d) $\frac{x+y}{x-y}$;

e) $\frac{x-2y}{x^2-y}$;

f) $\frac{x}{x-2}+\frac{y}{y(x-3)}$;

g) $\frac{x-1}{x}+\frac{y}{3x-y}$

h) $\frac{y}{x-y}-\frac{x}{x+y}$;

i) $\frac{3x+y}{x^3-y^3}-\frac{y}{3x-3}$;

j) $x+y+\frac{x}{y-4}$;

k) $xy+x^2y-\frac{y}{x+3}$;

l) $1+x^3y+x^4y^2$;

m) $13-2x^2+(x-y)^2$.

5.6. Kasrni qisqartiring:

a) $\frac{21a^3-6a^2b}{12ab-42a^2}$;

b) $\frac{6m^3-3mn^2}{2m^3n+mn^2}$;

d) $\frac{x^2-2mx+3x-6m}{x^2+2mx+3x+6m}$;

e) $\frac{8ab+2a-20b-5}{4ab-8b^2+a-2b}$;

f) $\frac{16a^2-8ab+b^2}{16a^2-b^2}$;

g) $\frac{9x^2-25y^2}{9x^2+30xy+25y^2}$;

h) $\frac{a^2-3a}{a^2+3a-18}$;

i) $\frac{4x^2-8x+3}{4x^2-1}$;

j) $\frac{m^2+4m-5}{m^2+7m-10}$;

k) $\frac{x^2+10x+25}{(x+5)^2}$;

l) $\frac{(x-2)^2}{(2-x)^2}$;

m) $\frac{x^6+x^4}{x^4+x^2}$.

Quyida keltirilgan ifodalar ratsional ifodalarmi:

5.7. a) $3x^2+y$;

b) $3x^2+\frac{1}{y}$;

d) $3x^2+\frac{1}{2}$;

e) $4a^2-x(a-3x)$;

f) $\frac{x^2}{x-4}$;

g) $\frac{x^3}{4}$;

h) $6x-\frac{1}{2}$;

i) $\dfrac{x^2+y}{1\frac{1}{2}-0,(5)x}$;

j) $\dfrac{xyz-\frac{1}{z}}{3-1\frac{1}{4}}$;

k) $xy+\sqrt{z}-\frac{z^2}{14}$?

Amallarni bajaring (**5.8 − 5.10**):

5.8. a) $\frac{a-2}{2}-1-\frac{a-3}{3}$;

b) $\frac{a+x}{4}-a+x$;

d) $4a-\frac{a-1}{4}-\frac{a+2}{3}$;

e) $\frac{(a-x)^2}{2a}+x$;

f) $c-\frac{(x+c)^2}{2x}$;

g) $a+x\frac{a^2+x^2}{a-x}$;

h) $\frac{a}{4x}+\frac{5}{12y}-\frac{c}{9xy^2}$;

i) $1-\frac{x}{x-y}-\frac{1}{x+y}$.

5.9. a) $\frac{a^2}{ax-x^2}+\frac{x}{x-a}$;

b) $\frac{x^2-4xy}{2y^2-xy}-\frac{4y}{x-2y}$;

d) $\frac{x}{2a^2-ax}+\frac{4a}{2ax-x^2}$;

e) $\frac{4y}{3x^2+2xy}+\frac{9x}{3xy+2x^2}$;

f) $\frac{x-25}{5x-25}=\frac{3x+5}{5x-x^2}$;

g) $\frac{12-y}{6y-36}+\frac{6}{6y-y^2}$;

h) $3x\frac{x-y}{2-x}+\frac{x+y}{4}$;

i) $\frac{x-12a}{x^2-16a^2}-\frac{4a}{4ax-x^2}$.

5.10. a) $\frac{a^2+3a}{ax-5x+8a-40}$;

b) $\frac{y}{3x-2}-\frac{3y}{6xy+9x-4y-6}$;

d) $\frac{x^2}{3ax-2-x+6a}-\frac{x}{3a-1}$;

e) $\frac{3x}{2y+3}+\frac{x^2+3x}{4xy-3-2y+6x}$.

5.11. Kasr ko‘rinishida ifodalang:

a) $\frac{x^2-xy}{y}\cdot\frac{y^2}{x^3}$;

b) $\frac{3a}{b^2}\cdot\frac{ab+b^2}{9}$;

d) $\frac{x-y}{xy}\cdot\frac{2xy}{xy-y^2}$;

e) $\frac{4ab}{cx+bx}\cdot\frac{ax+bx}{2ab}$;

f) $\frac{xa-xy}{3c^2}\cdot\frac{2x}{cy-ca}$;

g) $\frac{ax-ay}{5x^2y^2}\cdot\frac{5xy}{by-bx}$;

h) $\frac{kx+k^2}{k^2}\cdot\frac{x}{x+k}$;

i) $\frac{ax+ay}{xy^2}\cdot\frac{x^2y}{3x+3y}$;

j) $\frac{xy}{a^2+a^3}\cdot\frac{a+a^2}{x^2y^2}$;

k) $\frac{6a}{x^2-x}\cdot\frac{2x-2}{3ax}$;

l) $\frac{x^2-y^2}{2xy}\cdot\frac{2x}{x+y}$;

m) $\frac{4x^2}{x^2-9}\cdot\frac{3a-ax}{4x}$.

5.12. Soddalashtiring:

a) $\frac{x^2-4x}{x^2+7x}:\frac{24-6x}{49-x^2}$;

b) $\frac{y^3-16y}{2y+18}:\frac{4-y}{y^2+9y}$;

d) $\frac{(a+b)^2-2ab}{4a^2}:\frac{a^2+b^2}{ab}$;

e) $\frac{5c^3-5}{c+2}:\frac{(c+1)^2-c}{13c+26}$;

f) $\frac{(x+3)^2}{2x-4}:\frac{3x+9}{x^2-4}$;

g) $\frac{(x-3)^2}{x-8}:\frac{4x-12}{3x-24}$;

h) $\frac{a+b}{(a-b)^2}:\frac{(a+b)^2}{(a-b)^3}$;

i) $\frac{(3c-b)^2}{3c+b}:\frac{3c-b}{(3c+b)^2}$.

5.13. Ifodani soddalashtiring:

a) $\left(\frac{7(m-2)}{m^3-8}-\frac{m+2}{m^2+2m+4}\right)\cdot\frac{2m^2+4m+8}{m-3}$;

b) $\frac{a+5}{a^2-9}:\left(\frac{a+2}{a^2-3a+9}-\frac{2(a+8)}{a^3+27}\right)$;

d) $\left(\frac{x+2}{3x}-\frac{2}{x-2}-\frac{x-14}{3x^2-6x}\right):\frac{x+2}{6x}\cdot\frac{1}{x-5}$;

e) $\frac{1}{2}+\left(\frac{3m}{1-3m}+\frac{2m}{3m+1}\right)\cdot\frac{9m^2-6m+1}{6m^2+10m}$;

f) $\left(\frac{1}{x+y}-\frac{y^2}{xy^2-x^3}\right):\left(\frac{x-y}{x^2+xy}-\frac{x}{x^2+xy}\right)-\frac{x}{x-y}$;

g) $\frac{2a+3}{2a-3}\cdot\left(\frac{2a^2+3a}{4a^2+12a+9}-\frac{3a+2}{2a+3}\right)+\frac{4a-1}{2a-3}-\frac{a-1}{a}$;

h) $\left(\frac{a+3}{a^2+2a+1}+\frac{a-1}{a^2-2a-3}\right)\cdot\frac{a^2-2a-3}{a+2}-1$;

i) $\frac{3(m+3)}{m^2+3m+9}+\frac{m^2-3m}{(m+3)^2}\cdot\left(\frac{3m}{m^3-27}+\frac{1}{m-3}\right)$.

5.14. Ifodani soddalashtiring:

a) $\left(\frac{a}{a-b}-\frac{b}{a+b}\right):\left(\frac{a+b}{b}:\frac{a-b}{a}\right)$;

b) $\left(2x+1-\frac{1}{1-2x}\right):\left(2x-\frac{4x^2}{2x-1}\right)$;

d) $\left(p-q+\frac{4q^2-p^2}{p+q}\right):\left(\frac{p}{p^2-q^2}+\frac{2}{q-p}+\frac{1}{p+q}\right)$;

e) $\left(\frac{2}{2x+y}-\frac{1}{2x-y}-\frac{3y}{y^2-4x^2}\right)\cdot\left(\frac{y^2}{8x^2}-\frac{1}{2}\right)$;

f) $\left(\frac{5x+y}{x^2-5xy}+\frac{5x-y}{x^2+5xy}\right)\cdot\frac{x^2-25y^2}{x^2+y^2}$;

g) $\frac{9a^2-16b^2}{7a}\cdot\left(\frac{3b-4a}{4b^2-3ab}-\frac{3b+4a}{4b^2+3ab}\right)$;

h) $\frac{4xy}{y^2-x^2}:\left(\frac{1}{y^2-x^2}+\frac{1}{x^2+2xy+y^2}\right)$;

i) $\frac{a-2}{a^2+2a}:\left(\frac{a}{a^2-2a}-\frac{a^2+4}{a^3-4a}-\frac{1}{a^2+2a}\right)$;

j) $\frac{4a-5}{a^2 9}+\frac{9(a-3)}{15-27a+4a^2}\cdot\frac{4a^2-17a+15}{a-2}-\frac{7}{a+3}$;

k) $\left(a^2-y^2-x^2+2xy\right) : \frac{a+y-x}{a+y+x}$;

l) $\frac{a^2-1}{x^2+ax}\cdot\left(\frac{x}{x-1}-1\right)\cdot\frac{a-ax^3-x^4+x}{1-a^2}$, $(x=-1)$;

m) $\frac{x}{ax-2a^2}-\frac{2}{x^2+x-2ax-2a}\cdot\left(1+\frac{3x+x^2}{x+3}\right)$.

5.15. Kasrni qisqartiring:

a) $\frac{x^2-x+1}{x^4+x^2+1}$;

b) $\frac{x^{14}-x^7+1}{x^{21}+1}$;

d) $\frac{x(y-a)-y(x-a)}{x(y-a)^2-y(x-a)^2}$;

e) $\frac{x^{33}-1}{x^{33}+x^{22}+x^{11}}$.

5.16. k ning qanday qiymatlarida $\frac{(k-3)^2}{k}$ ifoda natural qiymatlar qabul qiladi?

5.17. Ifodani soddalashtiring va o‘zgaruvchilarnung ko‘rsatilgan qiymatlarida ifodaning qiymatini hisoblang:

a) $\left(\frac{x-2y}{x^3+y^3}+\frac{y}{x^3-x^2y+xy^2}\right)\cdot\frac{x^3-xy^2}{x^2+y^2}+\frac{2y^2}{x^3+x^2y+xy^2+y^3}$;

$x=0{,}2;\ y=0{,}8$;

b) $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}$;

$a=\frac{1}{3};\ b=\sqrt{3};\ c=\frac{\sqrt{3}}{2}$.

5.18. $m=a-\frac{1}{a}$ bo‘lganda $a^4+\frac{1}{a^4}=m^2(m^2+4)+4$ bo‘lishini isbot qiling.

5.19. Ratsional ifodalarni kanonik ko‘rinishga keltiring:

a) $\dfrac{2x-\dfrac{x+2}{x+1}}{\dfrac{x(x+1)}{x-1}-1}$; b) $\dfrac{\dfrac{x+1}{x^2+x+1}-\dfrac{x-1}{x^2+x+1}}{\dfrac{x-1}{x^2-x+1}+\dfrac{x+1}{x^2-x+1}}$;

d) $\dfrac{1-\dfrac{1-\dfrac{1-x}{1+2x}}{1+2\cdot\dfrac{1-x}{1+2x}}}{1+2\cdot\dfrac{1-\dfrac{1-x}{1+2x}}{1+2\dfrac{1-x}{1+2x}}}$; e) $\dfrac{\dfrac{x-1}{x^2-x+1}+\dfrac{x+1}{x^2-x+1}}{1+\dfrac{1-2x}{x^2+x+1}}$;

f) $\dfrac{(x+1)^2-x^4}{x^2-\left(x^2-1\right)^2}-\dfrac{\left(x^2+1\right)^2-x^2}{1-(x(x-1))^2}-\dfrac{1-(x(x+1))^2}{(x+1)^2-x^4}$.

2- §. Irratsional ifodalarni ayniy almashtirishlar

1. Arifmetik ildiz. Ratsional ko‘rsatkichli daraja. $a \geq 0$ sonning *n*- darajali *arifmetik ildizi* deb ($n \in N$), *n*- darajasi *a* ga teng bo‘lgan $b \geq 0$ songa aytiladi va $b=\sqrt[n]{a}$ orqali belgilanadi. Ta’rif bo‘yicha: $\left(\sqrt[n]{a}\right)^n = a$.

$a>0$, $m \in Z$ va $n \in N$ bo‘lsa, $\sqrt[n]{a^m}$ soni *a* ning $r=\frac{m}{n}$ *ratsional ko‘rsatkichli darajasi* deb ataladi, ya’ni $a^r = a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

Xususan, $\sqrt[n]{a} = a^{\frac{1}{n}}$.

Ratsional ko‘rsatkichli darajaning x o s s a l a r i butun ko‘rsatkichli daraja xossalariga o‘xshash. *a*, *b* — ixtiyoriy musbat sonlar, *r* va *q* — ixtiyoriy ratsional sonlar bo‘lsin. U holda:

1) $$(ab)^r = a^r b^r \qquad (1').$$

Haqiqatan, $r = m/n,\ n \in N,\ m \in Z$ bo‘lsin. U holda:

$$\left((ab)^r\right)^n = \left((ab)^{\frac{m}{n}}\right)^n = \left(\sqrt[n]{(ab)^m}\right)^n = (ab)^m = a^m b^m =$$

$$= \left(\sqrt[n]{a^m}\right)^n \left(\sqrt[n]{b^m}\right)^n = \left(a^{\frac{m}{n}} \cdot b^{\frac{m}{n}}\right)^n = \left(a^r b^r\right)^n,$$

demak, (1′) o‘rinli.

Xususan,

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}. \qquad (2')$$

2) $a^r \cdot a^q = a^{r+q}$, bunda

$$r = \frac{k}{n}, q\frac{m}{n}. \qquad (3')$$

Haqiqatan,

$$\left(a^{\frac{k}{n}} \cdot a^{\frac{m}{n}}\right)^n = \left(a^{\frac{k}{n}}\right)^n \cdot \left(a^{\frac{m}{n}}\right)^n = \left(\sqrt[n]{a^k}\right)^n \left(\sqrt[n]{a^m}\right)^n = a^k \cdot a^m = a^{k+m} =$$

$$= \left(a^{\frac{k+m}{n}}\right)^n = \left(a^{\frac{k}{n}+\frac{m}{n}}\right)^n.$$

3) $$\frac{a^r}{a^q} = a^{r-q} \qquad (4').$$

((2′) kabi isbotlanadi).

4) $(a^r)^q = a^{rq}$, bunda

$$r = p/k,\ q = m/n. \qquad (5')$$

Haqiqatan,

$$((a^{p/k})^{m/n})^{nk} = (((a^{p/k})^{m/n})^n)^k = ((a^{p/k})^m)^k = (a^p)^m = a^{pm} = \left(a^{\frac{pm}{kn}}\right)^{kn}.$$

Bundan (5′) ning o‘rinli ekani ma’lum bo‘ladi.

M i s o l. $5\sqrt{\frac{3}{5}} - 2^{-1} \cdot 60^{0,5} + 6$ ni hisoblang.

Y e c h i s h. $5 \cdot 0{,}6^{0{,}5} - 0{,}5 \cdot 10 \cdot 0{,}6^{0{,}5} + 6 = 5 \cdot 0{,}6^{0{,}5} - 5 \cdot 0{,}6^{0{,}5} + 6 = 6.$

Mashqlar

5.20. Ifodalar ma'noga egami:

a) $3^{-\frac{3}{4}}$; b) $(-3)^{-\frac{1}{3}}$; d) $4^{-\frac{1}{9}}$; e) $(-3)^{-\frac{2}{3}}$; f) $\left(\sqrt[3]{-4}\right)^{\frac{1}{2}}$;

g) $\left(\sqrt{4}\right)^{\frac{2}{5}}$; h) $(x-1)^{\frac{1}{3}}, (x<1)$; i) $(x+2)^{\frac{1}{4}}, (x \geq -2)$?

5.21. O'zgaruvchining ifoda ma'noga ega bo'ladigan barcha qiymatlarini toping.

a) $4{,}5^{\frac{x}{2}}$, bunda $x \in Q$; b) $(-4{,}5)^{\frac{x}{2}}$, bunda $x \in Q$;

d) $(3+x)^{\frac{1}{5}}$; e) $\left(x^2+1\right)^{\frac{1}{3}}$; f) $\left(\frac{x}{2}\right)^{-\frac{1}{4}}$;

g) $\left(|x|+1\right)^{\frac{2}{3}}$; h) $\left(1-|x|\right)^{\frac{4}{5}}$; i) $\left(1-|x|\right)^{-3}$.

5.22. Hisoblang:

a) $49^{\frac{1}{2}}$; b) $1000^{\frac{1}{3}}$; d) $4^{-\frac{1}{2}}$; e) $8^{-\frac{2}{3}}$;

f) $9^{2\frac{1}{2}}$; g) $0{,}16^{-1\frac{1}{6}}$; h) $0{,}008^{1\frac{1}{3}}$; i) $\left(3\frac{3}{8}\right)^{-\frac{4}{3}}$;

j) $9^{-1{,}5}$; k) $\left(\frac{1}{8}\right)^{-\frac{3}{4}}$; l) $\left(\frac{1}{64}\right)^{-\frac{4}{3}}$; m) $(25)^{-\frac{3}{2}}$;

n) $27^{-\frac{5}{6}} \cdot 3^{2{,}5}$; o) $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$; p) $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$; q) $\left(\frac{4}{9}\right)^{-\frac{3}{4}}$.

5.23. Ifodaning qiymatini toping:

a) $\left(\left(\frac{3}{4}\right)^{0}\right)^{0{,}5} - 7{,}5 \cdot 4^{-\frac{2}{3}} - (-2)^{-4} + 81^{0{,}25}$;

b) $0{,}027^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} - 256^{0{,}75} - 3^{-1} - (5{,}5)^{0}$;

d) $\left(\frac{9}{16}\right)^{-\frac{1}{10}} : \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left(\left(\frac{4}{3}\right)^{-\frac{1}{2}}\right)^{-\frac{2}{5}} \cdot \left(\frac{6}{5}\right)^{-3};$

e) $\left(9^{-\frac{2}{3}}\right)^{\frac{3}{4}} : \left(25^{2,5}\right)^{-0,1} + \left(\left(\frac{3}{4}\right)^{-1} \cdot \left(\frac{2}{9}\right)^{\frac{6}{7}}\right)^{0} : 36^{-\frac{1}{2}} + \frac{1}{\sqrt{5}};$

f) $\left(4^{-\frac{1}{4}} + \left(\frac{1}{2^{-\frac{2}{3}}}\right)^{-\frac{1}{2}}\right) : \left(4^{-0,25} - \left(2\sqrt{2}\right)^{-\frac{1}{2}}\right);$

g) $(0,04)^{-1,5} \cdot (0,125)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}};$

h) $\dfrac{2 \cdot 4^{-2} + \left(81^{-\frac{1}{2}}\right)^{3} \cdot \left(\frac{1}{9}\right)^{-3}}{125^{-\frac{1}{3}} \cdot \left(\frac{1}{5}\right)^{-2} + \left(\sqrt{3}\right)^{0} \cdot \left(\frac{1}{2}\right)^{-2}}.$

5.24. Amallarni bajaring:

a) $c^{\frac{1}{3}} \cdot c^{\frac{1}{4}} \cdot c^{\frac{1}{12}};$

b) $b^{-0,2} : b^{-0,7};$

d) $(m^{0,4})^{2,5};$

e) $y^{0,8} \cdot y^{-5} \cdot y^{7,2};$

f) $x^{\frac{1}{2}} \cdot x^{\frac{3}{14}} \cdot x^{\frac{2}{7}};$

g) $(m^{0,3})^{1,2} \cdot (m^{-0,4})^{0,4};$

h) $4^{\frac{1}{3}} \cdot 2^{1\frac{2}{3}} \cdot 8^{-\frac{1}{9}};$

i) $4^{-\frac{1}{3}} \cdot 16^{\frac{1}{3}} \cdot \sqrt[3]{4}.$

2. Ildiz. Yuqorida arifmetik ildizga ta’rif berilgan edi. $a \geq 0$ da $x = \sqrt[n]{a}$ son $x^n = a$ tenglamaning yagona nomanfiy yechimi ekanligi, shuningdek, $a \in R$ va n — toq natural son bo‘lsa, $x^n = a$ tenglamaning yagona yechimga ega ekanligi quyida isbotlanadi.

$x^n = a$ tenglamaning (bu yerda $a \in R$, $n \in N$) har qanday ildizi a sonining n- darajali ildizi deyiladi.

1- t e o r e m a. ***Har qanday $a \geq 0$ haqiqiy son uchun har doim $x^n = a$ tenglikni qanoatlantiruvchi yagona $x \geq 0$ haqiqiy son mavjud.***

I s b o t. Nomanfiy butun sonlarning $0, 1^n, 2^n, ..., k^n, ...$ n- darajalari ketma-ketligini qaraylik. Unda, albatta, n- darajasi a dan katta butun sonlar mavjud bo'ladi. Ulardan eng kichigi $(p+1)$ soni bo'lsin: $p^n \leq a < (p+1)^n$.

Endi $[p; p+1]$ oraliqni koordinatalari p; p, 1; p, 2; ..., p, 9; $p+1$ bo'lgan nuqtalar bilan teng o'n bo'lakka ajratamiz. Bu sonlar ichida a dan kattalaridan eng kichigi p, (q_1+1) bo'lsin.

$(p,\ q_1)^n \leq a \leq (p,\ (q_1+1))^n$, bunda q_1 — o'ndan birlar raqami. Bu oraliq x ning qiymatini $[p; p+1]$ oraliqqa nisbatan aniq ifodalaydi. Endi bu oraliqni o'nga bo'lamiz va ikkinchi yaqinlashishni topamiz: $(p,\ q_1q_2) \leq a \leq (p,\ q_1(q_2+1))^n$, q_2 — yuzdan birlar raqami. Shu yo'l bilan m — qadamdan so'ng $(p,\ q_1q_2 \ldots q_m)^n \leq a \leq (p,\ q_1q_2 \ldots (q_m+1))^n$ yoki $a_1^n \leq a \leq a_2^n$ ga ega bo'lamiz, bunda a_1 orqali a ning quyi (kami bilan olingan) va a_2 orqali a ning yuqori (ortig'i bilan olingan) chegaraviy qiymatlari, ya'ni o'nli yaqinlashishlari belgilangan.

Ikkinchi tomondan, ko'paytirish qoidasiga muvofiq $a_1^n \leq x^n \leq a_2^n$ tengsizlikni qanoatlantiruvchi yagona x haqiqiy son mavjud. Demak, $x^n = a$. Boshqacha bo'lishi, ya'ni x dan farqli biror y uchun $y^n = a$ bo'lishi mumkin emas. Masalan, $y < x$ bo'lsa, ko'paytirishning monotonlik xossasiga muvofiq $y^n < x^n$, ya'ni $y^n < a$ bo'lardi. Shu kabi, $y > x$ bo'lganda $y^n > a$ ga ega bo'lar edik. Demak, teorema to'g'ri.

2- t e o r e m a. ***Agar A natural son hech bir natural sonning n- darajasi bo'lmasa, $\sqrt[n]{A}$ soni irratsional sondir.***

I s b o t. Shart bo'yicha A soni nomanfiy sonlarning

$$0^n,\ 1^n,\ 2^n,\ \ldots,\ k^n,\ \ldots .$$

n- darajalar ketma-ketligida uchramaydi, demak, $\sqrt[n]{A}$ butun son emas. U kasr ham emas. Haqiqatan, $\sqrt[n]{A} = \frac{p}{q}$ bo'lsin, deb faraz qilaylik, bunda p va q lar o'zaro tub va $q \neq 1$, $q \neq 0$. U holda

$A = \frac{p^n}{q^n}$ va p^n va q^n — o‘zaro tub, $q^n \neq 1$ bo‘lganidan A soni qisqarmas kasr bo‘ladi. Bu esa shartga zid. Demak, $\sqrt[n]{A}$ soni faqat irratsionaldir. Teorema isbot qilindi.

3- t e o r e m a. ***Agar p/q, q ≠ 1, qisqarmas kasrning surati va maxraji aniq n- daraja bo‘lmasa,*** $\sqrt[n]{\frac{p}{q}}$ ***ildiz irratsional sondir.***

I s b o t. Teskaricha, ildiz ratsional son, deb faraz qilaylik, ya’ni $\sqrt[n]{\frac{p}{q}} = \frac{a}{b}$, $B(a, b) = 1$. U holda $\frac{p}{q} = \frac{a^n}{b^n}$, $B(a^n, b^n) = 1$ va bundan $p = a^n$, $q = b^n$ bo‘lshi kelib chiqadi. Lekin shart bo‘yicha p va q n- daraja emas. Demak, $\sqrt[n]{\frac{p}{q}}$ — irratsional son. Isbot qilindi.

4- t e o r e m a. ***Haqiqiy sonlar sohasida toq darajali ildiz faqat bir qiymatli va uning uchun ushbu tenglik o‘rinli:***

$$\sqrt[2n+1]{-a} = -\sqrt[2n+1]{a}.$$

I s b o t. $x^{2n+1} = a$, $a \geq 0$, (1) tenglama $\forall a \in R$ uchun yagona yechimga ega ekanligini ko‘rsatamiz:

a) $a \geq 0$ bo‘lsin. U holda $\forall x < 0$ son uchun $x^{2n+1} < 0 \leq a$. Demak, (1) ning, mavjudligi 1- teoremadan ko‘rinadigan, $x = \sqrt[2n+1]{a} \geq 0$ ildizi uning yagona haqiqiy ildizidir;

b) $a < 0$ bo‘lsa, (1) ni $(-x)^{2n+1} = -a$ ko‘rinishda yozib olish mumkin. $-a > 0$ bo‘lgani uchun, a) holga ko‘ra, oxirgi tenglama va, demak, (1) tenglama ham yagona $x = \sqrt[2n+1]{-a}$ yechimga egadir.

$\forall a \in R$ uchun $x_1 = -\sqrt[2n+1]{a}$ va $x_2 = \sqrt[2n+1]{-a}$ sonlari (1) ning ildizlari bo‘ladi. Yuqorida isbotlanganlarga ko‘ra, $x_1 = x_2$. Teorema isbot qilindi.

Teoremadan ko‘rinadiki, $\sqrt[n]{a^n} = a$ ayniyat n ning 1 dan katta toq natural qiymatlarida, ixtiyoriy $a \in R$ uchun o‘rinli. Agar $n = 2m$ (bu yerda $m \in N$) bo‘lsa, $\sqrt[2m]{a^{2m}} = \sqrt[2m]{|a|^{2m}} = |a|$ bo‘ladi.

Demak, $a \geq 0$ bo‘lsa, $\sqrt[2m]{a^{2m}} = a$ tenglik, $a < 0$ bo‘lganda esa $\sqrt[2m]{a^{2m}} = -a$ tenglik o‘rinli.

1- m i s o l.

$$\sqrt{(-7)^2} = \sqrt{|-7|^2} = |-7| = 7, \sqrt{(-7)^2} = \sqrt{49} = 7.$$

Agar $a \leq 0$, $b \leq 0$ bo‘lsa, $ab \geq 0$ va $\sqrt{ab} = \sqrt{|a||b|} = \sqrt{|a|} \cdot \sqrt{|b|}$ bo‘ladi.

2- m i s o l. $\sqrt{(-3)(-12)} = \sqrt{|-3||-12|} = \sqrt{36} = 6.$

M a s h q l a r

5.25. Ifodalar ma’noga egami:

a) $\sqrt[3]{-9}$;
b) $\sqrt{-9}$;
d) $\sqrt[3]{9}$;
e) $\sqrt{9}$;
f) $\sqrt[6]{-0,25}$;
g) $\sqrt{0,25}$;
h) $\sqrt[4]{-81}$;
i) $\sqrt[7]{-2}$;
j) $\sqrt[3]{i}$;
k) $\sqrt[3]{-i}$;
l) $\sqrt[4]{i}$;
m) $\sqrt[4]{-i}$;
n) $\sqrt[8]{x-y}$, bunda $x < y$;
o) $\sqrt[7]{x-y}$, bunda $x \leq y$;
p) $\sqrt[8]{y-x}$, bunda $x \leq y$;
q) $\sqrt[9]{y-x}$, bunda $x \geq y$?

5.26. Ifodalar o‘zgaruvchining qanday qiymatlarida ma’noga ega:

a) $\sqrt{-x}$;
b) $\sqrt[4]{x^2}$;
d) $\sqrt[6]{x^2+4}$;
e) $\sqrt[8]{(x+4)^2}$;
f) $\sqrt[3]{x-1}$;
g) $\sqrt[5]{(x+1)^2}$;
h) $\sqrt[7]{16x}$;
i) $\sqrt[3]{-x+2}$;
j) $\sqrt[4]{-x^2} + \sqrt[4]{x^2-1}$;
k) $\sqrt{x^2-6x+9}$;
l) $\sqrt{x^2+2x+2}$;
m) $\sqrt[6]{-(x-3)^2}$?

5.27. Tengliklar o'zgaruvchining qanday qiymatlarida to'g'ri:

a) $\sqrt{(x-2)^2} = 2 - x$;

b) $\sqrt{(x+3)^2} = x + 3$;

d) $\sqrt{(x-3)^2} = x - 3$;

e) $\sqrt{(x-4)^2} = 4 - x$;

f) $\sqrt[3]{x-3} = \sqrt[3]{3-x}$;

g) $\sqrt[3]{x-3} = 0$;

h) $\sqrt{x^2 - 1} = -1$;

i) $\sqrt{x} = 1$;

j) $\sqrt[3]{-x} = 2$;

k) $\sqrt[3]{-x} = -2$;

l) $\sqrt{x^2 - 6x + 9} = 1$;

m) $\sqrt[3]{x-2} = 1$?

3. Arifmetik ildizlarni shakl almashtirish. Ko'paytmaning *n*-darajali ildizi ko'paytuvchilar *n*- darajali ildizlarining ko'paytmasiga teng:

$$\sqrt[n]{ab...c} = \sqrt[n]{a} \cdot \sqrt[n]{b}...\sqrt[n]{c}, \qquad (1)$$

bu yerda $a \geq 0$, $b \geq 0$, ... , $c \geq 0$.

Haqiqatan,

$$\sqrt[n]{ab...c} = (ab...c)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} ... c^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b} ... \sqrt[n]{c}. \qquad (2)$$

Xususan, $\sqrt[n]{a^n b} = \left[\begin{array}{l} |a|\sqrt[n]{b}, \text{ agar } n - \text{ juft bo'lsa,} \\ a\sqrt[n]{b}, \text{ agar } n - \text{ toq bo'lsa.} \end{array}\right.$

Ko'paytuvchini ildiz ishorasi ostiga kiritish:

$$a\sqrt[n]{b} = \sqrt[n]{a^n b}, \qquad (a \geq 0,\ b \geq 0). \qquad (3)$$

Kasrdan ildiz chiqarish:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \qquad (a \geq 0,\ b \geq 0). \qquad (4)$$

Ildizni darajaga ko‘tarish uchun ildiz ostidagi ifodani shu darajaga ko‘tarish kifoya:

$$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}, \qquad (a \geq 0). \qquad (5)$$

Haqiqatan, $\left(\sqrt[n]{a}\right)^m = \left(a^{\frac{1}{n}}\right)^m = a^{m \cdot \frac{1}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}.$

a sonning *m*- darajasining *n*- darajali ildizini topish uchun *a* ning *n*- darajali ildizini *m*- darajaga ko‘tarish kifoya, ya’ni

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m, \ (a > 0). \qquad (6)$$

Ildizdan ildiz chiqarish uchun ildiz ostidagi ifoda o‘zgartirilmay qoldiriladi, ildizlar ko‘rsatkichlari esa ko‘paytiriladi:

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}, \ (a \geq 0). \qquad (7)$$

Haqiqatan, $\sqrt[n]{\sqrt[m]{a}} = \left((a)^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m}\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[nm]{a}.$

Har xil ko‘rsatkichli $\sqrt[n]{a}$, $\sqrt[m]{b}$, ... , $\sqrt[k]{c}$ ildizlarni bir xil ko‘rsatkichli ildizlarga aylantirish uchun *n*, *m*, ..., *k* sonlarining umumiy karralisi (bo‘linuvchisi) bo‘lgan α soni topiladi. $\alpha = nu = mv = ... = kw$ bo‘lsin, bunda *u*, *v*, ... , *w* – qo‘shimcha ko‘paytuvchilar. Natijada ildizlar quyidagi ko‘rinishga keladi:

$$\sqrt[\alpha]{a^u}, \sqrt[\alpha]{b^v}, ..., \sqrt[\alpha]{c^w}.$$

M i s o l. $\sqrt[8]{10} > \sqrt[4]{3}$, chunki $\sqrt[8]{10} > \sqrt[8]{3^2}, 10 > 9$.

M a s h q l a r

5.28. Ko‘paytmadan ildiz chiqaring:

a) $\sqrt{16 \cdot 121}$; b) $\sqrt[3]{-125 \cdot 27}$;

d) $\sqrt[4]{16\cdot 81}$;   e) $\sqrt[5]{32\cdot 243}$;

f) $\sqrt{9\cdot 25\cdot 36\cdot 49}$;   g) $\sqrt[3]{8\cdot 27\cdot 64\cdot 125}$;

h) $\sqrt[4]{81\cdot 625\cdot 256}$;   i) $\sqrt{0,01\cdot 0,09\cdot 0,25}$.

5.29. Bo‘linmadan ildiz chiqaring:

a) $\sqrt{\frac{36}{49}}$;   b) $\sqrt[3]{-\frac{64}{27}}$;   d) $\sqrt[4]{\frac{16}{81}}$;   e) $\sqrt[5]{\frac{243}{32}}$;

f) $\sqrt{\frac{25}{64}}$;   g) $\sqrt[3]{\frac{64}{125}}$;   h) $\sqrt[4]{\frac{81}{625}}$;   i) $\sqrt{\frac{0,01}{0,09}}$.

5.30. Darajadan ildiz chiqaring:

a) $\sqrt[4]{15^8}$;   b) $\sqrt[4]{(-15)^8}$;   d) $\sqrt[3]{-5^6}$;   e) $\sqrt{\left(\frac{1}{3}\right)^4}$;

f) $\sqrt[4]{x^4}$, bunda, $x\geq 0$;   g) $\sqrt[3]{x^6}$, bunda $x\in R$;

h) $\sqrt{(x^2+1)^2}$, bunda$x\in R$;

i) $\sqrt{x^6}$, bunda $x\geq 0$.

5.31. Ildizdan ildiz chiqaring:

a) $\sqrt[3]{\sqrt[3]{16}}$;   b) $\sqrt[4]{\sqrt[3]{76}}$;   d) $\sqrt[5]{\sqrt[3]{4}}$;   e) $\sqrt[7]{\sqrt[3]{25}}$;

f) $\sqrt[7]{\sqrt[3]{x^2}}$, bunda $x\geq 0$;   g) $\sqrt[3]{\sqrt{x}}$, bunda $x\geq 0$;

h) $\sqrt[3]{\sqrt[4]{x}}$, bunda $x\geq 0$;   i) $\sqrt[3]{\sqrt[3]{x}}$, bunda $x\in R$.

5.32. Ildizni darajaga ko‘taring:

a) $\left(\sqrt[4]{2}\right)^3$;   b) $\left(\sqrt[6]{16}\right)^3$;   d) $\left(\sqrt[3]{-2}\right)^5$;   e) $\left(\sqrt[4]{4}\right)^2$;

f) $\left(\sqrt[4]{x}\right)^3$;   g) $\left(\sqrt[4]{x^2}\right)^6$;   h) $\left(\sqrt[4]{x+2}\right)^5$;   i) $\left(\sqrt[3]{x^4}\right)^6$.

5.33. Berilgan ildizlarni bir xil ko‘rsatkichli ildizlarga aylantiring:

a) $\sqrt{3}$va$\sqrt[3]{4}$; b) $\sqrt[3]{2}$ va $\sqrt[4]{4}$;

d) $\sqrt{5}$va$\sqrt[4]{6}$; e) $\sqrt[5]{2}$ va $\sqrt[3]{3}$;

f) $\sqrt{x}$ va $\sqrt[8]{y}$; g) $\sqrt[3]{x+1}$ va $\sqrt[7]{y}$;

h) $\sqrt{x^2+1}$ va $\sqrt[6]{y^2-1}$; i) $\sqrt[5]{x-y}$ va $\sqrt[4]{y}$.

5.34. Ko'paytuvchini ildiz belgisi ostidan chiqaring:

a) $\sqrt{12}$; f) $\sqrt{98}$; j) $\sqrt{\left(x^2-2\right)^2 \cdot y}$;

b) $\sqrt[4]{1250}$; g) $\sqrt[3]{375}$; k) $\sqrt[4]{x^4y^3}$;

d) $\sqrt[3]{81}$; h) $\sqrt[4]{48}$; l) $\sqrt[7]{(x-1)^7z^2}$;

e) $\sqrt[3]{24}$; i) $\sqrt{243}$; m) $\sqrt[5]{(y+1)^{10}x^2}$.

5.35. Ko'paytuvchini ildiz belgisi ostiga kiriting:

a) $4\sqrt{5}$; h) $x^2\sqrt[4]{y^3}$, bunda $x \le 0$;

b) $-3\sqrt[3]{2}$; i) $x^3\sqrt[4]{y^5}$, bunda $x \le 0$;

d) $-3\sqrt[4]{2}$; j) $(x-1)^2\sqrt[4]{y-2}$, bunda $x \le 1$;

e) $2\sqrt[5]{3}$; k) $(x-1)^3\sqrt[4]{y-2}$, bunda$x \le 1$;

f) $x\sqrt{y^3}$, bunda $x \le 0$; l) $-x\sqrt[4]{y}$, bunda $x \ge 0$;

g) $x\sqrt[5]{y^3}$, bunda $x \le 0$; m) $(\sqrt{3}-2)\sqrt{xy^3}$.

5.36. Hisoblang:

a) $\sqrt{18}+\sqrt{50}-\sqrt{98}$;

b) $\sqrt[3]{81}-\sqrt[3]{24}+\sqrt[3]{375}$;

d) $2\sqrt{3}-\sqrt{27}+3\sqrt{12}-2\sqrt{243}$;

e) $\sqrt{50}-5\sqrt{8}+\sqrt{2}+\sqrt{128}$;

f) $\sqrt{2}+3\sqrt{32}+0,5\sqrt{128}-6\sqrt{18}$;

g) $\sqrt[3]{2}+\sqrt[3]{250}-\sqrt[3]{686}-\sqrt[3]{16}$;

h) $20\sqrt{245}-\sqrt{5}+\sqrt{125}-2,5\sqrt{180}$;

i) $2\sqrt{3}+\sqrt{192}-2\sqrt{75}+\sqrt[4]{128}$.

5.37. Soddalashtiring:

a) $\sqrt[3]{16\sqrt{2}}$; e) $\sqrt[4]{12\sqrt{9\sqrt[3]{4}}}$; h) $\sqrt{\frac{a+1}{a-1}\sqrt{\frac{a-1}{a+1}}}$;

b) $\sqrt{5\sqrt[3]{625}}$; f) $\sqrt[5]{2\sqrt[4]{4\sqrt[3]{8}}}$; i) $\sqrt[3]{2\sqrt{2\sqrt[3]{2}}}$.

d) $\sqrt[3]{3\sqrt[4]{3\sqrt[5]{3}}}$; g) $\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}}$;

5.38. Sonlarni taqqoslang:

a) $2\sqrt{3}$ va $3\sqrt{2}$; f) $\sqrt{2}$ va $\sqrt[3]{3}$;

b) $2\sqrt[3]{3}$ va $3\sqrt[3]{2}$; g) $\sqrt[3]{12}$ va $\sqrt{5}$;

d) $5\sqrt{7}$ va $8\sqrt{3}$; h) $\sqrt{8}$ va $\sqrt[3]{19}$;

e) $3\sqrt[3]{4}$ va $3\sqrt[3]{2}$; i) $\sqrt[12]{2}$ va $\sqrt[15]{3}$.

5.39. Ifodaning qiymatlarini toping:

a) $\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{40}$; f) $\sqrt{2}\cdot\sqrt{6}\cdot\sqrt{3}$;

b) $\sqrt[4]{2}\cdot\sqrt[6]{32}$; g) $\sqrt{7}\cdot\sqrt[3]{6}\cdot\sqrt[6]{2}$;

d) $\sqrt[5]{a^2}\cdot\sqrt[15]{a^4}$, $a=3$; h) $\sqrt[3]{a}\cdot\sqrt{5}$, $a=2$;

e) $\sqrt[3]{a^2}\cdot\sqrt[4]{a}$, $a=2$; i) $\sqrt[4]{x}\cdot\sqrt{y}$, $x=3$, $y=2$.

5.40. Ifodani soddalashtiring:

a) $\frac{\sqrt[3]{4}}{\sqrt{2}}$; b) $\frac{\sqrt[3]{8}}{\sqrt[3]{2}}$; d) $\frac{\sqrt{24}}{\sqrt{4}}$;

e) $\frac{\sqrt[3]{2}}{\sqrt[4]{3}}$; f) $\sqrt[12]{a^2} : \sqrt[4]{a}$; g) $\sqrt[9]{a^8} : \sqrt[6]{a^5}$;

h) $\frac{\sqrt[4]{2^7}}{\sqrt[3]{2^4}}$; i) $\frac{\sqrt[14]{3^9}}{\sqrt[9]{3^2}}$.

5.41. Darajaga ko'taring:

a) $\left(\sqrt[3]{4x^2}\right)^2$; f) $\left(a^2x\sqrt[3]{3a^2x}\right)^4$;

b) $\left(2\sqrt[3]{3x^2}\right)^3$; g) $\left(\sqrt[3]{2+xy^2}\right)^2$;

d) $\left(3\sqrt{4x^2-1}\right)^2$; h) $\left(\sqrt{xy+z}\right)^3$;

e) $\left(\sqrt[3]{x^8}\right)^6$; i) $\left(\sqrt[6]{xy}\right)^2$.

5.42. Kasr maxrajidagi irratsionallikni yo'qoting:

a) $\frac{2}{\sqrt{3}}$; g) $\frac{1}{\sqrt{5}}$; l) $\frac{2}{\sqrt{a}+\sqrt{x}}$;

b) $\frac{5}{\sqrt[3]{12}}$; h) $\frac{2}{\sqrt[3]{75}}$; m) $\frac{a}{\sqrt[3]{a}+\sqrt[3]{x}}$;

d) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$; i) $\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$; n) $\frac{x-y}{\sqrt{x+y}}$;

e) $\frac{4}{1+\sqrt{3}-\sqrt{2}}$; j) $\frac{12}{3+\sqrt{2}-\sqrt{5}}$; o) $\frac{1-a}{\sqrt{1-\sqrt{a}}}$;

f) $\frac{\sqrt[3]{5}+\sqrt[3]{3}}{\sqrt[3]{5}-\sqrt[3]{3}}$; k) $\frac{15}{\sqrt[3]{3}+\sqrt[3]{7}}$; p) $\frac{x+y}{\sqrt{x-y}}$.

5.43. Hisoblang:

a) $\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{37}+\sqrt{36}}$;

b) $\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{9}+\sqrt{8}}+\ldots+\frac{1}{\sqrt{23}+\sqrt{22}}$;

d) $\frac{1}{\sqrt{3}-\sqrt{2}}+\frac{1}{2-\sqrt{3}}-\sqrt{2}-2\sqrt{3}$;

e) $\frac{3}{\sqrt{5}-\sqrt{2}}+\frac{5}{\sqrt{7}+\sqrt{2}}-\sqrt{7}-\sqrt{5}$.

5.44. Tenglik to‘g‘rimi:

a) $\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{4}{\sqrt{7}+\sqrt{3}}=\frac{1}{\sqrt{7}-\sqrt{6}}$;

b) $-\frac{2}{\sqrt{8}+\sqrt{6}}+\frac{5}{\sqrt{11}+\sqrt{6}}=-\frac{3}{\sqrt{8}+\sqrt{11}}$;

d) $\frac{8\sqrt{7}}{\sqrt{5\sqrt{7}}-\sqrt{2\sqrt{7}}}+\frac{4\sqrt{7}}{\sqrt{5\sqrt{7}}+\sqrt{8\sqrt{7}}}=-4\sqrt[4]{175}$;

e) $\frac{4\sqrt{5}}{\sqrt{3\sqrt{5}}-\sqrt{2\sqrt{5}}}-\frac{5\sqrt{5}}{4\sqrt{2\sqrt{5}}-3\sqrt{3\sqrt{5}}}=-\sqrt[4]{45}$?

4. Irratsional ifodalarni soddalashtirish. Sonlar, harflar va algebraik amallar (qo‘shish, ayirish, ko‘paytirish, bo‘lish, darajaga ko‘tarish va ildiz chiqarish) bilan tuzilgan ifoda *algebraik ifoda* deyiladi. Ildiz chiqarish amali qatnashgan ifoda shu argumentga nisbatan *irratsional ifoda* deyiladi. Masalan, $3-\sqrt{5}, \sqrt{5+\sqrt{a}}$, $\sqrt{a^2-\sqrt{ab}}$ ifodalar irratsional ifodalardir.

Irratsional ifodalar ustida amallar arifmetik amallar qonunlariga va ildizlar ustida amal qoidalariga muvofiq bajariladi.

1- m i s o l. Darajani ildiz ostidan chiqarishda daraja ko‘rsatkichi ildiz ko‘rsatkichiga bo‘linadi. Chiqqan bo‘linma va qoldiq mos tartibda ildiz ostidan chiqqan va ildiz ostida qolgan sonlarning daraja ko‘rsatkichlarini beradi, $\sqrt[5]{a^7b^9c^{-10}} = abc^{-2}\sqrt[5]{a^2b^4}$.

2- misol. $a^ub^v \dots c^w$ ifodali maxrajni m- darajali ildiz ostidan chiqarish (kasrni irratsionallikdan qutqazish) uchun ildiz ostidagi kasrning surat va maxraji $a^{m-u}b^{m-v} \dots c^{m-w}$ ga ko‘paytirilishi kifoya:

$$x = \sqrt[3]{\frac{a^5}{c^ud^v}} = \sqrt[3]{\frac{a^5 \cdot c^{3-u}d^{3-v}}{c^ud^v \cdot c^{3-u}d^{3-v}}} = \sqrt[3]{\frac{a^5 \cdot c^{3-u}d^{3-v}}{c^3d^3}} = \frac{1}{cd}\sqrt[3]{a^5c^{3-u}d^{3-v}}.$$

3- m i s o l. $\sqrt[n]{a}(a \geq 0)$ ildizni m- darajaga ko‘taramiz: $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$. Agar $m = kn + l$ bo‘lsa, $\sqrt[n]{a^{kn+l}} = a^k\sqrt[n]{a^l}$ bo‘ladi.

4- m i s o l. O‘xshash ildizlarni keltiramiz:

$$a\sqrt[n]{A} + b\sqrt[m]{B} + c\sqrt[n]{A} + d\sqrt[n]{A} = (a + c + d)\sqrt[n]{A} + b\sqrt[m]{B}.$$

5- m i s o l. Ildizlarni ko‘paytirish va bo‘lish:

$$\sqrt[m]{A} \cdot \sqrt[n]{B} = \sqrt[mn]{A^n} \cdot \sqrt[nm]{B^m} = \sqrt[mn]{A^nB^m}; \quad \frac{\sqrt[m]{A}}{\sqrt[n]{B}} = \sqrt[mn]{\frac{A^n}{B^m}}.$$

6- m i s o l. Murakkab kvadrat ildizni almashtirish

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}, \qquad (1)$$

$$A > 0, \ B > 0, \ A^2 > B$$

formulasini isbotlaymiz.

I s b o t. $x = \sqrt{A + \sqrt{B}} + \sqrt{A - \sqrt{B}}$ belgilashni kiritib, uni kvadratga ko‘tarsak: $x^2 = 2A + 2\sqrt{A^2 - B}$, $x = \sqrt{2A + 2\sqrt{A^2 - B}}$. U holda $\sqrt{A + \sqrt{B}} + \sqrt{A - \sqrt{B}} = \; = 2\sqrt{\frac{A + \sqrt{A^2 - B}}{2}}$. Shu kabi

$\sqrt{A+\sqrt{B}}-\sqrt{A-\sqrt{B}}=2\sqrt{\frac{A-\sqrt{A^2-B}}{2}}$. Keyingi ikki tenglikni qo‘shsak va ayirsak, (1) formula hosil bo‘ladi.

$S=\sqrt[3]{A}+\sqrt[3]{B}$ irratsional ifodadagi ildizlarni yo‘qotish uchun $x^3+y^3=(x+y)(x^2-xy+y^2)$ ayniyatdan foydalanish mumkin. Bizda $x=\sqrt[3]{A}, y=\sqrt[3]{B}$. Shunga ko‘ra S ni $M=\sqrt[3]{A^2}-$ $-\sqrt[3]{AB}+\sqrt[3]{A^2}$ ifodaga ko‘paytirish kerak bo‘ladi.

7- m i s o l. $x=\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}$ ifodani soddalashtiramiz.

Y e c h i s h. Oldin kvadrat ildizlar ostidagi ifodalarning musbat ekanini, ya’ni ildizlar haqiqiy sonlar sohasida ma’noga egaligini bilishimiz kerak.

a) $29-12\sqrt{5}>0$ (?) $\Rightarrow 29>12\sqrt{5}$ (?) $\Rightarrow$

$\Rightarrow 841>144\cdot 5$ (?) $\Rightarrow 841>720$ (!);

$3-\sqrt{29-12\sqrt{5}}>0$ (?) $\Rightarrow 3>\sqrt{29-12\sqrt{5}}$ (?) $\Rightarrow 9>29-$

$-12\sqrt{5}$ (?) $\Rightarrow 12\sqrt{5}>29-9=20$ (?) $\Rightarrow 720>400$ (!);

$\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}>0$ (?) $\Rightarrow$

$\Rightarrow 5>3-\sqrt{29-12\sqrt{5}}$ (?) $\Rightarrow 2+\sqrt{29-12\sqrt{5}}>0$ (!)

Demak, haqiqiy sonlar sohasida almashtirishlarni bajarish mumkin;

b) murakkab ildiz formulasidan foydalanamiz:

$$\sqrt{29-12\sqrt{5}}=\sqrt{29-\sqrt{720}}=\sqrt{\frac{29+\sqrt{841-720}}{2}}-$$

$$-\sqrt{\frac{29-\sqrt{841-720}}{2}}=\sqrt{20}-3;$$

$$\sqrt{3-\left(\sqrt{20}-3\right)}=\sqrt{6-\sqrt{20}}=\sqrt{\frac{6+\sqrt{36-20}}{2}}-$$

$$-\sqrt{\frac{6-\sqrt{36-20}}{2}}=\sqrt{5}-1,\ x=\sqrt{5}-\left(\sqrt{5}-1\right)=1.$$

8- m i s o l. x ning qanday qiymatlarida $\sqrt{(x-8)^2}=x-8$ tenglik oʻrinli boʻlishini aniqlaymiz.

Y e c h i s h. $\sqrt{(x-8)^2}=|x-8|$ boʻlgani uchun, berilgan tenglik $x-8\geq 0$ boʻlganda, ya'ni $x\in[8;+\infty)$ larda oʻrinli boʻladi.

9- m i s o l. x ning qanday qiymatlarida $\sqrt{x-3}\sqrt{x+3}=$ $=\sqrt{x^2-9}$ tenglik oʻrinli boʻlishini aniqlaymiz.

Y e c h i s h. x ning $x-3<0$ yoki $x+3<0$ boʻladigan qiymatlarida tenglikning chap tomoni ma'noga ega emas. Shu sababli x ning $x-3\geq 0$ va $x+3\geq 0$ tengsizliklar bajariladigan qiymatlarini, ya'ni $x\geq 3$ boʻlgan holni qaraymiz.

$x\geq 3$ boʻlsa, arifmetik ildizlarni koʻpaytirish qoidasi (3-band, (1) tenglik) ga asosan, $\sqrt{x-3}\cdot\sqrt{x+3}=\sqrt{(x-3)(x+3)}=$ $=\sqrt{x^2-9}$ tenglikka ega boʻlamiz. Shunday qilib, berilgan tenglik $x\in[3;+\infty)$ lar uchun oʻrinli.

M a s h q l a r

5.45. Murakkab ildiz formulalaridan foydalanib, ifodalarni soddalashtiring:

a) $\sqrt{5+2\sqrt{6}}$; d) $\sqrt{10-2\sqrt{21}}$;

b) $\sqrt{6-\sqrt{20}}$; e) $\sqrt{4\sqrt{2}+2\sqrt{6}}$.

5.46. Darajaga ko‘taring:

$$\left(\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}\right)^2.$$

5.47. Ifodani soddalashtiring:

a) $\left(\sqrt{ab}-\frac{ab}{a+\sqrt{ab}}\right):\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}$;

b) $\left(\frac{\left(\sqrt{a}+1\right)^3-a\sqrt{a}+2}{\left(\sqrt{a}+1\right)^2-\frac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}\right)$;

d) $\left(\frac{\sqrt{a+1}}{\sqrt{1+a}-\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2}+a-1}\right)\cdot\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)$;

e) $\frac{\left(\sqrt{a}-\sqrt{b}\right)^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}}+\frac{3\sqrt{ab}-3b}{a-b}$;

f) $\frac{\frac{a+x}{\sqrt[3]{a^2}-\sqrt[3]{x^2}}+\frac{\sqrt[3]{ax^2}-\sqrt[3]{a^2x}}{\sqrt[3]{a^2}-2\sqrt[3]{ax}+\sqrt[3]{x^2}}}{\sqrt[6]{a}-\sqrt[6]{x}}$;

g) $\left(\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}+\frac{a-4+\frac{3}{a}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right)^2$;

h) $\left(\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}}-\frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}\right)^{-1}-\left(\frac{1-2x}{3x-2}\right)^{-1}$;

i) $\left(a+b^{\frac{3}{2}}:\sqrt{a}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}$.

5.48. $x=\frac{\sqrt{3}}{2}$ bo'lsa, $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$ ifodaning qiymatini toping.

5.49. $x=13,\ y=5$ bo'lsa, $\left(x+y^{\frac{3}{2}}:\sqrt{x}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)$ ifodaning qiymatini toping.

5.50. Ayniyatni isbotlang:

a) $\frac{a^{\frac{1}{2}}+1}{a+a^{\frac{1}{2}}+1}:\frac{1}{a^{\frac{3}{2}}-1}-a=-1;$

b) $\left(\frac{\left(a+\sqrt[3]{a^2x}\right):\left(x+\sqrt[3]{ax^2}\right)-1}{\sqrt[3]{a}+\sqrt[3]{x}}-\frac{1}{\sqrt[3]{x}}\right)^6=\frac{a^2}{x^4}.$