VIII bob

KO'RSATKICHLI VA LOGARIFMIK FUNKSIYALAR

1- §. Ko'rsatkichli funksiya

1. Irratsional ko'rsatkichli daraja. $a > 0$, $a \neq 1$ soni va $x > 0$ irratsional son berilgan bo'lsin. r_n ratsional sonlar x ga kami bilan, s_m ratsional sonlar ortig'i bilan (o'nli) yaqinlashsin, $r_n < x < s_m$, $n, m \in N$. U holda $a > 1$ da $a^{r_n} < x < a^{s_m}$ bo'ladi. Bu esa barcha a^{r_n} sonlarning A to'plami a^{s_m} sonlar B to'plamining chap tomonida yotishini va bu to'plamlarni hech bo'lmasa bitta son ajratishini bildiradi. Bu son irratsional ko'rsatkichli a^x darajaning qiymati sifatida qabul qilinadi.

$0 < a < 1$ holi ham shunday qaraladi. Faqat bunda A va B to'plamlarning rollari almashadi.

Irratsional ko'rsatkichli a^x darajaning *xossalari* ratsional ko'rsatkichli darajaning xossalariga o'xshash (a, b lar musbat, α va β lar haqiqiy sonlar):

1) $(ab)^\alpha = a^\alpha b^\alpha$; 2) $\left(\frac{a}{b}\right)^\alpha = \frac{a^\alpha}{b^\alpha}$; 3) $a^\alpha a^\beta = a^{\alpha+\beta}$;

4) $\frac{a^\alpha}{a^\beta} = a^{\alpha-\beta}$; 5) $(a^\alpha)^\beta = a^{\alpha\beta}$.

Darajalarni taqqoslashda ushbu ta'kiddan ham foydalaniladi:

Agar $a > 1$ va $m \in N$ bo'lsa, $a^m > 1$ yoki $\sqrt[n]{a^m} = a^{\frac{m}{n}} > 1$, shu kabi $a > 1$ va ixtiyoriy $r > 0$ da $a^r > 1$ bo'ladi. Agar $a > 1$, $r < s$ bo'lsa, $a^r < a^s$ bo'ladi. Haqiqatan, $a^s = a^r \cdot a^{s-r} > a^r \cdot 1 = a^r$. Aksincha, $a > 1$ va $0 < a^r < a^s$ bo'lsa, $r < s$ bo'ladi (isbot qiling). Shuningdek, $0 < a < 1$ va $r < s$ bo'lgan holda $a^r > a^s$ bo'lishi ham shu kabi isbotlanadi.

M i s o l. $0{,}5^\alpha > 0{,}5^\beta$ bo'lsa, α kattami yoki β mi?

Y e c h i s h. $a = 0{,}5$, ya'ni $0 < a < 1$ bo'lgani uchun $\beta > \alpha$.

Mashqlar

8.1. Nolga teng bo'lmagan a va b sonlari uchun $(ab)^{\alpha} = a^{\alpha}b^{\alpha}$, $a \in R$ munosabatni isbot qiling.

8.2. Quyidagi sonlardan qaysi biri katta:

a) $2^{1,41}$ mi yoki $0,125^{-\frac{\sqrt{2}}{3}}$ mi; b) $3^{\sqrt{5}}$ mi yoki $3^{\sqrt[3]{9}}$ mi?

8.3. Sonlarni o'sib borish tartibida joylashtiring:

$\pi^{\sqrt{3}}$, $(\sqrt{3})^{\pi}$, $\sqrt{\pi^{3}}$.

8.4. $\left(\frac{3}{7}\right)^{2\sqrt{2}} - 1$ ayirmaning ishorasini aniqlang.

8.5. Agar: a) $\left(\frac{2}{3}\right)^{\alpha} = 2$ bo'lsa, α ning; b) $a + 4^{0,3\sqrt{2}} = 5$ bo'lsa, $a - 1$ ning ishorasini aniqlang.

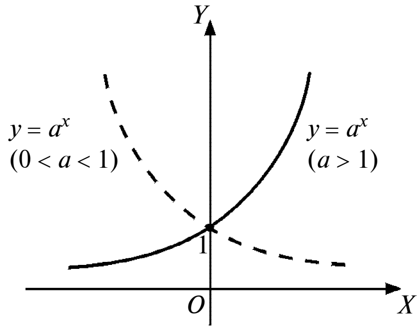
8.6. $3^{\sqrt{3}} < 7$ tengsizlikni isbotlang.

2. Ko'rsatkichli funksiya va uning xossalari. $a > 0$, $a \neq 1$ bo'lsin. $f(x) = a^{x}$ tenglik bilan aniqlangan funksiya a asosli *ko'rsatkichli funksiya* deyiladi. Bu funksiya barcha haqiqiy sonlar to'plamida aniqlangan, $D(f) = R$, chunki $a > 0$ bo'lganda a^{x} daraja barcha $x \in R$ uchun ma'noga ega. x ning istalgan haqiqiy qiymatida $a^{x} > 0$ bo'lgani uchun va ixtiyoriy $b > 0$ sonda $a^{x} = b$ bo'ladigan birgina $x \in R$ soni mavjud bo'lgani uchun $E(f) = R_{+}$ bo'ladi.

Xossalari:

1) $a > 1$ bo'lsa, $f(x) = a^{x}$ funksiya R da o'sadi. $0 < a < 1$ bo'lsa, $f(x) = a^{x}$ funksiya R da kamayadi.

Isbot. $a > 1$ holni qarash bilan cheklanamiz. $a > 1$ va $\alpha < \beta$ bo'lsin, bu yerda α, β sonlari ixtiyoriy haqiqiy sonlar. U holda $\beta - \alpha > 0$, $a > 1$ bo'lgani uchun $a^{\beta - \alpha} > a^{0}$ yoki $a^{\beta - \alpha} > 1$ tengsizlikka ega bo'lamiz. Bundan, $a^{\beta - \alpha} \cdot a^{\alpha} > 1 \cdot a^{\alpha}$ yoki $a^{\beta} > a^{\alpha}$ hosil bo'ladi. Demak, $\alpha < \beta$ dan $a^{\alpha} < a^{\beta}$ ekani kelib chiqadi. Bu esa a^{x} funksiya o'suvchi ekanligini bildiradi.



70- rasm.

70- rasmda $y = a^x$ ko‘rsatkichli funksiyaning sxematik grafigi tasvirlangan.

Agar $a > 1$ bo‘lsa, $x \to +\infty$ da a^x cheksiz ortadi, $x \to -\infty$ da a^x nolgacha kamayadi. Demak, a^x grafigi $y = 0$ to‘g‘ri chiziqqa tomon cheksiz yaqinlashadi, ya’ni Ox o‘qi funksiya grafigining *gorizontal asimptotasi*. Shu kabi $0 < a < 1$ bo‘lganda, a^x funksiya $+\infty$ dan 0 gacha kamayadi, Ox o‘qi – gorizontal asimptota;

2) f funksiya juft ham, toq ham emas. Haqiqatan,

$$f(-x) = a^{-x} = \frac{1}{a^x} \neq \begin{cases} a^x, \\ -a^x; \end{cases} \quad f(-x) \neq \begin{cases} f(x), \\ -f(x); \end{cases}$$

3) f davriy funksiya emas, chunki ixtiyoriy $T \neq 0$ da $a^x \neq a^{x+T}$;

4) x ning hech qanday qiymatida a^x nolga aylanmaydi;

5) *funksionallik xossasi:* har qanday x va z da $f(x+z) = f(x) \cdot f(z)$ tenglik o‘rinli. Chunki $a^{x+z} = a^x \cdot a^z$. Xuddi shunday $f(x)/f(z) = f(x-z)$ ekanligi isbotlanadi.

M i s o l. $f(x) = a^x$ $(a > 0,\ a \neq 1)$ ko‘rinishdagi uzluksiz funksiyaning ayrim qiymatlari jadvalda berilgan:

x	1	2	3	4
y	3	9	27	81

Funksiyaning analitik ifodasini tuzamiz.

Y e c h i s h. $f(1) = 3$, $f(2) = 9$, $f(1+2) = f(3) = 27$ va $f(1) \cdot f(2) = 3 \cdot 9 = 27$, ya’ni (5) xossa bajarilmoqda. Qolgan qiymatlar ham shu natijani beradi. Demak, $f(x)$ bog‘lanish ko‘rsatkichli funksiya. Uning asosi a ni aniqlaymiz: $y = a^x$ tenglikdagi x va y o‘rniga jadval qiymatlaridan biror juftni, masalan, (1; 3) ni qo‘ysak, $a^1 = 3$, ya’ni $a = 3$ olinadi. Demak, izlanayotgan ifoda $y = 3^x$.

Mashqlar

8.7. 1, q, q^2, ..., q^n, ... geometrik progressiyaning $u_k = \sqrt{u_{k-j}u_{k+j}}$ asosiy xossasi $f(x) = a^x$ ko‘rsatkichli funksiyaning $f(x)\cdot f(y) = f(x+y)$ xossasidan foydalanib isbot qilinsin. Bu yerda $k,\ j\in N,\ k > j$.

8.8. Quyidagi funksiyalar grafiklarini [–2; 1] oraliqda yasang:

a) $y = 4^x$; b) $y = 3^x$; d) $y = 2^x$;
e) $y = -3\cdot 3^x$; f) $y = -2\cdot 3^x$.

8.9. Tenglamalarni yeching:

a) $5^x = 125$; b) $3^{1+x} = 81$; d) $0{,}01^x = 100$.

8.10. Ifodalarni soddalashtiring:

a) $(9^x)^2 - 3\cdot 9^{2x} + 9^{2x+1} = 0$; b) $2^{8x}\cdot 3^x + 12^x - 2^{8x+1}\cdot 6^x$;
d) $a^{2x} + 2a^xb^x + b^{2x} - (a^x - b^x)^2$.

8.11. Jadvalda $y = f(x)$ uzluksiz funksiyaning bir necha qiymati keltirilgan. Ular $y = A\cdot a^x$ ($A\in R,\ a > 0,\ a\neq 1$) ko‘rinishdagi funksiyani ifodalashini tushuntiring, funksiyaning analitik ifodasini tuzing:

a)

x	1	2	3
y	0,2	0,04	0,008

b)

x	1	3	5	7
y	–2	–8	–32	–128

8.12. a) Bankka 1000 so‘m pul har yili 10% ga o‘sish sharti bilan qo‘yilgan. Mablag‘ning o‘sish tenglamasini tuzing. Tenglamadan foydalanib, mablag‘ning 3, 5, 10 yildan keyin qanchaga teng bo‘lishini toping.

b) Korxonaning har t yilda pul qadr-qiymati o‘zgarishi ham e’tiborga olingan, ya’ni diskontlangan D_t daromadini bilish uchun $D_t = D\cdot K_d$ tenglikdan foydalaniladi, bunda D — mo‘ljal bo‘yicha har yilgi daromad, K_d — diskontlash koeffitsiyenti, $K_d = \frac{1}{(1+k)^t}$, k — pul qiymatining o‘zgarish sur'ati (odatda bank kreditlari bo'yicha o'rtacha % larda). Bank $k = 10\%$ hisobidan kredit bergan bo'lsin.

1) $t = 1, 2, 3, 4$- yillar uchun K_d koeffitsiyentlarni toping.

2) Kredit uchun to'lovi bo'lmagan korxona daromadi 1- yilda 30000 so'm, 2- yilda 40000 so'm, 3- yilda 50000 so'm, 4- yilda 60000 so'm bo'lganda uning $t = 1, 2, 3, 4$- yillardagi diskontlangan daromadi qanday bo'ladi?

3) Agar korxona bankdan 100000 so'm kredit olgan bo'lsa, uni qancha vaqtdan keyin qaytara oladi?

8.13. Radioaktiv moddaning massasi 1- yilda 8 g ga, 4- yilda 1 g ga teng bo'lgan. Bu massa bir yilda qancha marta o'zgargan? Massaning boshlang'ich, undan 4 yil oldingi, 7,5- yildagi qiymati qancha bo'lgan?

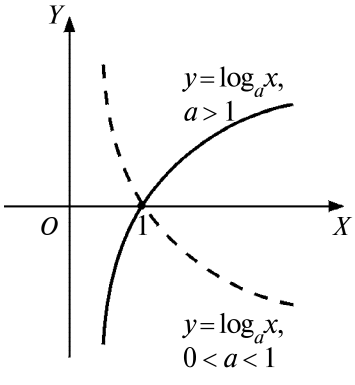
8.14. 10 sm uzunlikdagi xira muhitdan o'tishda yorug'lik kuchi uch martaga kamaygan. U 5, 20, 25 sm uzunlikdagi oraliqlarda necha marta kamayadi?

2- §. Logarifmik funksiya

1. Logarifmlar. Logarifmik funksiya. $a > 0$, $a \neq 1$ bo'lsin. N sonining a asos bo'yicha *logarifmi* deb, N sonini hosil qilish uchun a sonini ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi va $\log_a N$ bilan belgilanadi. Ta'rifga ko'ra, $a^x = N$ $(a > 0,\ a \neq 1)$ tenglamaning x yechimi $x = \log_a N$ sonidan iborat. Ifodaning logarifmini topish amali shu ifodani *logarifmlash*, berilgan logarifmiga ko'ra shu ifodaning o'zini topish esa *potensirlash* deyiladi. $x = \log_a N$ ifoda potensirlansa, qaytadan $N = a^x$ hosil bo'ladi. $a > 0$, $a \neq 1$ va $N > 0$ bo'lgan holda $a^x = N$ va $\log_a N = x$ tengliklar teng kuchlidir.

Shu tariqa biz o'zining aniqlanish sohasida uzluksiz va monoton bo'lgan $y = \log_a x$ $(a > 0,\ a \neq 1)$ funksiyaga ega bo'lamiz. Bu funksiya *a asosli logarifmik funksiya* deyiladi. $y = \log_a x$ funksiya $y = a^x$ funksiyaga teskari funksiyadir. Uning grafigi $y = a^x$ funksiya grafigini $y = x$ to'g'ri chiziqqa nisbatan simmetrik almashtirish bilan hosil qilinadi (71- rasm). Logarifmik funksiya ko'rsatkichli funksiyaga teskari funksiya bo'lganligi sababli, uning xossalarini ko'rsatkichli funksiya xossalaridan foydalanib hosil qilish mumkin.

Jumladan, $f(x) = a^x$ funksiyaning aniqlanish sohasi $D(f) = \{-\infty < x < < +\infty\}$, o'zgarish sohasi $E(f) = \{0 < < y < +\infty\}$ edi. Shunga ko'ra $f(x) = \log_a x$ funksiya uchun $D(f) = \{0 < x < < +\infty\}$, $E(f) = \{-\infty < y < +\infty\}$ bo'ladi. $a > 1$ da $\log_a x$ funksiya $(0; +\infty)$ nurda uzluksiz, o'suvchi, $0 < x < 1$ da manfiy, $x > 1$ da musbat, $-\infty$ dan $+\infty$ gacha o'sadi. Shu kabi $0 < a < 1$ da funksiya $(0; +\infty)$ da uzluksiz, $+\infty$ dan 0 gacha kamayadi, $0 < x < 1$ oraliqda musbat, $x > 1$ da manfiy qiymatlarni qabul qiladi. Ordinatalar o'qi $\log_a x$ funksiya uchun *vertikal asimptota.*



71- rasm.

Logarifmik funksiyaning qolgan xossalarini isbotlashda ushbu *asosiy logarifmik ayniyatdan* ham foydalaniladi:

$$a^{\log_a N} = N \quad (N > 0,\ a > 0,\ a \neq 1. \qquad (1)$$

(1) ayniyat $a^x = N$ tenglikka $x = \log_a N$ ni qo'yish bilan hosil qilinadi. O'zgaruvchi qatnashgan $a^{\log_a x} = x$ tenglik x ning $x > 0$ qiymatlaridagina o'rinli bo'ladi. $x \leq 0$ da $a^{\log_a x} = x$ ifoda ham o'z ma'nosini yo'qotadi. $y = x$ va $y = a^{\log_a x}$ munosabatlar o'rtasidagi farqni 72- rasmdan tushunish mumkin.

1) $\log_a 1 = 0$, chunki $a^0 = 1$;

2) $\log_a a = 1$, chunki $a^1 = a$;

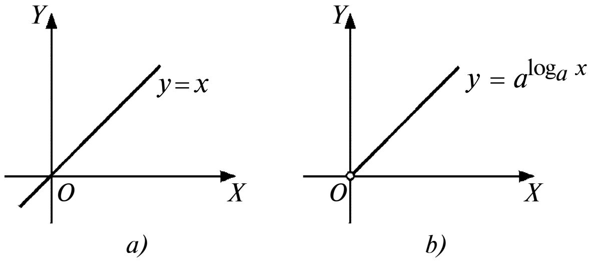
3) $\log_a N = \dfrac{\log_c N}{\log_c a}$ $(c > 0,\ c \neq 1)$. (2)

Bu tenglik $N = a^c$ tenglikka $N = c^{\log_c N}$, $a = c^{\log_c a}$, $c = \log_a N$ larni qo'yish va almashtirishlarni bajarish orqali hosil bo'ladi;

4) $\log_a (NM) = \log_a N + \log_a M$. (3)

Haqiqatan, $NM = a^{\log_a N} \cdot a^{\log_a M} = a^{\log_a N + \log_a M}$. Ikkinchi tomondan, $NM = a^{\log_a NM}$. Tengliklarning o'ng qismlari tenglashtirilsa, (3) tenglik hosil bo'ladi.

Agar N va M bir vaqtda manfiy bo'lsa, u holda:



72- rasm.

$$\log_a(NM) = \log_a|N| + \log_a|M|;$$

5) $\log_a \frac{1}{N} = -\log_a N.$ (4)

Haqiqatan, $N \cdot \frac{1}{N} = 1$ tenglikni logarifmlasak:

$$\log_a\left(N \cdot \frac{1}{N}\right) = \log_a 1 \text{ yoki } \log_a N + \log_a \frac{1}{N} = 0,$$

bundan (4) tenglik hosil bo‘ladi;

6) $\log_a \frac{N}{M} = \log_a N - \log_a M.$ (5)

Haqiqatan, $\log_a \frac{N}{M} = \log_a N + \log_a \frac{1}{M} = \log_a N - \log_a M$;

7) $\log_a N^\beta = \beta \log_a N$, β — haqiqiy son. (6)

Haqiqatan, $x = \log_a N^\beta$ va $y = \log_a N$ bo‘lsin. Ta’rifga ko‘ra $N^\beta = a^x$ va $N = a^y$ yoki $N^\beta = a^{\beta y}$. Bulardan $a^x = a^{\beta y}$ yoki $x = \beta y$ va (6) tenglik hosil bo‘ladi;

8) $\log_{a^\beta} N = \frac{1}{\beta}\log_a N.$ (7)

Haqiqatan, a^β asosdan a asosga o‘tilsa,

$$\log_{a^\beta} N = \frac{1}{\log_a a^\beta} \cdot \log_a N = \frac{1}{\beta \log_a a}\log_a N = \frac{1}{\beta}\log_a N;$$

9) agar $a > 1$ bo‘lsa, $M < N$ dan $\log_a M < \log_a N$ kelib chiqadi (va aksincha). Haqiqatan, $(M < N) \Rightarrow (a^{\log_a M} < a^{\log_a N}) \Rightarrow$ (darajaning xossasi) $(\log_a M < \log_a N)$ (va aksincha). Shu kabi,

agar $0 < a < 1$ bo‘lsa, $\log_a M < \log_a N$ bo‘lganda $M > N$ bo‘ladi (va aksincha);

10) agar $\log_a M = \log_a N$ bo‘lsa, $M = N$ bo‘ladi (va aksincha).

Haqiqatan, $(\log_a M = \log_a N) \Rightarrow (a^{\log_a M} = a^{\log_a N}) \Rightarrow (M = N)$.

1- m i s o l. $A = \log_3 9 - \log_{\sqrt{3}} 9 - \log_{\frac{\sqrt{3}}{2}}\left(\frac{64}{9}\right) - \log_{\frac{1}{3}} 9$ ifodaning son qiymatini toping.

Y e c h i s h. Logarifmning yuqorida isbotlangan xossalaridan foydalanib, ifodadagi har bir logarifmning qiymatini topib olamiz:

$$\log_3 9 = \log_3 3^2 = 2\log_3 3 = 2 \cdot 1 = 2;$$

$$\log_{\sqrt{3}} 9 = \log_{3^{\frac{1}{2}}} 3^2 = \frac{2}{\frac{1}{2}} \cdot \log_3 3 = 2 \cdot 2 \cdot 1 = 4;$$

$$\log_{\frac{1}{3}} 9 = \frac{\log_3 9}{\log_3 \frac{1}{3}} = \frac{2}{-1} = -2;$$

$$\log_{\frac{\sqrt{3}}{2}}\left(\frac{64}{9}\right) = \frac{\log_3\left(\frac{64}{9}\right)}{\log_3 \frac{\sqrt{3}}{2}} = \frac{\log_3 64 - \log_3 9}{\log_3 \sqrt{3} - \log_3 2} = \frac{6\log_3 2 - 2\cdot 1}{\frac{1}{2}\log_3 3 - \log_3 2} =$$

$$= \frac{4(3\log_3 2 - 1)}{1 - 2\log_3 2}.$$

Demak, $A = \frac{4(3\log_3 2 - 1)}{1 - 2\log_3 2}$.

Amaliyotda asosi 10 bo‘lgan (*o‘nli* logarifmlar) va asosi $e = 2{,}7182818...$ ga teng bo‘lgan (*natural* logarifmlar) logarifmlar keng qo‘llaniladi. Ularni mos ravishda $\lg N$ va $\ln N$ ko‘rinishda belgilash qabul qilingan. Son o‘nli logarifmining butun qismi logarifmning *xarakteristikasi*, kasr qismi logarifmning *mantissasi* deyiladi. Masalan, $\lg 2 = 0{,}3010$ da xarakteristika 0 ga, mantissa 0,3010 ga teng. $\lg 2000 = \lg 2 \cdot 10^3 = 3\lg 10 + \lg 2 = 3{,}3010$ da xarakteristika 3 ga, mantissa 0,3010 ga teng. $\lg 0{,}2 = \lg 2 \cdot 10^{-1} = \lg 2 - 1 = 0{,}3010 - 1 = -1 + 0{,}3010$ da xarakteristika -1, mantissa 0,3010. Odatda, mantissa musbat qiymatlarda yoziladi.

Agar logarifm qiymati manfiy bo‘lsa, mantissani musbat qilish uchun shu qiymatga 1 qo‘shiladi, umumiy qiymat o‘zgarmasligi uchun xarakteristikadan 1 olinadi va logarifm qiymati *sun’iy* ko‘rinishda yoziladi. Masalan,

$$\lg 0{,}2 = -0{,}6990 + 1 - 1 = \overline{1}{,}3010,$$

bunda xarakteristika –1 ga, mantissa esa 0,3010 ga teng.

2- m i s o l. a) $\ln 10 = \frac{\lg 10}{\lg e} = \frac{1}{\lg e} = 2{,}30259...;$

$$\lg e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10} = 0{,}43429... .$$

3- m i s o l. a) $\lg 1000^{67}$; b) $\ln e^{4,8}$ larni hisoblang.

Y e c h i s h: a) $\lg 1000^{67} = \lg 10^{3\cdot 67} = \lg 10^{201} = 201\cdot \lg 10 = 201\cdot 1 = 201;$

b) $\ln e^{4,8} = 4{,}8\ln e = 4{,}8\cdot 1 = 4{,}8.$

4- m i s o l. Jadvalda $\lg 3 = 0{,}4771$ ekanligi berilgan. a) $\lg 270$ ni; b) 3^{1000} ni toping.

Y e c h i s h: a) $\lg 270 = \lg 3^3\cdot 10 = 3\lg 3 + \lg 10 = 3\cdot 0{,}4771 + 1 = 2{,}4313.$

b) $3^{1000} = x$ deb, bu tenglikni logarifmlasak, $\lg x = 1000\lg 3 \approx 477{,}1$ yoki bundan $x \approx 10^{477,1}$ hosil bo‘ladi. Demak, $3^{1000} = 10^{477,1} \approx 1\underbrace{000...0}_{477 \text{ ta}}$.

5- m i s o l. Ushbu $X = \sqrt[3]{\frac{(x^3+1)^4(y^6+1)^7}{(x^4+y^2)^5}}\cdot c^{3\sin x}\cdot\sqrt{c}$ ifodani c asos bo‘yicha logarifmlang.

Y e c h i s h. $\log_c X = \log_c\left(\frac{(x^3+1)^{\frac{4}{3}}(y^6+1)^{\frac{7}{3}}}{(x^4+y^2)^{\frac{5}{3}}}\cdot c^{3\sin x}\cdot c^{\frac{1}{2}}\right) =$

$$= \frac{4}{3}\log\ (x^3+1) + \frac{7}{3}\log_c(y^6+1) - \frac{5}{3}(x^4+y^2) + 3\sin x + \frac{1}{2}.$$

6-misol. $\lg X = \frac{3}{4}\lg(x^2+4y-1) - \frac{3\,\mathrm{tg}4x}{4} - 2\lg(x-3)$ ifoda bo‘yicha X ni toping.

Yechish. Logarifmning xossalaridan va $\lg 4x = \lg 10^{\mathrm{tg}4x}$ ekanligidan foydalanib, $\lg X = \frac{3}{4}\lg(x^2+4y-1) - \frac{3}{4}\mathrm{tg}10^{\mathrm{tg}4x} - \frac{8}{4}\lg(x-3) = \frac{1}{4}\lg\frac{(x^2+4y-1)^3}{10^{3\mathrm{tg}4x}\cdot(x-3)^8}$ ga ega bo‘lamiz. Bundan, $X = \sqrt[4]{\frac{(x^2+4y-1)^3}{10^{3\mathrm{tg}4x}\cdot(x-3)^8}}$ hosil bo‘ladi.

Mashqlar

8.15. $a > 0,\ a \neq 1$ bo‘lsa, ifodaning qiymatini toping:

a) $\log_{a^3} a$; b) $\log_{a^4} a^{\frac{1}{3}}$; d) $\log_{\frac{1}{a}} a^7$;

e) $\log_{\sqrt{a}} \sqrt[3]{a}$; f) $\log_{a^{-1}} \sqrt{a}$; g) $\log_{a^2} a^{-5}$.

8.16. x ni toping:

a) $\log_{0,1} x = -2$; b) $\log_{36} x = \frac{1}{2}$;

d) $\log_x 9 = -1$; e) $\log_{\sqrt{x}} 8 = 3$.

8.17. $a > 0,\ a \neq 1$ va $x_1 > 0,\ x_2 > 0,\ \ldots,\ x_n > 0$ bo‘lsa, $\log_a(x_1x_2\ldots x_n) = \sum_{i=1}^{n}\log_a x_i$ ni isbotlang.

8.18. $\log_a x^{2n} = 2n\log_a|x|$ $(a > 0,\ a \neq 1,\ n \in N)$ munosabatni isbotlang.

8.19. $\frac{\log_a x}{\log_b x} = \log_a b$ tenglikni isbotlang ($a > 0,\ a \neq 1,\ b > 0,\ b \neq 1,\ x > 0,\ x \neq 1$).

8.20. Hisoblang:

a) $\log_{\sqrt[4]{3}} 81$; b) $\log_{16} \sqrt{2}$; d) $\log_{0,001} \sqrt[6]{10}$;

e) $\log_{\sqrt{2}} \frac{1}{64}$; f) $\frac{9^{\log_9 48}}{8^{\log_8 16}}$;

g) $(\log_2 \log_4 \log_8 16) \cdot 10^{\frac{1}{2}\lg 4 - \lg 2 + \lg 0,1}$; h) $\frac{\lg 81 + \lg 64}{2\lg 3 + 3\lg 2}$;

i) $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 5 \cdot \log_5 4 \cdot \log_4 3 \cdot \log_3 2$;

j) lg6; k) lg72.

8.21. Agar: a) $\log_6 8 = c$ bo'lsa, $\log_{24} 72$;

b) $\log_{36} 8 = b$ bo'lsa, $\log_{36} 9$;

d) $\log_{1000} 9 = a$ va $\log_{1000} 4 = b$ bo'lsa, $\log_5 6$ ni toping.

8.22. Tengsizlik a ning qanday qiymatlarida o'rinli:

a) $\log_5 a < \log_5 3a$; b) $\log_{0,6} a > \log_{0,6} \frac{a}{2}$;

d) $\log_a \sqrt{8} < \log_a 2,2$?

8.23. Funksiyalarning va ularga teskari funksiyalarning aniqlanish sohalarini toping:

a) $y = \lg(x^2 + 6x)$; b) $y = \lg(10^{3x} + 3)$;

d) $y = 10^{x^2 + 2x}$; e) $y = \frac{1}{\lg \sqrt{x+2}}$;

f) $y = \log_2(x - 8) + \log_2(8 - x)$.

8.24. $x \to +\infty$ da qaysi funksiya tezroq o'sadi:

a) $\log_4 x$ mi yoki $\log_2 x$ mi;

b) $\log_{\frac{1}{5}} x$ mi yoki $\log_{\frac{1}{2}} x$ mi?

Ulardan qaysilari $0 < x < 1$ da ikkinchisidan katta?

8.25. Funksiyalarning grafigini yasang:

a) $\log_{0,5}|x|$; b) $|\log_3 x|$; d) $|\lg(x + 1)|$.

8.26. Quyidagi ifodalar bilan berilgan chiziqlarni chizing:

a) $|y| = \lg(x + 3)$; b) $|y| = |\lg(x + 1)|$.

8.27. Agar $a^2 + b^2 + 18ab$, $a > b$ bo'lsa, $\lg \frac{a-b}{4} = \frac{1}{2}(\lg a + \lg b)$ bo'lishini isbot qiling.

8.28. Ikki N va M sonning istalgan asos bo‘yicha logarifmlari nisbatlari teng, ya’ni

$$\frac{\log_a N}{\log_a M} = \frac{\log_b N}{\log_b M} = \ldots = \frac{\log_c N}{\log_c M}$$

bo‘lishini isbot qiling.

8.29. Agar biror $y = f(x)$ funksiyaning teng qadamli jadvalida funksiyaning yonma-yon turgan qiymatlari nisbatlari teng bo‘lsa, jadval $y = A \cdot a^x$ funksiyani ifodalaydi. Shuni isbot qiling (bunda logarifmlarning xossalaridan foydalaning).

8.30. Tebrangich x (sm) erkin tebranish amplitudasining tebranish boshlangandan o‘tgan t (s) ga bog‘liqligi kuzatilib, ushbu jadval tuzilgan:

x	0	1	2	3	4	5
t	30,3	15,0	7,50	3,75	1,875	0,9375

$x = f(t)$ bog‘lanish grafigini chizing va analitik ifodasini tuzing.

2. Ko‘rsatkichli va logarifmik ifodalarni ayniy almashtirishlar. Oldingi bandlarda logarifmning va logarifmik funksiyaning, shuningdek, darajaning va ko‘rsatkichli funksiyaning xossalari bilan tanishgan edik. Bu xossalardan logarifmik va ko‘rsatkichli ifodalarni shakl almashtirishlarda foydalaniladi.

1- m i s o l. $3^{2+\log_3 2}$ ni hisoblang.

Y e c h i s h. $3^{2+\log_3 2} = 3^2 \cdot 3^{\log_3 2} = 9 \cdot 2 = 18.$

2- m i s o l. $a^{\log_b c} = c^{\log_b a}$ $(a > 0,\ a \neq 1,\ b > 0,\ b \neq 1,\ c > 0)$ tenglikni isbotlang.

I s b o t. Logarifmning $\log_a b^p = p \cdot \log_a b$ $(a > 0,\ a \neq 1,\ b > 0,$ $p \in R)$ xossasidan foydalansak, $\log_b a \cdot \log_b c = \log_b a \cdot \log_b c$ tenglikdan $\log_b (a^{\log_b c}) = \log_b (c^{\log_b a})$ tenglikni hosil qilamiz. Logarifmik funksiyaning monotonlik xossasidan $a^{\log_b c} = c^{\log_b a}$ ekanligi kelib chiqadi.

3- m i s o l. $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}}$ ifodani soddalashtiring.

Yechish. $a^{\sqrt{\log_a b}}$ ifodada shakl almashtirish bajaramiz:

$$a^{\sqrt{\log_a b}} = a^{\frac{\log_a b}{\sqrt{\log_a b}}} = (a^{\log_a b})^{\frac{1}{\sqrt{\log_a b}}} = b^{\frac{1}{\sqrt{\log_a b}}} = b^{\sqrt{\log_b a}}.$$

Demak, $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}} = 0$.

4- misol. $A = \log_4 \frac{x^2}{4} - 2\log_4(4x^4)$ ifodani soddalashtiring va uning $x = -2$ dagi qiymatini toping.

Yechish. $\log_a b^{2n} = 2n\log|b|$ $(a > 0,\ a \neq 1,\ b \neq 0,\ n \in N)$ bo‘lgani uchun $\log_4 \frac{x^2}{4} = \log_4 x^2 - \log_4 4 = 2\log_4|x| - 1$ va $\log_4(4x^4) = \log_4 4 + \log_4 x^4 = 1 + 4\log_4|x|$ tengliklar o‘rinli.

U holda, $A = 2\log_4|x| - 1 - 2(1 + 4\log_4|x|) = -3 - 6\log_4|x|$. $x = -2$ bo‘lsa, $A = -3 - 6\log_4|-2| = -3 - 6\log_4 2 = -6$.

5- misol. $A = \dfrac{(\lg b \cdot 2^{\log_2(\lg b)^{\frac{1}{2}}})\lg^{\frac{1}{2}} b^2}{\sqrt{\frac{\lg^2 b+1}{2\lg b}+1-10^{0,5\lg(\lg^{\frac{1}{2}})}}}$ ifodani soddalashtiring.

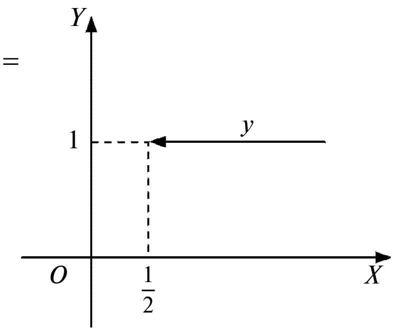
Yechish. Musbat sonlargina logarifmga ega bo‘lgani uchun $\lg b > 0$ yoki $b > 1$ munosabatga ega bo‘lamiz. Darajaning va logarifmning tegishli xossalaridan foydalanib, shakl almashtirishlar bajaramiz:

$$A = \frac{(\lg b \cdot \lg b)^{\frac{1}{2}} \lg^{\frac{1}{2}} b^2}{\sqrt{\frac{(\lg b+1)^2}{2\lg b} - \sqrt{\lg\sqrt{b}}}} = \frac{\lg b \cdot \frac{1}{\sqrt{\lg b^2}}}{\frac{\lg b+1-\sqrt{2\lg b}\cdot\sqrt{\frac{1}{2}\lg b}}{\sqrt{\lg b^2}}} = \frac{\lg b}{\lg b+1-\lg b} = \lg b.$$

6- misol. $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2\sqrt{4x^2 - 4x + 1}$ funksiyaning grafigini yasang.

Yechish. Funksiya ifodasini soddalashtirmay, grafikni yasashga harakat qilish maqsadga muvofiq emas ekanligi ko‘rinib turibdi. Shu sababli dastlab funksiyaning ifodasini soddalashtiramiz:

$$\log_2 \sqrt{4x^2 - 4x + 1} = \log_2 \sqrt{(2x-1)^2} =$$
$$= \log_2 |2x - 1| = \log_2 \left(2 \cdot \left|x - \frac{1}{2}\right|\right) =$$
$$= 1 + \log_2 \left|x - \frac{1}{2}\right|$$



73- rasm.

tenglik o‘rinlidir. Bu yerda funksiyaning aniqlanish sohasi $\left(\frac{1}{2};\, +\infty\right)$ oraliqdan iboratligini ko‘ramiz. $x > \frac{1}{2}$ da esa $\log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) = -\log_2\left(x - \frac{1}{2}\right)$ bo‘lgani uchun

$$y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2 \sqrt{4x^2 - 4x + 1} = -\log_2\left(x - \frac{1}{2}\right) +$$
$$+\left(1 + \log_2\left|x - \frac{1}{2}\right|\right) = -\log_2\left(x - \frac{1}{2}\right) + 1 + \log_2\left|x - \frac{1}{2}\right| = 1$$

ga ega bo‘lamiz.

Endi funksiya grafigini yasash (73- rasm) qiyinchilik tug‘dirmaydi.

Mashqlar

8.31. Ifodani soddalashtiring:

a) $\sqrt{25^{\frac{1}{\log_6 5}} + 49^{\frac{1}{\log_8 7}}}$; b) $81^{\frac{1}{\log_9 3}} + 27^{\log_9 36} + 3^{\frac{4}{\log_7 9}}$;

d) $\left(b^{\frac{\log_{100} a}{\lg a}} + a^{\frac{\log_{100} b}{\lg b}}\right)^{\log_{\sqrt{a}+\sqrt{b}}(a+b)}$;

e) $\left(\left(\log_b^4 a + \log_a^4 b + 2\right)^{\frac{1}{2}} + 2\right)^{\frac{1}{2}} - \log_b a - \log_a b$.

8.32. *x* ni toping:

a) $\log_3 x = 2\log_3 (a+b) - \frac{2}{3}\log_3 (a-b) + \frac{1}{2}\log_3 a$;

b) $\log_4 x = \log_4 (a-b) + \frac{1}{3}(2\log_4 a + 3\log_4 b)$;

d) $\log_5 x = 5\log_5 m + \frac{1}{2}\Big(\log_5 (m+n) + \frac{1}{3}\log_5 (m-n) -$
$- \log_5 m - \log_5 n\Big)$;

e) $\log_6 x = -\log_6 (a+b) + \frac{2}{5}\Big[2\log_6 a + \frac{1}{2}\log_6 b -$

$-\frac{1}{3}(\log_6 a - \log_6 b) - \log_6 a\Big]$.

8.33. Sonning musbat yoki manfiy ekanini aniqlang:

a) $\lg 2 + \lg 3 + \lg 0{,}16$;

b) $\log_{\frac{1}{5}} 7 - \frac{1}{2}\log_{\frac{1}{5}} 1{,}2 - 3\log_{\frac{1}{5}} 2$;

d) $\frac{1}{2}\log_{11} 5 + \frac{1}{2}\log_{11} 3 - \log_{11} 4{,}5$;

e) $\lg 4 + \lg 12 - 2\lg 7$;

f) $\log_3 3 + \log_3 1{,}4 - \frac{1}{2}\log_3 16$;

g) $1 + 2\lg 2 - 3\lg 5 + \lg 3$.

8.34. Hisoblang:

a) $\frac{\log_3 12}{\log_{36} 3} - \frac{\log_3 4}{\log_{108} 3}$; b) $\lg\mathrm{tg}1^\circ \cdot \lg\mathrm{tg}2^\circ \cdot \ldots \cdot \lg\mathrm{tg}89^\circ$;

d) $\lg 5 \cdot \lg 20 + (\lg 2)^2$; e) $\lg\sin 1^\circ \cdot \lg\sin 2^\circ \cdot \ldots \cdot \lg\sin 90^\circ$;

f) $\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$; g) $\lg\mathrm{tg}1^\circ + \lg\mathrm{tg}2^\circ + \ldots + \lg\mathrm{tg}89^\circ$;

h) $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$; i) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$;

j) $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})} - 6\log_8 (\sqrt{3} - \sqrt{2})$;

k) $2^{\log_{2\sqrt{2}}(5-\sqrt{10})} + 8\log_{\frac{1}{4}} (\sqrt{5} - \sqrt{2})$.

8.35. Funksiya grafigini yasang:

a) $y = x^{\frac{1}{\lg x}}$;   b) $y = 9^{\log_{\sqrt{3}}\left|x^2-5x+6\right|}$;

d) $y = 3^{2\log_3(x-1)}$;   e) $y = x + x^{\frac{1}{\lg x}}$.

8.36. $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$ ni jadvalsiz hisoblang.

8.37. $\lg 2 = a$, $\log_2 7 = b$ bo‘lsa, lg56 ni toping.

8.38. $\lg 3 = a$, $\lg 2 = b$ bo‘lsa, $\log_5 6$ ni toping.

8.39. $\log_3 7 = a$, $\log_7 5 = b$, $\log_5 4 = c$ bo‘lsa, $\log_3 12$ ni toping.

8.40. Agar $b = 8^{\frac{1}{1-\log_8 a}}$ va $c = 8^{\frac{1}{1-\log_8 b}}$ bo‘lsa, $\log_8 a$ ni $\log_8 c$ orqali ifodalang.

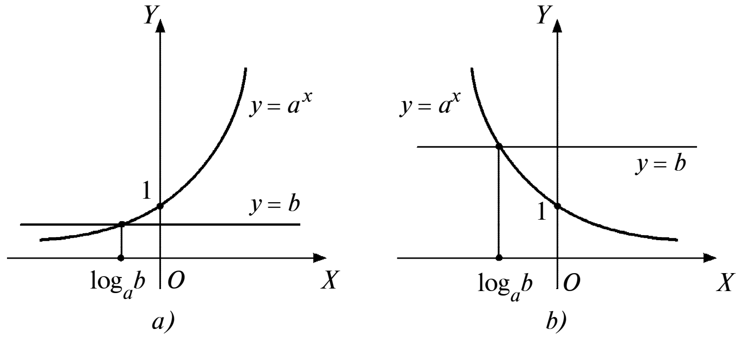
3- §. Ko‘rsatkichli va logarifmik tenglamalar, tengsizliklar

1. Ko‘rsatkichli tenglamalar va tengsizliklar. $a^x = b$ $(a,\ b\in R)$ tenglama *eng sodda ko‘rsatkichli* tenglamadir, bu yerda $a > 0$, $a \neq 1$.

Ko‘rsatkichli funksiyaning qiymatlar to‘plami $(0; +\infty)$ oraliqdan iborat bo‘lgani uchun $b \leq 0$ bo‘lganda qaralayotgan tenglama yechimga ega bo‘lmaydi. Agar $b > 0$ bo‘lsa, tenglama yagona yechimga ega va bu yechim $x = \log_a b$ sonidan iborat bo‘ladi (74-rasm).

T e o r e m a. ***Agar*** $a > 0$, $a \neq 1$ ***bo‘lsa,***

$$a^{f(x)} = a^{g(x)} \quad \textbf{(1)}$$



74- rasm.

va

$$\boldsymbol{f(x) = g(x)} \qquad \textbf{(2)}$$

tenglamalar teng kuchlidir.

Isbot. Agar α soni (2) tenglamaning ildizi bo‘lsa, $f(\alpha) = g(\alpha)$ bo‘ladi. U holda, $a^{f(\alpha)} = a^{g(\alpha)}$. Aksincha, α (1) tenglamaning ildizi bo‘lsa, $a^{f(\alpha)} = a^{g(\alpha)}$ va a^x funksiyaning monotonligidan $f(\alpha) = g(\alpha)$ bo‘ladi. Teorema isbot qilindi.

1-misol. $8^{5x^2-46} = 8^{2(x^2+1)}$ tenglamani yeching.

Yechish. Tenglama (1) ko‘rinishda berilgan. Unga teng kuchli (2) ko‘rinishga o‘tamiz: $5x^2 - 46 = 2(x^2 + 1)$, bundan $x = -4$, $x = 4$ aniqlanadi.

Agar tenglama

$$a^{f(x)} = b^{g(x)} \qquad (3)$$

(bu yerda $a > 0$, $a \neq 1$, $b > 0$, $b \neq 0$) ko‘rinishda bo‘lsa, $b^{g(x)} = a^{\log_a(b^{g(x)})} = a^{g(x)\log_a b}$ ekanidan foydalanib, tenglamani

$$a^{f(x)} = a^{g(x)\log_a b}$$

ko‘rinishga keltiramiz. Bundan unga teng kuchli $f(x) = g(x)\log_a b$ tenglamaga o‘tiladi.

2-misol. $5^{3x-1} = 3^x$ tenglamani yechamiz.

Yechish. $5^{3x-1} = 5^{x\log_5 3} \Rightarrow 3x - 1 = x\log_5 3 \Rightarrow x = \frac{1}{3-\log_5 3}$.

Agar tenglama $f(a^x) = 0$ ko‘rinishda bo‘lsa, $a^x = t$ almashtirish orqali $f(t) = 0$ tenglamaga o‘tiladi. Har vaqt $a^x > 0$ bo‘lgani uchun $f(t) = 0$ tenglamaning musbat ildizlarigina olinadi, so‘ng $a^x = t$ bog‘lanish yordamida berilgan tenglama ildizlari topiladi.

3-misol. $4^x + 2^x - 6 = 0$ tenglamani yechamiz.

Yechish. $2^x = t$ almashtirish $(2^x)^2 + 2^x - 6 = 0$ tenglamani $t^2 + t - 6 = 0$ kvadrat tenglamaga keltiradi. Uning yechimlari $t = -3$, $t = 2$. Musbat yechim bo‘yicha $2^x = 2$ ni tuzamiz. Bundan $x = 1$.

Ko‘rsatkichli tengsizliklarni yechishda $y = a^x$ funksiyaning monotonligidan foydalaniladi. $a^{f(x)} > a^{g(x)}$ tengsizlik, $a > 1$ bo‘lsa, $f(x) > g(x)$ tengsizlikka, $0 < a < 1$ bo‘lganda esa $f(x) < g(x)$ tengsizlikka teng kuchli.

4- m i s o l. $0{,}5^{x^2+3x+7} < 0{,}5^{x^2+1}$ tengsizlikni yeching.

Y e c h i s h. $0 < 0{,}5 < 1$ bo‘lgani uchun tengsizlik $x^2 + 3x + 7 >$ $> x^2 + 1$ algebraik tengsizlikka teng kuchli. Undan $x > -2$ aniqlanadi.

5- m i s o l. $4^{0{,}75x^2-2x+1} > 16^{x^2}$ tengsizlikni yechamiz.

Yechish. $4^{0{,}75x^2-2x+1} > 16^{x^2}$ tengsizlikni $4^{0{,}75x^2-2x+1} > 4^{2x^2}$ ko‘rinishida yozib olamiz. $a = 4 > 1$ bo‘lgani uchun, tengsizlik o‘ziga teng kuchli bo‘lgan $0{,}75x^2 - 2x + 1 > 2x^2$ tengsizlikka keladi.

J a v o b: $-2 < x < 0{,}4$.

Agar tengsizlik $f(a^x) < 0$ ko‘rinishda bo‘lsa, $a^x = t$ almashtirish uni $f(t) < 0$ ko‘rinishga keltiradi.

6- m i s o l. $9^x - 3^{x+1} - 4 < 0$ tengsizligini yechamiz.

Y e c h i s h. $3^x = t$ almashtirish tengsizlikni $t^2 - 3t - 4 < 0$ tengsizlikka keltiradi. Oxirgi tengsizlikning yechimi $(-1; 4)$ bo‘yicha $-1 < 3^x < 4$ tengsizligini tuzamiz va yechamiz.

J a v o b: $-\infty < x < \log_3 4$.

7 - m i s o l. $a^{x-1} < a^{2x}$ $(a > 0)$ tengsizlikni yechamiz.

Y e c h i s h. $a > 1$, $a = 1$ va $0 < a < 1$ bo‘lgan hollarni alohida-alohida qaraymiz.

$0 < a < 1$ bo‘lsa, berilgan tengsizlik $x - 1 > 2x$ tengsizlikka yoki $x < -1$ tengsizlikka teng kuchli. Demak, bu holda, $(-\infty; -1)$ oraliqdagi barcha sonlar va faqat shu sonlar tengsizlikning yechimi bo‘ladi.

$a = 1$ bo‘lsa, $1^{x-1} < 1^{2x}$ tengsizlikka ega bo‘lamiz. Bu tengsizlik yechimga ega emas.

$a > 1$ bo‘lsa, berilgan tengsizlik $x - 1 < 2x$ yoki $x > -1$ tengsizlikka teng kuchlidir. Demak, $a > 1$ bo‘lsa, $(-1; +\infty)$ oraliqdagi barcha sonlar va faqat shu sonlar tengsizlikning yechimi bo‘ladi.

J a v o b: $0 < a < 1$ bo‘lsa, $x \in (-\infty; -1)$; $a = 1$ bo‘lsa, $\varnothing$; $a > 1$ bo‘lsa, $x \in (-1; +\infty)$.

Mashqlar

8.41. Ko‘rsatkichli tenglamalarni yeching:

a) $4^{x-1} - 2^x = 0$; b) $5^x - 125 \cdot 5^{-x} = 20$;

d) $3\cdot\left(\frac{5}{6}\right)^{2x}-2\cdot\left(\frac{5}{6}\right)^{x}-1=0$;

e) $9^{-|x-2|}-4\cdot3^{-|x-2|}-a=0,\ a\in R$;

f) $0{,}5^{x^2-20x-23{,}5}=\frac{8}{\sqrt{2}}$; g) $9^x+4^{x-0{,}5}=9^{x+0{,}5}+2^{2x}$;

h) $4^{\sqrt{x-8}}+16=10-2^{\sqrt{x-8}}$;

i) $4^{1+3+5+\ldots+(2x-1)}=0{,}25^{-64}$;

j) $a^x=|x+2|$, a — parametr;

k) $3^{2x-3}\cdot5^{3x-2}=\frac{5}{3}$; l) $3^x\cdot5^{x-1}=1$;

m) $2^{x+4}+2^{x+1}+3\cdot2^{x+2}=120$;

n) $4^x-7^{x+2}=7^{x+1}-2\cdot4^{x+1}$;

o) $\frac{1}{2^{x^2-1}}+2^{1-x^2}=2$; p) $9\cdot4^x-13\cdot6^x+4\cdot9^x=0$;

q) $10\cdot4^x-9\cdot2^x(4^x+1)+2(16^x+2\cdot4^x+1)=0$;

r) $4^x-2\cdot6^x=9^{x+\frac{1}{2}}$; s) $3\cdot4^x+2\cdot25^x=5\cdot10^x$;

t) $\left(\sqrt{4+\sqrt{15}}\right)^x+\left(\sqrt{4-\sqrt{15}}\right)^x=8$;

u) $\left(\frac{3}{4}\right)^{x-2}\cdot\sqrt{\frac{4}{3}}=\frac{1}{2}\sqrt{3^{2x-7}}$; v) $\left(\frac{2}{5}\right)^x+\left(\frac{3}{5}\right)^x=1$;

x) $3^{x+1}=11-2^x$; y) $2^x+5^x=7^x$;

z) $2^{x+2}=\frac{5x+3}{x}$; o‘) $2^{|x|+1}=2-x^2$.

8.42. Ko‘rsatkichli tengsizliklarni yeching:

a) $\left(\frac{1}{2}\right)^{x^4-5x^2}>2^{-8x^2+6}$; b) $4^x-4\cdot2^x+3>0$;

d) $3^{2(x+1)}-5\cdot3^x+2<0$;

e) $|5^x-5|-|5^x-4|\geq|5^x+4|-8$;

f) $a^{\frac{x-3}{x+1}}>a^{\frac{2x-1}{x+1}}$; g) $2^{x^2+4x+4}>2$;

h) $2^x \cdot 3^{x-2} \geq \frac{2}{3}$; i) $\left(\frac{2}{3}\right)^x - 2^{x+1} \geq 3^{-x} - 2$;

j) $2^{x+4} + 3 \cdot 2^{x-2} \geq 67$; k) $4^x - 5 \cdot 2^x + 4 \geq 0$;

l) $0{,}1^{4x^2-2x-2} \leq 0{,}1^{2x-3}$; m) $\frac{2^{x-1}-1}{2^{x+1}+1} < 2$;

n) $(0{,}3)^{2+4+6+\ldots+2x} > (0{,}3)^{72}$, $x \in N$;

o) $4^x - 2 \cdot 5^{2x} - 10^x > 0$; p) $\sqrt{9^x - 3^{x+2}} > 3^x - 9$;

q) $x^2 \cdot 2^{2x} + 9(x+2) \cdot 2^x + 8x^2 \leq (x+2) \cdot 2^{2x} + 9x^2 \cdot 2^x + 8x + 16$;

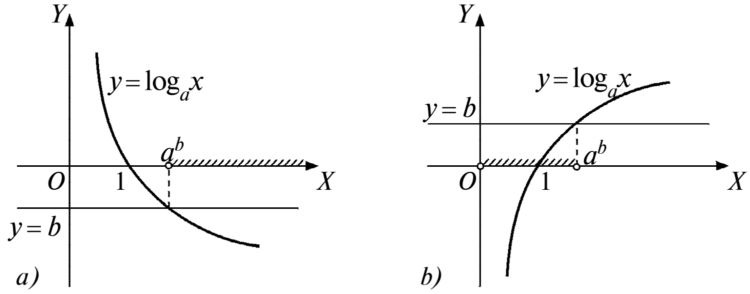
r) $\left(\frac{1}{3}\right)^{x+\frac{1}{2}-\frac{2}{x}} = \frac{1}{\sqrt{27}}$; s) $2^x + 2^{|x|} \geq 2\sqrt{2}$;

t) $0{,}2^{\frac{2x-3}{x-2}} > 5$; u) $\left(\frac{1}{5}\right)^{\frac{2x+1}{1-x}} > \left(\frac{1}{5}\right)^{-3}$.

8.43. a ning qanday qiymatlarida $|x+1| - |3x+15| = a^x$ tenglama: a) yagona yechimga ega? b) bittadan ortiq yechimga ega bo‘ladi? d) yechimga ega bo‘lmaydi?

2. Logarifmik tenglamalar va tengsizliklar. $\log_a x = b$ ($a > 0$, $a \neq 1$) tenglamani qaraymiz. Bu tenglama eng sodda logarifmik tenglama deyiladi. $x = a^b$ son qaralayotgan tenglamaning ildizi bo‘lishini ko‘rish qiyin emas.

Berilgan tenglama $x = a^b$ dan boshqa ildizga ega emasligini $y = \log_a x$ logarifmik funksiyaning monotonligidan foydalanib isbotlash mumkin (75- rasm).



75- rasm.

$\log_x N = b$ ko'rinishdagi tenglamani qaraymiz. Bu tenglamaning aniqlanish sohasi x ning $x > 0$, $x \neq 1$ munosabatlarni qanoatlantiruvchi barcha qiymatlaridan tashkil topadi. Agar $N \leq 0$ bo'lsa, bu tenglama yechimga ega bo'lmaydi. $N > 0$ bo'lsa, $x = N^{\frac{1}{b}}$ dan iborat yagona yechimga ega bo'ladi.

$\log_a x < b$, $\log_a x > b$, $\log_a x \leq b$, $\log_a x \geq b$ ko'rinishdagi (bu yerda $a > 0$, $a \neq 1$) tengsizliklar eng sodda logarifmik tengsizliklardir. Ularni yechishda $y = \log_a x$ funksiyaning monotonligidan foydalaniladi.

$\log_a x < b$ logarifmik tengsizlikni qaraymiz. Agar $0 < a < 1$ bo'lsa, bu tengsizlikning barcha yechimlari to'plami $(a^b;\ +\infty)$ oraliqdan iborat bo'ladi (75- *a* rasm). Agar $a > 1$ bo'lsa, qaralayotgan tengsizlikning barcha yechimlari to'plami $(0;\ a^b)$ oraliqdan iborat bo'ladi (75- *b* rasm).

$\log_a x > b$, $\log_a x \leq b$, $\log_a x \geq b$ tengsizliklar ham shunga o'xshash yechiladi.

1- m i s o l. a) $\log_3 x = 9$; b) $\log_x 64 = 2$ tenglamalarni yechamiz.

Y e c h i s h. a) Tenglamani potensirlaymiz. Natijada: $x = 3^9$;

b) tenglamani potensirlaymiz: $x^2 = 64$, bundan $x = 8$.

2- m i s o l. a) $\log_3 x < 9$; b) $\log_{\frac{1}{3}} x < 9$ tengsizliklarni yechamiz.

Y e c h i s h. a) oldingi misolda $\log_3 x = 9$ tenglamaning $x = 3^9$ ildizi topilgan edi. Asos $a = 3 > 1$, $b = 9$.

Y e c h i m: $(0;\ 3^9)$ yoki $0 < x < 3^9$;

b) $a = \frac{1}{3} \in (0;\ 1)$ bo'lgani uchun yechim $(3^{-9};\ +\infty)$ oraliqdan iborat.

1- t e o r e m a. $\boldsymbol{\log_a f(x) = \log_a g(x)}$ $\boldsymbol{(a > 0,\ a \neq 1)}$ ***tenglama***

$$\begin{cases} f(x) = g(x), \\ f(x) > 0 \end{cases} \qquad (1)$$

sistemaga teng kuchlidir.

Isbot. $y = \log_a t$ ($a > 0$, $a \neq 1$) logarifmik funksiya monoton. Shunga ko‘ra $\log_a f(x) = \log_a g(x)$ tengligining bajarilishi uchun $f(x) = g(x)$ bo‘lishi kerak. Demak, $f(x) > 0$ bo‘lganda $\log_a f(x) = \log_a g(x)$ tenglama $f(x) = g(x)$ tenglamaga teng kuchli.

1′-teorema. $\boldsymbol{\log_a f(x) = \log_a g(x)}$ $\boldsymbol{(a > 0,\ a \neq 1)}$ ***tenglama***

$$\begin{cases} f(x) = g(x), \\ g(x) > 0 \end{cases}$$

sistemaga teng kuchlidir.

Bu teoremani isbotlashda 1- teoremaning isbotidagi kabi mulohazalar yuritiladi (1′- teoremani mustaqil isbotlang).

2-teorema. ***Agar*** $\boldsymbol{0 < a < 1}$ ***bo‘lsa,*** $\boldsymbol{\log_a f(x) > \log_a g(x)}$ ***tengsizlik*** $\boldsymbol{0 < f(x) < g(x)}$ ***qo‘sh tengsizlikka,*** $\boldsymbol{a > 1}$ ***bo‘lsa,*** $\boldsymbol{f(x) > g(x) > 0}$ ***qo‘sh tengsizlikka teng kuchlidir.***

Bu teoremaning isboti logarifmik funksiyaning monotonligidan kelib chiqadi.

3-misol. $\frac{\lg\sqrt{x+7} - \lg 2}{\lg 8 - \lg(x-5)} = -1$ tenglamani yechamiz.

Yechish. 1) Tenglamaning aniqlanish sohasini topamiz:

$$\begin{cases} x + 7 > 0, \\ x - 5 > 0, \\ \lg 8 - \lg(x-5) \neq 0 \end{cases} \Rightarrow \begin{cases} x > -7, \\ x > 5, \\ x - 5 \neq 8 \end{cases} \Rightarrow \begin{cases} x > 5, \\ x \neq 13; \end{cases}$$

2) ifodani sodda ko‘rinishga keltirish maqsadida ayniy almashtirishlarni bajaramiz:

$$\lg\sqrt{x+7} - \lg 2 = \lg(x-5) - \lg 8 \Rightarrow \lg\frac{\sqrt{x+7}}{2} = \lg\frac{x-5}{8} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{x+7}}{2} = \frac{x-5}{8} \Rightarrow \left(\sqrt{x+7}\right)^2 = \left(\frac{x-5}{4}\right)^2 \Rightarrow x^2 - 26x + 87 = 0.$$

Bundan $x = 29$ ekani aniqlanadi.

4-misol. $\log_x \frac{3x+5}{x-3} < 0$ tengsizlikni yeching.

Yechish. Tengsizlikni $\log_x \frac{3x+5}{x-3} < \log_x 1$ ko‘rinishda yozib olamiz va quyidagi hollarni qaraymiz:

1) $0 < x < 1$ bo‘lsin. U holda $\frac{3x+5}{x-3} > 1$ tengsizlikka yoki $\frac{x+4}{x-3} > 0$ tengsizlikka ega bo‘lamiz. Bu tengsizlik (0; 1) oraliqda yechimga ega emas.

2) $x > 1$ bo‘lsin. U holda $0 < \frac{3x+5}{x-3} < 1$ qo‘sh tengsizlikka ega bo‘lamiz. Bu qo‘sh tengsizlik $x > 1$ shartni qanoatlantiruvchi yechimga ega emas. Shunday qilib, berilgan tengsizlik yechimga ega emas.

5 - m i s o l. $\log_{\frac{x}{3}} x^2 - 18\log_{81x} x^3 + 20\log_{9x}\sqrt{x} = 0$ tenglamani yeching.

Y e c h i s h. Logarifmni boshqa asosga o‘tkazish formulasidan foydalanib, barcha logarifmlarni 3 asosga o‘tkazamiz:

$$\frac{2\log_3 x}{\log_3 x - 1} - 18\cdot\frac{3\log_3 x}{4+\log_3 x} + 20\cdot\frac{\frac{1}{2}\log_3 x}{2+\log_3 x} = 0.$$

Bu tenglamada $\log_3 x = t$ almashtirish bajaramiz va $\frac{t(7t^2+2t-14)}{(t-1)(t+4)(t+2)} = 0$ tenglamaga ega bo‘lamiz. Uni yechib, $t_1 = 0$, $t_2 = \frac{-1-3\sqrt{11}}{7}$, $t_3 = \frac{-1+3\sqrt{11}}{7}$ yechimlarni topamiz. $\log_3 x = t$ bog‘lanish yordamida berilgan tenglamaning ildizlari topiladi:

$$x_1 = 0,\ x_2 = 3^{\frac{-1-3\sqrt{11}}{7}},\ x_3 = 3^{\frac{-1+3\sqrt{11}}{7}}.$$

M a s h q l a r

8.44. Tenglamani yeching:

a) $2\cdot\ln(x-3) = \ln x - \ln 4$;

b) $x^{\lg x} = x^{10}$;

d) $0{,}1\cdot x^{\lg x - 4} = 100^3$;

e) $4^{\frac{1}{\log_{16} x}} = \frac{1}{64}$;

f) $x^{2\log_a x} = ax,\ a > 0$;

g) $\log_{25}(x^2 - 10x + 9) = 2$;

h) $\sqrt{\log_x \sqrt{3x}}\,\log_{\frac{1}{2}} x = 1$;

i) $\log_{\frac{x^3}{a}} a^{-2} + \log_{a^2} x = a$; j) $2\log_x x^4 + \log_2 x = 4$;

k) $(1 + \log_c a)\log_a x \log_b c = \log_b x \log_a x \log_a c$;

l) $\log_4(2\log_3(1 + \log_2(1 + 3\log_3 x))) = \frac{1}{2}$;

m) $\log_3(1 + \log_3(2^x - 7)) = 1$; n) $\log_3(3^x - 8) = 2 - x$;

o) $\log_3(x + 1) + \log_3(x + 3) = 1$;

p) $3^{\log 3 \lg\sqrt{x}} - \lg x + \lg^2 x - 3 = 0$;

q) $9^{\log 3(1-2x)} = 5x^2 - 5$; r) $x^{\log 3\, x} = 9$;

s) $3\left(\log_x \sqrt{5}\right)^2 - 3\log_x \sqrt{5} + 1 = 0$;

t) $\log_3(4 \cdot 3^x - 1) = 2x + 1$;

u) $1 + 2\log_{(x+2)}5 = \log_5(x + 2)$;

v) $\lg(\lg x) + \lg(\lg x^3 - 2) = 0$; x) $\lg_2 x = 6 - x$;

y) $\log_2 x = 3^{-x} + \frac{8}{9}$; z) $\log_3(x + 5) = \log_{\frac{1}{2}} x + 4$;

o‘) $\log_2(3^x + 4) = 2 - 5^x$; g‘) $x\log_2 x = 24$.

8.45. Tengsizlikni yeching:

a) $\lg^2 x^2 + 5\lg x > -1,25$;

b) $\log_x(\sqrt{9 - x^2} - x - 1) \geq 1$;

d) $(\log_x 2)(\log_{2x} 2)(\log_2 4x) > 1$;

e) $\log_{\frac{49-x^2}{16}} \frac{46-4x-x^2}{14} > 1$;

f) $x^{(\lg x)^2 - 3\lg x + 1} > 1000$; g) $\log_x(24 - 2x - x^2) < 1$;

h) $\log_{x-1}9 < \log_x 3$; i) $\log_{\frac{1}{3}}((x + 5)(x - 6)) > 2$;

j) $(\log_{2x}0,5)^2 \leq \log_{2x}(2x^2)$;

k) $2^x\log_3 x + \log_3 x \le 2^{x+1} + 2$;

l) $\log_{\frac{1}{3}}(5x-1) > 0$; m) $\log_5(3x-1) < 1$;

n) $\log_2 x \le \frac{2}{\log_2 x - 1}$; o) $\log_{3x+5}(9x^2 + 8x + 8) > 2$;

p) $\log_{0,2}(x^2 - x - 2) > \log_{0,2}(-x^2 + 2x + 3)$;

q) $\log_x(\log_9(3^x - 9)) < 1$; r) $\log_{2x}(x^2 - 5x + 6) < 1$;

s) $\log_{x^2}(2 + x) < 1$; t) $(0,5)^{\log_3\log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)} < 1$;

u) $\frac{1-\log_4 x}{1+\log_2 x} \le \frac{1}{2}$.

3. Ko‘rsatkichli va logarifmik tenglamalar sistemalari. Bu tur sistemalarni yechishda oldingi bandlarda bayon qilingan algebraik qo‘shish, o‘rniga qo‘yish, yangi o‘zgaruvchi kiritish, ko‘paytuvchilarga ajratish, grafik yechish usullaridan, shuningdek, funksiyalarning xossalaridan foydalaniladi.

1- m i s o l.
$$\begin{cases} \log_{\sqrt{3}} x + \log_3 y = \log_{\sqrt[5]{3}} 3, \\ \log_3 x - \log_{\sqrt{3}} y = -\log_3 243 \end{cases} \qquad (1)$$
ni yeching.

Y e c h i s h . Logarifmlarni bir asosga ($a = 3$ ga) keltirilib, potensirlashlar va soddalashtirishlar bajariladi:

$$\log_{\sqrt{3}} x = 2\log_3 x;\ \ \log_3 3 = 1;\ \ \log_{\sqrt[5]{3}} 3 = 5\log_3 3;$$

$$\log_3 x = u;\ \ \log_3 y = v.$$

$$(1) \Rightarrow \begin{cases} 2u + v = 5, \\ u - 2v = -5, \end{cases} \Rightarrow \begin{cases} y = 3^3 = 27, \\ x = 3. \end{cases}$$

2- m i s o l.

$$\begin{cases} 2^{1+2\log_2(y-x)} = 32, \\ 2\log_5(2y - x - 12) = \log_5(y - x) + \log_5(y + x) \end{cases} \qquad (2)$$

ni yeching.

Y e c h i s h . Birinchi tenglamadan $(y-x)^2 = 16$ tenglamani va bundan $y-x>0$ ekanligini e'tiborga olib, $y-x=4$ ni olamiz. Sistema quyidagi ko'rinishga keladi:

$$\begin{cases} y-x=4, \\ 2\log_5(2y-x-12)=\log_5(y-x)+\log_5(y+x). \end{cases} \qquad (2')$$

(2′) sistemadagi 1- tenglamadan $y=4+x$ ni topib, 2- tenglamaga qo'ysak, faqat x noma'lum qatnashadigan tenglama hosil bo'ladi, uni yechib, x ni topamiz:

$$2\log_5(x-4)=\log_5 4\log_5(4+2x) \Rightarrow \log_5(x-4)^2=$$

$$=\log_5 4(4+2x) \Rightarrow (x-4)^2=4(4+2x) \Rightarrow x^2-16x=0 \Rightarrow$$

$$\Rightarrow \{x_1=0,\ x_2=16\}.$$

Bu tenglamani faqat $x=16$ soni qanoatlantiradi. $y=4+x$ dan $y=20$ ekani kelib chiqadi.

J a v o b : (16; 20).

3- m i s o l . $\begin{cases} y-2^x=1, \\ \log_{\frac{1}{2}} x-y=0 \end{cases}$ sistemani grafik usulda yeching.

Y e c h i s h . Koordinatalar sistemasida $y=2^x+1$ va $y=\log_{\frac{1}{2}} x$ funksiyalar grafiklarini yasaymiz (76- rasm).

Ikkala grafik taqriban $A(0,5;\ 2,2)$ nuqtada kesishadi.

J a v o b : $x\approx 0,5,\ y\approx 2,2.$

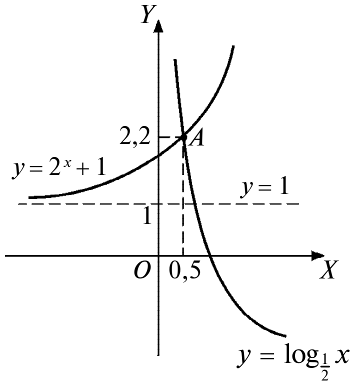
4- m i s o l . $n>0,\ n\neq 1,$

$\frac{\lg n}{10^{2m}-1}>0$ bo'lganda

$$\begin{cases} \lg x-\lg y=m, \\ 10^{x^2-y^2}=n \end{cases}$$

sistemani yeching.

Y e c h i s h . Logarifmik funksiya ta'rifiga ko'ra $x>0,\ y>0$. Ikkinchi tenglamadan $(x^2-y^2)\lg 10=$ $=\lg n$; $x^2-y^2=\lg n$. Berilgan sistema



76- rasm.

$$\begin{cases} \dfrac{x}{y} = 10^m, \\ x^2 - y^2 = \lg n, \\ x > 0, \;\; y > 0 \end{cases}$$ tenglama va tengsizliklar sistemasiga keladi.

Bundan $x = 10^m y$, $10^{2m}y^2 - y^2 = \lg n$, $y^2 = \dfrac{\lg n}{10^{2m}-1}$,

$y = \sqrt{\dfrac{\lg n}{10^{2m}-1}}$, $x = 10^m \sqrt{\dfrac{\lg n}{10^{2m}-1}}$.

Mashqlar

Quyidagi tenglamalar sistemasini yeching (**8.46–8.55**):

8.46. $\begin{cases} x + y = 6, \\ \log_2 x + \log_2 y = 3. \end{cases}$

8.47. $\begin{cases} x - y = 1, \\ 4^x + 2^y = 18. \end{cases}$

8.48. $\begin{cases} x^2 + y^2 = 68, \\ \log_2 x - \log_2 y = 2. \end{cases}$

8.49. $\begin{cases} 3^{x+y} = 9, \\ \log_2 (x+1) + \log_2 (y+1) = 2. \end{cases}$

8.50. $\begin{cases} \log_3 (x - y) = 1, \\ 10 \cdot 25^y - 5^{x-1} = 125. \end{cases}$

8.51. $\begin{cases} \log_2 x + 2\log_4 y = 3, \\ 3\log_8 (x+1) - \log_{\sqrt{2}} (y-1) = \log_{\frac{1}{2}} 3. \end{cases}$

8.52. $\begin{cases} \log_{x-2} (xy - x - 2y + 2) + \frac{1}{2}\log_{y-1} (x^2 - 4x + 4) = 3, \\ \log_{x+1} (y + x - 2) - \log_{y+2} (x^2 + y^2) = -1. \end{cases}$

8.53. $\begin{cases} \log_{x-2} (xy + x + y + 1) + \log_{x+y} (y+2) = -4, \\ 2\log_{x+1} (y+1) - \log_{x+y} (y^2 + 2x + xy + 2y) = 2. \end{cases}$

8.54. $\begin{cases} x^y = y^{2x}, \\ x^3 = y^2, \end{cases}$ $(x > 0, \;\; y > 0)$.

8.55. $\begin{cases} x^y = y^{4x}, \\ x^x = y^y, \end{cases}$ $(x > 0,\ y > 0)$.

8.56. Tenglamalar sistemasini yeching:

a) $\begin{cases} 5^{2x} - 2^y = 21, \\ 2\log_4 x + \log_4 y = 2; \end{cases}$ b) $\begin{cases} 4^{3x} - 3^y = -26, \\ 4^x - 3^{\frac{y}{3}} = -2; \end{cases}$

d) $\begin{cases} \frac{1}{\lg u + 1} = -2^{-v} + \frac{1}{\lg u - 1}, \\ \lg^2 u = 2^v + 5; \end{cases}$

e) $\begin{cases} \lg|x| + \lg|y| = 1 + \lg 4, \\ |x|^{\lg|y|} = 4. \end{cases}$

8.57. Tenglamalar sistemasini yeching:

a) $\begin{cases} 3^x \cdot 2^y = 9, \\ \log_{\sqrt{3}}(x - y) = 2; \end{cases}$ b) $\begin{cases} \log_{a^2} x + \log_a y = \frac{3}{2}, \\ \log_b x + \log_{b^2} y = 1; \end{cases}$

d) $\begin{cases} \log_3 x^2 + \log_3 y^2 = 2, \\ y - 5x = -2. \end{cases}$

8.58. a va b parametrlarning qanday qiymatlarida $\begin{cases} a^x + a^y = 2^{-1}, \\ x + y = -\log_a 16 \end{cases}$ sistema yechimga ega bo‘ladi?

8.59. Tenglamalar sistemasini yeching:

$$\begin{cases} \log_{0,3} x^3 + \log_{0,3} y^2 = -2, \\ x - 3y = 0,1. \end{cases}$$

8.60. Agar $3^{y+5} = 9^x$ va $x + y = 1$ bo‘lsa, $x - y$ ni toping.

8.61. Taxta, bo‘yoq va sement xarid qilindi. Agar 1 m^3 taxta sotuvdagi narxidan to‘rt marta arzon, 1 quti bo‘yoq ikki marta qimmat, 1 t sement uch marta arzon bo‘lganda qilingan xarid 750 so‘m turgan bo‘lardi. Agar taxta besh marta arzon, boyoq to‘rt marta arzon, sement ikki marta arzon bo‘lganda xarid uchun 400 so‘m to‘langan bo‘lardi. Necha so‘mlik xarid qilingan?

8.62. Jami 228 so‘mga 3, 5, 7 so‘mlik uch xil qalam keltirilgan. 7 so‘mliklari 3 so‘mliklaridan 6 dona kam, 3 so‘mliklari 5 so‘mliklaridan 2,2 marta ko‘p, 3 so‘mlik va 5 so‘mliklarining umumiy soni 7 so‘mliklari sonidan ikki marta ortiq. Har qaysi qalamdan qanchadan keltrilgan?

Takrorlashga doir mashqlar

8.63. Ko‘rsatkichli funksiyaning xossalaridan foydalanib, sonlarni taqqoslang:

a) $\left(\frac{5}{7}\right)^{0,8}$ va 1; b) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$ va 1;

d) $\left(\frac{4}{5}\right)^{3}$ va $\left(\frac{4}{5}\right)^{5}$; e) $(0,4)^{-2}$ va $(0,4)^{3}$;

f) $(2,56)^{0}$ va $(0,312)^{0}$; g) $(1,7)^{-3}$ va $(1,7)^{-2}$;

h) $\left(\frac{1}{3}\right)^{2,7}$ va $\left(\frac{1}{3}\right)^{5,2}$; i) $\left(\frac{8}{5}\right)^{-3}$ va $\left(\frac{8}{5}\right)^{\frac{1}{2}}$;

j) $(0,2)^{-6,5}$ va $5^{5,6}$; k) $3^{-1,2}$ va $\left(\frac{1}{3}\right)^{2,8}$;

l) $\mathrm{tg}\left(\frac{\pi}{3}\right)^{-1}$ va 1; m) $\left(\sqrt{3}\right)^{-2}$ va $\left(\frac{1}{3}\right)^{2}$.

8.64. Agar:

a) $a^{-\frac{2}{3}} > a^{\frac{5}{3}}$; b) $a^{\frac{7}{8}} > a^{\frac{11}{8}}$; d) $a^{\frac{3}{5}} > a^{0,6}$; e) $a^{-\frac{1}{3}} > a^{0,2}$

bo‘lsa, a o‘zgaruvchi qanday qiymatlarni qabul qilishi mumkinligini aniqlang.

8.65. Agar:

a) $1,34\alpha < 1,34\beta$; b) $\sqrt{0,364^{\alpha}} < \sqrt{0,364^{\beta}}$;

d) $\sqrt[20]{1,6^{\alpha}} < \sqrt[20]{1,6^{\beta}}$ bo‘lsa, α va β larni taqqoslang.

8.66. $\alpha^{0,4} < \alpha^{0,5}$ bo‘lsa, 1 va α sonlarini taqqoslang.

8.67. a) $\pi^{1,5}$ va $3,14^{1,5}$; b) $2,71828...^{-0,8}$ va $2,72^{-0,8}$ sonlarini taqqoslang.

8.68. x o‘zgaruvchi –5 dan 0 gacha o‘zgarsa, $y=\left(\frac{1}{5}\right)^x$ funksiya qanday o‘zgaradi?

8.69. Funksiyaning aniqlanish sohasini toping:

a) $y=16^{\frac{1}{9-x}}$; b) $f(x)=\left(\frac{1}{18}\right)^{\sqrt{x^2-9}}$;

d) $g(x)=\frac{19}{14^{x^4}}$; e) $\varphi(x)=\frac{1}{2^{x^2}-4}$.

8.70. Funksiyaning qiymatlar sohasini toping:

a) $y=3^{|x|}$; b) $y=-9^x$;

d) $y=|13^x-13|$; e) $y=\frac{1}{\left|4^{x^2}+1\right|}$.

8.71. Logarifmik funksiyaning xossalaridan foydalanib, sonlarni taqqoslang:

a) $\log_4 5$ va $\log_4 9$; b) $\log_{\frac{1}{5}} 8$ va $\log_{\frac{1}{5}} 15$;

d) $\log_9 7$ va $\log_8 7$; e) $\log_{\frac{1}{3}} 7$ va $\log_{\frac{1}{9}} 7$.

8.72. Ifodaning ishorasini aniqlang:

a) $\log_{0,8} 4-\log_{\frac{1}{2}} 5$; b) $\log_3 10-2$;

d) $\log_{0,2} 18-\log_{0,2} 17$; e) $\log_4 8-1$.

8.73. Funksiyaning aniqlanish sohasini toping:

a) $y=\log_3(3x+10)$; b) $y=\log_{30}(-12x)$;

d) $y=\log_5 x^2$; e) $y=\log_3(x^2-\sqrt{3})$;

f) $y=\log_{11}(9-x^2)$; g) $y=\log_{13}(13x^2+11)$;

h) $y=\log_4\sqrt{9x^2-16)}$; i) $y=\log_5|x^2-3x+10|$.

8.74. Agar barcha $x>0$ sonlar uchun

a) $\log_a(x^2+3)>\log_a x$; b) $\log_a(x^2+3)<\log_a x$

bo‘lsa, a qanday qiymatlar qabul qilishi mumkin?

8.75. Hisoblang:

a) $15^{1+\log_{15} 2}$; b) $4^{2+\log_4 9}$;

d) $17^{3\log_{17} 2}$; e) $8^{1-\log_2 3}$;

f) $\log_{\sqrt{5}} \sqrt{625}$; g) $\log_2 0{,}125 + \log_{\sqrt{3}} 9$;

h) $\log_{\sqrt[3]{7}} \sqrt{49}$; i) $\log_2 \log_2 \sqrt{\sqrt[4]{2}}$.

8.76. $\log_4 125 = a$ bo‘lsa, lg64 ni toping.

8.77. Ifodani soddalashtiring: $a^{\frac{\lg(\lg a)}{\lg a}} + \lg b^2 + \log_{100} a$.

8.78. Agar $y = 10^{\frac{1}{1-\lg x}}$ va $z = 10^{\frac{1}{1-\lg y}}$ bo‘lsa, $x = 10^{\frac{1}{1-\lg z}}$ bo‘lishini isbotlang.

Tenglamani yeching (**8.79−8.102**):

8.79. $(2(2^{\sqrt{x}+3})^{\frac{1}{2\sqrt{x}}})^{\frac{2}{\sqrt{x}-1}} = 4$. **8.80.** $\sqrt{2^{x^2-2x-10}} = \sqrt{33+\sqrt{128}} - 1$.

8.81. $\sqrt[x-1]{5^{x+3}}\,\sqrt[x^2-1]{5^{2(x-1)}} = \sqrt[x+1]{25^{x+4}}$.

8.82. $3^x + \sqrt{3^{x+2}\cdot 7^x} = 3\cdot 7^x + \sqrt{21^x}$.

8.83. $8(4^x + 4^{-x}) - 54(2^x + 2^{-x}) + 101 = 0$.

8.84. $0{,}5\lg(x+3) - 2\lg 2 = 1 - \lg\sqrt{25x+375}$.

8.85. $\lg^2(100^x) + \lg 2(10x) + \lg^2 x = 14$.

8.86. $\log_{2x^2-2}(3x^2 + x - 4) = \log_8 16 - \log_{27} 3$.

8.87. $\log_3(3^x = 8) = 2 - x$.

8.88. $2x + 1 = 2\log_2(9^x + 3^{2x-1} - 2^{x+3,5})$.

8.89. $x(1 - \lg 5) = \lg(2^x + x - 1)$.

8.90. $2(\lg 2 - 1) + \lg(5^{\sqrt{x}} + 1) = \lg(5^{1-\sqrt{x}} + 5)$.

8.91. $\log_3(3^x - 1)\cdot\log_3(3^{x+1} - 3) = 6$.

8.92. $x + \lg(1 + 2^x) = x\lg 5 + \lg 6$.

8.93. $\log_6(2^{\sqrt{x+1}} - 3) = \log_6 \log_{3\sqrt{3}} 9^{\frac{1}{3}} - \frac{\sqrt{x}}{2}\log_6 4$.

8.94. $7^{\lg x} - 5^{\lg x+1} = 3\cdot 5^{\lg x-1} - 13\cdot 7^{\lg x-1}$.

8.95. $\log_{2-2x^2}(2-x^2-x^4) = 2 - \dfrac{1}{\log_{\frac{3}{4}}(2-2x^2)}$.

8.96. $x^2\log_6\sqrt{5x^2-2x-3} - x\log_{\frac{1}{6}}(5x^2-2x-3) = x^2+2x$.

8.97. $\log_3 2 + \log_3\log_3(4-x) = \log_3\log_3(19-6x)$.

8.98. $\sqrt{2\log_8(-x)} - \log_8\sqrt{x^2} = 0$.

8.99. $\log_{3x+7}(9+12x+4x^2) + \log_{2x+3}(6x^2+23x+21) = 4$.

8.100. $\log_{1-2x}(6x^2-5x+1) + \log_{1-3x}(4x^2-4x+1) = 2$.

8.101. $\log_{x+1}(1-3x) = \log_{\sqrt{1-3x}}(1-2x-3x^2) - 1$.

8.102. $\sqrt{4-x}\cdot 4^{\log_2 x} + \log_3(x-2) = 9$, x – butun son.

Tengsizlikni yeching (**8.103–8.110**):

8.103. $\frac{1}{4}\cdot\left(\frac{1}{8}\right)^{x-2} < 3\left(\frac{1}{2}\right)^{x-1} + 2^x$.

8.104. $|3^x - 2| \le 1$.

8.105. $2^{|x+2|} > 16$.

8.106. $(\sqrt{5}+2)^{x-1} \ge (\sqrt{5}-2)^{\frac{x-1}{x+1}}$.

8.107. $\log_3\sqrt{x^2+x-2} < 1$.

8.108. $\sqrt{\log_2\left(\frac{3x-1}{2-x}\right)} < 1$.

8.109. $\left(\frac{1}{2}\right)^{\log_3\log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)} > 1$.

8.110. $(3^{x+3}+3^{-x})^{3\lg x - \lg(2x^2+3x)} < 1$.

Tenglamalar sistemasini yeching (**8.111–8.114**):

8.111. $\begin{cases} y^2 = 4^x + 8, \\ 2^{x+1} + y + 1 = 0. \end{cases}$

8.112. $\begin{cases} 3^x\cdot 5^y = 75, \\ 3^y\cdot 5^x = 45. \end{cases}$

8.113. $\begin{cases} \lg^2 x = \lg^2 y + \lg^2(x\cdot y), \\ \lg^2(x-y) + \lg x\cdot\lg y = 0. \end{cases}$

8.114. $\begin{cases} \log_5 x + 3^{\log_3 y} = 7, \\ x^y = 5^{12}. \end{cases}$

8.115. Tengsizliklar sistemasini yeching:

$$\begin{cases} x + 2{,}25y - \sqrt{x} - 1{,}5\sqrt{y} + 0{,}5 \le 0, \\ \sqrt{\log_x y} + \sqrt{\log_y x} \ge 2. \end{cases}$$