UZLUKSIZ BO'LMAGAN FUNKSIYALARNING FURYE ALMASHTIRISHI HAQIDA

Barakayev Azamat, Boboyev Birodar

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1-2-bosqich

Mazkur maqolada cheksiz tipdagi uzilishga ega bo'lgan funksiyalar Furye almashtrishining cheksizdagi xarakteri haqidagi asimptotik baho olingan.

$$f(x) = \frac{\varphi(x)}{q(x)} \qquad (1)$$

ko'rinishdagi funksiya berilgan bo'lsin, bunda $\varphi(x) \in C_0^\infty(R^3)$ va $q(x) \in C_0^\infty(R^3)$ bo'lib $q(0) = 0$ hamda $x = 0$ nuqta $q(x)$ funksiya uchun minimum nuqtasi bo'lsin, ya'ni

$$\nabla q(0) = 0 \ \text{ va } \ detD^2q(0) \neq 0$$

shartlar bajarilsin.

Bu holda $x = 0$ nuqta $q(x)$ funksiyaning xosmas (aynimagan) minimum nuqtasi deyiladi.

Biz $\varphi(x)$ funksiya $x = 0$ nuqtaning yetarlicha kichik atrofidan tashqarida aynan nolga teng deb faraz qilamiz.

Ushbu maqolada (1) shaklda berilgan $\hat{f}(\xi)$ funksiyaning $\xi \to \infty$ dagi asimptotikasining bosh hadini topish masalasi bilan shug'ullanamiz va qoldiq hadni baholaymiz.

Osongina ko'rsatish mumkinki, agar $\varphi(x)$ ning havzasi koordinatalar boshining yetarli kichik atrofida tayinlangan bo'lsa, $f \in L^1(R^3)$ bo'ladi. Shuning uchun funksiyaning Furye almashtirishi $\widehat{f}(\xi)$ Lebeg ma'nosidagi integral orqali yozish mumkin, ya'ni

$$\hat{f}(\xi) = \int_{R^3} e^{i(\xi,x)} \frac{\varphi(x)}{q(x)} dx \qquad (2)$$

Ma'lumki Lebeg teoremasiga ko'ra [1] $\xi \to \infty$ da $\hat{f}(\xi) \to 0$ munosabat o'rinli bo'ladi. Ammo Lebeg teoremasi $\hat{f}(\xi)$ funksiyaning cheksizlikdagi xarakteri haqida hech qanday ma'lumot berolmaydi.

Biz $\hat{f}(\xi)$ ning cheksizlikdagi asimptotikasining bosh hadini hisoblab, qoldiq hadini baholaymiz. $q(x)$ funksiya uchun Makloren formulasidan foydalanib, uni

$$q(x) = \frac{1}{2}(D^2q(0)x, x) + R(x)$$

shaklda yozib olamiz, bunda $R(x) \in C^{\infty}(R^3)$ bo'lib uning ikkinchi tartibgacha xususiy hosilalari nolga teng. $q(x)$ funksiyaning yuqoridagi yoyilmasini (2) ga qo'yib f ning Furye almashtirishi uchun

$$\hat{f}(\xi) = \int_{R^3} \frac{e^{i(\xi,x)}\varphi(x)dx}{\frac{1}{2}(D^2q(0)x, x) + R(x)} \qquad (3)$$

ifodani hosil qilamiz.

Algebra kursidan ma'lumki, [2] q funksiya $x = 0$ da lokal aynimagan minimumga ega bo'lgani uchun, shunday B matritsa topiladiki,

$$\frac{1}{2}(D^2q(0)By, By) = y_1^2 + y_2^2 + y_3^2$$

tenglik o'rinli bo'ladi.

(3) integralda $x = By$ almashtirish olsak

$$\hat{f}(\xi) = |detB| \int_{R^3} \frac{e^{i(B^t\xi,y)}\varphi(By)dy}{y_1^2 + y_2^2 + y_3^2 + R(By)}$$

tenglik o'rinli. Bu yerda B^t transponirlangan matritsa.

Dastlab quyidagi

$$\frac{1}{y_1^2+y_2^2+y_3^2+R(By)} - \frac{1}{y_1^2+y_2^2+y_3^2} = \frac{-R(By)}{(y_1^2+y_2^2+y_3^2)(y_1^2+y_2^2+y_3^2+R(By))}$$

tenglikdan foydalanib

$$I_0 = \int_{R^3} \frac{e^{i(B^t\xi,y)}\varphi(By)dy}{y_1^2 + y_2^2 + y_3^2}$$

integralni qaraymiz. Buning uchun $\varphi(By)$ funksiyani quyidagicha yozib olamiz:

$$\varphi(By) = X_0(y)(\varphi(0,0) + \sum_{j=1}^{3} a_j y_j) + \sum_{j,k=1}^{3} y_j y_k R_{jk}(y).$$

Bu yerda $X_0(y)$ sferik simmetrik va koordinatalar boshining $\varphi(By)$ funksiyaning havzasini saqlovchi atrofida aynan birga teng bo'lib uning havzasi (support) koordinatalar boshining kichik atrofida joylashgan. Endi

$$I_0^0(\xi) = \int_{R^3} \frac{e^{i(B^t\xi,y)}X_0(y)dy}{y_1^2+y_2^2+y_3^2}$$

integralni baholaymiz. Soddalik uchun $\lambda = B^t\xi$ belgilash olamiz.

Osongina ko'rish mumkinki $I_0^0(\lambda)$ sferik simmetrik funksiya bo'ladi. Demak,

$$I_0^0(\lambda_1,\lambda_2,\lambda_3) = I_0^0(|\lambda|,0,0)$$

tenglik o'rinli.

$$I_0^0(|\lambda|,0,0) = \int_{R^3} \frac{e^{i|\lambda|y_1}X_0(y)\tilde{X}(y_2,y_3)dy_1dy_2dy_3}{y_1^2+y_2^2+y_3^2}$$

Bu yerda $\tilde{X}(y_2,y_3)$ havzasi kompakt va $X_0(y)$ funksiyaning havzasida aynan birga teng funksiya.

Bir karrali J_1 integralni hisoblab

$$J_1(|\lambda|,y_2,y_3) = \int_{R^1} \frac{e^{i|\lambda|y_1}X_0(y)dy_1}{y_1^2+y_2^2+y_3^2} = \int_{R^1} \frac{e^{i|\lambda|y_1}dy_1}{y_1^2+y_2^2+y_3^2} +$$

$$+\int_{R^1} \frac{e^{i|\lambda|y_1}dy_1}{y_1^2+y_2^2+y_3^2} + \int_{R^1} \frac{e^{i|\lambda|y_1}(X_0(y)-1)dy_1}{y_1^2+y_2^2+y_3^2} =$$

$$= \frac{\pi e^{-|\lambda|\sqrt{y_2^2+y_3^2}}}{\sqrt{y_2^2+y_3^2}} + O(|\lambda|^{-\infty})$$

tenglikka ega bo'lamiz, bunda O munosabat (y_1,y_2) ixtiyoriy kompakt to'plamda o'zgarganda tekis bo'ladi.Demak,

$$I_0^0(|\lambda|,0,0) = \int_{R^2} \tilde{X}(y_2,y_3)\frac{\pi e^{-|\lambda|\sqrt{y_2^2+y_3^2}}dy_2dy_3}{\sqrt{y_2^2+y_3^2}} + O(|\lambda|^{-\infty})$$

tenglikka kelamiz. Oxirgi integralda qutb koordinatalar sistemasiga o'tib,

$$I_0^0(|\lambda|,0,0) = \pi\int_0^1 \tilde{X}(r,0)e^{-\lambda r}dr + O(|\lambda|^{-\infty})$$

munosabatni hosil qilamiz. Bo'laklab integrallash formulasini qo'llab

$$I_0^0(|\lambda|,0{,}0)=\frac{\pi}{|\lambda|}-\pi\int_0^1 \widetilde{X_0}'(r,0)\,e^{-\lambda r}dr+O(|\lambda|^{-\infty})$$

munosabatga ega bo'lamiz.

$\widetilde{X_0}$ funksiya nolning yetarli kichik atrofida aynan birga teng ekanligini hisobga olsak,

$$I_0^0(|\lambda|,0{,}0)=\frac{\pi}{|\lambda|}+\ O(|\lambda|^{-\infty})$$

munosabatga ega bo'lamiz. Yoyilmaning qolgan hadlari $\frac{\pi}{|\lambda|}$ ga nisbatan cheksiz kichik miqdor bo'ladi.

Shunday qilib, quyidagi teorema o'rinli bo'ladi.

Teorema: Agar $q(x)>0$, $x\in R^3$ funksiya (0,0,0) nuqtada xosmas minumumga ega bo'lsa va $\varphi(x)$ koordinatalar boshining biror atrofidan tashqarida aynan nolga teng bo'lgan silliq funksiya bo'lsa, u holda shunday B matritsa topiladiki $\hat{f}$ funksiya uchun

$$\hat{f}(\xi)=\frac{\pi|\det B|\varphi(0{,}0{,}0)}{|B^t\xi|}+O(|\xi|^{-2})\ (|\xi|\to+\infty)$$

tenglik o'rinli bo'ladi.

Endi qoldiq hadni baholaymiz. Agar $j=2{,}3$ bo'lsa

$$\int_{R^3}\frac{e^{i|\lambda|y_1}y_jX_0(y)dy_1}{y_1^2+y_2^2+y_3^2}=0$$

tenglamalar ixtiyoriy λ uchun o'rinli.

$$\int_{R^3}\frac{e^{i|\lambda|y_1}y_1X_0(y)dy_1}{y_1^2+y_2^2+y_3^2}$$

integralni qaraymiz.

$$J_1=\int_R\frac{e^{i|\lambda|y_1}y_jX_0(y)dy_1}{y_1^2+y_2^2+y_3^2}$$

integral uchun bo'laklab integrallash formulasini qo'llaymiz va

$$\widetilde{J_1} = \frac{1}{i|\lambda|}\int_R \left(\frac{y_1 X_0(y)}{y_1^2+y_2^2+y_3^2}\right)'_{y_1} e^{i|\lambda|y_1}\,dy_1$$

tenglikka kelamiz. Haqiqatan ham, bo'laklab integrallash formulasini qo'llab,

$$\widetilde{J_1} = \int_R \frac{e^{i|\lambda|y_1}X_0(y)dy_1}{\left(y_1^2+y_2^2+y_3^2\right)^2} = \int_{-\infty}^{\infty}\frac{e^{i|\lambda|y_1}dy_1}{\left(y_1^2+y_2^2+y_3^2\right)^2} + O(|\lambda|^{-\infty}) = \left|y_1 = \sqrt{y_2^2+y_3^2}z\right| =$$

$$= \frac{1}{(y_2^2+y_3^2)^{\frac{3}{2}}}\int_{-\infty}^{\infty}\frac{e^{i|\lambda|\sqrt{y_2^2+y_3^2}y_1}dy_1}{\left(\sqrt{y_1^2+1}\right)^2} + O(|\lambda|^{-\infty}) = \frac{\pi}{2}\frac{|\lambda|\sqrt{y_2^2+y_3^2}}{(y_2^2+y_3^2)^{\frac{3}{2}}}e^{-|\lambda|\sqrt{y_2^2+y_3^2}} +$$

$$+\frac{\pi}{4}\frac{e^{-|\lambda|\sqrt{y_2^2+y_3^2}}}{(y_2^2+y_3^2)^{\frac{3}{2}}} + O(|\lambda|^{-\infty}).$$

tengliklarga ega bo'lamiz.

Demak,

$$J_1 = O(|\lambda|^{-2})$$

munosabat o'rinli bo'ladi.

$$J_R = \int_{R^3}\frac{y_i y_j R_{jk}(y)e^{i|\lambda|y_1}X_0(y)\widetilde{X_0}(y_2 y_3)dy}{y_1^2+y_2^2+y_3^2}$$

integralda ikki marta y_1 o'zgaruvchi bo'yicha bo'laklab integrallash formulasini qo'llasak,

$$J_R = O(|\lambda|^{-2})$$

munosabatga ega bo'lamiz. Haqiqatan ham

$$a(y) = \frac{y_i y_j R_{jk}(y)X_0(y)\widetilde{X_0}(y_2 y_3)}{y_1^2+y_2^2+y_3^2}$$

belgilashni olamiz. U holda $j = 0{,}1{,}2$ uchun

$$\left|\frac{\partial^j a(y)}{\partial y_1^j}\right| \le \frac{C_j}{|y|^j}$$

tengsizlik bajariladi. Shuning uchun $a(y)$ kompakt havzaga ega bo'lganligi sababli

$$\frac{\partial^j a(y)}{\partial y_1^j}$$

funksiya R^3 da Lebeg ma'nosida integrallanuvchi bo'ladi. Shuning uchun y_1 o'zgaruvchi bo'yicha ikki marta bo'laklab integrallash formulasini qo'llash mumkin va izlangan asimptotik bahoga kelamiz.

Shu bilan teorema to'liq isbot bo'ldi.

Adabiyotlar

1. Федорюк М. В. Метод перевала.-М: Наука, 1977.

2. Курош А.Г. Олий алгебра курси.-Т.:Ўкитувчи, 1976.

3. Гельфанд И.М. Шилов Г.Е. Обобщенные функции и действия над ними.-М.Наука, 1958,(1-часть).