O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA`LIM VAZIRLIGI

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«Ommaviy xizmat ko'rsatish sistemasidagi ba'zi limit teoremalarining yaqinlashish tezligini baholash »

(Bakalavr akademik daraja olish uchun bajarilgan bitiruv malakaviy ishi**)**

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Masalaning qo'yilishi.

Bir kanalli $M|G|1|\infty$ modelli ommaviy xizmat ko'rsatish sistemasi qaraladi. Bu sistemaning ba'zi xarakteristkalari (sistemaning bandlik davrida va sistemadagi talablarning kutish vaqti) o'rganiladi. $M|G|1|\infty$ sistemaning tavsifi quyida keltiriladi. Bu sistemada qaralayotgan uchta limit teoramalar ya'ni sistemaning bandlik davri va statsionar kutish vaqti (sistemadagi talablarga xizmat ko'rsatish tartibi «FIFO» va «RS» bo'lgan hollar, bu hollar quyida keltirilgan) uchun kritik zagruzkada isbot qilingan limit teoremalar o'rganiladi. Bu limit teoremalarning yaqinlashish tezliklari baholanadi.

Mavzuning dolzarbligi.

Keyingi yillarda $M|G|1|\infty$ modelli ommaviy xizmat ko'rsatish sistemasining limit teoremalari va bu limit teoremalarning yaqinlashish tezligini baholash bo'limi rivojlanmoqda.

Ommaviy xizmat ko'rsatish sistemalarining xarakteristkalarini amalga tadbiq qilishda ularning ba'zi shartlar asosida limit teoremalari va bu limit teoramalar yaqinlashish tezligining baholari ahamiyatlidir. Shu maqsadda ushbu malakaviy bitiruv ishida $M|G|1|\infty$ ommaviy xizmat ko'rsatish sistemasining ba'zi xarakteristkalari uchun olingan limit teoramalarning yaqinlashish tezligini baholash mavzusining dolzarbligidan dalolatdir.

Ishning maqsadi.

Bir kanalli ommaviy xizmat ko'rsatish sistemasidagi sistemaning bandlik davri va talablarning statsionar kutish vaqti xarakteristkalari uchun olingan limit teoremalarning yaqinlashish tezligini kritik zagruzkada baholash bitiruv ishining maqsadidir.

Ilmiy tadqiqot usuli.

Bitiruv ishida keltirilgan limit teoremalrning yaqinlashish tezligini baholashda sistemaning talablarga xizmat ko'rsatish vaqtining uzunligi uchun

Laplas-Stelts almashtirishining asimptotok yoyilmasidan foydalaniladi. Bundan tashqari yaqinlashish tezligini baholashda Berri-Eseen tengsizligidan foydalanilgan.

Ishning tuzilishi.

Bir kanalli $M|G|1|\infty$ modelli ommaviy xizmat ko'rsatish sistemasi quyidagicha ifodalaniladi:

Sistemaga a parametrli Puasson taqsimotli bog'liqsiz talablar kelib tushadi. Talablarga xizmat ko'rsatish vaqtining uzunligi ixtiyoriy $B(x)$, $B(+0)=0$ taqsimot funksiyaga ega.

Limit teoremalar $B(x)$ taqsimot funksiyaning birinchi ikkita β_1 va β_2 momentlarning chekliligi shartida ya'ni $S\to 0$, $(\operatorname{Re} S\geq 0)$ da

$$\beta(x)=\int_0^\infty e^{-sx}dB(x)=1-\beta_1\cdot s+\frac{1}{2}\beta_2\cdot s^2(1+0_s(1)) \qquad (1)$$

yoyilma o'rinli bo'lgan sharti ostida isbot qilingan. Shu bilna birga bu limit teoremaning yaqinlashish tezligini baholashda limit teoremalarga qo'yilganshartlar qanoatlantiriladi.

Mazkur birtiruv malakaviy ishda ba'zi qo'shimcha shartlar bajarilganda limit teoremalarning yaqinlashish tezligi baholanadi.Yuqorida keltirilgan (1) shart o'rniga quyidagi kuchliroq shart bajarilishi faraz qilinadi: $S\to 0$, $(\operatorname{Re} S\geq 0)$ da

$$\beta(s)=1-\beta_1\cdot s+\frac{1}{2}\beta_2\cdot s^2-\varepsilon s^{\delta}(1+0_s(1)) \qquad (2)$$

yoyilma o'rinli bo'lsin, bu yerda $2<\delta\leq 3$, ε -musbat o'zgarmas son.

Biz ko'rib o'tayotgan $M|G|1|\infty$ ommaviy xizmat ko'rsatish sistemasida quyidagi xarakteristkalar asosiy rol o'ynaydi.

$\pi-$, $M|G|1|\infty$ sistemaning bandlik davri, $\pi(u)$, $(u\geq 0)$ -u zagruzkali badlik davri, ω- talablarning sistemada statsionar kutish vaqti, $\rho=1-a\cdot\beta$- sistemadagi talablarning zagruzkasi, ya'ni bir birlik vaqt ichida sistemaga kelib tushgan talablarga o'rtacha xizmat ko'rsatish vaqti.

Statsionar kutish vaqti sistemada qabul qilingan xizmat ko'rsatish tartibiga bog'liq bo'ladi:

"FIFO "- birinchi kelgan talablarga birinchi xizmat ko'rsatiladi.

"RS" – xizmat ko'rsatish uchun talablar tasodifiy tanlanadi.

Mazkur bitiruv malakavi ishida ba'zi limit teoremalar o'rganiladi va bu limit teoremalarning yaqinlashish tezligi kritik zagruzkada baholanadi.

Bitiruv malakaviy ishi kirish qismi, uchta paragraf, xulosa va foydalanilgan adabiyotlardan iborat.

Kirish qismida quyida keltirilgan natijalar uchun qo'yilgan shartlar, $M|G|1|\infty$ sistemaning tavsifi va bitiruv ishida qo'llaniladigan ba'zi belgilashlar keltirilgan.

Birinchi paragrafda $M|G|1|\infty$ sistemaning u- zagruzkali bandlik davri uchun limit teorema keltirilgan va shu limit teoremaning yaqinlashish tezligini kritik zagruzkadagi bahosi olingan.

Ikkinchi va uchunchi paragraflarda mos ravishda " FIFO" va "RS" tartibli $M|G|1|\infty$ modelli ommaviy xizmat ko'rsatish sistemasidagi talablarning statsionar kutish vaqti uchun olingan limit teoremalar keltirilgan va bu teoremalarning kritik zagruzkada yaqinlashish tezligi baholangan.

1-§. Sistemaning bandlik davri uchun yaqinlashish tezligining tahlili

Bu paragrafda $M|G|1|\infty$ sistemaning u zagruzkali bandlik davri uchun olingan limit teoremaning yaqinlashish tezligining (1) va (2) shartlar bajarilgandagi bahosi olingan.

Keyinchalik qulaylik uchun quyidagicha belgilashlar kiritamiz. $\pi(u)$, $(u \ge 0)$ - $M|G|1|\infty$ sistemaning u zaderjkali bandlik davri, $\pi - M|G|1|\infty$ sistemaning bandlik davri,

$$\rho_1 = a\beta_1, \quad \rho_2 = \frac{a\beta_2}{2}, \quad \beta_k = \int_0^\infty x^k dB(x),$$

$$\Pi^{(u)}(x) = P\{\pi(u) < x\}, \ (x \ge 0), \quad \Pi(x) = P\{\pi < x\}, \ (x \ge 0),$$

$$\pi^{(u)}(s) = \int_0^\infty e^{-sx} d\Pi^{(u)}(x), \quad (s \ge 0), \qquad \pi(s) = \int_0^\infty e^{-sx} d\Pi(x), \quad (s \ge 0)$$

$\Gamma(\cdot)$ - Eylerning gamma funksiyasi,

$$\mu(s) = s + a - a\pi(s), \quad (s \ge 0),$$
$$v(s) = s - a + a\beta(s), \quad (s \ge 0).$$

deb, olamiz.

<u>**Teorema 1.1.**</u> ([4]) (1) va $u \to +\infty$, $\rho \downarrow 0$ da $\frac{\rho u}{\rho_2} = \tau \to \tau^0$, $(0 < \tau^0 < +\infty)$ shartlar bajarilsin. U holda

$$\lim P\left\{\frac{\rho^2 \pi(u)}{\rho_2} < x\right\} = F(x), \quad (x \ge 0) \qquad (1.1)$$

limit mavjud, bu yerda

$$\left.\begin{aligned} F'(x) &= \frac{\tau^0}{2x\sqrt{\pi x}} \exp\left\{-\frac{(\tau^0 - x)^2}{4x}\right\}, \\ f(s) &= \int_0^\infty e^{-sx} dF(x) = \exp\{-\tau^0 \Delta(s)\}, (s \ge 0), \\ \Delta(s) &= \sqrt{s + 1/4} - 1/2 \end{aligned}\right\} \qquad (1.2)$$

Keyinchalik $M|G|1|\infty$ sistemadagi kritik zagruzkada (1.1) limit teoramaning yaqinlashish tezligi baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 1.1. Yuqoridagi (2) shart bajarilsin. U holda $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli

$$\mu(\frac{\rho^2 s}{\rho_2}) = \frac{\rho}{\rho_2}\left[\Delta(s) + A\frac{\Delta^{\delta}(s)}{1+2\Delta(s)}\rho^{\delta-2}(1+0(1))\right] \qquad (1.3)$$

bu yerda $A = 2\varepsilon / \beta_2 \rho_2^{\delta-2}$

Lemma 1.2. (2) va $u \to +\infty$, $\rho \downarrow 0$ da $\frac{\rho u}{\rho_2} = \tau \to \tau^0$, $(0 < \tau^0 < +\infty)$ shartlar bajarilsin. U holda $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli

$$\begin{aligned} &\pi^{(u)}(\rho^2 \rho_2^{-1} s) = \exp\left\{-\tau^0 \Delta(s)\right\} + \frac{\Delta(s)}{\exp\left\{\tau^0 \Delta(s)\right\}}\varepsilon_1(1+0(1)) - \\ &\tau^0 A \frac{\Delta^{\delta}(s)}{(1+2\Delta(s))\exp\left\{\tau^0 \Delta(s)\right\}}\rho^{\delta-2}(1+0(1)) \end{aligned} \qquad (1.4)$$

bu yerda $\varepsilon_1 = \left|\tau^0 - \tau\right|$.

Teorema1.2 (2) va $\rho \downarrow 0$, $u \to +\infty$ da $\frac{\rho u}{\rho_2} = \tau \to \tau^0$ $(0 < \tau^0 < +\infty)$ shartlar bajarilsin. Agar $\lim\left(|\varepsilon_1| / \rho^{\delta-2}\right) = d$, $(\varepsilon_1 = \tau^0 - \tau)$ limit mavjud bo'lsa, u holda

$$\begin{aligned} &\sup_{x \geq 0}\left|\Pi^{(u)}(\rho_2 x / \rho^2) - F(x)\right| \leq C_1 \cdot \chi(d = +\infty) + \\ &+ C_2 \chi(0 \leq d < +\infty)\rho^{\delta-2} \end{aligned} \qquad (1.5)$$

tenglik o'rinli bo'ladi, bu yerda

$$C_2 = C_1 \cdot d + \sqrt{2}A(\tau^0)^{-(\delta-2)/2} \cdot b \cdot \Gamma(\delta/2) \cdot (1 + \frac{2\delta}{\tau^0} + \frac{\delta(\delta+2)}{(\tau^0)^2}),$$

$$b > (2\pi)^{-1} - o'zgarmas\ son \qquad \chi(A) = \begin{cases} 1, agar\ A\ bajarilsa \\ 0,\ agar\ A\ bajarilmasa \end{cases}$$

Lemma 1.1 isboti:

Berilgan $\delta(2<\delta\leq 3)$ va $\theta\geq 0$ lar uchun quyidagi

$$\underset{\theta}{F}(z,s)=0 \tag{1.6}$$

tenglamani qaraymiz, bu yerda

$$\underset{\theta}{F}(z,s)=z+z^2-\theta z^\delta-s.$$

$F_\theta(z,s)$ funksiya $O(0,0)$ nuqta atrofida analitik funksiya bo'lib quyidagi

$$F_\theta(0,0)=0, \qquad \left.\frac{\partial F(z,s)}{\partial z}\right|_{\substack{z=0\\ s=0}}=1$$

shartlarni qanoatlantiradi. Shuning uchun oshkormas funksiya haqidagi teoremaga ko'ra ([11]) $z=D_\theta(s)$ funksiya yuqorida qaralayotgan (1.6) tenglamaning yechimi bo'lib, $s=0$ nuqta atrofida $D_\theta(0)=0$ boshlang'ich shartni qanoatlantiruvchi yagona analitik funksiya bo'ladi. Bundan quyidagi ayniyatga ega bo'lamiz.

$$s\equiv D_\theta(s)+[D_\theta(s)]^2-\theta[D_\theta(s)]^\delta \tag{1.7}$$

$s>0,\ \theta\downarrow 0$ bo'lsin. Quyidagi

$$E(s,\theta)=D_\theta(s)-\Delta(s)$$

belgilashni kiritamiz.

$$D_\theta(s)=E(s,0)+\Delta(s)$$

Ifodani (1.7) tenglikka qo'ysak quyidagini hosil qilamiz

$$\begin{aligned} s\equiv \Delta(s)+E(s,\theta)+[\Delta(s)+E(s,\theta)]^2 - \\ -\theta[\Delta(s)+E(s,\theta)]^\delta. \end{aligned} \tag{1.8}$$

Chekli θ va s larda $\Delta(s)$ va $D_\theta(s)$ lar chegaralangan bo'lganligi uchun $E(s,\theta)$ ham chegaralanganligi kelib chiqadi. U holda $\theta\downarrow 0$ da (1.7) tenglikdan

$$s\equiv D_0(s)+[D_0(s)]^2$$

tenglik o'rinli. Yoki xuddi shuningdek

$$D_0(s)=\lim_{\theta\downarrow 0}D_\theta(s)=\Delta(s)$$

shuning uchun

$$\lim_{\theta\downarrow 0} E(s,\theta)=0.$$

Endi $\theta\downarrow 0$ da (1.8) tenglikni quyidagi ko'rinishda yozamiz:

$$E(s,\theta)(1+2\Delta(s))=\theta\Delta^{\delta}(s)+0_{\theta}(E(s,\theta)).$$

Bundan $\theta\downarrow 0$ da osongina ko'rsatish mumkinki

$$E(s,\theta)=\theta\cdot\frac{\Delta^{\delta}(s)}{1+2\Delta(s)}(1+O_{\theta}(1)) \qquad (1.9)$$

$s^{*}\downarrow 0$ da quyidagi $v(s^{*})=s^{*}-a+a\beta(s^{*})$ funksiyani qaraymiz. (2) munosabatni e'tiborga olsak

$$v(s^{*})=\rho s^{*}+\rho_2(s^{*})^2-\frac{2\varepsilon}{\beta_2}\cdot\rho_2(s^{*})^{\delta}(1+o_{s^{*}}(1))$$

ega bo'lamiz.

Yuqoridagi tenglikdan $s^{*}=\rho\cdot\rho_2^{-1}s$ deb olsak, $\rho\downarrow 0$ da quyidagi

$$v(\rho\rho_2^{-1}s)=\rho^2\rho_2^{-1}[s+s^2-\frac{2\varepsilon}{\beta_2\rho_2^{\delta-2}}\rho^{\delta-2}\cdot s^{\delta}(1+\varphi)]$$

munosabatni hosil qilamiz, bu yerda $\rho\downarrow 0$ da $\varphi\to 0$. Agar $A_{\rho}=\dfrac{2\varepsilon\rho^{\delta-2}}{\beta_2\rho_2^{\delta-2}}$ deb belgilasak u holda quyidagi asimptotik munosabatni hosil qilamiz:

$$v(\rho\rho_2^{-1}s)=\rho^2\rho_2^{-1}[s+s^2-A_{\rho}s^{\delta}(1+\varphi)] \qquad (1.10)$$

[4] da $\rho_1\le 1$ bo'lganda $\mu(s)$ va $v(s)$ funksiyalar uchun quyidagi

$$\mu(v(s))=s \qquad (1.11)$$

Ifoda o'rinli ekanligi isbot qilingan. (1.10) va (1.11) larga asosan

$$s^{*}=\mu(\rho^2\rho_2^{-1}(s+s^2-A_{\rho}s^{\delta}(1+\varphi))) \qquad (1.12)$$

hosil qilamiz.

O'rta qiymat haqidagi teoremaga ko'ra ([10])

$$|\mu(u+v)-\mu(u)|=|\mu'(u+mv)|\cdot|v|$$

tenglik o'rinli, bu yerda $u+v>0,\ u>0\quad 0<m<1$.

$\mu'(s)$ $(s>0)$ funksiya barcha musbat u va v lar uchun monoton kamayuvchi bo'lganligi uchun.

$$\mu'(u+mv)\le\mu'(0)=\rho^{-1}$$

Bundan

$$\left|\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta(1+\varphi)))-\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta))\right|\le$$
$$\le\rho\rho_2^{-1}A_\rho s^\delta|\varphi|.$$

Bu tengsizlikdan (1.12) tenglikni e'tiborga olsak

$$\left|1-\frac{\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta))}{\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta(1+\varphi)))}\right|\le A_\rho s^{\delta-1}|\varphi|$$

tengsizlikni hosil qilamiz.

Bundan quyidagini hosil qilamiz.

$$\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta))=\mu(\rho^2\rho_2^{-1}(s+s^2-A_\rho s^\delta(1+\psi)))(1+\psi)\quad(1.13)$$

bu yerda ψ, $s>0$ da quyidagi

$$\lim_{\rho\downarrow 0}\frac{|\psi|}{|\varphi|A_\rho s^\delta}\le 1\qquad(1.14)$$

shartni qanoatlantiradi.

Quyidagi

$$s+s^2-A_\rho s^\delta=\tau\qquad(\tau\ge 0)$$

tenglamani qaraymiz. Bu tenglamaning yechimi $s=D_{A_\rho}(\tau)$ funksiyadan iborat. (1.13) tenglikdagi s o'rniga $D_{A_\rho}(\tau)$ ni qo'ysak va (1.14) shartni e'tiborga olsak

$$\mu(\rho^2\rho_2^{-1})=\rho\rho_2^{-1}(\Delta(\tau)+A_\rho\cdot\frac{\Delta^\delta(\tau)}{1+2\Delta(\tau)}(1+O_\rho(1)))$$

munosabatni hosil qilamiz. Bu 1.2. Lemmaning isbotidir.

1.2. Lemmaning isboti

Ma'lumki

$$\pi^{(u)}(\rho^2 s/\rho_2) = \exp\{u\mu(\rho^2 s/\rho_2)\} \tag{1.15}$$

(1.3) ifodani (1.15) ga qo'ysak

$$\pi^{(u)}(\rho^2 s/\rho_2) = \exp\left\{-u\rho\rho_2^{-1}\left[\Delta(s) + A\frac{\Delta^\delta(s)}{1+2\Delta(s)}\rho^{\delta-2}(1+0(1))\right]\right\} \tag{1.16}$$

(1.16) –munosabatda $\tau = \frac{u\rho}{\rho_2}$, $u \to +\infty$ va $\rho \downarrow 0$ da $\tau \to \tau^0$, $\varepsilon_1 = \tau^0 - \tau$ larni e'tiborga olsak

$$\pi^{(u)}(\rho^2 s/\rho_2) = \exp\{\tau\Delta(s)\}\cdot\exp\left\{-\tau A\frac{\Delta^\delta(s)}{1+2\Delta(s)}\cdot\rho^{\delta-2}(1+0(1))\right\} =$$

$$= \exp\{\tau\Delta(s)\}\cdot\left[1-\tau A\frac{\Delta^\delta(s)}{1+2\Delta(s)}\cdot\rho^{\delta-2}(1+0(1))\right] =$$

$$= \exp\{(\varepsilon_1-\tau^0)\Delta(s)\}\cdot\left[1-\tau^0 A\frac{\Delta^\delta(s)}{1+2\Delta(s)}\cdot\rho^{\delta-2}(1+0(1))\right] =$$

$$= \exp\{\tau^0\Delta(s)\}\cdot[1+\varepsilon_1\Delta(s)\cdot(1+0(1))]\cdot\left[1-\tau^0 A\frac{\Delta^\delta(s)}{1+2\Delta(s)}\cdot\rho^{\delta-2}(1+0(1))\right]$$

Bundan

$$\pi^{(u)}(\rho^2 s/\rho_2) = \exp\{\tau\Delta(s)\} + \frac{\Delta(s)}{\exp\{\tau^0\Delta(s)\}}\varepsilon_1(1+0(1)) -$$

$$-\tau^0 A\frac{\Delta^\delta(s)}{(1+2\Delta(s))\exp\{\tau^0\Delta(s)\}}\rho^{\delta-2}(1+0(1))$$

1.2 Lemma isbot bo'ldi.

1.2. Teoremaning isboti. 1.1 Teoremada aniqlangan

$\Pi^{(u)}(\frac{\rho_2 x}{\rho^2})$ va $F(x)$ funksiyalar uchun bizga ma'lum bo'lgan Berri- Esseen tengsizligini ([5]) qo'llasak

$$\nabla \underset{=\!=}{def} \sup_{x\ge 0}\left|\Pi^{(u)}\left(\frac{\rho_2 x}{\rho^2}\right)-F(x)\right| \le$$
$$\le b_0\int_{-T}^{T}\left|\frac{\pi^{(u)}(-\rho^2 ti/\rho_2)-f(-ti)}{ti}\right|dt+r(b_0)C\frac{1}{T}, \tag{1.17}$$

bu yerda $b_0 > (2\pi)^{-1}$, $T-$ ixtiyoriy musbat son, $r(b_0)-$ faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, $i-$ mavhum birlik, $\pi^{(u)}(-\rho^2 ti/\rho_2)$ va $f(-ti)$ lar mos ravishda $\Pi^{(u)}(\rho_2 x/\rho^2)$ va $F(x)$ taqsimot funksiyalarning mos xarakteristik funksiyalari, o'zgarmas C esa $\sup\limits_{x\ge 0}|F'(x)|\le C$ shartdan aniqlanadi.

Shuni takidlab, o'tamizki $F'(x)$ funksiya musbat yarim o'qda uzluksiz va $x_0=\sqrt{9+(\tau^0)^2}-3$ nuqtada eng katta qiymatga erishadi. Shuning uchun C o'zgarmas sifatida

$$C=\frac{\tau^0}{2\sqrt{\pi}}\exp\left\{-\frac{(\tau^0-x_0)}{4x_0}\right\}\cdot\frac{1}{x_0\sqrt{x_0}}$$

qiymatni olish mumkin.

1.2 Lemmaning tasdig'ini va $f(-ti)=\exp\{-\tau^0\Delta(-ti)\}$ ni (1.17) ga qo'ysak

$$\nabla \le b\left[\int_{-T}^{T}\frac{|\Delta(-ti)|}{|t|\left|\exp\{\tau^0\Delta(-ti)\}\right|}dt\cdot|\varepsilon_1|\cdot(1+0(1)+\right.$$
$$\left.+\tau^0 A\int_{-T}^{T}\frac{|\Delta(-ti)|^{\delta}}{|t|\cdot|1+2\Delta(-ti)|\cdot\left|\exp\{\tau^0\Delta(-ti)\}\right|}dt\cdot\rho^{\delta-2}(1+0(1))\right]+r(b)CT^{-1} \tag{1.18}$$

tengsizlikni hosil qilamiz.

Quyidagi $(t\in[-T,+T],\quad v(t))=\sqrt{1+16t^2})$

$$\left|\Delta(-ti)\right| = \frac{1}{2}\sqrt{1+v(t)-\sqrt{2(1+v(t))}},$$

$$\left|1+2\Delta(-ti)\right| = \sqrt{v(t)},$$

$$\left|\exp\left\{\tau^0\Delta(-ti)\right\}\right| = \exp\left\{-\frac{\tau^0}{2}[1-\sqrt{\frac{1+v(t)}{2}}]\right\},$$

tengliklarni e'tiborga olsak (1.18) tengsizlikni

$$\nabla \le r(b_0)\frac{C}{T} + b_0 e^{\tau^0/2}[R(T)\cdot\left|\varepsilon_1\right|(1+0(1)) + \\ + \frac{A\tau^0}{2^{\delta-1}}\cdot L(T)\rho^{\delta-2}(1+0(1))], \tag{1.19}$$

ko'rinishida yozish mumkin, bu yerda

$$R(T) = \int_0^T \frac{\sqrt{1+v(t)-\sqrt{2(1+v(t))}}}{t\exp\left\{\frac{\tau^0}{2\sqrt{2}}\cdot\sqrt{1+v(t)}\right\}}dt \tag{1.20}$$

$$L(T) = \int_0^T \frac{[1+v(t)-\sqrt{2(1+v(t))}]^{\delta/2}}{t\cdot\sqrt{v(t)}\cdot\exp\left\{\frac{\tau^0}{2\sqrt{2}}\cdot\sqrt{1+v(t)}\right\}}dt \tag{1.21}$$

Quyidagi $x=\sqrt{1+v(t)}$ yangi o'zgaruvchini kiritib va ba'zi almashtirishlarni bajarsak (1.20) ifoda

$$R(T) = 2\left[R_1 - R_2(t)\cdot e^{\frac{\tau^0}{2\sqrt{2}}\cdot\sqrt{1+v(t)}}\right], \tag{1.22}$$

ko'rinishini oladi, bu yerda

$$R_1 = \int_{\sqrt{2}}^{\infty}\frac{x^2-1}{x^2-2}\sqrt{\frac{x-\sqrt{2}}{x}}\cdot e^{-\frac{\tau^0 x}{2\sqrt{2}}}dx \tag{1.23}$$

$$R_2(T) = \int_0^{\infty}\frac{\omega^2(T,x-\frac{1}{T})}{\omega^2(T,x)-\frac{2}{T}}\cdot\sqrt{\frac{\omega(T,x)-\frac{2}{T}}{\omega(T,x)}}\cdot \ell^{-\frac{\tau^0}{2\sqrt{2}}x}dx \tag{1.24}$$

$$\omega(T,x)=\frac{x}{\sqrt{T}}+\sqrt{\frac{1}{T}+\sqrt{\frac{1}{T}+16}}, \qquad (x\in(0,+\infty)).$$

Xuddi shunga o'xshash (1.21) munosabatdan

$$L(T)=2\left[L_1-L_2(T)T^{\frac{\delta-2}{2}}\cdot e^{\frac{\tau^0}{2\sqrt{2}}\cdot\sqrt{1+\nu(t)}}\right] \tag{1.25}$$

ni hosil qilamiz, bu yerda

$$L_1=\int_{\sqrt{2}}^{\infty}\frac{[x(x-\sqrt{2})]^{(\delta-2)/2}}{x+\sqrt{2}}\cdot\sqrt{x^2-1}\cdot e^{-\frac{\tau^0}{2\sqrt{2}}}dx \tag{1.26}$$

$$L_2(T)=\int_0^{\infty}\frac{[\omega(T,x)]^{(\delta-2)/2}\cdot[\omega(T,x)-\sqrt{2/T}]^{\delta/2}}{\omega^2(T,x)-(2/T)}\cdot$$
$$\cdot\sqrt{\omega^2(T,x)-(1/T)}\cdot e^{-\frac{\tau^0}{2\sqrt{2}}}dx \tag{1.27}$$

Veyershtrass teoremasiga asosan (1.24) va (1.27) munosabatlardan $R_2(\rho^{-1})$ va $L_2(\rho^{-1})$ funksiyalar $\rho\in[0,1]$ da tekis yaqinlashadi. U holda $\rho\downarrow 0$ (1.22) va (1.25) larga ko'ra mos ravishda

$$R(\rho^{-1})=2R_1(1+o_\rho(1)) \tag{1.28}$$

$$L(\rho^{-1})=2L_1(1+o_\rho(1)) \tag{1.29}$$

munosabatlar o'rinli.

Shuning uchun $d\neq+\infty$ hol uchun (1.19), (1.28) va (1.29) lardan $\rho\downarrow 0$ da quyidagi $(b>b_0>(2\pi)^{-1})$

$$\nabla\le 2be^{\tau^0/2}\left[R_1d+\frac{A\tau^0}{2^{\delta-1}}\cdot L_1\right]\rho^{\delta-2} \tag{1.30}$$

baho kelib chiqadi, $d=+\infty$ hol uchun (1.19),(1.28) va (1.29) larga ko'ra

$$\nabla\le 2be^{\tau^0/2}R_1\cdot|\varepsilon_1| \tag{1.31}$$

tengsizlik o'rinli .

(1.30)va (1.31) - tengsizliklar shuni ko'rsatadiki ∇ baho uchun (1.23) va (1.26– integrallarni qarab chiqishga to'g'ri keladi. R_1 uchun quyidagi yuqori integral bahoni yozish mumkin.

$$R_1 \le \frac{1}{2^{7/4}} \int_{\sqrt{2}}^{\infty} x^2 (x-\sqrt{2})^{-1/2} e^{-\tau^0 x/2\sqrt{2}} dx$$

Bu tengsizlikning o'ng tomonidagi integral jadval integralidir ([12]) ya'ni uning uchun quyidagi munosabat o'rinli

$$R_1 \le \sqrt{\frac{\pi}{2}} \cdot e^{-\tau^0/2} \cdot \psi(\frac{1}{2};\frac{7}{2};\frac{\tau^0}{2}) \qquad (1.32)$$

Bu yerda $\psi(x;y;z)$ funksiya umumlashgan giper geometrik funksiya

$${}_1F_1(x;y;z) = \sum_{k\ge 0} \frac{(x)_k}{(y)_k k!} \cdot z^k,$$

orqali ifodalanadi, bu yerda $(x)_k = x(x+1)...(x+k-1)$. Ya'ni bizning hol uchun

$$\psi(\frac{1}{2};\frac{7}{2};\frac{\tau^0}{2}) = \frac{3}{4}(\frac{2}{\tau^0})^{5/2} {}_1F_1(-2;-\frac{3}{2};\frac{\tau^0}{2})$$

ko'rinishni oladi. Ba'zi hisoblardan so'ng

$$\psi(\frac{1}{2};\frac{7}{2};\frac{\tau^0}{2}) = \sqrt{\frac{2}{\tau^0}}(1+\frac{2}{\tau^0}+\frac{3}{(\tau^0)^2}) \qquad (1.33)$$

qiymatni hosil qilamiz.

(1.33) ni (1.32) qo'ysak R_1 uchun quyidagi bahoni topamiz.

$$R_1 \le \sqrt{\frac{\pi}{\tau^0}} \cdot e^{-\frac{\tau^0}{2}} \cdot (1+\frac{2}{\tau^0}+\frac{3}{(\tau^0)^2}) \qquad (1.34)$$

Huddi shunday jadval integralidan foydalanib L_1 uchun quyidagi yuqori bahoni topamiz.

$$L_1 \le \frac{1}{2^{(10-\delta)/4}} \int_{\sqrt{2}}^{\infty} x^2 (x-\sqrt{2})^{(\delta/2)-1} e^{-\tau^0 x/2\sqrt{2}} dx,$$

Ya'ni ([12] ga qarang)

$$L_1 \le \frac{(\sqrt{2})^{\delta-3}\Gamma(\delta/2)}{\exp\{\tau^0/2\}} \psi(\frac{\delta}{2};3+\frac{\delta}{2};\frac{\tau^0}{2}) \qquad (1.35)$$

Keyingi hisoblashlar hatijasida

$$\psi(\frac{\delta}{2},3+\frac{\delta}{2};\frac{\tau^0}{2}) = (\sqrt{\frac{2}{\tau^0}})^{\delta}(1+\frac{2\delta}{\tau^0}+\frac{\delta(\delta+2)}{(\tau^0)^2}),$$

ifodani (1.35) qo'ysak L_1 uchun quyidagi

$$L_1 \le \frac{\Gamma(\delta/2)2^{\delta-(3/2)}}{(\tau^0)^{\delta/2}\exp\{\tau^0/2\}}\left[1+\frac{2\delta}{\tau^0}+\frac{\delta(\delta+2)}{(\tau^0)^2}\right] \qquad (1.36)$$

bahoni hosil qilamiz. (1.30), (1.31), (1.34) va (1.36) tengsizliklar 1.2 Teoremaning isbotini tasdiqlaydi.

2-§. « FIFO» tartibli kutish vaqti uchun yaqinlashish tezligining tahlili.

Mazkur paragrafda « FIFO» tartibli $M|G|1|\infty$ sistemada talablarning statsionar kutish vaqti uchun olingan limit teoremaning yaqinlashish tezligi (1) va (2) – shartlar bajarilganda bahosi olingan.

Statsionar kutish vaqti ω sistemaning talablarga xizmat qilish tartibiga bog'liq.

Faraz qilaylik, talablarga « FIFO» tartibli xizmat ko'rsatilayotgan bo'lsin, ya'ni birinchi kelgan talabga birinchi xizmat ko'rsatilsin.

Qulaylik uchun quyidagicha belgilashlar kiritaylik.

$$W^F(x) = P\{\omega^F < x\} \ (x \geq 0), \quad \omega^F(s) = \int_0^\infty e^{-sx} dW^F(x) \ (s \geq 0),$$

bu yerda yuqorgi F indeks « FIFO» tartibli xizmat ko'rsatilayotganligini bildiradi.

Teorema 2.1 [8]. (1) -munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da quyidagi

$$\lim P\{\rho \cdot \omega^F / \rho_2) < x\} E(x), \ (x \geq 0) \qquad \textbf{(2.1)}$$

limit mavjud, bu yerda

$$E(x) = 1 - e^{-x} \ (x \geq 0), \quad e(s) = \int_0^\infty e^{-sx} dE(x) = \frac{1}{1+s} \ (s \geq 0)$$

Bu paragrafda $M|G|1|\infty$ sistemadagi kritik zagruzkada 2.1-limit teoremaning yaqinlashish tezligi baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 2.1 Yuqoridagi (2) munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da $s \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli.

$$\omega(\rho s / \rho_2) = \frac{1}{1+s} + A\frac{s^{\delta-1}}{(1+s)^2}\rho^{\delta-2}(1 + 0_\rho(1)) \qquad (2.2)$$

Teorema 2.2. (2)- munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da

$$\sup_{x\ge 0}\left|W^F(\rho_2 x/\rho)-E(x)\right|\le C_3\chi(\delta<3)\rho^{\delta-2}+ \\ +C_4\chi(\delta=3)\rho\ln\rho \qquad (2.3)$$

tengsizlik o'rinli bo'ladi, bu yerda

$$C_3=\frac{b\pi A}{\sin\frac{\delta-1}{2}\pi},\quad C_4=\frac{2b\beta_3}{3\beta_2\rho_2},\quad b>(2\pi)^{-1}.$$

2.1.Lemmaning isboti.

Ma'lumki [1],

$$\omega^F(s^*)=\frac{\rho s^*}{s^*-a(1-\beta(s^*))} \qquad (2.4)$$

(2.4) da $s^*=\rho\cdot\rho_2^{-1}s$ deb olsak

$$\omega^F(\rho s/\rho_2)=\frac{\rho^2 s/\rho_2}{(\rho s/\rho_2)-a+a\beta(\rho s/\rho_2)}. \qquad (2.5)$$

hosil qilamiz.

(2) –munosabatni e'tiborga olsak quyidagi

$$\beta(\rho s/\rho_2)=1-\beta_1\rho_2^{-1}\rho s+\beta_2\rho^2 s^2/(2\rho^2)-\varepsilon\rho_2^{-\delta}s^\delta(1+0_\rho(1)) \qquad (2.6)$$

asimptotik yoyilmani hosil qilamiz. (2.6) munosabatni (2.5) ga qo'ysak

$$\omega^F(\rho s/\rho_2)=$$

$$=\frac{\rho^2 s/\rho_2}{(\rho s/\rho_2)+(a\beta_1 s\rho/\rho_2)+(a\beta_2 s^2\rho^2/(2\rho_2^2))-(a\varepsilon s^\delta\rho^\delta/\rho_2^\delta)(1+0_\rho(1))}=$$

$$=\frac{\rho^2 s/\rho_2}{((1-a\beta_1)\rho s/\rho_2)+(\rho^2 s^2/\rho_2)-(2a\varepsilon\beta_2\rho^\delta s/(2\rho_2^\delta\beta_2))(1+0_\rho(1))}=$$

$$=\frac{\rho^2 s/\rho_2}{(\rho^2 s/\rho_2)+(\rho^2 s^2/\rho_2)-(2\varepsilon s^\delta\rho^\delta/(\beta_2\rho_2^{\delta-1}))(1+0_\rho(1))}=$$

$$=\frac{\rho^2 s/\rho_2}{(\rho^2(s+s^2)/\rho_2)+[1-(2\varepsilon s^\delta\rho^{\delta-2}/\beta_2\rho_2^{\delta-2}(s+s^2))(1+0_\rho(1))]}.$$

Bundan

$$\omega^F(\rho s/\rho_2) = \frac{1}{1+s} + \frac{2\varepsilon}{\beta_2 \rho_2^{\delta-2}} \cdot \frac{s^{\delta-1}}{(1+s)^2} \rho^{\delta}(1+0_{\rho}(1)).$$

Bu yerda agar $A = 2\varepsilon / \beta_2 \rho_2^{\delta-2}$ deb belgilasak (2.2) ni hosil qilamiz. 2.1. lemma isbot bo'ldi.

2.2 Teoremaning isboti. 2.1 Teoramada aniqlangan $W^F(\frac{\rho_2 x}{\rho})$ va $E(x)$ Funksiyalar uchun bizga ma'lum bo'lgan Berri-Esseen tengsizligini ([5]) qo'llasak

$$\begin{aligned} \nabla^F &\underset{\overline{\overline{}}}{def} \sup_{x\ge 0} \left| W^F(\rho_2 x/\rho) - E(x) \right| \le \\ &\le b_0 \int_{-T}^{T} \left| \frac{\omega^F(-\rho ti/\rho_2) - e(-ti)}{ti} \right| dt + r(b_0) C^{(1)} T^{-1} \end{aligned} \tag{2.7}$$

bu yerda $b_0 > (2\pi)^{-1}, T$ -ixtiyoriy musbat son, $r(b_0)$ -faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, i -mavhum birlik, $\omega^F(-\rho ti/\rho_2)$ va $e(-ti)$ lar mos ravishda $W(\rho_2 x/\rho)$ va $E(x)$ taqsimot funksiyalarning mos xarakteristik funksiyalari, o'zgarmas $C^{(1)}$ esa $\sup_{x\ge 0} |E'(x)| \le C^{(1)}$ shartdan aniqlanadi. Shuni takidlab o'tamizki $E'(x)$ funksiya musbat yarim o'qda uzluksiz va $x_0 = 0$ nuqtada eng katta qiymatga erishadi. Shuning uchun $C^{(1)}$ o'zgarmas sifatida $C^{(1)} = 1$ qiymatni olishi mumkin.

$s = -ti$ ni hisobga lsak, 2.1.Lemmaning tasdig'ini va $\ell(-ti) = \dfrac{1}{1-ti}$ ni (2.7) ga qo'ysak

$$\nabla^F \le b_0 A \int_{-T}^{T} \frac{|t|^{\delta-2}}{|1-ti|^2} dt \rho^{\delta-2}(1+0_{\rho}(1)) + r(b_0) T^{-1},$$

yoki

$$\nabla^F \le 2 b_0 A \Phi(T) \rho^{\delta-2}(1+0_{\rho}(1)) + r(b_0) \cdot T^{-1} \tag{2.8}$$

tengsizlikni hosil qilamiz, bu yerda

$$\Phi(T)=\int_0^T \frac{t^{\delta-2}}{1+t^2}dt\,.$$

Endi bu integralni hisoblaymiz.

$$\Phi(T)=\int_0^T \frac{t^{\delta-2}}{1+t^2}dt=\frac{1}{2}\int_0^{T^2}\frac{x^{(\delta-3)/2}}{1+x}dx=$$

$$=\frac{1}{2}\left[\chi(\delta<3)\int_0^{T^2}\frac{x^{(\delta-3)/2}}{1+x}dx+\chi(\delta=3)\int_0^{T^2}\frac{dx}{1+x}\right]=$$

$$=\frac{1}{2}\Big[\chi(\delta=3)\ln(1+T^2)+\chi(\delta<3)$$

$$\left(\int_0^\infty \frac{y^{(\delta-3)/2}}{y+T^{-2}}dy-\int_1^\infty \frac{y^{(\delta-3)/2}dy}{y+T^{-2}}\right)$$

Bundan

$$\Phi(T)=\frac{\chi(\delta<3)\pi}{2\sin\frac{\delta-1}{2}\pi}+\frac{1}{2}\chi(\delta=3)\ln(1+T^2)-$$
$$-\frac{1}{2}\chi(\delta<3)\Phi_1(T)T^{\delta-3} \tag{2.9}$$

bu yerda

$$\Phi_1(T)=\int_1^\infty \frac{y^{(\delta-3)/2}}{y+T^{-2}}dy \tag{2.10}$$

(2.9) ni (2.8) tengsizlikka qo'ysak

$$\nabla^F \le b_0A\left[\chi(\delta<3)\left(\frac{\pi}{\sin\frac{\delta-1}{2}\pi}-T^{\delta-3}\Phi_1(T)\right)+\right.$$
$$\left.+\chi(\delta=3)\ln(1+T^2)\right]\overline{\underline{\rho}}^{\delta-2}(1+0_\rho(1))+r(b_0)T^{-1} \tag{2.11}$$

hosil qilamiz.

Veyershtrass teoremasiga asosan (2.10) munosabatdan $\Phi_1(\rho^{-1})$ funksiya $\rho\in[0,1]$ da tekis yaqinlashadi. U holda $T=\rho^{-1}$ deb olsak, $\rho\downarrow 0$ da (2.11) dan quyidagi $(b>b_0>(2\pi)^{-1})$

$$\nabla^F \leq \frac{\chi(\delta<3)}{\sin\frac{\delta-1}{2}\pi}\rho^{\delta-2}+\chi(\delta=3)2Ab\rho\ln\rho^{-1}$$

Baho kelib chiqadi. Bu tasdiq 2.2 teoremaning isbotidir.

§3. " RS" tartibli kutish vaqti uchun yaqinlashish tezligining tahlili.

Bu paragrafda " RS" tartibli $M|G|1|\infty$ sistemada talablarning statsionar kutish vaqti uchun olingan limit teoremaning yaqinlashish tezligini (1)- va (2) – shartlar bajarilganda bahosi olingan.

Statsionar kutish vaqti ω, sistemaning talablarga xizmat qilish tartibiga bog'liqdir.

Faraz qilaylik, talablarda " RS" tartibli xizmat ko'rsatilayotgan bo'lsin, ya'ni talablarni tasodifiy ravishda tanlab xizmat ko'rsatilsin.

$$W^R = P\{\omega^R < x\}(x \geq 0)\omega^R(s) = \int_0^\infty e^{-Sx} dW^R(x), \quad (S \geq 0),$$

bu yerda yuqori R indeks " RS" tartibli xizmat ko'rsatilayotganligini bildiradi.

Teorema 3.1([3]).

(1) – munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da quyidagi

$$\lim P\{(\rho \cdot \omega^R / \rho_2) < x\} = R(x), (x \geq 0) \qquad (3.1)$$

limit mavjud. Bu yerda

$$R(x) = 1 - \int_0^\infty \exp\{-\sqrt{t^2 + x}\} dt$$

$$r(S) = \int_0^\infty e^{-Sx} dR(x) = \int_0^\infty \frac{e^{-z}}{1 + Sz} dz.(\ \mathrm{Re}\, S \geq 0)$$

Bu paragrafda yuqoridagi 3.1 limit teoremaning yaqinlashish tezligi kritik zagruzkada baholanadi.

Quyidagi tasdiqlarni keltiramiz:

Lemma 3.1

Yuqoridagi (2) munosabat o'rinli bo'lsin. U holda $\rho \downarrow 0$ da $S \geq 0$ uchun quyidagi asimptotik yoyilma o'rinli.

$$\omega^R(\rho S/\rho_2)=\int_0^\infty \varphi(S,Z)dz+$$
$$+\rho^{\delta-2}\int_0^\infty \varphi(S,Z)dz(1+0_\rho(1)), \qquad (3.2)$$

bu yerda $\varphi(S,Z)=\dfrac{e^{-z}}{1+SZ}$;

$$\varphi(s,z)=\chi(\delta=3)[\frac{1}{6}\cdot\frac{s}{1+sz}\cdot z^3e^{-z}-$$
$$-\frac{s}{1+sz}ze^{-z}]+\frac{2\varepsilon}{\rho_2\rho_2^{\delta-2}}\cdot\frac{(sz)^{\delta-1}}{(1+sz)^2}e^{-z} \qquad (3.3)$$

Teorema 3.2.

(2)- munosabat o'rinli bo'lsin. U holda $\rho\downarrow 0$ da

$$\sup_{x\geq 0}|W^R(\rho_2 x/\rho)-R_0(x)|\leq C_5\chi(\delta<3)\cdot$$
$$\cdot\rho^{\delta-2}+C_6\chi(\delta=3)\rho\ln\rho \qquad (3.4)$$

bu yerda $C_5=\dfrac{b\pi A}{\sin\frac{\delta-1}{2}}\pi,\qquad C_6=-\dfrac{2b\beta_3}{3\beta_2\rho_2}-\dfrac{8}{3}b$

$$A=2\varepsilon/\beta_2\rho_2^{\delta-2},\ \ b>(2\pi)^{-1}$$

3.1. Lemmaning isboti.

Ma'lumki ([6])

$$\omega^R(S)=\rho\cdot\int_0^{a(1-\pi(s))}[\frac{sz}{zs-a(1-\beta(zs))}-$$
$$-\frac{(1+z)s}{zs-a(1-\beta(1+z)s)}]\exp(\int_0^z\frac{sdv}{sv-a(1-\beta(s(1+v)))})dz$$

(2) –munosabatga ko'ra osongina keltirish munkin.

$$a(1-\beta(\frac{z\rho s}{\rho_2}))=\frac{zs}{\rho_2}\rho-(\frac{zs}{\rho_2}-\frac{(zs)^2}{\rho_2})\rho^2+$$
$$+\frac{2\varepsilon}{\beta_2}\cdot\frac{(zs)^\delta}{\rho_2^{\delta-1}}\rho^\delta(1+0(1)) \qquad (3.6)$$

U holda (3.6) ga ko'ra quyidagini hosil qilamiz.

$$\frac{sz\rho^2/\rho_2}{(zs\rho/\rho_2)-a(1-\beta(z\rho s/\rho_2))}=$$

$$=\frac{sz\rho^2/\rho_2}{[(zs/\rho_2)+(z^2\rho^2/\rho_2)]\rho^2-(2\varepsilon z^\delta s^\delta/\beta_2\rho_2^{\delta-1})\rho^\delta(1+0_\rho(1))}=$$

$$=\frac{sz\rho^2/\rho_2}{1+zs-(2\varepsilon(zs)^{\delta-1}/\beta_2\rho_2^{\delta-2})\rho^{\delta-2}(1+0_\rho(1))}=\frac{1}{1+zs}\cdot$$

$$\cdot[1+(2\varepsilon(zs)^{\delta-1}/\beta_2\rho_2^{\delta-2}(1+z))\rho^{\delta-2}(1+0_\rho(1))].$$

bundan $\rho\downarrow 0$ da

$$\frac{zs\rho^2/\rho_2}{(zs\rho/\rho_2)-a(1-\beta(zs\rho/\rho_2))}=\frac{1}{1+zs}+A\frac{(zs)^{\delta-1}}{(1+zs)^2}\rho^{\delta-2}(1+0_\rho(1))$$

bu yerda $A=2\varepsilon/\beta_2\rho_2^{\delta=2}$ ni hosil qilamiz.

Huddi shunday o'xshash (3.6) munosabatni qo'llab $\rho\downarrow 0$ da osongina quyidagilarni hosil qilamiz.

$$\begin{aligned}&\frac{s\rho/\rho_2}{(zs\rho/\rho_2)-a[1-\beta((1+v)s\rho/\rho_2]}=\\&=-[1+(1+v+(1+v)^2s(\rho(1+0_\rho(1))))]\end{aligned}\qquad(3.8)$$

yoki

$$\frac{(1+z)s\rho^2/\rho_2}{(zs\rho/\rho_2)-a[1-\beta((1+z)s\rho/\rho_2]}=(1+z)\rho(1+0_\rho(1))\qquad(3.9)$$

(3.5) formuladagi s o'rniga $s^*=\rho s/\rho_2$ ifodani qo'ysak, kritik zagruzkada (3.7)-(3.9) munosabatlardan osongina quyidagilarni keltirib chiqaramiz.

$$\omega^R(\rho s/\rho_2)=\int_0^{a(1-\pi(s^*))/s^*}[\frac{1}{1+zs}+A_1\frac{(zs)^{\delta-1}}{(1+zs)^2}\rho^{\delta-2}(1+0_\rho(1))+$$

$$+(1+z)\rho(1+0_\rho(1))]\cdot\exp\{-z-\int_0^z(1+v+(1+v)^2s)dv\cdot\rho(1+0_\rho(1))\}dt=$$

$$=\int_0^{a(1-\pi(s^*))/s^*}[\frac{1}{1+zs}+(A_1\frac{(zs)^{\delta-1}}{(1+zs)^2}+(1+z)\chi(\delta=3)\rho^{\delta-2}(1+0_\rho(1))]\cdot$$

$$\cdot\exp\{-z-[(1+s)z+(\frac{1}{2}+s)z^2+\frac{1}{3}sz^3]\rho^{\delta-2}(1+0_\rho(1))\}dz=$$

$$=\int_0^{a(1-\pi(s^*))/s^*}[\frac{1}{1+sz}+(\chi(\delta=3)(1+z)+A_1\frac{(zs)^{\delta-1}}{(1+zs)^2})\rho^{\delta-2}(1+0(1))]\cdot$$

$$\cdot[e^{-z}-((1+s)ze^{-z}+(\frac{1}{2}+s)z^2e^{-z}+\frac{1}{3}sz^3e^{-z})\cdot\rho(1+0_\rho(1))]dz=$$

$$=\int_0^{a(1-\pi(s^*))/s^*}\{\frac{e^{-z}}{1+zs}-[\chi(\delta=3)(\frac{1+s}{1+sz}ze^{-z}+\frac{1}{2}\cdot\frac{1+2s}{1+zs}z^2e^{-z}+\frac{1}{3}\cdot\frac{s}{1+zs}\cdot$$

$$z^3e^{-z}-(1+z)e^{-z}-A_1\frac{(zs)^{\delta-1}e^{-z}}{(1+zs)^2}]\rho^{\delta-2}(1+0_\rho(1))\}dz.$$

Bundan $\rho\downarrow 0$ da

$$\omega^R(\rho s/\rho_2)=\int_0^{a(1-\pi(s^*))/s^*}\varphi(zs)dz-\int_0^{a(1-\pi(s^*))/s^*}\varphi^{(1)}(zs)dz\cdot\rho^{\delta-2}(1+0_\rho(1))$$

yoki

$$\omega^R(\rho s/\rho_2)=\int_0^{\infty}\varphi(zs)dz-\int_{a(1-\pi(s^*))/s^*}^{\infty}\varphi(z,z)dz-[\int_0^{\infty}\varphi^{(1)}(s,z)dz-$$
$$-\int_{a(1-\pi(s^*))/s^*}^{\infty}\varphi^{(1)}(s,z)dz]\cdot\rho^{\delta-2}(1+0_\rho(1)) \tag{3.10}$$

bu yerda $\varphi(s,z)=\dfrac{e^{-z}}{1+sz}$,

$$\varphi^{(1)}(s,z)=\chi(\delta=3)[\frac{1}{3}\cdot\frac{s}{1+zs}z^3e^{-z}+\frac{1}{2}\cdot\frac{1+2s}{1+zs}z^2e^{z}+$$
$$+\frac{1+s}{1+zs}ze^{-z}-(1+z)e^{-z}]-A_1\frac{(zs)^{\delta-1}}{(1+zs)^2}e^{-z}$$

Bundan keyin quyidagini ko'rsatamiz.

$$\lim_{\rho\downarrow 0}\frac{a(1-\pi(s^*))}{s^*}=\infty \qquad (3.11)$$

(2)– munosabatni e'tiborga olib va $\pi(s)=\beta(s+a-a\pi(s^*))$ ni qo'llab quyidagiga ega bo'lamiz.

$$1-\pi(s^*)=\beta_1\cdot(s^*+a-a\pi(s^*))-\frac{1}{2}\beta_2(s^*+a-$$
$$-a\pi(s^*))^2+\varepsilon(s^*+a-a\pi(s^*))^{\delta}(1+0(1)) .$$

Bundan

$$\frac{a(1-\pi(s^*))}{s^*}=1-\rho+\frac{a(1-\pi(s^*))}{s^*}(1-\rho)-$$
$$-\rho_2 s^*(1+\frac{a(1-\pi(s^*))^2}{s^*})+a\varepsilon(s^*)^{\delta-1}(1+\frac{a(1-\pi(s^*))}{s^*})^{\delta}(1+0(1)),$$

yoki

$$\frac{a(1-\pi(s^*))}{s^*}\rho=1-\rho-\rho_2 s^*[1+2\frac{a(1-\pi(s^*))^2}{s^*}+$$
$$+(\frac{a(1-\pi(s^*))}{s^*})^{\delta}]^{\delta}(1+0(1)) \qquad (3.12)$$

Quyidagi

$$L(s^*)=a(1-\pi(s^*))/s^*$$

belgilash kiritsak, (3.12) tenglikdan

$$(2\rho_2 s^*+\rho)L(s^*)=1-\rho-\rho_2 s^*-\rho_2 s^* L^2(s^*)+$$
$$+a\varepsilon(s^*)^{\delta-1}(1+L(s^*))^{\delta}(1+0(1)) \qquad (3.13)$$

hosil qilamiz.

(3.11) munosabatning to'g'ligini isbotlash uchun

$$\lim_{\rho\downarrow 0}L(s^*)=+\infty$$

ni ko'rsatish yetarlidir.

$$\lim_{\rho\downarrow 0} L(s^*) = A < +\infty .$$

U holda shunday $\{\rho_n\}_{n\ge 1}$ lar mavjudki $\lim_{n\to\infty}\rho_n = 0$ va $\lim_{n\to\infty} L^{(n)} = A$ bu yerda

$$L^{(n)} = L(s^*)\,|_{\rho=\rho_n}$$

(3.13) munosabatda ρ o'rniga ρ_n ni qo'ysak va s^* munosabatni e'tiborga olsak

$$\rho_n(2s+1)L^{(n)} = 1-\rho_n-\rho_n s-\rho_n s\cdot[L^{(n)}]^2+\frac{\varepsilon}{\rho_2\beta_1}\cdot$$

$$\cdot(1-\rho_n)\rho_n^{\delta-1}s^{\delta-1}(1+L^{(n)})^{\delta}\cdot(1+0(1))$$

ni hosil qilamiz.

Bunday ziddiyatga kelib qoldik. Haqiqatdan ham oxirgi tenglikning chap tomoni nolga intiladi, o'ng tomoni esa birga intiladi.

Quyidagi

$$\lim_{\rho\downarrow 0}\frac{1}{\rho^{\delta-2}}\cdot\int_{a(1-\pi(s^*))/s^*}^{\infty}\varphi(s,z)=0 \qquad (3.14)$$

tenglik o'rinli ekanini ko'rsatamiz.

Haqiqatdan

$$0\le\frac{1}{\rho^{\delta-2}}\int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty}\varphi(s,z)=\frac{1}{\rho^{\delta-2}}\int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty}\frac{e^{z}dz}{1+sz}=\frac{1}{\rho^{\delta-2}}\cdot$$

$$\cdot e^{-\frac{a(1-\pi(s^*))}{s^*}}\cdot\int_0^{\infty}\frac{e^{-x}}{1+\dfrac{as(1-\pi(s^*))}{s^*}+sx}=\frac{1}{\rho^{\delta-2}}e^{-\frac{a(1-\pi(s^*))}{s^*}}\cdot$$

$$\cdot\int_0^{\infty}\frac{e^{-x}}{1+\dfrac{a\rho_2(1-\pi(\rho s/\rho_2))}{\rho}}\le\frac{1}{\rho^{\delta-2}}\cdot e^{-\frac{a(1-\pi(s^*))}{s^*}}\cdot$$

$$\cdot\int_0^{\infty}\frac{e^{-x}dx}{1+sx}\le\frac{1}{\rho^{\delta-2}}\cdot e^{-\frac{a(1-\pi(s^*))}{s^*}}.$$

Bundan

$$0 \le \frac{1}{\rho^{\delta-2}} \int_{\frac{a(1-\pi(s^*))}{s^*}}^{\infty} \frac{e^{-z}dz}{1+sz} \le \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} \qquad (3.15)$$

osongina keltirib chiqarish munkinki

$$\lim_{\rho \downarrow 0} \frac{1}{\rho^{\delta-2}} \cdot e^{-\frac{a(1-\pi(s^*))}{s^*}} = 0 .$$

U holda (3.15) dan (3.14) kelib chiqadi. Bundan keyin (3.11) ni e'tiborga olsak quyidagini hosil qilamiz.

$$\lim \frac{\int_{a(1-\pi(s^*))/s^*}^{\infty} \varphi^{(1)(s,z)dz}}{\int_0^{\infty} \varphi^{(1)}(s,z)dz} = 0 \qquad (3.16)$$

(3.10), (3.14),(3.16) ladan va murakkab bo'lmagan almashtirishlar ni bajarsak (3.2) ni hosil qilamiz.

3.1.Lemma isbot bo'ldi.

<u>3.2. Teoremaning isboti.</u>

Berri Essen tengsizligini ([5]) 3.1. teoremada aniqlangan $W^R(\rho_2 x/\rho)$ va $R(x)$ taqsimot funksiyalar uchun qo'llab quyidagini hosil qilamiz.

$$\begin{aligned} \nabla_1^R \underline{\underline{\det}} \sup_{x \ge 0} | W^R(\rho_2 x/\rho) - R_0(x) | \le \\ \le b_0 \int_{-T}^{T} | \frac{\omega^R(-\rho ti/\rho_2) - \tau_0(-ti)}{ti} | dt + \\ + \tau(b_0) c^{(3)} T^{-1} \end{aligned} \qquad (3.17)$$

bu yerda $b_0 > (2\pi)^{-1}$, $T-$ixtoyoriy musbat son, $T(b_0)$-faqat b_0 ga bog'liq bo'lgan musbat o'zgarmas, $i-$mavhum birlik, $\omega^R(-\rho ti/\rho_2)$ va $r(-ti)$ lar mos ravishda $\omega^R(\rho_2 x/\rho)$ va $R(x)$ taqsimot funksiyalarga mos xarakteristik funksiyalar, o'zgarmas $C^{(3)}$ esa $\sup_{x \ge 0} | R'(x) | C^{(3)}$ shartdan aniqlanadi.

Shuni ta'kidlab o'tamizki, $R'(x)$ funksiya musbat yarim o'qda musbat, uzluksiz va $R'(0)=0,\ R'(+\infty)=0$. Shuning uchun $R'(x)$ funksiya o'zining eng katta qiymatiga erishadi.

$s=-ti$ ni hisobga olsak, 3.1. lemmaning tasdig'ini va

$r(-ti)=\int\limits_0^\infty \frac{e^{-z}}{1-tzi}dz$ ni (3.17) ga qo'ysak, quyidagini hosil qilamiz.

$$\nabla_1^R \le b_0 \int\limits_{-T}^{T} [\int\limits_0^\infty \frac{|\varphi-(ti,z)|}{|t/\backslash} dz]dt\cdot \tag{3.18}$$
$$\cdot\rho^{\delta-2}(1+0\rho(1))+\tau(b_0)C^{(3)}T^{-1}$$

(3.3) ni

$$bR_1^{(1)}(T)=\int\limits_{-T}^{T}[\int\limits_0^\infty \frac{|\varphi(-ti,z)}{|t|}dt$$

ga qo'ysak osongina quyidagini hosil qilamiz.

$$R_1^{(1)}(T)\le 2\chi(\delta=3)\{\frac{1}{6}\int\limits_0^T[\int\limits_0^\infty \frac{z^3e^{-z}}{\sqrt{1+(zt)^2}}]dt+$$
$$+\int\limits_0^T[\int\limits_0^\infty \frac{z^3e^{-z}}{\sqrt{1+(zt)^2}}dt]dt\}+\frac{4\varepsilon}{\beta_2\rho_2^{\delta-2}}\cdot \tag{3.19}$$
$$\cdot+\int\limits_0^T[\int\limits_0^\infty \frac{t^{\delta-2}z^{\delta-1}e^{-z}}{1+(zt)^2}dz]dt.$$

Shuni ko'rsatamizki,

$$\int\limits_0^\infty \frac{t^{\delta-2}z^{\delta-1}e^{-z}}{1+(zt)^2}dt$$

Integral $[0,T]$ kemada $t-$ lar bo'yicha tekis yaqinlashuvchi.

Haqiqatdan

$$0\le\int\limits_0^\infty \frac{t^{\delta-2}z^{\delta-1}e^{-z}}{1+(zt)^2}dz\le T^{\delta-2}\int\limits_0^\infty z^{\delta-1}e^{-z}dz=T^{\delta-2}\Gamma(\delta)$$

Huddi shunday (3.19) tengsizlikning o'ng tomonidagi qolgan integrallar ham $[0,T]$ kesmada t lar bo'yicha tekis yaqinlashishini osongina ko'rsatish mumkin.

Bularni e'tiborga olsak, integralning tartibini almashtirish teoremasiga ko'ra ([6]) (3.19) tengsizliklarni quyidagicha ifodalash mumkin.

$$R_1^{(1)}(T) \le 2\chi(\delta=3)\{\frac{1}{6}\int_0^{\infty}[\int_0^T \frac{dt}{\sqrt{1+(zt)^2}} \cdot$$

$$\cdot z^3 e^{-z} dz + \int_0^{\infty}[\int_0^T \frac{t^{\delta-2}dt}{\sqrt{1+(zt)^2}}]ze^{-z}dz\} + \tag{3.20}$$

$$+\frac{4\varepsilon}{\beta_2\rho_2^{\delta-2}}\int_0^{\infty}[\int_0^T \frac{dt}{1+(zt)^2}]z^{\delta-2}e^{-z}dt$$

Quyidagi integrallarni hisoblaymiz.

$$R_2^{(1)}(T,z) = \int_0^T \frac{dt}{\sqrt{1+(zt)^2}}$$

va

$$R_3^{(1)}(T,z) = \int_0^T \frac{t^{\delta-2}}{1+(zt)^2}dt$$

$$R_2^{(1)}(T,z) = \frac{1}{2}\int_0^{zt}\frac{du}{\sqrt{1+u^2}} = \frac{1}{z}\ln|u+\sqrt{1+u^2}\,\|_0^{zt}$$

Bundan,

$$R_2^{(1)}(T,z) = \frac{1}{z}\ln(2T+\sqrt{1+(zT)^2} \tag{3.21}$$

$$R_3^{(1)}(T,z)=\frac{1}{2z^{\delta-1}}\int_0^{(zt)^2}\frac{u^{(\delta-3)/2}du}{1+u}=$$

$$=\frac{1}{2z^{\delta-1}}[\chi(\delta<3)\int_0^{(zt)^2}\frac{u^{(\delta-3)/2}du}{1+u}+$$

$$+\chi(\delta=3)\int_0^{(zt)^2}\frac{du}{1+u}+]=\frac{\chi(\delta<3)T^{\delta-3}}{2z^2}\cdot$$

$$\cdot\int_0^1\frac{x^{(\delta-3)/2}}{x+(zt)^2}+\frac{\chi(\delta=3)}{2z^2}\ln(1+(zT)^2)=$$

$$=\frac{\chi(\delta<3)}{2z^2}T^{\delta-3}[\int_0^\infty\frac{x^{(\delta-3)/2}dx}{x+(zt)^{-2}}-$$

$$-\int_1^\infty\frac{x^{(\delta-3)/2}dx}{x+(zt)^{-2}}]+\frac{\chi(\delta=3)}{2z^2}\ln(1+(zt)^2)\quad.$$

Bundan quyidagi osongina lelib chiqadi.

$$R_3^{(1)}(T,z)=\frac{\chi(\delta=3)}{2z^2}\ln(1+(zt)^2)+$$

$$+\frac{\chi(\delta<3)}{2z^{\delta-1}}\cdot\frac{\pi}{\sin\frac{\delta-1}{2}\pi}-R_4^{(1)}(T,z)\frac{\chi(\delta<3)}{2z^2}T^{\delta-3}\qquad(3.22)$$

bu yerda

$$R_4^{(1)}(T,z)=\int_1^\infty\frac{x^{(\delta-3)/2}dx}{x+(zT)^{-2}}$$

Agar $T=\rho^{-1}$deb olsak va

$$\ln(zt+\sqrt{1+(z\pi)^2}=\ln(z+\sqrt{\rho^2+z^2}-\ln\rho$$

$$\ln(1+(zt)^2=\ln(\rho^2+z^2)-2\ln\rho$$

larni hisobga olsak (3.21) va (3.22) lardan quyidagilarni hosil qilamiz.

$$R_2^{(1)}(\rho^{-1},z)=\frac{1}{z}[\ln(z+\sqrt{\rho^2+z^2}-\ln\rho)]\qquad(3.23)$$

$$R_3^{(1)}(\rho^{-1},z)=\frac{\chi(\delta=3)}{2z^2}[\ln(\rho^2+z^2)-2\ln\rho]+$$
$$+\frac{\chi(\delta<3)}{2z^{\delta-1}}\cdot\frac{\pi}{\sin\frac{\delta-1}{2}}-R_4^{(1)}(\rho^{-1},z)\cdot\frac{\chi(\delta<3)}{2z^2}\cdot\rho^{-(\delta-3)} \qquad (2.24)$$

U holda (3.23) va (3.21) larni (3.20) ga qo'ysak quyidagini hosil qilamiz.

$$R_3^{(1)}(\rho^{-1},z)\le\frac{1}{3}\chi(\delta=3)[R_5^{(1)}(\rho)+$$
$$+\frac{\beta_3}{\beta_2\rho_2}R_6^{(1)}(\rho)-(8+\frac{\beta_2}{\beta_2\rho_2})\ln\rho]+ \qquad (3.25)$$
$$+\frac{2\varepsilon\chi(\delta<3)}{\beta_2\rho_2^{\delta-2}}[\frac{\pi}{\sin\frac{\delta-1}{2}\pi}-R_7^{(1)}(\rho)\rho^{3-\delta}]$$

bu yerda

$$R_5^{(1)}(\rho)=\int_0^\infty(z^2+6)e^{-z}\ln(z+\sqrt{\rho^2+z^2})dz$$
$$R_6^{(1)}(\rho)=\int_0^\infty e^{-z}\ln(\rho^2+z^2)dz$$
$$R_7^{(1)}(\rho)=\int_0^\infty R_4(\rho^{-1},z)z^{\delta-3}e^{-z}dz$$

Yuqoridagi $R_5^{(1)},R_6^{(1)}(\rho)$ va $R_7^{(1)}(\rho)$ integrallarning $0\le\rho\le1$ kesmada tekis yaqinlashishini ko'rsatamiz.

Haqiqatdan

$$0\le\int_0^\infty(z^2+6)e^{-z}\ln(z+\sqrt{\rho^2+z^2})dz\le$$
$$\le\int_0^\infty(z^2+6)e^{-z}\ln(z+\sqrt{1+z^2})dz=$$
$$=\int_0^\infty(z^2+6)e^{-z}[\int_0^z\frac{dx}{\sqrt{1+x^2}}]dz\le$$
$$\le\int_0^\infty z(z^2+6)e^{-z}dz=12$$

Huddi shunday $R_6^{(1)}(\rho)$ va $R_7^{(1)}(\rho)$ intervallar ham $0 \le \rho \le 1$ kesmada tekis yaqinlashadi.

U holda (3.25) ni e'tiborga olsak quyidagini hosil qilamiz.

$$
\begin{aligned}
& R_1^{(1)}(\rho^{-1})\rho^{\delta-2}(1+0_\rho(1)) \le \frac{2\varepsilon\chi(\delta<3)}{\beta_2\rho_2^{\delta-2}} \cdot \\
& \cdot \frac{\pi}{\sin\frac{\delta-1}{2}\pi}\rho^{\delta-2}(1+0_\rho(1)) - (\frac{8}{3}+\frac{2\beta_3}{3\beta_2\rho_2}) \cdot \qquad (3.26) \\
& \cdot \chi(\delta=3)\rho\ln(1+0_\rho(1))
\end{aligned}
$$

(3.26) ni (3.18) ga qo'ysak 3.2 teoremaning tasdigini $(b<b_0>(2\pi)^{-1})$ uchun hosil qilamiz.

XULOSA

Bitiruv malakaviy ishida “FIFO” va “RS” tartibli $M|G|1|\infty$ sistemaning bandlik davri va statsionar kutish vaqti uchun kritik zagruzkada olingan limit teoremalarning yaqinlashish tezligi baholangan.

Yuqorida keltirilgan 1.2., va 3.2. teoremalarning isboti davomida baholar olishga erishdim.

Bir kanalli $M|G|1|\infty$ modelli ommaviy xizmat ko’rsatish sistemasi quyidagicha ifodalaniladi:

Sistemaga a parametrli Puasson taqsimotli bog’liqsiz talablar kelib tushadi. Talablarga xizmat ko’rsatish vaqtining uzunligi ixtiyoriy $B(x)$, $B(+0)=0$ taqsimot funksiyaga ega.

Limit teoremalar $B(x)$ taqsimot funksiyaning birinchi ikkita β_1 va β_2 momentlarning chekliligi shartida ya’ni $S\to 0$, $(\mathrm{Re}\,S\geq 0)$ da

$$\beta(x)=\int_0^\infty e^{-sx}dB(x)=1-\beta_1\cdot s+\frac{1}{2}\beta_2\cdot s^2(1+0_s(1)) \qquad (1)$$

yoyilma o’rinli bo’lgan sharti ostida isbot qilingan. Shu bilna birga bu limit teoremaning yaqinlashish tezligini baholashda limit teoremalarga qo’yilganshartlar qanoatlantiriladi.

Mazkur birtiruv malakaviy ishda ba’zi qo’shimcha shartlar bajarilganda limit teoremalarning yaqinlashish tezligi baholanadi.Yuqorida keltirilgan (1) shart o’rniga quyidagi kuchliroq shart bajarilishi faraz qilinadi: $S\to 0$, $(\mathrm{Re}\,S\geq 0)$ da

$$\beta(s)=1-\beta_1\cdot s+\frac{1}{2}\beta_2\cdot s^2-\varepsilon s^{\delta}(1+0_s(1)) \qquad (2)$$

yoyilma o’rinli bo’lsin, bu yerda $2<\delta\leq 3$, ε -musbat o’zgarmas son.

Biz ko’rib o’tayotgan $M|G|1|\infty$ ommaviy xizmat ko’rsatish sistemasida quyidagi xarakteristkalar asosiy rol o’ynaydi.

$\pi-$, $M|G|1|\infty$ sistemaning bandlik davri, $\pi(u)$, $(u\geq 0)$ -u zagruzkali badlik davri, ω- talablarning sistemada statsionar kutish vaqti, $\rho=1-a\cdot\beta$-

sistemadagi talablarning zagruzkasi, ya’ni bir birlik vaqt ichida sistemaga kelib tushgan talablarga o’rtacha xizmat ko’rsatish vaqti.

Statsionar kutish vaqti sistemada qabul qilingan xizmat ko’rsatish tartibiga bog’liq bo’ladi:

“FIFO ”- birinchi kelgan talablarga birinchi xizmat ko’rsatiladi.

“RS” – xizmat ko’rsatish uchun talablar tasodifiy tanlanadi.

Mazkur bitiruv malakavi ishida ba’zi limit teoremalar o’rganiladi va bu limit teoremalarning yaqinlashish tezligi kritik zagruzkada baholanadi.

Bitiruv malakaviy ishi kirish qismi, uchta paragraf, xulosa va foydalanilgan adabiyotlardan iborat.

Bitiruv malakaviy ishida $M|G|1|\infty$ sistemaning u- zagruzkali bandlik davri uchun limit teorema keltirilgan va shu limit teoremaning yaqinlashish tezligini kritik zagruzkadagi bahosi olingan.

Ushbu ishda mos ravishda “ FIFO” va “RS” tartibli $M|G|1|\infty$ modelli ommaviy xizmat ko’rsatish sistemasidagi talablarning statsionar kutish vaqti uchun olingan limit teoremalar keltirilgan va bu teoremalarning kritik zagruzkada yaqinlashish tezligi baholangan.

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