O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

TOSHKENT DAVLAT SHARQSHUNOSLIK INSTITUTI

MATEMATIKA VA INFORMATIKA KAFEDRASI

MOLIYAVIY MATEMATIKA
fanidan ma`ruzalar matni

Ta’lim sohasi: 340000 – biznes va boshqaruv
Bakalavriat yo’nalishlari: 5341000 – mintaqashunoslik
5341100 – Jahon iqtisodi va xalqaro iqtisodiy munosabatlar

MARUZA 1.

Asosiy tushunchalar.Foiz, proportchiy va oddiy foiz hisoblash.

Tayanch iboralar: Foiz va proportchiy, moliyaviy hisoblar.

Kirish

Biznes bilan yuqori saviyada shug`ullanish, har qanday moliyaviy amaliyotni amalga oshirishda moliyaviy natijalarini baholay olishni talab qiladi. Boshlang`ich asosiy tushunchalar va moliyaviy hisoblar uslubiyatini chuqur egallamasdan turib yuksak iqtisodiy natijalarga erishish mushkul. Shu maqsadda moliyaviy hisoblarga tegishli bo`lgan turli mavzu va muammolarni qamrab oluvchi fanlar o`rganilmoqda, moliyaviy hisoblarni to`liq sistemaga soluvchi, umumiy metodologiyaga tayanuvchi fanlardan biri bu moliyaviy matematika fanidir.

Oddiy protsentlar

1.1 **Oddiy protsentlar formulasi:**

$S(t_0)$ -boshlang‘ich moliyaviy mablag‘ miqdori bo‘lsin va bir davr uchun hisoblanadigan foiz stavka miqdori i foizni tashkil etsa, u holda bir davr mobaynida moliyaviy mablag‘ning o‘sishi $i \cdot S(t_0)$ miqdorni tashkil etadi.

$$S(t_0+1) = S(t_0) + i \cdot S(t_0) \Rightarrow S(t_0+1) = S(t_0)\,(1+i)$$

Oddiy foiz hisoblash to‘lovlarida foiz boshlang‘ich moliyaviy miqdor bo‘yicha hisoblanadi. Shu tufayli, keyingi davrdagi, ya’ni t_0+2 davrdagi foiz to‘lov miqdori yana $i \cdot S(t_0)$ ga teng bo‘ladi. Shu tufayli:

$$S(t_0+2) = S(t_0+1) + i \cdot S(t_0) = S(t_0)(1+i) + S(t_0)i = S(t_0)(1+2i)$$

Ushbu jarayonni davom ettirish natijasida t_0+n davr uchun

$$S(t_0+n) = S(t_0)(1+n \cdot i)$$

kelib chiqadi, bu yerda

$S(t_0)$ - boshlang‘ich moliyaviy mablag‘

i - bir davr uchun foiz stavkasi

n - davrlar soni

Demak, oddiy protsentlar ushbu formula yordamida hisoblanadi:

$$S(t_0+n)=S(t_0)(1+n\cdot i)$$

Agar muddatlar birinchi va oxirgi kunlarning kalendardagi sanalari orqali berilsa, u holda, oddiy protsent formulasidan foydalanish uchun muddat sifatida ssudalar kunining sonini yildagi kunlar soniga nisbatini olish mumkin:
Ya`ni

$$n=\frac{t}{Y}$$

yildagi kunlar soni Y – 360 ga teng deb olinsa, bunday holda odatdagi protsentlar, agar 365 deb olinsa aniq protsentlar sxemasi asosida hisoblanadi.

T_1 va T_2 sanalarga mos yildagi kunlar nomerini $N(T_1)$ va $N(T_2)$ hamda ikkita sanalar orasidagi kunlar soni t bo'lsa :

$t=N(T_2)$ - $N(T_1)$ (odatdagi yil uchun)

$t=N(T_2)$ - $N(T_1)+1$ (kabisa yili uchun, 29 fevral sanalar orasiga tushsa) formulalar yordamida hisoblanadi.

Invistirlash muddati uchun kunlar soning aniq va taqribiy qiymatlarini hisobga olib, oddiy protsentlarni hisoblashda quyidagi usullardan foydalanamiz.

1. Kunlar soning aniq qiymatdagi odatdagi protsentlar (365/360)
2. Kunlar soning aniq qiymatdagi aniq protsentlar (365/365)
3. Kunlar soning taqribiy qiymatdagi aniq protsentlar (360/365)

Misol:

$$\left.\begin{aligned} &S(t_0)=5\,mln\\ &i=20\%\ yillik\\ &n=3\,yil\\ &S(t_0+3)=?\end{aligned}\right\}$$

$$S(t_0+3)=S(t_0)(1+3\cdot 0.2)=5\,mln\cdot 1.6=8\,mln$$

$$\left.\begin{aligned} &i=yillik\,\%\\ &n=oylar\ hisobi\end{aligned}\right|\Rightarrow i_{oy}=\frac{i_{yil}}{12}$$

$i = yil\,\%$

$n = kunlar\ soni$

$$Y\begin{cases}365\\366\end{cases} \qquad i_{kun} = \frac{i_{yil}}{Y}$$

Oy bo‘yicha hisoblash

(2) $S(t_0 + m) = S(t_0)\left(1 + \frac{i_{yil}}{12}\cdot m\right)$ $m - oy$ lar soni

$S(t_0 + t) = S(t_0)\left(1 + \frac{i_{yil}}{Y}\cdot t\right)$ $t - kun$ lar soni

Misol (2):

$$\left.\begin{array}{l} S(t_0) = 10\,mln \\ i = 20\%\ \ yil \\ a)\, m = 15\,oy \\ b)\, t = 200\,kun \\ a)\, S(t_0 + m) = ? \\ b)\, S(t_0 + t) = ? \end{array}\right\}$$

$Y = 360$

$$S(t_0 + m) = S(t_0)\left(1 + \frac{i}{12}\cdot m\right)$$

$$S(t_0 + 15) = 10\,mln\left(1 + \frac{0.2}{12}\cdot 15\right) = 12.5\ mln$$

$$S(t_0 + t) = S(t_0)\left(1 + \frac{i}{Y}\cdot t\right)$$

$$S(t_0 + 200) = 10\,mln\left(1 + \frac{0.2}{360}\cdot 200\right) = 10\,mln\left(1 + \frac{1}{9}\right) = 11\frac{1}{9}\,mln$$

Agar berilgan muddat ichida operatsiyalar oddiy protsent bo’yicha bir necha marta o’zgaruvchi stavkalar bilan protsent oshirilsa, u holda jamg’arilgan umumiy mablag’:

$$S(T+n_1+n_2+...+n_i)=S(T)(1+n_1i_1)(1+n_2i_2)...(1+n_ii_i)$$

Bu yerda i_i – stavkalar; n_i – ssuda muddatlari ($n_i=D_i/U$ D_i – ssuda kunlarining soni, U-360 yoki U- 365).
Agar protsentning muddatlari va vaqt bo’yicha protsent stavkalar o’zgarmasa, u holda

$$S(t_0+t_1+t_2+...+t_m)=S(t_0)(1+i_1t_1)(1+i_2t_2)...(1+i_mt_m) \quad (4)$$

Misol (1):

$$\left.\begin{array}{l} S(t_0)=10\,mln \\ i=24\%\,yil \\ a)\,S(t_0+4)=? \\ b)S(t_0+18\,oy)=? \\ c)S(t_0+300\,kun)=? \\ Y=360\,kun \end{array}\right\}$$

$$S^1(t_0+n)=S(t_0)(1+n\cdot i)=S(t_0+4)=10\,mln\cdot(1+0.24\cdot 4)=$$
$$=10\,mln\cdot(1+0.96)=19600000$$

$$S(t_0+m)=S(t_0)\left(1+\frac{i}{12}\cdot m\right)=S(t_0)\left(1+\frac{0.24}{12}\cdot 18\right)=13.6\ mln$$

$$S(t_0+300\,kun)=S(t_0)\left(1+\frac{i}{Y}\cdot t\right)=10\,mln\cdot\left(1+\frac{0.24}{360}\cdot 300\right)=12\,mln$$

Misol (2):

$$S(t_0)=10\,mln$$
$$t_1^{(1)}=2\,yil,\ i_1^{(1)}=24\%$$
$$t_2^{(1)}=3\,yil,\ i_2^{(1)}=30\%$$
$$t_1^{(2)}=1\,yil,\ i_1^{(2)}=20\%$$
$$t_2^{(2)}=2\,yil,\ i_2^{(2)}=26\%$$
$$t_3^{(2)}=2\,yil,\ i_3^{(2)}=32\%$$
$$a)\,S(t_0+t_1^{(1)}+t_2^{(1)})=?$$
$$b)\,S(t_0+t_1^{(2)}+t_2^{(2)}+t_3^{(2)})=?$$

$$S(t_0+t_1^{(1)}+t_2^{(1)})=S(t_0)(1+i_1^{(1)}t_1^{(1)})(1+i_2^{(1)}t_2^{(1)})=10\,mln\cdot(1+0.24\cdot 2)\cdot(1+0.3\cdot 3)=$$
$$10\,mln\cdot 1.48\cdot 1.9=28120000$$

$$S(t_0+t_1^{(2)}+t_2^{(2)}+t_3^{(2)})=S(t_0)(1+i_1^{(2)}t_1^{(2)})(1+i_2^{(2)}t_2^{(2)})(1+i_3^{(2)}t_3^{(2)})=$$
$$10\,mln\cdot(1+0,2\cdot 1)\cdot(1+0,26\cdot 2)\cdot(1+0,32\cdot 2)=10\,mln\cdot 1.9\cdot 1.52\cdot 1.64=29913600$$

$S(t_0)=18\,mln=10000\$,\ \ t_0=01.06.$

1) $a(t_0)=1\$=1800\,so`m$

$i_{oy}^{(1)}=1\%$

$i_1^{(1)}=i_2^{(1)}=1\%, i_3^{(1)}=i_4^{(1)}=1.5\%,\ i_5^{(1)}=i_6^{(1)}=0.5\%$

2) $m=6\,oy$

$i_{oy}^{(2)}=2\%$

1) $S^{(1)}(t_0+6)=S(t_0)(1+i\cdot 6)=10000\$(1+0.01\cdot 6)=10600\$$

2) $S^{(2)}(t_0+6)=18\,mln(1+0.02\cdot 6)=18\,mln\cdot 1.12=20.16\,mln$

3) $a(t_0+6)=a(t_0)(1+i_1\cdot 1)(1+i_2\cdot 1)(1+i_3\cdot 1)(1+i_4\cdot 1)(1+i_5\cdot 1)(1+i_6\cdot 1)=$
$=1800\cdot(1+0.01)\cdot(1+0.01)\cdot(1+0.015)\cdot(1+0.015)\cdot(1+0.05)\cdot(1+0.05)=$
$1800\cdot 1.0201\cdot 1.0302\cdot 1.010025=1910\,so`m$

4) $\dfrac{S^{(2)}(t_0+6)}{a(t_0+6)}=\dfrac{20.16\,mln}{1910\,so`m}=10555\$$

$a(t_0)=1570+200=1770$

$\dfrac{20.16}{1770}=12800$

Misol (3):

$$\left.\begin{array}{l} S(t_0+36)=10\,mln \\ i=2\%\,oy \\ S(t_0)=? \end{array}\right\}$$

$$S(t_0)=\frac{S(t_0+n)}{i+n\cdot i}=\frac{S(t_0+36)}{1+n\cdot i}=\frac{10\,mln}{1+36\cdot 0.02}=\frac{10\,mln}{1.72}=5812\,mln$$

Misol (4):

$$\left.\begin{array}{l} S(t_0+36)=10\,mln \\ d=2\%\,oyiga \\ S(t_0)=? \end{array}\right\}$$

$$S(t_0) = S(t_0+n)(1-n\cdot d) = 10\,mln\cdot(1-36\cdot 0.02) = 10\,mln\cdot 0.28 = 2.8\,mln$$

Misol (5):

$$\left.\begin{array}{l} S(t_0+36) = 10\,mln \\ d = 1\% \\ S(t_0) = 5\,mln \\ n = ? \end{array}\right\}$$

$$S(t_0) = S(t_0+n)(1-n\cdot d)$$
$$5\,mln = 10\,mln\cdot(1-n\cdot 0.01)$$
$$0.5\,mln = 1-n\cdot 0.01$$
$$0.01n = 0.5$$
$$n = 50\,oy$$

Uy ishi.

$$S(t_0+n) = (5k+6m)\,mln = 23\,mln$$
$$d = 1km\%\,oy = 1.3\%$$
$$S(t_0) = (2k+3m)\,mln = 11\,mln$$
$$n = ?$$

Diskont masalasi

Ba'zi bir hollarda moliyaviy amaliyotda ushbu amaliyotning ba'zi ishtirokchilari o'z ulushlarini ushbu amaliyotning boshlanish chog'ida olishga erishadi. Bunday holda, ularning daromadlari *diskont daromad* deyiladi. Diskontlash ikki xil usulda amalga oshiriladi:

1.usul: Matematik diskontlash;

2.usul: Bank diskonti deb ataladi.

Matematik diskontlash – bu oshirilgan moliyaviy mablag' asosida boshlang'ich moliyaviy mablag' miqdorini aniqlash masalasi, ya'ni

$$S(t_0+n) = S(t_0)(1+n\cdot i) \qquad (1)$$

$$S(t_0) = \frac{S(t_0+n)}{(1+n\cdot i)} \qquad (5)$$

Bank diskonti. Moliyaviy amaliyotda bank tashkilotlari ushbu moliyaviy amaliyot moliyalashtirilayotganda o'z foydalarini ushbu amaliyot boshida chegirib olib qoladilar. Agarda bank diskonti kartasi d - yillik diskont stavka (miqdori)si, u holda, har bir davr uchun bank berilayotgan $S(t_0)$ mablag‘dan $d \cdot S(t_0)$ miqdoridagi o‘z foydasini chegirib oladi. Agarda moliyaviy mablag‘ n- davrdan so‘ng qaytarilishi ko‘zda tutilgan bo‘lsa, u holda bank foyda miqdori $n \cdot d \cdot S(t_0)$ teng bo‘ladi. Bu holda, $S(t_0)$ moliyaviy mablag‘dan bank chegirmasi olingandan so‘ng

$$S^{(d)}(t_0 + n) = S(t_0)(1 - n \cdot d) \qquad (6)$$

Misol:

$$\left.\begin{array}{l} S(t_0) = 10\, mln \\ n = 15\, oy \\ d = 1\%\, oyiga \\ S^{(d)}(t_0 + n) = ? \end{array}\right\}$$

$$S^{(d)}(t_0 + n) = S(t_0)(1 - 0.01 \cdot 10) = 10\, mln \cdot 0.9 = 9\, mln$$

Amaliy mashg‘ulot

$$S(t_0 + n) : S(t_0)(1 - n \cdot i) \qquad (1)$$

$$S(t_0 + m) = S(t_0)\left(1 + \frac{i}{12} \cdot m\right) \qquad (2)$$

$$S(t_0 + t) = S(t_0)\left(1 + \frac{i}{Y} \cdot t\right) \qquad (3)$$

Turli molyaviy operatsiyalarda qimmatbaho qog’oz kursini tovar narxidan chiqarib tashlashdan hosil bo’lgan pul miqdori diskont deyiladi.

$$D = S - P$$

D-diskont, S-to’lov uchun berilgan qarz miqdori, P - sotib olish narxi.

Berilgan muddat uchun hisob stavkasi deb, diskontni qimmatbaho qog’ozning to’la narxiga nisbatiga aytiladi:

$$d = \frac{D}{S}$$

Ma`lum muddat uchun hisob stavkasi va yillik stavka orasidagi bog’lanish ushbu formuladan topiladi:

$$d_1 = d \cdot t$$

1-masala. Investor 5 mln. naqd pulga yoki bir yildan so'ng 5400000 so'mga kvartira sotib olishi mumkin. Agar investor hisobida 5000 000 somdan kam bo'lmagan pul bo'lsa va bank yiliga 7% to'lasa, u holda qaysi muqobil afzal bo'ladi?

Yechish. 5000 000 so'm va 5400 000 so'mlar vaqtning turli davrlarga tegishli bo'lganligi uchun ularni biz taqqoslay olmaymiz. Ammo yil oxirida 5400000 so'mni olish uchun yil boshida bankka p=540000/ 1.7 =5046730 so'm qo'yish kerak.

Shunday qilib joriy qiymat naqd pul to'lanadigan qiymatdan katta
yani 5046730 > 5000 000.

Demak bunday holda naqd pul to'lab kvartirani olish afzal ekan.

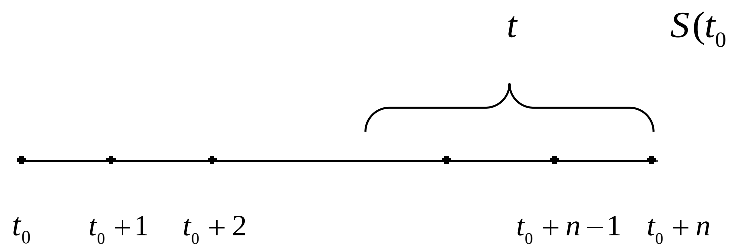
MARUZA 2.

Qarz to`lov sxemalari,diskont va veksellar.

Tayanch iboralar: Qarz to`lovi,teng to`lov,diskont,veksel.

Qimmatli qog'oz (veksel) qiymatini aniqlash.

$S(t_0+n)$ qimmatli qog'oz n muddatga chiqarilgan bo`lsin. Agar s davr uchun diskont foiz d bo'lsa, u holda to'lov vaqtiga t davr qolgandagi qimmatli qog'ozning qiymatini aniqlang.



$S^0(t_0+(n-t+1)$

$S^1(t_0+(n-t))$

Qimmatli qog'oz qiymatini aniqlash.

$$S(t_0) = S(t_0+n)(1-n\cdot d) \qquad (6)$$

$$S(t_0+1) = S(t_0+n)(1-(n-1)\cdot d)$$

$$S(t_0+(n-t)) = S(t_0+n)(1-(n-t)\cdot d)$$

Misol (1):

$$\left.\begin{aligned}
&t_0 = 1.04\ (1\ aprel)\\
&t_0 + n = 30.12\\
&Y = 360\, kun\\
&d = 18\%\ oy\\
&t = 90\, kun\\
&t_0 + t = 1.10\\
&S(t_0 + n) = 15\, mln\\
&S(t_0 + t) = ?
\end{aligned}\right\}$$

$$S(t_0 + t) = S(1.10) = S(30.12)\left(1 - (270 - 90)\cdot\frac{0.18}{360}\right) =$$

$$= 15\, mln\left(1 - 180\cdot\frac{0.18}{360}\right) = 15\, mln \cdot 0.91 = 13.55\, mln$$

Misol (2):

$$\left.\begin{aligned}
&d = 18\%\ yil\\
&Y = 360\, kun\\
&t_0 = 01.01.08\\
&t_0 + n = 01.09.09\\
&t_0 + t = 01.03.09\\
&S(t_0 + n) = 10\, mln\\
&S(t_0 + t) = ?
\end{aligned}\right\}$$

$n = 600\, kun$; $t = 180$

$$S(t_0 + t) = S(01.03.09) = S(01.09.09)\left(1 - (600 - 180)\cdot\frac{0.18}{360}\right) =$$

$$= 10 mln\left(1 - 420\cdot\frac{0.18}{360}\right) = 7.9\, mln$$

Mavzu: Qarz to‘lov sxemasi.

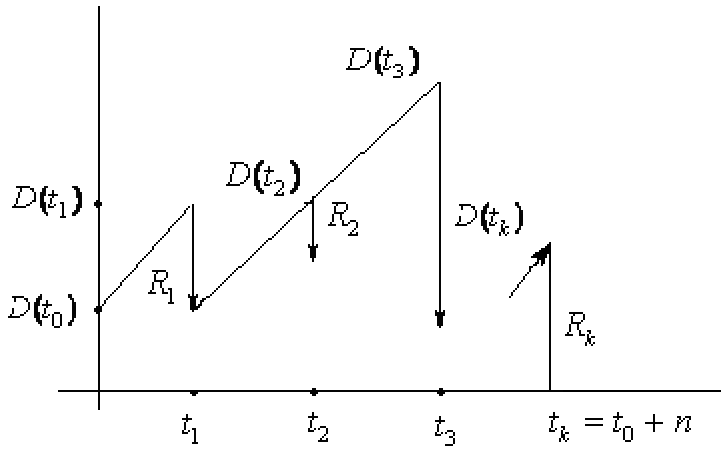
Aktuar va “savdogar” usullari.

$D(t_0)$ miqdordagi qarz t_0 davrda olingan bo‘lib, $(t_0 + n)$ davrda to‘liq to‘lanishi kerak. Qarz uchun foiz stavkasi bir davr uchun i foizni tashkil etsin. Ushbu qarz bo‘yicha turli davrlarda, turli to‘lovlar amalga oshirilgan bo‘lsin. Ya’ni t_j-davrda, R_j- to‘lov , $j = \overline{1, k-1}$.

Ushbu to‘lovlar miqdori umumiy qarz va uning foizini qoplamagan taqdirda, yagona oxirgi k inchi to‘lov orqali qarzdan qutulish masalasi vujudga keladi. Ushbu yagona to‘lov miqdori qanday aniqlanishi kerak?

1-usul. *Aktuar*.

Aktuar usulda har bir davrdagi to‘langan to‘lov miqdoridan eng avvalo shu davrga kelib hosil bo‘lgan qarz uchun hisoblangan foiz miqdori chegirib olib qolinadi. Qolgan qoldiq esa asosiy qarz miqdorini kamaytirishga sarf qilinadi. Agarda to‘lov miqdori qarz foizidan kichik bo‘lsa, u holda to‘lov sxemasida hech qanday o‘zgartirish qilinmaydi va bu to‘lov miqdori, kelgusidagi to‘lov miqdoriga qo‘shib hisoblanadi.



$R_1 \geq D(t_1) - D(t_0)$

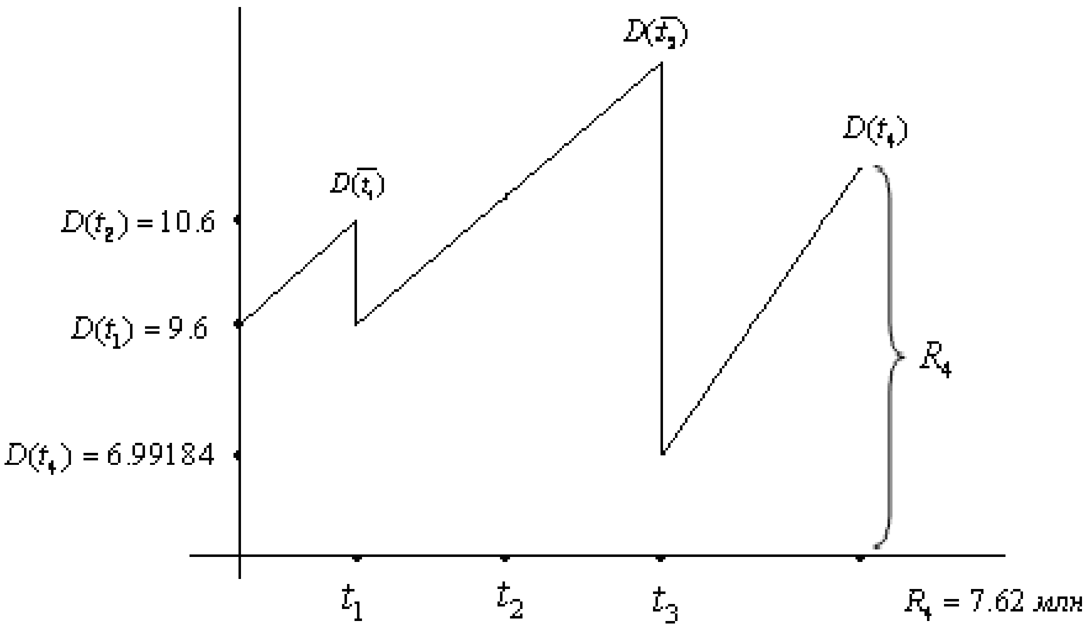
$R_2 < D(t_2) - D(t_1)$

$R_2 + R_3 > D(t_3) - D(t_1)$

$R_k = D(t_k)$

Misol:

$$\left.\begin{aligned}
&D(t_0)=10mln\\
&i=36\%\\
&Y=360\,kun\\
&n=300\ kun\\
&t_0=01.01\ \ t_0+n=01.11\\
&t_1=01.03\ \ R_1=1mln\\
&t_2=01.05\ \ R_2=0.1mln\\
&t_3=01.08\ \ R_3=4\,mln\\
&t_n=t_0+n=01.11\ \ \ R_n=?
\end{aligned}\right\}$$



$$D(t_1)=D(t_0)\left(1+\frac{0.36}{360}\cdot 60\right)=10.6\,mln$$

$$R_1=1mln>10.6-10=0.6\,mln$$

$$D(\overline{t_1})=D(t_2)-R_1=10.6\,mln-1mln=9.6\,mln$$

$$D(t_2)=D(\bar{t})\left(1-\frac{0.36}{360}\cdot 60\right)=9.6\,mln\cdot 1.06=10.176\,mln$$

$$R_2=0.1<10.176-9.6=0.576$$

$$D(t_3)=D(t_2)\left(1+\frac{0.36}{360}\cdot 90\right)=10.176\,mln\cdot 1.09=11.09184\,mln$$

$$R_2+R_3=4.1>11.091840-9.6=1.491840$$

$$D(\overline{t_3})=\big(D(t_3)-(R_2+R_3)\big)=6.991840$$

$$D(t_4) = 6.99184\left(1 - \frac{0.36}{360}\cdot 90\right) = 6.99184\cdot 1.09 = 7.62$$

$$R_4 = 7.62\,mln$$

Aktuar va savdogar usullari amaliyoti.

$$\left.\begin{array}{l} D(t_0) = 20mln \quad t_0 = 01.01.09 \\ i = 24\%\,yillik \quad t_0 + n = 01.03.10 \\ Y = 360\,kun \\ t_1 = 01.06.09 \;\; R_1 = 3\,mln \\ t_2 = 01.09.09 \;\; R_2 = 0.5\,mln \\ t_3 = 20.02.10 \;\; R_3 = 8\,mln \\ t_4 = 01.03.10 \;\; R_4 = ? \end{array}\right\}$$

I. Aktuar.

$$D(t_1) = D(t_0)\left(1 + \frac{i}{360}\cdot t_1\right) = 20\,mln\left(1\cdot\frac{0.24}{360}\cdot 150\right) = 20\,mln\cdot 1.1 = 22\,mln$$

$$D\overline{(t_1)} = D(t_1) - R_1 = 22\,mln - 3\,mln = 19\,mln$$

$$D(t_2) = D(\overline{t_1})\left(1 + \frac{0.24}{360}\cdot 90\right) = 19\,mln\cdot 1.06 = 20.14\,mln$$

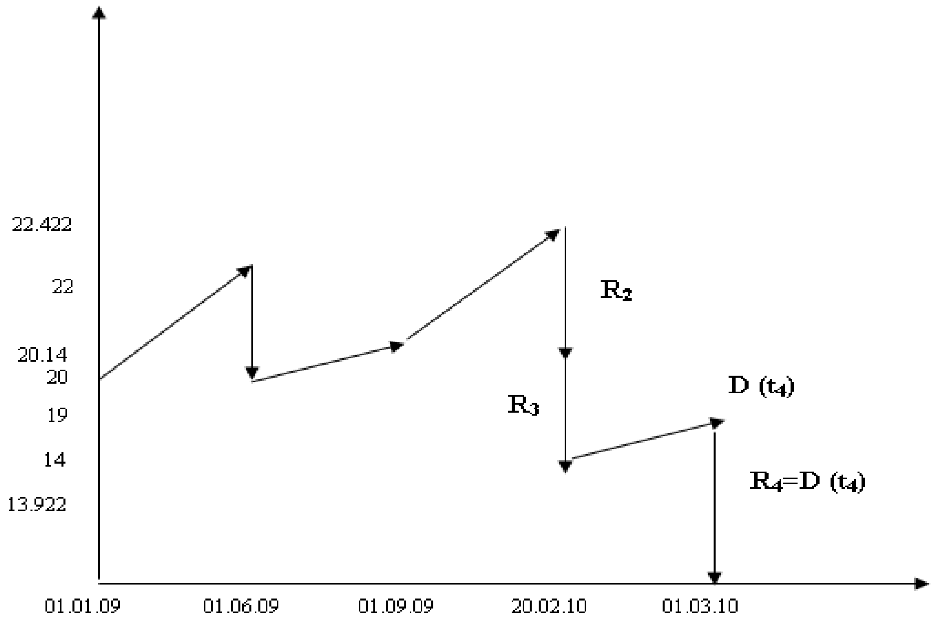
$$D(t_3) = D(t_2)\left(1 + \frac{0.24}{360}\cdot 170\right) = 20.14\cdot\left(\frac{3.34}{3}\right) = 20.14\cdot 1.11(3) = 22.422\,mln$$

$$D\overline{(t_3)} = \left(D(t_3) - (R_2 + R_3)\right) = 22.422 - (0.5 + 8) = 13.922$$

$$D(t_4) = D(\overline{t_3})\cdot\left(1 + \frac{0.24}{360}\cdot 10\,kun\right) = 13.922\cdot\left(\frac{3.02}{2}\right) \approx 14\,mln$$

$$R_4 = D(t_4) = 14\,mln$$

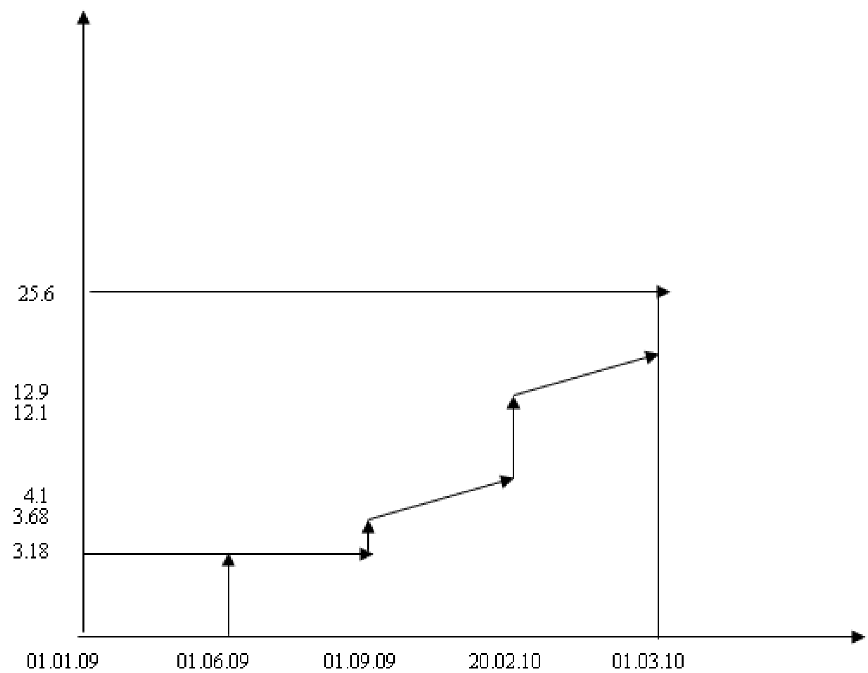
$$R_{AKTUAR} = R_1 + R_2 + R_3 + R_4 = 3\,mln + 0.5\,mln + 8\,mln + 14mln = 25.5\,mln$$



Savdogar usuli.

Savdogar usulida, qarz miqdori o‘zgarishi bilan birgalikda shu qarz bo‘yicha amalga oshirilgan qarz to‘lovlari ham xuddi shu asosida o‘sib boradi va davr oxiriga kelib aniqlanadigan yagona to‘lov miqdori, oshirilgan qarz bilan jamlangan to‘lov miqdorlari orasidagi farqga teng bo‘ladi.

$$D(t_0+n)=D(t_0)\left(1+\frac{i}{Y}\cdot t_n\right)=20\cdot\left(1+\frac{0.24}{360}\cdot 420\right)=25.6\,mln$$



$$R(t_2) = R_1\left(1+\frac{0.24}{360}\cdot 90\right) = 3mln\cdot 1.06 = 3.18mln$$

$$R(\overline{t_2}) = R(t_2) + R_2 = 3.18 + 0.5 = 3.68mln$$

$$R(t_3) = R(\overline{t_2})\left(1+\frac{0.24}{360}\cdot 170\right) = 3.68\cdot\left(\frac{3.34}{3}\right) \approx 4.097 \approx 4.1$$

$$R(\overline{t_3}) = R(t_3) + R_3 = 4.1mln + 8mln = 12.1mln$$

$$R(t_4) = R(\overline{t_3})\cdot\left(1+\frac{0.24}{360}\cdot 10\,kun\right) = 12.1\cdot\left(\frac{3.02}{6}\right) \approx 12.9mln$$

$$R_4 = D(t_0+n) - R(t_4) = 25.6 - 12.9 = 12.7\,mln$$

$$R_{SAVDOGAR} = 3mln + 0.5mln + 8mln + 12.7mln = 24.2mln$$

$$R^{akt} = 25.5mln \qquad R^{akt} - R^{sav} = 1.3mln$$

Vekselning narxini $P = S(1 - d\cdot t)$ formuladan topamiz.

Hisob va protsent stavkalarining ekvivalentligini ifodalovchi tenglama quyidagidan iborat:

$$(1+i_1)(1-d_1) = 1$$

Veksel belgilangan muddatdan oldin sotishda uning narxini belgilash uchun quyidagi formuladan foydalanish zarur:

$$S(t) = S(1)[1-(1-t)d] \qquad 0 \leq t \leq 1$$

$[1-(1-t)d]$ – diskont koeffisenti deyiladi.

Pul resurslarining konversiya bilan va konversiyasiz protsentlarini jamg’arishning quyidagi to’rtta varianti mavjud bo’lishi mumkin :

Konversiyasiz : dollar-dollar ;

Konversiyali : dollar-so’m-so’m-dollar ;

Konversiyasiz : so’m-so’m ;

Konversiyali : so’m-dollar-dollar-so’m

1-variant: dollar-so’m-so’m-dollar

Quyidagi belgilarni kiritamiz:

P_v – dollardagi depozit mablag’ (dipozit-kredit muassasalariga saqlash uchun joylashtirilgan pul mablag’lari yoki qimmatli qog’ozlar)

P_v – so’mlardagi depozit mablag’lari;

S_v – dollardagi jamg’arilgan pul miqdori;

S_c – so'mlardagi jamg'arilgan pul miqdori;

K_0 –operatsiya boshidagi almashtirish kursi (dollar kursining so'mdagi qiymati);

K_1 – operatsiya oxiridagi almashtirish kursi;

T – depozit muddati ;

i – so'mlar miqdori uchun jamg'arma stavkasi ;

j – dollar miqdori uchun jamg'arma stavkasi ;

Operatsiya uchta qadamni ko'zda tutadi : valyutani so'mga almashtirish, bu summaga qo'yilgan protsentlar va nihoyat, dastlabki valyutaga konvertatsiya qilish, oxirgi jamg'arilgan valyutadagi pul miqdori quyidagicha aniqlanadi :

$$S_v = \frac{P_v K_0}{K_1}(1+iT)$$

Operatsiya samaradorligi

$$i_{sam} = \frac{S_v - P_v}{P_v T}$$

Buni yuqoridagi formulaga qo'yib

$$i_{sam} = \frac{\left[K_0/K_1(1+iT)-1\right]}{T}$$

hosil qilamiz.

2- variant. So'm-dollar-dollar-so'm.

$$S_c = \frac{P_0 K_1}{K_0}(1+iT)$$

$$i_{sam} = \frac{S_c - P_c}{P_c T} \quad \text{yoki} \quad -\frac{\left[K_1/K_0(1+iT)-1\right]}{T}$$

Bir qancha davlatlarda berilgan protsentlardan soliq olinadi, bu o'z navbatida jamg'arilgan pul miqdorini va protsent operatsiyalarining daromadliligini kengayishiga olib keladi.

2-masala. Veksel 2003-yil 10-yanvarda chiqarilgan bo'lib, uni yopish muddati 2003 yil 10 oktyabrgacha. Veksel bo'yicha yillik kirish 12%. Agar veksel 2003-yil 10-mayda 10% hisob stavkasi bo'yicha hisobga olingan bo'lsa, u holda vekselni sotib olish narxi qancha?

Vekselning belgilangan muddat uchun narxi 100 000 so'm,

Yechish: 2003-yil 10-yanvarda 1-oktyabrgacha kunlar soning aniq qiymati (1-ilova) jadval bo'yicha 283-10=273 kun. To'lov muddatining yillardagi uzunligi T=273/360=0.75yil.

U holda vekselning umumiy narxi $S = P(1+iT) = 100(1+0.12\ 0.75) = 109$ ming so'm.

Agar veksel 10-maydan hisobga olinsa, u holda to'lovgacha (yopishgacha) bo'lgan muddat 283-130=153 kun yoki T=152/360=0.42. bunday holda d=10%.

Hisob stavkasi bo'yicha vekselning yillik hisob narxi $P = S(1 - dT) = 109(1 - 0.1\ 0.42) = 104.422$ ming so'm.

1.2 Oddiy protsentlar bo'yicha masalalar

1.2.1 Vekselni hisobga olish. Valyuta ayriboshlash

1. 3 oy ichida 97500 so'mning oddiy protsentda yillik 10% stavkadagi jamg'arilgan pul miqdorini toping?
2. 1 oyda 97500 so'mning oddiy protsentning yillik 24% stavkadagi jamg'arilgan pul miqdorini toping?
3. 97800 so'mning 90 kun ichida yillik 15% stavkadagi odatdagi va aniq oddiy protsentlarni toping?
4. Yillik 8% stavkada 972 000 so'mning 190 kun ichida jamg'arilgan qiymati, aniq oddiy protsentlarini toping?
5. 6 oy mabaynida yillik 5% stavkada oddiy protsentda jamg'arilgan mablag'ning qiymati $ 2050 bo'lsa, omonatning boshlang'ich qiymatini toping?
6. Yillik shkalada (3,$500) hodisa uchun unga ekvivalent bo'lgan hodisani 0,2,u. momentlar uchun ikki mintaqaga va 20% yillik stavkaga nisbatan toping?

MARUZA 3.

Murakkab protsentlar

Tayanch iboralar: ssuda, jamg'arma koeffisenti, jamg'arilgan summa, yillik protsent stavkasi.

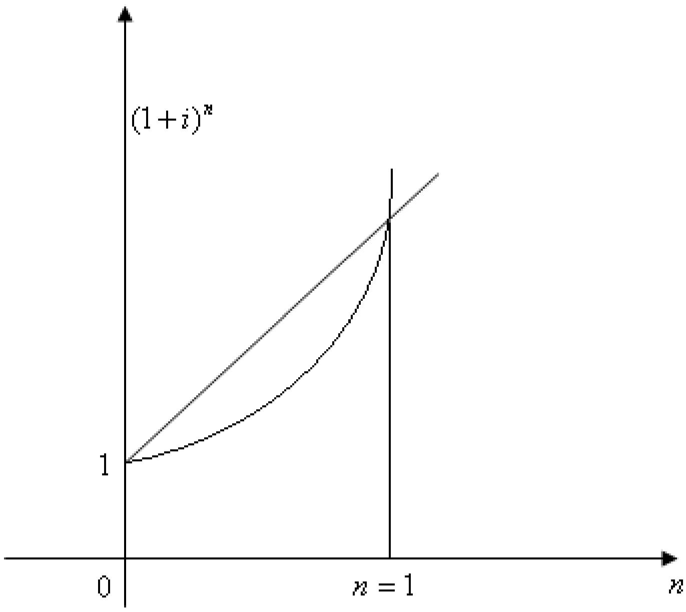
Samarali foiz stavkalari.

$t_0 \quad S(t_0) \quad t_0 + n \quad i\%$

$S(t_0+1) = S(t_0)(1+i)$

$S(t_0+2) = S(t_0+i)(1+i) = S(t_0)(1+i)(1+i) = S(t_0+2) = S(t_0)(1+i)^2$

$S(t_0+n) = S(t_0)(1+i)^n \quad (1)$



$(1+i)^n > (1+n\cdot i) \quad n>1 \quad i \neq 0$

Misol:

$$\left.\begin{array}{l} S(t_0) = 3mln \\ i = 30\%\ yil \\ n = 3\ yil \\ S(t_0+3) = ? \end{array}\right\}$$

$S(t_0+3) = S(t_0)(1+i)^3 = 3(1+0.3)^3 = 3\cdot 1.3^3 = 3\cdot 2.197 = 6.591$

$$i_{yil} = \frac{S(t_0+3)-S(t_0)}{S(t_0)\cdot 3} = \frac{6.591-3mln}{3\cdot 3}\cdot 100\% = \frac{3.591}{9}\cdot 100\% = 39.9\%$$

$$(1+0.02)^{12} = 1+12\cdot 0.02+\frac{12\cdot 11}{2}\cdot 0.0004 = 1.24+0.0264 \approx 1.2664 \approx 1.(27)$$

i foiz stavkasi bir davr uchun aniqlangan foiz stavkasi bir davr m ta davr ostiga bo‘lingan bo‘lsin, $i_m = \frac{i}{m}$ davr osti foiz stavkasi va har bir davr ostida oshirilgan mablag‘lar, kapitalizasiya qilinsin, u holda,

$$S(t_0+n) = S(t_0)\left(1+\frac{i}{m}\right)^{m\cdot n} \qquad (2)$$

$$(a+b)^n = a^n + n\cdot a^{n-1}\cdot b + \frac{n(n-1)}{2}\cdot a^{n-2}\cdot b^2 + \frac{n(n-1)}{2}\cdot a^2\cdot b^{n-2} + n\cdot a\cdot b^{n-1} + b^n$$

kelib chiqadiki, $(1+i)^n \geq 1+ni$ dan

Misol:

$$\left.\begin{array}{l} S(t_0) = 10\,mln \\ i = 2\%\ oylik \\ n = 3\ yil \\ S(t_0+n) = S(t_0+36) = ? \end{array}\right\}$$

$$S(t_0+36) = S(t_0)\left(1+\frac{0.24}{12}\right)^{12\cdot 3} = 10\,mln\cdot 1.02^{36} = 10\,mln\cdot 2.028 = 20.280\,mln$$

Samarador foiz stavkasi

Boshlang‘ich moliyaviy mobest n davr mobaynida har bir davr ostida $i_m = 4m\%$ foiz bo‘yicha hisoblansin, u holda $S(t_0+n) = S(t_0)\left(1+\frac{i}{m}\right)^{m\cdot n}$ ga teng bo‘ladi.

$$S(t_0+n) = S(t_0)(1+i_c)^n$$

$$i_c = ?\ \ S(t_0+n) = S_m(t_0+n)$$

$$S(t_0)(1+i_c)^n = S(t_0)\left(1+\frac{i}{m}\right)^{mn} \quad : S(t_0)\neq 0$$

$$(1+i_c)^n = \left(1+\frac{i}{m}\right)^{mn} \Rightarrow 1+i_c = \left(1+\frac{i}{m}\right)^m$$

$$i_c = \left(1 + \frac{i}{m}\right)^m - 1$$

Misol:

$$\left.\begin{array}{l} i_m = 2\%\ oylik \\ i_{yil} = ? \end{array}\right\}$$

$$i_{yil} = (1 + i_m)^m - 1 = (1 + 0.02)^{12} - 1 = (1 + 12 \cdot 0.02 + \frac{12 \cdot 11}{2} \cdot 0.0004) - 1 = 26.64\%$$

Murakkab foiz stavkasi bilan oddiy foiz stavkasini solishtirish

$$S(t_0 + n) = S(t_0)(1 + n \cdot i_{od}) \qquad (1)$$

$$S^M(t_0 + n) = S(t_0)(1 + i^m)^n \qquad (2)$$

Agar $S(t_0 + n) = S^M(t_0 + n)$ $\quad i_{od} = ? \quad i^M = ?$

$$S(t_0)(1 + n \cdot i_{od}) = S(t_0)\left(1 + i^M\right)^n : S(t_0) \neq 0$$

$$(1 + n \cdot i_{od}) = \left(1 + i^{(m)}\right)^n$$

$$i_{od} = \frac{\left(1 + i^{(m)}\right) - 1}{n} \qquad (4)$$

$$i^{(M)} = \sqrt[n]{1 + n \cdot \log} - 1 \qquad (5)$$

Misol (1):

$$\left.\begin{array}{l} S(t_0) = 10mln \\ 1) i_{oy}^M = 2\% \ \ m = 24\,oy \\ 2) i_{od} = 25\%\,yil \ \ n = 2\,yil \\ a) S^m(t_0 + n) = ? \ S^0(t_0 + n) \\ b) i_{od}^c = ? \ i_c^M = ? \end{array}\right\}$$

$$S^m(t_0 + n) = S(t_0)(1 + 0.02)^{24} = 10\,mln \cdot 1.5904 = 15.904mln$$

$$(1 + 0.02)^{24} = 1 + 24 \cdot 0.02 + \frac{24 \cdot 23}{2} + 0.02^2 = 1.5904$$

$$S^0(t_0 + n) = S(t_0)(1 + n \cdot i_{od}) = 10\,mln \cdot (1 + 2 \cdot 0.25) = 15\,mln$$

$$S^m(t_0+n) > S^0(t_0+n)$$

$$i_{od}^c = \frac{\left(1+i^m\right)-1}{n} = (1+0.02)^{12}-1 = 26.64\%$$

$$i_m^c = \sqrt[n]{1+0.25}-1 = (1+0.25)^{\frac{1}{12}}-1 = \frac{12.25}{12}-1 = \frac{0.25}{12}\cdot 100\% = 2.08\%$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0)$$

$$a = 1.25:\ f(x) = 1.25^x \qquad 1.25^{\frac{1}{12}} =$$

$$x_0 = 0;\ x = \frac{1}{12}$$

Murakkab foiz amaliyoti

$$S(t_0+n) = S(t_0)(1+i)^n \qquad (1)$$

$$S_m(t_0+n) = S(t_0)(1+\frac{i}{m})^{mn} \qquad (2)$$

$$S(t_0+t_1+t_2+...+t_n) = S(t_0)\left(1+\frac{i_1}{m_1}\right)^{m_1t_1}\cdot\left(1+\frac{i_2}{m_2}\right)^{m_2t_2}...\left(1+\frac{i_k}{m_k}\right)^{m_kt_k} \qquad (3)$$

$$i_c = (1+i_m)^m - 1 \qquad (4)$$

$$i_m = \sqrt[m]{1+i_c} - 1 \qquad (5)$$

$$i_0 = \frac{(1+i_c)-1}{n} \qquad (6)$$

$$i_c = \sqrt[n]{1+ni_0} - 1 \qquad (7)$$

Misol (1):

$$\left.\begin{array}{l} S(t_0) = 5\,mln \\ i = 26\% \\ n = 3\,yil \\ a)\,S(t_0+3) = ? \\ b)\,m = 2 \\ S_m(t_0+3) = ? \end{array}\right\}$$

$$a)\, S(t_0+3)=S(t_0)(1+3)^n=S(t_0)(1+0.26)^3=5\cdot 2=10\,mln$$

$$b)\, S_m(t_0+3)=S(t_0)\left(1+\frac{0.26}{2}\right)^{2\cdot 3}=5\cdot 2.081=10.405\,mln$$

Misol (2):

$$\left.\begin{array}{l} S(t_0)=10\,mln \\ t_1^{(1)}=1\,yil \quad i_1^{(1)}=24\% \ m_1^{(1)}=4 \\ t_2^{(1)}=2\,yil \quad i_2^{(1)}=28\% \ m_2^{(1)}=4 \\ t_1^{(2)}=2\,yil \quad i_1^{(2)}=26\% \ m_1^{(2)}=2 \\ t_2^{(2)}=1\,yil \quad i_2^{(2)}=30\% \ m_2^{(2)}=6 \\ S^{(1)}(t_0+t_1^{(1)}+t_2^{(1)})=? \\ S^{(2)}(t_0+t_1^{(2)}+t_2^{(2)})=? \end{array}\right\}$$

$$S^{(1)}(t_0+t_1^{(1)}+t_2^{(1)})=S(t_0)\left(1+\frac{0.24}{4}\right)^{4\cdot 1}\cdot\left(1+\frac{0.28}{4}\right)^{2\cdot 4}=10\cdot(1.06)^4\cdot(1.07)^8=21.6\,mln$$

$$S^{(2)}(t_0+t_1^{(2)}+t_2^{(2)})=S(t_0)\left(1+\frac{0.26}{2}\right)^{2\cdot 2}\cdot\left(1+\frac{0.3}{6}\right)^{6\cdot 1}=10\cdot(1.12)^4\cdot(1.05)^6=21.842\,mln$$

$$S(t_0+t_1^{(1)}+t_2^{(1)})<S(t_0+t_1^{(2)}+t_2^{(2)})$$

Misol (3):

$$\left.\begin{array}{l} S(t_0)=10\,mln \\ S(t_0+n)=20\,mln \\ i=26\% \\ a)\, n=3\,yil \ i_c=? \\ b)\, i_m=2\% \ m=2 \\ n=? \end{array}\right\}$$

$$S(t_0+n)=S(t_0)(1+i)^n$$

$$20=10\cdot(1+i)^3$$

$$2=(1+i)^3$$

$$i=26\%$$

$$a) i=\sqrt[3]{2}-1=1.26-1=0.26$$

$$S_m(t_0+n)=S(t_0)\left(1+\frac{i}{m}\right)^{mn}$$

$$20=10\cdot(1+0.02)^{12\cdot n}$$

$$2=1.02^{mn}$$

$$\log_{1.02}\cdot 2=12\cdot n$$

$$n=\frac{1}{12}\log_{1.02}\cdot 2\approx(36)\,oy$$

Misol (4):

$$\left.\begin{array}{l} S(t_0)=10\,mln \\ S(t_0+n)=24\,mln \\ i=26\% \\ a)\,n=4\ \ i_c=? \\ b)\,i_m=?\ m=4\ n=4 \\ c)\,i_m=?\ m=12\ n=4 \end{array}\right\}$$

1) $S(t_0+n)=S(t_0)(1+i)^n$

$$2.4=10\cdot(1+i)^4$$

$$\sqrt[4]{2.4}=1+i$$

$$i=\sqrt[4]{2.4}-1$$

2) $i_m=\sqrt[4]{1+\sqrt[4]{2.4}-1}=\sqrt[16]{2.4}-1$

3) $i_m=\sqrt[12]{1+\sqrt[4]{2.4}-1}=\sqrt[48]{2.4}-1=?$

$$2.4^{\frac{1}{16}}=(2.56-0.16)^{\frac{1}{16}}=\left(\frac{2}{100}^{8}-\frac{2}{100}^{4}\right)^{\frac{1}{16}}=\left(\frac{1}{10}\right)^{\frac{1}{8}}\left(2^8-2^4\right)^{\frac{1}{16}}=?$$

Misol (5):

$$\left.\begin{aligned}
&S(t_0)=6\,mln\\
&a)\,t_1=2\,yil,\ i_1=2\%, m_1=12\\
&t_2=1\,yil,\ i_2=8\%, m_2=4\\
&b)\,n=3\,yil, i_0=30\%\\
&S_m(t_0+t_1+t_2)=?\\
&S^0(t_0+3)=?
\end{aligned}\right\}$$

$$S_m(t_0+t_1+t_2)=S(t_0)(1+0.02)^{12\cdot 2}(1+0.08)^{4\cdot 1}=6\cdot 1.02^{24}\cdot 1.08^4=6\cdot 1.6\cdot 1.36=13.056\,mln$$

$$S^0(t_0+3)=S(t_0)(1+3\cdot 0.3)=6\cdot 1.9=11.4\,mln$$

Murakkab diskont

$$S(t_0+n)=S(t_0)(1+n)^n \quad (1)$$

$$S(t_0)=\frac{S(t_0+n)}{(1+n)^n} \qquad (2)$$

Misol (1):

$$\left.\begin{aligned}
&S(t_0+n)=5\,mln\\
&n=5\,yil, i=20\%\\
&S(t_0)=?
\end{aligned}\right\}$$

$$S(t_0)=\frac{S(t_0+n)}{(1+n)^n}=\frac{5}{(1+0.2)^5}=\frac{5}{2.48}\approx 2\,mln$$

$D-s$ davr uchun diskont stavkasi bo‘lsa, u holda hozirgi qiymati $S(t_0+n)$ t_0+n davrdagi miqdori $S^d(t_0)=S(t_0+n)(1-d)^n$ (3)

Misol (2):

$$\left.\begin{aligned}
&S(t_0+n)=5\,mln\\
&n=5\,yil\\
&d=20\%\\
&S(t_0)=?
\end{aligned}\right\}$$

$$S^d(t_0)=S(t_0+n)(1-d)^n=5\cdot(1-0.2)^5=5\cdot 0.32768=1.6384$$

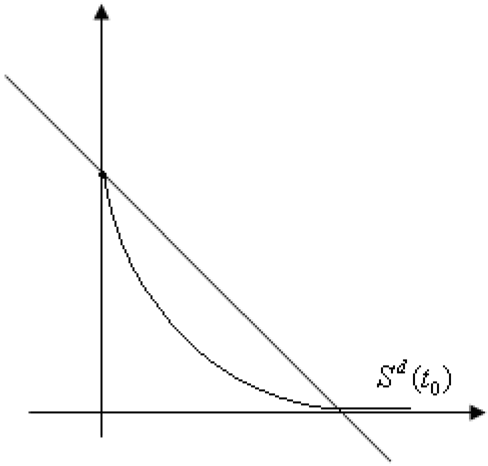
$$D = S(t_0 + n) - S^d(t_0) = 5 - 1.6384 = 3.3616\, so`m$$

$$\log = \frac{3.3616}{5mln \cdot 5} \cdot 100\% = 13.44\%$$

$$S(t_0) = S(t_0 + n)(1-d)^n$$

$$n^* = \frac{1}{d}$$

$$S^0(t_0) = S(t_0 + n)(1-nd)^n$$



d m marta $d_m = \dfrac{d}{m}$ diskontlansin, u holda bu miqdor

$$S_m^d(t_0) = S(t_0 + m)\left(1 - \frac{d}{m}\right)^{mn} \quad (4)$$

$$S(t_0) = \frac{S(t_0 + n)}{(1+n)^n}$$

$$S^d(t_0) = S(t_0 + n)(1-d)^n$$

$$S(t_0) = S^d(t_0) \Leftrightarrow \frac{S(t_0 + n)}{(1+i)^n} = S(t_0 + n)(1-d)^n$$

$$(1-d)^n = \frac{1}{(1+i)^n} \to d_i = 1 - \frac{1}{1+i} = \frac{i}{1+i}$$

$$d_i = \frac{i}{1+i} \qquad i_d = \frac{d}{1-d}$$

$1 - d_m = \left(1 - \dfrac{d}{m}\right)^n$ - yil bo‘yicha

Misol (3):

$$\left.\begin{array}{l} S(t_0+n)=5\,mln \\ n=5\,yil \\ i=20\% \\ a)S^m(t_0)=? \\ b)S^0(t_0)=? \\ c)d_i=? \end{array}\right\}$$

$$S^m(t_0)=\frac{S(t_0+n)}{(1+n)^n}=\frac{5\,mln}{(1+0.2)^5}\approx 2\,mln$$

$$S^0(t_0)=\frac{S(t_0+n)}{1+n\cdot i}=\frac{5\,mln}{1+5\cdot 0.2}=2.5\,mln$$

$$d_i=\frac{i}{1+i}=\frac{0.2}{1.2}=\frac{1}{6}\cdot 100\%=16.6\%$$

Misol (4):

$$\left.\begin{array}{l} S(t_0+n)=5\,mln \\ n=5\,yil \\ i=20\% \\ a)S^d(t_0)=? \\ b)S^0(t_0)=? \\ c)i_d=? \end{array}\right\}$$

$$S^d(t_0)=5\,mln\cdot(1-0.2)^5=5\,mln\cdot 0.32768=1.6384$$

$$S^0(t_0)=5mln\cdot(1-0.2\cdot 5)=0$$

$$i_d=\frac{d}{1-d}=\frac{0.2}{1-0.2}=\frac{0.2}{0.8}\cdot 100\%=25$$

Misol (5):

$$\left.\begin{array}{l} S(t_0+5)=5\,mln \\ m=4\,martaga \\ d=20\% \\ S^d_{t_0}=? \end{array}\right\}$$

$$S^d_m(t_0)=S(t_0+m)\left(1-\frac{d}{m}\right)^{mn}=5\cdot\left(1-\frac{0.2}{4}\right)=4.75$$

$$S_m^d = S(t_0+n)\left(1-\frac{d}{m}\right)^{mn} = 5\cdot\left(1-\frac{0.2}{4}\right)^{4\cdot 5} = 5\cdot(0.95)^{20} = 5\cdot 0.34 \approx 1.7mln$$

$$S^d(t_0) = 5mln(1-0.2)^5 = 5\cdot(0.8)^5 \approx 1.63mln$$

Misol (6):

$$\left.\begin{array}{l} S(t_0+3)=10\,mln \\ S(t_0)=6\,mln \\ a)\,n=3 \\ i=?\ d=? \\ b)\,n=3, m=2 \\ i_m=?\ d_m=? \\ c)\,n=3,\ m=6 \\ i_m=?\ d_m=? \end{array}\right\}$$

$$a)\,S(t_0+n)=S(t_0)(1+i)^n$$

$$10\,mln = 6(1+i)^3$$

$$(1+i)^3=\frac{5}{3}$$

$$1+i=\sqrt[3]{\frac{5}{3}}$$

$$i=\sqrt[3]{\frac{5}{3}}-1=19\%$$

$$S(t_0)=S(t_0+n)(1-d)^n$$

$$6=10(1-d)^n$$

$$(1-d)^n=\frac{3}{5}$$

$$1-d=\sqrt[3]{0.6}$$

$$d=1-\sqrt[3]{0.6}=1-0.85=15\%$$

$$S_m^d = S(t_0+n)\left(1-\frac{d}{m}\right)^{mn} = 10\,mln\left(1-\frac{d}{2}\right)^6$$

$$\left(1-\frac{d}{2}\right)^6=0.6$$

$$1-\frac{d}{2}=\sqrt[6]{0.6}$$

$$\frac{d}{2} = 1 - \sqrt[6]{0.6}$$

$$d = 2(1 - \sqrt[6]{0.6}) = 0.16$$

Murakkab diskont.

$$\left.\begin{array}{l} S(t_0 + n) = 10\,mln \\ n = 4\,yil \\ a) i = 20\%\,yil,\ S(t_0) = ? \\ b)\ d = 20\%\,yil,\ S^d(t_0) = ? \\ c) d_m = \dfrac{d}{4},\ m = 4,\ S_m^d(t_0) = ? \end{array}\right\}$$

$$S(t_0 + n) = S(t_0)(1 + i)^n$$

$$S^d(t_0) = S(t_0 + n)(1 - d)^n$$

$$S_m^d(t_0) = S(t_0 + n)\left(1 - \frac{d}{m}\right)^{mn}$$

$$a) 10\,mln = S(t_0)(1 + 0.2)^4$$

$$S(t_0) = \frac{10\,mln}{(1.2)^4} = \frac{10\,mln}{2.07} = 4830000$$

$$b) S^d(t_0) = 10\,mln(1 - 0.2)^4 = 10\,mln \cdot 0.4096 = 4096000$$

$$c) S_m^d(t_0) = 10\,mln\left(1 - \frac{d}{m}\right)^{mn}$$

$$S_m^d(t_0) = 10\,mln\left(1 - \frac{0.2}{4}\right)^{4 \cdot 4} = 10\,mln \cdot (0.95)^{16} = 10 \cdot 0.44 = 4.4\,mln$$

Quyidagi belgilarni kiritamiz :

$D(t_0)$ -qarzning boshlang’ich miqdori (ssudalar,kreditlar va boshqalar);
$D(t_0 + n)$-ssuda miqdorining oxirida jamg’arilgan summa;
n -jamg’arma muddati;
i -O’nli kasrlarda ifodalangan yillik protsent stavkasi;

Murakkab protsent bo’yicha jamg’arilgan pul miqdori
$D(t_0 + n) = D(t_0)(1 + i)^n$

(1)

formuladan topiladi.

$a_{n,i} = (1+i)^n$ -jamg'arma koeffisenti deyiladi.

Agar muddat kasr sondan iborat bo'lsa, jamg'arilgan pul miqdorini quyidagi formuladan topamiz.

$$n = [n] + \{n\}$$

$$S(t_0 + n) = S(t_0)(1+i)^{[n]}\ (1+i\{n\}) \qquad (2)$$

Bunda $[n]$ - n ning butun qismi, $\{n\}$ -kasr qismi. Agar jamg'arma (1) formula yordamida hisoblansa, uni umumiy metod, (2) formula bilan hisoblansa, biz uni aralash metod deb yuritamiz, agar yillik $i^{(m)}$ protsent stavkaga mos keluvchi i_m protsent stavkasi davr uzunligi $T = \frac{1}{m}$ yildan iborat bo'lgan ustama haq to'lash davri uchun

$$i_m = \frac{i^{(m)}}{m}, \quad m = 1, 2, 3...$$

ni tashkil etsa,u holda bunday yillik $i^{(m)}$ protsent stavkasi nominal (nominal rate) protsent stavkasi deyiladi.

Nominal stavka bo'yicha jamg'arma miqdori

$$S^{(m)}(t_0 + n) = S(t_0)\left(1 + \frac{i^{(m)}}{m}\right)^{mT}$$

Formula yordamida hisoblanadi. Nominal stavka bo'yicha jamg'arma koeffisenti

$$a_{n,i}^{m} = \left(1 + \frac{i^{(m)}}{m}\right)^{mT}$$

m karrali ustama protsentlardagi nominal stavka va yillik murakkab stavkalar uchun ushbu tenglik bajariladigan bo'lsa

$$(1+i)^n = \left(1 + \frac{i^{(m)}}{m}\right)^{mn}$$

bu tenglikdan topiladigan i stavka samarali stavka deyiladi.

Yuqoridagi formuladan,

$$i_{sam} = \left(1 + \frac{i^{(m)}}{m}\right)^{m} - 1$$

Agar shartnoma tuzishda i_{sam} va m berilgan bo'lsa, u holda i^m –nominal stavka

$$i^{(m)} = m\left[(1+i_{sam})^{\frac{1}{m}} - 1\right]$$

formuladan topiladi.

Agar ikkita yillik nominal stavkalarga yillik samarali stavkalar mos tushsa, bunday yillik nominal stavkalar ekvivalent deyiladi.

$$\left(1+\frac{i^{(m_1)}}{m_1}\right)^{m_1} = \left(1+\frac{i^{(m_2)}}{m_2}\right)^{m_2} = 1+i_{sam}$$

$\frac{i^{(m)}}{m} \leq$ *0.10, m*$\leq$ *6* larda quyidagi taqribiy formuladan foydalanish mumkin

$$i_{sam} = i^m + \frac{m-1}{2m}\left[i^{(m)}\right]^2$$

Agar oldindan belgilangan yillik murakkab stavka d ning qiymati murakkab protsent bo’yicha i yillik stavka ekvivalent bo’lsa, u holda d ni yillik murakkab hisob stavkasi deb yuritamiz.

$$(1-d)^n \cdot (1+i)^n = 1 \text{ yoki } (1-d)\ (1+i) = 1$$

Murakkab hisob stavkasi bo’yicha diskontirlash quyidagi formula bo’yicha amalga oshiriladi.

$$P = S(1-d)^n$$

Bunda *d*-murakkab yillik hisob stavka.

Oddiy va murakkab hisob stavkalari bo’yicha diskont ko’paytuvchilari mos ravishda

$$W_0 = (1-dn) \text{ va } W_m = (1-d_m)^n$$

Formula yordamida hisoblanadi.

Nominal stavka bo’yicha diskontirlash quyidagiga teng:

$$P = S\left(1-\frac{f}{m}\right)^{mn}$$

$(1-d)^n = \left(1-\frac{f}{m}\right)^{mn}$ tenglikdan topiladigan *d* ni samarali hisob stavkasi deb yuritamiz

$$d_{sam} = 1-\left(1-\frac{f}{m}\right)^m$$

O’z navbatida

$f = m\left(1-\sqrt[m]{1-d}\right)$, bunda *f* nominal hisob stavkasi.

O'zgaruvchi stavkalar bo'yicha jamg'rma miqdori quyidagi formuladan topiladi:

$$S = P(1+i_1)^{t_1} \cdot (1+i_2)^{t_2} \cdot .... \cdot (1+i_k)^{t_k}$$

Murakkab protsent bo'yicha i va $i^{(m)}$ nominal stavkalar bo'yicha jamg'arish davrlari ushbu formuladan topiladi:

1 Masalalar yechish namunalari

1– masala. Birinchi yil kvartalda ssuda bo'yicha protsent stavkasi 30% va plus 2% majinal xarajat (komission xarajatlar uchun to'lov) va ikkinchi yilning birinchi yarm yilida 40% va plus marjinal xarajat 3% ni tashkil etsa, u holda jamg'arma koefisenti quyidagiga teng bo'ladi :

$$(1+0.3+0.02)^4(1+0.4+0.03)^2= 1.32^4 \cdot 1.43^2=6.208$$

2– masala. Mijoz bankka 2.5 mln. so'mni 9.5% protsent stavkasi bo'yicha qo'ydi. U 2 yilu 270 kundan so'ng qancha pul olishini aralash metotdan foydalanib hisoblang.

Yechim. $S=P(1+i)^{[T]}(1+\{T\}i)=2.5 \cdot 1.095^2(1+270/365 \cdot 0.095)=3.21$ mln. so'm.

3 – masala. Yarim yillik ustama protsent to'lash bo'yicha nominal protsent stavkasini, unga ekvivalent bo'lgan har oyda ustama protsent to'lanadigan 24% ni nominal stavka bo'yicha hisoblang.
Yechish: shartga ko'ra $i^{(12)}$ =0.24, unga ekvivalent bo'lganni topish kerak. Ekvalent tarifiga ko'ra

$$\left(1+\frac{i^{(2)}}{2}\right)=\left(1+\frac{0,24}{12}\right)^{12}=1,2682$$

bundan $i^{(2)}=\left(\sqrt{1,2682}-1\right)\cdot 2=0,25$ yoki 25%

4 – masala. ustama protsentga soliq stavkasi 10% yillik protsent stavkasi 30% protsentni qo'yish muddati 3 yil. Ssudaning boshlang'ich qiymati 5 mln. so'm bo'lsa, soliq to'lagan hollar uchun murakkab protsent stavkasi bo'yicha hisoblang.
Yechish. Soliq to'langan holda
$S=5(1+0.3)^3=5 \cdot 1.3^3=10.985$ mln. so'm
Soliq to'langandan so'ng
$S^1=5[(1-0.1)(1+0.3)^3+0.1]=10.3865$ mln. so'm.
5-masala. Agar birinchi ikki yil uchun ssudaga 15% stavka, qolgan 3 yil uchun u 20% ni tashkil etsa, ssudaning berilgan muddati uchun o'rtacha stavkani toping.

$$i=\sqrt[5]{1{,}15^2\cdot 1{,}2^3}-1\approx 0{,}18$$

MARUZA 4.

Teng kuchli foiz stavkalari.

Tayanch iboralar: Nominal stavka, samarali stavka , stavka ekvivalenti.

Nominal stavka bo'yicha jamg'arma miqdori

$$S^{(m)}(t_0+n)=S(t_0)\left(1+\frac{i^{(m)}}{m}\right)^{mn}$$

Formula yordamida hisoblanadi. Nominal stavka bo'yicha jamg'arma koeffisenti

$$a_{n,i}^{(m)}=\left(1+\frac{i^{(m)}}{m}\right)^{mn}$$

m karrali ustama protsentlardagi nominal stavka va yillik murakkab stavkalar uchun ushbu tenglik bajariladigan bo'lsa

$$(1+i)^n=\left(1+\frac{i^{(m)}}{m}\right)^{mn}$$

bu tenglikdan topiladigan i stavka samarali stavka deyiladi.
Yuqoridagi formuladan,

$$i_{sam}=\left(1+\frac{i^{(m)}}{m}\right)^{m}-1$$

Agar shartnoma tuzishda i_{sam} va m berilgan bo'lsa , u holda i^m –nominal stavka

$$i^{(m)}=m\left[(1+i_{sam})^{\frac{1}{m}}-1\right]$$

formuladan topiladi.

Agar ikkita yillik nominal stavkalarga yillik samarali stavkalar mos tushsa, bunday yillik nominal stavkalar ekvivalent deyiladi

$$\left(1+\frac{i^{(m_1)}}{m_1}\right)^{m_1}=\left(1+\frac{i^{(m_2)}}{m_2}\right)^{m_2}=1+i_{sam}$$

$\frac{i^{(m)}}{m} \leq 0.10, m \leq 6$ larda quyidagi taqribiy formuladan foydalanish mumkin

$$i_{sam} = i^m + \frac{m-1}{2m}\left[i^{(m)}\right]^2$$

Agar oldindan belgilangan yillik murakkab stavka d ning qiymati murakkab protsent bo'yicha i yillik stavka ekvivalent bo'lsa, u holda d ni yillik murakkab hisob stavkasi deb yuritamiz.

$$(1-d)^n \cdot (1+i)^n = 1 \text{ yoki } (1-d)\ (1+i) = 1$$

Murakkab hisob stavkasi bo'yicha diskontirlash quyidagi formula bo'yicha amalga oshiriladi.

$$P = S(1-d)^n$$

Bunda d-murakkab yillik hisob stavka.

Oddiy va murakkab hisob stavkalari bo'yicha diskont ko'paytuvchilari mos ravishda

$$W_0 = (1-dn) \text{ va } W_m = (1-d_m)^n$$

Formula yordamida hisoblanadi.

Nominal stavka bo'yicha diskontirlash quyidagiga teng:

$$P = S\left(1-\frac{f}{m}\right)^{mn}$$

$(1-d)^n = \left(1-\frac{f}{m}\right)^{mn}$ tenglikdan topiladigan *d* ni samarali hisob stavkasi deb yuritamiz

$$d_{sam} = 1-\left(1-\frac{f}{m}\right)^m$$

O'z navbatida

$f = m\left(1-\sqrt[m]{1-d}\right)$, bunda *f* nominal hisob stavkasi.

O'zgaruvchi stavkalar bo'yicha jamg'rma miqdori quyidagi formuladan topiladi:

$$S = P(1+i_1)^{t_1} \cdot (1+i_2)^{t_2} \cdot \ldots \cdot (1+i_k)^{t_k}$$

Murakkab protsent bo'yicha *i* va $i^{(m)}$ nominal stavkalar bo'yicha jamg'arish davrlari ushbu formuladan topiladi:

$$T = \frac{\log(S/P)}{\log(1+i)}$$

$$T = \frac{\log(S/P)}{m\log\left(1+\frac{i^{(m)}}{m}\right)}$$

Murakkab protsent bo'yicha d hisob stavkasi va f nominal stavkalari bo'yicha diskontlash davrlari ushbu formuladan topiladi:

$$T = \frac{\log(P/S)}{\log(1-d)}$$

$$T = \frac{\log(P/S)}{m\log\left(1+\frac{f}{m}\right)}$$

Agar moliyaviy operatsiyalar protsent stavkalari vaqt bo'yicha o'zgaradigan bo'lsa, u holda o'rta qiymat yordamida stavkalarning barcha qiymatlarini umumlashtirish mumkin.

$t_1, t_2, ..., t_k$ ketma-ket davrlar uchun $i_1, i_2, \ldots i_k$ oddiy protsent stavkalari bo'yicha ustama protsent to'lanadigan bo'lsa, o'rta qiymat quyidagi tenglikdan topiladi:

$$1+Ni = 1+\sum_{1}^{k} t_t i_t$$

Bundan

$$i = \frac{\sum_{1}^{k} t_t i_t}{N}$$,bunda $N= T_t$

Hisob stavkasining o'rta qiymati

$$D = \frac{T_t i_t}{N}$$

Agar stavkalar vaqt bo'yicha murakkab protsent stavkalari uchun o'zgaradigan bo'lsa, u holda jamg'arma koeffisentlari tengligiga asosan

$$(1+i)^N = (1+i_1)^{t_1} \cdot (1+i_2)^{t_2} \ldots\ldots\ldots\ldots$$

bunda

$$i = \sqrt[N]{(1+i_1)^{T_1} \cdot (1+i_2)^{T_2} \cdot \ldots} - 1$$

Masalalar.

1. - *m* karrali ustama protsentning yillik nominal stavkasi *i(m)*, *p*-karrali ustama protsentning yillik stavkasi *i(p)* bo'lsin. Qanday shartda bu stavkalar bir xil samarali stavkani ifodalaydi?
2. 5% yillik nominal stavkaga ekvivalent bo'lgan samarali stavkani quyidagi hollarda toping: a) har yarim yilda ustama protsent to'lanadigan; b)har kvartalda protsent to'lanadigan ; c)har oyda protsent to'lanadigan.
3. $300 omonat uchun jamg'arilgan mablag' miqdorini beshinchi yil oxirida toping, agar yiliga 300% stavkada ustama protsent to'lanadigan bo'lsa?
4. Har kvartalda bir marotaba amalgam oshiriladigan 12% yilik nominal stavkaga ekvivalent bo'lgan samarali yillik stavkani toping.
5. Uch yil muddatga 12% yillik stavkada qo'yilgan va bir yilda 2 marta protsent to'lanadigan $300 summaning jamg'arilgan qiymatini toping.
6. 5 yil mobaynida omonatga o'z mablag'ini 3 marta ko'paytiradi. Protsentlar har yarim yilda amalgam oshiriladi. Yillik nominal stavka qancha bo'lgan?
7. A. bank oyiga 6% dan, B. bank yiliga 80% bo'lgan ustama protsentga har yarim yilda to'laydi. Bir yilga pul qo'yuvchi omonatchi uchun qaysi bank qulay?
8. Har yarim yilda yiliga 26% nominal stavkada ustama protsent to'lanadi. 6 yil mobaynida $2000 summani jamg'arish uchun depozitga qancha pul qo'yish mumkin?

MARUZA 5.

Moliyaviy oqimlar. Moliyaviy teng kuchlilik.

Tayanch iboralar: moliyaviy oqim, moliyaviy teng kuchlilik,renta.

Moliyaviy oqimlar.

***Ta`rif.*** Agar barcha $R_k$ to`lovlar bir xil ishoraga ega va vaqtning bir xil $t_k - t_{k-1}$, $k = 1, 2, 3, ..., n$ oralig`ida sodir bo`lsa, u holda bu to`lovlarni belgilashga bog`liq bo`lmagan holda bunday to`lovlar ketma-ketligi moliyaviy renta yoki annuitet (annuity) deyiladi. Rentani to`lash bir yilda bir marta yoki $l$ marta, ustama protsent to`lovi esa yiliga bir marta yoki m marta amalga oshirilishi mumkin. Bu – diskret (l, m) karrali renta bo`lib, biz faqat $l = m$ bo`lgan holni qaraymiz. Agar to`lov yoki ustama protsent tez-tez (masalan, bir haftada) amalga oshiriladigan bo`lsa, ularni uzluksiz deb tahlil qilish va *uzluksiz* renta deb nomlash mumkin. Agar to`lov har bir davr oxirida amalga oshirilsa, u holda bunday rentani *postnumerando* yoki *odatdagi* (ordinary annuity), agar davr boshida amalga oshirilsa, u holda *prenumerendo* yoki *oldindan to`lash* (annuitydue) deyiladi.

Moliyaviy oqimlarning teng kuchliligi.

$$\{x_t\}_{t=1}^{n} = (x_1, x_2, ..., x_n)$$

$$\{y_t\}_{t=1}^{k} = (y_1, y_2, ..., y_k)$$

$$f\left(\{x_t\}\right) \qquad f\left(\{y_t\}\right)$$

Moliyaviy oqimlarni solishtirish uchun bu oqimlarning barchasi bir davrda akslantiriladi va shu davrdagi akslantirilgan miqdorlar o'zaro solishtirishi kerak.

$$(1)\begin{cases} R: f_1\left(\{x_t\}\right) \to f_1^T \\ R: f_2\left(\{y_t\}\right) \to f_2^T \end{cases}$$

$$f_1^T = ?, \ \ f_2^T = ?$$

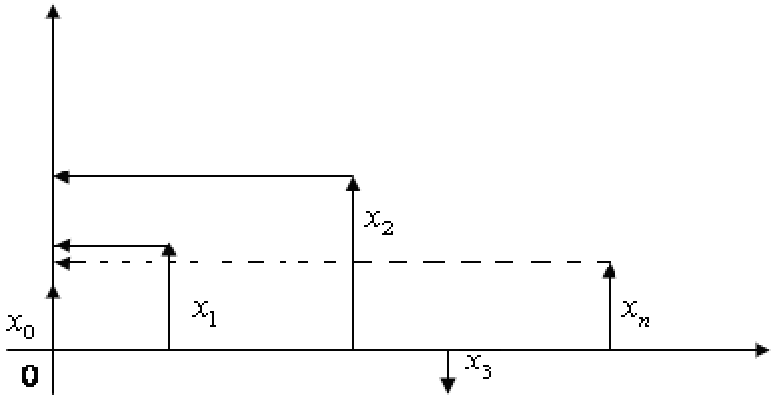
Moliyaviy tahlilda moliyaviy oqimlar asosan ikki davrda akslantiriladi:

1) Ko'rilayotgan davr boshiga;

2) Davr oxiriga.

Bundan tashqari ba'zi hollarda katta o'zgarishlar ro'y berishdan oldingi qabul qilingan davrga akslantiriladi.

Butun ko‘rilayotgan davr mobaynida i davr stavkasi amal qilsa, bir davr uchun u holda



$SV_n\left(\{x_t\}\right)$

$$PV\left(\{x_t\}\right) = x_0 + \frac{x_1}{1+i} + ... + \frac{x_n}{(1+i)^n} \qquad (2)$$

$$SV_n\left(\{x_t\}\right) = x_0(1+i)^n + x_1(1+i)^{n-1} + ... + x_{n-1}(1+i)^n + x_n \quad (3)$$

Ta’rif 1: $\{x_t\}_{t=1}^{n}$ va $\{y_t\}_{t=1}^{k}$ moliyaviy oqimlar T - davrga nisbatan *o‘zaro teng kuchli moliyaviy oqim* deyiladi, agarda $f_1^T\left(\{x_t\}\right) = f_2^T\left(\{y_t\}\right)$

Ta’rif 2: Agarda moliyaviy oqimda barcha miqdorlar bir xil ishorali bo‘lsa, u holda bunday oqim *renta* deyiladi.

Renta quyidagi turlarga bo‘linadi:

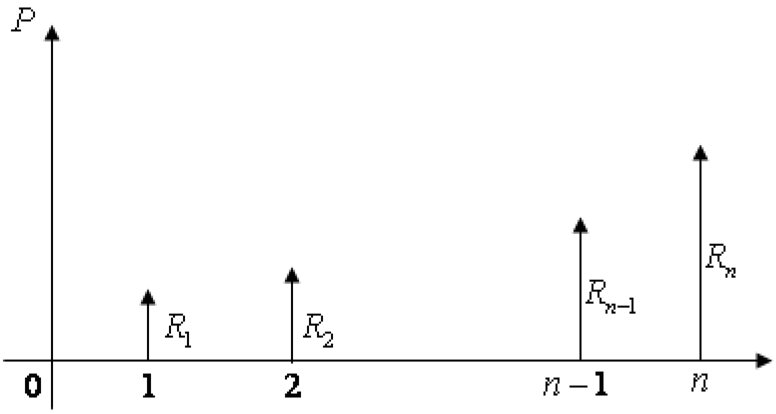
1. O‘zgarmas renta;
2. Shartli renta;
3. Chekli va cheksiz renta;
4. Diskret va uzluksiz.

Renta to‘lovini amalga oshirish vaqtiga qarab, renta ikki turga ajratiladi:

1. Oldindan to‘lov rentalari;
2. Davr oxiridagi to‘lov.

R - bir davr uchun to‘lov miqdori.

i - foiz stavkasi.



Постнумерандо

$$PV^{(1)} = \frac{R}{1+i} + \frac{R}{(1+i)^2} ... + \frac{R}{(1+i)^n} = (1+i)^{-1} \cdot R \cdot (1 + (1+i)^{-1} + ... + (1+i)^{-n(-1)}) =$$

$$= (1+i)^{-1} \cdot R \cdot \frac{1-(1+i)^{-n}}{1-(1+i)^{-1}} = R \cdot \frac{1+(1+i)^{-n}}{1+i-1} = R \cdot \frac{1-(1+i)^{-n}}{i} = R \cdot \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV^{(1)} = R \cdot \frac{(1+i)^n - 1}{i(1+i)^n} \qquad (4)$$

$$PV^{(0)} = R \cdot \frac{R}{1+i} + ... + \frac{R}{(1+i)^{n-1}} = R \cdot (1 + (1+i)^{-1} + ... + (1+i)^{n-1}) =$$

$$= R \cdot \frac{1-(1+i)^{-n}}{1-(1+i)^{-1}} = R(1+i) \cdot \frac{1-(1+i)^{-n}}{i} = (1+i) \cdot R \cdot \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV^0 = (1+i) \cdot R \cdot \frac{(1+i)^n - 1}{i(1+i)^n} \qquad (5)$$

$$SV^{(1)}\left(R\right) = R(1+i)^{n-1} + R(1+i)^{n-2} + ... + R(1+i) + R = R((1+i)^{n-1} + (1+i)^{n-2} +$$

$$+ ... + (1+i) + 1) = R \cdot \frac{1-(1+i)^n}{1-(1+i)} = R \cdot \frac{(1+i)^n - 1}{i}$$

$$SV^{(1)}\left(R\right) = R \cdot \frac{(1+i)^n - 1}{i} \qquad (6)$$

$$SV^0\left(R\right)=R(1+i)^n+R(1+i)^{n-1}+...+R(1+i)=(1+i)\cdot R((1+i)^{n-1}+(1+i)^{n-2}+$$

$$+...+(1+i)+i)=(1+i)\cdot R\cdot\frac{(1+i)^n-1}{i}$$

$$SV^0\left(R\right)=(1+i)\cdot R\cdot\frac{(1+i)^n-1}{i} \quad (7)$$

$$PV^0(R)=(1+i)PV^{(1)}(R) \quad (8)$$

$$SV^0\left(R\right)=(1+i)SV^{(1)}\left(R\right) \quad (9)$$

$$SV^0\left(R\right)=(1+i)^n PV\left(R\right) \quad (10)$$

$$SV^{(1)}\left(R\right)=(1+i)^n PV^{(1)}\left(R\right) \quad (11)$$

Amaliy mashg'ulot

$$PV(R^2)=(1+i)\cdot R\cdot\frac{1-(1+i)^{-n}}{i} \quad (1)$$

$$PV(R^{(2)})=R\cdot\frac{1-(1+i)^{-ni}}{i} \quad (2)$$

$$SV\left(R^0\right)=(1+i)\cdot R\cdot\frac{(1+i)^n-1}{i} \quad (3)$$

$$SV\left(R^1\right)=R\cdot\frac{(1+i)^n-1}{i} \quad (4)$$

$$PV(R^0)=(1+i)PV(R^{(1)}) \quad (5)$$

$$SV(R^0)=(1+i)SV(R^{(1)}) \quad (6)$$

$$SV(R^0)=(1+i)^n PV(R^0) \quad (7)$$

$$SV(R^1)=(1+i)^n PV(R^{(1)}) \quad (8)$$

Misol:

$$\left.\begin{array}{l} N=1 \\ R=2000000 \\ i=20\% \\ n=4\,yil \\ PV(R^0)=? \\ PV(R^{(1)})=? \\ SV(R^0)=? \\ SV(R^{(1)})=? \end{array}\right\}$$

$$PV(R^0)=(1+0.2)\cdot 2000000\cdot\frac{1-(1+0.2)^{-4}}{0.2}=1.2\cdot 2000000\cdot\frac{1-(1.2)^{-4}}{0.2}=6000000$$

$$PV(R^{(1)})=2000000\cdot\frac{1-(1+0.2)^{-4}}{0.2}=5000000$$

$$SV(R^0)=(1+0.2)\cdot 2000000\cdot\frac{(1+0.2)^{-4}-1}{0.2}=1.2\cdot 2000000\cdot\frac{(1.2)^{-4}-1}{0.2}=12000000$$

$$SV(R^{(1)})=2000000\cdot\frac{(1+0.2)^{-4}-1}{0.2}=10000000$$

Misol (2):

$$\begin{cases} R_1^{(0)}=3000000 \\ n_1=3\,yil \end{cases}$$

$$R_2^{(0)}=2000000$$

$$n_2=5\,yil$$

$$i=20\%$$

$$n=4\,yil$$

$$R^{(0)}=?$$

$$PV_3(R_1^{(0)})=?$$

$$PV_5(R_2^{(0)})=?$$

I) $PV_4(R^{(0)})=PV_3(R_1^{(0)})+PV_5(R_2^{(0)})$

II) $SV_4(R^{(0)})=(1+i)^{4-3}SV_3(R_1^{(0)})+(1+i)^{4-5}SV_5(R_2^{(0)})$

I) $PV_3(R_1^{(0)}) = (1+0.2)\cdot 3mln\cdot\dfrac{1-(1+0.2)^{-3}}{0.2} = 1.2\cdot 3mln\cdot\dfrac{1-(1.2)^{-3}}{0.2} = 7740000$

$$PV_5(R_2^{(0)}) = 1.2\cdot 3mln\cdot\frac{1-(1+0.2)^{-5}}{0.2} = 7200000$$

$$PV_4(R^{(0)}) = (1+0.2)\cdot R\cdot\frac{1-(1+0.2)^{-4}}{0.2} = 3.12\cdot R$$

$$3.12\cdot R = 7740000 + 7200000$$

$$R = \frac{14.940.000}{3.12} = 4788000$$

II) $SV_4(R^{(0)}) = (1+i)^{4-3}SV_3(R_1^{(0)}) + (1+i)^{4-5}SV_5(R_2^{(0)})$

$$SV_3(R_1^{(0)}) = (1+0.2)\cdot 3mln\cdot\frac{(1+0.2)^3-1}{0.2} = 13104000$$

$$SV_5(R_2^{(0)}) = 1.2\cdot 3mln\cdot\frac{(1+0.2)^5-1}{0.2} = 17850000$$

$$SV_4(R^{(0)}) = 1.2\cdot R\frac{1-(1+0.2)^4}{0.2} = 6.44R$$

$$6.44R = 13104000\cdot 1.2 + \frac{17850000}{1.2} = 15724800 + 14875000 = 30599800$$

$$R = \frac{30599800}{6.44} = 475152174$$

Muddat nuqtai nazaridan renta shartsiz (annuity certain), ya`ni oldindan birinchi va oxirgi to`lovlar sanasi kelishib olinadigan va shartli (contingent annuity), ya`ni birinchi va oxirgi to`lovlar sanasi qandaydir hodisaning yuz berishi yoki yuz bermasligiga bog`liq bo`lgan rentalarga bo`linadi. Shartli rentaga nafaqa misol bo`ladi, unga to`lov insonning ma`lum yoshga yetganidan keyin amalga oshiriladi va uning o`limidan so`ng to`xtatiladi. Shartli rentaning tahlili sug`urtaviy matematikaning fundamental bo`limlaridan biridir. Busiz sug`urta firmalarining qonuniy faoliyatlari va nafaqa fondlarini yo`lga qo`yish mumkin emas.

Agar $n=\infty$ bo`lsa, u holda bunga mos keluvchi renta *abadiy* (perpetuity) renta deyiladi.

3-Misol. Nafaqa fondi o`zining a`zolariga har oyda 100000 so`m pul to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yish lozim bo`ladi. qulaylik uchun nafaqa fondi o`zgarmas $i=0{,}5\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

Yechish. Abadiy renta modelidan foydalanamiz. Bunda oylik to`lov $R = 100000,\quad i = 0{,}005$. Oylik renta qanday usulda to`lanishi aytilmagan, shuning uchun biz har ikkala holni, ya`ni to`lovni har oy oxiri va har oy boshi uchun hisoblaymiz.

Birinchi holda

$$S_1(0) = \frac{R}{i} = \frac{100000}{0{,}005} = 2\cdot 10^7 = 20 \text{ mln. so`m.}$$

$$S_2(0) = \frac{R}{d} = \frac{R(1+i)}{i} = 2\cdot 10^7\cdot 1{,}005 = 20 \text{ mln 100 ming so`m.}$$

2-.Masala. Birinchi ishtirokchining renta olish muddatini hisoblashga keltiriladi, uni *n_1* bilan belgilaymiz. qolgan muddatda pulni ikkinchi ishtirokchi oladi. Shunday qilib, birinchi ishtirokchi dastlabki *n_1* muddat davomida, ikkinchisi esa $n_2 = n - n_1$ muddatdagi qoldirilgan rentani oladi. Yuqoridagi shartlarga asosan rentani bo`lishdan quyidagi kelib chiqadi: $S_1(0) = n_1 S_2(0)$, $Ra_{n_1|i} = Ra_{n_2|i}\cdot v^{n_1}$.

$n_2 = n - n_1$ hisobga olsak,

$$\frac{1-(1+i)^n}{i} = \frac{1-(1+i)^{-(n-n_1)}}{i}\cdot v^{n_1}.$$

Soddalashtirishlardan so`ng

$$n_1 = \frac{-\ln\left\{\left[1+(1+i)^{-n}\right]/2\right\}}{\ln(1+i)} \text{ hosil bo`ladi.}$$

2-Misol. Postnumerando yillik rentaning muddati 10 yil, $i = 20\%$. Renta yuqorida keltirilgan kabi ikkita ishtirokchi orasida bo`lingan bo`lsin. U holda

$$n_1 = \frac{-\ln\left[\left(1+1{,}2^{10}\right)/2\right]}{\ln 1{,}2} = 2{,}961 \approx 3 \text{ yil.}$$

Ikkinchi ishtirokchi qolgan 7 yil davomida oladi.

Masalalar.

1. Nafaqa fondi o`zining a`zolariga har oyda 200 ming so`m to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yish kerak bo`ladi. qulaylik uchun nafaqa fondi o`zgarmas $i = 2\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

2. Korxona 3 yil mobaynida 150 mln. so`mlik maxsus fondni tashkil etish maqsadida har yili bankka 15% yillik protsent stavka bo`yicha to`lovlar amalga oshiriladi. 3 yil mobaynida jamg`arilgan puldan to`lanadigan badal miqdori topilsin.

3. Firma 150 mln. so`m miqdoridagi fondni tashkil etish maqsadida har yili bankka 43,196 mln. so`m pulni 15% stavka bo`yicha to`lanadi. Fondni tashkil etish uchun qancha muddat zarur bo`lishi topilsin.

4. Renta har kvartalda, protsentlar ham har kvartalda bir kvartal uchun $i = 20\%$ stavka bo`yicha to`lanadigan bo`lsin. Postnumerando renta uchun $n = 4$ bo`lganda diskontirlash koeffisienti va jamg`arma koeffisienti topilsin.

5. $R = 2$ mln. so`mdan yiliga abadiy renta to`lanadigan bo`lsin. Yillik stavka 10% bo`lganda bu rentaning postnumerando va prenumerando qiymatlari topilsin.

6. Nafaqa fondi o`zining a`zolariga har oyda 1 mln. so`m to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yishi lozim bo`ladi. qulaylik uchun nafaqa fondi o`zgarmas $i = 0{,}1\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

MARUZA 6.

Moliyaviy rentalar. Moliyaviy teng kuchlilik.

Tayanch iboralar:m**oliyaviy oqim, moliyaviy teng kuchlilik,renta**.

Muddat nuqtai nazaridan renta shartsiz (annuity certain), ya`ni oldindan birinchi va oxirgi to`lovlar sanasi kelishib olinadigan va shartli (contingent annuity), ya`ni birinchi va oxirgi to`lovlar sanasi qandaydir hodisaning yuz berishi yoki yuz bermasligiga bog`liq bo`lgan rentalarga bo`linadi. Shartli rentaga nafaqa misol bo`ladi, unga to`lov insonning ma`lum yoshga yetganidan keyin amalga oshiriladi va uning o`limidan so`ng to`xtatiladi. Shartli rentaning tahlili sug`urtaviy matematikaning fundamental bo`limlaridan biridir. Busiz sug`urta firmalarining qonuniy faoliyatlari va nafaqa fondlarini yo`lga qo`yish mumkin emas.

Agar $n = \infty$ bo`lsa, u holda bunga mos keluvchi renta *abadiy* (perpetuity) renta deyiladi.

Diskontirlash koeffisientlari va rentalarni jamg`arish

Postnumerando va prenumerando rentalarning diskontirlash koeffisientlari mos ravishda

$$a_{n|i} = \frac{S_1(0)}{R} = v + v^2 + \cdots + v^n$$

$$\ddot{a}_{n|i} = \frac{S_0(0)}{R} = 1 + v + \cdots + v^{n-1} \qquad (2.2.1)$$

Bulardan

$$a_{n|i} = \frac{1 - v^n}{i}, \quad \ddot{a}_{n|i} = \frac{1 - v^n}{d} \qquad (2.2.2)$$

n momentdagi postnumerando va prenumerando rentalarning jamg`arma qiymatlarini mos ravishda quyidagicha belgilaymiz.

$$S_{n|i} = AV_{a|i} = (1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1, ,$$

$$\ddot{S}_{n|i} = AV_{\ddot{a}|i} = (1+i)^n + (1+i)^{n-1} + \cdots + (1+i) \qquad (2.2.3)$$

Bulardan

$$S_{n|i} = v^{-n}\,\frac{1 - v^n}{i}, , \quad \ddot{S}_{n|i} = v^{-n}\,\frac{1 - v^n}{d} \qquad (2.2.4)$$

$a_{n|i}$ va $\ddot{a}_{n|i}$ kattaliklar rentaning diskontirlash koeffisientlari, $S_{n|i}$ va $\ddot{S}_{n|i}$ kattaliklar esa rentaning jamg`arma koeffisientlari deb yuritiladi.

2-Misol. $R = 1$ mln. so`mdan yiliga abadiy renta to`lanadigan bo`lsin. Yillik stavka $i = 5,\ 10,\ 100\%$ bo`lganda bu rentaning postnumerando va prenumerando joriy qiymatlari topilsin.

Yechish. Bu yerda

$a_{\infty|0,05} = \dfrac{1}{0,05} = 20,\quad a_{\infty|0,10} = \dfrac{1}{0,10} = 10,\quad a_{\infty|1} = 1,\quad \ddot{a}_{\infty|i} = (1+i)a_{\infty|i}$ bo`lganligi uchun

$$\ddot{a}_{\infty|0,05} = 21,\quad \ddot{a}_{\infty|0,10} = 11,\quad \ddot{a}_{\infty|1} = 2.$$

Demak, abadiy ($n = \infty$) rentaning postnumerando va prenumerando joriy qiymatlari mos ravishda 20, 10 va 1 mln. so`m, 21, 11 va 2 mln. so`mni tashkil etar ekan.

$i > 0$ bo`lganda abadiy renta uchun

$$\ddot{a}_{\infty|i} - a_{\infty|i} = \frac{1}{d} - \frac{1}{i} = 1$$

3-Misol. Nafaqa fondi o`zining a`zolariga har oyda 100000 so`m pul to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yish lozim bo`ladi. qulaylik uchun nafaqa fondi o`zgarmas $i = 0,5\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

Yechish. Abadiy renta modelidan foydalanamiz. Bunda oylik to`lov $R = 100000,\quad i = 0,005$. Oylik renta qanday usulda to`lanishi aytilmagan, shuning uchun biz har ikkala holni, ya`ni to`lovni har oy oxiri va har oy boshi uchun hisoblaymiz.

Birinchi holda

$$S_1(0) = \frac{R}{i} = \frac{100000}{0,005} = 2\cdot 10^7 = 20 \text{ mln. so`m.}$$

$$S_2(0) = \frac{R}{d} = \frac{R(1+i)}{i} = 2\cdot 10^7 \cdot 1,005 = 20 \text{ mln 100 ming so`m.}$$

Qoldirilgan rentalar

Ayrim hollarda, masalan, qarzlarni qoplash maqsadida to`lovlar qandaydir muddatga qoldiriladi. Bunday turdagi to`lovlar oqimi qoldirilgan renta (deferred annuities) deb yuritiladi. 0 momentdagi joriy narx (qiymat) ni prenumerando to`lov uchun $h_1\ddot{a}_{n|i}$ bilan, postnumerando uchun esa $h_1 a_{n|i}$ kabi belgilaymiz. Bunda $n = 1,2,\ldots,\quad h > 0.$ $h = 0$ bo`lganda qoldirilgan renta tayanch renta bilan mos tushadi. (h-qoldirilgan muddat)

Darhaqiqat,

$$h_1\ddot{a}_{n|i} = v^h + v^{h+1} + \cdots + v^{h+n-1} = v^h\left(1 + v + \cdots + v^{n-1}\right) = v^h\ddot{a}_{n|i}$$

$$h_1 a_{n|i} = v^{h+1} + v^{h+2} + \dots + v^{h+n} = v^h a_{n|i}$$

qoldirilgan rentaning joriy qiymatini har qanday $h > 0$ uchun, butun $h = 1,2,\dots$ va $n = 1,2,\dots$ da

$$h_1 \ddot{a}_{n|i} = \left(1 + v + \dots + v^{h+n-1}\right) - \left(1 + v + \dots v^{h-1}\right) = \ddot{a}_{n+h|i} - \ddot{a}_{h|i},$$

$$h_1 a_{n|i} = \left(v + v^2 + \dots + v^{h+n}\right) - \left(v + v^2 + \dots + v^h\right) = a_{n+h|i} - a_{h|i}$$

1-Misol. Postnumerando rentaning yillik parametrlari $R = 4$ mln. so`m, $n = 5$, $i = 18{,}5\%$ lardan iborat bo`lib, renta baholash momentidan 1,5 yil o`tgandan keyin to`lanadigan bo`lsa, qoldirilgan rentaning joriy qiymati topilsin.

Yechish.

$$S(0) = Ra_{n|i} = 4 \cdot \frac{1 - 1{,}85^{-5}}{0{,}185} = 4 \cdot 3{,}092 = 12{,}368 \text{ mln. so`m.}$$

Endi qoldirilgan rentaning joriy qiymatini topamiz: $12{,}368 \cdot 1{,}85^{-1{,}5} = 9{,}588$ mln. so`m.

Aytaylik, chegaralangan yillik postnumerando renta vaqt bo`yicha ikkita ishtirokchi orasida bo`linadigan bo`lsin. Renta *R* va *n* parametrlarga ega. Bo`linish sharti: a) Har bir ishtirokchi renta jamg`arilgan qiymatining 50% ini oladi; b) Renta ketma – ket, avval birinchi ishtirokchiga, so`ngra ikkinchi ishtirokchiga to`lanadi.

2.Masala. Birinchi ishtirokchining renta olish muddatini hisoblashga keltiriladi, uni n_1 bilan belgilaymiz. qolgan muddatda pulni ikkinchi ishtirokchi oladi. Shunday qilib, birinchi ishtirokchi dastlabki n_1 muddat davomida, ikkinchisi esa $n_2 = n - n_1$ muddatdagi qoldirilgan rentani oladi. Yuqoridagi shartlarga asosan rentani bo`lishdan quyidagi kelib chiqadi: $S_1(0) = n_1 S_2(0)$, $Ra_{n_1|i} = Ra_{n_2|i} \cdot v^{n_1}$.

$n_2 = n - n_1$ hisobga olsak,

$$\frac{1 - (1+i)^n}{i} = \frac{1 - (1+i)^{-(n-n_1)}}{i} \cdot v^{n_1}.$$

Soddalashtirishlardan so`ng

$$n_1 = \frac{-\ln\left\{\left[1 + (1+i)^{-n}\right]/2\right\}}{\ln(1+i)} \text{ hosil bo`ladi.}$$

2-Misol. Postnumerando yillik rentaning muddati 10 yil, $i = 20\%$. Renta yuqorida keltirilgan kabi ikkita ishtirokchi orasida bo`lingan bo`lsin. U holda

$$n_1 = \frac{-\ln\left[\left(1 + 1{,}2^{10}\right)/2\right]}{\ln 1{,}2} = 2{,}961 \approx 3 \text{ yil.}$$

Ikkinchi ishtirokchi qolgan 7 yil davomida oladi.

Masalalar.

1. Nafaqa fondi o`zining a`zolariga har oyda 200 ming so`m to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yish kerak bo`ladi. qulaylik uchun nafaqa

fondi o`zgarmas $i = 2\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

2. Korxona 3 yil mobaynida 150 mln. so`mlik maxsus fondni tashkil etish maqsadida har yili bankka 15% yillik protsent stavka bo`yicha to`lovlar amalga oshiriladi. 3 yil mobaynida jamg`arilgan puldan to`lanadigan badal miqdori topilsin.

3. Firma 150 mln. so`m miqdoridagi fondni tashkil etish maqsadida har yili bankka 43,196 mln. so`m pulni 15% stavka bo`yicha to`lanadi. Fondni tashkil etish uchun qancha muddat zarur bo`lishi topilsin.

4. Renta har kvartalda, protsentlar ham har kvartalda bir kvartal uchun $i = 20\%$ stavka bo`yicha to`lanadigan bo`lsin. Postnumerando renta uchun $n = 4$ bo`lganda diskontirlash koeffisienti va jamg`arma koeffisienti topilsin.

5. $R = 2$ mln. so`mdan yiliga abadiy renta to`lanadigan bo`lsin. Yillik stavka 10% bo`lganda bu rentaning postnumerando va prenumerando qiymatlari topilsin.

6. Nafaqa fondi o`zining a`zolariga har oyda 1 mln. so`m to`lab turishi uchun nafaqa fondiga qanday miqdordagi pulni qo`yishi lozim bo`ladi. qulaylik uchun nafaqa fondi o`zgarmas $i = 0{,}1\%$ stavka bo`yicha o`z pulini har oyda investisiya qilishi mumkin deb hisoblaymiz.

7. Shartnoma shartiga asosan to`lovlar yil oxirida amalga oshiriladi, birinchi to`lov 2 mln. so`m bo`lib, uning qiymati har yili 0,2 mln. so`mga ko`payadi. To`lov muddati 4 yil, protsent stavkasi esa 8%. Muddat oxirida jamg`arilgan pul miqdori topilsin.

8. Firma 5 yil davomida har doim o`sib boruvchi yiliga 5 mln. so`m absolyut o`suvchi to`lovlar bo`yicha 150 mln. so`m kredit olgan. To`lovlar va ustama protsentlar yil oxirida amalga oshirilib, yillik protsent stavka 9%ga teng. Birinchi to`lov qiymati va umumiy to`lovlar summasi topilsin.

Vaqt bo`yicha hadlari bir xil absolyut o`zgaradigan rentalar

Rentaning birinchi hadi *R* va ayirmasi *a* bo`lgan arifmetik progressiya bo`yicha o`zgaradigan bo`lsin, ya`ni

$$R, R+a, R+2a, \dots, R+(n-1)a$$

Rentaning *t*-hadi $R+(t-1)a$ ga teng. Bu rentaning yig`indisi (joriy qiymati) yillik postnumerando renta uchun

$$S(0) = \left(R + \frac{a}{i}\right)a_{n|i} - \frac{nav^n}{i}$$

formulani $(1+i)^n$ ga ko`paytirib, jamg`arilgan mablag` qiymati formulasini yozamiz:

$$S(n) = \left(R + \frac{a}{i}\right)S_{n|i} - \frac{na}{i}$$

. O`z navbatida prenumerando renta uchun

$$S(0) = \left[\left(R + \frac{a}{i}\right)\ddot{a}_{n|i} - \frac{nav^n}{i}\right](1+i) = \left(R + \frac{a}{i}\right)\ddot{a}_{n|i} - \frac{nav^{n-1}}{i}$$

1-Misol. Postnumerando to`lovlari vaqt bo`yicha bir xil oqimni tashkil etib uning birinchi hadi 15 mln. so`m. Har bir keyingi to`lovlar har doim 2 mln. so`mga oshadi. Yillik protsent stavkasi 20% ni tashkil etib, to`lovlar muddati 10 yil. Shu ma`lumotlar asosida joriy va jamg`arilgan pul mablag`lari topilsin.

Yechish. Masalaning shartiga ko`ra $R=15$, $a=2$, $i=20\%$, $n=10$. Jadvaldan $a_{10|20}=4{,}192472$, $v^{10}=0{,}161505$ larni topamiz.

(5.1.1) formulani tadbiq etib,

$$S(0)=\left(15+\frac{2}{0{,}2}\right)\cdot 4{,}192472-\frac{10\cdot 2\cdot 0{,}161505}{0{,}2}=88{,}661 \text{ mln. so`m.}$$

ni topamiz. (5.1.3) va (5.1.4) formulalarni tadbiq etib ham shu natijalarga ega bo`lish mumkin.

$$S(0)=15a_{10|20}+\frac{a_{10|20}-10\cdot 1{,}2^{10}}{0{,}2}\cdot 2=62{,}887+25{,}774=88{,}661 \text{ mln. so`m,}$$

$$S(n)=15S_{10|20}+\frac{S_{10|20}-10}{0{,}2}\cdot 2=389{,}380+159{,}585=548{,}965 \text{ mln. so`m.}$$

Ba`zan o`zgaruvchi rentalarni tahlil qilishda teskari masalani yechishga to`g`ri keladi, ya`ni rentaning birinchi hadi $R$ yoki uning $a$ o`sishini boshqa parametrlar bo`yicha aniqlashga to`g`ri keladi.

Yillik postnumerando renta uchun yuqoridagilardan R ni topamiz.

$$R=\frac{S(0)+\frac{nav^n}{i}}{a_{n|i}}-\frac{a}{i},$$

MARUZA 7

Investision loyiha tahlili.

Investision loyihalarda ushbu loyiha bo‘yicha harajatlar va daromadlar turli davrlarda amalga oshirilishi mumkin. Shu tufayli bu miqdorlarni o‘zaro solishtirish ma’lum qiyinchilik tug‘diradi. Ushbu qiyinchilikni bartaraf etish uchun, bu miqdorlarning barchasi ma’lum bir davrga akslantirilib, shu davrda moliyaviy oqimlarning teng kuchlilik qoidasi asosida o‘zaro solishtiriladi. Investision loyihalar tahlilida moliyaviy oqimlar asosan davr boshiga akslantiriladi va shu davrda o‘zaro solishtiriladi.

Loyiha muddati N davrda $\left[0;\, N\right]$ bo‘lsa,

$\left\{x_t\right\}_{t=0}^{N}$ investision loyihalar xarajatlar vektori

$\left\{y_t\right\}_{t=0}^{N}$ investision loyihalar daromadlar vektori bo‘lsin,

Butun davr davomida i - foiz bank foiz stavkasiga amal qilsin. Umumiy xarajat va umumiy daromadlarni o‘zaro solishtirish uchun bu miqdorlarni davr boshiga quyidagicha akslantiramiz.

$$PV\left(\left\{x_t\right\}_{t=0}^{N}\right) = x_0 + \frac{x_1}{1+i} + \frac{x_2}{(1+i)^2} + \ldots + \frac{x_n}{(1+i)^n} \qquad (1)$$

$$PV\left(\left\{y_t\right\}_{t=0}^{N}\right) = y_0 + \frac{y_1}{1+i} + \frac{y_2}{(1+i)^2} + \ldots + \frac{y_n}{(1+i)^n} \qquad (2)$$

$$NPV\left(\left\{x_t\right\},\, \left\{y_t\right\}\right) = PV\left(\left\{y_t\right\}_{t=0}^{N}\right) - PV\left(\left\{x_t\right\}_{t=0}^{N}\right) \qquad (3)$$

Ta'rif (1): $\{x_t\},\{y_t\}$ investision loyiha samaradorligi deyiladi., agar

$NPV\left(\{x_t\},\{y_t\}\right)\geq 0$ bo'lsa.

Investision loyiha tahlili bir necha bosqichdan iborat bo'lib, birinchi bosqichda "Investision loyiha samaradorligi" tekshiriladi. Agar investision loyiha samarador bo'lsa, ikkinchi bosqichga o'tiladi. Ikkinchi bosqichda "Investision loyihaning ichki samaradorlik darajasi" $NPV\left(\{x_t\},\{y_t\},i^*\right)=0 \quad i^*=?\ \ i^*\geq i$. Agarda ushbu samaradorlik darajasi

Uchinchi bosqichda investision loyiha umumiy xarajatlariningqoplanish muddati aniqlanadi.

$PV\left(\{y_t\}_{t=0}^{T}\right)\geq PV\left(\{x_t\}_{t=0}^{N}\right) \quad T-?$

$R_1=300000$

$n_1=24oy$

$i=2\%$

$R_2=?$

$PV(R^{(0)})=?$

$R=360000$

$R_2=480\,oy$

$$PV(R^0)=(1+0.26)\cdot 300\cdot\frac{1-(1+0.26)^{-20}}{0.26}=$$

$$1.26\cdot 300\cdot\frac{1-(1+0.26)^{-20}}{0.26}=1453846\cdot 12=17446153.8$$

Agar ushbu muddat juda katta bo'lsa, bu loyihalar tahlildan o'tmaydi.

Ichki samaradorlik darajasini aniqlash

$\{x_t\},\{y_t\},\ i=\overline{0,N},\ i^*$- ichki samaradorlik darajasi aniqlansin.

$$NPV\left(\{x_t\},\{y_t\},i^*\right)=0$$

$$NPV\left(\{x_t\},\{y_t\},i^*\right)=(y_0-x_0)+\frac{y_1-x_1}{1+i^*}+\frac{y_2-x_2}{(1+i^*)^2}+\ldots+\frac{y_n-x_n}{(1+i^*)^n}=0 \quad (4)$$

$$(1+i^*)=z\Rightarrow(4)\cdot z^n\Rightarrow f(z)=(y_0-x_0)\cdot z^n+(y_1-x_1)\cdot z^{n-1}+\ldots+(y_n-x_n)=0$$

$f(z)$ uzluksiz funksiya bo'lgani uchun $z(-\infty;\infty)$

Shunday interval tanlab olindiki. $z \in [a,b]$ bu interval uchun $f(a) \cdot f(b) < 0$ dan $f(z)$ uzluksizidan Roll teoremasiga asosan, z^* $f(z^*) = 0$ bo‘ladi.

Ushbu z^* nuqtani aniqlash uchun oraliqlarni teng ikkiga bo‘lish usulidan foydalanamiz. Bu usulning g‘oyasi quyidagicha:

Chetki nuqtalarda funksiya qismatlari qarama-qarshi bo‘lgan oraliq uchun shu oraliqdan o‘rta nuqtasi aniqlanadi va ushbu nuqtada funksiya qiymati hisoblanib, kelgusi jarayon uchun aniqlangan nuqta va kesmaning qolgan qismi shunday aniqlanadiki, bu oraliqda funksiyaning chetki nqtadagi qiymatlari qarama-qarshi bo‘lsin. Shu usul bilan aniqlangan oraliq yana ikkiga bo‘linadi.

$z \in [a,b] \quad f(a) \cdot f(b) < 0$

$$z^{(1)} = \frac{a+b}{2}; \; f\left((z)^{(1)}\right)$$

$$f\left((z)^{(1)}\right) \cdot f(a) < 0 \Rightarrow (z)^{(2)} \in \left[a; (z)^{(1)}\right] \Rightarrow (z)^{(2)} = \frac{a+(z)^{(1)}}{2}$$

$$f\left((z)^{(1)}\right) \cdot f(b) < 0 \Rightarrow (z)^{(2)} \in \left[(z)^{(2)}; b\right] \Rightarrow (z)^{(2)} = \frac{(z)^{(1)}+b}{2}$$

Ushbu jarayon to‘xtatiladi, oldindan berilgan $\varepsilon > 0$ aniqlangani uchun $\left|z^{(k)} - z^{(k-1)}\right| < \varepsilon$

va $z^* = \dfrac{z^{(k)} + z^{(k-1)}}{2}$ agar, $f(z^k) \cdot f(z^{k-1}) < 0$

$z^* = 1 + i^* \Rightarrow i^* = z^* - 1$

$N = 3\,yil$

$\{x_t\} = (10, 6, 4, 0)$

$\{y_t\} = (0, 6, 8, 20)$

$i = 20\%$

Birinchi bosqich.

$$NPV\left(\{x_t\}, \{y_t\}, i\right) = (y_0 - x_0) + \frac{y_1 - x_1}{1+i} + \frac{y_2 - x_2}{(1+i)^2} + \frac{y_3 - x_3}{(1+i)^3} = (0-10) + \frac{6-6}{1+0.2} + \frac{8-4}{(1+(0.2)^2} +$$

$$+ \frac{20-0}{(1+(0.2)^3} = -10 + 0 + 2.8 + 11.4 = 4.3 > 0$$

Ikkinchi bosqich.

$$NPV\left(\{x_t\},\{y_t\},i^*\right)=(0-10)+\frac{6-6}{1+i^*}+\frac{8-4}{(1+i^*)^2}+\frac{20-0}{(1+i^*)^3}=0$$

$$f(z)=(0-10)\cdot z^3+(6-6)\cdot z^2+(8-4)\cdot z+\frac{20}{10}\Rightarrow$$

$$f(z)=-z^3+0.4z+2=0$$

$$f(0)=2$$

$$f(1.4)=-0.15<0$$

$$z^{(1)}\in(0;1.4)\Rightarrow z^{(1)}=\frac{0+1.4}{2}=0.7$$

$$f(z^1)=f(0.7)=-0.7^3+0.4\cdot 0.7+2>0$$

$$z^{(2)}\in\left[0.7;1.4\right]\Rightarrow z^{(2)}=\frac{0.7+1.4}{2}\approx 1.1$$

$$f(z^2)=f(1.1)=-1.1^3+0.4\cdot 1.1+2>0\Rightarrow z^{(3)}\in\left[z^{(2)};1.4\right]\Rightarrow$$

$$\Rightarrow z^{(3)}=\frac{-1.1+1.4}{2}=1.3$$

$$f(z^3)=f(1.3)=-1.3^3+0.4\cdot 1.3+2>0\Rightarrow z^4\in\left[1.3;1.4\right]\Rightarrow$$

$$\Rightarrow z^4=\frac{1.3+1.4}{2}=13.5$$

$$f(z^4)=f(1.35)$$

MARUZA 8.

Lizing amaliyoti

1. Moliyaviy lizing.

2. Joriy yoki operativ lizing.

Moliyaviy lizingda quyidagi shartlar bajarilishi shart:

a) Moliyaviy lizing lizing beruvchidan barcha xarajatlari to‘liq qoplanilishi shartdir.

b) Moliyaviy lizing muddatidan oldin to‘xtatilishi mumkin emas.

Lizing qoidalari turli mamlakatlarda turli bo‘lganligi bilan quyidagi qoidalar ko‘p mamlakatlarda qo‘llaniladi.

1. Lizing shartnomasida lizing oluvchi, albatta, ma’lum miqdorda o‘zining moliyaviy mablag‘i bilan ishtirok etishi shart va bu ishtirok kamida 20% mablag‘i bo‘lishi kerak. Mablag‘ $\geq$ 20%

2. Lizingga olinayotgan jihozning ishga yaroqlilik muddati lizing muddatidan qisqa bo‘lishi mumkin emas.

Ba’zi hollarda “qaytarma lizing” amaliyoti qo‘llaniladi, bu holda lizingga olinayotgan mahsulot yangi xaridorga to‘liq sotiladi va sotib olingan xaridor bilan lizing shartnomasi tuziladi.

Birinchi hol. Muntazam o‘zgarmas lizing to‘lovlar

K - lizing olinayotgan jihoz qiymati

n - muddat (oy, yil)

i - ushbu davr foiz stavkasi

R - muntazam to‘lov muddati

$K = PV\left(\{R\}\right)$ (1)

$$PV(R^0) = R_1^0 \cdot \frac{1-(1+i)^{-n}}{1-(1+i)^{-1}} = R^0(1+i)\frac{1-(1+i)^{-n}}{i}$$

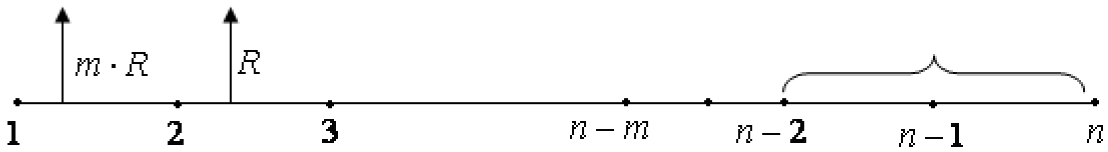
$$K = R^0(1+i)\frac{1-(1+i)^{-n}}{i} \qquad (2)$$

$$R^0 = \frac{K \cdot i}{(1+i)(1-(1+i)^{-n})} \qquad (3)$$

$K = R \cdot a_{n,i}$

$a_{n,i}$ - keltiruvchi koeffisient.

Ikkinchi hol. Birinchi to‘lov miqdori m karralisiga teng bo‘lgan hol.



$$K = (m-1)R \cdot \left(\frac{1}{1+i}\right) + a_{n-m+1,i} \cdot R \qquad (4)$$

$n-(m-1) = n-m+1$

$$K = (m-1)R \cdot \nu + a_{n-m+1,i} \cdot R \qquad (5)$$

$$R = \frac{K}{(m-1)\cdot \nu + a_{n-m+1,i}} \qquad (6)$$

$\nu = (1+i)^{-1}$

Uchinchi hol. Oldindan to‘lov (avans) — Lizing shartnomasi bo‘yicha A- oldindan avans to‘lovi miqdori bo‘lsa,

$$K = A + R_A \cdot a_{n,i} \qquad (7)$$

$$R_A = \frac{K-A}{a_{n,i}} \qquad (8)$$

Lizing to‘lovi kamayadi.

To‘rtinchi hol. Lizing mahsuloti qoldiq qiymati bo‘yicha sotib olinsin.

$$K = S \cdot K \cdot \nu^n + R_S \cdot a_{n,i} \qquad (9)$$

$$R_S = \frac{K - S \cdot K \cdot \nu^n}{a_{n/i}} \qquad (10)$$

Misol (1): Muntazam teng to‘lov

$$\left.\begin{array}{l} K = 1000 \\ n = 36\,oy \\ i = 2\% \\ R = ? \end{array}\right\}$$

$$K = R \cdot V \cdot \left(\{R\}\right) = R \cdot a_{n,i}$$

$$a_{36,2} = \frac{(1-(1+0.02))^{-36}}{0.02}$$

$$a_1 = \frac{0.02}{(1-(1+0.02))^{-36}}$$

$$R = \frac{K}{a_{36,2}} = 1000 \cdot a_1 = 1000 \cdot 0.03923 = 39.23$$

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