



IV bob KO'PHADLAR

1- §. Birhadlar va ko'phadlar

1. Algebraik ifoda. Natural ko'rsatkichli daraja. Birhad. Algebra qo'llaniladigan harfiy belgilashlar bir xil turdagi ko'plab masalalarni formulalar ko'rinishida berilgan umumiy qoida asosida yechishga imkoniyat yaratadi. Agar sonli ifodadagi ayrim yoki barcha sonlar harflar bilan almashtirilsa, *harfiy ifoda* hosil bo'ladi. Biz harfiy ifodalashdan matematika, fizika va boshqa fanlarni o'rganishda keng foydalanamiz.

To'rt matematik amal, butun darajaga ko'tarish va butun ko'rsatkichli ildiz chiqarish ishoralari orqali birlashtirilgan harflar va sonlardan iborat ifodalar *algebraik ifoda* deyiladi. Agar algebraik ifodada sonlar va harflarning ildiz ishoralari qatnashmasa, u *ratsional algebraik ifoda*, ildiz ishoralari qatnashsa, *irratsional algebraik ifoda* deyiladi. Agar ratsional ifodada harfli ifodaga bo'lish amali qatnashmasa, u *butun algebraik ifoda* deyiladi.

Misolalar. 1) $6b - 3a + dc$ — butun algebraik ifoda;

2) $\frac{bc+a}{c}$ — kasr algebraik ifoda;

3) $5 + \sqrt{c}$ — irratsional algebraik ifoda;

4) $(a - b)^2 = (b - a)^2$ — ayniyat.

Irratsional ifoda biror ratsional ifodaga aynan teng bo'lishi ham mumkin. Masalan, $\sqrt{(a^2 + 2)^2} - 2 = a^2$. Algebraik ifodalarni shakl almashtirishlar haqida V bobda alohida to'xtalamiz.

Har biri a ga teng bo'lgan $n(n \geq 2)$ ta ko'paytuvchining ko'paytmasi a sonining n -darajasi deyiladi va a^n deb belgilanadi. Shunday qilib,

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ marta}} \quad (n \geq 2).$$

Ta'rifga asosan $a^1 = a$. Natural ko'rsatkichli darajaning xossalari:

$$1^\circ. a^m \cdot a^n = a^{m+n}; m, n \in N.$$

$$2^\circ. a^m : a^n = a^{m-n}; m, n \in N, m > n.$$

$$3^\circ. (a^m)^n = a^{mn}; m, n \in N.$$

$$4^\circ. (ab)^n = a^n b^n; n \in N.$$

$$5^\circ. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}; a, b \in R, b \neq 0, n \in N.$$

3°- xossani isbotlaymiz (qolgan xossalar ham shu kabi isbotlanadi):

$$\begin{aligned} (a^m)^n &= \underbrace{a^m \cdot a^m \cdot \dots \cdot a^m}_{n \text{ marta}} = \underbrace{a \cdot \dots \cdot a}_{m \text{ marta}} \cdot \underbrace{a \cdot \dots \cdot a}_{m \text{ marta}} \cdot \dots \cdot \underbrace{a \cdot \dots \cdot a}_{m \text{ marta}} \dots = \\ &= \underbrace{a \cdot a \cdot \dots \cdot a}_{mn \text{ marta}} = a^{mn}. \end{aligned}$$

Butun musbat darajali harf, son yoki ulardan tuzilgan ko'paytuvchilar ko'paytmasidan iborat butun algebraik ifoda *birhad* deyiladi. Koeffitsiyentlari bilangina farq qiladigan birhadlar *o'xshash birhadlar* deyiladi. Masalan, $3ab$ va $-4,2ab$ lar o'xshash birhadlardir.

Har qanday birhad turli ko'rinishda yozilishi mumkin. Masalan, $7a^6 \cdot b^5 = 3,5 \cdot 2a^6 \cdot b^5 = 7a^2 \cdot b^3 \cdot a^2 \cdot a^2 \cdot b^2 = \dots$

Lekin $7a^6b^5$ birhadda sonli ko'paytuvchi birinchi o'rinda, harflar alfavit tartibida daraja ko'rsatkichi orqali bir marta yozilgan bo'lib, u *standart (kanonik)* ko'rinishda yozilgandir.

Birhaddagi barcha harflar darajalarining yig'indisi shu birhadning *darajasi* deyiladi.

Son yoki bitta harf ham birhaddir. Masalan, $x; y; \frac{3}{4}; 0; 3; (9)$ — birhadlardir.



M a s h q l a r

4.1. Ifodani x asosli daraja ko‘rinishida yozing:

- | | | |
|--------------------------------|--------------------------|------------------------------------|
| a) $x^3 \cdot x^5$; | f) $(x^2)^3$; | j) $x^3 \cdot x^a (a \in N)$; |
| b) $x^4 \cdot x^5 \cdot x^6$; | g) $(x^3)^2$; | k) $(x^2 \cdot x^3)^a (a \in N)$; |
| d) $-x^3 \cdot x^4$; | h) $(x^2 \cdot x^4)^3$; | l) $x^2 \cdot (x^3)^4$; |
| e) $-x^3 \cdot x^3$; | i) $((x^3)^4)^5$; | m) $(x^4)^2 \cdot (x^2)^4$. |

4.2. Ifodaning qiymatini toping:

- | | |
|--|--|
| a) $\frac{2^5 \cdot 11^8}{22^{10}} \cdot \frac{34^4 \cdot 2^{10}}{17^5 \cdot 8^4}$; | f) $\frac{12^8}{2^3 \cdot 3^4} \cdot \frac{10}{2^6 \cdot 5^7}$; |
| b) $\frac{2^8 \cdot 7^9}{14^{10}} \cdot \frac{26^5}{13^6 \cdot 8^4}$; | g) $\frac{12^5}{2^3 \cdot 3^4} \cdot \frac{10^5}{2^6 \cdot 5^7}$; |
| d) $\frac{14^{10}}{2^8 \cdot 7^9} \cdot \frac{13^6 \cdot 8^4}{26^5}$; | h) $\frac{10^5}{2^6 \cdot 5^7} \cdot \frac{12^5}{2^3 \cdot 3^4}$; |
| e) $\frac{12^5}{2^3 \cdot 4^4}$; | i) $\frac{10^5}{2^7 \cdot 5^6} \cdot \frac{2^4 \cdot 3^3}{12^5}$. |

4.3. Birhadning darajasini aniqlang:

- | | |
|--------------------|--|
| a) $3x^4xy^5$; | h) $13yz^{15}$; |
| b) $-31xy^4$; | i) $43x^2y^3z^{19}$; |
| d) $0,8x^2y^2$; | j) $15x^4y^3z$; |
| e) 15 ; | k) x^4y^2z ; |
| f) $3xy^9z$; | l) $x \cdot x^2 \cdot \dots \cdot x^9$; |
| g) $14x^2y^3z^4$; | m) $xyx^2y^2x^4y^4x^6y^6 \cdot \dots \cdot x^{20}y^{20}$. |

4.4. Birhadni standart shaklga keltiring:

- | | |
|------------------------------|--|
| a) $13xy \cdot 14x^2y^3$; | f) $3xy(-1,5)y^3$; |
| b) $x^2y^2xzy^4$; | g) $\frac{2}{3}ax^2y^2 \cdot 6,5x^3$; |
| d) $3x^2z^2y^2 \cdot xz^5$; | h) $a \cdot xy^2z \cdot y^4 \cdot x^5$; |
| e) $11x^2y \cdot 13x^3y^4$; | i) $a(x^2)^3yz^2x^3$. |

4.5. A^n ni toping:

- a) $A = 3x^2yz$, $n = 3$; f) $A = 2x^2yz^2$, $n = 4$;
b) $A = 13xy^2$, $n = 2$; g) $A = 3xz^4$, $n = 5$;
d) $A = x^2y^4z$, $n = 14$; h) $A = 4y^2z^3$, $n = 4$;
e) $A = 41xyz^2$, $n = 3$; i) $A = 14xy^3z^3$, $n = 2$.

4.6. Birhadning koeffitsiyentini aniqlang:

- a) $1,5xy^2\left(\frac{2}{3}\right)x^2$; f) $1,(51)x^2yz^2 \cdot \frac{3}{4}xy$;
b) $\frac{4}{7}xz \cdot \frac{13}{8}x^2y$; g) $1\frac{3}{7}xy^2 \cdot \frac{4}{10}z^2$;
d) $\frac{14}{15}x \cdot \frac{15}{28}y \cdot 2y^3$; h) $\frac{11}{13}x^2y^3z$;
e) $0,(3)xy \cdot \frac{1}{9}z$; i) $\frac{13}{14}xy \cdot \frac{17}{13}z^2$.

4.7. Ifodani soddalashtiring:

- a) $(13a + 15b) - (14a - 7b)$;
b) $(11x^3 - 12x^2) + (x^3 - x^2 + x^4)$;
d) $(3a^2x - 11x^2) - (3a^2x + 6x^2)$;
e) $(4x^2y + 8xy) - (3x^2y - 5xy)$;
f) $(23x - 11y + 10a) - (-15x + 10y - 15a)$;
g) $(7a^2 - 5ax - x^2) + (-2a^2 + ax - 2x^2)$;
h) $(13x^2 - 8xy - y^2) + (-11x^2 - 9xy)$;
i) $(11xy + 13y^2) - (9xy + x^2)$.

4.8. Amallarni bajaring:

- a) $a(a^2 + x) - x(a - x)$;
b) $13(x^2 + y) + 5(x^2 - y)$;
d) $2(a - 3x) + 3(a - 2x)$;
e) $13(2a - 3x) + 11(a + x)$;
f) $-3(a^2 - x^2) - 2(a^2 + x^2)$;
g) $-(3a - 2x) + 5(a - 2x)$;
h) $17(x^2 - y^2) - 15(y^2 - x^2)$;
i) $19(x^3y - xz^2) + 17(-x^3y + xz^2)$.

4.9. Ifodani soddalashtiring va o'zgaruvchining ko'rsatilgan qiymatida ifoda qiymatini toping:

- a) $(a - 4)(a - 2) - (a - 1)(a - 3)$; $a = 1,75$;
- b) $(2a - 5)(a + 1) - (a + 2)(a - 3)$; $a = -2,6$;
- d) $(a - 5)(a - 1) + (a - 2)(a - 3)$; $a = 1,3$;
- e) $(x + 1)(x + 2) + (x + 3)(x + 4)$; $x = -0,4$.

2. Ko'phadlar. Birhadlar yig'indisi *ko'phad* deyiladi.

Masalan, $3a^2b + 7b^2c, 9x^2y + xy^2$ ifodalarning har biri ko'phaddir.

Ko'phad tarkibidagi eng katta darajali birhadning darajasi shu *ko'phadning darajasi* deyiladi. Masalan, $P(x) = c + ax^2 + bx$, $R(x, y) = 3xy + z$ ikkinchi darajali ko'phaddir.

$P(x) = c + ax^2 + bx$ va $P(x) = ax^2 + bx + c$ ko'phadlarni qaraylik, ular bitta ko'phadning ikki ko'rinishli yozuvi. Ulardan ikkinchisi x o'zgaruvchi daraja ko'rsatkichlarining kamayib borishi tartibida, ya'ni *standart* ko'rinishdagi yozuvdir. Ko'p argumentli ko'phadlar ham standart ko'rinishda yozilishi mumkin. x, y, \dots, z — o'zgaruvchilar, a, b lar noldan farqli sonlar bo'lsin. $ax^{k_1}y^{k_2} \dots z^{k_n}$ va $bx^{m_1}y^{m_2} \dots z^{m_n}$ birhadlarni solishtiraylik. $k_1 = m_1, k_2 = m_2, \dots, k_i = m_i$, lekin $k_{i+1} > m_{i+1}$ bo'lsa, birinchi birhad ikkinchisidan katta, chunki ulardagi x va y lar daraja ko'rsatkichlari bir xil bo'lsa-da, z ning ko'rsatkichi birinchi birhadga katta.

Agar ko'p o'zgaruvchili ko'phadda har qaysi qo'shiluvchi o'zidan o'ngda turgan barcha qo'shiluvchilardan katta bo'lsa, qo'shiluvchilar *lug'aviy (leksikografik)* tartibda joylashtirilgan deyiladi. Masalan, $P(x, y, z) = 8x^5y^6z^2 - 5x^4y^8z + 16x^4y^5z^4$ ko'phadning qo'shiluvchilari lug'aviy tartibda joylashtirilgan.

Agar ko'phadning barcha hadlarida x, y, \dots, z o'zgaruvchilarning ko'rsatkichlari yig'indisi m ga teng bo'lsa, uni *m-darajali bir jinsli ko'phad* deyiladi. Masalan, $8x - 5y + z$ — birinchi darajali bir jinsli (bunda $m = 1$), $x^3 + y^3 + z^3 - 7xy^2 - 5xyz$ — uchinchi darajali ($m = 3$) bir jinsli ko'phad.

Agar $ax^{k_1} \dots z^{k_n}$ birhad $m = k_1 + \dots + k_n$ darajali bo'lsa, ixtiyoriy umumiy λ ko'paytuvchi uchun $a(\lambda x)$ ga ega bo'lamiz.

Agar ixtiyoriy λ soni uchun $f(\lambda x, \dots, \lambda z) = \lambda^m f(x, \dots, z)$ tenglik bajarilsa, $f(x, \dots, z)$ ko'phad (funksiya) m - darajali bir jinsli ko'phad (funksiya) bo'ladi. Masalan, $f(x, y) = y^3 + x^2 \sqrt{xy + \frac{x^3}{y}}$ funksiya 3- darajali bir jinsli funksiyadir, chunki

$$f(2x, 2y) = 8y^3 + 4x^2 \cdot \sqrt{4 \left(xy + \frac{x^3}{y} \right)} = 2^3 f(x; y).$$

Shu kabi, $f(x, y) = x^3 + 2x^2y - y^3 + x^2 \sqrt{xy + \frac{x^3}{y}}$ – uchinchi darajali ($m = 3$), $f(x, y, z) = \frac{y+z}{3x+y}$ nolinch darajali ($m = 0$), $f(x, y, z) = z \cdot \frac{y+z}{3x+y}$ birinchi darajali ($m = 1$) bir jinsli funksiyalardir. Agar $x^3y + xy^3$ ko'phadda x o'rniga y , y o'rniga x yozilsa (ya'ni x va y lar o'rin almashtirilsa), oldingi ko'phadning o'zi hosil bo'ladi.

Agar $P(x, y, \dots, z)$ ko'phad tarkibidagi harflarning har qanday o'rin almashtirilishida unga aynan teng ko'phad hosil bo'lsa, P ko'phad *simmetrik ko'phad* deyiladi. Simmetrik ko'phadda qo'shiluvchilar o'rin almashtirilganda yig'indi, ko'paytuvchilar o'rin almashtirilganda ko'paytma o'zgarmaydi.

Agar $(\lambda + x)(\lambda + y) \dots (\lambda + z)$ ifodadagi qavslar ochilsa, λ darajalarining koeffitsiyentlari sifatida x, y, \dots, z o'zgaruvchilarning simmetrik ko'phadlari turgan bo'ladi. Ular *asosiy simmetrik ko'phadlar* deyiladi. Masalan, o'zgaruvchilar soni $n = 2$ bo'lsa, $(\lambda + x)(\lambda + y) = \lambda^2 + (x + y)\lambda + xy$ bo'lib, asosiy simmetrik ko'phadlar $x + y$ va xy bo'ladi. Ularni $\sigma_1 = x + y$, $\sigma_2 = xy$ orqali ifodalaymiz. Shu kabi, $n = 3$ da $\sigma_1 = x + y + z$, $\sigma_2 = xy + xz + yz$, $\sigma_3 = xyz$ bo'ladi.

Bulardan tashqari, quyidagi ko'rinishdagi $\sigma_1 = x + y + \dots + z$ (n ta qo'shiluvchi), $\sigma_2 = x^2 + y^2 + \dots + z^2$, ..., $\sigma_k = x^k + y^k + \dots + z^k$ darajali yig'indilar ham simmetrik ko'phadlardir.

1-teorema. *Ixtiyoriy $s_k = x^k + y^k$ darajali yig'indi $\sigma_1 = x + y$ va $\sigma_2 = xy$ larning ko'phadi ko'rinishida tasvirlanishi mumkin.*

Isbot. Haqiqatan, $k = 1$ da $s_1 = x + y = \sigma_1$, $k = 2$ da $s_2 = x^2 + y^2 = (x + y)^2 - 2xy = \sigma_1^2 - 2\sigma_2$. Teorema s_{n-1} va s_n (bunda $1 \leq n \leq k$, $k \leq 2$) uchun to'g'ri bo'lsin. Uning s_{n+1} uchun to'g'riligini isbotlaymiz:

$$\begin{aligned} s_{n+1} &= x^{n+1} + y^{n+1} = (x^n + y^n)(x + y) - x^n y - xy^n = \\ &= (x^n + y^n)(x + y) - (x^{n-1} + y^{n-1})xy = s_n \sigma_1 - s_{n-1} \sigma_2. \end{aligned}$$

Faraz bo'yicha s_n va s_{n-1} lar uchun teorema to'g'ri edi. Demak, teorema s_{n+1} uchun ham to'g'ri.

2- teorema. x, \dots, z o'zgaruvchilari har qanday simmetrik P ko'phad yagona ravishda shu o'zgaruvchilardan tuzilgan asosiy simmetrik ko'phadlardan iborat bo'ladi.

Isbot. $n = 2$ bo'lgan holni qaraymiz. $P(x, y)$ simmetrik ko'phad $ax^m y^k$ qo'shiluvchiga ega bo'lsin. Agar $m = k$ bo'lsa, bu qo'shiluvchi $a(xy)^k$ ga, ya'ni $a\sigma^k$ ga teng, $k > m$ bo'lsa, $P(x, y)$ ning tarkibida $ax^m y^k$ bilan bir qatorda x va y larni o'rin almashtirishdan hosil bo'luvchi $ax^m y^k$ qo'shiluvchi ham bo'ladi: $ax^k y^m + ax^m y^k = a(xy)^m (x^{k-m} + y^{k-m}) = a\sigma_2^m s_{k-m}$. Lekin 1- teoremaga muvofiq ixtiyoriy s_{k-m} darajali yig'indi, demak, P simmetrik ko'phad ham har doim σ_1, σ_2 orqali ifodalanadi.

1- misol. $P(x, y) = x^3 + y^3 + 2x^2 y + 2xy^2$ simmetrik ko'phadni σ_1 va σ_2 lar orqali ifodalaymiz.

Yechish. $P(x, y) = (x + y)(x^2 - xy + y^2) + 2xy(x + y) = (x + y)(x^2 - xy + y^2 + 2xy) = (x + y)((x + y)^2 - xy) = \sigma_1(\sigma_1^2 - \sigma_2)$.

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ ($a_n \neq 0$) ko'rinishdagi butun ratsional ifoda bir o'zgaruvchili n - darajali ko'phad deyiladi. Har qanday son 0- darajali ko'phaddan iborat. 0 soni esa darajaga ega bo'lmagan ko'phad. $a_n x^n$ qo'shiluvchi ko'phadning bosh hadi, a_0 esa uning ozod hadi deyiladi.

3- teorema. O'zgaruvchi x bo'yicha tuzilgan har qanday butun ratsional ifoda

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (1)$$

ko‘rinishdagi ifodaga aynan tengdir, bunda a_n, \dots, a_0 – haqiqiy sonlar, $a_n \neq 0$.

Isbot. Teorema sonlar va x ifoda uchun har doim o‘rinli. U $A(x)$ va $B(x)$ ifodalar uchun o‘rinli, deylik: $A(x) = a_m x^m + \dots + a_0$ ($m > n$), $B(x) = b_n x^n + \dots + b_0$. U holda $A(x) + B(x) = (a_m x^m + \dots + a_0) + (b_n x^n + \dots + b_0) = (a_m x^m + \dots + a_0) + (0 \cdot x^m + \dots + 0 \cdot x^{n+1} + b_n x^n + \dots + b_0) = (a_m + 0)x^m + \dots + (a_0 + b_0)$ yig‘indi (1) ko‘rinishda bo‘ladi. Shu kabi,

$$A(x)B(x) = (a_m x^m + \dots + a_0)(b_n x^n + \dots + b_0) = \dots = \sum_{i=0}^{m+n} c_i x^i, \quad (2)$$

$c_i = a_i b_0 + a_{i-1} b_1 + \dots + a_1 b_{i-1} + a_0 b_i$ (agar $i > m$ bo‘lsa, $a_i = 0$ bo‘ladi).

Shunday qilib, teorema barcha sonlar va x ifoda uchun o‘rinli, uning $A(x)$ va $B(x)$ uchun o‘rinli bo‘lganidan $A(x) + B(x)$ va $A(x) \cdot B(x)$ uchun o‘rinli bo‘lishi kelib chiqadi. Demak, teorema barcha ratsional ifodalar uchun o‘rinli.

(2) tenglikka qaraganda, ikki ko‘phad ko‘paytmasining bosh hadi ko‘payuvchilar bosh hadlarining ko‘paytmasiga, ozod hadi ozod hadlarining ko‘paytmasiga teng, ko‘paytmaning darajasi ko‘payuvchilar darajalarining yig‘indisiga teng. Bir xil darajali ko‘phadlarni qo‘shganda kichik darajali ko‘phad hosil bo‘lishi mumkin, turli darajali ko‘phadlarni qo‘shganda esa darajasi katta darajali qo‘shiluvchining darajasi bilan bir xil bo‘lgan ko‘phad hosil bo‘ladi. Masalan, $(4x^2 - x + 3) + (-4x^2 - 2x + 1) = -3x + 4$, $(4x^2 - x + 3) + (-2x + 1) = 4x^2 - 3x + 4$.

Ikki ko‘phadning aynan teng bo‘lish shartini ifodalovchi teoremani isbotsiz keltiramiz.

3-teorema. Agar $P(x)$ ko‘phadning hech bo‘lmaganda bitta koeffitsiyenti noldan farqli bo‘lsa, shunday $x_0 \in \mathbb{R}$ soni topiladiki, unda ko‘phad nolga aylanmaydi, ya‘ni $P(x) \neq 0$ bo‘ladi.

1-xulosa. Agar x ning har qanday qiymatida $P(x)$ ko‘phad nolga teng bo‘lsa, u holda uning barcha koeffitsiyentlari nolga teng bo‘ladi.

I s b o t. Barcha $x \in R$ uchun $P(x) = 0$ bo'lsin. Agar $P(x)$ ning biror koeffitsiyenti nolga teng bo'lmasa, 3- teoremaga muvofiq shunday $x = b$ soni topiladiki, unda $P(b) \neq 0$ bo'ladi. Bu esa $\forall x \in R$ uchun $P(x) = 0$ bo'lishlik shartiga zid. Demak, barcha koeffitsiyentlar nolga teng.

2- x u l o s a. *Aynan teng $P(x)$ va $Q(x)$ ko'phadlarda x ning bir xil darajalari oldidagi koeffitsiyentlari teng bo'ladi.*

I s b o t. $P(x) \equiv Q(x)$ bo'lgani uchun $P(x) - Q(x) \equiv 0$ bo'ladi. 1- xulosaga ko'ra, bu ayirmaning barcha koeffitsiyentlari nolga teng. Bundan, $P(x)$ va $Q(x)$ ko'phadlarning mos koeffitsiyentlari teng bo'lishi kelib chiqadi.

1- m i s o l. Agar $P(x) = (x^2 + 2)^3 - 6(x^2 - 2)^2 - 4x^3 - 36x^2 + 20$ va $Q(x) = (x^3 - 2)^2$ bo'lsa, $P(x) \equiv Q(x)$ bo'lishini isbot qilamiz.

I s b o t. $P(x) = (x^6 + 3 \cdot 2x^4 + 3 \cdot 4x + 8) - 6(x^4 - 4x^2 + 4) - 4x^3 - 36x^2 + 20 = x^6 - 4x^3 + 4$, $Q(x) = x^6 - 4x^3 + 4$. Demak, $P(x) \equiv Q(x)$.

Amalda (masalan, kalkulatorlarda hisoblashlar sonini kamaytirish maqsadida) butun ratsional ifodalarning quyidagi ko'rinishdagi yozuvidan foydalanish qulay:

$$(\dots((a_n x + a_{n-1})x + a_{n-2})x + \dots) + a_0. \quad (3)$$

2- m i s o l. $P(x) = 5x^4 + 4x^3 - 7x^2 - 2x + 4$ ifodaning $x = 3,89$ dagi son qiymatini hisoblash zarur bo'lsin. Shu yozuv bo'yicha jami 14 marta, $P(x) = (((5x + 4)x - 7)x - 2)x + 4$ ko'rinishi bo'yicha esa 9 marta amal bajariladi.

3- m i s o l. $P(x) = (3x - 1)^{99} \cdot (2x - 1)^{100} + x^2$ ko'phad koeffitsiyentlarining yig'indisini va ozod hadini toping.

Y e c h i s h. $P(x)$ ko'phad koeffitsiyentlarining yig'indisi $P(1) = (3 \cdot 1 - 1)^{99} \cdot (2 \cdot 1 - 1)^{100} + 1^2 = 2^{99} + 1$ ga, ozod hadi esa $P(0) = (3 \cdot 0 - 1)^{99} \cdot (2 \cdot 0 - 1)^{100} + 0^2 = -1$ ga teng.



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4.10. Ko'phadni ko'paytuvchilarga ajrating:

a) $7ax + 14ay$; b) $3a^2x + 6a^4x^3$;

- g) $x^3 + y^3 + 2xy(x + y)$;
 h) $x^3 - y^3 - 5x(x^2 + xy + y^2)$;
 i) $a^4 + ax^2 - a^3x - x^4$;
 j) $(x + y)(x^2 + y^2) - x^3 - y^3$;
 k) $36a^2 - (a^2 + 9)^2$;
 l) $8x^3 - 27y^{18}$;
 m) $(x - y)(x^3 + y^3)(x^2 + xy + y^2) - (x^6 - y^6)$.

4.14. k ning istalgan natural qiymatida

- a) $(k + 1)^2 - (k - 1)^2$ ning qiymati 4 ga;
 b) $(2k + 3)^2 - (2k - 1)^2$ ning qiymati 8 ga;
 d) $k^3 - k$ ning qiymati 6 ga;
 e) $(3k + 1)^2 - (3k - 1)^2$ ning qiymati 12 ga bo'linishini isbotlang.

4.15. Agar $a + b + c = 0$ bo'lsa, $a^3 + b^3 + c^3 = 3abc$ bo'lishini isbotlang.

4.16. Sonlarni taqqoslang:

- a) $45^2 - 31^2$ va $44^2 - 30^2$; d) $297 \cdot 299$ va 298^2 ;
 b) $26^3 - 24^3$ va $(26 - 24)^3$; e) $(17 + 13)^2$ va $17^3 + 13^3$.

4.17. $ab = 0$ bo'lsa, $|a + b|$ ning qiymati nimaga teng bo'lishi mumkin? ($\sqrt{x^2} = |x|$ dan foydalaning.)

4.18. $|a|^2 + |b|^2 + |c|^2 = 0$ bo'lsa, $(a + b + c)^2$ ning qiymatini toping.

4.19. $(x + y + z)^2 - 2xy - 2xz$ ni soddalashtiring.

4.20. $(x - y - z)^2$ ni ko'phadga aylantiring.

4.21. $f(x) = x^3 - 3x^2 + 2x - 1$ ko'phad berilgan. Quyidagilarni hisoblang:

- a) $f(2)$; f) $f(1 - i)$; j) $f(x - 1)$;
 b) $f(i)$; g) $f(i + 2)$; k) $f(a)$;
 d) $f(i + 1)$; h) $f(-i)$; l) $f(2^n)$;
 e) $f(\sqrt{2})$; i) $f(\sqrt{3} - 1)$; m) $f\left(\frac{1}{\sqrt{3}}\right)$.

4.22. Ko'phad koeffitsiyentlarining yig'indisini toping:

a) $f(x) = (4x - 1)^{1999}(2x - 1)^{2000} + (8x - 1)^2(4x - 1)$;

b) $f(x) = (3x - 2)^{2000}(3x - 1)^{199} + (8x + 1)^2 + 2$;

d) $f(x) = (x - 2)^{200}(2 - x) + (4 - x)^{99}(x - 1)^{20} + 3$;

e) $f(x) = (x - 1)(x - 2)^{20} + (4 - 4x)^{18}(x + 3)^2 + 17$.

4.23. $f(x)$ ko'phad koeffitsiyentlarining yig'indisi m ga teng. a ni toping:

a) $f(x) = x^3 + ax^2 + 3x + 1$; $m = 5$;

b) $f(x) = 7x^3 + 2x^2 + ax + 2$; $m = 4$;

d) $f(x) = 12x^4 + 2x^3 + ax^2 + 1$; $m = 12$;

e) $f(x) = ax^2 + 4x^4 + 8x + 1$; $m = -4$.

4.24. Ko'phadning ozod hadini toping:

a) $f(x) = (3x^2 - 1)^{20}(4x + 1)^{15} - x^{20} + 15$;

b) $f(x) = (3x - 4)^{18}(13x - 1)^{16} + x^{17} - 15$;

d) $f(x) = (2x + 1)^{15}(3x^2 + 2)^4 + (x - 2)^2 + 17$;

e) $f(x) = (3x + 1)^2(3x + 4)^3(x + 1)^{200} + (x - 1)^{20} + 19$.

4.25. $f(x)$, $g(x)$ lar teng ko'phadlar bo'lsa, a , b larni toping:

a) $f(x) = ax^7 + 3x^6 + x^2 + 1$, $g(x) = 3x^6 + bx^2 + 1$;

b) $f(x) = ax^3 + bx^2 + 3x + 2$, $g(x) = x^3 + bx^2 + 3x + 2$;

d) $f(x) = ax^3 + 2x + 3$, $g(x) = 4x^3 + bx + 3$;

e) $f(x) = ax^8 + bx^3 + 9$, $g(x) = ax^{10} + 4x^3 + ax^2 + 9$.

4.26. $x + 5 = a(x - 2)(x - 3) + b(x - 1)(x - 3) + c(x - 1)(x - 2)$ tenglik ayniyat bo'lsa, a , b , c larni toping.

4.27. Ko'phadlar yig'indisini toping:

a) $f(x) = x^{88} + 3x^{77} + 4x^2 + 1$, $g(x) = 4x^{88} + 3x^{65} + 15$;

b) $f(x) = x^4 - 5x^3 + 4x^2 - 1$, $g(x) = -x^4 + 6x^3 + x + 2$;

d) $f(x) = x^6 + 5x^2 + 11x + 4$, $g(x) = 2x^6 + x^4 + 3x^3 + 5$;

e) $f(x) = x^7 + x^6 + 5x^4 + 12$, $g(x) = 7x^3 + 8x^2 - 11$.

4.28. Ko'phadlar yig'indisining darajasini toping:

- a) $f(x) = (x - 1)^7(x - 2)^5 + 3x$, $g(x) = (2x - 4)^{12} + 4x^2$;
- b) $f(x) = (2x + 5)^{15} + 3x^4 + 4$, $g(x) = (2x + 3)^{16} - 4x^3 + x + 1$;
- d) $f(x) = (3x + 5)^{15} + 31x^5 + 2$, $g(x) = -(3x + 11)^{15} + 33x^6 + 4$;
- e) $f(x) = x^7 + x^6 + 3x^2 + x + 3$, $g(x) = -x^7 + 2x^6 + 4x^5 + 2$.

4.29. 4.27- misoldagi ko'phadlar uchun $f(x) - g(x)$ ni toping.

4.30. Ko'phadlarni ko'paytiring:

- a) $f(x) = 5x^4 + 4x^2 + x + 2$, $g(x) = 4x$;
- b) $f(x) = 4x^4 + 3x^3 + 2$, $g(x) = 4x^3 + 7x + 1$;
- d) $f(x) = 11x^4 + 3x^2 + 3x + 5$, $g(x) = 5x^6 + 7x^2 + 4x + 2$;
- e) $f(x) = 13x^3 + 4x^2 + x + 2$, $g(x) = 2x^2 + 5x + 6$.

4.31. Ayniyatlarni isbotlang:

- 1) $(x^2 + y^2 + z^2)(u^2 + v^2 + w^2) = (xu + yv + zw)^2 + (zv - yw)^2 + (xw - zu)^2 + (xv + yu)^2$;
- 2) $(y - z)^5 + (z - x)^5 + (x - y)^5 = 5(x - y)(y - z)(z - x)(x^2 + y^2 + z^2 - xy - yz - xz)$.

4.32. a) x , y , z ning s_2 , s_3 , s_4 darajali yig'indilarini σ_1 va σ_2 asosiy simmetrik ko'phadlar orqali ifodalang;

b) $x^4 + y^4 = \sigma_1^4 - 4\sigma_1^2\sigma_2 + 2\sigma_1^2$ tenglikni isbot qiling.

4.33. a) $x^2 - 4x + 3 = 0$ kvadrat tenglamani yechmay,

1) shunday yangi kvadrat tenglama tuzingki, uning ildizlari berilgan tenglama x_1 , x_2 ildizlari kvadratlaridan iborat bo'lsin;

2) yangi kvadrat tenglama ildizlari $\alpha_1 = x_1 + 2x_2$ va $\alpha_2 = x_2 + 2x_1$ bo'lsin;

b) $x^2 + x - 2 = 0$ tenglamani yechmasdan, uning ildizlarining uchinchi darajali yig'indisini toping.

4.34. 1) $x^3 + 4x^2y + 4xy^2 + y^3$; 2) $x^4 - 5x^4y + 6x^3y^2 + 6x^2y^3 - 5xy^4 + y^5$ simmetrik ko'phadlarni α_1 va α_2 lar orqali ifodalang.

4.35. σ_1 va σ_2 lardan iborat ko'paytuvchilarga ajrating:

a) $x^4 - 12x^3y + 15x^2y^2 - 12xy^3 + y^4$;

$$b) 16x^4 + 13x^3y + 8x^2y^2 + 13xy^3 + 16y^4;$$

d) butun koeffitsiyentli $P(x,y) = Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ex + F$ ko'phad ratsional koeffitsiyentli $ax + by + c$ uchhadning aniq kvadrati bo'lishi uchun A, B, C, D, E, F koeffitsiyentlarga nisbatan qanday shartlar qo'yilishi kerak?

3. Qisqa ko'paytirish formulalarining umumlashmalari. Agar ko'phadni ko'phadga ko'paytirish qoidalaridan foydalanib, zarur soddalashtirishlarni bajarsak, quyidagi formulalar hosil bo'ladi:

$$(x \pm a)^2 = x^2 \pm 2ax + a^2,$$

$$(x \pm a)^3 = x^3 \pm 3x^2a + 3xa^2 \pm a^3,$$

$$(x + a)(x - a) = x^2 - a^2,$$

$$(x + a)(x^2 - ax + a^2) = x^3 + a^3,$$

$$(x - a)(x^2 + ax + a^2) = x^3 - a^3,$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$$

va hokazo.

Endi $x + a$ ikkihadni m natural ko'rsatkichli darajaga ko'tarish qonuniyati bilan tanishamiz. Shu maqsadda $(x + a)$, $(x + a)^2$, $(x + a)^3$, $(x + a)^4$ va hokazo darajalarga ko'tarishlarni bajarib, hosil bo'lgan *yoyilmaning* koeffitsiyentlarini kuzataylik:

$$(x + a)^1 = 1x + 1a,$$

$$(x + a)^2 = 1x^2 + 2ax + 1a^2,$$

$$(x + a)^3 = 1x^3 + 3x^2a + 3xa^2 + 1a^3.$$

Yoyilmalardan bosh koeffitsiyentlar 1 ga tengligini ko'ramiz. Oxirgi ko'phadni $x + a$ ga ko'paytirib,

$$(x + a)^4 = 1x^4 + 4x^3a + 6x^2a^2 + 4a^3x + 1a^4$$

ni hosil qilamiz. Shu kabi,

$$(x + a)^5 = 1x^5 + 5x^4a + 10x^3a^2 + 10x^2a^3 + 5xa^4 + 1a^5$$

va hokazolarni hosil qilamiz.

$(x + a)^n$ uchun quyidagiga ega bo'lamiz:

Masalan:

$$\begin{aligned}(x+y)^6 &= x^6 + 6x^5y + \frac{6 \cdot 5}{1 \cdot 2}x^4y^2 + \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}x^3y^3 + \\ &+ \frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4}x^4y^2 + \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}xy^5 + \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}y^6 = \\ &= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6.\end{aligned}$$

(1) ni matematik induksiya metodidan foydalanib isbotlaymiz.

$n = 1$ da $a + b = a + b$, ya'ni (1) tenglik to'g'ri.

$n = m$ da (1) tenglik to'g'ri, ya'ni $(a + b)^m = a^m + ma^{m-1}b + \dots + b^m$ tenglik o'rinli deb faraz qilamiz.

U holda $n = m + 1$ uchun

$$\begin{aligned}(a+b)^{m+1} &= (a+b)^m \cdot (a+b) = a^m + ma^{m-1}b + \frac{m(m-1)}{1 \cdot 2}a^{m-2} + b^2 + \\ &+ \dots + \frac{m(m-1)(m-2) \cdot \dots \cdot (m-(k-1))}{1 \cdot 2 \cdot 3 \cdot \dots \cdot k}a^{m-k} \cdot b^k + \dots + \\ &+ mab^{m-1} + b^m)(a+b) = a^{m+1} + (m+1)a^mb + \frac{(m+1)m}{1 \cdot 2}a^{m-1}b^2 + \dots + \\ &+ \frac{(m+1)m \cdot (m-1) \cdot \dots \cdot (m-k)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (k+1)}a^mb^{k+1} + \dots + (m+1)ab^m + b^{m+1}\end{aligned}$$

bo'ladi. Demak, (1) formula o'rinli.



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4.36. Ko'phad shaklida yozing:

- | | | |
|--------------------------|----------------------|--------------------|
| a) $(x + y + z)^2$; | e) $(x + y - z)^2$; | h) $(a + b)^7$; |
| b) $(x + y + z)^3$; | f) $(x + y - z)^3$; | i) $(2x + 3y)^8$; |
| d) $(a + b + c + d)^2$; | g) $(x + y)^6$; | j) $(5x - 4y)^6$. |

4.37. Ko'paytuvchilarga ajrating:

- | | |
|----------------------------------|-----------------------------------|
| a) $a^4 - 1$; | b) $a^{12} - 2a^6 + 1$; |
| d) $a^2 - 2a^3b - 2ab^3 + b^2$; | e) $a^3 - 7a^3 - 7a + 15$; |
| f) $a^3 - 5a^2 - a + 5$; | g) $a^4 - 10a^2 + 169$; |
| h) $a^{10} + a^5 + 1$; | i) $(x + 3)^4 + (x + 5)^4 - 16$; |
| j) $a^3 + b^3 + c^3 - 3abc$. | |

4.38. Ayniyatlarni isbot qiling:

- a) $(x^2 - 1)(x^2 + 1)(x^4 + 1) = x^8 - 1$;
- b) $(x^2 + x + 1)(x^2 - x + 1) = x^4 + x^2 + 1$;
- d) $(x^2 - 3x + 1)^2 - 1 = (x - 3)(x - 2)(x - 1)x$;
- e) $x^5 + 1 = (x + 1)[x(x - 1)(x^2 + 1) + 1]$.

4.39. Ifodalarni soddalashtiring:

- 1) $(a^2 + a + 1)(a^2 - a + 1)(a^4 - a^2 + 1)$;
- 2) $(x + y + z)^2 - (x + y - z)^2 - (y + z - x)^2 + (z + x - y)^2$.

4.40. Ayniyatlarni isbot qiling:

- a) $(x^2 - y^2)(a^2 - b^2) = (ax + by)^2 - (ay + bx)^2$;
- b) $x^4 - 8x + 63 = (x^2 + 4x + 9)(x^2 - 4x + 7)$;
- d) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$;
- e) $x^4 + 2x^3 + 4x^2 + 3x - 10 = (x - 1)(x + 2)(x^2 + x + 5)$;
- f) $x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1)$;
- g) $x^6 - 2x^5 + 4x^4 + 2x^3 - 5x^2 = x^2(x - 1)(x + 1)(x^2 - 2x + 5)$.

4. Ko'phadlarni bo'lish. Bir o'zgaruvchili $A(x)$ va $B(x)$ ko'phadlar uchun

$$A(x) = B(x) \cdot Q(x) \quad (1)$$

tenglik o'rinli bo'ladigan $Q(x)$ ko'phad mavjud bo'lsa, $A(x)$ ko'phad $B(x)$ ko'phadga bo'linadi (yoki qoldiqsiz bo'linadi) deyiladi. Bunda $A(x)$ ko'phad bo'linuvchi, $B(x)$ ko'phad bo'luvchi, $Q(x)$ ko'phad esa bo'linma deyiladi.

$x^3 - 1 = (x^2 + x + 1)(x - 1)$ ayniyatdan, $A(x) = x^3 - 1$ ko'phadning $B(x) = x^2 + x + 1$ ko'phadga (qoldiqsiz) bo'linishini va bo'linma $Q(x) = x - 1$ ko'phadga tengligini ko'ramiz.

Butun sonni butun songa (butun) bo'lish amali kabi, ko'phadni ko'phadga qoldiqsiz bo'lish amali hamma vaqt ham bajarilavermaydi. Shu sababli ko'phadni ko'phadga qoldiqsiz bo'lishga nisbatan yanada umumiyroq bo'lgan amal — ko'phadni ko'phadga qoldiqli bo'lish amali kiritiladi.

$A(x)$ ko'phadni $B(x)$ ko'phadga qoldiqli bo'lish deb, uni quyidagicha ko'rinishda tasvirlashga aytiladi:

$$A(x) = B(x) \cdot Q(x) + R(x). \quad (2)$$

(2) tenglikdagi $Q(x)$ va $R(x)$ lar bir o'zgaruvchili ko'phadlar bo'lib, $R(x)$ ko'phadning darajasi $B(x)$ ko'phadning darajasidan kichik yoki $R(x) = 0$.

(2) tenglikdagi $A(x)$ ko'phad *bo'linuvchi*, $B(x)$ ko'phad *bo'luvchi*, $Q(x)$ ko'phad *bo'linma* (yoki to'liqsiz bo'linma), $R(x)$ ko'phad esa *qoldiq* deyiladi.

Agar (2) tenglikda $R(x) = 0$ bo'lsa, (1) tenglik hosil bo'ladi, ya'ni $A(x)$ ko'phad $B(x)$ ko'phadga qoldiqsiz bo'linadi. Shu sababli qoldiqsiz bo'lishni qoldikli bo'lishning xususiy holi sifatida qaraymiz.

Oliy matematika kursida, har qanday $A(x)$ ko'phadning har qanday $B(x)$ ko'phadga (bu yerda $B(x) \neq 0$) qoldikli bo'linishi haqidagi quyidagi teorema isbotlanadi.

Teorema. *$A(x)$ va $B(x)$ ko'phadlar haqiqiy koeffitsiyentli va $B(x) \neq 0$ bo'lsin. U holda shunday $Q(x)$ va $R(x)$ ko'phadlar topiladiki, ular uchun $A(x) = B(x) \cdot Q(x) + R(x)$ tenglik o'rinli bo'ladi va bunda $R(x)$ ning darajasi $B(x)$ nikidan kichik yoki $R(x) = 0$ bo'ladi hamda $Q(x)$, $R(x)$ ko'phadlar bir qiymatli aniqlanadi.*

Bu teorema ko'phadni ko'phadga bo'lishning amaliy usulini bermaydi. Ko'phadni ko'phadga bo'lishning amaliy usullari – «aniqmas koeffitsiyentlar usuli» va «burchakli bo'lish» usulini misollarda qaraymiz.

1- m i s o l. $A(x) = x^3 + x + 1$ ko'phadni $B(x) = x^2 + x + 1$ ko'phadga aniqmas koeffitsiyentlar usuli bilan bo'lamiz.

Y e c h i s h. $A(x)$ ko'phad 3- darajali, $B(x)$ esa 2-darajali ko'phad bo'lgani uchun $Q(x)$ ko'phad 1- darajali ko'phad bo'lishi kerak. $A(x)$ ko'phadni $B(x)$ ko'phadga bo'lishdagi qoldiqning darajasi ko'pi bilan 1 ga teng bo'ladi. Shu sababli $Q(x)$ ni $Q(x) = ax + b$ ko'rinishda, $R(x)$ ni esa $R(x) = px + q$ ko'rinishda izlaymiz. Bu yerdagi a, b, p, q lar topilishi kerak bo'lgan aniqmas koeffitsiyentlardir.

$A(x) = B(x) \cdot Q(x) + R(x)$ tenglikni $x^3 + x + 1 = (x^2 + x + 1) \cdot (ax + b) + (px + q)$ ko'rinishda yozib, uning o'ng tomonidagi amallarni bajaramiz. Ixchamlashtirishlardan so'ng,

$x^3 + x + 1 = ax^3 + (a + b)x^2 + (a + b + p)x + (b + q)$ tenglikni hosil qilamiz. Ko'phadlarning tenglik shartiga ko'ra,

$$\begin{cases} a = 1, \\ a + b = 0, \\ a + b + p = 1, \\ b + q = 1 \end{cases} \text{ sistemaga ega bo'lamiz. Bundan } a = 1,$$

$b = -1, p = 1, q = 2$ ekanligi aniqlanadi.

Demak, $Q(x) = x - 1, R(x) = x + 2$.

2- misol. Ushbu

$$A(x) = \frac{3x^4 - 10ax^3 + 22a^2x^2 - 24a^3x - 10a^4}{x^2 + 22ax - 3a^2}$$

ifodadan butun qism ajratamiz. Buning uchun suratdagi ko'phadni maxrajdagi ko'phadga bo'lish lozim. Bo'lishni «burchakli bo'lish» usulida bajaramiz:

$$\begin{array}{r} \underline{3x^4 - 10ax^3 + 22a^2x^2 - 24a^3x + 10a^4} \quad \left| \begin{array}{l} x^2 - 2ax + 3a^2 \\ \hline 3x^2 - 4ax + 5a^2 \end{array} \right. \\ \underline{3x^4 - 6ax^3 + 9a^2x^2} \\ -4ax^3 + 13a^2x^2 - 24a^3x \\ \underline{-4ax^3 + 8a^2x^2 - 12a^3x} \\ 5a^2x^2 - 12a^3x + 10a^4 \\ \underline{5a^2x^2 - 10a^3x + 15a^4} \\ -2a^3x - 5a^4. \end{array}$$

$$\text{Demak, } A(x) = 3x^2 - 4ax + 5a^2 + \frac{-2a^3x - 5a^4}{x^2 - 2ax + 3a^2}.$$

n - darajali $A(x)$ va m - ($m \leq n$) darajali $B(x)$ ikkita ko'phad berilgan bo'lib, ularning eng katta umumiy bo'luvchisini topish talab qilinsin. Uni topishda *Yevklid algoritmidan* foydalanamiz: oldin $A(x)$ ni $B(x)$ ga bo'lamiz, so'ng $B(x)$ ni birinchi $r_1(x)$ qoldiqqa, undan so'ng $r_1(x)$ ni ikkinchi $r_2(x)$ qoldiqqa bo'lamiz va hokazo. Bo'linmalarni q_k orqali belgilaylik, bunda $k = 1, 2, 3, \dots$. Quyidagiga ega bo'lamiz:

$$\begin{aligned}
A(x) &= B(x) \cdot q_1(x) + r_1(x), \\
B(x) &= r_1(x) \cdot q_2(x) + r_2(x), \\
r_1(x) &= r_2(x) \cdot q_3(x) + r_3(x), \\
&\dots\dots\dots \\
r_{n-2}(x) &= r_{n-1}(x) \cdot q_n(x) + r_n(x), \\
r_{n-1}(x) &= r_n(x) \cdot q_{n+1}(x).
\end{aligned}$$

Agar $A(x)$ va $B(x)$ lar umumiy bo'luvchiga ega bo'lmasa (ya'ni eng katta umumiy bo'luvchi doimiy son bo'lsa), ular *o'zaro tub ko'phadlar* deyiladi.

Tenglamalarning karrali ildizlarini topish kabi masalalarni hal qilishda Yevklid algoritmidan foydalanadilar. Ketma-ket bo'lishlardan qoladigan qoldiqlarning darajalari (ular natural sonlar) kamayib, bir necha qadamdan so'ng 0 ga teng bo'ladi ($r_{n+1}(x) = 0$).

Undan oldingi noldan farqli $r_n(x) \neq 0$ qoldiq $A(x)$ va $B(x)$ ning eng katta umumiy bo'luvchisi bo'ladi.

3- misol. $A(x) = x^3 - 3x^2 + 3x - 1$ va $B(x) = x^2 - x$ ko'phadlarning eng katta umumiy bo'luvchisini topamiz.

$$\begin{array}{r|l}
\text{Yechish. 1)} & x^3 - 3x^2 + 3x - 1 \quad | \quad x^2 - x \\
& \underline{x^3 - x^2} \quad | \quad x - 2 \\
& -2x^2 + 3x \quad | \\
& \underline{-2x^2 + 2x} \\
& r_1 = x - 1
\end{array}$$

$$\begin{array}{r|l}
2) & x^2 - x \quad | \quad x - 1 \\
& \underline{x^2 - x} \quad | \quad x \\
& r_2 = 0
\end{array}
\quad \text{Eng katta umumiy bo'luvchi:} \quad x - 1.$$

4- misol. $A(x) = x^3 - 3x^2 + 3x - 1$ va $B(x) = x^2 - x - 1$ larning eng katta umumiy bo'luvchisini topamiz.

Yechish. Ketma-ket bo'lishlar natijasida quyidagi oraliq natijalarni topamiz: $r_1(x) = 2x - 3$, $r_2 = -0,25 \neq 0$. Demak, $A(x)$ va $B(x)$ ko'phadlar umumiy bo'luvchiga ega emas, ya'ni ular o'zaro tubdir.



M a s h q l a r

4.41. $P(x)$ ni $D(x)$ ga qoldiqli bo'lishni bajaring:

a) $P(x) = x^3 + 5x^2 + 5x + 3$, $D(x) = x^2 + 4x + 1$;

b) $P(x) = x^3 + 5x^2 + 5x + 3$, $D(x) = x + 1$;

d) $P(x) = x^4 + 5x^3 + 9x^2 + 11x + 6$, $D(x) = x^2 + 3x + 1$;

e) $P(x) = x^4 + 5x^3 + 9x^2 + 11x + 6$, $D(x) = x^2 + 2x + 1$;

f) $P(x) = 3x^5 + 2x^4 - 10x^3 + 5x^2 + x + 10$, $D(x) = x^3 - x^2 + x - 1$;

g) $P(x) = 3x^5 + 2x^4 - 10x^3 + 5x^2 + x + 10$, $D(x) = x^2 + 3x - 4$;

h) $P(x) = 4x^6 + 3x^5 - 15x^2 + 4x + 5$, $D(x) = x^3 + 4x^2 - 1$;

i) $P(x) = 4x^6 + 3x^5 - 15x^2 + 4x + 5$, $D(x) = x^4 - 4x + 2$;

j) $P(x) = 3x^4 + 3x^2 + 5x + 4$, $D(x) = x^2 + 3x + 2$;

k) $P(x) = x^5 + 3x^4 + 9x^3 + 12x^2 + 20x$, $D(x) = x^3 + 4x$;

l) $P(x) = x^5 + 3x^4 + 9x^3 + 12x^2 + 20x$, $D(x) = x^2 + 3x + 5$;

m) $P(x) = 4x^4 + 5x^2 + 6x + 11$, $D(x) = x^2 + 5x - 4$.

4.42. Yevklid algoritmi yordamida ko'phadlarning eng katta umumiy bo'luvchisini toping:

a) $x^4 + x^3 + 3x^2 - 4x - 1$; $x^3 + x^2 - x - 1$;

b) $x^5 + x^4 - x^3 - 2x - 1$; $3x^4 + 2x^3 + x^2 + 2x - 2$;

d) $x^6 - 7x^4 - 8x^3 - 7x + 7$; $3x^5 - 7x^3 + 3x^2 - 7$;

e) $x^5 - 2x^4 + x^3 - 7x^2 - 12x + 10$; $3x^4 - 6x^3 + 5x^2 + 2x - 2$;

f) $x^6 + 2x^4 - 4x^3 - 3x^2 + 8x - 5$; $x^5 + x^2 - x + 1$;

g) $x^5 + 3x^4 - 12x^3 - 52x^2 - 52x - 12$; $x^4 + 3x^3 - 6x^2 - 22x - 12$;

h) $x^5 + x^4 - x^3 - 3x^2 - 3x - 1$; $x^4 - 2x^3 - x^2 - 2x + 1$;

i) $x^4 - 4x^3 + 1$; $x^3 - 3x^2 + 1$.

4.43. a va b ning qanday qiymatlarida $x^4 - 4x^3 - x^2 + ax - b$ ko'phad $x^2 - 5x + 4$ uchhadga qoldiqsiz bo'linadi?



V b o b

ALGEBRAIK IFODALAR

1- §. Ratsional ifodalar

1. Butun ko'rsatkichli daraja. Har qanday a haqiqiy sonning α butun ko'rsatkichli darajasi yoki α -darajasi deb, a^α songa aytilishini bilamiz, bunda a – daraja asosi, α – daraja ko'rsatkichi,

$$a^\alpha = \begin{cases} a, & \text{agar } \alpha = 1 \text{ bo'lsa,} \\ \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ marta}}, & \text{agar } \alpha = n, n \in N, n \geq 2 \text{ bo'lsa.} \end{cases}$$

Har qanday $a \neq 0$ haqiqiy sonning nolinch darajasi 1 ga teng, $a^0 = 1$. Nolning nolinch darajasi, ya'ni 0^0 ma'noga ega emas.

Ixtiyoriy $a \neq 0$ haqiqiy sonning butun manfiy ko'rsatkichli darajasi $\frac{1}{a^n}$ sonidan iborat, $a^{-n} = \frac{1}{a^n}$. 0^{-n} ifoda ma'noga ega emas.

Butun ko'rsatkichli darajaning xossalari (a, b – noldan farqli haqiqiy sonlar, α, β – butun sonlar):

$$1) \quad (ab)^\alpha = a^\alpha b^\alpha. \quad (1)$$

Haqiqatan, $\alpha = n \in N$ bo'lsa, haqiqiy sonlarni ko'paytirishning asosiy qonunlariga muvofiq: $(ab)^\alpha = (ab)^n = \underbrace{(ab)(ab)\dots(ab)}_{n \text{ ta}} =$

$$= \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ ta}} \cdot \underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ ta}} = a^n \cdot b^n = a^\alpha \cdot b^\alpha; \text{ agar } \alpha = 0 \text{ bo'lsa,}$$

$$(ab)^\alpha = (ab)^0 = 1 = 1 \cdot 1 = a^0 b^0 = a^\alpha b^\alpha; \text{ agar } \alpha = -n, n \in N \text{ bo'lsa,}$$

$$(ab)^\alpha = (ab)^{-n} = \frac{1}{(ab)^n} = \frac{1}{a^n b^n}. \text{ Xususan,}$$

$$\left(\frac{a}{b}\right)^\alpha = \frac{a^\alpha}{b^\alpha}, \quad (2)$$

$$2) \quad a^\alpha a^\beta = a^{\alpha+\beta}. \quad (3)$$

Haqiqatan, agar $\alpha = n$, $\beta = m$, $n \in N$, $m \in N$ bo'lsa, u holda:

$$\begin{aligned} a^\alpha \cdot a^\beta &= a^n \cdot a^m = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ ta}} \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ ta}} = \\ &= \underbrace{a \cdot a \cdot \dots \cdot a}_{m+n \text{ ta}} = a^{m+n} = a^{\alpha+\beta}. \end{aligned}$$

$\alpha = n$, $\beta = -m$ va $\alpha = -n$, $\beta = m$ bo'lgan hollar ham shu kabi isbotlanadi. $\alpha = -n$, $\beta = -m$ holning isbotini quyidagicha bajarish mumkin:

$$\begin{aligned} a^\alpha a^\beta &= a^{-n} a^{-m} = \frac{1}{a^n} \cdot \frac{1}{a^m} = \frac{1}{a^n a^m} = \frac{1}{a^{n+m}} = a^{-(n+m)} = \\ &= a^{-n-m} = a^{(-n)+(-m)} = a^{\alpha+\beta}. \end{aligned}$$

$$3) \quad \frac{a^\alpha}{a^\beta} = a^{\alpha-\beta}. \quad (4)$$

$$4) \quad (a^\alpha)^\beta = a^{\alpha\beta}. \quad (5)$$

Xususan, $\alpha = n$, $\beta = m$, $n, m \in N$ bo'lganda: $(a^\alpha)^\beta = (a^n)^m = \underbrace{a^n \cdot a^n \cdot \dots \cdot a^n}_{m \text{ ta}} = \underbrace{a a \dots a}_{n m \text{ ta}} = a^{nm} = a^{\alpha\beta}$.

M i s o l. $A = \frac{116^8 \cdot 87^4}{58^9 \cdot 174^3}$ ni hisoblang.

$$\text{Y e c h i s h. } A = \frac{(2 \cdot 58)^8 \cdot 87^4}{58^9 \cdot (2 \cdot 87)^3} = \frac{2^8 \cdot 87}{58 \cdot 2^3} = \frac{2^5 \cdot 3 \cdot 29}{2 \cdot 29} = 48.$$



M a s h q l a r

5.1. Ifodani soddalashtiring:

$$\text{a) } (0,25x^{-1}y^{-3})^2 \cdot \left(\frac{x^{-3}}{4y^2}\right)^{-3}; \quad \text{b) } \left(\frac{a^{-3}b^4}{9}\right) \cdot \left(\frac{3}{a^{-2}b^3}\right)^{-3};$$

$$d) \left(\frac{c^{-4}}{10a^5b^2}\right)^{-2} \cdot (5a^3bc^2)^{-2}; \quad e) \left(\frac{x^{-2}y^{-3}}{6z}\right)^{-3} \cdot \left(\frac{x^{-2}y^{-2}}{9z}\right)^2.$$

5.2. O'zgaruvchilarning istagan qiymatida ifoda ayni bir qiymat qabul qilishini isbotlang ($m, n \in \mathbb{Z}$):

$$a) \frac{2^m \cdot 3^{n-1} - 2^{m-1} \cdot 3^n}{2^m \cdot 3^n}; \quad d) \frac{5^m \cdot 4^n}{5^{m-2} \cdot 2^{2n} + 5^m \cdot 2^{2n-1}};$$

$$b) \frac{5^{n+1} \cdot 2^{n-2} + 5^{n-2} \cdot 2^{n-1}}{10^{n-2}}; \quad e) \frac{21^n}{3^{n-1} \cdot 7^{n+1} \cdot 3^n \cdot 7^n}.$$

2. Ratsional ifodalarni ayniy shakl almashtirish. Biror $X(x_1, \dots, x_n)$ algebraik ifodani *aynan almashtirish* deb, uni, umuman olganda, X ga o'xshamaydigan shunday $Y(x_1, \dots, x_n)$ algebraik ifodaga almashtirish tushuniladiki, barcha x_1, \dots, x_n qiymatlarda

X va Y qiymatlari teng bo'lsin. Masalan, $A(x) = \frac{(x^2+1)(x-1)}{x^2-1}$,

$$B(x) = \frac{x^2+1}{x+1}, \quad C(x) = \frac{(x^2+1)(x-1)(x+3)}{(x^2-1)(x+3)}$$

$x \neq -1$, $x \neq 1$ qiymatlarda, $B(x)$ ifoda $x \neq -1$ qiymatlarda, $C(x)$ esa $x \neq -1$, $x \neq 1$, $x \neq -3$ qiymatlarda aniqlangan. Ularning umumiy mavjudlik sohasi $x \neq \pm 1$, $x \neq -3$ qiymatlardan iborat, unda ular bir xil qiymatlar qabul qilishadi, ya'ni *aynan tengdir*. Umumiy mavjudlik sohasida bir ratsional ifodani unga aynan teng ifoda bilan almashtirish shu ifodani *ayniy almashtirish* deyiladi. Ayniy almashtirishlardan tenglamalarni yechish, teoremlar va ayniyatlarni isbotlash kabi masalalarni yechishda foydalaniladi. Ayniy almashtirishlar kasrlarni qisqartirish, qavslarni ochish, umumiy ko'paytuvchini qavsdan tashqariga chiqarish, o'xshash hadlarni ixchamlash va shu kabilardan iborat bo'ladi. Ayniy almashtirishlarda arifmetik amallarning xossalaridan foydalaniladi. Quyidagi ayniyatlar o'rinli:

$$1) (AB)^n = A^n B^n;$$

$$2) A^m A^n = A^{m+n};$$

- 3) $(A^m)^n = A^{mn}$;
- 4) $\frac{A}{B} + \frac{C}{D} = \frac{AD+BC}{BD}$, $B \neq 0$, $D \neq 0$;
- 5) $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$, $B \neq 0$, $D \neq 0$;
- 6) $\frac{A}{B} : \frac{C}{D} = \frac{AD}{BC}$, $B \neq 0$, $C \neq 0$, $D \neq 0$;
- 7) $\frac{AC}{BD} = \frac{A}{B}$, $B \neq 0$, $C \neq 0$;
- 8) $\frac{A^m}{A^n} = \begin{cases} A^{m-n}, & m > n \\ 1, & m = n, A \neq 0 \text{ da;} \end{cases}$
- 9) $|AB| = |A| \cdot |B|$;
- 10) $|A^n| = |A|^n$.

Ratsional ifodalarning kanonik shakli qisqarmas $\frac{P(x)}{Q(x)}$ kasrdan iborat bo'ladi. Bu yerda $P(x)$ va $Q(x)$ lar ko'phadlar bo'lib, $Q(x)$ ko'phadning bosh koeffitsiyenti esa 1 ga teng.

M i s o l. $\frac{16-x^2}{2x^4+9} : \left(\frac{1}{x-3} - \frac{1}{x-3} \cdot \frac{x-3}{2x+1} \right)$ ratsional ifodani kanonik

ko'rinishga keltiring.

Y e c h i s h. $\frac{1}{x-3} - \frac{1}{x-3} \cdot \frac{x-3}{2x+1} = \frac{x+4}{(x-3)(2x+1)}$,

$$\frac{16-x^2}{2x^4+9} \cdot \frac{x+4}{(x-3)(2x+1)} = \frac{(4-x)(4+x)(x-3)(2x+1)}{(2x^4+9)(x+4)} =$$

$$= \frac{-2x^3+13x^2-17x-12}{2x^4+9} = \frac{-x^3+\frac{13}{2}x^2-\frac{17}{2}x-6}{x^4+\frac{9}{2}}.$$



M a s h q l a r

5.3. O'zgaruvchining ifoda ma'noga ega bo'lmaydigan barcha qiymatlari to'plamini toping:

- a) $\frac{5-x}{x-2}$; b) $\frac{x^2+3}{x^2+4}$; d) $\frac{x+3}{(x-1)(x-2)}$;

e) $\frac{x^2-4}{x^2-9}$;	f) $\frac{3a}{3+2a}$;	g) $\frac{a-4}{5}$;
h) $\frac{a^2-5}{a-4,5}$;	i) $\frac{13a+2}{26-2a}$;	j) $\frac{3x}{x(x+2)}$;
k) $\frac{x-2}{a^2-x^2}$;	l) $\frac{x}{x^2-16}$;	m) $\frac{y}{3y(y-5)}$;
n) $x^2 + x + 2$;	o) $\frac{x-1}{x} + \frac{7}{x-3}$;	p) $\frac{4x}{x+5} - \frac{8x^2}{x-9}$;
q) $\frac{31x^2}{9x-9} + x^2 - x$.		

5.4. O'zgaruvchining ifoda ma'noga ega bo'ladigan barcha haqiqiy qiymatlari to'plamini tuzing:

a) $\frac{3}{x+2}$;	j) $\frac{x+4}{x-3} + \frac{1}{x+2}$;
b) $\frac{x^3+13}{x^2+5}$;	k) $\frac{7x-4}{x^2-16} + x + 2$;
d) $\frac{x+5}{x^2-9}$;	l) $\frac{x+2}{7x-7} + \frac{13}{x-2}$;
e) $\frac{3x+5}{4x^2-9}$;	m) $\frac{x^2+x-3}{x^2-5x} + \frac{1}{x}$;
f) $\frac{11a}{13-a^2}$;	n) $x^2 - x - 1$;
g) $\frac{a+5}{4-a}$;	o) $\frac{x-2}{x^2-a^2}$;
h) $\frac{3a+13}{4a^2-1}$;	p) $\frac{7}{x^2+x+1} + x^2$;
i) $\frac{17a}{(a-1)(a-2)(a-3)}$;	q) $x^2 - \frac{1}{(x-1)(x-4)}$.

5.5. Ifodaning aniqlanish sohasini toping:

a) $\frac{2x-y}{x(x-y)}$;	b) $\frac{x}{x^2-y^2}$;
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