O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

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ANIZOTROP SILINDRIK QATLAM VA QOBIQLARNING O’QQA NISBATAN SIMMETRIK TEBRANISHLARI

5A440204 – Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi

Deformatsiyalanuvchi qattiq jism mexanikasi mutaxassisligi bo’yicha magistr darajasini olish uchun

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M U N D A R I J A

KIRISH

Hozirgi zamon texnikasining juda tez sur’atlar bilan rivojlanishi deformatsiyalanuvchi qattiq jismlar mexanikasi oldiga yangidan-yangi, vaqt o’tgan sari tobora murakkablashib borayotgan masalalarni qo’ymoqda. Shu paytgacha ishlatib kelingan an’anaviy materiallar yuqori bosimli va yuqori haroratli o’ta murakkab sharoitlarda ishlatilmoqda. Bu esa o’z navbatida materiallar ustidagi tadqiqotlarni ularning reologik xususiyatlarini hisobga olgan holda o’tkazishni taqozo qiladi. Ana shunday reologik xususiyatlarga qovushoq-elastiklik kiradi.

Qovushoq-elastiklik nazariyasining intensiv ravishda rivojlanishi va amaliyotga keng miqyosda qo’llanilishi keyingi bir necha o’n yilliklarga to’g’ri keladi. Bu sohadagi aktivlik, taxminan 1950 yillarda boshlanib, birinchi navbatda raketa texnikasining ehtiyojlari va polimer materiallarning raketa qattiq yoqilg’isi sifatida ishlatilishi hamda polimerlarning boshqa sohalarda ham keng qo’llana boshlanganligi bilan bog’liq. Mana shu yangi plastmassa va polimer moddalari shunday mexanik xususiyatlarga egaki, bu xususiyatlari klassik elastiklik nazariyasi yoki yopishqoq suyuqlikning harakati nazariyasi doiralarida tavsiflab bo’lmaydi. Natijada yangi, umumiyroq nazariya yaratish zaruriyati paydo bo’ldi.

Ma’lumki, elastik jism unga sarf qilingan energiyani to’liq qaytaradi, ya’ni o’zida tashqaridan berilayotgan energiyani to’plash xususiyatiga ega. Tashqi ta’sir yoki tashqi energiya uzatish jarayoni tugagandan keyin, elastik jism to’plangan energiyani to’liq qaytaradi va o’zining asl holiga qaytadi. Ta’kidlash kerakki, bunda energiyani qisman bo’lsada tarqatish jarayoni kuzatilmaydi. Ikkinchi tomondan Nyutonning yopishqoq suyuqligi nogidrostatik kuchlanganlik holatida tashqaridan berilgan energiyani tarqatib yuborish xususiyatiga ega. Lekin uni to’plash xususiyatiga ega emas. Shuning uchun ham bu ikki klassik nazariya o’zlarining deformasiyalanishlari uchun sarflangan ishni qisman (to’liqmas) qaytarish xususiyatiga ega bo’lgan moddalarning mexanik xulqining tavsiflashga ojizlik qiladilar. Bunday materiallar mexanik energiyani to'plash xususiyati bilan birgalikda, uni tarqatish xususiyatiga ham egadirlar.

Bunday materiallarni tavsiflashning boshqa bir usuli, namuna sirtiga to'satdan qo'yilgan tekis tarqalgan zo'riqish ta'sirida ularning mexanik xulqini tavsiflashdan iboratdir. Bunda to'satdan qo'yilgan zo'riqish deganda namunaga dinamik kuch qo'yilgan deb tushunmaslik kerak, ya'ni to'satdan qo'yilgan zo'riqish namunaning dinamik deformasiyalanishiga olib keluvchi tezliklarda amalga oshirilmasligi kerak.

Shunday to'satdan qo'yilgan va keyinchalik o'zgarmay qoladigan kuchlanish ta'sir ettirilgan elastik jism, oniy deformasiyalanadi, qaysiki keyinchalik bu deformasiyalar o'zgarmasdan qoladi. Agar Nyuton tipidagi yopishqoq suyuqlikda bir jinsli urinma kuchlanish to'satdan qo'yilsa stasionar oqim vujudga keladi.

Shunday materiallar mavjudki, to'satdan qo'yilib keyin o'zgarmas holda ushlab turilgan kuchlanganlik holati ularda oniy deformasiyani vujudga keltiradi va uning orqasidan oqish jarayoni boshlanadi, qaysiki vaqtning o'sib borishi bilan chegaralangan yoki chegaralanmagan oqish jarayoni bo'lishi mumkin. O'zini mana shunday tutuvchi materiallar haqida bir vaqtda elastiklik va polzuchest xususiyatiga ega material deb yuritadilar. Bunday xususiyat, ko'rinib turibdiki, elastik yoki yopishqoq modellar yordamida tavsiflanishi mumkin emas, balki har ikkala xususiyat belgilarini o'zida mujassamlashtiradi.

Qo'yilgan sirt kuchlarini bir marta to'satdan o'zgartirganda material mexanik xulqining umumlashmasi bo'lgan holni qarab chiqish foydalidir. Faraz qilaylik yuqoridagi oniy elastiklik va polzuchest xususiyatlarga egan bulgan material bir kuchlanganlik holatining har xil vaqtlarda sodir bo'luvchi va biri ikkinchisining ustiga tushuvchi ikkta o'zgarishiga tortilsin. Kuchlanishlar birinchi marta qo'yilganidan keyin to ikkinchi o'zgarishgacha o'tgan vaqt davomidagi materialning mexanik xulqi vaqtdan va boshlang'ich qo'yilgan kuchlanish miqdoridan bog'liq bo'ladi. Endi ikkinchi kuchlanganlik holati to'satdan qo'yilgandan keyin o'tgan juda kichik vaqt intervalidan keyin vujudga keluvchi holatni qaraymiz. Materialning mexanik xulqi nafaqat tashqi zo'riqishlarning ikkinchi o'zgarishidan, balki birinchi qo'yilgan kuchlanishlar sathining davom etayotgan va vaqtdan bog'liq ta'siriga ham bog'liq bo'ladi. Shu yerda shuni ta'kidlash lozimki elastik materialning mexanik xulqi vaqtning istalgan payti uchun faqat kuchlanishlarning umumiy yig'indi sathiga bog'liq bo'ladi.

Shunday qilib, qaralayotgan elastik va yopishqoq materiallarga nisbatan umumiyroq tipdagi material, xotira effekti deb atalishi mumkin bo'lgan xususiyatga egadir. Bunda materialning mexanik xulqi nafaqat kechayotgan kuchlanganlik holati bilan, balki o'tgan barcha kuchlanganlik holatlari bilan aniqlanadi. Umumiyroq qilib aytganda material o'tgan holatlarni «eslab qoladi». Shunga o'xshash holat deformasiyalar qaralganda ham vujudga keladi: bu holda kechayotgan kuchlanish butun deformasiyalanish tarixidan bog'liq bo'ladi.

Adabiyotlar ro'yxatida keltirilgan [12] monografiyada ana shu oxirgi xususiyatning, ya'ni materiallarning eslab qolish xususiyati faktining oddiy, lekin fundamental matematik tavsifi beriladi va izotermik sharoitlarda kuchlanishlar va deformasiyalar oralidagi chiziqli qayishqoq-elastiklik munosabatlarini qurish uchun Stiltyes integralidan foydalaniladi. Bunda «Xotira» terminidan foydalanish ancha qat'iylashadi; shu bilan birga ta'kidlash lozimki xotirali materiallarning mexanik xulqi haqidagi boshqa nazariyalar ham mavjud, lekin ular bu yerda keltirilayotgan nazariyadan tubdan farq qiladilar.Masalan inkremental plastiklik nazariyasi xotira effektini hisobga oladi, lekin shu manodaki oxirgi deformasiya holati nafaqat oxirgi kuchlanganlik holatidan bog'liq bo'ladi, balki kuchlanishlar fazosidan oxirgi kuchlanganlik holatiga olib kelgan yo'ldan ham bog'liq bo'ladi.

Ta'kidlash lozimki, plastiklik nazariyasida yuklash va yuksizlash programmasida foydalanilgan vaqt masshtabi hisobga olinmaydi, qayishqoq-elastiklik nazariyasida esa vaqtdan xarakterli bog'lanish yoki deformasiyalanish tezligidan bog’lanish mavjud. Xuddi ana shu narsa bu ikki nazariya o’rtasidagi farqni tashkil etadi.

Biz bundan keyingi hamma xulosalarni chiqarishda va tadbiqda materialni bir jinsli deb hisoblaymiz. Juda ko’p hollarda ba’zi birjinslimas materiallar uchun umumlashmalarni yengil chiqarish mumkin, lekin shunday materiallar ham mavjudki ular uchun bunday umumlashmalar qilish juda qiyin yoki umuman mumkin emas. Bu yerdan xulosa qilib shuni aytish mumkin: bir jinsli materiallar uchun olingan va rivojlantirilgan metodlarni birjinslimas materiallar uchun umumlashtirish har bir holda individual yondashishni talab etadi.

Qovushoq-elastiklik nazariyasining juda ko’p natijalari oxirgi vaqtlarda olingan bo’lsada, chiziqli izotermik hol uchun qovushoq-elastiklik nazariyasi juda ko’p vaqtdan buyon ma’lum. Bu Maksvell, Kelvin, Foxt kabi olimlar nomi bilan bog’liq.

Birinchi marta 1874 yilda Bolsman uch o’lchovli izotrop qovuhoq-elastiklik nazariyasining tenglamalarini berdi. 1909 yilga kelib Volterra ushbu munosabatlarni anizotrop jismlar uchun yaratdi.

Mavzuning dolzarbligi. Elastik jismlarda nostatsionar to’lqinlarning tarqalish jarayonlarini va qonuniyatlarini o’rganish mexanikaning dolzarb masalalaridan biridir. Shu nuqtai nazardan qaraganda tebranish jarayonlarining paydo bo’lish qonuniyatlarini o’rganish ham muhim ahamiyatga ega. Doiraviy silindrik qatlam va qobiqlarda va boshqa muhandislik inshootlari elementlarida (sterjenlar, plastinalar va boshqalar) to’lqinlar tarqalish jarayonlari va qonuniyatlarini tadqiq etish yuqorida ko’rsatilgan ana shunday dolzarb masalalarning eng muhimlaridan hisoblanadi.

Doiraviy silindrik qatlam va qobiqlarining buralma, bo’ylama-radial va ko’ndalang tebranishlari jarayonida, deformatsiyalanuvchi muhit bilan o’zaro nostatsionar ta’sirlashuvini o’rganish haqidagi masalalar ham ko’rsatib o’tilgan dolzarb masalalar qatoriga kiradi. Shu nuqtai-nazardan qaraganda dissertatsiya mavzusi doirasida tadqiq etilishi lozim bo’lgan anizotrop silindrik qatlam va qobiqlarning nostatsionar tebranishlari tenglamalarini keltirib chiqarish hamda ular asosida hal etilgan amaliy masalalarning natijalarini o'rganish va shunday masalalarni yechish deformatsiyalanuvchi qattiq jism mexanikasining aktual masalasidir.

Dissertatsiyaning asosiy maqsadi yuqorida aytilganlardan kelib chiqqan holda, quyidagilardan iboratqilib belgilangan:

Tashqi dinamik yuklar ta'siri ostidagi doiraviy silindrik anizotrop qatlam va qobiqlarning nostatsionar tebranishlari taqribiy va aniqlashtirilgan tenglamalarni keltirib chiqarishni o'rganish; o'rganilgan tenglamalar asosida doiraviy silindrik elastik va qovushoq elastik qatlam, qobiq va sterjenlarning o'qqa nisbatan simmetrik tebranishlarini ular materialining anizotropik, xususan, transversal-izotrop xususiyatini hisobga olgan holda tadqiq etish; tadqiqot natijalari asosida ilmiy xulosalar chiqarish .

Olingan asosiy natijalarning ilmiy yangiligi quyidagilardan iborat:

- doiraviy silindrik elastik qobiqning nostatsoinar tebranishlari texnik nazariyasi taqribiy tenglamalari tadqiqi;
- olingan taqribiy tenglamalar asosida deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi anizotrop silindrik qatlamning buralma tebranishlari;
- aylanish inersiyasi va ko'ndalang siljish deformatsiyasining anizotrop silindrik qatlamning buralma tebranishlariga ta'siri haqidagi masalaning tahlili;
- anizotrop sterjenning uchidan berilgan zarba haqidagi masalaning tadqiqi.

Dissertatsiya himoyasiga quyidagi natijalar olib chiqilmoqda:

- doiraviy silindrik qatlam, qobiq va sterjenlarning o'qqa nisbatan simmetrik tebranish taqribiy va aniqlashtirilgan tenglamalari;
- olingan taqribiy tenglamalar asosida deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi anizotrop silindrik qatlamning buralma tebranishlari tadqiqi;
- aylanish inersiyasi va ko'ndalang siljish deformatsiyasining anizotrop silindrik qatlamning buralma tebranishlariga ta'siri haqidagi masalaning tahlili;
- anizotrop sterjenning uchidan berilgan zarba haqidagi masala natijalari.

Tadqiqotlarning ilmiy ahamiyati doiraviy silindrik qovushoq-elastik va elastik transversal-izotrop qatlamning buralma tebranishlari haqidagi masalaning yechilshi bo'lib, olingan natijalar tebranish jarayonidagi sodir bo'lishi mumkin bo'lgan holatlarni aniqlashda, sistema nuqtalaridagi kuchlangan-deformatsiyalangan holatini hisoblashda va boshqa hollarda ishlatilishi mumkin.

Tadqiqotlarning amaliy ahamiyati doiraviy **s**ilindrik anizotrop qatlamning o'qqa nisbatan simmetrik tebranishlari texnik nazariyasi tenglamalarining boshqa masalalarni, xususan silindrik qatlam, qobiq va sterjenlarning majburiy tebranishlari haqidagi masalalarni ham yechishga qo'llash mumkin ekanligidir.

Dissertatsiya ishi kirish, 2 ta bob, asosiy xulosalar va foydalanilgan adabiyotlar ro'yxatidan iborat.

Birinchi bobda dissertatsiya ishini bajarishda lozim bo'ladigan asosiy tenglama va munosabatlar bayon etilgan.

Birinchi paragraph doiraviy silindrik qatlam, qobiq va sterjenlarning deformatsiyalanuvchi muhit bilan o'zaro nostatsionar ta'siriga bag'ishlangan ilmiy adabiyotlar sharhiga bagishlangan bo'lib, avvalo unda qanday adabiyotlarga sharh berilishi kerakligi asoslangan. Xususan, Shunday masalalarga materiallarning nostatsionar xaraktirdagi modellarining mukammallashtirish va ularning reologik, anizotrop va boshqa xususiyatlarni hisobga olgan holda haqiqiy jismlarga yaqin konstruksiyalarining tebranishlari masalalari kiradi. Materiallarning deformatsiyalanuvchanlik va mustaxkamlik xususiyatlarini ma'lum va sinovdan o'tgan hamda mukammallashtirgan modellarning qurshab turgan muhit tashqi dinamik (siyslik,portlash,akustik vaboshqa) ta'sirlar doirasida materialning deformatsiyalanuvchi va mustaxkamlik xususiyatlarini effektiv aniqlash usullarini mukammallashtirish muhim ahamiyat kasb etadi. Bu masalalarning yechimlariga juda ko'p tadqiqotlar bag'ishlangan. Shuning uchun to'g'ridan-to'g'ri yoki ko'chma ma'noda ushbu disertatsiyaga aloqador ishlarning qisqacha sharhi keltirilgan.

Ikkinchi paragraf elastiklik nazariyasining geometrik, dinamik va fizik tenglamalari, bu nazariyaning asosiy masalalarining shakllantirilishiga bag'ishlangan. Elastiklik nazariyasining asosiy tenglamalari juda ko'p darsliklarda bayon etilgan. Ular ichidan faqat [22] darslikdan foydalandik va shu kitobda keltirilgan ma'lumotlardan ba'zilarini keltirilgan. Bu tenglamalar yopiq sistemani tashkil etadi va jismga ta'sir etgan tashqi kuchlarni hisobga olgan holda uning kuchlangan-deformatsialangan holatini aniqlashga imkon beradi.

Uchinchi paragraf silindrik (r,θ,z) koordinatalar sistemasida bir jinsli va izotrop, ichki va tashqi radiuslari mos ravishda r_1 va r_2 bo'lgan doiraviy silindrik qovushoq-elastik qatlam qaralgan [7]. Tebranish tenglamalarini chiqarishda silindrik qatlamni uch o'lchovli matematik elastiklik nazariyaga qat'iy bo'ysinuvchi uch o'lchovli jism sifatida olingan

Ikkinchi bob doiraviy silindrik qovushoq-elastik, transversal-izotrop qatlamning va transversal-izotrop doiraviy sterjenning buralma va bo'ylama-radial tebranishlari haqidagi masalalarni tadqiq qilishga bag'ishlangan. Bu bob 4 ta paragrafdan iborat.

Birinchi paragraf silindrik qatlamning o'qqa nisbatan simmetrik tebranishlari texnik nazariyasi tenglamalari keltirib chiqarishga bag'ishlangan. Eslatilgan matematik nazariyaning deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi silindrik qovushoq-elastik qatlamning o'qqa nisbatan simmetrik tebranishlari tenglamalari juda umumiy ko'rinishga ega bo'lganliklari uchun amaliy masalalarni yechishda foydalanish mumkin bo'lgan tebranish tenglamalari va KDH ni hisoblash formulalarini chiqarilgan. Pirovardida bu tenglama va formulalarni silindrik qatlam o'qqa nisbatan simmetrik tebranishlari texnik nazarisining asosiy tenglama va munosabatlari deb atalgan.

Ikkinchisida esa deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi anizotrop silindrik qatlamning buralma tebranishlari tadqiq etilgan. O'rab turuvchi deformatsiyalanuvchi muhitning uzunligi chegaralanmagan transversal-izotrop qatlamning buralma tebranishlariga ta'siri o'rganilgan. Fazaviy tezlikka nisbatan oltinchi tartibli dispersion tenglama olingan va u sonly yechilgan. Qatlamning erkin buralma tebranishlariga tashqi muhitning ta'siri baholangan. Qatlamni o'rab turuvchi tashqi muhit parametrlari sifatida ba'zi tuproq va tog' jinslarining fizik-mexanik xarakteristikalari olingan.

Uchinchi paragrafda aylanish inersiyasi va ko'ndalang siljish deformatsiyasining anizotrop silindrik qatlamning buralma tebranishlariga ta'siri o'rganilgan. Aniqrog'i ikkala cheti ham buralishdan qistirib mahkamlangan doiraviy silindrik anizotrop qatlamning xususiy buralma tebranishlariga aylanish inersiyasi va ko'ndalang siljish deformatsiyasining ta'sirini baholash haqidagi masala qaralgan.

Bunday ikkala cheti ham buralishdan qistirib mahkamlangan doiraviy silindrik transversal-izotrop qatlamning xususiy buralma tebranishlari uchun asosiy, hal qiluvchi tenglamalalar sifatida [10] tadqiqot ishida taklif etilgan aniqlashtirilgan tebranish tenglamalaridan foydalanilgan.

Dissertatsiya ishining oxirgi bo'limi asosiy xulosalar deb ataladi va bu bo'limda dissertatsiya ishida olingan eng muhim natijalar asosida chiqarilgan xulosalar keltirilgan. Dissertatsiya ishi foydalanilgan adabiyotlar ro'yxati bilan yakunlanadi.

I BOB

SILINDRIK QATLAM VA QOBIQLAR HARAKATINING ASOSIY TENGLAMALARI

1.1. Silindrik qobiq va qatlamlarning tebranishlari bo'yicha olib borilgan tadqiqotlar to'g'risida.

Qurilish va tabiatshunoslikning turli sohalari va ilmiy texnik jarayonning talablari doimo yangidan-yangi nazariy va amaliy qurilish mexanikasi masalalarini olg'a suradi. Shunday masalalarga materiallarning nostatsionar xaraktirdagi modellarining mukammallashtirish va ularning reologik, anizotrop va boshqa xususiyatlarni hisobga olgan holda haqiqiy jismlarga yaqin konstruksiyalarining tebranishlari masalalari kiradi. Materiallarning deformatsiyalanuvchanlik va mustaxkamlik xususiyatlarini ma'lum va sinovdan o'tgan hamda mukammallashtirgan modellarning qurshab turgan muhit tashqi dinamik (siyslik,portlash,akustik vaboshqa) ta'sirlar doirasida materialning deformatsiyalanuvchi va mustaxkamlik xususiyatlarini effektiv aniqlash usullarini mukammallashtirish muhim ahamiyat kasb etadi .Bu masalalarning yechimlariga juda ko'p tadqiqotlar bag'ishlangan. Shuning uchun biz to'g'ridan-to'g'ri yoki ko'chma ma'noda ushbu disertatsiyaga aloqador ishlarning qisqacha ro’yxatini keltiramiz.

Nostatsionar xarakterdagi tutash muhut bilan bog'liq muammolar borasida dastlabki tadqiqotlar A.G.Gorshkov[16], E.I.Grigol yuk[13], G.I.Petrashen [18] va boshqalar. Bu ishlar mexanikaning turli yunalishlari bo’yicha olib borilgan,ular orasida ayniqsa muhitli konstruksiya elementlarining nostatsionar ta’sirlari haqidagi masalalar alohida kasb etadi.

Sterjen,plastina,qobuqning deformatsiyalanuvchi muhit bilan nostatsionar ta’siriga bag’ishlangan nazariy va amaliy tadqiqotlar har xil matematik modellari va boshqa savollar E.I.Grigoryuk va A.G.Gorshkov[13,16], V.D.Kubenko[15] ishlarida ko'satilgan. Deformatsiyalanuvchi jismlarning o'zlarini qurshab turgan deformatsiyalanuvchi muhit bilan o'zaro nostatsionar ta'siriga bag'ishlangan to'liqroq ma'lumotlar A.V.Vestyak, A.G.Gorshkov va D.V.Tarlakovskiy [13]ishlarida ko'rsatilgan. To'lqinlarning qayishqoq va elastik qayishqoq muhitda tarqarilishi shuningdek sterjen,plastina va qobiqlarning tebranishlari I.G.Filippov,O.A.Egorchev [19] va Markus Stefan [14] va boshqalar ishlarida ko'rsatilgan. Shuni aytib o'tish kerakki, 1972 yilgacha chop etilgan va sterjen,plastina,qobiqning tebranishlariga bag'ishlangan ishlarning to'liq shakli E.I.Grigolyuk va I.T.Selezov[13] va boshqalarning monografiyalarida keltirilgan.

Konstruksion elementli dinamik masalalarni yechishda tadqiqotchilar turli usullarni qo'llaydi. Ular orasida bu sistemalarning dinamik xaraktirlarini taqribiy va aniq tebranish tenglamalarini o'rganish asosiy o'rin tutadi. Bu tadqiqot statikaning shunga o'xshash masalalaridagi oddiyroq matematik dinamika masalalari uchun bir o'lchamli va ikki o'lchamli masalalari taqriban integrallashga asoslangan.

[13]dan ma'lumki,klassik tenglamani kichik chastotali tebranishlar jarayonlarini qanoatlantirish uchun yozilgan. Bir muncha katta chastotali jarayonlar uchun yetarli emas. Qisqa muddatli dinamik yuklanish ta'siridagi mexanik sistema uchun yozilganda unchalik to'g'ri bo'lmaydi. Bir qancha tadqiqotchilar bu klassaik nazariya kamchiligi ustida bir qancha urinishlarni doiraviy qobiq ko'ndalang kesimi tebranish tenglamalarini aniqlashtirgan .

Ta'kidlash kerakki , bunday ishlarni aniqlashtirilgan klassik tebranishalar nazariyasi bilan ko'p oilmlar aniqlashtirilgan nazariyasi ishlagan.Tadqiqot qilishda mexanik sistema tebranishlar masalalarni yechish uchun har xil qabul qilingan gipotezalar , chaklashlar tebranish tenglamalarini qobiqlar nazariyasi uchun Krirxgaf – Zyav gipotezasi qabul qilinganligi barchaga ma'lum .Yana biri Temoshenko tipidagi nazariyaham qabul qilingan .

Silindrik qobuq va qatlamning klassik tebranishlar ustida juda katta ishlar [7] ishlarda analiz qilingan. [7,13] ishlar natijasiga ko’ra bu nazariyasinini kamchiligi ixtiyoriy qalinlikdagi silindrik qatlamlardagi to’lqin jarayonlarini matematik nuqtaiy nazardan tavsiflashga imkon bermaydi. Qatlam materialining aniq ko’rinib turgan xarakteristikalarini hisobga olmaydi . Kuchalishi deformasiyalangan holarlar uchun va deformasiyalar tenzorlarning barcha kompanintlarini aniqlashga imkon bermaydi.

Shuning uchun , kelgusuda aytib o’tolayotgan ishlarga [13] monografiya va shartlariga asosan ushbu dissertatsion ishda qaralayotgan masalalarga bevosita aloqador bo’lgan ayrim tadqiqotlarni ko’satib , o’tamiz [1,4,6,7,18,19,20,21] ishlarga binoan G.I.Petrashenya, I.G. Filippov va X.Xudoynazarov ishlarida yoritilgan qayishqoqlikning dinamik nazariyasining uch o’lchamli masalalarining umumiy yechimidan foydalinish usuli qo’llanilgan nazariya kamchiliklardan holi nazariya deb topildi. Bu usulning ajoyib va omadli qila olgan I.G.Filippov va uning shogirdlari stejenlar, plastinalar, turli reologik anizotrop temperaturaviy bir bir jinsli emslik va hakoza xossalarini hisobga olgan holda , shuning dek kesimning o’zgartirayotgan va boshqa faktorlarni hisobga olgan holda ixtiyoriy kesimga qalinlikdagi slindrik qobuq va qatlamlar uchun umumiy tebranish tenglamalarni oldilar.

Ular qayishqoqlikning , qayishqoq yoki deyarli qayishqoq sistema sirtining chegarilida berilgan dinamik shartlarni qanotlantiruvchi darajali qatorlarga yoyilagan taqribiy yechimnidan topishga keltiriluvchi uch o’lchovlimasalarning umumiy yechimidan foydalanib va integral almashtirishlarni vaqt va koordinata bo’yicha qo’llshga asoslangan. Ushbu usulning qulay shuki , turli yechim yordamida ko’chish va kuchlanishlarni ixtiyoriy kesimda ixtiyoriy vaqt momentiuchun topish mumkin .

Bundan tshqari bu metoqga asoslangan nazariya silindrik qatlam va qobiqning chegaralarida berilgan chegaraviy shartlarni , shuningdek muhitni qursgab olgan muhit chegarasida ta’sirlashish shartlarini yanada to’g’riroq talqin qilish mumkin.Bu nuqtaiay nazardan [7] ishda yoritilgan o'zi qurshab olgan muhit bilan nostasionar ta'sirlashuvchi doiraviy silindrik elastik qayishqoq qatlamning tebranishlarining matematik nazariyasi muhim ahamyatga ega .

Unda elastik qayishqoqlik anizotropiya , qalinlikning o'zgaruvchi qatlam materialining bir qancha xossalarini hisobga olib defoamsiyalanuvchi qattiq va suyuq muhit ta'sirida silindrik qatlamning o'qqa simmetrik bo'lgan tebranishlarining matematik nazariyasi yaratilgan .

Dissertatsiyaning 2-3- boblarida bu nazariyaning tushunchalari va tenglamalari berilgan. Kuchalanganlik deformasiyalanganlik holti silindrik qobiqning ko'ndalang va bo'ylama kesimi tebranish tenglamalari mutaxasislar tekshirgan. Ayniqsa , [20] ishda German – Mirski nazariyasi doiraviy silindrik qobiqning deformasiyasi tekshirilgan. Elastik silindrik qovushoq qobiqning bo'ylama zarba ta'siri haqidagi asosiy aniqlangan tebranish tenglamalari aniq masalalar , [1] ishda yechigan , bu yerda aylanish inersiyasi inobatga oinadi va ko'ndalang siljish deformasiyasi inobatga olinadi .

Deformasiyaning o'tish jarayoni yopiq doiraviy silindrik qobiqda o'qqa nisbatan simmetrik bo'lmagan chetki yuklanishlar S.P.Timoshenko nazariyasi asosida [9] ishda ko'rsatilgan . Silindrik qobiq va qatlamiga tebranish deformatsiyasining o'tish jarayoni texnikada eng muhim ahamiyatga ega, bu masalaning yechimi nostasionar akustik atrof va zarbali to'lqinlar bilan bog'liq.

1.2. Elastiklik nazariyasining geometrik, dinamik va fizik tenglamalari. Elastiklik nazariyasining asosiy masalalari.

Elastiklik nazariyasining asosiy tenglamalari juda ko'p darsliklarda bayon etilgan. Biz ular ichidan faqat [22] darslikdan foydalandik va shu kitobda keltirilgan ma'lumotlardan ba'zilarini quyida keltiramiz. Bu tenglamalar yopiq sistemani tashkil etadi va jismga ta'sir etgan tashqi kuchlarni hisobga olgan holda uning kuchlangan-deformat-sialangan holatini aniqlashga imkon beradi. Bu tenglamalarni uch turga bo'ladilar: *geometrik, statik (dinamik) va fizik tenglamalar.*

1^0. Geometrik tenglamalar.

Elastik jismning deformatsialangan holati deformatsia tenzori $\varepsilon_{ij} = \varepsilon_{ij}(x_k)$ komponentalari, yoki $u_i = u_i(x_k)$ ko'chishlar bilan to'lqin aniqlanadi. Deformatsia tenzori komponentalari va ko'chishlar o'zaro Koshining differensial munosabatlari bilan bog'langan:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (1.2.1)$$

hamda Gen-Venanning differensial munosabatlari bilan o'zaro bog'langan:

$$\varepsilon_{ik,j\ell} + \varepsilon_{j\ell,ik} - \varepsilon_{i\ell,jk} - \varepsilon_{jk,i\ell} = 0. \qquad (1.2.2)$$

Ma'lumki, bu munosabatlar deformatsialarning uzviylik tenglamasi deb ham yuritiladi.

2^0. Dinamik (Statik) tenglamalar.

Elastik jismning kuchlanganlik holati kuchlanish tenzorining $\sigma_{ij} = \sigma_{ij}(x_k)$ oltita komponentasi bilan to'liq aniqlanadi. Ushbu komponentalar *uchta muvozanat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j} + \rho f_i = 0. \qquad (1.2.3)$$

Agar jism harakatda bo'lsa, simmetrik kuchlanish tenzorining olti komponentasi *uchta harakat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j} + \rho f_i = \rho \frac{\partial^2 u_i}{\partial t^2} . \qquad (1.2.4)$$

Ushbu (1.2.3) - tenglamalar *statik,* (1.2.4) - tenglamalar *dinamik* tenglamalar deb yuritiladi.

3⁰. Fizik tenglamalar.

Kuchlanish tenzorining σ_{ij} komponentalari deformatsiya tenzorining ε_{ij} komponentalari bilan Guk qonuni vositasida bog‘langan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \qquad (1.2.5)$$

yoki u_i ko‘chish komponentalari bilan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \qquad (1.2.6)$$

ko‘rinishda bog‘langan. Bu yerda

$$\theta = \varepsilon_{ii} = u_{i,i} = div\vec{u} \, .$$

Ba’zi hollarda Guk qonunini (1.2.5) ga teskari shaklda, ya’ni ε_{ij} larga nisbatan yechilgan ko‘rinishda ishlatishga to‘g‘ri kelishi mumkin:

$$\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sum\right], \qquad (1.2.7)$$

bu yerda:

$$\sum = \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}.$$

Yuqorida sanab o‘tilgan (1.2.1) (1.2.3), (1.2.5), (1.2.7) formulalar elastiklik nazariyasi *statik masalalarining asosiy tenglamalari deb* yuritiladi. Elastiklik nazariyasi *dinamik masalalarining asosiy tenglamalari deb* (1.2.1) - (1.2.2), (1.2.4) (1.2.5) va (1.2.7) tenglamalarga aytiladi.

Elastiklik nazariyasining asosiy masalalari. Chiziqli-elastik jismning holatini uning V hajmining ichki nuqtalarida aniqlovchi asosiy tenglamalarga uning S sirtidagi shartlarni qo‘shish kerak. Bu shartlar *chegaraviy shartlar* deyiladi va ular tashqi berilgan F_i sirt kuchlari bilan yoki jism sirti nuqtalarining berilgan $u_i|_S$ ko‘chishlari bilan aniqlanadi.

Chegaraviy shartlarning berilishiga qarab elastiklik nazariyasining uch asosiy masalalarini bir-biridan farqlaydilar.

1^0. Birinchi tur asosiy masala.

Birinchi tur asosiy masalada f_i massaviy va F_i sirt kuchlari jismning butun sirtida berilganda jism egallagan V hajmning ichki nuqtalarida kuchlanish tenzori komponentalari $\sigma_{ij}(x_k)$ larni hamda V - hajmning ichki nuqtalari va jism S sirti nuqtalarida ko‘chish vektorining $u_i(x_k)$ komponentalarini aniqlash talab etiladi. Demak, bu holda chegaraviy shartlar:

$$\sigma_{ij} n_j|_S = F_i \tag{1.2.8}$$

ko‘rinishda bo‘ladi. Bu yerda F_i - sirt kuchi $\vec{F}$ ning komponentalari; $n_j - S$ sirtning qaralayotgan nuqtasida tashqi normali bo‘yicha yo‘nalgan birlik $\vec{n}$ vektori komponentalari.

Bu holda izlanayotgan to‘qqiz noma’lumlar (oltita σ_{ij} kuchlanishlar va uchta u_i ko‘chishlar) (1.2.3) yoki (1.2.4), (1.2.5) tenglamalarni, hamda (1.2.8) chegaraviy shartlarni qanoatlantirishlari kerak.

2^0. Ikkinchi tur asosiy masala.

Ikkinchi tur asosiy masalada f_i massaviy kuchlar va jismning S sirtida $u_i(x_k)|_S$ ko‘chishlar ma’lum bo‘lganda, jism egallangan V hajm ichidagi nuqtalarda $u_i(x_k)$ ko‘chishlarni va kuchlanish tenzori komponentalari $\sigma_{ij}(x_k)$ larni aniqlash talab etiladi. Demak, bu holda chegaraviy shartlar

$$u_i = u_i|_S \tag{1.2.9}$$

ko‘rinishida bo‘ladi. Izlanuvchi $u_i(x_k)$ va $\sigma_{ij}(x_k)$ funksiyalar (1.2.3) yoki (1.2.4), (1.2.5) hamda (1.2.9) chegaraviy shartlarni qanoatlantirishlari kerak.

3^0. Uchinchi tur asosiy masala.

Chegaraviy shartlar aralash xarakterga ega bo‘lishlari mumkin. Birinchi tur asosiy masalada jismning butun sirtida kuchlanishlar, ikkinchi tur asosiy masalada jismning butun sirtida ko‘chishlar beriladi. Shunday masalalar ham uchrashi mumkinki. Bunda jism sirtining ma’lum qismida kuchlanishlar, qolgan qismida esa ko‘chishlar berilishi mumkin. bunday holda *masala aralash masala* deyiladi. Faraz qilaylik, jism S sirtining S_σ qismida kuchlanishlar, S_u qismida esa ko‘chishlar berilgan bo‘lsin. Tabiiyki,

$$S = S_\sigma + S_u \, .$$

Uchinchi tur asosiy masalada jism sirtining S_σ qismida berilgan tashqi sirt kuchlari -F_i, va qolgan S_u qismida berilgan $u_i(x_k)\big|_S$ ko‘chishlar, hamda umumiy holda, berilgan f_i massaviy kuchlar bo‘yicha jism egallagan V sohaning ichki nuqtalarida $u_i(x_j)$ ko‘chishlarni hamda $\sigma_{ij}(x_j)$ kuchlanishlarni aniqlash talab etiladi.

Izlanayotgan to‘qqiz noma’lum funksiyalar bu holda (1.2.3) yoki (1.2.4), (1.2.5) tenglamalarni hamda

$$\begin{aligned} \sigma_{ij} n_j \Big|_{S_\sigma} &= F_i \\ u_i \Big|_{S_u} &= u_i \end{aligned} \qquad (1.2.10)$$

chegaraviy shartlarni qanoatlantirishlari kerak.

Yuqoridagi uch asosiy masaladan tashqari, elastiklik nazariyasining *to‘g‘ri va teskari masalalarini* ham farqlaydilar.

Elastiklik nazariyasining to‘g‘ri masalasida yuqorida keltirilgan uch asosiy masaladan birini tashqi kuchlar berilgan holda yechish, ya’ni jismning kuchlangan - deformatsialangan holatini aniqlovchi $u_i(x_k)$ va $\sigma_{ij}(x_k)$ funksialarni jism egallagan V sohaning ichki nuqtalari uchun aniqlash talab etiladi. Ammo ta’kidlash lozimki, elastiklik nazariyasining to‘g‘ri masalasini yechish juda katta matematik qiyinchiliklarga olib keladi.

Elastiklik nazariyasining teskari masalasida $u_i = u_i(x_k)$ ko‘chishlar yoki $\sigma_{ij} = \sigma_{ij}(x_k)$ kuchlanishlar uzluksiz funksiyalar sifatida beriladi. Asosiy (1.2.1) (1.2.2), (1.2.3) yoki (1.2.4) hamda (1.2.5) tenglamalardan qolgan funksiyalar va berilgan u_i ko‘chishlarni yoki kuchlanishlarni yuzaga keltiruvchi tashqi kuchlarni aniqlash talab etiladi.

Teskari masalani yechish to‘g‘ri masalani yechishga nisbatan ancha oson kechadi. Agar bunda ko‘chishlar berilgan bo‘lsa masala nisbatan juda oson yechiladi. σ_{ij} kuchlanishlar berilgan holda u_i ko‘chishlarni aniqlash uchun (1.2.1) tenglamalarni integrallashga to‘g‘ri keladi va σ_{ij} kuchlanishlarni uzviylik tenglamalari qanoatlanadigan qilib berishga to‘g‘ri keladi. Lekin baribir bunday masalani yechish to‘g‘ri masalani yechishga nisbatan oson.

Lame tenglamalari. Elastiklik nazariyasining to‘g‘ri masalasini, asosiy o‘zgarmaslar sifatida birinchi navbatda, yoki $u_i(x_k)$ ko‘chishlarni yoki $\sigma_{ij}(x_k)$ larni qabul qilib yechish qulay. To‘g‘ri masalani yechishning ana shu ikki yo‘li *ko‘chishlarga nisbatan yechim* yoki *kuchlanishlarga nisbatan yechim* deyiladi. Bunday holatlarda asosiy tenglamalar ham ko‘chishlarga nisbatan yoki kuchla-nishlarga nisbatan yozilishlari kerak.

Quyida biz asosiy tenglamani (muvozanat tenglamalarini) ko‘chishlarga nisbatan keltirib chiqaramiz. Buning uchun (1.2.6) Guk qonuni yordamida (1.2.3) kuchlanishlarga nisbatan muvozanat tenglamalaridan kuchlanish tenzorining $\sigma_{ij}(x_k)$ komponentalarini chiqarib tashlash zarur bo‘ladi.

Guk qonunining (1.2.6) ifodasidan x_j koordinata bo‘yicha hosila olamiz:

$$\sigma_{ij,j} = \lambda\delta_{ij}\theta_{,j} + \mu(u_{i,jj} + u_{j,ij}), \tag{1.2.11}$$

lekin

$$\theta_{,j} \cdot \delta ij = \theta_{,i}; \quad u_{j,ij} = u_{j,ji}\left(\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\partial^2 u_j}{\partial x_j \partial x_i}\right) \tag{1.2.12}$$

$$u_{j,j} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = div\vec{u} = \theta$$

hamda

$$u_{i,jj} = \frac{\partial^2 u_i}{\partial x_1^2} + \frac{\partial^2 u_i}{\partial x_2^2} + \frac{\partial^2 u_i}{\partial x_3^2} = \nabla^2 u_i \tag{1.2.13}$$

bu yerda ∇^2 orqali Laplas operatori belgilangan. Endi (1.2.12) va (1.2.13) ifodalardan foydalanib (1.2.11) - Guk qonunini quyidagicha yozish mumkin:

$$\sigma_{ij,j} = \lambda\delta_{ij}\theta_{,j} + \mu(\nabla^2 u_i + \theta_{,i})$$

yoki

$$\sigma_{ij,j} = \mu\nabla^2 u_i + (\lambda + \mu)\theta_{,i}. \tag{1.2.14}$$

Lekin

$$\lambda + \mu = \frac{Ev}{(1+v)(1-2v)} + \frac{E}{2(1+v)} = \frac{E}{2(1+v)(1-2v)} = \frac{\mu}{1-2v},$$

$$\sigma_{ij,j} = \mu\nabla^2 u_i + \frac{\mu}{1-2v}\theta_{,i}.$$

yoki nihoyat:

$$\sigma_{ij,j} = \mu\left(\nabla^2 u_i + \frac{1}{1-2v}\theta_{,i}\right) \tag{1.2.15}$$

olingan (1.2.15) ifodani (1.2.4) muvozanat tenglamalariga qo‘ysak,

$$\nabla^2 u_i + \frac{1}{1-2v}\theta_{,i} = -\frac{\rho}{\mu} f_i \tag{1.2.16}$$

elastik muvozanatning ko‘chishlarga nisbatan tenglamalariga ega bo‘lamiz. Bu tenglamalar uchta differensial tenglamalar sistemasini aniqlaydi:

$$\nabla^2 u_1 + \frac{1}{1-2\nu}\frac{\partial\theta}{\partial x_1} = -\frac{\rho}{\mu} f_1;$$
$$\nabla^2 u_2 + \frac{1}{1-2\nu}\frac{\partial\theta}{\partial x_2} = -\frac{\rho}{\mu} f_2;$$
$$\nabla^2 u_3 + \frac{1}{1-2\nu}\frac{\partial\theta}{\partial x_3} = -\frac{\rho}{\mu} f_3.$$

(1.2.17)

olingan (1.2.16) yoki undan kelib chiquvchi (1.2.17) tenglamalar *Lame tenglamalari deyiladi.*

Lame tenglamalarini bitta vektor tenglama ko‘rinishida ishlatish ancha qulay. Buning uchun (1.2.16) tenglamani $\vec{э}_i$ bazis vektoriga ko‘paytirish yetarli.

$$\nabla^2 u_i \cdot \vec{э}_i + \frac{1}{1-2\nu}\theta_{,i}\vec{э}_i = -\frac{\rho}{\mu} f_i\vec{э}_i,$$

lekin

$$\theta_{,i}\vec{э} = \frac{\partial\theta}{\partial x_1}\vec{э}_1 + \frac{\partial\theta}{\partial x_2}\vec{э}_2 + \frac{\partial\theta}{\partial x_3}\vec{э}_3 = grad\theta,$$

hamda

$$\theta = div\vec{u}, \qquad \vec{u}_i\vec{э}_i = \vec{u}$$

bo‘lganliklari uchun

$$\nabla^2\vec{u} + \frac{1}{1-2\nu} grad\, div\vec{u} = -\frac{\rho}{\mu}\vec{f}_i \tag{1.2.18}$$

vektor tenglamaga ega bo‘lamiz. Ushbu tenglamani

$$\nabla^2\vec{u} = grad\, div\vec{u} - rot\, rot\vec{u}$$

ekanligini hisobga olib

$$\frac{2(1-\nu)}{1-2\nu} grad\, div\vec{u} - rot\, rot\,\vec{u} = -\frac{\rho}{\mu}\vec{f} \tag{1.2.19}$$

kabi ba‘zida ishlatish qulay bo‘lgan shaklda yozish mumkin. Ko‘p masalalarda massaviy kuchlarni hisobga olmaslik yoki nolga teng deb hisoblash mumkin. Bu holda Lamening (1.2.16) tenglamalari

$$\nabla^2 u_i + \frac{1}{1-2\nu}\theta_{,i} = 0 \tag{1.2.20}$$

ko‘rinishni oladi. Bu tenglamani x_i koordinata bo‘yicha differensiallab,

$$\nabla^2 u_{i,i} + \frac{1}{1-2\nu}\theta_{,ii} = 0$$

tenglamaga ega bo‘lamiz. Lekin

$$u_{i,i} = \theta \text{ va } \theta_{,ii} = \nabla^2\theta$$

bo‘lganliklari uchun bu tenglamadan

$$\nabla^2\theta + \frac{1}{1-2\nu}\nabla^2\theta = \frac{2(1-\nu)}{1-2\nu}\nabla^2\theta = 0$$

yoki

$$\nabla^2\theta = 0 \tag{1.2.21}$$

ifodani olamiz. Bu ifoda massaviy kuchlar nolga teng yoki o‘zgarmas bo‘lganlarida θ - hajmiy deformatsia Laplas tenglamasini qanoatlantirishini, va demak, garmonik funksiya ekanligini ko‘rsatadi.

Endi (1.2.20) ga ∇^2 - Laplas operatori bilan ta’sir etamiz

$$\nabla^2\nabla^2 u_i - \frac{1}{1-2\nu}\nabla^2\theta_{,i} = 0,$$

lekin

$$\nabla^2\theta_{,i} = (\nabla^2\theta)_{,i}$$

bo‘lganligidan

$$\nabla^2\nabla^2 u_i = 0, \tag{1.2.22}$$

ya’ni ko‘chish vektorining u_i komponentalari bigarmonik funksiyasi ekan. Lekin bu narsa u_i ko‘chishlar ixtiyoriy bigarmonik funksiya ekanligini anglatmaydi, chunki ular avvalo tartibi past bo‘lgan Lame tenglamalarini ham qanoatlantirishlari kerak.

Agar masala ko‘chishlarga nisbatan yechilayotgan bo‘lsa, chegaraviy shartlar ham ko‘chishlar orqali ifodalanishi kerak. Ikkinchi tur asosiy masalada bu sohada muammo yo‘q. Ammo birinchi tur masalada (1.2.8) chegaraviy shartlarni ko‘chishlar orqali yozish kerak.

Bu ishni uddalash uchun yana (1.2.6) Guk qonunidan foydalanamiz. Uning ikkala tomonini ham n_j ga ko‘paytiramiz:

$$\sigma_{ij}n_j = \lambda\delta_{ij}n_j\theta + \mu(u_{i,j}n_j + u_{j,i}n_j)$$

lekin

$$n_j\delta_{ij} = n_i\,;$$

$$u_{i,j}n_j = \frac{\partial u_i}{\partial x_1}n_1 + \frac{\partial u_i}{\partial x_2}n_2 + \frac{\partial u_i}{\partial x_3}n_3 = \frac{\partial u_i}{\partial n},$$

ya’ni $u_{i,j}n_j$ ko‘paytma $u_i(x_k)$ funksiyadan jism sirtining shu nuqtasidagi normali bo‘yicha hosilasidan iborat bo‘lganligi uchun yuqoridagi tenglik:

$$\sigma_{ij}n_j = \lambda\,\theta\,n_i + \mu\left(\frac{\partial u_i}{\partial n} + u_{j,i}n_j\right)$$

ko‘rinishni oladi. U holda (1.2.8) chegaraviy shartlarni quyidagicha yozish mumkin:

$$\left[\lambda\theta\,n_i + \mu\left(\frac{\partial u_i}{\partial n} + \frac{\partial u_j}{\partial x_i}n_j\right)\right]_S = F_i \qquad (1.2.23)$$

Shunday qilib (1.2.16) Lame tenglamalari, birinchi tur asosiy masalada (1.2.23) chegaraviy shartlar bilan va ikkinchi tur asosiy masalada (1.2.9) chegaraviy shartlar bilan, ko‘chish vektorining hamma uchta u_i komponentalarini to‘liq aniqlaydi. Ko‘chish komponentalari u_i lar ma’lum bo‘lgach, (5.1) formulalar bilan deformatsiya tenzorining ε_{ij} komponentalari va (1.2.6) Guk qonuni formulalari bilan kuchlanish tenzorining hamma σ_{ij} komponentalari hisoblanadi. Boshqacha aytganda, jism istalgan nuqtasining kuchlangan-deformatsiyalangan holati to‘liq aniqlanadi.

Elastiklik nazariyasining uchinchi asosiy masalasi ko‘chishlarga nisbatan yechilayotgan bo‘lsa, (5.10) chegaraviy shartlarni

$$\left[\lambda\theta n_i + \mu\left(\frac{\partial u_i}{\partial n} + \frac{\partial u_j}{\partial x_i} n_j\right)\right]_{S_\sigma} = F_i \tag{1.2.24}$$

$$u_i \Big|_{S_u} = u_i$$

ko‘rinishda foydalanish zarur bo‘ladi. Agar jism muvozanatda emas balki harakatda bo‘lsa, uning harakat tenglamalari (1.2.4) ko‘rinishda bo‘ladi. Ko‘rinib turibdiki, ushbu harakat tenglamalari ko‘chishlarga nisbatan quyidagi uch shaklda yozilishi mumkin:

$$\begin{gathered}\nabla^2 u_i + \frac{1}{1-2\nu}\theta_{,i} = -\frac{\rho}{\mu}\left(f_i - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right);\\ \nabla^2 \vec{u} + \frac{1}{1-2\nu} grad\, div\vec{u} = -\frac{\rho}{\mu}\left(\vec{f} - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right)\end{gathered} \tag{1.2.25}$$

$$\frac{2(1-\nu)}{1-2\nu} grad\, div\vec{u} - rot\, rot\, \vec{u} = -\frac{\rho}{\mu}\left(\vec{f} - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right).$$

Keltirilgan ushbu tenglamalarda, tabiiyki, u_i ko‘chishlar x_k koordinatalardan tashqari, t vaqtning ham funksiyalari bo‘lishlari, ya‘ni

$$u_i = u_i(x_k, t)$$

bo‘lishi kerak.

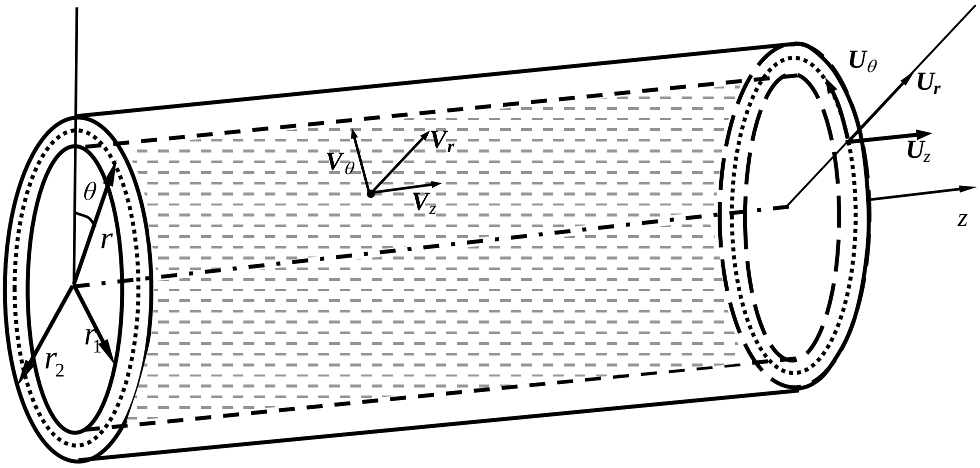
Elastik jismning harakati (1.2.25) tenglamalardan biri yordamida yechilayotganda (1.2.23), (1.2.9) yoki (1.2.24) chegaraviy shartlardan tashqari vaqtning $t = 0$ payti uchun boshlang‘ich shartlar ham qo‘yilishi kerak. Boshqacha aytganda, $t = 0$ bo‘lganda u_i ko‘chishlar va ularning vaqt bo‘yicha birinchi tartibli hosilalari $\left(\frac{\partial u_i}{\partial t} - tezliklar\right)$ berilgan qiymatlarga ega bo‘lishlari kerak:

$$u_i(x_k,t)\big|_{t=0} = u_i^{(0)}; \quad \frac{\partial u_i(x_k,t)}{\partial t}\Big|_{t=0} = u_i^{(01)}. \qquad (1.2.26)$$

Shunday qilib elastiklik nazariyasining asosiy tenglamalari, formulalari va masalalari bizga bundan keyin ishlatishga qulay tarzda keltirildi.

1.3. Doiraviy silindrik qovushoq-elastik qatlam va qobiqlarning nostatsionar tebranishlari to’lqin tenglamalari

Silindrik (r,θ,z) koordinatalar sistemasida bir jinsli va izotrop, ichki va tashqi radiuslari mos ravishda r_1 va r_2 bo’lgan doiraviy silindrik qovushoq-elastik qatlamni olamiz [7]. Tebranish tenglamalarini chiqarishda silindrik qatlamni uch o’lchovli matematik elastiklik nazariyaga qat’iy bo’ysinuvchi uch o’lchovli jism sifatida qaraymiz (1-rasm).



1-rasm.

Avvalo bundan keyin zarur bo'ladigan asosiy belgilashlar, tenglama va munosobatlarni keltiramiz. Elastik strejen nuqtalarining kichik ko'chish komponentalarini

$$U_r(r,\theta,z,t),\ U_\theta(r,\theta,z,t),\ U_z(r,\theta,z,t), \tag{1.3.1}$$

lar bilan, kuchlanish va deformasiya tenzori komponentalarini esa

$$\begin{aligned}
&\sigma_{rr}=\sigma_{rr}(r,\theta,z,t),\ \sigma_{zz}=\sigma_{zz}(r,\theta,z,t),\ \sigma_{\theta\theta}=\sigma_{\theta\theta}(r,\theta,z,t),\\
&\sigma_{r\theta}=\sigma_{r\theta}(r,\theta,z,t),\ \sigma_{rz}=\sigma_{rz}(r,\theta,z,t),\ \sigma_{\theta z}=\sigma_{\theta z}(r,\theta,z,t),\\
&\varepsilon_{rr}=\varepsilon_{rr}(r,\theta,z,t),\quad \varepsilon_{\theta\theta}=\varepsilon_{\theta\theta}(r,\theta,z,t),\ \varepsilon_{zz}=\varepsilon_{ij}(r,\theta,z,t),\\
&\varepsilon_{r\theta}=\varepsilon_{r\theta}(r,\theta,z,t),\quad \varepsilon_{\theta z}=\varepsilon_{\theta z}(r,\theta,z,t),\ \varepsilon_{rz}=\varepsilon_{rz}(r,\theta,z,t),
\end{aligned} \tag{1.3.2}$$

lar bilan belgilaymiz [2] .

Ko'chish vektori $\vec{U}$ ni Φ - boylama va $\vec{\Psi}$ -kondalang to'lqinlar potensiallari orqali quydagi formula bilan kiritamiz [15]

$$\vec{U}=grad\,\Phi+rot\,\left[\vec{e}_z\Psi_1+rot(\vec{e}_z\,\Psi_2)\right] \tag{1.3.3}$$

Ko'chish vektori (1.3.3) ko'rinishida tasvirlanganda $\vec{\Psi}$ -ko'ndalang to'lqinlar potensiali vektori maydonining solenoidallik sharti

$$div\vec{\Psi}=0 \tag{1.3.4}$$

bo'lishi kerak. Agar $\vec{\Psi}$ -vektor

$$\vec{\Psi}=\vec{e}_z\Psi_1+rot(\vec{e}_z\Psi_2). \tag{1.3.5}$$

$\vec{e}_z$ – z o'qi bo'yicha yo'nalgan birlik vektori. Endi [17]

$$\begin{aligned}
&grad\Phi=\frac{\partial\Phi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{e}_\theta+\frac{\partial\Phi}{\partial z}\vec{e}_z,\\
&rot(\vec{\psi})=rot(\psi_1\vec{e}_z+rot(\psi_2\vec{e}_z))=\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}+\frac{\partial^2\psi_2}{\partial r\partial z}\right)\vec{e}_r+\left(\frac{1}{r}\frac{\partial^2\psi_2}{\partial\theta\partial z}-\frac{\partial\psi_1}{\partial r}\right)\vec{e}_\theta+\\
&+\left(-\frac{\partial^2\psi_2}{\partial r^2}-\frac{1}{r^2}\frac{\partial\psi_2}{\partial\theta^2}-\frac{1}{r}\frac{\partial\psi_2}{\partial r}\right)\vec{e}_z,
\end{aligned} \tag{1.3.6}$$

ifodalarni hisobga olib ko'chish vektorini

$$\vec{U}=\frac{\partial\Phi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{e}_\theta+\frac{\partial\Phi}{\partial z}\vec{e}_z+\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}+\frac{\partial^2\psi_2}{\partial r\partial z}\right)\vec{e}_r+\left(\frac{1}{r}\frac{\partial^2\psi_2}{\partial\theta\partial z}-\frac{\partial\psi_1}{\partial r}\right)\vec{e}_\theta+$$
$$+\left(-\frac{\partial^2\psi_2}{\partial r^2}-\frac{1}{r^2}\frac{\partial\psi_2}{\partial\theta^2}-\frac{1}{r}\frac{\partial\psi_2}{\partial r}\right)\vec{e}_z \qquad (1.3.7)$$

kabi yozamiz. Ko'chish vektorining oxirgi ifodasidagi $\vec{e}_r,\vec{e}_\theta,\vec{e}_z$-birlik vektorlari oldidagi koeffisientlar mos ravishda U_r,U_θ,U_z- komponentalardan iboratdir, ya'ni

$$U_r=\frac{\partial\Phi}{\partial r}+\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}+\frac{\partial^2\psi_2}{\partial r\partial z};$$
$$U_\theta=\frac{1}{r}\frac{\partial\Phi}{\partial\theta}+\frac{1}{r}\frac{\partial^2\psi_2}{\partial\theta\partial z}-\frac{\partial\psi_1}{\partial r}; \qquad (1.3.8)$$
$$U_z=\frac{\partial\Phi}{\partial z}+-\frac{\partial^2\psi_2}{\partial r^2}-\frac{1}{r^2}\frac{\partial\psi_2}{\partial\theta^2}-\frac{1}{r}\frac{\partial\psi_2}{\partial r};$$

Deformasiya tenzori kompanentalari va ko'chish vektori kompanentalari bilan o'zaro Koshi munosabatlari orqali bog'langan [7]

$$\varepsilon_{rr}=\frac{\partial U_r}{\partial r},\ \varepsilon_{\theta\theta}=\frac{1}{r}\frac{\partial U_\theta}{\partial\theta}+\frac{U_r}{r},\ \varepsilon_{zz}=\frac{\partial U_z}{\partial z},$$
$$2\varepsilon_{rz}=\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r},\ 2\varepsilon_{r\theta}=\frac{1}{r}\frac{\partial U_r}{\partial\theta}+r\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right),\ 2\varepsilon_{\theta z}=\frac{\partial U_\theta}{\partial z}+\frac{1}{r}\frac{\partial U_z}{\partial\theta}, \qquad (1.3.9)$$

Bu munosabatlardan foydalanib $\varepsilon_{ij}=\varepsilon_{ij}(r,\theta,z,t),\ (i,j=r,\theta,z)$ deformatsiyalarni Φ,ψ_1,ψ_2 potensiallar orqali ifodalaymiz. Buning uchun (1.3.9) formulalarga (1.3.8) ifodalarni qo'yamiz. Natijada ushbu formulalarga ega bo'lamiz

$$\varepsilon_{rr}=\frac{\partial^2}{\partial r^2}\Phi-\frac{1}{r}\left(\frac{1}{r}-\frac{\partial}{\partial r}\right)\frac{\partial}{\partial\theta}\Psi_1+\frac{\partial^3}{\partial r^2\partial z}\Psi_2,$$
$$\varepsilon_{\theta\theta}=\frac{1}{r}\left[\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}+\frac{\partial}{\partial r}\right)\Phi+\left(\frac{1}{r}\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial r\partial\theta}\right)\Psi_1+\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}+\frac{\partial}{\partial r}\right)\Psi_2\right],$$

$$\varepsilon_{zz}=\frac{\partial^2}{\partial z^2}\Phi-\frac{\partial}{\partial z}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2,$$

$$\varepsilon_{rz}=2\frac{\partial^2}{\partial r\partial z}\Phi+\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\Psi_1+\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)-\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Psi_2, \tag{1.3.10}$$

$$\varepsilon_{r\theta}=\frac{2}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\Phi+\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}-r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\Psi_1+\frac{2}{r}\frac{\partial^2}{\partial z\partial\theta}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\Psi_2,$$

$$\varepsilon_{z\theta}=\frac{\partial}{\partial z}\left(\frac{2}{r}\frac{\partial}{\partial\theta}\Phi-\frac{\partial}{\partial r}\Psi_1\right)-\frac{\partial}{\partial\theta}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^3}\frac{\partial^2}{\partial\theta^2}-\frac{1}{r}\frac{\partial^2}{\partial z^2}\right)\Psi_2,$$

Oxirgi (1.3.10) tengliklardan foydalanib

$$\varepsilon=\varepsilon_{rr}+\varepsilon_{\theta\theta}+\varepsilon_{zz}=\Delta\Phi$$

munosabatni isbotlash qiyin emas. Bunda ε - hajmiy deformasiya; Δ – uch o'lchamli Laplas operatori

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2} \tag{1.3.11}$$

Tebranishlar o'qqa nisbatan simmetrik bo'lganida ko'chishlar va deformatsiyalarning (1.3.8),(1.3.10) ifodalari quyidagi ko'rinishni oladi

$$U_r=\frac{\partial}{\partial r}\left(\Phi+\frac{\partial}{\partial z}\Psi_2\right),\quad U_\theta=-\frac{\partial}{\partial r}\Psi_1,\quad U_z=\frac{\partial}{\partial z}\Phi-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\Psi_2 \tag{1.3.12}$$

va

$$\varepsilon_{rr}=\frac{\partial^2}{\partial r^2}\left(\Phi+\frac{\partial}{\partial z}\Psi_2\right),\qquad \varepsilon_{\theta\theta}=\frac{1}{r}\frac{\partial}{\partial r}\left(\Phi+\frac{\partial}{\partial z}\Psi_2\right),$$

$$\varepsilon_{zz}=\frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\Phi-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\Psi_2\right), \tag{1.3.13}$$

$$\varepsilon_{rz}=2\frac{\partial^2}{\partial r\partial z}\Phi+\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2,$$

$$\varepsilon_{r\theta}=-r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_1,\quad \varepsilon_{z\theta}=-\frac{\partial^2}{\partial r\partial z}\Psi_1.$$

Qovushoq elastik jism nuqtalarida kuchlanish va defomatsiya tenzorlari kompanentalari Boltsman-Volter munosobatlari bilan bog'langan deb hisoblaymiz

$$\sigma_{rr}=L_1(\varepsilon)+2M\varepsilon_{rr},\ \sigma_{\theta\theta}=L_1(\varepsilon)+2M\varepsilon_{\theta\theta},\ \sigma_{zz}=L_1(\varepsilon)+2M\varepsilon_{zz},$$
$$\sigma_{r\theta}=2M\varepsilon_{r\theta},\ \sigma_{rz}=2M\varepsilon_{rz},\ \ \sigma_{z\theta}=2M\varepsilon_{z\theta}, \tag{1.3.14}$$

Bu yerda L_1,M - qovushoqlik teskarilanuvchi operatorlari [20]

$$(L_1,M)\varsigma=(\lambda,\mu)\left[\varsigma(t)-\int_0^t[f_1(t-\tau),f_2(t-\tau)]\varsigma(\tau)d\tau\right];$$

λ,μ – Lame koeffisentlari; $f_1(t),f_2(t)$ – qovushoq-elastiklik operatorlarining ixtiyoriy yadrolari.

U holda $\sigma_{ij}=\sigma_{ij}(r,\theta,z,t),\ (i,j=r,\theta,z)$ kuchlanishlar uchun (1.3.10) ifodalardan foydalanib quyidagilarga ega bo'lamiz .

$$\sigma_{rr}=L_1(\Delta\Phi)+2M\left(\frac{\partial^2}{\partial r^2}\Phi-\frac{1}{r}\left(\frac{1}{r}-\frac{\partial}{\partial r}\right)\frac{\partial}{\partial\theta}\Psi_1+\frac{\partial^3}{\partial r^2\partial z}\Psi_2\right),$$
$$\sigma_{\theta\theta}=L_1(\Delta\Phi)+2M\left(\frac{1}{r}\left[\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}+\frac{\partial}{\partial r}\right)\Phi+\left(\frac{1}{r}\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial r\partial\theta}\right)\Psi_1+\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2}+\frac{\partial}{\partial r}\right)\Psi_2\right]\right),$$
$$\sigma_{zz}=L_1(\Delta\Phi)+2M\left(\frac{\partial^2}{\partial z^2}\Phi-\frac{\partial}{\partial z}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2\right),$$
$$\sigma_{r\theta}=M\left(\frac{2}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\Phi+\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}-r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\Psi_1+\frac{2}{r}\frac{\partial^2}{\partial z\partial\theta}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\Psi_2\right),$$
$$\sigma_{rz}=M\left(2\frac{\partial^2}{\partial r\partial z}\Phi+\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\Psi_1+\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)-\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Psi_2\right), \tag{1.3.15}$$
$$\sigma_{\theta z}=M\left(\frac{\partial}{\partial z}\left(\frac{2}{r}\frac{\partial}{\partial\theta}\Phi-\frac{\partial}{\partial r}\Psi_1\right)-\frac{\partial}{\partial\theta}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^3}\frac{\partial^2}{\partial\theta^2}-\frac{1}{r}\frac{\partial^2}{\partial z^2}\right)\Psi_2\right),$$

Sterjenning bo'ylama-radial tebranish tenglamalari ψ_1-potensialga bog'liq bo'lmaganligi uchun va tebranishlar o'qqa nisbatan simmetrik bo'lganida kuchlanishlarning yuqoridagi ifodasi ushbu ko'rinishni oladi

$$\sigma_{rr}=L_1(\Delta\Phi)+2M\left(\frac{\partial^2}{\partial r^2}\Phi_1+\frac{\partial^3}{\partial r^2\partial z}\Psi_2\right),$$
$$\sigma_{\theta\theta}=L_1(\Delta\Phi)+2M\left(\frac{1}{r}\left[\left(\frac{\partial}{\partial r}\right)\Phi+\frac{\partial^2}{\partial z\partial\partial}\Psi_2\right]\right), \tag{1.3.16}$$
$$\sigma_{zz}=L_1(\Delta\Phi)+2M\left(\frac{\partial^2}{\partial z^2}\Phi-\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\Psi_2\right),$$

$$\sigma_{rz} = M\left(2\frac{\partial^2}{\partial r\partial z}\Phi + \frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)\Psi_2\right),$$

$$\sigma_{\theta z} = 0, \qquad \sigma_{r\theta} = 0.$$

Silindrik koordinatalar sistemasida uch o'lchovli qovushoq-elastik jism sifatida sterjen nuqtalarining harakat tenglamalari quyidagicha bo'ladi [22]

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\frac{\partial^2 U_r}{\partial t^2};$$

$$\frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\sigma_{z\theta}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho\frac{\partial^2 U_\theta}{\partial t^2}; \qquad (1.3.17)$$

$$\frac{\partial\sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{z\theta}}{\partial\theta} + \frac{\partial\sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 U_z}{\partial t^2};$$

Sterjen nuqtalarining harakat tenglamalariga kuchlanishlarning (1.2.15) ifodasini va ko'chishlarning (1.27) ifodalarini qo'yib, soddalashtirishlardan so'ng quyidagiga ega bo'lamiz

$$\frac{\partial}{\partial r}(L_1 + 2M)\Delta\Phi + \frac{1}{r}\frac{\partial}{\partial\theta}M(\Delta\psi_1) + \frac{\partial^2}{\partial r\partial z}M(\Delta\psi_2) = \rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial\Phi}{\partial r} + \frac{1}{r}\frac{\partial\psi_1}{\partial\theta} + \frac{\partial^2\psi_2}{\partial r\partial z}\right);$$

$$\frac{1}{r}\frac{\partial}{\partial\theta}(L_1 + 2M)\Delta\Phi - \frac{\partial}{\partial r}M(\Delta\psi_1) + \frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}M(\Delta\psi_2) = \rho\frac{\partial^2}{\partial t^2}\left(\frac{1}{r}\frac{\partial\Phi}{\partial\theta} - \frac{\partial\psi_1}{\partial r} + \frac{1}{r}\frac{\partial^2\psi_2}{\partial\theta\partial z}\right);$$

$$\frac{\partial}{\partial z}(L_1 + 2M)\Delta\Phi - M\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\Delta\psi_2 = \rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial\Phi}{\partial z} + -\frac{\partial^2\psi_2}{\partial r^2} - \frac{1}{r^2}\frac{\partial\psi_2}{\partial\theta^2} - \frac{1}{r}\frac{\partial\psi_2}{\partial r}\right);$$

(1.3.18)

Bu tenglamalarni mos ravishda $\vec{e}_r, \vec{e}_\theta, \vec{e}_z$ - birlik vektorlariga ko'paytirib qo'shamiz. Natijada

$$\left(\frac{\partial}{\partial r}(L_1+2M)\Delta\Phi-\rho\frac{\partial^2}{\partial t^2}\frac{\partial\Phi}{\partial r}\right)\bar{e}_r+\left(\frac{1}{r}\frac{\partial}{\partial\theta}(L_1+2M)\Delta\Phi-\rho\frac{\partial^2}{\partial t^2}\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right)\bar{e}_\theta+$$
$$+\left(\frac{\partial}{\partial z}(L_1+2M)\Delta\Phi-\rho\frac{\partial^2}{\partial t^2}\frac{\partial\Phi}{\partial z}\right)\bar{e}_z+\left(\frac{\partial}{\partial z}M(\Delta\psi_1)-\rho\frac{\partial^2}{\partial t^2}\frac{\partial\psi_1}{\partial z}\right)\bar{e}_z+$$
$$+\left(\frac{1}{r}\frac{\partial}{\partial\theta}M(\Delta\psi_1)-\rho\frac{\partial^2}{\partial t^2}\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}\right)\bar{e}_\theta+\left(-\frac{\partial}{\partial r}M(\Delta\psi_1)+\rho\frac{\partial^2}{\partial t^2}\frac{\partial\psi_1}{\partial r}\right)\bar{e}_r+$$
$$+\left(\frac{\partial^2}{\partial r\partial z}M(\Delta\psi_2)-\rho\frac{\partial^2}{\partial t^2}\frac{\partial^2\psi_2}{\partial r\partial z}\right)\bar{e}_r+\left(\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}M(\Delta\psi_2)-\rho\frac{\partial^2}{\partial t^2}\frac{1}{r}\frac{\partial^2\psi_2}{\partial\theta\partial z}\right)\bar{e}_\theta+$$
$$+\left(-M\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\psi_2+\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2\psi_2}{\partial r^2}+\frac{1}{r^2}\frac{\partial\psi_2}{\partial\theta^2}+\frac{1}{r}\frac{\partial\psi_2}{\partial r}\right)\right)\bar{e}_z=0$$

(1.3.19)

Vektor tenglamani hosil qilamiz. Bu tenglamada katta bo'lmagan soddalashtirishladan so'ng Φ,Ψ_1,Ψ_2 potensiallarga nisbatan to'lqin tenglamalarini keltirib chiqaramiz.

$$\begin{aligned}&L(\Delta\Phi)=\rho\ddot{\Phi},\quad L=L_1+2M,\\&M(\Delta\ddot{\Psi}_1)=\rho\ddot{\psi}_1,\\&M(\Delta\ddot{\Psi}_2)=\rho\ddot{\psi}_2;\end{aligned}\tag{1.3.20}$$

bunda ρ – sterjen materialining zichligi ;

Endi doiraviy silindrik qovushoq-elastik qatlamning o'qqa nisbatan simmetrik tebranishlari haqidagi dinamik chegaraviy masalani qaraymiz. Buning uchun tebranish tenglamalarini aniqlash va chegaraviy shartlarni qo'ymoq zarur. Yuqorida murojaat qilingan [7] monografiyada silindrik qatlam, qobiq yoki sterjenlarning bo'ylama-radial tebranishlari (1.3.20) sistemaning faqat Φ *va* Ψ_2 larga nisbatan tenglamalari ya'ni

$$L(\Delta\Phi)=\rho\ddot{\Phi},\qquad M(\Delta\ddot{\Psi}_2)=\rho\ddot{\psi}_2\tag{1.3.21}$$

tenglamalar bilan tavsiflanishi keltirilgan.

Qaralayotgan sterjenning bo'ylama-radial tebranishlari uning sirtida $f_r^{(i)}(z,t),\quad f_{rz}^{(i)}(z,t)\ \ (i\ =\ 1,2)$ kuchlanishlarning ta'siri natijasida vujudga keladi deb hisoblanadi [19]. U holda masala uchun chegaraviy shartlar quyidagi ko’rinishda yoziladi.

$$\begin{aligned} \sigma_{rr}(r_0, z, t) &= f_r\ (z,t)\,, \\ \sigma_{rz}(r_0, z, t) &= f_{rz}(z,t), \end{aligned} \qquad (1.3.22)$$

Boshlang’ich shartlarni nolga teng deb hisoblaymiz.

Shunday qilib doiraviy qovushoq-elastik sterjenning bo’ylama-radial tebranishlari haqidagi masala (1.3.21) tenglamalarni (1.3.22) chegaraviy va nolga teng boshlang’ich shartlarda yechishga keltirililadi.

II BOB
ANIZOTROP SILINDRIK QATLAM VA QOBIQLARNING O'QQA NISBATAN SIMMETRIK TEBRANISHLARI

2.1. Silindrik qatlamning o'qqa nisbatan simmetrik tebranishlari texnik nazariyasi tenglamalari

Yuqorida 1 Bob doirasida ta'kidlanganidek [7] monografiyada deformatsiyalanuvchi muhityda joylashgan doiraviy silindrik qovushoq – elastic qatlam va qobiqlarning o'qqa nisbatan simmetrik tebranishlari matematik nazariyasi yaratilgan. Ushbu nazariya doirasida shunday qatlam o'qqa nisbatan simmetrik tebranishlari umumiy (matematik ma'noda aniq) tenglamalari keltirib chiqarilgan. Silindrik qovushoq – elastik qatlam va qobiqlarning ixtiyoriy kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatini (KDH) vaqtning istalgan payti uchun izlanuvchi funksiyalar maydoni orqali aniqlashga imkon beradigan algoritm yaratilgan. Ammo, muallifning o'zi ta'kidlaganidek, olingan integro-differensial tenglamalar hamda kuchlanishlar va deformatsiyalar uchun chiqarilgan formulalarning tartiblari juda katta bo'lganliklari sababli ularni amaliy masalalarni yechishda qo'llash mushkul.

Bundan kelib chiqadiki, ulardan amaliy masalalarni yechishda foydalanish mumkin bo'lgan tebranish tenglamalari va KDH ni hisoblash formulalarini chiqarish uchun bu tenglamalar va formulalar tarkibiga kiruvchi cheksiz qatorlar hadlari sonini chegaralash zarur. Boshqacha aytganda bu cheksiz yig'indilarda nolinchi, birinchi, ikkinchi va hokazo yaqinlashishlar bilan chegaralanish kerak. Bu ishni bajarish uchun o'sha cheksiz qatorlarning yaqinlashishi zarur va yetarlidir [18]. Ushbu qatorlarning yaqinlashishlari qaralayotgan monografiyada isbotlangan, qatorlarning yaqinlashish oraliqlarini topish orqali olinadigan tabranish tenglamalarining qo'llanilish sohalari ham aniqlangan. Lekin, taqribiy tebranish tenglamalari va sistema KDH uchun mos formulalar har bir yaqinlashishlar uchun alohida-alohida keltirib chiqarilmagan.

Shuning uchun ushbu paragraph doirasida amaliy masalalarni yechishda foydalanish mumkin bo'lgan tebranish tenglamalari va KDH ni hisoblash formulalarini chiqarish bilan shug'ullanamiz. Pirovardida bu tenglama va formulalarni silindrik qatlam o'qqa nisbatan simmetrik tebranishlari texnik nazarisining asosiy tenglama va munosabatlari deb ataymiz.

Eslatilgan matematik nazariyaning deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi silindrik qovushoq-elastik qatlamning o'qqa nisbatan simmetrik tebranishlari tenglamalari quyidagi ko'rinishga egalar

$$d_{11}U_{r,0}^{(0)} - d_{21}U_{z,0}^{(0)} - \xi[d_{31}U_{r,1}^{(0)} - d_{41}U_{z,1}^{(0)}] = M_0^{-1}[f_r^{(1)}];$$

$$(d_{12} + R_1 g_1)U_{r,0}^{(0)} - (d_{22} - R_1 g_2)U_{z,0}^{(0)} -$$

$$-\xi[(d_{32} + R_1 g_3)U_{r,1}^{(0)} - (d_{42} - R_1 g_4)U_{z,1}^{(0)}] = F; \qquad (2.1.1)$$

$$e_{1i}U_{r,0}^{(0)} + e_{2i}U_{z,0}^{(0)} - \xi[e_{3i}U_{r,1}^{(0)} + d_{4i}U_{z,1}^{(0)}] = M_0^{-1}[\delta_i f_{rz}^{(1)}]; \quad (i=1,2),$$

$$\delta_1 = 1, \qquad \delta_2 = 0,$$

bu yerda $U_{r,0}^{(0)}$, $U_{r,1}^{(0)}$, $U_{z,0}^{(0)}$, $U_{z,1}^{(0)}$ — funksiyalar qatlam nuqtalarining $U_r^{(0)}$-radial va $U_я^{(0)}$ bo'ylama ko'chishlari bosh qismlari, bunda darajadagi "0" indeksi qatlam nuqtalariga tegishlilikni anglatadi; $f_r^{(i)}$, $f_{rz}^{(i)}$ - tashqaridan ta'sir etuvchi kuchlanishlar funksiyalari; R_1 va R_2 lar – tashqi o'rab turuvchi muhitning qatlam bo'ylama-radial tebranishlariga bo'lgan reaksiyasini aniqlaydilar; ξ - qatlam oraliq sirtining radiusi [1,3] ; d_{ij} , e_{ij} , g_{ij} (i=1,2), (j=1,2) - operatorlar quyidagi formulalar vositasida aniqlanadilar

$$d_{1i} = \sum_{n=0}^{\infty}\left\{[n(1-q_1) - q_1]\lambda_{02}^n - [\lambda_{01} - (n+1)(\lambda_{02} - \frac{\partial^2}{\partial z^2})q_1 Q_n\right\}\frac{(r_i/2)^{2n}}{n!(n+1)!};$$

$$d_{2i} = \sum_{n=0}^{\infty}\left\{(n+1)(1+q_1)\frac{\partial}{\partial z}\lambda_{02}^{n} + \frac{\partial}{\partial z}[\lambda_{01} - (n+1)(\lambda_{02} - \frac{\partial^2}{\partial z^2})q_1 Q_n\right\}\frac{(r_i/2)^{2n}}{n!(n+1)!};$$

$$d_{3i} = \sum_{n=0}^{\infty}\lambda_{02}^{n}\left\{\eta_{3,n}(r_i)[\lambda_{02}^{n} + (\lambda_{02} - \frac{\partial^2}{\partial z^2})q_2 Q_n] - \frac{2}{r_i^2} - \right.$$

$$\left. - \frac{(r_i/2)^{2n}}{(n!)^2}[\lambda_{02}{}^{n} + Q_{n+1}q_2]\frac{\eta_{1,n}^{(m)}(r_i)}{n+1}\right. ;$$

$$d_{4i} = \sum_{n=0}^{\infty}\left\{\eta_{3,n}(r_i)\frac{\partial}{\partial z}[\lambda_{02}^{n} - (\lambda_{02} - \frac{\partial^2}{\partial z^2})q_2 Q_n] - \frac{(r_i/2)^{2n}}{(n!)^2}\frac{\partial}{\partial z}Q_{n+1}q_2\right\}\frac{\eta_{1,n}^{(m)}(r_i)}{n+1}$$

$$e_{1i} = \sum_{n=0}^{\infty}\frac{\partial}{\partial z}\left\{\lambda_{02}^{n} + q_1[Q_{n+1} + Q_n\lambda_{01}^{n}]\right\}\frac{(r_i/2)^{2n+1}}{n!(n+1)!}; \qquad (2.1.2)$$

$$e_{2i} = \sum_{n=0}^{\infty}\lambda_{01}\left\{(1-q_1)\lambda_{02}^{n} + 2\frac{\partial^2}{\partial z^2}q_1 Q_n\right\}\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$e_{3i} = \sum_{n=0}^{\infty}\eta_{1,n}(r_i)\frac{\partial}{\partial z}[\lambda_{02}^{n+1} + \lambda_{02}2q_2 Q_n]\frac{(r_i/2)^{2n+1}}{n!(n+1)!} + \frac{\partial}{\partial z}\frac{1}{r_i};$$

$$e_{4i} = \sum_{n=0}^{\infty}\eta_{1,n}(r_i)[\lambda_{02}^{n+1} + 2q_2 Q_{n+1}\frac{\partial}{\partial z}]\frac{(r_i/2)^{2n+1}}{n!(n+1)!} - \frac{1}{r_i};$$

$$g_1 = \sum_{n=0}^{\infty}[\lambda_{02}^{n} + \lambda_{01}q_1 Q_n]\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$

$$g_2 = \sum_{n=0}^{\infty}\lambda_{01}q_1 Q_n\frac{\partial}{\partial z}\frac{(r_2/2)^{2n+1}}{n!(n+1)!};$$

$$g_3 = \sum_{n=0}^{\infty}\eta_{1,n}(r_2)[\lambda_{02}^{n+1} + \lambda_{02}q_2 Q_{n+1}]\frac{(r_i/2)^{2n+1}}{n!(n+1)!} + \frac{1}{r_2};$$

$$g_4 = \sum_{n=0}^{\infty}\eta_{1,n}(r_2)q_2 Q_{n+1}\frac{\partial}{\partial z}\frac{(r_2/2)^{2n+1}}{n!(n+1)!};$$

$$F = M_0^{-1}\left[f_r^{(2)}\right] - R_1\left(f_0\right) - R_2\left[M_1^{-1}\left(\frac{\partial f_{rz}^{(2)}}{\partial z}\right)\right];$$

$$Q_n=\sum_{k=1}^{n-1}\lambda_{01}^{n-k-1}\lambda_{02}^{k};\qquad Q_0\equiv 0;\qquad Q_1\equiv 1;$$

$$\eta_{1,n}^{(m)}(r_i\ )=\ln\frac{r_i}{\xi}+\frac{n}{2(n+1)}-\sum_{k=1}^{n}\frac{1}{k};\qquad \eta_{3,n}^{(m)}(r_i)=\ln\frac{r}{\xi}+\frac{1}{2}-\sum_{k=1}^{n}\frac{1}{k};$$

$$q_1=1-L_0M_0^{-1};\qquad q_2=L_0M_0^{-1}-1;$$

λ_{01}^{n} λ_{02}^{n} -operatorlar (z, t) o’zgaruvchilarda quyidagi ko’rinishga egalar

$$\lambda_{01}{}^{n}=\left[L_0{}^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\frac{\partial^2}{\partial z^2}\right]^n,\qquad \lambda_{02}{}^{n}=\left[\rho M_0{}^{-1}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right]^n,$$

L_0, M_0- ko’rinishi quyidagicha bo’lgan qovushoq-elastiklik operatorlari

$$(L_0,M_0)\zeta=(\lambda,\mu)\left[\zeta(t)-\int_0^t\left[f_{10}(t-\tau),f_{20}(t-\tau)\right]\zeta(\tau)d\tau\right];$$

λ,μ- Lame koeffitsientlari; $f_{10}(t), f_{20}(t)$- qovushoq-elastiklik operatorlari. Yuqoridagi (2.1.1) tenglamalar sistemasi hosilalar bo’yicha juda yuqori tartibga va katta tenglamalar soniga ega. Ushbu faktorlar bu sistemani yechishni qiyinlashtiradi va uni qatlam va qobiqlarning bo’ylama-radial tebranishlari haqidagi amaliy masalalarni yechish uchun qo’llash imkoniyatini yo’qotadi. Odatda, bunday hollarda (2.1.1) tenglamalarning chap tomonlarida hadlar sonini chegaralaydilar va “kesilgan” tebranish tenglamalaridan foydalanadilar [18,20] hamda hosil bo’lgan sistemani bitta yoki ikkita tenglamaga keltiradilar.

Ushbu tebranish umumiy tenglamalari (2.1.1) sistemasini bundan keying tadqiqotlar uchun qulay bo’lgan shaklga keltiramiz. Buning uchun $n=0, R_1=0, R_2=0$ deb hisoblaymiz, ya’ni o’rab turuvchi muhitni yo’q, qatlamning ichki va nashqi sirtlariga tashqi yuklanishlar ta’sir etmaydi, qatlam matyerialining Puasson koeffitsiyenti o’zgarmas ($\nu=const$) hamda tenglamalarda nolinchi yaqinlashish bilan chegaralanish kifoya deb hisoblaymiz.

U holda $f_{10}(t)=f_{20}(t)=f_0(t)$ va $\lambda_{01},\lambda_{02},q_1,q_2,L_0,M_0$ operatorlar quyidagi ko’rinishlarni oladilar

$$L=(\lambda+2\mu)\tilde{N};\ M=\mu\tilde{N};\ K(t)=f_0(t);\ q_1=\frac{\lambda+\mu}{\mu};q_2=\frac{\lambda+\mu}{\lambda+2\mu};$$

$$\lambda_{01}{}^{n}=\left[L_0{}^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\frac{\partial^2}{\partial z^2}\right]^n,\qquad \lambda_{02}{}^{n}=\left[\rho M_0{}^{-1}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right]^n,$$

L_0, M_0- qovushoq-elastiklik operatorlari

$$(L_0,M_0)\zeta=(\lambda,\mu)\left[\zeta(t)-\int_0^t\left[f_{10}(t-\tau),f_{20}(t-\tau)\right]\zeta(\tau)d\tau\right];$$

Kiritilgan o'zgarishlar hisobga olinganda (2.1.1) tenglamalar sistemasi quyidagi ko'rinishga keladi

$$\begin{aligned}
&d_{11}U_{r,0}^{(0)}-d_{21}U_{z,0}^{(0)}-\xi d_{31}U_{r,1}^{(0)}+\xi d_{41}U_{z,1}^{(0)}]=0,\\
&d_{12}U_{r,0}^{(0)}-d_{22}U_{z,0}^{(0)}-\xi d_{32}U_{r,1}^{(0)}+\xi d_{42}U_{z,1}^{(0)}=0,\\
&e_{11}U_{r,0}^{(0)}+e_{21}U_{z,0}^{(0)}-\xi e_{31}U_{r,1}^{(0)}-\xi d_{41}U_{z,1}^{(0)}=0,\\
&e_{12}U_{r,0}^{(0)}+e_{22}U_{z,0}^{(0)}-\xi e_{32}U_{r,1}^{(0)}-\xi d_{42}U_{z,1}^{(0)}=0
\end{aligned}\qquad (2.1.3)$$

(2.1.2) formulalar boyicha n=0 hol uchun operatorlarning soda ifodalariga ega bo'lamiz

$$d_{1i}=q_1;d_{2i}=(1+q_1)\frac{\partial}{\partial z};e_{1i}=\frac{r_i}{2}(1+q_1)\frac{\partial}{\partial z};e_{2i}=(1-q_1)\frac{\partial}{\partial z}\frac{r_i}{2};$$

$$d_{3i}=(\frac{1}{2}-q_2\ln\frac{r_i}{\xi})\lambda_{02}-\frac{2}{r_i^2};\quad d_{4i}=\frac{\partial}{\partial z}[(1+q_2)\ln\frac{r_i}{\xi}+\frac{1}{2}];$$

$$e_{3i}=\frac{\partial}{\partial z}\frac{r_i}{2}[(1+2q_2)\lambda_{02}\ln\frac{r_i}{\xi}+\frac{2}{r_i^2}];\ e_{4i}=\frac{r_i}{2}[(2q_2\frac{\partial^2}{\partial z^2}+\lambda_{02})\ln\frac{r_i}{\xi}-\frac{2}{r_i^2}];$$

$$g_1=\frac{r_2}{2};\quad g_2=0;\quad g_3=\frac{r_2}{2}(1+q_2)\lambda_{02}\ln\left(\frac{r_i}{\xi}\right)+\frac{1}{r_2};\quad g_4=\frac{r_2}{2}q_2\frac{\partial}{\partial z}\ln\left(\frac{r_i}{\xi}\right);$$

Oxirgi ifodalarning qiymatlarini (2.1.3) tenglamalarga qo'yib

$$-q_1U_{r,0}^{(0)}-(1+q_1)\frac{\partial U_{z,0}^{(0)}}{\partial z}+\xi[(q_2\ln\frac{r_1}{\xi}-\frac{1}{2})\lambda_{02}+\frac{2}{r_1^2}]U_{r,1}^{(0)}+$$

$$+\xi[(1+q_2)\ln\frac{r_1}{\xi}+\frac{1}{2}]\frac{\partial U_{z,1}^{(0)}}{\partial z}=0,$$

$$-q_1U_{r,0}^{(0)}-(1+q_1)\frac{\partial U_{z,0}^{(0)}}{\partial z}+\xi[(q_2\ \ln\frac{r_2}{\xi}-\frac{1}{2})\lambda_{02}+\frac{2}{r_2^2}]U_{r,1}^{(0)}+$$

$$+\xi[(1+q_2)\ln\frac{r_2}{\xi}+\frac{1}{2}]\frac{\partial U_{z,1}^{(0)}}{\partial z}=0, \qquad (2.1.4)$$

$$(1+q_1)\frac{\partial U_{r,0}^{(0)}}{\partial z}-(1-q_1)\lambda_{01}U_{z,0}^{(0)}-\xi[(1+2q_2)\ln\frac{r_1}{\xi}\lambda_{02}+\frac{2}{r_1^2}]\frac{\partial U_{r,1}^{(0)}}{\partial z}-$$

$$-\xi[(\lambda_{02}+2q_2\frac{\partial^2}{\partial z^2})\ln\frac{r_1}{\xi}+\frac{2}{r_1^2}]U_{z,1}^{(0)}=0,$$

$$(1+q_1)\frac{\partial U_{r,0}^{(0)}}{\partial z}-(1-q_1)\lambda_{01}U_{z,0}^{(0)}-\xi[(1+2q_2)\ln\frac{r_2}{\xi}\lambda_{02}+\frac{2}{r_2^2}]\frac{\partial U_{r,1}^{(0)}}{\partial z}-$$

$$-\xi[(\lambda_{02}+2q_2\frac{\partial^2}{\partial z^2})\ln\frac{r_2}{\xi}+\frac{2}{r_2^2}]U_{z,1}^{(0)}=0.$$

Bu tenglamalardan $U_{r,0}^{(0)}, U_{z,0}^{(0)}, \xi U_{r,1}^{(0)}, \xi U_{z,1}^{(0)}$ funksiyalarga nisbatan alohida-alohida tenglamalar olish uchun unga Kramer qoidasini qo’llaymiz. Olingan (2.1.4) sistema determinantini hisdoblasak uning

$$\Delta_1=d_{11}(l_1d_{22}+l_2d_{42}+l_3d_{32})+d_{21}(-l_1d_{12}+l_4d_{32}+l_5d_{42})-d_{31}(l_3d_{12}+l_4d_{22}+$$

$$+l_6d_{42})-d_{41}(l_2d_{12}+l_5d_{22}+l_6d_{32})=\sum_{i=1}^{6}l_im_i \qquad (2.1.5)$$

tengligini toppish qiyin emas. Bu yerda

$$l_1=e_{41}e_{32}-e_{31}e_{42}, \quad l_2=e_{31}e_{22}-e_{21}e_{32},$$

$$l_3=e_{41}e_{22}-e_{21}e_{42}, \quad l_4=e_{41}e_{12}-e_{42}e_{11},$$

$$l_5=e_{12}e_{31}-e_{32}e_{11}, \quad l_6=e_{11}e_{22}-e_{12}e_{21}, \qquad (2.1.6)$$

$$m_1=d_{11}d_{22}-d_{12}d_{21}, \quad m_2=d_{11}d_{42}-d_{12}d_{41},$$

$$m_3=d_{11}d_{32}-d_{12}d_{31}, \quad m_4=d_{21}d_{32}-d_{31}d_{22},$$

$$m_5=d_{21}d_{42}-d_{22}d_{41}, \quad m_6=d_{41}d_{32}-d_{42}d_{31}$$

Endi (2.1.2) formuladan foydalanib ushbu ifodalarni chiqarish oson

$$l_1 = (\lambda_{02} - \frac{\partial^2}{\partial z^2})\frac{\partial}{\partial z} q_2 \left[\ln\left(\frac{r_2}{\xi}\right)\frac{r_2}{r_1} - \frac{r_1}{r_2}\ln\left(\frac{r_1}{\xi}\right)\right],$$

$$l_2 = \lambda_{01}\frac{\partial}{\partial z}(1-q_1)\left[\frac{r_2^{\,2} - r_1^{\,2}}{2r_2 r_1} - \frac{r_1 r_2}{4}(1+2q_2)\lambda_{02}\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$l_3 = \lambda_{01}(1-q_1)\left[\frac{r_2^{\,2} - r_1^{\,2}}{2r_2^{\,2} r_1^2} + \frac{r_1 r_2}{4}(\lambda_{02} + 2q_2\frac{\partial^2}{\partial z^2})\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$l_4 = \frac{1}{2}(1+q_1)\frac{\partial}{\partial z}\left[\frac{r_2^{\,2} - r_1^{\,2}}{r_2 r_1} + \frac{r_1 r_2}{2}(\lambda_{02} + 2q_2\frac{\partial^2}{\partial z^2})\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$l_5 = \frac{1}{2}(1+q_1)\frac{\partial^2}{\partial z^2}\left[\frac{r_2^{\,2} - r_1^{\,2}}{r_2 r_1} + \frac{r_1 r_2}{2}(1+2q_2)\lambda_{02}\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$l_6 = 0, \qquad m_1 = 0, \qquad m_2 = -q_1(1+q_2)\frac{\partial}{\partial z}\ln\left(\frac{r_1}{r_2}\right), \tag{2.1.7}$$

$$m_3 = q_1\left[2\frac{r_2^{\,2} - r_1^{\,2}}{2r_2^{\,2} r_1^2} - q_2\lambda_{02}\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$m_4 = (1+q_1)\frac{\partial}{\partial z}\left[2\frac{r_2^{\,2} - r_1^{\,2}}{r_2^{\,2} r_1^2} + \lambda_{02} q_2\ln\left(\frac{r_1}{r_2}\right)\right],$$

$$m_5 = (1+q_1)(1+q_2)\frac{\partial^2}{\partial z^2}\ln\left(\frac{r_1}{r_2}\right),$$

$$m_6 = \frac{\partial}{\partial z}\left\{(\frac{1}{2} + q_2)\lambda_{02}\ln\left(\frac{r_1}{r_2}\right) + 2(1+q_2)\left[\frac{1}{r_1^2}\ln\left(\frac{r_2}{\xi}\right) - \frac{1}{r_2^2}\ln\left(\frac{r_2}{\xi}\right)\right] + \frac{r_2^{\,2} - r_1^{\,2}}{r_2 r_1}\right\}$$

Oxirgi (2.1.7) ifodalarni determinantning (2.1.5) ifodasiga qo'yamiz

$$\Delta_1 = \ln^2\left(\frac{r_1}{r_2}\right)\frac{r_1 r_2}{4}\left[(1+q_1)^2\frac{\partial^2}{\partial z^2} - q_1(1-q_1)\lambda_{01}\right]\cdot\Big[-\omega_1^2 + \omega_1\Big[(1+q_2)\lambda_{02} -$$

$$-(1-q_2)\frac{\partial^2}{\partial z^2}\quad\Big] + \lambda_{02}\big(q_2\lambda_{02} - (1+3q_2)\partial/\partial z\big)\quad\Big], \tag{2.1.8}$$

bu yerda $\omega_1 = 2\dfrac{r_2^{\,2} - r_1^{\,2}}{r_2^{\,2} r_1^2 \ln(r_1 / r_2)}$.

Ana shu (2.1.8) asosida quyidagi xulosani chiqarish mumkin: $\boldsymbol{U}_{r,0}^{(0)}, \boldsymbol{U}_{z,0}^{(0)}$ funksiyalar sterjen nazariyasi bo'yicha tebranish tenglamasini, ya'ni

$$\left[(1+q_1)^2 \frac{\partial^2}{\partial z^2} - q_1(1-q_1)\lambda_{01}\right](\boldsymbol{U}_{r,0}^{(0)}, \boldsymbol{U}_{z,0}^{(0)})=0 \qquad (2.1.9)$$

va silindrik qatlamning bo'ylama tebranishlari aniqlashtirilgan nazariyasi tenglamasini, ya'ni

$$\Delta_2(\boldsymbol{U}_{r,0}^{(0)}, \boldsymbol{U}_{z,0}^{(0)})=0 \qquad (2.1.10)$$

qanoatlantiradilar.

Endi (2.1.9) tenglama va (2.1.10) operatorning yoyilgan korinishlarini taqdim etamiz

$$\left[\rho_0 \mathrm{M}_0^{-1} L_0\left(\mathrm{M}_0^{-1} - L_0^{-1}\right)\frac{\partial^2}{\partial t^2} - \left(3\mathrm{M}_0^{-1} - 4L_0^{-1}\right)L_0\left(\frac{\partial^2}{\partial z^2}\right)\right](U_{r,0}^{(0)}, U_{z,0}^{(0)}) = 0$$

и $\Delta_2 = \rho_0^2\left(\mathrm{M}_0^{-1} - L_0^{-1}\right)\mathrm{M}_0^{-1}\left(\dfrac{\partial^4}{\partial t^4}\right) - p_0\left(4\mathrm{M}_0^{-1} - 5L_0^{-1}\right)\dfrac{\partial^4}{\partial t^2 \partial z^2} + \left(3\mathrm{M}_0^{-1} - 4L_0^{-1}\right)\times$

$$x\mathrm{M}_0^{-1}\left(\frac{\partial^4}{\partial t^4}\right) + \omega_1\left[\rho_0 L_0^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - 2\frac{\partial^2}{\partial z^2}\right] + \omega_1^2. \qquad (2.1.11)$$

Yuqoridagi (2.1.4) tenglamalar sistemasida birinchi tenglamadan ikkinchi tenglamani, uchinchi tenglamadan esa to'rtinchisini ayiramiz. Natijada ikki tenglamadan iborat sistemaga ega bo'lamiz

$$(\omega_1 + q_2\lambda_{02})U_{r,1}^{(0)} + (1+q_1)\frac{\partial U_{z,1}^{(0)}}{\partial z} = 0,$$

$$\left[\,(1+2q_2)\lambda_{02} + \omega_1\,\right]\frac{\partial U_{r,1}^{(0)}}{\partial z} + \left((\lambda_{02} + 2q_2\frac{\partial^2}{\partial z^2}) + \omega_1\right)U_{z,1}^{(0)} = 0. \qquad (2.1.12)$$

Bu sistemaning birinchi tenglamasidan topamiz

$$(1+q_2)\frac{\partial U_{z,1}^{(0)}}{\partial z}=-(\omega_1+q_2\lambda_{02})U_{r,1}^{(0)} \qquad (2.1.13)$$

Ushbu sistemaning ikkinchi tenglamasini z bo'yicha differensiallab quyidagiga ega bo'lamiz

$$\left[\ (1+2q_2)\lambda_{02}+\omega_1\ \right]\frac{\partial^2 U_{r,1}^{(0)}}{\partial \text{я}^2}+\left((\lambda_{02}+2q_2\frac{\partial^2}{\partial z^2})+\omega_1\right)\frac{\partial U_{z,1}^{(0)}}{\partial z}=0$$

Va ushbu oxirgi tenglamaga $\frac{\partial U_{z,1}^{(0)}}{\partial z}$ ning ifodasini (2.1.13) formula bo'yicha qoyamiz. U holda

$$\left[\ (1+2q_2)\lambda_{02}+\omega_1\ \right]\frac{\partial^2 U_{r,1}^{(0)}}{\partial z^2}-$$

$$\left((\lambda_{02}+2q_2\frac{\partial^2}{\partial z^2})+\omega_1\right)\frac{1}{1+q_2}(\omega_1+q_2\lambda_{02})U_{r,1}^{(0)}=0,$$

yoki

$$\Delta_2 U_{r,1}^{(0)}=0, \qquad (2.1.14)$$

bu yerda

$$\Delta_2=\omega_1{}^2+\omega_1\left[\ (1+2q_2)\lambda_{02}-(1-q_2)\frac{\partial^2}{\partial z^2}\ \right]+\lambda_{02}\left(q_2\lambda_{02}-(1+3q_2)\frac{\partial^2}{\partial z^2}\right)$$

Oxirgi (2.1.14) tenglamaning yechimini U_1 funksiya orqali belgilaymiz, va uni (2.1.13) tenglamaga qo'yib z bo'yicha integrallaymiz. Natijada $U_{z,1}^{(0)}$ funksiyani topamiz

$$U_{z,1}^{(0)}=-\int\frac{1}{1+q_2}(\omega_1+q_2\lambda_{02})U_1\,dz$$

Nihoyat $U_{r,1}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalarning topilgan qiymatlarini (2.1.4) sistemaning birinchi va uchinchi tenglamalariga qo'yib, quyidagini olamiz

$$-q_1 U_{r,0}^{(0)} - (1+q_1)\frac{\partial U_{z,0}^{(0)}}{\partial z} = F_2;$$

$$(1+q_1)\frac{\partial U_{r,0}^{(0)}}{\partial z} + (1-q_1)\lambda_{01} U_{z,0}^{(0)} = F_3, \qquad (2.1.15)$$

bunda

$$F_2 = -\xi[(q_2 \ln\frac{r_1}{\xi} - \frac{1}{2})\lambda_{02} + \frac{2}{r_1^2}]U_{r,1}^{(0)} - \xi[(1+q_2)\ln\frac{r_1}{\xi} + \frac{1}{2}]\frac{\partial U_{z,1}^{(0)}}{\partial z};$$

$$F_3 = \xi[(1+2q_2)\ln\frac{r_1}{\xi}\lambda_{02} + \frac{2}{r_1^2}]\frac{\partial U_{r,1}^{(0)}}{\partial z} + \xi[(\lambda_{02} + 2q_2\frac{\partial^2}{\partial z^2})\ln\frac{r_1}{\xi} + \frac{2}{r_1^2}]U_{z,1}^{(0)}.$$

Oxirgi (2.1.15) sistemadan $U_{z,0}^{(0)}$ funksiya uchun ushbu ikkinchi tartibli integro-differensial tenglamaga ega bo'lamiz

$$\rho_0\left(\mathrm{M}_0^{-1} - L_0^{-1}\right)\mathrm{M}_0^{-1}\left(\frac{\partial^2 U_{z,0}^{(0)}}{\partial t^2}\right) - \left(3\mathrm{M}_0^{-1} - 4L_0^{-1}\right)L_0\left(\frac{\partial^2 U_{z,0}^{(0)}}{\partial z^2}\right) =$$

$$= -\left(\mathrm{M}_0^{-1} - L_0^{-1}\right)F_3 + \left(\mathrm{M}_0^{-1} - 2L_0^{-1}\right)\frac{\partial}{\partial z}F_2 \;. \qquad (2.1.16)$$

Ana shu oxirgi tenglamadan $U_{z,0}^{(0)}$ funksiyani aniqlaymiz. Bu holda (2.1.15) tenglamalar sistemasining birinchi tenglamasidan $U_{r,0}^{(0)}$ funksiyaning qiymatini toppish qiyinchilik tug'dirmaydi

$$q_1 U_{r,0}^{(0)} = -(1+q_1)\frac{\partial U_{z,0}^{(0)}}{\partial z} - F_2. \qquad (2.1.17)$$

Olingan (2.1.13), (2.1.14), (2.1.16) (2.1.17) tenglamalar sistemalarida o'lchamsiz koordinatalarga quyidagi formulalar yordamida o'tamiz

$z = z^*\xi;$ $\quad bt = t^*\xi;$ $\quad r = r^*\xi;$ $\quad U_{r,1} = U_{r,1}^*\xi;$ $\quad r_2 = r_2^*\xi;$

$U_{z,0} = U_{z,0}^*\xi;$ $\quad U_{z,1} = U_{z,1}^*;$ $\quad U_{r,0} = U_{r,0}^*;$ $\quad r_1 = r_1^*\xi;$

va qulaylik uchun bundan keyin belgilashlarni odatdagidek "*" belgilashni tashlab yuboramiz .

U holda tashqi ta'sir yuklari bo'lmagan va Puasson koeffitsiyenti o'zgarmas bo'lgan hol uchun

$$(\nu = const, \mathrm{M}_0 = \lambda \tilde{\mathrm{M}}_0, L = (\lambda + 2\mu)\tilde{\mathrm{M}}_0, \tilde{\mathrm{M}}_0 = 1 - \int_0^t k(t-\tau)d\tau)$$

qovushoq-elastik silindrik qatlam bo'ylama radial tebranishlari aniqlashtirilgan tenglamalari sistemasi quyidagi ko'rinishni oladi

$$\begin{gathered}
\Delta_2 U_{r,1}{}^{(0)} = 0, \\
\left(1 + q_2^{(0)}\right)\frac{\partial Uz,1^{(0)}}{\partial z} = \left(\omega_1 - q_2^{(0)} \cdot \partial_2\right) U_{r,1}{}^{(0)} \\
\Delta_3 U_{z,0}{}^{(0)} = 0, \\
q_1 U_{r,0}^{(0)} = -(1+q_1)\frac{\partial U_{z,0}^{(0)}}{\partial z} - F_2 ,
\end{gathered} \qquad (2.1.19)$$

bu yerda

$$\Delta_2 = a_1 \tilde{\mathrm{M}}_0^{-2}\left(\frac{\partial^4}{\partial t^4}\right) - a_2 \tilde{\mathrm{M}}_0^{-1}\left(\frac{\partial^4}{\partial t^2 \partial z^2}\right) + a_3 \frac{\partial^4}{\partial z^4} +$$

$$+ \omega_1 \cdot a_3 \mathrm{M}_0^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - 2\omega_1 \frac{\partial^2}{\partial z^2} - \omega_1^2 .$$

$$\lambda_1 = \frac{1-2\nu}{2(1-\nu)}\tilde{\mathrm{M}}^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}; \ \lambda_2 = \partial_2 = \tilde{\mathrm{M}}^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2};$$

$$\lambda_3 = \tilde{\mathrm{M}}_0^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - c_0 \frac{\partial^2}{\partial t^2}; \ F_4 = -a_4 \frac{\partial F_2}{\partial z} + F_3;$$

$$F_2 = \left[\left(q_2 \ln r_1 - \frac{1}{2}\right)\partial^2 + \frac{2}{r_1^2}\right] U_{\gamma,1}^{(0)} + \left[\left(1+q_2\right)\ln r_1 + \frac{1}{2}\right]\frac{\partial Uz,_1^{(0)}}{\partial z};$$

$$F_3 = \left[\left(1+2q_2\right)\ln r_1 \cdot \partial_2 + \frac{2}{r_1^2}\right]\frac{\partial Uz,_1^{(0)}}{\partial z} + \left[\left(\partial^2 + 2q_2 \frac{\partial^2}{\partial z^2}\right)\ln r_1 + \frac{2}{r_1^2}\right] U_{z,1}^{(0)};$$

$$a_1 = 1 - \frac{b_0^2}{a_0^2}; \qquad a_2 = 4 - 5\frac{b_0^2}{a_0^2}; \qquad a_3 = 3 - 4a_5; \qquad a_4 = \frac{a_0{}^2 - 2b_0{}^2}{a_0{}^2 - b_0{}^2};$$

$$q_1 = 1 - \frac{\lambda + 2\mu}{\mu}; \qquad q_2 = -- \frac{\lambda + \mu}{\lambda + 2\mu}; \qquad a_5 = \frac{b_0^{\ 2}}{a_0^{\ 2}}; \qquad c_0^2 = \frac{3a^2 - 4b_0^{\ 2}}{a_0^{\ 2} - b_0^{\ 2}};$$

$$a_0^2 = \frac{\lambda + 2\mu}{\rho}; \qquad b_0^2 = \frac{\mu}{\rho}$$

λ, μ - Lame koeffitsiyentlarini ν - Puasson koeffitsiyenti orqali ifodalab quyidagi formulalarga ega bo'lamiz

$$a_1 = \frac{1}{2(1-\nu)}; \qquad a_2 = \frac{3+2\nu}{2(1-\nu)}; \qquad a_3 = \frac{1+\nu}{1-\nu}; \qquad a_4 = 2\nu; \qquad a_5 = \frac{1-2\nu}{2(1-\nu)};$$

$$\Delta_2 = \frac{1}{2(1-\nu)} \tilde{\mathrm{M}}_0^{-2} \left(\frac{\partial^4}{\partial t^4} \right) - \frac{3+2\nu}{2(1-\nu)} \tilde{\mathrm{M}}_0^{-1} \left(\frac{\partial^4}{\partial t^2 \partial z^2} \right) + \frac{1+\nu}{1-\nu} \frac{\partial^4}{\partial z^4} +$$

$$+\omega_1 \frac{1-2\nu}{2(1-\nu)} \tilde{\mathrm{M}}_0^{-1} \left(\frac{\partial^2}{\partial t^2} \right) - 2\omega_1 \frac{\partial^2}{\partial z^2} - \omega_1^2; \qquad c_0 = 2(1+\nu);$$

Agar qatlamning materiali elastic bo'lsa, u holda integrodifferesial operatorlar

$$M = \mu; \qquad L = \lambda + 2\mu,$$

ko'rinishga ega bo'lib, ularning yadrolari nolga teng bo'ladilar, ya'ni $K(t) = 0$ hamda λ_1 va λ_2 operatorlari esa

$$\lambda_1 = \frac{1-2\nu}{2(1-\nu)} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}; \qquad \lambda_2 = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}.$$

kabi bo'ladilar.

U holda (2.1.19) tenglamalar sistemasi o'lchamsiz o'zgaruvchilarda, qulaylik uchun balandgi "0" indeks tashlab yozilgan holda ushbu ko'rinishni oladilar

$$\frac{\partial^4 U_{r,1}}{\partial t^4} - (3+2\nu) \frac{\partial^4 U_{r,1}}{\partial t^2 \partial z^2} + 2(1+\nu) \frac{\partial^4 U_{r,1}}{\partial z^4} + \omega_1 (1-2\nu) \frac{\partial^2 U_{r,1}}{\partial t^2} -$$

$$-4(1-\nu)\omega_1 \frac{\partial^2 U_{r,1}}{\partial z^2} - 2(1-\nu)\omega_1^2 U_{r,1} = 0;$$

$$\frac{\partial U_{z,1}}{\partial z}-\frac{1}{1-2\nu}[\lambda_2-2(1-\nu)\omega_1]U_{r,1}=0; \qquad (2.1.20)$$

$$2\nu\frac{\partial U_{z,0}}{\partial z}-(1-2\nu)\left[\left(\frac{\ln r_1}{2(1-\nu)}-\frac{1}{2}\right)\lambda_2+\frac{2}{r_1^2}\right]U_{r,1}+$$

$$+(1-2\nu)\left[\frac{1-2\nu}{2(1-\nu)}\ln r_1+\frac{1}{2}\right]\frac{\partial U_{z,1}}{\partial z}+U_{r,0}=0,$$

$$\frac{\partial^2 U_{z,0}}{\partial t^2}-2(1+\nu)\frac{\partial^2 U_{z,0}}{\partial z^2}=F(z,t),$$

bu yerda

$$F(z,t)=-2\nu\frac{\partial}{\partial z}\left\{\left[\frac{2}{r_1^2}-\frac{1}{2}\left(1+\frac{\ln r_1}{1-\nu}\right)\lambda_2\right]U_{r,1}+\frac{1}{2}(1+\frac{1-2\nu}{1-\nu}\ln r_1)\frac{\partial U_{z,1}}{\partial z}\right\}+$$

$$+\left[\frac{2}{r_1^2}-\frac{\nu\ln r_1}{1-\nu}\lambda_2\right]\frac{\partial U_{r,1}}{\partial z}+\left[\left(\lambda_2-\frac{1}{1-\nu}\frac{\partial^2}{\partial z^2}\right)\ln r_1+\frac{2}{r_1^2}\right]U_{z,1}.$$

Agar tadqiqotlarda elastik materialdan yasalgan qatlam ishlanayotgan holat bo'lsa, (2.1.20) tenglamalar sistemasidan foydalaniladi. Bu sistemani yechish algoritmi quyidagicha: $U_{r,1}$ ga nisbatan birinchi tenglamani yechib $U_{r,1}$ funksiyaning topilgan ifodasini ikkinchi tenglamaga qo’yamiz. Hosil qilingan tenglamadan $U_{z,1}$ ni topamiz. $U_{r,1}$ va $U_{z,1}$ funksiyalarning topilgan qiymatlarini to’rtinchi tenglamaga qo’yib $U_{z,0}$ ni topamiz. Nihoyat $U_{r,1}$, $U_{z,1}$, $U_{z,0}$ funksiyalarnig olingan ifodalarini sistemaning uchinchi tenglamasiga qo’yib oxirgi $U_{r,0}$ funksiyani aniqlaymiz.

2.2. Deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi anizotrop silindrik qatlamning buralma tebranishlari

Mazkur ishda o'rab turuvchi deformatsiyalanuvchi muhitning uzunligi chegaralanmagan transversal-izotrop qatlamning buralma tebranishlariga ta'siri o'rganilgan. Fazaviy tezlikka nisbatan oltinchi tartibli dispersion tenglama olingan va u sonly yechilgan. Qatlamning erkin buralma tebranishlariga tashqi muhitning ta'siri baholangan. Qatlamni o'rab turuvchi tashqi muhit parametrlari sifatida ba'zi tuproq va tog' jinslarining fizik-mexanik xarakteristikalari olingan.

Muhandislik qurilmalari elementlarining, xususan, silindrik qatlamning, nostatsionar tebranishlarini o'rganishga bir muncha sondagi ilmiy ishlar bag'ishlangan [1,3,6]. Ularda asosiy tebranish tenglamalari sifatida Kirxgoff-Love nazariyasiga asoslangan tenglamalar qabul qilingan. Ammo bunday masalalarni aylanish inersiyasi va ko'ndalang siljish deformatsiyasini o' strukturalarida hisobga oluvchi tebranish tenglamalari asosida yechish ham juda muhim ahamiyatga ega.

Quyida o'rab turuvchi deformatsiyalanuvchi muhitning uzunligi chegaralanmagan transversal-izotrop qatlamning buralma tebranishlariga ta'siri o'rganilgan. Masalani yechishda asosiy tebranish tenglamalari sifatida silindrik qatlamning aylanish inersiyasi va ko'ndalang siljish deformatsiyasini hisobga oluvchi va (2.1.20) tenlamalardan xususiy holda kelib chiqadigan tebranish tenglamalari qo'llanilgan [4]

$$\begin{gathered}
\frac{r_1^2}{4}\gamma_0 U_{\theta,0} + \xi\left[\frac{1}{2}\left(\gamma_0 - \frac{4}{r_1^2}\right) + \frac{r_1^2}{8}\left(\ln\frac{r_1}{\xi} - \frac{1}{2}\right)\gamma_0^2\right]U_{\theta,1} = 0 \\
\left[\frac{r_2^2}{4}\gamma_0 - r_2 R_0\right]U_{\theta,0} + \xi\left[\frac{1}{2}\left(\gamma_0 - \frac{4}{r_2^2}\right) + \frac{r_2^2}{8}\left(\ln\frac{r_2}{\xi} - \frac{1}{2}\right)\gamma_0^2 - R_0\left[\frac{1}{r_2} + \gamma_0\frac{r_2}{2}\ln\frac{r_2}{\xi}\right]\right]U_{\theta,1} = 0
\end{gathered} \tag{2.2.1}$$

bu yerda r_1 va r_2- qatlam radiuslari ($r_1 < r_2$); $U_{\theta,0}, U_{\theta,1}$- buralma U_θ - ko'chishning bosh qismlari.Bunda, ta'kidlash lozimki, umumiy tenglamalarda birinchi yaqinlashish bilan chegaralanilgan, ya'ni

$$U_\theta = \xi U_{\theta,0} + U_{\theta,1}; \tag{2.2.2}$$

ξ-qatlam oraliq sirtining [7] mohografiyada kiritilgan radiusi

$$\xi = \frac{r_1}{2}\left(\chi - \frac{r_1}{r_2}\right), \quad 2 + \frac{r_1}{r_2} \le \chi \le 2 + \frac{r_2}{r_1} + \frac{r_1}{r_2}; \tag{2.2.3}$$

R_0-o'rab turuvchi muhitning reaksiyasini ifodalovchi differensial operator va juda tez sodir bo'luvchi to'lqinli jarayonlar uchun quyidagicha aniqlanadi

$$\boldsymbol{R}_0 = \frac{\mu_1}{b_1 \mu_0} \frac{\partial}{\partial t}; \tag{2.2.4}$$

γ_0-differensial operator

$$\gamma_0 = \frac{1}{b_0^2}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\mu_0^*}{\mu_0} \frac{\partial^2}{\partial z^2}; \tag{2.2.5}$$

$b_0 = \sqrt{\dfrac{\mu_0}{\rho_0}}$ - qatlam materialida ko'ndalag to'lqinlar tarqalish tezligi

$$\mu_0 = \frac{E_0}{2(1+\nu_0)}; \quad \mu_1 = \frac{E_1}{2(1+\nu_1)}; \quad \mu_0' = \frac{E}{1 + 2\nu' + E/E_0}; \tag{2.2.6}$$

ρ_0-qatlam materialining zichligi;

$E,\ E_0, E_1$-модули упругости слоя и среды;

ν', ν_0, ν_1-коэффициенты Пуассона.

Yuqoridagi (2.2.1) tenglamalar sistemasida $U_{\theta,0}$ *va* $U_{\theta,1}$ fuksiyalar qoshidagi differensial operatorlarning chiziqliligidan foydalanib, hamda (2.2.2) va (2.2.4) tengliklarni e'tiborga olib, U_θ buralma ko'chish funksiyasi , xuddi $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalar kabi

$$DU_\theta = 0 \tag{2.2.7}$$

differensial tenglamani qanoatlantiradi degan xulosaga kelamiz, bu yerda D differensial operatori quyidagicha aniqlanadi

$$D=\frac{\xi}{2}\left[\frac{1}{b_0^2}\left(\frac{\partial^2}{\partial t^2}\right)-\frac{\mu'}{\mu_0}\frac{\partial^2}{\partial z^2}\right]\left\{\frac{r_2^4-r_1^4}{r_1^2r_2^2}-\frac{r_2^2-r_1^2}{4}\gamma_0+\frac{r_2^2r_1^2}{16}\ln\frac{r_2}{r_1}\gamma_0\right\}-$$

$$-\xi\frac{\mu_1}{b_1\mu_0}\frac{\partial}{\partial t}\left[\frac{2r_2}{r_1^2}-\frac{2r_2^2-r_1^2}{4r_2}\gamma_0+\frac{r_1^2r_2}{8}\left(\ln\frac{r_2}{r_1}+\frac{1}{2}\right)\gamma_0^2\right] \tag{2.2.8}$$

Endi (2.2.8) ifodada o'lchamsiz koordinatalarga ushbu formulalar vositasida o'tamiz

$$z=\xi\, z^*,\ r_1=\xi r_1^*,\ r_2=\xi r_2^*,\ U_\theta=\xi U_\theta^*,\ t=\frac{\xi}{b_{0,1}}t^*$$

Va yozuvning qulayligi uchun keying joylarda harflar peshonasidagi yulduzchalarni tashlab yozamiz va ushbularga ega bo'lamiz

$$\frac{r_1^2r_2^2}{32}\ln\frac{r_2}{r_1}\frac{\partial^6U_\theta}{\partial t^6}-3\frac{r_1^2r_2^2}{32}\ln\frac{r_2}{r_1}\mu_0^*\frac{\partial^6U_\theta}{\partial t^4\partial z^2}+3\frac{r_1^2r_2^2}{32}\ln\frac{r_2}{r_1}\mu_0^{*2}\frac{\partial^6U_\theta}{\partial t^2\partial z^4}-$$
$$-\frac{r_1^2r_2^2}{32}\ln\frac{r_2}{r_1}\mu_0^{*3}\frac{\partial^6U_\theta}{\partial z^6}-\frac{r_2^2-r_1^2}{8}\frac{\partial^4U_\theta}{\partial t^4}+\frac{r_2^2-r_1^2}{4}\mu_0^*\frac{\partial^4U_\theta}{\partial t^2\partial z^2}-\frac{r_2^2-r_1^2}{8}\mu_0^{*2}\frac{\partial^4U_\theta}{\partial z^4}+$$
$$+\frac{r_2^4-r_1^4}{2r_1^2r_2^2}\frac{\partial^2U_\theta}{\partial t^2}-\frac{r_2^4-r_1^4}{2r_1^2r_2^2}\mu_0^*\frac{\partial^2U_\theta}{\partial z^2}-\frac{\mu_1b_0}{b_1\mu_0}\frac{2r_2}{r_1^2}\frac{\partial U_\theta}{\partial t}+\frac{\mu_1b_0}{b_1\mu_0}\frac{2r_2^2-r_1^2}{4r_2}\frac{\partial^3U_\theta}{\partial t^3}-$$
$$-\frac{\mu_1b_0}{b_1\mu_0}\frac{2r_2^2-r_1^2}{4r_2}\mu_0^*\frac{\partial^3U_\theta}{\partial z^2\partial t}-\frac{\mu_1b_0}{b_1\mu_0}\frac{r_1^2r_2}{8}\left(\ln\frac{r_2}{r_1}+\frac{1}{2}\right)\frac{\partial^5U_\theta}{\partial t^5}+$$
$$\frac{\mu_1b_0}{b_1\mu_0}\frac{r_1^2r_2}{4}\left(\ln\frac{r_2}{r_1}+\frac{1}{2}\right)\mu_0^*\frac{\partial^5U_\theta}{\partial t^3\partial z^2}-\frac{\mu_1b_0}{b_1\mu_0}\frac{r_1^2r_2}{8}\left(\ln\frac{r_2}{r_1}+\frac{1}{2}\right)\mu_0^{*2}\frac{\partial^5U_\theta}{\partial t\partial z^4}=0;$$

(2.2.9)

Qatlam uzunligi chegaralanmagan bo'lganligi uchun (2.2.9) tenglamaning yechimini quyidagi ko'rinishda izlaymiz

$$U_\theta=A\exp(ikz+\omega t)\ , \tag{2.2.10}$$

bu yerda A - tebranishlar amplitudasi.

Ko'chishning (2.2.10) ifodasini (2.2.9) ga qo'yib quyidagi chastota tenglamasini olamiz

$$f_1\omega^6+3f_1\mu_0^*\omega^4k^2+3f_1\mu_0^{*2}\omega^2k^4+f_1\mu_0^{*3}k^6-f_2\omega^4-2f_2\mu_0^*\omega^2k^2-$$

$$-f_2\mu_0^{*2}k^4+f_3\omega^2+f_3\mu_0^{*}k^2-f_4f_5\omega+f_4f_6\omega^3+f_4f_6\mu_0^{*}k^2\omega-$$

$$-f_4f_7\omega^5-2f_4f_7\mu_0^{*}\omega^3k^2-f_4f_7\mu_0^{*2}k^4\omega=0 \qquad (2.2.11)$$

Bu yerda ω-tebranishlar chastotasi; k – to'lqin soni;

$$f_1=\frac{r_1^2r_2^2}{32}\left(\ln\frac{r_2}{r_1}\right);\quad f_2=\frac{r_2^2-r_1^2}{8};\quad f_3=\frac{r_2^4-r_1^4}{2r_1^2r_2^2};\quad f_4=\frac{\mu_1b_0}{b_1\mu_0};$$
$$f_5=\frac{2r_2}{r_1^2};\quad f_6=\frac{2r_2^2-r_1^2}{4r_2};\quad f_7=\frac{r_1^2r_2}{8}\left(\ln\frac{r_2}{r_1}-\frac{1}{2}\right);\quad \mu_0^*=\frac{\mu_0^{'}}{\mu_0}. \qquad (2.2.12)$$

Chiqarilgan (2.2.11) tenglamadan $C=\omega/k$ fazaviy tezlikka nisbatan tenglamani keltirib chiqarish qiyin emas. Bu tenglama ushbu ko'rinishga ega bo'ladi

$$f_1C^6-\frac{f_4f_7}{k}C^5+\left(3f_1\mu_0^*-\frac{f_2}{k^2}\right)C^4+\left(\frac{f_4f_6}{k^3}-\frac{2f_4f_7\mu_0^*}{k}\right)C^3+\left(3f_1\mu_0^{*2}-\frac{2f_2\mu_0^*}{k^2}+\frac{f_3}{k^4}\right)C^2+$$

$$+\left(\frac{f_4f_6\mu_0^*}{k^3}-\frac{f_4f_5}{k^5}-\frac{f_4f_7\mu_0^{*2}}{k}\right)C+\left(f_1\mu_0^{*3}-\frac{f_2\mu_0^{*2}}{k^2}+\frac{f_3\mu_0^*}{k^4}\right)=0 \qquad (2.2.13)$$

Bu holda tashqi o'rab turuvchi muhitning ta'siri (2.2.13) tenglamada fazaviy tezlikning toq darajalari oldidagi koeffitsiyentlar bilan hisobga olinadi.

Tashqi o'rab turuvchi muhitning anizotrop silindrik qatlam erkin tebranishlariga ko'rsatgan ta'sirini baholash uchun (2.2.13) tenglama sonly yechildi. Hisoblashlar fizik-mexanik xarakteristikalari quyidagicha bo'lgan transversal-izotrop qatlam uchun olib borildi [2]

$$E_0=5\cdot10^5\ кг/см^2,\quad E=1\cdot10^5\ кг/см^2,\quad \nu^{'}=0.33,\quad \nu=0.1.$$

Tashqi muhitning fizik-mexanik xarakteristikalari uchun suglinok, loy, aralashma, qum va argillitlarning xarakteristikalari qabul qilindi [5].

Suglinok $E_1=19\,M$МПа $\nu_1=0.35,\quad \rho_1=1.75\ г/см^3,$

Loy $E_1=21\,M$МПа $\nu_1=0.42,\quad \rho_1=1.93\ г/см^3,$

Aralashma $E_1=16\,M$МПа $\nu_1=0.3,\quad \rho_1=1.72\ г/см^3,$

Qum $E_1=30\,M$МПа $\nu_1=0.27,\quad \rho_1=1.96\ г/см^3,$

Argillit $E_1=32\,M$МПа $\nu_1=0.29,\quad \rho_1=2.41\ г/см^3.$

O’tkazilgan hisoblashlar natijalari shuni ko’rsatadiki, qatlamni o’rab turubchi muhitning ta’siri fazaviy tezlikning pasayishiga olib keladi

Qatlam qalinligining oshishi, to’lqin soning qabul qilgan qiymatidan qat’i nazar, fazaviy tezlikning kamaytiradi.

Olingan sonly natijalarni taqqoslah va uning tahlili shuni ko’rsatadiki tashqi o’rab turuvchi muhitning borligi fazaviy tezlikning turlicha kamayishiga olib keladi va har xil muhitlar uchun turlicha bo’ladi. Masalan bu Argillitda 45% gacha, loyda - 31% gacha, suglinokda- 30% gacha, aralashma va qumda - 22% gacha bo’ladi.

2.3. Aylanish inersiyasi va ko’ndalang siljish deformatsiyasining anizotrop silindrik qatlamning buralma tebranishlariga ta’siri

Yupqa devorli qobiqlarning xususiy tebranishlarini hisoblash muammosi birinchi marta A. Love tomonidan qo’yilgan. Chetlari bilan erkin tayangan silindrik qobiqning xususiy tebranishi haqidagi masala birinchi marta F.Flugge tomonidan yechilgan bo’lsa qobiq chetlari qistirib mahkamlangan holda masala A.P.Filippov tomonidan yechilgan [7,13].

Quyida ikkala cheti ham buralishdan qistirib mahkamlangan doiraviy silindrik anizotrop qatlamning xususiy buralma tebranishlariga aylanish inersiyasi va ko’ndalang siljish deformatsiyasining ta’sirini bahilash haqidagi masala qaralgan.

Bunday ikkala cheti ham buralishdan qistirib mahkamlangan doiraviy silindrik transversal-izotrop qatlamning xususiy buralma tebranishlari uchun asosiy, hal qiluvchi tenglamalalar sifatida [10] tadqiqot ishida taklif etilgan aniqlashtirilgan tebranish tenglamalaridan foydalanamiz

$$\begin{aligned}
&\frac{r_1^2}{4}\gamma_0 U_{\theta,0} + \xi\left[\frac{1}{2}\left(\gamma_0 - \frac{4}{r_1^2}\right) + \frac{r_1^2}{8}\left(\ln\frac{r_1}{\xi} - \frac{1}{2}\right)\gamma_0^2\right]U_{\theta,1} = 0\\
&\frac{r_2^2}{4}\gamma_0 U_{\theta,0} + \xi\left[\frac{1}{2}\left(\gamma_0 - \frac{4}{r_2^2}\right) + \frac{r_2^2}{8}\left(\ln\frac{r_2}{\xi} - \frac{1}{2}\right)\gamma_0^2\right]U_{\theta,1} = 0
\end{aligned} \tag{2.3.1}$$

Bu yerda , xuddi oldingi paragrafdagidek $U_{\theta,0}$ va $U_{\theta,1}$ - qatlam nuqtalari buralma ko'chishlarining bosh qismlari

$$U_\theta = \xi U_{\theta,0} + U_{\theta,1}$$

ξ - qatlam "oraliq" sirtining quyidagi formula bilan aniqlanuvchi radiusi [4]

$$\xi = \frac{r_1}{2}\left(\chi - \frac{r_1}{r_2}\right), \qquad 2 + \frac{r_1}{r_2} \le \chi \le 2\frac{r_2}{r_1} + \frac{r_1}{r_2};$$

r_1, r_2- qaralayotgan silindrik qatlam ko'ndalang kesimining radiusi:

γ_0- quyidagi formula bilan aniqlanuvchi differensial operator

$$\gamma_0 = \frac{1}{b^2}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\mu'}{\mu}\frac{\partial^2}{\partial z^2}$$

b-qatlam materialida ko'ndalang to'lqinlar tarqalish tezligi

$$b^2 = \frac{\mu}{\rho};$$

μ-siljish moduli;

μ' - izotropiya tekisliklari va ularga perpendikulyar yo'nalishlar uchun siljish moduli;

ρ - qatlam matyerilining zichligi.

Yuqoridagi (2.3.1) tenglamalar sistemasida $U_{\theta,0}$ va $U_{\theta,1}$ fuksiyalar qoshidagi differensial operatorlarning chiziqliligidan foydalanib, hamda (2.2.2) va (2.2.4) tengliklarni e'tiborga olib, U_θ buralma ko'chish funksiyasi , xuddi $U_{\theta,0}$ va $U_{\theta,1}$ funksiyalar kabi

$$D_0 U_\theta = 0, \tag{2.3.2}$$

differensial tenglamani qanoatlantiradi degan xulosaga kelamiz, bu yerda D differensial operatori quyidagicha aniqlanadi

$$D_0 = \frac{\xi}{2}\left[\frac{1}{b^2}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}\right]\left[\frac{r_2^4 - r_1^4}{r_1^2 r_2^2} - \frac{r_2^2 - r_1^2}{4}\gamma_0 + \frac{r_2^2 r_1^2}{32}\gamma_0^2\right].$$

Keltirilgan (2.3.2) tenglamadan ikkita ikkinchi va to'rtinchi tartibli differensial tenglamalar kelib chiqadi. Shulardan ikkinchi tartibli tenlama silindrik anizotrop qatlamning sterjen nazariyasi bo'yicha buralma tebranishlari tenglamasidan iboratdir. Shuning uchun faqat tarkibida aylanish inersiyasi va ko'ndalang siljish deformatsiyasini hisobga olubchi hadlar qatnashuvchi to'rtinchi tartibli tenglamani qaraymiz. Bu tenglamani uncha qiyin bo'lmagan ba'zi matematik almashtirishlardan so'ng, keying o'rinlarda foydalanish qulay bo'lgan shaklga keltiramiz

$$\frac{\partial^4 U_\theta}{\partial t^4} - 2b^2\frac{\partial^4 U_\theta}{\partial t^2 \partial z^2} + b^4\frac{\partial^4 U_\theta}{\partial z^4} - f_1 b^2\frac{\partial^2 U_\theta}{\partial t^2} + f_1 b^4\frac{\partial^2 U_\theta}{\partial t^2} + f_2 U_\theta = 0, \qquad (2.3.3)$$

bu yerda

$$f_1 = 4\frac{r_2^2 - r_1^2}{r_1^2 r_2^2 \ln\left(\frac{r_2}{r_1}\right)}, \quad f_2 = 16\frac{r_2^4 - r_1^4}{r_1^4 r_2^4 \ln\left(\frac{r_2}{r_1}\right)},$$

O'lchamsiz o'zgaruvchlarga ushbu formulalar orqali o'tamiz

$$z = \xi z^*; \quad r_i = \xi r_i^*; \quad U_\theta = \xi U_\theta^*; \quad t = \left(\frac{\xi}{b}\right)t^*; \quad l = \xi l^* \qquad (2.3.4)$$

va yozuvning qulayligi uchun keying o'rinlarda harflar qoshidagi "yulduzcha" belgisini tashlab yozamiz. Ushbu (2.3.4) almashtirishlardan so'ng (2.3.3) tenglamani quyidagi ko'rinishda yozamiz:

$$N_1 a_1\frac{\partial^4 U_\theta}{\partial t^4} + \frac{E}{\mu}\mu^{*2}\frac{\partial^4 U_\theta}{\partial z^4} - \left(a_2 N_1 + \frac{E}{\mu}a_1\right)\mu^*\frac{\partial^4 U_\theta}{\partial t^2 \partial z^2} - S_1 S_4\frac{\partial^2 U_\theta}{\partial t^2} +$$

$$+\frac{S_3}{r_2^2}\frac{E}{\mu}\mu^*\frac{\partial^2 U_\theta}{\partial z^2} + S_2\frac{E}{\mu}U_\theta = 0, \qquad (2.3.5)$$

Bu yerda a_1 va a_2 koeffitsiyentlar ko'ndalang siljish deformatsiyasi (a_1) va aylanish inersiyasi (a_2) hisobga olinish yoki olinmasligiga qarab 0 yoki 1 qiymatlarni qabul qiladilar;

$N_1 = 2(1+\nu)$;

ν -Puasson koeffitsaiyenti;

E- elastiklik moduli;

$$S_1 = 2N_1 \frac{r_2^4 - r_1^4}{r_1^2 r_2^2 \ln\left(\frac{r_2}{r_1}\right)}; \quad S_2 = 16 \frac{r_2^4 - r_1^4}{r_1^2 r_2^2 \ln\left(\frac{r_2}{r_1}\right)};$$

$$S_3 = 4 \frac{r_2^2 - r_1^2}{r_1^2 \ln\left(\frac{r_2}{r_1}\right)}; \quad S_4 = \frac{2}{r_1^2 + r_2^2}.$$

Masalaning shartiga ko'ra qatlamning ikkala uchi han buralishdan qistirib mahkamlangan, ya'ni qatlam uchlarida buralma ko'chish nolga teng. Qatlam uchlarining mahkamlanishi simmetrikligini e'tiborga olib, koordinatalar boshini qatlamning uzunligi bo'yicha ortasiga qo'yamiz. U holda masalaning chegaraviy shartlari quyidagi ko'rinishni oladi

$$U_\theta\left(\pm\frac{\ell}{2}\right) = 0; \qquad U_\theta''\left(\pm\frac{\ell}{2}\right) = 0 \tag{2.3.6}$$

(2.3.5) tenglamaning yechimini

$$U_\theta(z,t) = \overline{U}_\theta(z)\cos\omega t \tag{2.3.7}$$

ko'rinishda izlaymiz. Ushbu yechimni (2.3.5) tenglamaga qo'yib quyidagini olamiz

$$\overline{U}_\theta^{IV} + \eta^2 \overline{U}_\theta^{II} + \psi^2 \overline{U}_\theta = 0 \tag{2.3.8}$$

где

$$\eta^2 = \left(N_1 \frac{\mu}{E} a_2 \omega^2 + a_1 \omega^2 + \frac{S_3}{r_2^2}\right)\frac{1}{\mu^*};$$

$$\psi^2 = \left(N_1 \frac{\mu}{E} a_1 \omega^4 + S_1 S_4 \frac{\mu}{E} \omega^2 + S_2 \right) \frac{1}{\mu^{*2}}. \tag{2.3.9}$$

Ma'lumki (2.3.7) tenglamaning umumiy yechimi

$$\overline{U}_\theta(z) = C_1 Sinmz + C_2 Cosmz + C_3 Chnz + C_4 Chnz \tag{2.3.10}$$

ko'rtinishga ega, bu yerda

$$m = \pm\sqrt{-\frac{\eta^2}{2} + \sqrt{\frac{\eta^4}{4} + \psi^2}}; \quad n = \pm\sqrt{\frac{\eta^2}{2} + \sqrt{\frac{\eta^4}{4} + \psi^2}}; \tag{2.3.11}$$

Bu yerdagi m va n miqdorlar orasida ushbu bo'lanish bor:

$$n^2 - m^2 = \eta^2$$

Bundan tashqari (2.3.11) dan

$$\psi^2 = m^2\left(m^2 + \eta^2\right) \tag{2.3.12}$$

kelib chiqadi. Yana (2.3.9) belgilashlarni ham e'tiborga olib oxirgi tenglikni quyidagicha yozamiz:

$$a_1\omega^4 + \left(\frac{S_1 S_4}{N_1} - a_2 m^2 \mu^* - a_1 m^2 \mu^* \right)\omega^2 + S_2 -$$

$$- m^4 \mu^{*2} - m^2 \mu^* \frac{S_3}{r_2^2} = 0. \tag{2.3.13}$$

Bu esa ω - chastotaga nisbatan bikvadrat tenglamadir. Yuqoridagi (2.3.10) yechim ko'rinishidan kelib chiqqan holda bo'lishi mumkin bo'lgan ikki xil simmetrik va antisimmetrik tebranishlarni qaraymiz.

Simmetrik tebranishlar holida (2.3.10) umumiy yechimning toq funksiyalari oldidagi koeffitsiyentlar nolga teng bo'ladi, ya'ni $C_1 = C_3 = 0$. U holda

$$\overline{U}_\theta(z) = C_2 Cosmz + C_4 Chnz \tag{2.3.14}$$

Bu yechimni (2.3.6) chegaraviy shartlarga qoyib quyidagiga ega bo'lamiz

$$m = \frac{2k+1}{\ell}\pi; \quad k = 0,1,2,\ldots \tag{2.3.15}$$

Antisimmetrik tebranishlar holatida, xudi shunga o'xshash, olamiz

$$m = \frac{2k\pi}{\ell}; \quad k = 0,1,2,... \qquad (2.3.16)$$

Erishilgan (2.3.15) va (2.3.16) natijalar turli uzulikdagi qatlamlar uchun tebranishlar chastotasini aniqlashga imkon beradi. O'tkazilgan hisoblashlar shuni ko'rsatadiki:

- aylanish inersiyasini hisobga olish xususiy chastotalarning qiymatlarini 3% gacha oshishiga olib keladi. Ammo qatlam uzunligining oshib borishi bilan bu ko'rsatkich kamayib boradi va yetarli darajadagi uzun qatlamlar buralma tebranishlarida aylanish inersiyasining ta'sirini hisobga olmasli mumkin;
- ko'ndalang siljish deformatsiyasining ta'siri esa xususiy tebranishlar chastotasining qiymatlari sezilarli darajada 55% dan 65% gacha oshishiga olib keladi;
- qatlam qalinligining oshishi xususiy chastotalarning qiymatlarini kamayishiga olib keladi. Umuman olganda ushbu effect masalaning fizikma'nosiga mos keladi;
- masalani yechish uchun qabul qilingan nazariya ko'ndalang kesimining o'lchamlari uzunligi bilan solishtirilishi mumkin bo'lgan qatlamlar uchun, hamda to'lqin paydo bolish shakli juda yuqori bo'lgan tebranish jarayonlari uchun yaroqli emas.

Olingan natijalarga o'xshash natijalarni antisimmetrik tebranishlar uchun ham olish mumkin va ular yuqoridagi natijalarning to'g'riligini tasdiqlaydilar.Bundan tashqari, yana shuni ham ta'kidlash lozimki, yuqorida olingan natijalarning xususiy holi – yupqa devorli silindrik izotrop qobiqning xususiy chastotalari uchun bog'lanishlar [6] ishda olingan.

ASOSIY XULOSALAR

1. Deformatsiyalanuvchi muhit bilan o'zaro ta'sirlashuvchi anizotrop silindrik qatlamning buralma tebranishlari masalasining yechimi va o'tkazilgan hisoblashlar natijalari shuni ko'rsatadiki:

- qatlamni o'rab turubchi muhitning ta'siri fazaviy tezlikning pasayishiga olib keladi;

- qatlam qalinligining oshishi, to'lqin soning qabul qilgan qiymatidan qat'i nazar, fazaviy tezlikning kamaytiradi.

- olingan sonly natijalarni taqqoslah va uning tahlili shuni ko'rsatadiki tashqi o'rab turuvchi muhitning borligi fazaviy tezlikning turlicha kamayishiga olib keladi va har xil muhitlar uchun turlicha bo'ladi. Masalan bu Argillitda 45% gacha, loyda - 31% gacha, suglinokda- 30% gacha, aralashma va qumda - 22% gacha bo'ladi.

2. Aylanish inersiyasi va ko'ndalang siljish deformatsiyasining anizotrop silindrik qatlamning buralma tebranishlariga ta'siri haqidagi masalani yechib o'tkazilgan hisoblashlar shuni ko'rsatadiki:

- erishilgan natijalar turli uzulikdagi qatlamlar uchun tebranishlar chastotasini aniqlashga imkon beradi;

- aylanish inersiyasini hisobga olish xususiy chastotalarning qiymatlarini 3% gacha oshishiga olib keladi. Ammo qatlam uzunligining oshib borishi bilan bu ko'rsatkich kamayib boradi va yetarli darajadagi uzun qatlamlar buralma tebranishlarida aylanish inersiyasining ta'sirini hisobga olmasli mumkin;

- ko'ndalang siljish deformatsiyasining ta'siri esa xususiy tebranishlar chastotasining qiymatlari sezilarli darajada 55% dan 65% gacha oshishiga olib keladi;

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