O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

Z.M. BOBUR NOMIDAGI ANDIJON DAVLAT UNIVERSITETI

Qo’lyozma xuquqida

UDK 512.83

G’oymatova Dilafruz

ЙИРИК МАСШТАБЛИ СИСТЕМАЛАР АБСОЛЮТ ТУРҒУНЛИГИНI TEKSHIRISH

5A130102- “Differentsial tenglamalar ” mutaxasisligi bo’yicha
magistr akademik darajasini olish
uchun magistrlik dissertatsiyasi

Ilmiy raxbar :
Fizika-matematika fanlari nomzodi

Andijon-2014

MUNDARAJA

KIRISH

Mavzuning dolzarbligi: Ma’lumki xayot xarakatdan iborat, shuning uchun xarakat bilan bog’liq bo’lgan masalalarni o’rganish va xal qilish katta axamiyatga ega.Bundan tashqari ko’plab murakkab jarayonlar yirik masshtabli sistemalar orqali modellashtiriladi .Bundan qaralayotgan mavzuning dolzarbligi kelib chiqadi.

Ishning maqsadi: Yirik masshtabli sistemalar turg’unligining umumiy masalasini va Lyapunov matritsa funktsiyasi usulini o’rganib , ba’zi yirik masshtabli sistemalar , Giroskopik stablizatsiya xarakatidagi orbital observatoriya turg`unligining , Neyron to`rlar turg`unligining yetarli shartlarini xosil qilish.

Tadqiqot usuli: Yirik masshtabli sistemalarni dekompozitsiya qilish usuli, Lyapunovning to’g’ri usuli , Lyapunovning matritsa funktsiyasi usuli.

Ilmiy yangiliklar: Dissertatsiyada quyidagi natijalar olindi:

1)yirik masshtabli sistemalar vertikal va gorizontal o’qlarga nisbatan dekompozitsiya qilindi ;

2)yirik masshtabli sistemalar o’zaro muvozanatlashuvchi qism sistemalarga nisbatan dekompozitsiya qilindi;

3) giroskopik stablizatsiya xarakatidagi orbital observatoriya turg`unligining etarli shartlari xosil qilindi ;

4) neyron to`rlar turg`unligining etarli shartlari xosil qilindi;

Amaliy va nazariy ahamiyati: Ишнинг назарий жихатдан ахамияти шундан иборатки , унда олинган назарий натижалар янги бўлиб , диссертациядан илмий изланишлар олиб боришда , шу ва бунга яқинроқ бўлган мавзулар бўйича махсус курслар ўқишда фойдаланиш мумкин.

Ishning aprobatsiyasi: **E'lon qilingan ishlari**: Mavzu bo'yicha 2013 yil Namangan shahrida bo'lib o'tgan respublika konferentsiyasida, 2013 yil Andijon shahrida bo'lib o'tgan hududiy konferentsiyada va matematika kafedrasi seminarlarida maruzalar qilingan.

Dissertatsiyaning tuzilishi va xajmi: Dissrtatsiya kirish, 3 ta bob, hulosa va foydalanilgan adabiyotlar ro'yhatidan tuzilgan bo'lib,umumiy hajmi 70 sahifadan iborat.

Dissertatsiyaning qisqacha mazmuni. Ushbu magistlik dissertatsiyasining kirish qismida mavzuning dolzarbligi, ishning maqsadi, tadqiqot usullari, ilmiy yangiliklari, amaliy va nazariy ahamiyati, aprobatsiyasi, dissertatsiyaning tuzilishi, hajmi va qisqacha mazmuni bayon qilingan.

1 -bob yirik macshtabli sistemalar turg'unligining umumiy masalasini o^rganishga bg^ishlangan bo^lib , unda masalaning qo'yilishi ,yirik macshtabli sistemalarning dekompozisiyasi,Lyapunovning matrisa funksiyasi usuli qarab chiqilgan.

Ko'plab jarayonlar yirik mastabli sistemalar (Y.M.S.) orqali modellashtiriladi. Bunday sistemalarning xarakterli belgilari quyidagilardir:

1. Ko'p o'lchovlilik;

2.Sistema strukturasining ko'p karraliligi (to'rlar, daraxtlar,ierarxik strukturalar va boshqalar);

3.Sistema elementlarining ko'p bog'lamliligi (qism sistemalar orasidagi bitta darajadagi va xar xil darajali ierarxiyalar orasidagi bog'lanishlar);

4.Elementlari tabiatining ko'pxilliligi (mashinalar, avtomatlar, robotlar, odam-operatorlar);

5.Sistema xolati va tarkibi o'zgarishining ko'pkarraliligi (sistema strukturasi, tarkibi va bog'lanishlarning o'zgaruvchanligi);

6.Sistemaning ko'p kriteriyaliligi (qism sistemalar uchun har-xil lokal kriteriyalar va to'la sistema uchun global kriteriyalar, ularning qarama-qarshililigi).

7.Sistema yoki uning elementlari tabiatiga xos bo'lgan ba'zi bir xususiy belgilar.

Bu belgilar sistema dinamik xossalarini (turg'unlik, boshqaruvchanlik, kuzatuvchanlik, optimallik) o'rganishda ma'lum qiyinchiliklarga olib keladi. SHuning uchun ko'plab mutaxassis olimlar tomonidan bir qancha usullar yaratildi. Jumladan, Lyapunov vektor funktsiyasi usuli , mini – maks usuli , Lyapunov matritsa – funktsiyasi usuli va boshqalar.

Ma'lumki, Lyapunov funktsiyasini qurishning umumiy algaritmi mavjud bo'lmaganligi uchun yuqoridagi xarakterli belgilarga ega bo'lgan sistemalar turg'unligini taxlili qilishda xosil bo'ladigan qiyinchiliklar Lyapunovning to'g'ri usulini effektiv ravishda qo'llash imkonini bermaydi.

Shuning uchun qaralayotgan sistemani soddalashtirish yoki boshqa sistema bilan almashtirish bilan bog'liq bo'lgan bir qancha yo'nalishlar paydo bo'lib, bularda turg'unlik (turg'unmaslik) masalasi bevosita berilgan sistemani tekshirish yo'li bilan emas, balki bu oralik sistemalarni tekshirish yo'li bilan xal etiladi.

Yirik masshtabli sistemalar turg'unligining umumiy masalasi quyidagicha ikki xil ko'rinishda qo'yiladi.

A muammo: Y.M.S. o'zaro bog'liq bo'lgan qism sistemalarga dekompazitsiya qilingan (ajratilgan) bo'lsin. Shunday shartlarni aniqlashimiz kerakki, bularda sistemaning Lyapunov bo'yicha turg'unligi o'zaro bog'liq bo'lmagan qism sistemalarning turg'unligidan va ular orasidagi bog'lanishlarning xossalaridan kelib chiqsin. Boshqacha aytganda, masalani bunday qo'yilishida, avval Y.M.S. ni tashkil qiluvchi har bir erkin qism sistema uchun Lyapunov funktsiyasi tuzilib, ularni turg'unligi (turg'unmasligi)ning yetarli shartlari xosil qilinadi. So'ngra bu qism sistemalar orasidagi bog'lanishlarning xossalaridan va erki n qism sistemalar turg'unligining yetarli shartlaridan foydalanib, berilgan sistema turg'unligining yetarli shartlari xosil qilinadi.

B muammo: Y.M.S. o'zaro bog'liq bo'lgan qism sistemalarga dekompazitsiya qilingan (ajratilgan) bo'lsin. Sistema tartibi pasayishini taminlaydigan shunday shartlarni aniqlashimiz kerakki, unda qaralayotgan sistemaning turg'unligi o'zaro bog'liq bo'lgan qism sistemalar xossalaridan kelib chiqib, bunda erkin qism sistemalar turg'unligi xaqidagi ma'lumotlardan foydalanilmaydi. Boshqacha aytganda masalaning bunday qo'yilishida, har bir qism sistema uchun Lyapunov funktsiyasi tanlanib, ular yordamida qaralayotgan sistema uchun Lyapunov funktsiyasi tuziladi va qaralayotgan sistema turg'unligining yetarli shartlari xosil qilinadi.

Bu ko'rsatilgan muammolar Y.M.S. turg'unligini tekshirish bilan shug'ulanuvchi mutaxassislar uchun bosh yo'nalishlarni aniqlab berdi. Ularni xal etish yo'lida bir qancha matematik usullar yaratildi yoki rivojlantirildi. Jumladan, Lyapunovning vektor-funktsiyasi usuli, vektorli normalar usuli, minimaksli usul, Lyapunov matritsa funktsiyasi usuli va boshqalar.

Lyapunovdan keyin olimlar (N.G.Chetaev, I.G.Malkin, N.N.Krasovskiy, V.M.Matrosov, A.A.Martinyuk va boshqalar) tomonidan Lyapunovning to`g`ri usuli rivojlantirildi va takomillashtirildi. Jumladan, V.M.Matrosov tomonidan 1962 yili Lyapunovning vektor funktsiyasi usuli yaratilgan bo`lib, A.A.Martinyuk tomonidan 1979 yili Lyapunovning matritsa-funktsiyasi usuli yaratildi. Xar ikkala usulning maqsadi murakkab sistemalar uchun tanlanadigan Lyapunovning skalyar funktsiyasini qurishni qisman bo`lsa xam alogritmlashdan iborat. Bunda qaralayotgan murakkab sistema bir nechta sodda qism sitesmalarga dekompozitsiya qilinadi, so`ngra xar bir qism sistema uchun Lyapunovning skalyar funktsiyasi tuzilib, ular yordamida umumiy sistema uchun Lyapunovning skalyar funktsiyasi tuziladi. Lyapunovning matritsa – funktsiyasi usuli Lyapunovning vektor – funktsiyasi usuliga qaraganda umumiyroq bo`lgani uchun Lyapunov matritsa-funktsiyasi usulini ko`rib chiqamiz.

Bu usulning moxiyati quyidagicha: Avval qaralayotgan sistema dekompozitsiya qilinib, uning erkli qism sistemalari va bu erkin qism sistemalarni o`zaro bog`lovchi funtsiyalar ajiratiladi, ya'ni qaralayotgan sistema

$$\dot{x}_i = f_i(x_i) + f_i^*(x) \qquad i = 1,2,..,s \qquad (1)$$

bu erda

$$x^T=(x_1,x_2,..,x_s),\ x_i \in R^{ni}, \quad x \in R^n,\ n_1+n_2+\ldots+n_s=n$$

ko`rinishga keltiriladi va bu sistemaning $0 \in R^{n_i}$, $i=1,\ldots,s$ xolatlari uchun $N_{ix} \subseteq R^{n_i}$ –ochiq bog`langan atroflar xamda x=0 nuqtaning $N_x \subseteq N_{1x} \times N_{2x} \times \ldots \times N_{sx}$ bog`langan atrofi mavjud bo`lsin deb olamiz.

$$\dot{x}_i = f_i(x_i), \qquad i=1,2,..s \qquad (2)$$

tenglamalar erkin qism sistemalarni, $f_i^*(x)$ fuktsiyalar esa bu erkin qism sistemalar orasidagi bog`lanishlarni ifodalaydi. So`ngra (1) va (2) sistemalar bilan birga quyidagi matritsa funktsiyani qaraymiz.

$$U(x)=\begin{bmatrix} v_{11}(x_1)\ v_{12}(x_1,x_2)\ v_{13}(x_1,x_3)\ldots\ldots\ldots v_{1s}(x_1,x_s) \\ v_{21}(x_1,x_2)\quad v_{22}(x_2)\quad v_{23}(x_2,x_3)\ldots\ldots v_{2s}(x_2,x_s) \\ \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots \\ v_{s1}(x_s,x_1)\quad v_{s2}(x_s,x_2)\quad v_{s3}(x_s,x_3)\ \ldots\ldots v_{ss}(x_s) \end{bmatrix} \qquad (3)$$

bu erda $v_{ij}(x_i,x_j)=v_{ji}(x_i,x_j)$ bo`lib, $v_{ii}(x_i)$ funktsiyalar (2) erkin qism sistemalarga qarab tanlanadi. $v_{ij}(x_i,x_j)$ funktsiyalar esa, $f_i^*(x)$ funktsiyalarga qarab shunday tanlanadiki, unda U(x) matritsa funktsiya musbat aniqlanganlik shartlarini qanoatlantiradi.

Endi U(x) matritsa-funktsiya va $\eta=(\eta_1,\eta_2,\ldots,\eta_s)$ o`zgarmas vektor yordamida quyidagi skalyar funktsiyani tuzamiz.

$$V(x)=\eta^T U(x)\eta \qquad (4)$$

Bu skalyar funktsiya musbat aniqlangan bo`lishi uchun U(x) matritsa-funktsiya elementlarini quyidagi shartlarni qanoat lantiradigan qilib tanlaymiz.

$$a)\ v_{ii}(x_i) \geq a_{ii}\|x_i\|^2, \quad \forall(t,x_i) \in \Im_0 \times N_{ix},\ i=1,2,..s \qquad \Im_0=[0,+\infty]$$

(*)

$$b) \quad v_{ij}(x_i, x_j) \geq a_{ij}\|x_i\|\|x_j\|, \qquad \forall(t, x_i, x_j) \in \Im_0 \times N_{ix} \times N_{jx},$$
$$i, j = 1,2,\ldots,s, \; i \neq j$$

Bu shartlar bajarilganda V(x) skalyar funktsiya uchun quyidagi tengsizlik o`rinli bo`ladi.

$$V(x) \geq u^T H^T AHu \qquad (5)$$

bu erda

$$u = (\|x_1\|, \|x_2\|, \ldots, \|x_s\|)^T, \quad H = diag(\eta_1, \eta_2, \ldots, \eta_s), \quad A = (a_{ij})_{i,j=1}^{s} \qquad (5)$$

tengsizlikdan ko`rinadiki,V(x) funktsiya musbat aniqlangan bo`lishi uchun A o`zgarmas matritsa musbat aniqlangan bo`lishi etarli. Shundan so`ng U(x) matritsa-funktsiyaning $v_{ij}(x_i, x_j)$ elementlaridan (1) sistema yordmida xosilalar olib, ularni yuqoridan baxolaymiz. Faraz qilaylik ular quyidagicha baxolangan bo`lsin.

$$a) \quad (D^+_{x_i} v_{ii}) f_i(x) \leq \rho_{1,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ j\neq i}}^{s} \rho_{1,i,j}\|x_i\|\|x_j\|, \;\; \forall(x_i, x_j) \in N_{ix0} \times N_{jx0}, \;\; i, j \in [1, s].$$

$$b) \quad (D^+ v_{ii})^T f_i^*(x) \leq \rho_{2,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ j\neq i}}^{s} \rho_{2,i,j}\|x_i\|\|x_j\|, \quad \forall x_i \in N_{ix0}, \; i \in [1, s],$$

$$c) \quad (D^+_{x_i} v_{ij})^T f_i(x) \leq \rho_{3,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ j\neq i}}^{s} \rho_{3,i,j}\|x_i\|\|x_j\| + \rho_{4,j}\|x_j\|^2, \;\; \forall(x_i, x_j) \in N_{ix0} \times N_{jx0}, \; i, j \in [1, s], \; i < j.$$

$$d) \quad (D^+_{x_i} v_{ij})^T f_i^*(x) \leq \rho_{5,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ i\neq j}}^{s} \rho_{4,i,j}\|x_i\|\|x_j\| + \rho_{6,j}\|x_j\|^2, \;\; \forall x_i \in N_{ix0}, \; i, j \in [1, s], \; i < j.$$

$$e) \quad (D^+_{x_j} v_{ij})^T f_i(x) \leq \rho_{7,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ j\neq i}}^{s} \rho_{5,i,j}\|x_i\|\|x_j\| + \rho_{8,j}\|x_j\|^2, \forall(x_i, x_j) \in N_{ix0} \times N_{jx0}, \; i, j \in [1, s], \; i < j$$

$$f) \quad (D^+_{x_j} v_{ij})^T f_i^*(x) \leq \rho_{9,i}\|x_i\|^2 + \sum_{\substack{j=1 \\ i\neq j}}^{s} \rho_{6,i,j}\|x_i\|\|x_j\| + \rho_{10,j}\|x_j\|^2, \forall x_i \in N_{ix0}, \; i, j \in [1, s], \; i < j.$$

(4) skalyar funktsiyadan (1) sistema yordamida yuqoridagi tengsizliklarni qanoatlantiruvchi xosila uchun quyidagi tengsizlik xosil bo`ladi.

$$\dot{V}(x) \le u^T G u \qquad (6)$$

bu erda

$$G = \{g_{i,j}\}_{i,j=1}^{\overline{s}}, \quad g_{i,j} = g_{i,j}.$$

$$g_{ii} = [\eta_i^2(\rho_{1,i} + \rho_{2,i}) + 2\sum_{\substack{j=1 \\ i<j}}^{s} \eta_i \eta_j (\rho_{3,i} + \rho_{4,j} + \rho_{5,i} + \rho_{6,j} + \rho_{7,i} + \rho_{8,j} + \rho_{9,i} + \rho_{10,j})].$$

$$g_{ij} = \frac{1}{2}\eta_i^2(\rho_{1,i,j} + \rho_{2,i,j}) + \eta_i \eta_j (\rho_{3,i,j} + \rho_{4,i,j} + \rho_{5,i,j} + \rho_{6,i,j})$$

(6) tengsizlikdan ko`rinadiki, $\dot{V}(x)$ xosila manfiy aniqlangan bo`lishi uchun G-o`zgarmas matritsani manfiy aniqlangan bo`lishi etarli.

Yuqoridagilarga asoslanib quyidagi teoremani keltiramiz.

Teorema 1. Agar (1) sistema uchun elementlari (*) shartlarni qanoatlantiruvchi U(x) matritsa – funktsiya mavjud bo`lib, quyidagi shartlar bajarilsa

A-matritsa musbat aniqlangan,

G- matritsa yarim manfiy aniqlangan.

U xolda (1) sistemaning x=0 echimi (toyimagan xarakat) turg`un bo`ladi.

2 – bobda Giroskopik stablizatsiya xarakatidagi orbital observatoriya turg`unligini taxlil qilinib , unda sistemaning matematik modeli Lyapunov matritsa funktsiyasini qurish sistema turg`unligining etarli shartlari qarab chiqilgan

Avval qaralayotgan sistemaning matematik modelini quyidagi ko^ rinishga keltirib olamiz

$$
\begin{aligned}
&\mu_2 \dot{x}_1 = S_1 x_2 \\
&\dot{x}_2 = -S_2 x_2 - MAx_3 + m + m_P \\
&\dot{x}_3 = -Hx_3 + MGx_2 + x_4 + m_\alpha \\
&\dot{x}_4 = -Dx_4 + f(\alpha)(N_1 x_1 + N_2 x_2) - N_3 x_3 + r \\
&\dot{\alpha} = x_3
\end{aligned}
\tag{7}
$$

Bu erda S_1, S_2 matritsalar va $\mu_j \quad j = 1,2,..,6$ o`zgarmaslar (2) soxada quyidagi shartlarni qanoatlantiradi

$$\underline{S}_1 \le S_1 \le \overline{S}_1, \quad \underline{S}_2 \le S_2 \le \overline{S}_2, \qquad \mu_j \in (0,1]$$

$$\underline{S}_1 = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \overline{S}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \underline{S}_2 = \begin{pmatrix} 0 & 0 & \Delta a_1 P_2^* - a_{12} \\ \Delta a_2 P_3^* - a_{23} & 0 & 0 \\ 0 & \Delta a_3 P_1^* - a_{31} & 0 \end{pmatrix}$$

$$\overline{S}_2 = \begin{pmatrix} 0 & a_{13} & \Delta a_1 P_1^* \\ \Delta a_2 P_3^* & 0 & a_{21} \\ a_{32} & \Delta a_3 P_1^* & 0 \end{pmatrix}$$

(7) sistema uchun

$$U(x_1, x_2, x_3, x_4, \alpha, \mu_2, M) = \left[v_{ij}(\cdot)\right], \qquad v_{ij} = v_{ji}, \quad i, j = 1,2,3,\ldots,5 \tag{8}$$

matritsa–funktsiya elementlarini quyidagi ko`rinishda quramiz.

$$
\begin{aligned}
&v_{11}(x_1, \mu_2) = \mu_2 x_1^T B_{11} x_1, && v_{12}(x_1, x_2, \mu_2) = \mu_2 x_1^T B_{12} x_2, \\
&v_{13} = 0, && v_{14}(x_1, x_4, \mu_2, M) = \mu_2 x_1^T B_{14}(Mx_2), \\
&v_{15} = 0, && v_{22}(x_2) = x_2^T B_{22} x_2, \\
&v_{23}(x_2, x_3) = x_2^T B_{23} x_3, && v_{24}(x_2, x_4, M) = x_2^T B_{24}(Mx_4), \\
&v_{25}(x_2, \alpha) = x_2^T B_{25} \alpha, && v_{33}(x_3) = x_3^T B_{33} x_3, \\
&v_{34}(x_{31}, x_{41}, M) = x_3^T B_{34}(Mx_4), && v_{35}(x_3, \alpha) = x_3^T B_{35} \alpha, \\
&v_{44}(x_4, M) = (Mx_4)^T B_{44} x_4, \quad v_{45} = 0, && v_{55}(\alpha) = \alpha^T B_{55} \alpha
\end{aligned}
\tag{9}
$$

Bu erda $B_{ii,}$ i=1,2,3,4,5- musbat aniqlangan simmetrik matritsalar, B_{11}, B_{12}, B_{13},B_{23}, B_{24}, B_{25}, B_{34}, B_{35}-o`zgarmas matritsalar.

(8) matritsa–funktsiya va $\eta = (1,1,1,1,1)^T$ o`zgarmas vektor yordamida quyidagi skalyar funktsiyani tuzamiz.

$$V(x_1, x_2, x_3, x_4, \alpha, \mu_2, M) = \eta^T U(x_1, x_2, x_3, x_4, \alpha, \mu_2, M)\eta \qquad (10)$$

Sistemaning tug^unligi masalasini xal qilish uchun (10) funksiyani quyidan baholab , uni musbat aniqlanganlik shartlarini xosil qilamiz. So^ngra bu funksiyadan (7) sistema yordamida olingan xosilani yuqoridan baholab ,uni manfiy aniqlanganlik shartlarini xosil qilamiz.

3 – bob neyron to`rlar matematik modelining taxliliga bag’ishlangan bo’lib, unda quyidagi chiziqsiz, avtonom oddiy differentsial tenglamani qaraymiz:

$$\dot{x} = -H(x)(-Tx + S(x) - I) \qquad (11)$$

bu erda

$$x = (x_1, x_2, \ldots, x_n)^T \in (-1,1)^n, \qquad \dot{x} = \frac{dx}{dt},$$

$H(x) - \forall x \in (-1,1)^n$ da aniqlangan $n \times n$ tartibli matritsa funktsiya, $T = (T_{ij})\ n \times n$ o`lchovli o`zgarmas matritsa

$S(x) = (S_1(x_1), \ldots, S_n(x_n))^T$ *bu yerda* $S_1 : (-1,1) \to R, \quad i = \overline{1,n}$

$I = (I_1, .., I_n)^T$ o`zgarmas xaqiqiy vektor.

(1.1) sistema quyidagi ko`rinishda yoziladi.

$$\dot{x}_i = \sum_{j=1}^{S} H_{ij}(x_j)(T_{ij}x_j - S_j(x_j) + I_j) = f_i(x) \qquad (12)$$

bu erda

$H(x) = (H_{ij}(x_j))_{i,j=1}^{S}, \quad H_{ij}(x) - n_i \times n_j$ o`lchovli matritsa funktsiya,

$T = (T_{ij})_{i,j=1}^{S}, \quad T_{ij} - n_i \times n_j$ o`lchovli o`zgarmas matritsa,

$S(x) = (S_1^T(x_1),....,S_s^T(x_s))^T$, $S_i(x) - n_i$ o`lchovli vektor funktsiya,

$I = (I_1^T, I_2^T, ..., I_S^T)^T$, $I_i - n_i$ o`lchovli o`zgarmas vektor,

$x = (x_1^T, x_2^T, ..., x_s^T)^T$, $n_1 + n_2 + n_S = n$

(12) sistemadan erkin qism sistemalarni ajratib olish uchun

$$x^i = (0^T, 0^T, .., x_i^T, ..., 0^T) \in (-1,1)^n$$

belgilashni kiritamiz. U xolda i chi erkin qism sistema quyidagi

$$x_i = -H_{ii}(x^i)(-T_{ii}x_i + S_i(x^i) - I_i) = f_i(x^i) \qquad (13)$$

(bu erda $x_i \in (-1,1)^n$, $n_1 + n_2 + ..n_S = n$) ko`rinishda bo`lib,

$$f_i^*(x) = f_i(x) - f_i(x^i)$$

tenglik bilan aniqlangan $f_i^*(x)$ funktsiya (12) sistemadagi erkin qism sistemalarni o`zaro aloqalarni ifodalaydi. Natijada (12) sistemani quyidagicha yozishimiz mumkin.

$$\dot{x} = f_i(x^i) + f_i^*(x), \quad i = \overline{1,S} \qquad (14)$$

Agar $x = €$ echimning turg`unligini aniqlash kerak bo`lsa, (14) sistemada $x = x - €$ almashtirish qilib, uni x=0 echimga keltirish mumkin ekanligini xisobga olsak, (14) sistemaning x=0 echimini turg`unlik shartlarini xosil qilishning umumiy nazariyasini quramiz. Buning uchun quyidagi farazlarni kiritamiz.

Faraz 1: (14) sistema uchun U(x) matritsa -funktsiya mavjud bo`lib, bu matritsa-funktsiya va $\eta = (\eta_1, \eta_2, ..., \eta_S)^T$ o`zgarmas vektor yordamida tuzilgan V(x) skalyar funktsiya uchun

$$V(x) \geq U^T H^T AHU$$

tengsizlik o`rinli bo`lsin.

Faraz 2: U(x) matritsa-funktsiya elementlaridan (13) va (14) sistemalar bo`yicha olingan xosilalar uchun quyidagi tengsizlik o`rinli bo`ladigan $\rho_{ii}, \quad i=\overline{1,S} \quad va \quad \rho_{ij}, \; i,j=\overline{1,S}, \quad i<j$ o`zgarmaslar mavjud bo`lsin:

$$\sum_{i=1}^{S}\eta_i^2 \left\{D_t^+ v_{ii} + (D_{x_i}^+ v_{ii})^T f_i(x^i) + (D_{x_i}^+ v_{ii})^T f_i^*(x^i)\right\} + 2\sum_{i=1}^{S-1}\sum_{j=2}^{S}\eta_i\eta_j \left\{D_t^+ v_{ij} + (D_{x_i}^+ v_{ij})^T f_i(x^i) + (D_{x_j}^+ v_{ij})^T f_j(x^j) + \right.$$

$$\left. + (D_{x_i}^+ v_{ij})^T f^*(x) + (D_{x_j}^+ v_{ij})^T f_j^*(x)\right\} \le \sum_{i=1}^{S}\rho_{ii}\|x_i\|^2 + 2\sum_{i=1}^{S}\sum_{\substack{j=2\\ j>i}}^{S}\rho_{ij}\|x_i\|\|x_j\|$$

Lemma : Agar faraz 2 dagi shartlar bajarilsa V(x) skalyar funktsiyadan (3.6) sistema yordamida olingan xosila uchun quyidagi tengsizlik o`rinli bo`ladi.

$$D^+ v(x,\eta) \le U^T G U \qquad (15)$$

bu erda $U^T = (\|x_1\|, \|x_2\|, \ldots, \|x_S\|), \quad G = \left[\rho_{ij}\right], \; \rho_{ij} = \rho_{ji}, \; i,j=\overline{1,S}.$

Teorema 2: Agar (14) sistema uchun faraz 1 va faraz 2 dagi shartlar bajarilib, A matritsa musbat aniqlangan , G matritsa manfiy aniqlangan bo`lsa, u xolda bu sistemaning x=0 echimi asimptotik turg`un bo`ladi.

1-BOB

YIRIK MASSHTABLI SISTEMALAR TURG'UNLIGINING UMUMIY MASALASI.

Ko'plab jarayonlar yirik mastabli sistemalar (Y.M.S.) orqali modellashtiriladi. Bunday sistemalarning xarakterli belgilari quyidagilardir:

2. Ko'p o'lchovlilik;

2.Sistema strukturasining ko'p karraliligi (to'rlar, daraxtlar,ierarxik strukturalar va boshqalar);

3.Sistema elementlarining ko'p bog'lamliligi (qism sistemalar orasidagi bitta darajadagi va xar xil darajali ierarxiyalar orasidagi bog'lanishlar);

4.Elementlari tabiatining ko'pxilliligi (mashinalar, avtomatlar, robotlar, odam-operatorlar);

5.Sistema xolati va tarkibi o'zgarishining ko'pkarraliligi (sistema strukturasi, tarkibi va bog'lanishlarning o'zgaruvchanligi);

6.Sistemaning ko'p kriteriyaliligi (qism sistemalar uchun har-xil lokal kriteriyalar va to'la sistema uchun global kriteriyalar, ularning qarama-qarshililigi).

7.Sistema yoki uning elementlari tabiatiga xos bo'lgan ba'zi bir xususiy belgilar.

Bu belgilar sistema dinamik xossalarini (turg'unlik, boshqaruvchanlik, kuzatuvchanlik, optimallik) o'rganishda ma'lum qiyinchiliklarga olib keladi. SHuning uchun ko'plab mutaxassis olimlar tomonidan bir qancha usullar yaratildi. Jumladan, Lyapunov vektor funktsiyasi usuli , mini – maks usuli , Lyapunov matritsa – funktsiyasi usuli va boshqalar. Biz bu bobda Lyapunov matritsa – funktsiyasi usulini qarab chiqamiz.

§1.1. Masalaning qo'yilishi

Ma'lumki, Lyapunov funktsiyasini qurishning umumiy algaritmi mavjud bo'lmaganligi uchun yuqoridagi xarakterli belgilarga ega bo'lgan sistemalar turg'unligini taxlili qilishda xosil bo'ladigan qiyinchiliklar Lyapunovning to'g'ri usulini effektiv ravishda qo'llash imkonini bermaydi.

Shuning uchun qaralayotgan sistemani soddalashtirish yoki boshqa sistema bilan almashtirish bilan bog'liq bo'lgan bir qancha yo'nalishlar paydo bo'lib, bularda turg'unlik (turg'unmaslik) masalasi bevosita berilgan sistemani tekshirish yo'li bilan emas, balki bu oralik sistemalarni tekshirish yo'li bilan xal etiladi.

Yirik masshtabli sistemalar turg'unligining umumiy masalasi quyidagicha ikki xil ko'rinishda qo'yiladi.

A muammo: Y.M.S. o'zaro bog'liq bo'lgan qism sistemalarga dekompazitsiya qilingan (ajratilgan) bo'lsin. Shunday shartlarni aniqlashimiz kerakki, bularda sistemaning Lyapunov bo'yicha turg'unligi o'zaro bog'liq bo'lmagan qism sistemalarning turg'unligidan va ular orasidagi bog'lanishlarning xossalaridan kelib chiqsin. Boshqacha aytganda, masalani bunday qo'yilishida, avval Y.M.S. ni tashkil qiluvchi har bir erkin qism sistema uchun Lyapunov funktsiyasi tuzilib, ularni turg'unligi (turg'unmasligi)ning yetarli shartlari xosil qilinadi. So'ngra bu qism sistemalar orasidagi bog'lanishlarning xossalaridan va erki n qism sistemalar turg'unligining yetarli shartlaridan foydalanib, berilgan sistema turg'unligining yetarli shartlari xosil qilinadi.

B muammo: Y.M.S. o'zaro bog'liq bo'lgan qism sistemalarga dekompazitsiya qilingan (ajratilgan) bo'lsin. Sistema tartibi pasayishini taminlaydigan shunday shartlarni aniqlashimiz kerakki, unda qaralayotgan sistemaning turg'unligi o'zaro bog'liq bo'lgan qism sistemalar xossalaridan kelib chiqib, bunda erkin qism sistemalar turg'unligi xaqidagi ma'lumotlardan foydalanilmaydi. Boshqacha aytganda masalaning bunday qo'yilishida, har bir qism sistema uchun Lyapunov funktsiyasi tanlanib, ular yordamida qaralayotgan sistema uchun Lyapunov funktsiyasi tuziladi va qaralayotgan sistema turg'unligining yetarli shartlari xosil qilinadi.

Bu ko'rsatilgan muammolar Y.M.S. turg'unligini tekshirish bilan shug'ulanuvchi mutaxassislar uchun bosh yo'nalishlarni aniqlab berdi. Ularni xal etish yo'lida bir qancha matematik usullar yaratildi yoki rivojlantirildi. Jumladan, Lyapunovning vektor-funktsiyasi usuli, vektorli normalar usuli, minimaksli usul, Lyapunov matritsa funktsiyasi usuli va boshqalar.

§1.2. Yirik masshtabli sistemalarning dekompozitsiyasi.

Faraz qilaylik (S). Y.M.S. xolati quyidagi differentsial tenglama bilan ifodalansin.

$$\frac{dx}{dt} = f(t, x), \tag{1.1}$$

bu yerda $t \in T = [t_0, \quad \infty) \subset R, \qquad x \in R^n, f \in F$ bo'lib, F oila quyidagicha aniqlanadi:

$$F = \{f^1, f^2, \dots, f^N\}, f^k: TxR^n \to \mathbf{R}^n, \tag{1.2}$$

N-haqiqiy son.

Agar (S). Y.M.S. s ta qism sistemalardan tashkil topgan bo'lsa, u xolda (S). Y.M.S.ni ($S_i i$) o'zaro bog'liq bo'lgan qism sistemalardan tashkil topgan deb qarash mumkin bo'lib, ($S_i i$) o'zaro bog'liq bo'lgan qism sistemalar quyidagi tenglamalar bilan ifodalanadi:

$$\forall i = 1,2, \dots, s \tag{1.3}$$

bu yerda x va f vektorlar quyidagicha aniqlanadi:

$$x = (x_1^T, x_2^T, \dots, x_s^T)^T, \qquad f = (f_1^T, f_2^T, \dots, f_s^T)^T,$$

shuningdek,

$$f_i \in F_i, \qquad F_i = (f_i^1, \qquad f_i^2, \dots, \qquad f_i^N), \qquad f_i^k : \mathrm{T}x R^n \to R^{n_i},$$

$$\forall\, k = 1,2,\dots,N, \qquad n_1 + n_2 + \cdots n_s = n, \qquad i = 1,2,\dots,s.$$

$\mathbf{f}_i$ funktsiyalar barcha $t \in R$ da, faqat va faqat x=0 dagina

$$f(t,x)=0$$

shartni qanoatlantiradi.

$$x^i = (O^T, O^T, \dots, x_i^T, O^T, \dots, O^T)^T, \qquad i = 1,2,\dots,s$$

belgilash kiritamiz,

$$g_i : \mathrm{T}x R^{n_i} \to R^{n_i},$$

funktsiyani

$$g_i(t,x_i) = f_i(t,x^i)$$

tenglik bilan aniqlaymiz.

U xolda $[(\hat{S}]_i)$ i- erkin qism sistema

(1.4)

bu yerda $x_i \in R^{n_i}$,

$$f_i^*(t,x) = f_i(t,x) - g_i(t,x_i), \qquad \forall\, i = 1,2,\dots,s \qquad (1.5)$$

ifoda yordamida aniqlangan f_i^* funktsiya (S) sistemani o'zining $[(S]_i)$ i- qism sistemasiga ta'sirini ifodalaydi.

(1.4) va (1.5) ga asosan (1.3) ni quyidagicha yozish mumkin:

(1.6)

Agar

$$g=(g_1^T, g_2^T, \dots, g_S^T)^T, \qquad f^{\bullet}=(f_1^{\bullet T}, f_2^{\bullet T}, \dots, f_S^{\bullet T})^T,$$

deb olsak, (1.6) ni quyidagicha yozish mumkin:

$$\frac{dx}{dt}=g(t,x)+f^{\bullet}(t,x), \tag{1.7}$$

(S) Y.M.S. ni (1.6) ko'rinishda dekompozitsiya qilish nazariy bo'lib, amaliy tadbiqlarda bu bir muncha noqulaydir. Masalan (1.5) tenglik yordamida $f_i^{\bullet}$ funktsiyalarni har doim ham aniqlash qulay bo'lavermaydi. SHuning uchun (S) Y.M.S. ni dekompozitsiya qilishni qulaylashtirish maqsadida har bir s ta $\left([\hat{S}]_i\right)$ qism sistemalardan iborat bo'lgan Y.M.S. bilan sxs tartibli matritsa o'rtasida quyidagicha o'zaro bir qiymatli mosli o'rnatamiz:

1. (S) Y.M.S.ning $\left([\hat{S}]_i\right)$ erkin qism sistemalarini matritsaning bosh dioganaliga mos qo'yamiz;

2. qism sistemalarni $\left([\hat{S}]_j\right)$ qism sistemaga ta'sirini ifodalovchi funktsiyani i-satr va j-ustun kesishgan joyga mos qo'yamiz. Natijada matritsaning bosh dioganalida erkin qism sistemalar bo'lib, bosh dioganaldan tashqarida bu erkin qism sistemalar orasidagi to'g'ri va teskari bog'lanishlar bo'ladi.

Bunday moslik o'rnatilgandan keyin (1.1) sistema quyidagi ko'rishlarning biriga keladi:

$$\frac{dx}{dt}=Ax, \tag{1.8}$$

$$\frac{dx}{dt}=A(t)x, \tag{1.9}$$

$$\frac{dx}{dt}=A(x), \tag{1.10}$$

$$\frac{dx}{dt}=A(t,x), \tag{1.11}$$

bu yerda A, A(t), A(x), A(t,x) - bosh dioganalga nisbatan simmetrik bo'lgan sxs tartibli kvadrat matritsalar.

(1.8), (1.9), (1.10), (1.11) sistemalarni dekompozitsiya qilish masalasi mos ravishda A, A(t), A(x), A(t,x)-matritsalarni blok matritsalarga ajratish masalasi bilan teng kuchli bo'lgani uchun (S) Y.M.S.ni dekompozitsiya qilish masalasi matritsani blok matritsalarga ajratish masalasiga keladi.

Misol tariqasida (1.8) sistemani ba'zi xususiy xollarda dekompazitsiya qilish usullarini qarab chiqamiz. Faraz qilaylik, A matritsa markazga nisbattan simmetrik bo'lgan sxs o'lchovli kvadrat matritsa bo'lsin. U xolda (1.8) sistemani quyidagicha usullarda dekompaziya qilish mumkin:

1.Vertikal va gorizantal simmetriya o'qlariga nisbatan dekompazitsiya qilish. Bu xolda (1.8) sistema n=2k da,

$$\begin{aligned} \dot{y} &= A_1 y + B_1 z, \\ \dot{z} &= B_1^0 y + A_1^0 z, \end{aligned} \tag{1.12}$$

ko'rinishga , n=2k+1 da esa

$$\begin{aligned} \dot{y} &= A_1 y + a_2 x_{k+1} + B_1 z, \\ \dot{x}_{k+1} &= a_1^T y + a_{k+1,k+1} x_{k+1} + (a_1^T)^0 z \\ \dot{z} &= B_1^0 y + a_2^0 x_{k+1} + A_1^0 z, \end{aligned} \tag{1.13}$$

ko'rinishga keladi. Bu yerda erkin qism sistemalar

$$\dot{y} = A_1 y, \tag{1.14}$$

$$\dot{z} = A_1^0 z, \tag{1.15}$$

$$\dot{x}_{k+1} = a_{k+1,k+1} x_{k+1}. \tag{1.16}$$

ko'rinishlarda bo'lib, A_1, B_1- k-tartibli kvadrat matritsalar, A_1^0, B_1^0 -mos ravishda ularni markazga nisbatan transponirlangani [21], vektorlar esa

$$a_1 = (a_{k+1,1}, a_{k+1,2}, ..., a_{k+1,k})^T,\ a_2 = (a_{k+1,k+2}, a_{k+1,k+3}, ..., a_{k+1,n})^T,$$

$$y = (x_1, x_2, ..., x_k)^T,\ z = (x_{k+2}, x_{k+3}, ..., x_n)^T,\ x = (y^T, x_{k+1}, z^T)^T.$$

ko'rinishda aniqlangan.

(1.8) sistema muvozanat xolati turg'unligining yetarli shartlarini xosil qilish uchun, (1.12) sistema va (1.14), (1.15) qism sistemalarga mos Lyapunov matritsa funktsiyasi quyidagi ko'rinishda tanlanadi:

$$U_1(y,z)=\begin{pmatrix} v_{11}(y) & v_{12}(y,z) \\ v_{21}(y,z) & v_{22}(z)\end{pmatrix},\quad v_{12}=v_{21}\ , \tag{1.17}$$

bu yerda

$$v_{11}(y)=y^T P_1 y,\ \ v_{22}(z)=z^T P_1^0 z,\ \ v_{12}(y,z)=v_{21}(y,z)=y^T P_2 z,$$

$P_1,\ P_1^0$ - bosh dioganalga nisbatan simmetrik bo'lgan musbat aniqlangan matritsalar, P_2 - o'zgarmas matritsa bo'lib, bularning barchasi k-tartibli matritsalardir.

(1.13)sistema va (1.14), (1.15), (1.16) qism sistemalarga mos Lyapunov matritsa funktsiyasi esa, quyidagicha tanlanadi.

$$U_2(y,x_{k+1},z)=(v'_{ij}),\quad i,j=1,2,3,\quad v'_{ij}=v'_{ji} \tag{1.18}$$

bu yerda

$$v'_{11}=v_{11}(y),\ \ v'_{12}=\alpha_{12}x_{k+1}y,\ \ v'_{13}=v_{12}(y,z),\ \ v'_{21}=v'_{12},\quad v'_{22}=\alpha_{22}x_{k+1}^2,$$

$$v'_{23}=\alpha_{23}x_{k+1}z\ ,\ \ v'_{33}=v_{22}(z),\ \ v'_{31}=v'_{13}, v'_{32}=v'_{23}\quad ,$$

$P_1, P_1^0,\ P_2$ -matritsalar (1.17) dagidek aniqlanadi, $\alpha_{22}>0,\ \alpha_{12},\alpha_{23}$ -lar xaqiqiy sonlar

2.O'zaro muvozanatlashuvchi qism sistemalarga nisbatan dekompazitsiya qilish. Bu xolda (1.8) sistema n=2k da

$$\dot{y}_i=A_{ii}y_i+\sum_{\substack{j=1\\ i\neq j}}^{k}A_{ij}y_j,\quad i=1,2,\ldots,k=\frac{n}{2} \tag{1.19}$$

ko'rinishga, n=2k+1 da esa,

$$\begin{aligned} &\dot{y}_i=A_{ii}y_i+\sum_{\substack{j=1\\ i\neq j}}^{k}A_{ij}y_j+a_i x_{k+1},\quad i=1,2,\ldots,k=\frac{n-1}{2}, \\ &\dot{x}_{k+1}=a_{k+1,k+1}x_{k+1}+\sum_{i=1}^{k}a_i y_i, \end{aligned} \tag{1.20}$$

ko'rinishga keladi. Bu yerda

$$A_{ii}=\begin{pmatrix} a_{ii} & a_{i,n+1-i} \\ a_{i,n+1-i} & a_{ii} \end{pmatrix}, \qquad A_{ij}=\begin{pmatrix} a_{ij} & a_{i,n+1-j} \\ a_{i,n+1-j} & a_{ij} \end{pmatrix},$$

$$a_i=(a_{i,k+1},a_{i,k+1})^T, \qquad a_j=(a_{k+1,j},a_{k+1,j})^T,$$

$$y_i=(x_i,x_{n+1-i})^T, \qquad y=(y_1^T,y_2^T,\ldots,y_k^T)^T,\ i,j=1,2,\ldots,k,\ i\neq j,$$

bo'lib, A_{ii} va A_{ij} lar matritsa markaziga nisbatan simmetrik bo'lgan matritsalardir. Bu xolda i- erkin qism sistemalar

$$\dot{y}_i=A_{ii}y_i \tag{1.21}$$

ko'rinishda bo'lib, ularning muvozanat xolati turg'unligini yetarli shartlari

$$a_{ii}<0,\ |a_{ii}|>|a_{i,n+1-i}|,\ i=1,2,\ldots,k \tag{1.22}$$

shartlardan,

$$\dot{x}_{k+1}=a_{k+1,k+1}x_{k+1} \tag{1.23}$$

qism sistema uchun muvozanat xolat turg'unligining yetarli sharti

$$a_{k+1,k+1}<0. \tag{1.24}$$

dan iborat bo'ladi. (1.19) va (1.20) sistemalar, hamda (1.21), (1.23) qism sistemalar uchun yuqoridagi kabi Lyapunov matritsa funktsiyalarini tuzish mumkin.

§ 2.3 Lyapunovning matritsa-funktsiyasi usuli

Lyapunovdan keyin olimlar (N.G.Chetaev, I.G.Malkin, N.N.Krasovskiy, V.M.Matrosov, A.A.Martinyuk va boshqalar) tomonidan Lyapunovning to`g`ri usuli rivojlantirildi va takomillashtirildi. Jumladan, V.M.Matrosov tomonidan 1962 yili Lyapunovning vektor funktsiyasi usuli yaratilgan bo`lib, A.A.Martinyuk tomonidan 1979 yili Lyapunovning matritsa-funktsiyasi usuli yaratildi.

Ma'lumki, Lyapunov funktsiyasini qurish algoritmi mavjud emas, bu xol murakkab sistemalar uchun Lyapunov funktsiyasini tanlashni qiyinlashtiradi.

Sodda sistemalar uchun bunday qiyinchilik bo`lmaydi. Yuqorida aytilgan xar ikkala usulning maqsadi murakkab sistemalar uchun tanlanadigan Lyapunovning skalyar funktsiyasini qurishni qisman bo`lsa xam alogritmlashdan iborat. Bunda qaralayotgan murakkab sistema bir nechta sodda qism sitesmalarga dekompozitsiya qilinadi, so`ngra xar bir qism sistema uchun Lyapunovning skalyar funktsiyasi tuzilib, ular yordamida umumiy sistema uchun Lyapunovning skalyar funktsiyasi tuziladi. Lyapunovning matritsa – funktsiyasi usuli Lyapunovning vektor – funktsiyasi usuliga qaraganda umumiyroq bo`lgani uchun Lyapunov matritsa-funktsiyasi usulini ko`rib chiqamiz.

Bu usulning moxiyati quyidagicha: Avval qaralayotgan sistema dekompozitsiya qilinib, uning erkli qism sistemalari va bu erkin qism sistemalarni o`zaro bog`lovchi funtsiyalar ajiratiladi, ya'ni qaralayotgan sistema

$$\dot{x}_i = f_i(x_i) + f_i^*(x) \qquad i = 1,2,..,s \qquad (1.25)$$

bu erda

$$x^T = (x_1, x_2, .., x_s), \ x_i \in R^{ni}, \quad x \in R^n, n_1 + n_2 + ... + n_s = n$$

ko`rinishga keltiriladi va bu sistemaning $0 \in R^{n_i}, \ i = 1,...,s$ xolatlari uchun $N_{ix} \subseteq R^{n_i}$ –ochiq bog`langan atroflar xamda x=0 nuqtaning $N_x \subseteq N_{1x} \times N_{2x} \times ... \times N_{sx}$ bog`langan atrofi mavjud bo`lsin deb olamiz.

$$\dot{x}_i = f_i(x_i), \qquad i = 1,2,..s \qquad (1.26)$$

tenglamalar erkin qism sistemalarni, $f_i^*(x)$ fuktsiyalar esa bu erkin qism sistemalar orasidagi bog`lanishlarni ifodalaydi. So`ngra (1.25) va (1.26) sistemalar bilan birga quyidagi matritsa funktsiyani qaraymiz.

$$U(x) = \begin{bmatrix} v_{11}(x_1) \ v_{12}(x_1, x_2) \ v_{13}(x_1, x_3) .... v_{1s}(x_1, x_s) \\ v_{21}(x_1, x_2) \quad v_{22}(x_2) \quad v_{23}(x_2, x_3) ..... v_{2s}(x_2, x_s) \\ .... \quad .... \quad .... \quad .... \\ v_{s1}(x_s, x_1) \quad v_{s2}(x_s, x_2) \quad v_{s3}(x_s, x_3) \ .... v_{ss}(x_s) \end{bmatrix} \qquad (1.27)$$

bu erda $v_{ij}(x_i,x_j)=v_{ji}(x_i,x_j)$ bo`lib, $v_{ii}(x_i)$ funktsiyalar (1.26) erkin qism sistemalarga qarab tanlanadi. $v_{ij}(x_i,x_j)$ funktsiyalar esa, $f_i^*(x)$ funktsiyalarga qarab shunday tanlanadiki, unda U(x) matritsa funktsiya musbat aniqlanganlik shartlarini qanoatlantiradi.

Endi U(x) matritsa-funktsiya va $\eta=(\eta_1,\eta_2,\ldots,\eta_s)$ o`zgarmas vektor yordamida quyidagi skalyar funktsiyani tuzamiz.

$$V(x)=\eta^T U(x)\eta \qquad (1.28)$$

Bu skalyar funktsiya musbat aniqlangan bo`lishi uchun U(x) matritsa-funktsiya elementlarini quyidagi shartlarni qanoat lantiradigan qilib tanlaymiz.

$$a)\ \ v_{ii}(x_i)\geq a_{ii}\|x_i\|^2,\ \ \forall(t,x_i)\in\Im_0\times N_{ix},\ i=1,2,..s \qquad \Im_0=[0,+\infty]$$

(*)

$$b)\quad v_{ij}(x_i,x_j)\geq a_{ij}\|x_i\|\|x_j\|,\qquad \forall(t,x_i,x_j)\in\Im_0\times N_{ix}\times N_{jx},$$
$$i,j=1,2,\ldots,s,\ i\neq j$$

Bu shartlar bajarilganda V(x) skalyar funktsiya uchun quyidagi tengsizlik o`rinli bo`ladi.

$$V(x)\geq u^T H^T AHu \qquad (1.29)$$

bu erda

$$u=(\|x_1\|,\|x_2\|,\ldots,\|x_s\|)^T,\quad H=diag(\eta_1,\eta_2,\ldots,\eta_s),\quad A=\{a_{ij}\}_{i,j=1}^{s} \qquad (1.29)$$

tengsizlikdan ko`rinadiki,V(x) funktsiya musbat aniqlangan bo`lishi uchun A o`zgarmas matritsa musbat aniqlangan bo`lishi etarli. Shundan so`ng U(x) matritsa-funktsiyaning $v_{ij}(x_i,x_j)$ elementlaridan (1.25) sistema yordmida xosilalar olib, ularni yuqoridan baxolaymiz. Faraz qilaylik ular quyidagicha baxolangan bo`lsin.

$$a)\quad (D^{+}_{x_i} v_{ii}) f_i(x) \le \rho_{1,i}\|x_i\|^2 + \sum_{\substack{j=1\\ j\ne i}}^{s} \rho_{1,i,j}\|x_i\|\|x_j\|,\quad \forall (x_i, x_j) \in N_{ix0}\times N_{jx0},\quad i, j\in[1,s].$$

$$b)\quad (D^{+} v_{ii})^T\ f_i^{*}(x) \le \rho_{2,i}\|x_i\|^2 + \sum_{\substack{j=1\\ j\ne i}}^{s} \rho_{2,i,j}\|x_i\|\|x_j\|,\quad \forall x_i \in N_{ix0},\ i\in[1,s],$$

$$c)\quad (D^{+}_{x_i} v_{ij})^T\ f_i(x) \le \rho_{3,i}\|x_i\|^2 + \sum_{\substack{j=1\\ j\ne i}}^{s} \rho_{3,i,j}\|x_i\|\|x_j\| + \rho_{4,j}\|x_j\|^2,\ \ \forall (x_i, x_j)\in N_{ix0}\times N_{jx0},\ i, j\in[1,s],\ i<j.$$

$$d)\quad (D^{+}_{x_i} v_{ij})^T\ f_i^{*}(x) \le \rho_{5,i}\|x_i\|^2 + \sum_{\substack{j=1\\ i\ne j}}^{s} \rho_{4,i,j}\|x_i\|\|x_j\| + \rho_{6,j}\|x_j\|^2,\ \ \forall x_i \in N_{ix0},\ i, j\in[1,s],\ i<j.$$

$$e)\quad (D^{+}_{x_j} v_{ij})^T\ f_i(x) \le \rho_{7,i}\|x_i\|^2 + \sum_{\substack{j=1\\ j\ne i}}^{s} \rho_{5,i,j}\|x_i\|\|x_j\| + \rho_{8,j}\|x_j\|^2,\ \forall (x_i, x_j)\in N_{ix0}\times N_{jx0},\ i, j\in[1,s],\ i<j$$

$$f)\quad (D^{+}_{x_j} v_{ij})^T\ f_i^{*}(x) \le \rho_{9,i}\|x_i\|^2 + \sum_{\substack{j=1\\ i\ne j}}^{s} \rho_{6,i,j}\|x_i\|\|x_j\| + \rho_{10,j}\|x_j\|^2,\ \forall x_i \in N_{ix0},\ i, j\in[1,s],\ i<j.$$

(1,28) skalyar funktsiyadan (1,25) sistema yordamida yuqoridagi tengsizliklarni qanoatlantiruvchi xosila uchun quyidagi tengsizlik xosil bo`ladi.

$$\dot{V}(x) \le u^T G u \qquad (1.30)$$

bu erda

$$G = [g_{i,j}]_{i,j=1}^{s},\qquad g_{i,j} = g_{i,j}.$$

$$g_{ii} = [\eta_i^2(\rho_{1,i}+\rho_{2,i}) + 2\sum_{\substack{j=1\\ i<j}}^{s}\eta_i\eta_j(\rho_{3,i}+\rho_{4,j}+\rho_{5,i}+\rho_{6,j}+\rho_{7,i}+\rho_{8,j}+\rho_{9,i}+\rho_{10,j})].$$

$$g_{ij} = \frac{1}{2}\eta_i^2(\rho_{1,i,j}+\rho_{2,i,j}) + \eta_i\eta_j(\rho_{3,i,j}+\rho_{4,i,j}+\rho_{5,i,j}+\rho_{6,i,j})$$

(1.30) tengsizlikdan ko`rinadiki, $\dot{V}(x)$ xosila manfiy aniqlangan bo`lishi uchun G-o`zgarmas matritsani manfiy aniqlangan bo`lishi etarli.

Yuqoridagilarga asoslanib quyidagi teoremani keltiramiz.

Teorema 1.1. Agar (1.25) sistema uchun elementlari (*) shartlarni qanoatlantiruvchi U(x) matritsa – funktsiya mavjud bo`lib, quyidagi shartlar bajarilsa A-matritsa musbat aniqlangan. G- matritsa yarim manfiy aniqlangan.

U xolda (1.25) sistemaning x=0 echimi (toyimagan xarakat) turg`un bo`ladi.

Isbot: $\eta^T = (\eta_1, \eta_2, ..., \eta_s)$ vektor yordamida (1,27) matritsa–funktsiyadan (1,29) tengsizlikni qanoatlantiruvchi (1,28) skalyar funktsiyani tuzamiz. Bu skalyar funktsiya xosilasi uchun (1,30) baxolashni xosil qilamiz. (1,29) tengsizlikdan va A matritsaning musbat aniqlanganligidan (1,28) funktsiyaning musbat aniqlanganligi xosil bo`ladi, (1,30) baxolashdan va G matritsaning yarim manfiy aniqlanganligidan esa (1,29) skalyar funktsiyadan (1,25) sistema yordamida olingan xosila manfiy aniqlangan yoki aynan nolga teng bo`ladi. Bu shartlar esa sistema muvozanat xolatining turg`un bo`lishi uchun etarli.

Teorema 1.2. Agar (1.25) sistema uchun elementlari (∗) shartlarni qanoatlantiruvchi U(x) matritsa – funktsiya mavjud bo`lib, quyidagi shartlar bajarilsa

A-matritsa musbat aniqlangan.

G- matritsa manfiy aniqlangan.

U xolda (1.25) sistemaning x=0 echimi (toyimagan xarakat) asimptotik turg`un bo`ladi.

Isbot: $\eta^T = (\eta_1, \eta_2, ..., \eta_s)$ vektor yordamida (1,27) matritsa–funktsiyadan (1,29) tengsizlikni qanoatlantiruvchi (1,28) skalyar funktsiyani tuzamiz. Bu skalyar funktsiya xosilasi uchun (1,30) baxolashni xosil qilamiz. (1,29) tengsizlikdan va A matritsaning musbat aniqlanganligidan (1,28) funktsiyaning musbat aniqlanganligi xosil bo`ladi, (1,30) baxolashdan va G matritsaning manfiy aniqlanganligidan esa (1,29) skalyar funktsiyadan (1,25) sistema yordamida olingan xosila manfiy aniqlangan bo`ladi. Bu shartlar esa sistema muvozanat xolatining asimptotik turg`un bo`lishi uchun etarli.

Misol 1.1.

Ikkinchi tartibli ikkita qism sistemalardan tuzilgan to`rtinchi tartibli sistemani qaraymiz.

$$\frac{dx_1}{dt}=\begin{pmatrix}-1 & 0{,}5\\ -0{,}5 & -2\end{pmatrix}x_1+\begin{pmatrix}0{,}5 & 1\\ -1 & 0{,}5\end{pmatrix}x_2$$

$$\frac{dx_2}{dt}=\begin{pmatrix}-2 & -1\\ 0{,}5 & -3\end{pmatrix}x_2+\begin{pmatrix}0{,}1 & -1\\ 1 & 0{,}1\end{pmatrix}x_1,\quad x_i\in R^2,\ i=1{,}2$$

Berilgan sistema uchun elementlari quyidagicha bo`lgan matritsa-funktsiyani tuzamiz.

$$v_{ii}(x_i)=x_i^T diag\ \{2{,}2\}x_i,\ i=1{,}2,\quad v_{12}=(x_1,x_2)=v_{21}(x_2,x_1)=x_i^T diag\ \{0{,}1{,}0{,}1\}x_2$$

Bu funktsiya uchun quyidagilar o`rinli bo`ladi.

$$v_{ii}(x_i)\geq 2\|x_i\|^2\quad \forall x_i\in N_{ix_0},\ i=1{,}2;\quad v_{12}(x_1,x_2)\geq -0{,}1\|x_1\|\|x_2\|\quad \forall(x_1,x_2)\in N_{1x_0}\times N_{2x_0}$$

Agar $\eta^T=(1{,}1)$ bo`lsa, u xolda A matritsaning ko`rinishi

$$A=\begin{pmatrix}2 & -0{,}1\\ -0{,}1 & 2\end{pmatrix}$$

kabi bo`lib, u musbat aniqlangan bo`ladi. G matritsa elementlari esa

$$g_{11}=-2,\quad g_{22}=-3{,}59,\quad g_{12}=0{,}214$$

ko`rinishida bo`ladi. G matritsaning bundek elementlaridan quyidagiga ega bo`lamiz.

$$G=\begin{pmatrix}-2 & 0{,}214\\ 0{,}214 & -3{,}59\end{pmatrix}$$

G matritsa manfiy aniqlanganligidan sistemaning x=0 muvozanat xolati asimptotik turg`un bo`ladi.

Misol 1.2.. Quyidagi yirik masshtabli sistemani qaraymiz

$$\frac{dx}{dt} = A_1 x + A_2 y + A_3 z$$
$$\frac{dy}{dt} = B_1 x + B_2 y + B_3 z$$
$$\frac{dz}{dt} = C_1 x + C_2 y + C_3 z$$

bu erda $x \in R^{n_1}$, $y \in R^{n_2}$, $z \in R^{n_3}$, $n_1 + n_2 + n_3 = n$, $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$ – o`zgarmas matritsalar. Bu sistemaga mos erkin qism sistemalar quyidagicha bo`ladi

$$\frac{dx}{dt} = A_1 x$$
$$\frac{dy}{dt} = B_2 y$$
$$\frac{dz}{dt} = C_3 z$$

Elementlari quyidagi ko`rinishda bo`lgan

$$U(z, y, z) = [v_{ij}(\cdot)],\ i, j = 1,2,3$$

matritsa funktsiyani tuzamiz

$$\begin{aligned} &v_{11}(x) = x^T P_{11} x,\ \ v_{22}(x) = y^T P_{22} y,\ \ v_{33}(x) = z^T P_{33} z,\ v_{12}(x,y) = v_{21}(x,y) = x^T P_{12} y, \\ &v_{13}(x,z) = v_{31}(x,z) = x^T P_{13} z,\ v_{23}(y,z) = v_{32}(y,z) = y^T P_{23} z \end{aligned} \tag{$*$}$$

bu erda P_{ii}, $i = 1,2,3$ – musbat aniqlangan simmetrik matritsa,

P_{12}, P_{13}, P_{23} – o`zgarmas matritsalar.

$(*)$ funktsiyalar uchun quyidagilar o`rinli bo`ladi.

$$\begin{aligned} &v_{11}(x) \ge \lambda_m(P_{11})\|x\|^2,\ \forall x \in N_{x0},\ N_{x0} = \{x \in N_x, x \ne 0\}, \\ &v_{22}(y) \ge \lambda_m(P_{22})\|y\|^2,\ \forall y \in N_{y0},\ N_{y0} = \{y \in N_y, y \ne 0\}, \\ &v_{33}(z) \ge \lambda_m(P_{33})\|z\|^2,\ \forall z \in N_{z0},\ N_{z0} = \{z \in N_z, z \ne 0\}, \\ &v_{12}(x,y) \ge -\lambda_M^{\frac{1}{2}}(P_{12}P_{12}^T)\|x\|\|y\|,\ \forall (x,y) \in N_{x0} \times N_{y0}, \\ &v_{13}(x,z) \ge -\lambda_M^{\frac{1}{2}}(P_{13}P_{13}^T)\|x\|\|z\|,\ \forall (x,z) \in N_{x0} \times N_{z0}, \\ &v_{23}(y,z) \ge -\lambda_M^{\frac{1}{2}}(P_{23}P_{23}^T)\|y\|\|z\|,\ \forall (y,z) \in N_{y0} \times N_{z0}, \end{aligned}$$

bu erda $\lambda_m(P_{ii}) - P_{ii}$ matritsaning minimal xos qiymati, i=1,2,3.

$\lambda_M^{\frac{1}{2}}(P_{12}P_{12}^T), \lambda_M^{\frac{1}{2}}(P_{13}P_{13}^T)$, $\lambda_M^{\frac{1}{2}}(P_{23}P_{23}^T) - P_{12}, P_{13}, P_{23}$ matritsalarning normalari. $U(x,y,z)$ matritsa funktsiya va $\eta \in R^3$, $\eta_i > 0$, $i = 1,2,3$ o`zgarmas vektordan foydalanib quyidagi skalyar funktsiyani tuzamiz.

$$V(x,y,z) = \eta^T U(x,y,z)\eta$$

Bu skalyar funktsiya uchun quyidagi baxolash o`rinli bo`ladi.

$$V(x,y,z) \geq u^T H^T P H u$$

bu erda $u^T = (\|x\|, \|y\|, \|z\|)$, $H^T = H = diag(\eta_1, \eta_2, \eta_3)$

$$P = \begin{bmatrix} \lambda_m(P_{11}) & -\lambda_M^{\frac{1}{2}}(P_{12}P_{12}^T) & -\lambda_M^{\frac{1}{2}}(P_{13}P_{13}^T) \\ -\lambda_M^{\frac{1}{2}}(P_{12}P_{12}^T) & \lambda_m(P_{22}) & -\lambda_M^{\frac{1}{2}}(P_{23}P_{23}^T) \\ -\lambda_M^{\frac{1}{2}}(P_{13}P_{13}^T) & -\lambda_M^{\frac{1}{2}}(P_{23}P_{23}^T) & \lambda_m(P_{33}) \end{bmatrix}$$

Endi berilgan sistema yordamida $(*)$ funktsiyalardan olingan xosilalarni baxolab chiqamiz:

$$\dot{v}_{11} = \dot{x}^T P_{11} x + x^T P_{11}\dot{x} = (A_1 x + A_2 y + A_3 z)^T P_{11} x + x^T P_{11}(A_1 x + A_2 y + A_3 z) = x^T P_{11} A_1^T P_{11} x +$$

$$+ y^T A_2^T P_{11} x + z^T A_3 P_{11} x + x^T P_{11} A_1 x + x^T P_{11} A_2 y + x^T P_{11} A_3 z = x^T (P_{11}A_1 + A_1^T P_{11})x + 2x^T P_{11} A_2 y +$$

$$+ 2x^T P_{11} A_3 z \leq \lambda_m (P_{11}A_1 + A_1^T P_{11})\|x\|^2 + 2\lambda_M^{\frac{1}{2}} P_{11} A_2 (P_{11}A_2)^T \|x\|\|y\| + 2\lambda_M^{\frac{1}{2}} P_{11} A_3 (P_{11} A_3)^T \|x\|\|z\|$$

$$\dot{v}_{22} = \dot{y}^T P_{22} y + y^T P_{22}\dot{y} = (B_1 x + B_2 y + B_3 z)^T P_{22} y + y^T P_{22}(B_1 x + B_2 y + B_3 z) = B_1^T x^T P_{22} y +$$

$$+ B_2^T y^T P_{22} y + B_3^T z^T P_{22} y + y^T P_{22} B_1 x + y^T P_{22} B_2 y + y^T P_{22} B_3 z = y^T (P_{22}B_1 + B_1^T P_{22}) y +$$

$$+ 2x^T P_{22} B_1 y + 2z^T P_{22} B_3 y \leq \lambda_m (P_{22}B_1 + B_1^T P_{22})\|y\|^2 + 2\lambda_M^{\frac{1}{2}} (P_{22}B_1 (P_{22}B_1)^T)\|x\|\|y\| +$$

$$+ 2\lambda_M^{\frac{1}{2}} (P_{22} B_3 (P_{22} B_3)^T)\|z\|\|y\|$$

$$\dot{v}_{33} = \dot{z}^T P_{33} z + z^T P_{33} \dot{z} = (C_1 x + C_2 y + C_3 z)^T P_{33} z + z^T P_{33} (C_1 x + C_2 y + C_3 z) = C_1^T x^T P_{33} +$$

$$+ C_2^T y^T P_{33} z + C_3^T z^T P_{33} z + z^T P_{33} C_1 x + z^T P_{33} C_2 y + z^T P_{33} C_3 z = z^T (C_3^T P_{33} + C_3 P_{33}) z +$$

$$+ 2x^T C_1 P_{33} z + 2z^T C_2 P_{33} y \le \lambda_m (C_3^T P_{33} + C_3 P_{33}) \|z\|^2 + 2\lambda_M^{\frac{1}{2}} (P_{33} C_1 (P_{33} C_1)^T) \|x\| \|z\| +$$

$$+ 2\lambda_M^{\frac{1}{2}} (P_{33} C_3 (P_{33} C_3)^T) \|z\| \|y\|$$

$$\dot{v}_{12} = \dot{x}^T P_{12} y + x^T P_{12} \dot{y} = (A_1 x + A_2 y + A_3 z)^T P_{12} y + x^T P_{12} (B_1 x + B_2 y + B_3 z) = x^T A_1^T P_{12} y +$$

$$+ y^T A_2^T P_{12} y + z^T A_3^T P_{12} y + x^T P_{12} B_1 x + x^T P_{12} B_2 y + x^T P_{12} B_3 z = x^T \frac{1}{2} (P_{12} B_1 + (P_{12} B_1)^T) x +$$

$$+ y^T \frac{1}{2} (A_2^T P_{12} + (A_2^T P_{12})^T) y + x^T (A_1^T P_{12} + P_{12} B_2) y + y^T P_{12}^T A_3 z + x^T P_{12} B_3 z =$$

$$\frac{1}{2} x^T (P_{12} B_1 + B_1^T P_{12}^T) x + \frac{1}{2} y^T (A_2^T P_{12} + P_{12}^T A_2) y + x^T (A_1^T P_{12} + P_{12} B_2) y + y^T P_{12}^T A_3 z + x^T P_{12} B_3 z \le$$

$$\le \frac{1}{2} \lambda_m (P_{12} B_1 + B_1^T P_{12}^T) \|x\|^2 + \frac{1}{2} \lambda_m (A_2^T P_{12} + P_{12}^T A_2) \|y\|^2 + \lambda_M^{\frac{1}{2}} ((A_1^T P_{12} + P_{12} B_2)(A_1^T P_{12} + P_{12} B_2)^T) \|x\| \|y\| +$$

$$+ \lambda_M^{\frac{1}{2}} (P_{12}^T A_3 (P_{12}^T A_3)^T) \|y\| \|z\| + \lambda_M^{\frac{1}{2}} (P_{12} B_3 (P_{12} B_3)^T) \|x\| \|z\|$$

$$\dot v_{13} = \dot x^T P_{13} z + x^T P_{13} z = (A_1x + A_2y + A_3z)^T P_{13}z + x^T P_{13}(C_1x+C_2y+C_3z) = x^T A_1^T P_{13} z +$$

$$+ y^T A_2^T P_{13} z + z^T A_3^T P_{13} z + x^T P_{13} C_1 x + x^T P_{13} C_2 y + x^T P_{13} C_3 z = \frac{1}{2} x^T (P_{13}C_1 + (P_{13}C_1)^T)x +$$

$$+\frac{1}{2} z^T (A_2^T P_{13} + (A_2^T P_{13})^T)z + x^T (A_2^T P_{13} + P_{13}C_3)z + y^T A_2^T P_{13} z + x^T P_{13} C_2 y = \frac{1}{2} x^T (P_{13}C_1 + P_{13}^T C_1^T)x +$$

$$+\frac{1}{2} z^T(A_2^T P_{13} + A_2 P_{13}^T)z + x^T(A_2^T P_{13}+P_{13}C_3)z + y^T A_2^T P_{13} z + x^T P_{13}C_2 y \le \lambda_m (P_{13}C_1 + P_{13}^T C_1^T)\|x\|^T +$$

$$+\lambda_m (A_2^T P_{13} + A_2 P_{13}^T)\|z\|^2 + \lambda_M^{\frac{1}{2}}((A_2^T P_{13}+P_{13}C_3)(A_2^T P_{13} + P_{13}C_3)^T)\|x\|\|z\| + \lambda_M^{\frac{1}{2}}(A_2^T P_{13}(A_2^T P_{13})^T)\|y\|\|z\| +$$

$$+\lambda_M^{\frac{1}{2}}(P_{13}C_2(P_{13}C_2)^T)\|x\|\|y\|$$

$$\dot v_{23} = \dot y^T P_{23} z + y^T P_{23}\dot z = (B_1x+B_2y+B_3z)^T P_{23}z + y^T P_{23}(C_1x+C_2y+C_3z) = B_1^T x^T P_{23} z +$$

$$+B_2^T y^T P_{23} z + B_3^T z^T P_{23} z + y^T P_{23}C_1 x + y^T P_{23} C_2 y + y^T P_{23} C_3 z = y^T (B_2 P_{23} + P_{23}C_3)z +$$

$$+x^T B_1^T P_{23} z + z^T B_3^T P_{23} z + y^T P_{23} C_1 x + y^T P_{23}C_2 y \le \lambda_m P_{23} B_3^T \|z\|^2 + \lambda_m P_{23} C_2\|y\|^2 +$$

$$+\lambda_M^{\frac{1}{2}}(B_2P_{23}+P_{23}C_3)\|y\|\|z\| + \lambda_M^{\frac{1}{2}} B_1^T P_{23}\|x\|\|z\| + \lambda_M^{\frac{1}{2}} P_{23}C_1\|x\|\|y\|$$

Bu baxolashlardan foydalanib

$$\dot V(x,y,z) = \eta^T \dot U(x,y,z)\eta$$

xosilani yuqoridan baxolaymiz

$$\dot V = \begin{pmatrix}\eta_1 & \eta_2 & \eta_3\end{pmatrix} \begin{pmatrix} \dot v_{11} & \dot v_{12} & \dot v_{13} \\ \dot v_{21} & \dot v_{22} & \dot v_{23} \\ \dot v_{31} & \dot v_{32} & \dot v_{33} \end{pmatrix} \begin{pmatrix}\eta_1 \\ \eta_2 \\ \eta_3\end{pmatrix} =$$

$$= \begin{pmatrix}\eta_1\dot v_{11} + \eta_2\dot v_{21} + \eta_3\dot v_{31} & \eta_1\dot v_{12} + \eta_2\dot v_{22} + \eta_3\dot v_{32} & \eta_1\dot v_{13} + \eta_2\dot v_{23} + \eta_3\dot v_{33}\end{pmatrix} \begin{pmatrix}\eta_1 \\ \eta_2 \\ \eta_3\end{pmatrix} =$$

$$= \eta_1^2 \dot{v}_{11} + \eta_1\eta_2 \dot{v}_{21} + \eta_1\eta_3 \dot{v}_{31} + \eta_1\eta_2 \dot{v}_{12} + \eta_2^2 \dot{v}_{22} + \eta_2\eta_3 \dot{v}_{32} + \eta_1\eta_3 \dot{v}_{13} + \eta_2\eta_3 \dot{v}_{23} + \eta_3^2 \dot{v}_{33} =$$

$$= \eta_1^2 \dot{v}_{11} + \eta_2^2 \dot{v}_{22} + \eta_3^2 \dot{v}_{33} + \eta_1\eta_2 \dot{v}_{12} + \eta_1\eta_3 \dot{v}_{13} + \eta_2\eta_3 \dot{v}_{23} = \eta_1^2 (x^T (P_{11}A_1 + A_1^T P_{11})x + 2x^T P_{11}A_2 y +$$

$$+ 2x^T P_{11}A_3 z) + \eta_2^2 (y^T (P_{22}B_1 + B_1^T P_{22})y + 2x^T P_{22}B_1 y + 2z^T P_{22}B_3 y) + \eta_3^2 (z^T (C_3^T P_{33} + C_3 P_{33})z +$$

$$= \eta_1^2 \dot{v}_{11} + \eta_1\eta_2 \dot{v}_{21} + \eta_1\eta_3 \dot{v}_{31} + \eta_1\eta_2 \dot{v}_{12} + \eta_2^2 \dot{v}_{22} + \eta_2\eta_3 \dot{v}_{32} + \eta_1\eta_3 \dot{v}_{13} + \eta_2\eta_3 \dot{v}_{23} + \eta_3^2 \dot{v}_{33} =$$

$$= \eta_1^2 \dot{v}_{11} + \eta_2^2 \dot{v}_{22} + \eta_3^2 \dot{v}_{33} + \eta_1\eta_2 \dot{v}_{12} + \eta_1\eta_3 \dot{v}_{13} + \eta_2\eta_3 \dot{v}_{23} = \eta_1^2 (x^T (P_{11}A_1 + A_1^T P_{11})x + 2x^T P_{11}A_2 y +$$

$$+ 2x^T P_{11}A_3 z) + \eta_2^2 (y^T (P_{22}B_1 + B_1^T P_{22})y + 2x^T P_{22}B_1 y + 2z^T P_{22}B_3 y) + \eta_3^2 (z^T (C_3^T P_{33} + C_3 P_{33})z +$$

$$+ 2x^T C_1 P_{33} z + 2z^T C_2 P_{33} y) + \eta_1\eta_2 (\frac{1}{2} x^T (P_{12}B_1 + B_1^T P_{12}^T)x + \frac{1}{2} y^T (A_2^T P_{12} + P_{12}^T A_2)y +$$

$$x^T (A_1^T P_{12} + P_{12}B_2)y + y^T P_{12}^T A_3 z + x^T P_{12}B_3 z) + \eta_1\eta_3 (\frac{1}{2} x^T (P_{13}C_1 + P_{13}^T C_1^T)x +$$

$$+ \frac{1}{2} z^T (A_2^T P_{13} + A_2 P_{13}^T)z + x^T (A_2^T P_{13} + P_{13}C_3)z + y^T A_2^T P_{13} z + x^T P_{13}C_2 y) + \eta_2\eta_3 (y^T (B_2 P_{23} + P_{23}C_3)z +$$

$$+ x^T B_1^T P_{23} z + z^T B_3^T P_{23} z + y^T P_{23}C_1 x + y^T P_{23}C_2 y) \le$$

$$\le \eta_1^2 (\lambda_m (P_{11}A_1 + A_1^T P_{11})\|x\|^2 + 2\lambda_M^{\frac{1}{2}} P_{11}A_2 (P_{11}A_2)^T \|x\|\|y\| + 2\lambda_M^{\frac{1}{2}} P_{11}A_3 (P_{11}A_3)^T \|x\|\|z\|) +$$

$$\eta_2^2 (\lambda_m (P_{22}B_1 + B_1^T P_{22})\|y\|^2 + +\lambda_M^{\frac{1}{2}} (P_{22}B_1 (P_{22}B_1)^T)\|x\|\|y\| + 2\lambda_M^{\frac{1}{2}} (P_{22}B_3 (P_{22}B_3)^T)\|z\|\|y\|) +$$

$$\eta_3^2 (\lambda_m (C_3^T P_{33} + C_3 P_{33})\|z\|^2 + 2\lambda_M^{\frac{1}{2}} (P_{33}C_1 (P_{33}C_1)^T)\|x\|\|z\| + 2\lambda_M^{\frac{1}{2}} (P_{33}C_3 (P_{33}C_3)^T)\|z\|\|y\|) +$$

$$+ \eta_1\eta_2 (\frac{1}{2} \lambda_m (P_{12}B_1 + B_1^T P_{12}^T)\|x\|^2 + \frac{1}{2} \lambda_m (A_2^T P_{12} + P_{12}^T A_2)\|y\|^2 + \lambda_M^{\frac{1}{2}} ((A_1^T P_{12} + P_{12}B_2)(A_1^T P_{12} + P_{12}B_2)^T)\|x\|\|y\| +$$

$$\lambda_M^{\frac{1}{2}} (P_{12}^T A_3 (P_{12}^T A_3)^T)\|y\|\|z\| + \lambda_M^{\frac{1}{2}} (P_{12}B_3 (P_{12}B_3)^T)\|x\|\|z\|) + \eta_1\eta_3 (\lambda_m (P_{13}C_1 + P_{13}^T C_1^T)\|x\|^T +$$

$$+ \lambda_m (A_2^T P_{13} + A_2 P_{13}^T)\|z\|^2 + \lambda_M^{\frac{1}{2}} ((A_2^T P_{13} + P_{13}C_3)(A_2^T P_{13} + P_{13}C_3)^T)\|x\|\|z\| + \lambda_M^{\frac{1}{2}} (A_2^T P_{13} (A_2^T P_{13})^T)\|y\|\|z\| +$$

$$+ \lambda_M^{\frac{1}{2}} (P_{13}C_2 (P_{13}C_2)^T)\|x\|\|y\|) + \eta_2\eta_3 (\lambda_m P_{23}B_3^T \|z\|^2 + \lambda_m P_{23}C_2 \|y\|^2 + \lambda_M^{\frac{1}{2}} (B_2 P_{23} + P_{23}C_3)\|y\|\|z\| +$$

$$\lambda_M^{\frac{1}{2}} B_1^T P_{23} \|x\|\|z\| + \lambda_M^{\frac{1}{2}} P_{23}C_1 \|x\|\|y\|) =$$

$$= (\eta_1^2 (\lambda_m (P_{11}A_1 + A_1^T P_{11}) + \eta_1\eta_2 (\frac{1}{2} \lambda_m (P_{12}B_1 + B_1^T P_{12}^T) + \eta_1\eta_3 (\lambda_m (P_{13}C_1 + P_{13}^T C_1^T))\|x\|^2 +$$

$$+ (\eta_2^2 \lambda_m (P_{22}B_1 + B_1^T P_{22}) + \eta_1\eta_2 (\frac{1}{2} \lambda_m (A_2^T P_{12} + P_{12}^T A_2)) + \lambda_m P_{23}C_2)\|y\|^2 + (\eta_3^2 \lambda_m (C_3^T P_{33} + C_3 P_{33}) +$$

$$+ \lambda_m (A_2^T P_{13} + A_2 P_{13}^T) + \eta_2\eta_3 \lambda_m P_{23}B_3^T)\|z\|^2 + (\eta_1^2 2\lambda_M^{\frac{1}{2}} P_{11}A_2 (P_{11}A_2)^T + \eta_2^2 \lambda_M^{\frac{1}{2}} (P_{22}B_1 (P_{22}B_1)^T) +$$

$$\eta_1\eta_2 \lambda_M^{\frac{1}{2}} ((A_1^T P_{12} + P_{12}B_2)(A_1^T P_{12} + P_{12}B_2)^T) + \eta_1\eta_3 \lambda_M^{\frac{1}{2}} (P_{13}C_2 (P_{13}C_2)^T) + \eta_2\eta_3 \lambda_M^{\frac{1}{2}} P_{23}C_1)\|x\|\|y\| +$$

$$+ (\eta_1^2 2\lambda_M^{\frac{1}{2}} P_{11}A_3 (P_{11}A_3)^T + \eta_3^2 2\lambda_M^{\frac{1}{2}} (P_{33}C_1 (P_{33}C_1)^T + \eta_1\eta_2 \lambda_M^{\frac{1}{2}} (P_{12}B_3 (P_{12}B_3)^T +$$

$$+ \eta_1\eta_3 \lambda_M^{\frac{1}{2}} ((A_2^T P_{13} + P_{13}C_3)(A_2^T P_{13} + P_{13}C_3)^T + \eta_2\eta_3 \lambda_M^{\frac{1}{2}} B_1^T P_{23})\|x\|\|z\| + (\eta_2^2 2\lambda_M^{\frac{1}{2}} (P_{22}B_3 (P_{22}B_3)^T$$

$$+ \eta_3^2 2\lambda_M^{\frac{1}{2}} (P_{33}C_3 (P_{33}C_3)^T) + \eta_1\eta_2 \lambda_M^{\frac{1}{2}} (P_{12}^T A_3 (P_{12}^T A_3)^T) + \lambda_M^{\frac{1}{2}} A_2^T P_{13} (A_2^T P_{13})^T + \eta_2\eta_3 (\lambda_M^{\frac{1}{2}} (B_2 P_{23} + P_{23}C_3))\|y\|\|z\|$$

quyidagicha belgilash kiritamiz:

$$\rho_{11}=\eta_1^2(\lambda_m(P_{11}A_1+A_1^TP_{11})+\eta_1\eta_2(\frac{1}{2}\lambda_m(P_{12}B_1+B_1^TP_{12}^T)+\eta_1\eta_3(\lambda_m(P_{13}C_1+P_{13}^TC_1^{\ T})$$

$$\rho_{22}=\eta_2^2\lambda_m(P_{22}B_1+B_1^TP_{22})+\eta_1\eta_2(\frac{1}{2}\lambda_m(A_2^TP_{12}+P_{12}^TA_2))+\lambda_mP_{23}C_2$$

$$\rho_{33}=\eta_3^2\lambda_m(C_3^TP_{33}+C_3P_{33})+\lambda_m(A_2^TP_{13}+A_2P_{13}^{\ T})+\eta_2\eta_3\lambda_mP_{23}B_3^T$$

$$\rho_{12}=\rho_{21}=\eta_1^2\lambda_M^{\frac{1}{2}}P_{11}A_2(P_{11}A_2)^T+\eta_2^2\lambda_M^{\frac{1}{2}}(P_{22}B_1(P_{22}B_1)^T)+$$

$$+\frac{1}{2}\eta_1\eta_2\lambda_M^{\frac{1}{2}}((A_1^TP_{12}+P_{12}B_2)(A_1^TP_{12}+P_{12}B_2)^T)+\eta_1\eta_3\lambda_M^{\frac{1}{2}}(P_{13}C_2(P_{13}C_2)^T)+\frac{1}{2}\eta_2\eta_3\lambda_M^{\frac{1}{2}}P_{23}C_1$$

$$\rho_{13}=\rho_{31}=\eta_1^2 2\lambda_M^{\frac{1}{2}}P_{11}A_3(P_{11}A_3)^T+\eta_3^2 2\lambda_M^{\frac{1}{2}}(P_{33}C_1(P_{33}C_1)^T+\eta_1\eta_2\lambda_M^{\frac{1}{2}}(P_{12}B_3(P_{12}B_3)^T+$$

$$+\eta_1\eta_3\lambda_M^{\frac{1}{2}}((A_2^TP_{13}+P_{13}C_3)(A_2^TP_{13}+P_{13}C_3)^T+\eta_2\eta_3\lambda_M^{\frac{1}{2}}B_1^TP_{23}$$

$$\rho_{23}=\rho_{32}=\eta_2^2\lambda_M^{\frac{1}{2}}(P_{22}B_3(P_{22}B_3)^T+\eta_3^2\lambda_M^{\frac{1}{2}}(P_{33}C_3(P_{33}C_3)^T)+\frac{1}{2}\eta_1\eta_2\lambda_M^{\frac{1}{2}}(P_{12}^TA_3(P_{12}^TA_3)^T)+$$

$$+\frac{1}{2}\lambda_M^{\frac{1}{2}}A_2^TP_{13}(A_2^TP_{13})^T+\eta_2\eta_3\lambda_M^{\frac{1}{2}}(B_2P_{23}+P_{23}C_3)$$

U xolda xosila funktsiya uchun quyidagi tengsizlik o`rinli bo`ladi

$$\dot V\le\rho_{11}\|x\|^2+\rho_{22}\|y\|^2+\rho_{33}\|z\|^2+\rho_{12}\|x\|\|y\|+\rho_{13}\|x\|\|z\|+\rho_{23}\|y\|\|z\|$$

Bu tengsizlikni quyidagicha yozishimiz mumkin

$$\dot V\le u^TSu$$

bu erda $S=[\rho_{ij}]$, i,j=1,2,3, $\rho_{ij}=\rho_{ji}$

S matritsa manfiy aniqlangan bo`lishi uchun

$a)\quad \rho_{11}<0$

$b)\quad \begin{vmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{vmatrix}=\rho_{11}\rho_{22}-\rho_{12}^2>0 \qquad (**)$

$c)\quad |S|=\rho_{11}\rho_{22}\rho_{33}+2\rho_{12}\rho_{23}\rho_{13}-\rho_{13}^2\rho_{22}-\rho_{23}^2\rho_{11}-\rho_{12}^2\rho_{33}<0$

shartlarning bajarilishi etarli. Bundan xulosa qilib shuni aytishimiz mumkinki, agar P matritsa musbat aniqlangan bo`lsa, $(**)$ shartlar bajarilganda S matritsa manfiy aniqlangan bo`lib, teorema 1.2 ning barcha shartlari bajariladi, ya'ni berilgan sistema muvozanat xolati asimptotik turg`un bo`ladi.

Misol 1.3. Quyidagi Lure sistemasini qaraylik

$$\frac{dx}{dt} = A_{11}x + A_{12}y + A_{13}z + q_1 f_1(\sigma_1) = f_1^*$$
$$\frac{dy}{dt} = A_{21}x + A_{22}y + A_{23}z + q_2 f_2(\sigma_2) = f_2^*$$
$$\frac{dz}{dt} = A_{31}x + A_{32}y + A_{33}z + q_3 f_3(\sigma_3) = f_3^*$$

bu erda

$$\sigma_i = c_{i1}^T x + c_{i2}^T y + c_{i3}^T z, \quad \frac{f_i(\sigma_i)}{\sigma_i} \in [0, k_i],\ i = 1,2,3,\ \ \sigma_i \in (-\infty, +\infty).$$

Faraz qilaylik bu sistema uchun xam yuqorida qaralgan misoldagi kabi matritsa–funktsiya tuzilgan bo`lib, u erdagi baxolashlar o`rinli bo`lsin. Bu funktsiyalardan berilgan sistema yordamida olingan to`la xosila quyidagi tengsizlikni qanoatlantiradi:

$$\dot{V}(x, y, z) \le u^T S u$$

bu erda $S = [\sigma_{ij}]$, i,j=1,2,3, $\sigma_{ij} = \sigma_{ji}$, σ_{ii}, $i = 1,2,3$ lar mos ravishda quyidagi matritsalarning maksimal xos qiymatlari.

$$\eta_1^2\{A_{11}^T P_{11} + P_{11}A_{11} + P_{11}(q_1 k_1^* c_{11}^T) + (q_1 k_1^* c_{11}^T)^T P_{11}\} + 2\eta_1\eta_2\{P_{12}A_{21} + P_{12}(q_2 k_2^* c_{21}^T)\} + 2\eta_1\eta_3\{P_{13}A_{31} +$$
$$P_{13}(q_3 k_3^* c_{31}^T)\}$$
$$\eta_2^2\{A_{22}^T P_{22} + P_{22}A_{22} + P_{22}(q_2 k_2^* c_{22}^T) + (q_2 k_2^* c_{22}^T)^T P_{22}\} + 2\eta_1\eta_2\{A_{12}^T P_{12} + (q_1 k_1^* c_{12}^T)^T P_{12}\} + 2\eta_2\eta_3\{P_{23}A_{32} +$$
$$P_{23}(q_3 k_3^* c_{32}^T)\}$$
$$\eta_3^2\{A_{33}^T P_{33} + P_{33}A_{33} + P_{33}(q_3 k_3^* c_{33}^T) + (q_3 k_3^* c_{33}^T)^T P_{33} + 2\eta_1\eta_3\{A_{13}^T P_{13} + (q_1 k_1^* c_{13}^T)^T P_{13}\} + 2\eta_2\eta_3\{A_{23}^T P_{23} +$$
$$(q_2 k_2^* c_{23}^T)^T P_{23}\}$$

, $\sigma_{ij}, i \ne j,\ i, j = 1,2,3$ lar mos ravishda quyidagi matritsalarning normalari.

$$\eta_1^2\{P_{11}A_{12}+P_{11}(q_1k_1^*c_{12}^T)+\eta_2^2\{\{P_{22}A_{21}+P_{22}(q_2k_2^*c_{21}^T)\}+\eta_1\eta_2\{A_{11}^TP_{12}+P_{12}A_{22}+(q_1k_1^*c_{11}^T)^TP_{12}+$$
$$+P_{12}(q_2k_2^*c_{22}^T)\}+\eta_1\eta_3\{P_{13}A_{32}+P_{13}(q_3k_3^*c_{32}^T)+\eta_2\eta_3\{P_{23}A_{31}+P_{23}(q_3k_3^*c_{31}^T)\}$$
$$\eta_1^2\{P_{11}A_{13}+P_{11}(q_1k_1^*c_{13}^T)\}+\eta_3^2\{P_{33}A_{31}+P_{33}(q_3k_3^*c_{31}^T)\}+\eta_1\eta_2\{P_{12}A_{23}+P_{12}(q_2k_2^*c_{23}^T)\}+$$
$$+\eta_1\eta_3\{P_{13}A_{33}+A_{11}^TP_{13}+P_{13}(q_3k_3^*c_{33}^T)+(q_1k_1^*c_{11}^T)P_{13}\}+\eta_2\eta_3\{A_{21}^TP_{23}+(q_2k_2^*c_{21}^T)^TP_{23}\}$$
$$\eta_2^2\{P_{22}A_{23}+P_{22}(q_2k_2^*c_{23}^T)\}+\eta_3^2\{P_{33}A_{32}+P_{33}(q_3k_3^*c_{31}^T)\}+\eta_1\eta_2\{A_{13}^TP_{12}+(q_1k_1^*c_{13}^T)^TP_{12}\}$$
$$+\eta_1\eta_3\{A_{12}^TP_{13}+(q_1k_1^*c_{12}^T)P_{13}\}+\eta_2\eta_3\{A_{22}^TP_{23}+P_{23}A_{22}+(q_2k_2^*c_{22}^T)^TP_{23}+P_{23}(q_3k_3^*c_{33}^T)\}$$

bu erda

$$k_i^*=\begin{cases}\sigma_iq_i^TP_{ij}x>0,\ (yoki\ \ \sigma_iq_i^TP_{ij}y>0,\ \ yoki\ \ \sigma_iq_i^TP_{ij}z>0)\quad da\quad k_i\\ qo\lg an\ hollarda\quad 0\end{cases}$$

Bu baxolashlar quyidagicha xulosa qilishimiz imkonini beradi: Agar P matritsa musbat aniqlangan bo`lib, S matritsa manfiy aniqlangan bo`lsa, u xolda Lure tipidagi sistema muvozanat xolati asimptotik turg`un bo`ladi.

Misol 1.4. Misol 1.11 dagi matritsalarning ko`rinishi quyidagicha bo`lsin

$$A_{11}=\begin{pmatrix}-3&0\\0&-3\end{pmatrix};\quad A_{12}=\begin{pmatrix}-5&0\\0&-5\end{pmatrix};\quad A_{13}=\begin{pmatrix}1&0\\0&1\end{pmatrix};\quad A_{21}=\begin{pmatrix}5&0\\0&5\end{pmatrix};\quad A_{22}=\begin{pmatrix}0,1&0\\0&0,1\end{pmatrix};\quad A_{23}=\begin{pmatrix}-2&0\\0&-2\end{pmatrix};$$

$$A_{31}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix};\quad A_{32}=\begin{pmatrix}2,3&0\\0&2,3\end{pmatrix};\quad A_{33}=\begin{pmatrix}0&1\\-1&0\end{pmatrix};\quad q_1=\begin{pmatrix}0,1\\0\end{pmatrix};\quad q_2=\begin{pmatrix}-0,1\\0\end{pmatrix};\quad q_3=\begin{pmatrix}0\\0,1\end{pmatrix};$$

$$c_{11}=\begin{pmatrix}-0,1\\0\end{pmatrix};\ c_{12}=\begin{pmatrix}-0,01\\0\end{pmatrix};\ c_{13}=\begin{pmatrix}-0,1\\0,1\end{pmatrix};\ c_{21}=\begin{pmatrix}0,1\\0\end{pmatrix};\ c_{22}=\begin{pmatrix}0,01\\0\end{pmatrix};\ c_{23}=\begin{pmatrix}0,1\\0,1\end{pmatrix};$$

$$c_{31}=\begin{pmatrix}-0,1\\-0,1\end{pmatrix};\quad c_{32}=\begin{pmatrix}0\\-0,01\end{pmatrix};\quad c_{33}=\begin{pmatrix}0\\-0,1\end{pmatrix}.\quad k_i=1;\quad i=1,2,3.$$

$U(x,y,z)$ matritsa – funktsiyaning elementlarini

$$v_{11}(x)=x^Tdiag[1,1]x;\qquad v_{22}(y)=y^Tdiag[1,1]y;\quad v_{33}(z)=z^Tdiag[1,1]z$$

$$v_{12}(x,y)=x^Tdiag[0,1;0,1]y;\ \ v_{13}(x,z)=x^Tdiag[0,1;0,1]z;\ \ v_{23}(y,z)=y^Tdiag[0,1;0,1]z.$$

ko`rinishda tanlasak, ular uchun

$$v_{11}(x) \geq \|x\|^2; \quad v_{22}(y) \geq \|y\|^2; \quad v_{33}(z) \geq \|z\|^2 \;;$$

$$v_{12}(x,y) \geq -0{,}1\|x\|\|y\|; \quad v_{13}(x,z) \geq -0{,}1\|x\|\|z\|; \quad v_{23}(y,z) \geq -0{,}1\|y\|\|z\|.$$

baxolar o`rinli bo`lib,

$$P = \begin{bmatrix} 1 & -0{,}1 & -0{,}1 \\ -0{,}1 & 1 & -0{,}1 \\ -0{,}1 & -0{,}1 & 1 \end{bmatrix}$$

matritsa musbat aniqlangan bo`ladi.

Agar $\eta = (1{,}1{,}1)$ bo`lsa u xolda tanlab olingan $v_{ij}(\cdot);\ i,j \in [1{,}3]$ elementlarga ko`ra S matritsa quyidagi qiymatlarni qabul qiladi

$$S = \begin{cases} k_i^* = 0 \quad \partial a & \begin{bmatrix} -5{,}2 & 0{,}16 & 0{,}2 \\ 0{,}16 & -0{,}34 & 0{,}15 \\ 0{,}2 & 0{,}16 & -0{,}2 \end{bmatrix} \\ \\ k_i^* = k_i = 1 \quad \partial a & \begin{bmatrix} -5{,}202 & 0{,}18 & 0{,}03 \\ 0{,}18 & -0{,}3402 & 0{,}012 \\ 0{,}03 & 0{,}012 & -0{,}202 \end{bmatrix} \end{cases}$$

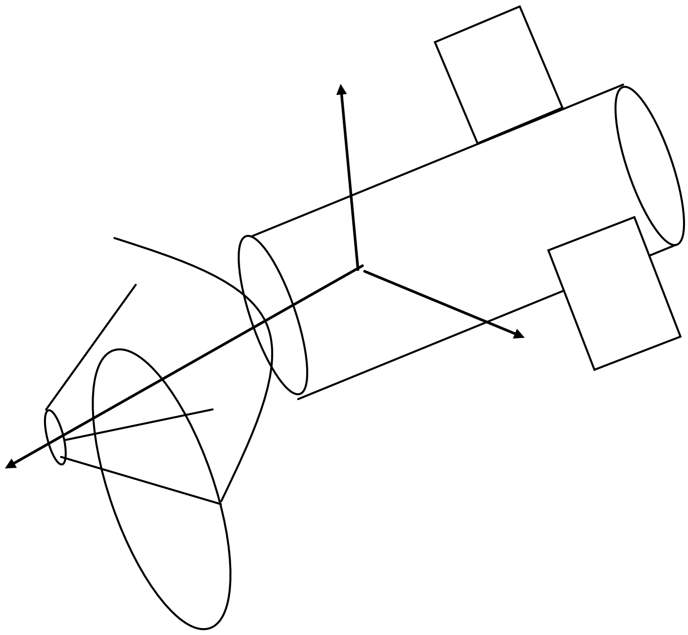
Ikkala xolda xam S matritsa manfiy aniqlangan bo`ladi. Bundan esa teorema 1.2 ga asosan qaralayotgan sistemaning x=y=z=0 muvozanat xolati asimptotik turg`un bo`ladi.

2–BOB.

GIROSKOPIK STABLIZASIYA XARAKATIDAGI ORBITAL OBSERVATORIYA TURG^UNLIGINING TAXLILI.

Qaralayotgan mexanik sistema murakkab sistemalar turkumiga kiradi. Bunday sistemalar turg`unligini taxlil qilish uchun Lyapunovning skalyar funktsiyasini qurish va tadbiq etish ancha qiyinchiliklar tug`diradi. Berilgan sistema [1] monogirafiyada Lyapunovning vektor funktsiyasi usulida analiz qilingan bo`lib, bir qator natijalar olingan.

Bu erda biz Lyapunovning matritsa-funktsiyasi usulini qo`llab kosmik apparat (KA) uchun turg`unlikning yangi etarli shartlarni xosil qilamiz.



1–rasm

§2.1. Sistemaning matematik modeli.

Kosmik apparat (1 - rasm)(KA) J_1,J_2,J_3 bosh markaziy inertsiya momentli qattiq jismni ifodalaydi, parraklar pretsessiya o`qlari uning bosh o`qlari bo`yicha yo`nalgan, girostablizatorning (GS) elementlari absolyut qattiq, xar bir juft parraklari aynan teng va o`zgarmas aylanish tezligiga ega deb faraz qilinadi

Quyidagicha belgilashlar kiritiladi [1]:

γ_j–kosmik aparatning orientattsiyasini aniqlovchi uchish burchagi, p_i-kosmik apparat burchak tezligini o`qdagi proektsiyasi,

α_i –pretsessiya burchagi,

A_i'-xar bir girokojuxning aylanish o`qiga nisbatan olingan inertsiya momenti,

β_i' - xar bir giroskopning pretsetsiya o`qiga nisbatan olingan inertsiya momenti.

C_i'-xar bir girokojuxning ekvatorial inertsiya momenti,

H_i'-giroskopning kinetik momenti.

$A_i' = C_i'$ deb faraz qilib, «KAQGS» sistema uchun xarakat tenglamasini quyidagi ko`rinishda olamiz [1].

$$I_1 \dot{p}_1 + (I_3 + I_2) p_2 p_3 + H_1 \dot{\alpha}_1 \cos\alpha_1 + H_3 p_2 \sin\alpha_3 + H_2 p_3 \sin\alpha_2 = M_1 + M_{P_1} \quad (123)$$

$$B_i \dot{\alpha}_1 - H_i p_1 \cos\alpha_i + b_i \dot{\alpha}_i = M_{yi} + M_{\alpha i}, \quad i = 1,2,3$$

$$\dot{\gamma}_1 = (p_1 \cos\gamma_3 - p_2 \sin\gamma_3)/\cos\gamma_2, \quad (2.1)$$

$$\dot{\gamma}_2 = p_1 \sin\gamma_3 + p_2 \cos\gamma_3$$

$$\dot{\gamma}_3 = p_3 + (p_1 \sin\gamma_3 - p_2 \cos\gamma_3)/\cos\gamma_2$$

Bu erda (123) belgi indekslarni siklik almashuvini bildiradi

$$I_1 = J_1 + B_2 + A_1 + A_3 \quad (123)$$
$$B_i = 2B_i', \quad A = 2A', \quad H_i = 2H_i'$$

M_i –(KA) da xarakatlanuvchi qo`zg`alish momentini ox o`qidagi proektsiyasi,

M_π –sistema yordamida giroskoplarni bo`shatishni yuzaga keltirish momenti,

b_i- pretsessiya o`qidagi ishqalanish koefitsienti,

M_{yi}-dadchik momenti (MD) bilan yuzaga keltiriluvchi boshqarish momenti,

M_α-pretsessiya o`qi bo`ylab qo`zg`alish momenti.

Yuqoridagi farazlar xaqiqiy konstruktsiyada aniq bajarilmasligi mumkin, bu (1) sistemada qo`zg`ovchi faktorlar sifatida qo`shimcha inertsiyali, giroskopik va boshqa momentlarni yuzaga kelishiga olib keladi. Bularning xarakatlari qandaydir usul bilan $M_i$, $M_{\alpha i}$ larning ifodalariga qo`shiluvchi momentlarga kiritilgan va qoniqarli baxolanadi deb faraz qilinadi. Sanab o`tilganlardan tashqari M_i momentga tashqi kuchlar momentlari, reaktiv momentlar, GS rotorning

debalansini yuzaga keltiruvchi momentlar; $M_{\alpha i}$ ga esa ishqalanish momentlari, rotorlarning dinamik debalansi va MD dagi boshqa momentlar kiradi.

Uzoq vaqt davomida KA ning berilgan yo`nalishi talab qilinganda GS bo`shatilishini(razgruzka) xisobga olish zaruriyati kelib chiqadi. Bo`shatilish (razgruzka) quyidagi qonun bo`yicha yuzaga keltiriladi deb qabul qilinadi.

$$M_{\pi}(\alpha_i(t)) = \begin{cases} 0, \; t \in [t_k, \tau_k) \\ -M_{\pi}^0 sign\alpha_i(\tau_k), \quad t \in [\tau_k, t_{k+1}), \; |\alpha_i(\tau_k)| = \alpha_i^0 \end{cases}$$

bu erda

α_i^0-bo`shatilish boshlangandagi erishilgan pretsessiya burchagi,

τ_k-ulash momenti (k=0,1...)

$t_k = \tau_{k-1} + T_{\pi}(k+1)$-bo`shatilish (razgruzka)ning tugash vaqti,

T_{π}-o`zgarmas miqdorlar

M_{π}^0 shunday tanlanadiki, unda $|\alpha_1(t_{k+1})|$ etarlicha kichik bo`lsin.

M_{yi}–boshqarish momenti γ_i va p_i, $\dot{\alpha}_i$ lardan olingan axborotlar asosida shakillantiriladi.

$$M_{yi} = f_i(\alpha_i)(K_{1i}\gamma_i + K_{2i}p_i) - K_{3i}\dot{\alpha}_i$$

Datchiklardagi real xarakteristikalarni xisobga olish bilan M_{yi} ni quyidagi tenglamalardan aniqlash mumkin

$$T_i \dot{M}_{yi} + M_{yi} = F_i(\sigma_i),$$

$$\sigma_i = f_i(\alpha_i)(F_{1i}(\gamma_i) + F_{2i}(p_i)) - F_{3i}(\dot{\alpha}_i)$$

bu erda T_i- Γ_i parraklarni boshqarish zanjiridagi o`zgarmas vaqt $f_i(\alpha_i)=\sec\alpha_i$ *ёки* $f_i(\alpha_i)\equiv 1;$ $F_i(\sigma_i),\ F_{ji}(x_i)$-lar chiziqsiz xarakteristikalar bo`lib,

$$|\gamma_i|\le\gamma_i^*<\frac{\pi}{2};\quad |p_i|\le p_i^*$$
$$\left|\dot{\alpha}_i\right|\le q_i^*=\alpha_i^*,\qquad |\alpha_i|\le\alpha_i^*\le\frac{\pi}{2}\tag{2.2}$$

soxada vaqt bo`yicha bir qiymatlimas o`zgaradi va

$$\left|F_{ij}(x_j)-k_{ji}x_j\right|\le x_{ji}^0$$
$$\min(\sigma_i^*,\sigma_i-\sigma_i^0)\le F_i(\sigma_i)\le\max(-\sigma_i^*,\sigma_i+\sigma_i^*)$$

shartni qanoatlantiradi.

$x_{ji}^0(x_{1i}^0=\gamma_i^*,x_{2i}^0=p_i^0,x_{3i}^0=\dot{\alpha}_i^0=q_i^0)$ miqdorlar sezgirlik soxasida,eng sezgir elementlar sifatidagi shovqunlar, shuningdek signallarni kuchaytirgichlar va boshqa chiziqchiz xarakteristikalardan aniqlanadi. σ_i^0 sezmaslik soxasi va boshqa chiziqsiz statistik xarakteristikalardan aniqlanadi. σ_i^* datchik momentlari yoki kuchaytiruvchi qurilmalarni to`ydirilishi bilan bog`liq bo`lgan miqdor. [1] monografiyaga asosan quyidagi belgilashlarni kiritamiz:

$$x_{1i}=\gamma_i,\ x_{2i}=p_i,\ x_{3i}=q_i=\dot{\alpha}_i,\ x_{4i}=\frac{M_{yi}}{B_i}=u_i;\quad \alpha_i=\frac{H_i}{I_i};\ a_{ij}=\frac{H_j}{I_i}\quad (i\ne j),$$
$$\Delta a_1=(I_3-I_2)I_1\quad (123)$$
$$\psi_1=a_{13}p_2\sin\alpha_3-a_{12}p_3\sin\alpha_2+\Delta a_1p_2p_3\quad (123)$$
$$\theta_1=\frac{[p_1(\cos\gamma_3-\cos\gamma_2)-p_2\sin\gamma_3]}{\cos\gamma_2}\qquad \theta_2=p_2(\cos\gamma_3-1)+p_1\sin\gamma_3$$
$$\theta_3=(p_2\sin\gamma_3-p_1\cos\gamma_3)tg\gamma_2;\qquad m_i=\frac{M_i}{I_i};\ m_{p1}=\frac{M_{ai}}{B_i},\ n_{ji}=\frac{k_{ji}}{(B_iT_i)};\ g_i=\frac{H_i}{B_i},$$
$$h_i=\frac{b_i}{B_i},\quad d_i=\frac{1}{T_i},\quad v_i=\frac{\sigma_i}{(B_iT_i)},\quad \varphi_i(v_i)=\frac{F_i(B_iT_iv_i)}{(B_iT_i)}-v_i,$$
$$r=v_i-f(\alpha_i)(n_{1i}\gamma_i+n_{2i}p_i)+n_{3i}g_i$$

Bu belgilashlarda (2.1) sistema quyidagi ko`rinishga keladi.

$$
\begin{aligned}
&\dot{x}_{1i} = x_{2i} + \theta_i \\
&\dot{x}_{2i} = -a_i x_{3i}\cos\alpha_i + m_i + m_{Pi} - \psi_i \\
&\dot{x}_{3i} = -h_i x_{3i} + g_i x_{2i}\cos\alpha_i + x_{4i} + m_{1i} \\
&\dot{x}_{4i} = -d_i x_{4i} + f_i(\alpha_i)(n_{1i}x_{1i} + n_{2i}x_{2i}) - n_{3i}x_{3i} + r_i + \varphi_i(v_i) \\
&\dot{\alpha} = x_{3i}, \qquad i = 1,2,3.
\end{aligned}
\tag{2.3}
$$

[45,46] maqolalardagi tekshirish metodlariga mos belgilashlarni kiritamiz

$x_j = (x_{j1}, x_{j2}, x_{j3})^T$, $j = 1,2,3,4;$ $\quad \alpha = (\alpha_1, \alpha_2, \alpha_3)^T;$ $\quad \mu_i = \cos t_i;$ $i = 1,2,3,$

$\mu_k = \cos\alpha_{k-3}, k = 4,5,6;$ $\quad m = (m_1, m_2, m_3)^T,$ $\quad m_\alpha = (m_{\alpha 1}, m_{\alpha 2}, m_{\alpha 3})^T,$ $\quad m_P = (m_{P1}, m_{P2}, m_{P3})^T,$

$A = diag(a_1, a_2, a_3),$ $G = diag(g_1, g_2, g_3),$ $H = diag(h_1, h_2, h_3),$ $D = diag(d_1, d_2, d_3),$

$N_i = diag(n_{i1}, n_{i2}, n_{i3}), i = 1,2,3;$ $\quad M = diag(\mu_4, \mu_4, \mu_6),$ $f(\alpha) = diag(f_1(\alpha_1), f_2(\alpha_2), f_3(\alpha_3)),$

$r = (r_1 + \varphi_1(v_1), r_2 + \varphi_2(v_2), r_3 + \varphi_3(v_3))^T,$

$$
S_1 = \begin{pmatrix} \mu_3 & -\mu_3' & 0 \\ -\mu_2\mu_3' & \mu_2\mu_3' & 0 \\ -\mu_2^2\mu_3' & \mu_2^2\mu_3' & \mu_2 \end{pmatrix}; \qquad \mu_3' = \sqrt{1-\mu_3^2}: \quad S_2 = \begin{pmatrix} 0 & a_{13}\mu_5' & \Delta a_1 p_2 - a_{12}\mu_6' \\ \Delta a_2 p_3 - a_{23}\mu_6' & 0 & a_{21}\mu_4' \\ a_{32}\mu_5' & \Delta a_3 p_1 - a_{31}\mu_4' & 0 \end{pmatrix}
$$

$\mu_k' = \sqrt{1-\mu_k^2} \qquad\qquad k = 4,5,6.$

Bu belgilashlarda (2.3) sistema quyidagi ko`rinishga keladi

$$
\begin{aligned}
&\mu_2 \dot{x}_1 = S_1 x_2 \\
&\dot{x}_2 = -S_2 x_2 - MAx_3 + m + m_P \\
&\dot{x}_3 = -Hx_3 + MGx_2 + x_4 + m_\alpha \\
&\dot{x}_4 = -Dx_4 + f(\alpha)(N_1 x_1 + N_2 x_2) - N_3 x_3 + r \\
&\dot{\alpha} = x_3
\end{aligned}
\tag{2.4}
$$

Bu erda S_1, S_2 matritsalar va μ_j $j = 1,2,..,6$ o`zgarmaslar (2) soxada quyidagi shartlarni qanoatlantiradi

$$
\underline{S}_1 \le S_1 \le \overline{S}_1, \quad \underline{S}_2 \le S_2 \le \overline{S}_2, \qquad \mu_j \in (0,1]
$$

bu erda

$$\underline{S}_1 = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \overline{S}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \underline{S}_2 = \begin{pmatrix} 0 & 0 & \Delta a_1 P_2^* - a_{12} \\ \Delta a_2 P_3^* - a_{23} & 0 & 0 \\ 0 & \Delta a_3 P_1^* - a_{31} & 0 \end{pmatrix}$$

$$\overline{S}_2 = \begin{pmatrix} 0 & a_{13} & \Delta a_1 P_1^* \\ \Delta a_2 P_3^* & 0 & a_{21} \\ a_{32} & \Delta a_3 P_1^* & 0 \end{pmatrix}$$

§ 2.2. Lyapunov matritsa funktsiyasini qurish.

(2.4) sistema uchun

$U(x_1, x_2, x_3, x_4, \alpha, \mu_2, M) =$ [redacted] $_{ij}(\cdot)$, $\quad v_{ij} = v_{ji}, \quad i, j = 1,2,3,\dots,5 \qquad (2.5)$

matritsa–funktsiya elementlarini quyidagi ko`rinishda quramiz.

$$\begin{aligned}
&v_{11}(x_1,\mu_2) = \mu_2 x_1^T B_{11} x_1, && v_{12}(x_1,x_2,\mu_2) = \mu_2 x_1^T B_{12} x_2, \\
&v_{13} = 0, && v_{14}(x_1,x_4,\mu_2,M) = \mu_2 x_1^T B_{14}(Mx_2), \\
&v_{15} = 0, && v_{22}(x_2) = x_2^T B_{22} x_2, \\
&v_{23}(x_2,x_3) = x_2^T B_{23} x_3, && v_{24}(x_2,x_4,M) = x_2^T B_{24}(Mx_4), \\
&v_{25}(x_2,\alpha) = x_2^T B_{25}\alpha, && v_{33}(x_3) = x_3^T B_{33} x_3, \\
&v_{34}(x_{31},x_{41},M) = x_3^T B_{34}(Mx_4), && v_{35}(x_3,\alpha) = x_3^T B_{35}\alpha, \\
&v_{44}(x_4,M) = (Mx_4)^T B_{44} x_4, \quad v_{45} = 0, && v_{55}(\alpha) = \alpha^T B_{55}\alpha
\end{aligned} \qquad (2.6)$$

Bu erda $B_{ii,}$ i=1,2,3,4,5- musbat aniqlangan simmetrik matritsalar, B_{11}, B_{12}, B_{13},B_{23}, B_{24}, B_{25}, B_{34}, B_{35}-o`zgarmas matritsalar.

(2.6) funktsiyalar uchun quyidagi baxolar o`rinli

$$\mu_2\lambda_m(B_{11})\|x_1\|^2 \le v_{11}(x_1,\mu) \le \mu_2\lambda_M(B_{11})\|x_1\|^2 : \quad \lambda_m(B_{22})\|x_2\|^2 \le v_{22}(x_2,\mu) \le \lambda_M(B_{22})\|x_2\|^2$$
$$\lambda_m(B_{33})\|x_3\|^2 \le v_{33}(x_3,\mu) \le \lambda_M(B_{33})\|x_3\|^2 : \quad \underline{\mu}\lambda_m(B_{44})\|x_4\|^2 \le v_{44}(x_4,M) \le \overline{\mu}\lambda_M(B_{44})\|x_4\|^2$$
$$-\mu_2\lambda_m^{\frac{1}{2}}(B_{12}B_{12}^T)\|x_1\|\|x_2\| \le v_{12}(x_1,x_2,\mu_2) \le \mu_2\lambda_M^{\frac{1}{2}}(B_{12}B_{12}^T)\|x_1\|\|x_2\|$$
$$-\mu_2\overline{\mu}\lambda_m^{\frac{1}{2}}(B_{14}B_{14}^T)\|x_1\|\|x_4\| \le v_{14}(x_1,x_4,\mu_2,M) \le \mu_2\overline{\mu}\lambda_M^{\frac{1}{2}}(B_{14}B_{14}^T)\|x_1\|\|x_4\| \tag{2.7}$$
$$-\lambda_m^{\frac{1}{2}}(B_{23}B_{23}^T)\|x_2\|\|x_3\| \le v_{23}(x_2,x_3) \le \lambda_M^{\frac{1}{2}}(B_{23}B_{23}^T)\|x_2\|\|x_3\|$$
$$-\overline{\mu}_2\lambda_m^{\frac{1}{2}}(B_{24}B_{24}^T)\|x_2\|\|x_4\| \le v_{24}(x_2,x_4) \le \overline{\mu}_2\lambda_M^{\frac{1}{2}}(B_{24}B_{24}^T)\|x_2\|\|x_4\|$$
$$-\lambda_M^{\frac{1}{2}}(B_{25}B_{25}^T)\|x_2\|\|\alpha\| \le v_{25}(x_2,\alpha) \le \lambda_M^{\frac{1}{2}}(B_{25}B_{25}^T)\|x_2\|\|\alpha\|$$
$$-\overline{\mu}_2\lambda_M^{1/2}(B_{34}B_{34}^T)\|x_3\|\|x_4\| \le v_{34}(x_3,x_4,M) \le \overline{\mu}_2\lambda_M^{1/2}(B_{34}B_{34}^T)\|x_3\|\|x_4\|$$
$$-\lambda_M^{1/2}(B_{35}B_{35}^T)\|x_3\|\|\alpha\| \le v_{35}(x_3,\alpha) \le \lambda_M^{1/2}(B_{35}B_{35}^T)\|x_3\|\|\alpha\|$$
$$-\lambda_M^{1/2}(B_{55}B_{55}^T)\|\alpha\|^2 \le v_{55}(\alpha) \le \lambda_M^{1/2}(B_{55}B_{55}^T)\|\alpha\|^2$$

bu erda $\underline{\mu} = \min(\mu_4,\mu_5,\mu_6)$, $\overline{\mu} = \max(\mu_4,\mu_5,\mu_6)$, $\lambda_m(B_{ii}) - B_{ii}$ matritsaning minimal xos qiymati, $\lambda_M(B_{ii}) - B_{ii}$ matritsaning maksimal xos qiymati. i=иймати. i=1,2,3,4,5, $\lambda_M^{1/2}(B_{ij},B_{ij}^T) - B_{ij}$ –matritsa normasi.

(2.5) matritsa–funktsiya va $\eta = (1,1,1,1,1)^T$ o`zgarmas vektor yordamida quyidagi skalyar funktsiyani tuzamiz.

$$V(x_1,x_2,x_3,x_4,\alpha,\mu_2,M) = \eta^T U(x_1,x_2,x_3,x_4,\alpha,\mu_2,M)\eta \tag{2.8}$$

Agar (2.5) matritsa–funktsiya elementlari (2.7) shartlarni qanoatlantirsa, u xolda (2.8) funktsiya uchun quyidagi ikki tamonlama baxo o`rinli bo`ladi.

$$u^T \underline{B}u \le V(x_1,x_2,x_3,x_4,\alpha,\mu_2,M) \le u^T \overline{B}u \tag{2.9}$$

bu erda

$$u = (\|x_1\|,\|x_2\|,\|x_3\|,\|x_4\|,\|\alpha\|)^T$$

$\underline{B} = \{\underline{b}_{ij}\}_{ij=1}^{5}, \quad \underline{b}_{ij} = \underline{b}_{ji}, \qquad \overline{B} = \{\overline{b}_{ij}\}_{ij=1}^{5}, \quad \overline{b_{ij}} = \overline{b}_{ji},$

$\underline{b}_{11} = \mu_2 \lambda_m(B_{11}), \quad \underline{b}_{22} = \lambda_m(B_{22}), \quad \underline{b}_{33} = \lambda_m(B_{33})$

$\underline{b}_{44} = \lambda_m(B_{44}), \quad \underline{b}_{55} = \lambda_m(B_{55}), \quad \overline{b}_{11} = \mu_2 \lambda_M(B_{11}),$

$\overline{b}_{22} = \lambda_M(B_{22}), \quad \overline{b}_{33} = \lambda_M(B_{33}),$

$\overline{b}_{44} = \mu\lambda_M(B_{44}), \quad \overline{b}_{55} = \lambda_M(B_{55}), \quad \overline{b}_{12} = -\underline{b}_{12} = \mu_2 \lambda_M^{1/2}(B_{12}B_{12}^T),$

$\overline{b}_{13} = -\underline{b}_{13} = 0, \quad \overline{b}_{14} = -\underline{b}_{14} = \mu_2 \overline{\mu}\lambda_M^{1/2}(B_{14}B_{14}^T),$

$\overline{b}_{15} = -\underline{b}_{15} = 0, \quad \overline{b}_{23} = -\underline{b}_{23} = \lambda_M^{1/2}(B_{23}B_{23}^T),$

$\overline{b}_{24} = -\underline{b}_{24} = \overline{\mu}\lambda_M^{1/2}(B_{24}B_{24}^T), \quad \overline{b}_{25} = -\underline{b}_{25} = \lambda_M^{1/2}(B_{25}B_{25}^T),$

$\overline{b}_{34} = -\underline{b}_{34} = \overline{\mu}\lambda_M^{1/2}(B_{34}B_{34}^T), \quad \overline{b}_{35} = -\underline{b}_{35} = \lambda_M^{1/2}(B_{35}B_{35}^T),$

$\overline{b}_{45} = -\underline{b}_{45} = 0.$

§ 3. Sistema turg`unligining etarli shartlari.

Sistema turg`unligining etarli shartlarini xosil qilish uchun avval $\underline{B}$ matritsaning musbat aniqlangan bo`lishlik shartlarini aniqlaymiz, so`ngra (2.4) sistema yordamida (2.8) funktsiyadan olingan xosilani yuqoridan baxolaymiz. Bu baxolash quyidagi ko`rinishda bo`ladi.

$$\frac{d}{dt}V(x_1, x_2, x_3, x_4, \alpha, \mu_2, M) = \sum_{i=1}^{5}\frac{dv_{11}}{dt} + 2\sum_{i=1}^{5}\sum_{j=2}^{5}\frac{dv_{ii}}{dt} \le$$

$$\le \sum_{i=1}^{4} x_i^T K_{ii} x_i + 2\sum_{i=1}^{4}\sum_{j=2}^{4} x_i^T K_{ij} x_j + \sum_{i=1}^{4} x_i^T K'_{i1}\alpha + \sum_{i=1}^{4} x_i^T K'_{i2}(m + m_p) +$$

$$+ \sum_{i=1}^{4} x_i^T K'_{i3} m_\alpha + \sum_{i=1}^{4} x_i^T K'_{i4} r + m_\alpha^T B_{35}\alpha + (m + m_p)^T B_{25}\alpha,$$

bu erda

$$K_{11}=\mu_2^*(B_{14}N_1+(B_{14}N_1)^T),$$
$$K_{22}=-\tilde{S}_2^T B_{22}-B_{22}\tilde{S}_2+B_{12}+B_{23}M^*G+(B_{23}M^*G)^T+B_{24}N_2+(B_{24}N_2)^T$$
$$K_{33}=-H^T B_{33}-B_{33}H-(M^*A)^T B_{23}-B_{23}M^*A-B_{34}M^*N_3-(B_{34}M^*N_3)^T$$
$$K_{44}=-(M^*D)^T B_{44}M^*D+B_{34}M^*+(B_{34}M^*)^T$$
$$K_{12}=B_{11}\tilde{S}_1+\mu_2^*B_{12}\tilde{S}_2+\mu_2^*B_{14}N_2,\qquad K_{13}=-\mu^*B_{12}M^*A-\mu_2^*B_{14}N_3+N_1^T B_{34}$$
$$K_{14}=N_1^T B_{44}-\mu_2^*B_{14}M^*D+N_1^T B_{24}^T,$$
$$K_{23}=-B_{22}M^*A+(M^*G)^T B_{33}-\tilde{S}_2^T B_{23}-B_{23}H-B_{24}M^*N_3+N_2^T B_{34}+B_{25}$$
$$K_{24}=B_{33}+N_2^T B_{44}+B_{23}-B_{24}M^*D+\mu_2^*\tilde{S}_2^T B_{14}-\tilde{S}_2B_{24}M^*+(M^*G)^T B_{34}$$
$$K_{34}=-(M^*N_3)^T B_{44}-(M^*A)^T B_{24}-B_{34}M^*D-H^T B_{34}M^*,\qquad K'_{11}=0,$$
$$K'_{21}=2(\tilde{S}_2^T B_{25}+(M^*G)^T B_{35}),\quad K'_{31}=2(B_{55}-(M^*A)^T B_{25}-H^T B_{35}),\qquad K'_{41}=2B_{35},$$
$$K'_{12}=2\mu_2^*B_{22},\quad K'_{22}=2B_{22},\quad K'_{23}=2B_{23},\quad K'_{42}=2M^*B_{24}^T,\quad K'_{13}=0,\quad K'_{23}=2B_{23},$$
$$K'_{33}=2B_{33},\quad K'_{43}=2M^*B_{34}^T,\quad K'_{14}=2\mu_2B_{14}M^*,\quad K'_{24}=2B_{24}M^*,\quad K'_{34}=2B_{34}M^*,$$
$$K'_{44}=2B_{44}M^*$$

$$\mu_2^*=\begin{cases}\cos\gamma_2^* & agar\ mos\ ko'\ paytuvchi\ manfiy\ bo'lsa\\ 1, & agar\ mos\ ko'\ paytuvchi\ musbat\ bo'lsa\end{cases},\qquad M^*=diag(\mu_4^*,\mu_5^*,\mu_6^*),$$

$$\mu_k^*=\begin{cases}\cos\alpha_{k-3}^*, & agar\ mos\ ko'\ paytuvchi\ manfiy\ bo'lsa\\ 1, & agar\ mos\ ko'\ paytuvchi\ musbat\ bo'lsa.\end{cases}\qquad k=4,5,6$$

$$\tilde{S}_1=\begin{pmatrix}\mu_3^* & -\mu_3^* & 0\\ -\mu_2^*\mu_3^{*\prime} & \mu_2^*\mu_3^{*\prime} & 0\\ -(\mu_2^*)^2\mu_3^{*\prime} & (\mu_2^*)^2\mu_3^{*\prime} & \mu_2^*\end{pmatrix},\qquad \mu_3^{*\prime}=\sqrt{1-(\mu_3^*)^2},$$

$$\tilde{S}_2=\begin{pmatrix}0 & a_{13}\mu_6^{*\prime} & \Delta a_1\overline{P}_2-a_{12}\mu_5^{*\prime}\\ \Delta a_2\overline{P}_3-a_{23}\mu_6^{*\prime} & 0 & a_{21}\mu_3^{*\prime}\\ a_{32}\mu_5^{*\prime} & \Delta a_3\overline{P}_1-a_{31}\mu_4^{*\prime} & 0\end{pmatrix},\quad \mu_k^*=\sqrt{1-(\mu_k^*)^2}\qquad k=4,5,6.$$

$$\overline{P}_i=\begin{cases}-P_i^*, & agar\ mos\ ko'\ paytuvchi\ manfiy\ bo'lsa\\ P_i^*, & agar\ mos\ ko'\ paytuvchimusbat\ bo'lsa.\end{cases}$$

Bir qator xisoblashlardan so`ng quyidagilarga ega bo`lamiz.

$$\frac{d}{dt}V(x_1,x_2,x_3,x_4,\alpha,\mu_2,M)\le w^T Kw+\beta_1\|\alpha\|w+\beta_2\|m+m_p\|w+$$
$$\beta_3\|m_\alpha\|w+\beta_4\|r\|w+\lambda_M^{1/2}(B_{35}B_{35}^T)\|m_\alpha\|\|\alpha\|+\lambda_M^{1/2}(B_{25}B_{25}^T)\|m+m_p\|\|\alpha\|. \qquad (2.10)$$

bu erda

$$w=(\|x_1\|,\|x_2\|,\|x_3\|,\|x_4\|)^T,$$
$$K=\|\rho_{ij}\|_{i,j=1}^{4},\quad \rho_{ij}=\rho_{ji},\quad \rho_{ii}=\lambda_M(K_{ii}),$$
$$\rho_{ij}=\lambda_M(K_{ij}K_{ij}^T),\ i,j=1,2,3,4.\ i\ne j,\quad \beta_i=(\beta_{1j},\beta_{2j},\beta_{3j},\beta_{4j})^T,\ j=1,2,3,4.$$
$$\beta_{ij}-K'_{ij}\quad matritsaning\ normasi.$$

Faraz qilaylik

$\|m_\alpha\|\le\overline{m}_\alpha,\quad \|m+m_p\|\le\overline{m}+\overline{m}_p,\quad \|r\|<\overline{r},$ u xolda (2) soxada (10) ifoda

$$\frac{d}{dt}V(x_1,x_2,x_3,x_4,\alpha,\mu_2,M)\le\lambda_M\|w\|^2+l\|w\|+f \qquad (2.11)$$

ko`rinishni oladi bu erda

$$l=\|\beta_1\|\|\alpha^*\|+\|\beta_2\|(\overline{m}+\overline{m}_p)+\|\beta_3\|\overline{m}_\alpha+\|\beta_4\|\overline{r},$$
$$f=\lambda_M^{1/2}(B_{35}B_{35}^T)\|m_\alpha\|\|\alpha\|+\lambda_M^{1/2}(B_{25}B_{25}^T)\|\overline{m}+\overline{m}_p\|\|\alpha^*\|$$

(2.11) dan

$$\lambda_M(K)<0,$$
$$\|w\|>\frac{l+\sqrt{l^2+4f\lambda_M(K)}}{-2\lambda_M(K)} \qquad (2.12)$$

shart bajarilgandagina $\frac{d}{dt}V(x_1,x_2,x_3,x_4,\alpha,\mu_2,M)$ ifoda manfiy aniqlangan bo`ladi.

Ammo (2.2) soxada

$$\|w\|\le\left\{\sum_{i=1}^{3}\left[(\gamma_i^*)^2+(P_i^*)^2+(q_i^*)^2+(U_i^*)^2\right]\right\}^{1/2} \qquad (2.13)$$

tengsizlik bajariladi, bu erda $U_i^*>|U_i|,\quad i=1,2,3.$

Teorema. 2.1. (2.4) sistema uchun elementlari (2.6) ko`rinishda bo`lgan (2.5) LMF tuzilgan bo`lib, (2.2) soxada bu sistema uchun (2.13) shart bajarilsin. Agar

$$\lambda_m(\underline{B}) > 0, \quad \lambda_M(K) < 0$$

shart bajarilib, w vektor (2.12) tengsizlikni qanoatlantirsa, u xolda (2.4) sistemaning toyimagan xarakati asimptotik turg`un bo`ladi.

Isbot: $\lambda_m(\underline{B}) > 0$ shartdan (2.8) skalyar funktsiyani Lyapunov ma'nosida musbat aniqlanganligi kelib chiqadi. Agar $\lambda_M(K) < 0$ shart va (2.12) tengsizlik bajarilsa u xolda (2.8) funktsiyadan (2.4) sistema bo`yicha olingan xosila manfiy aniqlangan bo`ladi.

[19] da ko`rsatilganidek bu shartlar (2.4) sistema toyimagan xarakatini asimptotik turg`un bo`lishi uchun etarlidir.

Misol 2.1:

(2.4) sistemaning parametrlari quyidagi qiymatlarni qabul qilsin.

$$I_1 = I_2 = I_3 = 1{,}4 \cdot 10^4 \, кгмс^2, \quad H = 70 кгмс^2, \;\; B = 0{,}2\, кгмс^2,$$

$$k_1 = 8 \cdot 10^4 \, кгмс^2, \; k_2 = 1{,}6 \cdot 10^4 \, кгмс^2, \; k_3 = 4\, кгмс^2, \;\; b = 2\, кгмс^2, \;\; T = 0{,}01 c.$$

U xolda xar bir kanal uchun xarakat tenglamalari sistemasi quyidagi ko`rinishga keladi:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -0{,}5 \cdot 10^{-2} \cos\alpha x_3 + m \\
\dot{x}_3 &= -10 x_3 + 350 \cos\alpha x_2 + x_4 + m_\alpha \\
\dot{x}_4 &= -10^2 x_4 + 4 \cdot 10^7 f(\alpha) x_1 + 8 \cdot 10^6 f(\alpha) x_2 - 2 \cdot 10^3 x_3
\end{aligned} \tag{2.14}$$

bu erda $f(\alpha) = \sec\alpha, \quad 0 \le \alpha \le \alpha^* < \frac{\pi}{2}$ (2.14) sistemaning echimi

$$x'(t) = \underset{j=\overline{1,4}}{col}(x'_j(t)), \qquad x'(t_0) = 0$$

bo`lsin. U xolda $y_j = x_j - x'_j, \quad j = \overline{1,4}$ almashtirish bu sistemani quyidagi ko`rinishga keltiradi:

$$\begin{aligned}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -0{,}5 \cdot 10^{-2} \cos\alpha y_3 \\
\dot{y}_3 &= -10 y_3 + 350 \cos\alpha y_2 + y_4 \\
\dot{y}_4 &= -10^2 y_4 + 4 \cdot 10^7 f(\alpha) y_1 + 8 \cdot 10^6 f(\alpha) y_2 - 2 \cdot 10^3 y_3
\end{aligned} \qquad (2.15)$$

(2.15) sistema uchun elementlari

$$\begin{aligned}
&v_{11}(y_1,\alpha) = (800 f^2(\alpha) + 1{,}75) y_1^2, \quad && v_{12}(y_1,y_2,\alpha) = 30 f^2(\alpha) y_1 y_2, \\
&v_{13}(y_1,y_3,\alpha) = -5 \cdot 10^{-3} f(\alpha) y_1 y_3, \quad && v_{14}(y_1,y_4,\alpha) = -0{,}5 \cdot 10^{-4} f(\alpha) y_1 y_4, \\
&v_{22}(y_2,\alpha) = (13{,}8 f^2(\alpha) + 0{,}0525) y_2^2, && v_{23}(y_2,y_3,\alpha) = -1{,}4 \cdot 10^{-3} f(\alpha) y_2 y_3, \\
&v_{24}(y_2,y_4,\alpha) = -10^{-5} f(\alpha) y_2 y_4, && v_{33}(y_3,\alpha) = 0{,}75 \cdot 10^{-6} y_3^2, \\
&v_{34}(y_3,y_4) = 0{,}5 \cdot 10^{-8} y_3 y_4, && v_{44}(y_4) = 0{,}125 \cdot 10^{-9} y_4^2.
\end{aligned} \qquad (2.16)$$

lardan iborat bo`lgan quyidagi matritsa funktsiyani tuzamiz.

$$V_1(y_1,y_2,y_3,y_4,\alpha) = [v_{ij}(\cdot,\cdot,\cdot)]_{ij=1}^{\overline{4}}, \quad v_{ij} = v_{ji} \qquad (2.17)$$

(2.16) funktsiya uchun quyidagi baxolar bajariladi.

$$\begin{aligned}
&v_{11} = (800 f^2(\alpha) + 1{,}75)|y_1|^2, \qquad && v_{22} = (13{,}8 f^2(\alpha) + 0{,}0525)|y_2|^2, \\
&v_{33} = 0{,}75 \cdot 10^{-6} |y_3|^2, && v_{44} = 0{,}125 \cdot 10^{-9} |y_4|^2.
\end{aligned}$$

$$-30f^2(\alpha)|y_1||y_2| \le v_{12} \le 30f^2(\alpha)|y_1||y_2|$$
$$-5\cdot10^{-3} f(\alpha)|y_1||y_3| \le v_{13} \le 5\cdot10^{-3} f(\alpha)|y_1||y_3|$$
$$-0,5\cdot10^{-4} f(\alpha)|y_1||y_4| \le v_{14} \le 0,5\cdot10^{-4} f(\alpha)|y_1||y_4|$$
$$-1,4\cdot10^{-3} f(\alpha)|y_2||y_3| \le v_{23} \le 1,4\cdot10^{-3} f(\alpha)|y_2||y_3|$$
$$-10^{-5} f(\alpha)|y_2||y_4| \le v_{24} \le 10^{-5} f(\alpha)|y_2||y_4|$$
$$-0,5\cdot10^{-8}|y_3||y_4| \le v_{34} \le 0,5\cdot10^{-8}|y_3||y_4|$$

(2.17) matritsa–funktsiya va $\eta = (1,1,1,1)^T$ vektor yordamida

$$V_1(y_1,y_2,y_3,y_4,\alpha) = \eta^T U_1(y_1,y_2,y_3,y_4,\alpha)\eta \qquad (2.18)$$

skalyar funktsiyani kiritamiz. (2.18) funktsiya

$$u^T A_1 u \le v_1(y_1,y_2,y_3,y_4,\alpha) \le u^T B_1 u \qquad (2.19)$$

shartni qanoatlantirishini tekshirib ko`rish qiyin emas.

bu erda $u^T = (|y_1|,|y_2|,|y_3|,|y_4|)$.

$$A_1 = \begin{pmatrix} 800f^2(\alpha)+1,75 & -30f^2(\alpha) & -5\cdot10^{-3} f(\alpha) & -0,5\cdot10^{-4} f(\alpha) \\ -30f^2(\alpha) & 13,8f^2(\alpha)+0,0525 & -1,4\cdot10^{-3} f(\alpha) & -10^{-5} f(\alpha) \\ -5\cdot10^{-3} f(\alpha) & -1,4\cdot10^{-3} f(\alpha) & 0,75\cdot10^{-6} & -0,5\cdot10^{-8} \\ -0,5\cdot10^{-4} f(\alpha) & -10^{-5} f(\alpha) & -0,5\cdot10^{-8} & 0,125\cdot10^{-9} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 800f^2(\alpha)+1,75 & 30f^2(\alpha) & 5\cdot10^{-3} f(\alpha) & 0,5\cdot10^{-4} f(\alpha) \\ 30f^2(\alpha) & 13,8f^2(\alpha)+0,0525 & 1,4\cdot10^{-3} f(\alpha) & 10^{-5} f(\alpha) \\ 5\cdot10^{-3} f(\alpha) & 1,4\cdot10^{-3} f(\alpha) & 0,75\cdot10^{-6} & 0,5\cdot10^{-8} \\ 0,5\cdot10^{-4} f(\alpha) & 10^{-5} f(\alpha) & 0,5\cdot10^{-8} & 0,125\cdot10^{-9} \end{pmatrix}$$

Ma'lumki, agar A_1 matritsa musbat aniqlangan bo`lsa u xolda (2.18) funktsiya xam musbat aniqlangan bo`ladi. Shuning uchun A_1 matritsaning musbat aniqlanganligini tekshiramiz:

1) $$800f^2(\alpha)+1{,}75>0,\quad 13{,}8f^2(\alpha)+0{,}0525>0,$$
$$0{,}75\cdot10^{-6}>0,\quad 0{,}125\cdot10^{-9}>0$$

2) $$\begin{vmatrix} 800f^2(\alpha)+1{,}75 & -30f^2(\alpha) \\ -30f^2(\alpha) & 13{,}8f^2(\alpha)+0{,}0525 \end{vmatrix}=11040f^4(\alpha)+90{,}3f^2(\alpha)+$$
$$0{,}18375-900f^4(\alpha)==10140f^4(\alpha)+90{,}3f^2(\alpha)+0{,}18375-900f^4(\alpha)>0$$

3)

$$\begin{vmatrix} 800f^2(\alpha)+1{,}75 & -30f^2(\alpha) & -5\cdot10^{-3}f(\alpha) \\ -30f^2(\alpha) & 13{,}8f^2(\alpha)+0{,}0525 & -1{,}4\cdot10^{-3}f(\alpha) \\ -5\cdot10^{-3}f(\alpha) & -1{,}4\cdot10^{-3}f(\alpha) & 0{,}75\cdot10^{-6} \end{vmatrix}=$$
$$=8280f^4(\alpha)\cdot10^{-6}+49{,}6125f^2(\alpha)\cdot10^{-6}+0{,}0689062\cdot10^{-6}-420f^4(\alpha)\cdot10^{-6}-$$
$$-345f^4(\alpha)\cdot10^{-6}-1120f^4(\alpha)\cdot10^{-6}-675f^4(\alpha)\cdot10^{-6}-1{,}3125f^2(\alpha)\cdot10^{-6}-$$
$$-3{,}45f^2(\alpha)\cdot10^{-6}=5720f^4(\alpha)\cdot10^{-6}+44{,}87f^2(\alpha)\cdot10^{-6}+0{,}0689062\cdot10^{-6}>0$$

4)

$$\begin{vmatrix} 800f^2(\alpha)+1{,}75 & -30f^2(\alpha) & -5\cdot10^{-3}f(\alpha) & -0{,}5\cdot10^{-4}f(\alpha) \\ -30f^2(\alpha) & 13{,}8f^2(\alpha)+0{,}0525 & -1{,}4\cdot10^{-3}f(\alpha) & -10^{-5}f(\alpha) \\ -5\cdot10^{-3}f(\alpha) & -1{,}4\cdot10^{-3}f(\alpha) & 0{,}75\cdot10^{-6} & -0{,}5\cdot10^{-8} \\ -0{,}5\cdot10^{-4}f(\alpha) & -10^{-5}f(\alpha) & -0{,}5\cdot10^{-8} & 0{,}125\cdot10^{-9} \end{vmatrix}=$$

$$=\begin{vmatrix} 780f^2(\alpha)+1{,}75 & -34f^2(\alpha) & -7\cdot10^{-3}f(\alpha) & 0 \\ -34f^2(\alpha) & 13f^2(\alpha)+0{,}0525 & -1{,}8\cdot10^{-3}f(\alpha) & 0 \\ -7\cdot10^{-3}f(\alpha) & -1{,}8\cdot10^{-3}f(\alpha) & 0{,}55\cdot10^{-6} & 0 \\ -0{,}5\cdot10^{-4}f(\alpha) & -10^{-5}f(\alpha) & -0{,}5\cdot10^{-8} & 0{,}125\cdot10^{-9} \end{vmatrix}=$$

$$
= 0,125\cdot 10^{-9}\begin{vmatrix} 780 f^2(\alpha)+1,75 & -34 f^2(\alpha) & -7\cdot 10^{-3} f(\alpha) \\ -34 f^2(\alpha) & 13 f^2(\alpha)+0,0525 & -1,8\cdot 10^{-3} f(\alpha) \\ -7\cdot 10^{-3} f(\alpha) & -1,8\cdot 10^{-3} f(\alpha) & 0,55\cdot 10^{-6} \end{vmatrix} =
$$

$$
= 5577\cdot 10^{-6} f^4(\alpha) + 35,035\cdot 10^{-6} f^2(\alpha) + 0,050531210^{-6} - 856,810^{-6} f^4(\alpha) - 63710^{-6} f^4(\alpha) -
$$

$$
635,810^{-6} f^4(\alpha) - 2527,210^{-6} f^4(\alpha) - 2,572510^{-6} f^2(\alpha) - 5,7610^{-6} f^2(\alpha) =
$$

$$
920,210^{-6} f^4(\alpha) + 26,702510^{-6} f^2(\alpha) + 0,050531210^{-6} > 0
$$

Endi (2.18) funktsiyadan (2.15) sistema orqali olingan xosilani yuqoridan baxolaymiz.

$$
\dot{V}_1(y_1,y_2,y_3,y_4,\alpha) = \sum_{i=1}^{4}\dot{v}_{ii}(y_i,\cdot) + 2\sum_{i=1}^{4}\sum_{\substack{j=2\\ i<j}}^{4}\dot{v}_{ij}(y_i,y_j,\cdot) \le 2(800 f^2(\alpha)+1,75)y_1y_2 +
$$

$$
+2(13,8 f^2(\alpha)+0,0525)\cdot(-0,5\cdot 10^{-2}\cos\alpha)y_2y_3 + (-1,5\cdot 10^{-5})y_3^2 + 2\cdot 0,5\cdot 10^{-6}\cdot 350\cos\alpha y_2y_3 +
$$

$2\cdot 0,75\cdot 10^{-6} y_3y_4 + 2$ [redacted] $0 f^2(\alpha)y_2^2 - 30 f^2(\alpha)\cdot 0,510^{-2}\cos\alpha y_1y_3 - 5\cdot 10^{-3} f(\alpha)y_2y_3 +$

$$
5\cdot 10^{-2} f(\alpha)y_1y_3 - 5\cdot 10^{-3} f(\alpha)\cdot 350\cos\alpha y_1y_2 - 5\cdot 10^{-3} f(\alpha)y_1y_4 - 0,5\cdot 10^{-4} f(\alpha)y_2y_4 +
$$

$$
0,5\cdot 10^{-2} f(\alpha)y_1y_4 - 2\cdot 10^3 f^2(\alpha)y_1^2 - 4\cdot 10^2 f^2(\alpha)y_1y_2 + 10^{-1} f(\alpha)y_1y_3 +
$$

$$
+1,4\cdot 10^{-3}\cdot 0,5\cdot 10^{-2} f(\alpha)\cos\alpha y_3^2 + 1,4\cdot 10^{-2} f(\alpha)y_2y_3 - 1,4\cdot 10^{-2} f(\alpha)350\cos\alpha y_2^2 -
$$

$$
-1,4\cdot 10^{-3} f(\alpha)y_2y_4 + 0,5\cdot 10^{-7} f(\alpha)\cos\alpha y_3y_4 + 10^{-3} f(\alpha)y_2y_4 - 4\cdot 10^2 f^2(\alpha)y_1y_2 -
$$

$$
-8\cdot 10 f^2(\alpha)y_2^2 + 2\cdot 10 f(\alpha)y_2y_3 - 0,5\cdot 10 f^{-7}(\alpha)y_3y_4 + 175\cdot 10\cos\alpha y_2y_4 + 0,5\cdot 10 f^{-8}(\alpha)y_4^2 -
$$

$$
-0,5\cdot 10^{-6} y_3y_4 + 2\cdot 10^{-1} f(\alpha)y_1y_3 + 4\cdot 10^{-2} f(\alpha)y_2y_3 - 10^{-5} y_3^2 \;=
$$

$$
= -4\cdot 10^{-3} f^2(\alpha)y_1^2 - (160 f^2(\alpha) + 0,98 - 60 f^2(\alpha))y_2^2 - (1,5\cdot 10^{-5} + 2\cdot 10^{-5} - 1,4\cdot 10^{-5})y_3^2 -
$$

$-(0,25\cdot 10^{-7} - \cdot 10^{-8})y_4^2 + 2$ [redacted] $00 f^2(\alpha) + 1,75 - 5\cdot 10^{-3}350 - \cdot 400 f^2(\alpha) - 400 f^2(\alpha))y_1y_2 +$

$$
+(-0,15 f(\alpha) + 0,05 f(\alpha) + 0,1 f(\alpha) + 0,2 f(\alpha))y_1y_3 + (0,005 f(\alpha) - 0,005 f(\alpha) +
$$

$$
+0,005 f(\alpha))y_1y_2 + (-0,069 f(\alpha) - 0,02625\cdot 10^{-2}\cos\alpha + 262,5\cdot 10^{-6}\cos\alpha - 0,005 f(\alpha) +
$$

$$
+0,014 f(\alpha) + 0,02 f(\alpha) + 0,04 f(\alpha))y_2y_3 + (0,001 f(\alpha) - 0,00005 f(\alpha) - 0,0014 f(\alpha) +
$$

$$
+0,001 f(\alpha) + 1,75\cdot 10^{-8}\cos\alpha)y_2y_3 + (0,75\cdot 10^{-6} - 0,25\cdot 10^{-6} + 0,5\cdot 10^{-7} - 0,5\cdot 10^{-7} -
$$

$$
-0,5\cdot 10^{-6})y_3y_4 \;= -4000 f^2(\alpha)y_1^2 - (100 f^2(\alpha) + 0,98)y_2^2 - 2,1\cdot 10^{-5} y_3^2 - 0,15\cdot 10^{-7} y_4^2 +
$$

$+2$ [redacted] $,2 f(\alpha)y_1y_3 + 0,005 f(\alpha)y_1y_4 + (0,00055 f(\alpha) + 1,75\cdot 10^{-6}\cos\alpha)y_2y_4$

Demak (2.18) funktsiyadan (2.15) sistema orqali olingan xosila uchun quyidagi baxoga ega bo`lamiz.

$$\dot{V}_1(y_1, y_2, y_3, y_4, \alpha) \le u^T G_1 u \qquad (2.20)$$

bu erda $u^T = (|y_1|, |y_2|, |y_3|, |y_4|)$

$$G_1 = \begin{pmatrix} -4000 f^2(\alpha) & 0 & 0{,}2 f(\alpha) & 0{,}005 f(\alpha) \\ 0 & -100 f^2(\alpha) - 0{,}98 & 0 & 0{,}00055 f(\alpha) + 1{,}75 \cdot 10^{-6} \cos\alpha \\ 0{,}2 f(\alpha) & 0 & -2{,}1 \cdot 10^{-5} & 0 \\ 0{,}005 f(\alpha) & 0{,}00055 f(\alpha) + 1{,}75 \cdot 10^{-6} \cos\alpha & 0 & -0{,}15 \cdot 10^{-7} \end{pmatrix}$$

G_1 matritsaning ishorasini tekshiramiz.

1) $-4000 f^2(\alpha) < 0, \ -100 f^2(\alpha) - 0{,}98 < 0, \ -2{,}1 \cdot 10^{-5} < 0, \ -0{,}15 \cdot 10^{-7} < 0$

2) $$\begin{vmatrix} -4000 f^2(\alpha) & 0 \\ 0 & -100 f^2(\alpha) - 0{,}98 \end{vmatrix} = 4000 f^2(\alpha)(100 f^2(\alpha) - 0{,}98\ ) > 0$$

3) $$\begin{vmatrix} -4000 f^2(\alpha) & 0 & 0{,}2 f(\alpha) \\ 0 & -100 f^2(\alpha) - 0{,}98 & 0 \\ 0{,}2 f(\alpha) & 0 & -2{,}1 \cdot 10^{-5} \end{vmatrix} = -(100 f^2(\alpha) +$$

$$+ 0{,}98)(0{,}084 f^2(\alpha) - 0{,}04 f^2(\alpha)) = -0{,}044 f^2(\alpha)(100 f^2(\alpha) + 0{,}98) < 0$$

4) $$\begin{vmatrix} -4000 f^2(\alpha) & 0 & 0{,}2 f(\alpha) & 0{,}005 f(\alpha) \\ 0 & -100 f^2(\alpha) - 0{,}98 & 0 & \begin{matrix} 0{,}00055 f(\alpha) + \\ + 1{,}75 \cdot 10^{-6} \cos\alpha \end{matrix} \\ 0{,}2 f(\alpha) & 0 & -2{,}1 \cdot 10^{-5} & 0 \\ 0{,}005 f(\alpha) & 0{,}00055 f(\alpha) + 1{,}75 \cdot 10^{-6} \cos\alpha & 0 & -0{,}15 \cdot 10^{-7} \end{vmatrix} =$$

$$
= \begin{vmatrix} -4000 f^2(\alpha) & 0 & 0 & 0{,}0105 f(\alpha) \\ 0 & -100 f^2(\alpha) - 0{,}98 & 0 & 0{,}00055 f(\alpha) + 1{,}75 \cdot 10^{-6}\cos\alpha \\ 0{,}2 f(\alpha) & 0 & -2{,}1\cdot 10^{-5} & 0 \\ 0{,}005 f(\alpha) & 0{,}00055 f(\alpha) + 1{,}75\cdot 10^{-6}\cos\alpha & 0 & -0{,}15\cdot 10^{-7} \end{vmatrix} =
$$

$$
= 2{,}1\cdot 10^{-5} \begin{vmatrix} -4000 f^2(\alpha) & 0 & 0{,}005 f(\alpha) \\ 0 & -100 f^2(\alpha) - 0{,}98 & 0{,}00055 f(\alpha) + 1{,}75\cdot 10^{-6}\cos\alpha \\ 0{,}005 f(\alpha) & 0{,}00055 f(\alpha) + 1{,}75\cdot 10^{-6}\cos\alpha & -0{,}15\cdot 10^{-7} \end{vmatrix}
$$

$$
= -2{,}1\cdot 10^{-5}(-0{,}0066 f^4(\alpha) - 646{,}8\cdot 10^{-7} f^2(\alpha) + 0{,}00525 f^4(\alpha) + 514\cdot 10^{-7} f^2(\alpha) + \\ + 0{,}00133 f^4(\alpha) + 84{,}7\cdot 10^{-7} f^2(\alpha) + 0{,}13475\cdot 10^{-7}) = -2{,}1\cdot 10^{-5}(-0{,}19\cdot 10^{-4} f^4(\alpha) - \\ - 38{,}1\cdot 10^{-7} f^2(\alpha) + 0{,}13475\cdot 10^{-7}) = 2{,}1\cdot 10^{-9}(0{,}19 f^4(\alpha) + 48{,}1\cdot 10^{-3} f^2(\alpha) - \\ - 0{,}13475\cdot 10^{-3}) > 0
$$

$$
f(\alpha) = \sec\alpha = \frac{1}{\cos\alpha} \geq 1
$$

Demak, G_1 matritsa manfiy aniqlangan ekan, shuning uchun (2.15) sistemaning muvozanat xolati asimptotik turg`un bo`ladi.

Parametrlarning yuqorida keltirilgan qiymatlarida to`la sistema

$$
y_i = x_j - x'_j, \quad j = \overline{1,4}
$$

almashtirishdan so`ng quyidagi ko`rinishga keladi:

$$
\begin{aligned}
&\dot{y}_1 = I\, y_2, \\
&\dot{y}_2 = -0{,}5\cdot 10^{-2}\cos\alpha\, I\, y_3 \\
&\dot{y}_3 = -10\, I\, y_4 + 350\cos\alpha\, I\, y_2 + I\, y_4 \\
&\dot{y}_4 = -10^2 I\, y_4 + 4\cdot 10^7 f(\alpha) I\, y_1 + 8\cdot 10^6 f(\alpha) I\, y_2 - 2\cdot 10^3 I\, y_3
\end{aligned} \qquad (2.21)
$$

bu erda $y_j^T = (y_{j1}, y_{j2}, y_{j3}),\ \ j=\overline{1,4},\ \ I = diag(1,1,1)$ (2.21) sistema uchun elementlari

$$\begin{aligned}
&v_{11}(y_1,\alpha) = y_1^T(800 f^2(\alpha)+1,75)Iy_1, \quad v_{12} = y_1^T 30 f(\alpha) y_2,\\
&v_{13}(y_1,y_3,\alpha) = y_1^T(-5\cdot 10^{-3} f(\alpha) I) y_3 \quad v_{14}(y_1,y_4,\alpha) = y_1^T(-0,5\cdot 10^{-4} I) y_4,\\
&v_{22}(y_2,\alpha) = y_2^T(13,8 f^2(\alpha)+0,0525)Iy_2, v_{23}(y_2,y_3,\alpha) = y_2^T(-1,4\cdot 10^{-3} f(\alpha) I) y_3,\\
&v_{24}(y_2,y_4,\alpha) = y_2^T(-10^{-5} f(\alpha) I) y_4, \qquad v_{33}(y_3,\alpha) = y_3^T(0,75\cdot 10^{-6} I) y_3,\\
&v_{34}(y_3,y_4,\alpha) = y_3^T(0,5\cdot 10^{-8} I) y_4, \ \ v_{44}(y_4,\alpha) = y_4^T(0,125\cdot 10^{-9} I) y_4
\end{aligned} \tag{2.22}$$

ko`rinishda bo`lgan

$$U(y_1,y_2,y_3,y_4,\alpha) = [v_{ij}(\cdot,\cdot,\cdot)]_{ij=1}^{\overline{4}}, \quad v_{ij} = v_{ji} \tag{2.23}$$

matritsa – funktsiyani tuzamiz.

Bevosita tekshirish yo`li bilan

$$V(y_1,y_2,y_3,y_4,\alpha) = \eta U(y_1,y_2,y_3,y_4,\alpha)\eta\,, \tag{2.24}$$

bu erda $\eta = (1,1,1,1)^T$, skalyar funktsiya uchun

$$u^T A_1 u \le V(y_1,y_2,y_3,y_4,\alpha) \le u^T B_1 u \tag{2.25}$$

shartning bajarilishini ko`rish qiyin emas.

(2.24) funktsiyadan (2.21) sistema orqali olingan xosila uchun quyidagi baxo o`rinli bo`ladi.

$$\dot V(y_1,y_2,y_3,y_4,\alpha) \le u^T G_1 u \tag{2.26}$$

Tekshirib ko`rganimizdek $A_1$ matritsa musbat aniqlangan, $G_1$ matritsa esa manfiy aniqlangan, shuning uchun (2.21) sistemaning muvozanat xolati asimptotik turg`un bo`ladi.

3 - BOB

NEYRON TO'RLARNING TURG'UNLIGI

Markaziy nerv sistemasining funktsiyasi xamma organ va to`qimalardagi retseptorlar ta'sirlanganda paydo bo`lgan afferent (markazga intiluvchi) impulslarni qabul qilish, shu ta'surotlarni analiz va sintez qilish, hamda periferik organlarga ta'sir etuvchi efferent (markazdan qochuvchi) impuls oqimlarini vujudga keltirishdan iborat.

Markaziy nerv sistemasi organizmni tashqi muxitga moslanishini, xamma organlarni faoliyatini boshqarishni va birlashtirilishini ta'minlaydi.

Nerv markazlari bilan organlar o`rtasida ikki tomonlama aloqa borligi uchun, markaziy nerv sistemasi turli organlarni faoliyatini idora qiladi.

Markaziy nerv sistemasining tuzilishi va funktsiyasi xaqidagi xozirgi zamon tasavvurlarining asosini neyron nazariyasi tashkil etadi. Bu nazariyaning rivojlanishida ispan neyrogistologi R.Kaxal va ingiliz fiziologi Ch.Sheringtonlar ishlarining axamiyati katta.

Nerv sistemasi 2 xil xujayralar: nerv va glial xujayralardan tuzilgan. Glial xujayralar nerv xujayralariga nisbatan 8,9 barobar ko`p, lekin bu xujayralarda faqat neyronlar orqali informatsiyani o`tkazish bilan bog`liq bo`lgan xamma jarayonlar bajariladi.

Bundan kelib chiqadiki ,nerv sistemasining asosiy strukturaviy elementi nerv xujayrasi yoki neyrondir.

Nerv xujayrasi ta'sirni qabul qilib, uni qayta ishlab navbatdagi xujayraga yoki organga beradi. Xar bir neyronning somasi yoki tanasi bilan o`siklari bor. Akson- uzun o`sik bo`lib, qo`zg`alishni nerv xujayrasining tanasidan boshqa xujayralarga yoki organlarga o`tkazadi. Aksionning xususiyati shuki, xujayra tanasidan faqat bitta uzun o`sik chiqadi. Nerv xujayrasining tanasidan aksion chiqqan joy akson tepachasi deyiladi. Nervning kalta o`siklari dentritlar deyiladi. Ular boshqa neyronlardan kelgan impulslarni qabul qiladi va qo`zg`alishni nerv xujayrasining tanasiga o`tkazadi.

Markaziy nerv sistemasida neyronlarning tanalari bosh miya katta yarim sharlari po`stlog`ida, po`stloq ostidagi tuzilmalar, miya sopi, miyacha va orqa miyaning kulrang moddasida to`plangan. Neyronlarning melin bilan qoplangan o`siklari bosh miya bilan orqa miyaning turli bo`limlaridagi oq moddani xosil qiladi. Nerv xujayrasining tanasi va o`siklari membrana bilan qoplangan.

Retseptor neyronlari qo`zg`alishni qabul qilib periferik retseptorlardan markaziy nerv sistemasiga o`tkazadi. Bu neyronlar xar xil sezgilarni xosil qiluvchi impulslarni etkazib bergani uchun ko`pincha sensor yoki sezuvchi neyronlar deyiladi.

Effektor neyronlar periferik organ va to`qimalarga impuls yuboradi. Bu neyronlarni moto`r yoki xarakat neyronlari deyiladi.

Kontakt, orqali yoki kiritma neyronlar markaziy nerv sistemasida eng katta gurux bo`lib, retseptor va effekto`r xujayralarini bir biriga bog`laydi. Ular yuzaga chiqaradigan effektni xususiyatiga qarab qo`zg`atuvchi va to`rmozlovchi neyronlarga bo`linadi. Umuman olganda inson faoliyatining rivojlanishida neyron to`rlar katta axamiyatga ega bo`lib, ularning dinamik xossalarini, jumladan turg`unligi masalasini o`rganish xozirgi zamonning aktual masalalaridan biridir.

§3.1. Neyron to`rlar matematik modelining taxlili.

Quyidagi chiziqsiz, avtonom oddiy differentsial tenglamani qaraymiz:

$$\dot{x} = -H(x)(-Tx + S(x) - I) \qquad (3.1)$$

bu erda

$$x = (x_1, x_2, \ldots, x_n)^T \in (-1,1)^n, \qquad \dot{x} = \frac{dx}{dt},$$

$H(x) - \forall x \in (-1,1)^n$ da aniqlangan $n \times n$ tartibli matritsa funktsiya, $T = \{T_{ij}\}_{n \times n}$ o`lchovli o`zgarmas matritsa

$S(x) = (S_1(x_1),\dots,S_n(x_n))^T$ *bu yerda* $S_1 : (-1,1) \to R, \quad i = \overline{1,n}$

$I = (I_1,..,I_n)^T$ o`zgarmas xaqiqiy vektor.

(3.1) tenglama Hopfield [76] modelining umumlashganidan tashkil topgan bo`lib, neyron to`rlarning matematik modeli sifatida qaraladi. Bu sistema uchun quyidagi masalalar taxlil qilinadi:

a) (3.1) sistema ixtiyoriy t>0 uchun yagona echimga ega.

b) (3.1) sistema uchun E energetik funktsiyani

$$E(x) = -\frac{1}{2}x^T Tx + \sum_{i=1}^{n}\int_0^x S_i(\rho)d\rho - x^T I \qquad (3.2)$$

bu erda

$E(-1,1)^n \to R, \quad DE : (-1,1)^n \to L(R,R^n), \quad DE(x,y) = \nabla E(x)^T y$

($\nabla E(x)$ E ning gradienti bo`lib,

$\nabla E(x) = (\frac{\partial E(x)}{\partial x_1},\dots\dots,\frac{\partial E(x)}{\partial x_n})^T = -Tx + S(x) - I$ *ва* $E(x)$ dan (3.1) sistema yordamida olingan xosila

$D_{(1)}E(x) = \nabla E(x)^T(-H(x)(-Tx + S(x) - I)) = -\nabla E(x)^T H(x)\nabla E(x)$ ga teng bo`ladi, ko`rinishida olinadi va t ortganda E(x) funktsiya monoton kamayadi.

v) (3.1) sistema uchun muvozanat nuqtalar chekli sonda bo`ladi.

g) Agar (3.1) sistema uchun $\tilde{x}$ muvozanat xolat turg`un bo`lsa, u xolda u asimptotik turg`un bo`ladi.

d) $\tilde{x}$ turg`un (asimptotik) muvozanat xolat E(x) funktsiyaning minimum nuqtasi bo`ladi.

e) n o`lchovli fazoda $2^n$ ta soxalarning xar birida (3.1) sistemaning asimptotik turg`un muvozanat nuqtalari mavjud.

Bu keltirilgan masalalarni qarab chiqish uchun quyidagi farazlarni kiritamiz:

Faraz (A): (3.1) sistema uchun quyidagilar bajarilsin:

a) Ixtiyoriy $x \in (-1,1)^n$ uchun H(x) matritsa funktsiya simmetrik va musbat aniqlangan.

b) T matritsa simmetrik.

s) Ixtiyoriy $i,\ 1 \le i \le n$ uchun $S_i(-1,1) \to R$ funktsiya monoton o`suvchi bo`lib,

$S_i(0) = 0,\ -S_i(-x_i)$ *ва* $S_i^{-1} : R \to (-1,1),$ $S_i''(x_i) = \dfrac{d^2(S_i(x_i))}{dx_i^2}$ funktsiyalar mavjud.

Bu farazlardan quyidagilar kelib chiqadi:

1) $x \neq 0$ *да* $x_i S_i(x_i) > 0$ bo`ladi.

2) $\lim\limits_{\rho \to 1} S_i(\rho) = \infty,$ $\lim\limits_{\rho \to -1} S_i(\rho) = -\infty$

3) $x_i \in (-1,1)$ *учун* $S_i'(x_i) = \dfrac{dS_i(x_i)}{dx_i} > 0$ *ва* $S_i'(x_i) = -S_i'(-x_i)$ bo`ladi.

Lemma 3.1. Agar (3.1) sistema (A) farazdagi shartlarni qanoatlantirsa va E(x) energetik funktsiya (3.2) ko`rinishida aniqlangan bo`lsa, u xolda $\{x_m\} \subset (-1,1)^n$ uchun $m \to \infty$ *да* $x_m \to \partial(-1,1)^n,$ $(\partial(-1,1)^n -\ (-1,1)^n$ soxaning yopig`i va $m \to \infty$ *да* $E(x) \to \infty$ bo`ladi.

Isbotш:

$a = \sup\left|\{ x^T T x - x^T I \right| : x \in (-1,1)^n$ bo`lsin. U xolda $a \le \|T\| + |I| < \infty$ ga ega bo`lamiz.

$f_i(\xi)=\int_0^{\xi} S_i(\rho)d\rho, \quad \xi\in(-1,1),\ i=\overline{1,n}$ bo`lsin. (A) farazdan foydalanib har bir i uchun $f_i(\xi)\geq 0,\ \lim_{\xi\to 1} f_i(\xi)=\infty$ ва $\lim_{\xi\to -1} f_i(\xi)=\infty$ ifodalarni xosil qilamiz. $f(x)=\max_{1\leq i\leq n}\{f_i(x_i)\}$ bo`lsin va $E(x)\geq f(x)-a$ ekanligidan lemmani quyidagicha yozish mumkin $x_m\to\partial(-1,1)^n \quad \partial a \quad f(x_m)\to\infty$ bo`ladi. Lemma isbot bo`ldi.

Lemma 3.2. Agar sistema (A) farazlarni qanoatlan– tirsa, u xolda faqat va faqat $\nabla E(x)=0$ bo`lgandagini $x\in(-1,1)^n$ nuqta (3.1) sistemaning muvozanat nuqtasi bo`ladi. E(x) funktsiyaning kritik nuqtalar to`plami (3.1) sistemaning muvozanat nuqtalari to`plami bilan ustma -ust tushadi.

Isbot: x nuqta E ning kiritik nuqtasi deyiladi agar va faqat agar $\nabla E(x)=-Tx+S(x)-I=0$ bo`lsa. Bundan esa x nuqta (3.1) sistemaning muvozanat nuqtasi deyiladi agar va faqat agar $-H(x)(-Tx+S(x)-I)=0$ bo`lsa. Bu teoremani isbotlaydi.

Faraz (B.1). (A) farazdagi shartlar bajarilsin.

a) Quyidagi shartlarni bir vaqtda qanoatlantiruvchi $x\in(-1,1^n)$ mavjud emas.

1) $\nabla E(x)=0$ 2) $\det(\Im_E(x))=0$ 3) $\Im_E(x)\geq 0$ 4) $(S_1''(x_1),\ldots,S_n''(x_n))^T\perp N$

bu erda

$$N=\{z=(y_1^3,\ldots,y_n^3)^T\in R^n : \Im_E(x)(y_1,\ldots,y_n)^T=0\}$$

$$\Im_E(x)=\left[\frac{\partial^2 E(x)}{\partial x_i x_j}\right]=-T+diag(S_1''(x_1),\ldots,S_n''(x_n))$$

$D^2E(x,y,z)=y^T\,\Im_E(x)z$ bo`lib, $\Im_E(x)$ E funktsiyaning yakobiyan matritsasi va

$$D^2E:(-1,1)^n\to L^2(R^n;R),\qquad D^3E(x,y,z,u)=\sum_{i=1}^{n} S_i''(x_i)y_iz_iu_i;$$

b) (3.1) sistemaning muvozanat nuqtalar to`plami diskret bo`ladi.

Faraz (B.2). (A) farazdagi shartlar bajarilsin. $\nabla E(x) = 0$ *ва* $\det(\Im_E(x)) = 0$ tenglamalarni bir vaqtda qanoatlantiruvchi x mavjud emas.

(B.2) faraz (B.1) farazning birinchi qismiga mos kelib, undagi talablarni biroz yumshatadi, shuning uchun undan foydalanish qulayroq. (B.2) farazdan ko`rinadiki $\nabla E(x)$ funktsiyaning xar bir noli ajralganligi kelib chiqadi. Bundan Lemma 1.2 ga asosan (3.1) sistemaning muvozanat nuqtalarining ajralganligi kelib chiqadi. Umuman olganda (B.1) faraz (B.2) farazdan kelib chiqadi.

Lemma 3.3. Agar (A) farazdagi shartlar (3.1) sistema bilan T o`zgarmas matritsa uchun bajarilsa, u xolda (B.2) faraz Lebeg o`lchovidagi $I \in R^n$ lar uchun o`rinli bo`ladi.

Isbot: T doimiy uchun $K \in C'$ $K : (-1,1)^n \to R^n$ $K(x) = \nabla E(x) + I = -Tx + S(x)$ funktsiyani aniqlaymiz Sard ning teoremasiga ko`ra shunday $Q \in R^n$ mavjudki agar $K(x) \in R^n - Q$ bo`lsa $\det(DK(x)) \neq 0$ bo`ladi. Bundan $I \in R^n - Q$ da agar $\nabla E(x) = 0$ bo`lsa u xolda $K(x) = 0 + I = I \in R^n - Q$ *ва* $\det(J_E(x)) = \det(DK(x)) \neq 0$ bo`ladi.

Faraz (S). (A) farazdagi barcha shartlar bajarilib, ixtiyoriy

$$i\ (1 \le i \le n)\ \text{да} \qquad S_i(x) : (-1,1) \to R$$

funktsiya uchun

$$S_i''(\rho) > 0, \ \ \rho \in (0,1)$$

bo`lsin.

Teorema 3.1 Agar (3.1) sistema (A) va (B.1) farazlardagi shartlarni qanoatlantirsa, u xolda

1) ixtiyoriy $x \in (-1,1)$ uchun (3.1) sistemani $\varphi(\cdot, 0.x)$ yagona echimi mavjud.

2) (3.1) sistemaning muvozanat xolatidan tashqari echimlarida E energetik funktsiya monoton kamayadi. Shuningdek, (3.1) sistemani o`zgaruvchi davrli bo`lmagan echimlari mavjud.

3) $[0,\infty)$ da (3.1) sistemaning echimi mavjud.

4) (3.1) sistemaning xar bir muvozanat xolatiga mos kelmagan echimlari $t \to \infty$ da yaqinlashadi.

5) (3.1) sistema uchun chekli limitga ega bo`lgan muvozanat nuqtalar mavjud.

Isbot: E ning ixtiyoriy $\varphi:[0,\tilde{t}) \to (-1,1)^n$ echimidan t bo`yicha olingan xosilasi quyidagicha bo`lsin

$$D_{(L)}E(\varphi(t)) = -\nabla E(\varphi(t))^T H(\varphi(t))\nabla E(\varphi(t)), \quad t \in [0,\tilde{t})$$

$t \in [0,\tilde{t})$ lar uchun quyidagilarga egamiz $H(\varphi(t))\nabla E(\varphi(t)) \neq 0$ ва $\nabla E(\varphi(t)) \neq 0$ Bundan esa $H(\varphi(t))$ ni musbat aniqlanganlig ya'ni $\nabla E(\varphi(t))^T H(\varphi(t))\nabla E(\varphi(t)) > 0$ ekanligi kelib chiqadi. Bundan tashqari $D_{(L)}E(\varphi(t)) < 0$ ekanligidan $0 \le t_1 < t_2 < \tilde{t}$ da quyidagi munosabatga ega bo`lamiz $E(\varphi(t_2)) - E(\varphi(t_1)) = \int_{t_1}^{t_2} D_{(L)}E(\varphi(t))dt < 0$. Faraz qilaylik teoremaning 3–qismi noto`g`ri, ya'ni $\varphi$ faqat $[0,\tilde{t})$, $\tilde{t} > 0$ da mavjud. U xolda 3.1 lemmaga asosan shunday $t$, $0 < t < \tilde{t}$ topladiki $E(\varphi(0)) < E(\varphi(t))$ munosabat o`rinli bo`ladi. Teoremaning ikkinchi qismi va 3.1 lemmaga ko`ra (3.1) sistemaning ixtiyoriy $\varphi(\cdot,\tilde{x}):[0,+\infty) \to (-1,1)^n$ echimi uchun shunday $\sigma > 0$ topiladiki $\varphi([0,+\infty)) \subset C$ bo`ladi, bu erda $C = (-1+\sigma, 1-\sigma)^n$.

$\Omega(\varphi) = \{x \in (-1,1)^n : \{t_m\} \subset [0,+\infty),\ t_m \to +\infty,\ x = \lim_{m\to\infty} \varphi(t_m)\}$ bo`lsin. $\Omega(\varphi)$ ni xar bir elementi φ ning $\Omega-$limit nuqtasi deyiladi. $\Omega(\varphi) \subset \varphi([0,+\infty)) \subset \overline{C} \subset (-1,1)^n$ ga egamiz, $\overline{C}$ kompakt. Oddiy differentsial tenglama uchun invariantlik teoremasidan $\varphi(t)$ ni $\Omega(\varphi)$ ga yaqinlashishi kelib chiqadi va xar bir $x \in \Omega(\varphi)$ uchun quyidagi munosabat o`rinli bo`ladi $D_{(L)}E(x) = -\nabla E(x)^T H(x)\nabla E(x) = 0$ Bundan $H(x)$ ni musbat aniqlanganligi kelib chiqadi.Bundan tashqari φ ning $\Omega-$limit nuqtasi (3.1) sistemaning muvozanat xolati bo`ladi. B.1 farazga asosan (3.1) ning muvozanat nuqtalar to`plami diskret, ya'ni $\Omega(\varphi)$. $\Omega(\varphi) \subset \overline{C}$ va $\overline{C}$ ning kompaktligidan $\Omega(\varphi)$ chegaralanganligi kelib chiqadi. $\Omega(\varphi)$ o`zida faqat bitta nuqtani saqlasin deb olaylik. Boshqacha qilib aytganda $x \in \Omega(\varphi)$ nuqtani olamiz va $\varepsilon > 0$ x dan

$\Omega(\varphi)-\{x\}$ nuqtagacha bo`lgan eng qisqa masofa bo`lsin. U xolda yuqoridagilarga asosan shunday $t>0$ topiladiki $t>t$ lar uchun $x_t \in \Omega(\varphi)$ quyidagi munosabatni qanoatlantiradi. $|\varphi(t)-x_t|<\frac{\varepsilon}{3}$. $B_1=(x,\frac{\varepsilon}{3})$, ва $B_2=(x,\frac{\varepsilon}{3})$ bo`lsin, bu erda $x \in \Omega(\varphi)-\{x\}$. $B_1$ va $B_2$ lar bir biriga bog`liq emas. Bundan biz quyidagi munosabatlarni xosil qilamiz $\varphi((t,+\infty)) \subset B_1 \cup B_2$ va $\Omega(\varphi)$ ni berilishidan $\varphi((t,+\infty)) \cap B_1 \neq \varnothing$ va $\varphi((t,+\infty)) \cap B_2 \neq \varnothing$. Ammo bu $\varphi((t,+\infty))$ ni bog`liqliligiga zid.

$b=\sup\{|Tx-I|: x\in(-1,1)^n\}$ bo`lsin, u xolda $b \le \|T\|+|I|<+\infty$ munosabat o`rinli. A farazga ko`ra xar bir $i$ uchun $\rho \to \pm 1$ da $s_i(\rho) \to \pm\infty$ bo`ladi. Demak shunday $\delta,\ 0<\delta<\frac{1}{2}$ topiladiki $C(-1+\delta,1-\delta)^n$ dan tashqarida $\nabla E(x) \neq 0$ bo`ladi. 3.2 lemmaga ko`ra (3.1) sistemaning xamma muvozanat nuqtalari $\overline{C}$ kompakt to`plamda bo`ladi. $\overline{C}$ ning kompaktligidan va V.1 farazning b qismidan (3.1) sistemaning muvozanat nuqtalar to`plami chekli bo`ladi. Teorema isbotlandi.

Teorema 3.2. (3.1) sistema uchun (A) va (B.1) farazlarning shartlari bajarilsin. Agar $\tilde{x}$ (3.1) sistemaning muvozanat xolatini ifodalovchi echimi bo`lsa, u xolda quyidagilar ekvivalent.

a) $\tilde{x}$ echim turg`un.

b) $\tilde{x}$ -E(x) funktsiyaning minimum nuqtasi.

c) $\Im_E(x)>0$ d) $\tilde{x}$ echim asimptotik turg`un.

Isbot: *a)* ($(1)\Rightarrow(2)$) Faraz qilamiz $\tilde{x}$ (3.1) sistemaning turg`un muvozanat xolati va E ni minimum nuqtasi bo`lmasin. U xolda shunday $\{x_m\} \subsetneq (-1,1)^n$ ketma–ketlik mavjudki $0<|x_m-\tilde{x}|<\frac{1}{m}$ va $E(x_m)<E(\tilde{x})$ bo`ladi. B.1 farazga ko`ra shunday $\varepsilon>0$ mavjudki $B(\tilde{x},\varepsilon)-\{\tilde{x}\}$ da muvozanat nuqtalar mavjud bo`lmaydi. U xolda ixtiyoriy $\delta,\ \varepsilon>\delta>0$ uchun m ni shunday tanlash mumkinki $\frac{1}{m}<\delta$ bo`ladi. Bundan esa $x_m \in B(\tilde{x},\delta)-\{\tilde{x}\} \subset B(\tilde{x},\varepsilon)-\{\tilde{x}\}$ va x_m muvozanat nuqtasi bo`lmasligi kelib chiqadi. Teorema 3.1 ning 4–qismga ko`ra $\varphi(\cdot, x_m)$ echim (3.1) sistemaning x muvozanat nuqtasiga intiladi. Teorema 3.1 ning 2–qismiga ko`ra esa $E(x) < E(x_m) < E(\tilde{x}), \quad x \neq \tilde{x}$ bo`ladi. Demak $B(\tilde{x}, \varepsilon)$ o`zida $x$ ni saqlamaydi va $\varphi(t, x_m)$ $t \to \infty$ da $B(\tilde{x}, \varepsilon)$ dan chiqib ketadi. Bundan tashqari $\tilde{x}$ turg`unmas bo`ladi. Biz qarama–qarshilikka keldik, demak $\tilde{x}$ E ning minimumi bo`lar ekan.

b) ($(2) \Rightarrow (3)$) Faraz qilamiz $\tilde{x}$ E energetik funktsianing minimumi. Ikki xolni ko`rib chiqamiz 1) $J_E(\tilde{x})$ musbat aniqlanmagan ammo u yarim musbat aniqlangan. 2) $J_E(\tilde{x})$ yarim musbat aniqlanmagan.

Birinchi xol: biz $\nabla E(\tilde{x}) = 0$ va $\det(J_E(\tilde{x})) = 0$ munosabatlarga egamiz. B.1 farazning a) qismiga ko`ra shunday $y \in R^n, \quad y \neq 0$ mavjudki

$J_E(\tilde{x})y = 0, \quad D^3 E(\tilde{x}, y, y, y) = (s_1''(\tilde{x}_1), \ldots, s_n''(\tilde{x}_n))(y_1^3, \ldots, y_n^3)^T \neq 0$ munosabat o`rinli bo`ladi. E ni $\tilde{x}$ nuqta atrofida teylor qatoriga yoyib quyidagini xosil qilamiz

$E(\tilde{x} + ty) = E(\tilde{x}) + t\nabla E(\tilde{x})y + (\frac{t^2}{2})y^T J_E(\tilde{x})y + (\frac{t^3}{6})D^3(\tilde{x}, y, y, y) + o(t^3). \;\; t \in [-1,1]$ bu erda $\lim\limits_{t \to 0} \frac{o(t^3)}{t^3} = 0$

bo`ladi. $\nabla E(\tilde{x}) = 0$ va $J_E(\tilde{x})y = 0$ ekanligidan

$E(\tilde{x} + ty) - E(\tilde{x}) = (\frac{t^3}{6})D^3(\tilde{x}, y, y, y) + o(t^3) < 0. \;\; t \in [-1,1]$ kelib chiqadi. $D^3(\tilde{x}, y, y, y) \neq 0$ ekanligidan shunday $\delta > 0$ topiladiki agar $D^3(\tilde{x}, y, y, y) > 0$ bo`lsa u xolda

$E(\tilde{x} + ty) - E(\tilde{x}) = (\frac{t^3}{6})D^3(\tilde{x}, y, y, y) + o(t^3) < 0. \;\; t \in [-\delta, 0]$ bo`ladi va agar $D^3(\tilde{x}, y, y, y) < 0$

bo`lsa u xolda $E(\tilde{x} + ty) - E(\tilde{x}) = (\frac{t^3}{6})D^3(\tilde{x}, y, y, y) + o(t^3) < 0. \;\; t \in [0, \delta]$ bo`ladi.

Bundan tashqari $\tilde{x}$ E ning minimumi bo`lmaydi

Ikkinchi xol: $J_E(\tilde{x})$ yarim musbat aniqlanmagan. U xolda shunday $y \in R^n$ mavjud bo`ladiki $y \neq 0, \; y^T J_E(\tilde{x})y < 0$ munosabat o`rinli bo`ladi. E ni $\tilde{x}$ nuqta atrofida Teylor qatoriga yoyib quyidagini xosil qilamiz

$$E(\tilde{x} + ty) = E(\tilde{x}) + t\nabla E(\tilde{x})y + (\frac{t^2}{2})y^T J_E(\tilde{x})y + o(t^2). \;\; t \in [0,1]$$

bu erda $\lim_{t\to 0}\frac{o(t^2)}{t^2}=0$ bo`ladi. $\nabla E(\tilde{x})=0$ ekanligidan

$$E(\tilde{x}+ty)=E(\tilde{x})+(\frac{t^2}{2})y^T J_E(\tilde{x})y+o(t^2).\quad t\in[0,1]$$

kelib chiqadi.

$$y^T J_E(\tilde{x})y<0$$

dan shunday $\delta>0$ topiladiki

$$E(\tilde{x}+ty)-E(\tilde{x})=(\frac{t^2}{2})y^T J_E(\tilde{x})y+o(t^2)<0.\quad t\in[0,\delta]$$

bo`ladi. Yana bir bor $\tilde{x}$ E ning minimumi emasligi kelib chiqdi. 1 va 2 xollardagi ziddiyatlardan $J_E(\tilde{x})>0$ ekanligi kelib chiqadi.

c) ($(3)\Rightarrow(4)$) Faraz qilamiz $J_E(\tilde{x})$ musbat aniqlangan. U xolda $\tilde{x}$ ning shunday ochiq U atrofi topiladiki U atrofda funktsiya quyidagi $v(x)=E(x)-E(\tilde{x})$ ko`rinishda musbat aniqlangan bo`ladi va $x\neq\tilde{x}$ lar uchun $D_{(1)}v(x)=-\nabla v(x)^T H(x)\nabla v(x)<0$ bo`ladi. Bundan Lyapunovning turg`unlik nazariyasiga ko`ra $\tilde{x}$ asimptotik turg`un bo`ladi.

d) ($(4)\Rightarrow(1)$) Faraz qilaylik $\tilde{x}$ asimptotik turg`un. U xolda ta'rifga ko`ra $\tilde{x}$ turg`un bo`ladi. Teorema to`liq isbot bo`ldi.

Endi $(-1,1)^n$ ni 2^n ta soxalarini

$$\Lambda(\xi_1,\ldots,\xi_n)=\{x\in(-1,1)^n:\xi_i x_i>0,\ 1\le i\le n\}$$

bu erda $\xi_i=1$ *ёки* $\xi_i=-1,\quad 1\le i\le n$ ko`rinishida aniqlaymiz. Masalan, agar $n=2$ bo`lsa $2^2=4$ ta soxani qaraymiz, ular $\Lambda(1,1)$, $\Lambda(1,-1)$, $\Lambda(-1,1)$ ва $\Lambda(-1,-1)$ ko`rinishda bo`ladi.

Teorema 3.3 Agar (3.1) sistema uchun (A), (B.1) va (S) farazlardagi shartlar bajarilsa , u xolda

$$\Lambda(\xi_1,\ldots,\xi_n) = \{x \in (-1,1)^n : \xi_i x_i > 0,\ 1 \le i \le n\}$$

bu erda $\xi_i = 1$ *ёки* $\xi_i = -1,\quad 1 \le i \le n$ ko`rinishdagi $2^n$ ta soxalarning xar birida (3.1) sistemaning turg`un (asimptotik) muvozanat xolatga mos keluvchi echimlari mavjud.

Isbot: $F:(-1,1)^n \to R^n$ funktsiyani $F(x) = \nabla E(x) = -Tx + S(x) - I$ ko`rinishida olamiz. Uning xosilasi $DF:(-1,1)^n \to L(R^n, R^n)$ quyidagicha bo`ladi

$DF(x) = J_E(x) = -T + diag(s_1'(x_1),\ldots,s_n'(x_n))$. Avval $\Lambda = \Lambda(1,\ldots,1) = (0,1)^n$ soxada tekshiramiz. S farazga ko`ra xar bir i uchun $s_{i+}' = s_i' : (0,1) \to (\sigma_i, +\infty)$ bu erda $\sigma_i = s_i'(0)$ ko`rinishida aniqlangan funktsiya teskarilanuvchi bo`ladi. $s_i'$ ning teskarisini $p_i = (s_i')^{-1} : (\sigma_i, +\infty) \to (0,1)$ ko`rinishida aniqlaymiz. U xolda s_i' va p_i larning ikkisi xam C^1–funktsiyalar. $P : \prod_{i=1}^{n} (\sigma_i, +\infty) \to \Lambda$ funktsiyani

$P(u) = (p_1(u_1),\ldots,p_n(u_n))^T$ ko`rinishida aniqlaymiz. Biz bilamizki P teskarilanuvchi bo`lib P va P^{-1} ikkisi xam C^1 –funktsiyalar bo`ladi. $Q : \prod_{i=1}^{n} (\sigma_i, -\infty) \to R^n$

$Q(u) = F(P(u))$ ko`rinishida tanlaylik va

$D = \{x \in \Lambda : DF(x) > 0\}$ *ва* $C = P^{-1}(D)$ deb olamiz.

§3. 2. Neyron to`r turg`unligining Lyapunov matritsa funktsiyasi usuli yordamida tekshirish.

Yuqoridagi paragrafda ko`rilgan sistema turg`unligini tekshirishda E(x) energetik funktsiyadan foydalanish neyron to`r modeli turg`unligining etarli shartlarini xosil qilmaydi. Endigi paragrafda yirik masshtabli neyron to`r modelining turg`unligini etarli shartlarini xosil qilishda Lyapunovning matritsa-funktsiyasi usulidan foydalanish mumkin ekanligini ko`rsatamiz. Buning uchun avval (3.1) sistemani S ta qism sistemalarga ajratamiz, ya'ni (3.1) sistema quyidagi ko`rinishda yoziladi.

$$\dot{x}_i = \sum_{j=1}^{S} H_{ij}(x_j)(T_{ij}x_j - S_j(x_j) + I_j) = f_i(x) \qquad (3.4)$$

bu erda

$H(x) = (H_{ij}(x_j))_{i,j=1}^{S}$, $H_{ij}(x) - n_i \times n_j$ o`lchovli matritsa funktsiya,

$T = (T_{ij})_{i,j=1}^{S}$, $T_{ij} - n_i \times n_j$ o`lchovli o`zgarmas matritsa,

$S(x) = (S_1^T(x_1),....,S_s^T(x_s))^T$, $S_i(x) - n_i$ o`lchovli vektor funktsiya,

$I = (I_1^T, I_2^T,...,I_S^T)^T$, $I_i - n_i$ o`lchovli o`zgarmas vektor,

$x = (x_1^T, x_2^T,...,x_s^T)^T$, $n_1 + n_2 + n_S = n$

(3.4) sistemadan erkin qism sistemalarni ajratib olish uchun

$$x^i = (0^T, 0^T,..,x_i^T,...,0^T) \in (-1,1)^n$$

belgilashni kiritamiz. U xolda i chi erkin qism sistema quyidagi

$$x_i = -H_{ii}(x^i)(-T_{ii}x_i + S_i(x^i) - I_i) = f_i(x^i) \qquad (3.5)$$

(bu erda $x_i \in (-1,1)^n$, $n_1 + n_2 + ..n_S = n$) ko`rinishda bo`lib,

$$f_i^*(x) = f_i(x) - f_i(x^i)$$

tenglik bilan aniqlangan $f_i^*(x)$ funktsiya (3.4) sistemadagi erkin qism sistemalarni o`zaro aloqalarni ifodalaydi. Natijada (3.4) sistemani quyidagicha yozishimiz mumkin.

$$\dot{x} = f_i(x^i) + f_i^*(x), \quad i = \overline{1,S} \qquad (3.6)$$

Agar $x = \text{€}$ echimning turg`unligini aniqlash kerak bo`lsa, (3.6) sistemada $x = x - \text{€}$ almashtirish qilib, uni x=0 echimga keltirish mumkin ekanligini xisobga olsak, (3.6) sistemaning x=0 echimini turg`unlik shartlarini xosil qilishning umumiy nazariyasini quramiz. Buning uchun quyidagi farazlarni kiritamiz.

Faraz 1: (3.6) sistema uchun U(x) matritsa -funktsiya mavjud bo`lib, bu matritsa-funktsiya va $\eta=(\eta_1,\eta_2,...,\eta_S)^T$ o`zgarmas vektor yordamida tuzilgan V(x) skalyar funktsiya uchun

$$V(x)\geq U^T H^T AHU$$

tengsizlik o`rinli bo`lsin.

Faraz 2: U(x) matritsa-funktsiya elementlaridan (3.5) va (3.6) sistemalar bo`yicha olingan xosilalar uchun quyidagi tengsizlik o`rinli bo`ladigan $\rho_{ii},\quad i=\overline{1,S}\quad va\quad \rho_{ij},\ i,j=\overline{1,S},\quad i<j$ o`zgarmaslar mavjud bo`lsin:

$$\sum_{i=1}^{S}\eta_i^2\left\{D_t^+v_{ii}+(D_{x_i}^+v_{ii})^T f_i(x^i)+(D_{x_i}^+v_{ii})^T f_i^*(x^i)\right\}+2\sum_{i=1}^{S-1}\sum_{j=2}^{S}\eta_i\eta_j\left\{D_t^+v_{ij}+(D_{x_i}^+v_{ij})^T f_i(x^i)+(D_{x_j}^+v_{ij})^T f_j(x^j)+\right.$$
$$\left.+(D_{x_i}^+v_{ij})^T f^*(x)+(D_{x_j}^+v_{ij})^T f_j^*(x)\right\}\leq\sum_{i=1}^{S}\rho_{ii}\|x_i\|^2+2\sum_{i=1}^{S}\sum_{\substack{j=2\\ j>i}}^{S}\rho_{ij}\|x_i\|\|x_j\|$$

Lemma 3.4: Agar faraz 2 dagi shartlar bajarilsa V(x) skalyar funktsiyadan (3.6) sistema yordamida olingan xosila uchun quyidagi tengsizlik o`rinli bo`ladi.

$$D^+v(x,\eta)\leq U^T GU \qquad (3.7)$$

bu erda $U^T=(\|x_1\|,\|x_2\|,...,\|x_S\|),\quad G=\left[\rho_{ij}\right],\ \rho_{ij}=\rho_{ji},\ i,j=\overline{1,S}.$

Teorema 3.4: Agar (3.6) sistema uchun faraz 1 va faraz 2 dagi shartlar bajarilib, A matritsa musbat aniqlangan G matritsa manfiy aniqlangan bo`lsa, u xolda bu sistemaning x=0 echimi asimptotik turg`un bo`ladi.

Исбот: $\eta^T=(\eta_1,\eta_2,...,\eta_s)$ vektor yordamida U(x) matritsa–funktsiyadan faraz 1 dagi tengsizlikni qanoatlantiruvchi V(x) skalyar funktsiyani tuzamiz. Bu skalyar funktsiya xosilasi uchun faraz 2 dagi baxolashni xosil qilamiz. faraz 1 dagi tengsizlikdan va A matritsaning musbat aniqlanganligidan V(x) funktsiyaning musbat aniqlanganligi xosil bo`ladi, faraz 2 dagi baxolashdan va G matritsaning manfiy aniqlanganligidan esa V(x) skalyar funktsiyadan (3.5) sistema yordamida olingan xosila manfiy aniqlangan bo`ladi. Bu shartlar esa sistema muvozanat xolatining asimptotik turg`un bo`lishi uchun etarli.

Xulosa

Xulosa qilib shuni aytishimiz mumkinki, ushbu magistrlik dissertatsiyasi, xozirgi kunda dolzarb muammolardan biri bo`lgan dinamik sistemalarning turg`unligi masalasini xal qilishga qaratilgan bo`lib, bunda dinamik sistemalar turg`unligini tekshirishda eng effektiv usullardan biri bo`lgan Lyapunovning to`g`ri usulidan foydalandik. Ishda asosan Lyapunovning to`g`ri usulini rivojlantirish natijasida xosil bo`lgan matritsa–funktsiya usulini qo`lladik. Bu usul yirikmasshtabli sistemalarning turg`unligini tekshirishda Lyapunov funktsiyasini qurishni ma'lum bir algoritimini beradi. Bu usul qanchalik qulay ekanligini ko`rsatish maqsadida, dissertatsiyaning II bobida giroskopik stablizatsiya xarakatidagi orbital observatoriya turg`unligini tekshirib chiqdik. III bobda esa bu usul Neyron to`rlar turg`unligini tekshirishda boshqa usullarga nisbatan qulayroq ekanligini nazariy berib o`tdik. Keyinchalik ushbu magistrlik dissertatsiyani aynan shu bobini davom ettirib, bu sistema uchun bu usul xaqiqattan xam qulay ekanligini ko`rsatishga xarakat qilamiz.

Foydalanilgan adabiyotlar ro`yxati.

1. Abdullin R.Z.,Anapolskiy L.Yu. i dr. Metod vektornix funktsiy Lyapunova v teorii ustoychivosti.–M.: Nauka, 1987.–312 s.

2. Barbashin E.A. Vvedeniya v teorii ustoychivosti dvijeniya. M.: Nauka, 1967.

3. Barbashin E.A. Funktsiya Lyapunova. M.Nauka. 1970.

4. Bellman R. Vvedenie v teoriyu matrits: Per. s. angl.–M.: Nauka, 1969.

5. Gantmaxer F.R. Teoriya matrits.–3–e izd.–M.: Nauka, 1967.

6. Gruyich L.T., Marto`nyuk A.A., Ribbens–Pavella M. Ustoychivost krupnomasshtabnix sistem pri strukturnix i singulyarno`x vozmuheniyax.–Kiev: Nauk, dumka, 1984.–307 s

7. Demidovich B.P. Lektsii po matematicheskoy teorii ustoychivosti.– M.: Nauka, 1967.

8. Karimov I.A. Yuksak malakali mutaxasislar taraqqiyot omili–T.: O`zb.1995

10. Karimov I.A. Barkamol avlod orzusi, Sharq,T. 1999.

11. Krasovskiy N.N. Nekotorie zadachi teorii ustoychivosti dvijeniya.–M.: Fizmatgiz, 1959.

37. Lyapunov.A.M.Obshaya zadacha ob ustoychivosti dvijeniya.M: Gost.dat.1950

12. Malkin I.G. Teoriya ustoychivosti dvijeniya..M.:Nauka, 1966.

13. Martinyuk A.A., Miladjonov V.G. Ob ustoychivosti krupnomasshtabnix sistem pri strukturnix vozmuheniyax.Problema A , Elektronnoe modelirovanie.–1992.

14. Martinyuk A.A., Miladjonov V.G. Ob ustoychivosti krupnomasshtabnix sistem pri strukturnix vozmuheniyax.Problema B. Elektronnoe modelirovanie.–1992.

15. Martinyuk A.A., Miladjonov V.G. K teorii ustoychivosti orbitalnoy observatorii pri giroskopicheskoy stablizatsii dvijeniya., Prikl. mexanika., 2000. Tom 36, №5.

16. Mullajonov R.V., Abdugapparova Sh.N, G'oymatova D.G., Yirik masshtabli sistemalarning dekompozitsiyasi., Yosh matematiklarning yangi teoremalari – 2013 Respublika ilmiy amaliy anjuman materiallari., Namangan 2013 y.

17. Miladjonov V.G', G'oymatova D.G., Deformatsiyalanishi va bikirligi oshkor holda vaqtga bog'liq bo'lib, chiziqsiz bo'lgan sistema asimptotik turg'unligining yetarli shartlari., Andijon 2013 y.

18. Муллажонов Р.В. Динамик системалар турғунлигини Ляпунов матрица – функция усули ёрдамида текшириш // Аспирант, доктарант ва тадқиқотчиларнинг республика илмий – амалий анжумани тезислари тўплами. − Тошкент, 2007. −С. 276 −277.

19. Муллажонов Р.В., Каримжонов А. Анализ устойчивости нестационарные линейных КМС. //Труды международной конференции «Актуальные проблемы прикладной математики и информационных технологий-Аль Хорезми 2009». − Ташкент, 2009. −том №1. −С.251−255.

20. Муллажонов Р.В. Обобщенное транспонирование матриц и структуры линейных крупномасштабных систем // Украинский журнал «Доп.НАН Украини». −Киев, 2009. - №11.С. 27 −35.

21. Муллажонов Р.В. Анализ устойчивости линейных крупномасштабных систем. // Проблема механики. − Ташкент, 2010. −№2.−С. 4− 7.

22. Miladjonov V.G., Mullajonov V.R. K Teorii ustoychivosti orbitalnoy observatorii pri giroskopicheskoy stablizatsii. And.konf.2001.

23. Mullajonov V.R. Neyron setlar modeli., Tez. –And. 2003.

24. Rush N., Abets P., Lalua M. Pryamoy metod Lyapunova v teorii ustoychivosti.–M: Mir, 1980.

25. Saloxitdinov M.S., Nasritdinov G.N. Oddiy differentsial tenglamalar. –T. Ukituvchi. 1982.

26. www.sciencearea.com.ua.

27. www.stcu.kiev.ua.

28. www.attrasoft.com/abm/champ27_1.html.

29. www.emsl.pnl.gov:2080/proj/neuron/neural/what.html.

30. www.math.ucla.edu/nykamp/285j.1.02s/.