O’ZBEKISTON RESPUBLIKASI

OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI

SAMARQAND DAVLAR UNIVERSITETI

Qo’lyozma huquqida

UDK:517.5

USANOV KAMOLIDDIN HOLBOYEVICH

TEKISLIKDA ELASTIK TENGLAMALAR SISTEMASI UCHUN CHEGARALANMAGAN SOHADA KOSHI MASALASINI YECHISH

5A 460 107-Matematik fizika

Magistr

akademik darajasini olish uchun yozilgan

DISSERTATSIYA

Ish ko’rib chiqildi va himoyaga

ruxsat berildi.

“Matematik fizika va funksianal

analiz” kafedrasi mudiri:

dots. A.M.Xalxujaev______

Ilmiy rahbar:

dots. I.E.Niyozov____

SAMARQAND-2013

M u n d a r i j a

K i r i sh

Ishning umumiy tavsifi. Bu ishda elastiklik nazariyasi tenglamalari sistemasi yechimini tekislikda uning berilgan qiymatlari va chegaraning qismida uning kuchlanishi qiymatlari bo'yicha davom ettirish masalasi, ya'ni elastiklik nazariyasi tenglamalari sistemasi uchun Koshi masalasi o'rganiladi.

Elastiklik nazariyasi masalalari yechimi uchun sohaning butun chegarasida u yoki bu chegaraviy shartlarning berilishi talab etiladi. Klassik masalalarda bu ko'chish vektorning berilishi, sohaning butun chegarasida kuchlanish vektorining berilishi yoki chegaraning bir qismida ko'chish, ikkinchi qismida esa kuchlanish vektorining berilishidan iborat. Boshqa masalalarda chegaraning har bir qismida ko'chish va kuchlanishlar komponentalarining kerakli miqdori kombinatsiyasi berilgan. Ammo ko'plab masalalarda chegaraning qismi na ko'chishni na kuchlanishni o'lchashga qodir yoki faqat ba'zi integral xarakteristikalar ma'lum bo'ladi. Tajribani izlanishda tabiiy jismning kuchlanishli deformatsiyalangan holatini o'rganishda o'lchashlar sirtning faqat mumkin bo'lgan qismida o'tkazilishi mumkin. Bunday masalalar yechimi tajribaviy ma'lumotlar bo'lmaganligi sababli ma'lum qiyinchiliklarni keltirib chiqaradi. Shunga o'xshash vaziyatlar, masalan, geomexanika masalalarida vujudga keladi.

$D - R^2$ da chegaralangan bir bog'lamli soha, ∂D - uning chegarasi, $S - \partial D$ ning silliq qismi.

D elastik muhitda vektor ko'rinishdagi quyidagi tenglamani qaraymiz:

$$\mu \Delta u(y) + (\lambda + \mu) grad\ div\ u(y) = 0 \qquad (0.1)$$

bu yerda $u=(u_1;u_2)$ - ko'chish vektori, λ,μ - qaralayotgan elastik muhitning Lame doimiylari; Δ - Laplas operatori.

$u(y)-D$ sohada (0.1) sistemaning regulyar yechimi bo'lsin va S da quyidagi Koshi shartlarini qanoatlantirsin:

$$\left.\begin{array}{ll} u(y)=f(y), & y\in S \\ T(\partial_y,n)u(y)=g(y), & y\in S \end{array}\right\} \qquad (0.2)$$

bunda $f=(f_1,f_2)$, $g=(g_1,g_2)-S$ da berilgan uzluksiz vektor – funksiyalar; $T(\partial_y,n)$ - kuchlanish operatori.

Berilgan f va g lardan kelib chiqib, $u(y)\in D$ ni davom ettirish talab etiladi.

Garmonik funksiyalar nazariyasida Laplas operatori qanday rol o'ynasa,

$$\mu\Delta+(\lambda+\mu)grad\ div$$

operatori elastiklik nazariyasida xuddi shunday rol o'ynaydi va xususiy holda, $\mu=1,\ \lambda=-1$ da Laplas operatoriga aylanadi. Ixtiyoriy $f(y)$ va $g(y)$ larda (0.1) masala yechimga ega emas. Agar $f(y)$ va $g(y)$ lar S (S - analitik yoy) da analitik va D da analitik davom ettiriladigan bo'lsa, u holda xususiy hosilali tenglamalar nazariyasining fundamental teoremasiga asosan davom ettirish mumkin va yagonadir. Biroq (0.1) va (0.2) masala Adamar bo'yicha nokorrekt, ya'ni nokorrektligi xuddi Laplas tenglamasi uchun Koshi masalasidagidek.

f va g faqat deyarli yaqin qilib berilishi mumkinligi sababli masalaning yechimiga yaqin yechim to'g'risida gap ketish mumkin. Lekin yechimda turg'unlikning bo'lmaganligidan qo'shimcha ma'lumotlarsiz yaqin yechimning bo'lishi mumkin emas.

A.N.Tixonovning ishlarida, keyin M.M.Lavrentyevning ishlarida rivojlangan bunday masalalar qo'yilishining shartli korrektligi nokorrekt masalalar izlanishi uchun prinsiplarni berdi. Nokorrekt masalalarning shartli korrektligi izlanishida o'rnatilgan yagonalik va turg'unlik teoremalaridan so'ng yechimni qurishning effektiv metodi, ya'ni regulyarizatsiyalanuvchi operatorlarni qurish tug'iladi.

M.M.Lavrentyev o'zining mashhur monografiyasida ko'rsatib bergan edi.

M.M.Lavrentyevdan kelib chiqib, $\varepsilon > 0$ parametrdan bog'liq $\Phi_\varepsilon(y,x)$ Laplas tenglamasining fundamental yechimini $x \in D$ nuqta, S chegara qismining Karleman funksiyasi deb ataymiz, agar quyidagi tengsizlik o'rinli bo'lsa;

$$\int\limits_{\partial D \setminus S} \left[|\Phi_\varepsilon(y,x)| + \left| T(\partial_y, n)\Phi_\varepsilon(y,x) \right| \right] dS_y \le \varepsilon .$$

Karleman formulasining ko'p o'lchovli analogi bilan A.A.Ayzenberg, elliptik sistemalar uchun Karleman matritsasini qurish bilan N.N.Tarxanov shug'ullangan.

M.M.Lavrentyev ideyasidan foydalanib, Sh.Ya.Yarmuxammedov Laplas tenglamasi uchun Koshi masalasining regulyarizatsiyalangan yechimini aniq ko'rinishda qurdi.

Ma'lumki, elastiklik nazariyasining tekislikdagi masalalari yaxshi o'rganilgan. Kompleks o'zgaruvchili analitik funksiyalarning ma'lum metodidan tashqari elastiklik nazariyasining qator statistik masalalari potensiallar nazariyasi va integral tenglamalar metodi bilan ham Fredgolm va Laurachel tomonidan qarab chiqilgan. Ular bilan korrekt masalalar o'rganilgan edi.

f va g o'rniga $\delta, \delta \in (0,1)$ aniqlik (C metrikada) berilgan bo'lsin, qaysiki yechim mavjud bo'lgan sinfga qarashli bo'lmasligi mumkin. Ushbu ishda σ parametrdan bog'liq $u(x, f_\delta, g_\delta) = u_{\sigma\delta}$ vektor funksiyalar oilasi quriladi va biror shart va maxsus $\gamma(\delta), \delta \to 0$ parametr tanlanganda $u_{\sigma\delta}(x)$ oila odatdagi ma'noda (0.1) – (0.2) masalaning yechimiga yaqinlashishi isbotlanadi.

A.N.Tixonovdan kelib chiqib, $u_{\sigma\delta}(x)$ funksiyani (0.1) – (0.2) masala, ya'ni elastiklik nazariyasi sistemasi uchun Koshi masalasining regulyarizatsiyalangan yechimi deb ataymiz. Regulyarizatsiyalangan yechim masalaning metodining turg'unligini aniqlaydi.

Maxsus sohalarda chegaralangan analitik funksiyalarni davom ettirish masalasi chegaraning qismida aniq berilgan holni T.Karleman qarab chiqqan edi. T.Karlemanning izlanishlarini G.M.Goluzin va V.I.Krilovlar davom ettirishdi. Soha yo'lakdan iborat bo'lganda Laplas tenglamasi uchun Koshi masalasining regulyarizatsiyalangan yechimi V.K.Ivanov tomonidan qurilgan. Laplas tenglamasi uchun Koshi masalasining regulyarizatsiyalangan yechimini qurish uchun foydalanilgan.

Ushbu ishning maqsadi ikki o'lchamli bo'lgan hol uchun nazariyani ishlab chiqishdan iborat. Ikkala nazariya ko'plab umumiylikka ega va parallel rivojlanadi.

Ishning maqsadi. Ushbu dissertatsion ishda Karleman matrisasini aniq ko'rinishda qurish keltiriladi va uning asosida (0.1) – (0.2) masalaning regulyarizatsiyalangan yechimi quriladi. Bunda (0.1) sistema chegaralangan sohalarda va yo'lak tipidagi chegaralanmagan sohalarda qaraladi, soha chegarasi ikkita qismdan tashkil topgan, Koshi shartlari esa chegaraning bir qismida beriladi.

Shunday qilib, Karleman matritsasining effektiv qurilishi (0.1) – (0.2) masalaning regulyarizasiyalangan yechimini aniq ko'rinishda yozishga imkon beradi.

Ilmiy yangiligi. Tekis sohalarda elastiklik nazariyasi sistemasining Koshi masalasi oldin qo'yilmagan va o'rganilmagan. Dissertatsiyada maxsus sohalar sinfi uchun (0.1) – (0.2) masalaning regulyarizatsiyalangan yechimi aniq ko'rinishda topilgan.

Tadbiqi. Qo'yilgan masalaning yechimini aniq ko'rinishda qurish katta nazariy va amaliy qiziqish uyg'otadi. Dissertatsiyada olingan natijalar elastiklik nazariyasining matematik masalalari izlanilishida, geomexanikaning masalalarida foydalanilishi mumkin.

§ 1.1 da o'rganiladigan Koshi masalasining qo'yilishi, yordamchi tasdiqlar, mulohazalar keltirilgan. Qaraladigan masala uchun Adamar misoli berilgan.

§ 1.2 – 1.3 paragraflar elastiklik nazariyasi sistemasining fundamental yechimlari matritsasini qurish keltirilgan. Maxsus sohalar sinfi uchun yuqorida ta'kidlangan xossalarga ega elastiklik nazariyasi sistemasining fundamental yechimlari matritsasi aniq ko'rinishda qurilgan. § 2.1 – 2.2 da maxsus chegaralanmagan sohalar sinfi uchun (0.1) – (0.2) masalaning Somilian - Bettining umumlashgan formulasi keltirilgan. 2.3-§. Chegaralanmagan tekis sohada elastiklik nazariyasi sistemalari uchun Koshi masalasi qaralgan.

I-Bob

1.1-§ Tekislikda elastiklik nazariyasi sistemasi yechimi uchun Somilian-Betti formulasi

$x=(x_1,x_2)$, $y=(y_1,y_2)$ R^2 tekislik nuqtasi va elastik D muhit R^2da bo'lakli--siliq ∂D bilan chegaralangan bo'lsin.

Quyidagi vektor ko'rinishida yoziladigan

$$\mu\Delta U(x)+(\lambda+\mu) graddiv U(x)=0 \qquad (1.1)$$

bir jinsli izotropik elastik muhitning asosiy tenglamasini qarayiz. Bu yerda $U=(U_1,U_2)$- siljish vektori, $\lambda,\mu>0$ qaralayotgan elastik muhitning Lame doimiylari; Δ-lanlas operatori.

Qaralayotgan (1.1) tenglama sistema ko'rinishida quyidagicha yoziladi.

$$\left.\begin{aligned}&\mu\Delta U_1(x)+(\mu+\lambda)\left[\frac{\partial^2 U_1(x)}{\partial x^2}+\frac{\partial^2 U_2(x)}{\partial x_1\partial x_2}\right]=0\\&\mu\Delta U_2(x)+(\lambda+\mu)\left[\frac{\partial^2 U_2(x)}{\partial x_2^2}+\frac{\partial^2 U_1(x)}{\partial x_1\partial x_2}\right]=0\end{aligned}\right\} \qquad (1.2)$$

(1.2) sistemani matritsa ko'rinishida yozish qulay shu maqsadda $A(\partial_x)=\left|A_{ij}(\partial_x)\right|_{2x2}$ matritsali differensial operatorni kiritamiz.

Bu yerda $A_{ij}(\partial_x)=\delta_{ij}\mu\Delta+\dfrac{(\lambda+\mu)\partial_x^2}{\partial x_i\partial x_j}$ bunda δ_{ij}-Kroniker belgisi. u holda (1.2) sistemani quyidagicha yozish mumkin $A(\partial_x)U(x)=0$

1.1-ta'rif. D da aniqlangan φ funksiyani ($\varphi=\|\varphi_{ij}\|_{nxm}$ matritsali) D da regulyar deymiz, agar $\varphi\in C^1(\overline{D})\cap C^2(D)$ $(\varphi_{ij}\in C^1(D)\cap C^2(D),\ ixj=nxm)$ va φ

funksiyaning $(\varphi_{ij}, ixj = nxm)$ dekard koordinatalari bo'yicha barcha ikki tartibli xosilalari D sohada integrallanuvchi bo'lsa kelgusida ∂D - D chekli sohani chegaralovchi yopiq egri chiziq deb hisoblaymiz.

Endi mazkur hamda o'rganiladigan elastik nazaryasi sistemasi uchun Koshi masalasining qo'shilishini bayon qilish mumkin. $U(y) = (U_1(y); U_2(y))$ (1.2) sistemaning D sohadagi regulyar yechimi bo'lsinm, y'ani

$$A(\partial_y)U(y) = 0 \qquad (1.3)$$

Bu yechimning $S \subset \partial D$ dagi

$$\left.\begin{array}{ll} U(y) = f(y), & y \in S \\ T(\partial_x, n)U(y) = g(y), & y \in S \end{array}\right\} \qquad (1.4)$$

berilganlariga ko'ra yechimni sohaning ichida aniqlash masalasi, ya'ni Koshi masalasi qaraladi.

Bu yerda $S - \partial D$ ning qismi, $f(y) = (f_1(y); f_2(y))$ va $g(y) = (g_1(y); g_2(y))$ S da berilgan uzluksiz vektor funksiyalar, $n(n_1; n_2)$ - y nuqtadagi S-ga o'kazilgan D ga nisbatan tashqi ortonormal.

$$T(\partial_y, n(y)) = \left\| T_{ij}(\partial_y, n(y)) \right\|_{2x2} = \left\| \lambda n_i \frac{\partial}{\partial y_j} + \mu n_j \frac{\partial}{\partial y_i} + \mu \delta_{ij} \frac{\partial}{\partial n(y)} \right\|_{2x2}$$

Bu yerda $T(\partial_y, n(y))$ ni kuchlanish operatori deb ataymiz.

(1.3) sistema elliptikdir. uning elliptikligi shundan iboratki, $\xi_1^2 + \xi_2^2 \neq 0$ tenglikni qanoatlantiruvchi ixtiyoriy haqiqiy ξ_1 va ξ_2 lar uchun $\det A(\xi)$ determinant noldan farqli bo'lishi lozim, bu yerda $\det A(\xi) = \mu(\lambda + 2\mu)(\xi_1^2 + \xi_2^2)$.

Shuning uchun $\lambda, \mu > 0$ ekanligini hisobga olib (elastiklik nazariyasida shunday faraz qilinadi) (1.3) elliptik sistemadan iborat ekanligini tasdiqlash mumkin. Malumki, elliptik tenglamalar uchun Koshi masalasi Adamar bo'yicha nokorrekt. Demak (1.3)-(1.4) masala nokorrekt masalalar qatoriga kiradi. (1.3)-(1.4) masala yechimi mavjud bo'lsa u yagona lekin turgun emas. (1.3)-(1.4) masalalarning noturg'unligi quyidagi misoldan kelib chiqadi.

1.1-Misol. $U(y)=(U_1(y),U_2(y))$ vektor funksiya $y>0$ yarim tekisligida elastiklik nazariyasi sistemasining regulyar yechimi bo'lsin, yani $A(\partial_y)U(y)=0$ va Koshi shartlarini qanoatlantirsin

$$U(y_1,0)=\left(0,\qquad \frac{cos\,(\sigma\,\mathrm{y}_1)}{\sigma^2}\right),$$

$$T\left(\partial_y,n(y)\right)U(y_1,0)=\left(0,\qquad \frac{2(\lambda+\mu)cos\,(\sigma\,\mathrm{y}_1)}{\sigma}\right)$$

Bu masalaning echimi

$$U(\mathrm{y})=\left(\frac{\sin(\sigma\mathrm{y}_1)\,sh\,(\sigma\mathrm{y}_2)}{\sigma^2},\qquad \frac{\cos(\sigma\mathrm{y}_1)\,ch\,(\sigma\mathrm{y}_2)}{\sigma^2}\right)$$

ga teng.

Ko'rish qiyin emaski yetarli katta σ da $y_2=0$ chegarada

$$|U(y)|=\frac{|cos\,(\sigma\,\mathrm{y}_1)|}{\sigma^2}\ ,\qquad \left|T\left(\partial_y,n(y)\right)U(y)\right|=\frac{2(\lambda+\mu)|cos\,(\sigma\,\mathrm{y}_1)|}{\sigma}\,.$$

boshlang'ich shartlar yetarlicha kichik bo'ladi. Yechimning o'zi esa $y_1\neq 0,\ y_2\neq 0,\ y_2>0$ da yetarlicha katta bo'ladi.

Qaralayotgan masala shartli nokorrekt masalalar jumlasiga kiradi. Ya'ni masalani qandaydir funksional fazolarda qaralsa masala korrekt yechiladigan masalaga kelib qoladi. Bunday korrektlik sharti qaralayotgan yechimlar sinfini kompaktgacha qisqartirtishdan iborat. Nokkorekt masalalarda odatda yechimning mavjudligi va yagonaligi oldindan faraz qilinib uning yechimini topish masalasi o'rganiladi.

Kelgusida (1.3) sistemaning logorifmik maxsuslikka ega bazi yechimlari muhim rol o'ynaydi.

1.2–ta'rif.

$$\Gamma(y-x) = \left\| \lambda' \delta_{kj} \ln\frac{1}{|x-y|} + \frac{\mu'(y_k - x_k)(y_j - x_j)}{r^2} \right\|_{2\times 2}, \qquad (1.5)$$

matritsa (1.2) elastiklik nazariyasi sistemaning fundamental yechimlari matrisasi deb ataladi ya'ni $A(\partial_x)\Gamma(y-x)=0, \quad x \neq y$. Bu yerda $k,j=1,2;$

$$\lambda' = \frac{\lambda+3\mu}{4\pi\mu(\lambda+2\mu)}; \quad \mu' = \frac{\lambda+\mu}{4\pi\mu(\lambda+2\mu)}.$$

Fundamental yechimlar matritsasi $\Gamma(y-x)$ quyidagi xossalarga ega.

1. $\Gamma(y-x)=\Gamma(x-y)$.

2. $|\Gamma(y-x)| \le C_1(\lambda,\mu)\ln\frac{1}{r}$.

3. $|T(\partial_y,n)\Gamma(y-x)| \le C_2\frac{(\lambda,\mu)}{r^2}$

Bularda $C_k(\lambda,\mu),\ k=1,2$ faqat λ va μ larga bog'liq musbat koifesentlar $r=|x-y|$.

$$|\Gamma(y-x)| = \left\{\sum_{k,j=1}^{2}\Gamma_{kj}^2(x-y)\right|^{1/2}$$

Bu xossalar $\Gamma(y-x)$ matritsaning ta'rifiga ko'ra oson isbotlanadi. Xususiy xosilali differensial tenglamalar nazariyasida Grin, Gaus, Stoks- Ostogradskiy formulalari tipidagi bazi integral ayniyatlar muhim rol o'ynaydi. Bu ayniyatlat elastik nazariyasida bazida Somilian-Betti formulasi deb ataladi.

Betti formulasining [14] natijasini keltiramiz.

1.1-Teorema. $U=(U_1,U_2)$ va $V=(V_1,V_2)$ D soxada regulyar vektor-funksiyalar bo'lsa, u holda

$$\int_D [V(x)\{A(\partial_x)U(x)\}-U(x)\{A(\partial_x)V(x)\}]dx =$$

$$=\int_{\partial D}[V(y)\{T(\partial_y,n)U(y)\}-U(y)\{T(\partial_y,n)V(y)\}]dS_y \quad (1.6)$$

Bundan U va V vektorlar (1.2) sistemaning regulyar yechimlari bo'lsa, quyidagi tenglikni oson ko'rish mumkin, ya'ni

$$\int_D [V(y)\{T(\partial_y,n)U(y)\}-U(y)\{T(\partial_y,n)V(y)\}]dS_y =0$$

1.2 –Teorema. (1.1) tenglamaning D sohadagi ixtiyoriy U regulyar yechimi uchun

$$U(x)=\int_{\partial D}[\Gamma(y-x)\{T(\partial_y,n)U(y)\}-U(y)\{T(\partial_y,n)\Gamma(y-x)\}]\,dS_y \quad (1.7)$$

formula o'rinli.

Isbot: $U=(U_1,U_2)$-D sohada aniqlangan ixtiyoriy vector –funksiya, z- bu soxaning ixtiyoriy nuqtasi bo'lsin. $\varepsilon>0$ni shunday tanlaymizki $\overline{Ш(z,\varepsilon)}\subset D$ bo'lsin va (1.6) formulaning $D\setminus Ш(z,\varepsilon)$ sohada qo'llaymiz. V- funksiya sifatida (1.6) formulada $\Gamma(y-x)$fundamental yechimlar matritsasining j-ustunini $\Gamma^j(y-x)$ olamiz. Ravshanki, $\Gamma^j(y-x)$

$$A(\partial_x)V(x)=0$$

tenglamaning ixtiyoriy $\varepsilon>0$ uchun D\Ш(z,ε) sohadagi regulyar yechimi bo'ladi. Demak,

$$\int_{D\backslash Ш(z,\varepsilon)} \Gamma^j(\mathrm{y}-z)\mathrm{A}(\partial_x)U(x)\,dx =$$

$$=\int_{\partial D}\left[\Gamma^j(\mathrm{y}-z)\left\{T\left(\partial_y,n(y)\right)U(y)\right\}-U(y)\left\{T\left(\partial_y,n(y)\right)\Gamma^j(\mathrm{y}-z)\right\}\right]ds_y \; +$$

$$+\int_{C(z,\varepsilon)}\left[\Gamma^j(\mathrm{y}-z)\left\{T\left(\partial_y,n(y)\right)U(y)\right\}\right.$$

$$\left.-\,U(y)\left\{T\left(\partial_y,n(y)\right)\Gamma^j(\mathrm{y}-z)\right\}\right]ds_y\,, \qquad (1.8)$$

bu yerda $Ш(z,\varepsilon)$ markizi z nuqtada bo'lgan radiusi ε ga teng shar. Bu tenglik ixtiyoriy $\varepsilon>0$ uchun o'rinli .Shuning uchun,

$$\lim_{\varepsilon\to 0}\int_{D\backslash Ш(z,\varepsilon)} \Gamma^j(\mathrm{y}-z)\mathrm{A}(\partial_x)U(x)\,dx = \int_D \Gamma^j(\mathrm{y}-z)\mathrm{A}(\partial_x)U(x)\,dx \qquad (1.9)$$

$$\lim_{\varepsilon\to 0}\int_{C(z,\varepsilon)}\left[\Gamma^j(\mathrm{y}-z)\left\{T\left(\partial_y,n(y)\right)U(y)\right\}\right]ds_y=0 \qquad (1.10)$$

$$\lim_{\varepsilon\to 0}\int_{C(z,\varepsilon)}\left[U(y)\left\{T\left(\partial_y,n(y)\right)\Gamma^j(\mathrm{y}-z)\right\}\right]ds_y = U_j(z) \qquad (1.11)$$

formulalardan foydalanib

$$U_j(z)=-\int_D \Gamma^j(\mathrm{y}-z)\mathrm{A}(\partial_x)U(x)\,dx +$$

$$+\int_{\partial D}\left[\Gamma^{j}(\mathrm{y}-z)\left\{T\left(\partial_{y},n(y)\right)U(y)\right\}-\right.$$

$$\left.-U(y)\left\{T\left(\partial_{y},n(y)\right)\Gamma^{j}(\mathrm{y}-z)\right\}\right]ds_{y}\,. \quad (1.12)$$

ni olamiz. Endi (1.9),(1.10)va (1.11) formulalarning o'rinlarini ko'rsatamiz.

$$J_1=\int_{C(z,\varepsilon)}\left[\Gamma^{j}(\mathrm{y}-z)\left\{T\left(\partial_{y},n(y)\right)U(y)\right\}\right]ds_{y}\,,$$

$$J_2=\lim_{\varepsilon\to 0}\int_{C(z,\varepsilon)}\left[U(y)\left\{T\left(\partial_{y},n(y)\right)\Gamma^{j}(\mathrm{y}-z)\right\}\right]ds_{y}\,,$$

$$|J_1|\le\int_{C(z,\varepsilon)}\left|\Gamma^{j}(\mathrm{y}-z)\right|\left|T\left(\partial_{y},n(y)\right)U(y)\right|ds_{y}\le$$

$$\le\max_{y\in C(z,\varepsilon)}\left|T\left(\partial_{y},n(y)\right)U(y)\right|\int_{C(z,\varepsilon)}C(\lambda,\mu)\ln\frac{1}{|x-y|}ds_{y}=$$

$$2\pi\varepsilon\max_{y\in C(z,\varepsilon)}\left|T\left(\partial_{y},n(y)\right)U(y)\right|\mathrm{C}(\lambda,\mu)\,ln\,(1/\varepsilon)$$

larni qaraymiz, bundan (1.10) kelib chiqadi.Fundamental yechim matritsasi xossasidan

$$\left|\int_{C(z,\varepsilon)}[U(y)-U(z)]\left\{T\left(\partial_{y},n(y)\right)\Gamma^{j}(\mathrm{y}-z)\right\}ds_{y}\right|\le$$

$$\le C\max_{y\in \text{Ш}(z,\varepsilon)}|U(y)-U(z)|\,, \quad (1.13)$$

ni olamiz, bu yerda $C=const$. Demak (1.11) ni isbotlash uchun

$\lim_{\varepsilon\to 0} U(z) \int_{C(z,\varepsilon)} \left\{T\left(\partial_y, n(y)\right)\Gamma^j(\mathrm{y}-z)\right\} ds_y \;= U_j(z)$ ni isbotlash yetarli.

$$\int\limits_{C(z,\varepsilon)} T(\partial_y, n)T^j(y-z)dS_y = \begin{cases} (1{,}0), & agar \quad j=1 \\ (0{,}1) & agar \quad j=2 \end{cases} \qquad (1.14)$$

haqiqatdan, agar $y \in C(z,\varepsilon)$ bo'lsa, u holda

$$n_k(y) = -(y_k - z_k)|y-z|^{-1}, \; n_k(y)(y_j - z_j) - n_j(y)(y_k - z_k) = 0$$

$$|y-z|^2 \sum_{i=1}^{z} n_i(y)(y_i - z_i) = -\varepsilon^{-1}$$

va $T(\partial_y, n)\Gamma^j(y-z)$ ifodadan (1.14) kelib chiqadi. (1.13) va (1.14) dan (1.11) kelib chiqadi. (1.19) bevosita $|\Gamma(y-x)| \le C\ln\frac{1}{|x-y|}$ dan kelib chiqadi.

Endi agar, xususan (1.3) sistemaning D sohadagi regulyar yechimi bo'lsa, u holda biz (1.7) ni olamiz. Shu bilan teorema isbotlandi. (1.7) formulani Somilian-Betti formulasi deb ataymiz.

1.2-§. Elastiklik nazriyasi sistemasi

fundamental yechimini qurish

Laplas tenglamasi uchun Koshi masalasini yechishda Karleman funksiyasini tushunchasini birinchi bo'lib M.M. Lavrentev [18] kiritgam. Shu ta'rifdan foydalanib biz Karleman matrisasi tarifini beramiz.

1.3- Ta'rif. $(2x2)$ o'lchovli $\Pi_\sigma(y,x)$, $\sigma > 0$ parametrga bog'liq $y \neq x$ da aniqlangan matritsaga $x \in D$ va $\partial D \setminus S$ qism uchun Karleman matritsasiqaraymiz, agar u quyidagi shartlarni qanoatlantirsa

1) $\Pi_\sigma(y,x) = \Gamma(\mathrm{y}-x) + G_\sigma(\mathrm{y},x)$ (1.15)

Ko'rinishida ifodalansa, bu holda $G_\sigma(\mathrm{y},x)$-D da $y = x$ ni qo'shgan holda (1.3) sistemani qanoatlantiruvchi $(2x2)$ o'lchovli matritsa, ya'ni

$$A(\partial_y)G_\sigma(x,y) = 0 \qquad (1.16)$$

2) Tayinlangan $x \in D$ da $\Pi_\sigma(y,x)$ matritsa

$$\int_{\partial D \setminus S} \left[\left| \Pi_\sigma(\mathrm{y,x}) \right| + \left| T(\partial_y, n)\Pi_\sigma(\mathrm{y,x}) \right| \right] dS_y \leq \varepsilon(\sigma) \qquad (1.17)$$

tengsizlikni qanoatlantiradi, bu yerda $\varepsilon(\sigma) \to 0$, $\sigma \to \infty$.

D soxa va $\partial D \setminus S$ qismi uchun Karleman matritsasi ma'lum bo'lsin. Karleman matritsasi yordamida (1.3)-(1.4) masala yechimining turg'unligini oson keltirib chiqarish mumkin, hamda bu masalani yechish usulini ko'rsatish mumkin.

1.3-Teorema. (1.3) tenglamaning D da regulyar yechimi

$$U(x) = \int_{\partial D} \left[\Pi_\sigma(\mathrm{y,x}) \{ T(\partial_y, n)U(y) \} - U(y) \{ T(\partial_y, n)\Pi_\sigma(\mathrm{y,x}) \} \right] dS_y \qquad x \in D \quad (1.18)$$

ko'rinishida ifodalash mumkin, bu yerda $\Pi_\sigma(y,x)$ - D soxa uchun Karleman matritsasi.

Isbot. (1.15) ga ko'ra Somilian- Betti formulasini qo'llab, bunda V sifatida $G_\sigma(y,x)$ regulyar yechimlar matritsasining ixtiyoriy $G_\sigma^j(y,x)$ ustunini olamz va (1.16) ga ko'ra $G_\sigma(y,x)$ (1.3) tenglamaning regulyar yechimi bo'ladi. u holda

$$\int_{\partial D}\left[G_\sigma(y,x)\{T(\partial_y,n)U(y)\}-U(y)\{T(\partial_y,n)G_\sigma(y,x)\}\right]dS_y=0.$$

Bu ma'lumot asosida (1.7) Somilian-Betti integral formulasidan $\Gamma(y-x)$ matritsasini $\Pi_\sigma(y,x)$ Karleman matritsasi bilan almashtirish mumkin. Demak, (1.7) formula faqat shu holda saqlanadiki, agar $\Gamma(y-x)$ ga ixtiyoriy $G_\sigma(y,x)$ regulyar yechimlar matritsasini qo'shsak, ya'ni (1.18) o'rinli bo'sa.

Elastiklik nazariyasi sistemasining maxsus ko'rinishdagi fundamental yechimlari matritsasini qurishga o'tamiz.

$K(\omega)$, $\omega = u + iv$ ($u, v -$ haqiqiy)- haqiqiy ω da haqiqiy butun funksiya,

$$K(u) \neq 0, \qquad \sup_{v\geq 1}\left| v^p K^{(p)}(u+iv)\right| \leq M(p,u) < \infty, \quad p=0,1,2, \quad u \in R^1 \qquad (1.19)$$

shartlarni qanoatlantiruvchi funksiya bo'lsin, $\alpha = |y_1 - x_1|$ deb olamiz. $\alpha > 0$, $y \neq x$ da $\Phi(y,x)$ ni

$$-2\pi K(x_2)\Phi(y,x) = \int_0^\infty Im\,\frac{K(\omega)}{\omega - x_2}\,\frac{udu}{\sqrt{u^2+\alpha^2}} \qquad (1.20)$$

tenglik bilan aniqlaymiz. Bu yerda $\omega = i\sqrt{u^2+\alpha^2} + y_2$.

[3] ishda quyidagi Lemma isbotlanadi.

1.1- Lemma. (1.20) formula bo'yicha aniqlangan $\Phi(\mathrm{y}, x)$ funksiya

$$\Phi(\mathrm{y}, x) = \ ln\left(\frac{1}{|x-\mathrm{y}|}\right) + \ g(\mathrm{y}, x)$$

ko'rinishida ifodalanadi. Bu yerda $g(y,x)$- funksiya barcha x,y qiymatlari uchun aniqlangan va butun R^2 da y o'zgaruvchi bo'yicha garmonik bo'lgan biror funksiya.

(1.20) formula bilan aniqlangan $\Phi(\mathrm{y}, x)$ funksiya yordamida quyidagi matritsani tuzamiz

$$\Pi(\mathrm{y}, x) = \left\|\Pi_{kj}(\mathrm{y}, x)\right\|_{2x2} = \left\|\lambda' \delta_{kj}\Phi(\mathrm{y}, x) - \mu'\left(\mathrm{y}_j - x_j\right)\frac{\partial\Phi(\mathrm{y}, x)}{\partial \mathrm{y}_k}\right\|_{2\times 2} \qquad (1.21)$$

Endi quyidagi asosiy teoremani isbotlaymiz.

1.4- Teorema. (1.21) formula bilan aniqlangan $\Pi(\mathrm{y}, x)$ matritsa

$$\Pi(\mathrm{y}, x) \ = \ \Gamma(\mathrm{y} - x) \ + \ G(\mathrm{y}, x)$$

ko'rinishida ifodalanadi. Bu yerda $\Gamma(\mathrm{y} - x)$ - elastiklik nazariyasi sistemasining fundamental yechimlar matritsasi, $G(y,x) - x,y$ ning barcha qiymatlari uchun aniqlangan va butun R^2 da y o'zgaruvchi bo'yicha (1.2) sistemani qanoatlantiruvchi $(2x2)$ matritsa ya'ni, $\mathrm{A}(\partial_y)\ G\ (\mathrm{y}, x) \ = \ 0.$

Isbot. 1.1 –Lemma va $\Pi(\mathrm{y}, x)$ ning tta'rifiga asosan

$$\Pi(\mathrm{y},x) = \left\| \lambda'\delta_{kj}\Phi(\mathrm{y},x) - \mu'(\mathrm{y}_j - x_j)\frac{\partial\Phi(\mathrm{y},x)}{\partial \mathrm{y}_k} \right\|_{2\times 2} =$$

$$= \left\| \lambda'\delta_{kj}\left[\, ln\,\frac{1}{|x-y|} + g(\mathrm{y},x)\,\right] - \right.$$

$$\left. -\mu'\left(\mathrm{y}_j - x_j\right)\frac{\partial}{\partial \mathrm{y}_k}\ \left[\, ln\,\frac{1}{|x-y|} + g(\mathrm{y},x)\,\right] \right\|_{2\times 2} =$$

$$= \left\| \lambda'\delta_{kj}\ ln\,\frac{1}{|x-y|} - \mu'\left(\mathrm{y}_j - x_j\right)\frac{(\mathrm{y}_k - x_k)}{|x-y|^2} \right\|_{2\times 2} =$$

$$= \left\| \lambda'\delta_{kj}\ g(\mathrm{y},x) - \mu'\left(\mathrm{y}_j - x_j\right)\frac{\partial}{\partial \mathrm{y}_k}\ g(\mathrm{y},x) \right\|_{2\times 2} = \Gamma(\mathrm{y}-x) + G(\mathrm{y},x)$$

bundan (1.22) lelib chiqadi. Endi,

$$G(\mathrm{y},x) = \left\| \lambda'^{\delta_{kj}}\, g(\mathrm{y},x) - \mu'(\mathrm{y}_j - x_j)\frac{\partial}{\partial \mathrm{y}_k}\ g(\mathrm{y},x) \right\|_{2\times 2}$$

(1.3) tenglamaning y o’zgaruvchi bo’yicha $y = x$ ni ham hisobga olganda regulyar yechimdan iborat.

$\Delta_y\, g(\mathrm{y},x) = 0,$

$$\Delta_y = \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2} \quad va\ \ divG^j(y,x) = \frac{1}{2\pi(\lambda+2\mu)}\frac{\partial}{\partial y_j} g(y,x)$$

bo’lgani uchun $\mathrm{A}(\partial_y)\ G^{\,j}(\mathrm{y},x)$ vektorning k -koordinatasi uchun

$$\sum_{i=1}^{2} A_{ki}(\partial_y)G_{ij}(y,x) = \mu\,\Delta\left[\lambda'\delta_{kj}\ g(\mathrm{y},x) - \mu'\left(\mathrm{y}_j - x_j\right)\frac{\partial}{\partial \mathrm{y}_k}\ g(\mathrm{y},x)\right] +$$

$$+(\lambda+\mu)\frac{\partial}{\partial \mathrm{y}_k}\ \ divG^j(y,x)$$

$$=-\frac{(\lambda+\mu)}{2\pi(\lambda+2\mu)}\frac{\partial^2}{\partial y_j^2}g(\mathrm{y},x)+\frac{(\lambda+\mu)}{2\pi(\lambda+2\mu)}\frac{\partial^2}{\partial y_j^2}g(\mathrm{y},x)=0.$$

ga ega bo'lamiz. Demak $G(y,x)$ matritsaning hadi bir ustuni y o'zgaruvchi bo'yicha (1.3) tenglamaning butun R^2 ga qanoatlantiradi. Shunday qilib biz tuzgan (1.21) va (1.22) formulalar bilan aniqlangan $\Pi(y,x)$ matritsa Karleman matritsasi ekan. Teorema to'liq isbotlandi.

1.3-§. Maxsus sohalarda elastiklik nazariyasi sistemasining Karleman matritsasi

(1.20) va (1.21) formulalarda $K(w)$ funksiyani aniq ko'rinishda tanlab, elastiklik nazariyasi sistemasi uchun Koshi masalasining Karleman matritsasini sohalarning keng sinfi uchun aniq ko'rinishda quramiz.

1. D elastik muhit R^2 da chegaralanmagan bir bog'lamli, chegarasi $y_2 = 0$ haqiqiy o'qning l bo'lagi va $y_2 > 0$ yarim tekislikda yotuvchi silliq S egri chiziqdan iborat bo'lsin, ya'ni D qalpoq shaklidagi soha.

$\sigma > 0$ da (1.20) formulada

$$K(w) = \exp \sigma w,\ w = i\sqrt{u^2 + \alpha^2}$$

ifodani qo'yamiz va quyidagiga ega bo'lamiz:

$$-2\pi K(x_2)\Phi_\sigma(y,x) = \int_0^\infty \operatorname{Im} \frac{\exp(\sigma w)}{w - x_2} \cdot \frac{udu}{\sqrt{u^2 + \alpha^2}} \qquad (1.23)$$

Shuni ta'kidlaymizki, bu yerda tanlangan $K(w)$ (1.19) shartlarni qanoatlantirmaydi, ammo $\Phi_\sigma(y,x)$ uchun lemma 1.1 ning tasdig'i to'g'ridir.

(1.23) formuladan ko'rinadiki, l da $\Phi_\sigma(y,x)$ funksiya, uning $\nabla\Phi_\sigma(y,x)$ gradiyenti va uning ikkinchi tartibli xususiy xosilalari

$$\frac{\partial^2 \Phi_\sigma(y,x)}{\partial y_i \partial y_j},\quad i, j = 1,2,...$$

$\sigma \to \infty$ da barcha y va $x \in R^2$ larda eksponental nolga intiladi. (1.21) ga

$$\Phi(y,x)=\Phi_\sigma(y,x)$$

ni qo'yib, quyidagini olamiz:

$$\Pi_\sigma(y,x)=\left\| \lambda'\delta_{kj}\Phi_\sigma(y,x)-\mu'(y_j-x_j)\frac{\partial\Phi_\sigma(y,x)}{\partial y_k}\right\|_{2\times 2} \qquad (1.24)$$

1.5-Teorema (1.24) formula bilan aniqlangan $\Pi_\sigma(y,x)$ matrisa. D soha va l qismi uchun (1.3) va (1.4) Koshi masalasining Karleman matritsasidan iborat bo'ladi.

Isbot. $\Pi_\sigma(y,x)=\Gamma(y-x)-G_\sigma(y,x)$ ifoda (1.24) va lemma 1.1 dan birdan kelib chiqadi.

$A(\partial_y)G(y,x)=0$ tenglikning isboti xuddi teorema 1.4 dagidek bajariladi.

$$\int_l \left[\left|\Pi(y,x,\sigma)\right|+\left|T(\partial_y,n)\Pi(y,x,\sigma)\right|\right]dS_y \le c(\lambda,\mu,x)\delta\exp(-\sigma,x_2) \qquad (1.25)$$

ekanligini isbotlaymiz .

$$\frac{\partial\Phi_\sigma(y,x)}{\partial y_1}=\frac{\exp\sigma(y_2-x_2)\left[(y_2-x_2)\sin\sigma|y_1-x_1|+|y_1-x_1|\cos\sigma(y_2-x_2)\right]}{2\pi|x-y|^2},$$

$$\frac{\partial\Phi_\sigma(y,x)}{\partial y_2}=\frac{\exp\sigma(y_2-x_2)\left[(y_2-x_2)\cos\sigma|y_1-x_1|+|y_1-x_1|\sin\sigma(y_2-x_2)\right]}{2\pi|x-y|^2}$$

va $\quad \left|\Phi_\sigma(y,x)\right|\le\dfrac{const\left(\dfrac{1}{|x-y|}\right)}{\exp\sigma(y_2-x_2)}$ bo'lgani uchun quyidagiga ega bo'lamiz:

$$\left|\Pi(y,\lambda,\sigma)\right| \leq \frac{c_1(\lambda,\mu)}{\ln\left(\dfrac{1}{|x-y|}\right)} \exp\sigma\left(y_2 - x_2\right)$$

$$\left|T(\partial,n)\Pi\left(y,x,\sigma\right)\right| \leq \frac{c_2(\lambda,\mu)}{\left(\dfrac{\sigma}{|x-y|}\right)} \exp\sigma\left(y_2 - x_2\right)$$

Ushbu oxirgi tengsizliklardan (1.25) ni olamiz. Teorema isbotlandi.

II-Bob

2.1-§. Chegaralanmagan sohada elastiklik nazariyasi sistemasi yechimi uchun Somilian-Betti formulasi

$D \subset \mathbf{R}^2$, $D -$ chekli bog'lamli soha, $\partial D -$ uning bo'lakli silliq chegarasi bo'lsin. через $\boldsymbol{A}(D)$ -deb elastiklik nazariyasi sistemasining regulyar echimlari fazosini qaraymiz, ya'ni

$$\boldsymbol{A}(D) = \{u \in C^2(D) \cap C^1(\overline{D}): \mu \vartriangle u + (\lambda + \mu)\, grad\, div\, u = 0\} \qquad (2.1)$$

Ma'lumki, agar D-chegaralangan bo'lsa va $u \in \boldsymbol{A}(D)$, bu holda

$$\int\limits_{\partial D} [\,\Pi(y,x)\{T\,(\partial_y, n)u(y)\} - u(y)\{T\,(\partial_y, n)\Pi(y,x)\}]ds_y =$$

$$= \begin{cases} u(x), & x \epsilon D \\ 0\ , & x \notin \overline{D} \end{cases} \qquad (2.2)$$

bunda $T(\partial_y, n)$ –kuchlanish операторi.

$u \in \boldsymbol{A}(D)$ bo'lsin va $D -$ chekli bog'lamli soha chegaralanmagan bo'lsin. Quyidagi belgilashlarni kritamiz

$$D_R = \{y \epsilon D : |y| < R\}, \quad D_R^\infty = D \backslash D_R, \quad R>0.$$

Agar

$$\lim_{R\to\infty} \int\limits_{\partial D_R^\infty} \left[\Pi(y,x)\left\{T\,(\partial_y, n)u(y)\right\} - u(y)\left\{T\,(\partial_y, n)\Pi(y,x)\right\}\right]ds_y = 0 \qquad (2.3)$$

bo'lsa, u holda $x \in D$ uchun

$$\int\limits_{\partial D} \left[\Pi(y,x)\left\{T\,(\partial_y, n)u(y)\right\} - u(y)\left\{T\,(\partial_y, n)\Pi(y,x)\right\}\right]ds_y = u(x) \qquad (2.4)$$

o'rinli bo'ladi. Haqiqatdan ham $x \in D \quad (|x| < R)$ uchun

$$\int_{\partial D} [\Pi(y,x)\{T(\partial_y,n)u(y)\} - u(y)\{T(\partial_y,n)\Pi(y,x)\}]ds_y =$$

$$= \int_{\partial D_R} [\Pi(y,x)\{T(\partial_y,n)u(y)\} - u(y)\{T(\partial_y,n)\Pi(y,x)\}]ds_y +$$

$$+ \int_{\partial D_R^{\infty}} [\Pi(y,x)\{T(\partial_y,n)u(y)\} - u(y)\{T(\partial_y,n)\Pi(y,x)\}]ds_y =$$

$$= u(x) + \int_{\partial D_R^{\infty}} [\Pi(y,x)\{T(\partial_y,n)u(y)\} - u(y)\{T(\partial_y,n)\Pi(y,x)\}]ds_y .$$

Endi $R \to \infty$ da limitga o'tib (4) ni olamiz.

(2.4) formulaning xususiy hollarini qaraymiz. Chegaralanmagan D-soha quyidagi qatlamda joylashgan bo'lsin $0 < y_2 < h, h = \frac{\pi}{\rho}$, $\rho > 0$, bunda tashqari ∂D cheksizgacha jo'zilgan bo'lib, ayrim $b_0 > 0$ uchun ∂D chegara quyidagi integral bahoga ega bo'lsin

$$\int_{\partial D} \exp[-b_0 \mathrm{ch}\rho_0 |y_1|] Ds_y < \infty, \quad 0 < \rho_0 < \rho . \qquad (2.5)$$

$u \in \boldsymbol{A}(D)$ quyidagi o'sish shartini qanoatlantirsin

$$|u(y)| + |T(\partial y,n)u(y)| \le C \exp(exp\rho_2\, |y_1|), \quad \rho_2 < \rho, \ \ y \in D. \ (2.5^0)$$

$\Phi(y,x)$ –Karleman funksiyasini quramiz. Buning uchun quyidagi yordamchi funksiy kerak bo'ladi.

$$K(\omega) = exp\left[-b\, chi\, \rho_1 \left(\omega - \frac{h}{2}\right) - b1\, chi\, \rho_0 \left(\omega - \frac{h}{2}\right)\right], \qquad (2.6)$$

bunda $\omega = i\sqrt{u^2 + \alpha^2} + y_2, \quad 0 < \rho_1 < \rho, \quad 0 < x_2 < h, \quad b > 0,$

$b_1 > b_0 \cos\dfrac{\rho_0\, h}{2} - 1\,, \ \ \alpha = |y_1 - x_1|.$

Endi $\Phi(y,x)$ –Karleman funksiyasini quramiz

$$-2\pi\, K(x_2)\Phi_\sigma\,(y,x) = \int_0^\infty Im\frac{\exp(\sigma,\omega)}{\omega - x_2}\frac{u\,du}{\sqrt{u^2+\alpha^2}}\ . \qquad (2.7)$$

(2.4) formulani o'rinli ekanligini ko'rsatish uchun

$$\lim_{n\to\infty}\int_{\partial D_R^\infty}[\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y = 0$$

tenglikni tekis ($\forall x \in D$) bajarilishini isbotlashimiz kerak

Quyidagi integralni baholaymiz

$$\int_{\partial \mathrm{R}^2}\left[\Pi(y,x)\{\,\mathrm{T}\left(\partial_y,n\right)u(y)\} - \mathrm{u}(y)\{\,\mathrm{T}\left(\partial_y,n\right)\Pi(y,x)\}\right]ds_y\,,$$

$$\left|\int_{\partial \mathrm{R}^2}\left[\Pi(y,x)\{\,\mathrm{T}\left(\partial_y,n\right)u(y)\} - \mathrm{u}(y)\{\,\mathrm{T}\left(\partial_y,n\right)\Pi(y,x)\}\right]ds_y\right| \le$$

$$\le \int_{\partial \mathrm{R}^2}\left|\left[\Pi(y,x)\{\,\mathrm{T}\left(\partial_y,n\right)u(y)\} - \mathrm{u}(y)\{\,\mathrm{T}\left(\partial_y,n\right)\Pi(y,x)\}\right]\right|ds_y \le$$

$$\le \int_{\partial \mathrm{R}^2}\left|\Pi(y,x) + \mathrm{T}\left(\partial_y,n\right)\Pi(y,x)\right|\left|\mathrm{T}\left(\partial_y,n\right)u(y) + \mathrm{u}(y)\right|ds_y \qquad (2.8)$$

dan $\Phi(\mathrm{y},\mathrm{x})\,,\frac{\partial\Phi}{\partial \mathrm{y_i}},\frac{\partial^2\varphi}{\partial \mathrm{y_i}\,\partial \mathrm{y_j}}$ $\ i,j = 1{,}2$ larni baholashga to'g'ri keladi.

$$\Phi(\mathrm{y},\mathrm{x}) = -\frac{1}{2\pi \mathrm{K}(\mathrm{x}_2)}\int_\alpha^\infty \frac{\mathrm{K}(\mathrm{i}\sqrt{\mathrm{u}^2+\alpha^2}+\mathrm{y}_2)}{\mathrm{i}\sqrt{\mathrm{u}^2+\alpha^2}+\mathrm{y}_2-\mathrm{x}_2}\,\mathrm{d}\sqrt{\mathrm{u}^2+\alpha^2} =$$

$$= -\frac{1}{2\pi \mathrm{K}(\mathrm{x}_2)}\mathrm{Im}\int_\alpha^\infty \frac{\mathrm{K}(\mathrm{it}+\mathrm{y}_2)}{(\mathrm{it}+\mathrm{y}_2)-\mathrm{x}_2}\,\mathrm{dt}$$

$\mathrm{y}_1 > \mathrm{x}_1$ bo'lsa, bu holda

$$\frac{\partial\Phi}{\partial \mathrm{y}_1} = -\frac{1}{2\pi \mathrm{K}(\mathrm{x}_2)}\mathrm{Im}\frac{\mathrm{K}(\mathrm{i}(\mathrm{y}_1-\mathrm{x}_1)+\mathrm{y}_2}{\mathrm{i}(\mathrm{y}_2+\mathrm{x}_1)+\mathrm{y}_2-\mathrm{x}_2}$$

$y_1 < x_1$ bo'lsa, bu holda

$$\frac{\partial \Phi}{\partial y_1} = -\frac{1}{2\pi K(x_2)} \mathrm{Im} \frac{K(i(y_1 - x_1) + y_2}{i(x_1 - y_1) + y_2 - x_2} \tag{2.9}$$

$$\frac{\partial \Phi}{\partial y_2} = -\frac{1}{2\pi K(x_2)} \mathrm{Im} \frac{K(i|y_1 - x_1| + y_2}{i(y_2 - x_2) - |y_1 - x_1|} = -\frac{1}{2\pi K(x_2)} \mathrm{Re} \frac{K(i|y_1 - x_1| + y_2}{i|y_1 - x_1| + y_2 - x_2} \tag{2.10}$$

Yuqoridagi ifodalarga asosan $y_1 > x_1$ bo'lsa, bu holda

$$\frac{\partial^2 \Phi}{\partial y_1^2} = \frac{1}{2\pi K(x_2)} \mathrm{Im} \frac{\partial}{\partial y_1} \frac{K(\omega_0)}{\omega_0 - x_2}$$

$$== \frac{1}{2\pi K(x_2)} \mathrm{Im} \left[\frac{K'(\omega_0) \dfrac{\partial \omega_0}{\partial y_1}}{\omega_0 - x_2} - \frac{K(\omega) \dfrac{\partial}{\partial y_1} (\omega_0 - x_2)}{(\omega_0 - x_2)^2} \right] =$$

$$= \frac{1}{2\pi K(x_2)} \mathrm{Im} \left[\frac{iK'(\omega_0)}{\omega_0 - x_2} - \frac{iK(\omega_0)}{(\omega_0 - x_2)^2} \right] =$$

$$= \frac{1}{2\pi K(x_2)} \mathrm{Re} \left[\frac{iK'(\omega_0)}{\omega_0 - x_2} - \frac{iK(\omega_0)}{(\omega_0 - x_2)^2} \right],$$

bu erda $\omega_0 = i(y_1 - x_1) + y_2$.

$y_1 < x_1$ bo'lsa, bu holda

$$\frac{\partial^2 \Phi}{\partial y_1^2} = -\frac{1}{2\pi K(x_2)} \mathrm{Im} \frac{\partial}{\partial y_1} \frac{K(\widetilde{\omega_0})}{\widetilde{\omega_0} - x_2} =$$

$$= -\frac{1}{2\pi K(x_2)} \mathcal{J}_m \left[\frac{K'(\widetilde{\omega_0}) \dfrac{\partial \omega_0}{\partial y_1}}{\widetilde{\omega_0} - x_2} - \frac{K(\widetilde{\omega_0}) \dfrac{\partial}{\partial y_1} (\widetilde{\omega_0} - x_2)}{(\widetilde{\omega_0} - x_2)^2} \frac{\partial}{\partial y_1} (\widetilde{\omega_0} - x_2) \right] =$$

$$= -\frac{1}{2\pi K(x_2)} \mathcal{J}_m \left[\frac{K'(\widetilde{\omega_0})(-i)}{\widetilde{\omega_0} - x_2} - \frac{K(\widetilde{\omega_0})(-i)}{(\widetilde{\omega_0} - x_2)^2} \right] =$$

$$= \frac{1}{2\pi K(x_2)} R_e \left[\frac{K'(\widetilde{\omega_0})}{\widetilde{\omega_0} - x_2} - \frac{K(\widetilde{\omega_0})}{(\widetilde{\omega_0} - x_2)^2} \right],$$

bu erda $\widetilde{\omega_0} = i(x_1 - y_1) + y_2$.

Bu ikki ifodalarni birlashtirib yozsak

$$\frac{\partial^2 \Phi}{\partial y_1^2} = \frac{1}{2\pi K(x_2)} R_e \left[\frac{K'(i|y_1 - x_1| + y_2)}{i|y_1 - x_1| + y_2 - x_2} - \frac{K(i|y_1 - x_1| + y_2}{(|y_1 - x_1| + y_2 - x_2)^2} \right] \quad (\partial_1^2)$$

Ma’lumki,

$$\frac{\partial \Phi}{\partial y_2} = -\frac{1}{2\pi K(x_2)} \operatorname{Im} \frac{K(i|y_1 - x_1| + y_2)}{i(y_2 - x_2) - |y_1 - x_1|} =$$

$$= -\frac{1}{2\pi K(x_2)} \operatorname{Re} \frac{K(i|y_1 - x_1| + y_2}{i|y_1 - x_1| + y_2 - x_2} \quad \Rightarrow$$

$$\frac{\partial^2 \Phi}{\partial y_2^2} = -\frac{1}{2\pi K(x_2)} R_e \frac{\partial}{\partial y_2} \frac{K(\omega_0)}{\omega_0 - x_2} =$$

$$= \frac{1}{2\pi K(x_2)} R_e \left[\frac{K'(\omega_0) \frac{\partial \omega_0}{\partial y_2}}{\omega_0 - x_2} - \frac{K(\omega_0) \frac{\partial}{\partial y_1} (\omega_0 - x_2)}{(\omega_0 - x_2)^2} \right] =$$

$$= \frac{1}{2\pi K(x_2)} R_e \left[\frac{K'(\omega_0)}{\omega_0 - x_2} - \frac{K(\omega_0)}{(\omega_0 - x_2)^2} \right] \quad (\partial_2^2)$$

(∂_1^2) va (∂_2^2) dan $\Phi(x, y) -$ funksiyaning garmonik ekanligi ko’rinadi.

Endi $\frac{\partial^2 \Phi}{\partial y_1 \partial y_2} = \frac{\partial^2 \Phi}{\partial y_2 \partial y_1}$ ekanligini ko’rsatamiz

$y_1 > x_1$ da

$$\frac{\partial \Phi}{\partial y_1} = -\frac{1}{2\pi K(x_2)} \operatorname{Im} \frac{K(i(y_1 - x_1) + y_2}{i(x_1 - y_1) + y_2 - x_2} \quad \frac{\partial^2 \Phi}{\partial y_2 \partial y_1} =$$

$$=\frac{1}{2\pi K(x_2)} \mathrm{Im} \frac{\partial}{\partial y_2} \frac{K(\omega_0)}{\omega_0 - x_2} =$$

$$=\frac{1}{2\pi K(x_2)} \mathrm{Im} \left[\frac{K'(\omega_0)\frac{\partial \omega_0}{\partial y_2}}{\omega_0 - x_2} - \frac{K(\omega_0)}{(\omega_0 - x_2)^2} \frac{\partial \omega_0}{\partial y_2} \right] =$$

$$=\frac{1}{2\pi K(x_2)} \mathrm{Im} \left[\frac{K'(\omega_0)}{\omega_0 - x_2} - \frac{K(\omega_0)}{(\omega_0 - x_2)^2} \right],$$

$$\frac{\partial \Phi}{\partial y_2} = -\frac{1}{2\pi K(x_2)} \mathrm{R_e} \frac{K(i(y_1 - x_1) + y_2}{i(x_1 - y_1) + y_2 - x_2} \ ,$$

$$\frac{\partial^2 \Phi}{\partial y_1 \, \partial y_2} = \frac{1}{2\pi K(x_2)} \mathrm{R_e} \frac{\partial}{\partial y_2} \frac{K(\omega_0)}{\omega_0 - x_2} =$$

$$= -\frac{1}{2\pi K(x_2)} \mathrm{R_e} \left[\frac{K'(\omega_0)\frac{\partial \omega_0}{\partial y_1}}{\omega_0 - x_2} - \frac{K(\omega_0)}{(\omega_0 - x_2)^2} \frac{\partial \omega_0}{\partial y_1} \right] =$$

$$= -\frac{1}{2\pi K(x_2)} \mathrm{R_e} \left[\frac{K'(\omega_0) i}{\omega_0 - x_2} - \frac{K(\omega_0) i}{(\omega_0 - x_2)^2} \right] =$$

$$= \frac{1}{2\pi K(x_2)} \mathcal{J}_m \left[\frac{K'(\omega_0)}{\omega_0 - x_2} - \frac{K(\omega_0)}{(\omega_0 - x_2)^2} \right] \qquad (2.11)$$

Xuddi shunday $y_1 < x_1$ da

$$\frac{\partial^2 \Phi}{\partial y_1 \, \partial y_2} = \frac{\partial^2 \Phi}{\partial y_2 \, \partial y_1}$$

$$|\Phi(y, x)| = \frac{1}{2\pi K(x_2)} \left| \int_0^\infty \mathrm{Im} \frac{K(\omega)}{\omega - x_2} \frac{u du}{\sqrt{u^2 - \alpha^2}} \right| =$$

$$= \frac{1}{2\pi |K(x_2)|} \left| \mathrm{Im} \int_\alpha^\infty \frac{K(it + y_2)}{it + y_2 - x_2} dt \right| \leq$$

$$\leq \frac{1}{2\pi|K(x_2)|}\int_{\alpha}^{\infty}\frac{|K(it+y_2)|}{|it+y_2-x_2|}dt = \frac{1}{2\pi|K(x_2)|}\int_{\alpha}^{\infty}\frac{|K(it+y_2)|}{\sqrt{t^2+(y_2-x_2)^2}}dt \leq$$

$$\leq \frac{1}{2\pi r|K(x_2)|}\int_{\alpha}^{\infty}|K(it+y_2)|dt \ .$$

Endi $K(\omega) -$ funksiyaning ko'rinishini e'tiborga olsak

$$|K(it+y_2)| = \left|\left[\frac{\frac{\rho}{2}(it+y_2-x_2)}{\sin\frac{\rho}{2}(it+y_2-x_2)}\right]^p (it+y_2+x_2+2h)^{-k} \times\right.$$

$$\times \exp\left[\sigma(it+y_2) - -bchi\rho_1\left(it+y_2-\frac{h}{2}\right) - b_1 chi\rho_0\left(it+y_2-\frac{h}{2}\right)\right] =$$

$$= \frac{\left(\frac{\rho}{2}\right)^p\left(\sqrt{t^2+(y_2-x_2)^2}\right)^p}{\left|ish\frac{\rho t}{2}\cos\frac{\rho}{2}(y_2-x_2)+\sin\frac{\rho}{2}(y_2-x_2)\cos\frac{\rho t}{2}\right|}\times$$

$$\times \exp\left[\sigma y_2 - bchi\rho_1 t\cos\rho_1\left(y_2-\frac{h}{2}\right) - b_1 chi\rho_0 t\cos\rho_0\left(y_2-\frac{h}{2}\right)\right] \leq$$

$$\leq C\exp\left[\sigma y_2 - bchi\rho_1 t\cos\rho_1\left(y_2-\frac{h}{2}\right) - b_1 chi\rho_0 t\cos\rho_0\left(y_2-\frac{h}{2}\right)\right]. \quad (2.12)$$

Bunda C=const, $0 < y_2 < h$ $\quad h = \frac{\pi}{\rho}$, $t \in [\alpha,\infty)$, $\alpha = |y_1 - x_1|$,

σ, b, b_1>0, $-\frac{\pi}{2} < \frac{\rho_1 h}{2} \leq \rho_1\left(y_2-\frac{h}{2}\right) \leq \frac{\rho_1 h}{2} < \frac{\pi}{2}$, $-\frac{\pi}{2} < \rho_0\left(y_2-\frac{h}{2}\right) < \frac{\pi}{2}$,

$$|\Phi(y,x)| \leq \frac{C}{K(x_2)r}\int_{\alpha}^{\infty}\exp\left[-bch\rho_1 t\cos\rho_1\left(y_2-\frac{h}{2}\right)-\right.$$

$$\left.-b_1 ch\rho_0 t\cos\rho_0\left(y_2-\frac{h}{2}\right)\right]dt \leq$$

$$\leq \frac{C}{K(x_2)r}\int_{\alpha}^{\infty}\exp\left[-(b-\varepsilon)ch\rho_1 t\cos\rho_1\left(y_2-\frac{h}{2}\right)-\right.$$

$$-(b_1-\varepsilon)\mathrm{ch}\rho_0 t\cos\rho_0\left(y_2-\frac{h}{2}\right)\Big]\times$$

$$\times\exp\left[-\varepsilon\mathrm{ch}\rho_1 t\cos\rho_1\left(y_2-\frac{h}{2}\right)+\mathrm{ch}\rho_0 t\cos\rho_0\left(y_2-\frac{h}{2}\right)\right]dt\leq$$

$$\leq\frac{C}{K(x_2)r}\exp[-(b-\varepsilon)\mathrm{ch}\rho_1|y_1|-(b_1-\varepsilon)\mathrm{ch}\rho_0|y_1|]\int_\alpha^\infty\exp(-t)\,dt\leq$$

$$\leq\tilde{C}\exp[-(b-\varepsilon)\mathrm{ch}\rho_1|y_1|-(b_1-\varepsilon)\mathrm{ch}\rho_0|y_1|]\ ,\qquad(2.13)$$

$0<\varepsilon\ll b,\ \varepsilon\ll b_1.$

(2.9) , (2.10) va (2.12) lardan

$$\left|\frac{\partial\Phi}{\partial y_i}\right|\leq C[-b\mathrm{ch}\rho_1|y_1|-b_1\mathrm{ch}\rho_0|y_1|]\qquad(2.14)$$

(∂_1^2) , (∂_2^2) va (2.11) dan

$$\left|\frac{\partial^2\varphi}{\partial y_i\,\partial y_j}\right|\leq C\exp[-b\mathrm{ch}\rho_1|y_1|-b_1\mathrm{ch}\rho_0|y_1|]+$$

$$+C_0[b_1\rho_1\mathrm{sh}\rho_1|y_1|+b_1\rho_0\mathrm{sh}\rho_0|y_1|]\exp(-b\mathrm{ch}\rho_1|y_1|-b_1\mathrm{ch}\rho_0|y_1|)\leq$$

$$\leq C_1\exp(-(b-\varepsilon)\mathrm{ch}\rho_1|y_1|-(b_1-\varepsilon)\mathrm{ch}\rho_0|y_1|)\qquad(2.15)$$

$\varepsilon>0$ ni (2.13) dan tanlaymiz.

$$|\Pi(y,x)|+\left|T\left(\partial_y,n\right)\Pi(y,x)\right|\leq$$

$$\leq C\exp(-(b-\varepsilon)\mathrm{ch}\rho_1|y_1|-(b_1-\varepsilon)\mathrm{ch}\rho_0|y_1|).\qquad(2.16)$$

Endi $\rho_2<\rho_1<\rho$ va (2.5^0) shartdan quyidagi baholashlarni olamiz

$$\left[|\Pi(y,x)|+\left|T\left(\partial_y,n\right)\Pi(y,x)\right|\right]\left[\left|T\left(\partial_y,n\right)u(y)\right|+\ |u(y)|\right]\leq$$

$$\leq\mathrm{const}\exp(-b_0\mathrm{ch}\rho_0|y_1|),\qquad(2.17)$$

bunda $b_1 > b_0(\cos\rho_0 \frac{h}{2})^{-1} + \varepsilon.$

Yuqoridagi shartlardan

$$\lim_{n\to\infty} \int_{\partial D_R^{\infty}} [\,\Pi(y,x)\{T\left(\partial_y, n\right)u(y)\} - u(y)\{T\left(\partial_y, n\right)\Pi(y,x)\}]ds_y = 0$$

$A_\rho(D) = \{U(y): U(y)\epsilon A(D),\ |U(y)| + |TU(y)| \le \exp[\exp\rho|y_1|]\,, \rho > 0\}$

2.1-Теорема. Agar $u(y)\epsilon A(D)$

$|\,u(y)\,| + |T(\partial_y, n)u(y)\,| \le C\ exp[a\ cos\ \rho_1(y_2 - \frac{h}{2})\ exp\ \rho_1\ |y_1|],$ (2.17*)

$a \ge 0, y\epsilon\,\partial D$. Bu holda agar $0 < \rho_1 < \rho$ bo'lsa,

$$u(x) = \int_{\partial D} \Big[\Pi(y,x)\Big\{T\Big(\partial_y, n(y)\Big)u(y)\Big\} -$$

$$-u(y)\Big\{T\Big(\partial_y, n(y)\Big)\Pi(y,x)\Big\}ds_y\,,\ \ x \in D$$

bu erda $a = b$.

Isbot. D sohani $y_2 = \frac{h}{2}$ chiziq bilan kesib $D_1 = \left\{y\epsilon D: 0 < y_2 < \frac{h}{2}\right\}$ va $D_2 = \left\{y\epsilon D: \frac{h}{2} < y_2 < 0\right\}$ sohalarga ajratamiz. D_1 sohada

$$K_1(\omega) = K(\omega)\exp\left[-\delta_1 \mathrm{ch}\tau\left(\omega - \frac{h}{4}\right) - \delta_2 \mathrm{ch}\rho\left(\omega - \frac{h}{2}\right)\right]$$

$\omega = i\sqrt{u^2+\alpha^2} + y_2$, $\alpha = |y_1 - x_1|$, $u\epsilon(0;+\infty)$, $\delta_1 > 0\,, \delta_2 > 0$, $\rho < \tau < 2\rho$, $0 \le y_2 \le \frac{h}{2}$, $0 < x_2 < \frac{h}{2}$, deb $K(\omega)$ ni (2.6) formuladan olamiz. $\Phi_1(y,x)$ funksiyani quyidagicha quramiz

$$\Phi_1 = -\frac{1}{2\pi K_1(x_2)}\int_0^{\infty} \mathrm{Im}\frac{K_1(\omega)}{\omega - x_2}\frac{udu}{\sqrt{u^2+\alpha^2}}$$

$K_1(\omega) -$ funksiya uchu quyidagi baholash o'rinlidir

$$|\mathrm{K}_1(\omega)| \le |\mathrm{K}(\omega)| \exp(-\delta_1 \mathrm{ch}\tau\sqrt{u^2+\alpha^2}\left(y_2-\frac{h}{4}\right)-$$

$$-\delta_1\, \mathrm{ch}\rho\sqrt{u^2+\alpha^2}\cos\rho\left(y_2-\frac{h}{2}\right), \qquad (2.18)$$

$$-\frac{\pi}{2} < -\frac{\tau h}{4} < \tau\left(y_2-\frac{h}{4}\right) < \frac{\tau h}{4} < \frac{\pi}{2}\,,\ \cos\tau(y_2-\frac{h}{4}) \ge \delta_0 > 0,$$

$\cos\rho(y_2-\frac{h}{2}) \ge \delta_0 > 0.$

Φ_1 , $\frac{\partial\Phi_1}{\partial y_2}$, $\frac{\partial^2\Phi_1}{\partial y_i\,\partial y_j}$, larni (2.13), (2.14), (2.15) dagiday baholab quydagiga ega bo'lamiz

$$|\Pi_1(y,x)| + |T\Pi_1(y,x)| \le O(\exp(-\delta_0 \mathrm{ch}\tau|y_1|) \qquad (2.19)$$

$x\epsilon D_1$, $y\epsilon D_1 \cup \partial D_1$, $y \to \infty$.

$u\epsilon A_{(2\rho-\varepsilon)}(D_1)$, $\varepsilon > 0$ bo'lsin, (2.18) da $\tau\epsilon(2\rho-\varepsilon;\ 2\rho)$ deb, quyidagiga ega bo'lamiz

$$u(x) = \int_{\partial D_1}\left[\Pi_1(y,x)\left\{T\left(\partial_y, n(y)\right)u(y)\right\}-\right.$$

$$\left.-u(y)\left\{T\left(\partial_y, n(y)\right)\Pi_1(y,x)\right\}ds_y\,,\ \ x \in D_1 \qquad (2.20)$$

Agar $u(x) \in A(D_2) \cap A_{2\rho-\varepsilon}(D_2)$, bu holda $\tau \in (2\rho-\varepsilon, 2\rho)$ bo'lganda quyidagiga ega bo'lamiz

$$u(x) = \int_{\partial D_2}\left[\Pi_2(y,x)\left\{T\left(\partial_y, n(y)\right)u(y)\right\}-\right.$$

$$\left.-u(y)\left\{T\left(\partial_y, n(y)\right)\Pi_2(y,x)\right\}ds_y\,,\ \ x \in D_2 \qquad (2.21)$$

bu erda $\mathrm{K}_2(\omega) \le \mathrm{K}(\omega)\, exp\left[-\delta_1\, chi\, \tau\,\left(\omega-\frac{3h}{4}\right)-\delta_2\, chi\, \rho\left(\omega-\frac{h}{2}\right)\right]$

$$-2\pi\, K_2(x_2)\, \Phi_2(y,x) = \int_0^\infty Im\, \frac{K_2(\omega)}{\omega - x_2} \frac{udu}{\sqrt{u^2+\alpha\alpha^2}}$$

$$\Pi_2(y,x) = \left\| \lambda' \delta_{kj} \Phi_2(y,x) - \mu'(y_j - x_i) \frac{\partial \Phi_2(y,x)}{\partial y_k} \right\|_{2x2}$$

$$\Phi_2(y,x) = -\frac{1}{2\pi K_2(x_2)} \int_0^\infty \mathrm{Im} \frac{K_2(\omega)}{\omega - x_2} \frac{udu}{\sqrt{u^2+\alpha^2}}$$

$$K_2(\omega) = K(\omega) \exp(-\delta_1 \mathrm{ch}\tau \left(y_2 - \frac{3h}{4}\right) - \delta_1 \mathrm{chi}\rho \left(\omega - \frac{h}{2}\right))$$

Endi $u \epsilon A_\rho(D)$ va (17^*) shartni qanoatlantirsin, bu holda (20), (21) formulalar o'rinli va bu integrallar tekis yaqinlashadi barcha $x \in D$.

2.2-§. Maxsus soha uchun integral formula

Endi ∂D: $y_2 = f(y_1)$ formula yordamida berilsin. Bunda

$$\left|\frac{\partial f}{\partial y_1}\right| \leq c < \infty, \quad (2,22)$$

$$K(\omega) = \frac{\exp(-\varepsilon(\omega+1)^{\rho_1})}{(\omega+x_2)^k} \quad (2.23)$$

$$\omega = i\sqrt{u^2+\alpha^2} + y_2,\ \varepsilon > 0,\ y_2 \geq 0,\ 0 < \rho_1 < 1,\ k\epsilon\mathbb{N}.$$

Quyidagi funksiyalar sinfini qaraymiz

$$B_\rho(D) = \{U(y): U\epsilon A(D),\ |U(y)| + |TU(y)| \leq \exp(|y|^\rho),\ \rho > 0\}. \quad (2.24)$$

ρ_1, ρ ni $\rho < \rho_1 < 1$ shartdan tanlab, $u(y) \in B_\rho(D)$ uchun

$$|K(\omega)| = \frac{\exp R_e(-\varepsilon(\omega+1)^{\rho_1})}{|\omega+x_2|^k},$$

$$\omega + 1 = i\sqrt{u^2+\alpha^2} + y_2 + 1 = \sqrt{u^2+\alpha^2+(y_2+1)^2}(\cos\varphi + i\sin\varphi)$$

$$\cos\varphi = \frac{y_2+1}{\sqrt{u^2+\alpha^2+(y_2+1)^2}},\ \sin\varphi = \frac{\sqrt{u^2+\alpha^2}}{\sqrt{u^2+\alpha^2+(y_2+1)^2}},$$

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < -\frac{\pi\rho_1}{2} < \rho_1\varphi < -\frac{\pi\rho_1}{2} < \frac{\pi}{2} \quad (2.25)$$

Shuning uchun

$$(\omega+1)^{\rho_1} = (\sqrt{u^2+\alpha^2+(y_2+1)^2})^{\rho_1}(\cos\rho_1\varphi + i\sin\rho_1\varphi)$$

va

$$|K(\omega)| = \frac{\exp\left[-\varepsilon(\sqrt{u^2+\alpha^2+(y_2+1)^2})^{\rho_1}\cos\rho_1\varphi\right]}{(u^2+\alpha^2+(y_2+1)^2)^K} \quad (2.26)$$

(2.25) ga ko'ra $\cos\rho_1\varphi \geq \delta_0 > 0$, (2.26) dan

$$|K(\omega)| = \frac{\exp[-\varepsilon|\omega|^{\rho_1}\cos\rho_1\varphi]}{|\omega|^k}$$

$\Phi(y,x)$ funksiyani quramiz

$$\Phi(y,x) = -\frac{1}{2\pi K_2(x_2)}\int_0^\infty \mathrm{Im}\frac{K_2(\omega)}{\omega - x_2}\frac{udu}{\sqrt{u^2+\alpha^2}}$$

$$|\Phi(y,x)| \le -\frac{1}{2\pi|K(x_2)|}\int_0^\infty \frac{\exp\left[-\varepsilon_0\left[\sqrt{t^2+(y_2+1)^2}\right]^{\rho_1}\right]}{\left[\sqrt{t^2+(y_2-x_2)^2}\right]^k\sqrt{t^2+(y_2-x_2)^2}}dt \le$$

$$\le \frac{1}{2\pi|K(x_2)|}\frac{\exp\left[-\varepsilon_0\left[\sqrt{t^2+(y_2+1)^2}\right]^{\rho_1}\right]}{\left[\sqrt{t^2+(y_2-x_2)^2}\right]^k}\int_0^\infty \frac{dt}{\sqrt{t^2+(y_2-x_2)^2}} \le$$

$$\le \tilde{C}\frac{\exp\left[-\varepsilon_0\left[\sqrt{t^2+(y_2+1)^2}\right]^{\rho_1}\right]}{r^k}\ . \qquad (2.27)$$

(2.9), (2.10), (∂_1), (∂_2), (2.11) larga ko'ra

$$\left|\frac{\partial\Phi}{\partial y_i}\right| \le \tilde{C}\frac{|K|}{2} \le \tilde{C}\frac{\exp\left[-\varepsilon_0\left[\sqrt{\alpha^2+(y_2+1)^2}\right]^{\rho_1}\right]}{r^{k+1}} \qquad (2.28)$$

$$\left|\frac{\partial^2\Phi}{\partial y_i\,\partial y_j}\right| \le \tilde{C}\frac{\exp\left[-\varepsilon_0\left[\sqrt{\alpha^2+(y_2+1)^2}\right]^{\rho_1}\right]}{r^{k+2}} \qquad (2.29)$$

$$|\Pi(x,y)| + |T\Pi(x,y)| \le C\frac{\exp\left[-\varepsilon_0\left[\sqrt{\alpha^2+(y_2+1)^2}\right]^{\rho_1}\right]}{r^{k+2}} \qquad (2.30)$$

$u\epsilon B_\rho(D)$ ligidan

$$\int_{D_R^\infty}[\Pi(TU) - U(T\Pi)]ds_y \to 0\,,\quad R\to\infty.$$

Tasdiq isbot bo’ldi.

(2.7), (2.8), (2,9) lardan

$$|\Pi(x,y)| =\circ\left(\frac{1}{y^2}\right),\quad |T\Pi(x,y)| =\circ\left(\frac{1}{y^3}\right),\quad y\to\infty,\qquad y\epsilon\,\partial D \quad (2.31)$$

2.2-Teorema. $u(y)\epsilon B_\rho(D)$, D soha $y_2>0$ dan, bo’lib chegarasi cheksizgacha cho’zilib boradigan va ∂D: $y_2=f(y_1)$ tenglama bilan berilgan bo’lib, f −differensiallanuvchi va (2.22) shartni qanoatlantirishi bilan birga

$0\le f\le y_0<\infty$ shartni qanoatlantirsin. Bundan tashqari $u(y)$ quyidagi chegaraviy shartlarni qanoatlantirsin

$$\int\limits_{\partial D}\frac{|u(y)|}{1+|y|^2}ds<\infty,\qquad \int\limits_{\partial D}|Tu(y)|\frac{ds}{1+|y|}<\infty,\qquad (2.32)$$

Agar $\rho<1$ bo’lsa, bu holda

$$u(x)=\int\limits_{\partial D}[\,\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\}-u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y\ \ ,x\epsilon D$$

formula o’rinli bo’lib,

$$\Pi(y,x)=\left\|\lambda'\delta_{kj}\Phi(y,x)-\mu'(y_j-x_j)\frac{\partial}{\partial y_k}\Phi(y,x)\right\|\qquad (2.33)$$

$$\Phi(y,x)=\frac{1}{4\pi}\ln\frac{(y_1-x_1)^2+(y_2+x_2)^2}{(y_1-x_1)^2+(y_2-x_2)^2}\qquad (2.34)$$

Isbot. (2.31) va (2.32) lardan (2.4) integralning tekis yaqinlashishi kelib chiqadi. $K(\omega)$ ning formulasida $\varepsilon=0$ qo′yibquyidagiga ega bo′lamiz

$$\Phi(y,x)=$$

$$= -\frac{x_2}{\pi}\int_0^{\infty} \mathrm{Im}\frac{udu}{\left(i\sqrt{u^2+\alpha^2}+y_2+x_2\right)\left(i\sqrt{u^2+\alpha^2}+y_2-x_2\right)\sqrt{u^2+\alpha^2}} =$$

$$= -\frac{x_2}{\pi}\int_{\alpha}^{\infty} \mathcal{J}_{\mathrm{m}}\frac{dt}{(it+y_2+x_2)(it+y_2-x_2)} =$$

$$= -\frac{x_2}{\pi}\int_{\alpha}^{\infty} \mathcal{J}_{\mathrm{m}}\frac{(y_2+x_2-it)(y_2-x_2-it)}{(t^2+(y_2+x_2)^2)(t^2+(y_2-x_2)^2)}dt =$$

$$= -\frac{x_2y_2}{\pi}\int_{\alpha}^{\infty} \frac{tdt}{(t^2+(y_2+x_2)^2)(t^2+(y_2-x_2)^2)} =$$

$$= \frac{1}{4\pi}\ln\frac{\alpha^2+(y_2-x_2)^2}{\alpha^2+(y_2+x_2)^2} ,$$

bu erda $\alpha^2 = (y_1-x_1)^2$.

2.3-Teorema. $D = \{(y_1, y_2):\ y_2 > 0\}$ -yuqori yarim tekislik, $u(y) \in B_\rho(D)$ fuksiya

$$\int_{\partial D} \frac{|u(y)|}{1+|y|^2}ds < \infty , \qquad \int_{\partial D} |Tu(y)|\frac{ds}{1+|y|} < \infty , \quad (2.35)$$

shartlarni qanoatlantirsin, bu holda agar $\rho < 1$ bo'lsa, $x_2 > 0$ da

$$u(x) = \int_{y_2=0} [\Pi(y,x)\{T\left(\partial_y, n\right)u(y)\} - u(y)\{T\left(\partial_y, n\right)\Pi(y,x)\}]ds_y \ , x\epsilon D$$

formula o′rinlidir.

Isbot. (2.35) formulani quyidagicha yozish mumkin

$$\int_{-\infty}^{\infty} \frac{u(y_1,0)}{1+|y_1|^2}dy_1 < \infty , \qquad \int_{-\infty}^{\infty} \frac{|Tu(y_1,0)|}{1+|y_1|}dy_1 < \infty.$$

$K(\omega)$ ni (2.23) formuladagiday olamiz va mos ravishda $\Phi(x,y)$, $\Pi(x,y)$ larni quramiz.

$D_R = \{x \epsilon D: |x| \leq R\}$, $\partial D_R^{\infty} = D \setminus D_R$ deb olsak, $x \epsilon D_R$

$$\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y =$$

$$= \int_{\partial D_R} [\,\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y +$$

$$+ \int_{\partial D_R^{\infty}} [\,\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y =$$

$$= u(x) + \int_{\partial D_R^{\infty}} [\,\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y\,.$$

$$\int_{\partial D_R^{\infty}} [\,\Pi(y,x)\{T\left(\partial_y,n\right)u(y)\} - u(y)\{T\left(\partial_y,n\right)\Pi(y,x)\}]ds_y \rightrightarrows 0, \qquad R \to \infty.$$

2.3-§. Chegaralanmagan tekis sohada elastiklik nazariyasi sistemalari uchun Koshi masalasi

$D \subset \mathbb{R}^2$ soha chegarasi $y_2 = 0$ to'g'ri chiziq va

$$0 < f(y_1) < h, \qquad |f(y_1)| \leq L < \infty, \qquad y_1 \in \mathbb{R}. \qquad (2.36)$$

shartlarini qanoatlantiruvchi $y_2 = f(y_1)$ egri chiziqdan iborat $0 < y_2 < h,\ h = \frac{\pi}{\rho}$, $\rho > 0$ yo'lak ichida yotsin. Faraz qilaylik, $u(x) \in A(D)$ ∂D da o'zining normal xosilasi bilan chegaralangan bo'lsin.

$$|u(y)| + \left|\mathrm{T}\left(\partial_y, n\right)u(y)\right| \leq \mathrm{M}, \qquad \mathrm{y} \in \partial D. \qquad (2.37)$$

S - $y_2 = f(y_1),\ y_1 \in \mathbb{R}$ tenglama bilan berilgan ∂D chegaraning qismi bo'lsin. Bu shartlarda Somilian- Betti formulasi o'rinli

$$u(x) = \int_{\partial D} [\Pi_\sigma(y,x)\{\mathrm{T}\left(\partial_y, n\right)u(y)\} - \mathrm{u}(y)\{\mathrm{T}\left(\partial_y, n\right)[\Pi_\sigma(y,x)\}]ds_y,\ \ x \in D$$

bu holda $K(\omega) = [\omega - x_2 + 2h]^{-1} exp\sigma\omega,\ \ \omega = i\sqrt{u^2 + \alpha^2} + y_2\ ,\ h = \frac{\pi}{\rho}$,

$$K(x_2) = [2h]^{-1} exp\sigma x_2,\ \ 0 < x_2 < h,$$

deb olamiz va quyidagi $u_\sigma(x)$ belgilashni kritamiz

$$u_\sigma(x) = \int_S [\Pi_\sigma(y,x)\{\mathrm{T}\left(\partial_y, n\right)u(y)\} - \mathrm{u}(y)\{\mathrm{T}\left(\partial_y, n\right)[\Pi_\sigma(y,x)\}]ds_y,\ \ x \in D.$$

Quyidagi teorema o'rinli

2.4- Teorema. $u(x) \in A_\rho(D)$ (2.37) chegaraviy shartni qanoatlantirsin. u holda

$$\left|u(x) - u_\sigma(x)\right| \leq M\, C_\rho(x)\, exp(-\sigma x_2), \quad x \in D\,,$$

$C_\rho(x)$- faqat ρ va x_2 ga bog'liq kattalik.

Quyida yozuvning qulayligi uchun ρ ga bog'liq barcha doimiylarni C_ρ deb belgilaymiz. Turli tenglamalarda C_ρ turli doimiylarini bildiradi.

Isbot: Somilian- Betti formulasi (1.18) ga asosan

$$u(x) - u_\sigma(x) = \int\limits_{y_2=0} [\Pi_\sigma(y,x)\{\mathrm{T}\left(\partial_y, n\right)u(y)\} - \mathrm{u}(y)\{\mathrm{T}\left(\partial_y, n\right)[\Pi_\sigma(y,x)\}]ds_y\,.$$

(2.37) ko'ra esa,

$$|u(x) - u_\sigma(x)| \leq M \int\limits_{-\infty}^{+\infty} \left[|\Pi_\sigma(y,x)| + \left|\mathrm{T}\left(\partial_y, n\right)[\Pi_\sigma(y,x)\right|\right]ds_y\,.$$

Oldingi teoremalar isbotlaridagicha o'xshash $|\Phi_\sigma|$, $|\partial\Phi_\sigma / \partial y_i|$, $\left|\partial^2\Phi_\sigma / \partial y_i\, \partial y_j\right|$, $i,j = 1{,}2$

larni boholaymiz.

$$K(\omega) = [\omega - x_2 + 2h]^{-1} exp\sigma\omega, \quad \omega = i\sqrt{u^2 + \alpha^2} + y_2\,, \qquad h = \frac{\pi}{\rho},$$

deb,

$$\Phi_\sigma(\mathrm{y},x) = -\frac{h}{\pi}\exp(-\sigma\, x_2)\int_0^\infty Im\ \frac{K(\omega)}{\omega - x_2}\ \frac{udu}{\sqrt{u^2+\alpha^2}} =$$

$$= -\frac{h}{\pi}\exp\sigma(y_2 - \ x_2)\int_\alpha^\infty \frac{[(y_2 - x_2 + 2h)(y_2 - x_2) - t^2]sin\sigma t}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]}\ dt +$$

$$+\frac{2h}{\pi}\exp\sigma(y_2 - \ x_2)\int_\alpha^\infty \frac{(y_2 - x_2 + h)tcos\sigma t}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]}\ dt.$$

ni olamiz.

$\alpha \le u\ ,-h \le y_2 - x_2 \le h\ ,\quad h \le y_2 - x_2 + 2h \le 3h\ ,\ \ 0 \le y_2 \le h,\ \ 0 \le x_2 \le h$

lardan

$$\left|\int_\alpha^\infty \frac{[(y_2 - x_2 + 2h)(y_2 - x_2)]sin\sigma t}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]}\ dt\right| \le$$

$$\le 3h|y_2 - x_2|[(y_2 - x_2 + 2h)^2 + \alpha^2]^{-1} \times$$

$$\times \int_\alpha^\infty \frac{1}{(y_2 - x_2)^2 + t^2}\ dt \le \frac{3h\pi}{2(\alpha^2 + h^2)}\ ,$$

$$\left|\int_\alpha^\infty \frac{[(y_2 - x_2)t^2]sin\sigma t}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]}\ dt\right| \le$$

$$\leq |y_2 - x_2| \left| \int_{\alpha}^{\infty} \frac{t^2}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]} \, dt \right| =$$

$$= |y_2 - x_2| \left| \int_{\alpha}^{\infty} \frac{dt}{(y_2 - x_2 + 2h)^2 + t^2} - \frac{(y_2 - x_2)^2}{[(y_2 - x_2)^2 - (y_2 - x_2 + 2h)^2]} \times \right.$$

$$\left. \times \int_{\alpha}^{\infty} \left[\frac{1}{(y_2 - x_2 + 2h)^2 + t^2} - \frac{1}{(y_2 - x_2)^2 + t^2} \right] dt \right| \leq \frac{\pi}{2} \left| \frac{y_2 - x_2}{y_2 - x_2 + 2h} + \right.$$

$$\left. + \frac{(y_2 - x_2)^2}{4h(y_2 - x_2 + h)} + \frac{(y_2 - x_2)^2}{4h(y_2 - x_2 + h)(y_2 - x_2 + 2h)} \right| = C_\rho \,,$$

$$\left| (y_2 - x_2 + h) \int_{\alpha}^{\infty} \frac{t cos \sigma t}{((y_2 - x_2)^2 + t^2)[(y_2 - x_2 + 2h)^2 + t^2]} \, dt \right|$$

$$\leq C_\rho \ln(1 + 8h^2 r^{-2}). \quad (2.61)$$

ni olamiz. Yuqoridagilarga asosan

$$|\Phi_\sigma(\mathrm{y}, x)| \leq C_\rho \exp \sigma(y_2 - x_2) \ln(1 + 8h^2 r^{-2})$$

ni olamiz. $\Phi_\sigma(\mathrm{y}, x)$ funksiyalarning xuddi shunday xosilalari formulalariga ko'ra (1.4-§) quyidagicha bo'ladi.

$$|\partial \Phi_\sigma(y, x) / \partial y_i| \leq C_\rho \exp \sigma(y_2 - x_2) \left[\frac{1}{(y_2 - x_2 + 2h)^2 + \alpha^2} + r^{-2} \right] ,$$

$$\left| \partial^2 \Phi_\sigma(y, x) / \partial y_i \, \partial y_j \right| \leq C_\rho \sigma \exp \sigma(y_2 - x_2) \left[\frac{1}{r(\alpha^2 + 4h^2)} + \right.$$

$$+\frac{1}{r\sqrt{\alpha^2+4h^2}}+\frac{1}{r^2\sqrt{\alpha^2+4h^2}}\Bigg].$$

Shundan

$$|\Pi_\sigma(y,x)|+\left|\mathrm{T}\left(\partial_y,n\right)\Pi_\sigma(y,x)\right|\leq$$

$$\leq C_\rho\sigma\, exp\, \sigma(y_2-\ x_2)\left[\frac{\sigma^{-1}}{(y_2-x_2+2h)^2+\alpha^2}+\right.$$

$$+\frac{1}{r(\alpha^2+4h^2)}+\frac{1}{r\sqrt{\alpha^2+4h^2}}+\frac{1}{r^2\sqrt{\alpha^2+4h^2}}+$$

$$\left.+\sigma^{-1}\ln(1+8h^2r^{-2})+\frac{\sigma^{-1}}{r^2}\right].$$

ni olamiz. $y_2=0$ da $u(x)-\ u_\sigma(x)$ uchun

$$|u(x)-\ u_\sigma(x)|\leq MC_\rho\sigma\exp(-\sigma x_2)\left[\int\limits_{-\infty}^{\infty}\ln\left(1+\frac{8h^2}{\alpha^2+x_2^2}\right)d\alpha+\right.$$

$$+\int\limits_{-\infty}^{\infty}\left(\frac{1}{(2h-x_2)^2+\alpha^2}+\frac{1}{\alpha^2+x_2^2}\right)d\alpha+\int\limits_{-\infty}^{\infty}\frac{d\alpha}{\sqrt{\alpha^2+4h^2}\sqrt{\alpha^2+x_2^2}}+$$

$$\left.+\int\limits_{-\infty}^{\infty}\frac{d\alpha}{(\alpha^2+4h^2)\sqrt{\alpha^2+x_2^2}}+\int\limits_{-\infty}^{\infty}\frac{d\alpha}{\sqrt{\alpha^2+4h^2}(\alpha^2+x_2^2)}\right]=$$

$$=\ MC_\rho\sigma\exp\sigma(y_2-\ x_2)\left[\int\limits_{0}^{\infty}\ln\left(1+\frac{8h^2}{t^2+x_2^2}\right)dt+\right.$$

$$+\int_0^\infty \left(\frac{1}{(2h-x_2)^2+t^2}+\frac{1}{t^2+x_2^2}\right)dt+\int_0^\infty \frac{dt}{\sqrt{t^2+4h^2}\sqrt{t^2+x_2^2}}+$$

$$+\int_0^\infty \frac{dt}{(t^2+4h^2)\sqrt{t^2+x_2^2}}+\int_0^\infty \frac{dt}{\sqrt{t^2+4h^2}(t^2+x_2^2)}\Bigg].$$

ga ega bo'lamiz, oxirgi tengsizlikning o'ng tomonidagi har bir integralni boholaymiz.

$$\int_0^\infty \ln\left(1+\frac{8h^2}{t^2+x_2^2}\right)dt=16h^2\int_0^\infty \frac{dt}{t^2+x_2^2+8h^2}-$$

$$-16h^2x_2^2\int_0^\infty \frac{dt}{(t^2+x_2^2+8h^2)(t^2+x_2^2)}=$$

$$=(16h^2+2x_2^2)\int_0^\infty \frac{dt}{t^2+x_2^2+8h^2}-2x_2^2\int_0^\infty \frac{dt}{t^2+x_2^2}=\frac{\pi(16h^2+2x_2^2)}{2\sqrt{x_2^2+8h^2}}-\pi x_2\,,$$

$$\int_0^\infty \left(\frac{1}{(2h-x_2)^2+t^2}+\frac{1}{t^2+x_2^2}\right)dt=\frac{\pi}{2}\left[\frac{1}{2h-x_2}+\frac{1}{x_2}\right],$$

$$\int_0^\infty \frac{dt}{\sqrt{t^2+4h^2}\sqrt{t^2+x_2^2}}\le\int_0^\infty \frac{dt}{\sqrt{t^2+x_2^2}\sqrt{t^2+x_2^2}}=\frac{\pi}{2x_2},$$

$$\int_0^\infty \frac{dt}{(t^2+4h^2)\sqrt{t^2+x_2^2}}=\int_0^1 \frac{dt}{(t^2+4h^2)\sqrt{t^2+x_2^2}}+\int_1^\infty \frac{dt}{(t^2+4h^2)\sqrt{t^2+x_2^2}}\le$$

$$\leq (4h^2x_2)^{-1} + \int_1^{\infty} \frac{dt}{t^2+4h^2} = \frac{1}{4h^2x_2} - \frac{\pi}{4h},$$

$$\int_0^{\infty} \frac{dt}{\sqrt{t^2+4h^2}(t^2+x_2^2)} = \int_0^{1} \frac{dt}{\sqrt{t^2+4h^2}(t^2+x_2^2)} + \int_1^{\infty} \frac{dt}{\sqrt{t^2+4h^2}(t^2+x_2^2)} \leq$$

$$\leq \frac{1}{4hx_2} + \frac{1}{2\sqrt{1+4h^2}x_2}.$$

Bu tengsizliklarni birlashtirib talab qilingan tengsizliklarni olamiz. Teorema isbotlandi.

1. Turg'unlik bahosini keltiramiz.

$u(x) \in A_\rho(D)$ funksiya $y_2 = 0$ to'g'ri chiziqda (2.37) chegaraviy shartni qanoatlantirsin. S sohada esa

$$|u(y)| + \left|T(\partial_y, n)u(y)\right| \leq \delta, \qquad y \in S,\ \ 0 < \delta < 1.$$

shartni qanoatlantirsin. Bunday shartlarda

$$\left|\ u(x)\right| \ \leq\ M\, C_\rho(x)\delta^{x_2/h}\, ln\left(\frac{\mathrm{M}}{\delta}\right), \qquad x \in D$$

tengsizlik o'rinli ekanligini ko'rsatamiz.

Haqiqatdan ham

$$\left|\ u(x)\right| \ \leq \delta \int_S \left[|\Pi_\sigma(y,x)| + \left|\mathrm{T}\left(\partial_y, n\right)\Pi_\sigma(y,x)\right|\right] ds_y +$$

$$+M\int_{-\infty}^{+\infty}\left[|\Pi_\sigma(y,x)|+\left|\mathrm{T}\left(\partial_y,n\right)\Pi_\sigma(y,x)\right|\right]dy_1=$$

$$=\delta\int_{-\infty}^{+\infty}\left[|\Pi_\sigma(y,x)|+\left|\mathrm{T}\left(\partial_y,n\right)\Pi_\sigma(y,x)\right|\right]\sqrt{1+(f'(y))^2}dy_1+$$

$$+M\int_{-\infty}^{+\infty}\left[|\Pi_\sigma(y,x)|+\left|\mathrm{T}\left(\partial_y,n\right)\Pi_\sigma(y,x)\right|\right]dy_1.$$

Yuqoridagidan

$$\left|\,u(x)\,\right|\ \le\ C_\rho\sigma\exp(-\sigma x_2)[M+\delta exp\sigma h]$$

ga ega bo'lamiz. $\sigma=h^{-1}ln\frac{M}{\delta}$ deb olsak, bu holda isbotlanishi kerak bo'lgan ifodani olamiz.

2. Endi faraz qilaylik $u(y)$ va $T\left(\partial_y,n\right)u(y)$ o'rniga S da ularning uzluksiz taqribiy qiymatlari mos ravishda $f_\delta(y)$ va $g_b(y)$ berilgan bo'lsin.

$$\max_S|u(y)-f_\delta(y)|+\max_S\left|T\left(\partial_y,n\right)u(y)-g_\delta(\mathrm{y})\right|<\ \delta.$$

Faraz qilaylik, $u(x)\in A_\rho(D)$ funksiya $y_2=0$ to'g'ri chiziqda (2.37) chegaraviy shartni qanoatlantirsin. Bu shartlarda

$$u_{\sigma\delta}(x)=\int_S\left[\Pi_\sigma(y,x)g_\delta(y)-f_\delta(y)\left\{T(\partial_y,n)\Pi_\sigma(y,x)\right\}\right]ds_y\ ,\qquad x\in D,$$

deb olamiz, bu holda $K(\omega)$ yuqoridagidek aniqlanadi. u holda

$$\left| \, u(x) - u_{\sigma\delta}(x) \, \right| \leq \; M \, C_\rho(x) \delta^{x_2/h} \, ln\left(\frac{\mathrm{M}}{\delta}\right), \qquad x \in D,$$

bu yerda $\sigma = h^{-1} ln \frac{M}{\delta}$, $C_\rho(x)$ (2.57) dan aniqlaniladi.

(2.1)-Natija.

$$\lim_{\sigma\to\infty} u_\sigma(x) = u(x), \qquad \lim_{\delta\to 0} u_{\sigma\delta}(x) = u(x),$$

limitlar D soxadagi har bir kompaktda tekis bajariladi.

3. Yuqoridagi olingan natijalarni biroz umumlashtirish maqsadida S egri chiziqqa nisbatan quyidagi shartlarni qo'yamiz

$$\int_S \exp[-\mathrm{b}_0 \mathrm{ch} \rho_0 |\mathrm{y}_1|] \mathrm{Ds}_\mathrm{y} < \infty, \quad 0 < \rho_0 < \rho \, .$$

$u(x) \in A_\rho(D)$ chegaraviy shartlarni qanopatlantirsin

$$|u(y)| + \left|T\left(\partial_y, n\right) u(y)\right| \leq C \; exp[acos\rho_1 (y_2 - h/2) exp\rho_1 |y_1|], \;\; y \in \partial D.$$

Faraz qilaylik S da berilgan uzluksiz $f_\delta(y)$ va $g_\delta(y)$ funksiyalar

$$|f_\delta(y)| + \; |g_\delta(y)| \leq exp[acos\rho_1 (y_2 - h/2) exp\rho_1 |y_1|], \quad y \in S$$

$$|u(y) - f_\delta(y)| + \left|T\left(\partial_y, n\right) u(y) - g_\delta(y)\right| \leq$$

$$\leq \; \delta \; exp[acos\rho_1 (y_2 - h/2) exp\rho_1 |y_1|], \;\; y \in S$$

tengsizlikni qanoatlantirsin, bu yerda $0 < \; \delta < 1, \; 0 < \rho_1 < \rho \, , \; a \geq 0$.

$$K(\omega) = exp[\sigma\omega - bchi\rho_1(\omega - h/2) - b_1 chi\rho_0(\omega - h/2)|]$$

bu yerda $\omega = i\sqrt{u^2 + \alpha^2} + y_2$, $0 < \rho_1 < \rho$, $0 < x_2 < h$, $b > 0,\ b_1 \geq$

$$\geq b_0[cos(\rho_0 h/2)]^{-1} + \varepsilon,$$

$$\varepsilon > 0, \qquad \alpha = |y_1 - x_1|, \quad b = 2aexp\rho_1 x_1, \quad \sigma = h^{-1}ln\delta^{-1}$$

deb olamiz. Bu shartlarda

$$u_{\sigma\delta}(x) = \int_S \left[\Pi_\sigma(y,x)g_\delta(y) - f_\delta(y)\{T(\partial_y, n)\Pi_\sigma(y,x)\}\right]ds_y \ , \qquad x \in D.$$

deb olamiz.

2.5- Teorema: Yuqorida keltirilgan formulalardan

$$\left|u(x) - u_{\sigma\delta}(x)\right| \leq C(x)\delta^{x_2/h}\, ln\left(\frac{M}{\delta}\right) exp[2acos\rho_1(y_2 - h/2)exp\rho_1|y_1|], \ x \in D \ (2.72)$$

bu yerda

$$C(x) = C_\rho \int_{\partial D} exp[-b_0 ch\rho_0\alpha](r^{-1} + r^{-2})ds_y \, .$$

Isbot: $u(x) - u_{\sigma\delta}(x)$ ni qaraymiz va uni baholaymiz

$$\left|u(x) - u_{\sigma\delta}(x)\right| =$$

$$= \left| \iint_S \left[\Pi_\sigma(y,x)\{T(\partial_y,n)u(y) - g_\delta(y)\} - \right.\right.$$

$$-\{u(y) - f_\delta(y)\}\{T(\partial_y,n)\Pi_\sigma(y,x)\}\big] ds_y +$$

$$\left. + \int\limits_{y_2=0} \left[\Pi_\sigma(y,x)\{T(\partial_y,n)u(y)\} - u(y)\{T(\partial_y,n)\Pi_\sigma(y,x)\}\right] ds_y \right| \le$$

$$\le \int_S \left[|\Pi_\sigma(y,x)| + |T(\partial_y,n)\Pi_\sigma(y,x)|\right] \times$$

$$\times \left[|T(\partial_y,n)u(y) - g_\delta(y)| + |u(y) - f_\delta(y)|\right] ds_y +$$

$$+ \int_{y_2=0} \left[|\Pi_\sigma(y,x)| + |T(\partial_y,n)\Pi_\sigma(y,x)|\right]\left[|T(\partial_y,n)u(y)| + |u(y)|\right] ds_y \le$$

$$\le \delta \int\limits_S exp[acos\rho_1(y_2 - h/2)exp\rho_1|y_1|] \times$$

$$\times \left[|\Pi_\sigma(y,x)| + |T(\partial_y,n)\Pi_\sigma(y,x)|\right] ds_y +$$

$$+C \int\limits_S exp[acos\rho_1(-h/2)exp\rho_1|y_1|] \, [|\Pi_\sigma(y,x)| +$$

$$+|T(\partial_y,n)\Pi_\sigma(y,x)|] ds_y \,.$$

Yana $|\Pi_\sigma(y,x)|$ va $|T(\partial_y,n)\Pi_\sigma(y,x)|$ larni baholaymiz. (2.21) ga ko’ra

$$K(\omega) = exp\left[\sigma y_2 - bcos\rho_1(y_1 - h/2)\, ch\rho_1\sqrt{u^2 + \alpha^2} - \right.$$

$$-b_1 cos\rho_0(y_2-h/2)\, ch\rho_0\sqrt{u^2+\alpha^2}$$

$$|\Phi_\sigma(y,x)| \leq [2\pi|K(x_2)|]^{-1}\int_0^\infty \frac{|K(\omega)|}{|\omega-x_2|}\frac{udu}{\sqrt{u^2+\alpha^2}} \leq$$

$$\leq C\, exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\int_\alpha^\infty exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\, ch\,\rho_1 t-$$

$$-(b_0[cos(\rho_0 h/2)]^{-1}+\varepsilon)cos\rho_0(y_2-h/2)\, ch\rho_0 t]\frac{dt}{\sqrt{(y_2-x_2)^2+t^2}}\leq$$

$$\leq Cr^{-1}\, exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\, exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\, ch\rho_1\alpha-$$

$$-b_0 ch\rho_0\alpha]\int_\alpha^\infty exp[-\varepsilon ch\rho_0 t]dt\leq$$

$$\leq Cr^{-1}\, exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\, exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\, ch\rho_1(|y_1|-|x_1|)-b_0 ch\rho_0\alpha]\times$$

$$\times\int_\alpha^\infty exp[-\varepsilon ch\rho_0 t]dt\leq Cr^{-1}\, exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\, exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\times$$

$$\times\, ch\rho_1(|y_1|-|x_1|)-b_0 ch\rho_0\alpha]\,\frac{\exp(-\varepsilon\rho_0\alpha)}{\varepsilon\rho_0}.$$

$\frac{\partial\Phi_\sigma(y,x)}{\partial y_i}$, $\frac{\partial^2\Phi_\sigma(y,x)}{\partial y_i\partial y_j}$, $i,j=1{,}2$ lar uchun quyidagi tengsizlikni olamiz

$$\left|\frac{\partial\Phi_\sigma(y,x)}{\partial y_i}\right|\le[2\pi|K(x_2)|r]^{-1}\,|K(\omega)|\le$$

$$\times\, Cr^{-1}\,exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\, exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\,ch\rho_1(|y_1|-|x_1|)-b_0 ch\rho_0\alpha],$$

$$K'(\omega)=[\sigma-bi\rho_1 shi\rho_1(\omega-h/2)-b_1 i\rho_0 shi\rho_0(\omega-h/2)|]\times$$

$$exp[\sigma\omega-bchi\rho_1(\omega-h/2)-b_1 chi\rho_0(\omega-h/2)|]$$

bo’lgani uchun

$$|K'(\omega)|\le C\sigma exp[\sigma y_2-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|\,-b_0 ch\rho_0\alpha]]$$

Shuning uchun

$$\left|\frac{\partial^2\Phi_\sigma(y,x)}{\partial y_i\partial y_j}\right|\le C[r^{-1}+r^{-2}]\sigma\, exp[\sigma(y_2-\ x_2)+bcos\rho_1(x_2-h/2)]\times$$

$$\times\, exp[-2acos\rho_1(y_2-h/2)exp\rho_1|x_1|-b_0 ch\rho_0\alpha]\,.$$

Endi yuqoridagi tengsizliklarni hisobga olsak

$$|\Pi_\sigma(y,x)| + \left|T(\partial_y,n)\Pi_\sigma(y,x)\right| \le$$

$$\le C[r^{-1}+r^{-2}]\sigma\, exp[\sigma(y_2 - \ x_2) + bcos\rho_1(x_2 - h/2)] \times$$

$$\times\, exp[-2acos\rho_1(y_2 - h/2)exp\rho_1|x_1| - b_0 ch\rho_0\alpha]\ .$$

ni olamiz. u holda

$$\left|u(x) - u_{\sigma\delta}(x)\right| \le C\sigma\delta \int_S exp[acos\rho_1(y_2 - h/2)exp\rho_1|x_1|] \times$$

$$\times\, exp[\sigma(y_2 - \ x_2) + bcos\rho_1(x_2 - h/2)] \times$$

$$\times\, exp[-2acos\rho_1(y_2 - h/2)exp\rho_1|x_1| - b_0 ch\rho_0\alpha][r^{-1}+r^{-2}]ds_y +$$

$$+C\sigma \int_{y_2=0} exp[acos\rho_1(y_2 - h/2)exp\rho_1|x_1|] \times$$

$$\times\, exp[-\,\sigma x_2 + bcos\rho_1(x_2 - h/2)] \times$$

$$\times\, exp[-2acos\rho_1(y_2 - h/2)exp\rho_1|x_1| - b_0 ch\rho_0\alpha][r^{-1}+r^{-2}]ds_y \le$$

$$\le [1+\delta exp\sigma h]C\sigma\, exp(-\,\sigma x_2)exp[bcos\rho_1(x_2 - h/2)]\exp\left(\frac{acos(\rho_1 h}{2}\right) \times$$

$$\times \int_{\partial D} \exp(-b_0 ch\rho_0\alpha)[r^{-1}+r^{-2}]ds_y\ .$$

Bu yerda $b = 2aexp\rho_1|x_1|$, $b_0 = acos\rho_1(y_2 - h/2)$.

Agar $\sigma = h^{-1}ln\delta^{-1}$ va $C(x) = C_\rho \int_{\partial D} exp[-b_0 ch\rho_0\alpha](r^{-1} + r^{-2})ds_y$

deb olsak teorema isbotlanadi.

Xulosa

Bu ish elastiklik nazariyasi tenglamalari sistemasi yechimini tekislikdagi chegaralanmagan sohada chegaraning qismida uning berilgan qiymatlari va uning kuchlanishi qiymatlari bo'yicha davom ettirish masalasi, ya'ni elastiklik nazariyasi tenglamalari sistemasi uchun Koshi masalasi o'rganiladi.

Bu ishda asosan chegaralanmagan sohada mos ravishda chegaralanmagan yechim holida Karleman matritsasini qurish yo'llari o'rganilgan. Karleman matritsasini qurishda bundan oldin qaralgan holdan farqli ravishda maxsuslik tartibi katta bo'lgan yadro holida Karleman matritsasini mustaqil ravishda qurildi va shu holda regulyarlashgan yechim bilan aniq yechim orasidagi farq baholangan.

ADABIYOTLAR

1. Tixonov A. N. O reshenii nekorrektno postavlennых zadach i metode regulyarizasii // DAH SSSR.-1963-T. 151.-№3. st-501-504
2. Lavrentyev M. M. O nekotorых nekorrektnых zadachax matematicheskoy fiziki. Novosibirsk: VS. SO. AN. SSSR, 1962.92 s.
3. Lavrentyev M. M. O zadache Koshi dlya uravneniya Laplasa // Izv. AN. SSSR Ser. Matem. 1956. T.20.№6. S. 819-842.
4. Yarmuxamedov Sh. O zadache Koshi dlya urovneniya Laplasa. DAN SSSR, 1977, t. 235, №2, st.281-283.
5. Yarmuxamedov Sh. Ya. , Ishankulov T. I., Maxmudov O. I. O zadache Koshi dlya sistemы uravneniy teorii uprugosti v prostranstve // Sib. matem. jurn. 1992. T. 33. №1. S. 186-190.
6. Ishankulov T. I., Maxmudov O. I. Zadacha Koshi dlya sistemы uravneniy termouprugosti v prostranstve. // Matem. zametki. 1998. T 64. №2. S. 210-217.
7. Maxmudov O. I. Zadacha Koshi dlya sistemы uravneniy prostranstvennoy teorii uprugosti v peremesheniyax // Izv. Vuzov. Matematika, 1994. № 1 (380). S. 54-61.
8. Maxmudov O. I., Niyozov I. E. Regulyarizasiya resheniya zadachi Koshi dlya sistemы uravneniy teorii uprugosti v peremeщyeniyax // Sib. matem. jurn. 1998. T. 39. № 2. S. 369-376.
9. Maxmudov O. I., Niyozov I. E. Ob odnoy zadache Koshi dlya sistemы uravneniy teorii uprugosti. // Differens. uravneniya .2000. T. 36. №5. S. 674-678.
10. Maxmudov O. I., Niyozov I. E. Regulyarizasiya resheniya zadachi Koshi dlya sistemы teorii uprugosti. // Matem. zametki. 2000. T. 68. №4. S. 548-553.
11. Maxmudov O. I., Niyozov I. E. Zadacha Koshi dlya sistemы teorii uprugosti v beskonechnoy oblasti. // Uzbekskiy matem. jurnal. 1999. №2. S. 34-39.

12. Kupradze V. D., Burchuladze T. V., Gegeliya T. G. i dr. Tryoxmernыye zadachi matematicheskoy teorii uprugosti i termouprugosti. Klassicheskaya i mikropolyarnaya teoriya. Statika, garmonicheskiye kolebaniya, dinamika. Osnovы i metodы resheniya. M.: Nauka, 1976.
13. Fixtengols G. M. Kurs differensialnogo i integralnogo ischisleniye. M. 1969. T. 2. St. 600.