenglamalarga nisbatan aniqlashtirilgan bo'lib, aylanish inertsiyasi va ko'ndalang siljish diformatsiyasi kabi effektlarni avtomatik tarzda hisobga oladilar.

Tenglamalar chiqarishning yuqorida tavsiflangan uslubining boshqa usullarga nisbatan yana bir afzalligi shundan iboratki, bu usul yordamida tebranishlar tenglamalari bilan bir qatorda ko'chishlar va kuchlanishlar maydonlarining hamma komponentalari uchun analitik formulalarni chiqarish mumkin. Ushbu formulalar yordamida silindrik qatlamning ixtiyoriy ravishda tanlangan nuqtadagi kuchlanganlik-deformatsiyalanganlik holatini vaqtning istalgan payti uchun aniqlash mumkin. Ammo bunday ishni klassik va S. P.Timo-shenko tipidagi aniqlashtirilgan nazariyalar asosida bajarish mushkul [26].

Suv osti tekis zarbali to'lqinining oralaridagi fazo ideal siqiluvchi suyuqlik bilan to'ldirilgan ikkita silindrik qobiqlardan iborat konstruksiya bilan o'zaro tasirlashuvi masalasi H. Huang [17] ishida o'rganilgan.Masala vaqt bo'yicha Laplasning integral almashtirishi asosida o'zgaruvchilarni ajratish metodi bilan yechilgan. Laplas almashtirishida originallarda o'tishda integrallarning asimtotik va aniq baholashlaridan foydalanilgan, qaytarish integralini aniq baholashlar uchun tebranishning har bir shakli uchun Volteraning II-jins integral tenglamasini sonli ravishda yechishga to'g'ri keladi, hisoblashlar tushirilgan po'lat qobiqlar uchun bajarilgan.

Xuddi shunga o'xshash masalalarni yechishga professor X.Xudoynazarov va A. Abdurashidovlarning [15] monografiyasi ham bag'ishlangan. Bu ilmiy ishlar majmuasida deformatsiyalanuvchi qattiq jismlar va konstruksiyalarning qobiqsimon elementlarining muhit bilan xususan suyuqlik bilan konstruksiyaga tashqi impulsli gidrodinamik va seysmik zo'riqishlar ta'siri davomidagi o'zaro ta'sirlashuviga oid masalalar tadqiqotlari natijalari keltirilgan. Bu masalalarning har biri muhandislik konstruksiyalari va inshoatlari elementlarining mustahkamligini ta'minlash bilan bog'liq.

Konstruksiyalar deformatsiyalanuvchi qattiq va buzuluvchi qobiqsimon elementlarining yuqori darajada qizdirilgan kriogen hamda pufakchali suyuqliklar bilan o'zaro nostatsionar ta'sirlashuviga oid ba'zi bir holatlar o'rganilgan [5]. Prizma, plastina va qobiq ko'rinishdagi konstruksiya elementlarining elastoplastik deformatsiyalanishi, ularning suyuq muhit bilan o'zaro ta'sirlashuviga qanday ta'sir ko'rsatishi taxlil qilingan. Gidrodinamik effektlarni hisobga olgan holda uch muhit (qattiq, suyuq va gaz) o'zaro nostatsionar ta'sirlashuviga oid dinamik masalalarni yechishning sonli usuli ishlab chiqilgan.

Shunday qilib, ilmiy adabiyotlar tahlili shuni ko'rsatadiki doiraviy elastik silindrik qatlam, qobiq va sterjenlarlarning tebranishlari masalalari yaxshi o'rganilmagan. Shu nuqtai nazardan ushbu tebranish masalalaridan bir nechtasini magistrlik disertatsiyasi doirasida yechish amaliy ahamiyatga ega bo'ladi .

§1.2 Elastik jismning dinamik deformatsiyalangan holati va uni aniqlovchi munosabatlar.

Ma'lumki elastiklik nazariyasi kursidaelastik jism uchun asosiy tenglamalar ва munosabatlar chiqarilgan. Bu tenglamalar yopiq sistemani tashkil etadi va jismga ta'sir etgan tashqi kuchlarni hisobga olgan holda uning kuchlangan-deformatsialangan holatini aniqlashga imkon beradi. Bu tenglamalar odatda uch turga bo‘linadir: *geometrik, statik (dinamik) va fizik tenglamalar.* Biz quyidagi dissertatsiya ishida zarur bo'ladigan tenglamalar va munosabatlarni avvalo tenzor yozuvi ko'rinishida hamda ularning silindrik koordinatalar sistemasidagi ko'rinishlarini keltiramiz [20].

1^0. Geometrik tenglamalar.

Elastik jismning deformatsialangan holati deformatsialar tenzori $\varepsilon_{ij} = \varepsilon_{ij}(x_k)$ komponentalari, yoki $u_i = u_i(x_k)$ ko‘chishlar bilan to‘liq aniqlanadi. Deformatsialar tenzori komponentalari va ko‘chishlar o‘zaro Koshining differensial munosabatlari bilan bog‘langan:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

Ushbu munosabatlar silindrik koordinatalari sistemasida quyidagi ko'rinishda yoziladi

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r};\quad \varepsilon_{\theta\theta}=\frac{1}{2}\cdot\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r};\quad \varepsilon_{zz}=\frac{\partial u_z}{\partial z};\quad \varepsilon_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\cdot\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right);$$
$$\varepsilon_{\theta z}=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\theta}\right);\quad \varepsilon_{zr}=\frac{1}{2}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right).\tag{1.1}$$

bu yerda hajmiy deformatsiya

$$\varepsilon=\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{zz}=\frac{\partial u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}+\frac{u_r}{r}\tag{1.2}$$

Kichik burilish tenzori va burilish vektori komponentalari esa

$$\omega_{z\theta}=\omega_r=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta}-\frac{\partial u_\theta}{\partial z}\right);\quad \omega_{rz}=\omega_\theta=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right);$$
$$\omega_{\theta r}=\omega_z=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial r}-\frac{1}{r}\cdot\frac{\partial u_r}{\partial\theta}+\frac{u_\theta}{r}\right).\tag{1.3}$$

kabi bo’ladi. Deformatsiyalar Sen-Venanning differensial munosabatlari bilan o‘zaro bog‘langan:

$$\varepsilon_{ik,j\ell}+\varepsilon_{j\ell,ik}-\varepsilon_{i\ell,jk}-\varepsilon_{jk,i\ell}=0.$$

Ushbu munosabatlar silindrik koordinatalari sistemasida quyidagi ko’rinishda yoziladi

$$\frac{\partial^2\varepsilon_{\theta\theta}}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\varepsilon_{rr}}{\partial\theta^2}-\frac{2}{r}\frac{\partial^2\varepsilon_{r\theta}}{\partial r\partial\theta}+\frac{1}{r}\left(2\frac{\partial\varepsilon_{\theta\theta}}{\partial r}-\frac{\partial\varepsilon_{rr}}{\partial r}-\frac{2}{r}\frac{\partial\varepsilon_{r\theta}}{\partial\theta}\right)=0,$$
$$\frac{1}{r^2}\frac{\partial^2\varepsilon_{zz}}{\partial\theta^2}+\frac{\partial^2\varepsilon_{\theta\theta}}{\partial z^2}+\frac{1}{r}\frac{\partial\varepsilon_{zz}}{\partial r}-\frac{2}{r}\left(\frac{\partial^2\varepsilon_{\theta z}}{\partial\theta\partial z}+\frac{\partial\varepsilon_{zr}}{\partial z}\right)=0,$$
$$\frac{\partial^2\varepsilon_{zz}}{\partial r^2}+\frac{\partial^2\varepsilon_{rr}}{\partial z^2}-2\frac{\partial^2\varepsilon_{zr}}{\partial z\partial r}=0,\tag{1.4}$$
$$\frac{1}{r}\frac{\partial^2\varepsilon_{rr}}{\partial\theta\partial z}+\frac{\partial^2\varepsilon_{\theta z}}{\partial r^2}-\frac{\partial^2\varepsilon_{r\theta}}{\partial z\partial r}-\frac{1}{r}\frac{\partial^2\varepsilon_{zr}}{\partial z\partial\theta}+\frac{1}{r}\left(\frac{\partial\varepsilon_{\theta z}}{\partial r}+\frac{1}{r}\frac{\partial\varepsilon_{r\theta}}{\partial z}-\frac{\varepsilon_{\theta z}}{\partial r}\right)=0,$$
$$\frac{\partial^2\varepsilon_{\theta\theta}}{\partial z\partial r}+\frac{1}{r^2}\frac{\partial^2\varepsilon_{zr}}{\partial\theta^2}-\frac{1}{r}\frac{\partial^2\varepsilon_{r\theta}}{\partial\theta\partial z}-\frac{1}{r}\frac{\partial^2\varepsilon_{\theta z}}{\partial r\partial\theta}+\frac{1}{r}\left(\frac{\partial\varepsilon_{\theta\theta}}{\partial z}-\frac{1}{r}\frac{\partial\varepsilon_{\theta z}}{\partial\theta}-\frac{\partial^2\varepsilon_{rr}}{\partial z}\right)=0,$$
$$\frac{1}{r}\frac{\partial^2\varepsilon_{zz}}{\partial r\partial\theta}+\frac{\partial^2\varepsilon_{r\theta}}{\partial z^2}-\frac{\partial^2\varepsilon_{\theta z}}{\partial z\partial r}-\frac{1}{r}\frac{\partial^2\varepsilon_{zr}}{\partial\theta\partial z}+\frac{1}{r}\left(\frac{\partial\varepsilon_{\theta z}}{\partial z}-\frac{1}{r}\frac{\partial\varepsilon_{zz}}{\partial\theta}\right)=0.$$

Ma’lumki, bu munosabatlar deformatsialarning uzviylik tenglamasi deb ham yuritiladi.

2^0. Statik (dinamik) tenglamalar.

Elastik jismning kuchlanganlik holati kuchlanish tenzorining $\sigma_{ij}=\sigma_{ij}(x_k)$ oltita komponentasi bilan to‘liq aniqlanadi. Ushbu komponentalar *uchta muvozanat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j}+\rho f_i=0.$$

Ushbu kuchlanishlarga nisbatan muvozanat tenglamalari silindrik koordinatalari sistemasida [4]

$$\begin{gathered}\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+\rho f_r=0,\\ \frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{z\theta}}{\partial z}+\frac{2}{r}\sigma_{r\theta}+\rho f_\theta=0,\\ \frac{\partial\sigma_{zr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{zr}}{r}+\rho f_z=0.\end{gathered}\tag{1.5}$$

ko’rinishda yoziladi

Agar jism harakatda bo‘lsa, simmetrik kuchlanish tenzorining olti komponentasi *uchta harakat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j}+\rho f_i=\rho\frac{\partial^2 u_i}{\partial t^2}.$$

yoki

$$\begin{gathered}\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+\rho f_r=\rho\frac{\partial^2 u_r}{\partial t^2},\\ \frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{z\theta}}{\partial z}+\frac{2}{r}\sigma_{r\theta}+\rho f_\theta=\rho\frac{\partial^2 u_\theta}{\partial t^2},\\ \frac{\partial\sigma_{zr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{zr}}{r}+\rho f_z=\rho\frac{\partial^2 u_z}{\partial t^2}.\end{gathered}\tag{1.6}$$

Ushbu (1.5) - tenglamalar *statik*, (1.6)- tenglamalar *dinamik* tenglamalar deb yuritiladi.

Ko‘chishlarga nisbatan muvozanat va harakat tenglamalari (Lame tenglamalari) quyidagi ko’rinishga ega [7]:

$$\nabla^2 u_r+\frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}-\frac{u_r}{r^2}+\frac{1}{1-2\nu}\frac{\partial\varepsilon}{\partial r}+\frac{\rho}{\mu}f_r=0\left(\frac{\rho}{\mu}\frac{\partial^2 u_r}{\partial t^2}\right)$$

$$\nabla^2 u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2} + \frac{1}{1-2\nu}\frac{\partial\varepsilon}{r\partial\theta} + \frac{\rho}{\mu} r\ f_\theta = 0\left(\frac{\rho}{\mu}\frac{\partial^2 u_\theta}{\partial t^2}\right)$$

$$\nabla^2 u_z + \frac{1}{1-2\nu}\frac{\partial\varepsilon}{\partial z} + \frac{\rho}{\mu} f_z = 0\left(\frac{\rho}{\mu}\frac{\partial^2 u_z}{\partial t^2}\right). \tag{1.7}$$

3^0. Fizik tenglamalar.

Izotrop jism uchun kuchlanish tenzorining σ_{ij} komponentalari deformatsiya tenzorining ε_{ij} komponentalari bilan Guk qonuni vositasida bog‘langan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \tag{1.8}$$

yokiu_i ko‘chish komponentalari bilan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \tag{1.9}$$

ko‘rinishda bog‘langan. Bu yerda

$$\theta = \varepsilon_{ii} = u_{i,i} = div\vec{u}\ .$$

Ushbu munosabatlar silindrik koordinatalari sistemasida quyidagi ko’rinishda yoziladi [7]

$$\begin{aligned}
&\sigma_{rr} = \lambda\varepsilon + 2\mu\varepsilon_{rr}; \quad \sigma_{r\theta} = 2\mu\varepsilon_{r\theta};\\
&\sigma_{\theta\theta} = \lambda\varepsilon + 2\mu\varepsilon_{\theta\theta}; \quad \sigma_{\theta z} = 2\mu\varepsilon_{\theta z};\\
&\sigma_{zz} = \lambda\varepsilon + 2\mu\varepsilon_{zz}; \quad \sigma_{zr} = 2\mu\varepsilon_{zr}.
\end{aligned} \tag{1.10}$$

Ba’zi hollarda Guk qonunini (1.8) ga teskari shaklda, ya’ni ε_{ij} larga nisbatan yechilgan ko‘rinishda ishlatishga to‘g‘ri kelishi mumkin:

$$\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sum\right], \tag{1.11}$$

bu yerda:

$$\sum = \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}.$$

hajmiy kuchlanish. Ushbu kuchlanishlar Beltrami tenglamalarini qanoatlantirishlari kerak

$$\nabla^2\sigma_{rr} - \frac{4}{r^2}\frac{\partial\sigma_{r\theta}}{\partial\theta} - \frac{2}{r^2}(\sigma_{rr} - \sigma_{\theta\theta}) + \frac{1}{1+\nu}\frac{\partial^2\Sigma}{\partial r^2} = 0,$$

$$\nabla^2\sigma_{\theta\theta} + \frac{4}{r^2}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{2}{r^2}(\sigma_{rr} - \sigma_{\theta\theta}) + \frac{1}{1+\nu}\left(\frac{1}{v}\frac{\partial\Sigma}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Sigma}{\partial\theta^2}\right) = 0\ ,$$

$$\nabla^2\sigma_{r\theta}+\frac{2}{r^2}\frac{\partial}{\partial\theta}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)-\frac{4}{r^2}\sigma_{r\theta}+\frac{1}{1+\nu}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Sigma}{\partial\theta}\right)=0,\ \nabla^2\sigma_{zz}+\frac{1}{1+\nu}\frac{\partial^2\Sigma}{\partial z^2}=0, \quad (1.12)$$

$$\nabla^2\sigma_{\theta z}+\frac{2}{r^2}\frac{\partial\sigma_{zr}}{\partial\theta}-\frac{\sigma_{\theta z}}{r^2}+\frac{1}{1+\nu}\frac{1}{r}\frac{\partial^2\Sigma}{\partial\theta\,\partial z}=0,\ \nabla^2\sigma_{zr}-\frac{2}{r^2}\frac{\partial\sigma_{\theta z}}{\partial\theta}-\frac{\sigma_{zr}}{r^2}+\frac{1}{1+\nu}\frac{\partial^2\Sigma}{\partial z\,\partial r}=0.$$

Yuqorida sanab o‘tilgan (1.1) - (1.12) formulalar elastiklik nazariyasi *statik va dinamik masalalarining asosiy tenglamalari deb* yuritiladi.

§1.3Doiraviy elastik sterjenning bo’ylama tebranishlari haqidagi masalaning qo’yilishi.

Bir jinsli va izotrop, radiusi r_0 bo’lgan doiraviy silindrik elastik sterjenni silindrik koordinatalar sistemasida qaraymiz. Tebranish tenglamalarini chiqarishda sterjenni uch o’lchovli jism deb hisoblaymiz.Ushbu dissertatsiya doirasida balkaning bo’ylama tebranishlari tadqiq etiladi.Balkaning yoki sterjenning bo’ylama tebranishlari uning o’qiga nisbatan simmetrik tebranishlardan iboratdir [6]. Shuning uchun sterjen nuqtalarining $\vec{U}(r,\theta,z,t)$ ko’chish vektorini [14] monografiya xulosalari asosida quyidagicha tasvirlaymiz

$$\vec{U}=grad\Phi+rot\left(\vec{e}_z\Psi_2\right), \quad (1.13)$$

bu yerda Φ - bo’ylama va Ψ_2 -ko’ndalang to’lqin potensiallari, $\vec{e}_z - z$ o’qi bo’yicha yo’nalgan birlik vektori [23].

$$\vec{U}=\frac{\partial\Phi}{\partial r}\vec{e}_r+\frac{\partial\Phi}{\partial z}\vec{e}_z+\frac{\partial^2\Psi_2}{\partial r\partial z}\vec{e}_r-\left(\frac{\partial^2\Psi_2}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi_2}{\partial r}\right)\vec{e}_z$$

kabiifodalaymiz.

Ko’chish vektorining oxirgi ifodasidagi $\vec{e}_r,\vec{e}_z$-birlik vektorlari oldidagi koeffisientlar mos ravishdaU_r,U_z- ko’chish komponentalarini aniqlaydi

$$U_r=\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Psi_2}{\partial r\partial z};$$

$$U_z=\frac{\partial\Phi}{\partial z}+-\frac{\partial^2\Psi_2}{\partial r^2}-\frac{1}{r}\frac{\partial\Psi_2}{\partial r}; \quad (1.14)$$

Deformasiya tenzori va ko'chish vektori komponentalari orasidagi munosabatlar

$$\varepsilon_{rr} = \frac{\partial U_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{U_r}{r}, \quad \varepsilon_{zz} = \frac{\partial U_z}{\partial z}, \; 2\varepsilon_{rz} = \frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}$$

ya'niKoshi munosabatlaridan $\varepsilon_{ij} = \varepsilon_{ij}(r,\theta,z,t), \; (i,j = r,\theta,z)$ deformatsiyalarni Φ, Ψ_2 potensiallar orqali ifodalaymiz

$$\varepsilon_{rr} = \frac{\partial^2}{\partial r^2}\Phi + \frac{\partial^3}{\partial r^2 \partial z}\Psi_2, \; \varepsilon_{\theta\theta} = \frac{1}{r}\left[\frac{\partial}{\partial r}\Phi + \frac{\partial^2}{\partial z \partial r}\Psi_2\right],$$

$$\varepsilon_{zz} = \frac{\partial^2}{\partial z^2}\Phi - \frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2, \; \varepsilon_{rz} = 2\frac{\partial^2}{\partial r \partial z}\Phi + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2, \tag{1.15}$$

$$\varepsilon = \varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz} = \Delta\Phi. \; \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

bunda ε - hajmiy deformasiya; Δ –Laplas operatori. Elastik jism uchun Guk qonuniga

$$\sigma_{rr} = \lambda\varepsilon + 2\mu\varepsilon_{rr}, \; \sigma_{\theta\theta} = \lambda\varepsilon + 2\mu\varepsilon_{\theta\theta},$$
$$\sigma_{zz} = \lambda\varepsilon + 2\mu\varepsilon_{zz}, \; \sigma_{rz} = 2\mu\varepsilon_{rz},$$

asosan $\sigma_{ij} = \sigma_{ij}(r,z,t), \; (i,j = r,z)$ kuchlanishlar

va $\varepsilon_{ij} = \varepsilon_{ij}(r,z,t), \; (i,j = r,z)$ deformatsiyalar orasidagi quyidagi munosabatga ega bo'lamiz

$$\sigma_{rr} = \lambda(\Delta\Phi) + 2\mu\left(\frac{\partial^2}{\partial r^2}\Phi + \frac{\partial^3}{\partial r^2 \partial z}\Psi_2\right),$$

$$\sigma_{\theta\theta} = \lambda(\Delta\Phi) + 2\mu\left(\frac{1}{r}\left[\frac{\partial}{\partial r}\Phi + \frac{\partial^2}{\partial z \partial r}\Psi_2\right]\right),$$

$$\sigma_{zz} = \lambda(\Delta\Phi) + 2\mu\left(\frac{\partial^2}{\partial z^2}\Phi - \frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2\right), \tag{1.16}$$

$$\sigma_{rz} = \mu\left(2\frac{\partial^2}{\partial r \partial z}\Phi + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2\right),$$

bu yerda λ,μ – Lame koeffisentlari.

Harakat tenglamalari - (1.6) ga kuchlanishlarning ushbu ifodalarini va ko'chishlarning (1.14) ifodalarini qo'yib, soddalashtirishlardan so'ng quyidagiga ega bo'lamiz

$$(\lambda + 2\mu)\Delta\Phi - \rho\ddot{\Phi} = 0, \quad \mu(\Delta\Psi_2) - \rho\ddot{\Psi}_2 = 0,$$

Bundan esa quyidagi to'lqin tenglamalari kelib chiqadi [4]

$$(\lambda + 2\mu)(\Delta\Phi) = \rho\ddot{\Phi}, \quad \mu(\Delta\Psi_2) = \rho\ddot{\Psi}_2;$$

bunda ρ– sterjen materialining zichligi.

Doiraviy elastik sterjenning bo'ylama-radial tebranishlari haqidagi masala quyidagicha qo'yiladi:

a) Bo'ylama va ko'ndalang to'lqin potensiallariga nisbatan harakat tenglamalari

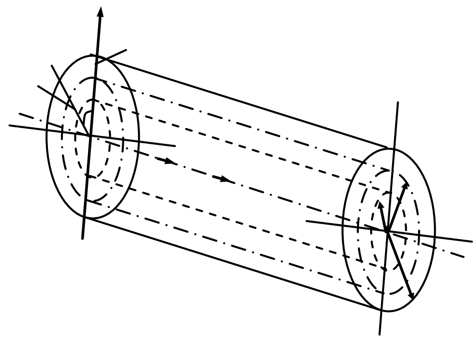
$$(\lambda + 2\mu)(\Delta\Phi) = \rho\ddot{\Phi}, \quad \mu(\Delta\Psi_2) = \rho\ddot{\Psi}_2 \tag{1.17}$$

ko'rinishida qabul qilinadi;

b) sterjenning bo'ylama-radial tebranishlari uning sirtida ta'sir etuvchi normal $f_r\,(z,t)$ va urinma $f_{rz}(z,t)$ kuchlanishlar ta'siri natijasida vujudga keladi deb hisoblanadi, ya'ni sterjen sirtidagi chegaraviy shartlar quyidagi ko'rinishga ega, ya'ni

$$\begin{aligned} \sigma_{rr}(r_0, z, t) &= f_r(z,t), \\ \sigma_{rz}(r_0, z, t) &= f_{rz}(z,t), \end{aligned} \tag{1.18}$$

Boshlang'ich shartlarni nolga teng deb hisoblaymiz.



1.1–Chizma.Doiraviy sterjenning bo'ylama – radial tebranishlari uchun ko'chish vektori komponentalari va silindrik koordintalar sistemasi.

Shunday qilib doiraviy elastik sterjenning bo'ylama-radial tebranishlari haqidagi masala (1.17) - bo'ylama va ko'ndalang potensiallarga nisbatan to'lqin tenglamalarini (1.18) chegaraviy shartlar va nolga teng boshlang'ch shartlarda integrallashga keltiriladi.

§1.4 Doiraviy elastik sterjenning bo'ylama tebranishlari tenglamalarini keltirib chiqarish umumiy usuli.

Tebranishlar tenglamalarini keltirib chiqarishning elastiklik dinamik nazariyasiga asoslangan usullari uchta asosiy yo'nalishlarga bo'linadilar. Dinamikada variatsion prinsiplardan foydalanishga asoslangan usullarni bu yo'nalishlarning birinchisini tashkil etadi deb hisoblash mumkin [16]. Ammo ta'kidlash lozimki, dinamikaning nostatsionar masalalarini yechishda variatsion prinsiplarga asoslangan usullarning mantiqiy jihatdan asoslanganligi [8] ishda jiddiy ishonchsizliklarga olib kelishi isbotlangan.

Ikkinchi asosiy yo'nalishga elastik ko'chishlar komponentalarini qatorlarga (shu jumladan darajali qatorlarga) yoyishga asoslangan usullarni kiritish mumkin. Bu usulning statik masalalar uchun ishlab chiqilgan umumiy sxemasi birinchi marta N.A. Kilchevskiy [24] tomonidan taklif etilgan.Dinamik masalalarning juda keng sinflari uchun bu usul P. Epshteyn tomonidan ishlab chiqilgan[24]. Bu usul keyingi o’n yilliklar dsavomida o’zining sezilarli taraqqiyotiga erishdi. Ushbu usul asosida V.Z.Vlasov qobiqli sistemalarga nisbatan o’zining boshlang’ich funksiyalar usulini ishlab chiqdi. Tekis deformatsiya holati uchun elastik ko’chishlarni darajali qatorlarga yoyish usulining qat’iy matematik asoslarini qatlam haqidagi dinamik masala misolida G.I.Petrashen [8] berdi.

Uchinchi asosiy yo’nailishga elastiklik dinamik nazariyasining uch o’lchovli masalalarining aniq yechimlaridan foydalanish usuli kiradi, va bu usul [8,10,13,14] ishlarda rivojlantirilgan. Ushbu usul I.G.Filippov va uning o’quvchilari [10-15,21-23,25-27] ishlarida dinamik masalalarni yechishga muvaffaqiyatli qo’llanildi va usul o’zining yangi, sezilarli rivojiga erishdi. Ushbu tadqiqotlarda qobiq, plastina va sterjenlarning umumiy tebranish tenglamalari ularning materiallarining har xil xususiyatlarini, xususan turli reologik, anizotropik, bir jinslimaslik va boshqa xususiyatlarini hamda bikrlik o’zgaruvchanligi, kesim geometriyasining doimiymasligi va boshqa faktorlarni hisobga olgan holda ishlab chiqildi.

Ushbu dissertatsiya doirasida keltirib chiqariladigan tenglamalarning barchasi [2,3] ana shu usul asosida olingan. Tenglamalardan tashqari, ushbu usul yordamida qaralayotgan sistema (bizning holda doiraviy brus) kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatlarini bir qiymatli aniqlashga imkon beruvchi algoritm ham ishlab chiqilgan bo’lib, quyida biz ularning [13] monografiyada keltirilgan variantlaridan ham foydalanamiz.

Tebranish tenglamasini keltirib chiqarishda, sterjenning bo’ylama-radial tebranishlarini uyg’otuvchi tashqi dinamik kuchlanish funksiyalari $f_r(z,t)$, $f_{rz}(z,t)$ larni xuddi [8]maqoladagi kabi quyidagicha tasvirlaymiz

$$f_r(z,t)=\int_0^{\infty}\left.\begin{matrix}\sin kz\\ -\cos kz\end{matrix}\right\}dk\int_{(l)} f_r^{(0)}e^{pt}\,dp,\qquad f_{rz}(z,t)=\int_0^{\infty}\left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\}dk\int_{(l)} f_{rz}^{(0)}e^{pt}\,dp, \tag{1.19}$$

bunda ❨ $p$tekislikdagi $(-i\omega_0,\ i\omega_0)$ oraliqdan o’ng tomonda joylashgan chiziq. Bundan tashqari, $f_r^{(0)}, f_{rz}^{(0)}$ funksiyalar shunday tanlanadiki, bunda $k \le k_0$ bo’lganda,

$f_r\,(z,t),\ \ f_{rz}(z,t)$ funksiyalar juda kichik bo'lishlari kerak. Endi $\Phi\ ,\Psi_2$ potensiallarni ham quyidagicha tasvirlaymiz

$$\Phi=\int_0^\infty \left.\begin{matrix}\sin kz\\ -\cos kz\end{matrix}\right\}dk\int_{(l)}\Phi_0 e^{pt}dp,\quad \Psi_2=\int_0^\infty \left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\}dk\int_{(l)}\Psi_{20} e^{pt}dp\ . \tag{1.20}$$

(1.20) formulada tasvirlangan $\Phi\ ,\Psi_2$ potensiallar *r*, *z*va*t* bo'yicha differensiallanuvchi funksiyalardir. Endi (1.20)tasvirlarni (1.17) harakat tenglamasiga qo'yamiz va quyidagiga ega bo'lamiz

$$\Omega_\alpha\Phi_0=0,\quad \Omega_\beta\Psi_{20}=0, \tag{1.21}$$

bu yerda

$$\Omega_{\alpha,\beta}=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\left(\alpha^2,\beta^2\right);\quad \alpha^2=k^2+\rho\ p^2;\ \ \beta^2=k^2+\rho\ p^2. \tag{1.22}$$

Olingan (1.21) tenglamalar sistemasi Besselning qutb koordinatalari sistemasidagi ikkinchi tartibli oddiy differensial tenglamalaridan iboratdir. Bu tenglamalarning umumiy yechimlari ma'lum va ular Bessel funksiyalari

$$\Phi_0(r)=A\ I_0\left(\alpha_0 r\right),\ \ \Psi_{20}(r)=B\ I_0\left(\beta_0 r\right) \tag{1.23}$$

orqali ifodalanadi, bunda I_0 – Besselning modifitsirlangan funksiyasi; A, B- integrallash o'zgarmaslari.

Ko'chishlarni quyidagicha tasvirlaymiz

$$U_r=\int_0^\infty \left.\begin{matrix}\sin kz\\ -\cos kz\end{matrix}\right\}dk\int_{(l)}U_r^{(0)} e^{pt}dp,\quad U_z=\int_0^\infty \left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\}dk\int_{(l)}U_z^{(0)} e^{pt}dp.$$

U holda kiritilgan $U_r^{(0)},U_z^{(0)}$ ko'chishlar uchun

$$U_r^0=\frac{d}{dr}\Phi_0+k\frac{d}{dr}\Psi_{20},\ \ U_z^0=k\Phi_0-\left(\frac{1}{r}\frac{d}{dr}+\frac{d^2}{dr^2}\right)\Psi_{20}.$$

munosabatlarni hisobga olsak

$$U_r^{(0)} = \alpha_0 \left[I_1(\alpha_0 r) A \right] - k\beta_0 \left[I_1(\beta_0 r) B \right], \quad U_z^{(0)} = k \left[I_0(\alpha_0 r) A \right] - \beta_0^2 \left[I_0(\beta_0 r) B \right] \quad (1.24)$$

ifodalarga ega bo’lamiz.

Ko’chishning ushbu ifodasidagi Besselning modifitsirlangan funksiyalarini *r* radial koordinatabo’yicha qatorga yoyamiz. U holda

$$U_r^{(0)} = \sum_{n=0}^{\infty} \left[\alpha_0^{2n+2} A - k\beta_0^{2n+2} B \right] \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!\,(n+1)!} \quad U_z^{(0)} = \sum_{n=0}^{\infty} \left[kA\alpha_0^{2n} - B\beta_0^{2n+2} \right] \frac{\left(\frac{r}{2}\right)^{2n}}{(n!)^2} \quad (1.25)$$

ifodalarga ega bo’lamiz.

Ko’chishlarning bu ifodasida n=0 deb quyidagiga ega bo’lamiz

$$\frac{r}{2} U_{r,0}^{(0)} = \frac{r}{2} \alpha_0^2 A - \frac{r}{2} k\beta_0^2 B \;; \quad U_{z,0}^{(0)} = kA - \beta_0^2 B; \quad (1.26)$$

(1.26) ni (1.25) ga qoyamiz

$$\begin{aligned} U_{r,}^{(0)} &= \sum_{n=0}^{\infty} \left[\beta_0^{2n} U_{r,0}^{(0)} + \alpha_0^2 q_1^{(0)} Q_n (U_{r,0}^{(0)} - kU_{z,0}^{(0)}) \right] \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!} \\ U_z^{(0)} &= \sum_{n=0}^{\infty} \left[\alpha_0^{2n} U_{z,0}^{(0)} + q_1^{(0)} Q_n (kU_{r,0}^{(0)} - \alpha_0^2 U_{z,0}^{(0)}) \right] \frac{\left(\frac{r}{2}\right)^{2n}}{(n!)^2}; \end{aligned} \quad (1.27)$$

bu yerda $q_1^{(0)} = 1 - \frac{\lambda + 2\mu}{\mu}; \; Q_n = \frac{\alpha_0^{2n} - \beta_0^{2n}}{\alpha_0^2 - \beta_0^2} = \sum_{k=1}^{n-1} \alpha_0^{2(n-k-1)} \beta_0^{2k}; \quad Q_0 \equiv 0; \quad Q_1 \equiv 1.$

Chegaraviy shartlarni kiritilgan $U_r^{(0)}, U_z^{(0)}$ ko’chishlar orqali ifodalaymiz. Buning uchun (1.18) chegaraviy shartlarga kuchlanishlarning (1.16) ifodalarini qo’yamiz

$$\Delta\Phi + \frac{\mu}{\lambda+2\mu}\left(\frac{\partial^2}{\partial r^2}\Phi + \frac{\partial^3}{\partial r^2\partial z}\Psi_2\right) = \frac{1}{\lambda+2\mu} f_r(r_0,z,t),$$
$$2\frac{\partial^2}{\partial r\partial z}\Phi + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2 = \frac{1}{\mu} f_{rz}(r_0,z,t),$$

bu ifodaga potensiallarning (1.20) ifodalarni va tashqi ta’sir funksiyalarining (1.19) tasvirlarini qo’yamiz va hosil qilingan formuladan z bo’yicha hosila olsak u quyidagi ko’rinishga ega bo’ladi.

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-k^2\right]\Phi_0(r) + 2\frac{\mu}{\lambda+2\mu}\frac{\partial^2}{\partial r^2}\left(\Phi_0(r) - k\,\psi_{20}(r)\right) = \frac{1}{\lambda+2\mu} f_r^{(0)},$$
$$2\frac{\partial}{\partial r}\Phi_0(r) - \frac{\partial}{\partial r}\left(-k^2\psi_{20}(r) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\psi_{20}(r)\right) = \frac{1}{\mu} f_{rz}^{(0)}$$

Olingan ushbu tenglamaga Φ_0 va ψ_{20} potensiallarning (1.23) qiymatlarini qo’yib

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-k^2\right]A\,I_0(\alpha_0 r_0) + 2\frac{\partial^2}{\partial r^2}\left(A\,I_0(\alpha_0 r_0) - k\,B\,I_0(\beta_0 r_0)\right) = \frac{1}{\lambda+2\mu} f_r^{(0)},$$
$$2\frac{\partial}{\partial r}A\,I_0(\alpha_0 r_0) + \frac{\partial}{\partial r}\left(-k^2 B I_0(\beta_0 r_0) - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]B\,I_0(\beta_0 r_0)\right) = \frac{1}{\mu} f_{rz}^{(0)},$$

tenglamalarga ega bo’lamiz. Bu tenglamalarda zarur soddalashtirishlarni bajarib v

$$\begin{aligned}&\left[(\beta_0^2+k^2)I_0(\alpha_0 r_0) - \frac{2\alpha_0}{r_0}I_1(\alpha_0 r_0)\right]A - 2k\left[\beta_0^2 I_0(\beta_0 r_0) - \frac{\beta_0}{r}I_1(\beta_0 r_0)\right]B = \frac{1}{\lambda+2\mu} f_r^{(0)}\\ &2\alpha_0 k\, I_1(\alpha_0 r_0)A - \beta_0(\beta_0^2+k^2)I_1(\beta_0 r_0)B = -\frac{1}{\mu} f_{rz}^{(0)}\end{aligned} \qquad (1.28)$$

tenglamalarni olamiz. Bu yerda modifitsirlangan Bessel funksiyasining standart yoyilmasidan, ya’ni

$$I_0(\alpha_0 r_0) = \sum_{n=0}^{\infty}\alpha_0^{2n}\frac{(r_0/2)^{2n}}{(n!)^2};\quad I_1(\alpha_0 r_0) = \sum_{n=0}^{\infty}\alpha_0^{2n+1}\frac{(r_0/2)^{2n+1}}{n!(n+1)!}$$

dan foydalanib hamda, (1.26) tenglamalardan A va B larni topib

$$A=\frac{U_{r,0}^{(0)}-kU_{z,0}^{(0)}}{\alpha_0^2-k^2};\qquad B=-\frac{kU_{r,0}^{(0)}+\alpha_0^2U_{z,0}^{(0)}}{\beta_0^2(\alpha_0^2-k^2)};$$

oxirgi (1.28) tenglamalarga qo'yamiz va qatorga yoyamiz

$$\left[(\beta_0^2+k^2)\sum_{n=0}^{\infty}\alpha_0^{2n}\frac{(r_0/2)^{2n}}{(n!)^2}-\frac{2\alpha_0}{r_0}\sum_{n=0}^{\infty}\alpha_0^{2n+1}\frac{(r_0/2)^{2n+1}}{n!(n+1)!}\right]\frac{U_{r,0}^{(0)}-kU_{z,0}^{(0)}}{\alpha_0^2-k^2}+$$
$$+2k\left[\beta_0^2\sum_{n=0}^{\infty}\beta_0^{2n}\frac{(r_0/2)^{2n}}{(n!)^2}-\frac{\beta_0}{r}\sum_{n=0}^{\infty}\beta_0^{2n+1}\frac{(r_0/2)^{2n+1}}{n!(n+1)!}\right]\frac{kU_{r,0}^{(0)}+\alpha_0^2U_{z,0}^{(0)}}{\beta_0^2(\alpha_0^2-k^2)}=\frac{1}{\lambda+2\mu}f_r^{(0)}$$
$$2\alpha_0k\left[\sum_{n=0}^{\infty}\alpha_0^{2n+1}\frac{(r_0/2)^{2n+1}}{n!(n+1)!}\frac{U_{r,0}^{(0)}-kU_{z,0}^{(0)}}{\alpha_0^2-k^2}\right]+$$
$$+\beta_0(\beta_0^2+k^2)\left[\sum_{n=0}^{\infty}\beta_0^{2n+1}\frac{(r_0/2)^{2n+1}}{n!(n+1)!}\frac{kU_{r,0}^{(0)}+\alpha_0^2U_{z,0}^{(0)}}{\beta_0^2(\alpha_0^2-k^2)}\right]=\frac{1}{\mu}f_{rz}^{(0)}$$

yoki

$$\sum_{n=0}^{\infty}\left[\frac{\alpha_0^{2n}(\beta_0^2+k^2)+2k^2\beta_0^{2n}}{\alpha_0^2-k^2}U_{r,0}^{(0)}+\frac{2\alpha_0^2k\beta_0^{2n}-k\alpha_0^{2n}(\beta_0^2+k^2)}{\alpha_0^2-k^2}U_{z,0}^{(0)}\right]\frac{(r_0/2)^{2n}}{(n!)^2}+$$
$$+\sum_{n=0}^{\infty}\left[\left(\frac{\alpha_0^{2n+1}\alpha_0\beta_0^2+k^2\beta_0^2\beta_0^{2n+1}}{\alpha_0^2-k^2}\right)U_{r,0}^{(0)}+\left(\frac{k\beta_0\beta_0^{2n+1}-k\alpha_0\beta_0^2\alpha_0^{2n+1}}{\alpha_0^2-k^2}\right)U_{z,0}^{(0)}\right]\frac{(r_0/2)^{2n}}{n!(n+1)!}=\frac{1}{\lambda+2\mu}f_r^{(0)}$$
$$\frac{1}{\alpha_0^2-k^2}\sum_{n=0}^{\infty}\left[\left(2\alpha_0k\alpha_0^{2n+1}+k\beta_0^{2n+1}(\beta_0^2+k^2)\right)U_{r,0}^{(0)}+\left((\beta_0^2+k^2)\beta_0^{2n}\alpha_0^2-2\alpha_0k^2\alpha_0^{2n+1}\right)U_{z,0}^{(0)}\right]\times$$
$$\times\frac{(r_0/2)^{2n+1}}{n!(n+1)!}=\frac{1}{\mu}f_{rz}^{(0)}$$

Yoki

$$\sum_{n=0}^{\infty}\left[\beta_0^2(\alpha_0^{2n}(\alpha_0^2+n+1)+k^2(\alpha_0^{2n}+\beta_0^{2n}))U_{r,0}^{(0)}\frac{(r_0/2)^{2n}}{n!(n+1)!}+\right.$$
$$+\sum_{n=0}^{\infty}\left[\left((2\alpha_0^2\beta_0^{2n}-\alpha_0^{2n}\beta_0^2-\alpha_0^{2n}k^2)(n+1)+\beta_0^2(\beta_0^{2n}-\alpha_0^{2n+2})\right)U_{z,0}^{(0)}\frac{(r_0/2)^{2n}}{n!(n+1)!}=\right.$$
$$=\frac{\alpha_0^2-k^2}{\lambda+2\mu}f_r^{(0)}\qquad(1.29)$$

$$\sum_{n=0}^{\infty}\left[\left(2\alpha_0k\alpha_0^{2n+1}+k\beta_0^{2n+1}(\beta_0^2+k^2)\right)U_{r,0}^{(0)}+\left((\beta_0^2+k^2)\beta_0^{2n}\alpha_0^2-2\alpha_0k^2\alpha_0^{2n+1}\right)U_{z,0}^{(0)}\right]\frac{(r_0/2)^{2n+1}}{n!(n+1)!}=\frac{\alpha_0^2-k^2}{\lambda+2\mu}f_{rz}^{(0)}$$

tenglamalarga ega bo'lamiz.

Olingan oxirgi tenglamalarni Fur'e va Laplaslar bo'yicha teskari almashtirib

$$d_1 U_{r,0} + d_2 U_{z,0} = \frac{1}{\lambda + 2\mu} f_r \,;\; d_3 U_{r,0} + d_4 U_{z,0} = \frac{1}{\mu} f_{rz}; \qquad (1.30)$$

tenglamalar sistemasiga ega bo'lamiz. Bu yerda

$$\begin{aligned}
d_1 &= \sum_{n=0}^{\infty} \left\{ \left[n(1-q_1) - q_1 \right] \lambda_2^n - \left[\lambda_1 - (n+1)(\lambda_2 - \frac{\partial^2}{\partial z^2}) \right] q_1 Q_n \right\} \frac{(r_0/2)^{2n}}{n!(n+1)!}; \\
d_2 &= \sum_{n=0}^{\infty} \left\{ (n+1)(1+q_1) \frac{\partial}{\partial z} \lambda_2^n + [\frac{\partial}{\partial z} \lambda_1 - (n+1) \frac{\partial}{\partial z} \lambda_2 - \frac{\partial^3}{\partial z^3}] q_1 Q_n \right\} - \frac{(r_i/2)^{2n}}{n!(n+1)!}; \\
d_3 &= \sum_{n=0}^{\infty} \frac{\partial}{\partial z} \left(\lambda_2^n + q_1 (Q_{n+1} + Q_n \lambda_1) \right) \frac{(r_0/2)^{2n+1}}{n!(n+1)!}; \\
d_4 &= \sum \lambda_1 \left\{ (1-q_1) \lambda_2^n + 2 \frac{\partial^2}{\partial z^2} q_1 Q_n \right\} \frac{(r_0/2)^{2n+1}}{n!(n+1)!};
\end{aligned} \qquad (1.31)$$

$$\lambda_1 = \left[\frac{\rho}{\lambda + 2\mu} \left(\frac{\partial^2}{\partial t^2} \right) - \frac{\partial^2}{\partial z^2} \right]^n; \;\; \lambda_1 = \left[\frac{\rho}{\mu} \left(\frac{\partial^2}{\partial t^2} \right) - \frac{\partial^2}{\partial z^2} \right]^n; \qquad n = 0,1,2,\ldots$$

$$Q_n = \sum_{k=1}^{n-1} \lambda_1^{(n-k-1)} \lambda_2^k; \qquad Q_0 \equiv 0; \qquad Q_1 \equiv 1; \quad q_1 = 1 - \frac{\lambda + 2\mu}{\mu}$$

Keltirib chiqarilgan (1.30) tenglamalar sistemasi d_i $(i = \overline{1,4})$ operatorlarning (1.31) ifodalariga ko'ra tartibi cheksiz bo'lgan integrodifferensial tenglamalar sistemasidan iboratdir.

Xuddi yuqoridagidek ishlarni ko'chishlar uchun ham takrorlab ushbu

$$\begin{aligned}
U_r(r,z,t) &= a_1 U_{r,0} + a_2 U_{z,0}; \\
U_z(r,z,t) &= a_3 U_{z,0} + a_4 U_{r,0}.
\end{aligned} \qquad (1.32)$$

fo'rmulaga ega bo'lamiz. Bu yerda

$$a_1 = \sum_{n=0}^{\infty} [\lambda_2 + \lambda_1 q_1 Q_n] \frac{(r/2)^{2n+1}}{n!(n+1)!}; \quad a_2 = \sum_{n=0}^{\infty} \lambda_1 q_1 Q_n \frac{\partial}{\partial z} \frac{(r/2)^{2n+1}}{n!(n+1)!};$$
$$a_3 = \sum_{n=0}^{\infty} [\lambda_1^n - \lambda_1 q_1 Q_n] \frac{(r/2)^{2n}}{(n!)^2}; \quad a_4 = \sum_{n=0}^{\infty} q_1 Q_n \frac{\partial}{\partial z} \frac{(r/2)^{2n}}{(n!)^2}. \tag{1.33}$$

Kuchlanishlardan faqat σ_{rr} va σ_{rz} lar uchungina formulalarni keltiramiz

$$\sigma_{rr} = \frac{1}{\mu} [d_1 U_{r,0} - d_2 U_{z,0}],$$
$$\sigma_{rz} = \frac{1}{\mu} [d_3 U_{r,0} + d_4 U_{z,0}], \tag{1.34}$$

Hosil qilingan (1.31) va (1.33) formulalardan amaliyotda va boshlang’ich shartlarni shakllantirishda foydalaniladi.

II BOB.

DOIRAVIY ELASTIK BALKANING MAJBURIY BO’YLAMA TEBRANISHLARINI AYLANISH INERSIYASI VA KO’NDALANG SILJISH DEFORMATSIYASINI HISOBGA OLIB TADQIQ ETISH.

Ushbu bob doiraviy qovushoq-elastik balka va sterjenlarning bo’ylama tebranishlarini uning aniqlashtirilgan tenglamasi asosida tatqiq etishga bag’ishlangan va Ko’ndalang kesimi doiraviysilindrik qobiqning bo’ylama tebranish umumiy tenglamalaridan, qobiqning ichki radiusi nolga teng bo’lgan xususiy holi uchun doiraviy qovushoq-elastik sterjenning umumiy tenglamalari keltirib chiqarilgan. Olingan umumiy tebranish tenglamalaridan xususiy hollarda klassik va aniqlashtirilgan tenglamalar keltirib chiqarilgan.Unda doiraviy elastik balkaning bo’ylama tebranishlari klassik va aniqlashtirilgan tenglamalari keltirib chiqarilgan. Shuningdek aniqlashtirilgan tenglamalarning turli xil ko’rinishlari muhokama etilgan. Aniqlashtirilgan tenglamalarning har bir hadi qanday mexanik ma’noga ega ekanligi ochib berilgan.Olingan tenglamalarning boshqa mualliflar tomonidan olingan shunday tenglamalar bilan taqqoslash natijalari bayon etilgan.Suyuqlik bilan ta’sirlashuvchi doiraviy silindrik sterjenning bo’ylama tebranishlarihaqidagi masalavauzunligi bir tomonlama chegaralangan va bir uchi qistirib mahkamlangan doiraviy balkaning bo’ylama tebranishlarida uning kuchlangan-deformatsiyalangan holatini aniqlash haqidagi masala yechilgan.

§2.1. Doiraviy elastik balkaning bo’ylama tebranishlari klassik va aniqlashtirilgan tenglamalari.

Oldingi paragrafda keltirib chiqarilgan (1.30) tenglamalar doiraviy qovushoq-elastik sterjenning bo’ylama tebranishi umumiy tenglamalar sistemasi hisoblanadi.Bu tenglamalar ko’chishlarning bosh qismlariga nisbatan keltirilib chiqarildi. Sterjenning sirti unga ta’sir etuvchi kuchlanishlardan xoli deb hisoblab, sterjen bo’ylama tebranishining klassik va aniqlashtirilgan tenglamalarini keltirib chiqaramiz.

Avvalo qovushoq-elastik sterjenning umumiy (1.30) tenglamasidan qayishqoq-elastik sterjen uchun klassik bo’ylama tebranish tenglamasiga yaqin, lekin undan umumiy bo’lgan tenglamani chiqaramiz. Buning uchun (1.31) formulada $n=0$ deb quyidagiga ega bo’lamiz

$$
\begin{aligned}
d_1 &= -q_1 + \left[(1-2q_1)\lambda_2 - (\lambda_1 - 2\lambda_2 + 2\frac{\partial^2}{\partial z^2})q_1\right]\frac{r_0}{8};\\
d_2 &= (1+q_1)\frac{\partial}{\partial z} + \frac{\partial}{\partial z}\left[2(1+q_1)\lambda_2 + (\lambda_1 - 2\lambda_2 + 2\frac{\partial^2}{\partial z^2})q_1\right]\frac{r_0}{8};\\
d_3 &= \frac{r_0}{2}(1+q_1)\frac{\partial}{\partial z} + \frac{r_0^3}{16}\frac{\partial}{\partial z}\left[\lambda_2 + q_1(\lambda_2+\lambda_1)\right] = \frac{r_0}{2}(1+q_1)\frac{\partial}{\partial z} + \frac{r_0^3}{16}\frac{\partial}{\partial z}\left[(1+q_1)\lambda_2 + q_1\lambda_1\right]\frac{\partial}{\partial z};\\
d_4 &= \frac{r_0}{2}(1-q_1)\lambda_1 + \frac{r_0^3}{16}\left[(1-q_1)\lambda_2 + 2q_1\frac{\partial^2}{\partial z^2}\right]\lambda_1.
\end{aligned}
\tag{2.1}
$$

bu yerda

$$
q_1 = 1 - \frac{\lambda+2\mu}{\mu} \qquad \lambda_1^n = \left[\frac{1}{a^2}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}\right]^n; \qquad n = 0.1.2.3...
$$

$$
\lambda_2^n = \left[\frac{1}{b^2}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}\right]^n; \qquad n = 0.1.2.3...
$$

r_0-sterjenningradiusi; $a=\sqrt{\frac{\lambda+2\mu}{\rho}}$- bo’ylamato’lqintarqalishtezligi; $b=\sqrt{\frac{\mu}{\rho}}$- ko’ndalangto’lqintarqalishtezligi; ρ-sterjenmaterialiningzichligi; λ,μ- sterjenmaterialiuchunLamekoeffitsiyentlari.

$$q_1 = 1 - \frac{\lambda + 2\mu}{\mu} = -\frac{\lambda + \mu}{\mu}; \qquad 1 - q_1 = 1 + \frac{\lambda + \mu}{\mu} = \frac{a^2}{b^2};$$
$$1 + q_1 = 1 - \frac{\lambda + \mu}{\mu} = -\frac{\lambda}{\mu}; \qquad 1 - 2q_1 = 1 + 2\frac{\lambda + \mu}{\mu} = \frac{2\lambda + 3\mu}{\mu};$$

Bundan keyin yozuvning soddaligi uchun (1.31) formuladan $U_{r.0} = U_r$, $U_{z,0} = U_z$ va $f_{rz} = 0$ deb hisoblaymiz. U holda qayishqoq-elastik sterjenning bo'ylama-radial tebranishlari umumiy tenglamalari sistemasi quyidagi ko'rinishga ega bo'ladi

$$\begin{cases} d_1 U_r - d_2 U_z = F; \\ d_3 U_r + d_4 U_z = 0; \qquad F = \dfrac{1}{\mu} M^{-1}(f_r), \end{cases} \tag{2.2}$$

bu yerda U_z, U_r - sterjen nuqtalarining bo'ylama va radial ko'chishlari; f_r - sterjen birlik sirtiga qo'yilgan tashqi ta'sir kuchlari.

Ushbu (2.2) tenglamalar sistemani yechish uchun uning asosiy determinantini hisoblaymiz. Buning uchun

$$\Delta_0 = d_1 d_4 + d_2 d_3 \tag{2.3}$$

ifodani hisoblash kerak. Bu ifodaga kirgan qo'shiluvchilarni alohida-alohida topamiz:

1)
$$d_{14} = \left\{ -q_1 + \frac{r_0^2}{8}\left[(1-2q_1)\lambda_2 - q_1(\lambda_1 - 2\lambda_2 + 2\frac{\partial^2}{\partial z^2})\right]\right\} *$$
$$* \left\{ \frac{r_0}{2}(1-q_1)\lambda_1 + \frac{r_0^3}{16}\left[(1-q_1)\lambda_2 + 2q_1\frac{\partial^2}{\partial z^2}\right]\lambda_1 \right\} =$$
$$= -\frac{r_0}{2} q_1(1-q_1)\lambda_1 + \frac{r_0^3}{16}\left[(1-2q_1)\lambda_2 - (\lambda_1 - 2\lambda_2 + 2\frac{\partial^2}{\partial z^2})q_1\right](1-q_1)\lambda_1 -$$
$$- \frac{r_0^3}{16} q_1 \left[(1-q_1)\lambda_2 + 2q_1\frac{\partial^2}{\partial z^2}\right]\lambda_1 + \frac{r_0^5}{128}\left[(1-2q_1)\lambda_2 - q_1(\lambda_1 - 2\lambda_2 + 2\frac{\partial^2}{\partial z^2})\right] *$$
$$* \left[(1-q_1)\lambda_2 + 2q_1\frac{\partial^2}{\partial z^2}\right]\lambda_1;$$

$$2)\qquad d_{23}=\left\{(1+q_1)\frac{\partial}{\partial z}+\frac{r_0^2}{8}\left[2(1+q_1)\lambda_2+(\lambda_1-2\lambda_2+2\frac{\partial^2}{\partial z^2})q_1\right]\frac{\partial}{\partial z}\right\}*$$

$$*\left\{\frac{r_0}{2}(1+q_1)\frac{\partial}{\partial z}+\frac{r_0^3}{16}[(1+q_1)\lambda_2+q_1\lambda_1]\frac{\partial}{\partial z}\right\}=$$

$$=\frac{r_0}{2}(1+q_1)^2\frac{\partial^2}{\partial z^2}+\frac{r_0^3}{16}(1+q_1)\left[2(1+q_1)\lambda_2+(\lambda_1-2\lambda_2+2\frac{\partial^2}{\partial z^2})q_1\right]\frac{\partial^2}{\partial z^2}+$$

$$+\frac{r_0^3}{16}(1+q_1)[(1+q_1)\lambda_2+q_1\lambda_1]\frac{\partial^2}{\partial z^2}+$$

$$+\frac{r_0^5}{128}\left[2(1+q_1)\lambda_2+(\lambda_1-2\lambda_2+2\frac{\partial^2}{\partial z^2})q_1\right][(1+q_1)\lambda_2+q_1\lambda_1]\frac{\partial^2}{\partial z^2};$$

Ushbu ifodalardagi oxirgi qo'shiluvchilarni yuqori tartibli cheksiz kichik miqdorlar sifatida tashlab yuboramiz va kerakli matematik amallarni bajarishda davom etamiz. Demak,

$$1)\qquad d_{14}=-\frac{r_0}{2}q_1(1-q_1)\lambda_1+\frac{r_0^3}{16}\lambda_1\left\{\begin{array}{l}(1-q_1)(1-2q_1)\lambda_2-q_1(1-q_1)(\lambda_1-2\lambda_2+\\+2\frac{\partial^2}{\partial z^2})-q_1(1-q_1)\lambda_2-2\frac{\partial^2}{\partial z^2}q_1^2\end{array}\right\}=$$

$$=-\frac{r_0}{2}q_1(1-q_1)\lambda_1+\frac{r_0^3}{16}\lambda_1\left\{\begin{array}{l}(1-q_1)(1-3q_1)\lambda_2-q_1(1-q_1)(\lambda_1-2\lambda_2)-\\-2q_1(1-2q_1)\frac{\partial^2}{\partial z^2}\end{array}\right\};\qquad(2.4)$$

$$2)\qquad d_{23}=-\frac{r_0}{2}\ (1+q_1)^2\frac{\partial^2}{\partial z^2}+\frac{r_0^3}{16}(1+q_1)\left\{\begin{array}{l}\left[2(1+q_1)\lambda_2+(\lambda_1-2\lambda_2+2\frac{\partial^2}{\partial z^2})q_1\right]+\\+(1+q_1)\lambda_2+q_1\lambda_1\end{array}\right\}=\qquad(2.5)$$

$$=\frac{r_0}{2}\ (1+q_1)^2\frac{\partial^2}{\partial z^2}+\frac{r_0^3}{16}(1+q_1)\left[3(1+q_1)\lambda_2+2(\lambda_1-\lambda_2+\frac{\partial^2}{\partial z^2})q_1\right]\frac{\partial^2}{\partial z^2}.$$

Endi (2.4) va (2.5) ifodalarni (2.3) tenglikka qo'yamiz. U holda

$$\Delta_0=\frac{r_0}{2}\left[-q_1(1-q_1)\lambda_1+(1+q_1)^2\frac{\partial^2}{\partial z^2}\right]+\frac{r_0^3}{16}\left\{\begin{array}{l}(1-q_1)(1-3q_1)\lambda_1\lambda_2-q_1(1-q_1)(\lambda_1^2-2\lambda_1\lambda_2)-\\-2q_1(1-2q_1)\lambda_1\frac{\partial^2}{\partial z^2}+3(1+q_1)^2\lambda_2\frac{\partial^2}{\partial z^2}+\\+2q_1(1+q_1)(\lambda_1-\lambda_2+\frac{\partial^2}{\partial z^2})\frac{\partial^2}{\partial z^2}\end{array}\right\}=\qquad(2.6)$$

$$=\frac{r_0}{2}\nabla_0+\frac{r_0^3}{16}\nabla_{1,}$$

bu yerda

$$\nabla_0 = -q_1(1-q_1)\lambda_1 + (1+q_1)^2\frac{\partial^2}{\partial z^2}; \tag{2.7}$$

$$\nabla_1 = -q_1(1-q_1)\lambda_1^2 + (1-q_1)^2\lambda_1\lambda_2 - \frac{\partial^2}{\partial z^2}\left[\begin{array}{l} 2q_1(1-2q_1)\lambda_1 - 3(1+q_1)^2\lambda_2 - \\ -2q_1(1+q_1)(\lambda_1-\lambda_2+\dfrac{\partial^2}{\partial z^2}) \end{array}\right] =$$

$$= -q_1(1-q_1)\left[\frac{1}{a^4}M^{-2}\frac{\partial^4}{\partial t^4} - \frac{2}{a^2}M^{-1}\left(\frac{\partial^4}{\partial z^2\partial t^2}\right) + \frac{\partial^4}{\partial z^4}\right] +$$

$$+(1-q_1)^2\left[\frac{1}{a^2b^2}M^{-2}\frac{\partial^4}{\partial t^4} - \left(\frac{1}{a^2}+\frac{1}{b^2}\right)M^{-1}\left(\frac{\partial^4}{\partial z^2\partial t^2}\right) + \frac{\partial^4}{\partial z^4}\right] - \tag{2.8}$$

$$-\frac{\partial^2}{\partial z^2}\left[\begin{array}{l} 2q_1(1-2q_1)\dfrac{1}{a^2}M^{-1}\left(\dfrac{\partial^2}{\partial t^2}\right) - 2q_1(1-2q_1)\dfrac{\partial^2}{\partial z^2} - 3\dfrac{(1+q_1)^2}{b^2}M^{-1}\left(\dfrac{\partial^2}{\partial t^2}\right) + \\ +3(1+q_1)^2\dfrac{\partial^2}{\partial z^2} + 2q_1(1+q_1)\left(\dfrac{1}{a^2}-\dfrac{1}{b^2}\right)M^{-1}\left(\dfrac{\partial^2}{\partial t^2}\right) - 2q_1(1+q_1)\dfrac{\partial^2}{\partial z^2} \end{array}\right];$$

yoki

$$\nabla_1 = \left[-\frac{q_1(1-q_1)}{a^4} + \frac{(1-q_1)^2}{a^2b^2}\right]M^{-2}\frac{\partial^4}{\partial t^4} + \left[\frac{2q_1(1-q_1)}{a^2} - \frac{(1-q_1)^2(b^2-a^2)}{a^2b^2}\right]*$$

$$*M^{-1}\left(\frac{\partial^4}{\partial z^2\partial t^2}\right) + \left[-q_1(1-q_1)+(1-q_1)^2\right]\frac{\partial^4}{\partial z^4} - \tag{2.9}$$

$$-\frac{\partial^2}{\partial z^2}\left\{\begin{array}{l} \left[\dfrac{2q_1(1-2q_1)}{a^2} - 3\dfrac{(1+q_1)^2}{b^2} + \dfrac{2q_1(1+q_1)}{a^2b^2}(b^2-a^2)\right]M^{-1}\left(\dfrac{\partial^2}{\partial t^2}\right) - \\ -\left[2q_1(1-2q_1) - 3(1+q_1)^2 + 2q_1(1+q_1)\right]\dfrac{\partial^2}{\partial z^2} \end{array}\right\}$$

Endi U_z ni aniqlash uchun $\begin{vmatrix} d_1 & F \\ d_3 & o \end{vmatrix} = e_{12}F$ ni hisoblaymiz va uni $\Delta_z(f_r)$ bilan belgilaymiz. U holda

$$\Delta_z(f_r) = \frac{r_0}{2\mu}(1+q_1)M^{-1}\left(\frac{\partial f_r}{\partial z}\right) + \frac{r_0^3}{16\mu}\left[(1+q_1)\lambda_2 + q_1\lambda_1\right]\underline{\overline{M}}^{-1}\left(\frac{\partial f_r}{\partial z}\right)$$

va demak (2.2) tenglamalar sistemasini yechish

$$\Delta_0 U_z = \Delta_z(f_r) \tag{2.10}$$

differensial tenglamani yechishga keltiriladi. Faraz qilaylik $f_r \equiv 0$ bo'lsin. U holda

$$\Delta_0 U_z = 0 \tag{2.11}$$

bir jinsli tenglamaga ega bo'lamiz.

Avvalo bu tenglamaning quyidagi xususiy ($\frac{r_0}{2}$ ning oldidagi koeffitsiyentni nolga tenglashtirilgan hol) holini qaraymiz:

$$-q_1(1-q_1)\lambda_1 U_z + (1+q_1)^2 \frac{\partial^2 U_z}{\partial z^2} = 0$$

yoki

$$-q_1(1-q_1)\frac{1}{a^2} M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right) + \left[q_1(1-q_1) + (1+q_1)^2\right] \frac{\partial^2 U_z}{\partial t^2} = 0 \qquad (2.12)$$

lekin

$$q_1 = \frac{b^2 - a^2}{b^2}; \quad 1 - q_1 = \frac{a^2}{b^2}; \quad 1 + q_1 = \frac{a^2 - 2b^2}{b^2};$$

$$(1+q_1)^2 + q_1(1-q_1) = -\frac{3a^2 - 4b^2}{b^2}$$

bo’lganligi uchun bu tenglamani

$$-\frac{q_1(1-q_1)}{a^2} = \frac{a^2(a^2-b^2)}{a^2 b^4} = \frac{(a^2-b^2)}{b^4}$$

ga bo’lib yuborib

$$\frac{\partial^2 U_z}{\partial t^2} - \frac{3a^2 - 4b^2}{a^2 - b^2} b^2 M\left(\frac{\partial^2 U_z}{\partial z^2}\right) = 0 \qquad (2.13)$$

tenglamaga ega bo’lamiz. Bu yerda $c^2 = \frac{3a^2 - 4b^2}{a^2 - b^2} b^2$ - bo’ylama to’lqinlarning sterjenlarda tarqalish klassik tezligidan iboratdir. Agar jism qayishqoq elastikmas balki elastik bo’lsa M=1 bo’ladi va (2.12) tenglama elastik sterjen tebranishlari klassik tenglamasiga aylanadi.

$$\frac{\partial^2 U_z}{\partial t^2} - \frac{3a^2 - 4b^2}{a^2 - b^2} b^2 \frac{\partial^2 U_z}{\partial z^2} = 0 \qquad (2.14)$$

Bundan keyin (2.7) ni quyidagi ko’rinishda ishlatamiz

$$\nabla_0 = -\frac{q_1(1-q_1)}{a^2}\left[M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right) - \frac{3a^2 - 4b^2}{a^2 - b^2} b^2 \frac{\partial^2 U_z}{\partial z^2}\right]$$

Lekin $\frac{3a^2 - 4b^2}{a^2 - b^2} = 2(1+\nu); \quad b^2 = \frac{\mu}{\rho}; \qquad va \qquad \mu = \frac{E}{2(1+\nu)}$ ekanligini hisobga olsak

$$\nabla_0 = -\frac{q_1(1-q_1)}{\rho a^2}\left[\rho M^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - E\frac{\partial^2}{\partial z^2}\right], \tag{2.15}$$

bu yerda E-sterjen materialining elastiklik moduli.

Endi ∇_1ni hisoblaymiz. Buning uchun ∇_1ning (2.6) chi ifodasini quyidagi ko'rinishda yozib olamiz

$$\nabla_1 = \frac{q_1(1-q_1)}{a^2}\left\{\frac{1-3q_1+q_1^2}{q_1 a^2}M^{-2}\left(\frac{\partial^4}{\partial t^4}\right) - \frac{6q_1^3+q_1^2-2q_1-3}{q_1(1-q_1)}M^{-1}\left(\frac{\partial^4}{\partial t^2 \partial z^2}\right) + \frac{7q_1^2-q_1+4}{q_1(1-q_1)}a^2\frac{\partial^4}{\partial z^4}\right\} \tag{2.16}$$

yoki

$$\nabla_1 = \frac{q_1(1-q_1)}{\rho a^2}\left\{\frac{a_1}{\mu}\rho^2 M^{-2}\left(\frac{\partial^4}{\partial t^4}\right) - (1+a_2)M^{-1}\left(\frac{\partial^4}{\partial t^2 \partial z^2}\right) + a_3\frac{\partial^4}{\partial z^4}\right\}, \tag{2.17}$$

bu yerda $a_1 = \dfrac{1-3q_1+q_1^2}{q_1(1-q_1)}$; $\quad a_2 = \dfrac{6q_1^3+2q_1^2-3q_1-3}{q_1(1-q_1)}$; $\quad a_3 = \dfrac{7q_1^2-q_1+4}{2q_1(1+\nu)}$;

Nihoyat (2.17) operatorni quyidagicha yozishimiz mumkin

$$\nabla_1 = \frac{q_1(1-q_1)}{\rho a^2}a_3\left\{\frac{a_4}{\mu}\rho^2 M^{-2}\left(\frac{\partial^4}{\partial t^4}\right) - (1-a_5)\rho M^{-1}\left(\frac{\partial^4}{\partial t^2 \partial z^2}\right) + aE\frac{\partial^4}{\partial z^4}\right\}, \tag{2.18}$$

bu yerda

$$q_1 = -\frac{1}{1-2\nu}; \quad 1-q_1 = \frac{2(1-\nu)}{1-2\nu}; \qquad \frac{3a^2-4b^2}{a^2-b^2} = 2(1+\nu);$$

$$a_4 = \frac{a_1}{a_3}; \qquad a_5 = 1 - \frac{2(1+\nu)(6q_1^3+q_1^2-2q_1-3}{(1-q_1)(7q_1^2-q_1+4)} \quad yoki$$

$$a_5 = \frac{-24\nu^4+4\nu^3+70\nu^2-27\nu+23}{-16\nu^3+34\nu^2-30\nu+12}:$$

Shunday qilib $\Delta_0 U_z = 0$ tenglamani (2.16) ga asosan umumiy holda quyidagicha yozishimiz mumkin ekan

$$\frac{r_0}{2}\nabla_0 U_z + \frac{r_0^3}{16}\nabla_1 U_z = 0 \tag{2.19}$$

yoki (2.15) va (2.18) larga asosan

$$\frac{r_0^3}{16}\left\{\frac{a_4}{\mu}\rho^2 M^{-2}\left(\frac{\partial^4 U_z}{\partial t^4}\right) - (\rho - a_5\rho)M^{-1}\left(\frac{\partial^4 U_z}{\partial t^2 \partial z^2}\right) + E\frac{\partial^4 U_z}{\partial z^4}\right\} - \frac{1}{a_3}\frac{r_0}{2}\left\{\rho M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right) - E\frac{\partial^2 U_z}{\partial z^2}\right\} = 0 \tag{2.20}$$

Tenglamani $\left(\pi r_0\right)$ ga ko'paytiramiz va $\pi r_0 = F$ - sterjen ko'ndalang kesimining yuzasi hamda $\frac{\pi r_0^2}{4} = I$ - sterjen ko'ndalang kesimi inertsiya momenti ekanligini hisobga olib oxirgi tenglamani quyidagicha yozamiz

$$EI\frac{\partial^4 U_z}{\partial z^4} - \frac{2}{a_3}\rho F M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right) + \frac{8}{a_3 r_0^2}EI\frac{\partial^2 U_z}{\partial z^2} - \left(\rho I - EI\frac{\rho}{\frac{2(1+\nu)}{a_5}\mu}\right)M^{-1}\left(\frac{\partial^4 U_z}{\partial t^2 \partial z^2}\right) +$$

$$+ a_4\frac{\rho^2 I}{\mu}M^{-2}\left(\frac{\partial^4 U_z}{\partial t^4}\right) = 0,$$

demak,

$$EI\frac{\partial^4 U_z}{\partial z^4} - \frac{2}{a_3}\rho F M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right) + \frac{8}{a_3 r_0^2}EI\frac{\partial^2 U_z}{\partial z^2} - \left(\rho I - EI\frac{\rho}{k_1\mu}\right)M^{-1}\left(\frac{\partial^4 U_z}{\partial t^2 \partial z^2}\right) +$$
$$+ \frac{\rho^2 I}{k_2\mu}M^{-2}\left(\frac{\partial^4 U_z}{\partial t^4}\right) = 0, \qquad (2.21)$$

bu yerda

$$k_1 = \frac{2(1+\nu)}{a_5}; \quad k_2 = \frac{1}{a_4}. \qquad (2.22)$$

Olingan (2.21) tenglamadan ko'rinadiki $\rho I M^{-1}\left(\frac{\partial^4 U_z}{\partial t^2 \partial z^2}\right)$ had aylanish inertsiyasini hisobga oladi, k_1 va k_2 koeffitsiyentlari bor hadlar ko'ndalang siljish deformatsiyasini, $\frac{1}{a_3}\rho F M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right)$ had bo'ylama inertsiya kuchlarining ta'sirini, $EI\frac{\partial^4 U_z}{\partial z^4}$ had bo'ylama normal kuchlanishlarni (taqsimlangan kuchlarni) hisobga oladilar. Nihoyat $\frac{8}{a_3 r_0^2}EI\frac{\partial^2 U_z}{\partial z^2}$ had esa vujudga keluvchi reaktiv momentlarni xarakterlaydi. Olingan (2.21) umumiy tenglamadan xususiy holda quyidagilar olinishi mumkin:

a)aylanish inertsiyasi hisobga olingan tenglama:

$$EI\frac{\partial^4 U_z}{\partial z^4}-\frac{2}{a_3}\rho M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right)-\frac{8}{a_3 r_0^2}EI\frac{\partial^2 U_z}{\partial z^2}+\rho I M^{-1}\left(\frac{\partial^4 U_z}{\partial t^2\partial z^2}\right)=0 \qquad (2.23)$$

b) ko'ndalang siljish deformatsiyasi hisobga olingan tenglama (reaktiv moment hisobga olinmagan)

$$EI\frac{\partial^4 U_z}{\partial z^4}-\frac{2}{a_3}\rho F M^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right)+\frac{\rho}{k_1\mu}EIM^{-1}\left(\frac{\partial^4 U_z}{\partial t^2\partial z^2}\right)+\frac{\rho^2 I}{k_2\mu}M^{-2}\left(\frac{\partial^4 U_z}{\partial t^4}\right)=0 \qquad (2.24)$$

c) sterjen elastik, aylanish inertsiyasi va reaktiv moment hisobga olinmagan hol.

Bunda (2.24) tenglamada M=1 deb hisoblash kifoya, ya'ni

$$EI\frac{\partial^4 U_z}{\partial z^4}-\frac{2}{a_3}\rho F\frac{\partial^2 U_z}{\partial t^2}+\frac{\rho}{k_1\mu}EI\frac{\partial^4 U_z}{\partial t^2\partial z^2}+\frac{\rho^2 I}{k_2\mu}\frac{\partial^4 U_z}{\partial t^4}=0 \qquad (2.25)$$

bu yerda

$$a_1=\frac{1-3q_1+q_1^2}{q_1(1-q_1)};\qquad a_3=\frac{7q_1^2-q_1+4}{2q_1(1+\nu)};\qquad a_5=\frac{-24\nu^4+4\nu^3+70\nu^2-27\nu+23}{-16\nu^3+34\nu^2-30\nu+12};$$

$$k_1=\frac{2(1+\nu)}{a_5};\qquad k_2=\frac{1}{a_4}$$

Oxirgi (2.25) tenglama qo'llanilganda 4 ta boshlang'ich va 4 ta chegaraviy shart qo'yilishi kerak.

Yuqorida d_i $(i=1.2.3.4)$ operatorlarni topish uchun (2.1) formulalarni chiqardik. Ular vositasida kuchlanishlar uchun formulalar quyidagi ko'rinishni oladi:

$$\sigma_{rr}(r,z,t)=M_0\left\{\begin{array}{l}-q_1U_r+\dfrac{r^2}{8}\left[(1-2q_1)\lambda-(\lambda_1-2\lambda_2+2\dfrac{\partial^2}{\partial z^2})q_1\right]U_r-\\ -(1+q_1)\dfrac{\partial U_z}{\partial z}+\dfrac{r^2}{8}\left[2(1+q_1)\lambda_2+(\lambda_1-2\lambda_2+2\dfrac{\partial^2}{\partial z^2})q_1\right]\dfrac{\partial U_z}{\partial z}\end{array}\right\};$$

$$\sigma_{rz}(r,z,t)=M_0\left\{\begin{array}{l}\dfrac{r_0}{2}\left[(1+q_1)\dfrac{\partial U_r}{\partial z}+(1-q_1)\lambda_1U_z\right]+\\ +\dfrac{r^3}{16}\left[\left(\lambda_1q_1+\lambda_2+\lambda_2q_1\right)\dfrac{\partial U_r}{\partial z}+(\lambda_2-\lambda_2q_1+2q_1\dfrac{\partial^2}{\partial z^2})\lambda U_z\right]\end{array}\right\} \qquad (2.26)$$

Xuddi shunday o'xshash ifodalarni ko'chishlar uchun ham olmoq uchun ularning ifodalarida $n=0$ deb hisoblash kerak. U holda

$$a_1 = \frac{r}{2}\lambda_2; \qquad a_2 = 0; \qquad a_3 = 1; \qquad a_4 = 0,$$

demak ,

$$\begin{aligned} U_r(r,z,t) &= \frac{r}{2}\lambda_2 U_r; \\ U_z(r,z,t) &= U_z. \end{aligned} \tag{2.27}$$

Amaliy masalalarni yechish uchun (2.23), (2.24) yoki (2.25) tenglamalardan birortasi ishlatilganda chegaraviy va boshlang'ich shartlarni shakllantirish hamda sterjen kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatini aniqlash uchun (2.26) va (2.27) formulalardan foydalanish zarur bo'ladi.

§2.2. Suyuqlik bilan ta'sirlashuvchi doiraviy silindrik sterjenning bo'ylama tebranishlari.

Doiraviy silindrik sterjenning uchiga qo'yilgan yuklanish ta'sirida uyg'otilgan bo'ylama deformatsiyalanish jarayonini tadqiq qilamiz.Bu jarayon sterjenning bir holatdan (tinch holatdan) ikkinchi (tebranish holatiga) holatga o'tish jarayoni sifatida muhim ahamiyatga ega. Odatda [2,24] qobiqlar,plastinalar, sterjenlar va bolkalardagi bunday o'tish jarayonlari haqidagi masalalarning yechimlari taqribiy tebranish tenglamalari asosida topiladi.Taqribiy tebranish tenglamalarining doiraviy silindrik sterjen va balkalardagi yuqoridagi ma'noda deformatsiyaning o'tish jarayoniga qo'llanilishi masalasi, uning asosiy muammolari va tahlili [14,23,24] ishlarda keltirilgan.

Qobiqlar, plastinalar, sterjenlar va balkalarning turli xil tebranish tenglamalari va ularni to'lqin dinamikasi masalalarini yechishda, deformatsiyaning o'tish jarayonlarini ham qo'shgan holda qo'llash tahlili [5,16,24] ishlarida keltirilgan. Bundan tashqari yana qator [24] ishlar mavjudki, ularda silindrik jismlardagi deformatsiyaning o’tish jarayoni haqidagi masalalarning yechimlari turli taqribiy nazariyalarni jalb qilish asosida topilgan.

Yuqorida aytilganlardan kelib chiqqan holda doiraviy silindrik sterjenning bo’ylama tebranishlari haqidagi masalani, unga ta’sir etuvchi suyuqlikni hisobga olgan holda, yechishda asosiy tebranish tenglamalari sifatida I-bobda ishlab chiqilgan va ushbu bobning birinchi paragrafida soddalashtirilgan variantlari keltirilgan tenglamalardan foydalanamiz.Bunday tenglamani keltirib chiqarish uchun asosiy tenglama sifatida (1.30) tenglamani va uning tarkibidagi (1.31) differensial aperatorlarni qabul qilamiz. Masalani yechishda qo’llaniladigan tenglamani (1.30) tenglamalar sistemasidan keltirib chiqarish uchun ushbu bobning birinchi paragrafidagi $m=0$ usuldan foydalanamiz. Buning uchun (1.31) formulalardan $m=0$ bo’lgan hol bilan chegaralanamiz va d_i $(i=1,2,3,4)$ differensial operatorlarning (2.1) qiymatlarini (1.30) ga qo’yib quyidagi sistemani hosil qilamiz.

$$\begin{cases} \frac{2-2\nu}{1-2\nu}(c^2\cdot\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2})u+2v=0 \\ (c^2\cdot\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2})u-\frac{1-2\nu}{2(1-\nu)}(1-\frac{\partial^2}{\partial z^2})v=0 \end{cases} \quad (2.28)$$

bu yerda (1.30) dagi $u_{z,0}$ va $u_{z,1}$ lar o’rniga u va v belgilar qo’llaniladi va u-bo’ylama ko’chish, v-balka o’qidan uzoqlikdagi nuqtalarning bo’ylama ko’chishlari; ν-puasson koeffitsiyenti; $c^2=\frac{b^2}{a^2}$; b-sterjen materialida ko’ndalang to’lqinlar tarqalish tezligi;

a-sterjen materialida bo’ylama to’lqinlar tarqalish tezligi;

t-vaqt; z-bo’ylama koordinata;

Sterjen sirtida ta’sir etuvchi tashqi kuchlanishlar funksiyalari $f_r(z,t)$ va $f_{rz}(z,t)$ nolga teng deb hisoblanadi.

Bundan tashqari bo'ylama to'lqinlar tarqalish jarayonida sterjenning ko'ndalang kesimi o'zgarmasdan qoladi deb hisoblanadi, ya'ni unda radial ko'chishlar paydo bo'lmaydi deb hisoblanadi.Bunday holat, masalan sterjen uchiga o'ta bikr, deformatsiyalanmaydigan xalqa kiydirish yo'li bilan erishilishi mumkin.

Sterjenga ta'sir etayotgan suyuqlikni qovushoq suyuqlik deb hisoblaymiz. U holda uning ta'siri [23] tatqiqot natijalariga ko'ra

$$D = \frac{b^2 \cdot \rho_0'}{r_0 \mu} \frac{a_0^2}{b^2} \left(\frac{\partial^2}{\partial z^2} + \frac{4\nu'}{3br_0} \frac{\partial^3}{\partial t \partial z^2} - \frac{\partial^2}{\partial t^2} \right) \qquad (2.29)$$

differensial operator yordamida hisobga olinishi mumkin. Bu yerda

ρ_0'-suyuqlik zichligi;

r_0-balka yoki sterjenning ko'ndalang kesimi radiusi;

μ-sterjen materialining siljish koeffitsienti;

ν' - suyuqlikning kinematik qovushoqlik koeffitsienti;

a_0-suyuqlikda tovushning tarqalish tezligi.

Ushbu mulohazalarni hisobga olganda (2.28) sistemani,ba'zi matematik soddalashtirishlardan so'ng quyidagicha yozish mumkin:

$$\left(\frac{2-2\nu}{1-2\nu} \lambda - D \right) U - 2 \left[1 + D \frac{b^2 \rho_0'}{2r_0 \mu} \frac{\partial^2}{\partial t^2} \right] V = 0 \qquad (2.30)$$

$$\lambda U - \frac{1-2\nu}{2(1-\nu)} V = 0,$$

bu yerda λ-differensial operator

$$\lambda = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial t^2} \qquad (2.31)$$

Suyuqlikning sterjen tebranishlariga reaksiyasining ifodasi bo'lgan (2.29) fo'rmulada va (2.30) tenglamalarda o'lchamsiz o'zgaruvchilar quyidagi fo'rmulalar bilan kiritilgan:

$$r^* = \frac{r}{r_0};\ t^* = \frac{bt}{r_0};\ U^* = \frac{U}{z};\ V^* = V;\ z^* = \frac{z}{z_0}.$$

Suyuqlik reaksiyasining (2.29) ifodasidagi ikkinchi had suyuqlik reaksiyasining qovushoqlik hisobidan paydo bo'lgan. Bunda, suyuqliklarning ko'pchiligi uchun qovushoqlikning kinematik koeffitsienti 10^{-4} va undan pastroq tartibga ega. Bundan tashqari ushbu qo'shiluvchining maxrajida Lame koeffitsiyentining mavjudligi va uning tartibi 10^{10} va undan yuqori ekanligini hisobga olsak, ushbu hadning suyuqlik reaksiyasiga qo'shadigan hissasi juda kam ekanligini ko'rish mumkin. Shu sababli xuddi ana shu hadni tashlab yuborish mumkin deb hisoblaymiz.

Shunday qilib quyidagi masalani yechish talab etiladi.

"Tebranish jarayoni (2.30) tenglamalar sistemasi bilan xarakterlanuvchi, doiraviy silindrik, sterjen uchiga to'satdan qo'yilgan bo'ylama ko'chish natijasida vujudga kelgan bo'ylama deformatsiya jarayoni tadqiq etilsin. Sterjen ixtiyoriy kesimidagi bo'ylama ko'chishlar va kuchlanishlar aniqlansin. Bundan sterjen uchiga ensiz, deformatsiyalanmaydigan xalqa kiygizilgan deb hisoblansin".

Qo'yilgan masala shartlariga asosan

$z = 0$ bo'lganda $U = g(t)$; $z \to \infty$ bo'lganda $U = 0$, (2.32)

bu yerda $g(t)$-berilgan ko'chishning impulsi.

Masalaning boshlang'ich shartlarini no'lga teng deb hisoblaymiz.

Chegaraviy shartlarning berilishidan ko'rinib turibdiki (2.30) tenglamalar sistemasidan ikkinchi funksiyani chiqarish kerak. Buning uchun (2.30) ning ikkinchi tenglamasidan

$$V=\frac{1}{2}(1-q_1)\lambda U \tag{2.33}$$

bu yerda

$$q_1=-\frac{\lambda+\mu}{\mu}.$$

Bu yerda va bundan keyin ham, xuddi chiziqli elastiklik nazariyasida bo'lgani kabi, sterjenning bo'ylama ko'chishlarini kichik ko'chishlar deb hisoblaymiz. U holda U – bo'ylama ko'chishning nafaqat o'zi, balki uning yuqori tartibli hisoblari (4-chi va 5-chi tartibli hosilalari) ham juda kichik miqdorlar bo'ladilar. Ushbu holatni hisobga olsak tebranish tenglamalari sistemasi ushbu

$$\frac{\partial^2 u}{\partial t^2}-2\frac{1-\nu}{1-2\nu}\frac{\partial^2 u}{\partial z^2}+\frac{1}{r^2-1}\left[\alpha_1\frac{\partial^2}{\partial z^2}-\alpha_2\frac{\partial^2}{\partial t^2}\right]U=0,$$

ko'rinishga keladi. Bu yerda

$$\alpha_1=\frac{\alpha_0^2\rho_0'}{r_0\mu};\ \alpha_2=\frac{b^2\rho_0'}{r_0\mu}.$$

quyidagicha belgilashlar kiritamiz.

$$b_0^2=\left[\frac{1-2\nu}{2(1-\nu)}\right]^{-1};\ a_1=\frac{\alpha_1}{r^2-1};\ a_2=\frac{\alpha_2}{r^2-1}-1.$$

U holda oxirgi tenglama quyidagi ko'rinishni oladi:

$$(a_1-b_0^2)\frac{\partial^2 u}{\partial z^2}=a_2\frac{\partial^2 u}{\partial t^2} \tag{2.34}$$

Shunday qilib shakllantirilgan masalani yechish (2.34)-differensial tenglamani (2.32) chegaraviy va nolga boshlang'ich shartlarda integrallashga keltirildi.

Oxirgi (2.34) tenglama va (2.32) shartlarga Laplas almashtirishini t-vaqt bo'yicha qo'llaymiz. Boshlang'ich shartlarning no'lga teng ekanligi, yani

$t=0$ bo'lganda $u=\frac{\partial u}{\partial t}=0.$

ekanligidan foydalanib quyidagilarga ega bo'lamiz:

$$\frac{d^2\overline{u}}{dz^2}=\frac{a_2 p^2}{a_1-b_0^2}\overline{u} \qquad (2.35)$$

$z=0$ bo'lganda $\overline{u}=G(p);$

$z\to\infty$ bo'lganda $\overline{u}=0.$ (2.36)

bu yerda $u\div\overline{u}$; $g(t)\div G(p)$; p -almashtirish parametri.

Laplas almashtirishi qo'llanilgan (2.35) tenglamaning umumiy yechimini

$$\overline{u}=Ae^{-kz} \qquad (2.37)$$

ko'rinishda izlaymiz. Bu yerda A-const, ko'chish amplitudasi.

Endi (2.37) ni (2.35) ga qoyib va Ae^{-kz} ga qisqartirib

$$k=\pm\sqrt{\frac{a_2 p}{a_1-b_0^2}} \qquad (2.38)$$

qiymatlarga ega bo'lamiz. Ammo, chegaraviy shartlarning (2.36) ifodasiga ko'ra. $z\to\infty$ bo'lganda $\overline{u}=0$ bo'lishi kerak.Ana shuni hisobga olsak (2.37) yechimda

$$k=\sqrt{\frac{a_2 p}{a_1-b_0^2}}$$

ildizni tashlab yuborishga to'g'ri kelishini ko'rish qiyin emas. Ushbu holatni va (2.33) tenglikni hisobga olsak

$$\overline{V}=\frac{r^2}{2}\left(p^2-b_0^2\frac{d^2}{dz^2}\right)\overline{u}$$

Ifodaga ega bo'lamiz. Endi (2.38) da faqat manfiy ildiz bilan cheklanib (2.37) umumiy yechim asosida

$$\overline{V}=\frac{r^2}{2}\frac{a_1 b_0^2(1+a_2)}{a_1-b_0^2}p^2 A e^{-\sqrt{\frac{a_2}{a_1-b_0^2}}p\cdot z}$$

ifodaga ega bo'lamiz. U holda $\bar{u}$-almashtirilgan ko'chish uchun

$$\bar{u}=Ae^{-\sqrt{\frac{a_2}{a_1-b_0^2}}p\cdot z}-\frac{r^2}{4}\frac{a_1 b_0^2(1+a_2)}{a_1-b_0^2}p^2 A e^{-\sqrt{\frac{a_2}{a_1-b_0^2}}p\cdot z} \qquad (2.39)$$

formulani chiqarish qiyin emas.

Olingan yechimni (2.36) chegaraviy shartlarning birinchisiga qo'yib (ikkinchisi biz ishlatib bo'ldik) A-integrallash o'zgarmasi uchun

$$A=\frac{a_3^2 G(p)}{p^2+a_3^2}$$

ifodani olamiz. Bu yerda

$$a_3^2=-\frac{4(a_1-b_0^2)}{\left[a_1-b_0^2(1+a_2)\right]}$$

belgilash kiritamiz $\beta^2=\dfrac{a_2}{a_1-b_0^2}$

u holda A ning yuqoridagi qiymatini $\bar{u}$ ning (2.39) formulasiga qo'yib, oxirgi belgilashni hisobga olgan holda $\bar{u}=\dfrac{a_3^2 G(p)}{p^2+a_3^2}e^{-\beta p z}$

oxirgi ifodani ariginalini topish qiyin emas. Bu ariginal $G(p)$ va $\dfrac{e^{-\beta p z}}{p^2+a_3^2}$

tablitsa ariginallari integral tugunidan iborat bo'ladi, ya'ni

$$u = a_3 \int_{\beta z}^{t} g(\xi - \beta z) \sin[a_3(t-\xi)] d\xi \qquad (2.40)$$

u holda V funksiya uchun

$$V = 2a_3 \int_{\beta z}^{t} g(\xi - \beta z) \sin[a_3(t-\xi)] d\xi - 2g(t-\beta z) \qquad (2.41)$$

formulaga ega bo'lamiz.

Endi topilgan(2.40) va (2.41) formulalar yordamida $\sigma_{zz}(z,t)$ va $\sigma_{rr}(z,t)$ kuchlanishlarni va umumiy bo'ylama u_z-ko'chishni aniqlash qiyin emas.Buning uchun yana (1.31) formulalarga murojat qilamiz. Ularga ko'ra

$$\sigma_{zz}(z,t) = \mu\left\{(1-q_1)\frac{\partial u}{\partial z} + \frac{1-2q_2}{2}\frac{\partial (r_0 v)}{\partial z}\right\};$$

$$\sigma_{rr}(z,t) = \frac{r_0\mu}{2}\left\{(1-q_1)\lambda u - \frac{2}{t^2}(r_0 v)\right\}, \qquad (2.42)$$

$$q_2 = -\frac{1}{(1-2\nu)}.$$

Ushbu formulalarda o'lchamsiz koordinatalarga paragraf boshida kiritilgan formulalar orqali o'tamiz. Olingan ifodalarda u va v lar o'rniga ularning (2.40) va (2.41) qiymatlarini qo'yib, kuchlanishlar uchun

$$\sigma_{zz}(z,t) = q_1 a_3 \beta\mu\left[\int_{\beta z}^{t} g'(\xi-\beta z)\sin[a_3(t-\xi)]d\xi + \frac{g(0)}{a_3}\sin[a_3(t-\beta z)]\right] - 2(1-2q_2)\beta g'(t-\beta z);$$

$$\sigma_{rr}(z,t) = \beta\mu g'(t-\beta z) + q_1\beta\mu a_3\left[\int_{\beta z}^{t} g'(\xi-\beta z)\sin a_3(t-\xi)d\xi + \frac{g(0)}{a_3}\sin a_3(t-\beta z)\right]. \qquad (2.43)$$

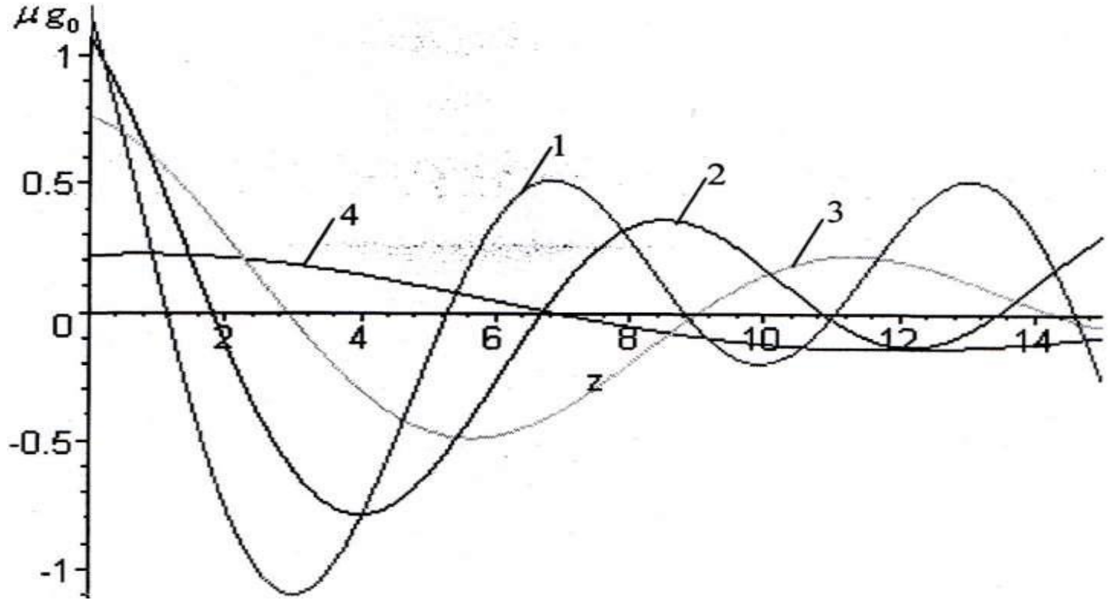
formulalarniolamiz.

Masalani yechish pirovardida olingan natijalar (2.40),(2.41) va (2.43) formulalar ko'rinishida qayd etiladi. Ushbu formulalar yordamida ko'chish va kuchlanishlarni sterjenning ixtiyoriy kesimidagi istalgan nuqta uchun hisoblash mumkin. Boshqacha aytganda yuqorida olingan analitik natijalar sterjen kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatini kvadratura topishga imkon beradi.

Sterjen kesimlaridagi kuchlanishlarning vaqt va koordinata bo'yicha o'zgarishlarini sonli tadqiq qilish uchun ular (2.43) formulalar bo'yicha hisoblandi. Ushbu hisoblashlarda sterjen kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatiga suyuqlikning va uning zichligining ta'sirini o'rganishga alohida e'tibor berildi.

Sterjen materiali elastik xarakteristikalari $\nu = 0.3$ va $E = 2 \cdot 10^{11} pa$ bo'lgan po'lat olindi.

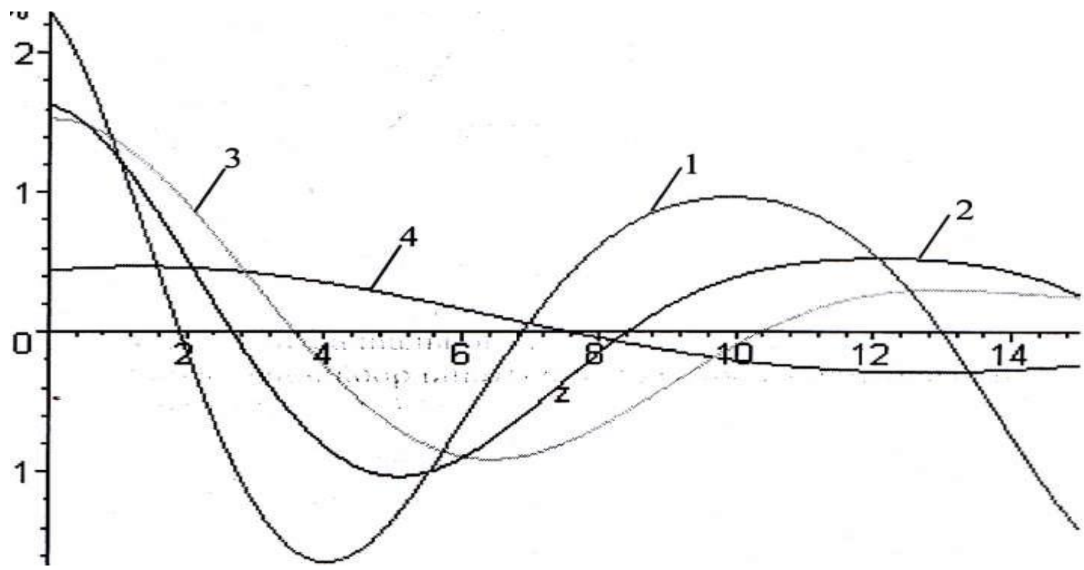
Suyuqlikning o'lchamsiz zichligi 200 dan 500 gacha o'zgaradi deb hisoblandi.Shuningdek suyuqlikda (tinch holatdagi) tarqalish tezligi $a_0 = 1500 \frac{metr}{sekund} = 1500 \frac{m}{s}$ ga teng deb olinadi.Tashqi yuklama $g(t) = g_0 \sin(\frac{\pi t}{t_1})$ qonuniyat bo'yicha o'zgaradi deb hisoblanadi.



2.1-chizma

$$"1"-\rho_0' = 200;"2"-\rho_0' = 700;"3"-\rho_0' = 1100;"4"-\rho_0' = 1500;$$

$t = 5,\ t_1 = 2$ bo'lganda σ_{rr} ning koordinata bo'yicha o'zgarishlari



2.2-chizma

$$"1"-\rho_0' = 200;"2"-\rho_0' = 700;"3"-\rho_0' = 1100;"4"-\rho_0' = 1500;$$

$t = 5,\ t_1 = 2$ bo'lganda σ_{zz} ning koordinata bo'yicha o'zgarishlari

Hisoblash natijalar 2.1-2.2 chizmalarda keltirilgan. Xususan 2.1-2.2-chizmalarda narmal radial va bo'ylama kuchlanishlarning, vaqtning fiksirlangan $t=0$ payti uchun $t_1=2$,bo'ylama koordinatadan bog'lanish grafiklari keltirilgan. Keltirilgan bog'lanishlardan ko'rinadiki, σ_{zz} va σ_{rr} kuchlanishlar ta'sirlashuvchi suyuqlikning zichligidan kuchli ravishda bog'liq. Masalan, $z=0,2$ kesimda σ_{rr} kuchlanishning (2.1-chizma) $\rho_0'=200$ bo'lgandagi qiymati $\rho_0'=1500$ bo'lgandagi qiymatidan 5 barobarga yaqin katta bo'lgani holda, $\rho_0'=1000$ bo'lgandagi qiymatlarini 50% dan ko'proqqa katta bo'ladi.

Bundan tashqari suyuqlikning tasiri natijasida σ_{rr} kuchlanish, katta kinematik qovushoqlikni hisobga olmagan taqdirda ham bo'ylama koordinata o'sib borishi bilan juda tez so'nadi.Xuddi shunday holat normal bo'ylama kuchlanishlar uchun ham kuzatiladi.(2.2-chizma).Ammo,bu yertda suyuqlik zichligining ta'siri yanada kuchliroq nomoyon bo'ladi.Masalan sterjenning $z=0$ kesimida σ_{zz} kuchlanish $\rho_0'=200$ bo'lganda o'zining $\rho_0'=1500$ bo'lgandagi qiymatidan deyarli 10 barobarga katta bo'ladi.Xuddi shunday holatda normal radial kuchlanishda bu ko'rsatkich 5 ga teng. Xuddi shunday $\rho_0'=200$ va $\rho_0'=1000$ bo'lgan hollarda qiymatlar 72% (53%ga qarama-qarshi)ga farq qiladilar. Bo'ylama kuchlanishlarning kardinata bo'yicha so'nishi yanada kuchliroq kuzatiladi.Masalan $z=13$ kesimdagi σ_{zz} kuchlanish $z=0$ kesimga nisbatan qariyb 5 barobar (σ_{rr} dagi 4 ga qarama-qarshi) kamayadi.

Shunday qilib quyidagi xulosalarni chiqarish mumkin.Suyuqlik bilan ta'sirlashuvchi doiraviy elastik sterjenning bo'ylama tebranishlari haqidagi masalaning sonli yechish natijalari quyidagilarni ko'rsatdi.

- σ_{zz} va σ_{rr} kuchlanishlar ta'sirlashuvchi suyuqlik zichligidan kuchli bog'liq suyuqlikning zichligi oshib borishi bilan kuchlanishlarning qiymatlari bir necha barobar, hatto bitta tartibgacha kamayadi;

- bu kuchlanishlar koordinata bo'yicha intensiv so'nadilar. Bunda bo'ylama kuchlanishlardagi so'nish darajasi radial kuchlanishlardagiga nisbatan ancha qariyb 2 barobar, kichik.

Paragraf nihoyasida xuddi shunday masalani elastik silindrik qobiqlar uchun B.F.Yalg'ashev [18] yechganligini takidlaymiz.Olingan natijalar, bir-biriga juda yaqin ekanligini, hatto ba'zi hollarda ustma-ust tushushini e'tirof etamiz.

§2.3. Bir uchi qistirib mahkamlangan doiraviy balkaning bo'ylama tebranishlari.

Ushbu paragrafda uzunligi bir tomonlama chegaralangan va bir uchi qistirib mahkamlangan doiraviy balkaning bo'ylama tebranishlarida uning kuchlangan-deformatsiyalangan holatini aniqlash haqidagi masala yechiladi.

Tashqi yuklanish bo'lmagan holda, ya'ni doiraviy silindrik sterjenning sirti tashqi yuklardan xoli bo'lgan holda va Puasson koeffitsiyenti o'zgarmas bo'lganda, ya'ni

$$\left(M = \mu\tilde{M}, \quad L = (\lambda + 2\mu)\tilde{M}, \quad \tilde{M} = 1 - \int_0^t k(t-\tau)d\tau \right)$$

tengliklar o'rinli bo'lganida, bunday balkaning tebranish tenglamalari sistemasi, [3] ishnatijalarigaquyidagi ko'rinishda bo'ladi

$$\tilde{M}^{-1}\left(\frac{\partial^3 U_{z,0}}{\partial t^2 \partial z}\right) - \frac{2(1-\nu)}{1-2\nu}\frac{\partial^3 U_{z,0}}{\partial z^3} - q_1\frac{\partial^2 U_{r,0}}{\partial z^2} + \frac{1}{r_i^2}\frac{\partial U_{z,1}}{\partial z} -$$

$$-q_1 d_2\left(\ln r_i - \frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}\left(\tilde{M}^{-1}\left(\frac{\partial^4 U_{r,1}}{\partial t^2 \partial z^2}\right) - \frac{\partial^4 U_{r,1}}{\partial z^4}\right) + \frac{1}{r_i^2}\frac{\partial^2 U_{r,1}}{\partial z^2}$$

$$+\left(\ln r_i - \frac{1}{2}\right)\left(\tilde{M}^{-1}\left(\frac{\partial^3 U_{z,1}}{\partial z \partial t^2}\right) - \frac{\partial^3 U_{z,1}}{\partial z^3}\right) = 0; \qquad (i = 1, 2) \tag{2.44}$$

$$\frac{2\nu}{1-2\nu}\cdot\frac{\partial U_{z,0}}{\partial z} + \frac{1}{1-2\nu}\cdot U_{r,0} - \left(\ln r_i + \frac{1}{2}\right)\frac{\partial U_{z,1}}{\partial z} - \frac{2U_{r,1}}{r_i^2} +$$

$$+\left(\frac{\ln r_i}{1-2\nu}+\frac{1}{2}\right)\cdot\frac{1-2\nu}{2(1-\nu)}\left(d_2\tilde{M}^{-1}\left(\frac{\partial^2 U_{r,1}}{\partial t^2}\right)-\frac{\partial^2 U_{r,1}}{\partial z^2}\right)=0,\ (i=1,2)$$

Bu yerda ν - Puasson koeffitsiyenti; o'lchamsiz kattaliklar (2.31) formulalar bilan kiritilgan;

$$q_1=\frac{1-2\nu}{2\nu}-\frac{4(1-\nu)^2}{1-2\nu};\ d_1=\frac{1}{(1+c_2)^2(1+a_0)^2};\ d_2=\frac{1}{(1+c_2)^3(1+a_0)}.$$

Balka kesimlarining kuchlangan-deformatsiyalangan holati quyidagi formulalar bilan aniqlanadi

$$U_z=\frac{1+c_2}{1+a_0}\left(U_{z,0}+\ln r U_{z,1}\right);$$

$$U_r=\frac{1}{r}U_{r,1}+\frac{r}{2}\left[U_{r,0}-G_1\lambda_4\lambda_1^{-1}\frac{\partial U_{z,0}}{\partial z}\right]+\left(\ln r-\frac{1}{2}\right)\left[\lambda_1 U_{r,1}-\frac{\partial}{\partial z}G_1 U_{z,1}\right]\frac{r}{2};$$

$$\sigma_{rz}\cdot\tilde{M}^{-1}=(1+c_2)\left\{\frac{r}{2}\cdot\left[(1+c_2)^2\tilde{M}^{-1}\left(\frac{\partial^2 U_{z,0}}{\partial t^2}\right)-\frac{2(1-\nu)}{1-2\nu}\cdot\frac{\partial^2 U_{z,0}}{\partial z^2}\right]-\frac{r}{2}q_1\cdot\frac{\partial U_{r,0}}{\partial z}+\right.$$

$$+\frac{r}{2}\left(\ln r-\frac{1}{2}\right)\left[(1+c_2)^2\cdot\tilde{M}^{-1}\left(\frac{\partial^2 U_{z,1}}{\partial t^2}\right)-q_2\frac{\partial^2 U_{z,1}}{\partial z^2}\right]+\frac{1}{r}U_{z,1}-\frac{r}{2}\left(\ln r-\frac{1}{2}\right)q_1\times$$

$$\left.\times\left[(1+c_2)(1+a_0)\frac{1-2\nu}{2(1-\nu)}\cdot\tilde{M}^{-1}\left(\frac{\partial^3 U_{r,1}}{\partial t^2\partial z}\right)-\frac{1-2\nu}{2(1-\nu)}\cdot\frac{\partial^3 U_{r,1}}{\partial z^3}\right]+\frac{1}{r}\cdot\frac{\partial U_{r,1}}{\partial z}\right\};\tag{2.45}$$

$$\sigma_{rr}=\tilde{M}(1+c_2)\left\{\frac{2\nu}{1-2\nu}\cdot\frac{\partial U_{z,0}}{\partial z}+\frac{1}{1-2\nu}\cdot U_{r,0}-\left(\ln r+\frac{1}{2}\right)\frac{\partial U_{z,1}}{\partial z}-\right.$$

$$\left.-\frac{2}{r^2}\cdot U_{r,1}+\left(\frac{\ln r}{1-2\nu}+\frac{1}{2}\right)\cdot\frac{1-2\nu}{2(1-\nu)}\left[(1+c_2)(1+a_0)\tilde{M}^{-1}\left(\frac{\partial^2 U_{r,1}}{\partial t^2}\right)-\frac{\partial^2 U_{r,1}}{\partial z^2}\right]\right\},$$

$$\lambda_1=\tilde{M}^{-1}(1+c_2)(1+a_0)\cdot\frac{1-2\nu}{2(1-\nu)}\cdot\frac{\partial^2}{\partial t^2}-\frac{1-2\nu}{2(1-\nu)}\cdot\frac{\partial^2}{\partial z^2}.$$

Masalaning boshlang'ich shartlari nolga teng. Balkaning uchlaridagi chegaraviy shartylar quyidagi ko'rinishga ega

$$z=0\ \text{bop'lganda}\ U_z=f(t),\quad U_r=0,\tag{2.46}$$

$$z\to\infty\ \text{bo'lganda}\ U_{z,0}=U_{z,1}=0,\ U_{r,0}=U_{r,1}=0$$

Masalani yechish uchun Laplas almashtirishlarini qo'llaymiz. Shu maqsadda (2.44) tenglamalar sistemasiga Laplas almashtirishlarini qo'llash natijasida ushbu, almashtirishlardagi tenglamalar sistemasiga ega bo'lamiz

$$d_1Q^2(p)\cdot\frac{\partial\overline{U}_{z,0}}{\partial z}-\frac{2(1-\nu)}{1-2\nu}\cdot\frac{\partial^3\overline{U}_{z,0}}{\partial z^3}-q_1\frac{\partial^2\overline{U}_{r,0}}{\partial t^2}+\left(\ln r_1-\frac{1}{2}\right)\left[d_1\cdot Q^2(p)\cdot\frac{\partial\overline{U}_{z,1}}{\partial z}-\frac{\partial^3\overline{U}_{z,1}}{\partial z^3}\right]+$$

$$+\frac{1}{r_1^2}\frac{\partial\overline{U}_{z,1}}{\partial z}-\left(\ln r_1-\frac{1}{2}\right)q_1\frac{1-2\nu}{2(1-\nu)}\left[d_2Q^2(p)\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}-\frac{\partial^4\overline{U}_{z,1}}{\partial z^4}\right]+\frac{1}{r_1^2}\frac{\partial\overline{U}_{r,1}}{\partial z^2}=0,$$

$$d_1Q^2(p)\cdot\frac{\partial\overline{U}_{z,0}}{\partial z}-\frac{2(1-\nu)}{1-2\nu}\cdot\frac{\partial^3\overline{U}_{z,0}}{\partial z^3}-q_1\frac{\partial^2\overline{U}_{r,0}}{\partial t^2}+\left(\ln r_2-\frac{1}{2}\right)\left[d_1\cdot Q^2(p)\cdot\frac{\partial\overline{U}_{z,1}}{\partial z}-\frac{\partial^3\overline{U}_{z,1}}{\partial z^3}\right]+$$

$$+\frac{1}{r_2^2}\frac{\partial\overline{U}_{z,1}}{\partial z}-\left(\ln r_2-\frac{1}{2}\right)q_1\frac{1-2\nu}{2(1-\nu)}\left[d_2Q^2(p)\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}-\frac{\partial^4\overline{U}_{z,1}}{\partial z^4}\right]+\frac{1}{r_2^2}\frac{\partial\overline{U}_{r,1}}{\partial z^2}=0, \quad (2.47)$$

$$\frac{2\nu}{1-2\nu}\cdot\frac{\partial U_{z,0}}{\partial z}+\frac{1}{1-2\nu}\overline{U}_{r,0}-\left(\ln r_1+\frac{1}{2}\right)\frac{\partial\overline{U}_{z,1}}{\partial z}-\frac{2}{r_1^2}\cdot\overline{U}_{r,1}+$$

$$+\left(\ln r_1\frac{1}{1-2\nu}+\frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}\left\{Q^2(p)\overline{U}_{r,1}-\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}\right\}=0;$$

$$\frac{2\nu}{1-2\nu}\cdot\frac{\partial U_{z,0}}{\partial z}+\frac{1}{1-2\nu}\overline{U}_{r,0}-\left(\ln r_2+\frac{1}{2}\right)\frac{\partial\overline{U}_{z,1}}{\partial z}-\frac{2}{r_2^2}\cdot\overline{U}_{r,1}+$$

$$+\left(\ln r_2\frac{1}{1-2\nu}+\frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}\left\{Q^2(p)\overline{U}_{r,1}-\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}\right\}=0;$$

Buyerda

$$Q^2(p)=\frac{p^2}{1-\overline{f}_{20}(p)};\quad \overline{f}_{20}(p)\div f_{20}(t);\ \overline{U}_{r,1}=U_{r,1};\quad \overline{U}_{z,1}=U_{z,1};\quad \overline{U}_{r,0}=U_{r,o}.$$

Yuqoridagi (2.47) tenglamalar sistemasining birinchi va ikkinchi tenglamalarini qaraymiz. Sistemaning birinchi tenglamasidan ikkinchisini ayiramiz, natijada quyidagi tenglamani olamiz

$$\ln\frac{r_1}{r_2}\left[d_1Q^2(p)-\frac{\partial^2}{\partial z^2}\right]\frac{\partial\overline{U}_{z,1}}{\partial z}+\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\left(\frac{\partial\overline{U}_{z,1}}{\partial z}+\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}\right)-$$

$$-\ln\frac{r_1}{r_2}\frac{1-2\nu}{2(1-\nu)}\left(d_2Q^2(p)-\frac{\partial^2}{\partial z^2}\right)\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}=0\,. \quad (2.48)$$

Endi (2.47) sistemaning uchinchi va to'rtinchi tenglamalarini qaraymiz. Bu tenglamalarining birini ikkinchisidan ayirish ushbu tenglamani beradi

$$\frac{\partial \overline{U}_{z,1}}{\partial z}=\left[\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2\left(r_1^2-r_2^2\right)}{r_1^2 r_2^2}+\frac{1}{2(1-\nu)}d_2Q^2(p)-\frac{1}{2(1-\nu)}\frac{\partial^2}{\partial z^2}\right]\overline{U}_{r,1} \qquad (2.49)$$

Ushbu ifodani (2.48) ga qo'ysak natijada to'rtinchi tartibli bir jinsli differensial tenglama kelib chiqadi

$$A\cdot\frac{\partial^4\overline{U}_{r,1}}{\partial z^4}+B\cdot\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}+C\cdot\overline{U}_{r,1}=0, \qquad (2.50)$$

buyerda

$$A=\ln\frac{r_1}{r_2}\frac{1}{2(1-\nu)}\left(+(1-2\nu)q_1\right);\ B=-\left(d_1+d_2q_1(1-2\nu)\right)\ln\frac{r_1}{r_2}\frac{1}{2(1-\nu)}Q^2(p)+\frac{r_1^2-r_2^2}{r_1^2r_2^2}\left(3-\frac{1}{2(1-\nu)}\right);$$

$$C=\frac{r_1^2-r_2^2}{r_1^2r_2^2}\left\{\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2(r_2^2-r_1^2)}{r_1^2r_2^2}+\frac{d_2}{2(1-\nu)}Q^2(p)+d_1Q^2(p)\times\right.$$
$$\left.\times\left[\frac{2(r_1^2-r_2^2)}{r_1^2r_2^2}+\ln\frac{r_1}{r_2}\frac{d_2}{2(1-\nu)}Q^2(p)\right].\right.$$

olingan (2.50) tenglmaning cheksizlikda qo'zgolishlarning so'nishi shartini qanoatlantiruvchi umumiy yechimi

$$\overline{U}_{r,1}=A_1e^{-\alpha_1 z}+A_2e^{-\alpha_2 z} \qquad (2.51)$$

ga teng, bu yerda α_1,α_2 - mos xarakteristik

$$A\alpha^4+B\alpha^2+C=0$$

tenglamaning ildizlari va ular

$$\alpha_{1,2}=\sqrt{\left(-B\pm\sqrt{B^2-4AC}\right)/2A}.$$

formula bilan aniqlanadilar.

$\overline{U}_{r,1}$ funksiyaning (2.51) qiymatlarini (2.49) ga qo'yib, quyidagiga ega bo'lamiz

$$\frac{\partial\overline{U}_{z,1}}{\partial z}=\left\{\frac{1}{\ln(r_1/r_2)}\cdot\frac{2\left(r_1^2-r_2^2\right)}{r_1^2\cdot r_2^2}+\frac{d_2}{2(1-\nu)}Q^2(p)-\frac{\alpha_1^2}{2(1-\nu)}\right\}\cdot A_1e^{-\alpha_1 z}-$$
$$-\left\{\frac{1}{\ln(r_1/r_2)}\cdot\frac{2\left(r_1^2-r_2^2\right)}{r_1^2\cdot r_2^2}+\frac{d_2}{2(1-\nu)}Q^2(p)-\frac{\alpha_2^2}{2(1-\nu)}\right\}\cdot A_2e^{-\alpha_2}.$$

Butengliknizbo'yichaintegrallabhamdaqo'zg'olishlarningcheksizlikdaso'nishsharti nihisobgaolib, ushbuifodagaegabo'lamiz

$$\overline{U}_{z,1}=\left\{\frac{1}{\ln(r_1/r_2)}\cdot\frac{2\left(r_1^2-r_2^2\right)}{r_1^2\cdot r_2^2}+\frac{d_2}{2(1-\nu)}Q^2(p)-\frac{\alpha_1^2}{2(1-\nu)}\right\}\cdot A_1\frac{e^{\alpha_1 z}}{-\alpha_1}-$$

$$-\left\{\frac{1}{\ln(r_1/r_2)}\cdot\frac{2\left(r_1^2-r_2^2\right)}{r_1^2\cdot r_2^2}+\frac{d_2}{2(1-\nu)}Q^2(p)-\frac{\alpha_2^2}{2(1-\nu)}\right\}\cdot A_2\frac{e^{-\alpha_2}}{-\alpha_2}. \quad (2.52)$$

Yuqoridagi (2.47) tenglamalar sistemasining uchinchi tenglamasidan

$$\overline{U}_{r,0}=(1-2\nu)\cdot\left\{\frac{2\nu}{1-2\nu}\cdot\frac{\partial\overline{U}_{z,0}}{\partial z}+\left(\ln r_1+\frac{1}{2}\right)\cdot\frac{\partial\overline{U}_{z,1}}{\partial z}+\frac{2}{r_1^2}\overline{U}_{r,1}-\right.$$

$$\left.-\left(\ln\frac{r_1}{1-2\nu}+\frac{1}{2}\right)\cdot\frac{1-2\nu}{2(1-\nu)}\left[d_2Q^2(p)\overline{U}_{r,1}-\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}\right]\right\}. \quad (2.53)$$

Endi $\overline{U}_{r,0}$ funksiyaning oxirgi ifodasini (2.47) sistemaning birinchi tenglamasiga qo'yib, quyidagi bir jinslimas tenglamaga ega bo'lamiz

$$\frac{\partial^2\overline{U}_{z,0}}{\partial z^2}+\left[2q_1\nu-\frac{2(1-\nu)}{1-2\nu}\right]^{-1}\cdot d_1Q^2(p)\overline{U}_{z,0}=F_1, \quad (2.54)$$

bu yerda

$$\overline{F}_1=q_4\left[\overline{g}_1(p)A_1e^{-\alpha_1}+\overline{g}_2(p)A_2e^{-\alpha_2}\right],$$

$$\overline{g}_i(p)=q_8\alpha_i+q_9\frac{Q^2(p)}{\alpha_i}+q_{10}Q^2(p)\alpha_i-\left(q_6-\frac{q_5}{2(1-\nu)}\right)\times$$

$$\times\alpha_i^{\,3}+\frac{q_{11}}{\alpha_i}+q_{12}\frac{Q^4(p)}{\alpha_i},\quad (i=1,2)$$

$$q_4=\left(2\nu q_1-\frac{2(1-\nu)}{1-2\nu}\right)^{-1};\ q_5=q_1(1-2\nu)\left(\ln r_1+1/2\right)+\left(\ln r_1-1/2\right);$$

$$q_6=\frac{1-2\nu}{2(1-\nu)}q_1\left[\ln r_1+(1-2\nu)/2\right]-\left(\ln r_1-1/2\right);$$

$$q_7=\frac{1}{r_1}\left[q_1(1-2\nu)-1\right];$$

$$q_8=-q_5/\ln(r_1/r_2)\left[2\left(r_1^2-r_2^2\right)/r_1^2r_2^2-q_7-\frac{1}{r_1}\cdot\frac{1}{1-2\nu}\right];$$

$$q_9=\left(\ln r_1-\frac{1}{2}\right)\frac{d_1}{\ln(r_1/r_2)}\frac{2(r_1^2-r_2^2)}{r_1^2r_2^2}+\frac{1}{r_1^2}\frac{d_2}{2(1-\nu)};$$

$$q_{10}=\frac{d_2}{2(1-\nu)}\left[q_5+q_1(1-2\nu)(1-\nu)\right]-d_1\left(\ln r_1-\frac{1}{2}\right)\frac{1}{2(1-\nu)},$$

$$q_{11}=\frac{1}{r_1^2}\frac{1}{\ln(r_1/r_2)}\frac{2(r_1^2-r_2^2)}{r_1^2 r_2^2};\quad q_{12}=\left(\ln r_1-\frac{1}{2}\right)\frac{d_1 d_2}{2(1-\nu)}.$$

Bu tenglamaning umumiy yechimini integrallash o'zgarmaslarini variatsiyalash usuli bilan yechamiz. Ungako'ra

$$\overline{U}_{z,0}=\overline{A}_3 e^{-\beta z}-\frac{q_4}{2\beta}\left\{\frac{A_1\overline{g}_1(p)}{\beta-\alpha_1}\left(e^{-\alpha_1 z}-e^{-\beta z}\right)+\frac{A_2\overline{g}_2(p)}{\beta-\alpha_2}\times\right.$$

$$\left.\times\left(e^{-\alpha_2 z}-e^{-\beta z}\right)\frac{A_1\overline{g}_1(p)}{\beta+\alpha_1}e^{-\alpha_1 z}+\frac{A_2\overline{g}_2(p)}{\beta+\alpha_2}e^{-\alpha_2 z}\right\},\tag{2.55}$$

bu yerda

$$\beta=\sqrt{q_4 d_1 Q^2(p)}$$

birjinslimas (2.47) tenlamaga mos bir jinsli differensial tenlamaning xarakteristik tenglamasinig musbat ildizi.

Yuqoridagi (2.51), (2.52) ifodalarni (2.53) ga qo'ysak

$$\overline{U}_{r,0}=2\nu A_3\beta e^{-\beta z}+\sum_{i=1}^{2}A_i\left\{-\left[\left(\frac{\alpha_i}{\beta-\alpha_i}+\frac{\alpha_i}{\beta+\alpha_i}\right)e^{-\alpha_1 z}-\frac{\beta}{\beta-\alpha_i}e^{-\beta z}\right]\times\right.$$

$$\times\frac{q_4\overline{g}_i(p)}{\beta}+e^{-\alpha_1 z}\left[\left[\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2\left(r_1^2-r_2^2\right)}{r_1^2 r_2^2}+\frac{1}{2(1-\nu)}d_2 Q^2(p)-\frac{1}{2(1-\nu)}\alpha_i^2\right]\times\right.$$

$$\times\left(\ln r_1+\frac{1}{2}\right)+\frac{2}{r_1^2}-\left(\frac{1}{1-2\nu}\ln r_1+\frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}d_2 Q^2(p)+$$

$$\left.\left.+\left(\frac{1}{1-2\nu}\ln r_1+\frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}\alpha_i^2\right]\right\}.\tag{2.56}$$

Shakllantirilgan (2.45) chegaraviy shartlarga Laplas almashtirishlarini qo'llab, olamiz

$$\overline{U}_z=-\overline{f}(p),\qquad \overline{U}_r=0$$

yoki

$$\begin{aligned}&\overline{U}_{z,0}+\ln r\,\overline{U}_{z,1}=-\overline{f}(p),\\&\frac{1}{r}\overline{U}_{r,1}+\frac{r}{2}\overline{U}_{r,0}=0,\\&(1-2\nu)\left[(1+c_2)(1+a_0)Q^2(p)\overline{U}_{r,1}-\frac{\partial^2\overline{U}_{r,1}}{\partial z^2}\right]-\frac{\partial\overline{U}_{z,1}}{\partial z}=0.\end{aligned}\tag{2.57}$$

Masalaniyechishjarayonidaolingan (2.51), (2.52), (2.55) va (2.56) ifodalarniLaplasbo'yichaalmashtirilgan (2.57) chegaraviyshartlargaqo'yib A_1, A_2, A_3 o'zgarmaslarganisbatanalgebraictenglamalarsistemasigaegabo'lamiz

$$\begin{aligned} &a_{11}A_1 + a_{12}A_2 + a_{13}A_3 = -\bar{f}(p); \\ &a_{21}A_1 + a_{22}A_2 + a_{23}A_3 = 0; \\ &a_{31}A_1 + a_{32}A_2 = 0 \end{aligned} \tag{2.58}$$

yoki

$$a_{ij}A_j = -\delta_{ij}\bar{f}(p), \qquad (i, j = 1, 2, 3),$$

bu yerda

$$a_{13} = 1; \quad a_{23} = r\nu\beta; \quad a_{33} = 0;$$

$$a_{1i} = -\frac{q_4}{2\beta}\cdot\frac{\bar{g}_i(p)}{\beta+\alpha_i} - \ln r\cdot\frac{1}{\alpha_i}\times$$

$$\times\left\{\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2(r_1^2 - r_2^2)}{r_1^2 r_2^2} + \frac{1}{2(1-\nu)}d_2Q^2(p) - \frac{1}{2(1-\nu)}\alpha_i^2\right\}; \quad (i = 1,2)$$

$$a_{2i} = \frac{r}{2}\left\{-\frac{\nu q_4\bar{g}_i(p)}{\beta}\left[\frac{2\beta\alpha_i}{\beta^2 - \alpha_i^2} - \frac{\beta}{\beta - \alpha_i}\right] + \left(\ln r_1 + \frac{1}{2}\right)\times\right.$$

$$\times\left[\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2(r_1^2 - r_2^2)}{r_1^2 r_2^2} + \frac{1}{2(1-\nu)}d_2Q^2(p) - \frac{1}{2(1-\nu)}\alpha_i^2\right] +$$

$$\left. + \frac{2}{r_1^2} - \left(\frac{1}{1-2\nu}\ln r_1 + \frac{1}{2}\right)\frac{1-2\nu}{2(1-\nu)}(d_2Q^2(p) + \alpha_i^2)\right\} + \frac{1}{r}, (i=1,2)$$

$$a_{3i} = (1-2\nu)(d_2Q^2(p) - \alpha_i^2) -$$

$$-\left[\left(\ln\frac{r_1}{r_2}\right)^{-1}\frac{2(r_1^2 - r_2^2)}{r_1^2 r_2^2} + \frac{1}{2(1-\nu)}d_2Q^2(p) - \frac{1}{2(1-\nu)}\alpha_i^2\right], (i = 1, 2)$$

$\bar{U}_{z,0}, \bar{U}_{z,1}, \bar{U}_{r,0}, \bar{U}_{r,1}$ funksiyalarning olingan qiymatlarini p-parametrning katta qiymatlari uchun originallarini topamiz. Demak, p ni yetarlicha katta deb hisoblaymiz. U holda α_i va β larning ifodalarini quyidagicha tasvirlash mumkin

$$\alpha_1 = \gamma_5\left\{Q^2(p) + \frac{3\gamma_3}{\gamma_4}\right\}^{\frac{1}{2}}; \quad \alpha_2 = \gamma_7\left\{Q^2(p) + \frac{3\gamma_3}{\gamma_6}\right\}^{\frac{1}{2}}; \ \beta = \gamma_{10}Q(p)$$

yoki

$$\alpha_1=\gamma_5\left(p^2+\gamma_8^2\right)^{\frac{1}{2}};\qquad \alpha_2=\gamma_7\left(p^2+\gamma_9^2\right)^{\frac{1}{2}};\qquad \beta=\gamma_{10}\left(p^2+\gamma_0^2\right)^{\frac{1}{2}}$$

buyerda

$$\gamma_0=\sqrt{-\frac{c_1^2}{4}-c_2}\;;\; c_1=\sum_{m=1}^{n}\frac{\alpha_m}{\tau_m};\qquad c_2=\sum_{m=1}^{n-1}\sum_{l=m+1}\alpha_m\alpha_l\left(\frac{1}{\tau_m}-\frac{1}{\tau_l}\right)^2;$$

τ_m,τ_l – fiksirlangan vaqtlar; α_m,α_l - muhitningparametrlari

$$\gamma_1=\ln\frac{r_1}{r_2}\cdot\frac{1}{2(1-\nu)}\left(\;+(1-2\nu)q_1\right);\quad \gamma_2=-\ln\frac{r_1}{r_2}\cdot\frac{1}{2(1-\nu)}\left[d_1+d_2+q_1d_2(1-2\nu)\right];$$

$$\gamma_3=\frac{r_1^2-r_2^2}{r_1^2\cdot r_2^2};\quad \gamma_4=-\gamma_2+\sqrt{\gamma_2^2-4\gamma_1\frac{d_1d_2}{2(1-\nu)}\ln\frac{r_1}{r_2}};$$

$$\gamma_8=\sqrt{\frac{3\gamma_3}{\gamma_4}-\gamma_0^{\,2}};\quad \gamma_9=\sqrt{\frac{3\gamma_3}{\gamma_6}-\gamma_0^2};\quad \gamma_{10}=\sqrt{\frac{d_1}{2q_1\nu-\frac{2(1-\nu)}{1-2\nu}}}.$$

Olingan oxirgi tenglamalar sistemasini A_1, A_2, A_3 larga nisbatan yechib quyidagilarga ega bo'lamiz

$$A_1=\bar{f}_1(p)\cdot\frac{\gamma_{50}}{Q(p)},\quad A_2=-\bar{f}_1(p)\cdot\frac{\gamma_{51}}{Q(p)},\quad A_3=-\bar{f}_1(p)\cdot\frac{\gamma_{52}}{Q(p)},\tag{2.59}$$

bu yerda

$$\gamma_{11}=-\frac{q_4\gamma_5}{2(\gamma_{10}+\gamma_5)};\;\gamma_{12}=q_8+\frac{q_9}{\gamma_5^2}-\frac{3\gamma_3}{\gamma_4}\gamma_5^2\cdot\left(q_6-q_5\frac{1}{2(1-v)}\right);$$

$$\gamma_{13}=q_{10}-\gamma_5^2\cdot\left(q_6-q_5\frac{1}{2(1-v)}\right)+\frac{q_{12}}{\gamma_5^2};\quad \gamma_{15}=\frac{d_2-\gamma_5^2}{2(1-v)};$$

$$\gamma_{16}=-\frac{q_4\gamma_7}{2(\gamma_{10}+\gamma_7)};\quad \gamma_{18}=q_{10}+\frac{q_9}{\gamma_7^2}-\left(q_6-q_5\frac{1}{2(1-v)}\right)\cdot\gamma_7^2;$$

$$\gamma_{20}=\frac{d_2-\gamma_7^2}{2(1-v)};\;\gamma_{22}=\frac{r}{2}\left\{\frac{vq_4\gamma_5\gamma_{13}}{\gamma_5+\gamma_{10}}+\left(\ln r_1+\frac{1}{2}\right)\gamma_{15}-\frac{1-2v}{2(1-v)}\left(\frac{\ln r_1}{1-2v}+\frac{1}{2}\right)\left(d_2-\gamma_5^2\right)\right\};$$

$$\gamma_{25}=\frac{r}{2}\cdot\frac{vq_4\gamma_7\gamma_{18}}{\gamma_7+\gamma_{10}}+\left(\ln r_1+\frac{1}{2}\right)\gamma_{20}-\frac{1-2v}{2(1-v)}\left(\frac{\ln r_1}{1-2v}+\frac{1}{2}\right)\cdot\left(d_2-\gamma_7^2\right);$$

$$\gamma_{27}=\left(d_2-\gamma_5^2\right)\cdot\frac{2(1-v)(1-2v)-1}{2(1-v)};\quad \gamma_{29}=\left(d_2-\gamma_7^2\right)\cdot\frac{2(1-v)(1-2v)-1}{2(1-v)};$$

$$\gamma_{30}=\frac{3\gamma_3}{\gamma_6}\cdot\frac{2(1-v)(1-2v)-1}{2(1-v)};\qquad \gamma_{32}=\gamma_{16}\cdot\gamma_{18}-\ln r\cdot\frac{\gamma_{10}\cdot\gamma_{20}}{\gamma_5};$$

$$\gamma_{34}=\gamma_{29}\cdot\left(\gamma_{11}\gamma_{13}-\gamma_{15}\cdot\ln r\cdot\frac{\gamma_{10}}{\gamma_5}\right);\quad \gamma_{37}=\gamma_{22}\gamma_{29}+\gamma_{25}\gamma_{27};$$

$$\gamma_{40} = r \cdot \nu \left(\gamma_{32}\gamma_7 - \gamma_{34}\right) + \gamma_{22}\gamma_{29} ;$$

$$\gamma_{50} = \frac{r \cdot \nu \gamma_{10}\gamma_{29}}{\gamma_{40}} ; \quad \gamma_{51} = \frac{r \cdot \nu \gamma_{10}\gamma_{27}}{\gamma_{40}} ; \quad \gamma_{52} = \frac{\gamma_{37}}{\gamma_{40}} .$$

A_1, A_2, A_3 larning (2.59) qiymatlarini (2.51), (2.52), (2.55) va (2.56) formulalarga qo'yib ushbu ifodalarga ega bo'lamiz

$$\bar{U}_{r,1} = \gamma_5 \cdot \gamma_{50} \cdot \bar{f}_1(p) \cdot \frac{e^{-\alpha_1 z}}{\alpha_1} - \gamma_{51} \cdot \gamma_7 \cdot \frac{e^{-\alpha_2 z}}{\alpha_2} \bar{f}(p), \tag{2.60}$$

$$\bar{U}_{z,1} = \gamma_{51} \bar{f}(p) \frac{e^{-\alpha_2 z}}{\alpha_2} \left\{ \frac{\gamma_{59}}{p + c_1/2} + \left(p + \frac{c_1}{2}\right) \cdot \left(d_2 - \frac{\gamma_7^2}{2(1-\nu)}\right) - \frac{3\gamma_3\gamma_7^2 / 2(1-\nu)\gamma_6}{p + c_1/2} \right\} -$$

$$- \gamma_{50} f(p) \frac{e^{-\alpha_1 z}}{\alpha_1} \left\{ \frac{\gamma_{59} - 3\gamma_3 \cdot \gamma_5^2 / 2\gamma_9 (1-\nu)}{p + c_1/2} + \left(d_2 - \frac{\gamma_5^2}{2(1-\nu)}\right)\left(p + \frac{c_1}{2}\right) \right\}; \tag{2.61}$$

$$\bar{U}_{z,0} = \bar{f}_1(p) \frac{e^{-\beta z}}{\beta} \left\{ \gamma_{60} \cdot \left(p + c_1/2\right) + \gamma_{61} \frac{1}{p + c_1/2} \right\} - q_4 \left\{ \frac{e^{-\alpha_1 z}}{\alpha_1} \bar{f}_1(p) \left(\gamma_{62} \cdot \left(p + c_1/2\right) + \right.\right.$$

$$\left.\left. + \gamma_{63} / \left(p + c_1/2\right)\right) + \frac{e^{-\alpha_2 z}}{\alpha_2} \bar{f}_1(p) \left(\gamma_{64} \cdot \left(p + c_1/2\right) + \gamma_{65} \frac{1}{p + c_1/2} \right) \right\}, \tag{2.62}$$

$$\bar{U}_{r,0} = \bar{f}_1(p) \frac{e^{-\beta z}}{\beta} \left(\gamma_{70} p^2 + \gamma_{71} p + \gamma_{72}\right) + \gamma_{50} \bar{f}_1(p) \frac{e^{-\alpha_1 z}}{\alpha_1} \left(\gamma_{73} p^2 + c_1\gamma_{73} p + \gamma_{76}\right)$$

$$- \gamma_{51} \bar{f}_1(p) \frac{e^{-\alpha_2 z}}{\alpha_2} \left(\gamma_{76} p^2 + c_1\gamma_{76} p + \gamma_{78}\right) \tag{2.63}$$

bu yerda

$$\gamma_{59} = \frac{2\left(r_1^2 - r_2^2\right)}{r_1^2 \cdot r_2^2 \cdot \ln r_1 / r_2} ; \; \gamma_{74} = c_1\gamma_{73};$$

$$\gamma_{60} = -\gamma_{52}\gamma_{10} + q_4 \left\{ \frac{\gamma_{50}}{2\left(\gamma_{10} - \gamma_5\right)} \left[q_{10}\gamma_5 - \left(q_6 - \frac{q_5}{2(1-\nu)}\right) \cdot \gamma_5^3 + \frac{q_{12}}{\gamma_5} \right] - \right.$$

$$\left. - \frac{\gamma_{51}}{2\left(\gamma_{10} - \gamma_7\right)} \left[q_{10}\gamma_7 - \left(q_6 - q_5 \frac{1}{2(1-\nu)}\right) \cdot \gamma_7^3 + \frac{q_{12}}{\gamma_7} \right] \right\};$$

$$\gamma_{61} = \frac{q_4\gamma_{50}}{2\left(\gamma_{10} - \gamma_5\right)} \left(q_8\gamma_5 + \frac{q_9}{\gamma_5} \right) + \frac{q_4\gamma_{51}}{2\left(\gamma_{10} - \gamma_7\right)} \left(q_8\gamma_7 + \frac{q_9}{\gamma_7} \right);$$

$$\gamma_{62} = -\frac{\gamma_{50}}{\gamma_{10} - \gamma_5^2} \cdot \left(\gamma_5^2 q_{10} - \left(q_6 - q_5 \frac{1}{2(1-\nu)} \right) \cdot \gamma_5^4 + \frac{q_2}{\gamma_5} \right);$$

$$\gamma_{63} = -\frac{\gamma_{50}}{\gamma_{10} - \gamma_5^2} \cdot \left(q_8\gamma_5^2 + q_9 + \gamma_5^2 q_{10} \cdot \frac{3\gamma_3}{\gamma_4} - 2\gamma_5^4\gamma_8^2 \left(q_6 - q_5 \frac{1}{2(1-\nu)} \right) \right);$$

$$\gamma_{64} = \frac{\gamma_{51}}{\gamma_{10} - \gamma_7^2} \cdot \left(\gamma_7^2 q_{10} - \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \cdot \gamma_7^4 + \frac{q_2}{\gamma_7} \right);$$

$$\gamma_{65} = \frac{\gamma_{51}}{\gamma_{10} - \gamma_7^2} \cdot \left(\gamma_7^2 q_8 + q_9 + \gamma_7^2 q_0 \frac{3\gamma_3}{\gamma_6} - 2\gamma_9^2 \gamma_7^4 \cdot \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \right);$$

$$\gamma_{70} = -2\nu\gamma_{52}\gamma_{10}^2 - \left[\left(\gamma_5^3 \left(q_6 - \frac{q_5}{2(1-\nu)} \right) - \frac{q_{12}}{\gamma_5} \right) \cdot \frac{\gamma_{10} \cdot \gamma_{50}}{\gamma_{10} - \gamma_5} + \frac{\gamma_{10} \cdot \gamma_{51}}{\gamma_{10} - \gamma_7} \times \right.$$

$$\left. \times \left(\gamma_7^3 \left(q_6 - \frac{q_5}{2(1-\nu)} \right) - \frac{q_{12}}{\gamma_7} \right) \right] \cdot \nu \cdot q_4 \,;$$

$$\gamma_{71} = -2\nu c_1 \gamma_{52} \gamma_{10}^2 + \nu q_4 \left[\frac{\gamma_{10} \cdot \gamma_{50}}{\gamma_{10} - \gamma_5} \left(\gamma_5 q_{10} - \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \cdot \gamma_5^3 c_1 + \frac{q_{12} c_1}{\gamma_5} \right) - \right.$$

$$\left. - \frac{\gamma_{10} \cdot \gamma_{51}}{\gamma_{10} - \gamma_5} \left(\gamma_7 q_{10} - \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \cdot c_1 \right) \cdot \gamma_7^3 + \frac{q_{12} c_1}{\gamma_7} \right];$$

$$\gamma_{72} = 2\nu c_2 \gamma_{10}^2 \gamma_5 + \nu q_4 \gamma_{10} \left[\frac{\gamma_{50}}{\gamma_{10} - \gamma_5} \left(q_8 \gamma_5 + \frac{q_9}{\gamma_5} + \frac{c_1 \gamma_5 q_{10}}{2} - \gamma_5^3 \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \times \right. \right.$$

$$\left. \times \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) + \frac{q_{11}}{\gamma_5} - \frac{q_{12} c_2}{\gamma_5} \right) - \frac{\gamma_{51}}{\gamma_{10} - \gamma_7} \left(q_8 \gamma_7 + \frac{q_9}{\gamma_7} + \frac{c_1 \gamma_7 q_{10}}{2} - \gamma_7^3 \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \times \right.$$

$$\left. \left. \times \left(\frac{3\gamma_3}{\gamma_6} - c_2 \right) + \frac{q_{11}}{\gamma_7} - \frac{q_{12} c_2}{\gamma_7} \right) \right];$$

$$\gamma_{73} = -\nu q_4 \frac{2\gamma_5^2}{\gamma_{10}^2 - \gamma_5^2} \left[\gamma_5 q_{10} - \left(q_6 - q_5 \frac{1}{2(1-\nu)} \right) \cdot \gamma_5^3 + \frac{q_{12}}{\gamma_5} \right] +$$

$$+ \gamma_5 \left[\eta_1 \left(\frac{d_2}{2(1-\nu)} - \gamma_5^2 \right) - \gamma_{58} d_2 + \gamma_{58} \gamma_5^2 \right];$$

$$\gamma_{75} = -\nu q \frac{2\gamma_5^2}{\gamma_{10}^2 - \gamma_5^2} \left[(q_8 + q_9) \gamma_5 - q_{10} \gamma_5 c_2 - \gamma_5^3 \left(q_6 - \frac{q_5}{2(1-\nu)} \right) \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) - \frac{\gamma_{12} c_2}{\gamma_5} \right] +$$

$$+ \gamma_5 \left[\eta_1 \left(\gamma_{59} - \frac{d_2 c_2}{2(1-\nu)} - \gamma_5^2 \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) \right) + \frac{2}{r_1^2} + \gamma_{58} d_2 c_2 + \gamma_{58} \gamma_5^2 \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) \right];$$

$$\gamma_{76} = -\nu q_4 \frac{2\gamma_7^2}{\gamma_{10}^2 - \gamma_7^2} \left[\gamma_5 q_{10} - \left(q_6 - q_5 \frac{1}{2(1-\nu)} \right) \cdot \gamma_7^3 + \frac{q_{12}}{\gamma_7} \right] +$$

$$+ \gamma_7 \left[\eta_1 \left(\frac{d_2}{2(1-\nu)} - \gamma_7^2 \right) - \gamma_{58} d_2 + \gamma_{58} \gamma_7^2 \right];$$

$$\gamma_{78} = -\nu q_4 \frac{2\gamma_7^2}{\gamma_{10}^2 - \gamma_7^2} \left[\gamma_7 (q_8 + q_9) - c_2 \gamma_5 q_{10} - \gamma_7^3 \left(q_6 - q_5 \frac{1}{2(1-\nu)} \right) \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) - \frac{\gamma_{12} c_2}{\gamma_7} \right] +$$

$$+ \gamma_7 \left[\eta_1 \left(\gamma_{59} - \frac{d_2 c_2}{2(1-\nu)} - \gamma_7^2 \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) \right) + \frac{2}{r_1^2} + \gamma_{58} d_2 c_2 + \gamma_{58} \gamma_7^2 \left(\frac{3\gamma_3}{\gamma_4} - c_2 \right) \right];$$

$$\gamma_{80}=\gamma_{59}-\frac{3\gamma_3\gamma_7^2}{2(1-\nu)\gamma_6};\quad \gamma_{81}=d_2-\frac{\gamma_7^2}{2(1-\nu)};$$

$$\gamma_{82}=\gamma_{59}-\frac{\gamma_3\gamma_5^2}{2(1-\nu)\gamma_4};\quad \gamma_{84}=d_2-\frac{\gamma_5^2}{2(1-\nu)}.$$

Laplasning teskari almashtirishlarini (2.60)-(2.63) ifodalarga qo'llaymiz. Bunda kechikish teoremasi va Borelning tugun haqidagi teoremalaridan foydalanamiz. Izlanayotgan funksiyalarning, masalaning yechimi bo'lgan quyidagi qiymatlariga ega bo'lamiz

$$U_{z,1}=\frac{\gamma_{51}}{\gamma_7}\int_{\gamma_7 z}^{t} e^{-\frac{c_1}{2}\tau}F_1(t-\tau)I_0\left(\gamma_9\sqrt{\tau^2-\gamma_7^2 z^2}\right)d\tau-\frac{\gamma_{50}}{\gamma_5}\int_{\gamma_5 z}^{t} e^{-\frac{c_1}{2}\tau}F_2(t-\tau)I_0\left(\gamma_8\sqrt{\tau^2-\gamma_5^2 z^2}\right)d\tau, \quad (2.64)$$

$$U_{r,1}=\gamma_{50}\int_{\gamma_5 z}^{t} f(t-\tau)e^{-\frac{c_1}{2}\tau}I_0\left(\gamma_8\sqrt{\tau^2-\gamma_5^2 z^2}\right)d\tau-$$

$$-\gamma_{51}\int_{\gamma_7 z}^{t} f(t-\tau)e^{-\frac{c_1}{2}\tau}I_0\left(\gamma_9\sqrt{\tau^2-\gamma_7^2 z^2}\right)d\tau; \quad (2.65)$$

$$U_{z,0}=\frac{1}{\gamma_{10}}\int_{\gamma_{10} z}^{t}\left[\gamma_{60}F_0(t-\tau)+\gamma_{61}\int_0^{t-\tau} f(\xi)d\xi\right]\cdot e^{-\frac{c_1}{2}\tau}I_0\left(\gamma_{10}\sqrt{\tau^2-\gamma_{10}^2 z^2}\right)d\tau-$$

$$-q_4\left\{\frac{1}{\gamma_5}\int_{\gamma_5 z}^{t} e^{-\frac{c_1}{2}\tau}I_0\left(\gamma_8\sqrt{\tau^2-\gamma_5^2 z^2}\right)\cdot\left[\gamma_{62}F_0(t-\tau)+\gamma_{63}\int_0^{t-\tau} f(\xi)d\xi\right]d\tau+\right.$$

$$\left.+\frac{1}{\gamma_7}\int_{\gamma_7 z}^{t}\left[\gamma_{64}F_0(t-\tau)+\gamma_{65}\int_0^{t-\tau} f(\xi)\,d\xi\right]\cdot e^{-\frac{c_1}{2}\tau}\,I_0\left(\gamma_9\sqrt{\tau^2-\gamma_7^2 z^2}\right)d\tau\right\}, \quad (2.66)$$

$$U_{r,0}=\frac{1}{\gamma_{10}}\int_0^t\left[\gamma_{70}f''(t-\tau)+\gamma_{71}f'(t-\tau)+\gamma_{72}f(t-\tau)\right]I_0\left(\gamma_{10}\sqrt{\tau^2-\gamma_{10}^2 z^2}\right)e^{-\frac{c_1}{2}\tau}\,d\tau+$$

$$+\frac{\gamma_{50}}{\gamma_5}\int_0^t\left[\gamma_{73}f''(t-\tau)+\gamma_{73}c_1 f'(t-\tau)+\gamma_{75}f(t-\tau)\right]I_0\left(\gamma_8\sqrt{\tau^2-\gamma_5^2 z^2}\right)e^{-\frac{c_1}{2}\tau}\,d\tau-$$

$$-\frac{\gamma_{51}}{\gamma_{10}}\int_{\gamma_7 z}^{t}\left[\gamma_{76}f''(t-\tau)+\gamma_{76}c_1 f'(t-\tau)+\gamma_{78}\,f(t-\tau)\right]I_0\left(\gamma_9\sqrt{\tau^2-\gamma_7^2 z^2}\right)e^{-\frac{c_1}{2}}\,d\tau. \quad (2.67)$$

$$F_1(t)=\gamma_{80}\int_0^t f(\xi)\,d\xi+\gamma_{81}\left(f'(t)-\frac{c_1}{2}f(t)\right);\quad F_2(t)=\gamma_{82}\int_0^t f(\xi)\,d\xi+\gamma_{84}\left(f'(t)+\frac{c_1}{2}f(t)\right);$$

$$F_0=f'(t)+\frac{c_1}{2}f(t).$$

Balka kesimlaridagi ko'chish va kuchlanishlar (2.45) formulalar bilan hisoblandi, bunda balka materialining fizik parametrlari uchun quyidagi qiymatlar qabul qilindi [2]:

$$\alpha_1 = 0{,}78;\ \alpha_2 = 0{,}18;\ \alpha_3 = 0{,}1;\ \tau_1 = 80\,mks;\ b = 1040\ m/s,$$

$$\tau_2 = 0{,}14\cdot 10^6\,mks;\ \tau_3 = 0{,}11\cdot 10^7\,mks;\ K(t) = \sum_{n=1}^{3}\alpha_n \tau_n^{-1} e^{-t/\tau_n}\ ,$$

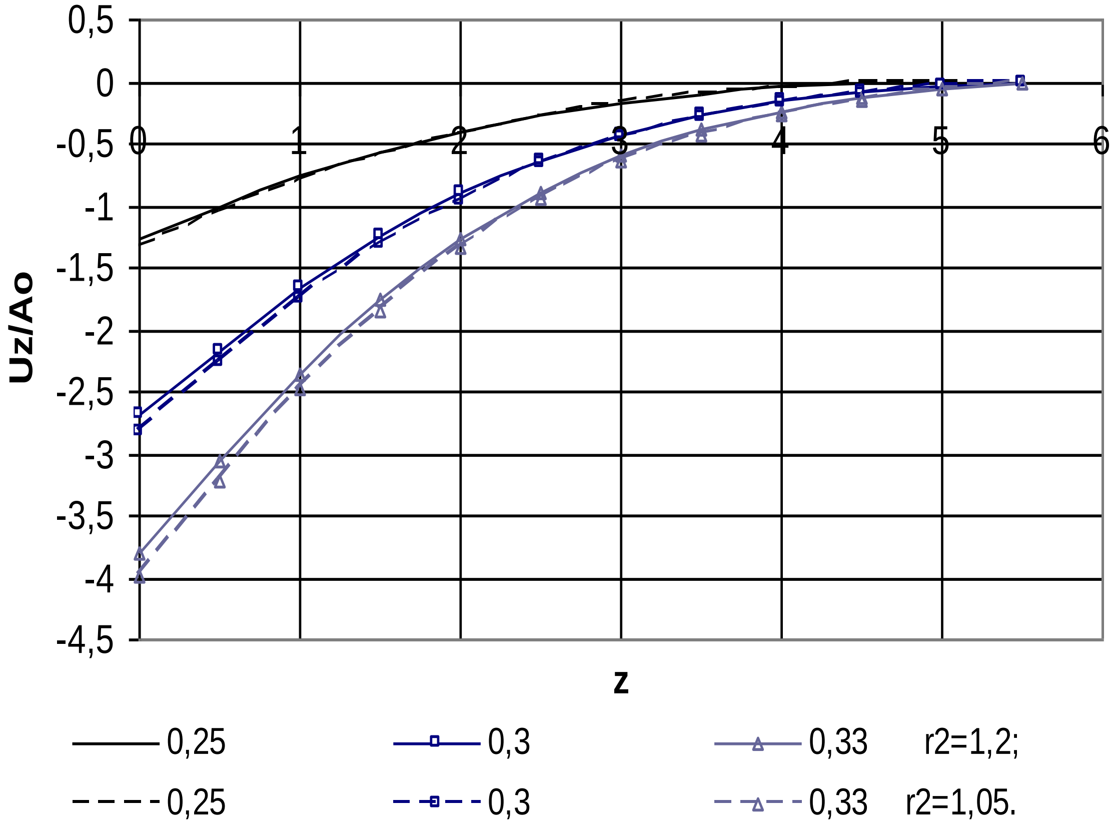
Balka sirti nuqtalari uchun o'tkazilgan hisoblashlar natijalari 2.3-2.4 chizmalarda keltirilgan. 2.3- chizmada balka kesimlaridagi bo'ylama ko'chishning bo'ylama koordinatadan bog'liq o'zgarish grafigi keltirilgan. 2.4 – chizmada esa normal radial kuchlanishning balka sirtidagi nuqtalarda bo'ylama koordinatadan bog'liq ravishda o'zgarish grfiklari keltirilgan.
Ushbu sonli natijalar hisoblanishida tashqi yuklanish funksiyasi

$$f(t) = A_0 H(t)$$

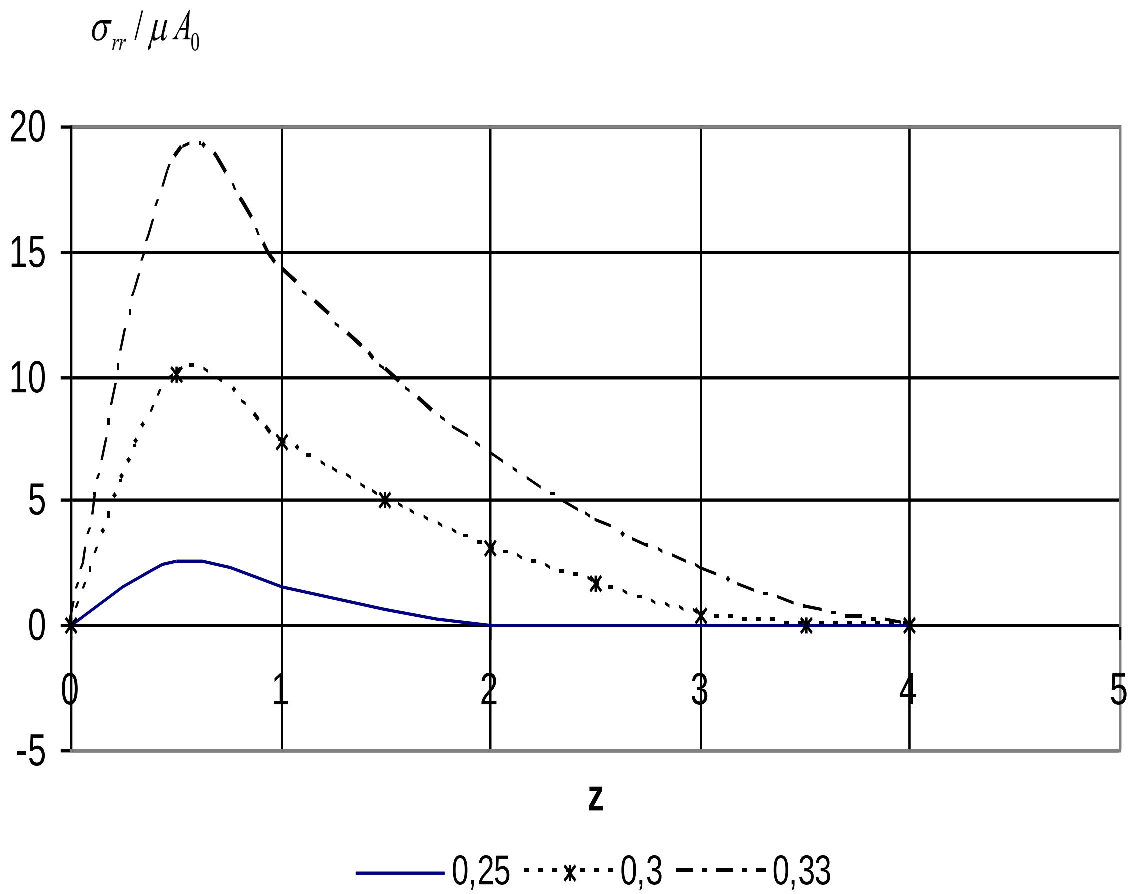
ko'rinishdaqabulqilingan. Bu yerda $H(t)$- Hevisaydning birlik funksiyasi; A_0- tebranishlar amplitudasi. Hisoblashlar vaqtning t=10 fiksirlangan payti va a_0 ning har xil qiymatlari uchun o'tkazildi.

2.3-chizmadabalka sirti nuqtalari bo'ylama ko'chishlarining Puasson koeffitsientining har xil qiymatlarida (0,25; 0,30; 0,33) bo'ylama koordinatadan bog'liqliq grafiklari tasvirlangan. 2.4 – chizma. Balka sirti nuqtalari normal radial kuchlanishlarining Puasson koeffitsientining har xil qiymatlarida (0,25; 0,30; 0,33)bo'ylama koordinatadan bog'liqliqgrafiklari tasvirlangan.

Keltirilgan grafiklardan ko'rinadiki Puasson koeffitsientining qiymatlari o'sib borishi bilan bo'ylama ko'chishning mos qiymatlari orasidagi farq oshib boradi. Xususan, balkaningz=0,5kesimida Puasson koeffitsientining $\nu = 0{,}25$qiymatidabufarq 5,2 % ni, $\nu = 0{,}3$bo'lganda - 5,4 % ni, $\nu = 0{,}33$bolganda -5.7 % nitashkil etadi.



2.3-chizma.Balka sirti nuqtalari bo'ylama ko'chishlarining Puasson koeffitsientining har xil qiymatlarida (0,25; 0,30; 0,33) bo'ylama koordinatadan bog'liqligi.



2.4 – chizma. BalkasirtinuqtalarinormalradialkuchlanishlariningPuasson koeffitsientining har xil qiymatlarida (0,25; 0,30; 0,33) bo'ylama koordinatadan bog'liqligi.

Normal radial kuchlanishlar uchun keltirilgan grafiklardan ko'rinadiki Puasson koeffitsientining qiymatlari o'sib borishi bilan radial kuchlanishlarning mos qiymatlari orasidagi farq kamayib boradi. Xususan, balkaningz=0,5kesimidagi kuchlanish qiymati Puasson koeffitsientining $\nu = 0{,}25$qiymatida-0,5 ga teng bo'lgani holda $\nu = 0{,}3$bo'lganda -2,2 ga (oldingi qiymatga nisbatan 340% ga kam), $\nu = 0{,}33$bolganda esa -3 ga (oldingi qiymatga nisbatan 36% ga kam) teng bo'ladi.

Keltirilgan grafiklar ko'chishlarning balkaning taxminan $z = 5{,}5$kesimida kuchlanishlar esa $z = 4$kesimida batamom so'nishlarini ko'rsatadi.

ASOSIY XULOSALAR

Shunday qilib, magistrlik dissertatsiyasi doirasida bajarilgan ishlarni yakunlab quyidagi xulosalarni chiqarish mumkin:

1. Doiraviy silindrik elastik balka va sterjenlarning professorlar I. G. Filippov va X. Xudoynazarovlar taklif etgan, tebranish tenglamalarini chiqarish metodikasi va bu yerda ishlatilgan usul ham silindrik sterjen nostatsionar tebranishlari haqidagi uch o'lchovli masalaga Fure va Laplas almashtirishlarini qo'llashga va almashtirishlardagi masalaning umumiy yechimlaridan foydalanishga hamda bu yechimlarning darajali qatorlarga standart yoyilmalarini qo'llashga asoslangan;

2. Doiraviy elastik sterjenning bo'ylama tebranishlari haqidagi masala qoy-ildi. Bir jinsli va izotrop, radiusi r_0 bo'lgan doiraviy silindrik elastik sterjenningsilindrik koordinatalar sistemasida tebranish tenglamalarini chiqarishda sterjen uch o'lchovli jism deb hisoblandi. Balkaning yoki sterjenning bo'ylama tebranishlari o'qga nisbatan simmetrik tebranishlardan iborat bo'lganligi uchun sterjen nuqtalarining $\vec{U}(r,\theta,z,t)$ ko'chish vektorining faqat ikkita tuzuvchisi hisobga olindi.

3. Doiraviy elastik balka va sterjenlarning bo'ylama tebranishlari tenglamalarini keltirib chiqarish umumiy usuli o'rganib chiqildi. Ushbu dissertatsiya doirasida keltirib chiqariladigan tenglamalarning barchasi ana shu usul asosida olingan. Tenglamalardan tashqari, ushbu usul yordamida qaralayotgan sistema (bu holda doiraviy balka) kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatlarini bir qiymatli aniqlashga imkon beruvchi algoritm ham ishlab chiqildi.

4. Konkret amaliy masalalarni yechish uchun qo'llaniladigan tenglamalar qarab chiqilgan va doiraviy elastik balkaning bo'ylama tebranishlari klassik va aniqlashtirilgan tenglamalarikeltirildi.Olingan tenglamalarning boshqa avtorlar tomonidan olingan shunday tenglamalar bilan taqqoslash ishlari amalga

oshirildi.Shuningdek aniqlashtirilgan tenglamalarning turli xil ko'rinishlari muhokama etildi. Aniqlashtirilgan tenglamalarning har bir hadi qanday mexanik ma'noga ega ekanligi ochib berildi.

5. Doiraviy sterjenning uchiga qo'yilgan zarba ta'sirida uyg'otilgan bo'ylama deformatsiyalanish jarayoni haqidagi masala, sterjen bilan ta'sirlashuvchi suyuqlikni hisobga olgan holda, yechildi. Olingan sonli natijalar asosida quyidagi xulosalar chiqarildi:

- Normal radial va bo'ylama kuchlanishlar ta'sirlashuvchi suyuqlik zichligidan kuchli bog'liq: suyuqlikning zichligi oshib borishi bilan kuchlanishlar qiymatlari keskin ravishda kamayib boradi;

- Bunday jarayonda kuchlanishlar bo'ylama koordinata bo'yicha intensive ravishda so'nadilar. Bunda bo'ylama kuchlanishlar radial kuchlanishlarga nisbatan qariyb ikki marta sekin so'nadi;

6. Doiraviy silindrik balkaning sirti tashqi yuklardan xoli bo'lgan holda va Puasson koeffitsiyenti o'zgarmas bo'lganda, uzunligi bir tomonlama chegaralangan va bir uchi qistirib mahkamlangan doiraviy balkaning bo'ylama tebranishlari haqidagi masala yechildi. Olingan sonli natijalar quyidagi xulosalarni chiqarishga imkon berdi:

-Puasson koeffitsientining qiymatlari o'sib borishi bilan bo'ylama ko'chishning mos qiymatlari orasidagi farq oshib boradi. Xususan, balkaningz=0,5kesimida Puasson koeffitsientining $\nu = 0{,}25$qiymatidabufarq5,2 % ni, $\nu = 0{,}3$bo'lganda - 5,4 % ni, $\nu = 0{,}33$bolganda -5.7 % ni tashkil etadi;

- Puasson koeffitsientining qiymatlari o'sib borishi bilan radial kuchlanishlarning mos qiymatlari orasidagi farq kamayib boradi. Xususan, balkaningz=0,5kesimidagi kuchlanish qiymati Puasson koeffitsientining $\nu = 0{,}25$qiymatida-0,5 ga teng bo'lgani holda $\nu = 0{,}3$bo'lganda -2,2 ga, $\nu = 0{,}33$bolganda esa -3 ga teng bo'ladi;

7. Olingan sonly natijalar ko’chishlarning balkaning taxminan $z = 5{,}5$ kesimida kuchlanishlar esa $z = 4$ kesimida batamom so’nishlarini ko’rsatadi.

FOYDALANILGANADABIYOTLARRO'YXATI

1. Abdiyev B.Doiraviy elastik sterjenning bo'ylama tebranishlarini klassik va aniqlashtirilgan tenglamalar asosida sonli tadqiq etish. Magistr. Dissert. Samarqand, 2011 – 66 bet.
2. Амиркулова Ф. А. Уточненные уравнения продольно-радиальных колебаний предварительно напряженных вязкоупругих цилиндрических слоев // Узб. журнал Проблемы Механики, Ташкент, 2002, №6, стр.27-32.
3. Амиркулова Ф.А. Колебания предварительно -напряжен-ного вязкоупругого кругового цилиндрического слоя с защемленным торцом // ДАН РУз, 2004, №4, стр. 41-44.
4. Амензаде Ю.А. Теория упругости. – М: Высшая школа, 1976. – 272с.
5. Вестяк А.В., Горшков А.Г., Тарлаковский Д.В. Нестационарное взаимодействие деформируемых тел с окружающей средой // В.кн. Итоги науки и техники. Механика деформируемого твердого тела, том 15. – М.: ВИНИТИ, 1983.-С. 69-148.
6. Кубенко В.Д. Нестационарное взаимодействие элементов конструкций со средой.-Киев: Наук. думка, 1999.-252 с.
7. Ляв А. Математическая теория упругости. – М. – Л.: ОНТИ, 1935. – 674с.
8. Петрашень Г.И. Проблемы инженерной теории колебаний вырожденных систем // Исследования по упругости и пластичности.- Л.:»Изд-во ЛГУ», 2006. №4.-С. 23-51.
9. Sartayev A. Doiraviy qovushoq-elastik sterjenlarning bo'ylama tebranishlarini klassik va aniqlashtirilgan tenglamalar asosida sonli tadqiq etish. Magistr. Dissert. Samarqand, 2011 – 69 bet.
10. Филиппов И.Г, Худойназаров Х.Х. Уточнение уравнений продольно-радиальных колебаний круговой цилиндрической вязкоупругой оболочки // Прикл. мех.-1990.-26,№2.-с.63-71.
11. ХудойназаровХ., Ш. Буркутбоев Продольно-радиальные колебания вращающегося цилиндрического слоя под действием импульсной

нагрузки//Современний проблеми механики грунтов и сложних реогологических систем. Материалы международной научно-технической конфренции. Самарканд-2013. 254-258

12. Худойназаров Х. Поперечние колебания кругових цилиндрических оболочек и стерженей, взаимодействуюших с деформируемой средой. SamDU Ilmiy tadqiqotlar axborotnomasi 2013 yil №1 son. 26-32 bet.
13. Худойназаров Х.Х., Амиркулова Ф.А. Взаимодействие цилиндрических слоев и оболочек со связанными полями. – Ташкент.Изд-во «Навруз», 2011 – 336с.
14. Худойназаров Х.Х. Нестационарное взаимодействие круговых цилиндрических упругих и вязкоупругих оболочек и стержней с деформируемой средой. – Ташкент: «Изд-во им. Абу Али ибн Сино», 2003.-325с.
15. Худойназаров Х.Х., Абдирашидов А. Нестационарное взаимодействие упругопластически деформируемых элементов конструкций с жидкостью. – Ташкент: «ФАН», 2005. – 220с.
16. Худойназаров Х.Х., Исматова Ф.И., Диёров Н.Н. Основные направления развития уточненных теорий колебаноя цилиндрических оболочек // Илм. мақ. тўплами «Механика ва математиканинг замонавий масалалари». – Самарканд: «СамДУ нашриёти», 1997. – С. 184 – 194.
17. Huang H. Interaction of acoustic shock waves with a cylindrical elastic shell immersed near a hard surface// Wave Motion, 2001, 3, № 5.- pp. 269 – 278.
18. Khudaynazarov Kh., Yalgashev B.F. Axisimmetrical oscillations of cyclic cylindrical laer with the viscous compressible liquid // Materials of the international conference dedicated to M.A.Lavrentyev on the occasion of his birthday centenary. Ukraine, Kiev, 31 oktober-3 november 2000.
19. Markus Stefan.The mechanics of cylindrical shells.-Amsterdam: «Elsevier», 1988.-195p.

20. Xolmurodov R.I., Xudoynazarov X.X. Elastiklik nazariyasi. – Tashkent: «FAN», 2003. – ч.I. – 159с, ч.II. – 163с.
21. Худойназаров Х.Х., Буркутбоев Ш.М. Математическая модель поперечных колебаний кругового цилиндрического слоя, взаимодействующего с внутренним и внешним потоками жидкостей. // VI Казахтанско-Российская международная практическая конференция «Математическое моделирование научно-технологических и экологических проблем нефте-газедобывающей промышленности» 11-13 октября 2007г. Астана.
22. Худайназаров Х.Х., Бабажанов Б.Б.Расчет трехслойного стержня под действием импульсной нагрузки. Всероссийская научно-практическая конференция «Инженерные системы - 2008». Москва, 7-10 апреля 2008 г. с 50.
23. Худойназаров Х.Х., Ялгашев Б.Ф. Уточненные уравнения продольно-радиальных колебаний круговых цилиндрических слоев и оболочек, заполненных вязкой сжимаемой жидкость. Труды межд. конф. «Современные проблемы газовой и волновой динамики», Москва, 21 апреля 2009 г. – Москва, Изд. МГУ 2009. – С.
24. Григолюк Э.И., Селезов И.Т. Неклассические теории колебаний стержней, пластин и оболочек// Итоги науки и техники. Сер. Механика деформ. Твердых тел. – Т. 5 – М.: ВИНИТИ, 1973. – 272с.
25. Худойназаров Х. У Нишонов.«Nonlinear modelling elastic deformation of ribbed plate». 2011йил 4-6 июнь, Самарқанд шаҳри.
26. Khudoynazarov Kh., Burkutboyev Sh. Modelling and analysis of torsional vibrations of rotating cylindrical shell// Samarkand State University and Malaysian Mathematical Sciences Society Joint Mathematics Meeting, 2011 Sciences.-Samarkand, 02-03.06.2011.-2 p.
27. Khudoynazarov Kh., Karshiev A. Nishonov U. Nonlinear modelling elastic deformation of ribbed plate// Samarkand State University and Malaysian

Mathematical Sciences Society Joint Mathematics Meeting, 2011 Sciences.-Samarkand, 02-03.06.2011.-2 p.

28. Худойназаров Х.Х., Гадоев А., Бегжанов А. Уравнения технической теории продольно-радиальных колебаний цилиндрического слоя.//Научно-технический журнал «Проблемы архитектуры и строительства», 2014, №1.-С.77-82.
29. Бегжанов А. Суюклик билан таъсирлашувчи доиравий цилиндрик стерженнинг буйлама тебранишлари // СамДУ Магистрантлари илмий ишлари туплами, 2014. – СамДУ нашри.