MIRZO ULUG'BEK NOMIDAGI O'ZBEKISTON MILLIY UNIVERSITETI

MEXANIKA-MATEMATIKA FAKULTETI MATEMATIKA YO'NALISHI

3-KURS TALABASI SULTONOVA DILRABONING

MATEMATIK-FIZIKA FANIDAN

Kurs ishi

Mavzu: Parabolik tipdagi tenglamalar uchun qo'yilgan Koshi masalasini Puasson formulasidan foydalanib yechish.

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Quyidagi tenglamalar uchun qo’yilgan Koshi masalasini Puasson formula-sidan foydalanib yeching.

1-misol

$$U_t = U_{xx} + 2t$$
$$U(x,0) = 1$$

Yechish: Puasson formulasidan foydalanib yechamiz:

Puasson formulasi: $U(x,t) = \dfrac{1}{2a\sqrt{\pi t}} \displaystyle\int_{-\infty}^{+\infty} \varphi(\xi) 2e^{-\frac{|x-\xi|^2}{4a^2 t}} d\xi + \int_0^t \int_{-\infty}^{+\infty} \dfrac{1}{2a\sqrt{\pi(t-\tau)}} \cdot e^{-\frac{(x-\xi)^2}{4a^{2(t-\tau)}}} f(\xi,\tau) d\xi d\tau$

$a = 1;\ f(x,t) = 2t; \varphi(x) = 1$

$$U(x,t) = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\frac{|x-\xi|^2}{4t}} d\xi + \int_0^t \int_{-\infty}^{+\infty} \frac{2\tau e^{\frac{|x-\xi|^2}{4(t-\tau)}}}{2\sqrt{\pi(t-\tau)}} d\xi d\tau = \frac{1}{2\sqrt{\pi t}} 2\sqrt{t}\sqrt{\pi} + \int_0^t \frac{\tau}{\sqrt{\pi(t-\tau)}} \int_{-\infty}^{+\infty} e^{\frac{|x-\xi|^2}{4(t-\tau)}} d\xi d\tau =$$

$$= 1 + \int_0^t \frac{\tau}{\sqrt{\pi\ \ t-\tau}} 2\sqrt{t-\tau}\sqrt{\pi} d\tau = 1 + \int_0^t 2\tau d\tau = 1 + t^2$$

Javob: $U(x,t) = 1 + t^2$.

2-misol

$$U_t = U_{xx} + \sin t$$
$$U(x,t)|_{t=0} = 0$$
$$\varphi(x) = 0$$
$$f(x,t) = \sin t$$

Yechim:

$$U(x,t) = \int_0^t \int_{-\infty}^{\infty} \frac{\sin\tau e^{-\frac{|x-\xi|^2}{4a^2(t-\tau)}}}{2a\sqrt{\pi(t-\tau)}} d\xi d\tau = \int_0^t \frac{\sin\tau}{2a\sqrt{\pi(t-\tau)}} \int_{-\infty}^{\infty} e^{-\frac{|x-\xi|^2}{4a^2(t-\tau)}} d\xi d\tau$$

Bizga ma’lumki:

$$\int_{-\infty}^{\infty} e^{-\rho^2} d\rho = \sqrt{\pi}$$

$$\rho^2 = \frac{|x-\xi|^2}{4a^2(t-\tau)}$$

$$\rho = \frac{x-\xi}{2a\sqrt{t-\tau}}$$

$$d\rho = -\frac{d\xi}{2a\sqrt{t-\tau}}$$

$$\xi = -\infty \qquad \rho = \infty$$

$$\xi = \infty \qquad \rho = -\infty$$

$$\int_{-\infty}^{\infty} e^{-\rho^2} 2a\sqrt{t-\tau}\, d\rho = 2a\sqrt{t-\tau}\sqrt{\pi}$$

$$U(x,t) = \int_0^t \frac{\sin\tau}{2a\sqrt{t-\tau}} 2a\sqrt{t-\tau}\sqrt{\pi}\, d\tau = \int_0^t \sin\tau = -\cos t\,|_0^t =$$

$$= -(\cos t - 1) = 1 - \cos t$$

$$Javob:\ U(x,t) = 1 - \cos t$$