**WO`ZBEKSTAN RESPUBLİKASI BAYLANIS,
İNFORMATİZACIYALASTIRIW HA'M
TELEKOMMUNİKACİYALIQ TEXNOLOGİYALARI
MA'MLEKETLİK KOMİTETİ**

**TASHKENT İNFORMACİYALIQ TEXNOLOGİYALARI
UNİVERSİTETİ NO`KİS FİLİALI**

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SIZIQLI ALGEBRANIN`

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Bul woqi’w-metodaikali’q qollanba Tashkent informaciyali’q texnologiyalari’ universiteti No’kis filiali’ni’n’ ilimiy-metodikali’q ken’esinin’ 2014 ji’l 7-sanli’ bayannamasi’ menen usi’ni’s yetildi.

Bul woqi’w qollanbada Maple integrallasqan sistemasi’nda programmalasti’ri’w haqqi’nda tu’sinikler ha’m si’zi’qli’ algebra boyi’nsha 7 a`meliy jumi’sti’n’ wori’nlani’wi’ berilgen. A`meliy jumi’slardag`i’ sheshilgen mi’sallardi’n` Maple integrallasqan sistemasi’nda sheshiliwi ko`rsetilgen. Ha`r bir jumi’sta jumi’sti’n` maqseti, ma`selenin` qoyi’li’wi’, studentlerdin` a`meliy jumi’slardi’ wori’nlawi’ ushi’n za`rurli metodikali’q ko`rsetpeler, paydalani’lg’an a’debiyatlar, paydalanatug`i’n u’skeneler, ma`seleni komp`yuterde shi’g`ari’w, jan`a materiallardi’ bekkemlew, tapsi’rmalardi’n` variantlari’ ha`m a`debiyatlar dizimi berilgen. Woqi’w qollanba joqari’ woqi’w wori’nlari’nda “Joqari’ matematika” kursi’n wo’tiwshi talabalar ushi’n arnalg’an.

MAZMUNI'

KİRİSİW

Ha`zirgi waqi'tlari' joqarg`i' woqi'w wori'nlari'ni'n` woqi'w processlerinde studentlerdin` joqari' matematikadan alg`an bilim da`rejesi joqari' ha`m ti'yanaqli' boli'wi' ushi'n komp`yuterden ha`m informaciyali'q texnologiyalardan paydalani'w u'lken a`hmiyetke iye. Bul studentlerimizdi lati'n grafikasi'nda jazi'lg`an, komp`yuterden ha`m informaciyali'q texnologiyalardan paydalani'wdi' u'yretetug`i'n metodikali'q woqiw qollanbalar ha`m metodikali'q ko`rsetpeler menen ta`miyinlewge tikkeley baylani'sli'.

Usi'ni'li'p ati'rg`an metodikali'q woqi'w qollanba lati'n grafikasi'nda jazi'li'p, wonda Maple integrallasqan sistemag'a qi'sqasha tu'sinik, si'zi'qli' algebra elementlerinin` 7 a`meliy jumi'slari'ni'n` wori'nlani'w ta`rtibi ha`m tapsi'rmalar berilgen. A`meliy jumi'slardag`i' sheshilgen mi'sallardi'n` Maple integrallasqan sistemada sheshiliwi ko`rsetilgen. Maple integrallasqan sistema [2] a'debiyatta ha'm si'zi'qli' algebranin` elementleri [1,3-6] a'debiyatlarda qaralg'an.

Usi' metodikali'q woqi'w qollanbada si'zi'qli' algebrani'n` elementleri yekinshi ha`m u'shinshi ta`rtipli determinantlardi' yesaplaw, determinantlardi'n` qa`siyetleri, *n*-shi ta`rtipli determinantlardi' yesaplaw usi'llari', matricalar u'stinde a`mellerdi wori'nlaw, matricani'n` rangi, keri matrica ha`m si'zi'qli' ten`lemeler sistemasi'n sheshiw temalari'ni'n` a`meliy jumi'slari' analitikali'q usi'ldan, komp`yuterden ha`m informaciyali'q texnologiyalardan paydalani'p wori'nlandi'.

Bul metodikali'q woqi'w qollanba joqari' woqi'w wori'nlari'ni'n` 1-kurs studentlerine arnalg`an.

Uli'wma aytqanda, metodikali'q qollanba joqari' matematika pa`nine qii'zi'g`i'wshi' student jaslardi'n` komp`yuterden ha`m informaciyali'q texnologiyalardan paydalani'wi'na, bilim wo`risinin` ken`eyiwine, sapali' woylawi'na, teren` pikirlewine ha`m alg`an bilimlerin qollana biliw qa`biletinin` arti'wi'na ja`rdem beredi degen u'mittemiz.

1.MAPLE İNTEGRALLASQAN SİSTEMADA PROGRAMMALASTIRIW

Maple paydalani’wshi’lardi’n’ ken’ wortali’g’i’na arnalg’an kompyuterli matematikani’n’ sistemasi. Usi’ waqi’tqa deyin wol kompyuterli algebrani’n’ sistemasi’ bolg’an. Yendi wog’an simvolli’ ha’m sanli’ yesaplawlarg’a grafikali’q vizualizaciyani’n’ qurallari’ ha’m elektron hujjetlerdi tayarlawi’ qosi’lg’an. Maple integrallasqan sistema wo’zine to’mendegilerdi biriktiredi:

- ku’shli programmalasti’ri’w tili;
- hu’jjet ha’m programmalardi’ tayarlaw ha’m redaktorlaw ushi’n redaktordin’ zamanogo’y ko’p aynali’ paydalani’wshi’ni’n’ interfeysi;
- ku’shli mag’li’wmatlar sistemasi’;
- algoritmler yadrosi’n ha’m matematikali’q an’latpalardi’ tu’rlendiriw qag’i’ydalari’;
- sanli’ ha’m simvolli’ processorlari’;
- diagnostikalar sistemasi’;
- keltirilgen ha’m qosi’msha funkciyalardi’n’ bibliotekalari’;
- islep shi’g’ari’wshi’lardi’n’ funkciyalar paketin ha’m ayi’ri’m basqa programmalasti’ri’w tillerin ha’m programmalari’n qollawi’.

Maple u’sh tildi qollaydi’: kiritiw, iske asi’ri’w ha’m programmalasti’ri’w. Kiriw tilinin’ alfaviti matematikali’q sistemada kishi lati’n ha’riplerinen a dan z ke shekem, 26 u’lken lati’n ha’riplerinen (A dan Z ke shekem), 10 arab cifrlari’nan (0 den 9 shekem) ha’m 32 ayri’qsha simvollardan (arifmetikali’q a’meller +,-,*,/,^ da’rejege ko’teriw belgisi ha’m t.b.lar).

Menshiklew belgisi retinde **:=**, a’piwayi’ ten’lik belgi = qollani’ladi’. Maple sistemasi’ altenativ simvollarg’a iye boli’wi’ mu’mkin . Bular kelesi simvollar: ^ ha’m **;**[ha’m **(**|;**]**ha’m|**)**; **{**ha’m**(*;}**ha’m ***)**. Bul sistemada ayri’qsha simvollar ha’m quramali’ belgiler qollani’ladi’:

: shi’g’ari’w yacheykasi’na yesaplaw na’tiyjesin shi’g’ari’wshi’ an’latpa fiksatori’;

; shi’g’ari’w yacheykasi’na yesaplaw na’tiyjesin beriwshi an’latpa fiksatori’;

programmali’q tu’sindiriwdin’ ko’rsetkishi;

‘ qatardi’ shegaralawshi’;

:= menshiklew operatori’;

;; bos operator;

:: wo’zgeriwshi tu’rinin’ ko’rsetkishi.

Maple da sanni’n’ na’tiyje cifrlari’ wonli’q noqattan keyin beriledi. Maple da wo’zgeriwshilerdin’ tu’rlerin ani’q ko’rsetiw ushin **name::type** du’zilisi ko’rsetiledi, bul jerde **name** wo’zgeriwshinin’ atamasi’ (identifikator), **type** – wo’zgeriwshin’ tu’ri, ma’selen pu’tin sanlar **integer**, haqi’yqi’y sanlar wo’zgermeli noqati’ menen **float** ha’m t.b.lar.

Maplede bes tiykarg’i’ operatorlardi’n’ tu’rleri bar:

binary –binarli’ operatorlar (yeki operandasi’ menen);

unary – unarli’ operatorlar (bir operanda menen);

nullary – nolarli’ operatorlar (operandasi’z –bul bir, yeki ha’m u’sh qawsi’rmalar);

precedence –u’lkenlik operatorlari’;

functional –funkcional operatorlar.

> simvoldan keyin kiritiwge **?** soraw belgisi qara fonda wornati’ladi’.

Kiritilgenler kiriw maydani’nda **Enter** klavishin basqanda kiritiw qatari’na wo’tkeriledi. Kiriw tikkeley kiriw qatari’nda yari’m qoyi’w ren’de su’wretlenedi ha’m qollani’latug’i’n an’latpani’n’ tamanlani’wi’n **;** simvoli’ menen tamamlaymi’z. **:** belgi an’latpani’n’ yesaplaw na’tiyjelerin shi’g’ari’wg’a ajrati’ladi’.

Tekstli tu’sindiriwdin’ kiritiw qatari’n ali’wshi’n a’sbaplar pa’nelindegi **T** ha’riptegi tu’yindi basi’w kerek, al matematikali’q an’latpani’n’ kiriw qatari’n ali’wshi’n **[>** simvoldag’i’ tu’yindi basi’w kerek. Tekstli bloklar ayri’qsha belgilerge iye bolmaydi’, al matematikali’q an’latpani’n’ kiriw qatari’ **>** belgige iye.

Programma **restart** buyri’g’i’ menen baslanadi’. Funcsiyalar **with(name)** so’zi ja’rdemi menen pa’ketti ju’klew dag’azalang’an son’ qollani’ladi’, bunda **name** –qollani’latug’i’n pakettin’ ati’.

Maple da 2700 shekem funkciyalar bar. Si’zi’qli’ algebrada **linalg** paketi paydalani’ladi’, wonda 100 den aslam matricali’q analizdin’ ha’m si’zi’qli’ algebrani’n’ funkciyalari’ berilgen. Solardan yen’ a’hmiyetlilerin to’mende ko’rsetemiz:

addcol –matricani’n’ bag’analari’nan si’zi’qli’ kombinacyalardi’ du’zedi;

addrow –matricani’n’ qatarlari’nan si’zi’qli’ kombinacyalardi’ du’zedi;

adj – tu’yinles matricag’a qaytaradi’;

angle – vektorlar arasi’ndag’i’ mu’yeshti yesaplaydi’;

augment – yeki yamasa ko’p matricalardi’ birgelikte gorizontal biriktiredi;

band – lentali’ matricani’ du’zedi;

basis – vektorli’ ken’islik ushi’n bazisti tabadi’;

BlockDiagonal –blok-diagonalli’ matricani’ du’zedi;

charpoly – matricani’n’ xarakteristikali’q polinomi’n qaytaradi’;

col – matricani’n’ bag’anasi’n vektorday yetip shi’g’aradi’;

coldim – matricani’n’ bag’anasi’ni’n’ wo’lshemin ani’qlaydi’;

crossprod – vektorli’q ko’beymeni yesaplaydi’;

curl –vektordi’n’ rotori’n yesaplaydi’;

delcols – matricani’n’ bag’analari’n ali’p taslaydi’;

delrows – matricani’n’ qatarlari’n ali’p taslaydi’;

det – matricani’n’ determinanti’n yesaplaydi’;

diag – blok-diagonalli’ matricani’ du’zedi;

dotprod – vektorlardi’n’ skalyar ko’beymesin yesaplaydi’;

eigenvals – matricani’n’ menshikli ma’nislerin yesaplaydi’;

eigenvects – matricani’n’ menshikli vektorlari’n yesaplaydi’;

entermatrix – matricani’ interaktivli kiriwin ta’miynleydi;

equal – yeki matricani’n’ ten’ligin ani’qlaydi’;

exponential – eksponencialli’ matricani’ du’zedi;

fibonacci –Fibonachchi matricasi’;

hermite –matricani’n’ Ermitli normal formasi’;

innerprod –vektorli’q ko’beymeni yesaplaydi’;

inverse – keri matricani’ du’zedi;

leastsqrs – yen’ kishi kvadratlar usi’li’ boyi’nsha ten’lemelerdi sheshiw;

linsolve –si’zi’qli’ ten’lemeler sistemasi’n sheshiw;

Ludecomp –LU ajrati’wdi’ a’melge asi’radi’;

matadd – matricalardi’ yamasa vektorlardi’ qosi’w;

matrix –matricani’ du’zedi;

norm – matricani’n’ yamasa vektordi’n’ normasi’;

rank – matricani’n’ rangin qaytaradi’;

singval – matricani’n’ singulyarli’ ma’nisin yesaplaydi’;

singularvals – matricani’n’ singulyarli ma’nislerin yesaplaydi’;

subvector – matricadan ko’rsetilgen vektordi’ shi’g’aradi’;

swapcol – matricada yeki bag’anani’ wo’zgertedi;

swaprow – matricada yeki qatardi’ wo’zgertedi;

trace – matricani’n’ izin qaytaradi’;

transpose –transponirlengen matricani’ yesaplaydi’;

vecdim –vektordi’n’ wo’lshemin qaytaradi’;

Linalg pakettegi barli’q funkciyalar menen Maple dag’i’ mag’li’wmatar sistemasi’ ja’rdemi menen tani’si’wg’a boladi’. **Linalg** paketinin’ bibliotekali’q fayli’nda vektorlar ha’m matricalardi’n’ beriliwi ushi’n kelesi funkciyalar bar.

vector (n,list) –list dizimde berilgenler menen berilgen **n** elementli vektor;

matrix (n,m,list) –list dizimde berilgenler menen **n** qatar sani’ ha’m **m** bag’ana sani’ndag’i’ elementleri menen berilgen matrica.

Qalegen funkciya tuwrali’ mag’li’wmat ali’w ushi’n **? name** buyri’g’i’ni’n’ ja’rdemin paydalanami’z.

№1 A`MELİY JUMIS
TEMA: YEKİNSHİ HA`M U`SHİNSHİ TA`RTİPLİ DETERMİNATLARDI YESAPLAW

1. **Jumi'sti'n' maqseti**: Yekinshi ha`m u`shinshi ta`rtipli determinantlardi' yesaplaw usi'llari'n ha'm wolardi' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.
2. **Ma`selenin` qoyi'li'wi':** Yekinshi ha`m u`shinshi ta`rtipli determinantlardi' yesaplaw.
3. **Paydalani'lg`an a`debiyatlar**: [1-6]
4. **Paydalani'lg`an u`skeneler**: kompyuter
5. **Metodikali'q ko`rsetpeler**:

1-Ani'qlama. *2 - shi ta`rtipli determinant*

$$\Delta = \begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \qquad (1)$$

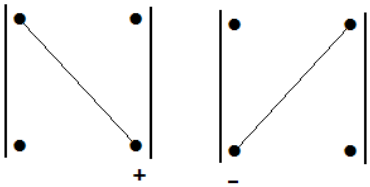
(1) ko`riniste beriledi.

a_1, a_2, b_1, b_2 sanlari' determinantti'n` elementleri dep ataladi. Wolar yeki qatarlarda ha`m yeki bag`analarda jaylasqan.

Determinantti' si'zi'qli' tu`rlendiriwdin` aniqlawshi'si' depte aytadi'. Bizler woni' aldag`i' waqi'tlari' determinant dep aytami'z.

(1) formula yekinshi ta`rtipli determinantti'n` «jayi'li'w» qag`i'ydasi'n beredi yekinshi ta`rtipli determinant, wonin` 1 - shi ha`m 2 - shi diagonallari' elementlerinin` ko`beymelerinin` ayi'rmasi'na ten`.

(1) formulani'n' won' ta'repindegi ko'beymelerdin' belgileri to'mendegi sxemadag'i' ko'rsetilgenindey qoyi'ladi':



2-Aniqlama. *3 – shi ta`rtipli determinantlar dep*

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \qquad (2)$$

an`latpag`a ayti'ladi'.

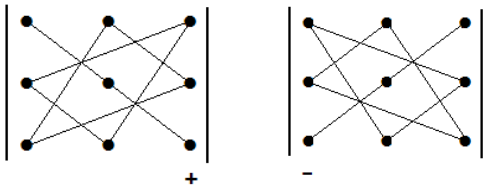
$a_i, b_i, c_i (i = 1,2,3)$ sanlarg`a determinantti'n` elementleri dep ayti'ladi'. Wolar u`sh qatar ha`m u`sh bag`anada jaylasqan.

(2) ni ashsaq

$$\Delta = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_1 b_3 c_2 - a_2 b_1 c_3 - a_3 b_2 c_1` \qquad (3)$$

alami'z, wonda u`shewi «+» belgi menen, al qalg`an u`shewi «–» ali'nadi'.

(3) formulanin' won' ta'repindegi qosi'li'wshi'lardi'n' belgilerin to'mendegi sxemadan paydalani'p ani'qlaw qolay:



(3) formuladag'i' u'sh ag'zalardan biri 3 – shi ta`rtipli determinantti'n' tiykarg'i' diagonali' elementlerinin' ko'beymesinen, qalg'an yeki ag'za bolsa tiykarg'i' diagonalg'a parallellerde jaylasqan ha'm wog'an qarsi' mu'yeshtegi elementlerdin' ko'beymesinen turadi'.

(3) formuladag'i' u'sh teris ag'zalari' yekinshi diagonalg'a sali'sti'rg'anda tap usi'nday usi'lda tabi'ladi'. Determinati' yesaplawdi'n' bul usi'li' u'shmu'yeshlik usi'li' delinedi.

Mi'sal. To'mendegi u'shinishi ta'rtipli determinantti' yesaplan'.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \qquad (4)$$

Sheshimi. (3) formulag'a muwapi'q to'mendegige iye bolami'z:

$$\Delta = 1\cdot 3\cdot 5 + 2\cdot 4\cdot 3 + 3\cdot 2\cdot 4 - 3\cdot 3\cdot 3 - 1\cdot 4\cdot 4 - 2\cdot 2\cdot 5 =$$
$$= 15 + 24 + 24 - 27 - 16 - 20 = 63 - 63 = 0$$

6. Kompyuterden paydalani'w:

> restart;

restart bu`yrigin xar bir programmada worinlatiw u`sinis yetiledi

> with(linalg);

Warning, the protected names norm and trace have been redefined and unprotected

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

> A:=Matrix([[1,2,3],[2,3,4],[3,4,5]]);

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

> B:=det(A);

$B := 0$

>

7. Jan’a materiallardi’ bekkemlew: Yesaplar shi’g’ari’w ha’m wolardi’ kompyuterdegi na’tiyje menen sali’sti’ri’w.

9. U’yge tapsi’rmalar:

TAPSIRMALAR

Determinatlardi’ yesaplan`

1. $\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}$
2. $\begin{vmatrix} 2 & 3 \\ 6 & -10 \end{vmatrix}$
3. $\begin{vmatrix} 3 & -2 \\ -4 & 5 \end{vmatrix}$
4. $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$
5. $\begin{vmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{vmatrix}$
6. $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$
7. $\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$
8. $\begin{vmatrix} tg\,\alpha & -1 \\ 1 & tg\,\alpha \end{vmatrix}$
9. $\begin{vmatrix} 1+\sqrt{2} & 2-\sqrt{5} \\ 2+\sqrt{5} & 1-\sqrt{2} \end{vmatrix}$
10. $\begin{vmatrix} \log_b a & 1 \\ 1 & \log_a b \end{vmatrix}$
11. $\begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix}$
12. $\begin{vmatrix} 3 & 2 \\ 8 & 5 \end{vmatrix}$
13. $\begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix}$
14. $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$
15. $\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$
16. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$
17. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$
18. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$
19. $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$
20. $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$
21. $\begin{vmatrix} 4 & 2 & -1 \\ 5 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}$
22. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$
23. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$
24. $\begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}$
25. $\begin{vmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{vmatrix}$
26. $\begin{vmatrix} 1 & 2 & 3 \\ 5 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}$
27. $\begin{vmatrix} -1 & 5 & 4 \\ 3 & -2 & 0 \\ -1 & 3 & 6 \end{vmatrix}$
28. $\begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix}$
29. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Juwabi':

1. 26	**2.** -38	**3.** 7	**4.** $2a$	**5.** 1
6. 5	**7.** 5	**8.** $1/\cos^2\alpha$	**9.** 0	**10.** 0
11. -1	**12.** -1	**13.** 0	**14.** 0	**15.** -1
16. 40	**17.** -3	**18.** 100	**19.** -5	**20.** 0
21. 1	**22.** 1	**23.** 2	**24.** 4	**25.** $2a^2(a+x)$
26. -8	**27.** -50	**28.** 16	**29.** $3abc - a^3 - b^3 - c^3$	

№2 A'MELİY JUMIS
TEMA: DETERMİNATLARDIN` QA`SİYETLERİ

1. U`yge berilgen tapsi'rmalardi' tekseriw.
**2. Jumi'sti'n` maqseti**: Determinantlardi' yesaplawda determiantlardin` qa`siyetlerin paydalani'wdi' ha'm wolardi' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.
3. Ma`selenin` qoyi'li'wi': Determinantlardi woni'n` qa`siyetleri boyi'nsha yesaplaw.
4. Paydalani'lg`an a`debiyatlar: [1-6].
5. Paydalani'lg`an u`skeneler: komp`yuter.
6. Metodikali'q ko`rsetpeler:

1-Aniqlama. *Berilgen determinant* d_{ij} *elementinin` minori' dep, bu`l element jaylasqan i-qatari' ha`m j-bag'anasi' bolg`an elementlerin wo`shiriwden qalg`an elementlerden du`zilgen ta`rtibi birge kemeygen determinantqa ayti'ladi' ha`m* M_{ij} *siyaqli' belgilenedi.*

Mi'sal. $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ c_2 elementinin` minori' $\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} = M_{23}$

2-Aniqlama. *Berilgen determinantti'n` $a_{ij}$ elementinin` algebrali'q toi'qlti'ri'wshi'si' dep* $Aij = (-1)^{i+j} M_{ij}$ *sang`a ayti'ladi'.*

Bul jerde i-element jaylasqan qatar nomeri, j-bag`ana nomeri, $M_{ij}$-elementtin` minori'. Determinantlardi'n` ayi'ri'm qa`siyetlerin ko`rip shig`ayi'q.

Usi

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + b_1c_2a_3 + c_1b_3a_2 - c_1b_2a_3 - b_1a_2c_3 - a_1c_2b_3 \quad (1)$$

3-shi ta`rtipli determinantti' ko`rip shi'g`ami'z:

**1-Qa`siyet.** *Determinantti'n` barli'q qatarlari' ha`m bag`analari' almasti'ri'lsa, woni'n` ma`nisi wo`zgermeydi, yag`ni'y*

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

boladi'.

**Da`lillew.** Won` ta`repindegi determinantti' yesaplap, bug`an iseniw mu`mkin.

$$a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - b_1a_2c_3 - a_1c_2b_3 = \Delta.$$

**<u>2-Qa`siyet.</u>** *Determinantti'n` yeki parallel qatarlari' almasti'ri'lsa, woni'n` ma`nisinin` absolyut mug`dari' saqlani'p, al belgisi kerige wo`zgeredi.*

Meyli,

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \tag{2}$$

nin` birinshi ha`m yekinshi qatari'n almasti'rami'z, sonda

$$\tilde{\Delta} = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \tag{3}$$

ni' alami'z.

$$\tilde{\Delta} = a_1(-A_1) + b_1(-B_1) + c_1(-C_1) = -\Delta$$

<u>Saldar-1</u>. *Determinantti'n` yeki parallel qatari' birdey bolsa, wonda wol nol`ge ten`.*

Meyli,

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

bolsi'n.

Almasti'rg`anli'qtan determinat $-\Delta$ g`a ten`, biraq determinant wo`zgermeydi. Sonli'qtan $-\Delta = \Delta$ ha`m u`si'nnan $\Delta = 0$.

<u>Saldar-2.</u> *Determinantti'n` qanday da bir qatar elementlerin parallel` qatardi'n` sa`ykes elementlerinin` algebrali'q toli'qti'ri'wshi'lari'na ko`beytkendegi qosi'ndi'si' nol`ge ten`, yag`ni'y*

$$\left.\begin{aligned} a_1A_2 + b_1B_2 + c_1C_2 = 0 \\ a_1A_3 + b_1B_3 + c_1C_3 = 0 \end{aligned}\right\} \quad (4)$$

Ja`ne

$$\left.\begin{aligned} a_1B_1 + a_2B_2 + a_3B_3 = 0 \\ a_1C_1 + a_2C_2 + a_3C_3 = 0 \end{aligned}\right\} \quad (5)$$

ha`m t.b.lar (barli'g`i' 12)

(4) ha`m (5) formuladag`i' an`latpalar yeki birdey parallel qatardan turadi, sonli'qtan nol`ge ten`.

Mi'sal.

$$a_1A_2 + b_1B_2 + c_1C_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

**3-Qa`siyet.** *Determinantti'n` qanday da bir qatar elementlerinin` uli'wma ko`beytiwshisin determinant belgisi aldi'na shi'g`ari'w mu`mkin.*

$$\begin{vmatrix} ka_1 & ka_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Saldar-3. *Yeger determinantti'n` qandayda bir qatari'ni'n` barli'q elementleri nolge ten` bolsa, wonda determinant nolge ten`.*

Saldar-4. *Yeger determinantti'n` qanday da bir qatari'n'in` elementleri woni'n` parallel qatari'ni'n` sa`ykes elementlerine proporcional bolsa, wonda determinant nol`ge ten`.*

Mi'sal.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ ka_1 & kb_1 & kc_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

**4-Qa`siyet.** *Determiantti'n` bir qatari'ni'n' (yaki bag`ana) barli'q elementlerin wolardi'n` algebrali'q toli'qti'ri'wshi'lari'na ko`beytip, na`tiyjeler qosi'lsa, qosi'ndi' determinantqa ten` boladi', yag`ni'y*

$$\Delta=(-1)^{3+1}a_3\begin{vmatrix}b_1 & c_1\\ b_2 & c_2\end{vmatrix}+(-1)^{3+2}b_3\begin{vmatrix}a_1 & c_1\\ a_2 & c_3\end{vmatrix}+(-1)^{3+3}c_3\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$$

**5-Qa`siyet.** *Yeger determinantti'n` bir qatari'ndag'i' (bag`anasi'ndag`i') elementler yeki qosi'li'wshi'lardan ibarat bolsa, wonda determinantti' yeki determinant qosi'ndi'si' ko`rinisinde jazi'w mu`mkin, yag`ni'y*

$$\begin{vmatrix}a_1+k_1 & b_1 & c_1\\ a_2+k_2 & b_2 & c_2\\ a_3+k_3 & b_3 & c_3\end{vmatrix}=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}+\begin{vmatrix}k_1 & b_1 & c_1\\ k_2 & b_2 & c_2\\ k_3 & b_3 & c_3\end{vmatrix}.$$

Saldar-5. *Determinantti'n` mug`dari' wo`zgermeydi, yegerde qanday da bir qatardi'n` elementlerine proporcional parallel qatardi'n` sa`ykes elementlerin proporcional koefficienttegi sandi' qossaq (yamasa alsaq). (Determinantti'n` elementtar tu`rleniwi delinedi).*

Meyli,

$$\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$$

bolsi'n.

Mi'sali ushi'n $\tilde{\Delta}=\begin{vmatrix}a_1\pm ka_2 & b_1\pm kb_2, & c_1\pm kc_2\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$ qarayi'q. 5 ha`m 3-qa`siyetler boyi'nsha

$$\tilde{\Delta}=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}\pm k\begin{vmatrix}a_2 & b_2 & c_2\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=\Delta\pm k\cdot 0=\Delta.$$

Mi'sal. $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$ simmetriyali' determinantti' yesaplan'.

Sheshimi. 2-qatardan 1-qatardi'n` 2 ge ko`beytkenin, al 3-qatardan 1-qatardi'n` 3ge ko`beytkenin alsaq,

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -4 \\ 0 & -4 & -8 \end{vmatrix} = 1 \cdot \begin{vmatrix} -3 & -4 \\ -4 & -8 \end{vmatrix} = 24 - 16 = 8$$

di alami'z.

7**. Kompyuterden paydalani'w:**

```
> restart;
```

restart buyri'g'i'n xa'r bir programmada wori'nlati'w usi'ni's yetiledi

```
> with(linalg):
> A:=Matrix([[1,2,3],[0,-3,-4],[0,-4,-8]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -4 \\ 0 & -4 & -8 \end{bmatrix}$$

```
> B:=det(A);
```

$$B := 8$$

```
>
```

8. Jan'a materyallardi' bekkemlew: Yesaplar shi'g'ari'w ha'm wolardi' kompyuterdegi na'tiyjesi menen sali'sti'ri'w.

9. U'yge tapsi'rmalar:

TAPSIRMALAR

Determinantti'n` qa`siyetleri boyi'nsha yesaplan`

1. $\begin{vmatrix} 3 & 5 & 7 & 2 \\ 1 & 2 & 3 & 4 \\ -2 & -3 & 3 & 2 \\ 1 & 3 & 5 & 4 \end{vmatrix}$

2. $\begin{vmatrix} 1 & 3 & 0 & 0 & 0 \\ 3 & 2 & 3 & 0 & 0 \\ 0 & 4 & 3 & 4 & 0 \\ 0 & 0 & 5 & 4 & 5 \\ 0 & 0 & 0 & 6 & 5 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix}$

4. $\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$

5. $\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$

6. $\begin{vmatrix} 10 & 2 & 0 & 0 & 0 \\ 12 & 10 & 2 & 0 & 0 \\ 0 & 12 & 10 & 2 & 0 \\ 0 & 0 & 12 & 10 & 2 \\ 0 & 0 & 0 & 12 & 10 \end{vmatrix}$

7. $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$

8. $\begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix}$

9. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$

10. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

11. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \end{vmatrix}$

12. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix}$

13. $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$

14. $\begin{vmatrix} 5 & 6 & 0 & 0 & 0 \\ 1 & 5 & 6 & 0 & 0 \\ 0 & 1 & 5 & 6 & 0 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 5 \end{vmatrix}$

15. $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a & b \\ 1 & a & 0 & c \\ 1 & b & c & 0 \end{vmatrix}$

16. $\begin{vmatrix} b_1 & b_2 & 0 & 0 & 0 \\ b_3 & b_3 & b_4 & 0 & 0 \\ b_5 & b_5 & b_5 & b_6 & 0 \\ b_7 & b_7 & b_7 & b_7 & b_8 \\ b_9 & b_9 & b_9 & b_9 & b_{10} \end{vmatrix}$

17. $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

18. $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$

19. $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$

20. $\begin{vmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{vmatrix}$

21. $\begin{vmatrix} x & 0 & -1 & 1 & 0 \\ 1 & x & -1 & 1 & 0 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix}$

22. $\begin{vmatrix} 1 & x & x^2 & x^3 \\ x^2 & x^2 & x & 1 \\ 1 & 2x & 3x^2 & 4x^3 \\ 4x^3 & 3x & 2x & 1 \end{vmatrix}$

23. $\begin{vmatrix} 3 & 6 & 5 & 6 & 4 \\ 5 & 9 & 7 & 8 & 6 \\ 6 & 12 & 13 & 9 & 7 \\ 4 & 6 & 6 & 5 & 4 \\ 2 & 5 & 4 & 5 & 3 \end{vmatrix}$

24. $\begin{vmatrix} 35 & 59 & 71 & 52 \\ 42 & 70 & 77 & 54 \\ 43 & 68 & 72 & 52 \\ 29 & 49 & 65 & 50 \end{vmatrix}$

25. $\begin{vmatrix} 27 & 44 & 40 & 55 \\ 20 & 64 & 21 & 40 \\ 13 & -20 & -13 & 24 \\ 46 & 45 & -55 & 84 \end{vmatrix}$

26. $\begin{vmatrix} 24 & 11 & 13 & 17 & 19 \\ 51 & 13 & 32 & 40 & 46 \\ 61 & 11 & 14 & 50 & 56 \\ 62 & 20 & 7 & 13 & 52 \\ 80 & 24 & 45 & 57 & 70 \end{vmatrix}$

27. $\begin{vmatrix} \frac{3}{2} & -\frac{9}{2} & -\frac{3}{2} & -3 \\ \frac{5}{3} & -\frac{8}{3} & -\frac{2}{3} & -\frac{7}{3} \\ \frac{4}{3} & -\frac{5}{3} & -1 & -\frac{2}{3} \\ 7 & -8 & -4 & -5 \end{vmatrix}$

28. $\begin{vmatrix} \frac{3}{4} & 2 & -\frac{1}{2} & -5 \\ 1 & -2 & -\frac{3}{2} & 8 \\ \frac{5}{6} & -\frac{4}{3} & \frac{4}{3} & -\frac{14}{3} \\ \frac{2}{5} & -\frac{4}{5} & \frac{1}{2} & \frac{12}{5} \end{vmatrix}$

29. $\begin{vmatrix} \sqrt{2} & \sqrt{3} & \sqrt{5} & \sqrt{3} \\ \sqrt{6} & \sqrt{21} & \sqrt{10} & -2\sqrt{3} \\ \sqrt{10} & 2\sqrt{15} & 5 & \sqrt{6} \\ 2 & 2\sqrt{6} & \sqrt{10} & \sqrt{15} \end{vmatrix}$

30. $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 7 & 10 & 13 \\ 3 & 5 & 11 & 16 & 21 \\ 2 & -7 & 7 & 7 & 2 \\ 1 & 4 & 5 & 3 & 10 \end{vmatrix}$

Juwabi'.

1. 70

2. 640

3. $(b-a)(c-a)(d-a)\times(c-b)(d-b)(d-c)$

4. 900

5. 12

6. 21280

7. $a^2\ b^2$

8. 48

9. 1

10. 160

11. 12

12. 900

13. 394

14. 665

15. $a^2+b^2+c^2-\\-2(bc+ca+ab)$

16. $(b_1-b_2)(b_3-b_4)\times\\\times(b_5-b_6)(b_7b_{10}-b_8b_9)$

17. $-2(x^3+y^3)$

18. x^2z^2

19. $(af-be+cd)^2$

20. $(a+b+c+d)(a+b+c+d)\times\\(a-b+c-d)(a-b-c+d)$

21. $(x+1)\left(x^2-x+1\right)^2$

22. $x^2\left(x^2-1\right)^4$

23. 5

24. 10

25. 1

26. 100

27. 1

28. 1

29. $9\sqrt{10}\left(\sqrt{3}-\sqrt{2}\right)$

30. 52

№3 A`MELİY JUMIS
TEMA: n-SHİ TA`RTİPLİ DETERMİNANTLARDI YESAPLAW USILLARI

1. U`yge berilgen tapsi'rmalardi' tekseriw

**2. Jumi'sti'n` maqseti** : $n$-shi ta`rtipli determinantlardi' yesaplawdi' an'satlas - ratug`i'n ha`r qi'yli' yesaplaw usi'llari'n ha'm wolardi' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.

3. Ma`selenin` qoyi'li'wi': n-shi ta`rtipli determinantlardi' ha`r qi'yli' yesaplaw usi'llari' menen yesaplan`.

4. Paydalani'lg`an a`debiyatlar:[1-6].

5. Paydalani'lg`an u`skeneler: komp`yuter.

6. Metodikali'q ko`rsetpeler:

Anaw ya mi'naw ayri'qsha tu`rdegi determinatlarg`a ju`da` a`piwayi' an`latpag`a keletu`g`i'n ha`r qi'yli' yesaplaw usi'llari' qollani'ladi'. Solardi' to`mende qaraymi'z:

U`shmu`yeshlik ko`rinisine keltiriw u`si'li'

Bul usi'l determinantti' tu`rlendiriw arqali' diagonaldi'n` bir ta`repinde jatqan barliq elementleri nolge ten`.

1 –Mi'sal. n-shi ta`rtipli determinantti' yesaplan`.

$$D=\begin{vmatrix} 1 & 1 & 1....1 \\ 1 & 0 & 1...\ 0 \\ 1 & 1 & 0....1 \\ ..... & & \\ 1 & 1 & 1....0 \end{vmatrix}$$

Sheshimi. Birinshi qatarda yesaplaymi'z:

$$D=\begin{vmatrix} 1 & 1 & 1..... & 1 \\ 0 & -1 & 0.... & 0 \\ 0 & 0 & -1.... & 0 \\ ..... & & & \\ 0 & 0 & 0....- & 1 \end{vmatrix}=(-1)^{n-1}.$$

2-Mi'sal. Determinantti' yesaplan`.

$$D=\begin{vmatrix} a_1 & x & x \;.....\; x \\ x & a_2 & x \;.... .x \\ x & x & a_3 ......x \\ \multicolumn{3}{c}{......................} \\ x & x & x......a_n \end{vmatrix}$$

Sheshimi. Birinshi qatardi' qalg\`anlari'nan alami'z

$$D=\begin{vmatrix} a_1 & x & x & .... & x \\ x-a_1 & a_2-x & 0 & .... & 0 \\ x-a_1 & 0 & a_3-x & .... & 0 \\ \multicolumn{5}{c}{...} \\ x-a_1 & 0 & 0 & .... & a_n-x \end{vmatrix}$$

Birinshi bag\`anadan $a_1-x$-ti', yekinshi bag\`anadan a_2-x-ti', ..., n-shi bag\`anadan $a_n-x$ ti shi'g\`arami'z.

$$D=(a_1-x)(a_2-x)...(a_n-x)\begin{vmatrix} \frac{a_1}{a_1-x} & \frac{x}{a_2-x} & \frac{x}{a_3-x} & .... & \frac{x}{a_n-x} \\ -1 & 1 & 0 & ..... & 0 \\ -1 & 0 & 1 & ..... & 0 \\ \multicolumn{5}{c}{..} \\ -1 & 0 & 0 & ..... & 1 \end{vmatrix}$$

$\frac{a_1}{a_1-x}=1+\frac{x}{a_1-x}$ ti' qoyami'z ha\`m barli'q bag\`analardi' birge qosami'z:

$$D=(a_1-x)(a_2-x)...(a_n-x)\times$$

$$\times\begin{vmatrix} 1+\frac{x}{a_1-x}+...+\frac{x}{a_n-x} & \frac{x}{a_2-x} & \frac{x}{a_3-x} & ... & \frac{x}{a_n-x} \\ 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ \multicolumn{5}{c}{..} \\ 0 & 0 & 0 & ... & 1 \end{vmatrix}=$$

$$=x(a_1-x)(a_2-x)...(a_n-x)\left(\frac{1}{x}+\frac{1}{a_1-x}+\frac{1}{a_2-x}+...+\frac{1}{a_n-x}\right)$$

Si’zi’qli’ ko`beytiwshilerge aji’rati’w u`si’li’

Determinantti’ bir yamasa bir neshe kiriwshi ha`riplerdin` ko`p ag`zali’si’ dep qaraymi’z. Woni’ tu`rlendire woti’ri’p, wol bir qatar si’zi’qli’ ko`beytiwshilerge bo`linetug`i’ni’n tabami’z, demek (bular wo`z-ara a`piwayi’ ko`beytiwshiler) wolardi’n`ko`beytiwshilerine bo`linedi.

3-misal. Determinantti’ yesaplan`.

$$D=\begin{vmatrix} 0 & x & y & z \\ x & 0 & z & y \\ y & z & 0 & x \\ z & y & x & 0 \end{vmatrix}$$

Sheshimi. Yeger birinshi bag`anag`a qalg`an bag`analardi qossaq, wonda determinant $x+y+z$ ge bo`linedi; yeger birinshi bag`anag`a yekinshini qosi’p, u`shinshi ha`m to`rtinshini alsaq, wonda $y+z-x$ ko`beytiwshi ajraladi’; yeger birinshi bag`anag`a u`shinshini qossaq, yekinshi ha`m to`rtinshini alsaq, wonda $x-y+z$ ko`beytiwshi ajraladi’ aqi’ri’nda, yegerde birinshi bag`anag`a to`rtinshini qosip, yekinshi ha`m u`shinshini alsaq, wonda $x+y-z$ ko`beytiwshi ajraladi’. $x,y,z$ g`a`rezsiz wo`zgeriwshiler dep yesaplasaq, bul to`rt ko`beytiwshilerdin` barli’g`i’n wo`z-ara a`piwayi’ boladi’ ha`m demek, determinatlar

$$(x+y+z)(y+z-x)(x-y+z)(x+y-z).$$

Bu`l ko`beyme z^4 ag`zasi’ –1 koefficient penen, al determinantta $z^4$ ag`za +1 koeffitsient penen boladi’. Demek,

$$D=-(x+y+z)(y+z-x)(x+z-y)(x+y-z)=$$
$$=x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2$$

4-Mi’sal. n-shi ta`rtipli Vandermond determinanti’n si’zi’qli’ ko`beytiwshilerge aji’rati’w u`si’li’ menen yesaplaw.

$$D_n = \begin{vmatrix} 1 & x_1 & x_1^2 \ldots x_1^{n-1} \\ 1 & x_2 & x_2^2 \ldots x_2^{n-1} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 1 & x_n & x_n^2 \ldots x_n^{n-1} \end{vmatrix}$$

$x_1,\ldots..,x_{n-1}$ lerge baylani'sli' koefficientler menen bir belgisizli x_n ag`zali' $D_n$ qaraymiz, wol $x_n = x_1, x_n = x_2,\ldots.,x_n = x_{n-1}$ de nolge aylanadi' ha`m sonli'qtan $x_n - x_1, x_n - x_2,\ldots.,x_n - x_{n-1}$ lerge bo`linedi.

Barli'q bu`l ko`beytiwshiler wo`z-ara apiwayi' (sebebi $x_1, x_2,\ldots,x_n$ algebrali'q g`a`rezsiz) Demek, $D_n$ wolardin` ko`beymesine bo`linedi, yag`ni'y

$$D_n = q(x_1, x_2,\ldots,x_n)(x_n - x_1)(x_n - x_2)\ldots(x_n - x_{n-1})$$

D_n di aqi'rg`i' qatar boyi'nsha jaysaq wol $x_n$ ge sali'sti'rg`anda $n=1$ da`rejeli ko`p ag`zali' boladi', sebebi koefficient $x_n^{n-1}$ de $x_1, x_2,\ldots,x_{n-1}$ belgisizlerden $D_{n-1}$ Vandermond determinanti'na ten`, tap sonday-aq aqi'rg`i' ten`liktin` won` bo`limindegi ko`beymesinin` qawsi'rmasi' $x_n^{n-1}$ den bir koefficient penen tursa, wonda $q(x_1, x_2,\ldots,x_n)$ ko`p ag`zali' $x_n$ di almaydi' ha`m x_n^{n-1} din` koefficientlerin sali'sti'ri'p, ten`liktin` yeki jag`i'ndag`i'si'n sali'sti'ri'p $D_{n-1} = q(x_1, x_2,\ldots,x_{n-1})$-di alami'z, sonli'qtan $D_n = D_{n-1}(x_n - x_1)(x_n - x_2)\ldots(x_n - x_{n-1})$. Bul ten`likte n-di $n-1$ ge almasti'ri'wg`a qollansaq,

$$D_{n-1} = D_{n-2}(x_{n-1} - x_1)\ldots(x_{n-1} - x_{n-2})$$

ge iye bolami'z.

Bu`l an`latpadag`i' $D_{n-1}$ di aldi'ng`i' D_n an`latpag`a qoyami'z. Bul ayti'mdi' qaytalap, aqi'ri'nda $x_2 - x_1$ ko`beytiwshini aji'ratami'z, son` birinshi ta`rtipli *Vandermond determinanti'na* $D_1 = 1$ ge kelemiz.

Solay yetip,

$$D_n = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)...(x_n - x_1)(x_n - x_2)...(x_n - x_{n-1}) = \prod_{i>j} (x_i - x_j)$$

Rekurrentli (rekursivli yamasa qayti'mli') qatnaslar u`si'li'

Bul u`si'lda berilgen determinantti' woni'n` qatari' yamasa bag`anasi' boyi'nsha tu`rlendirip ha`m tarqati'p sol tu`rdegi determinant arqali' to`men ta`rtipte an`lati'ladi' dep ju`wmaq shi'g`ari'wg`a boladi'. Ali'ng`an ten`lik reku`rrent qatnas delinedi. Sonnan keyin, determinantti'n` uli'wma ko`rinisi boyi'nsha tikkeley qansha to`men ta`rtiptegi determinant bolsa, sonshelli wolar rekurrent qatnasti'n' won` bo`leginde boladi'. Joqari'raq ta`rtiptegi determinant izbe-iz rekurrent qatnastan yesaplanadi'. Reku`rrentli qatnaslar usi'li' menen determinantti' yesaplaw mi'sali'nan buri'n, woni'n` bir dara jag`dayi'n qaraymi'z. Bu`l jerde rekurrent qatnas ma`seleni sheshiw ushi'n algoritmdi beredi. Meyli rekurrent qatnas

$$D_n = pD_{n-1} + qD_{n-2}, n > 2 \quad (1)$$

ko`riniske iye bolsi'n, bul jerde $p, q$ turaqli'lar, yag`ni'y n mug`dardan baylani'sli' yemes, $q = 0$ de $D_n$ geometriyali'q progressiyani'n` ag`zasi'nday yesaplanadi' $D_n = p^{n-1}D_1$, bu`l jerde $D_1 - 1$ ta`rtipli determinantti'n` berilgen tu`ri, yag`ni'y D_n determinantti'n` sol joqarg`i' mu`yeshinde turi'wshi' elementi.

Meyli $q \neq 0$ ha`m $\alpha, \beta$-lar $x^2 - px - q = 0$ kvadrat ten`lemenin` korenleri bolsi'n. Sonda $p = \alpha + \beta,\ q = -\alpha\beta$ ha`m (1) ten`likti to`mendegishe jazi'w mu`mkin:

$$D_n - \beta D_{n-1} = \alpha\left(D_{n-1} - \beta D_{n-2}\right) \quad (2)$$

yamasa

$$D_n - \alpha D_{n-1} = \beta\left(D_{n-1} - \alpha D_{n-2}\right) \quad (3)$$

Da`slep $\alpha \neq \beta$ dep uyg`arami'z. Geometriyali'q progressiyani'n` $(n-1)$ ag`zasi'ni'n` formulasi' boyi'nsha (2) ha`m (3) ten`liklerden

$$D_n - \beta D_{n-1} = \alpha^{n-2}(D_2 - \beta D_1)$$

ha`m

$$D_n - \alpha D_{n-1} = \beta^{n-2}(D_2 - \alpha D_1)$$

lerdi tabami'z, bunnan

$$D_n = \frac{\alpha^{n-1}(D_2 - \beta D_1) - \beta^{n-1}(D_2 - \alpha D_1)}{\alpha - \beta}$$

Yamasa

$$D_n = C_1\alpha^n + C_2\beta^n,$$

bul jerde

$$C_1 = \frac{D_2 - \beta D_1}{\alpha(\alpha - \beta)},\ C_2 = -\frac{D_2 - \alpha D_1}{\beta(\alpha - \beta)} \tag{4}$$

Aqi'rg`i' an`latpa D_n ushi'n an`sat yadta saqlanadi'. Wol $n > 2$ ushi'n shi'g`ari'ldi', biraq $n = 1$ ha`m $n = 2$ ushi'n tikkelley tekseriledi. $C_1$ ha`m C_2 ma`nisleri (4) keltirilgen an`latpadan tappay, al $D_1 = C_1\alpha + C_2\beta$ $D_2 = C_1\alpha^2 + C_2\beta^2$ baslang`ish sha'rtlerden tabi'ladi'.

Meyli $\alpha = \beta$ bolsi'n. (2) ha`m (3) ten`lemeler

$$D_n - \alpha D_{n-1} = \alpha(D_{n-1} - \alpha D_{n-2})$$

aylanadi', bunnan

$$D_n - \alpha D_{n-1} = A\alpha^{n-2}, \tag{5}$$

bu`l jerde

$$A = D_2 - \alpha D_1.$$

Bu`nda $n$di $n-1$ ge almasti'ri'p $D_{n-1} - \alpha D_{n-2} = A\alpha^{n-3}$ alami'z, bunnan

$$D_{n-1} = \alpha D_{n-2} + A\alpha^{n-3}$$

Bul an`latpani' (5) ke qoyi'p, $D_n - \alpha^2 D_{n-2} + 2A\alpha^{n-2}$ tabami'z.

Bul usi'ldi' bir neshe ma`rte qaytalap $D_n - \alpha^{n-1}D_1 + (n-1)A\alpha^{n-2}$ yamasa $D_n = \alpha^n\left[(n-1)C_1 + C_2\right]$ alami'z, bu`l jerde $C_1 = \frac{A}{\alpha^2}$, $C_2 = \frac{D_1}{\alpha}$ (bu`nda $\alpha \neq 0$, sebebi $q \neq 0$) .

5-Mi'sal. 2-mi'saldag`i' determinantti' rekurrentli qatnaslar usi'li' menen yesaplan`.

Sheshimi. D_n determinanttag`i' won` to`mendegi mu`yeshtegi elementti $a_n = x + (a_n - x)$ ko`riniste ali'p, D_n determinantti' yeki determinantlar qosi'ndi'si'na aji'ratami'z:

$$D_n = \begin{vmatrix} a_1 & x & \dots & x & x \\ x & a_2 & \dots & x & x \\ \dots & \dots & \dots & \dots & \dots \\ x & x & \dots & a_{n-1} & x \\ x & x & \dots & x & x \end{vmatrix} + \begin{vmatrix} a_1 & x & \dots & x & 0 \\ x & a_2 & \dots & x & 0 \\ \dots & \dots & \dots & \dots & \dots \\ x & x & \dots & a_{n-1} & 0 \\ x & x & \dots & x & a_n - x \end{vmatrix}.$$

Birinshi determinantta aqi'rg`i' bag`anani' qalg`anlari'nan alami'z, al yekinshi determinantti' aqi'rg`i' bag`ana boyi'nsha jayami'z

$$D_n = x(a_1 - x)(a_2 - x)\dots(a_{n-1} - x) + (a_n - x)D_{n-1}$$

Bul rekurrent qatnas boladi'. Bug`an $D_{n-1}$ ushi'n uqsas an`latpani' qoyi'p

$$D_n = x(a_1 - x)(a_2 - x)\dots(a_{n-1} - x) + x(a_1 - x)(a_2 - x)\dots(a_{n-2} - x)(a_n - x) + \\ + D_{n-2}(a_{n-1} - x)(a_n - x)$$

alami'z.

Pikirdi $n-1$ ma`rte qaytalap ha`m $D_1 = a_1 = x + (a_1 - x)$ yekenligin yeske ali'p,

$$D_n = x(a_1 - x)(a_2 - x)...(a_{n-1} - x) + x(a_1 - x)...(a_{n-2} - x)(a_n - x) + ...$$
$$+ x(a_2 - x)...(a_n - x) + (a_1 - x)(a_2 - x)...(a_n - x) =$$
$$= x(a_1 - x)(a_2 - x)...(a_n - x)\left(\frac{1}{x} + \frac{1}{a_1 - x} + ... + \frac{1}{a_n - x}\right)$$

alami'z. Na`tiyje 2 –mi'saldag`i' menen sa`ykes keledi.

6-Mi'sal.

$$D_n = \begin{vmatrix} 5 & 3 & 0 & 0 & ... & 0 & 0 \\ 2 & 5 & 3 & 0 & ... & 0 & 0 \\ 0 & 2 & 5 & 3 & ... & 0 & 0 \\ & & & ..... & & & \\ 0 & 0 & 0 & 0 & ... & 2 & 5 \end{vmatrix}$$

n-ta`rtipli determinantti' yesaplan`.

Sheshimi. Birinshi qatar boyi'nsha jayi'p, $D_n = 5D_{n-1} - 6D_{n-2}$ rekurrent qatnasti' tabami'z. $x^2 - 5x + 6 = 0$ ten`leme $\alpha = 2$ $\alpha = 3$ koren`lerge iye. (4) formula boyi'nsha $D_n = C_1\alpha^n + C_2\beta^n = 3^{n+1} - 2^{n+1}$.

Determinantti'n` determinantlar qosi'ndi'si' ko`rinisinde beriliw usi'li'

Gey bir determinantlar sol ta`rtiptegi qatarlarg`a (yamasa bag`analarg`a) sali'sti'rg`anda wolar determinantlardi'n` qosi'ndi'si'na aji'rali'w usi'li' menen an`sat yesaplanadi'.

7 –Mi'sal.

$$D_n = \begin{vmatrix} a_1 + b_1 & a_1 + b_2 & ... & a_1 + b_n \\ a_2 + b_1 & a_2 + b_2 & ... & a_2 + b_n \\ & ..... & & \\ a_n + b_1 & a_n + b_2 & ... & a_n + b_n \end{vmatrix}$$

determinantti' yesaplan`.

Sheshimi. Bul determinant birinshi qatarg`a sali'sti'rg`anda yeki determinantqa aji'raladi', wolardi'n` ha`r biri yekinshi qatarg`a sali'sti'rg`anda

ja`ne yeki determinantqa aji'raladi' ha`m t.b lar. Aqi'rg`i' qatarg`a shekem kelip, 2^n determinantlardi' alami'z.

Yeger ha`r bir jayi'w birinshi qosi'li'wshi' ushi'n $a_i$ sandi' qabi'l yetse, al yekinshi ushi'n $b_j$ qabi'l yetse, wonda determinantti'n` ali'ng`an qatarlari' yaki $a_i, a_i, ...a_i$ ko`riniste , yaki $b_1, b_2, ..., b_n$ ko`riniste boladi'. Yeki qatar birinshi tu`rdegi proporcional, al yekinshi tu`rdegi ten` $n > 2$ de ha`r bir ali'ng`an determinant keminde yeki qatari' bir tu`rdegi boladi' ha`m wol nolge aylanadi'. Demek, $D_n = 0 \quad n > 2$ de

Wonnan keyin,

$$D_1 = a_1 + b_1, \quad D_2 = \begin{vmatrix} a_1 & a_1 \\ b_1 & b_2 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 \\ a_2 & a_2 \end{vmatrix} = (a_1 - a_2)(b_2 - b_1)$$

Determinantti'n` elementlerin wo`zgertiw u`si'li'

Bul usi'l sonday jag`daylarda qollanadi', qashan determinantti'n` barli'q elementlerin wo`zgertiw usi'li' menen birdey sonda wol sonday ko`riniske keltiriledi, wonda barliq elementlerdin` algebrali'q toli'qti'ri'wshi'si'n an`sat yesaplaydi'. Usi'l kelesi qa`siyetke tiykarlang`an: yeger D determinantti'n` barli'q elementlerine birdey $x$ sani'n qossaq, wonda determinant $x$ sani'n determinantti'n` barli'q elementlerinin` algebrali'q toli'qti'ri'wshi'lari'ni'n` qosi'ndi'si'na ko`beytiwge artadi'. Haqi'yqati'nda da, meyli

$$D = \begin{vmatrix} a_{11} & \dots & a_{n1} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{vmatrix}, \quad D^1 = \begin{vmatrix} a_{11} + x & \dots & a_{n1} + x \\ \dots & \dots & \dots \\ a_{n1} + x & \dots & a_{nn} + x \end{vmatrix}$$

bolsi'n.

D^1 birinshi qatarg`a sali'sti'rg`anda yeki determinantqa aji'ratami'z, wolardi'n` ha`r birinen yekinshi qatarg`a sali'sti'rg`anda yeki determinant aji'ratami'z ha`m t.b. lar.

x qa ten` bir qatar elementlerinen ko`birek turi'wshi' qosi'latug`i'nlar nol`ge ten`.

x qa ten` bir qatar elementlerinen turi'wshi' qosi'latug`i'nlardi', bul qatar boyi'nsha aji'ratami'z. Sonda $D^1 = D + x\sum_{i,j=1}^{n} A_{ij}$ talap yetilgendi alami'z.

Solay etip, D^i determinantti' yesaplaw D determinantqa ha`m woni'n` algebrali'q toli'qti'ri'wshi'lari'ni'n` qosi'ndi'si'na keltiriledi.

8-Mi'sal. 2-misaldag`i' $D_n$ determinantti' yesaplaymi'z. Woni'n` barli'q elementlerinen x sani'n ali'p, determinantti' alami'z.

D ni'n` bas diagonalda jatpaytug`i'n elementlerinin` algebrali'q toliqtiri'wshi'lari' nolge ten`, al bas diagonaldag`i' ha`r bir element bas diagonaldag`i' qalg`an elementlerdin` ko`beymesine ten`. Sonli'qtan

$$D_n = (a_1 - x)\ldots(a_n - x) + x\sum_{i=1}^{n}(a_1 - x)\ldots(a_{i-1} - x)(a_{i+1} - x)\ldots(a_n - x) =$$

$$= x(a_1 - x)(a_2 - x)\ldots(a_n - x)\left(\frac{1}{x} + \frac{1}{a_1 - x} + \ldots + \frac{1}{a_n - x}\right).$$

7. Kompyuterden paydalani'w

```
> restart:
> with(linalg):
> F:=Matrix([[0,x,y,z],[x,0,z,y],[y,z,0,x],[z,y,x,0]]);
```

$$F := \begin{bmatrix} 0 & x & y & z \\ x & 0 & z & y \\ y & z & 0 & x \\ z & y & x & 0 \end{bmatrix}$$

```
> det(F);
```

$$x^4 - 2z^2x^2 - 2y^2x^2 + y^4 - 2y^2z^2 + z^4$$

```
>
```

8. Jan'a materiallardi' bekkemlew: Yesaplar shi'g'ari'w ha'm wolardi' kompyuterdegi na'tiyje menen sali'sti'ri'w.

9. U’yge tapsi’rmalar:

TAPSI’RMALAR

To`mendegi determinantlardi’ u`shmu`yeshlik ko`rinisine keltirip yesaplan`

1. $\begin{vmatrix} 1 & 2 & 3 \dots . n \\ -1 & 0 & 3 \dots n \\ -1 & -2 & 0 \dots n \\ \dots\dots\dots\dots\dots\dots \\ -1 & -2 & -3 \dots 0 \end{vmatrix}$,

2. $\begin{vmatrix} 1 & 2 & 3 \dots n-2 & n-1 & n \\ 2 & 3 & 4 \dots n-1 & n & n \\ 3 & 4 & 5 \dots n & n & n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ n & n & n \dots n & n & n \end{vmatrix}$,

3. $\begin{vmatrix} x_1 & a_{12} & a_{13} \dots a_{1n} \\ x_1 & x_2 & a_{23} \dots a_{2n} \\ x_1 & x_2 & x_3 \dots a_{3n} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_1 & x_2 & x_3 \dots x_n \end{vmatrix}$,

4. $\begin{vmatrix} 1 \dots\dots 1 & 1 & 1 \\ a_1 \dots\dots a_2 & a_1 - b & a_1 \\ a_2 \dots .. a_2 - b_2 & a_2 & a_2 \\ a_n - b_n \dots . a_n & a_n & a_n \end{vmatrix}$

5. $\begin{vmatrix} 3 & 2 & 2 \dots & 2 \\ 2 & 3 & 2 \dots & 2 \\ 2 & 2 & 3 \dots & 2 \\ \dots\dots\dots\dots\dots\dots \\ 2 & 2 & 2 \dots & 3 \end{vmatrix}$,

6. $\begin{vmatrix} a_0 & a_1 & a_2 \dots & a_n \\ -x & x & 0 \dots & 0 \\ 0 & -x & x \dots & 0 \\ \dots\dots\dots\dots\dots\dots \\ 0 & 0 & 0 \dots & x \end{vmatrix}$,

To`mendegi determinantlardi’ si’zi’qli’ ko`beytiwshilerge aji’ratiw usi’li’ menen yesaplan`

7. $\begin{vmatrix} 1 & 2 & 3 \dots & n \\ 1 & x+1 & 3 \dots & n \\ \dots\dots\dots\dots\dots\dots \\ 1 & 2 & 3 \dots & x+1 \end{vmatrix}$

8. $\begin{vmatrix} 1 & 1 & 1 & \dots\dots & 1 \\ 1 & 2-x & 1 & \dots\dots & 1 \\ 1 & 1 & 3-x & \dots\dots & 1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ 1 & 1 & 1 & \dots\dots & n+1-x \end{vmatrix}$,

9. $\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ a_0 & x & a_2 & \dots & a_n \\ a_0 & a_1 & x & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_0 & a_1 & a_2 & \dots & x \end{vmatrix}$

10. $\begin{vmatrix} -x & a & b & c \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix},$

11. $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix},$

12.* $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$

To`mendegi determinantlardi' reku`rrentli qatnaslar usi'li' menen yesaplan`

13*. $\begin{vmatrix} a_1b_1 & a_1b_2 & a_1b_3 & \dots & a_1b_n \\ a_1b_2 & a_2b_2 & a_2b_3 & \dots & a_2b_n \\ a_1b_3 & a_2b_3 & a_3b_3 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots & \dots \\ a_1b_n & a_2b_n & a_3b_n & \dots & a_nb_n \end{vmatrix},$

14. $\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ -y_1 & x_1 & 0 & \dots & 0 \\ 0 & -y_2 & x_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & x_n \end{vmatrix}$

Determinantti' basqasha birinshi qatar boyi'nsha jayi'w arqali' yesaplaw mu`mkin

15. $\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix},$

16. $\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & a_1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & a_2 & 0 & \dots & 0 & 0 \\ 1 & 0 & 1 & a_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 0 & \dots & 1 & a_n \end{vmatrix}$

17. $\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & 2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 2 \end{vmatrix},$

18. $\begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix}$

Determinantti’ determinantlar qosi’ndi’si’ ko`rinisinde beriliw usi’li’ menen yesaplan`

19*. $\begin{vmatrix} x+a_1 & a_2 & a_3 & \dots & a_n \\ a_1 & x+a_2 & a_3 & \dots & a_n \\ a_1 & a_2 & x+a_3 & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_1 & a_2 & a_3 & \dots & x+a_n \end{vmatrix}$

20*. $\begin{vmatrix} x_1 & a_2 & \dots & a_n \\ a_1 & x_2 & \dots & a_n \\ \dots & \dots & \dots & \dots \\ a_1 & a_2 & \dots & x_n \end{vmatrix}$

21*. $\begin{vmatrix} 0 & 1 & 1 & 1 & \dots & 1 & 1 \\ x_1 & a_1 & 0 & 0 & \dots & 0 & 0 \\ x_2 & x_2 & a_2 & 0 & \dots & 0 & 0 \\ x_3 & x_3 & x_3 & a_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_n & x_n & x_n & x_n & \dots & x_n & a_n \end{vmatrix}$

22. $\begin{vmatrix} x_1 & a_1b_2 & a_1b_3 & \dots & a_1b_n \\ a_2b_1 & x_2 & a_2b_3 & \dots & a_2b_n \\ a_3b_1 & a_3b_2 & x_3 & \dots & a_3b_n \\ \dots & \dots & \dots & \dots & \dots \\ a_nb_1 & a_nb_2 & a_nb_3 & \dots & x_n \end{vmatrix}$

Determinantlardi’ yesaplan`

23. $\begin{vmatrix} a_1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ a_2 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ a_n & x_n & x_n^2 & \dots & x_n^{n-1} \end{vmatrix}$, bul jerde $f(x)=(x-x_1)(x-x_2)\dots(x-x_n)$.

24. $\begin{vmatrix} a & a+x & a+2x & \dots & a+(n-2)x & a+(n-1)x \\ a+(n-1)x & a & a+x & \dots & a+(n-3)x & a+(n-2)x \\ a+(n-2)x & a+(n-1)x & a & \dots & a+(n-4)x & a+(n-3)x \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a+x & a+2x & a+3x & \dots & a+(n-1)x & a \end{vmatrix}$

25. $\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \binom{2}{1} & \binom{3}{1} & \dots & \binom{n}{1} \\ 1 & \binom{3}{2} & \binom{4}{2} & \dots & \binom{n+1}{2} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \binom{n}{n-1} & \binom{n+1}{n-1} & \dots & \binom{2n-2}{n-1} \end{vmatrix}$, bul jerde $\binom{n}{k} = C_n^k = \frac{n!}{k!(n-k)!}$

26. $\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1,2n-2} & a_{1,2n-1} & a_{1,2n} \\ 0 & a_{22} & a_{23} & \dots & a_{2,2n-2} & a_{2,2n-1} & 0 \\ 0 & 0 & a_{33} & \dots & a_{3,2n-2} & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & a_{2n-2,3} & \dots & a_{2n-2,2n-2} & 0 & 0 \\ 0 & a_{2n-1,2} & a_{2n-1,3} & \dots & a_{2n-1,2n-2} & a_{2n-1,2n-1} & 0 \\ a_{2n,1} & a_{2n,2} & a_{2n,3} & \dots & a_{2n,2n-2} & a_{2n,2n-1} & a_{2n,2n} \end{vmatrix}$

27. $\begin{vmatrix} a_{11} & 1 & a_{12} & 1 & \dots & a_{1n} & 1 \\ 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ a_{21} & x_1 & a_{22} & x_2 & \dots & a_{2n} & x_n \\ x_1 & 0 & x_2 & 0 & \dots & x_n & 0 \\ a_{31} & x_1^2 & a_{32} & x_2^2 & \dots & a_{3n} & x_n^2 \\ x_1^2 & 0 & x_2^2 & 0 & \dots & x_n^2 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & x_1^{n-1} & a_{n2} & x_2^{n-1} & \dots & a_{nn} & x_n^{n-1} \\ x_1^{n-1} & 0 & x_2^{n-1} & 0 & \dots & x_n^{n-1} & 0 \end{vmatrix}$

$2n$ ta`rtipli determinantti' yesaplan`

28. $$\begin{vmatrix} 1+x & x & \dots & x & x & \dots & x & 1+x \\ x & 1+x & \dots & x & x & \dots & 1+x & x \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x & x & \dots & 1+x & 1+x & \dots & x & x \\ x & x & \dots & 1+2x & 1+x & \dots & x & x \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x & 1+2x & \dots & x & x & \dots & 1+x & x \\ 1+2x & x & \dots & x & x & \dots & x & 1+x \end{vmatrix}$$

29. $$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} & a_{1n} & 0 & 0 & \dots & 0 & b_{1n} \\ a_{21} & a_{22} & \dots & a_{2,n-1} & 0 & 0 & 0 & \dots & b_{2,n-1} & b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & 0 & \dots & 0 & 0 & b_{n1} & b_{n2} & \dots & b_{n,n-1} & b_{nn} \\ c_{11} & c_{12} & \dots & c_{1,n-1} & c_{1n} & 0 & 0 & \dots & 0 & b_{1n} \\ c_{21} & c_{22} & \dots & c_{2,n-1} & 0 & 0 & 0 & \dots & b_{2,n-1} & b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_{n1} & 0 & \dots & 0 & 0 & b_{n1} & b_{n2} & \dots & b_{n,n-1} & b_{nn} \end{vmatrix},$$

30. $$\begin{vmatrix} 1 & 1 & \dots & 1 & x & a_1 & a_2-1 & \dots & a_{n-1}-1 & a_n-1 \\ 1 & 1 & \dots & x & 1 & a_1-1 & a_2 & \dots & a_{n-1}-1 & a_n-1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x & 1 & \dots & 1 & 1 & a_1-1 & a_2-1 & \dots & a_{n-1}-1 & a_n-1 \\ a_1-x & a_1 & \dots & a_1 & a_1 & -a_1 & -a_1 & \dots & -a_1 & x-a \\ a_2 & a_2-x & \dots & a_2 & a_2 & -a_2 & -a_2 & \dots & x-a_2 & -a_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_n & a_n & \dots & a_n & a_n-x & x-a_n & -a_n & \dots & -a_n & -a_n \end{vmatrix}$$

Juwabi'.

1. $n!$

2. $n\cdot(-1)^{\frac{n(n-1)}{2}}$

3. $x_1(x_2-a_{12})\dots(x_n-a_{n-1,n})$

4. $(-1)^{\frac{n(n-1)}{2}}b_1b_2\dots b_n$

5. $2n+1$

6. $(a_0+a_1+a_2+...+a_n)x^n$

7. $(x-1)(x-2)...(x-n+1)$

8. $(-1)^n(x-1)(x-2)...(x-n)$

9. $a_o(x-a_1)(x-a_2)...(x-a_n)$

10. $(x-a-b-c)(x-a+b+c)\times$
$\times(x+a-b+c)(x+a+b-c)$

11. $(x^2-1)(x^2-4)$

12*. x^2z^2

13. $a_1b_n\prod_{i=1}^{n-1}(a_{i+1}b_i-a_ib_{i+1})$

14. $a_ox_1x_2x_3...x_n+a_1y_1x_2x_3...x_n+$
$+a_2y_1y_2x_3...x_n+...+a_ny_1y_2...y_n$

15. $-a_1a_2...a_n\left(\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}\right)$

16. $a_1a_2...a_n-a_1a_2...a_{n-1}$
$+a_1a_2...a_{n-2}-...+(-1)^{n-1}a_1+(-1)^n$

17. $n+1$

18. $2^{n+1}-1$

19. $x^n+(a_1+a_2+...+a_n)x^{n-1}$

20. $(x_1-a_1)(x_2-a_2)...(x_n-a_n)$
$\times\left(\frac{a_1}{x_1-a_1}+\frac{a_2}{x_2-a_2}+...+\frac{a_n}{x_n-a_n}\right)$

21. $(a_1-x_1)(a_2-x_2)...(a_n-x_n)$
$-a_1a_2...a_n$

22. $(x_1-a_1b_1)(x_2-a_2b_2)...(x_n-a_nb_n)$
$\times\left(1+\frac{a_1b_1}{x_1}+\frac{a_2b_2}{x_2}+...+\frac{a_nb_n}{x_n}\right)$

23. $(-1)^{n-1}\prod_{i=1}^{n}x_i\times$
$\times\prod_{n\ge i>k\ge1}(x_i-x_k)\left[\sum_{i=1}^{n}\frac{a_i}{x_i f^1(x_i)}\right]$

24. $(-nx)^{n-1}\left[a+\frac{(n-1)x}{2}\right]$

25. 1

26. $\prod_{k=1}^{n}\left(a_{kk}a_{2n-k+1,2n-k+1}a_{k,2n-k+1}a_{2n-k+1,k}\right)$

27. $(-1)^n\prod_{n\ge i>k\ge1}(x_i-x_k)^2\cdot$

28. $(-1)^n(nx+1)x^n$

29. $b_{1n}b_{2,n-1}...b_{n1}(a_{1n}-c_{1n})\times$
$\times(a_{2,n-1}-c_{2,n-1})...(a_{n1}-c_{n1})$

30. $x^{2n}-x^{2n-2}(a_1+a_2+...+a_n)^2$

Ko'rsetpeler:

12. Yeki birinshi qatarlardi' ha`m yeki birinshi bag`analardi' almasti'ri'p, determinant x ti $-x$ wo`zgertkende wo`zgermeytug`i'ni' da`lillenedi. $x=0$ de determinant nol`ge aylanadi', $x^2$ qa bo`linedi, z ke de usi'nday.

13. $D_n=\frac{b_n}{b_{n-1}}(a_nb_{n-1}-a_{n-1}b_n)D_{n-1}$ qatnasti' ali'n`.

19. Bas diagonaldan shette turi'wshi' elementlerdi $a_i=0+a_i$ ko`riniste berin`.

20. $x_i = (x_i - a_i) + a_i$ dep ali'n'.

21. Sol joqarg`i' mu`yeshti $0 = 1 - 1$ dep ali'n' ha`m determinantti' birinshi qatarg`a sali'sti'rg`anda yeki determinant qosi'ndi'si' ko`rinisinde berin`.

23. Determinantti' birinshi bag`ana boyi'nsha jayi'n`.

24. Ha`r bir qatardan kelesin ali'n`, barli'q bag`analardi' kelesi bag`anag`a qosi'w, aqi'rg`i' qatarg`a barli'q aldi'ng`i'lari'n qosi'n` ha`m bul qatardi' barli'q aldi'ng`i' qatarlarg`a qosi'n`.

25. $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$, ha`r bir bag`anadan aldi'ng`i'si'n ali'n`, al son` ha`r bir qatardan aldi'ng`i'si'n ali'n`.

№4 A`MELİY JUMIS
TEMA: MATRİCALAR U`STİNDE A`MELLERDİ WORINLAW

1. U`yge berilgen tapsi'rmalardi' tekseriw.
**2. Jumi'sti'n` maqseti**: Matrica u`stinde a`mellerdi wori'nlawdi' ha'm wolardi' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.
3. Ma`selenin` qoyi'liwi': Berilgen matricalar u`stinde a`mellerdi wori'nlan`
4. Paydalani'lg`an a`debiyatlar: [1-6]
5. Paydalani'lg`an u`skeneler: kompyuter
6. Metodikali'q ko`rsetpeler:

1-Ani'qlama. *m×n sanlardi' m qatar ha`m n bag`ana tu`rinde*

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13}\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23}\cdots & a_{2n} \\ \cdots\cdots\cdots\cdots & & & \\ a_{m1} & a_{m2} & a_{m3}\cdots & a_{mn} \end{pmatrix}$$

jaziwdi' matrica deymiz.

i, j indeksler a_{ij} elementtin` qatar ha`m bag`ana nomerlerine sa`ykes keledi. Qi'sqasha $A=(a_{ij})$ dep jazami'z. A matricani'n` bag`anasi' qatar, al qatar bag`ana bolsa, wonday $A'$ matricani' $A$ matricani' transponirlewden ali'ng`an delinedi. Transponirlengen matrica to`mendegishe boladi':

$$A'=\begin{pmatrix} a_{11} & a_{21}\ldots & a_{m1} \\ a_{12} & a_{22}\ldots & a_{m2} \\ \cdots\cdots\cdots\cdots & & \\ a_{1n} & a_{2n}\ldots & a_{mn} \end{pmatrix},$$

$$a_{ij}^{'}=a_{ji}.$$

Yeger matrica $m=n$ bolsa, wonda matricani' kvadrat matrica deymiz. Birdey $m\times n$ wo'lchemli A ha'm B matricalar u`stinde to`mendegi a`meller wori'nlanadi':

$$\lambda A=\begin{pmatrix} \lambda a_{11} & \lambda a_{12}\ldots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22}\ldots & \lambda a_{2n} \\ \cdots\cdots\cdots\cdots & & \\ \lambda_{m1} & \lambda a_{m2}\ldots & \lambda a_{mn} \end{pmatrix}$$

$$A+B=\begin{pmatrix} a_{11} & a_{21}\ldots\ a_{m1} \\ a_{12} & a_{22}\ldots a_{m2} \\ \ldots\ldots\ldots\ldots\ldots \\ a_{1n} & a_{2n}\ldots. a_{mn} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12}\ldots\ b_{1n} \\ b_{21} & b_{22}\ldots\ b_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ b_{m1} & b_{m2}\ldots b_{mn} \end{pmatrix}=$$

$$=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \ldots\ a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \ldots\ a_{2n}+b_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots & & \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \ldots a_{mn}+b_{mn} \end{pmatrix}.$$

To'mendegi

$$E=\begin{pmatrix} 1 & 0 & 0\ldots\ 0 \\ 0 & 1 & 0\ldots\ 0 \\ 0 & 0 & 1\ \ldots\ 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \\ 0 & 0 & 0\ \ldots\ 1 \end{pmatrix}$$

matricag'a birlik matrica delinedi.

Yeger $A=\begin{pmatrix} a_{11} & a_{12}\ldots a_{1n} \\ a_{21} & a_{22}\ldots a_{2n} \\ \cdots\cdots\cdots\cdots & \\ a_{n1} & a_{n2}\ldots\ a_{nn} \end{pmatrix}$ bolsa, wonda $A\cdot E=E\cdot A=A.$

Yegerde $A\cdot B=B\cdot A=E$ bolsa, wonda B matrica A matricag' keri delinedi ha'm $B=A^{-1}$ dep belgileymiz. Demek, $A\cdot B=A\cdot A^{-1}=E.$

Determinant $D(AB)=D(A)D(B)=D(E)$,

$$d=D(A)=\begin{pmatrix} a_{11} & a_{12}\cdots a_{1n} \\ a_{21} & a_{22}\cdots a_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{n1}a_{n2}\cdots a_{nn} \end{pmatrix}.$$

Algebrali'q toli'qti'ri'wshi'si'nan jan`a $\tilde{A}$ matricasi'n du`zemiz.

$$\tilde{A}=\begin{pmatrix} A_{11} & a_{21}\ldots a_{n1} \\ A_{12} & a_{22}\ldots a_{n2} \\ \ldots\ldots\ldots\ldots\ldots & \\ A_{1n} & a_{2n}\cdot\ldots a_{nn} \end{pmatrix},\quad A\tilde{A}=\tilde{A}A=dE,$$

bunnan $A^{-1}=\frac{1}{d}\tilde{A}$ boladi'.

Determinantti' yesaplaw

$$D=\begin{vmatrix} a_{11} & a_{12} \dots a_{ij} & \dots & a_{1n} \\ a_{21} & a_{22} \dots a_{2j} & \dots & a_{2n} \\ \dots\dots\dots\dots\dots & & & \\ a_{i4} & a_{i2} \dots a_{ij} & \dots & a_{in} \\ \dots\dots\dots\dots\dots & & & \\ a_{m} & a_{n} \dots \cdot a_{nj} & \dots & a_{nn} \end{vmatrix}$$

ko'rinistegi n-ta`rtipli determinanttan $a_{ij}$ elementti aj'iratami'z. $i$ qatardi' ha`m j bag`anani' ali'p taslaymi'z, sonnan qalg`ani' minor boladi'.

$$\mathrm{M}_{ij}=\begin{vmatrix} a_{11} & a_{12} \dots a_{ij-1} & a_{ij+1} \dots & a_{in} \\ a_{21} & a_{22} \dots a_{2j-1} & a_{2j+1} \dots & a_{2n} \\ \dots\dots\dots\dots\dots & & & \\ a_{i-11} & a_{i-12} \dots a_{i-1j-1} & a_{i-1j+1} \dots a_{i-1n} & \\ a_{i+11} & a_{i+12} \dots a_{i+1j-1} & a_{i+1j+1} \dots a_{i+1n} & \\ \dots\dots\dots\dots\dots & & & \\ a_{n1} & a_{n2} \dots a_{nj+1} & a_{nj+1} \dots & a_{nn} \end{vmatrix}$$

$A_{ij}=(-1)^{i+j}M_{ij}$ algebrali'q toli'qti'ri'wshi'.

1-**Mi'sal.** $\Delta=\begin{vmatrix} 1 & 2 & 3. \\ 4 & 2 & -1 \\ 3 & 5 & 2 \end{vmatrix}$ ti yesaplan`.

Shehsimi.

$$\Delta=3A_{12}+2A_{22}+5A_{32},$$

$$\Delta=3(-1)^{1+2}\begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix}+2(-1)^{2+2}\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}+5(-1)^{3+2}\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix}=-3\cdot 11+2\cdot 1-5(-6)=-1$$

Matricadan paydalani’w

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= h \\ a_2x + b_2y + c_2z &= h_2 \\ a_3x + b_3y + c_3z &= h_3 \end{aligned}\right\} \qquad (1)$$

Bul sistemani’n` koefficientlerinen du`zilgen sanlar tablicasi’na

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

3-ta`rtipli (u`sh qatar ha`m 3 bag`ana) kvadratli’q matrica dep ataladi’.

Saltan` ag`zalari’n qosi’p du`zilgen

$$B = \left(\begin{array}{ccc|c} a_1 & b_1 & c_1 & h_1 \\ a_2 & b_2 & c_2 & h_2 \\ a_3 & b_3 & c_3 & h_3 \end{array}\right)$$

matricag`a ken`eyttirilgen matrica delinedi.

A matricani’n` sa`ykes qatarlari’n ha`m bag`analari’n almasti’ri’p

$$A^1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

matricani’ du`zemiz. $A$ dan $A^1$ ge wo`tiwdi matricani’ transponirlew delinedi.

Matricani’n` qa`siyetlerin ko`rip shig`ami’z:

1) A yaki A^1 matricani’n` yeki qatari’ni’n’ (yaki bag`anasi’n) wori’nlari’ almasti’ri’lsa, wolardi’n’ determinantlari’ belgisi qarama qarsi’g`a wo`zgeredi;

2) A yaki A^1 da yeki birdey qatarlar (bag`analar) bolsa, wolardi’n` determinatlari’ nol`ge ten` boladi’;

3) Transponirlew na`tiyjelerinde berilgen matrica determianti` ma`nisi wo`zgermeydi;

4) A ni’n` qandayda bir qatar (yaki bag`ana) dag`i barliq elementlerin $c$ g`a ko`beytsek, wonin` determianti’ c g`a ko`beyedi;

5) A da proporcional yeki qatar (yaki yeki bag`ana) bolsa, wonda wonin` determinanti’ nol`ge ten` boladi’;

6) A ni'n` qanday da bir qatari' elementleri qosi'ndi' tu`rinde bolsa, wonda

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2+a^1 & b_2+a^{11} & c_2+a^{111} \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_1 & c_1 \\ a^1 & a^{11} & \alpha^{111} \\ a_3 & b_3 & c_3 \end{vmatrix}$$

boladi'.

Matrica u'stinde elementar forma almasti'ri'wlar

2-Ani'qlama. *Amatricani' elementar forma wo'zgerttiriw degende to'mendegi a'meller tu'siniledi:*

1) A ni'n' qanday da bir qatari'n 0 *den wo'zgeshe sang'a ko'beytiw;*

2) qanday da bir qatar elementlerine qanday da bir sang'a ko'beyttirilgen basqa qatardi'n' sa'ykes elementlerin qosi'w;

3) yeki qatardi'n' wori'nlari'n almasti'ri'w.

1-3 a'mellerdi bag'analarda da wori'nlaw mu'mkin.

A matricadan elementar forma wo'zgerttiriwin qollap B matrica ali'nadi', woni' $A \to B$ dep jazami'z.

1-Teorema. *Yeger* $A \to B$ *bolsa, wonda* $B \to A$ *boladi', yag'ni'y elementar forma wo'zgerttiriw qaytadi'.*

2-Teorema. *Elementar forma wo'zgerttiriwler na'tiyjesinde* $A \to B$ *bolsa,* $r_A = r_B$ *boladi'.*

3-Teorema. (Kroneker-Kapelli teoremasi'). *Berilgen*

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= h_1 \\ a_2x + b_2y + c_2z &= h_2 \\ a_3x + b_3y + c_3z &= h_3 \end{aligned}\right\}$$

sistema birgelikli boli'wi' ushi'n bul sistema matricasi' ha'm woni'n' ken'eyttirilgen matricasi'ni'n' rangleri wo'z-ara ten' boli'wi' za'ru'r ha'm jeterli.

2-Mi'sal. $\begin{cases} 2x - y - z = 4 \\ 3x + 4y - 2z = 11. \\ 3x - 2y + 4z = 2 \end{cases}$

Sheshimi. A) $A=\begin{pmatrix}2&-1&-1\\3&4&-2\\3&-2&4\end{pmatrix}$ $\Delta=\begin{vmatrix}2&-1&-1\\3&4&-2\\3&-2&4\end{vmatrix}=60\neq 0$ $r_{\text{A}}=3$

B) $B=\begin{pmatrix}2&-1&-1&4\\3&4&-2&11\\2&-2&4&1\end{pmatrix}$, $\begin{pmatrix}0&0&0&0\\2&-1&-1&4\\3&4&-2&11\\2&-2&4&1\end{pmatrix}$, $r_{\text{B}}=3$ $x=3$ $y=1$ $z=1.$

Matricalar u'stinde a`meller

1. Matricalardi' ko'beytiw. Ha'r qanday matricalardi' ko'beyte beriw mu'mkin yemes. $A\cdot B$ a'mellerdi wori'nlaw ushi'n A ni'n' bag'analar sani' B ni'n' qatarlar sani'na ten' boli'wi' kerek.

$$\text{A}=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}\qquad \text{B}=\begin{pmatrix}1&1&1\\1&2&14\\1&3&9\end{pmatrix}$$

$$\text{A}\cdot\text{B}=\begin{pmatrix}1\cdot1+2\cdot1+3\cdot1&1\cdot1+2\cdot2+3\cdot3&1\cdot1+2\cdot4+3\cdot9\\4\cdot1+5\cdot1+6\cdot1&4\cdot1+5\cdot2+6\cdot3&4\cdot1+5\cdot4+6\cdot9\end{pmatrix}=\begin{pmatrix}6&16&36\\15&32&78\end{pmatrix}$$

2. Matricani' qosi'w ha'm ayi'ri'w. Birdey sandag'i' qatar ha'm bag'analar matricalar u'stinde g'ana qosi'w ha'm ayi'ri'w a'mellerin wori'nlaw mu'mkin.

<u>3-Ani'qlama.</u> *A ha'm B matricalardi'n' qosi'ndi'si' dep elementleri A ha'm B matricalardi'n' sa'ykes elementleri qosi'ndi'si'nan ibarat C matricag'a ayti'ladi' ha'm* $C=A+B$ *si'yaqli' jazi'ladi'.*

Mi'sallar.

1. $\begin{pmatrix}1&-2&3\\5&1&-3\end{pmatrix}=\begin{pmatrix}2&3&4\\1&3&5\end{pmatrix}=\begin{pmatrix}3&1&7\\6&4&2\end{pmatrix}.$

2. $\begin{pmatrix}1&7&5\\2&9&-7\end{pmatrix}-\begin{pmatrix}3&3&3\\5&-2&-9\end{pmatrix}=\begin{pmatrix}-2&4&2\\-3&11&2\end{pmatrix}$

Matricalardi’ qosi’w, ko’beytiw to’mendegi qa’siyetlerge iye.

1) $A + B = B + A$

2) $(A + B) + C = A + (B + C)$

3) $(A + B) \cdot C = A \cdot C + B \cdot C$

4) $D(A + B) = D \cdot A + D \cdot B$

7. Kompyuterden paydalani’w:

1.

```
> restart:
> with(linalg);
```

Warning, the protected names norm and trace have been redefined and unprotected

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

```
> A:=Matrix([[a,b,c],[d,e,f]]);
```

$$A := \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

```
> B:=Matrix([[j,h,k],[l,m,n],[o,p,s]]);
```

$$B := \begin{bmatrix} j & h & k \\ l & m & n \\ o & p & s \end{bmatrix}$$

> multiply (A, B);

$$\begin{bmatrix} a\,j + b\,l + c\,o & a\,h + b\,m + c\,p & a\,k + b\,n + c\,s \\ d\,j + e\,l + f\,o & d\,h + e\,m + f\,p & d\,k + e\,n + f\,s \end{bmatrix}$$

To'mendegi misaldi' qarasti'rayi'q.

> A:=Matrix([[1,2,3],[4,5,6]]);

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

> B:=Matrix([[1,1,1],[1,2,14],[1,3,9]]);

$$B := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 14 \\ 1 & 3 & 9 \end{bmatrix}$$

> multiply(A, B);

$$\begin{bmatrix} 6 & 14 & 56 \\ 15 & 32 & 128 \end{bmatrix}$$

>

2.

> restart:

> with(linalg):

Warning, the protected names norm and trace have been redefined and unprotected

> A:=Matrix([[a,b,c],[d,e,f]]);

$$A := \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

> B:=Matrix([[j,k,l],[m,n,p]]);

$$B := \begin{bmatrix} j & k & l \\ m & n & p \end{bmatrix}$$

> A+B

$$\begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+p \end{bmatrix}$$

```
> A:=Matrix([[1,-2,3],[5,1,-3]]);
```

$$A := \begin{bmatrix} 1 & -2 & 3 \\ 5 & 1 & -3 \end{bmatrix}$$

```
> B:=Matrix([[2,3,4],[1,3,5]]);
```

$$B := \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 5 \end{bmatrix}$$

```
> A+B;
```

$$\begin{bmatrix} 3 & 1 & 7 \\ 6 & 4 & 2 \end{bmatrix}$$

```
> B+A;
```

$$\begin{bmatrix} 3 & 1 & 7 \\ 6 & 4 & 2 \end{bmatrix}$$

```
>
```

8. Jan’a materiallardi’ bekkemlew: Yesaplar shi’g’ari’w ha’m wolardi’ kompyuterdegi na’tiyje menen sali’sti’ri’w.

9. U’yge tapsi’rmalar:

TAPSI’RMALAR

1. $\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$.

2. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

3. $\begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix}$.

4. $\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 & 5 \\ 4 & -1 & 3 \\ 9 & 6 & 5 \end{pmatrix}$.

5. $\begin{pmatrix} 2 & -1 & 3 & -4 \\ 3 & -2 & 4 & -3 \\ 5 & -3 & -2 & 1 \\ 3 & -2 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 7 & 8 & 6 & 9 \\ 5 & 7 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & 1 & 2 \end{pmatrix}$

6. $\begin{pmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{pmatrix}$.

7. $\begin{pmatrix} 2 & -3 \\ 4 & -6 \end{pmatrix} \cdot \begin{pmatrix} 9 & -6 \\ 6 & -4 \end{pmatrix}$.

8. $\begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$.

9. $\begin{pmatrix} 5 & 2 & -2 & 3 \\ 6 & 4 & -3 & 5 \\ 9 & 2 & -3 & 4 \\ 7 & 6 & -4 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & 2 & 2 \\ -1 & -5 & 3 & 11 \\ 16 & 24 & 8 & -8 \\ 8 & 16 & 0 & -16 \end{pmatrix}$.

10. $\begin{pmatrix} 0 & 2 & -1 \\ -2 & -1 & 2 \\ 3 & -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 70 & 34 & -107 \\ 52 & 26 & -68 \\ 101 & 50 & -140 \end{pmatrix} \cdot \begin{pmatrix} 27 & -18 & 10 \\ -46 & 31 & -17 \\ 3 & 2 & 1 \end{pmatrix}$.

11. $A = \begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}$ ha'm $B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$ matricalari'ni'n' qosi'ndi'si'n tabi'n'.

12. Yeger $A = \begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ bolsa, wonda $2A + 5B$ matricani' tabi'n'.

13. Yeger $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ bolsa, A^3 ti tabi'n'.

14. Yeger $\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}$ bolsa, A^3 ti tabi'n'

15. Yeger $\begin{pmatrix} 4 & -1 \\ 5 & -2 \end{pmatrix}$ bolsa, A^3 ti tabi'n'.

16. Yesaplan'. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^n$.

17. $\begin{pmatrix} \lambda_1 & 0 & .... & 0 & 0 \\ 0 & \lambda_2 & .... & 0 & 0 \\ ..... & & & & \\ 0 & 0 & .... & 0 & \lambda_n \end{pmatrix}^k$, bul jerde 0 ler, matricani'n' bas diagonali'ni'n' basqa barli'q elementlerdi bildiredi

18. $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}^n$ ti yesaplan'.

19. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n$ ti yesaplan'.

20. $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}^n$ ti yesaplan'.

21. $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$ ti yesaplan'.

Matricalardi'n' u'stinde ko'rsetilgen a'mellerdi wori'nlan'.

22. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & -\cos\beta \end{pmatrix}$

23. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 29 \\ 2 & 18 \\ 0 & -3 \end{pmatrix}$

24. $\begin{pmatrix} 1 & 5 & 3 \\ 2 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{pmatrix} \cdot$

25. $\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$

26. $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -3 & 1 \end{pmatrix}.$

27. $\begin{pmatrix} 3 & 0 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 2 & 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & 1 \\ -1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & -3 \\ 0 & 6 & -3 \\ 5 & -2 & 8 \end{pmatrix}$

28. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 .$

29. $\begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & -1 & 2 \\ -2 & -1 & 1 & 2 \\ 2 & 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -4 & 6 & 1 \\ 2 & 2 & -5 & -2 \\ 2 & -2 & 6 & 4 \\ 1 & 3 & 0 & 1 \end{pmatrix}$

30. $\left(\begin{pmatrix}2 & 1\\ 5 & 3\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\cdot\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}\right)^n$.

Juwabi'.

1. $\begin{pmatrix}5 & 2\\ 7 & 0\end{pmatrix}$

2. $\begin{pmatrix}a\alpha+b\gamma & a\beta+b\delta\\ c\alpha+d\gamma & c\beta+d\delta\end{pmatrix}$

3. $\begin{pmatrix}1 & 5 & -5\\ 3 & 10 & 0\\ 2 & 9 & -7\end{pmatrix}$

4. $\begin{pmatrix}11 & -22 & 29\\ 9 & -27 & 32\\ 13 & -17 & 26\end{pmatrix}$

5. $\begin{pmatrix}10 & 17 & 19 & 23\\ 17 & 23 & 27 & 35\\ 16 & 12 & 9 & 20\\ 7 & 1 & 3 & 10\end{pmatrix}$

6. $\begin{pmatrix}8 & 6 & 4 & 2\\ 5 & 0 & -5 & -10\\ 7 & 7 & 7 & 7\\ 10 & 9 & 8 & 7\end{pmatrix}$

7. $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

8. $\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$

9. $\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$

10. $\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 5\end{pmatrix}$

11. $\begin{pmatrix}4 & 7 & 11\\ 4 & 2 & -2\\ 3 & 3 & 3\end{pmatrix}\cdot$

12. $\begin{pmatrix}4 & 6 & 6\\ 1 & 7 & 3\\ 8 & 11 & 14\end{pmatrix}$

13. $\begin{pmatrix}47 & 78\\ 39 & 86\end{pmatrix}$

14. $\begin{pmatrix}13 & -14\\ 21 & -22\end{pmatrix}$

15. $\begin{pmatrix}304 & -61\\ 305 & -62\end{pmatrix}$

16. $\begin{pmatrix}\cos n\alpha & -\sin n\alpha\\ \sin n\alpha & \cos n\alpha\end{pmatrix}$

17. $\begin{pmatrix}\lambda_1^k & 0 & .... & 0 & 0\\ 0 & \lambda_2^k & .... & 0 & 0\\ ..... & & & & \\ 0 & 0 & .... & 0 & \lambda_n^k\end{pmatrix}^k$

18. $\begin{cases}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - n \text{ jup bolsa,}\\ \begin{pmatrix}2 & -1\\ 3 & -2\end{pmatrix} - n \text{ taq bolsa.}\end{cases}$

19. $\begin{pmatrix}1 & n\\ 0 & 1\end{pmatrix}$

20. $\begin{pmatrix}\lambda^n & n\lambda^{n-1}\\ 0 & \lambda^n\end{pmatrix}$

21. $\begin{pmatrix} 1 & n+m \\ 0 & 1 \end{pmatrix}$

22. $\begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}$

23. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$

24. $\begin{pmatrix} 6 & 14 & -2 \\ 10 & -19 & 17 \end{pmatrix}$

25. $\begin{pmatrix} 3 & 3 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 2 & -2 \end{pmatrix}$

26. $\begin{pmatrix} 2 & 4 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & -2 & 3 \end{pmatrix}$

27. $\begin{pmatrix} -1 & -4 & -1 \\ 2 & 9 & -7 \\ 13 & -9 & 15 \end{pmatrix}$

28. $\begin{pmatrix} 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

29. $\begin{pmatrix} 5 & 4 & 5 & 11 \\ 6 & 6 & -5 & -2 \\ 0 & 0 & 6 & 12 \\ 1 & 2 & 1 & 3 \end{pmatrix}$

30. $\begin{pmatrix} 3n+1 & -n \\ 9n & -3n+1 \end{pmatrix}$, birinshi ha’m u’shinshi matricalar wo’z-ara keri matricalar ha’m n ko’beytiwshiler ko’rinisinde da’rejeni jazami’z.

№5 A`MELİY JUMIS
TEMA: MATRİCANIN' RANGİ

1. U'yge berilgen tapsi'rmalardi' tekseriw
2. Jumi'sti'n' maqseti: Matricani'n' rangin tabi'wdi' ha'm woni' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.
3. Ma'selenin' qoyi'li'wi': Berilgen matricani'n' rangin tabi'n'.
4. Paydalani'lg'an a'debiyatlar: [1-6].
5. Paydalani'lg'an u'skeneler: kompyuter
6. Metodikali'q ko'rsetpeler:

Yeger A matricani'n' r ta'rtipli minorlari' ishinde hesh bolmag'anda birewi nolge ten' bolmay, $r+1$ ta'rtipli barli'q minorlari' 0 ge ten' bolsa, pu'tin r sani' A matricani'n' rangi delinedi.

1-Mi'sal'.

1. $A=\begin{pmatrix}1&2&3\\2&2&2\\3&3&3\end{pmatrix}$, $\begin{vmatrix}2&2\\3&3\end{vmatrix}=\begin{vmatrix}1&1\\3&3\end{vmatrix}=\begin{vmatrix}1&1\\1&2\end{vmatrix}=0$, $r_A=1$.

2. $B=\begin{pmatrix}1&0\\0&0\\0&0\end{pmatrix}$, $r_B=1$.

3. $A=\begin{pmatrix}1&2&3&4\\0&5&6&7\\0&0&0&0\end{pmatrix}$, $r_A=2$, $\begin{vmatrix}1&2\\0&5\end{vmatrix}\neq 0$.

4. $B=\begin{pmatrix}1&1&3&1\\2&1&4&3\\1&2&5&0\\5&4&13&6\end{pmatrix}$, $r_B=2$.

5. $A=\begin{pmatrix}1&1&1\\1&2&3\\1&4&9\end{pmatrix}$, $r_A=3$.

7. Kompyuterden paydalani'w:

```
> restart:
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

```
> A:=Matrix([[1,1,1],[1,2,3],[1,4,9]]);
```

$$A := \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

```
> rank(A);
```

3

>

8. Jan’a materiallardi’ bekkemlew: Yesaplar shi’g’ari’w ha’m wolardi’ kompyuterdegi na’tiyje menen sali’sti’ri’w.

9. U’yge tapsi’rmalar:

TAPSI’RMALAR

Matrisni’n’ rangin tabi’n’

1. $\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$

2. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$

3. $\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}$

5. $\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$

6. $\begin{pmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix}$

7. $\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$.

8. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 86 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$.

9. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$.

10. $\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}$.

11. $\begin{Vmatrix} 8 & 2 & 2 & -1 & 1 \\ 1 & 7 & 4 & -2 & 5 \\ -2 & 4 & 2 & -1 & 3 \end{Vmatrix}$

12. $\begin{Vmatrix} 1 & 7 & 7 & 9 \\ 7 & 5 & 1 & -1 \\ 4 & 2 & -1 & -3 \\ -1 & 1 & 3 & 5 \end{Vmatrix}$

13. $\begin{Vmatrix} 4 & 1 & 7 & -5 & 1 \\ 0 & -7 & 1 & -3 & -5 \\ 3 & 4 & 5 & -3 & 2 \\ 2 & 5 & 3 & -1 & 3 \end{Vmatrix}$

14. $\begin{Vmatrix} 8 & -4 & 5 & 5 & 9 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \\ 3 & -1 & 3 & 2 & 5 \end{Vmatrix}$

15. $\begin{Vmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{Vmatrix}$

16. $\begin{Vmatrix} 77 & 32 & 6 & 5 & 3 \\ 32 & 14 & 3 & 2 & 1 \\ 6 & 3 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 0 & 1 \end{Vmatrix}$,

17. $\begin{Vmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 \\ 1 & 4 & 1 & 1 \\ 5 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{Vmatrix}$

18. $\begin{Vmatrix} 3 & 1 & 1 & 2 & -1 \\ 0 & 2 & -1 & 1 & 2 \\ 4 & 3 & 2 & -1 & 1 \\ 12 & 9 & 8 & -7 & 3 \\ -12 & -5 & -8 & 5 & 1 \end{Vmatrix}$

19. $\begin{Vmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{Vmatrix}$

20. $\begin{Vmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{Vmatrix}$

21. $\begin{Vmatrix} 1 & 1 & 0 & 0...0 & 0 \\ 0 & 1 & 1 & 0...0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0...0 & 1 \end{Vmatrix}$,

22. $\begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 11 \end{pmatrix}$

23. $\begin{pmatrix} 1 & 0 & 2 & 0 & -3 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 2 & 0 & 4 & 0 & -6 & 0 \end{pmatrix}$

24. $\begin{Vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 3 \\ 0 & 0 & 0 & 3-\lambda \end{Vmatrix}$

λ parametrdin' ha'r qi'yli' ma'nislerinde kelesi matricalardi'n' rangi'n tabi'n'

24. $\begin{Vmatrix} 7-\lambda & -12 & 6 \\ 10 & -19-\lambda & 10 \\ 12 & -24 & 13-\lambda \end{Vmatrix}$

25. $\begin{Vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 3 \\ 0 & 0 & 0 & 3-\lambda \end{Vmatrix}$,

26. $\begin{Vmatrix} 3 & 4 & 2 & 2 \\ 3 & 17 & 7 & 1 \\ 1 & 10 & 4 & \lambda \\ 4 & 1 & 1 & 3 \end{Vmatrix}$

27. $\begin{Vmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{Vmatrix}$

28. $\begin{Vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-\lambda^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-\lambda^2 \end{Vmatrix}$

29. $\begin{Vmatrix} -\lambda & 1 & 2 & 3 & 1 \\ 1 & -\lambda & 3 & 2 & 1 \\ 2 & 3 & -\lambda & 1 & 1 \\ 3 & 2 & 1 & -\lambda & 1 \end{Vmatrix}$

30. $\begin{Vmatrix} \lambda & 1 & 2 & \ldots & n-1 & 1 \\ 1 & \lambda & 2 & \ldots & n-1 & 1 \\ 1 & 2 & \lambda & \ldots & n-1 & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 2 & 3 & \ldots & \lambda & 1 \\ 1 & 2 & 3 & \ldots & n & 1 \end{Vmatrix}$

Juwabi'.

1. 2 **2.** 3 **3.** 3 **4.** 2

5. $\lambda = 0$ de matrica rangi 2 ge ten', $\lambda \neq 0$ de wol 3 ge ten'.

6. $\lambda = 3$ de rang 2 ge ten', $\lambda \neq 3$ de rang 3 ge ten'.

7. 3 **8.** 2 **9.** 3 **10.** 2

11. 2 **12.** 3 **13.** 3 **14.** 3

15. 2 **16.** 4 **17.** 4 **18.** 4

19. 5 **20.** 5

21. Yeger n taq bolsa n, n jup bolsa $n=1$.

22. 2 **23.** 2

24. $\begin{cases} 1, \ yeger \ \lambda = 2, \\ 2, \ yeger \ \lambda = -1, \\ 3, \ yeger \ \lambda \neq -1. \end{cases}$

25. $\begin{cases} 2, \ yeger \ \lambda = 1, \\ 4, \ yeger \ \lambda = 2, \lambda = 3, \\ 4, \ yeger \ \lambda \neq -1, 2, 3. \end{cases}$

26. $\begin{cases} 2, \ yeger \ \lambda = 0, \\ 3, \ yeger \ \lambda \neq 0. \end{cases}$

27. $\begin{cases} 2, \ yeger \ \lambda = 3, \\ 3, \ yeger \ \lambda \neq 3. \end{cases}$

28. $\begin{cases} 3, \ yeger \ \lambda = \pm 1, \ \lambda = \pm 2, \\ 4, \ yeger \ \lambda \neq \pm 1, \ \lambda \neq \pm 2. \end{cases}$

29. $\begin{cases} 3, \ yeger \ \lambda = 0, \lambda = -2, \lambda = -4, \\ 4, \ yeger \ \lambda \neq 0, \lambda \neq -2, \lambda \neq -4. \end{cases}$

30. $\lambda = 1, 2, ... n$ bolg'anda n, qalg'an ma'nislerde $n+1$.

№6 A'MELİY JUMIS
TEMA: KERİ MATRİCA

1. **U'yge berilgen tapsi'rmalardi' tekseriw.**
2. **Jumi'sti'n' maqseti:** Keri matricani' yesaplawdi' ha'm woni' Maple integrallasqan sistemada programmalasti'ri'wdi' u`yreniw.
3. **Ma'selenin` qoyi'li'wi'**: Berilgen matricani'n' keri matricasi'n tabi'n'.
4. **Paydalani'lg'an a'debiyatlar**: [1-6].
5. **Paydalani'lg'an u'skeneler**: kompyuter.
6. **Metodikali'q ko'rsetpeler**:

Yeger $\Delta \neq 0$ bolsa, wonda A matricani'n' kerisi

$$A^{-1} = \begin{pmatrix} A_1/\Delta & A_2/\Delta & A_3/\Delta \\ B_1/\Delta & B_2/\Delta & B_3/\Delta \\ C_1/\Delta & C_2/\Delta & C_3/\Delta \end{pmatrix} \qquad (1)$$

boladi', bul jerde $A_i, B_i, C_i - (i = 1,2,3)$ sa'ykes a_i, b_i, c_i elementlerdi'n' algebrali'q toli'qti'ri'wshi'lari' boladi'. $A \cdot A^{-1} = E$ yekenligin tekseremiz.

$$AA^{-1} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \cdot \begin{pmatrix} A_1/\Delta & A_2/\Delta & A_3/\Delta \\ B_1/\Delta & B_2/\Delta & B_3/\Delta \\ C_1/\Delta & C_2/\Delta & C_3/\Delta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

7. Kompyuterden paydalani'w:

```
> restart:
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
> with(linalg):
> A := array( [[a,b],[c,d]] );
```

$$A := \begin{matrix} ? & a & & b & ? \\ ? & & & & ? \\ ? & & & & ? \\ ? & c & & d & ? \end{matrix}$$

```
> inverse(A);
```

$$\begin{bmatrix} \frac{d}{a\,d - b\,c} & -\frac{b}{a\,d - b\,c} \\ -\frac{c}{a\,d - b\,c} & \frac{a}{a\,d - b\,c} \end{bmatrix}$$

```
> A := array( [[5,-2],[-2,1]] );
```

$$A := \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

```
> inverse(A);
```

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

```
>
```

8. Jan’a materiallardi’ bekkemlew: Yesaplar shi’g’ari’w ha’m wolardi’ kompyuterdegi na’tiyje menen sali’sti’ri’w.

9. U’yge tapsi’rmalar:

TAPSI’RMALAR

Berilgen matricag’a keri bolg’an matricani’ tabi’n’

1. $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$

2. $\frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

3. $\begin{pmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 1 & -3 & 11 & -38 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

5. $\begin{pmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{pmatrix}$

6. $\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

7. $\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{Vmatrix}$

8. $\begin{Vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{Vmatrix}$

9. $\begin{Vmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{Vmatrix}$

10. $\begin{Vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{Vmatrix}$

11. $\begin{Vmatrix} 1 & 1 & \dots & 1 \\ 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{Vmatrix}$

12. $\begin{Vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1 \end{Vmatrix}$

13. $\begin{Vmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & -1 & 1 & 3 \\ 0 & 1 & 0 & 2 \end{Vmatrix}$

14. $\begin{Vmatrix} 2 & 3 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -2 \end{Vmatrix}$

15. $\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}$.

16. $\begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 2 \end{pmatrix}$.

17. $\begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 & 2 \end{pmatrix}$.

18. $\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \varepsilon & \varepsilon^2 & \dots & \varepsilon^{n-1} \\ 1 & \varepsilon^2 & \varepsilon^4 & \dots & \varepsilon^{2n-2} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \varepsilon^{n-1} & \varepsilon^{2n-2} & \dots & \varepsilon^{(n-1)^2} \end{pmatrix}$

19. $\frac{1}{n}\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \varepsilon^{-1} & \varepsilon^{-2} & \dots & \varepsilon^{-n+1} \\ 1 & \varepsilon^{-2} & \varepsilon^{-4} & \dots & \varepsilon^{-2n+2} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \varepsilon^{-n+1} & \varepsilon^{-2n+2} & \dots & \varepsilon^{-(n-1)^2} \end{pmatrix}$

20. $\begin{pmatrix} 2\cos & 1 & 0 & \dots & 0 \\ 1 & 2\cos x & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 2\cos x \end{pmatrix}$

21. Kvadrat $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$ matricani'n' kerisin tabi'n', bul jerde A, C-ayni'mag`an matricalar.

22. Kvadrat $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$ matricani'n' kerisin tabi'n', bul jerde A, C- ayni'mag`an matricalar

23. $\begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} X = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$

24. $X \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 6 & 2 \end{pmatrix}$

25. $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$

26. $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$

27. $X \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}$

28. $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} X \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

Sistemani' sheshin'

29. $\begin{cases} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} X + \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} Y = \begin{pmatrix} 2 & 8 \\ 0 & 5 \end{pmatrix} \\ \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix} X + \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} Y = \begin{pmatrix} 4 & 9 \\ -1 & -4 \end{pmatrix} \end{cases}$

30. $\begin{cases} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} X + \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} Y = \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} X + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} Y = \begin{pmatrix} 1 & 1 \\ 5 & 3 \end{pmatrix} \end{cases}$

Juwabi'.

1. $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$

2. $\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

3. $\begin{pmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 1 & -3 & 11 & -38 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

5. $\begin{pmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{pmatrix}$

6. $\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

7. $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

8. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

9. $\begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

10. $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/3 \\ 0 & 1/2 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$

11. $\begin{pmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & -1 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{pmatrix}$.

12. $\begin{pmatrix} 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & \ldots & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & -1 & 1 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 1 & -1 & 1 \end{pmatrix}$.

13. $\begin{pmatrix} -3 & 2 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 8 & -9/2 & 1 & -3/2 \end{pmatrix}$.

14. $\begin{pmatrix} -1 & 3 & -8 & 3 \\ 1 & -4 & 4 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

15. $\frac{1}{n-1}\begin{pmatrix} 2-n & 1 & 1 & \ldots & 1 \\ 1 & 2-n & 1 & \ldots & 1 \\ 1 & 1 & 2-n & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 1 & 1 & \ldots & 1 \end{pmatrix}$.

16. $\begin{pmatrix} n & n-1 & n-2 & \ldots & 1 \\ n-1 & n-2 & n-2 & \ldots & 1 \\ n-2 & n-2 & n-2 & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 1 & 1 & \ldots & 1 \end{pmatrix}$.

17. $\frac{1}{n+1}\begin{pmatrix} 1\cdot n & 1\cdot(n-1) & 1\cdot(n-2) & \ldots & 1\cdot 1 \\ 1\cdot(n-1) & 2\cdot(n-1) & 2\cdot(m-2) & \ldots & 2\cdot 1 \\ 1\cdot(n-2) & 2\cdot(n-2) & 3\cdot(n-2) & \ldots & 3\cdot 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1\cdot 1 & 2\cdot 1 & 3\cdot 1 & \ldots & n-1 \end{pmatrix}$.

18. $\frac{1}{n}\begin{pmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & \varepsilon^{-1} & \varepsilon^{-2} & \ldots & \varepsilon^{-n+1} \\ 1 & \varepsilon^{-2} & \varepsilon^{-4} & \ldots & \varepsilon^{-2n+2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \varepsilon^{-n+1} & \varepsilon^{-2n+2} & \ldots & \varepsilon^{-(n-1)^2} \end{pmatrix}$

19. $\frac{1}{1-a^2}\begin{pmatrix} 1 & -a & 0 & \ldots & 0 \\ -a & 1+a^2 & -a & \ldots & 0 \\ 0 & -a & 1+a^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1+a^2 & -a \\ 0 & 0 & \ldots & -a & 1 \end{pmatrix}$

20. $\frac{1}{\sin x \sin(n+1)x} \times \begin{pmatrix} \sin x \sin x & \sin x \sin(n-1)x & \ldots & \sin^2 x \\ \sin x \sin(n-1)x & \sin 2x \sin(n-1)x & \ldots & \sin 2x \sin x \\ \ldots & \ldots & \ldots & \ldots \\ \sin^2 x & \sin 2x \sin x & \ldots & \sin nx \sin x \end{pmatrix}$

21. $\begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$

22. $\begin{pmatrix} A^{-1} & -A^{1}BC^{-1} \\ 0 & C^{-1} \end{pmatrix}$

23. $\begin{pmatrix} a & b \\ 2a-1 & 2b-3 \end{pmatrix}, \ (a,b \in R)$

24. $\varnothing$

25. $\begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix}$

26. $\begin{pmatrix} 2 & -23 \\ 0 & 8 \end{pmatrix}$

27. $\begin{pmatrix} -3 & 2 & 0 \\ -4 & 5 & -2 \\ -5 & 3 & 0 \end{pmatrix}$

28. $\begin{pmatrix} 24 & 13 \\ -34 & -18 \end{pmatrix}$

29. $X = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}, \ Y = \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$

30. $X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \ Y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

№7 A’MELİY JUMIS
TEMA: SIZIQLI TEN’LEMELER SİSTEMALARIN SHESHİW

1. **U’yge berilgen tapsi’rmalardi’ tekseriw**
2. **Jumi’sti’n’ maqseti:** Si’zi’qli’ ten’lemeler sistemalari’n Kramer qag’i’ydasi’ ha’m Gauss usi’li’ menen sheshiwdi ha’m wolardi’ Maple integrallasqan sistemada programmalasti’ri’wdi’ u`yreniw.
3. **Ma’selenin` qoyi’li’wi’:** Berilgen si’zi’qli’ ten’lemeler sistemalari’n Kramer qag’i’ydasi’ ha’m Gauss usi’li’ menen sheshin’.
4. **Paydalani’lg’an a’debiyatlar:** [1-6].
5. **Paydalani’lg’an u’skeneler:** kompyuter.
6. **Metodikali’q ko’rsetpeler:**

Determinantti’n’ ja’rdemi menen yeki belgisizli yeki ten’lemelerdin’ si’zi’qli’ sistemasi’n sheshiw qolayli’.

$$\left.\begin{aligned} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{aligned}\right\} \qquad (1)$$

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ sistemani’n’ determinanti’n ja’ne qosi’msha

$$\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

determinantlardi’ kiritemiz.

(1) sistemani’n’ sheshimin tabi’w ushi’n joq yetiw usi’li’n paydalanami’z. (1) sistemani’n’ birinshi ten’lemesin b_2 ge, al yekinshisin $-b_1$ ko’beytip qossaq

$$(a_1b_2 - a_2b_1)x = c_1b_2 - c_2b_1 \qquad (2)$$

alami’z.

Usi’g’an uqsas (1) nin’ birinshi ten’lemesin $-a_2$ ge, al yekinshisin a_1 ge ko’beytip qossaq

$$(a_1b_2 - a_2b_1)y = a_1c_1 - a_2c_1 \qquad (3)$$

ti alami’z.

Belgilewimiz boyi’nsha (2) ha’m (3) formulalar

$$\Delta x = \Delta_x, \quad \Delta y = \Delta_y \qquad (4)$$

ko’rinisti aladi’.

Yeger $\Delta \neq 0$ bolsa, wonda (1) sistema

$$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta} \tag{5}$$

bir sheshimge iye.

(5) formulani’ Kramer formulasi’ dep aytadi’.

1-Mi’sal. Berilgen

$$\left.\begin{aligned} 7x - 6y = 5 \\ 8x - 7y = -10 \end{aligned}\right\} \tag{6}$$

sistemani’ sheshin’.

Sheshimi. $\Delta = \begin{vmatrix} 7 & -6 \\ 8 & -7 \end{vmatrix} = -49 + 48 = -1,$

$$\Delta x = \begin{vmatrix} 5 & -6 \\ -10 & -7 \end{vmatrix} = -35 - 60 = -95,\ \Delta y = \begin{vmatrix} 7 & 5 \\ 8 & -10 \end{vmatrix} = -70 - 40 = -110$$

Kramer formulasi’ boyi’nsha

$$x = \frac{\Delta x}{x} = \frac{-95}{-1} = 95,\ y = \frac{\Delta y}{\Delta} = \frac{-110}{-1} = 110$$

di’ alami’z.

U’sh belgisizli si’zi’qli’ ten’lemeler sistemasi’

U’sh belgisizli si’zi’qli’ ten’lemeler sistemasi’

$$\left.\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{aligned}\right\} \tag{7}$$

ko’riniske iye.

Berilgen

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \tag{8}$$

sistemani’n’ determinanti’na

$$\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},\ \Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\ \Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \tag{9}$$

qosi’msha determinantlardi’ kiritemiz.

Yekinshi ta’rtipli ten’lemeler sistemasi’na uqsas

$$\Delta x = \Delta_x,\ \Delta y = \Delta_y,\ \Delta z = \Delta_z \tag{10}$$

ali’nadi’.

Yeger sistema determinanti’ $\Delta \neq 0$ bolsa, wonda (7) sistema bir sheshimge iye.

$$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta} \tag{11}$$

(11) qatnasqa Kramer qag’i’ydasi’ delinedi.

Yeger sistema determinanti’ $\Delta = 0$ bolsa, wonda sistema (1) ha’m (7) ya birgelikli yemes, ya sheksiz ko’p sheshimge iye.

2-Mi’sal. $\left.\begin{array}{l} x + 2y + 3z = 1 \\ 2x + 3y + z = 0 \\ 3x + y + 2z = 0 \end{array}\right\}$ sistemani’ sheshin’.

Sheshimi.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1\cdot 3\cdot 2 + 2\cdot 1\cdot 3 + 3\cdot 2\cdot 1 - 3\cdot 3\cdot 3 - 1\cdot 1\cdot 1 - 2\cdot 2\cdot 2 =$$

$$= 6 + 6 + 6 - 27 - 1 - 8 = -18 \neq 0.$$

$$\Delta_x = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 1\cdot 3\cdot 2 + 2\cdot 1\cdot 0 + 3\cdot 0\cdot 1 - 0\cdot 3\cdot 3 - 1\cdot 1\cdot 1 - 2\cdot 0\cdot 2 = 6 - 1 = 5.$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 0 & 1 \\ 3 & 0 & 2 \end{vmatrix} = 1\cdot 0\cdot 2 + 1\cdot 1\cdot 3 + 3\cdot 2\cdot 0 - 3\cdot 0\cdot 3 - 1\cdot 1\cdot 0 - 1\cdot 2\cdot 2 = 3 - 4 = -1.$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 3 & 1 & 0 \end{vmatrix} = 1\cdot 3\cdot 0 + 2\cdot 0\cdot 3 + 1\cdot 2\cdot 1 - 1\cdot 3\cdot 3 - 1\cdot 0\cdot 1 - 2\cdot 0\cdot 2 = 2 - 9 = -7.$$

Kramer qag`iydasi'n qollani'p,

$$x = -\frac{5}{18},\ \ y = \frac{1}{18},\ \ z = \frac{7}{18}$$

lerdi alami'z.

Bir tekli u`sh si'zi'qli' ten'lemeler sistemasi'

$$\left.\begin{aligned} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3x + b_3y + c_3z = 0 \end{aligned}\right\} \qquad (12)$$

ko'rinistegi sistemag'a bir tekli delinedi.

(12) sistema $x = 0, y = 0, z = 0,$ qanaatlandi'radi' ha'm birgelikte boladi'.

1-Teorema. *Bir tekli u`sh si'zi'qli' ten'lemeler sistemasi' nol yemes sheshimge iye delinedi sonda, tek sonda g'ana, yeger wonin' determinanti' nolge ten' bolsa, yag'ni'y*

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \qquad (13)$$

Da'lillew. (12) nol yemes $(x_1y_1z_1)$ sheshimge iye bolsi'n.. Yeger $\Delta \neq 0$ bolsa, wonda Kramer qag'i'ydasi' boyi'nsha nol yemes sheshim boladi'. Bul ayti'mg'a qarsi'. Sonli'qtan $\Delta = 0$.

Meyli $\Delta = 0$ bolsi'n. Sonda (12) yamasa birgelikli yemes, yamasa sheksiz ko`p sheshimge iye. Bizin` sistemami'z birgelikli, sonli'qtan nolge ten' yemes sheshimge iye. Solay yetip, (12) sistema sheksiz ko'p sheshimge iye, soni'n' ishinde nolge ten' yemes sheshimge ha'm iye.

Ko’p wo’zgeriwshili si’zi’qli’ ten’lemeler sistemasi’. Gauss usi’li’

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{14}x_n &= a_{1,n+1} \\ a_2x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= a_{2,n+1} \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= a_{n,n+1} \end{aligned}\right\} \quad (14)$$

$i-$ten’leme nomeri, $j-$belgisiz wo’zgeriwshilerdin’ nomeri, saltan ag’za $a_{i,n+1}\ (i=1,2,\ldots\ldots,n)$. Gauss usi’li’ boyi’nsha (14) ten’lemeler sistemasi’n sheshemiz. Meyli $a_{11}\neq 0$ bolsi’n. (14) degi birinshi ten’lemeni a_{11}ge bo’lemiz sonda

$$a_{11}\neq 0 \quad x_1 + x_{12}x_2 + \ldots\ldots + x_{1n}\,x_n = x_{1,n+1} \quad (15)$$

$$x_{1j} = \frac{a_{1j}}{a_{11}}\ (j=1,2,\ldots\ldots,n+1) \quad (16)$$

payda boladi’. Yendi $i-$ ten’lemeni ko’remiz

$$a_{i1}x_1 + a_{i2}x + \cdots + a_{ij}x_n = a_{i,}n+1 \quad (17)$$

(15) di a_{ij} ge ko’beytemiz ha’m woni’ (17) ten alami’z. Sonda (18, 20) payda boladi’.

$$a_{i2}^{(1)}x_2 + \cdots + a_{in}^{(1)}x_n = a_i^{(1)}n+1 \quad (18)$$

$$a_{ij}^{(1)} = a_{ij} - a_{1j}a_{ij}/a_{11}\quad (j=2,3,\ldots,n+1) \quad (19)$$

$$\left.\begin{aligned} a_{22}^{(1)}x_2 + \cdots + a_{2n}^{(1)}{}_{2n}x_n &= a_{2,n+1}^{(1)}, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{n2}^{(1)}x_2 + \cdots + a_{nn}^{(1)}x_n &= a_{n,n+1}^{(1)} \end{aligned}\right\} \quad (20)$$

Bul process usi’lay $n-1$ ret dawam yete beredi. Solay yetip, Gauss usi’li’ni’n’ tuwri’ joli’ to’mendegi ko’riniste boladi’:

$$\left.\begin{aligned} a_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= a_{1,n+1} \\ x_2 + \cdots + a_{2n}^{(1)}x_n &= a_{2,n+1}^{(1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ x_n &= a_{n,n+1}^{(n-1)} \end{aligned}\right\} \quad (21)$$

Keri joli'nda aqi'rg'i' belgisizden baslap barli'q belgisizler to'mendegishe tabi'ladi':

$$\begin{cases} x_n = a_{n,n+1}^{(n-1)} \\ x_{n-1} = a_{n-1,n+1}^{(n-2)} - a_{n-1,n}^{(n-2)} \\ \cdots\cdots\cdots\cdots\cdots \\ x_1 = a_{1,n+1} - a_{1n}x_n \cdots - a_{1,n-1}x_{n-1} - \cdots - a_{12}x_2 \end{cases} \qquad (22)$$

3-Mi'sal. Ten'lemeler sistemasi'n sheshin'.

$$\left.\begin{aligned} 2x_1 - 3x_2 + 4x_3 &= 20, \\ 3x_1 + 4x_2 - 2x_3 &= -1, \\ 4x_1 + 2x_2 + 3x_3 &= 9 \end{aligned}\right\} \qquad (23)$$

Sheshimi.

$$\begin{aligned} x_3 &= 3 \\ x_2 - 0{,}625x_3 &= -3{,}875 \\ x_1 - 1{,}5x_2 - 2x_3 &= 10. \\ x_3 &= 3 \\ x_3 &= -3{,}875 + 0{,}625 \cdot 3 = -2 \\ x_1 &= 10 - 2 \cdot 3 + 1{,}5 \cdot (-2) = 1 \end{aligned}$$

7. Kompyuterden paydalani'w:

1.

```
> restart:
> with(linalg);
```

Warning, the protected names norm and trace have been redefined and unprotected

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith,*

stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

> A:=Matrix([[2,-1,-1],[3,4,-2],[3,-2,4]]);

$$A := \begin{bmatrix} 2 & -1 & -1 \\ 3 & 4 & -2 \\ 3 & -2 & 4 \end{bmatrix}$$

> Dt:=det(A);

$$Dt := 60$$

> R:=rank(A);

$$R := 3$$

> B:=vector([4,11,2]);

$$B := [4, 11, 2]$$

> X:=linsolve(A,B);

$$X := \left[\frac{21}{10}, \frac{17}{20}, \frac{-13}{20} \right]$$

2-usi'l. Si'zi'qli' ha'm si'zi'qli' yemes ten'lemeler sistemasi'n sheshiw

> eqs1:={2*x-y-z=4,3*x+4*y-2*z=11,3*x-2*y+4*z=2};

$$eqs1 := \{2x - y - z = 4, 3x + 4y - 2z = 11, 3x - 2y + 4z = 2\}$$

> a1:=solve(eqs1,{x,y,z});

$$a1 := \left\{ x = \frac{21}{10}, z = \frac{-13}{20}, y = \frac{17}{20} \right\}$$

>

2.

> restart:
> with(linalg):
> A:=Matrix([[7,-6],[8,-7]]);

$$A := \begin{bmatrix} 7 & -6 \\ 8 & -7 \end{bmatrix}$$

> B:=Matrix([[5],[-10]]);

$$B := \begin{bmatrix} 5 \\ \\ -10 \end{bmatrix}$$

```
> Ax:=Matrix([[5,-6],[-10,-7]]);
```

$$Ax := \begin{bmatrix} 5 & -6 \\ \\ -10 & -7 \end{bmatrix}$$

```
> Ay:=Matrix([[7,5],[8,-10]]);
```

$$Ay := \begin{bmatrix} 7 & 5 \\ \\ 8 & -10 \end{bmatrix}$$

```
> x:=det(Ax)/det(A);
```

$$x := 95$$

```
> y:=det(Ay)/det(A);
```

$$y := 110$$

3.

```
> restart:
> with(linalg):
> eqs1:={2*x-3*y+4*z=20,3*x+4*y-2*z=-1,4*x+2*y+3*z=9};
```

$$eqs1 := \{2x - 3y + 4z = 20, 3x + 4y - 2z = -1, 4x + 2y + 3z = 9\}$$

```
> a1:=solve(eqs1,{x,y,z});
```

$$a1 := \left\{ x = \frac{213}{43}, y = \frac{-186}{43}, z = \frac{-31}{43} \right\}$$

```
>
```

8. Jan’a materiallardi’ bekkemlew: Yesaplar shi’g’ari’w ha’m wolardi’ kompyuterdegi na’tiyje menen sali’sti’ri’w.

9. U’yge tapsi’rmalar:

TAPSIRMALAR

Si’zi’qli’ ten’lemeler sistemasi’n sheshin’

1. $\begin{cases} x+2y+3z=14 \\ y+2z+3z=20 \\ z+2t+3x=14 \\ t+2x+3y=12 \end{cases}$, $y-?$

2. $\begin{cases} y-3z+4t=-5 \\ x-2z+3t=-4 \\ 3x+2y-5t=12 \\ 4x+3y-5z=5 \end{cases}$

3. $\begin{cases} x-3y+5z-t=12 \\ 3x-5y+7z-t=0, \\ 5x-7y+z-3t=4 \\ 7x-y+3x-5t=16 \end{cases}$

4. $\begin{cases} x-2y=5 \\ 3y+4z=18 \\ 5z+6u=39 \\ 7u+8v=68 \\ 9v+10x=55 \end{cases}$

5. $\begin{cases} 2x+3y+2z=9 \\ x+2y-3z=14 \\ 3x+4y+z=16 \end{cases}$

6. $\begin{cases} 2x_1-x_2-x_3=4 \\ 3x_1+4x_2-2x_3=11 \\ 3x_1-2x_2+4x_3=11 \end{cases}$

7. $\begin{cases} x_1+x_2+2x_3=-1 \\ 2x_1-x_2+2x_3=-4 \\ 4x_1+x_2+4x_3=-2 \end{cases}$

8. $\begin{cases} 3x_1+2x_2+x_3=5 \\ 2x_1+3x_2+x_3=1 \\ 2x_1+x_2+3x_3=11 \end{cases}$

9. $\begin{cases} x_1+2x_2+4x_3=31 \\ 5x_1+x_2+2x_3=29 \\ 3x_1-x_2+x_3=10 \end{cases}$

10. $\begin{cases} x_1+x_2+2x_3+3x_4=4 \\ 3x_1-x_2-x_3-2x_4=-4 \\ 2x_1+3x_2-x_3-x_4=-6 \\ x_1+2x_2+3x_3-x_4=-4 \end{cases}$

11. $\begin{cases} x_1+2x_2+3x_3-2x_4=6 \\ 2x_1-x_2-2x_3-3x_4=8 \\ 3x_1+2x_2-x_3+2x_4=4 \\ 2x_1-3x_2+2x_3+x_4=-8 \end{cases}$

12. $\begin{cases} x_1+2x_2+3x_3+4x_4=5 \\ 2x_1+x_2+2x_3+3x_4=1 \\ 3x_1+2x_2+x_3+2x_4=1 \\ 4x_1+3x_2+2x_3+x_4=-5 \end{cases}$

13. $\begin{cases} 2x_1-x_2+3x_3+2x_4=4 \\ 3x_1+3x_2+3x_3+2x_4=6 \\ 3x_1-x_2-x_3+2x_4=6 \\ 3x_1-x_2+3x_3+20x_4=6 \end{cases}$

14. $\begin{cases} x_1+x_2+x_3+x_4=0 \\ x_2+x_3+x_4+x_5=0 \\ x_1+2x_2+3x_3=2 \\ x_2+2x_3+3x_4=-2 \\ x_3+2x_4+3x_5=2 \end{cases}$

15. $\begin{cases} x_1+2x_2-3x_3+4x_4-x_5=-1 \\ 2x_1-x_2+3x_3-4x_4+2x_5=8 \\ 3x_1+x_2-x_3+2x_4-x_5=3 \\ 4x_1+3x_2+4x_3+2x_4+2x_5=-2 \\ x_1-x_2-x_3+2x_4-3x_5=-3 \end{cases}$

16. $\begin{cases} 2x_1+2x_2-x_3+x_4=4 \\ 4x_1+3x_2-x_3+2x_4=6 \\ 8x_1+5x_2-3x_3+4x_4=12 \\ 3x_1+3x_2-2x_3+2x_3=6 \end{cases}$

17. $\begin{cases} 2x_1+3x_2+11x_3+5x_4=2 \\ x_1+x_2+5x_3+2x_4=1 \\ 2x_1+x_2+3x_3+2x_4=-3 \\ x_1+x_2+3x_3+4x_4=-3 \end{cases}$

18. $\begin{cases} 2x_1+5x_2+4x_3+x_4=20 \\ x_1+3x_2+2x_3+x_4=11 \\ 2x_1+10x_2+9x_3+7x_4=40 \\ 3x_1+8x_2+9x_3+2x_4=37 \end{cases}$

19. $\begin{cases} 3x_1+4x_2+x_3+2x_4+3=0 \\ 3x_1+5x_2+3x_3+5x_4+6=0 \\ 6x_1+8x_2+x_3+5x_4+8=0 \\ 3x_1+5x_2+3x_3+7x_4+8=0 \end{cases}$

20. $\begin{cases} 7x_1+9x_2+4x_3+2x_4-2=0 \\ 2x_1-2x_2+x_3+x_4-6=0 \\ 5x_1+6x_2+3x_3+2x_4-3=0 \end{cases}$

21. $\begin{cases} 6x+5y-2z+4t+4=0 \\ 9x-y+4z-t-13=0 \\ 3x+4y+2z-2t-1=0 \\ 3x-9y+2t-11=0 \end{cases}$

22. $\begin{cases} 2x-y-6z+3t+1=0 \\ 7x-4y+2z-15t+32=0 \\ x-2y-4z+9t-5=0 \\ x-y+2z-6t+8=0 \end{cases}$

23. $\begin{cases} 2x+y+4z+8t=-1 \\ x+3y-6z+2t=3 \\ 3x-2y+2z-2t=8 \\ 2x-y+2z=4 \end{cases}$

24. $\begin{cases} 2x-y+3z=9 \\ 3x-5y+z=-4 \\ 4x-7y+z=5 \end{cases}$

25. $\begin{cases} 2x-5y+3z+t=5 \\ 3x-7y+3z-t=-1 \\ 5x-9y+6z+2t=7 \\ 4x-6y+3z+t=8 \end{cases}$

26. $\begin{cases} 2x+3y+5z=10 \\ 3x+7y+4z=3 \\ x+2y+2z=3 \end{cases}$

27. $\begin{cases} 5x-6y+4z=3 \\ 3x-3y+2z=2 \\ 4x-5y+2z=1 \end{cases}$

28. $\begin{cases} 4x-3y+2z+4=0 \\ 6x-2y+3z+1=0 \\ 5x-3y+2z+3=0 \end{cases}$

Juwabi'.

1. $y=2$
2. $x=1,\ y=2,\ z=1,\ t=-1$
3. $x=1,\ y=1,\ z=0,\ t=-2$
4. $x=1,\ y=2,\ z=3,\ u=4,\ v=5$
5. $x=2,\ y=3,\ z=-2$
6. $x_1=3,\ x_2=x_3=1$
7. $x_1=1,\ x_2=2,\ x_3=-2$
8. $x_1=2,\ x_2=-2,\ x_3=3$

9. $x_1 = 3,\ \ x_2 = 4,\ \ \ \ x_3 = 5$

10. $x_1 = x_2 = -1,\ x_3 = 0,\ x_4 = 1$

11. $x_1 = 1,\ x_2 = 2, x_3 = -1, x_4 = -2$

12. $x_1 = -2, x_2 = 2,\ x_3 = -3,\ x_4 = 3$

13. $x_1 = 2,\ \ x_2 = x_3 = x_4 = 0$

14. $x_1 = 1,\ x_2 = -1, x_3 = 1,\ x_4 = -1,\ \ x_5 = 1$

15. $x_1 = 2,\ x_2 = 0,\ x_3 - 2,\ x_4 = -2, x_5 = 1$

16. $x_1 = x_2 = 1,\ x_3 = x_4 = -1$

17. $x_1 = -2,\ \ x_2 = 0,\ \ x_3 = 1,\ \ x_4 = -1$

18. $x_1 = 1,\ \ x_2 = x_3 = 2, x_4 = 0$

19. $x_1 = 2, x_2 = -2, x_3 = 1, x_4 - 1$

20. $x_1 = -0,\ 4,\ x_2 = -1, 2,\ x_3 = 3, 4,\ x_4 = 1$

21. $x = 2/3,\ y = -1,\ \ z = 3/2,\ \ t = 0$

22. $x = -3,\ \ y = 0,\ \ z = -1/2,\ \ t = 2/3$

23. $x = 2,\ \ y = -3,\ \ \ z = -3/2,\ \ t = 1/2$

24. $\varnothing$.

25. $\varnothing$

26. $x = 3,\ y = -2,\ z = 2$

27. $x = y = z = 1$

28. $x = 1,\ y = 2,\ \ z = -1$

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