O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

MEXANIKA-MATEMATIKA FAKULTETI

EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA YO'NALISHI

KURS ISHI

Mavzu: Navbatda bo'lish vaqti chegaralangan sistema

Gurux: 01

Bajardi: Ostonov M.

Tekshirdi: Husainov Y.

Samarqand - 2013

Navbatda bo'lish vaqti chegaralangan sistema

1. Sistemaning tavsifi

n ta bir xil asbobdan (kanaldan) iborat bo'lgan ommaviy xizmat qilish sistemasini qaraymiz. Sistemaga λ intensivlik bilan Puasson talablar oqimi kelsin. Har bir talabni xizmat qilish vaqti tasodifiy miqdor bo'lib, μ parametrli ko'rsatkichli qonun bo'yicha taqsimlangan. Agar hamma asboblar xizmat qilish bilan band bo'lgan paytda sistemaga yangi talab kelsa, u holda u navbatga turadi va xizmat qilinishini kutadi. Biroq kutishi T_{kut} vaqt bilan chegaralangan. Agar shu vaqt ichida talabga xizmat qilish boshlanmasa, u holda bu talab sistemadan chiqib ketadi. T_{kut}- tasodifiy miqdor bo'lib, ν parametrli ko'rsatkichli qonun bilan taqsimlangan.

Qaralayotgan sistemani biror fizik sistema sifatida qarasak, u quyidagi xolatlarda bo'lishi mumkin:

x_0 – hamma asboblar bo'sh;

x_1 – bitta asbob band, navbat yo'q;

. . . .

x_k – k ta asbob band, navbat yo'q;

. . . .

x_n – hamma asbob band, navbat yo'q;

x_{n+1}– hamma asbob band, navbatda bitta talab bor;

. . . .

x_{n+s}– hamma asbob band, navbatda s ta talab bor;

. . . .

Bizning shartlarimizda navbatda turgan talablar soni s keragicha katta bo'lishi mumkin. Shunday qilib, sistema holatlarning (sanoqli bo'lsada) cheksiz to'plamiga ega.

$P_k(t)$ orqali t momentda sistema x_k $(k = 0,1,2,...)$ holatda bo'lish ehtimolini belgilaymiz.

2. Tenglamalar tuzish

Hozir bizning masalamiz $P_k(t)$ ehtimollar aniqlanadigan tenglamalar tuzishdan iborat. Tenglamalardan biri ravshan, ya'ni har bir t uchun

$$\sum_{k=0}^{\infty} P_k(t) = 1 \tag{5.1}$$

$P_{x_0}(t)$ **ehtimol uchun tenglama.** $t+\Delta t$ momentda sistema x_0 holatda bo'lish ehtimolini topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_0 holatda bo'lgan, Δt vaqt ichida esa yangi talab kelmagan;

t momentda sistema x_1 holatda bo'lgan, Δt vaqt ichida asbob bo'shagan va yangi talab kelmagan;

Qolgan barcha imkoniyatlar, masalan, t momentda sistema x_2 yoki x_3 holatda bo'lgan va Δt vaqt ichida asboblarning bo'shashligi $0(\Delta t)$ ehtimolga teng ekanligicha ishonch hosil qilish qiyin emas.

Ko'rsatilgan hodisalardan birinchisining ehtimoli ehtimollarni ko'paytirish teoremasiga ko'ra

$$P_{x_0}(t)e^{-\lambda\Delta t} = P_{x_0}(t)(1-\lambda\Delta t + 0(\Delta t))$$

ga teng, ikkinchi hodisaning ehtimoli

$$P_{x_1}(t)e^{-\lambda\Delta t}\left(1-e^{-\mu\Delta t}\right) = P_{x_1}(t)\mu\Delta t + 0(\Delta t)$$

ga teng.

Shunday qilib,

$$P_{x_0}(t+\Delta t) = P_{x_0}(t)(1-\lambda\Delta t + 0(\Delta t)) + P_{x_1}(t)\mu\Delta t + 0(\Delta t)$$

yoki

$$P_{x_0}(t+\Delta t) = P_{x_0}(t)(1-\lambda\Delta t) + \mu P_{x_1}(t)\Delta t + 0(\Delta t).$$

Bundan

$$P_{x_0}(t+\Delta t) - P_{x_0}(t) = -\lambda P_{x_0}(t)\Delta t + \mu P_{x_1}(t)\Delta t + 0(\Delta t)$$

ni hosil qilamiz. Bu tenglikning har ikkala tomonini Δt ga bo'lsak, so'ng $\Delta t \to 0$ da limitga o'tsak, unda

$$\lim_{\Delta t \to 0} \frac{P_{x_0}(t+\Delta t) - P_{x_0}(t)}{\Delta t} = -\lambda P_{x_0}(t) + \mu P_{x_1}(t)$$

tenglikka kelamiz.

Funksiya hosilasining ta'rifiga ko'ra

$$\lim_{\Delta t \to 0} \frac{P_{x_0}(t+\Delta t) - P_{x_0}(t)}{\Delta t} = P'_{x_0}(t).$$

U holda quyidagi differensial tenglamaga ega bo'lamiz:

$$P'_{x_0}(t) = -\lambda P_{x_0}(t) + \mu P_{x_1}(t).$$

$P_{x_k}(t)$, $k \geq 1$ **ehtimollar uchun tenglama.** Bunda uchta $1 \leq k < n$, $k = n$ va $k > n$ holni alohida – alohida qarashga to'g'ri keladi.

$1 \leq k < n$ **bo'lsin**. $t + \Delta t$ momentda x_k holatga o'tadigan hollardan faqat muhimlarini sanab chiqamiz. Ular quyidagicha:

t momentda sistema x_k holatda bo'lgan, Δt vaqt ichida esa yangi talab kelmagan va hyech qaysi asbob ishini tugatmagan. Bu hodisaning ehtimoli

$$P_{x_k}(t) e^{-\lambda \Delta t} \left(e^{-\mu \Delta t}\right)^k = P_{x_k}(t)\left(1 - \lambda \Delta t - k\mu \Delta t + 0(\Delta t)\right)$$

ga teng;

t momentda sistema x_{k-1} holatda bo'lgan, Δt vaqt ichida esa bitta yangi talab kelgan, lekin oldingi bo'lgan talablarning hyech biri xizmat qilinib bo'linmagan. Bu hodisaning ehtimoli

$$P_{x_{k-1}}(t)\left(1 - e^{-\lambda \Delta t}\right)\left(e^{-\mu \Delta t}\right)^{k-1} = \lambda P_{x_{k-1}}(t)\Delta t + 0(\Delta t)$$

ga teng;

t momentda sistema x_{k+1} holatda bo'lgan, Δt vaqt ichida esa yangi talab kelmagan va faqat bitta talab xizmat qilingan. Bu hodisaning ehtimoli

$$P_{x_{k+1}}(t) e^{-\lambda \Delta t} C'_{k+1} \left(e^{-\mu \Delta t}\right)^k = (k+1)\mu P_{x_{k+1}}(t)\Delta t + 0(\Delta t)$$

ga teng;

Qolgan barcha mumkin bo'lgan imkoniyatlar $0(\Delta t)$ ehtimolga ega.

Topilgan ehtimollarni birga yig'ib, quyidagi tenglikni hosil qilamiz:

$$P_{X_k}(t+\Delta t) = P_{X_k}(t)(1-\lambda\Delta t - k\mu\Delta t) + \lambda P_{X_{k-1}}(t)\Delta t +$$
$$+(k+1)\mu P_{X_{k+1}}(t)\Delta t + 0(\Delta t)$$

orttirmaga ega bo'lamiz. Endi uni Δt ga bo'lib, $\Delta t \to 0$ da limitga o'tamiz:

$$\lim_{\Delta t\to 0}\frac{P_{x_k}(t+\Delta t) - P_{x_k}(t)}{\Delta t} = -(\lambda + k\mu)P_{x_k}(t) +$$
$$+\lambda P_{x_{k-1}}(t) + (k+1)\mu P_{x_{k+1}}(t)$$

Agar

$$\lim_{\Delta t\to 0}\frac{P_{X_k}(t+\Delta t) - P_{X_k}(t)}{\Delta t} = P'_{X_k}(t)$$

ekanligini e'tiborga olsak, unda yuqoridagi tenglikdan

$$P'_{x_k}(t) = -(\lambda + k\mu)P_{x_k}(t) + \lambda P_{x_{k-1}}(t) + (k+1)\mu P_{x_{k+1}}(t)$$

differensial tenglamani hosil qilamiz.

$k = n$ **bo'lsin**. $t+\Delta t$ momentda sistema x_n holatda bo'lish ehtimolini topamiz. Bu quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_n holatda bo'lgan, Δt vaqt ichida esa yangi talab kelmagan va asboblarning hyech biri bo'shamagan. Bu hodisaning ehtimoli

$$P_{x_n}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^n = P_{x_n}(t)(1-\lambda\Delta t - n\mu\Delta t + 0(\Delta t))$$

ga teng;

t momentda sistema x_{n-1} holatda bo'lgan, Δt vaqt ichida esa bitta yangi talab kelgan va asboblardan hyech biri bo'shamagan. Bu hodisaning ehtimoli

$$P_{x_{n-1}}(t)\cdot\left(1-e^{-\lambda\Delta t}\right)\cdot\left(e^{-\mu\Delta t}\right)^{n-1} = \lambda P_{x_{n-1}}(t)\Delta t + 0(\Delta t)$$

ga teng;

t momentda sistema x_{n+1} holatda bo'lgan, Δt vaqt ichida esa yangi talab kelmagan va yo asboblardan biri (qaysi biri bo'lishining ahamiyati yo'q) bo'shagan yoki bo'lmasa navbatdagi talabning kutish vaqti tugagan. Bu hodisaning ehtimoli

$$P_{x_{n+1}}(t)e^{-\lambda\Delta t}\left(C_n'\left(1-e^{-\mu\Delta t}\right)+\left(1-e^{-\nu\Delta t}\right)\right)=P_{x_{n+1}}(t)(n\mu+\nu)\Delta t+0(\Delta t)$$

ga teng.

Qolgan barcha mumkin bo'lgan imkoniyatlarning ehtimollari $0(\Delta t)$ ga teng.

Demak,

$$\begin{aligned}P_{x_n}(t+\Delta t)&=P_{x_n}(t)(1-\lambda\Delta t-n\mu\Delta t)+P_{x_{n-1}}(t)\lambda\Delta t+\\&+P_{x_{n+1}}(t)(n\mu+\nu)\Delta t+0(\Delta t).\end{aligned}$$

Bundan

$$\begin{aligned}P_{x_n}(t+\Delta t)-P_{x_n}(t)&=-(\lambda+n\mu)P_{x_n}(t)\Delta t+\lambda P_{x_{n-1}}(t)\Delta t+\\&+(n\mu+\nu)P_{x_{n+1}}(t)\Delta t+0(\Delta t)\end{aligned}$$

ni hosil qilamiz. Endi bu tenglikning har ikkala tomonini avval Δt ga bo'lib, so'ng $\Delta t\to 0$ da limitiga o'tsak, unda quyidagi differensial tenglamaga ega bo'lamiz:

$$P_{x_n}'(t)=-(\lambda+n\mu)P_{x_n}(t)+\lambda P_{x_{n-1}}(t)+(n\mu+\nu)P_{x_{n+1}}(t).$$

$k=n+s$, $s\geq 1$ **bo'lsin**. $t+\Delta t$ momentda sistema x_{n+s} holatda bo'lish ehtimolini topamiz. Bu hodisa quyidagi usullar bilan ro'y berishi mumkin:

t momentda sistema x_{n+s} holatda bo'lgan, Δt vaqt ichida esa talab kelmagan, asboblar bo'shamagan va navbatdagi talablarning kutish vaqtlari tugamagan. Bu hodisaning ehtimoli

$$P_{x_{n+s}}(t)e^{-\lambda\Delta t}\left(e^{-\mu\Delta t}\right)^n\left(e^{-\nu\Delta t}\right)^s=P_{x_{n+s}}(t)(1-(\lambda+n\mu+s\nu)\Delta t+0(\Delta t))$$

ga teng.

t vaqt momentda sistema x_{n+s-1} holatda bo'lgan, Δt vaqt ichida esa bitta talab kelgan, asboblar bo'shamagan va navbatdagi talablarning kutish vaqtlari tugamagan. Bu hodisaning ehtimoli

$$P_{x_{n+s-1}}(t)\left(1-e^{-\lambda\Delta t}\right)\left(e^{-\mu\Delta t}\right)^n\left(e^{-\nu\Delta t}\right)^{s-1}=\lambda P_{x_{n+s-1}}(t)\Delta t+0(\Delta t)$$

ga teng.

t momentda sistema x_{n+s+1} holatda bo'lgan, Δt vaqt ichida esa talab kelmagan va yo asboblardan biri (qaysi biri bo'lishi ahamiyati yo'q) bo'shagan yoki bo'lmasa navbatdagi talablardan birining (qaysi birining bo'lishi ahamiyati yo'q) kutish vaqti tugagan. Bu hodisaning ehtimoli

$$P_{x_{n+s+1}}(t)e^{-\lambda\Delta t}\left[C^1{}_n\left(1-e^{-\mu\Delta t}\right)+C^1{}_{s+1}\left(1-e^{-\nu\Delta t}\right)\right]=P_{x_{n+s+1}}(t)(n\mu+(s+1)\nu)\Delta t+0(\Delta t)$$

ga teng.

Qolgan barcha mumkin bo'lgan imkoniyatlar $0(\Delta t)$ ehtimolga ega bo'ladi.

Demak,

$$\begin{aligned}P_{x_{n+s}}(t+\Delta t)&=P_{x_{n+s}}(t)(1-(\lambda+n\mu+s\nu)\Delta t)+\\&+\lambda P_{x_{n+s-1}}(t)\Delta t+(n\mu+(s+1)\nu)P_{x_{n+s+1}}(t)\Delta t+0(\Delta t)\end{aligned}$$

Bundan

$$\begin{aligned}P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)&=-(\lambda+n\mu+s\nu)P_{x_{n+s}}(t)\Delta t+\\&+\lambda P_{x_{n+s-1}}(t)\Delta t+(n\mu+(s+1)\nu)P_{x_{n+s+1}}(t)\Delta t+0(\Delta t)\end{aligned}$$

ni hosil qilamiz. Bu tenglikning har ikkala tomonini Δt ga bo'lamiz:

$$\begin{aligned}\frac{P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)}{\Delta t}&=-(\lambda+n\mu+s\nu)P_{x_{n+s}}(t)+\\&+\lambda P_{x_{n+s-1}}(t)+(n\mu+(s+1)\nu)P_{x_{n+s+1}}(t)+\frac{0(\Delta t)}{\Delta t}\end{aligned}$$

Endi $\Delta t\to 0$ da limitga o'tamiz:

$$\begin{aligned}\lim_{\Delta t\to 0}\frac{P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)}{\Delta t}&=-(\lambda+n\mu+s\nu)P_{x_{n+s}}(t)+\\&+\lambda P_{x_{n+s-1}}(t)+(n\mu+(s+1)\nu)P_{x_{n+s+1}}(t)\end{aligned}$$

Agar

$$\lim_{\Delta t\to 0}\frac{P_{x_{n+s}}(t+\Delta t)-P_{x_{n+s}}(t)}{\Delta t}=P'_{x_{n+1}}(s)$$

va $\lim\limits_{\Delta t \to 0} \dfrac{0(\Delta t)}{\Delta t} = 0$ (chunki, $0(\Delta t) - \Delta t$ ga nisbatan yuqori tartibli cheksiz kichik miqdor) ekanini e'tiborga olsak, u holda yuqoridagi tenglikdan

$$P'_{x_{n+s}}(t) = -(\lambda + n\mu + s\nu)P_{x_{n+s}}(t) + \lambda P_{x_{n+s-1}}(t) + (n\mu + (s+1)\nu)P_{x_{n+s+1}}(t)$$

differensial tenglamaga ega bo'lamiz.

Shunday qilib, holatlar ehtimollari uchun cheksiz sondagi differensial tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} P'_{x_0}(t) = -\lambda P_{x_0}(t) + \mu P_{x_1}(t); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_k}(t) = -(\lambda + k\mu)P_{x_k}(t) + \lambda P_{x_{k-1}}(t) + (k+1)\mu P_{x_{k+1}}(t),\ 1 \le k \le n-1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_n}(t) = -(\lambda + n\mu)P_{x_n}(t) + \lambda P_{x_{n-1}}(t) + (n\mu + \nu)P_{x_{n+1}}(t) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ P'_{x_{n+s}}(t) = -(\lambda + n\mu + s\nu)P_{x_{n+s}}(t) + \lambda P_{x_{n+s-1}}(t) + (n\mu + (s+1)\nu)P_{x_{n+s+1}}(t),\ s \ge 1 \end{cases} \tag{5.2}$$

3. Stasionar yechimlar

Oldingi banddagi holatlar ehtimollari uchun differensial tenglamalar sistemasini

$$P_{x_0}(0) = 1,\ P_{x_1}(0) = P_{x_2}(0) = \dots = P_{x_k}(0) = \dots = 0$$

boshlang'ich shartlarda integrallab, $P_{x_k}(t)$ ehtimollarning ifodalarini t ning funksiyasi sifatida topish mumkin. Biroq uni amalda bajarish juda katta hajmdagi hisoblashlarni talab qiladi. Shu sababli ommaviy xizmat qilish nazariyasida stasionar yechimlar deb ataluvchi

$$P_{x_k} = \lim_{t \to \infty} P_{x_k}(t)$$

limit ehtimollar o'rganiladi. Bunday yechimlarning mavjudligi ergodik teoremalar yordamida ko'rsatiladi. Bizning masalamizda bu stasionar yechimlar (ehtimollar) mavjud bo'lar ekan [4]. Ana shu stasionar ehtimollarni topamiz. Buning uchun (5.2) tenglamalar sistemasida $t \to \infty$ da limitga o'tamiz. Bunda

$$\lim_{t\to\infty} P'_{x_k}(t) = 0$$

ekanligini tasavvur qilish qiyin emas. Aks holda t ning o'sishi bilan $P_{x_k}(t)$ ham cheksiz ortadi. Bu esa mumkin emas, chunki $P_{x_k}(t) \le 1$. Shuning uchun (5.2) differensial tenglamalar sistemasida $t \to \infty$ da limitga o'tib, quyidagi algebraik tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} -\lambda P_{x_0} + \mu P_{x_1} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ -(\lambda + k\mu)P_{x_k} + \lambda P_{x_{k-1}} + (k+1)\mu P_{x_k} = 0, \quad 1 \le k \le n-1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ -(\lambda + n\mu)P_{x_n} + \lambda P_{x_{n-1}} + (n\mu + \nu)P_{x_{n+1}} = 0; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ -(\lambda + n\mu + s\nu)P_{x_{n+s}} + \lambda P_{x_{n+s-1}} + [n\mu + (s+1)\nu]P_{x_{n+s+1}} = 0, \quad s \ge 1. \end{cases} \tag{5.3}$$

(5.1) tenglama esa

$$\sum_{k=0}^{\infty} P_{x_k} = 1 \tag{5.4}$$

ko'rinishda yoziladi.

Endi (5.3) tenglamalar sistemasini noma'lum $P_{x_1}, P_{x_2}, \dots, P_{x_k}, \dots$ larga nisbatan yechamiz.

Birinchi tenglamadan

$$P_{x_1} = \frac{\lambda}{\mu} P_{x_0} \tag{5.5}$$

ga ega bo'lamiz.

Ikkinchi tenglamadan

$$P_{x_2} = \frac{1}{2\mu}\left[(\lambda + \mu)P_{x_1} - \lambda P_{x_0}\right]$$

ni topamiz. Bunga P_{x_1} ning (5.5) dagi ifodasini qo'yib,

$$P_{x_2}=\frac{1}{2\mu}\left[(\lambda+\mu)\frac{\lambda}{\mu}P_{x_0}-\lambda P_{x_0}\right]=\frac{1}{2\mu}\cdot\frac{\lambda^2}{\mu}P_{x_0}=\frac{\left(\frac{\lambda}{\mu}\right)^2}{2!}p_{x_0} \qquad (5.6)$$

ga ega bo'lamiz.

Uchinchi tenglamadan

$$P_{x_3}=\frac{1}{3\mu}\left[(\lambda+2\mu)P_{x_2}-\lambda P_{x_1}\right]$$

ni topamiz. Bunga P_{x_1} va P_{x_2} larning (5.5) va (5.6) lardagi ifodalarini qo'yib,

$$P_{x_3}=\frac{1}{3\mu}\left[(\lambda+2\mu)\frac{\left(\frac{\lambda}{\mu}\right)^2}{2!}P_{x_0}-\lambda\cdot\frac{\lambda}{\mu}P_{x_0}\right]=\frac{\left(\frac{\lambda}{\mu}\right)^3}{6}P_{x_0}=\frac{\left(\frac{\lambda}{\mu}\right)^3}{3!}p_{x_0}$$

ga ega bo'lamiz.

Xuddi shunga o'xshash

$$P_{x_4}=\frac{\left(\frac{\lambda}{\mu}\right)^4}{4!}P_{x_0}$$

ga ega bo'lamiz.

Demak, har qanday $1\le k\le n$ uchun

$$P_{x_k}=\frac{\left(\frac{\lambda}{\mu}\right)^k}{k!}P_{x_0} \qquad (5.7)$$

bo'ladi.

Endi keyingi tenglamalarga o'tamiz. Ushbu

$$-(\lambda+n\mu)P_{x_n}+\lambda P_{x_{n-1}}+(n\mu+\nu)P_{x_{n+1}}=0$$

tenglamani $P_{x_{n+1}}$ ga nisbatan yechamiz:

$$P_{x_{n+1}}=\frac{1}{n\mu+\nu}\left[(\lambda+n\mu)P_{x_n}-\lambda P_{x_{n-1}}\right].$$

Bunga $P_{x_{n-1}}$ va P_{x_n} larning (5.7) dagi ifodalarini qo'yib,

$$P_{x_{n+1}}=\frac{1}{n\mu+\nu}\left[(\lambda+n\mu)\frac{\left(\frac{\lambda}{\mu}\right)^{n}}{n!}P_{x_0}-\lambda\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n-1}}{(n-1)!}P_{x_0}\right]=$$
$$=\frac{1}{n\mu+\nu}\cdot\lambda\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n}}{n!}P_{x_0}=\frac{1}{n+\frac{\nu}{\mu}}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n-1}}{n!}P_{x_0} \tag{5.8}$$

ni hosil qilamiz.

$$-(\lambda+n\mu+\nu)P_{x_{n+1}}+\lambda P_{x_n}+(n\mu+2\nu)P_{x_{n+2}}=0$$

tenglamani $P_{x_{n+2}}$ ga nisbatan yechamiz:

$$P_{x_{n+2}}=\frac{1}{n\mu+2\nu}\left[(\lambda+n\mu+\nu)P_{x_{n+1}}-\lambda P_{x_n}\right].$$

Bunga P_{x_n} va $P_{x_{n+1}}$ larning (5.7) va (5.8) lardagi ifodalarini qo'yib,

$$P_{x_{n+2}}=\frac{1}{n\mu+2\nu}\left[(\lambda+n\mu+\nu)\frac{1}{n+\frac{\nu}{\mu}}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+1}}{n!}P_{x_0}-\lambda\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n}}{n!}P_{x_0}\right]=$$
$$=\frac{1}{n\mu+2\nu}\cdot\lambda\cdot\frac{1}{n+\frac{\nu}{\mu}}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+1}}{n!}P_{x_0}=\frac{1}{\left(n+\frac{\nu}{\mu}\right)\left(n+2\frac{\nu}{\mu}\right)}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+2}}{n!}P_{x_0} \tag{5.9}$$

ni hosil qilamiz.

$$-(\lambda+n\mu+2\nu)P_{x_{n+2}}+\lambda P_{x_{n+1}}+(n\mu+3\nu)P_{x_{n+3}}=0$$

tenglamadan $P_{x_{n+3}}$ ni topamiz:

$$P_{x_{n+3}}=\frac{1}{n\mu+3\nu}\left[(\lambda+n\mu+2\nu)P_{x_{n+2}}-\lambda P_{x_{n+1}}\right].$$

Bunga $P_{x_{n+1}}$ va $P_{x_{n+2}}$ larning (5.8) va (5.9) lardagi ifodalarini qo'yib, quyidagini hosil qilamiz:

$$P_{x_{n+3}}=\frac{1}{n\mu+3\nu}\left[(\lambda+n\mu+2\nu)\cdot\frac{1}{\left(n+\frac{\nu}{\mu}\right)\left(n+2\frac{\nu}{\mu}\right)}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+2}}{n!}P_{x_0}-\lambda\frac{1}{n+\frac{\nu}{\mu}}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+1}}{n!}P_{x_0}\right]=$$

$$=\frac{1}{n\mu+3\nu}\cdot\frac{1}{\left(n+\frac{\nu}{\mu}\right)\left(n+2\frac{\nu}{\mu}\right)}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+2}}{n!}P_{x_0}=$$

$$=\frac{1}{\left(n+\frac{\nu}{\mu}\right)\left(n+2\frac{\nu}{\mu}\right)\left(n+3\frac{\nu}{\mu}\right)}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+3}}{n!}P_{x_0}$$

Demak, har qanday $s\geq 1$ uchun

$$P_{x_{n+s}}=\frac{1}{\left(n+\frac{\nu}{\mu}\right)\left(n+2\frac{\nu}{\mu}\right)\ldots\left(n+s\frac{\nu}{\mu}\right)}\cdot\frac{\left(\frac{\lambda}{\mu}\right)^{n+s}}{n!}P_{x_0}=\frac{\frac{\left(\frac{\lambda}{\mu}\right)^{n+s}}{n!}}{\prod_{i=1}^{s}\left(n+i\frac{\nu}{\mu}\right)}P_{x_0}\qquad(5.10)$$

bo'ladi.

(5.7) va (5.10) formulalarning har ikkalasida ham P_{x_0} ehtimol ko'paytuvchi sifatida qolayapti. Uni (5.4) tenglamadan topamiz. Unga (5.7) va (5.10) ifodalarni qo'yib quyidagini hosil qilamiz:

$$P_{x_0}\left[\sum_{k=0}^{n}\frac{\left(\frac{\lambda}{\mu}\right)^{k}}{k!}+\sum_{s=1}^{\infty}\frac{\frac{\left(\frac{\lambda}{\mu}\right)^{n+s}}{n!}}{\prod_{i=1}^{s}\left(n+i\frac{\lambda}{\mu}\right)}\right]=1,$$

bundan

$$P_{x_0}^{-1}=\sum_{k=0}^{n}\frac{\left(\frac{\lambda}{\mu}\right)^{k}}{k!}+\sum_{s=1}^{\infty}\frac{\frac{\left(\frac{\lambda}{\mu}\right)^{n+s}}{n!}}{\prod_{i=1}^{s}\left(n+i\frac{\lambda}{\mu}\right)}\qquad(5.11)$$

Ushbu belgilashlarni kiritamiz:

$$\frac{\lambda}{\mu}=\lambda MX_{xiz}=\alpha\,,$$

$$\frac{\nu}{\mu}=\nu MX_{xiz}=\beta\,,$$

bu yerda MX_{xiz} - talabga o'rtacha xizmat ko'rsatish vaqti. α va β parametrlar mos ravishda bitta talabga o'rtacha xizmat ko'rsatiladigan vaqtda keladigan talablarning o'rtacha sonini va navbatda turib ketadigan talablarning o'rtacha sonini ifodalaydi.

Bu belgilashlarda (5.7), (5.10) va (5.11) formulalar quyidagi ko'rinishni oladi:

$$P_{x_k} = \frac{\alpha^k}{k!} P_{x_0}, \quad (1 \le k \le n), \tag{5.12}$$

$$P_{x_{n+s}} = \frac{\frac{\alpha^{n+s}}{n!}}{\prod_{i=1}^{s}(n+i\beta)} P_{x_0}, \quad (s \ge 1), \tag{5.13}$$

$$P_{x_0} = \frac{1}{\sum_{k=0}^{n}\frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}} \tag{5.14}$$

(5.14) ni (5.12) va (5.13) larga qo'yib, holatlar ehtimollari uchun uzil - kesil quydagilarni hosil qilamiz:

$$P_{x_k} = \frac{\frac{\alpha^k}{k!}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}, \quad (1 \le k \le n) \tag{5.15}$$

$$P_{x_{n+s}} = \frac{\frac{\frac{\alpha^{n+s}}{n!}}{\prod_{i=1}^{s}(n+i\beta)}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}, \quad (s \ge 1). \tag{5.16}$$

Sistemaning barcha mumkin bo'lgan holatlari ehtimollarini bilsak, bizni qiziqtiradigan boshqa xarakteristikalarini ososngina aniqlashimiz mumkin, xususan, talabning xizmat qilinmasdan sistemadan ketishi ehtimolini. Uni quyidagi mulohazalardan aniqlaymiz: talabning xizmat qilinmasdan sistemadan ketish ehtimoli P_q bir birlik vaqt ichida navbatdan ketib qoladigan talablarning o'rtacha sonini bir birlik vaqt ichida sistemaga keladigan talablarning o'rtacha soniga bo'linmasi kabi bo'ladi. Bir birlik vaqt ichida navbatdan ketadigan talablarning o'rtacha sonini topamiz. Buning uchun avval navbatda turgan talablar sonining matematik kutilishini topamiz:

$$m_s = \sum_{s=1}^{\infty} s P_{x_{n+s}} = \frac{\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{s\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!}+\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}} \qquad (5.17)$$

P_q ni topish uchun m_s ni $\frac{\nu}{\mu}$ koeffisiyenta ko'paytirish kerak. Unda

$$P_q = \frac{\nu}{\mu}\cdot\frac{\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{s\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!}+\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}} =$$

$$= \frac{\frac{\nu}{\mu}}{\frac{\lambda}{\mu}}\cdot\frac{\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{s\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!}+\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}} =$$

$$= \frac{\beta}{\alpha}\cdot\frac{\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{s\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}{\sum_{k=0}^{n}\frac{\alpha^k}{k!}+\frac{\alpha^n}{n!}\sum_{s=1}^{\infty}\frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)}}$$

(5.18)

ni hosil qilamiz.

Sistemaning ishlash qobiliyati (xizmat qilish imkoniyati) sistemaga kelgan talab xizmat qilinishi ehtimoli bilan xarakterlanadi:

$$P_{xiz} = 1 - P_q \,.$$

O’sha λ va μ larda kutishli sistemaning ishlash qobiliyati qaytishli sistemaning ishlash qobiliyatiga qaraganda har doim yuqori bo’lishi ravshan.

(5.15), (5.16) va (5.18) formulalarda cheksiz yig’indilar qatnashganligi sababli, ulardan bevosita foydalanishi birmuncha qiyinchiliklar tug’diradi. Biroq bu yig’indilarning hadlari tez kamayib boradi. *)

---

*)_yig’indilarning r - hadidan boshlab barcha hadlarini tashlab yuborishidan sodir bo’ladigan xatoliklarini qo’pol baholash uchun

$$\sum_{s=r}^{\infty} \frac{\alpha^s}{\prod_{i=1}^{s}(n+i\beta)} < \frac{\left(\frac{\alpha}{\beta}\right)^r}{r!} e^{\frac{\alpha}{\beta}} \;,$$

$$\sum_{s=r}^{\infty} \frac{s\alpha^s}{\prod_{i=1}^{s}(n+i\beta)} < \frac{\left(\frac{\alpha}{\beta}\right)^r}{(r-1)!} e^{\frac{\alpha}{\beta}}$$

formulalardan foydalanish mumkin.

Endi $\beta \to \infty$ va $\beta \to 0$ da (5.15) va (5.16) formulalar nimaga aylanishini ko’raylik. Ravshanki, $\beta \to \infty$ kutishli sistema qaytishli sistemaga aylanadi (talab navbatdan darhol ketadi). Haqiqatan ham, $\beta \to \infty$ da (5.16) formula nolni beradi, (5.15) formula esa qaytishli sistema uchun Erlang formulasini beradi:

$$P_{x_k} = \frac{\frac{\alpha^k}{k!}}{\sum_{k=0}^{n} \frac{\alpha^k}{k!}} \;, \qquad (1 \le k \le n).$$

$\beta \to 0$ dagi holni qaraymiz. Bu holda kutishli sistema sof kutishli sistemaga aylanadi. Bunday sistemada talablar sisitemadan umuman ketmaydi, shuning uchun $P_q = 0$, chunki sistemaga kelgan har bir talab ertami, kechmi xizmat qilinishini kutadi. Lekin sof kutilish sistemasida $t \to \infty$da limitik stasionar rejim (tartib) har doim ham mavjud bo'lavermaydi. Bunday rejim faqat $\alpha < n$ bo'lgandagina mavjudligini ko'rsatish mumkin. Agarda $\alpha \geq n$ bo'lsa, u holda navbatda turadigan talablar vaqt o'tishi bilan cheksiz o'sib boradi.

Faraz qilaylik, $\alpha < n$ bo'lsin. Sof kutishli sistema uchun P_{x_k} $(0 \leq k \leq n)$ ehtimollarni topamiz. Buning uchun (5.14), (5.15) va (5.16) formulalarda $\beta \to 0$ deb quyidagini hosil qilamiz:

$$P_{x_0} = \frac{1}{\sum\limits_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^n}{n!} \sum\limits_{s=1}^{\infty} \frac{\alpha^s}{n^s}},$$

yoki progressiyani yig'sak (bu faqat $\alpha < n$ bo'lgandagina mumkin)

$$P_{x_0} = \frac{1}{\sum\limits_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n!(n-\alpha)}}.$$

Bundan, hamda (5.12) va (5.13) formulalardan foydalanib, quyidagilarni topamiz:

$$P_{x_k} = \frac{\frac{\alpha^k}{k!}}{\sum\limits_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n!(n-\alpha)}}, \quad (0 \leq k \leq n) \qquad (5.19)$$

$$P_{x_{n+s}} = \frac{\frac{\alpha^{n+s}}{n! n^s}}{\sum\limits_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n!(n-\alpha)}}, \quad (s \geq 1). \qquad (5.20)$$

(5.17) formuladan $\beta \to 0$ da navbatda turgan talablarning o'rtacha sonini topamiz:

$$m_s = \frac{\frac{\alpha^{n+1}}{n \cdot n! \left(1 - \frac{\alpha}{n}\right)^2}}{\sum\limits_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n!(n-\alpha)}}. \qquad (5.21)$$

Tushuntirish. Ushbu

$$\sum_{s=1}^{\infty} s\frac{\alpha^s}{n^s} = \sum_{s=1}^{\infty} s\left(\frac{\alpha}{n}\right)^s = \frac{\alpha}{n}\sum_{s=1}^{\infty} s\left(\frac{\alpha}{n}\right)^{s-1} =$$

$$= \frac{\alpha}{n}\left(1 + 2\frac{\alpha}{n} + 3\left(\frac{\alpha}{n}\right)^2 + \ldots + s\left(\frac{\alpha}{n}\right)^{s-1} + \ldots\right) = \frac{\alpha}{n}\cdot\frac{1}{\left(1-\frac{\alpha}{n}\right)^2}$$

ekanligini ko'rsatamiz. $\left(0 < \frac{\alpha}{n} < 1\right)$ bo'lgani uchun

$$S = 1 + \frac{\alpha}{n} + \left(\frac{\alpha}{n}\right)^2 + \left(\frac{\alpha}{n}\right)^3 + \ldots + \left(\frac{\alpha}{n}\right)^s + \ldots = \frac{1}{1-\frac{\alpha}{n}}$$

darajali qatorni ($\frac{\alpha}{n}$ ga nisbatan) hadma-had differensiallash mumkin va qator hadlarining hosilalari yig'indisi qator yig'indisining hosilasiga teng, ya'ni

$$S' = 1 + 2\frac{\alpha}{n} + 3\left(\frac{\alpha}{n}\right)^2 + \ldots + s\left(\frac{\alpha}{n}\right)^{s-1} + \ldots = \left(\frac{1}{1-\frac{\alpha}{n}}\right)' = \frac{1}{\left(1-\frac{\alpha}{n}\right)^2}\ .$$

***<u>M i s o l</u>*.** Uchta xizmat qiluvchi asbobdan iborat kutish vaqti chegaralanmagan sistemaga $\alpha = 4$ (talab soatiga) Puasson talablar oqimi kelsin. Bitta talabga ko'rsatiladigan o'rtacha xizmat vaqti $MX_{xiz.} = 30\min$.

Qarorlangan xizmat qilish tartib (stasionar rejim) mavjud bo'larmikan; agar u mavjud bo'lsa, u holda $P_{x_0}, P_{x_1}, P_{x_2}$ va P_{x_3} ehtimollarni hamda o'rtacha navbat uzunligi m_s ni toping.

***<u>Ye ch i sh</u>*.** Shartga ko'ra $n = 3$, $\mu = \frac{1}{MX_{xiz}} = \frac{1}{30\min} = 2\ soat$, $\alpha = \frac{\lambda}{\mu} = 2$.

$\alpha < n$ bo'lganidan stasionar rejim mavjud.

(5.19) formula bo'yicha

$$P_{x_0} = \frac{1}{\sum_{k=0}^{3}\frac{2^k}{k!} + \frac{2^{3+1}}{3!(3-2)}} = \frac{1}{9} \approx 0{,}111;$$

$$P_{x_1}=\frac{\frac{2}{1!}}{\sum_{k=0}^{3}\frac{2^k}{k!}+\frac{2^{3+1}}{3!(3-2)}}=\frac{2}{9}\approx 0{,}222;$$

$$P_{x_2}=\frac{\frac{2^2}{2!}}{\sum_{k=0}^{3}\frac{2^k}{k!}+\frac{2^{3+1}}{3!(3-2)}}=\frac{2}{9}\approx 0{,}222;$$

$$P_{x_3}=\frac{\frac{2^3}{3!}}{\sum_{k=0}^{3}\frac{2^k}{k!}+\frac{2^{3+1}}{3!(3-2)}}=\frac{8}{54}\approx 0{,}148;$$

larni topamiz.

(5.21) formulaga asosan o’rtacha navbat uzunligi

$$m_s=\frac{\frac{2^{3+1}}{3\cdot 3!\left(1-\frac{2}{4}\right)^2}}{\sum_{k=0}^{3}\frac{2^k}{k!}+\frac{2^{3+1}}{3!(3-2)}}=\frac{64}{81}\approx 0{,}79 \text{ (talab)}$$

.

Foydalanilgan adabiyotlar

1. Ventsel Ye.S. Teoriya veroyatnostey. Fiz-matiz 1962 g.
2. Gnedenko B.V., Kovalenko I.N. Vvedeniye v teoriyu massovogo obslujivaniya. M. Nauka, 1966 g.
3. Kovalenko I.N. O sisteme massovogo obslujivaniya ogranicheniyem na vremya ojidaniya. Kiyev, 1961 g.
4. Rozenberg V.Ya., Proxrov A.I. Chto takoye teorii massovogo obslujivaniya. Izdatelstvo Sovetskoye radio. 1965 g.
5. Novikov O.A., Petuxov S.I. Prikladn`ye vopros` teorii massovogo obslujivaniya. Izdatelstvo Sovetskoye radio. 1969 g.
6. Soati T.L. Element` teorii massovogo obslujivaniya i yeye prilojeniya. Izdatelstvo Sovetskoye radio. 1965 g.
7. Xinchin A.Ya. Rabot` po matematicheskoy teorii massovogo obslujivaniya. Fiz-matiz 1963 g.
8. Nurbekova N.G'. Aralash tipdagi xizmat ko'rsatish sistemasi. Magistrantlarning VIII ilmiy anjumani materiallari, 2009 y., 128-129 bet.