O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

MEXANIKA-MATEMATIKA FAKULTETI

EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA YO’NALISHI

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Mavzu: Navbatdagi uzunligi va talabning sistemada bo’lish vaqti chegaralanmagan ko’p kanalli sistema.

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Navbatdagi uzunligi va talabning sistemada bo'lish vaqti chegaralanmagan ko'p kanalli sistema.

1. Sistemaning tavsifi. n ta bir xil xizmat qiluvchi birlikdan (biz ularni "kanallar" deb ataymiz) tashkil topgan ommaviy xizmat qilish sistemasini qaraymiz. Sistemaga λ intensivlik bilan Puasson talablar oqimi kelsin. Agar talab kelgan momentda hamma kanal band bo'lsa, u holda bu talab navbatda m tadan kam talab turgan bo'lsa, navbatga turadi: agar navbatda turgan talablar soni m ga teng bo'lsa (undan ortiq bo'lmaydi), u holda bu talab navbatga turmaydi va sistemani tashlab ketadi. Har bir kanalning xizmat qilish vaqti (T_x) μ parametrli ko'rsatkichli qonun bo'yicha taqsimlangan tasodifiy miqdor. Sistemada qabul qilingan talablar (navbatda turganlar yoki xizmat qilinayotganlar) unda chegaralgan vaqt bo'lishi mumkin. Bu vaqtning tugashi bilanoq u xizmat qilinayotganligi yoki navbatda turganligidan qat'iy nazar sistemani tashlab ketadi. Sistemada bo'lish vaqti (T_δ) v parametrli ko'rsatkichli qonun bo'yicha taqsimlangan tasodifiy miqdor.

Qaralayotgan sistemani biror fizik sistema deb qarasak, ravshanki uning mumkin bo'lgan holatlari chekli bo'lib, ular quyidagilardan iborat:

x_0 - hamma kanal bo'sh, navbat yo'q;

x_1 - bitta kanal band, navbat yo'q;

……………………………………

x_k - k ta kanal band, navbat yo'q;

…………………………………....

x_n - hamma kanal band, navbat yo'q;

x_{n+1} - hamma kanal band, bitta talab navbatda turipti;

…………………………………………………………

x_{n+m} - hamma kanal band, m ta talab navbatda turipti.

2. Holatlar ehtimollari uchun differensial tenglamalar. t momentda sistema $x_k\ (k=0,1,2,\dots,n+m)$ holatda bo'lish hodisasining ehtimolini $P_k(t)$ orqali belgilaymiz va $P_k(t)$ ehtimollar uchun differensial tenglamalar sistemasini tuzamiz.

$P_0(t)$ uchun differensial tenglama tuzamiz. t vaqt momentini tayin qilib olamiz va $t+\Delta t$ momentda sistema x_0 holatda (hamma kanal bo'sh) bo'lishi hodisasining ehtimoli $P_0(t+\Delta t)$ ni topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_0 holatda bo'lgan, Δt vaqt ichida esa talab kelmagan;

t momentda sistema x_1 holatda bo'lgan, Δt vaqt ichida esa yo kanal xizmatni tugatgan yoki xizmat qilayotgan talab sistemada bo'lish vaqti tugashi munosabati bilan sistemadan ketgan va yangi talab kelmagan.

Qolgan barcha imkoniyatlar $0(\Delta t)$ ehtimolga ega ekanligiga ishonch hosil qilish qiyin emas.

Ko'rsatilgan hodisalardan birinchisining ehtimoli ehtimollarni ko'paytirish teoremasiga ko'ra

$$P_0(t)e^{-\lambda\Delta t}=P_0(t)[1-\lambda\Delta t+0(\Delta t)]$$

ga teng, ikkinchisining ehtimoli

$$\begin{aligned}
&P_1(t)[P(\xi_x<\Delta t)+P(\xi_\delta<\Delta t)]e^{-\lambda\Delta t}=\\
&=P_1(t)[P(1-e^{-\mu\Delta t})+P(1-e^{-\nu\Delta t})]e^{-\lambda\Delta t}=\\
&=P_1(t)[\mu\Delta t+0(\Delta t)+\nu\Delta t+0(\Delta t)]\cdot[1-\lambda\Delta t+0(\Delta t)]=\\
&=P_1(t)[(\mu+\nu)\Delta t+0(\Delta t)]\cdot[1-\lambda\Delta t+0(\Delta t)]
\end{aligned}$$

ga teng.

Izlanayotgan ehtimol birgalikda bo'lmagan hodisalarning ehtimollarini qo'shish teoremasiga ko'ra

$$\begin{aligned}
&P_0(t+\Delta t)=P_0(t)e^{-\lambda\Delta t}+P_1(t)[P(\xi_x<\Delta t)+P(\xi_\delta<\Delta t)]e^{-\lambda\Delta t}+0(\Delta t)=\\
&=P_0(t)[1-\lambda\Delta t+0(\Delta t)]+P_1(t)[(\mu+\nu)\Delta t+0(\Delta t)]\cdot[1-\lambda\Delta t+0(\Delta t)]=\\
&=P_0(t)(1-\lambda\Delta t)+P_1(t)(\mu+\nu)\Delta t+0(\Delta t)
\end{aligned}$$

Bu tenglikda $P_0(t)$ qo'shiluvchining chap tomonga o'tkazamiz, so'ng har ikkala tomonini Δt ga bo'lamiz. Natijada

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}=-\lambda P_0(t)+(\mu+\nu)P_1(t)+\frac{0(\Delta t)}{\Delta t}$$

ni hosil qilamiz.

Endi agar $\Delta t \to 0$ da limitga o'tsak, u holda quyidagi differensial tenglamani hosil qilamiz:

$$P_0'(t)=-\lambda P_0(t)+(\mu+\nu)P_1(t)$$

($0(\Delta t)$ miqdor Δt ga nisbatan yuqori tartibli cheksiz kichik miqdor bo'lgani uchun $\lim\limits_{\Delta t\to 0}\frac{0(\Delta t)}{\Delta t}=0$).

$P_k(t)$ $(0<k<n)$ ehtimollar uchun differensial tenglamalar tuzamiz. t vaqt momentini tayin qilib olamiz va $t+\Delta t$ momentda sistema x_k holatda bo'lish ehtimoli $P_k(t+\Delta t)$ ni topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_k holatda bo'lgan (k ta kanal band), Δt vaqt ichida esa undan chiqmagan (kanallardan hech biri bo'shamagan, xizmat qilayotgan talablardan hech biri ketmagan va talab kelmagan);

t momentda sistema x_{k-1} holatda bo'lgan, Δt vaqt ichida esa x_k holatga kelgan (bitta talab kelgan talablardan hech biri bo'shamagan va xizmat qilayotgan talablardan hech biri kelmagan);

t momentda sistema x_{k+1} holatda bo'lgan, Δt vaqt ichida esa x_{k+1} holatga kelgan (yo bitta kanal bo'shagan yoki xizmat qilayotgan talablardan biri vaqti tugashi munosabati bilan ketgan va talab kelmagan).

Qolgan barcha imkoniyatlar $0(\Delta t)$ ehtimolga ega ekanligini ko'rsatish qiyin emas.

Bu hodisalardan birinchisining ehtimoli ehtimollarni ko'paytirish haqidagi teoremaga ko'ra

$$
\begin{aligned}
&P_k(t)\left[P(\tau_x > \Delta t) + P(\tau_\delta > \Delta t)\right]^k \cdot e^{-\lambda \Delta t} = \\
&= P_k(t)\left[e^{-\mu \Delta t} \cdot e^{-\lambda \Delta t}\right]^k \cdot e^{-\lambda \Delta t} P_k(t)\left(e^{-k(\mu+\nu)\Delta t}\right) \cdot e^{-\lambda \Delta t} = \\
&= P_k(t)\, e^{-[\lambda + k(\mu+\nu)]\Delta t} = P_k(t)\left[1 - (\lambda + k(\mu+\nu))\Delta t + 0(\Delta t)\right]
\end{aligned}
$$

ga teng, ikkinchi hodisaning ehtimoli

$$
\begin{aligned}
&P_{k-1}(t)\left(1 - e^{-\lambda \Delta t}\right)\left[P(\tau_x > \Delta t) + P(\tau_\delta > \Delta t)\right]^{k-1} = \\
&= P_{k-1}(t)\left(1 - e^{-\lambda \Delta t}\right)\left[e^{-\mu \Delta t} \cdot e^{-\lambda \Delta t}\right]^{k-1} = \\
&= P_{k-1}(t)\left(1 - e^{-\lambda \Delta t}\right) e^{-(k-1)(\mu+\nu)\Delta t} = \\
&= P_{k-1}(t)\left[\lambda \Delta t + 0(\Delta t)\right]\left[1 - (k-1)(\mu+\nu)\Delta t + 0\Delta t\right] = \\
&= P_{k-1}(t)(k+1)(\mu+\nu)\Delta t + 0\Delta t
\end{aligned}
$$

ga teng, uchinchi hodisaning ehtimoli

$$
\begin{aligned}
&P_{k+1}(t)\left[c'_{k+1} P(\tau_x < \Delta t) + c'_{k+1} P(\tau_\delta < \Delta t)\right] e^{-\lambda \Delta t} = \\
&= P_{k+1}(t)\left[(k+1)\left(1 - e^{-\mu \Delta t}\right) + (k+1)\left(1 - e^{-\nu \Delta t}\right)\right] e^{-\lambda \Delta t} = \\
&= P_{k+1}(t)(k+1)\left[\mu \Delta t + 0(\Delta t) + \nu \Delta t + 0(\Delta t)\right]\left[1 - \lambda \Delta t + 0(\Delta t)\right] = \\
&= P_{k+1}(t)(k+1)\left[(\mu+\nu)\Delta t + 0(\Delta t)\right] \cdot \left[1 - \lambda \Delta t + 0(\Delta t)\right] = \\
&= P_{k+1}(t)(k+1)(\mu+\nu)\Delta t + 0(\Delta t)
\end{aligned}
$$

ga teng.

Topilgan ehtimollarni birga yig'ib, quyidagi tenglikni hosil qilamiz:

$$
\begin{aligned}
&P_k(t + \Delta t) = P_k(t)\left[1 - (\lambda + k(\mu+\nu))\Delta t + 0(\Delta t)\right] + \\
&P_{k-1}(t)\left[\lambda \Delta t + 0(\Delta t)\right] + P_{k+1}(t)(k+1)(\mu+\nu)\Delta t + 0(\Delta t)
\end{aligned}
$$

yoki

$$
\begin{aligned}
&P_k(t + \Delta t) = P_k(t)\left[1 - (\lambda + k(\mu+\nu))\Delta t\right] + P_{k-1}(t)\lambda \Delta t + \\
&+ P_{k+1}(t)(k+1)(\mu+\nu)\Delta t + 0(\Delta t)
\end{aligned}
$$

Bu tenglikda $P_k(t)$ qo'shiluvchini chap tomonga o'tkazamiz, so'ng har ikkala tomonini Δt ga bo'lamiz. Natijada

$$
\frac{P_k(t + \Delta t) - P_k(t)}{\Delta t} = -(\lambda + k(\mu+\nu))P_k(t) + \lambda P_{k-1}(t) + (k+1)(\mu+\nu)P_{k+1}(t) + \frac{0(\Delta t)}{\Delta t}
$$

ni hosil qilamiz. Bundan $\Delta t \to 0$ da limitga o'tib $P_k(t)$ ehtimol uchun quyidagi differensial tenglamaga ega bo'lamiz:

$$
\lim_{\Delta t} \frac{P_k(t + \Delta t) - P_k(t)}{\Delta t} = -(\lambda + k(\mu+\nu))P_k(t) + \lambda P_{k-1}(t) + (k+1)(\mu+\nu)P_{k+1}(t)
$$

yoki

$$
P'_k(t) = -(\lambda + k(\mu+\nu))P_k(t) + \lambda P_{k-1}(t) + (k+1)(\mu+\nu)P_{k+1}(t), \quad (k<0<n)
$$

($0(\Delta t)$ miqdor Δt ga nisbatan yuqori tartibli cheksiz kichik miqdor bo'lganidan $\lim\limits_{\Delta t\to 0}\frac{0(\Delta t)}{\Delta t}=0$ ga teng bo'ladi).

Endi $P_n(t)$ ehtimollar uchun differensial tenglamalar tuzamiz. t vaqt momentini tayin qilib olib, $t+\Delta t$ momentda sistema x_n holatda bo'lishi hodisasining ehtimoli $P_n(t+\Delta t)$ ni topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_n holatda bo'lgan, Δt vaqt ichida esa, undan chiqmagan (kanallardan hech biri bo'shamagan, xizmat qilayotgan talablardan hech biri ketmagan va talab kelmagan);

t momentda sistema x_{n-1} holatda bo'lgan, Δt vaqt ichida esa x_n holatga o'tgan (bitta talab kelgan, talablardan hech biri bo'shamagan va xizmat qilayotgan talablardan hech biri kelmagan);

t momentda sistema x_{n+1} holatda bo'lgan, Δt vaqt ichida esa x_n holatga kelgan (yo bitta kanal bo'shagan yoki xizmat qilayotgan talablardan biri vaqti tugashi munosabati bilan ketgan va talab kelmagan).

Qolgan barcha imkoniyatlar $0(\Delta t)$ ehtimolga ega ekanligini ko'rsatish qiyin emas.

Bu hodisalardan birinchisining ehtimoli ehtimollarni ko'paytirish haqidagi teoremaga ko'ra

$$P_n(t)\left[P(T_x>\Delta t)\cdot P(T_\delta>\Delta t)\right]^n\cdot e^{-\lambda\Delta t}=P_n(t)\left[e^{-\mu\Delta t}\cdot e^{-\lambda\Delta t}\right]^n e^{-\lambda\Delta t}=P_n(t)e^{-n(\mu+\nu)\Delta t}\cdot e^{-\lambda\Delta t}=$$
$$=P_n(t)e^{-[\lambda+n(\mu+\nu)]\Delta t}=P_n(t)\left\{1-[\lambda+n(\mu+\nu)]\Delta t+0(\Delta t)\right\}$$

ga teng, ikkinchi hodisaning ehtimoli

$$P_{n-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left[P(T_x>\Delta t)\cdot P(T_\delta>\Delta t)\right]^{n-1}=P_{n-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left[e^{-\mu\Delta t}\cdot e^{-\lambda\Delta t}\right]^{n-1}=$$
$$=P_{n-1}(t)\left(1-e^{-\lambda\Delta t}\right)e^{-(n-1)(\mu+\nu)\Delta t}=P_{n-1}(t)\left[\lambda\Delta t+0(\Delta t)\right]\left[1-(n-1)(\mu+\nu)\Delta t+0\Delta t\right]=$$
$$=P_{n-1}(t)(n+1)(\mu+\nu)\Delta t+0\Delta t$$

ga teng, uchinchi hodisaning ehtimoli

$$P_{n+1}(t)[c'_n P(\xi_x < \Delta t) + c'_{n+1} P(\xi_\delta < \Delta t)]e^{-\lambda\Delta t} =$$
$$= P_{n+1}(t)[n(1-e^{-\mu\Delta t}) + (n+1)(1-e^{-\nu\Delta t})]e^{-\lambda\Delta t} =$$
$$= P_{n+1}(t)[n(\mu\Delta t + 0(\Delta t)) + (n+1)(\nu\Delta t + 0(\Delta t))][1-\lambda\Delta t + 0(\Delta t)] =$$
$$= P_{n+1}(t)[(n\mu + (n+1)\nu)\Delta t + 0(\Delta t)][1-\lambda\Delta t + 0(\Delta t)] =$$
$$= P_{n+1}(t)[n\mu + (n+1)\nu]\Delta t + 0(\Delta t)$$

ga teng.

Demak,

$$P_n(t+\Delta t) = P_n(t)[1-(\lambda + n(\mu+\nu))\Delta t + 0(\Delta t)] +$$
$$P_{n-1}(t)\lambda\Delta t + 0(\Delta t) + P_{n+1}(t)[n\mu + (n+1)\nu]\Delta t + 0(\Delta t)$$

yoki

$$P_n(t+\Delta t) = P_n(t)[1-(\lambda + n(\mu+\nu))\Delta t] + P_{n-1}(t)\lambda\Delta t +$$
$$+ P_{n+1}(t)[n\mu + (n+1)\nu]\Delta t + 0(\Delta t)$$

Bu tenglikning o'ng tomonida turgan $P_n(t)$ qo'shiluvchini chap tomonga o'tkazamiz, so'ng hosil bo'lgan tengliklarning har ikkala tomonini Δt ga bo'lamiz. Natijada

$$\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = -(\lambda + n(\mu+\nu))P_n(t) + \lambda P_{n-1}(t) + [n\mu + (n+1)\nu]P_{n+1}(t) + \frac{0(\Delta t)}{\Delta t}$$

tenglikni hosil qilamiz. Bundan $\Delta t \to 0$ da limitga o'tib $P_n(t)$ ehtimol uchun quyidagi differensial tenglamaga ega bo'lamiz:

$$P'_n(t) = -(\lambda + n(\mu+\nu))P_n(t) + \lambda P_{n-1}(t) + [n\mu + (n+1)\nu]P_{n+1}(t),$$

Endi qolgan $P_{n+s}(t), s > 0$ ehtimollar uchun differensial tenglamalar tuzishga o'tamiz. Bunda $1 \le s < m$ va $s = m$ hollarni har birini alohida qarashimizga to'g'ri keladi. Avval $1 \le s < m$ bo'lsin. t vaqt momentini tayin qilib olamiz va $t+\Delta t$ momentda sistema x_{n+s} holatda bo'lishi hodisasining ehtimoli $P_{n+s}(t+\Delta t)$ ni topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_{n+s} holatda bo'lgan, Δt vaqt ichida esa, bu holat o'zgarmagan (talab kelmagan, kanallardan hech biri bo'shamagan va sistemada bo'lgan talablardan hech biri ketmagan);

t momentda sistema x_{n+s-1} holatda bo'lgan, Δt vaqt ichida esa x_{n+s} holatga o'tgan (bitta talab kelgan, hech biri kanal bo'shamagan va sistemada birorta ham talab kelmagan);

t momentda sistema x_{n+s+1} holatda bo'lgan, Δt vaqt ichida esa x_{n+s} holatga kelgan (yo kanallardan biri bo'shagan yoki sistemada bo'lgan talablardan biri vaqti tugashi munosabati bilan ketgan va talab kelmagan).

Δt vaqt ichida x_{n+s} holatga o'tish mumkin bo'lgan barcha imkoniyatlar $0(\Delta t)$ ga teng ehtimolga ega bo'ladi.

Bu hodisalardan birinchisining ehtimoli ehtimollarni ko'paytirish haqidagi teoremaga ko'ra

$$\begin{aligned}
&P_{n+s}(t)e^{-\lambda\Delta t}\left[P(\tau_x > \Delta t)\right]^n\left[P(\tau_\delta > \Delta t)\right]^{n+s} = \\
&= P_{n+s}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^n\left(e^{-\lambda\Delta t}\right)^{n+s} = \\
&P_{n+s}(t)e^{-\lambda\Delta t}\cdot e^{-n\mu\Delta t}\cdot e^{-(n+s)\nu\Delta t} = \\
&= P_{n+s}(t)e^{-[\lambda+n\mu+(n+s)\nu]\Delta t} = P_{n+s}(t)\left[1-(\lambda+n\mu+(n+s)\nu)\Delta t+0(\Delta t)\right]
\end{aligned}$$

ga teng, ikkinchi hodisaning ehtimoli

$$\begin{aligned}
&P_{n+s-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left[P(\tau_x > \Delta t)\right]^n\left[P(\tau_\delta > \Delta t)\right]^{n+s-1} = \\
&P_{n+s-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left(e^{-\mu\Delta t}\right)^n\cdot\left(e^{-\lambda\Delta t}\right)^{n+s-1} = \\
&= P_{n+s-1}(t)\left(1-e^{-\lambda\Delta t}\right)e^{-n\mu\Delta t}\cdot e^{-(n+s-1)\nu\Delta t} = \\
&= P_{n+s-1}(t)\left(1-e^{-\lambda\Delta t}\right)e^{-[n\mu+(n+s-1)\nu]\Delta t} = \\
&P_{n+s-1}(t)\left[\lambda\Delta t+0(\Delta t)\right]\left[1-(n\mu+(n+s-1)\nu)\Delta t+0\Delta t\right] = \\
&= P_{n+s-1}(t)\lambda\Delta t+0\Delta t
\end{aligned}$$

ga teng, uchinchi hodisaning ehtimoli

$$\begin{aligned}
&P_{n+s+1}(t)\left[c'_n P(\tau_x < \Delta t)+c'_{n+s+1}P(\tau_\delta < \Delta t)\right]e^{-\lambda\Delta t} = \\
&= P_{n+s+1}(t)\left[n\left(1-e^{-\mu\Delta t}\right)+(n+s+1)\left(1-e^{-\nu\Delta t}\right)\right]e^{-\lambda\Delta t} = \\
&= P_{n+s+1}(t)\left[n(\mu\Delta t+0(\Delta t))+(n+s+1)(\nu\Delta t+0(\Delta t))\right]\left[1-\lambda\Delta t+0(\Delta t)\right] = \\
&= P_{n+s+1}(t)\left[(n\mu+(n+s+1)\nu)\Delta t+0(\Delta t)\right]\cdot\left[1-\lambda\Delta t+0(\Delta t)\right] = \\
&= P_{n+s+1}(t)\left[n\mu+(n+s+1)\nu\right]\Delta t+0(\Delta t)
\end{aligned}$$

ga teng.

Topilgan ehtimollarni bir yerga yig'ib, quyidagi tenglikni hosil qilamiz:

$$\begin{aligned}
&P_{n+s}(t+\Delta t) = P_{n+s}(t)\left[1-(\lambda+n\mu+(n+s)\nu)\Delta t\right]+ \\
&P_{n+s-1}(t)\lambda\Delta t+P_{n+s+1}(t)\left[n\mu+(n+s+1)\nu\right]\Delta t+0(\Delta t)
\end{aligned}$$

Bu tenglikda $P_{n+s}(t)$ qo'shiluvchini chap tomonga o'tkazamiz, so'ng Δt ga bo'lamiz. Natijada

$$\frac{P_{n+s}(t+\Delta t)-P_{n+s}(t)}{\Delta t}=-\left[\lambda+n\mu+(n+s)\nu\right]P_{n+s}(t)+\lambda P_{n+s-1}(t)+$$
$$+\left[n\mu+(n+s+1)\nu\right]P_{n+s+1}(t)+\frac{0(\Delta t)}{\Delta t}$$

tenglikni hosil qilamiz.

Bundan $\Delta t\to 0$ da limitga o'tib, quyidagi differensial tenglamaga ega bo'lamiz:

$$P'_{n+s}(t)=-\left[\lambda+n\mu+(n+n)\nu\right]P_{n+s}(t)+\lambda P_{n+s-1}(t)+$$
$$+\left[n\mu+(n+s+1)\nu\right]P_{n+s+1}(t).$$

Endi oxirgi $P_{n+m}(t)$ ehtimollar uchun differensial tenglamalar tuzamiz. t vaqt momentini tayin qilib olamiz va $t+\Delta t$ momentda sistema x_{n+m} holatda bo'lishi hodisasining ehtimolini topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_{n+m} holatda bo'lgan, Δt vaqt ichida esa, bu holat o'zgarmagan (kanallardan hech biri bo'shamagan va sistemadagi talablardan hech biri ketmagan);

t momentda sistema x_{n+m-1} holatda bo'lgan, Δt vaqt ichida esa x_{n+m} holatga kelgan (bitta talab kelgan, hech biri kanal bo'shamagan va sistemadagi talablardan hech biri ketmagan);

Δt vaqt ichida x_{n+m} holatga o'tish mumkin bo'lgan barcha imkoniyatlar $0(\Delta t)$ ga teng ehtimolga ega bo'ladi.

Bu hodisalardan birinchisining ehtimoli

$$P_{n+m}(t)e^{-\lambda\Delta t}\left[P(T_x>\Delta t)\right]^{n}\left[P(T_\delta>\Delta t)\right]^{n+m}=$$
$$=P_{n+m}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^{n}\cdot\left(e^{-\lambda\Delta t}\right)^{n+m}=$$
$$P_{n+m}(t)e^{-\lambda\Delta t}\cdot e^{-n\mu\Delta t}\cdot e^{-(n+m)\nu\Delta t}=$$
$$=P_{n+m}(t)e^{-\left[\lambda+n\mu+(n+m)\nu\right]\Delta t}=P_{n+m}(t)\left[1-\left(\lambda+n\mu+(n+m)\nu\right)\Delta t+0(\Delta t)\right]$$

ga teng, ikkinchi hodisaning ehtimoli

$$
\begin{aligned}
& P_{n+m-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left[P(\xi_x>\Delta t)\right]^n\left[P(\xi_\delta>\Delta t)\right]^{n+m-1} = \\
& P_{n+m-1}(t)\left(1-e^{-\lambda\Delta t}\right)\left(e^{-\mu\Delta t}\right)^n\cdot\left(e^{-\lambda\Delta t}\right)^{n+m-1} = \\
& = P_{n+m-1}(t)\left(1-e^{-\lambda\Delta t}\right)e^{-n\mu\Delta t}\cdot e^{-(n+m-1)\nu\Delta t} = \\
& = P_{n+m-1}(t)\left(1-e^{-\lambda\Delta t}\right)e^{-\left[n\mu+(n+m-1)\nu\right]\Delta t} = \\
& P_{n+m-1}(t)\left[\lambda\Delta t+0(\Delta t)\right]\left[1-\left(n\mu+(n+m-1)\nu\right)\Delta t+0(\Delta t)\right] = \\
& = P_{n+m-1}(t)\lambda\Delta t+0\Delta t
\end{aligned}
$$

Topilgan ehtimollarni bir yerga yig'ib, quyidagi tenglikni hosil qilamiz:

$$
\begin{aligned}
& P_{n+m}(t+\Delta t) = P_{n+m}(t)\left[1-\left(n\mu+(n+m)\nu\right)\Delta t\right]+ \\
& P_{n+m-1}(t)\lambda\Delta t+0(\Delta t)
\end{aligned}
$$

Bu tenglikda $P_{n+m}(t)$ qo'shiluvchini chap tomonga o'tkazamiz, so'ng Δt ga bo'lamiz. Natijada

$$
\frac{P_{n+m}(t+\Delta t)-P_{n+m}(t)}{\Delta t} = -\left[n\mu+(n+m)\nu\right]P_{n+m}(t)+\lambda P_{n+m-1}(t)+\frac{0(\Delta t)}{\Delta t}
$$

tenglikni hosil qilamiz.

Bundan $\Delta t\to 0$ da limitga o'tib, $P_{n+m}(t)$ ehtimollar uchun quyidagi differensial tenglamaga ega bo'lamiz:

$$
P'_{n+m}(t) = -\left[n\mu+(n+m)\nu\right]P_{n+m}(t)+\lambda P_{n+m-1}(t)
$$

Shunday qilib, biz sistemaning holatlari ehtimollari uchun quyidagi differensial tenglamalarni hosil qildik:

$$
\begin{cases}
P_0'(t) = -\lambda P_0(t)+(\mu+\nu)P_1(t) \\
\dots\dots\dots\dots\dots\dots\dots\dots \\
P_k'(t) = -\left[\lambda+k(\mu+\nu)\right]P_k(t)+\lambda P_{k-1}(t)+(k+1)(\mu+\nu)P_{k+1}(t),\ (1\le k\le n-1) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_n'(t) = -\left[\lambda+n(\mu+\nu)\right]P_n(t)+\lambda P_{n-1}(t)+n(\mu+\nu)P_{n+1}(t), \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_{n+s}'(t) = -\left[\lambda+n\mu+(n+s)\nu\right]P_{n+s}(t)+\lambda P_{n+s-1}(t)+\left[n\mu+(n+s+1)\nu\right]P_{n+s+1}(t),\ (1\le s\le m-1), \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
P_{n+m}'(t) = -\left[n\mu+(n+m)\nu\right]P_{n+m}(t)+\lambda P_{n+m-1}(t).
\end{cases}
\tag{1.1}
$$

(1.1) tenglamalar sisremasini $P_0(0)=1, P_1(0)=,\dots P_{n+m}(0)=0$ boshlang'ich shartlarda integrallab $P_k(t)$ ehtimollarning ifodalarini toppish mumkin. Ammo bu juda katta hajmdagi hisoblashlarni talab qiladi, shu sababli bu yerda biz bunga to'xalmaymiz.

(1.1) tenglamalarni tuzishda biz λ, μ va ν parametrlar (miqdorlar)ni o’zgarmas bo’lsin degan farazdan hech qayerda foydalanganini ko’rmadik. Shuning uchun (1.1) tenglamalar vaqtga bog’liq bo’lgan $\lambda(t), \mu(t), \nu(t)$ lar uchun ham to’g’ri bo’lib qolaveradi, faqat sistemali holatdan holatga o’tkazuvchi hodisalar oqimi Puassonligicha qolsa bo’lgani (busiz jarayon Markov jarayoni bo’lmaydi).

3. Statsionar ehtimollar. Statsionarlik shartida, ya’ni $t \to \infty$ da barcha $P'_k(t)$ hosilalar $P_k(t)$ ehtimollar esa o’zgarmas $P(t)$ sonlarga intiladi. U holda (1.1) differensial tenglamalar sistemasini quyidagi algebraik tenglamalar sistemasiga aylantiradi:

$$
\begin{cases}
-\lambda P_0 + (\mu+\nu)P_1 = 0 \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
-[\lambda + k(\mu+\nu)]P_k + \lambda P_{k-1} + (k+1)(\mu+\nu)P_{k+1} = 0, 0 < k < n, \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
-[\lambda + n\mu + (n+s)\nu]P_{n+s} + \lambda P_{n+s-1} + [n\mu + (n+s-1)\nu]P_{n+s-1} = 0, 1 \le s \le m, \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
-[n\mu + (n+m)\nu]P_{n+m} + \lambda P_{n+m-1} = 0
\end{cases}
\tag{1.2}
$$

Bu tenglamalarga $\sum_{k=0}^{n+m} P_k = 1$ shartni qo’shish lozim.

(1.2) sistemani $P_0, P_1, \dots, P_{n+m}$ noma’lumlarga nisbatan yechamiz. Birinchi tenglamadan

$$P_1 = \frac{\lambda}{\mu+\nu} P_0 \tag{1.3}$$

ga ega bo’lamiz. Ikkinchi tenglamadan (4.3) hisobga olib,

$$
\begin{aligned}
P_2 &= \frac{1}{2(\mu+\nu)}\left[(\lambda+\mu+\nu)P_1 - \lambda P_0\right] = \frac{1}{2(\mu+\nu)}\left[(\lambda+\mu+\nu)\frac{\lambda}{(\mu+\nu)}P_0 - \lambda P_0\right] = \\
&= \frac{1}{2(\mu+\nu)}\left[\frac{\lambda}{(\mu+\nu)}P_1 + \lambda P_0 - \lambda P_0\right] = \frac{\lambda^2}{(\mu+\nu)^2}P_0
\end{aligned}
\tag{1.4}
$$

ni hosil qilamiz.

Uchinchi tenglamadan (4.3) va (4.4) larni hisobga olib, P_3ni topamiz:

$$\begin{aligned} P_3 &= \frac{1}{3(\mu+\nu)}\left[\left(\lambda+2(\mu+\nu)\right)P_2-\lambda P_1\right]= \\ &= \frac{1}{3(\mu+\nu)}\left[\left(\lambda+2(\mu+\nu)\right)\frac{\lambda^2}{(\mu+\nu)^2}P_0-\lambda\cdot\frac{\lambda}{(\mu+\nu)}P_0\right]= \\ &= \frac{1}{3(\mu+\nu)}\left[\frac{\lambda^3}{2(\mu+\nu)^2}P_0+\frac{\lambda^2}{\mu+\nu}P_0-\frac{\lambda^2}{\mu+\nu}P_0\right]= \\ &= \frac{\lambda^3}{2\cdot 3(\mu+\nu)^3}P_0=\frac{\lambda^3}{3!(\mu+\nu)^3}P_0 \end{aligned} \tag{1.5}$$

To'rtinchi tenglamadan (1.4) va (1.5) larni hisobga olib, P_4 ni topamiz:

$$\begin{aligned} P_4 &= \frac{1}{4(\mu+\nu)}\left[\left(\lambda+3(\mu+\nu)\right)P_3-\lambda P_2\right]= \\ &= \frac{1}{4(\mu+\nu)}\left[\left(\lambda+3(\mu+\nu)\right)\frac{\lambda^3}{2\cdot 3(\mu+\nu)^3}P_0-\lambda\cdot\frac{\lambda^2}{2(\mu+\nu)^2}P_0\right]= \\ &= \frac{1}{4(\mu+\nu)}\left[\frac{\lambda^4}{2\cdot 3(\mu+\nu)^3}P_0+\frac{\lambda^3}{2(\mu+\nu)^2}P_0-\frac{\lambda^3}{2(\mu+\nu)^2}P_0\right]= \\ &= \frac{\lambda^4}{2\cdot 3\cdot 4(\mu+\nu)^4}P_0=\frac{\lambda^4}{4!(\mu+\nu)^4}P_0 \end{aligned}$$

Bu jarayonni davom ettirib, ketma-ket quyidagilarni hosil qilamiz:

$$P_5=\frac{\lambda^5}{2\cdot 3\cdot 4\cdot 5(\mu+\nu)^5}P_0=\frac{\lambda^5}{5!(\mu+\nu)^5}P_0,$$

....

$$P_n=\frac{\lambda^n}{2\cdot 3\cdot\ldots\cdot n(\mu+\nu)^n}P_0=\frac{\lambda^n}{n!(\mu+\nu)^n}P_0.$$

Demak, har qanday $k\le n$ uchun

$$P_k=\frac{\lambda^k}{k!(\mu+\nu)^k}P_0=\frac{1}{k!}\cdot\left(\frac{\lambda}{(\mu+\nu)}\right)^k P_0 \tag{1.6}$$

Endi qolgan $P_{n+s}, s\ge 1$ ehtimollarni topishga o'tamiz. (1.2) sistemaning navbatdagi tenglamalardan (1.6)ni hisobga olib, P_{n+1}ni topamiz:

$$P_{n+1}=\frac{1}{n\mu+(n+1)\nu}\left[(\lambda+n(\mu+\nu))P_n-\lambda P_{n-1}\right]=$$

$$=\frac{1}{n\mu+(n+1)\nu}\left[(\lambda+n(\mu+\nu))\frac{1}{n!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^{n}P_0-\lambda\cdot\frac{1}{(n-1)!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^{n-1}\cdot P_0\right]=$$

$$=\frac{1}{n\mu+(n+1)\nu}\left[\frac{1}{n!}\cdot\lambda\left(\frac{\lambda}{\mu+\nu}\right)^{n}P_0+\frac{1}{(n-1)!}\cdot\frac{\lambda^{n}}{(\mu+\nu)^{n-1}}P_0-\frac{1}{(n-1)!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^{n-1}P_0\right]= \tag{1.7}$$

$$=\frac{\lambda^{4}}{n!(\mu+\nu)^{n}[n\mu+(n+1)\nu]}P_0$$

(4.2) sistemanig ushbu

$$-[\lambda+n\mu+(n+1)\nu]P_{n+1}+\lambda P_n+[n\mu+(n+2)\nu]P_{n+2}=0$$

tenglamasidan (4.6) va (4.7) larni hisobga olib, P_{n+2} ni topamiz:

$$P_{n+2}=\frac{1}{n\mu+(n+2)\nu}\left[(\lambda+n\mu+(n+1)\nu)P_{n+1}-\lambda P_n\right]=$$

$$=\frac{1}{n\mu+(n+2)\nu}\left[(\lambda+n\mu+(n+1)\nu)\frac{\lambda^{n+1}}{n!(\mu+\nu)^{n}(\lambda+n\mu+(n+1)\nu)}P_0-\lambda\cdot\frac{1}{n!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^{n}\cdot P_0\right]=$$

$$=\frac{1}{n\mu+(n+2)\nu}\left[\frac{\lambda^{n+2}}{n!(\mu+\nu)^{n}(n\mu+(n+1)\nu)}P_0+\frac{\lambda^{n+1}}{n!(\mu+\nu)^{n}}P_0-\lambda\frac{1}{n!}\cdot\left(\frac{\lambda}{\mu+\nu}\right)^{n}P_0\right]= \tag{.8}$$

$$=\frac{\lambda^{n+2}}{n!(\mu+\nu)^{n}[n\mu+(n+1)\nu][n\mu+(n+2)\nu]}P_0$$

navbatdagi ushbu

$$-[\lambda+n\mu+(n+2)\nu]P_{n+2}+\lambda P_{n+1}+[n\mu+(n+3)\nu]P_{n+3}=0$$

tenglamasidan (1.7) va (1.8) larni hisobga olib, P_{n+3} ni topamiz:

$$P_{n+3}=\frac{1}{n\mu+(n+3)\nu}\left[(\lambda+n\mu+(n+2)\nu)P_{n+2}-\lambda P_{n+1}\right]=$$

$$=\frac{1}{n\mu+(n+3)\nu}\left[(\lambda+n\mu+(n+2)\nu)\frac{\lambda^{n+2}}{n!(\mu+\nu)^n(\lambda+n\mu+(n+1)\nu)(\lambda+n\mu+(n+2)\nu)}P_0-\right.$$

$$\left.-\lambda\cdot\frac{\lambda^{n+2}}{n!(\mu+\nu)^n(\lambda+n\mu+(n+1)\nu)}\cdot P_0\right]=$$

$$=\frac{1}{n\mu+(n+3)\nu}\left[\frac{\lambda^{n+3}}{n!(\mu+\nu)^n(\lambda+n\mu+(n+1)\nu)(\lambda+n\mu+(n+2)\nu)}P_0+\right.$$

$$\left.+\frac{\lambda^{n+2}}{n!(\mu+\nu)^n(\lambda+n\mu+(n+1)\nu)}P_0-\lambda\frac{\lambda^{n+1}}{n!(\mu+\nu)^n(\lambda+n\mu+(n+1)\nu)}P_0\right]=$$

$$=\frac{\lambda^{n+2}}{n!(\mu+\nu)^n(n\mu+(n+1)\nu)(n\mu+(n+2)\nu)(n\mu+(n+3)\nu)}P_0=$$

$$=\frac{\lambda^{n+3}}{n!(\mu+\nu)^n\prod_{i=1}^{3}(n\mu+(n+i)\nu)}P_0.$$

Yuqoridagi topilgan ifodalardan kelib chiqib, har qanday $s(1\le s\le m)$ uchun

$$P_{n+s}=\frac{\lambda^{n+s}}{n!(\mu+\nu)^n\prod_{i=1}^{s}(n\mu+(n+i)\nu)}P_0 \qquad (1.9)$$

ga ega bo'lamiz.

(1.6) va (1.7) ifodalarning har ikkalasida ham P_0 noma'lum ko'paytuvchi sifatida qatnashayapti. Uni

$$\sum_{k=0}^{n+s}P_k=1$$

shakldan foydalanib topamiz. Bunga P_k va P_{k+s} larning (1.6) va (1.9) tengliklardagi ifodalarini qo'yamiz:

$$\sum_{k=0}^{n+s} P_k = \sum_{k=0}^{n} P_k + \sum_{s=1}^{m} P_{n+s} = P_0 + \sum_{k=1}^{n} P_k + \sum_{s=1} P_{n+s} =$$

$$= P_0 + \sum_{k=1}^{n} \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k P_0 + \sum_{s=1}^{m} \frac{\lambda^{n+s}}{n!(\mu+\nu)^n \prod_{i=1}^{s}\left[n\mu+(n+i)\nu\right]} P_0 =$$

$$= P_0\left[1 + \sum_{k=1}^{n} \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k + \frac{\lambda^n}{n!(\mu+\nu)^n} \sum_{s=1}^{m} \frac{\lambda^s}{\prod_{i=1}^{s}\left[n\mu+(n+i)\nu\right]} P_0\right] =$$

$$= P_0\left[\sum_{k=1}^{n} \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k + \frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n \sum_{s=1}^{m} \frac{\lambda^s}{\prod_{i=1}^{s}\left[n\mu+(n+i)\nu\right]}\right] = 1$$

Bundan

$$P_0 = \frac{1}{\sum_{k=1}^{n} \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k + \frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n \sum_{s=1}^{m} \frac{\lambda^s}{\prod_{i=1}^{s}\left[n\mu+(n+i)\nu\right]}} \tag{1.10}$$

ni hosil qilamiz.

Ushbu belgilashlarni kiritamiz:

$$\frac{\lambda}{\mu} = \alpha \text{ va } \frac{\nu}{\mu} = \beta$$

Ular mos ravishda bitta talabga o’rtacha xizmat ko’rsatiladigan vaqt ichida sistemaga keladigan talablarni o’rtacha soni va sistemadan ketadigan talablarning o’rtacha sonidir. Haqiqatdan ham,

$$\alpha = \frac{\lambda}{\mu} = \lambda \cdot \frac{1}{\mu} = \lambda \cdot MT_x, \quad \beta = \frac{\nu}{\mu} = \nu \cdot MT_x,$$

bu yerda T_x bitta talabni xizmat qilish vaqti, MT_x esa uning matematik kutilishi yangi belgilashlardan (4.6), (4.9) va (4.10) quyidagi ko’rinishlarni oladi:

$$P_k = \frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k P_0 = \frac{1}{k!}\left(\frac{\lambda/\mu}{1+\nu/\mu}\right)^k P_0 = \frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k P_0, \quad 1 \le k \le n; \tag{1.11}$$

$$P_{n+s}=\frac{\lambda^{n+s}}{n!(\mu+\nu)^n\prod_{i=1}^{s}[n\mu+(n+i)\nu]}P_0=$$

$$=\frac{1}{n!}\left(\frac{\lambda/\mu}{1+\nu/\mu}\right)^n\frac{\left(\lambda/\mu\right)^s}{\prod_{i=1}^{s}\left[n+(n+i)\frac{\nu}{\mu}\right]}P_0= \qquad (1.12)$$

$$\frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n\frac{(\alpha)^s}{\prod_{i=1}^{s}[n+(n+i)\beta]}P_0,\quad 1\le s\le m;$$

$$P_0=\frac{1}{\sum_{k=0}^{n}\frac{1}{k!}\left(\frac{\lambda}{\mu+\nu}\right)^k+\frac{1}{n!}\left(\frac{\lambda}{\mu+\nu}\right)^n\sum_{s=1}^{m}\frac{\lambda^s}{\prod_{i=1}^{s}[n\mu+(n+i)\nu]}}=$$

$$=\frac{1}{\sum_{k=0}^{n}\frac{1}{k!}\left(\frac{\lambda/\mu}{1+\nu/\mu}\right)^k+\frac{1}{n!}\left(\frac{\lambda/\mu}{1+\nu/\mu}\right)^n\sum_{s=1}^{m}\frac{\left(\lambda/\mu\right)^s}{\prod_{i=1}^{s}\left[n+(n+i)\frac{\nu}{\mu}\right]}}=$$

$$=\frac{1}{\sum_{k=0}^{n}\frac{1}{k!}\left(\frac{\alpha}{1+\beta}\right)^k+\frac{1}{n!}\left(\frac{\alpha}{1+\beta}\right)^n\sum_{s=1}^{m}\frac{(\alpha)^s}{\prod_{i=1}^{s}[n+(n+i)\beta]}} \qquad (1.13)$$

Shunday qilib (1.11), (1.12) va (1.13) formulalar qo'yilgan masalaning yechimini beradi. Ular yordamida bizni qiziqtiradigan yana boshqa xarakteristikalarni oson aniqlashimiz mumkin, xususan, sistemaga qabul qilingan talabning xizmat qilish ehtimoli P_{xiz} xizmat ehtimoli vaqt birligida xizmat qilinib kelinayotgan talabning o'rtacha soni, vaqt birligida sistemaga keladigan talabning o'rtacha soni bo'lgan nisbatini xizmat qilish bilan band kanallarning o'rtacha soniga ko'paytmasi sifatida aniqlanadi. Buning uchun avval xizmat qilish bilan band bo'lgan kanallarning matematik kutilishini topamiz:

$$m_x=\sum_{k=1}^{n+m}kP_k=\sum_{k=1}^{n}kP_k+\sum_{s=1}^{m}nP_{n+s}=\sum_{k=1}^{n}kP_k+n\sum_{s=1}^{m}P_{n+s}=$$

$$=\sum_{k=1}^{n}kP_k+n\left(1-\sum_{k=0}^{n}P_k\right)=n-\sum_{k=0}^{n-1}(n-k)P_k$$

m_x ni $\frac{\mu}{\nu}$ koeffisentga ko'paytirib P_{xiz} ni topamiz:

$$P_{xiz} = \frac{\mu}{\lambda} \cdot \left[n - \sum_{k=0}^{n-1} (n-k) P_k \right] = \frac{n - \sum_{k=0}^{n-1} (n-k) P_k}{\lambda / \mu} = \tag{1.14}$$

$$= \frac{n - \sum_{k=0}^{n-1} (n-k) P_k}{\alpha}$$

Sistemadagi talabning xizmat qilinmasdan ketish ehtimoli:

$$P_{qaytish} = 1 - P_{xiz} = 1 - \frac{n - \sum_{k=0}^{n-1} (n-k) P_k}{\alpha} = \frac{\alpha - n + \sum_{k=0}^{n-1} (n-k) P_k}{\alpha} \tag{1.15}$$

Agar (1.15) formulaga $m = 0$ va $\beta = 0$ larni qo'ysak, u holda rad qilishli sistema uchun bizga ma'lum bo'lgan Erlang formulasini hosil qilamiz:

$$P_{qaytish} = \frac{\frac{\alpha^n}{n!}}{\sum_{k=0}^{n} \frac{\alpha^k}{k!}}$$

hosil qilingan formulalardan foydalanishni misol bilan tushuntiramiz.

Misol. Elektron hisoblash qurilmasi vaqtning tasodifiy momentilarida 10guruh/min. intensivlik bilan keladigan ma'lumotlar ishlaydi. Ma'lumotlarning hajmi har birining ishlash qiyinchiligi har xilligini hisobga olib, ma'lumotlarning bor guruhini ishlashlik vaqtini $\mu = 10\, guruh/\min$. parametrli ko'rsatkichli qonun bo'yicha taqsimlangan tasodifiy miqdor deb qarash mumkun. Mashina kelayotgan ma'lumotlarni saqlash uchun 5 guruhgacha hajmdagi xotiraga ega. Agar ma'lumotlarning guruhi barcha xotiralar bo'lgani ustiga kelib qolsa, u holda u yo'qotiladi. Kelgan ma'lumotlar vaqt o'tishi bilan o'z ahamiyatini yo'qotadi va u o'z vaqtida ishlanmasa kelgandan keyin o'rtacha 1 min. o'tgach yaroqsiz bo'lib qoladi. Qancha foiz ma'lumot yo'qotilishini toping.

Yechilishi. Shartga ko'ra $n = 2, m = 5, \lambda = 10, \mu = 10, \nu = 1.$ 1) α va β parametrlarni topamiz:

$$\alpha = \frac{\lambda}{\mu} = \frac{10}{10} = 1, \quad \beta = \frac{\lambda}{\mu} = \frac{10}{10} = 1,$$

2) Ma'lumotlar o'z ahamiyatini yo'qotguncha ishlanmaganligi ehtimolini (1.15) formula bo'yicha topamiz:

$$P_{qaytish}=\frac{\alpha-n+\sum_{k=0}^{n-1}(n-k)P_k}{\alpha}=\frac{1-2+\sum_{k=0}^{1}(2-k)P_k}{1}=$$
$$=\frac{1-2+(2P_0+1P_1)}{1}.$$

P_0 va P_1 ehtimollarni (1.11) va (1.13) formulalar bo'yicha topamiz:

$$P_0=\frac{1}{A}=\frac{1}{2,6}\approx 0.38$$

bu yerda

$$A=\sum_{k=0}^{2}\frac{1}{k!}\left(\frac{1}{1+0,1}\right)^k+\frac{1}{2!}\left(\frac{1}{1+0,1}\right)^2\sum_{s=1}^{5}\frac{1^s}{\prod_{i=1}^{s}\left[2+(2+i)0,1\right]}=$$
$$=\sum_{k=0}^{2}\frac{1}{k!}\cdot\frac{1}{(1,1)^k}+\frac{1^2}{2!(1,1)^2}\sum_{s=1}^{5}\frac{1^s}{\prod_{i=1}^{s}\left[2+(2+i)0,1\right]}=$$
$$=1+\frac{1}{1,1}+\frac{1^2}{2!(1,1)^2}+\frac{1^2}{2!(1,1)^2}\left[\frac{1}{2+3\cdot 0,1}+\frac{1^2}{(2+3\cdot 0,1)(2+4\cdot 0,1)}+\right.$$
$$+\frac{1^3}{(2+3\cdot 0,1)(2+4\cdot 0,1)(2+5\cdot 0,1)}+\frac{1^4}{(2+3\cdot 0,1)(2+4\cdot 0,1)(2+5\cdot 0,1)(2+6\cdot 0,1)}+$$
$$\left.+\frac{1^5}{(2+3\cdot 0,1)(2+4\cdot 0,1)(2+5\cdot 0,1)(2+6\cdot 0,1)(2+4\cdot 0,1)}\right]=$$
$$=1+\frac{1}{1,1}+\frac{1}{2,42}+\frac{1}{2,42}\left(\frac{1}{2,3}+\frac{1}{2,3\cdot 2,4}+\frac{1}{2,3\cdot 2,4\cdot 2,5}+\right.$$
$$\left.+\frac{1}{2,3\cdot 2,4\cdot 2,5\cdot 2,6}+\frac{1}{2,3\cdot 2,4\cdot 2,5\cdot 2,6\cdot 2,7}\right)=1+\frac{1}{1,1}+$$
$$+\frac{1}{2,42}+\frac{1}{2,42}\left(\frac{1}{2,3}+\frac{1}{5,52}+\frac{1}{13,8}+\frac{1}{35,88}+\frac{1}{96,876}\right)\approx$$
$$\approx 1+0,91+0,41+0,41(0,43+0,18+0,07+0,03+0,01)=$$
$$=1,91+0,41+0,41\cdot 0,72=2,6$$
$$P_1=\frac{1}{1!}\left(\frac{\alpha}{1+\beta}\right)P_0=\frac{1}{1,1}\cdot 0,38\approx 0,35$$

Ma'lumot yo'qotilish ehtimoli

$$P_{qaytish}=\frac{1-2+0,76+0,35}{1}=0,11$$

ga teng bo'ladi.

Shunday qilib, elektron hisoblash qurilmasi ma'lumotlarni o'z vaqtida ishlayolmasligi natijasida taxminan 11% i yo'qotilar ekan.