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REFERAT

Matemaik fizika masalalani echilishi haqida

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Reja:

1. Issiqlik o'tkazuvchanlik tenglamasi uchun koshi masalasini yechish.
2. Issiqlik o'tkazuvchanlik tenglamasi uchun Koshi masalasini yeching.
3. $u_t = a^2 u_{xx}$ tenglamaning $t > 0, 0 < x < l$ yolakda $u_x(l,t) + h * u(l,t) = u,\ u(0,t) = T,\ U(x,0) = 0$ shartlarni qanoatlantiruvchi yechimini toping.

1. Issiqlik o'tkazuvchanlik tenglamasi uchun koshi masalasini yeching.

$4u_t = u_{xx}, u(x,0) = e^{-x^2}\sin x$.bu masalaning yechimini quyidagi formula orqali topamiz:

$$u(x,t) = \frac{1}{2a\sqrt{\pi t}}\int_{=\infty}^{+\infty}\varphi(\xi)e^{-\frac{(x-\xi)^2}{4a^2t}}d\xi$$,bunda a=1/2

$$u(x,t) = \frac{1}{\sqrt{\pi t}}\int_{=\infty}^{+\infty}e^{-\xi^2}\sin\xi * e^{-\frac{(x-\xi)^2}{t}}d\xi = \left\{-(\xi^2 + \frac{x^2+\xi^2-2x\xi}{t}) = -\frac{x^2}{t} - \frac{(t+1)}{t}(\xi^2 - \frac{2\xi x}{t+1} + (\frac{x}{t+1})^2 - \frac{x^2}{(t+1)^2}) = \frac{x^2(1-t-1)}{t(t+1)} - \frac{t+1}{t}(\xi - \frac{x}{t+1})^2 = -\frac{x^2}{t+1} - \frac{t+1}{t}(\xi - \frac{x}{t+1})^2\right\} =$$

$$= \frac{1}{\sqrt{\pi t}}e^{-\frac{x^2}{t+1}}\int_{-\infty}^{+\infty}e^{-\frac{t+1}{t}(\xi-\frac{x}{t+1})^2}\sin\xi d\xi$$ .integral va uning ostini I bilan belgilaymiz va bu integralni hisoblaymiz.Bu yerda quyidagicha belgilash kiritamiz: $\xi - \frac{x}{t+1} = s \to \xi = s + \frac{x}{t+1}, d\xi = ds$.

Demak,
$$I = \int_{-\infty}^{+\infty}e^{-\frac{t+1}{t}s^2}\sin(s+\frac{x}{t+1})ds = \int_{-\infty}^{+\infty}e^{-\frac{t+1}{t}s^2}(\sin s * \cos\frac{x}{t+1} + \sin\frac{x}{t+1}\cos s)ds = \cos\frac{x}{t+1}\int_{-\infty}^{+\infty}e^{-\frac{t+1}{t}s^2}\sin s ds + \sin\frac{x}{t+1}\int_{-\infty}^{+\infty}e^{-\frac{t+1}{t}s^2}\cos s ds =$$
$$\frac{\sqrt{\pi t}}{\sqrt{t+1}}e^{-\frac{t}{4(t+1)}}\sin\frac{x}{t+1}$$

Bu yerda 1-qo'shiluvchining qiymati 0 ga teng.Demak,javob quyidagiga teng:

$$u(x,t) = \frac{1}{\sqrt{\pi t}}\frac{\sqrt{\pi t}}{\sqrt{t+1}}e^{-\frac{t}{4(t+1)}}e^{-\frac{x^2}{t+1}}\sin\frac{x}{t+1} = \frac{e^{-\frac{x^2}{t+1}}}{\sqrt{t+1}}e^{-\frac{t}{4(t+1)}}\sin\frac{x}{t+1}.$$

2. Issiqlik o'tkazuvchanlik tenglamasi uchun Koshi masalasini yeching.

$$U_t = U_{xx} + 3t^2 \qquad U(x,t)|_{t=0} = \sin x$$

Masalaning yechimini quyidagi formuladan foydalanib topamiz:

$$U(x,t) = \frac{1}{2a\sqrt{\pi t}}\int_{-\infty}^{+\infty}\varphi(\xi)e^{-\frac{|x-\xi|^2}{4a^2t}}d\xi + \int_0^t\int_{-\infty}^{+\infty}\frac{1}{2a\sqrt{\pi(t-\tau)}}e^{-\frac{|x-\xi|^2}{4a^2(t-\tau)}}f(\xi,\tau)d\xi d\tau$$

$$a = 1 \qquad f(x,t) = 3t^2 \qquad \varphi(x) = \sin x$$

$$U(x,t) = \frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{+\infty}\sin\xi e^{-\frac{|x-\xi|^2}{4t}}d\xi + \int_0^t\int_{-\infty}^{+\infty}\frac{3\tau^2 e^{-\frac{|x-\xi|^2}{4(t-\tau)}}}{2\sqrt{\pi(t-\tau)}}d\xi d\tau$$

Ikkala qo'shiluvchini ham alohida – alohida hisoblaymiz:

$$I_1 = \int_{-\infty}^{+\infty}\sin\xi e^{-\frac{|x-\xi|^2}{4t}}d\xi = -\int_{-\infty}^{+\infty}\sin(x-\tau)e^{-\frac{\lambda^2}{4t}}d\tau = \begin{bmatrix} x-\xi=\lambda \\ \xi = x-\lambda \\ d\xi = -d\lambda \end{bmatrix} = -\int_{-\infty}^{+\infty}e^{-\frac{\lambda^2}{4t}}(\sin x\cos\lambda - \cos x\sin\lambda)d\lambda =$$

$$= -\sin x\int_{-\infty}^{+\infty}\cos\lambda e^{-\frac{\lambda^2}{4t}}d\lambda + \cos x\int_{-\infty}^{+\infty}\sin\lambda e^{-\frac{\lambda^2}{4t}}d\lambda = -\sin x\frac{\sqrt{\pi}}{\frac{1}{2\sqrt{t}}}e^{-\frac{1}{4\frac{1}{4t}}} = -2\sqrt{\pi t}\sin xe^{-t}$$

$$I_1^* = \int_{-\infty}^{+\infty}e^{-\frac{|x-\xi|^2}{4(t-\tau)}}d\xi = \begin{bmatrix} x-\xi=\lambda \\ \xi = x-\lambda \\ d\xi = -d\lambda \end{bmatrix} = e^{-\frac{\lambda^2}{4(t-\tau)}}d\lambda = 2\sqrt{t-\tau}\int_{-\infty}^{+\infty}e^{-(\frac{\lambda}{2\sqrt{t-\tau}})^2}d(\frac{\lambda}{2\sqrt{t-\tau}}) = -2\sqrt{\pi(t-\tau)}$$

Bu tenglik bizga mat.analizdan ma'lum.

$$I_2 = \int_{-\infty}^{+\infty}\frac{3\tau^2 e^{-\frac{|x-\xi|^2}{4(t-\tau)}}}{2\sqrt{\pi(t-\tau)}}d\tau = 3\int_0^t\tau^2 d\tau = 3\frac{t^3}{3} = t^3$$

Natijada quyidagi yechimni olamiz:

$$U(x,t) = \frac{1}{2\sqrt{\pi t}}(-2\sqrt{\pi t})\sin xe^{-t} + t^3 = e^{-t}\sin x + t^3$$

3. $u_t = a^2 u_{xx}$ tenglamaning $t>0, 0<x<l$ yolakda

$u_x(l,t) + h*u(l,t) = u,\ u(0,t) = T,\ U(x,0) = 0$ shartlarni qanoatlantiruvchi yechimini toping.

Tenglamaning chegaraviy shartlari bir jinsli bolmagani uchun yechimni quyidagi korinishda izlaymiz: $u(x,t) = v(x,t) + w(x,t)$ $v(0,t)+w(0,t) = T$, $V(0,t)=0$ bolsin.u holda

$w(0,t)=T$. $w(x,t) = (c_1x^2 + c_2x + c_3)T + (c_4x^2 + c_5x + c_6)u$.

$V_x(l,t) + w_x(l,t) + h*v(l,t) + h*w(l,t) = u$, $V_x(l,t) + h*v(l,t) = 0$ bolsin,u holda

$w_x(l,t) + h*w(l,t) = u$. $w(x,t) = (c_1x^2 + c_2x + c_3)T + (c_4x^2 + c_5x + c_6)u$ korinishda izlaymiz.

$c_3T + c_6u = T$

$(2c_1l + c_2)T + (2c_4l + c_5)u + h(c_1l^2 + c_2l + c_3)T + h(c_4l^2 + c_5l + c_6)u = u$.

Bundan esa $c_1 = 0, c_2 = -\frac{h}{1+hl}, c_3 = 1, c_4 = 0, c_5 = \frac{1}{1+hl}, c_6 = 0$ ekanligi kelib chiqadi.Demak u.

$u(x,t) = v(x,t) + \frac{u-hT}{1+hl}x + T$ ni bosh tenglamaga oborib qoyamiz.

$v_t = a^2 v_{xx}$ hosil boldi.tenglamani furye usulida yechamiz:

$$\begin{aligned} &X''(x) + \lambda^2 X(x) = 0 \\ &X(0) = 0, X'(l) + hX(l) = 0 \end{aligned}$$ shturm-luivill masalasini yechamiz:

$$\begin{aligned} &X(x) = c_1\cos\lambda x + c_2\sin\lambda x \\ &X(0) = c_1 = 0 \\ &-htg\lambda_n l = \lambda_n,\ c_2 = 1 \end{aligned}$$

Demak, $X_n(x) = \sin\lambda_n x$, umumiylikka ziyon keltirmagan holda c2=1deb olamiz.

$$\|X_n(x)\| = \int_0^l \frac{1-\cos 2\lambda_n x}{2}dx = \frac{l}{2} - \frac{1}{4\lambda_n}\sin 2\lambda_n x\Big|_0^l = \frac{l}{2} - \frac{2tg\lambda_n l}{4\lambda_n(1+tg^2\lambda_n l)} = \frac{l(h^2+\lambda_n^2)+h}{2(h^2+\lambda_n^2)}$$

$T'(t) + \lambda^2 a^2 T(t) = 0$ bu tenglamaning yechimi: $T_n(t) = e^{-\lambda_n^2 a^2 t}A_n$.

$$A_n = \frac{1}{\|X_n(x)\|^2}\int_0^l (Ax-T)\sin(\lambda_n x)dx$$

Bu integralni hisoblasak,quyidagi natija kelib chiqadi:

$$\int_0^l (Ax-T)\sin(\lambda_n x)dx = -\frac{\cos\lambda_n l}{\lambda_n}(Al-T) - \frac{T}{\lambda_n} + \frac{A}{\lambda_n^2}\sin\lambda_n l$$

$$\cos\lambda_n l = \frac{(-1)^n}{\sqrt{1+tg^2\lambda_n l}}, \quad \sin\lambda_n l = \frac{(-1)^{n+1}\lambda_n}{\lambda_n^2 + h^2}$$

natijada A_n quyidagiga teng boladi:

$$A_n = \frac{(-1)^n U}{\lambda_n\sqrt{\lambda_n{}^2 + h^2}} - \frac{T}{\lambda_n}$$

JAVOB: $u(x,t) = \sum_{n=1}^{\infty} \frac{2(\lambda_n{}^2 + h^2)}{l(\lambda_n{}^2 + h^2) + h} \frac{1}{\lambda_n} \left(\frac{(-1)^n U}{\lambda_n\sqrt{\lambda_n{}^2 + h^2}} - \frac{T}{\lambda_n} \right) e^{-\lambda_n{}^2 a^2 t} \sin\lambda_n x + \frac{U - Th}{1 + hl} x + T$

4.Issiqlik o'tkazuvchanlik tenglamasi uchun Koshi masalasini yeching.

$$u_t = \Delta u$$

$$u_t = u_{xx} + u_{yy}$$

$$u(x,y,t)|_{t=0} = \sin 2x \cos y$$

Tenglamaning yechimini quyidagi korinishda izlaymiz:

$$u(x,y,t) = \frac{1}{4a^2\pi t}\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} \varphi(\xi,\tau) e^{-\frac{|x-\xi|^2 + |y-\eta|^2}{4a^2 t}} d\eta$$

bu yerda $a = 1,\ \varphi(x,y) = \sin 2x \cos y.$

$$u(x,y,t) = \frac{1}{4\pi t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sin 2\xi \cos\eta e^{-\frac{|x-\xi|^2+|y-\eta|^2}{4t}} d\xi d\eta = \frac{1}{4\pi t}\int_{-\infty}^{+\infty} \sin 2\xi e^{-\frac{|x-\xi|^2}{4t}} d\xi \int_{-\infty}^{+\infty} \cos\eta e^{-\frac{|y-\eta|^2}{4t}} d\eta$$

$$I_1 = \int_{-\infty}^{+\infty} \sin 2\xi e^{-\frac{|x-\xi|^2}{4t}} d\xi = \begin{bmatrix} x - \xi = \lambda \\ \xi = x - \lambda \\ d\xi = -d\lambda \end{bmatrix} = -\int_{-\infty}^{+\infty} \sin(2x - 2\lambda) e^{-\frac{\lambda^2}{4t}} d\lambda = -\int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{4t}} (\sin 2x \cos 2\lambda - \cos 2x \sin 2\lambda) d\lambda =$$

$$= -\sin 2x \int_{-\infty}^{+\infty} \cos 2\lambda e^{-\frac{\lambda^2}{4t}} d\lambda + \cos 2x \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{4t}} \sin 2\lambda d\lambda = -\sin 2x \frac{2\sqrt{\pi t}}{1} e^{-\frac{4}{4\frac{1}{4t}}} = -2\sqrt{\pi t} \sin 2x e^{-4t}$$

$$I_2 = \int_{-\infty}^{+\infty} \cos\eta e^{-\frac{|y-\eta|^2}{4t}} d\eta = \begin{bmatrix} y - \eta = \lambda \\ \eta = y - \lambda \\ d\eta = -d\lambda \end{bmatrix} = -\int_{-\infty}^{+\infty} \cos(y - \lambda) e^{-\frac{\lambda^2}{4t}} d\lambda = \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{4t}} (\cos y \cos\lambda + \sin y \sin\lambda) d\lambda =$$

$$= -\cos y \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{4t}} \cos\lambda d\lambda - \sin y \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{4t}} \sin\lambda d\lambda = -\cos y 2\sqrt{\pi t} e^{-\frac{1}{4\frac{1}{4t}}} = -2\sqrt{\pi t} \cos y e^{-t}$$

Demak, javob:

$$u(x,y,t) = \frac{1}{4\pi t} 4\pi t \sin 2x e^{-4t} \cos y e^{-t} = \sin 2x \cos y e^{-5t}$$