**O’ZBEKİSTON RESPUBLİKASİ ALOQA, AXBOROTLASHTİRİSH VA TELEKOMMUNİKATSİYA TEXNOLOGİYALARİ DAVLAT QO’MİTASİ**

**TOSHKENT AXBOROT TEXNOLOGİYALARİ UNİVERSİTETİ NUKUS FİLİALİ**

**Komp`yuter injiniring fakul`teti**

**Axborot ta`lim texnologiyalari kafedrasi**

«Kasb ta`limi» yunalishining 4- kurs talabasi

ARTIKOVA JAMILA SAPARBAEVNANING

# «ODDİY DİFFERENTSİAL TENGLAMA UCHUN KOSHİ MASALASİNİ MATCAD SISTEMASIDA ECHİSH»

**mavzusidagi**

# BİTİRUV MALAKAVİY İSHİ

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Bitiruv malakaviy ishi kafedradan dastlabki himoyadan o’tdi
__ sonli bayonnamasi «_____» __________________ 2014 yil

Nukus-2014

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**KOMPYUTER INJINIRINGI FAKULTETI**

**«AXBOROT TA'LIM TEXNOLOGIYALARI » KAFEDRASI**

**TACDIQLAYMAN**
Kafedra mudiri________
«____» __________ 2014 .y.

**Artikova Jamila Saparbaevnaning**
**«Oddiy differentsial tenglama uchun koshi masalasini matcad sistemasida echish»** mavzusidagi bitiruv malakaviy ishiga

## VAZIFA

**Bitiruv malakaviy ishning mavzusi'** TATU NF-ning «____» __________ 2014y buyrug'i bilan tasdiqlandi.
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Kirish.
§1. Differentsial tenglama tushinchasi
§2. Koshi masalasi va o'ning quyilishi
§3. Oddiy differentsial tenglamaga qo'yilgan chegaraviy masalalarni sonli echish
Xulosa
Foydalanilgan adabiyotlar

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**Bitiruv malakaviy ishi paragraflar bo'yicha maslaxatchilar**.

| Paragraflar | Maslaxatchi | Imzo, sana | |
|---|---|---|---|
| | | Vazifani berdi | Vazifani qabulladi |
| 1 | Allamuratov Sh.Z | | |
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| 3 | Allamuratov Sh.Z | | |

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| 1. | Differentsial tenglama tushinchasi | 10.02.2014 | |
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| 3. | Oddiy differentsial tenglamaga qo'yil-gan chegaraviy masalalarni sonli echish | 10.04.2014 | |
| 4. | Titul beti , mundarija, adabiyotlar ruyxati, annotatsiya | 10.05.2014 | |
| 5. | POWER POINT dasturida dokladning slaydini yaratish | 10.06.2014 | |

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## АННОТАЦИЯ

В данной выпускной квалификационной работе рассмотрены обыкновенные дифференциальные уравнения первого порядка и постановка задачи Коши. Кроме того приведены приближенные методы решения обыкновенных дифференциальных уравнении с граничными условиями в системе MathCad.

## ANNOTATSIYA

Mazkur bitiruv malakaviy ishida birinchi tartibli oddiy differentsial tenglama va unga qo'yilgan Koshi masalasi qaralgan. Bundan tashqari oddiy differentsial tenglamaga qoyilgan chegaraviy shartlarda taqriybiy echish usullari MathCad tizimida keltirilgan.

## SUMMARY

In the given final qualifying work the ordinary differential equations of the first order and statement of a task Koshi are considered. Besides the approached methods of the decision the ordinary differential equation with boundary conditions in system MathCad are given.

Mundarija

## Kirish

Ta`limni tarbiyadan, tarbiyani esa ta`limdan ajratib bo'lmaydi-bu sharqona qarash, sharqona haet falsafasi.

Bu haqida fikr yuritganda, men Abdulla Avloniyning "Tarbiya biz uchun e haet-e mamot, e najot-e halokat, e saodat-e falokat masalasidir" degan chuqur ma`noli so'zlarini eslayman.

Shuning uchun ham mustaqillikning dastlabki yillaridanoq butun mamlakat miqesida ta`lim va tarbiya, ilm-fan, kasb-hunar o'rgatish tizimlarini tubdan isloh qilishga nihoyatda katta zarurat sezila boshladi. Kadrlar tayerlash milliy dasturini ishlab chiqish bilan bog`lik jaraen uzoq yillar mobaynida bu sohada talay muommalar yig`ilib qolganini ko'rsatdi. Shuning uchun ham bu og`ir, mas`uliyatli, ammo hal qilishni aslo paysalga solib bo'lmaydigan ishni qadamba-qadam, izchillik bilan bajarishga bel bog`ladik. [1]

**Mavzuning dolzarbligi.** Tabiatshunoslik va texnikaning kupgina masalalari qaralaetgan xodisa eki jaraenning urganaetgan noma`lum funktsiyani topishga keltiriladi. Bu uz navbatida berilgan boshlang'ich va chegaraviy shartlarda diffferentsial tenglama echishga olib keladi.

Ko`p hollarda esa differentsial tenglamalarga qo'yilgan chegaraviy shartlarda echim har doim ham analitik usılda olina vermaydi va echim taqriybiy sonli usullar yordamida olinadi. Taqriybiy usullar etarlicha urganilmagan. Shu boistan mavzu ilmiy va amaliy jihattan dolzarb deb hisoblash mumkin.

**Tadqiqot obyekti.** Oddiy differentsial tenglamalar va ularga qoyilgan chegaraviy masalalar bitiruv malakaviy ishining tadqiqot obyektidir. Oddiy differentsial tenglamalarni taqribiy yechish usullari yetarlicha mufassal [2,3,5,6,9-16] adabiyotlarda keltirilgan.

**Ishning amaliy ahamiyati.** Bitiruv malakaviy ishidan «Hisoblash matematikasi» va «Hisoblash usullari» fanlaridan bo'ladigan amaliy va laboratoriya mashg’ulotlarida, nochiziqli tenglamalarni sonli yechish bilan bog’liq tanlov fanlari mashg’ulotlarida foydalanish mumkin.

Masalan: Massasi *m* bulgan moddiy nuqta og’irlik kuchi ta`sirida erkin tushmoqda. Havoning qarshiligini hisobga olmay, bu moddiy nuqtaning harakat qonunini topish talab qilinsin [2].

Moddiy nuqtaning vaziyati $OM=S$ koordinata bilan aniqlanib, u $t$ vaqtga bog’lik xolda uzgaradi. N`yutonning ikkinchi qonuniga asosan:

$$ma = F$$

$m$ - moddiy nuqtaning massasi

$a$ - tezlanish

$F$ - ta`sir etuvchi kuch

Moddiy nuqtaga faqat og’irlik kuchi ta`sir etadi:

$$F = m\cdot g$$

$g$ – og’irlik kuchi tezlanishi

$a$ - tezlanish - ikkinchi tartibli xosila degani

$$m\frac{d^2S}{dt^2} = mg \text{ eki } \frac{d^2S}{dt^2} = g \qquad (1)$$

(1) noma`lum funktsiyani ikkinchi tartibli xosilasi qatnashgan tenglamadir.

Agar bu tenglamani $t$ buyicha 2 marta integrallasak, biz izlaetgan funktsiyani topishimiz mumkin:

$$\frac{dS}{dt} = gt + c_1 \qquad (2)$$

$$S = \frac{gt^2}{2} + c_1 t + c_2 \qquad (3)$$

Bu erda $s_1$ va $s_2$ integrallash doimiysi qatnashadi. Ularni nuqtaning boshlangich xolati va boshlangich tezligini bilgan holda aniqlash mumkin [10],

$t=0$ da moddiy nuqtaning tezligi $V_0$ ga, uning sanoq boshi 0 dan uzoqligi esa $S_0$ ga teng bulsin. $S_1=V_0$ ni (2) dan topamiz. $S_2=S_0$ ni (3) dan topamiz. U holda harakat konuni:

$$S = \frac{dt^2}{2} + V_0 t + S_0 \qquad \text{buladi.}$$

**Ishning tuzilishi.** Mazkur bitiruv malakaviy ish kirish uchta paragraf, xulosa va foydalanilgan adabiyotlar tizimidan iborat bo'lib jami 50 betdan iborat.

Birinchi paragrafta differentsial tenglama haqida umumiy tushinchalar, ikkinchi paragrafta Koshi masalasi va o'ning quyilishi, Adams ekstrapolyatsion va interpolyatsion metodlari, uchinchi paragrafta esa oddiy differentsial tenglamaga qo'yilgan chegaraviy masalalarni echish usullaridan kollokatsiya, haydash va Galerkin usullari qaralgan. 3.5 bandda Koshi masalasi uchun MathCad tizimida dasturiy ta'minot yaratilgan.

## §1. Differentsial tenglama tushinchasi

**Ta`rif:** Erkli uzgaruvchi va noma`lum funktsiya xamda uning xosilalari eki differentsiallarini bog'lovchi munosabat differentsial tenglama deyiladi.

Agar noma`lum funktsiya fakat bitta uzgaruvchiga boglik bulsa, bunday differentsial tenglama oddiy differentsial tenglama deyiladi.

Agar noma`lum funktsiya ikki eki undan ortik uzgaruvchiga boglik bulsa, bunday differentsial tenglamani xususiy xosilali differentsial tenglama deyiladi.

**Ta`rif:** Differentsial tenglamaga kirgan xosilalarning eng yuqori tartibi tenglamaning tartibi deyiladi.

$y'' - y' cosx - x^2 y = 0$ - ikkinchi tartibli differentsial tenglama.

$x(1 - y^2)dx + y(1 + x^2)dy = 0$ - birinchi tartibli differentsial tenglama.

$$F(x, y, y', y'', ..., y^{(n)}) = 0 \qquad (4)$$

bu erda $x$ - erkli uzgaruvchi, $y$ - noma`lum funktsiya

$y', y'', ..., y^{(n)}$ - noma`lum funktsiyaning xosilalari

**Ta`rif:** Differentsial tenglamaning echimi eki integrali deb, tenglamaga quyganda uni ayniyatga aylantiradigan har qanday differentsiallanuvchi $y = \varphi(x)$ funktsiyaga aytiladi.

3-Misol. $y=3e^x$ va $y=4e^{-x}$ funktsiyalarning differentsial tenglamaning echimi buladimi eki yuqmi?

a) $y=3e^x$; $y'=3e^x$ ; $y''=3e^x$ .

$y''-y=0$, $3e^x-3e^x = 0$ ; $0=0$ Demak, $y=3e^x$ tenglamaning echimi

b) $y=4e^{-x}$ ; $y'=-4e^{-x}$ ; $y''=4e^{-x}$.

$y''-y = 0 \Rightarrow 4e^{-x}-4e^{-x}=0; \Rightarrow 0=0$ Demak, $y=4e^{-x}$ funktsiya ham berilgan differentsial tenglamaning echimidir.

Ta`rif: Differentsial tenglama echimining grafigi integral egri chizigi deyiladi.

Tenglamani echimini topish jaraenini differentsial tenglamani integrallash deyiladi.

$$F(x,y,y')=0 \tag{5}$$

Birinchi tartibli differentsial tenglamaning umumiy kurinishi deyiladi.

Uni *y'* ga nisbatan echish mumkin bulsa,uni quydagicha ezish mumkin:

$$y'=f(x,y) \tag{6}$$

Eki

$$M(x,y)dx+N(x,y)dy \tag{7}$$

bunday ezuvni simmetrik ezuv deb ataladi,chunki bunda x va u uzgaruvchilar teng huquqlidir.

Differentsial tenglamani bitta funktsiya emas,balki funktsiyalarning butun bir tuplami qanoatlantirishi mumkin.

$$y_{\ x=x0} = y_0 \tag{8}$$

Differentsial tenglama uchun boshlangich shart deyiladi [3].

Ta`rif: Birinchi tartibli differentsial tenglamaning umumiy echimi deb quyidagi shartlarni kanoatlantiruvchi

$$y=\varphi(x,c)$$

(bunda c- ixtieriy uzgarmas son) funktsiyaga aytiladi:

a) *u* ixtieriy uzgarmas *c* ning xar qanday qiymatida differentsial tenglamani qanoatlantiradi.

b) boshlangich $y_{x=x0}=y_0$ shart har qanday bulganda xam, ixtieriy uzgarmas *s* ning shunday $s_o$ qiymatini topish mumkinki , $y=\varphi(x,c_0)$ funktsiya boshlangich shartni qanoatlantiradi, ya`ni

$$y_0=\varphi(x_0,c_0)$$

Ta`rif: Differentsial tenglamaning umumiy echimidan ixtieriy uzgarmasning mumkin bulgan qiymatlarida xosil qilinadigan echimlar xususiy echimlar deyiladi.

Bu ta`riflarni boshqacha qilib aytganda Koshi masalasi, ya`ni differentsial tenglamaning echimini mavjudligi deb va yagonaligi haqidadir.

### 1.1. Uzgaruvchilari ajraladigan va bir jinsli differentsial tenglamalar

Differentsial tenglamaning eng sodda turi bu uzgaruvchilari ajraladigan differentsial tenglamalardir.

$$P(x)dx+Q(y)dy=0 \tag{9}$$

Bu tenglamani echimini topish uchun tenglikning chap tomonini va ung tomonini integrallash natijasida topiladi:

$$\int (P(x)dx + Q(y)dy) = \int odx$$

$$\int P(x)dx + \int Q(y)dy = C$$

İxtieriy uzgarmasni berilgan tenglama uchun qulay bulgan istalgan kurinishda olish mumkin.

<u>Misol1</u>. $xdx+ydy = 0$

İntegrallab differentsial tenglamaning umumiy echimini topamiz:

$$\frac{x^2}{y} + \frac{y^2}{2} = C \,\Big|\, \cdot 2$$

$$x^2 + y^2 = 2C$$

Agar $2S = C_1^2$ desak, $x^2+y^2=C_1^2$ ga ega bulamiz. Bu markazi koordinata boshida etuvchi, radiusi $S_1$ ga teng bulgan kontsentrik aylonalar oilasidan iborat.

$$P_1(x)\cdot Q_1(y)dx + P_2(x)\cdot Q_2(y)dy=0 \tag{10}$$

kurinishdagi differentsial tenglamani noma`lumlari ajraladigan differentsial tenglama deyiladi.

Bu tenglamani umumiy integralini topish uchun tenglikning ikkala qismini $Q_1(y)\cdot P_2(x)\neq 0$ ga bulib, uni noma`lumlarini ajratib olamiz:

$$\frac{P^1(x)}{P^2(x)}dx + \frac{Q^2(y)}{Q^1(y)}dy = 0$$

Buni integrallab umumiy integral topiladi.

<u>Eslatma</u>: $y' = f_1(x) f_2(y)$ (11)

tenglama xam uzgaruvchilari ajraladigan tenglama deyiladi. Buni umumiy integralini topish uchun $y' = \frac{dy}{dx}$ shaklida ifodalab,

$$\frac{dy}{dx} = f_1(x) f_2(y)$$

kurinishdagi differentsial tenglama xosil qilamiz. Tenglikni $\frac{dx}{f^2(y)}$ ga kupaytrish natijasida uzgaruvchilarni ajratiladi: $\frac{dy}{f_2(y)} = f_1(x)dx$

Endi xar ikkala tomonini integrallab, umumiy integralini topamiz:

$$\int \frac{dy}{f_2(y)} = \int f_1(x)dx + c$$

1-misol:

$$x(1+y_2)dx - y_2(1+x_2)dy = 0$$

$$x(1+y_2)dx = y_2(1+x_2)dy$$

$$\frac{y^2}{1+y^2}dy = \frac{x}{1+x}dx$$

İntegrallab, $\int \frac{y^2+1-1}{1+y^2}dy = \int \frac{d(\frac{x^2}{2}+\frac{1}{2})}{1+x^2}$

$$\int dy - \int \frac{dy}{1+y^2} = \frac{1}{2}\int \frac{d(x^2+1)}{1+x^2}$$

$$y - arctgy = \frac{1}{2}\ln(1+x^2) + c$$

3-misol: $y' y = \dfrac{e^x}{1+e^x}$ ; differentsial tenglamani $y_{x=0} = \sqrt{2}$ boshlangich shartni qanoatlantiruvchi xususiy echimini toping.

$$y\frac{dy}{dx} = \frac{e^x}{1+e^x}$$

$$ydy = \frac{e^x}{1+e^x}dx$$

$$\frac{y^2}{2} = \ln\left|1+e^x\right| + \ln c$$

$$y = \sqrt{2\ln\left[c\cdot(1+e^x)\right]}$$ - umumiy echimidir

$$\sqrt{2} = \sqrt{2\ln\left[c(1+e^0)\right]}; c = \frac{e}{2}$$ ni topamiz.

U xolda $y = \sqrt{2\ln\frac{e}{2}(1+e^x)}$ - xususiy echimdir.

*Bir jinsli differentsial tenglamalar.*

Ta`rif: Agar *f(x,y)* funktsiyada x va y uzgaruvchilarni mos ravishda *tx* va *ty* ga almashtirilganda, $t^n$ ga kupaytrilgan yana usha funktsiyani xosil bulsa, ya`ni

$$f(tx,ty) = t^n f(x,y) \tag{12}$$

shart bajarilsa, *f(x,y)* funktsiyani *n* ulchovli bir jinsli funktsiya deyiladi [7]. Bu erda *t* ixtieriy parametr.

4-misol. $f(x,y) = \sqrt{x^2+y^2}$ funktsiya bir ulchovli bir jinsli funktsiya, chunki

$$f(tx,ty) = \sqrt{t^2x^2 + t^2y^2} = t\sqrt{x^2+y^2} = tf(x,y)$$

5-misol

$$t(x,y)=\frac{x-y}{x+y};\quad f(tx,ty)=\frac{tx-ty}{tx+ty}=\frac{t(x-y)}{t(x+y)}=\frac{x-y}{x+y}=t^0 f(x,y)$$

xosil buladi. Demak, bu nol ulchovli bir jinsli funktsiyadir.

*f(tx,ty)=f(x,y)* shartga buysunadigan nol ulchovli bir jinsli funktsiyani $f(x,y)=\varphi(\frac{y}{x})$ kurinishda ezilishi mumkin. t parametrni ixtieriy qilib tanlab olish mumkinligi uchun, $t=\frac{1}{x}$ deb olamiz. U xolda

$$f(x,y)=f(tx,ty)=f(1;\frac{y}{x})=\varphi(\frac{y}{x})$$

xosil kilamiz.

**Ta`rif**: Agar birinchi tartibli *y'=f(x,y)* differentsial tenglamaning ung tomoni *x* va *u* ga nisbatan nol ulchovli bulsa, bunday tenglama bir jinsli tenglama deyiladi.

$$y'=\varphi(\frac{y}{x}) \tag{13}$$

Bir jinsli differentsial tenglamani echish uchun $\frac{y}{x}=u$ deb belgilash kiritiladi.

$y=u\cdot x$; ni xosil kilamiz $y'=u'x+u$

*y* va *y'* ni (13) tenglamaga quyib, uzgaruvchilari ajraladigan tenglama xosil qilamiz:

$$\frac{du}{\varphi(u)-u}=\frac{dx}{x};$$

İntegrallab, $\int\frac{du}{\varphi(u)-u}=\int\frac{dx}{x}$

integrallashdan sung *u* ni $\frac{y}{x}$ ga nisbatan quyib, (13) tenglamaning umumiy integralini topamiz.

6-Misol: $y'=\frac{y+\sqrt{x^2-y^2}}{x}$ eki $y'=\frac{y}{x}+\sqrt{1-(\frac{y}{x})^2}$

bir jinsli differentsial tenglamaning umumiy echimini toping.

$$\frac{y}{x} = u.; y = ux; y' = u'x + u$$

$$u'x + u = u + \sqrt{1-u^2}; \frac{du}{dx} \cdot x = \sqrt{1-u^2}$$

$$\frac{du}{\sqrt{1-u^2}} = \frac{dx}{x};\ \arcsin u = \ln x + \ln c; u = Sin(\ln(cx));\ u = \frac{y}{x}$$

ligini xisobga olsak,

$$\frac{y}{x} = Sin(\ln Cx); y = x \cdot Sin(\ln Cx)$$

Bir jinsli tenglamalarga keltiriladigan differentsial tenglamalar.

$$\frac{dy}{dx} = \frac{ax + by + c}{a_1x + b_1y + c_1} \tag{14}$$

kurinishdagi tenglamalar bir jinsli tenglamalarga keltiriladigan differentsial tenglamalardir.

$s=s_1=0$ bulsa,

$$\left.\begin{aligned} x &= x_1 + \alpha \\ y &= y_1 + \beta \end{aligned}\right\} \tag{15}$$

almashtirish kiritamiz.

$dx=dx_1$; $dy=dy_1$ ; $\frac{dy}{dx} = \frac{dy_1}{dx_1}$ bularni (16) ga quyib,

$$\frac{dy_1}{dx_1} = \frac{(ax_1 + by_1 + (a\alpha + b\beta + c)}{(a_1x_1 + b_1y_1) + (a_1\alpha_1 + b_1\beta_1 + c_1)} \tag{16}$$

xosil buladi.

$$\left.\begin{aligned} a\alpha + b\beta + c &= 0 \\ a_1\alpha + b_1\beta + c_1 &= 0 \end{aligned}\right\} \tag{17}$$

$\alpha$ va $\beta$ ga nisbatan echamiz.

Agar $\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix} = 0$ bulsa (17) sistema echimga ega emas. (17) tenglama $z=ax+by$ almashtirish natijasida uzgaruvchilari ajraladigan differentsial tenglama xosil buladi.

7-misol: $y' = \dfrac{x+y-3}{x-y-1}$ differentsial tenglamaning umumiy echimini toping.

$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \neq 0$, $\begin{cases} x = x_1 + \alpha \\ y = y_1 + \beta \end{cases}$

almashtirish xosil qilamiz:

$$\frac{dy_1}{dx_1} = \frac{x_1 + y_1 + \alpha + \beta + 3}{x_1 - y_1 + \alpha - \beta - 1}$$

$$\begin{cases} \alpha + \beta + 3 = 0 \\ \alpha - \beta - 1 = 0 \end{cases}$$

ni echib, $\alpha=2$; $\beta=1$ ekanini topamiz.

Natijada bir jinsli $\dfrac{dy_1}{dx_1} = \dfrac{x_1 + y_1}{x_1 - y_1}$ tenglamaga ega bulamiz. İntegrallab,

$$arctgu - \frac{1}{2}\ln(x_1 + u^2) = \ln x_1 + \ln c; \quad cx_1\sqrt{1+u^2} = e^{arctg\frac{y_1}{x_1}}$$

$u = \dfrac{y_1}{x_1}$ ni quysak, $cx_1\sqrt{1 + \dfrac{y_2}{x_2}} = e^{arctg\frac{y-1}{x-2}}$

$x_1=x-2$; $y_1=y-1$ ligini xisobga olsak,

8-misol. $y' = \dfrac{2x+y-1}{4x+2y+5}$ Umumiy echimini toping.

$\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$ Demak, $\left.\begin{matrix} x = x_1 + \alpha \\ y = y_1 + \beta \end{matrix}\right\}$ almashtirish kiritish mumkin emas.

Bu tenglamani *2x+y=z* urniga quyish erdamida, uzgaruvchilari ajraladigan tenglama xosil qilamiz.

$y' = z' - 2; z' - 2 = \frac{z-1}{2z+5}$ eki $z' = \frac{5z+9}{2z+5}$ ni echib

$\frac{2}{5}z + \frac{7}{25}\ln|5z+9| = x + c$ ni topamiz

*z=2x+y* almashtirish bajarib,

$10y - 5x = c_1 = 7\ln|10x + 5y + 9|$ umumiy integralni topamiz.

$$c\sqrt{(x-2)^2 + (y-1)^2} = e^{arctg\frac{y-1}{x-2}}$$

## 1.2. Chiziqli va ularga keltiriladigan tenglamalar

Ta`rif: Noma`lum funktsiya va uning xosilasiga nisbatan chiziqli (birinchi darajali) bulgan tenglamalar birinchi tartibli chizikli tenglamalar deb ataladi [9].

$$y' + P(x)\,y = Q(x) \qquad (18)$$

birinchi tartibli chiziqli tenglamaning umumiy kurinishidir. Bu erda *P(x), Q(x)* - *x* ning funktsiyalari eki uzgarmas sonlardir.

1) *Q(x) = 0* bulsa, (18) - uzgaruvchilari ajraladigan differentsial tenglama xosil buladi. *Q(x)≠0* bulgan xolni kuraylik.

Bu tenglamani umumiy integralini topish uchun ikki xil usulda echishimiz mumkin:

1. Uzgarmas sonni variatsiyalash usuli

$$y' + P(x)y = Q(x)$$

y’+P(x)y=0 deb bir jinsli tenglama xosil qilamiz:

y’=-P(x)y

$$\frac{dy}{y} = -P(x)dx;\ \ln y = -\int P(x)dx + \ln c;\quad y = Ce^{-\int P(x)dx},$$

bu erda *s=s(x)* deb olamiz. U xolda

$$y' = ce^{-\int P(x)dx} \cdot (-\int P(x)dx)' + c' \cdot e^{-\int P(x)dx} = -cP(x)e^{-\int P(x)dx} + c'^{e-\int P(x)dx} = (c' - cp(x))e^{-\int P(x)dx}$$

*y* va *y'* ni (19) ga quyamiz

$$-cP(x)e^{-\int P(x)dx} + c'e^{-\int P(x)dx} + P(x)ce^{-\int P(x)dx} = Q(x)$$

$$c'e^{-\int P(x)dx} = Q(x)$$

$$dc = Q(x)e^{\int P(x)dx}dx$$

İntegrallab,

$$c = \int Q(x)e^{\int P(x)dx}dx$$ ni topamiz.

Demak, $y = \int Q(x)e^{\int P(x)dx}dx \cdot e^{-\int P(x)dx}$ umumiy integralni topamiz.

2 chi usul. (18) tenglama echimini

$$y=u(x)\cdot v(x) \qquad (19)$$

kurinishda izlaymiz:

$$y'=u'v+uv'$$

y va y'ni (19)ga quyamiz:

$$u'v+uv'+P(x)uv=Q(x)$$

$$u'v+u(v'+pv)=Q(x) \qquad (20)$$

funktsiyalardan birini ixtieriy tanlab olish mumkin bulgani uchun v funktsiyani kavs ichida turgan ifoda nolga teng buladigan qilib olamiz:

$$v'+pv=0 \qquad (21)$$

U xolda (20) u funktsiyani topish uchun (20) dan quyidagi tenglamani xosil qilamiz:

$$u'v = Q \qquad (22)$$

Avval (21) dan *v* ni topamiz .

$$\frac{dv}{dx}=-Pv; \quad \frac{dv}{v}=-Pdx; \quad \ln v=-\int Pdx; v=e-\int Pdx,$$

c=1 desak, $v=e^{-\int Pdx}$ buni (22) ga quyamiz

$$\frac{du}{dx}\cdot e^{-\int Pdx}=Q, \; du=Qe^{\int Pdx}dx, \; u=\int Qe^{\int Pdxdx}dx+c$$

u xolda (18) tenglamaning umumiy integrali:

$$y=u\cdot v=e^{-\int Pdx}(\int Qe^{\int Pdx}dx+c)$$

1-misol

$$y'+\frac{1}{x}y=\frac{sinx}{x}; \qquad y=uv; y'=u'v+uv'; \; u'v+uv'+\frac{1}{x}uv=\frac{sinx}{x}$$

$$u'v+u(v'+\frac{1}{x}v)=\frac{sinx}{x};$$

$$1)v'+\frac{1}{x}v=0; \; \frac{dv}{dx}=-\frac{1}{x}v; \frac{dv}{v}=-\frac{dx}{x}$$

$$\ln v=-\ln v; \; v=-\frac{1}{x};$$

$$2)-u'-\frac{1}{x}=\frac{sinx}{x}; \; \frac{du}{dx}=-sinx; \quad du=-sinxdx; \; u=cosx+c$$

Demak, $y=-\frac{1}{x}(cosx+c)$

<u>Bernulli tenglamasi</u>

$$y'+Py=Qy^{n} \qquad (23)$$

kurinishidagi tenglamani qaraymiz. *P,Q* – x ning uzluksiz funktsiyalardir.

(23) - Bernulli tenglamasi deyiladi.

$$y' + Py = Qy^n \;\Big/ : y^n$$

$$y^{-n}y' + Py^{-n+1} = Q$$

$z = y^{-n+1}$ almashtirish kiritamiz. U xolda

$z' = (-n+1)\, y^{-n}y'$ ;  $z' + (-n+1)Pz = (-n+1)Q$

chizikli tenglama xosil buladi.

Eslatma: $n=0$ bulsa, Bernulli tenglamasi chizikli tenglamani, $n=1$ bulsa, uzgaruvchilari ajraladigan differentsial tenglama xosil buladi. Bernulli tenglamani bevosita $y=uv$ urniga kuyish orqali echish ham mumkin.

Tuliq differentsialli tenglama

Ta`rif: Agar

$$P(x,y)dx + Q(x,y)dy = 0 \qquad (24)$$

tenglamaning chap kismi birorta $u(x,y)$ funktsiyaning tuliq diferentsiali bulsa, ya`ni

$$du = P(x,y)dx + Q(x,y)dy \qquad (25)$$

bulsa, (24) tenglama tuliq differentsialli tenglama funktsiyaning tuliq differentsiali

$$du = \frac{du}{dx}dx + \frac{du}{dy}dy \qquad (26)$$

formula bilan xisoblanadi. (25) va (26) ni taqqoslab,

$$\frac{du}{dx} = P(x,y); \frac{du}{dy} = Q(x,y)$$

ekanini topamiz.

$$\frac{dP}{dy} = \frac{d^2u}{dxdy}; \frac{dQ}{dx} = \frac{d^2u}{dydx}$$

bundan

$$\frac{dP}{dy}=\frac{dQ}{dx} \qquad (27)$$

ekanligi kelib chiqadi.

Demak, (24) tenglamani tuliq differentsialli tenglama bulishi uchun (27) shart bajariliish kerak.

Tuliq differentsialning umumiy echimini quyidagi kurinishda qidiriladi:

$$U(x,y)=\int_{x_0}^{x} P(t_1,y_0)dt+\int_{y_0}^{y} Q(x,t)dt \qquad (28)$$

Demak, differentsial tenglamaning umumiy integrali

$$\int_{x_0}^{x} P(t,y_0)dt+\int_{y_0}^{y} Q(x,t)dt=C \qquad (29)$$

$(x_0,y_0)$ nukta G soxaning tayin bir nuktasidir.

2-misol: $\frac{dy}{dx}=\frac{1-2xy}{3y^2+x^2}$ umumiy integralini toping.

$(2xy\text{-}1)dx+(3y^2+x^2)dy=0$

$P(x,y)=2xy\text{-}1;$ $Q(x,y)=3y^2+x^2$ ; $\frac{dP}{dy}=2x;\frac{dQ}{dx}=2x$

Demak, berilgan tuliq differentsialli tenglamadir.

$x_0=y_0=0$ deb (29) formula erdamida tenglamani umumiy integralini topamiz:

$$\int_{x_0}^{x}(2t\cdot 0-1)dt+\int_{0}^{y}(3t^2+x^2)dt=C$$

$$\left[-t\right]_0^x+\left[t^3+x^2t\right]_0^y=C$$

yoki $-x+y^3+x^2y=C$

## §2. Koshi masalasi va o'ning quylishi

Faraz qilaylik, bizga quyidagi Koshi masalasi berilgan bo'lsin

$$y' = f(x, y) \;, \tag{30}$$

$$y(x_0) = y_0 \,. \tag{31}$$

Bu masalani echish uchun differentsial tenglamalar kursidan ma`lum bo'lgan ketma-ket yaqinlashish usulini

$$y_n(x) = y_0 + \int_{x_0}^{x} f(t, y_{n-1}(t))dt \,,$$

$$y_0(t) \equiv y_0 \,, \quad n = 1,2,...$$

qo'llab yoki echimni $x_0$ nuqta atrofida qatorga (odatda Teylor qatoriga) yoyish usulini qo'llab, echimni taqribiy analitik ko'rinishini topishadi

$$y(x) \approx y_n(x) = \sum_{i=0}^{n} y^{(i)}(x_0) \frac{(x - x_0)^i}{i!} \,. \tag{32}$$

Bu ikkala sodda usullar ma`lum kamchiliklarga ega, xususan birinchisida har bir yangi yaqinlashishda integral hisoblanishi lozim bo'lsa, ikkinchisida esa $|x - x_0|$ kichik bo'lmasa (32) qator yaqinlashmasligi mumkin yoki juda sust yaqinlashuvchi bo'ladi. Shu bois ham (30)-(31) masalani boshqa usullar bilan echishni o'rganamiz.

*Ayirmali metodlar*

(30)-(31) masalani $[x_0, X]$ da echish talab etilgan bo'lsin. Berilgan kesmani $N$ ta teng bo'lakka bo'lamiz:

$$x_i = x_0 + ih, \; i = 1,2,...,N, \; N = \frac{X - x_0}{h}, \qquad h > 0 \,.$$

(30)-(31) masala – Koshi masalasini echish

$$y(x) = y_0 + \int_{x_0}^{x} f(t, y(t))dt \qquad (33)$$

integral tenglamani echish bilan teng kuchlidir. Biroq (33) da integral ostida noma`lum funktsiya qatnashishi masalani murakkablashtiradi. Ketma-ket ikkita nuqtadagi echimning orasida quyidagi munosabat o'rinlidir:

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} y'(x)dx. \qquad (34)$$

$y(x_{n+1})$ ni topish uchun $y(x_n)$ va $y'(x) = f(x, y(x))$ ni $[x_n, x_{n+1}]$ oraliqda bo'lish kerak. Shuning uchun ham (34) ni o'ng tomonidagi integralni taqribiy hisoblash lozim bo'ladi. Bu integralni qanday hisoblanishiga qarab, berilgan Koshi masalasini echadigan taqribiy u yoki bu metodlar hosil bo'ladi.

## 2.1. Adams ekstrapolyatsion metodi

Faraz qilaylik, $y(x)$ funktsiyani $x_n, x_{n-1}, ..., x_{n-k}$ nuqtalarda qiymati ma`lum bo'lsin. $x = x_n + uh$ almashtirish bajaramiz. U holda (34) quyidagi ko'rinishga keladi [4].

$$y(x_{n+1}) = y(x_n) + h\int_{x_n}^{x_{n+1}} y'(x_n + uh)du. \qquad (35)$$

$y'(x_n + uh)$ ni N`yutonning jadval oxirida interpolyatsiyalash formulasiga almashtiramiz

$$y'(x_n + uh) = y'(x_n) + \frac{u}{1!}\Delta y'(x_{n-1}) + \frac{u(u+1)}{2!}\Delta^2 y'(x_{n-2}) + ... +$$

$$+ \frac{u(u+1)...(u+k-1)}{k!}\Delta^k y'(x_{n-k}) + r_k(u), \qquad (36)$$

bu erda

$$r_k(u) = h^{k+1}\frac{u(u+1)...(u+k)}{(k+1)!} y^{(k+2)}(\xi), \quad x_{n-k} < \xi = \xi(u) < x_{n+1}, \qquad 0 \le u \le 1.$$

(36) ni (35) ga qo’yamiz va integrallash amalini bajarsak, quyidagi formulaga ega bo’lamiz:

$$y(x_{n+1}) = y(x_n) + h\left[ y'(x_n) + \frac{1}{2}\Delta y'(x_{n-1}) + \frac{5}{12}\Delta^2 y'(x_{n-2}) + \right.$$

$$\left. + \frac{3}{8}\Delta^3 y'(x_{n-3}) + ... + C_k \Delta^k y'(x_{n-k}) \right] + R_k, \qquad (37)$$

bu erda

$$C_k = \int_0^1 \frac{u(u+1)...(u+k-1)}{k!} du, \qquad (38)$$

$$R_k = h\int_0^1 r_k(u)du = h^{k+2}\int_0^1 \frac{u(u+1)...(u+k)}{(k+1)!} y^{(k+2)}(\xi)du.$$

Bu integraldagi $u(u+1)...(u+k)$ ishora saqlaganligi uchun, o’rta qiymat haqidagi teoremaga asosan va $y^{(k+2)}(x)$ ni uzluksizligidan

$$R_k = h^{k+2} C_{k+1} y^{(k+2)}(\bar{\xi})du, \qquad x_{n-k} \le (\bar{\xi}) \le x_{n+1}$$

kelib chiqadi. Agar $M_{k+2} = \max\limits_{x_{n-k} \le x \le x_n} \left| y^{(k+2)}(x) \right|$ desak va $C_k > 0$ ni e`tiborga olsak,

$$|R_k| = h^{k+2} C_{k+1} M_{k+2} \qquad (39)$$

bo’ladi. Agar $h > 0$ etarlicha kichik bo’lsa $R_k = O(h^{k+2})$ xatolikni e`tiborga olmasak, Adams ekstrapolyatsion metodining hisob formulasiga ega bo’lamiz:

$$y_{n+1} = y_n + h\left[ y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{5}{12}\Delta^2 y'_{n-2} + \frac{3}{8}\Delta^3 y'_{n-3} + ... + C_k \Delta^k y'_{n-k} \right]. \qquad (40)$$

Agar (40) da $k = 0$ bo’lsa, $y_{n+1} = y_n + hf(x_n, y_n)$ - Eyler usuli hosil bo’ladi.

Agar $k \ge 1$ bo’lganda $y_n, y_{n-1}, ..., y_{n-k}$ larni ma`lum deb (40) dan ketma-ket

$$y_{n+1}, y_{n+2}, ...$$

lar topiladi.

Bu metodni qiyinchiliksiz birinchi tartibli differentsial tenglamalar sistemasi uchun Koshi masalasiga qo'llash mumkin. Buni quyidagi masalada namoyish etamiz:

$$y' = f(x,y,z), \qquad y(x_0) = y_0,$$

$$z' = \varphi(x,y,z), \qquad z(x_0) = z_0,$$

$$y_{n+1} = y_n + h\left[ y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{5}{12}\Delta^2 y'_{n-2} + \frac{3}{8}\Delta^3 y'_{n-3} + \ldots + C_k \Delta^k y'_{n-k} \right],$$

$$z_{n+1} = z_n + \tau\left[ z'_n + \frac{1}{2}\Delta z'_{n-1} + \frac{5}{12}\Delta^2 z'_{n-2} + \frac{3}{8}\Delta^3 z'_{n-3} + \ldots + C_k \Delta^k z'_{n-k} \right].$$

## 2.2. Adams interpolyatsion metodi

(30)-(31) Koshi masalasini echimi $x_{n-k+1},\ldots,x_{n+1}$ nuqtalarda ma`lum deb faraz qilaylik (vohalanki $y_{n+1} = y(x_{n+1})$ noma`lum). Echimning $x_{n+1}$ nuqtadagi qiymatini $y(x_{n+1}) = y(x_n) + h\int_{-1}^{0} y'(x_{n+1} + uh)du$ deb yozib olib, $y'(x_{n+1} + uh)$ ni jadval oxirida interpolyatsiyalash (N`yuton) formulasiga almashtirib, integrallashni bajarsak, quyidagi hisob formulasiga ega bo'lamiz [6]:

$$y_{n+1} = y_n + h\left[ y'_{n+1} - \frac{1}{2}\Delta y'_n - \frac{1}{12}\Delta^2 y'_{n-1} - \frac{1}{24}\Delta^3 y'_{n-2} + \ldots + C_k \Delta^k y'_{n-k+1} \right], \quad (41)$$

bu erda

$$C_k = \int_{-1}^{0} \frac{u(u+1)\ldots(u+k-1)}{k!} du.$$

(41) formulaning xatoligi

$$R_k = h^{k+2} C_{k+1} y^{k+2}(\xi), \qquad x_{n-k+1} \le \overline{\xi} \le x_{n+1}.$$

Odatda (30)-(31) Koshi masalasini Adams metodi bilan echishda quyidagicha amal qilinadi.

Jadval boshidagi echimning qiymatlarini, ya`ni $x_{n-k}, x_{n-k+1},..., x_n$ nuqtalardagi echimning qiymatini qanaqa aniqlikda topilsa, shunday aniqlikka ega Adams formulasini tanlash kerak bo'ladi. Bundan tashqari (41) formula emas, u $y_{n+1}$ ga nisbatan $y_{n+1} = \varphi(y_{n+1})$ ko'rinishdagi chiziqsiz tenglamadir. Shuning uchun (40) dan $y^{э}_{n+1}$ topilib, uni (41) ning o'ng tomoniga qo'yamiz, $\varphi(y^{э}_{n+1}) = y^{u}_{n+1}$ deb belgilaymiz. Agar $y^{э}_{n+1}$ va $y^{u}_{n+1}$ lar berilgan aniqlik miqdorida ustma-ust tushsa $y_{n+1} = y^{э}_{n+1}$ deb hisobni (40) formula bilan davom ettiramiz. Bordi-yu $y^{э}_{n+1}$ va $y^{u}_{n+1}$ yuqorida nazarda tutilgan shartni qanoatlantirmasa, u holda $y_{n+1} = y^{u}_{n+1}$ deb jadvalni hozirgina topilgan yo'li qaytadan hisoblanadi va keyingi qadamda $y^{э}_{n+2}$ hisoblanib, $y^{u}_{n+2}$ bilan taqqoslanadi, bordi-yu yana qo'yilgan aniqlik bajarilmasa, unda hisoblashni $\frac{h}{2}$ qadam bilan $y^{э}_{n+2} = y_{n+2}$ deb davom ettiriladi.

Ko'pincha hisoblash jarayonini osonlashtirish uchun (40), (41) formulalarda ishtirok etuvchi chekli ayirmalarni funktsiyaning qiymatlari orqali ifodasiga almashtiriladi va $k$ ning berilgan qiymatiga mos ravishda quyidagi formulalarga ega bo'lamiz:

$$k = 0, \qquad y_{n+1} = hf(x_n, y_n),$$
$$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1}),$$

$$k = 1, \qquad y_{n+1} = y_n + h\left[\frac{3}{2} f_n - \frac{1}{2} f_{n-1}\right],$$
$$y_{n+1} = y_n + h\left[\frac{1}{2} f_{n+1} + \frac{1}{2} f_n\right],$$

$$k = 2, \qquad y_{n+1} = y_n + \frac{h}{12}[23 f_n - 16 f_{n-1} + 5 f_{n-2}],$$
$$y_{n+1} = y_n + \frac{h}{12}[5 f_{n+1} + 8 f_n - f_{n-1}],$$

$$k=3,\qquad y_{n+1}=y_n+\frac{h}{24}\left[55f_n-59f_{n-1}+37f_{n-2}-9f_{n-3}\right],$$

$$y_{n+1}=y_n+\frac{h}{24}\left[9f_{n+1}+19f_n-5f_{n-1}+f_{n-2}\right].$$

$x_{n+1}$ nuqtadagi echimni undan farqli va bittadan ko'p bo'lgan nuqtadagi echimning qiymatlari ishtirokida topilganligi uchun Adams metodlari ko'p qadamli metodlar deb ataladi.

### 2.3. Runge-Kutta usuli

(30), (31) masalani echish uchun (34) ni quyidagicha yozib olamiz

$$y(x+h)=y(x)+\int_x^{x+h}y'(t)dt=y(x)+h\int_0^1 f\left[(x+\alpha h),y(x+\alpha h)\right]d\alpha$$

yoki

$$y(x+h)-y(x)=\Delta y=h\int_0^1 f\left[(x+\alpha h),y(x+\alpha h)\right]d\alpha\,. \qquad (42)$$

(42) dagi integralni taqriban quyidagi kvadratur summaga o'xshash chekli summaga almashtiramiz:

$$\Delta y_i=\sum_{i=1}^{r}p_i\overline{\overline{K}}_i\,, \qquad (43)$$

bu erda

$$\overline{\overline{K}}_1=hf(x,y),\qquad \overline{\overline{K}}_2=hf(x+\alpha_2h,y+\beta_2\overline{\overline{K}}_1),\ldots$$

$$\ldots\overline{\overline{K}}_r=hf(x+\alpha_rh,y+\beta_r,\overline{\overline{K}}_1+\ldots+\beta_{2r-1}\overline{\overline{K}}_{r-1}).$$

Noma`lum $\alpha_i,\beta_{ii-1}$ $(i=2,3,...,r)$ va $p_i$ $\left(i=1,2,...,r\right)$ larni (43) ning chap va o'ng tomonlarini $h$ ning darajalari bo'yicha Teylor qatoriga yoyilmasining iloji boricha ko'p hadlarini ixtiyoriy $f(x,y)$ uchun bir xil bo'lish shartidan topiladi.

Quyida turli tartibli Runge-Kutta metodi bo'yicha hisoblash formulalarini keltiramiz [5]:

$r=1$. *Birinchi tartibli metod*

$$y(x+h)=y(x)+hf(x,y).$$

$r=2$. *İkkinchi tartibli metodlar*

1) $$y(x+h)=y(x)+\frac{h}{2}\left[f(x,y)+f(x+h,y+hf(x,y))\right],$$

2) $$y(x+h)=y(x)+hf\left[(x+\frac{h}{2},y+\frac{h}{2}f(x,y)\right],$$

3) $$y(x+h)=y(x)+\frac{h}{4}\left[f(x,y)+f\left(x+\frac{2}{3}h,y+\frac{2h}{3}f(x,y)\right)\right].$$

$r=3$. *Uchinchi tartibli metodlar*

1) $$y(x+h)=y(x)+\frac{1}{6}(k_1+4k_2+k_3),$$

$$k_1=hf(x,y),\ \ k_2=hf(x+\frac{h}{2},y+\frac{k_1}{2}),\ k_3=hf(x+h,y-k_1+2k_2).$$

2) $$y(x+h)=y(x)+\frac{1}{4}(k_1+3k_2),$$

$$k_1=hf(x,y),\ \ k_2=hf(x+\frac{h}{3},y+\frac{k_1}{3}),\ k_3=hf(x+\frac{2h}{3},y+\frac{2k_2}{3}).$$

$r=4$. *To'rtinchi tartibli metodlar*

1) $$y(x+h)=y(x)+\frac{1}{6}(k_1+2k_2+2k_3+k_4),$$

$$k_1=hf(x,y),\qquad k_2=hf(x+\frac{h}{2},y+\frac{k_1}{2}),$$

$$k_3=hf(x+\frac{h}{2},y+\frac{k_2}{20}),\qquad k_4=hf(x+h,y+k_3)$$

2) $$y(x+h)=y(x)+\frac{1}{3}\left[\frac{1}{2}k_1+\frac{3}{2}k_2+\frac{1}{2}k_3+\frac{1}{2}k_4\right],$$

$$k_1 = hf(x,y), \qquad k_2 = hf(x+\frac{h}{2}, y+\frac{k_1}{2}),$$

$$k_3 = hf(x+\frac{h}{2}, y-\frac{k_1}{2}+k_2), \qquad k_4 = hf\left[x+h, y+\frac{1}{2}k_2+\frac{1}{2}k_3\right].$$

**Misol.** $y' = \frac{2y}{x} + x$, $y(1) = 0$ Koshi masalasini echimini $x = 1{,}1;\ 1{,}2;\ 1{,}3$ nuqtalarda Runge-Kutta usuli bilan toping. $x = 1{,}4;\ 1{,}5$ nuqtalarda esa Adams usuli bilan toping.

**Echish.** $k = 3$ bo'lganligi uchun Adams usulida (40), (41) formulalarni qo'llaymiz. Runge-Kutta usulida $r = 4$ dagi birinchi formulani ishlatamiz. Odatda, jadval boshidagi echimni qiymatlarini topishda tanlangan metod tartibi Adams usuli tartibidan kichik bo'lmagan tartibli metod qo'llash kerak bo'ladi (yoki qadam $h$ Adams usullaridagi qadamdan kichikroq bo'lishi kerak).

Bizning misolda $h$ bir xil, shuning uchun $k = 3$ va $r = 4$ deb oldik.

$$y_{i+1} = y_i + \frac{1}{6}(k_1^{(i)} + 2k_2^{(i)} + 2k_3^{(i)} + k_4^{(i)}),$$

bu erda

$$k_1^{(i)} = hf(x_i, y_i), \qquad k_2^{(i)} = hf(x_i + \frac{h}{2}, y_i + \frac{k_1^{(i)}}{2}),$$

$$k_3^i = hf(x_i + \frac{h}{2}, y_i + \frac{k_2^{(i)}}{2}), \qquad k_4^{(i)} = hf(x_i + h, y_i + k_3^{(i)})$$

formula yordamida $i = 0{,}1{,}2$ deb $y_1, y_2 y_3$ larni hisoblaymiz, so'ngra (40) formula bilan

$$y_4^э = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$$

ni hisoblaymiz. Endi esa topilgan $y_4^э$ ni (41) formulani o'ng tomoniga qo'yamiz, natijani $y_4^u$ deb belgilaymiz.

$$y_4^u = y_3 + \frac{h}{24}\left[9f(x_4, y_4^э) + 19f(x_3, y_3) - 5f(x_2, y_2) + f(x_1, y_1)\right].$$

Agar berilgan aniqlikda $y_4^u \approx y_n^э$ bo'lsa, $y_4 = y_4^э$ deb, $y_5$ ni (40) formula bilan topamiz. Aks holda $y_4 = y_4^u$ deb, $y_5$ (40) formula bilan hisoblanadi.

Natija quyidagi jadvalda keltirilgan:

| $x_k$ | 1,0 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 |
|---|---|---|---|---|---|---|
| $y_k$ | 0 | 0,1153 | 0,2625 | 0,4434 | 0,6595 | 0,9123 |

## 2.4. Eyler va Eyler-Koshi usullari

(30), (31) masala echimini taqribiy qiymati $y(x_i) \cong y_i$ quyidagicha aniqlansa

$$y_{i+1} = y_i + hf(x_i, y_i), \quad i = 0,1,...$$

bu usul Eyler usuli bo'lar edi (Adams, Runge-Kutta usullarida $k = 0$ bo'lgan hol).

Eyler usulining quyidagi modifikatsiyalarini keltiramiz:

$$\text{I. } y_{i+1} = y_i + hf_{i+\frac{1}{2}}, \quad i = 0,1,... \qquad (44)$$

bu erda
$$f_{i+\frac{1}{2}} = f\left(x_{i+\frac{1}{2}}, y_{i+\frac{1}{2}}\right),$$

$$x_{i+\frac{1}{2}} = x_i + \frac{h}{2}, \qquad y_{i+\frac{1}{2}} = y_i + \frac{h}{2} f_i.$$

$$\text{II. } y_{i+1} = y_i + \frac{h}{2}\left(f_i + \bar{f}_{i+1}\right), \quad i = 0,1,... \qquad (45)$$

bu erda
$$\bar{y}_{i+1} = y_i + hf_i,$$

$$\bar{f}_{i+1} = f\left(x_{i+1}, \bar{y}_{i+1}\right).$$

Agar (30), (31) masalaning taqribiy echimini

$$y_{i+1}^{(0)} = y_i + hf\left(x_i, y_i\right)$$

deb uni quyidagi

$$y_{i+1}^{(k)} = y_i + \frac{h}{2}\Big(f\left(x_i, y_i\right) + f\Big(x_{i+1}, y_{i+1}^{(k-1)}\Big)\Big), \ \ k = 1,2,.... \tag{46}$$

iteratsion jarayon bilan berilgan aniqlik miqdorida topish mumkin, ya`ni $\left|y_{i+1}^{(k+1)} - y_{i+1}^{(k)}\right| < \varepsilon$ bo'lsa $y_{i+1} \approx y_{i+1}^{(k+1)}$ deyiladi, so'ng keyingi nuqtadagi echim uchun (46) iteratsion jarayon quriladi.

## §3. Oddiy differentsial tenglamaga qo'yilgan chegaraviy masalalarni echish

*3.1. Kollokatsiya usuli*

Faraz qilaylik, bizga $a \le x \le b$ da

$$L(y) \equiv y'' + p(x)y' + q(x)y = f(x) \tag{47}$$

differentsial tenglama uchun

$$\begin{aligned} l_a\left(y\right) &= \alpha_0 y(a) + \alpha_1 y'(a) = A, \\ l_b\left(y\right) &= \beta_0 y(b) + \beta_1 y'(b) = B \end{aligned} \tag{48}$$

chegaraviy shartlar berilgan bo'lsin [8], bu erda $\alpha_0^2 + \alpha_1^2 > 0, \quad \beta_0^2 + \beta_1^2 > 0$.

(47), (48) chegaraviy masala echimini quyidagi ko'rinishda izlaymiz:

$$y_n(x) = \varphi_0(x) + \sum_{k=1}^{n} c_k \varphi_k(x), \tag{49}$$

bu erda $\varphi_0(x)$

$$\begin{aligned} l_a(\varphi_0) &\equiv \alpha_1 \varphi_0'(a) + \alpha_0 \varphi_0(a) = A, \\ l_b(\varphi_0) &\equiv \beta_1 \varphi_0'(b) + \beta_0 \varphi_0(b) = B \end{aligned}$$

shartni, $\varphi_k(x)$ $(k = \overline{1,n})$ esa

$$l_a(\varphi_k) = l_b(\varphi_k) = 0$$

shartlarni qanoatlantiruvchi funktsiyalardir. Bundan tashqari $\{\varphi_k(x)\}_{k=0}^{n}$ funktsiyalar sistemasi quyidagi shartlarni bajarishini ta`minlash kerak:

1. $\{\varphi_k(x)\}_{k=0}^{n}$ - chiziqli bo\liqsiz;

2. $\varphi_k(x) \in C_2[a,b],\ \ k = \overline{0,n}$;

3. $\{\varphi_k(x)\}_{k=0}^{n}$ funktsiyalar sistemasi ikki marta uzluksiz differentsiallanuvchi funktsiyalar sinfida to'liq bo'lishi kerak, ya`ni

$$\left| y^{(s)}(x) - y_n^{(s)}(x) \right| < \varepsilon,\ s = 0,1,2; \quad x \in [a,b],\ \forall y(x) \in C_2[a,b], \quad \varepsilon > 0.$$

$\varphi_k(x)$ larni tanlanishiga ko'ra ixtiyoriy $c_k$ $\left(k = \overline{1,n}\right)$ lar uchun (49) (48) shartlarni qanoatlantiradi. (49) ni (47) ga qo'yib, quyidagini hosil qilamiz

$$R(x, c_1, c_2, ..., c_n) = L(y_n) - f(x) = L(\varphi_0) - f(x) + \sum_{k=1}^{n} c_k L(\varphi_k). \tag{50}$$

Endi $[a,b]$ ga tegishli kollokatsiya nuqtalari deb ataluvchi turli $x_1, x_2, ..., x_n$ nuqtalarni olib, $c_1, c_2, ..., c_n$ koeffitsientlarni topish uchun

$$\begin{aligned} R(x_1, c_1, c_2, ..., c_n) &= 0 \\ R(x_2, c_1, c_2, ..., c_n) &= 0 \\ \ldots \quad \ldots \quad \ldots \quad &\ldots \\ R(x_n, c_1, c_2, ..., c_n) &= 0 \end{aligned} \tag{51}$$

tenglamalar sistemasini hosil qilamiz. (51) dan $c_1, c_2, ..., c_n$ larni topib, echimning taqribiy analitik ko'rinishi (49) ni hosil qilamiz.

**Misol.**

$$\begin{gathered} y''(x) + 2xy'(x) - 2y(x) = 2, \\ y'(0) = -2, \\ y(1) + y'(1) = 0 \end{gathered}$$

chegaraviy masalani echish talab etilsin.

**Echish.** $\varphi_0(x)$ ni $\varphi_0(x) = b + cx$ ko'rinishida izlaymiz.

$$\varphi_0'(x)=c \text{ va } \varphi_0'(0)=-2 \quad \Rightarrow \quad c=-2.$$

$y(1)+y'(1)=0 \;\Rightarrow\; b-2-2=0 \;\Rightarrow\; b=4.$ Demak, $\varphi_0(x)=4-2x$.

$\varphi_k(x)$ $(k=1,2,...)$ funktsiyalar

$$\left.\begin{aligned}\varphi_k'(0)=0,\\ \varphi_k(1)+\varphi_k'(1)=0\end{aligned}\right\}$$

shartlarni qanoatlantirishi kerak.

$\varphi_k(x)=b_k+x^{k+1}$ desak, $b_k$ koeffitsientlarni ikkinchi tenglamadan topamiz.

Demak,

$$b_k=-(k+2) \;\Rightarrow\; \varphi_k(x)=x^{k+1}-(k+2).$$

$$\begin{gathered}\varphi_1(x)=x^2-3,\\ \varphi_2(x)=x^3-4,\\ \dots\\ \varphi_n(x)=x^{n+1}-(n+2).\end{gathered}$$

(49) da $n=2$ deb olaylik, unda

$$y_2(x)=4-2x+c_1(x^2-3)+c_2(x^3-4).$$

$L(\varphi_0),\quad L(\varphi_1),\quad L(\varphi_2)$ larni topib, $R(x,c_1,c_2)$ ni hosil qilamiz

$$\varphi_0(x)=4-2x,\quad \varphi_0'(x)=-2,\quad \varphi_0''(x)=0,$$

$$L(\varphi_0)=\varphi_0''(x)+2x\varphi_0'(x)-2\varphi_0(x)=2x\cdot(-2)-2(4-2x)=-4x-8+4x=-8,$$

$$\varphi_1(x)=x^2-3,\quad \varphi_1'(x)=2x,\quad \varphi_1''(x)=2,$$

$$L(\varphi_1)=\varphi_1''(x)+2x\varphi_1'(x)-2\varphi_1(x)=2+2x\cdot 2x-2(x^2-3)=2x^2+8,$$

$$\varphi_2(x)=x^3-4,\quad \varphi_2'(x)=3x^2,\quad \varphi_2''(x)=6x,$$

$$L(\varphi_2)=\varphi_2''(x)+2x\varphi_2'(x)-2\varphi_2(x)=6x+2x\cdot 3x^2-2(x^3-4)=$$

$$=6x+6x^3-2x^3+8=4x^3+6x+8.$$

$$R(x, c_1, c_2) = L(\varphi_0(x)) - f(x) + c_1 L(\varphi_1(x)) + c_2 L(\varphi_2(x)).$$

Kollokatsiya nuqtalarini $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{4}$ deb olaylik, u holda quyidagilar hosil bo'ladi:

$$L(\varphi_0(x_1)) = -8, \quad L(\varphi_0(x_2)) = -8,$$

$$L(\varphi_1(x_1)) = 2 \cdot \frac{1}{16} + 8 = \frac{1}{8} + 8 = 8\frac{1}{8},$$

$$L(\varphi_1(x_2)) = 2 \cdot \frac{9}{16} + 8 = \frac{9}{8} + 8 = 9\frac{1}{8},$$

$$L(\varphi_2(x_1)) = 4 \cdot \frac{1}{64} + 6 \cdot \frac{1}{4} + 8 = \frac{1}{16} + \frac{3}{2} + 8 = 9\frac{9}{16},$$

$$L(\varphi_2(x_2)) = 4 \cdot \frac{27}{64} + 6\frac{3}{4} + 8 = \frac{27}{16} + \frac{9}{12} + 8 = 14\frac{3}{16}.$$

Natijada quyidagi tenglamaga ega bo'lamiz:

$$\left.\begin{aligned} \frac{65}{8}c_1 + \frac{153}{16}c_2 &= 10 \\ \frac{73}{8}c_1 + \frac{227}{16}c_2 &= 10 \end{aligned}\right\} \Rightarrow \quad c_1 = 1{,}6510; \quad c_2 = -0{,}3570.$$

U holda $y_2(x) = 4 - 2x + 1{,}6510(x^2 - 3) - 0{,}3570(x^3 - 4)$.

## 3.2. Haydash usuli

Faraz qilaylik, $a \le x \le b$ kesmada

$$L(y) \equiv y'' + p(x)y' + q(x)y = f(x) \tag{52}$$

differentsial tenglamani

$$\begin{aligned} \alpha_0 y(a) + \alpha_1 y'(a) &= A, \\ \beta_0 y(b) + \beta_1 y'(b) &= B \end{aligned} \tag{53}$$

chegaraviy shartlarni qanoatlantiruvchi echimini topish talab etilsin [4,5]. Bu erda $\alpha_0^2+\alpha_1^2>0, \quad \beta_0^2+\beta_1^2>0$. Bu chegaraviy masalani echishning sodda usullaridan biri uni chekli ayirmali tenglamalar sistemasiga keltirishdir. Shu maqsadda $[a,b]$ kesmani $h$ qadam bilan $N$ ta teng bo’lakka bo’lamiz. Bo’linish nuqtalari ($\{x_k\}$ to’rning tugun nuqtalari) ning abstsissalari $x_k=a+kh \quad \left(h=\dfrac{b-a}{N},\ k=\overline{0,N}\right)$ formula bilan ifodalanadi.

Noma`lum funktsiya $y=y(x)$ va uning hosilalari $y'=y'(x),\ y''=y''(x)$ ni $\{x_k\}$ to’rdagi qiymatlarini mos holda

$$y_k=y(x_k), \quad y_k'=y'(x_k), \quad y_k''=y''(x_k)$$

deb belgilaymiz. Shuningdek,

$$p_k=p(x_k), \quad q_k=q(x_k), \quad f_k=f(x_k)$$

belgilashlardan ham foydalanamiz.

(52) differentsial tenglamani to’rning ichki nuqtalarida bir tomonlama o’ng ayirmali hosilalar yordamida

$$\frac{y_{k+2}-2y_{k+1}+y_k}{h^2}+p_k\frac{y_{k+1}-y_k}{h}+q_ky_k=f_k \quad \left(k=\overline{1,N-1}\right) \tag{54}$$

ko’rinishdagi yoki simmetrik ayirmali hosilalar bilan

$$\frac{y_{k+1}-2y_k+y_{k-1}}{h^2}+p_k\frac{y_{k+1}-y_{k-1}}{2h}+q_ky_k=f_k \quad \left(k=\overline{1,N-1}\right) \tag{55}$$

ko’rinishdagi $N-1$ ta tenglamadan iborat bo’lgan ayirmali tenglamalar sistemasi bilan approksimatsiya qilish mumkin.

Chegaraviy shartlarni esa

$$\begin{gathered}\alpha_0y_0+\alpha_1\frac{y_1-y_0}{h}=A,\\ \beta_0y_N+\beta_1\frac{y_N-y_{N-1}}{h}=B\end{gathered} \tag{*}$$

ayirmali tengliklar bilan approksimatsiya qilish mumkin. $N+1$ ta noma`lumni aniqlash uchun hosil bo'lgan $N+1$ ta ayirmali tenglamalardan iborat bo'lgan chiziqli algebraik tenglamalar sistemasini umumiy ko'rinishda quyidagicha yozish mumkin:

$$A_k y_{k+1} - B_k y_k + C_k y_{k-1} = -F_k \quad (k = \overline{1, N-1}), \tag{56}$$

$$\begin{aligned} y_0 &= \chi_1 y_1 + \mu_1, \\ y_N &= \chi_2 y_{N-1} + \mu_2. \end{aligned} \tag{57}$$

(56)-(57) – uch diagonalli chiziqli algebraik tenglamalar sistemasidir (bunda barcha $i = \overline{1, N-1}$ lar uchun $A_i \neq 0,\ B_i \neq 0$ deb faraz qilinadi). Shunday qilib, (52)-(53) chegaraviy masalani echish masalasi (56)-(57) chiziqli algebraik tenglamalar sistemasini echishga keltirildi. Bu tenglamalar sistemasini echish usuli - ***haydash usuli nomi bilan yuritiladi***. Haydash usulida echim to'rning ichki nuqtalarida

$$y_i = \alpha_{i+1} y_{i+1} + \beta_{i+1} \quad \left(i = \overline{N-1,\ 1}\right) \tag{58}$$

rekurrent formula orqali izlanadi. Bu erdagi $\alpha_i$, $\beta_i$ koeffitsientlar quyidagi tenglamadan aniqlanadi (bunda $C_i - \alpha_i A_i \neq 0$ deb faraz qilamiz):

$$\alpha_{i+1} = \frac{B_i}{C_i - \alpha_i A_i}, \qquad i = 1,2,\dots,N-1; \tag{59}$$

$$\beta_{i+1} = \frac{A_i \beta_i + F_i}{C_i - \alpha_i A_i}, \qquad i = 1,2,\dots,N-1. \tag{60}$$

$\alpha_i$, $\beta_i$ larni (59)-(60) bo'yicha aniqlash va hisoblashlarni boshlash uchun $\alpha_1$, $\beta_1$ larni qiymatini bilish kerak bo'ladi. Bu qiymatlarni quyidagi mulohazadan aniqlaymiz. (58) da $i = 0$ deb faraz qilsak,

$$y_0 = \alpha_1 y_1 + \beta_1$$

tenglik (chap chegaraviy shart)ga ega bo'lamiz va shu sababli bu tenglikni (57) dagi birinchi chegaraviy shart bilan qiyoslasak,

$$\begin{aligned} \alpha_1 &= \chi_1, \\ \beta_1 &= \mu_1 \end{aligned} \qquad (61)$$

ekanligini topamiz.

Endi aniqlangan $\alpha_i$, $\beta_i$ $\left(i=\overline{1, N-1}\right)$ larga ko'ra echimni (58) rekurrent formula orqali aniqlash va hisoblashni boshlash uchun $y_N$ ni qiymati ma`lum bo'lishi kerak. (58) da $i = N-1$ deb hisoblasak,

$$y_{N-1} = \alpha_N y_N + \beta_N$$

tenglikka ega bo'lish mumkin. Bu tenglikni (57) dagi ikkinchi chegaraviy shartga qo'yib va $1-\alpha_N \chi_2 \neq 0$ deb hisoblab,

$$y_N = \frac{\mu_2 + \chi_2 \beta_N}{1-\alpha_N \chi_2} \qquad (62)$$

tenglikka ega bo'lish mumkin. $y_N$ ning bu qiymatini bilgan holda qolgan $i = N-1, N-2, ..., 3, 2, 1, 0$ lar uchun barcha echimlar (58) rekurrent formula bo'yicha aniqlanadi.

Endi haydash usulining (56)-(57) masala uchun turg'unlik shartini keltiramiz:

$$|C_i| \geq |A_i| + |B_i|, \quad i = 1,2,...,N-1,$$

$$|\chi_\alpha| \leq 1, \;\; \alpha = 1,2, \qquad |\chi_1| + |\chi_2| < 2.$$

Bu shartlarning bajarilishi esa barcha $i = 1,2,...,N$ lar uchun

$$|\alpha_i| \leq 1$$

shartni bajarilishi bilan ekvivalentdir.

**Misol.**

$$\begin{gathered} y''(x) + 2xy'(x) - 2y(x) = 2, \\ y'(0) = -2, \\ y(1) + y'(1) = 0 \end{gathered}$$

chegaraviy masala haydash usuli bilan echilsin.

**Echish.** Differentsial tenglamani $\{x_k\}$ to'rning ichki nuqtalarida quyidagi ayirmali tenglamalar sistemasi bilan approksimatsiya qilamiz $\left(h = 0{,}1;\ \ N = 10\right)$:

$$\frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + 2x_k \frac{y_{k+1} - y_{k-1}}{2h} - 2y_k = 2 \quad \left(k = \overline{1, N-1}\right).$$

Chegaraviy shartlarni

$$\frac{y_1 - y_0}{h} = -2,$$

$$y_N + \frac{y_N - y_{N-1}}{h} = 0$$

ko'rinishidagi ayirmali tenglamalar bilan approksimatsiya qilish mumkin. Hosil qilingan ayirmali tenglamalar sistemasini o'xshash hadlarini gruppalab, (56)-(57) tenglamalar sistemasi ko'rinishiga keltiramiz:

$$(1 - x_k h) y_{k-1} - 2\left(1 + h^2\right) y_k + (1 + x_k h) y_{k+1} = -\left(-2h^2\right) \quad \left(k = \overline{1, N-1}\right),$$

$$y_0 = y_1 + 2h,$$

$$y_N = \frac{\beta_N}{1 + h - \alpha_N}.$$

Bu tenglamalar sistemasidan koeffitsientlar uchun quyidagi

$$A_k = 1 - x_k h, \quad C_k = 2\left(1 + h^2\right), \quad B_k = 1 + x_k h, \quad F_k = -2h^2 \quad \left(k = \overline{1, N-1}\right)$$

munosabatlarga ega bo'lish mumkin. Haydash usulining tur\unlik sharti

$$|C_k| \geq |A_k| + |B_k|, \quad k = 1, 2, \ldots, N-1,$$

bajarilishi ko'rinib turibdi. Demak, hosil qilingan algebraik tenglamalar sistemasini echish uchun haydash usulini qo'llash mumkin. $\alpha_1 = 1,\ \beta_1 = 2h$ ekanligi va $y_N$ ni qiymati ma`lum. Endi dastur tuzib, hisoblashlarni komp`yuterda bajarish mumkin.

quyida natijalarni jadval shaklida keltiramiz.

| | | |
|---|---|---|
| x[0]q0 | y[0]10.602483840 | ykol[0] 10.475000000 |
| x[1]10,1 | y[1]10.402483840 | ykol[1]10.291153000 |
| x[2]10,2 | y[2]10.234216193 | ykol[2]10.138184000 |
| x[3]10,3 | y[3]10.096747595 | ykol[3]10.013951000 |
| x[4]10,4 | y[4] -0.011417053 | ykol[4] 0.083688000 |
| x[5]10,5 | y[5] -0.092250134 | ykol[5] -0.156875000 |
| x[6]10,6 | y[6] -0.148094352 | ykol[6] -0.207752000 |
| x[7]10,7 | y[7] -0.181542892 | ykol[7] -0.238461000 |
| x[8]10,8 | y[8] -0.195316724 | ykol[8] -0.251144000 |
| x[9]10,9 | y[9] -0.192148446 | ykol[9] -0.247943000 |
| x[10]11 | y[10] -0.174680405 | ykol[10] -0.23100000 |

Bu erda $x[i]$ $\left(i=\overline{0,10}\right)$ - to'rning tugun nuqtalari, $y[i]$ $\left(i=\overline{0,10}\right)$ bilan masalaning haydash usuli bilan aniqlangan taqribiy qiymati va $ykol[i]$ $\left(i=\overline{0,10}\right)$ bilan esa berilgan chegaraviy masalaning kollokatsiya usuli bilan aniqlangan

$$y_2(x)=4-2x+1{,}6510\left(x^2-3\right)-0{,}3570\left(x^3-4\right)$$

taqribiy echimi solishtirish uchun keltirilgan.

### 3.3. Kichik kvadratlar usuli

Faraz qilaylik, $H$ - haqiqiy Gil`bert fazosi bo'lsin. $H$ da zich $D(A)$ to'plamda aniqlangan, qiymatlari $H$ ga tegishli chiziqli operatorni $A$ deb belgilaylik va

$$Ay=f \tag{63}$$

tenglamani ko'raylik. Bu erda $f\in H$ va echim $y\in D(A)$. (1) tenglama echimga ega bo'lsin deylik. (62) tenglamaga quyidagi

$$J(y)=\|Ay-f\|^2 \tag{64}$$

fuktsionalni mos qo'yamiz va (1) ni echish masalasini shu funktsionalni $D(A)$ da minimumga erishtiruvchi elementni aniqlash masalasiga almashtiramiz.

$$\min_{y\in D(A)} J(y)=J(y*)=0$$

tenglikdan ayonki, $y*$ $Ay=f$ tenglamaning echimi bo'ladi. $Ay=f$ tenglamani echish o'rniga (64) funktsionalni minimumini topishga asoslanib, $Ay=f$ tenglamani echish usuli *kichik kvadratlar usuli deyiladi.*

$J(y)$ funktsionalni quyidagicha minimumga erishtiramiz. $D(A)$ ga tegishli chiziqli erkli $\{\varphi_k(x)\}$ funktsiyalar sistemasini tanlaymiz va (63) tenglamaning echimiga $n-$chi yaqinlashishni

$$y_n(x)=\varphi_0(x)+\sum_{k=1}^{n} c_k\varphi_k(x) \tag{65}$$

ko'rinishda izlaymiz. Bu erda noma`lum $c_k$ koeffitsientlar

$$J(y_n)=\|Ay_n-f\|^2$$

funktsional minimumga erishsin deb topiladi.

Biz quyidagi

$$L(y)\equiv y''+p(x)y'+q(x)y=f(x) \tag{66}$$

$$\begin{aligned} l_a(y)&=\alpha_0 y(a)+\alpha_1 y'(a)=A,\\ l_b(y)&=\beta_0 y(b)+\beta_1 y'(b)=B \end{aligned} \tag{67}$$

chiziqli chegaraviy masalani kichik kvadratlar usuli bilan echish sxemasini keltiramiz.

Quyidagi shartlarni qanoatlantiruvchi $\varphi_k(x)$ $(k=0,1,...)$ funktsiyalar sistemasini tanlaymiz:

1. $\varphi_k(x)\in C_2[a,b]$;

2. $l_a(\varphi_0(x)) = A, \quad l_b(\varphi_0(x)) = B, \quad l_a(\varphi_k(x)) = l_b(\varphi_k(x)) = 0 \quad (k = 1,2,...)$;

3. İxtiyoriy chekli $n$ uchun $\varphi_k(x)$ funktsiyalar chiziqli bo\liqsiz;

4. $\varphi_k(x) \quad (k = 1,2,...)$ $C_2[a,b]$ da to'liq sistemani tashkil etadi.

(66)-(67) chegaraviy masalani taqribiy echimini (65) ko'rinishda olamiz.

(65) funktsiya (67) chegaraviy shartlarni ixtiyoriy $n$ va $c_1, c_2, ..., c_n$ lar uchun qanoatlantiradi. $c_1, c_2, ..., c_n$ larni shunday tanlaylikki,

$$J(y_n) = \|L(y_n) - f\|^2$$

minimal qiymatga ega bo'lsin.

Ma`lumki,

$$\|L(y_n) - f\|^2 = (L(y_n) - f, L(y_n) - f) =$$

$$= \int_a^b \left[ L(\varphi_0) + \sum_{k=1}^n c_k L(\varphi_k) - f \right]^2 dx = J(c_1, c_2, ..., c_n).$$

Bundan barcha $i = 1,2,...,n$ lar uchun

$$\frac{1}{2}\frac{\partial J}{\partial c_i} = \int_a^b \left[ L(\varphi_0) + \sum_{k=1}^n c_k L(\varphi_k) - f \right] \cdot L(\varphi_i) dx = 0 .$$

Buni quyidagicha yozamiz:

$$\sum_{k=1}^n A_{ik} c_k = -B_i, \quad i = 1,2,...,n, \tag{68}$$

bu erda

$$A_{ik} = A_{ki} = \int_a^b L(\varphi_k) L(\varphi_i) dx, \quad B_i = \int_a^b (L(\varphi_0) - f) L(\varphi_i) dx \quad (i,k = 1,2,...,n).$$

(68) tenglamalar sistemasining matritsasi $\{L(\varphi_k)\}_{k=1}^n$ funktsiyalar sistemasi uchun tuzilgan Gramm matritsasidir.

Ma`lumki , agar

$$L(y) = 0, \quad y(a) = 0, \quad y(b) = 0$$

chegaraviy masala nol echimga ega bo'lsa, (68) tenglamalar sistemasi yagona echimga ega bo'ladi.

**Misol.**

$$L(y) \equiv y''(x) + 2xy'(x) - 2y(x) = 2$$

$$l_0(y) \equiv y'(0) = -2,$$
$$l_1(y) \equiv y(1) + y'(1) = 0$$

chegaraviy masalani $n = 2$ da kichik kvadratlar usuli bilan eching.

**Echish.** Kollakatsiya usuli bilan $\varphi_0(x)$, $\varphi_1(x)$, $\varphi_2(x)$ lar topilib, $L(\varphi_0(x))$, $L(\varphi_1(x))$, $L(\varphi_2(x))$ lar hisoblangan edi, shu sababli ulardan foydalanamiz. (68) sistema koeffitsientlarini hisoblaymiz:

$$A_{11} = \int_0^1 L^2(\varphi_1(x))dx = \int_0^1 (2x^2 + 8)^2 dx = \int_0^1 (4x^4 + 32x^2 + 64)dx = \frac{4}{5} + \frac{32}{3} + 64 = \frac{1132}{15},$$

$$A_{12} = A_{21} = \int_0^1 (2x^2 + 8)(4x^3 + 6x + 8)dx = \int_0^1 (8x^5 + 16x^3 + 16x^2 + 48x + 64)dx =$$

$$= \frac{8}{6} + \frac{16}{4} + \frac{16}{3} + \frac{48}{2} + 64 = \frac{296}{3},$$

$$A_{22} = \int_0^1 (4x^3 + 6x + 8)^2 dx = \int_0^1 (16x^6 + 48x^4 + 64x^3 + 36x^2 + 96x + 64)dx =$$

$$= \frac{16}{7} + \frac{48}{5} + 16 + 12 + 48 + 64 = \frac{5316}{35},$$

$$B_1 = \int_0^1 (L(\varphi_0(x) - 2)L(\varphi_1(x))dx = \int_0^1 (-2-2)(2x^2 + 8)^2 dx = -4\left(\frac{2}{3} + 8\right) = -\frac{104}{3},$$

$$B_2 = \int_0^1 (L(\varphi_0(x) - 2)L(\varphi_2(x))dx = \int_0^1 (-4)(4x^3 + 6x + 8)^2 dx = -4(1 + 3 + 8) = -48.$$

Endi (68) tenglamalar sistemasini yozamiz.

$$\left.\begin{aligned}\frac{1132}{15}c_1+\frac{296}{3}c_2&=-\frac{104}{3}\\ \frac{296}{3}c_1+\frac{5316}{35}c_2&=-48\end{aligned}\right\}\quad\Rightarrow\quad\begin{aligned}c_1&\cong-0{,}30648\\ c_2&\cong-0{,}11693\end{aligned}\quad.$$

Shunday qilib, chegaraviy masalaning biz izlayotgan taqribiy echimi

$$y_2(x)=4-2x-0{,}30648\left(x^2-3\right)-0{,}11693\left(x^3-4\right)$$

ko'rinishda bo'ladi.

### 3.4. Galerkin usuli

Bu usulda ham kichik kvadratlar usulidagi (66)-(67) masalani echishdagi shartlar o'rinli deb hisoblaymiz. (65) ko'rinishdagi taqribiy echimning $c_1, c_2, ..., c_n$ parametrlari bu usulda quyidagi shartlardan topiladi:

$$\left(L(y_n(x))-f(x),\varphi_i(x)\right)=0\qquad\left(i=1,2,...,n\right).$$

Buni quyidagicha yozamiz

$$\sum_{k=1}^{n}c_k\left(L(\varphi_k(x)),\,\varphi_i(x)\right)=\left(f-L(\varphi_0(x)),\,\varphi_i(x)\right)\qquad\left(i=1,2,...,n\right).\qquad(69)$$

**Misol.** $y''+y+x=0$,

$y(0)=0,\quad y(1)=0$

chegaraviy masalani Galerkin usuli bilan eching.

**Echish.** $\varphi_i(x)$ funktsiyalar sifatida quyidagilarni olamiz:

$$\varphi_0(x)\equiv 0,\quad \varphi_1(x)\equiv x(1-x),\quad \varphi_2(x)\equiv x^2(1-x),\quad ...,\quad \varphi_n(x)\equiv x^n(1-x).$$

Taqribiy echim

$$y_n(x)=\sum_{k=1}^{n}c_k\varphi_k(x)=\sum_{k=1}^{n}c_kx^k(1-x)$$

ko'rinishda bo'ladi. Berilgan misolni $n=2$ da echamiz:

$$\varphi_1(x)=x-x^2,\quad \varphi_1'(x)=1-2x,\quad \varphi''(x)=-2,\quad L(\varphi_1(x))=-2+x-x^2,$$

$$\varphi_2(x)=x^2-x^3,\quad \varphi_1'(x)=2x-3x^2,\quad \varphi''(x)=2-6x,\quad L(\varphi_2(x))=2-6x+x^2-x^3,$$

$$L(y_2(x))=c_1L(\varphi_1(x))+c_2L(\varphi_2(x))=c_1\left(-2+x-x^2\right)+c_2\left(2-6x+x^2-x^3\right).$$

(69) tenglamalar sistemasini yozamiz:

$$c_1\left(\left(-2+x-x^2\right),\left(x-x^2\right)\right)+c_2\left(\left(2-6x+x^2-x^3\right),\left(x^2-x^3\right)\right)=\left(-x,\ x-x^2\right)$$

$$c_1\left(\left(-2+x-x^2\right),\left(x^2-x^3\right)\right)+c_2\left(\left(2-6x+x^2-x^3\right),\left(x^2-x^3\right)\right)=\left(-x,\ x^2-x^3\right).$$

Skalyar ko'paytmalarni hisoblab, $c_1$ va $c_2$ koeffitsientlarga bo\liq tenglamalarni quyidagi ko'rinishga keltiramiz

$$\left.\begin{array}{l} \frac{3}{10}c_1+\frac{3}{20}c_2=\frac{1}{12} \\[2mm] \frac{3}{20}c_1+\frac{13}{105}c_2=\frac{1}{20} \end{array}\right\}.$$

Bu sistemani echib $c_1$ va $c_2$ koeffitsientlarni aniqlaymiz:

$$c_1=\frac{71}{369},\quad c_2=\frac{7}{41}.$$

Demak, taqribiy echim quydagicha buladi.

$$y_2(x)=x(1-x)\left(\frac{71}{369}+\frac{7}{41}x\right)$$

### 3.5. Koshi masalasining MathCad da dasturiy ta`minotini yaratish

$$\frac{dx}{dt}=f(x,u,t),\ x\in R^n,\ u\in R^m,\ t\in[0,t], \tag{1}$$

Differentsial` tenglamaning echimini izlashdan oldin uning echimi bor yoki yuqligi haqidagi quydagi teoremani keltiramiz.

Teorema. Differentsial` tenglamaning ung tarafidagi $f(x)$ funktsiyasi Lipshits shartini qanoatlantirsin:

agar $|x_1 - x_2| < A,$ bwlsa $|f(x_1) - f(x_2)| < M,$ bwladi bunda M- Lipshits doimiysi; $x_1,\ x_2$- lar $[a,b]$ kesmadan olingan $f(x)$ funktsiyasining aniqlanish soxasiga tegishli nuqtalar.

U holda differentsial tenglama $[a,b]$ kesmada yolg`iz echimga ega bwladi. Yani funktsiya birinchi tur uzilish nuqtasiga ega bwlsa u holda echim bor, agar ikkinchi tur uzilish nuqtasiga ega bwlsa echim yuq.

## Koshi masalasini taqribiy echish

Koshi masalasi echimining bor bo'lishi haqidagi teorema quydagi iteratsion jarayonning yaqinlashuvchiligiga asoslangan, u (1) tenglamani integrallash asosida olingan.

$$x(t)_{k+1} = x(0) + \int_0^t f(x(\tau)_k, u(\tau))d\tau, \quad x(0) = x^0, \quad k = 0,1,.... \qquad (2)$$

Bu jarayon yuqoridagi shartlarda yaqinlashuvchi bwladi va oddiy differentsial tenglamani integrallash usuli Pikar usuli dep ataladi.

**MathCad dasturi yordamida echish**

Misol 1. Quydagi oddiy differentsial tenglamaga quyilgan Koshi masalasini echaylik.

$$\begin{cases} \dfrac{dx}{dt} = -x, \\ x(0) = 1. \end{cases} \Rightarrow \frac{dx}{x} = -dt, \quad \ln|x| = -t + \ln c, \quad x = ce^{-t}$$

$$1 = ce^{-0} \Rightarrow c = 1, \quad x = e^{-t}.$$

Bu aniq echim, taqribiy echimni (2) formula asosida tuzamiz.

Uning MathCad tizimidagi dasturi quydagicha. Bu erda aniq echim bilan taqribiy echimlar taqqoslangan.

$xn := 1 \qquad f(x,t) := -x \qquad x0 := 0$

$$x1(t) := xn + \int_0^t f(x0,\tau)\,d\tau \rightarrow 1$$

$$x2(t) := xn + \int_0^t f(x1(t),\tau)\,d\tau \rightarrow 1 - t$$

$$x3(t) := xn + \int_0^t f(x2(t),\tau)\,d\tau \rightarrow 1 - t + t^2$$

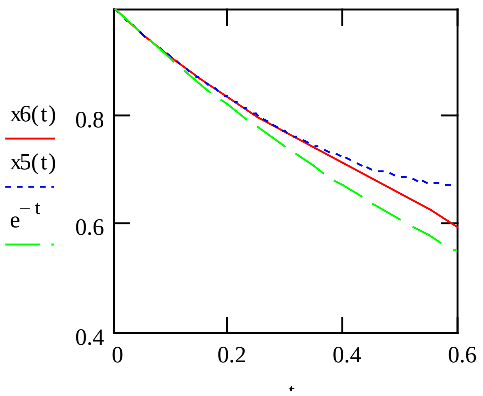
$$x4(t) := xn + \int_0^t f(x3(t),\tau)\,d\tau \rightarrow 1 - t + t^2 - t^3$$

$$x5(t) := xn + \int_0^t f(x4(t),\tau)\,d\tau \rightarrow 1 - t + t^2 - t^3 + t^4$$

$$x6(t) := xn + \int_0^t f(x5(t),\tau)\,d\tau \rightarrow 1 - t + t^2 - t^3 + t^4 - t^5$$

$t := 0, 0.05 .. .6$

| $x6(t) =$ | $x5(t) =$ | $e^{-t} =$ |
|---|---|---|
| 1 | 1 | 1 |
| 0.952 | 0.952 | 0.951 |
| 0.909 | 0.909 | 0.905 |
| 0.87 | 0.87 | 0.861 |
| 0.833 | 0.834 | 0.819 |
| 0.8 | 0.801 | 0.779 |
| 0.769 | 0.771 | 0.741 |
| 0.739 | 0.745 | 0.705 |
| 0.711 | 0.722 | 0.67 |
| 0.684 | 0.702 | 0.638 |
| 0.656 | 0.688 | 0.607 |
| 0.627 | 0.678 | 0.577 |
| 0.596 | 0.674 | 0.549 |



Misol 2. Chiziqli tenglamaga qoyilgan Koshi masalasini eching.

$$\begin{cases} \dfrac{dx}{dt} = t^2 - x^2, \\ x(0) = 0. \end{cases}$$

**Echimi:** Tenglamaning aniq echimi erikli doimiyni variyatsiyalash yoki Lagranj usulidan foydalanib echiladi. Uning uchun dastlab $\dfrac{dx}{dt} + x^2 = 0,$ birjinsli tenglama echimi topiladi. Birjinsli tenglama echimi $x = c(t)e^{-\int p(t)dt}$, kurinishda qidiriladi, bunda s(t) –topish kerak bwlgan t-ning uzluksiz funktsiyasi va $p(t) = 1, \ \ q(t) = t^2$.

Umumiy echim $x = e^{-\int p(t)dt}\left(C + \int q(t)e^{\int p(t)dt}dt\right),$ formulasidan foydalansak

$$x = e^{-t}\left(C + \int t^2 e^t dt\right) = \begin{vmatrix} u = t^2 & du = 2tdt \\ dv = e^t dt & v = e^t \end{vmatrix} =$$

$$= e^{-t}\left(C + t^2 e^t - 2\int te^t dt\right) = \begin{vmatrix} u = t & du = dt \\ dv = e^t dt & v = e^t \end{vmatrix} =$$

$$= e^{-t}\left(C + t^2 e^t - 2te^t + 2e^t\right) = Ce^{-t} + t^2 - 2t + 2,$$

$$x(0) = 0$$

$$0 = C + 2 \Rightarrow C = -2.$$

Aniq echim $x = -2e^{-t} + t^2 - 2t + 2,$ kurinishida bo'ladi.

MathCad tizimidagi dasturi quydagicha. Bu erda aniq echim bilan taqribiy echimlar taqqoslangan.

$xn := 0 \qquad f(x,t) := t^2 - x^2 \qquad\qquad x0 := 0$

$$x1(t) := xn + \int_0^t f(x0,\tau)\,d\tau \to \frac{1}{3}\cdot t^3$$

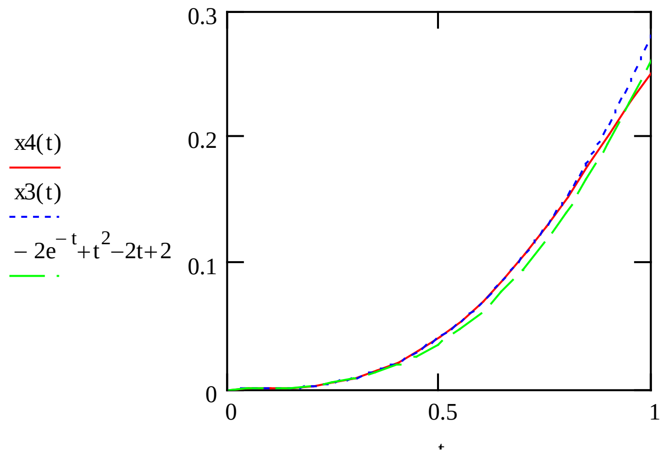
$$x2(t) := xn + \int_0^t f(x1(t),\tau)\,d\tau \to \frac{1}{3}\cdot t^3 - \frac{1}{9}\cdot t^7$$

$$x3(t) := xn + \int_0^t f(x2(t),\tau)\,d\tau \to \frac{1}{3}\cdot t^3 - \frac{1}{9}\cdot t^7 + \frac{2}{27}\cdot t^{11} - \frac{1}{81}\cdot t^{15}$$

$$x4(t) := xn + \int_0^t f(x3(t),\tau)\,d\tau \to \frac{1}{3}\cdot t^3 - \frac{1}{9}\cdot t^7 + \frac{2}{27}\cdot t^{11} - \frac{5}{81}\cdot t^{15} + \frac{2}{81}\cdot t^{19} - \frac{2}{243}\cdot t^{23} + \frac{4}{2187}\cdot t^{27} - \frac{1}{6561}\cdot t^3$$

$t := 0, 0.05 .. 1$

| $x4(t) =$ | $x3(t) =$ | $-2e^{-t} + t^2 - 2t + 2 =$ |
|---|---|---|
| 0 | 0 | 0 |
| $4.167\cdot 10^{-5}$ | $4.167\cdot 10^{-5}$ | $4.115\cdot 10^{-5}$ |
| $3.333\cdot 10^{-4}$ | $3.333\cdot 10^{-4}$ | $3.252\cdot 10^{-4}$ |
| $1.125\cdot 10^{-3}$ | $1.125\cdot 10^{-3}$ | $1.084\cdot 10^{-3}$ |
| $2.665\cdot 10^{-3}$ | $2.665\cdot 10^{-3}$ | $2.538\cdot 10^{-3}$ |
| $5.202\cdot 10^{-3}$ | $5.202\cdot 10^{-3}$ | $4.898\cdot 10^{-3}$ |
| $8.976\cdot 10^{-3}$ | $8.976\cdot 10^{-3}$ | $8.364\cdot 10^{-3}$ |
| 0.014 | 0.014 | 0.013 |
| 0.021 | 0.021 | 0.019 |
| 0.03 | 0.03 | 0.027 |
| 0.041 | 0.041 | 0.037 |
| 0.054 | 0.054 | 0.049 |
| 0.069 | 0.069 | 0.062 |
| 0.087 | 0.087 | 0.078 |
| 0.106 | 0.107 | 0.097 |
| 0.128 | 0.129 | 0.118 |



## Xulosa

Malakaviy bitiruv ishida birinshi ta`rtibli oddiy differentsial tenglamalar, Koshi masalasining quyilishi, shegaraviy sha`rtlarda oddiy differentsial tenglamalarni sonli echish usulları urganilgan.

Ko`p hollarda differentsial tenglamalarga qo'yilgan chegaraviy shartlarda echim har doim ham analitik usılda olina vermaydi. Shu boistan echim taqriybiy usullar yordamida olinadi.

Ushbu malakaviy bitiruv ishning maqsadi birinchi va ikkinchi ta`rtibli chiziqli va chiziqli emas differentsial tenglamalarni qo'yilgan chegaraviy shartlarda taqriybiy echishni urganish.

Bitiruv malakaviy ish kirish, uch paragraf, xulosa va foydalanilgan adabiyotlar tizimidan iborat bo'lıb birinchi paragrafta differentsial tenglama haqida umumiy tushinchalar, ikkinchi paragrafta Koshi masalasi va uning qo'yilishi, Adams ekstrapolyatsion va interpolyatsion usullari, uchinchi paragrafta oddiy differentsial tenglamalarga qo'yilgan chegaraviy masalalarni echish usullaridan kollokatsiya, haydash, kichik kvadratlar va Galerkin usullari keltirilgan.

Oxirgi punktta Koshi masalasining MathCad da dasturiy ta`minoti yaratilgan yani oddiy differentsial tenglamani taqriybiy echish usullaridan biri Pikar metodiga MathCad tizimida dastur tuzilgan. Aniq echim taqriybiy eshim bilan taqqoslangan.

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## Texnika xavfsizligi qoidalari

1.Informatika xonasini beton polli xonalarga yoki binoning erto'lalariga joylashtirish qat'iyan ta'qiqlanadi va sinf xonasining poli elektr tokini o'tkazmaydigan qilib (masalan taxtadan) yasalishi va barcha kompyuterlarning korpuslari yerga ulanishi (yoki elektr ta'minotining yevroandozalarga moslab ulanishi) talab qilinadi.

2. Yuqori kuchlanish (220 Volt) yuradigan simlarning barchasi, shu jumladan uzaytirgichlarning, kompyuter va boshqa elektr qurilmalarining elektr ta'minotiga ulanish simlarining ikki marta izolyasiya qilinganligi talab qilinadi.

3. Xonadagi kompyuterlarni devor bo'ylab yoki xonaning o'rtasiga ikki qator qilib joylashtirilishi kerak

4. Xonadagi barcha kompyuterlarni elektr tarmog'idan uzuvchi yagona uzgich ham bo'lishi kerak.

5. Kompyuter monitori o'tirgan o'quvchilarning ko'zlari darajasida bo'lib, o'quvchilar undan 40 sm dan 80 sm gacha bo'lgan masofada o'tirishlari imkoniyatiga ega bo'lishlari kerak.

6. Kompyuterning klaviaturasi o'tirgan o'quvchilarning bukilgan tirsaklari darajasida bo'lishi kerak. Sichqon uchun klaviaturaning ikkala tomonidan yetarlicha joy qoldirilishi va ular bir xil balandlikda bo'lishlari kerak.

7. Kompyuterda muttasil ishlash vaqti o'quvchilar uchun 60 minutdan ortmasligi kerak.

8. Kompyuter xonasining kvadrat metrlardagi sathi unga joylangan kompyuterlar sonidan kamida 6 marta ko'p bo'lishi kerak.

9. Kompyuter xonalari yetarli quvvatga ega ventilyasiya tizimiga ega bo'lishi kerak.

10. Klaviaturaning kompyuter ishlamayotgan paytda uzoq muddatga ochiq holda qolishi va unda chang yig'ilib qolishining oldini olish lozim