PARAMETRGA BOG’LIQ AYLANA AKSLANTIRISHLARINING BA’ZI XOSSALARI

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Bir parametrli birlik aylana akslantirishlari oilasini qaraylik:

$$T_\theta x = \{f(x,\theta)\},\ \ x \in S^1 := [0,1),\ \ \theta \in R.$$

Bunda $\{\cdot\}$ - sonning kasr qismi, $f_\theta(x) = f(x,\theta)$ esa har bir $\theta \in R$ da quyidagi shartlarni qanoatlantiradi:

1) $f_\theta(x)$ funksiya x bo’yicha R da uzluksiz;

2) $0 \le f_\theta(0) < 1$ va $x \in R$ uchun $f_\theta(x+1) = f_\theta(x) + 1$;

3) Shunday $x_c(\theta) \in S^1$ mavjudki $f_\theta \in C^2[x_c(\theta), x_c(\theta)+1)$, ixtiyoriy $x \in (x_c(\theta), x_c(\theta)+1)$, $f_\theta^{'}(x) \ge \text{const} > 0$ va

$$\sqrt{\frac{f_\theta^{'}(x_c - 0)}{f_\theta^{'}(x_c + 0)}} = c_\theta \ne 1.$$

p_θ orqali T_θ gomeomorfizmning burish sonini belgilaymiz ([1] ga qarang), ya’ni

$$\rho_\theta := \lim_{n\to\infty} \frac{f_\theta^n(x)}{n},$$

bu yerda $f_\theta^n(x) := f_\theta\left(f_\theta^{n-1}(x)\right)$ $- f_\theta$ funksiyaning n- iteratsiyasi.

Ta’rif 1. Ushbu songa

$$Cr(z_1, z_2, z_3, z_4) = \frac{(z_2 - z_1)(z_4 - z_3)}{(z_3 - z_1)(z_4 -_2)}$$

(z_1, z_2, z_3, z_4), $(z_1 < z_2 < z_3 < z_4)$ to’rtlikning ikkilangan munosabati deb aytiladi.

Ta’rif 2. Kesmalar uchligi $([z_1, z_2],[z_2, z_3],[z_3, z_4])$, $x_c(\theta)$ maxsus nuqtani qoplaydi deyiladi, agar $x_c(\theta) \in [z_1, z_4]$ bo’lsa. Agar

1) $x_c \in [z_1, z_2]$;

2) $(x_c - z_1)/(z_2 - z_1) \ge M$ bo’lsa,

bunday kesmalar uchligi maxsus nuqtani (to’g’ri ma’noda) M, $0<M\le 1$ konstantali qoplaydi deyiladi.

Ushbu ishda, agar kesmalar uchligi $([z_1,z_2],[z_2,z_3],[z_3,z_4])$ maxsus nuqta $x_c(\theta)$ ni qoplamasa u holda ixtiyoriy φ differensiallanuvchi funksiya uchun quyidagi natija o`rinli bo`ladi.

Teorema. Faraz qilaylik, biror $x_0\in S^1$ nuqtada $\varphi'(x_0)=p_0>0$ bo’lsin va qandaydir $R>1$ konstanta uchun quyidagi shartlar bajarilsin

a) $\frac{z_2-z_1}{R}\le z_3-z_2\le R(z_2-z_1)$, $\frac{z_2-z_1}{R}\le (z_4-z_3)\le R(z_2-z_1)$,

b) $\max\limits_{1\le i\le 4}|z_i-x_0|\le R(z_2-z_1)$.

U holda ixtiyoriy $\varepsilon>0$ son uchun shunday $\delta=\delta(\varepsilon)>0$ va $C>0$ sonlar mavjudki ixtiyoriy $z_i\in(x_0-\delta,x_0+\delta)$ $i=1,2,3,4$ sonlar uchun quyidagi tengsizlik

$$\left|\frac{Cr(\varphi(z_1)),(\varphi(z_2)),(\varphi(z_3)),(\varphi(z_4)),}{Cr(z_1,z_2,z_3,z_4)}-1\right|\le C_1\varepsilon$$

o’rinli bo’ladi. Bu yerda C_1 konstanta R va p_0 ga bog’liq bo’lib, ε ga bog’liq emas.

Isbot. Faraz qilaylik $\varphi'(x_0)=\rho_0>0$ bo’lsin. U holda hosila ta’rifiga ko’ra ixtiyoriy $\varepsilon>0$ son uchun shunday $\delta=\delta(x_0,\varepsilon)>0$ mavjudki, barcha $x\in(x_0-\delta,x_0+\delta)$ lar uchun

$$\rho_0-\varepsilon<\frac{\varphi(x)-\varphi(x_0)}{x-x_0}<\rho_0+\varepsilon \qquad (1)$$

tengsizlik o’rinli bo’ladi. Teoremaning a) va b) shartlarni qanotlantiruvchi $z_i<x_0$, (i=1,2,3,4) nuqtalarni olamiz. Aniqlik uchun $z_i<x_0$ deb faraz qilamiz. Qolgan hollarda teorema shunga o’xshash isbotlanadi. $x=z_i$ tenglikdan

$$(\rho_0-\varepsilon)(x_0-z_i)<\varphi(x_0)-\varphi(z_i)<(\rho_0+\varepsilon)(x_0-z_i) \qquad (2)$$

kelib chiqadi. Bundan quyidagi tengsizliklarni keltirib chiqaramiz

$$\rho_0-\varepsilon\frac{(x_0-z_2)+(x_0-z_1)}{z_2-z_1}\le\frac{\varphi(z_2)-\varphi(z_1)}{z_2-z_1}\le\rho_0+\varepsilon\frac{(x_0-z_2)+(x_0-z_1)}{z_2-z_1}, \qquad (3)$$

$$\rho_0-\varepsilon\frac{(x_0-z_3)+(x_0-z_1)}{z_3-z_1}\le\frac{\varphi(z_3)-\varphi(z_1)}{z_3-z_1}\le\rho_0+\varepsilon\frac{(x_0-z_3)+(x_0-z_1)}{z_3-z_1}, \qquad (4)$$

$$\rho_0-\varepsilon\frac{(x_0-z_4)+(x_0-z_2)}{z_4-z_2}\le\frac{\varphi(z_4)-\varphi(z_2)}{z_4-z_2}\le\rho_0+\varepsilon\frac{(x_0-z_4)+(x_0-z_2)}{z_4-z_2}, \qquad (5)$$

$$\rho_0-\varepsilon\frac{(x_0-z_4)+(x_0-z_3)}{z_4-z_3}\le\frac{\varphi(z_4)-\varphi(z_3)}{z_4-z_3}\le\rho_0+\varepsilon\frac{(x_0-z_4)+(x_0-z_3)}{z_4-z_3}. \qquad (6)$$

Teoremmaning a) va b) shartlaridan

$$\max_{1\le i\le 4}\max\left\{\frac{x_0-z_i}{z_2-z_1},\frac{x_0-z_i}{z_3-z_1},\frac{x_0-z_i}{z_4-z_2},\frac{x_0-z_i}{z_4-z_3}\right\}\le K_1 \qquad (7)$$

kelib chiqadi. Bu holda konstanta $K_1>0$ faqat R ga bog’liq va ε ga bog’liq emas.

Ra’vshanki

$$\frac{Cr(\varphi(z_1),\varphi(z_2),\varphi(z_3),\varphi(z_4))}{Cr(z_1,z_2,z_3,z_4)}=\frac{\varphi(z_2)-\varphi(z_1)}{z_2-z_1}\frac{\varphi(z_4)-\varphi(z_3)}{z_4-z_3}\frac{z_3-z_1}{\varphi(z_3)-\varphi(z_1)}\frac{z_4-z_2}{\varphi(z_4)-\varphi(z_2)}\ .$$

Bundan va (3)-(6) lardan lemmaning isboti kelib chiqadi.

Adabiyotlar

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