UDK 517.977.5

OPTIMAL BOSHQARUVNING BIR VA IKKI O'LCHOVLI MASALALARINI MAKSIMUM PRINSIPI VA OPTIMAL DASTURIY BOSHQARUV USULLARI BILAN YECHISH

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Mazkur ishda eng samarali va ko'p qo'llaniladigan maksimum prinsipi va optimal dasturiy boshqaruv usullaridan foydalanib, aniq amaliy masalalar yechilgan. Bunda diskret maksimum prinsipi va optimal dasturiy boshqaruv algoritmlari keltirilgan. Hisoblashlar natijalari trayektoriya, oprimal boshqaruv va funksionalning minimum qiymati shakli chiqarilgan.

In this paper present the solution of the concrete applied problems by using effective and widely used methods of maximum principia and optimum program management. Algorithms of a discrete principle of a maximum and optimum program management also are given. The results are submitted as a trajectory, optimum control and minimal pregnant meaning functional.

Kalit so'zlar: Lagranj masalasi; Boltsa masalasi; maksimum prinsipi; trayektoriya; optimal boshqaruv; model; algoritm; funksional.

Kirish. Biror ob'yektni boshqaruv modeliga ko'ra hosil qilingan funksionalning ekstremumini qo'shimcha shartlar asosida topish masalasi amaliy matematikaning dolzarb masalalaridan biri. Bunday masalalarni yechish uchun optimal boshqaruvning bir qator usullari mavjud [1,2]. Mazkur ishda bu usullardan eng samarali va ko'p qo'llaniladigan maksimum prinsipi va optimal dasturiy boshqaruv usullaridan foydalanib, aniq amaliy masalalar yechildi.

Masalaning qo'yilishi. Faraz qilaylik, bizga quyidagi

$$x(k+1) = f(k, x(k), u(k)), k = 0,1,\ldots,N-1, \tag{1}$$

ayirmali tenglama

$$x(0) = x_0, \tag{2}$$

boshlang'ich shart bilan berilganda quyidagi

$$\tilde{A}_i(x(N)) = 0, i = 1,\ldots,l, \tag{3}$$

shartni qanoatlantiruvchi $x(N)$ oxirgi holatdan ushbu

$$J = \sum_{k=0}^{1} f^0(k, x(k), u(k)) + F(x(N)), \tag{4}$$

funksionalning minimumini topish talab qilinsin, bu yerda x – sistema holatning vektori, $x \in R^n$; u – boshqaruv vektori, $u \in U(k) \subseteq R^q$; $U(k)$ – boshqaruvning mumkin bo'lgan qiymatlarining biror yopiq qovariq to'plami; k – diskret vaqt, $k \in T=[0,1,\ldots,N-1]$; N – qadamlar soni beriladi; $f(k,x,u)$: $T \times R^n \times U(k) \to R^n$ – uzluksiz differensiallanuvchi vektor-funksiya, $f(k,x,u) = (f_1(k,x,u),\ldots, f_n(k,x,u))^T$. Bu yerda ushbu $J(d^*) = \min_{d \in D(0,x_0)} J(d)$ masala (4) funksiomal bilan berilganda $d^* = (x^*(.), u^*(.)) \in D(0, x_0)$ juftlikni topish Boltsa masalasi deb ataladi. Xususan, agar bunda $F(x(N) = 0$ bo'lsa, u holda masala Lagranj masalasiga keladi. Bu yerda $x^*(.)$ - optimal trayektoriya; $u^*(.)$ - optimal boshqaruv.

Bunday masalalarni yechishning ko'plab usullari va algoritmlari mavjud [1,2].

Amaliyotda keng uchraydigan bir va ikki o'lchovli optimal boshqaruv masalalaridan namuna sifatida quyidagi ikki masalani maksimum prinsipi va optimal dasturiy boshqaruv usullari bilan yechamiz.

Masalani yechish algoritmlari.

1. Diskret maksimum prinsipi algoritmi.

1) Quyidagi gamiltonianni tuzish

$$H(k, \psi, x, u) = \sum_{i=0}^{1} \psi_i f_i(k, x, u) - f^0(k, x, u).$$

2) Quyidagi boshqaruv bo'yicha gamiltonianning maksimum shartidan optimal boshqaruv tuzilmasini topish

$$H(k, \psi(k+1), x^*(k), u^*(k)) = \max_{u \in U(k)} H(k, \psi(k+1), x^*(k), u.$$

3) Quyidagi shartlar bilan berilgan masalada kanonik tenglamalar sistemasini tuzish

$$x_j^*(k-1)= f_i(k,x^*(k),u^*(k)), x_j^*(0)=x_{0j}, j=1,\ldots,n, k=0,1,\ldots,N-1$$

$$\psi_i(k)=\frac{\partial H(k,\psi(k+1),x^*(k),u^*(k))}{\partial x_j}, j=1,\ldots,n, k=0,1,\ldots,N-1$$

$$\tilde{A}_i(x(N))=0, i=1,\ldots,l, \tag{5}$$

4) Transversallik sharti yoki undan kelib chiqadigan variatsiya formalaridan, xususiy holda masalaning qo'yilishidan (5) tenglamalar sistemasi uchun yetarli bolmagan boshlang'ich shartlarni aniqlab olish.

5) Hosil qilingan boshlang'ich masalani yechish. Natijada (4) funksionalni minimumga erishtiruvchi ushbu (x^*(.), u^*(.)) juftlikni aniqlash.

2. Optimal dasturiy boshqaruv algoritmi.

Faraz qilaylik boshqaruv obyektining modeli ushbu

$$x(k+1)=A(k)x(k)+B(k)u(k), \quad k=0,1,\ldots,N-1 \tag{6}$$

ayirmali tenglama va

$$x(0)=x_0 \tag{7}$$

boshlang'ich shart bilan berilgan, boshqaruv sifatining funksionali

$$J=\sum_{k=0}^{N-1}[x^T(k)S(k)x(k)+u^T(k)Q(k)u(k)]+x^T(N)\Lambda x(N), \tag{8}$$

kabi ifodalangan bo'lsin, bu yerda $S(k),\Lambda$ - nomusbat aniqlangan ($n\times n$) o'lchovli simmetrik matritsa; *Q(k)* – musbat aniqlangan ($q\times q$) o'lchovli simmetrik matritsa. (8) funksionalni minimallashtiruvchi to'la teskari bog'lanishli $u^*(k,x)$ boshqaruvni aniqlang.

Optimal boshqaruvni topishning asosiy munosabatlari quyidagilar.

(6)-(8) masala umumiy masalaning xususiy holi, buning uchun

$$f(k,x,u)=A(k)x+B(k)u,\ f^0(k,x,u)=x^TS(k)x+u^TQ(k)u, F(x)=x^T\Lambda x$$

deb olish kerak. U holda Bellman tenglamasi va uning boshlang'ich sharti quyidagicha yoziladi [1]:

$$B(k,x)=\min_u[x^TS(k)x+u^TQ(k)u+B(k+1,A(k)x+B(k)u)], \forall x\in R^n, k=0,1,\ldots,N-1,$$

$$B(N,x)=x^T\Lambda x, \forall x\in R^n \tag{9}$$

B(*k*,*x*) Bellman funksiyasini quyidagi kvadratik forma ko'rinishida izlash tavsiya etiladi:

$$B(k,x)=x^TP(k)x, \tag{10}$$

bu yerda *P*(*k*) – noma'lum nomusbat aniqlangan ($n\times n$) o'lchovli simmetrik matritsa.

(10) ni (9) ga qo'ysak, quyidagi tasdiqqa kelamiz:

Tasdiq. (6)-(8) masalada optimal boshqaruv quyidagi munosabatlardan aniqlanadi:

$$u^*(k,x)=-L(k)x, k=0,1,\ldots,N-1, \tag{11}$$

bu yerda *L*(*k*) - ($q\times n$) o'lchovli regulyatrni kuchaytirish matritsasining koeffisiyentlari,

$$L(k)=[Q(k)+B^T(k)P(k+1)B(k)]^{-1}B^T(k)P(k+1)A(k), \tag{12}$$

P(*k*) matritsa esa ($n\times n$) o'lchovli ushbu

$$P(k)=S(k)+L^T(k)Q(k)L(k)+[A(k)-B(k)L(k)]^TP(k+1)[A(k)-B(k)L(k)],$$

$$k=N-1,\ldots,0,1, \qquad P(N)=\Lambda, \tag{13}$$

tenglamani qanoatlantiruvchi matritsa.

Funksional minimumining miqdorini aniqlash formulasi quyidagicha:

$$\min J=x_0^TP(0)x_0, \forall x_0\in R^n, \tag{14}$$

Optimal dasturiy boshqatuv algoritmining bosqichlari.

1) (12) - (13) – sistemalarni tuzish.

2) *P*(*k*), *L*(*k*) matritsalarni aniqlash.

3) $u^*(k,x)=-L(k)x$ - optimal regulyatorni qurish.

4) (14) formula bo'yicha optimal boshqaruvdagi funksionalning qiymatini aniqlash.

Amaliy masalalar va ularning yechimlari.

1 – masala. Boshqaruv obyektining quyidagi modeli

$$x(k+1)=x(k)+u(k), \quad x(0)=2, \quad k=0,1$$

uchun ushbu

$$J=\sum_{k=0}^{1}[u^2(k)+x^2(k)]\rightarrow \min,$$

funksionalni minimumga erishtiruvchi ($x^*(.)$, $u^*(.)$) juftlikni aniqlang, bu yerda $x\in R$, $u\in R$.

Yechish. *1-usul* (*diskret maksimumprinsipi usuli*). Bu masalani umumiy masalaning qo'yilishi bilan taqqoslab, ushbu

$$f(k,x,u)=x+u,\ f^0(k,x,u)=u^2+x^2, F(x)=0, N=2.$$

tengliklarga kelamiz. Bu Lagranj masalasi, uni diskret maksimum prinsipi algoritmidan foydalanib yechamiz.

1) Quyidagi gamiltonianni tuzamiz

$$H(k,\psi,x,u)=\psi\cdot(x+u)-(u^2+x^2).$$

2) Boshqaruv bo'yicha gamiltoninanning maksimumini topamiz. Bu yerda boshqaruvga nisbatan cheklovlar yo'q, shuning uchun shartsiz ekstremum shartidan foydalanishimiz mumkin:

$$\frac{\partial H(k,\psi(k+1),x(k),u(k))}{\partial u}=\psi(k+1)-2u(k)=0$$

Bu yerdan $u^*(k)$=0,5$\psi(k+1)$. Topilgan boshqaruv ushbu $H(k,\psi(k+1),x(k),u)$ funksiyaning bosqaruv bo'yicha maksimumga erishishini ta'minlaydi, chunki quyidagi ekstremumning yetarli sharti bajariladi:

$$\frac{\partial^2 H(k,\psi(k+1),x(k),u(k))}{\partial u^2}=-2<0$$

3,4) Variatsiya formulasiga ko'ra (5) boshlang'ich masala tuziladi:

$$x^*(k+1)=x^*(k)+\frac{\psi(k+1)}{2}, x^*(0)=2, k=0{,}1 \quad \psi(k)=\psi(k+1)-2x^*(k), \psi(2)=0, k=0{,}1.$$

5) Boshlang'ich masalani yechamiz:

$$x^*(0)=2, u^*(0)=-1, x^*(1)=1, u^*(1)=0, \psi(1)=-2, x^*(2)=1, \psi(2)=0,$$

Hisob natijasiga ko'ra izlanayotgan juftlik: $x^*(.)=\{2;1;1\}$ - optimal trayektoriya; $u^*(.)=\{-1;0\}$ - optimal boshqaruv. Funksionalning minimumi quyidagicha:

$$\min J=\sum_{k=0}^{1}[u^2(k)+x^2(k)]=(-1)^2+2^2+0^2+1^2=6$$

2-usul (*optimal dasturiy boshqaruv usuli*). Bu masalani umumiy masalaning qo'yilishi bilan taqqoslab, ushbu

$$f(k,x,u)=x+u,\ f^0(k,x,u)=u^2+x^2, F(x)=0, N=2.$$

tengliklarga kelamiz. Bu Lagranj masalasi, uni optimal dasturiy boshqaruv algoritmidan foydalanib yechamiz, $A(k)=1, B(k)=1, S(k)=1, Q(k)=1, \Lambda=0.$

1) (12) - (13) – tenglamalarni tuzamiz.

$$L(k)=[1+P(k+1)]^{-1}P(k+1),$$
$$P(k)=1+L^2(k)+[1-L(k)]^2P(k+1), k=0{,}1; P(2)=0.$$

2) Bu tenglamalarning yechimlarini olamiz:

$$L(1)=[1+P(2)]^{-1}P(2)=[1+0]^{-1}\cdot 0=0,$$
$$P(1)=1+L^2(1)+[1-L(1)]^2P(2)=1+0^2+(1-0)^2\cdot 0=1,$$
$$L(0)=[1+P(1)]^{-1}P(1)=[1+1]^{-1}\cdot 1=\frac{1}{2},$$
$$P(0)=1+L^2(0)+[1-L(0)]^2P(1)=1+\left(\frac{1}{2}\right)^2+\left(1-\frac{1}{2}\right)^2\cdot 1=\frac{3}{2};$$

3) Optimal regulyatorni topamiz:

$$u^*(0,x)=-\frac{1}{2}x, u^*(1,x)=0;$$

4) Funksionalning optimal qiymatini aniqlaymiz:

$$\min J=\frac{3}{2}x^2(0), x(0)=2, \qquad \min J=\frac{3}{2}\cdot 2^2=6.$$

Shunday qilib, 1-masalaning har ikkala usul bilan olingan natijalari bir xilligi kelib chiqdi.

2 – masala. Boshqaruv obyektining quyidagi modeli

$$x_1(k+1)=x_2(k)+u(k), x_1(0)=1, k=0{,}1,$$
$$x_2(k+1)=-x_1(k)+x_2(k), \quad x_2(0)=1, \quad k=0{,}1,$$

uchun ushbu

$$J=\frac{1}{2}\sum_{k=0}^{1}[u^2(k)+x_1^2(k)+x_2^2(k)]+x_1(2)+x_2^2(2)\to\min$$

funksionalni minimumga erishtiruvchi ($x^*(.)$, $u^*(.)$) juftlikni aniqlang, bu yerda $x\in R$, $u\in R$

Yechish. *1-usul* (*diskret maksimumprinsipi usuli*). Bu masalani umumiy masalaning qo‘yilishi bilan taqqoslab, ushbu

$$f^0(k,x,u)=\frac{1}{2}(u^2+x_1^2+x_2^2), F(x)=x_1+x_2^2, N=2,$$
$$f_1(k,x,u)=x_2+u,\ f_2(k,x,u)=-x_1+x_2.$$

tengliklarga kelamiz. Bu Boltsa masalasi, uni diskret maksimum prinsipi algoritmidan foydalanib yechamiz.

1) Quyidagi gamiltonianni tuzamiz

$$H(k,\psi,x,u)=\psi_1[x_2+u]+\psi_2[-x_1+x_2]-\frac{1}{2}(u^2+x_1^2+x_2^2).$$

2) Boshqaruv bo‘yicha gamiltoninanning maksimumini topamiz. Bu yerda boshqaruvga nisbatan cheklovlar yo‘q, shuning uchun shartsiz ekstremum shartidan foydalanishimiz mumkin:

$$\frac{\partial H(k,\psi(k+1),x(k),u(k))}{\partial u}=\psi_1(k+1)-u(k)=0$$

Bu yerdan $u^*(k)=\psi_1(k+1)$. Topilgan boshqaruv ushbu $H(k,\psi(k+1),x(k),u)$ funksiyaning bosqaruv bo‘yicha maksimumga erishishini ta’minlaydi, chunki quyidagi ekstremumning yetarli sharti bajariladi:

$$\frac{\partial^2 H(k,\psi(k+1),x(k),u(k))}{\partial u^2}=-1<0.$$

3,4) Variatsiya formulasiga ko‘ra (5) boshlang’ich masala tuziladi:

$$x_1^*(k+1)=x_2^*(k)+\psi_1(k+1),\quad x_1^*(0)=1,\quad k=0,1,$$
$$x_2^*(k+1)=-x_1^*(k)+x_2^*(k),\quad x_1^*(0)=1,\quad k=0,1,$$
$$\psi_1(k)=-\psi_2(k+1)-x_1^*(k),\quad \psi_1(2)=-1,$$
$$\psi_2(k)=\psi_1(k+1)+\psi_2(k+1)-x_2^*(k),\quad \psi_2(2)=-2x_2^*(2).$$

5) Boshlang’ich masalani yechamiz:

$$x^*{}_1(0)=1,\ x^*{}_2(0)=1,\ u^*(0)=0{,}75,\ x^*{}_1(1)=-0{,}25,\ x^*{}_2(1)=0,\ u^*(1)=-1,$$
$$\psi_1(1)=0{,}75,\ \psi_2(1)=-1{,}5,\ x^*{}_1(2)=-1,\ x^*{}_2(2)=0{,}25,\ \psi_1(2)=-1,\ \psi_2(2)=-0{,}5.$$

Hisob natijasiga ko‘ra izlanayotgan juftlik: $x_1^*(.)=\{1;-0{,}25;-1\}, x_2^*(.)=\{1;0;0{,}25\}$ - optimal trayektoriya; $u^*(.)=\{0{,}75;-1\}$ - optimal boshqaruv. Funksionalning minimumi quyidagicha:

$$\min J=\frac{1}{2}\sum_{k=0}^{1}[u^2(k)+x_1^2(k)+x_2^2(k)]+x_1(2)+x_2^2(2)=0{,}8.$$

2-usul (*optimal dasturiy boshqaruv usuli*). Bu masalani umumiy masalaning qo‘yilishi bilan taqqoslab, ushbu

$$f^0(k,x,u)=\frac{1}{2}(u^2+x_1^2+x_2^2), F(x)=x_1+x_2^2, N=2,$$
$$f_1(k,x,u)=x_2+u,\ f_2(k,x,u)=-x_1+x_2.$$

tengliklarga kelamiz. Bu Boltsa masalasi, uni optimal dasturiy boshqaruv algoritmidan foydalanib yechamiz. $A(k)=1,\ B(k)=1,\ S(k)=1,\ Q(k)=1,\ \Lambda=1.$

1) (12) - (13) – tenglamalarni tuzamiz.

$$L(k)=[1+P(k+1)]^{-1}P(k+1),$$
$$P(k)=1+L^2(k)+[1-L(k)]^2P(k+1),\ \ k=0,1;$$

2) Bu tenglamalarning yechimlarini olamiz:

$$P(2)=1;$$
$$L(1)=[1+P(2)]^{-1}P(2)=[1+1]^{-1}\cdot 1=\frac{1}{2};$$
$$P(1)=1+L^2(1)+[1-L(1)]^2P(2)=1+\left(\frac{1}{2}\right)^2+\left(1-\frac{1}{2}\right)^2\cdot 1=\frac{3}{2};$$

$$L(0) = [1 + P(1)]^{-1} P(1) = [1 + \frac{3}{2}]^{-1} \cdot \frac{3}{2} = \frac{3}{5},$$
$$P(0) = 1 + L^2(0) + [1 - L(0)]^2 P(1) = 1 + \left(\frac{3}{5}\right)^2 + \left(1 - \frac{3}{5}\right)^2 \cdot \frac{3}{2} = \frac{8}{5}.$$

3) Optimal regulyatorni topamiz:

$$u^*(0, x) = -\frac{3}{5} x, \quad u^*(1, x) = -\frac{1}{2} x;$$

4) Funksionalning optimal qiymatini aniqlaymiz:

$$\min J = \frac{1}{2} \cdot \frac{8}{5} x^2(0), \quad x(0) = 1, \qquad \min J = \frac{4}{5} \cdot 1^2 = 0{,}8.$$

Ushbu 2-masalaning har ikkala usul bilan topilgan yechimlari ham bir chiqdi.

Xulosa. Demak, bu qo‘llanilayotgan usullarning har biri yetarlicha aniqlikdagi natijalarni berar ekan va ularning samarali usullar ekanligi yana bir bor isbotini topdi. Usbu ishda keltirilgan algoritmlar va olingan natijalardan shu kabi turdosh masalalarni yechishda samarali foydalanish mumkin [1].

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РЕШЕНИЕ ЗАДАЧ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ МЕТОДАМИ ПРИНЦИПА МАКСИМУМА И ОПТИМАЛНОГО ПРОГРАММНОГО УПРАВЛЕНИЯ

В работе решены конкретные прикладные задачи с эффективными и широко применяемыми методами принципа максимума и оптимального программного управления. Также приведены алгоритмы дискретного принципа максимума и оптимального программного управления. Результаты представлены в виде траектории, оптимального управления и минимального значения функционала.

Ключевые слова: задача Лагранжа; задача Больца; принцип максимума; траектория; оптимальное управление; модель; алгоритм; функционал.

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SOLUTION OF AN OPTIMUM CONTROL MANAGEMENT PROBLEM BY MAXIMUM PRINCIPIA AND OPTIMAL PROGRAM MANAGEMENT METHODS

In this paper present the solution of the concrete applied problems by using effective and widely used methods of maximum principia and optimum program management. Algorithms of a discrete principle of a maximum and optimum program management also are given. The results are submitted as a trajectory, optimum control and minimal pregnant meaning functional.

Key words: problems Lagrange; problem Bolts; maximum principia; trajectory; optimum control; model; algorithm; functional.