KO'P O'LCHOVLI TASODIFIY MIQDORLAR UCHUN BA'ZIBIR NATIJALAR.

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Mazkur maqolaga shartli bog'liq bo'lmagan tasodifiy miqdorlar uchun S. N. Bernshteyn tengsizligining analogi keltiriladi.

Faraz qilaylik, (Ω, F, P) ehtimolli fazo bo'lsin.

$$P^{F_1}: A \in F \to P^{F_1}(A) \epsilon S(\Omega, F_1)$$

$F_1 - \sigma -$algebra ostida nisbatan shartli ehtimol bo'lsin, bu yerda $S(\Omega, F_1)$ tasodifiy miqdorlar fazosidan iborat.

$\boldsymbol{\xi}$ va $\boldsymbol{\eta}$ tasodifiy miqdorlar F_1 ga nisbatan shartli bog'liq emas deyiladi, agar ixtiyoriy λ va ν lar uchun {$\boldsymbol{\xi}$<λ} va {$\boldsymbol{\eta}$<ν} hodisalar shartli bog'liq bo'lmasa, yani

$$P^{F_1}\{\xi < \lambda\ ,\ \eta < \mu\} = P^{F_1}\{\xi < \lambda\} * \quad P^{F_1}\{\eta < \mu\}\ .$$

$\{\xi_n\}$ tasodifiy miqdorlar $\boldsymbol{\xi}$ ga deyarli yaqinlashadi (d.ya.) P^{F_1} o'lchov bo'yicha deyiladi . $\xi_n \to \xi(d.ya.P^{F_1})$ va shundan yoziladi, agar ixtiyoriy $\varepsilon > 0$ uchun

$$P^{F_1}(sup_{k\geq n}\{\xi_n - \xi \geq \varepsilon\}) \to 0 \quad (d.ya.P)$$

Agar $X_1, X_2, \ \dots\ X_n$ lar n-o'lchovli shartli bo'g'liq bo'lmagan tasadifiy miqdorlar bo'lsa, $M^{F_1}X_i = 0$ var(X_i)=σ_1^2 va *t*-haqiqiy musbat son va $\sigma^2 = \sum \sigma_i^2$ bo'lsa, *y* vaqtiga $P^{F_1}[\sum X_i \geq t\sigma]$ ifoda uchun **X** chegaralangan holda, yuqori chegarani baholash mumkin.

Biz ikki o'lchovli holni qaraymiz, qaysiniki tasodifiy vektorlar

$$(X_1, Y_1)\ , (X_2, Y_2), \dots,\ (X_n, Y_n)$$

lar uchun $\qquad M^{F_1}X_i = M^{F_1}Y_i = 0$

$$\text{var}(X_i)=\text{var}(Y_i)=\sigma_1^2 \ , M^{F_1} X_i * Y_i = \rho\sigma_1^2$$

bo'lgan holda, tasodifiy vektorlar uchun quyidagi teoremani isbotlash mumkin.

Teorema. Faraz qilaylik $t, a \ va \ b$ lar haqiqiy musbat sonlar bo'lsin va shartli bog'liq va shartli bog'liq bo'lmagan tasodifiy miqdorlar uchun

$$(X_1, Y_1) \ , (X_2, Y_2), \ldots, \ (X_n, Y_n)$$

bu yerda

$$M^{F_1} X_i = M^{F_1} Y_i = 0$$

$$\text{var}(X_i)=\text{var}(Y_i)=\sigma_1^2 \ , M^{F_1} X_i * Y_i = \rho\sigma_1^2, \ \sigma^2 = \sum_{i=1}^{n} \sigma_i^2$$

bo'lmaganda, $|X_i| \le R$, $|Y_i| \le R$ shartga quyidagi

$$P^{F_1}\{\sum_i X_i \ge t\sigma, \sum_i Y_i \ge t\sigma\}, \exp\{\frac{-t^2(2-|P|)}{2(1+\frac{tR}{3\sigma})}\}$$

munosabat o'rinli.

Isbot. Quyidagi $e^{aX_i+bY_i}$ ifodani qaraymiz

$$e^{aX_i+bY_i} = \left\{1 + aX_i + \sum_{r=2}^{\infty} \frac{a^r X_i^r}{r!}\right\} \{1 + bY_i + \sum_{s=2}^{\infty} \frac{b^s Y_i^s}{s!}\}$$

bu ifodadan matematik kutilma olamiz, $M^{F_1} X_i = M^{F_1} Y_i = 0$ ekanligini e'tiborga olsak

$$M^{F_1} e^{aX_i+bY_i} = 1 + ab\rho\sigma_i^2 + \frac{a^2\sigma_i^2 A_i}{2} + \frac{b^2\sigma_i^2 B_i}{2} + ab\sigma_i^2(C_i + D_i + E_i)$$

bo'ladi, bu yerda

$$A_i = \sum_{r=2}^{\infty} \frac{a^{r-2} M^{F_1} X_i^r}{r!\sigma_i^2 * \frac{1}{2}}, \qquad B_i = \sum_{s=2}^{\infty} \frac{b^{s-2} M^{F_1} Y_i^s}{s!\sigma_i^2 * \frac{1}{2}},$$

$$C_i = \sum_{r=2}^{\infty} \frac{a^{r-1} M^{F_1} Y_i X_i^r}{r!\sigma_i^2}, \qquad D_i = \sum_{s=2}^{\infty} \frac{b^{s-1} M^{F_1} X_i Y_i^s}{s!\sigma_i^2}$$

$$C_i = \sum_{r=2}^{\infty} \sum_{s=2}^{\infty} \frac{a^{r-1} b^{s-1} M^{F_1} X_i^r Y_i^s}{s!\, r!\, \sigma_i^2}$$

iborat, shunday qilib quyiahdagini hosil qilamiz.

$$M^{F_1} e^{aX_i + bY_i} < \exp\left\{\left(\frac{a^2\sigma_i^2}{2} + \frac{b^2\sigma_i^2}{2}\right) M_i + ab\sigma_i^2 G_i\right\},$$

bu yerda,

$$M_i = \max(A_i, B_i), \;\; G_i = \rho + C_i + D_i + E_i \,.$$

Endi, agar S(X)=$\sum_{i=1}^{n} X_i$, S(Y)=$\sum_{i=1}^{n} Y_i$, deb olsak u vaqtda

$$M^{F_1} e^{aS(X) + BS(Y)} < \exp\left\{\left(\frac{a^2\sigma^2}{2} + \frac{b^2\sigma^2}{2}\right) M + ab\sigma_1^2 G\right\},$$

bu yerda $M = maxM_i, \;\; G = maxG_i.$

Shunday qilib,

$$P^{F_1}[S(X) \geq t\sigma, S(Y) \geq t\sigma] < \exp\{\frac{-t(a+b)}{2}\}$$

Shu jarayonni davom ettirib quyidagi ifodani hosil qilamiz.

$$P^{F_1}[S(X) \geq t\sigma, S(Y) \geq t\sigma] < \exp\{\frac{-t^2(2 - |\rho|)}{2(1 + \frac{tR}{3})}\}$$

Agar $|\rho|$=1 bo'lsa, biz bir o'lchovli tasodifiy miqdorlar uchun o'rinli ekanligini, $|\rho| = 0$ bo'lsa ikkita bir o'lchovli tasodifiy miqdorlar ko'paytmasi uchun o'rinli bo'ladi.

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Eksionentsial baholar

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Mazkur maqolaga bog'liq bo'lmagan tasodifiy miqdorlar ketma-ketligi berilgan bo'lib, matematik kutilmasi nolga teng va chekli dissersiyaga ega bo'lgan tasodifiy miqdorlar uchun quyi va yuqori baholar olinadi,

Faraz qilaylik $\{X_n; n = 1,2,..., \}$ tasodifiy miqdorlar berilgan bo'lsin va quyidagi belgilashlarni kiritamiz.

$S_n = \sum_{k=1}^{n} X_k, \quad \sigma_k^2 = MX_k^2, \quad B_n = \sum_{k=1}^{n} \sigma_k^2,$

$q_n(x) = P\{S_n \geq x\}.$

Umumiylikka halaqit bermasdan $\{H_n\}$ ketme-ketlikni kamaymaydigan deb olamiz.

Lemma1. Agar $0 \leq xH_n \leq B_n$ bo'lsa, u vaqtida

$$P\{S_n \geq x\} \leq \exp\{-\frac{x^2}{2B_n}(1 - \frac{xH_n}{2B_n})\}. \qquad (1)$$

Agar $xH_n \geq B_n$ bo'lsa, u vaqtda

$$P\{S_n \geq x\} \leq \exp\{-\frac{x}{4H_n}\} \qquad (2)$$

Isbot. Faraz qilaylik *t>0* va $tH_n \leq 1$ u vaqtda

$$|MX_n^k| \leq H_n^{k-2}\sigma_n^2$$

tengsizlikka ko'ra, ixtiyoriy $k \geq 2$ uchun quyidagiga ega bo'lamiz.

$$Me^{tX_n} = 1 + \sum_{k=2}^{\infty} \frac{t^k}{k!} MX_n^k \leq 1 + \frac{t^2\sigma_n^2}{2}\left(1 + \frac{t}{3}H_n + \frac{t^2H_n^2}{2} + \cdots\right) \leq$$

$$\leq 1 + \frac{t^2\sigma_n^2}{2}(1 + \frac{t}{2}H_n) \leq \exp\{\frac{t^2\sigma_n^2}{2}(1 + \frac{t}{2}H_n)\}.$$

$$Me^{tS_n} \leq \exp\{\frac{t^2}{2}B_n(1 + \frac{t}{2}H_n)\}.$$

Haqiqatan

$$P\{S_n \leq x\} \leq e^{-tx}Me^{tS_n} \leq \exp\{-tx + \frac{t^2}{2}B_n(1 + \frac{t}{2}H_n)\}$$

bu yerda $xH_n \leq B_n$ bo'lganda $t = \frac{x}{B_n}$ deb olsak, $xH_n \geq B_n$ bo'lganda $t = \frac{1}{M_n}$ deb olsak, (1) va (2) tengsizliklar kelib chiqadi.

Lemma2. Agar $x > 0, \frac{xH_n}{B_n} \to 0$ va $\frac{x^2}{B_n} \to \infty$, u vaqtda ixtiyoriy fiksizlangan ν>0 va hamma yetarli katta N lar uchun quyidagi tengsizlik o'rinli

$$P\{S_n \leq x\} \leq e^{-tx}Me^{tS_n} \leq \exp\{-\frac{x^2}{2B_n}(1 + \mu)\} \qquad (3)$$

Isbot. Ixtiyoriy $x \geq 0$ uchun $1 + x \geq e^{x(1-x)}$ bo'ladi.

Agar $0 \leq t + t_n \leq 1$ bo'lsa, u vaqtda

$$Me^{tX_n} \geq 1 + \frac{t^2\sigma_n^2}{2}\left(1 - \frac{t}{3}H_n - \frac{t^2H_n^2}{12} - \cdots\right) \geq$$

$$\geq 1 + \frac{t^2\sigma_n^2}{2}\left(1 - \frac{t}{2}H_n\right) \geq \exp\left\{\frac{t^2\sigma_n^2}{2}\left(1 - \frac{t}{2}H_n + \frac{t^2\sigma_n^2}{2}\right)\right\} \geq$$

$$\geq \exp\left\{\frac{t^2}{2}B_n(1 - tH_n)\right\}.$$

Agar $t = \frac{x}{(1-\sigma)B_n}$ deb olsak, bu yerda σ $-$kichik musbat son. U vaqtda $tH_n \to$, va ixtiyoriy fiksizlangan λ>0 uchun quyidagi tengsizlik o'rinli.

$$Me^{tS_n} \geq \exp\left\{\frac{t^2}{2}B_n(1-\lambda)\right\} \qquad (4)$$

yetarli katta N lar uchun. Keyin

$$Me^{tS_n} = -\int_{-\infty}^{\infty} e^{ty}\,dq_n(y) = t\int_{-\infty}^{\infty} e^{ty}q_n(y)\,dy = t\sum_{k=1}^{5} J_k, \qquad (5)$$

bu yerda $J_1, J_2, \ldots, J_5$ integrallarni $e^{ty}q_n(y)$ bo'yicha

$$(-\infty, 0), (0, t(1-\sigma)B_n), \ (t(1-\sigma)B_n, t(1+\sigma)B_n),$$

$(t(1+\sigma)B_n, 8tB_n)$ va $(8tB_n, \infty)$ intervallar bo'yicha mos ravishda integrallab, yetarli katta N lar uchun (3) tengsizlikka kelamiz yuqorida keltirilgan natijalarni shartli bog'liq bo'lmagan tasodifiy miqdorlar uchun keltirish mumkin.

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