**O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI**

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**MEXANIKA-MATEMATIKA FAKULTETI**

**EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA YO’NALISHI**

# KURS ISHI

**Mavzu: TALABNING SISTEMADA BO'LISH VAQTI CHEGARALANGAN SISTEMA**


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**Reja**

1. Sistemaning tavsifi.

2.Holatlar ehtimollari uchun differensial tenglamalar.

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**Sistemaning tavsifi.** Aytaylik, $n$ ta bir xil asbobdan (kanaldan) iborat bo'lgan ommaviy xizmat ko'rsatish sistemasiga $\lambda$ intensivlik bilan Puasson talablar oqimi kelsin. Har bir talabni xizmat qilish vaqti tasodifiy miqdor bo'lib, $\mu$ parametrli ko'rsatkichli qonun bo'yicha taqsimlangan bo'lsin. Agar hamma asboblar band bo'lgan paytda sistemaga yana talab kelsa, u holda u navbatga turib xizmat qilinishini kutadi. Sistemaga qabul qilingan har bir talablar (navbatda turgan yoki xizmat qilinayotgan) unda chegaralangan vaqt bo'lishi mumkin. Bu vaqt tugashi bilan talab, u xizmat qilingan yoki navbatda turgan bo'lishidan qat'iy nazar sistemadan ketadi. Sistemada bo'lish vaqti $\upsilon$ parametrli ko'rsatkichli qonun bo'yicha taqsimlangan tasodifiy miqdor. Sistemaning bo'lishi mumkin bo'lgan holatlari:

$x_0$ – hamma asbob bo'sh, navbat yo'q;

$x_1$ – bitta asbob band, navbat yo'q;

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$x_k$ – $k$ ta asbob band, navbat yo'q;

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$x_n$ – hamma asbob band, navbat yo'q;

$x_{n+1}$– hamma asbob band, navbatda bitta talab bor;

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$x_{n+s}$– hamma asbob band, navbatda $s$ ta talab bor;

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lar bo'ladi.

Bu sistemada ham navbatda turgan talablar soni $s$ keragicha katta bo'lishi mumkin.

**Tenglamalar tuzish.** $P_{x_k}(t)$ orqali $t$ momentda sistema $x_k$ $(k=0,1,2,...)$ holatda bo'lish ehtimolini belgilaymiz.

$P_{x_0}(t)$ **ehtimol uchun tenglama**. $t+\Delta t$ momentda sistema $x_0$ holatda bo'lish ehtimolini topamiz. Bu hodisa quyidagi hollarda ro'y berishi mumkin:

$t$ momentda sistema $x_0$ holatda bo'lgan, $\Delta t$ vaqt ichida talab kelmagan. Bu hodisaning ehtimoli

$$P_{x_0}(t)e^{-\lambda\Delta t} = P_{x_0}(t)(1-\lambda\Delta t + 0(\Delta t))$$

ga teng.

$t$ momentda sistema $x_1$ holatda bo'lgan, $\Delta t$ vaqt ichida esa talab kelmagan, yo asbob bo'shagan yoki talabning sistemada bo'lish vaqti tugagan. Bu hodisaning ehtimoli

$$P_{x_1}(t)e^{-\lambda\Delta t}\left(1-e^{-\mu\Delta t}+1-e^{-\upsilon\Delta t}\right)= P_{x_1}(t)(\mu+\upsilon)\Delta t + 0(\Delta t)$$

ga teng.

$t$ momentda sistema qolgan mumkin bo'lgan holatlarida bo'lib, $\Delta t$ vaqt ichida undan chiqib $x_0$ holatiga o'tish ehtimollari $0(\Delta t)$ ga teng.

Demak,

$$P_{x_0}(t+\Delta t) = P_{x_0}(t)(1-\lambda\Delta t + 0(\Delta t)) + P_{x_1}(t)(\mu+\upsilon)\Delta t + 0(\Delta t)$$

yoki

$$P_{x_0}(t+\Delta t) = P_{x_0}(t)(1-\lambda\Delta t) + P_{x_1}(t)(\mu+\upsilon)\Delta t + 0(\Delta t).$$

Bu tenglikdan

$$P_{x_0}(t+\Delta t) - P_{x_0}(t) = -\lambda P_{x_0}(t)\Delta t + (\mu+\upsilon)P_{x_1}(t)\Delta t + 0(\Delta t)$$

ga ega bo'lamiz. Endi bu orttirmani argument orttirmasi $\Delta t$ ga bo'lamiz. Natijada

$$\frac{P_{x_0}(t+\Delta t) - P_{x_0}(t)}{\Delta t} = -\lambda P_{x_0}(t) + (\mu+\upsilon)P_{x_1}(t) + \frac{0(\Delta t)}{\Delta t}$$

bo'lib, undan

$$\lim_{\Delta t\to 0}\frac{P_{x_0}(t+\Delta t) - P_{x_0}(t)}{\Delta t} = -\lambda P_{x_0}(t) + (\mu+\upsilon)P_{x_1}(t)$$

bo'lishi kelib chiqadi. Demak,

$$P'_{x_0}(t) = -\lambda P_{x_0}(t) + (\mu + \upsilon) P_{x_1}(t)$$

bo’ladi.

$P_{x_k}(t)$, $1 \le k \le n-1$ **ehtimollar uchun differensial tenglamalar**. $t + \Delta t$ momentda sistema $x_k$ holatda bo’lish ehtimolini topamiz. Bu quyidagi usullar bilan ro’y berishi mumkin:

$t$ momentda sistema $x_k$ holatda bo’lgan, $\Delta t$ vaqt ichida talab kelmagan, hyech qaysi asbob ishini tugatmagan va talablarning sistemada bo’lish vaqti tugamagan. Bu hodisaning ehtimoli

$$P_{x_k}(t) e^{-\lambda \Delta t} \left( e^{-\mu \Delta t} \right)^k \left( e^{-\upsilon \Delta t} \right)^k = P_{x_k}(t) [1 - (\lambda + k(\mu + \upsilon)) \Delta t + 0(\Delta t)]$$

ga teng.

$t$ momentda sistema $x_{k-1}$ holatda bo’lgan, $\Delta t$ vaqt ichida esa faqat bitta talab kelgan, lekin oldingi kelgan talablarning hyech biri xizmat qilinib bo’linmagan va ularning sistemada bo’lish vaqtlari ham tugamagan. Bu hodisaning ehtimoli

$$P_{x_{k-1}}(t) \left( 1 - e^{-\lambda \Delta t} \right) \left( e^{-\mu \Delta t} \right)^{k-1} \left( e^{-\upsilon \Delta t} \right)^{k-1} = \lambda P_{x_{k-1}}(t) \Delta t + 0(\Delta t)$$

ga teng.

$t$ momentda sistema $x_{k+1}$ holatda bo’lgan, $\Delta t$ vaqt ichida esa talab kelmagan, yo talablardan biri (qaysi biri bo’lishi ahamiyati yo’q) yo xizmat qilinib bo’lingan yoki sistemada bo’lish vaqti tugagan. Bu hodisaning ehtimoli

$$P_{x_{k+1}}(t) e^{-\lambda \Delta t} \left[ C'_{k+1} \left( 1 - e^{-\mu \Delta t} \right) + C'_{k+1} \left( 1 - e^{-\upsilon \Delta t} \right) \right] = P_{x_{k+1}}(t)(k+1)(\mu + \upsilon) \Delta t + 0(\Delta t)$$

ga teng.

Qolgan barcha mumkin bo’lgan imkoniyatlar $0(\Delta t)$ ehtimolga ega.

Demak,

$$P_{x_k}(t + \Delta t) = P_{x_k}(t) [1 - (\lambda + k(\mu + \upsilon)) \Delta t + 0(\Delta t)] +$$
$$+ P_{x_{k-1}}(t) \lambda \Delta t + P_{x_{k+1}}(t)(k+1)(\mu + \upsilon) \Delta t + 0(\Delta t)$$

Bu tenglikdan

$$P_{x_k}(t+\Delta t)-P_{x_k}(t)=-[\lambda+k(\mu+\upsilon)]P_{x_k}(t)\Delta t+\lambda P_{x_{k-1}}(t)\Delta t+$$
$$+(k+1)(\mu+\upsilon)P_{x_{k+1}}(t)\Delta t+0(\Delta t)$$

ni hosil qilamiz. Keyingi tenglikning har ikkala tomonini $\Delta t$ ga bo'lib, so'ngra $\Delta t\to 0$ da limitga o'tamiz:

$$\lim_{\Delta t\to 0}\frac{P_{x_k}(t+\Delta t)-P_{x_k}(t)}{\Delta t}=-[\lambda+k(\mu+\nu)]P_{x_k}(t)+$$
$$+\lambda P_{x_{k-1}}(t)+(k+1)(\mu+\nu)P_{x_{k+1}}(t)$$

Bundan $P_{x_k}(t)$ ehtimol uchun ushbu

$$P'_{x_k}(t)=-[\lambda+k(\mu+\upsilon)]P_{x_k}(t)+\lambda P_{x_{k-1}}(t)+(k+1)(\mu+\upsilon)P_{x_{k+1}}(t)$$

differensial tenglamaga ega bo'lamiz.

$P_{x_n}(t)$ **ehtimol uchun differensial tenglama**. $t+\Delta t$ momentda sistema $x_n$ holatda bo'lish ehtimolini topamiz. Bu quyidagi hollarda ro'y berishi mumkin:

$t$ momentda sistema $x_n$ holatda bo'lib, $\Delta t$ vaqt ichida esa talab kelmagan, hyech qaysi talabning xizmati ham sistemada bo'lish vaqti ham tugamagan. Bu hodisaning ehtimoli

$$P_{x_n}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^n\left(e^{-\upsilon\Delta t}\right)^n=P_{x_n}(t)[1-(\lambda+n(\mu+\upsilon))\Delta t+0(\Delta t)]$$

ga teng.

$t$ momentda sistema $x_{n-1}$ holatda bo'lgan, $\Delta t$ vaqt ichida esa bitta talab kelgan, lekin oldingi kelgan talablarning hyech biri xizmat qilinib bo'linmagan va ularning sistemada bo'lish vaqtlari tugamagan. Bu hodisaning ehtimoli

$$P_{x_{n-1}}(t)\cdot\left(1-e^{-\lambda\Delta t}\right)\cdot\left(e^{-\mu\Delta t}\right)^{k-1}\cdot\left(e^{-\upsilon\Delta t}\right)^{k-1}=P_{x_{n-1}}(t)\lambda\Delta t+0(\Delta t)$$

ga teng.

$t$ momentda sistema $x_{n+1}$ holatda bo'lgan, $\Delta t$ vaqt ichida esa talab kelmagan, yo bitta asbob (qaysi biri bo'lishi ahamiyati yo'q) xizmat qilib bo'lgan yoki bo'lmasa talablardan birining (qaysi biri bo'lishi ahamiyati yo'q) sistemada bo'lish vaqti tugagan. Bu hodisaning ehtimoli

$$P_{x_{n+1}}(t)e^{-\lambda\Delta t}\left[C_n'\left(1-e^{-\mu\Delta t}\right)+C_{n+1}'\left(1-e^{-\nu\Delta t}\right)\right]=P_{x_{n+1}}(t)[n\mu+(n+1)\upsilon]\Delta t+0(\Delta t)$$

ga teng.

Qolgan imkoniyatlarning ehtimollari $0(\Delta t)$ ga teng.

Demak,

$$P_{x_n}(t+\Delta t)=P_{x_n}(t)[1-(\lambda+n(\mu+\upsilon))\Delta t+0(\Delta t)+]+$$
$$+P_{x_{n-1}}(t)\lambda\Delta t+P_{x_{n+1}}(t)[n\mu+(n+1)\nu]\Delta t+0(\Delta t)$$

Bundan

$$P_{x_n}(t+\Delta t)-P_{x_n}(t)=-[\lambda+n(\mu+\nu)]P_{x_n}(t)\Delta t+\lambda P_{x_{n-1}}(t)\Delta t+$$
$$+[n\mu+(n+1)\nu]P_{x_{n+1}}(t)\Delta t+0(\Delta t)$$

ni hosil qilamiz. Endi bu tenglikning har ikkala tomonini $\Delta t$ ga bo'lamiz. Natijada

$$\frac{P_{x_n}(t+\Delta t)-P_{x_n}(t)}{\Delta t}=-[\lambda+n(\mu+\nu)]P_{x_n}(t)+\lambda P_{x_{n-1}}(t)+[n\mu+(n+1)]P_{x_{n+1}}(t)+\frac{0(\Delta t)}{\Delta t}$$

bo'lib, undan quyidagi differensial tenglamaga ega bo'lamiz:

$$P_{x_n}'(t)=-[\lambda+n(\mu+\nu)]P_{x_n}(t)+\lambda P_{x_{n-1}}(t)+[n\mu+(n+1)\mu]P_{x_{n+1}}(t).$$

$P_{n+s}(t)$ $s\geq 1$ **ehtmollar uchun differensial tenglamalar.** $t+\Delta t$ momentda sistema $x_{n+s}$ holatda bo'lish ehtimolini topamiz. Bu quyidagi hollarda ro'y berishi mumkin:

$t$ momentda sistema $x_{n+s}$ holatda bo'lgan, $\Delta t$ vaqt ichida esa talab kelmagan, hyech qaysi talabning xizmati ham sistemada bo'lish vaqti ham tugamagan. Bu hodisaning ehtimoli

$$P_{x_{n+s}}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^n\left(e^{-\nu\Delta t}\right)^{n+s}=P_{x_{n+s}}(t)[1-(\lambda+n\mu+(n+s)\nu)\Delta t+0(\Delta t)]$$

ga teng.

$t$ vaqt momentda sistema $x_{n+s-1}$ holatda bo'lgan, $\Delta t$ vaqt ichida esa bitta talab kelgan, lekin oldingi kelgan talablardan hyech biri xizmat qilinib bo'linmagan hamda sistemadagi barcha talablarning sistemada bo'lish vaqtlari tugamagan. Bu hodisaning ehtimoli

$$P_{x_{n+s-1}}(t)\left(1-e^{-\lambda\Delta t}\right)\left(e^{-\mu\Delta t}\right)^n\left(e^{-\nu\Delta t}\right)^{n+s-1}=P_{x_{n+s-1}}(t)\lambda\Delta t+0(\Delta t)$$

ga teng.

$t$ momentda sistema $x_{n+s+1}$ holatda bo'lgan, $\Delta t$ vaqt ichida esa talab kelmagan, yo bitta asbob xizmat qilib bo'lgan (qaysi birining bo'lishi ahamiyati yo'q) yoki bo'lmasa talablardan birining sistemada bo'lish vaqti (qaysi birining bo'lishi ahamiyati yo'q). Bu hodisaning ehtimoli

$$P_{x_{n+s+1}}(t)e^{-\lambda\Delta t}\left[C_n'\left(1-e^{-\mu\Delta t}\right)+C_{n+s+1}'\left(1-e^{-\nu\Delta t}\right)\right]=P_{x_{n+s+1}}(t)\left[n\mu+(n+s+1)\nu\right]\Delta t+0(\Delta t)$$

ga teng.

Qolgan barcha imkoniyatlarning ehtimollari $0(\Delta t)$ ga teng.

Demak,

$$P_{x_{n+s}}(t+\Delta t)=P_{x_{n+s}}(t)\left[1-(\lambda+n\mu+(n+s)\nu)\Delta t+0(\Delta t)\right]+$$
$$+P_{x_{n+s-1}}(t)\lambda\Delta t+P_{x_{n+s+1}}(t)\left[n\mu+(n+s+1)\nu\right]\Delta t+0(\Delta t)$$

Bundan quyidagini hosil qilamiz:

$$P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)=-\left[\lambda+n\mu+(n+s)\nu\right]P_{x_{n+s}}(t)\Delta t+$$
$$+\lambda P_{x_{n+s-1}}(t)\Delta t+\left[n\mu+(n+s+1)\nu\right]P_{x_{n+s+1}}(t)\Delta t+0(\Delta t)$$

Bu tenglikning har ikkala tomonini $\Delta t$ ga bo'lamiz:

$$\frac{P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)}{\Delta t}=-\left[\lambda+n\mu+(n+s)\nu\right]P_{x_{n+s}}(t)+$$
$$+\lambda P_{x_{n+s-1}}(t)+\left[n\mu+(n+s+1)\nu\right]P_{x_{n+s+1}}(t)+\frac{0(\Delta t)}{\Delta t}$$

Agar

$$\lim_{\Delta t\to 0}\frac{P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)}{\Delta t}=P_{x_{n+s}}'(t),$$

$$\lim_{\Delta t\to 0}\frac{0(\Delta t)}{\Delta t}=0$$

(chunki, $0(\Delta t)-\Delta t$ ga nisbatan yuqori tartibli cheksiz kichik miqdor) bo'lishini e'tiborga olsak, u holda yuqoridagi tenglikdan $P_{n+s}(t)$ ehtimollar uchun ushbu

$$P_{X_{n+s}}'(t)=-\left[\lambda+n\mu+(n+s)\nu\right]\cdot P_{X_{n+s}}(t)=\lambda P_{X_{n+s-1}}(t)+\left[n\mu+(n+s+1)\nu\right]P_{X_{n+s+1}}(t)$$

differensial tenglamalarga ega bo'lamiz.

Shunday qilib, holatlar ehtimollari aniqlanadigan differensial tenglamalar

$$\begin{cases} P'_{x_0}(t) = -\lambda P_{x_0}(t) + (\mu+\nu)P_{x_1}(t); \\ P'_{x_1}(t) = -(\lambda+\mu+\nu)P_{x_1}(t) + \lambda P_{x_0}(t) + 2(\mu+\nu)P_{x_2}(t); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_k}(t) = -[\lambda + k(\mu+\nu)]P_{x_k}(t) + \lambda P_{x_{k-1}}(t) + (k+1)(\mu+\nu)P_{x_{k+1}}(t), \quad 1 \le k \le n-1; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_n}(t) = -[\lambda + n(\mu+\nu)]P_{x_n}(t) + \lambda P_{x_{n-1}}(t) + [n\mu + (n+1)\nu]P_{x_{n+1}}(t); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_{n+s}}(t) = -[\lambda + n\mu + (n+s)\nu]P_{x_{n+s}}(t) + \lambda P_{x_{n+s-1}}(t) + [n\mu + (n+s+1)\nu]P_{x_{n+s+1}}(t), \quad s \ge 1 \end{cases} \tag{8.1}$$

ko'rinishda bo'ladi.

Bu tenglamalar sistemasini $P_{x_0}(0) = 1$, $P_{x_k}(0) = 0$, $k \ge 1$ boshlang'ich shartlarda $\sum_{k=0}^{\infty} P_{x_k}(t) = 1$ tenglama bilan birgalikda yechib, $P_{x_k}(t)$ ehtimollarning ifodalarini $t$ ning funksiyasi sifatida topish mumkin. Ammo bu juda katta hajmdagi hisoblashlarni talab qiladi, shu sababli unga to'xtalmaymiz.

**Stasionar yechimlar.** 7-ma'ruzada aytib o'tilganidek, limitik yechimlarni, ya'ni

$$\lim_{t \to \infty} P_{x_k}(t) = P_{x_k}$$

stasionar ehtimollarni topish bilan chegaralanamiz. Differensial tenglamalar sistemasida $t \to \infty$ da limitga o'tganda

$$\lim_{t \to \infty} P'_{x_k}(t) = 0$$

ligini eslatib o'tamiz. Aks holda $P_{x_k}(t)$ funksiya $t$ ning o'sishi bilan cheksiz o'sgan bo'lar edi, bu esa mumkin emas, chunki ma'nosiga ko'ra $P_{x_k}(t) \le 1$.

Shunga ko'ra differensial tenglamalar sistemasida limitga o'tib, quyidagi algebraik tenglamalar sistemasini hosil qilamiz:

$$\begin{cases}-\lambda P_{x_0} + (\mu+\nu)P_{x_1} = 1; \\ -(\lambda+\mu+\nu)P_{x_1} + \lambda P_{x_0} + 2(\mu+\nu)P_{x_2} = 0; \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ -[\lambda + k(\mu+\nu)]P_{x_k} + \lambda P_{x_{k-1}} + (k+1)(\mu+\nu)P_{x_{k+1}} = 0, \quad k \le n-1; \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ -[\lambda + n(\mu+\nu)]P_{x_n}(t) + \lambda P_{x_{n-1}} + [n\mu + (n+1)\nu]P_{x_{n+1}} = 0; \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ -[\lambda + n\mu + (n+s)\nu]P_{x_{n+s}} + \lambda P_{x_{n+s-1}} + [n\mu + (n+s+1)\nu]P_{x_{n+s+1}} = 0, \quad s \ge 1\end{cases} \tag{8.2}$$

Xizmat qiluvchi sistema har doim mumkin bo'lgan holatlarining birida bo'ladi. Shuning uchun bu tenglamalar sistemasiga yana

$$\sum_{k=0}^{\infty} P_{x_k} = 1 \tag{8.3}$$

tenglamani (shartni) qo'shish zarur, undan $P_{x_0}$ ehtimolni topishda foydalaniladi.

Birinchi tenglamasini $P_{x_1}$ ga nisbatan yechib,

$$P_{x_1} = \frac{\lambda}{\nu+\mu} P_{x_0}$$

ni topamiz.

Ikkinchisini $P_{x_2}$ ga nisbatan yechsak,

$$P_{x_2} = \frac{1}{2(\mu+\nu)}\left[(\lambda+\mu+\nu)P_{x_1} - P_{x_0}\right]$$

bo'ladi. Bunga $P_{x_1}$ ning ifodasini qo'yib, quyidagini hosil qilamiz:

$$P_{x_2} = \frac{1}{2(\mu+\nu)}\left[(\lambda+\mu+\nu)\frac{\lambda}{\mu+\nu}P_{x_0} - \lambda P_{x_0}\right] = \frac{1}{2(\mu+\nu)} \cdot \frac{\lambda^2}{\mu+\nu} P_{x_0} = \frac{1}{2}\left(\frac{\lambda}{\mu+\nu}\right)^2 P_{x_0}.$$

Uchinchisini $P_{x_3}$ ga nisbatan yechamiz:

$$-[\lambda + 2(\mu+\nu)]P_{x_2} + \lambda P_{x_1} + 3(\mu+\nu)P_{x_3} = 0,$$

$$P_{x_3} = \frac{1}{3(\mu+\nu)}\left[(\lambda + 2(\mu+\nu))P_{x_2} - \lambda P_{x_1}\right].$$

$P_{x_1}$ va $P_{x_2}$ larning yuqorida topilgan ifodalarini bu tenglikka qo'yib, quyidagiga ega bo'lamiz:

$$P_{x_3} = \frac{1}{3(\mu+\nu)}\left[(\lambda+2(\mu+\nu))\cdot\frac{1}{2}\left(\frac{\lambda}{\mu+\nu}\right)^2 P_{x_0} - \lambda\cdot\frac{\lambda}{\mu+\nu}P_{x_0}\right] =$$

$$= \frac{1}{3(\mu+\nu)}\cdot\frac{\lambda^3}{2(\mu+\nu)^2}P_{x_0} = \frac{1}{3!}\left(\frac{\lambda}{\mu+\nu}\right)^3 P_{x_0}.$$

Bu jarayonni davom ettirib, ketma-ket quyidagilarni hosil qilamiz:

$$P_{x_4} = \frac{1}{4!}\left(\frac{\lambda}{\mu+\nu}\right)^4 P_{x_0},$$

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$$P_{x_n} = \frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n P_{x_0}.$$

Demak, $k$ ning $1 \le k \le n$ tengsizlikni qanoatlantiradigan barcha qiymatlari uchun o'rinli bo'lgan ushbu

$$P_{x_k} = \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k P_{x_0} \qquad (8.4)$$

formulani hosil qildik.

Endi qolgan $P_{x_{n+s}}$, $s \ge 1$ ehtimollarni topishga o'tamiz.

(8.2) tenglamalar sistemasining

$$-[\lambda + n(\mu+\nu)]P_{x_n} + \lambda P_{x_{n-1}} + [n\mu + (n+1)\nu]P_{x_{n+1}} = 0$$

tenglamasini $P_{x_{n+1}}$ ga nisbatan yechib,

$$P_{x_{n+1}} = \frac{1}{n\mu + (n+1)\nu}\left[(\lambda + n(\mu+\nu))P_{x_n} - \lambda P_{x_{n-1}}\right]$$

tenglikka ega bo'lamiz. Bu tenglikning o'ng tomonida $P_{x_{n-1}}$ va $P_{x_n}$ larning o'rniga ularning (8.4) dagi ifodalarini qo'yib, quyidagini hosil qilamiz:

$$P_{x_{n+1}} = \frac{1}{n\mu + (n+1)\nu}\left[(\lambda + n(\mu+\nu))\cdot\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n P_{x_0} - \lambda\frac{1}{(n-1)!}\left(\frac{\lambda}{\mu+\nu}\right)^{n-1} P_{x_0}\right] =$$

$$= \frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n \cdot \frac{\lambda}{n\mu + (n+1)\nu}P_{x_0}.$$

(8.2) tenglamalar sistemasini $s=1$ bo'lgandagi tenglamasini $P_{x_{n+2}}$ ga nisbatan yechib,

$$P_{x_{n+2}}=\frac{1}{n\mu+(n+2)\nu}\left[(\lambda+n\mu(n+1)\nu)P_{x_{n+1}}-\lambda P_{x_n}\right]$$

tenglikka ega bo'lamiz. Bu tenglikning o'ng tomonida $P_{x_n}$ va $P_{x_{n+1}}$ larning o'rniga ularning yuqorida topilgan ifodalarini qo'yib, quyidagini hosil qilamiz:

$$P_{x_{n+2}}=\frac{1}{n\mu+(n+2)\nu}\left[(\lambda+n\mu(n+1)\nu)\cdot\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda}{n\mu+(n+1)\nu}P_{x_0}-\lambda\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n P_{x_0}\right]=$$

$$=\frac{\lambda}{n\mu+(n+2)\nu}\cdot\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda}{n\mu+(n+1)\nu}P_{x_0}=$$

$$=\frac{1}{n!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda^2}{[n\mu+(n+1)\nu][n\mu+(n+2)\nu]}P_{x_0}.$$

$s=2$ bo'lgandagi tenglamasini $P_{x_{n+3}}$ ga nisbatan yechib, quyidagini topamiz:

$$P_{x_{n+3}}=\frac{1}{n\mu+(n+3)\nu}\left[(\lambda+n\mu(n+2)\nu)P_{x_{n+2}}-\lambda P_{x_{n+1}}\right].$$

Bu tenglikning o'ng tomonida $P_{x_{n+1}}$ va $P_{x_{n+2}}$ larning o'rniga ularning yuqorida topilgan ifodalarini qo'yib, quyidagiga ega bo'lamiz:

$$P_{x_{n+3}}=\frac{1}{n\mu+(n+3)\nu}\left[\begin{array}{r}(\lambda+n\mu(n+2)\nu)\cdot\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda^2}{[n\mu+(n+1)\nu][n\mu+(n+2)\nu]}P_{x_0}-\\-\lambda\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda}{n\mu+(n+1)\nu}P_{x_0}\end{array}\right]=$$

$$=\frac{1}{n\mu+(n+3)\nu}\cdot\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda^2}{[n\mu+(n+1)\nu][n\mu+(n+2)\nu]}P_{x_0}=$$

$$=\frac{1}{n!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^n\cdot\frac{\lambda^3}{[n\mu+(n+1)\nu][n\mu+(n+2)\nu][n\mu+(n+3)\nu]}P_{x_0}.$$

Xuddi shunga o'xshash

$$P_{x_{n+4}} = \frac{1}{n!} \cdot \left( \frac{\lambda}{\mu+\nu} \right)^n \cdot \frac{\lambda^4}{[n\mu+(n+1)\nu][n\mu+(n+2)\nu][n\mu+(n+3)\nu][n\mu+(n+4)\nu]} P_{x_0}$$

ni topamiz.

Yuqorida topilgan $P_{x_{n+1}}$, $P_{x_{n+2}}$, $P_{x_{n+3}}$, $P_{x_{n+4}}$ larning ifodalaridan $s \geq 1$ uchun quyidagi xulosaga kelamiz:

$$P_{x_{n+s}} = \frac{1}{n!} \left( \frac{\lambda}{\mu+\nu} \right)^n \frac{\lambda^s}{\prod_{i=1}^{s} [n\mu + (n+i)\nu]} P_{x_0} .$$

Shunday qilib, sistema holatlarining ehtimollari uchun ikki turkum formulalarga ega bo'ldik.

Birinchi turkum formulalar

$$P_{x_k} = \frac{1}{k!} \left( \frac{\lambda}{\mu+\nu} \right)^k P_{x_0} , \ 1 \leq k \leq n; \qquad (8.5)$$

Ikkinchi turkum formulalar

$$P_{x_{n+s}} = \frac{1}{n!} \left( \frac{\lambda}{\mu+\nu} \right)^n \frac{\lambda^s}{\prod_{i=1}^{s} [n\mu + (n+i)\nu]} P_{x_0} . \qquad (8.6)$$

(8.5) va (8.6) formulalarning har ikkalasida ham $P_{x_0}$ noma'lum ko'paytuvchi sifatida qatnashayapti . Uni (8.2) shartdan foydalanib topamiz. (8.2) ga (8.5) va (8.6) larni qo'yib,

$$P_{x_0} = \left[ \sum_{k=0}^{n} \frac{1}{k!} \left( \frac{\lambda}{\mu+\nu} \right)^k + \sum_{s=1}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu+\nu} \right)^n \cdot \frac{\lambda^s}{\prod_{i=1}^{s} [n\mu + (n+i)\nu]} \right] = 1$$

ni hosil qilamiz. Bundan

$$P_{x_0} = \frac{1}{\sum_{k=0}^{n} \frac{1}{k!} \left( \frac{\lambda}{\mu+\nu} \right)^k + \sum_{s=1}^{\infty} \frac{1}{n!} \left( \frac{\lambda}{\mu+\nu} \right)^n \cdot \frac{\lambda^s}{\prod_{i=1}^{s} [n\mu + (n+i)\nu]}}$$

yoki

$$P_{x_0} = \frac{1}{\sum\limits_{k=0}^{n} \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k + \frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n \sum\limits_{s=1}^{\infty} \frac{\lambda^s}{\prod\limits_{i=1}^{s}\left[n\mu + (n+i)\nu\right]}} \qquad (8.7)$$

bo'ladi.

Ushbu

$$\frac{\lambda}{\mu} = \alpha \text{ va } \frac{\lambda}{\mu}$$

belgilashlarni kiritamiz. Ular mos ravishda bitta talabga o'rtacha xizmat ko'rsatish vaqtida keladigan talablarning o'rtacha sonidan va sistemadan ketadigan talablarning o'rtacha sonidan boshqa narsa emas. Haqiqatan ham,

$$\alpha = \frac{\lambda}{\mu} = \lambda MX_{xiz}, \quad \beta = \frac{\nu}{\mu} = \nu MX_{xiz},$$

bu yerda $X_{xiz}$ - bitta talabga xizmat ko'rsatish vaqti, $MX_{xiz}$ esa uning matematik kutilishi. Yangi belgilashlarda (8.5), (8.6) va (8.7) formulalar

$$P_{x_k} = \frac{1}{k!}\left(\frac{\frac{\lambda}{\mu}}{1+\frac{\nu}{\mu}}\right)^k P_{x_0} = \frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k P_{x_0}, \ 1 \le k \le n; \qquad (8.8)$$

$$P_{x_{n+s}} = \frac{1}{n!}\left(\frac{\frac{\lambda}{\mu}}{1+\frac{\nu}{\mu}}\right)^n \frac{\left(\frac{\lambda}{\mu}\right)^s}{\prod\limits_{i=1}^{s}\left[n + (n+i)\frac{\nu}{\mu}\right]} P_{x_0} =$$

$$= \frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n \frac{\alpha^s}{\prod\limits_{i=1}^{s}\left[n+(n+i)\beta\right]} P_{x_0} = \frac{\alpha^{n+s}}{n!(1+\beta)^n \prod\limits_{i=1}^{s}\left[n+(n+i)\beta\right]} P_{x_0}, \qquad s \ge 1; \qquad (8.9)$$

$$P_{x_0} = \frac{1}{\sum_{k=0}^{n} \frac{1}{k!}\left(\frac{\frac{\lambda}{\mu}}{1+\frac{\nu}{\mu}}\right)^k + \frac{1}{n!}\left(\frac{\frac{\lambda}{\mu}}{1+\frac{\nu}{\mu}}\right)^n \sum_{s=1}^{\infty} \frac{\left(\frac{\lambda}{\mu}\right)^s}{\prod_{i=1}^{s}\left[n+(n+i)\frac{\nu}{\mu}\right]}} =$$

$$= \frac{1}{\sum_{k=0}^{n} \frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k + \frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n \sum_{s=1}^{\infty} \frac{\alpha^s}{\prod_{i=1}^{s}[n+(n+i)\beta]}} \qquad (8.10)$$

ko'rinishlarni oladi.

(8.10) ni (8.8) va (8.9) larga qo'yib, sistemaning holatlari ehtimollari uchun uzil – kesil ifodalarni hosil qilamiz:

$$P_{x_k} = \frac{\frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k}{\sum_{k=0}^{n} \frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k + \frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n \sum_{s=1}^{\infty} \frac{\alpha^s}{\prod_{i=1}^{s}[n+(n+i)\beta]}}, \ 1 \le k \le n; \qquad (8.11)$$

$$P_{x_{n+s}} = \frac{\frac{\alpha^{n+s}}{n!(1+\beta)^n \prod_{i=1}^{s}[n+(n+i)\beta]}}{\sum_{k=0}^{n} \frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k + \frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n \sum_{s=1}^{\infty} \frac{\alpha^s}{\prod_{i=1}^{s}[n+(n+i)\beta]}}, \ s \ge 1; \qquad (8.12)$$

Sistemaning hamma holatlarining ehtimollarini bilsak, bizni qiziqtiradigan boshqa xarakteristikalarini osongina topishimiz mumkin, xususan, talabning xizmat qilinish ehtimolini $(P_x)$. $P_x$ ehtimolni hosil qilish uchun xizmat qilinayotgan talablar sonining matematik kutilishi $m_x$ ni xizmat qilish intensivligining sistemaga keladigan talablar intensivligiga bo'lgan nisbatiga, ya'ni

$$\frac{\mu}{\lambda} = \frac{1}{\frac{\lambda}{\mu}} = \frac{1}{\alpha}$$

koeffisiyentga ko'paytirish kerak. Demak,

$$P_x = \frac{1}{\alpha} m_x \tag{8.13}$$

Endi bu tenglikning o'ng tomonidagi $m_x$ni hisoblaymiz. Diskret tasodifiy miqdorning matematik kutilishi ta'rifiga ko'ra

$$m_x = \sum_{k=1}^{n} kP_{x_k} + \sum_{s=1}^{\infty} nP_{x_{n+s}} = \sum_{k=1}^{n} kP_{x_k} + n\sum_{s=1}^{\infty} P_{x_{n+s}} = \sum_{k=1}^{n} kP_{x_k} +$$
$$+ n\left(1 - \sum_{k=0}^{n} P_{x_k}\right) = n + \sum_{k=1}^{n-1} kP_x - n\sum_{k=0}^{n-1} P_{x_k} = n - \sum_{k=0}^{n-1} (n-k)P_{x_k}$$

bo'ladi.

$m_x$ ning ifodasini (8.13) ga qo'yib, quyidagiga ega bo'lamiz:

$$P_x = \frac{n - \sum_{k=0}^{n-1} (n-k)P_{x_k}}{\alpha},$$

bu yerda $P_{x_k}$ ehtimol (8.11) formuladan aniqlanadi.

Talabning sistemadan xizmat qilinmasdan ketish ehtimoli ushbu

$$P_q = 1 - P_x = 1 - \frac{n - \sum_{k=0}^{n-1} (n-k)P_{x_k}}{\alpha} = \frac{\alpha - n + \sum_{k=0}^{n-1} (n-k)P_{x_k}}{\alpha}$$

formula bo'yicha aniqlanadi.