O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS

TA’LIM VAZIRLIGI

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Fizika-matematika fakulteti

“Matematika” *kafedrasi*

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«Matrisaning Jordan shakli va kvadrat matrisalar to'plamida aniqlangan funksiyalar»mavzusida

“Matematika” ta’lim yo’nalishi bo’yicha bakalavr

darajasini olish uchun

BITIRUV MALAKAVIY ISHI

“ Ish ko’rildi va himoyaga ruxsat berildi ”

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«Himoya qilishga ruxsat berildi »

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KIRISH

O’z oldimizga qo’yilgan yuksak vazifa va marralarga erishishda zamonaviy bilim va tarbiya sohibi bo’lgan, yangicha fikrlaydigan yoshlarimiz bizning asosiy tayanchimiz va suyanchimizdir.

I.A. Karimov

Biz mustaqil O’zbekiston yoshlari har tomonlama yetuk barkamol avlod bo’lib yetishishimiz uchun Prezidentimiz Islom Abdug’aniyevich Karimov keng imkoniyatlar yaratib, yosh avlod tarbiyasiga juda katta e’tibor berib kelmoqdalar.

Keksalarning hayot darajasi va sifatini yanada yaxshilash, ularni moddiy va ma’naviy qo‘llab-quvvatlash ko‘lamini kengaytirish, yoshi ulug‘ insonlar, ayniqsa, 1941—1945 yillardagi urush va mehnat fronti faxriylariga ijtimoiy, pensiya ta’minoti va tibbiy xizmat ko‘rsatishni takomillashtirish, oila va jamiyatda, yoshlarni o‘zbek xalqining ko‘p asrlik qadriyat va an’analari ruhida tarbiyalashda keksalarning o‘rnini mustahkamlash maqsadida, shuningdek, 2015-yilning “Keksalarni e’zozlash yili” deb e’lon qilinishi munosabati bilan: «Keksalarni e’zozlash yili» Davlat dasturini amalga oshirishning ustuvor vazifa va yo‘nalishlari etib quyidagilar belgilansin: keksalarga g‘amxo‘rlik ko‘rsatish va e’tiborni kuchaytirish, ularning hayot darajasi va sifatini oshirish uchun qulay tashkiliy-huquqiy shart-sharoitlar yaratishga qaratilgan qonunchilik va me’yoriy-huquqiy bazani yanada takomillashtirish, pensiya ta’minoti va ijtimoiy qo‘llab-quvvatlash tizimini takomillashtirish, yoshi ulug‘ insonlarga davlat xizmatlari, shu jumladan, turli ma’lumotnoma va tasdiqlovchi hujjatlarni olish xizmatlari ko‘rsatishning eng qulay tizimini shakllantirish; keksalar, avvalambor, fashizm ustidan qozonilgan g‘alaba va Vatanimizni qayta tiklashga munosib hissa qo‘shgan 1941—1945 yillardagi urush va mehnat fronti faxriylarini manzilli ijtimoiy himoya qilish va qo‘llab-quvvatlashni kuchaytirish, yoshi ulug‘ insonlar, birinchi navbatda, yolg‘iz qariyalar va nogironlarga ko‘rsatiladigan ijtimoiy va maishiy xizmatlar ro‘yxatini kengaytirish, ularni farovon va munosib turmush sharoitlari bilan ta’minlash uchun mahallalar, «Nuroniy» jamg‘armasi, boshqa nodavlat tashkilotlari va ijtimoiy tuzilmalar tomonidan moddiy va ma’naviy qo‘llab-quvvatlash miqyosini oshirish; faxriylar va yoshi ulug‘ insonlarga tibbiy va ijtimoiy xizmat ko‘rsatish darajasi va sifatini oshirish, ularni tizimli asosda sog‘lomlashtirishni tashkil etish, profilaktika tadbirlarini ko‘paytirish, ko‘rish va eshitish organlari, tayanch-harakat tizimi va yurak-qon tomir kasalliklarini davolashning zamonaviy usullari bilan qamrab olish va ulardan foydalanishni kengaytirish, keksalar va nogironlarni yordamchi hamda texnik reabilitatsiya vositalari bilan, jumladan, imtiyozli asosda ta’minlash, keksalarga xizmat ko‘rsatishga ixtisoslashtirilgan sanatoriy-sog‘lomlashtirish va ijtimoiy xizmat ko‘rsatish muassasalarining moddiy-texnik bazasini yanada mustahkamlash; mamlakatimizning mudofaa qobiliyati, iqtisodiy, madaniy va intellektual salohiyatini oshirish, jamiyatimizda tinchlik, totuvlik va barqarorlik muhitini mustahkamlashga bebaho hissa qo‘shgan, sog‘lom va barkamol unib-o‘sayotgan avlodni tarbiyalashda faol ishtirok etayotgan keksa avlod vakillariga munosib e’tibor qaratish va ularni har tomonlama qo‘llab-quvvatlash bo‘yicha keng ko‘lamli chora-tadbirlarni tizimli asosda amalga oshirish, oila va jamiyatda yoshi ulug‘ insonlarning o‘rni va obro‘sini oshirish, bolalar va yoshlarni ota-onalarni, har bir nuroniyni chuqur hurmat qilish, e’zozlash va ularga g‘amxo‘rlik ko‘rsatish tuyg‘usi ruhida tarbiyalash bo‘yicha aniq tadbirlarni amalga oshirish; har bir keksaga har tomonlama e’tibor qaratish bo‘yicha amalga oshirilayotgan ishlar samaradorligini kuchaytirish va oshirish, keksalarning hayot faoliyati va dam olishini tashkil etish tizimini sifat jihatidan yangi bosqichga ko‘tarish, bunda ular uchun tuman va mahallalarda muloqot markazlari, qiziqishi bo‘yicha klublar tashkil etishga alohida e’tibor qaratish, jismoniy tarbiya va sport bilan shug‘ullanishlari uchun sharoit yaratish, televidenie, ommaviy axborot vositalari, kinoteatrlar, teatr va boshqa madaniy-ma’rifiy muassasalarda yoshi ulug‘ insonlarning qiziqishlariga mos mavzuda yangi maxsus ko‘rsatuvlar, spektakl va filmlar yaratish va efirga uzatish hajmini ko‘paytirish; mahallalar va fuqarolarning o‘zini o‘zi boshqarish organlari tashkiliy tuzilmasida keksalar va nogironlarga doimiy e’tibor qaratadigan sektorni mustahkamlash, bunda ularga xizmat ko‘rsatadigan idora va xizmatlar, birinchi navbatda, pensiya, ijtimoiy va tibbiy ta’minot xizmatlari faoliyatini muvofiqlashtirish va nazorat qilishni nazarda tutish. Respublika komissiyasiga (SH. Mirziyoev) «Keksalarni e’zozlash yili» Davlat dasturiga kiritilgan tadbirlarning to‘liq, o‘z vaqtida va sifatli bajarilishini tashkil etish va nazorat qilish yuklansin. Respublika komissiyasi: bir hafta muddatda Qoraqalpog‘iston Respublikasi, viloyatlar va Toshkent shahri, tuman va shaharlarda Qoraqalpog‘iston Respublikasi Vazirlar Kengashi Raisi va tegishli hududlar hokimlari boshchiligida Davlat dasturini amalga oshirish bo‘yicha tegishli komissiyalar tashkil etilishini ta’minlasin va ularning zimmasiga Davlat dasturida belgilangan vazifalarni so‘zsiz, to‘liq va o‘z vaqtida bajarilishi uchun shaxsiy javobgarlik yuklasin; ikki hafta muddatda vazirliklar, idoralar, kompaniyalar, uyushmalar va boshqa xo‘jalik birlashmalari, Qoraqalpog‘iston Respublikasi Vazirlar Kengashi, viloyatlar va Toshkent shahar hokimliklari tomonidan har bir vazirlik va idora, hudud, shahar, tuman va aholi punktlarining «Keksalarni e’zozlash yili» bo‘yicha hududiy va tarmoq dasturlari ishlab chiqilishi hamda qabul qilinishini ta’minlasin; respublika davlat va xo‘jalik boshqaruvi organlari, shuningdek, joylardagi davlat hokimiyati organlari darajasida tasdiqlangan Davlat dasturiga kiritilgan tadbirlarni o‘z vaqtida va sifatli bajarish bo‘yicha ishlarni muvofiqlashtirsin va ushbu tadbirlar amalga oshirilishi ustidan tizimli nazorat o‘rnatsin.

Bitiruv malakaviy ishimizning dolzarbligi. O’zbekiston Respublikasining mustaqilligini mustahkamlash yo’lida yosh avlodni chuqur bilimli, kelajakda yetuk kadrlar qilib yetishtirish uchun hozirgi kunda alohida e’tibor berilmoqda. Iqtidorli o’quvchilar bilan ishlash, ularning iqtidorini ro’yobga chiqarish va ularga zamonaviy ilmlarni berish hozirgi kunning asosiy vazifalaridan biridir. Shuning uchun faqat maktab dasturi bilan cheklanmasdan, o’quvchilarning matematik bilim va tafakkurini chuqurlashtirish, fikrlash doirasini kengaytirish, tasavvur qilish qobiliyatini o’stirish maqsadi mazkur malakaviy bitiruv ishida mujassamlashtirildi.

Hozirgi kunda ta’lim sohasida o’zgarishlar yuz bermoqda, ayniqsa, talabalarga bilimlarni chuqur o’rgatish hamda ularni ko’proq mustaqil ishlashga o’rgatish ta’lim tizimining dolzarb vazifalaridan biridir. Ma’lumki, matritsalar mavzusi Algebra va sonlar nazariyasida katta o’rin tutadi. Ayniqsa matritsaning Jordan shakli va kvadrat matritsalar to’plamida aniqlanadigan funksiyalar ustida ishlash dolzarb hisoblanadi.

Pedagogika institutining fizika-matematika fakultetida ilgari alohida-alohida o’qitilgan ,,Oliy algebra ” va ,,Sonlar nazariyasi” fanlari hozir yangi programa bo’yicha ,, Algebra va sonlar nazariyasi ” nomi bilan bitta fan sifatida o’qitiladi. Biz shu matritsalar halqasi matritsaviy funksiyalar va shu kabi matritsaga oid atama va tushunchalar bilan shu fan orqali tushunamiz.

Biz farzandlarimizning nafaqat jismoniy va ma’naviy sog’lom o’sishi, balki ularning eng zamonaviy intellektual bilimlarga ega bo’lgan, uyg’un rivojlangan insonlar bo’lib, XXI asr talablariga to`liq javob beradigan barkamol avlod bo`lib voyaga yetishi uchun zarur barcha imkoniyat va sharoitlarni yaratishni o`z oldimizga maqsad qilib qo`yganmiz.

Bitiruv malakaviy ishimizning maqsadi. Elementlari kompleks sonlardan tashkil topgan ixtiyoriy matritsalarni Jordan matritsasiga keltirish mumkin ekanligini ko’rsatish va ixtiyoriy Jordan katagi yoki Jordan matritsasini xarakteristik matritsasini topish mumkin.

Bitiruv malakaviy ishimizning vazifalari. Shuni alohida ta’kidlash lozimki, ta’lim-tarbiya sohasida islohotlar o’tkazish va ularning asosiy yo’nalishlari, talablari va maqsadlarini aniqlash, hamda tegishli xulosalarni chiqarishda, bugungi muhokama qilinadigan hujjatlarda keng jamoatchiligimizning fikr-mulohazalari, tarbiya va izohlari ifoda topgan desak, hech qanday mubolag’a bo’lmas. Shuning uchun ham amaldagi ta’lim-tarbiya tizimining zaif tomonlarini, zamon talablari, jamiyatimiz kelajagi va maqsadlariga javob bermaydigan jihatlarini chuqur tasavvur qilish, erkin, badavlat yashayotgan mamlakatlar tajribasini o’rganish, o’z o’lkamizga yuksak malakali, har jihatdan yetuk kadrlar tayyorlash dasturining asosiy sharti bo’lmog’i lozim. Shuning uchun ta’lim-tarbiya sohasida ham belgilanayotgan islohotlarni hayotga tadbiq qilishda islohotlarni bosqichma-bosqich o’tkazish prinsipi qo’yilgan.

Bitiruv malakaviy ishimizning o’rganilganlik darajasi. Mavzu misollar bilan, teorema va ta’riflar orqali keng o’rganilgan va hayotiy tatbiqlari o’rganilgan.

Bitiruv malakaviy ishimizning predmeti. Algebra va sonlar nazariyasi, Oliy algebra va Oliy matematika fanlari va bu fanlarning fizika, mexanika va boshqa tabiiy fanlardagi turli-tuman tatbiqlarini o’rganish, Oliy o’quv yurtlarida bu fan o’tilishini yo’lga qo’yish va bu fan ahamiyati haqida gapirib o’tish maqsadga muvofiq. Matritsaning Jordan shakli va kvadrat matritsalar to’plamida aniqlanadigan funksiyalar mavzusi orqali ba’zi bir differensial tenglamalarni yechishni talab qilinadi va keng qo’llaniladi

Bitiruv malakaviy ishimizning ob’yekti. Oliy o’quv yurtlarida bu fan o’tilishini yo’lga qo’yish va bu fan ahamiyati haqida gapirib o’tish maqsadga muvofiq. Matritsaning Jordan shakli va kvadrat matritsalar to’plamida aniqlanadigan funksiyalar mavzusi orqali ba’zi bir differensial tenglamalarni yechishni talab qilinadi va keng qo’llaniladi

Bitiruv malakaviy ishimizning ilmiy farazi. Bitiruv malakaviy ishi referativ-uslubiy xarakterga ega bo’lsada, bir qancha natijalarga erishilgan bo’lib, bitiruv malakaviy ishidan Oliy o'quv yurtlariga o'tiladigan Algebra va sonlar nazariyasi va Oliy matematika darslarida foydalaniladi.

Bitiruv malakaviy ishimizning yangiligi. Ish nazariy xarakterga ega bo'lib, olingan natijalar matematika, fizika va texnik masalalarni hal qilishda keng qo'llaniladi.

Bitiruv malakaviy ishimizning amaliy ahamiyati. Bitiruv malakaviy ishi referativ-uslubiy xarakterga ega bo'lsada, bir qancha natijalarga erishilgan bo'lib, bitiruv malakaviy ishidan Oliy o'quv yurtlariga o'tiladigan Algebra va sonlar nazariyasi va Oliy matematika darslarida foydalaniladi.

Bitiruv malakaviy ishimizning metodologik asosi. Bitiruv malakaviy ishidan Oliy o'quv yurtlariga o'tiladigan Algebra va sonlar nazariyasi va Oliy matematika darslarida foydalaniladi. Matritsaning Jordan shakli va kvadrat matritsalar to'plamida aniqlanadigan funksiyalar mavzusi misollar orqali aniq, ravshan tushuntirilgan. Ishning yaxshiligi shundaki, bu mavzu ba'zi bir differensial tenglamalarni yechishda keng qo'llaniladi.

Bitiruv malakaviy ishimizning metodlari. Oliy o'quv yurtlariga o'tiladigan Algebra va sonlar nazariyasi va Oliy matematika darslarida Matritsaning Jordan shakli va kvadrat matritsalar to'plamida aniqlanadigan funksiyalar mavzusi o'tilganda Bitiruv malakaviy ishini o'qib, metodik sifatida foydalanishlari mumkin.

Bitiruv malakaviy ishimizning tarkibi va hajmi. Mazkur bitiruv malakaviy ishi kirish, ikki bob, besh qism, xotima va foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib, I bobda mavzuning asosiy tushunchalari: Matritsalar halqasi, Matritsali funksiyalar haqida ma’lumotlar berilgan. Misollar orqali I bobning ikkala qismi ham keng yoritilgan. II bobda Jordan matritsasi, Jordan katagining xarakteristik matritsasi, Jordan matritsasining xarakteristik matritsasi haqida umumiy ma’lumotlar berilgan. Hamda ixtiyoriy matritsani Jordan matritsasiga keltirishga oid misollar ko’rsatilgan.

Hajmi____ sahifadan iborat.

I.Kvadrat matritsalar to’plamida aniqlangan funksiyalar

1.1.Matritsalar halqasi.

1.1.1-ta'rif: P maydonning mn ta a_{ij} sonlaridan tuzilgan

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko'rinishdagi ifoda P maydon ustidagi matritsa deyiladi.

Matritsa elementlarining gorizontal qatorlari uning satrlari deb, vertikal qatorlari ustunlari deb ataladi.

Shunday qilib, A matritsada m ta satr va n ta ustun bor. Matritsani tashkil etuvchi a_{ij} sonlari uning elementlari deyiladi.

Har bir a_{ij} elementning birinchi indeksi bu element turgan satrning nomerini, ikkinchi indeksi (bu element turgan satrning) esa ustunning nomerini bildiradi.

Demak, a_{ij} element i satr va j ustunda turadi. A matritsada satrlar soni ustunlar sonidan kichik, teng yoki katta (ya’ni $m < n, m = n, m > n$) bo'lishi mumkin. $m = n$ bo'lgan holda A ni n-tartibli kvadrat matritsa deyiladi. Kvadrat matritsada $a_{11}, a_{22}, ..., a_{nn}$ elementlar matritsaning birinchi (bosh) diagonalini $a_{1n}, a_{2n-1}, ..., a_{n-1,2}, a_{n-1,1}$ elementlar esa ikkinchi diagonalini tashkil etadi. $m \neq n$ bo'lgan holda A ni tog'ri burchakli matritsa deyiladi. To'g'ri burchakli matritsani m satrli va n ustunli matritsa yoki qisqaroq matritsa deb ham aytiladi.

Xususiy holda, matritsa bir satrli va n ustunli yoki m satrli va bir ustunli bo'lishi, ya'ni

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \end{pmatrix}$$

yoki

$$A = \begin{pmatrix} a_{11} \\ a_{12} \\ \ldots \\ a_{1n} \end{pmatrix}$$

ko'rinishdagi matritsani ifodalash mumkin.

Ikkalasi ham m satrli va n usunli A va B matritsalarni nomdosh (bir nomli) matritsalar deymiz.

Faqat nomdosh matritsalargina teng bo'lishi mumkin. A va B matritsalarnig tenglik sharti quyidagichadir: A ning har bir elementini B ning mos bij elementiga teng bo'lgandagina bu ikki matritsa teng, ya'ni $A = B$ bo'ladi. Demak, bironta i va j lar uchun $a_{ij} \neq b_{ij}$ bo'lsa, A va B lar teng bo'lmagan matritsalarni tasvirlaydi. Matritsalarning teng emasligi $A \neq B$ kabi belgilanadi.

A matritsaning satrlari m ta n o'lchovli gorizontal

$$\begin{aligned} a_1 &= (a_{11}, a_{12}, \ldots, a_{1n}) \\ a_1 &= (a_{21}, a_{22}, \ldots, a_{2n}) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ a_1 &= (a_{m1}, a_{m2}, \ldots, a_{mn}) \end{aligned}$$

vektorlarni, ustunlari esa n ta m o'lchovli vertikal vektorlarni tashkil etadi. Bu so'ngi vektorlarni tashkil etadi. Bu so'nggi vektorlarni gorizontal vektorlardan farq qilishi uchun

$$\begin{aligned} a^1 &= (a_{11}, a_{12}, ..., a_{m1}) \\ a^2 &= (a_{21}, a_{22}, ..., a_{m2}) \\ &.......................... \\ a^n &= (a_{m1}, a_{m2}, ..., a_{mn}) \end{aligned}$$

ko'rinishda belgilaymiz.

<u>Matritsani elementar almashtirishlar</u>

<u>1.1.2 - ta'rif:</u> Quyidagi almashtirishlarga matritsani elementar almashtirishlar deyiladi:

1) Satrlarni ustunlar qilib va ustunlarni satrlar qilib yozish (matritsani transponirlash)

2) Ikki satr (yoki ustun)ni o'zaro o'rin almashtirish

3) Satr (ustun) elementlarni istalgan songa ko'paytirib, boshqa satr (ustun) ning mos elementlariga qo'shish

4) Satr (ustun) elementlarini noldan farqli songa ko'paytirish;

<u>1.1.1 - teorema:</u> Elementar almashtirishlar matritsa rangini o'zgartirmaydi.

Isbot: Elementar almashtirishlarni; masalan, satrlarga tatbiq etaylik.

1) Mazkur holda matritsani transponirlash gorizontal vektorlar sistemasiga, demak, uning rangiga ta'sir qilmaydi.

2) Ikki satrning o'rnini almashtirish gorizontal vektorlar sistemasida ikki vektorni o'zaro o'rin almashishiga olib keladi. Bu esa sistemani, demak, uning rangini o'zgartirmaydi.

3) Satrning hamma elementlarini $c \neq 0$ songa ko'paytirish - gorizontal vektorlar sistemasining biror vektorni $c \neq 0$ ga ko'paytirishdan iborat. Buning natijasida sistema rangi o'zgarmaydi. Haqiqatan ham, umuman, qandaydir

$$b_1, b_2, ..., b_s \quad (1.1.1)$$

vektorlar sistemasi chiziqli bog'langan bo'lsa,

$$b_1, b_2, ...cb_i, ...b_s \quad (1.1.2)$$

sistema ham chiziqli bog'langan bo'ladi, chunki (1.1.1) chiziqli bo'lgani holda (1.1.2) ni chiziqli bog'langan desak, kamida bittasi noldan farqli $\alpha_1, \alpha_2, ...\alpha_i, ...\alpha_s$ sonlar uchun bajariladigan

$$\alpha_1 b_1 + ... + \alpha_i (cb_i) + ... + \alpha_s b_s = \alpha_1 b_1 + ... + (c\alpha_i) b_i + ... + \alpha_s b_s = 0$$

tenglik (1.1.1) ning chiziqli bog'langanligini ko'rsatadi. Agar (1.1.1) chiziqli bog'langan bo'lsa, kamida - bittasi noldan farqli $\alpha_1, \alpha_2, ...\alpha_s$ sonlar uchun

$$\alpha_1 b_1 + ... + \frac{\alpha_i}{c} \cdot cb_i + ... + \alpha_s b_s = \alpha_1 b_1 + ... + \alpha_i b_i + ... + \alpha_s b_s = 0$$

bajariladi.

4) Matritsaning j - satrini c songa ko'paytirib i - satriga qo'shish gorizontal

$$a_1, ..., a_i, ..., a_j, ..., a_m \quad (1.1.3)$$

sistemada a_{ij} vektorni c ga ko'paytirib, a_i vektorga qo'shishdan iborat. Buni bajarish natijasida

$$a_1,...,(a_i+ca_j),...,a_j,...,a_m \quad (1.1.4)$$

sistema hosil bo'ladi. (1.1.4) ning a_r+ca_j dan boshqa har bir a_k vektor (1.1.3) orqali chiziqli ifodalanadi.

$$a_k = 0\cdot a_1+...+1\cdot a_k+...0\cdot a_m$$

Endi a_i+ca_j vektor quyidagicha ifodalanadi:

$$a_i+ca_j = 0\cdot a_1+...+1\cdot a_i+...+c\cdot a_j+...+0\cdot a_m.$$

Aksincha (1.1.3) ning a_i dan boshqa har bir a_k vektorni (1.1.4) orqali

$$a_i = 0\cdot a_1+...+1\cdot a_i+...+0\cdot a_{j+1}+...+0\cdot a_m$$

ko'rinishda ifodalandi. a_i ning esa chiziqli ifodalanishi bunday bo'ladi:

$$a_i = 0\cdot a_1+...+1\cdot(a_i+c\cdot a_j)+...+c\cdot a_j+...+0\cdot a_m$$

Shunday qilib, (1.1.3) va (1.1.4) sistemalar ekvivalentdir.

Bunga asosan (1.1.3) va (1.1.4) sistemalarning ranglari teng ekanini ko'ramiz.

<u>1.1.2 - teorema:</u> Har bir matritsaning satriy rangi ustuniy rangiga teng.

Isboti. $(m\times n)$ matritsa berilgan bo'lsin.

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{pmatrix}$$

Matritsaning n o'lchovli gorizontal vektorlari va m o'lchovli vertikal vektorlari ushbulardan iborat:

$$a_1, a_2,...a_r,...a_m \quad (1.1.5)$$

$$a^1, a^2, ... a^s, ... a^n \quad (1.1.6)$$

(1.1.5) sistemaning satriy rangini aniqlovchi vektorlari $a_1, a_2, ..., a_r$ (1.1.6) deb va (1.1.5) sistemaning ustuniy rangini aniqlovchi vektorlarni $a^1, a^2, ... a^s$ (1.1.6) deb olamiz. (1.1.5) va (1.1.6) sistemalarning xuddi shu xilda joylashishiga A ning satrlarini o'zaro va ustunlarini o'zaro o'rin almashtirish natijasida erishish mumkin.

Demak, 1.1.1 - teoremaga ko'ra, A ning satriy va ustuniy ranglari o'zgarmaydi. Shunday qilib, A matritsaning satriy rangi r ga va ustuniy rangi s ga tengdir. Bu yerda $r = s$ ekanini ko'rsatish lozim. Biz $r < s$ deb faraz qilamiz. (1.1.5) vektorlar

$$a_i = (a_{i1}, a_{i2}, ..., a_{is}, ... a_{2n}) \quad \left(i = \overline{1,2}\right)$$

ko'rinishga ega. (1.1.6) vektorlar

$$a^j = (a_1, a_{2j}, ... a_{rj}, ... a_{mj})$$

ko'rinishga ega.

(1.1.5) vektorlarning birinchi s ta koordinatalaridan foydalanib, quyidagi s noma'lumli r ta bir jinsli chiziqli tenglamalar sistemasini tuzamiz:

$$a_{i1}x_1 + a_{i2}x_2 + ... + a_{is}x_s = 0 \quad \left(i = \overline{1,r}\right)$$

$r < s$ ga asosan, bu sistema $(\alpha_1, \alpha_2, ..., \alpha_s)$ nol bo'lmagan yechimga ega.

Demak,

$$a_{i1}\alpha_1 + a_{i2}\alpha_2 + ... + a_{is}\alpha_s = 0 \quad \left(i = \overline{1,r}\right) \quad (1.1.7)$$

tengliklar o'rinli, $(\alpha_1, \alpha_2, ..., \alpha_s)$ yechim

$$a_{k1}x_1 + a_{k2}x_2 + ... + a_{kn}x_s = 0 \quad \left(k = \overline{r+1, m}\right)$$

sistemani ham qanoatlantiradi. Haqiqatan,

$$\left(a_{r+1}, a_{r+2}, a_m\right)$$

gorizontal vektorlarning har qaysisi (1.1.5) sistema orqali chiziqli ifodalanishi, ya'ni $a_k = \mu_1 a_1 + \mu_2 a_2 + ... + \mu_r a_r$ bo'lishi bizga ma'lum.

Buni mukammal yozsak

$$\left(a_{k1}, a_{k2},..., a_{kn}\right) = \begin{pmatrix} \mu_1 a_{11} + \mu_2 a_{21} + ... + \mu_r a_{r1}, \mu_1 a_{12} + \mu_2 a_{22} + \\ + ... + \mu_r a_{r2},..., \mu_1 a_{1n} + \mu_2 a_{2n} + ... + \mu_r a_{rn} \end{pmatrix}$$

kelib chiqadi.

Demak, vektorlarning tenglik shartga muvofiq

$$\begin{aligned} &a_{k1} = \mu_{1k} a_{11} + \mu_{2k} a_{21} + ... + \mu_{rk} a_{r1}, \\ &a_{k2} = \mu_{1k} a_{12} + \mu_{2k} a_{22} + ... + \mu_{rk} a_{rk}, \\ &.., \\ &a_{ks} = \mu_{1k} a_{1s} + \mu_{2k} a_{2s} + ... + \mu_{rk} a_{rs}, \\ &a_{ks+1} = \mu_{1k} a_{1s+1} + \mu_{2k} a_{2s+1} + ... + \mu_{rk} a_{rs+1}, \\ &a_{kn} = \mu_{1k} a_{1n} + \mu_{2k} a_{3n} + ... + \mu_{rk} a_{rn} \end{aligned}$$

bo'ladi, bunda $\left(k = \overline{r+1, k}\right)$. Bu tengliklarning birinchisi s tasini ($s = n$ bo’lgani holda, hammasini) mos ravishda $\alpha_1, \alpha_2,..., \alpha_s$ larga ko'paytirib, natijalarni hadma - had qo'shsak, (1.1.7) ga asosan, ushbuni hosil qilamiz;

$$\begin{aligned} &a_{k1}\alpha_1 + a_{k2}\alpha_2 + ... + a_{ks}\alpha_s = \mu_{1k}\left(a_{11}\alpha_1 + a_{12}\alpha_2 + ... + a_{1s}\alpha_s\right) + \mu_{2n}\left(a_{21}\alpha_1 + a_{22}\alpha_2 + ... + a_{2s}\alpha_s\right) + .. \\ &.. + \mu_{rk}\left(a_{r1}\alpha_1 + a_{r2}\alpha_2 + ... + a_{rs}\alpha_s\right) = \mu_{1k} \cdot 0 + \mu_{2k} \cdot 0 + ... + \mu_{rk} \cdot 0 = 0 \end{aligned} \tag{1.1.8}$$

(1.1.7) va (1.1.8) tengliklar (1.1.6) sistemaning bog'langanligini ko'rsatadi. Chindan ham (1.1.7) va (1.1.8) larga ko'ra, quyidagiga ega bo'lamiz:

$$\alpha_1 a^1 + \alpha_2 a^2 + ... + \alpha_s a^s = \alpha_1(a_{11}, a_{12}, ..., a_{m1}) + \alpha_2(a_{21}, a_{22}, ..., a_{m2}) + \alpha_s, (a_{1s}, a_{2s}, ..., a_{ms}) =$$
$$= \begin{pmatrix} a_{11}\alpha_1 + a_{12}\alpha_2 + ... + a_{1s}\alpha_s; \\ a_{21}\alpha_1 + a_{22}\alpha_2 + ... + a_{2s}\alpha_s, ...a_{m1}\alpha_1 + a_{m1}\alpha_2 + ... + a_{ms}\alpha_s \end{pmatrix} = (0,0,...,0) = 0$$

Ammo, bu (1.1.6) ning chiziqliligiga ziddir shu sababli $r < s$ bo'lishi mumkin emas. Endi (1.1.3) va (1.1.4) sistemalarning rollarini almashtirib, yuqoridagi muhokamasini takrorlasak, $s < r$ ning mumkin emasligiga kelamiz.

Shunday qilib, $r = s$ ekanligi tasdiqlanadi.

Pog'onali matritsa.

Avvalo sistemalarning chiziqli bog'lanishi haqida tushuncha beramiz. Har qaysi P maydonning n ta sonidan tuzilgan m ta sistema berilgan bo'lsin:

$$\begin{cases} a_{11}, & a_{12}, & ... & a_{1n} \\ a_{21}, & a_{22}, & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1}, & a_{m2}, & ... & a_{mn} \end{cases} \quad (1.1.9)$$

Bunday sistemalar n o'lchovli vektorlarni, matritsaning satr yoki ustunlarini ifodalashi mumkin.

1.1.2-ta'rif. Kamida bittasi noldan farqli $C_1, C_2, ...C_m \in P$ sonlar uchun

$$c_1 a_{1i} + c_2 a_{2i} + ... + c_m a_{mi} = 0 \quad (i = \overline{1, n}) \quad (1.1.10)$$

tenglik bajarilsa, (1.1.9) ni chiziqli bog'langan sistemalar deyiladi. (1.1.9) tenglik $C = C_2 = ... = C_m$ shartdagina o'rinli bo'lsa, (1.1.9) ni chiziqli erkli sistemalar deyiladi.

1.1.3-ta'rif. (1.1.3) sistemalar va yana $b_1, b_2, ...b_n$ sistema berilgani holda $\alpha_1, \alpha_2, ..., \alpha_n$ sonlar uchun

$$b_i = \alpha_1 a_{1i} + \alpha_2 a_{2i} + ... + \alpha_m a_{mi} \quad (1.1.11)$$

tengliklar bajarilsa, (1.1.11) sistema (1.1.9) sistemalarning chiziqli kombinatsiyasi ((1.1.9) sistemalar orqali chiziqli ifodalanadi) deymiz.

1.1.1-misol.

$$A = \begin{pmatrix} 3 & 1 & -2 & -1 \\ 2 & -1 & 1 & -2 \\ -5 & -2 & 3 & 1 \end{pmatrix}$$

matritsada, birinchi satrni 2 ga va ikkinchi satrni -3 ga ko'paytirib, birinchini ikkinchiga qo'shsak, so'ngra yana birinchi satrni 5 ga va uchinchini 3 ga ko'paytirib, birinchini uchinchiga qo'shsak, ushbu

$$\begin{pmatrix} 3 & 1 & -2 & -1 \\ 0 & 1 & -3 & -4 \\ 0 & 3 & -1 & -2 \end{pmatrix}$$

matritsa hosil bo'ladi.

Bu matritsada ikkinchi satrni 3 ga va uchinchini -1 ga ko'paytirib, ikkinchini uchinchiga qo'shsak,

$$B=\begin{pmatrix} 3 & 1 & -2 & -1 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -8 & -10 \end{pmatrix}$$

matritsaga kelamiz. Yana

$$C=\begin{pmatrix} 2 & -3 & 3 & 0 \\ -4 & 2 & -4 & 5 \\ -2 & -1 & -1 & 5 \end{pmatrix}$$

matritsani olib, yuqoridagi singari almashtirishlarni bajarsak, ushbuni hosil qilamiz:

$$C\rightarrow\begin{pmatrix} 2 & -3 & 3 & 0 \\ 0 & -4 & 2 & 5 \\ 0 & -4 & 2 & 5 \end{pmatrix}\rightarrow\begin{pmatrix} 2 & -3 & 3 & 0 \\ 0 & -4 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}=\mathrm{D}$$

Umuman, m satrli matritsa berilgani holda birinchi va ikkinchi satrlarni, undan keyin birinchi va uchinchi satrlarni, nihoyat, birinchi va m -satrlarni shunday sonlarga ko'paytiramizki, birinchi satrni navbat bilan boshqa hamma satrlarga qo'shganimizda ikkinchi satrdan boshlab, birinchi ustun elementlari nollarga aylanadi. So'ngra ikkinchi satr yordamida keyingi hamma satrlar bilan shunday almashtirishlarni bajaramizki, uchinchi satrdan boshlab, ikkinchi ustun elementlari nollarga aylanadi.Undan keyin to'rtinchi satrdan boshlab, uchinchi ustun elementlari nollarga aylanadi va hokazo. Shu yo'sinda bu progressiyani oxirigacha davom ettiramiz.

Agar matritsaning ixtiyoriy satrlari boshqalari chiziqli ifodalangan bo'lsa, u holda mana shu almashtirishlar natijasida bunday satrlarning hamma elementlari nollarga (satrlar nol satrlarga) aylanadi.

Bironta elementi noldan farqli satrni nol bo'lmagan satr deb atasak, yuqoridagi almashtirishlardan keyin hosil bo'lgan satrlar soniga teng bo'ladi, chunki (chiziqli erkli gorizontal vektorlarni) bildiradi.

Yuqorida qo'llanilgan almashtirishlar matritsani elementar almashtirishlardan iborat bo'lgani uchun, ular matritsaning rangini o'zgartirmaydi.

<u>1.1.4 - ta'rif</u> Nol bo'lmagan satrlarga ega A matritsada har bir keyingi nol bo'lmagan satrning birinchi noldan farqli elementi avvalgi nol bo'lmagan satrning birinchi noldan farqli elementidan o'ngda turgan bo'lsa, A pog'onali matritsa deyiladi.

Masalan, mana bular pog'onali matritsalardir:

$$A = \begin{pmatrix} 1 & 0 & 2 & 3 & -5 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 7 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 8 & 3 & 0 & -7 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 1 & 5 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 6 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Yuqoridagi muhokamalardan bunday xulosaga kelamiz: pog'onali matritsaning rangi uning nol bo'lmagan satrlari soniga teng. Ixtiyoriy S matritsaning rangini aniqlash uchun, uni yuqorida ko'rsatilgan qoida bo'yicha elementar almashtirib, T pog'onali matritsaga kelamiz. U holda

$$r(S) = r(T)$$

bo'ladi.

Ba'zan matritsani pog'onali shaklga keltirish uchun uning satrlarini o'zaro yoki ustunlarini o'zaro almashtirish lozim.

1.1.2-misol.

$$A = \begin{pmatrix} 0 & 0 & 2 & 0 & 1 \\ 0 & 11 & 0 & 3 & 5 \end{pmatrix} \to \begin{pmatrix} 0 & 11 & 0 & 3 & 5 \\ 0 & 0 & 2 & 0 & 1 \end{pmatrix}$$

Demak, $r(A) = 2$;

$$B = \begin{pmatrix} 0 & 0 & 6 \\ 0 & -7 & 0 \\ 6 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 8 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

Demak, $r(B) = 3$.

Matritsalar ustida amallar

P maydon ustidagi istalgan ikki nomdosh matritsani qo'shish ushbu qoida bo'yicha bajariladi:

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \dots & a_{2n} + b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \dots & a_{mn} + b_{mn} \end{pmatrix}$$

Demak yig'indi matritsaning $a_{ij}+b_{ij}$ elementlari qo'shiluvchi matritsalarning mos a_{ij} va b_{ij} elementlari yig'indilariga teng bo'lib, yig'indi matritsa qo'shiluvchi matritsalar bilan nomdoshdir.

Matritsalarni qo'shish kommutativ va assotsiativ ekanligi ravshan, chunki bu amal matritsalarning elementlarini, ya'ni sonlarni qo'shishdan iborat.

Shunday qilib: istalgan $(m\times n)$ matritsalar uchun

$$A+B=B+A,\quad (A+B)+C=A+(B+C).$$

Hamma elementlari nollardan iborat

$$0=\begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

matritsani nol matritsa deb ataymiz. 0 bilan nomdosh har bir A matritsa uchun $A+0=A$ dir.

Ushbu

$$-A=-\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}=\begin{pmatrix} -a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & -a_{22} & \dots & -a_{2n} \\ \dots & \dots & \dots & \dots \\ -a_{m1} & -a_{m2} & \dots & -a_{mn} \end{pmatrix}$$

matritsa

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

matritsaga qarama - qarshi bo'lib, $A+(-A)=0$ dir. $A+(-B)$ yig'indini $A-B$ ko'rinishda yozib, uni A va B matritsalarning ayirmasi deymiz.

Xususiy holda, $A-A=0, A-0=A, 0-A=-A$.

Algebrada matritsalarni ko'paytirish qoidasi ham beriladi.

Faqat $(m\times n)$ matritsani $(n\times k)$ matritsaga ko'paytirish mumkin (boshqacha aytganda, faqat n ustunli matritsa n satrli matritsaga ko'paytirildi). Ko'paytmada $(m\times k)$ matritsa hosil bo'ladi. Buni quyidagi sxema bilan ifodalaymiz:

$$(m\times n)(n\times k)=(m\times k)$$

matritsalarni ko'paytirish qoidasi quyidagicha: $A\cdot B=C$ ko'paytmaning har bir c_{ij} elementini hosil qilish uchun A ning i -satridagi elementlarini B ning j -ustunlaridagi mos elementlariga ko'paytirib, natijalarni qo'shamiz:

$$c_i = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj} = \sum_{i=1}^{n} a_{ie}b_{ij}$$

$(m\times n)(n\times k)=(m\times k)$ sxema bo'yicha hosil qilingan ko'paytmani ma'noga ega ko'paytma deymiz.

Xususiy holda kvadrat matritsalarni ko'paytirish uchun ularning tartiblari bir xil bo'lishi talab qilinadi. Ko'paytma ham xuddi shu tartibdagi kvadrat matritsani ifodalaydi.

<u>1.1.3-misol:</u>

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & 1 & 3 \\ -7 & 2 & 0 & 5 \end{pmatrix} + \begin{pmatrix} -9 & 0 & 1 & -2 \\ 1 & 6 & 0 & 8 \\ 0 & 3 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -8 & 2 & 4 & 2 \\ 1 & 2 & 1 & 11 \\ -7 & 5 & 0 & 4 \end{pmatrix}$$

<u>1.1.4-misol:</u>

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & 1 & 3 \\ -7 & 2 & 0 & 5 \end{pmatrix} - \begin{pmatrix} -9 & 0 & 1 & -2 \\ 1 & 6 & 0 & 8 \\ 0 & 3 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 10 & 2 & 2 & 6 \\ -1 & -10 & 1 & -5 \\ -7 & -1 & 0 & 6 \end{pmatrix}$$

<u>1.1.5-misol.</u>

$$\begin{pmatrix} 2 & 1 & -2 & 0 \\ -3 & -6 & 1 & 4 \\ 0 & 3 & 5 & 7 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ -3 & 4 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 10 & -10 \\ -11 & -13 \\ -2 & -15 \end{pmatrix}$$

<u>1.1.6- misol:</u>

$$\begin{pmatrix} 1 & -2 & 3 \\ -3 & 1 & 2 \\ 0 & 4 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & 7 & -8 \\ 9 & 1 & 0 \\ -2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} -22 & 14 & 4 \\ -1 & -14 & 32 \\ 38 & 1 & -4 \end{pmatrix}$$

Ikkitadan ortiq matritsalarni ham ko'paytirish mumkin. Masalan, 3 ta matritsa quyidagi sxema bo'yicha ko'paytiriladi:

$$[(m\times n)(n\times k)](k\times p)=(m\times k)(k\times p)=(m\times p)$$

va demak, bunday ko'paytma ma'noga ega bo'ladi.

<u>1.1.7–misol.</u>

$$\left[\begin{pmatrix}1 & 2 & -3\end{pmatrix}\begin{pmatrix}3 & -1\\ 0 & 2\\ 1 & 4\end{pmatrix}\right]\cdot\begin{pmatrix}0 & 5 & -2\\ 1 & 3 & 8\end{pmatrix}=\begin{pmatrix}0 & -9\end{pmatrix}\begin{pmatrix}0 & 5 & -2\\ 1 & 3 & 8\end{pmatrix}=\begin{pmatrix}-9 & -27 & -72\end{pmatrix}$$

Ushbu 1.1.3-teorema matritsalarni ko'paytirish assotsiativ ekanini tasdiqlaydi.

<u>1.1.3 - teorema.</u> Uchta A,B,C matritsa uchun AB va ko'paytmalar ma'noga ega bo'lsa, $(AB)C$ va $A(BC)$ ko'paytmalar ham ma'noga ega bo'lib, $(AB)C=A(BC)$ tenglik bajariladi.

Isboti: A,B,C lar mos ravishda $(m\times n),(n\times k),(k\times p)$ (m×n) matritsalar bo'lsin. AB ko'paytma $(m\times n)(n\times k)=(m\times k)$ sxema bo'yicha va BC ko'paytma $(n\times k)(k\times p)=(n\times p)$ sxema bo'yicha aniqlanishi ravshan. U holda $(AB)C$ ko'paytma $(m\times k)(k\times p)=(m\times p)$ sxema bo'yicha aniqlanib, demak, ikkala ko'paytma ham ma'noga ega bo'ladi.

Endi $(AB)C=A(BC)$ tenglikning bajarilishini, demak, $(AB)C$ va $A(BC)$ matritsalarning umumiy u_{ij} va v_{ij} elementlari o'zaro teng ekanini isbotlaymiz.

Haqiqatan, AB ning umumiy elementi

$$l_{i\beta} = \sum_{\alpha=1}^{n} a_{i\alpha} b_{\alpha\beta} \quad (i = \overline{1,m} \text{ va } \beta = \overline{1,k}) \quad (1.1.12)$$

va BC ning umumiy elementi

$$l_{\alpha j} = \sum_{\beta=1}^{k} b_{\alpha\beta} c_{\beta j} \text{ bo'ladi } (i = \overline{1,p}) \quad (1.1.13)$$

(1.1.12) va (1.1.13) larga binoan $(AB)C$ va $A(BC)$ larning umumiy elementlari

$$u_{ij} = \sum_{\beta=1}^{k} c_{i\beta} c_{\beta i} = \sum_{\beta=1}^{k} \sum_{\alpha=1}^{n} a_{i\alpha} b_{\alpha\beta} c_{\beta j}$$

va

$$v_{ij} = \sum_{\alpha=1}^{n} a_{i\alpha} l_{\alpha j} = \sum_{\alpha=1}^{n} a_{i\alpha} \sum_{\beta=1}^{k} b_{\alpha\beta} c_{\beta i} = \sum_{\beta=1}^{k} \sum_{\alpha=1}^{n} a_{i\alpha} b_{\alpha\beta} c_{\beta j}$$

bo'ladi. Shunday qilib, $u_{ij} = v_{ij}$, ya'ni $(AB)C = A(BC)$.

Umuman, matritsalarni ko'paytirish kommutativ emas,

1.1.8-misol:

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 5 \end{pmatrix}$$

va

$$B=\begin{pmatrix}1 & 2\\ 1 & -2\\ 1 & 3\end{pmatrix}$$

matritsalar uchun:

$$AB=\begin{pmatrix}2 & -1\\ 8 & 9\end{pmatrix},\quad BA=\begin{pmatrix}2 & 7 & 9\\ 2 & -5 & -11\\ 2 & 10 & 14\end{pmatrix}$$

Bu yerda $AB \neq BA$

$$A=\begin{pmatrix}1 & -3\\ 4 & 5\end{pmatrix} \text{ va } B=\begin{pmatrix}2 & 1\\ 6 & 0\end{pmatrix}$$

matritsalar uchun

$$AB=\begin{pmatrix}20 & 1\\ -22 & 4\end{pmatrix},\quad BA=\begin{pmatrix}6 & -16\\ -6 & 18\end{pmatrix}$$

Bunda ham $AB \neq BA$

$$\alpha \in P \text{ sonni } A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{m1} & a_{m2} & \dots & a_{mn}\end{pmatrix} \quad (1.1.14)$$

matritsaga ko'paytirish deb bu sonni A ning hamma a_{ij} elementlariga ko'paytirishga aytiladi. Shunday qilib,

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \dots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \dots & \alpha a_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha a_{m1} & \alpha a_{m2} & \dots & \alpha a_{mn} \end{pmatrix} \quad (1.1.15)$$

Birinchidan $\alpha A = A\alpha$ ekanligi ravshan Ikkinchidan, (1.1.14) tenglik matritsadagi hamma elementlarning α umumiy ko'paytuvchisini matritsa belgisi tashqarisida chiqarish mumkinligini ko'rsatadi.

Nomdosh matritsalar uchun ushbu tengliklar o'rinli.

$$\alpha(A+B) = \alpha A + \alpha B \quad (\alpha+\beta)A = \alpha A + \beta A$$

$$\alpha(A-B) = \alpha A - \alpha B \quad (\alpha-\beta)A = \alpha A - \beta A$$

$$(\alpha\beta)A = \alpha(\beta A)$$

Elementer matritsalar

Bosh diagonali birlardan va qolgan hamma elementlari 0 lardan iborat

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

ko'rinishdagi n - tartibli kvadrat matritsa birlik matritsa deyiladi.

n - tartibli istalgan A matritsa uchun

$$AE = EA = A$$

ekanligiga ishonch hosil qilish oson.

1.1.9-misol.

$$\begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 5 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 5 \\ 6 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 5 \\ 6 & 2 & 3 \end{pmatrix}$$

Faqat, i - satr va j - ustundagi elementni 1 ga teng bo'lib, qolgan hamma elementlari 0 larga teng n - tartibli E_{ij} kvadrat matritsa elementar matritsa deb ataladi. n - tartibli kvadrat matritsalar orasida n^2 ta har xil elementar matritsalar mavjud.

Masalan, 2 - tartibli $2^2 = 4$ ta elementar matritsalar quyidagilardir:

$$E_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, E_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, E_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, E_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

E birlik matritsa n ta elementar matritsalar yig'indisiga yoyiladi:

$$E = E_{11} + E_{22} + ... + E_{nn}$$

Ushbu

$$\alpha E = \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ ... & ... & ... & ... \\ 0 & 0 & 0 & \alpha \end{pmatrix}$$

ko'rinishdagi matritsa skalyar matritsa deyiladi.

Ushbu tengliklar o'rinli:

$$\alpha E+\beta E=(\alpha+\beta)E;\alpha E\cdot\beta E=\alpha\beta E;(\alpha E)A=A(\alpha E)\alpha A$$

Istalgan n - tartibli kvadrat matritsa n^2 ta elementar matritsalar orqali chiziqli ifodalanadi:

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}=a_{11}E_{11}+a_{12}E_{12}+...+a_{1n}E_{1n}+a_{21}E_{21}+a_{22}E_{22}+$$
$$...+a_{2n}E_{2n}+a_{n1}E_{n1}+a_{n2}E_{n2}+...+a_{nn}E_{nn}$$

Xos va xosmas matritsalar

n - tartibli kvadrat matritsani olamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}$$

Bu matritsaning gorizontal

$$a_1=(a_{11},a_{12},...a_{1n})$$
$$a_2=(a_{21},a_{22},...a_{2n})$$
$$..........................$$
$$a_n=(a_{n1},a_{n2},...a_{nn})$$

vektorlari (yoki satrlari) chiziqli erkli yoki chiziqli bog'langan bo'lishi mumkin.

1.1.10-misol.

$$A=\begin{pmatrix} -4 & -2 & -1 \\ 6 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}$$

matritsaning satrlari chiziqli bog'langan chunki uchinchi satr qolgan ikki satrning chiziqli kombinatsiyasidan iborat;

$$2=1\cdot(-4)+1\cdot 6;$$
$$1=1\cdot(-2)+1\cdot 3$$
$$1=1\cdot(-1)+1\cdot 2$$

E birlik matritsaning satrlari chiziqli erkli, chunki ular V_n fazoning $\overline{e_1},\overline{e_2},...\overline{e_n}$ ortlarini tasvirlaydi.

1.1.5 - ta'rif. Satrlari chiziqli erkli matritsa xosmas matritsa deb va satrlari chiziqli bog'langan matritsa xos matritsa deb ataladi.

1.1.6 - ta'rif. A matritsa uchun $AB=E$ tenglikni qanoatlantiruvchi B matritsa A ga teskari matritsa deyiladi va u $B=A^{-1}$ ko'rinishda belgilanadi.

Shunday qilib, $AA^{-1}=E$

1.1.4 - teorema: A matritsaning satrlaridan biri qolganlari orqali chiziqli ifodalansa, A ni ixtiyoriy B matritsaga ko'paytirishdan hosil bo'lgan $C=AB$ ko'paytmaning ham xuddi o'sha satri qolganlari orqali chiziqli ifodalanadi.

Isboti: A ning birinchi satri qolgan satrlari orqali chiziqli ifodalanadi deb faraz qilamiz:

$$a_{1j} = a_2 a_{2j} + a_3 a_{3j} + \ldots + a_n a_{nj} = \sum_{i=1}^{n} a_i a_{ij} \quad \left(j = \overline{1,n}\right) \quad (1.1.15)$$

$C = AB$ ko'paytmaning ta'rifiga va (1.1.15) ga asosan:

$$c_{1j} = a_{11}b_{2j} + a_{12}b_{2j} + \ldots + a_{1n}b_{nj} = \left(\sum_{i=2}^{n} a_i a_{i1}\right) b_{ij} + \left(\sum_{i=2}^{n} a_i a_{i2}\right) b_{2j} + \ldots + \left(\sum_{i=2}^{n} a_i a_{in}\right) b_{nj} =$$
$$= a_2\left(a_{21}b_{1j} + a_{22}b_{2j} + \ldots + a_{2n}b_{nj}\right) + a_3\left(a_{31}b_{1j} + a_{32}b_{2j} + \ldots + a_{3n}b_{nj}\right) + \ldots + a_n\left(a_{n1}b_{1j} + a_{n2}b_{2j} + \ldots + a_{nn}b_{nj}\right) =$$
$$= a_2 c_{2j} + a_3 c_{3j} + \ldots + a_i c_{nj} = \sum_{i=2}^{n} a_i c_{ij}$$

1.1.5 - teorema. Xos A matritsaga teskari matritsa mavjud emas.

Isbot: A ning satrlari chiziqli bog'langanligi sababli, uning satrlaridan biri qolganlari orqali chiziqli ifodalanadi. U holda 1.1.4 - teoremaga muvofiq, $AA^{-1} = E$ ko'paytmaning ham o'sha satri qolganlari orqali chiziqli ifodalanadi. Bu esa E ning satrlari chiziqli erkli bo'lishiga zid keladi.

Demak, faqat xosmas kvadrat matritsalarga teskari matritsalar mavjud bo'lishi mumkin.

Matritsaviy tenglamalar

P sonlar maydoni ustidagi n noma'lumli n ta chiziqli

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \quad (1.1.16)$$

tenglamalar sistemasini olamiz. Sistema matritsasi ushbudan iborat:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$

Biz A xosmas matritsa bo'lgan holnigina qaraymiz. (1.1.16) ning chap tomonida A matritsani

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}$$

matritsaga ko'paytirishdan kelib chiqadigan n satrli va 1 ustunli matrisaning elementlari va o'ng tomonida

$$B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}$$

matritsaning elementlari turadi. Shu sababli ikki matritsaning tenglik shartiga asosan, (1.1.16) ni ushbu

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$

yoki qisqaroq,

$$AX = B \quad (1.1.17)$$

ko'rinishda yozish mumkin. Bu tenglama matritsaviy tenglama deyiladi.

A ga teskari A^{-1} matritsa mavjud bo'lganidan (1.1.17) quyidagicha yechamiz:

$$X = A^{-1}B$$

yoki

$$\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} - \begin{pmatrix} a'_{11} & a'_{12} & \dots & a'_{1n} \\ a'_{21} & a'_{22} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots \\ a'_{n1} & a'_{n2} & \dots & a'_{nn} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix} = \begin{pmatrix} a'_{11}b_1 + a'_{12}b_2 + \dots + a'_{1n}b_n \\ a'_{21}b_1 + a'_{22}b_2 + \dots + a'_{2n}b_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a'_{n1}b_1 + a'_{n2}b_2 + \dots + a'_{nn}b_n \end{pmatrix}$$

Bundan ikki matritsaning tenglik shartlariga muvofiq (1.1.17) yoki (1.1.16) ning yechimga kelamiz: $x_i = a'_{i1}b_1 + a'_{i2}b_2 + \dots + a'_{in}b_n, \ \left(i = \overline{1,n}\right)$ (1.1.18)

<u>1.1.11–misol.</u> Ushbu

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 7 \\ 2 & -3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix}$$

tenglamani yechaylik. A xosmas matritsa bo'lgani uchun, unga teskari A^{-1} matritsa mavjuddir. A^{-1} ni topamiz.

$$A^{-1} = \begin{pmatrix} 6 & -1 & -1 \\ -1 & 1 & -1 \\ -3 & 1 & 0 \end{pmatrix}$$

Demak, (1.1.18) ga ko'ra

$$\begin{aligned} x_1 &= 6\cdot 2 - 1\cdot 9 - 1\cdot 2 = 1 \\ x_2 &= -1\cdot 2 + 1\cdot 9 - 1\cdot 2 = 5 \\ x_3 &= -3\cdot 2 + 1\cdot 9 + 0\cdot 2 = 3 \end{aligned}$$

Kvadrat matritsa determinant

1.1.7 - ta'rif: n - tartibli

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

kvadrat matritsaning determinanti deb $n!$ ta hadlarning

$$\sum^{n!}(-1)^{\nu} a_{1\beta_1}a_{2\beta_2}...a_{n\beta_n} \quad (1.1.19)$$

ko'rinishdagi yig'indisiga aytiladiki, bu yig'indi ushbu talablarni qanoatlantiradi:

1) (1.1.19) yig'indidagi har bir $(-1)^{\nu} a_{1\beta_1}a_{2\beta_2}...a_{n\beta_n}$ had A matritsaning har bir satri va har bir ustunidan faqat bittadan olingan $a_{1\beta_1}, a_{2\beta_2},...a_{n\beta_n}$ elementlar ko'paytmasiga teng.

2) $(-1)^{\nu} a_{1\beta_1}a_{2\beta_2}...a_{n\beta_n}$ hadning birinchi $1,2,...n$ indekslari $a_{1\beta_1}, a_{2\beta_2},...a_{n\beta_n}$ elementlar turgan satrlar nomerlarini, ikkinchi $\beta_1, \beta_2,...\beta_n$ indekslari bu elementlar turgan ustunlar nomerlarini bildiradi va shu bilan birga, ikkinchi indekslar $1,2,3,...n$ raqamlarning qandaydirpermutatsiyasini ifodalaydi.

3) (1.1.19) yig'indiga hamma $n!$ ta hadlarning ikkinchi $\beta_1, \beta_2,...\beta_n$ indekslari, bir haddan ikkinchi hadga o'tib borish bilan $1,2,...n$ raqamlardan barcha $n!$ ta permutatsiyasi tuzib boradi.

4) $(-1)^{\nu}$ hadning birinchi $1,2,...n$ va ikkinchi $\beta_1, \beta_2,...\beta_n$ indekslari $\begin{pmatrix} 1,2,...n \\ \beta_1, \beta_2,...\beta_n \end{pmatrix}$ o'rniga qo'yishni tuzgani holda, ν ko’rsatkich bu o’rniga qo’yishdagi pastki satr inversiyalari sonini ifodalaydi.

Shunday qilib, (1.1.19) yig'indida ikkinchi indekslari juft permutatsiyalarni tashkil etuvchi $\frac{n!}{2}$ ta hadlar uchun $(-1)^{\nu} = +1$ bo'lganidan, bu hadlar o'z ishoralari bilan olinib, ikkinchi indekslar toq permutatsiyalarni tashkil etuvchi $\frac{n!}{2}$ ta hadlar uchun esa $(-1)^{\nu} = -1$ bo'lganidan bunday hadlar teskari ishoralar bilan olinadi.

(1.1.19) yig'indi n - tartibli determinanti deyiladi va u

$$D=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

shaklda belgilanadi. Uning gorizontal qatorlari satrlar deb, vertikal qatorlari esa ustunlar deb ataladi a_{ij} lar determinant elementlari deyiladi, bunda birinchi i indeks a_{ij} element turgan satrning nomerini ikkinchi j indeks ustunning nomerini bildiradi;

$a_{11}, a_{22}, \dots a_{nn}$ elementlar determinantning birinchi (bosh) diagonalini, $a_{1n}, a_{2,n-1}, \dots, a_{n-1,2} a_{n1}$ elementlar esa uning ikkinchi diagonalini tashkil etadi; n - tartibli determinanti n^2 ta elementdan tuziladi.

<u>1.1.5 – teorema</u> A kvadrat matritsaning D determinantini

$$\sum^{n!} (-1)^p a_{\alpha_1 1} a_{\alpha_2 2} \dots a_{\alpha_n n} \quad (1.1.20)$$

yig'indi shaklida ham ifodalash mumkin, bunda hamma hadlarning ikkinchi $1,2,\dots n$ nomerlari normal holda joylashgan bo'lib, birinchi $\alpha_1, \alpha_2, \dots \alpha_n$ nomerlari esa, hamma $n!$ permutatsiyalarni tuzadi va ρ ko'rsatkich $\alpha_1, \alpha_2, \dots \alpha_n$ permutatsiyalarning inversiyalari sonini bildiradi.

3-tartibli determinantlar uchun hisoblash usuli kelib chiqadi: (1.1.19) yig'indiga ko'ra:

$$D=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum^{3!} (-1)^{\nu} a_{1\beta_1} a_{2\beta_2} a_{3\beta_3} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$
$$- a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

<u>Matritsa minorlari</u>

$(m \times n)$ matritsa berilgan bo'lsin:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

<u>**1.1.8 - ta'rif.**</u> A matritsada r ta ixtiyoriy satr va r ta ustun ajratib, ularning kesishgan joylarida turuvchi elementlardan tuzilgan r - tartibli D determinant A ning r - tartibli minori deyiladi. 1.1.8 - ta'rifda $i \le r \le m$ va $1 \le r \le n$ dir.

<u>**1.1.13-misol:**</u>

$$A = \begin{pmatrix} 1 & -3 & 5 & 2 & -8 \\ 4 & 0 & -2 & 1 & 7 \\ 8 & -9 & 1 & 0 & -6 \end{pmatrix}$$

matritsada birinchi, ikkinchi, uchinchi satrlari va birinchi, to'rtinchi, beshinchi ustunlarni ajratib, ushbu 3 - tartibli minorni hosil qilamiz.

$$D = \begin{vmatrix} 1 & 2 & -8 \\ 4 & 1 & 7 \\ 8 & 0 & -6 \end{vmatrix}$$

n - tartibli A kvadrat matritsaning eng yuqori tartibli minori shu matritsaning determinantidan iboratligi ravshan. Bunday determinant $|A|$ ko'rinishda ham belgilandi. Matritsaning elementlari uning birinchi eng kichik tartibli minorlarini ifodalaydi.

1.1.6 - teorema. A matritsaning rangi uning noldan farqli minorlardan eng yuqori tartiblisining tartibiga teng.

Isboti: Noldan farqli eng yuqori tartibli D minor A matrisaning yuqori chap burchagida joylashgan deb faraz qilamiz:

$$A = \left(\begin{array}{c|c} a_{11}...a_{12} & a_{1,r+1}...a_{1n} \\ .....D.... & .... \\ a_{r1}...a_{rr} & a_{r,r+1}...a_{rn} \\ \multicolumn{2}{c}{a_{r+1,1}...a_{r+1,r}a_{r+1,r+1},...a_{r+1,n}} \\ \multicolumn{2}{c}{a_{m1},...a_{mr}a_{mr+1}...a_{mn}} \end{array} \right)$$

aks holda satrlarni o'zaro va ustunlarni o'zaro o'rin almashtirib, D ni shu aytilgan joyga keltirish mumkin, bundan A ning rangi o'zgarmaydi.

A ning birinchi r ta satrlari - chiziqli erkli, chunki ular chiziqli bog'langan bo'lsa, bu satrlardan kamida bittasi boshqalari orqali chiziqli ifodalanadi va bunday satrni nol - satrga aylantirish mumkin bo'ladi. U holda almashtirilgan matrisaning birinchi r ta satrlaridan tuzilgan r - tartibli minorlar, demak, determinantlarning xossalaridan kelib chiqadigan natijaga asosan D ham nolga teng bo'lib qoladi.

Shunday qilib, A ning birinchi r ta gorizontal vektorlari chiziqli erklidir.

A matritsaning S - satri $\left(S = \overline{r+1,m}\right)$ birinchi r ta satrlari orqali chiziqli ifodalanadi. Buni isbotlash maqsadida quyidagi $(r+1)$ - tartibli determinantni qaraymiz.

$$\Delta i = \begin{vmatrix} a_{11}...a_{1r} & a_{1i} \\ a_{21}...a_{2r} & a_{2i} \\ .... & ... \\ a_{r1}...a_{rr} & a_{ri} \\ a_{s1}...a_{sr} & a_{si} \end{vmatrix}$$

bunda $i = 1,2,...,n$; $S = r+1, r+2,...m$

Hamma Δi determinantlar nolga teng. Haqiqatan, $i \le r$ qiymatlarda Δi ning ikki satri teng bo'lib, $\Delta i = 0$ kelib chiqadi; $i > r$ qiymatlarda esa, Δi determinantlar A matritsaning $(r+1)$ - tartibli minorlarini ifodalab, demak, bu holda ham nolga teng bo'ladi.

Δi ni so'nggi ustun elementlari bo'yicha yoyamiz.

$$A_{s1}a_{1i} + A_{sr}a_{si} + ... + A_{sr}a_{ri} + Da_{1i} = 0 \quad (1.1.21)$$

bunda $a_{1i}, a_{2i},...a_{ri}$ elementlarining algebraik to'ldiruvchilari S ga bog'liq bo'lgani uchun, ularni $A_{s1}, A_{s2},...A_{sr}$ bilan belgiladik.

$D \ne 0$ ga muvofiq, (1.1.21) tengliklarni a_{si} ga nisbatan yecha olamiz:

$$\begin{gathered} a_{si} = B_{si}a_{1i} + A_{sr}a_{2i} + ... + A_{sr}a_{ri} \\ \left(i = \overline{1,n}, s = \overline{r+1,m}\right) \end{gathered} \quad (1.1.22)$$

(1.1.22) tengliklar A ning S - satri birinchi r ta satrlari orqali chiziqli ifodalanganini ko'rsatadi.

Demak, A matritsaning gorizontal vektorlar sistemasida chiziqli erkli vektorlarning maksimal soni r ga teng bo'lganidan, A ning rangi ham r ga tengdir.

1.2.Matritsali funksiyalar.

Xarakteristik ko'phadning (funksiyalar) ta'rifi.

n - tartibli A kvadrat matritsa bilan birga $A-\lambda I$ ko'rinishdagi matritsalar oilasini qaraymiz, bunda λ sonli parametr bo'lib, umumiy holda komplektdir.

1.2.1 - ta'rif: A matritsaning xarakteristik ko'phadi deb, λ o'zgaruvchining $d_A(\lambda)=\det(A-\lambda I)$ ko'phadga aytiladi.

Ko'rinib turibdiki, $d_A(\lambda)$ n - darajali ko'phad bo'ladi.

Haqiqatan ham,

$$d_A(\lambda)=\begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n1} & \dots & a_{nn}-\lambda \end{vmatrix} \quad (1.2.1)$$

Shuning uchun $\det(A-\lambda I)$ dagi faqat bitta had λ ni n - va $n-1$ - hadlar bilan o'zida saqlaydi.

Aynan $A-\lambda I$ matrisaning bosh diagonaldagi elementlar ko'paytmasidan iborat had. Qolgan barcha hadlar 2 ta bosh diagonalga teng bo'lmagan elementlarga ega bo'lib, darajasi $n-2$ dan oshmaydi.

Shunday qilib, $d_A(\lambda)$ ko'phadga λ^n va λ^{n-1} yuqori hadlar oldidagi mos koeffitsiyentlar $(a_{11}-\lambda)(a_{22}-\lambda)\dots(a_{nn}-\lambda)$ haddagi kabi bo'lib, mos ravishda $(-1)^n$ va $(-1)^{n-1}\sum_{k=1}^{n} a_{kk}=(-1)^{n-1}$ transponirlangan matritsa $d_A(\lambda)$ ko'phadning ozod hadi

$d_A(0)$ qiymat bilan bir xil bo'lib, $\det(A)$ ga teng.

Shunday qilib,

$$d_A(\lambda) = \det(A - \lambda I) = \delta_n \lambda^n + \delta_{n-1}\lambda^{n-1} + ... + \delta_1 \lambda + \delta_0, \quad \delta_n = (-1)^n; \delta_{n-1} = (-1)^{n-1} TrA; \delta_0 = \det A \qquad (1.2.2)$$

qolgan koeffitsiyentlar uchun ifodalar ancha murakkab bo'lgani uchun ularni keltirmaymiz.

Kvadrat matritsaning xos qiymatlari.

Algebraning asosiy teoremasiga muvofiq $A \in M^n$ matritsaning xarakteristik ko'phadni

$$d_A(\lambda) = (-1)^n (A - \lambda_1)(A - \lambda_2)....(A - \lambda_n) \quad (1.2.3)$$

ko'rinishda ifodalash mumkin bo'lib, bunda $\lambda_1, \lambda_2, ...\lambda_n$ $d_A(\lambda)$ ning ildizlari, umumiy holda $d_A(\lambda)$ ko'phadning kompleks ildizlari bo'lib, A matritsaning xos qiymatlari deyiladi, ular orasida o'zaro tenglari ham bo'lishi mumkin.

Bunday holda nomerlashning boshqa usullari ham qo'llaydilar: $\mu_1,...\mu_\rho$ $\rho \le n$; sonlar $d_A(\lambda)$ turli barcha ildizlari $\sigma_1,...\sigma_\rho$ esa ularning karralisini ifodalasa $d_A(\lambda) = (-1)^n (\lambda - \mu_1)^{\sigma_1} (\lambda - \mu_2)^{\sigma_2} ....(\lambda - \mu_p)^{\sigma_p}, \sigma_1 + \sigma_2 + ... + \sigma_p = n$

Agar $\sigma_k = 1$ bo'lsa, u holda μ_k xos qiymat oddiy ildiz deyiladi.

Xarakteristik ko'phadning koeffitsiyentlarni xos qiymatlar orqali ifodalash mumkin.

Jumladan (1.2.3) dan ko'rinadiki λ^{n-1} oldidagi δ_{n-1} koeffitsiyent $(-1)^{n-1}\sum_{k=1}^{n}\lambda_k$ ga teng bo'lib δ_0 ozod had esa $\prod_{k=1}^{n}\lambda_k$ (1.2.2) tenglikni e'tiborga olib, $TrA = \sum_{k=1}^{n}\lambda_k$; $\det A = \prod_{k=1}^{n}\lambda_k$ hosil qilamiz.

Natija sifatida ushbu kelib chiqadi. Natijada A matritsa xosmas matritsa bo'ladi. Faqat va faqat shu holdaki agar uning barcha xos qiymatlari 0 dan farqli bo'lsa A matritsaning $\mu_1,...\mu_n$ xos qiymatlari to'plami (ularning $\sigma_1,\sigma_2...\sigma_\rho$ karraliklari bilan birga) A matritsaning spektori deyiladi.

Shunday qilib, A - xosmas matritsa bo'lishi uchun 0 soni uning spektoriga tegishli bo'lmasligi zarur va yetarlidir.

Matritsaning spektorini izlash n - darajali $d_A(\lambda) = 0$ algebraik tenglamani yechishni talab etadi.

Ba'zan bu masala osonlashtiriladi. Xususan, A matritsa uchburchakli bo'lsa, u holda uning xos qiymatlari bosh diagonal elementlari bilan teng bo'ladi. Bu bevosita d_A ning ta'rifidagi (1.2.1) dan ko'rinadi.

Kvadrat matritsaning funksiyalari

Ta'rifga asosan

$$A^0 = I, A^\delta = E, A^2 = A \cdot A, A^3 = A^2 \cdot A, ...A^m = A^{m-1}A$$

Ravshanki,

$$A^m = A^{m-1}A = A^{m-2}A^2 = A^{m-3}AA^2 = ...$$

Shuning uchun, $A^m = A^r A^s$ bunda $r + s = m$.

Shunday qilib, $A^r A^s = A^s A^r$, boshqacha aytganda matritsaning darajalari o'zaro o'rin almashinadi, bu yerda m, r, s, manfiy bo'lmagan butun sonlar.

1.2.2-ta'rif: $P_m(x) = p_0 + p_1 x + ... + p_m x^m$ ko'phad berilgan bo'lsin. U vaqtda A matritsa argumentli P_m ko'phadning qiymati deb, $P_m(A) = p_0 I + p_1 A + ... + p_m A^m$ matritsaga aytiladi.

Matritsa darajalarning bir - biri bilan o'rin almashishi xossasiga asosan

$$(P+Q)(A) = P(A) + Q(A)$$
$$(PQ)(A) = P(A)Q(A)$$

tenglik o'rinli.

Agar A xosmas matritsa bo'lsa, u vaqtda teskari A^{-1} matritsa tushunchasini kiritish mumkin.

A matritsaning manfiy darajasi deb, A^{-1} matritsaning musbat darajasini tushunamiz, ya'ni $A^{-m} = \left(A^{-1}\right)^m$

Bu ta'rif λ soni A matritsaning spektriga tegishli bo'lmagan holda $(A - \lambda E)^{-m}$ matritsani qarash imkonini beradi.

Endi $P(x); Q(x)$ ko'phadlar bo'lib, Q ko'phadning ildizlari A matritsaning spektriga tegishli emas.

$R(x) = \dfrac{P(x)}{Q(x)}$ ratsional funksiyani qaraymiz va uning eng sodda kasrlarga yoyilmasidan foydalanamiz.

Bu yoyilma $R(x)$ uchun ko'phadning chiziqli kombinatsiyasi va $(x-x_k)^{-1}$ funksiyaning natural darajalari ko'rinishida ifodalaydi, bunda $x_k - Q$ ko'phadning ildizlari.

Bunga asoslanib, $R(A)$ matritsani aniqlash mumkin.

Ikkita R_1, R_2 ratsional funksiya uchun

$$\begin{aligned} R_1(A)+R_2(A)&=(R_1+R_2)(A) \\ R_1(A)R_2(A)&=(R_1R_2)(A) \end{aligned} \qquad (1.2.4)$$

tengliklar o'rinli.

Xususiy holda $R_1(A)$ va $R_2(A)$ o'rin almashinuvchi $\forall A \in M^n$ uchun $f(A)$ ni aniqlash mumkindir, agar $f(x)$ funksiya $\forall x$ uchun Teylorning yaqinlashuvchi qatori

$$f(x)=\sum_{k=0}^{\infty} f_k x^k \; ; \quad f_k = \frac{f^{(k)}(0)}{k}$$

yoyilsa, u vaqtda ta'rifga ko'ra $f(A)=\sum_{k=0}^{\infty} f_k A^k$ bo'lib, bunda qatorning yaqinlashishi har bir element bo'yicha

$$[f(A)]_{ij} = \sum_{k=0}^{\infty} f_k \left[A^k\right]_{ij}, \quad \left(ij=\overline{1,\dots n}\right)$$

$f_1(A), f_2(A)$ funksiyalar uchun (1.2.4) munosabatga o'xshash munosabatlar o'rinli

$$f_1(A)+f_2(A)=(f_1+f_2)(A);$$
$$f_1(A)f_2(A)=(f_1\cdot f_2)(A)$$

<u>1.2.1-misol</u>:

1) $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ bo'lsin. U vaqtda $A^m=\begin{pmatrix}1 & m\\ 0 & 1\end{pmatrix}$

Haqiqatdan ham bu tenglik $m=0$ va $m=1$ da o'rinli bo'lib, m ning qolgan qiymatlari uchun induksiya bo'yicha

$$A^m=A^{m-1}A=\begin{pmatrix}1 & m-1\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & m\\ 0 & 1\end{pmatrix}$$

u vaqtda

$$\exp(A)=\sum_{k=0}^{\infty}\frac{1}{k!}\begin{pmatrix}1 & k\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\sum\limits_{k=0}^{\infty}\frac{1}{k!} & \sum\limits_{k=0}^{\infty}\frac{k}{k!}\\ 0 & \sum\limits_{k=0}^{\infty}\frac{1}{k!}\end{pmatrix}=\begin{pmatrix}e & e\\ 0 & e\end{pmatrix}.$$

2) $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, $f(x)=\dfrac{x+1}{x-1}$ bo'lsin, u vaqtda

$$f(A)=(A+I)(A-I)^{-1}=\begin{pmatrix}2 & 2\\ 3 & 5\end{pmatrix}\begin{pmatrix}0 & 2\\ 3 & 3\end{pmatrix}^{-1}=-\frac{1}{6}\begin{pmatrix}2 & 2\\ 3 & 5\end{pmatrix}\begin{pmatrix}3 & -2\\ -3 & 0\end{pmatrix}=-\frac{1}{6}\begin{pmatrix}0 & -4\\ -6 & -6\end{pmatrix}=\begin{pmatrix}0 & \frac{2}{3}\\ 1 & 1\end{pmatrix}.$$

Boshqa tomondan $f(x)=1+2(x-1)$ bo'lganligi uchun bo'lib,

$$f(A)=I+2\begin{pmatrix}0 & 2\\ 3 & 3\end{pmatrix}^{-1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\frac{1}{3}\begin{pmatrix}3 & -2\\ -3 & 0\end{pmatrix}=\begin{pmatrix}0 & \frac{2}{3}\\ 1 & 1\end{pmatrix}$$

1) A va B lar 2 ta xosmas matritsalar bo'lsin. U vaqtda $B^{-1}-A^{-1}=B^{-1}(A-B)A^{1}=A^{-1}(A-B)B^{-1}$

2) Agar A, B o'rin almashinuvchi bo'lsa, u holda ularning istalgan natural darajalari ham o'rin almashinuchi bo'ladi.

Agar bunda A va B xosmas matritsalar bo'lsa, u holda ularning istalgan butun darajalari ham o'rin almashinuvchi bo'ladi.

Keli ayniyati.

1.2.1 - teorema: $A\in M^{n}$ va $d_A - A$ matritsaning xarakteristik ko'phadi bo'lsin, u holda $d_A(\lambda)=0$.

Isboti: $A-\lambda I$ matritsaga birlashtirilgan matritsa $(A-\lambda I)^{-1}$ bo'lsin. (7,8) ga asosan

$$(A-\lambda I)(A-\lambda I)=\det(A-\lambda I)I=d_A(\lambda)I \quad (1.2.5)$$

dan foydalanib $(A-\lambda I)^{1}$ matritsaning elementlari ishorasi aniqligagacha $A-\lambda I$ matritsaning $n-1$ - tartibli minorlari bilan bir xil bo'ladi, shunga ko'ra λ ning darajasi $n-1$ dan oshmaydigan ko'phadlardan iborat bo'ladi

$$\left[(A-\lambda I)^{1}\right]_{ij}=b_{0,ij}+b_{1,ij}\lambda+....+b_{n-1,ij}\lambda^{n-1}$$

ya'ni

$$\left(A-\lambda I\right)^{1}=B_0+B_1\lambda+\ldots+B_{n-1}\lambda^{n-1}$$

(1.1.5) munosabatga ko'ra

$$\left(A-\lambda I\right)\left(B_0+B_1\lambda+\ldots+B_{n-1}\lambda^{n-1}\right)=d_A(\lambda)I=\left(\delta_0+\delta_1\lambda+\ldots.+\delta_n\lambda^n\right)I \quad (1.2.6)$$

ning chap va o'ng tomonlaridagi a sonning bir xil darajalari oldidagi koeffitsiyentlarni tenglashtirib, ushbu

$$\begin{cases} AB_0=\delta_0 I \\ AB_1-B_0=\delta_1 I \\ AB_2-B_1=\delta_2 I \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ AB_{n-1}-B_{n-2}=\delta_{n-1}I \\ -B_{n-1}=\delta_n I \end{cases} \qquad (1.1.7)$$

tengliklarni hosil qilamiz.

Endi (1.1.7) dagi k - tenglikni A^{k-1} ga ko'paytiramiz.

$$\begin{cases} AB_0=\delta_0 I \\ A^2B_1-AB_0=\delta_1 A \\ A^3B_2-A^2B_1=\delta_2 A^2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A^nB_{n-1}-A^{n-1}B_{n-2}=b_{n-1}A^{n-1} \\ -A^nB_{n-1}=\delta_n A^n \end{cases} \qquad (1.1.8)$$

(1.1.8) tengliklarni qo'shamiz.

U vaqtda $\delta_0 I+\delta_1 A+....+\delta_n A^n=d_A(A)$ bo'lib, (1.2.5) ga teng.

Keli ayniyatining muhim natijalaridan biri shundan iboratki A^n matritsa $I, A, A^2,....A^{n-1}$ matritsalar chiziqli kombinatsiyadan iborat.

Shuning uchun istalgan $P(A)$ ko'phadni hisoblash A ning darajasi $n-1$ dan yuqoro bo'lmagan ko'phadni hisoblashga keltiriladi.

Haqiqatan ham, $P(x)=s(A)d_A(A)+r_{n-1}(A)=r_{n-1}(A)$

<u>1.2.2-misol</u>

$$A=\begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix} \quad A^{10}$$

matritsani topamiz.

$$d_A(\lambda)\begin{vmatrix}1-\lambda & 1\\ -2 & 4-\lambda\end{vmatrix}=\lambda^2-5\lambda+6=(\lambda-2)(\lambda-3)$$

$$\frac{x^{10}}{(x-2)(x-3)}=x^{10}\left(\frac{1}{x-3}-\frac{1}{x-2}\right)=\frac{x^{10}-3^{10}}{x-3}-\frac{x^{10}-2^{10}}{x-2}+\frac{3^{10}}{x-3}-\frac{2^{10}}{x-2}$$

$$x^{10}=d_A(x)S(x)+\left(3^{10}(x-2)-2^{10}(x-3)\right)$$

$$S(x)=\left(x^{10}-3^{10}\right)(x-3)^{-1}-\left(x^{10}-2^{10}\right)(x-2)^{-1}$$

Demak, $A^{10}=3^{10}(A-2I)-2^{10}(A-3I)$

<u>Matritsalar o'xshashligi</u>

Matritsalar o'xshashligi va diagonallashuvchi tushunchalarni qaraymiz.

1.2.2-ta'rif: $A, B \in M^n$ matritsalar o'xshash deyiladi, agar shunday xosmas $X \in M^n$ matritsa mavjud bo'lsaki, $B = X^{-1}AX$ (u vaqtda $A = \lambda BX^{-1}$).

Bunday holda X matritsa o'xshashligini amalga oshiruvchi matritsa, shuningdek affinitet ham deyiladi.

1^o O'xshash matritsalarning xarakteristik ko'phadlar (demak,determinanti, izi, spektri) bir xil.

Ta'kidlaymizki, o'xshash matritsalar, ularning determinantlari tengligi bois faqat bir vaqtda xosmas bo'lishlari mumkin.

2^o Agar A va B matritsalar o'xshash bo'lsa u holda ularning darajalari ham o'xshash bo'lib, o'xshashlik o'sha X matritsa bilan amalga oshiriladi.

Haqiqatan ham, Agar m natural son bo'lsa, u holda

$$B^m = \left(X^{-1}AX\right)^m = X^{-1}AXX^{-1}AX \ldots X^{-1}AX = X^{-1}A^m X$$

Agar A va B xosmas bo'lsa, u holda

$$B^{-1} = \left(X^{-1}AX\right)^{-1} = X^{-1}A^{-1}\left(X^{-1}\right)^{-1} = X^{-1}AX$$

demak, $B^m = X^{-1}A^m X$

Natija: O'xshash matritsalarga bog'liq ko'phadlar, ratsional funksiyalar ham o'sha affinitet bo'yicha o'xshashdir.

3^o Agar A va B matritsalar o'xshash bo'lsa, u holda A^t va B^t hamda A^*

va B^* lar ham o'xshash bo'ladi.

Haqiqatan ham, $B^t = \left(X^{-1}AX\right)^t = X^t A^t \left(X^t\right)^{-1}, B^* = X^* A^* \left(X^*\right)^{-1}$.

Lekin bunda affinitet almashadi.

<u>1.2.3-ta'rif:</u> Matritsa diagonallanadigan deyiladi, agar u biror diagonal matritsaga o'xshash bo'lsa;

Diagonal $A = diag\{a_{11}, a_{22}, ...a_{nn}\}$ matritsaning spektri $a_{11}, a_{22}, ...a_{nn}$ sonlar bilan ustma – ust tushgani uchun diagonallanadigan $B = X^{-1}AX$ matritsa uchun $a_{11}, a_{22}, ...a_{nn}$ sonlar uning xos qiymatlari bo'ladi.

Hamma kvadrat matritsalar ham diagonallanadigan bo'lavermaydi.

Misol tariqasida $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ matritsani qaraymiz.

Uning $d_A(\lambda) = \begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix} = \lambda^2$ xarakteristik ko'phadi bo'lib, u aslida 0 matritsaning xarakteristik ko'phadidir.

Ammo 0 matritsa biror boshqa matritsaga o'xshash emas. Chunki, agar $A = X^{-1}OX$ bo'lsa, u holda $A = 0$. Bu misoldan ko'rinadiki matritsalar xarakteristik ko'phadlarning tengligi ular o'xshashligining zaruriy sharti bo'lib, yetarlilik sharti bo'lmaydi.

Shu bilan birga diagonallanadigan matritsalar ko'p. Matritsa diagonallanishining yetarli shartini keltiramiz.

<u>1.2.2-teorema:</u> Barcha oddiy spektrli matritsalar, barcha ermit matritsalar va barcha unitar matritsalar diagonallanadigandir. Ta'kidlash joizki, bu sinflar birgalikda barcha diagonallanadigan matritsalar to'plamini qamrab ololmaydi. $A = diag\{\lambda_1, ...\lambda_n\}$ uchun $m = 0$ va m naturla qiymatlar qabul qilganda

$A^m = diag\{\lambda^m{}_1,...\lambda^m{}_n\}$, agar barcha λ_j lar noldan farqli bo'lsa, u holda bu munosabat barcha butun m sonlar uchun o'rinli bo'ladi.

Shuning uchun, $R(x)$ ratsional funksiya bo'lsa, u holda $R(A) = diag\{R(\lambda_1),...R(\lambda_n)\}$.

Endi, tabiiyki istalgan $f(x)$ funksiya uchun A diagonal matritsaning $f(A)$ funksiyasi ta'rifi sifatida $f(A) = diag\{f(\lambda_1),...f(\lambda_n)\}$ olish mumkin.

$d_A(A) = diag\{d_A(\lambda_1),...d_A(\lambda_n)\} = diag\{0,...0\} = 0$ tengliklarga asosan diagonal matritsalar uchun Keli ayniyati o'rinli ekanligi kelib chiqadi.

Haqiqatan ham, agar

$$A = diag\{\lambda_1,...\lambda_n\} \text{ va } B = X^{-1}AX$$

bo'lsa, u holda istalgan $p(x)$ ko'phad uchun $P(B) = X^{-1}P(A)X$ bo'lib bundan

$$d_\beta(B) = d_A(B) = X^{-1}d_A(A)X = 0 .$$

Tabiiyki $f(B) = X^{-1}diag\{f(\lambda_1),...f(\lambda_n)\}X$ formulani diagonallashinuvchi B matritsaning ixtiyoriy $f(x)$ funksiyasi ta'rifi sifatida qabul qilish mumkin.

Xulosa

I bobga asosiy tushunchalar ifodalangan bo'lib, unda matritsalar halqasi: matritsa haqida, matritsani elementar almashtirishlar, pog'onali matritsa, matritsalar ustida amallar, elementar matrisalar, xos va xosmas matritsalar, matritsaviy tenglamalar, kvadrat matritsa determinanti haqida asosiy tushunchalar, ta'riflar va teoremalar berilgan va misollar orqali tushuntirilgan.

Matritsali funksiyalar haqidagi asosiy tushunchalarga to'xtalib o'tilgan.

Xususan xarakteristik ko'phadning tarifi, Kvadrat matritsaning xos qiymatlari haqida asosiy tushunchalar misollar orqali tushuntirilgan.

II. Matritsaning Jordan shakli

2.1 Jordan matritsasi.

Bu rejada kompleks sonlar maydonidagi n -tartibli kvadrat matritsalarni tekshiramiz.

$$A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{Vmatrix}$$

bunda, a_{ik} - kompleks sonlar.

A matritsani λ matritsa deb qarash mumkin.

Haqiqatan, $a_{ik} \neq 0$ elementlar λ ga nisbatan nolinchi darajali ko'phadlarni, $a_{ik} = 0$ elementlar esa nol ko'phadlarni ifodalaydi.

Sonlardagi tuzilgan A matritsani ya'ni λ - matritsaning xususiy holini o'zgarmas matritsa deb ataymiz.

O'zgarmas matritsalarning eng sodda shakli - diagonal matritsa ya'ni ekanligi ma'lum, bunda

$$\begin{Vmatrix} a_1 & 0 & 0 & \dots & 0 \\ 0 & a_2 & 0 & \dots & 0 \\ 0 & 0 & a_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_4 \end{Vmatrix}$$

$a_1, a_2, \dots a_n$ - kompleks sonlar.

Bu mavzuda asosan diagonal matritsalarga qaraganda murakkabroq bo'lsa ham, lekin ko'pgina hollarda ko'rinishi sodda bo'lgan maxsus bir xil shakldagi o'zgarmas matritsalar tekshiriladi.

Biz avval Jordan katagi deb yuritiladigan o'zgarmas matritsani ko'rib o'tamiz.

2.1.1-ta' rif: Ushbu

$$J_0 = \begin{Vmatrix} \lambda_0 & 1 & 0 & \dots & 0 & 0 \\ 0 & \lambda_0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \lambda_0 & 1 \\ 0 & 0 & 0 & \dots & 0 & \lambda_0 \end{Vmatrix}$$

shakldagi n -tartibli o'zgarmas matritsa λ_0 ga oid n -tartibli Jordan katagi deyiladi. Bu matritsada: bosh diagonal bo'yicha elementlar biror λ_0 o'zgarmas songa, bosh diagonal ustidagi elementlar 1 ga va qolgan hamma elementlar nolga teng.

2.1.1-misol: Ushbu matritsa

$$\begin{Vmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{Vmatrix}$$

1 ga oid 5- tartibli Jordan katagi. Shunga o'xshash

$$\begin{Vmatrix} -3 & 1 \\ 0 & -3 \end{Vmatrix}$$

matritsa -3 ga oid 2- tartibli Jordan katagi. Biror songa oid 1- tartibli Jordan katagi ham bo'lishi mumkin.

2.1.2-misol. $\|4-7i\|$ matritsa $4-7i$ ga oid shunday katakdir.

Endi Jordan matritsasi tushunchasiga o'tamiz.

2.1.2- ta'rif: Bosh diagonali bo'ylab $\lambda_1, \lambda_2, ...\lambda_s$ sonlarga oid $m_1, m_2, ...m_s$ - tartibli $J_1, J_2, ...J_s$ kataklar joylashgan va qolgan hamma elementlari nollardan iborat bo'lgan n - tartibli o'zgarmas matritsa Jordanning normal shakldagi matritsasi yoki qisqacha Jordan matritsasi deb ataladi.(Bu matritsani Jordan shakli deb ham ataladi)

Jordan matritsasini yozganda odatda kataklar tashqarisidagi nollarga teng elementlarni tushirib qoldirib matritsani

$$J = \begin{Vmatrix} [J_1] & & \\ & [J_2] & \\ & & [J_s] \end{Vmatrix}$$

ko'rinishda yozadilar, bunda J_1 bilan λ_1 ga oid m_1 - tartibli katak, J_2 bilan λ_2 ga oid m_2 - tartibli katak,…, J_s bilan λ_s ga oid m_s - tartibli katak belgilangan bo'lib, $\lambda_1, \lambda_2, ...\lambda_s$ sonlarning hammasi bosh diagonal bo'yicha joylashgan.

$\lambda_1, \lambda_2, ...\lambda_s$ sonlar, shuningdek $m_1, m_2, ...m_s$ sonlar ham har xil yoki bir-biriga teng bo'lishi mumkin. Bunda

$$m_1 + m_2 + ... + m_s = n$$

2.1.3-misol: 6 - tartibli

$$J = \begin{Vmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} & & \\ & \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} & \\ & & [8] \end{Vmatrix}$$

Jordan matritsasi: 2 ga oid 2 - tartibli -1 ga oid 3 - tartibli va 8 ga oid 1 - tartibli kataklardan tuzilgan.

J_0 Jordan katagining λ_0 ga oid bitta 2 - tartibli kataklardangina tuzilgan Jordan matritsasi deb aytish mumkin.

2.1.4-misol:

$$\begin{Vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{Vmatrix}$$

matritsa 1 ga oid bitta 3 - tartibli katakdan iborat Jordan matritsasi.

2.2 Jordan katagining xarakteristik matritsasi.Jordan matritsasining xarakteristik matritsasi.

J_0 Jordan katagining xarakteristik matritsasini hosil qilish uchun bosh diagonaldagi λ_0 sondan λ ni ayirish kifoya ekanligi ma'lum. Demak, bu xarakteristik matritsa

$$J_0 - \lambda E = \begin{Vmatrix} \lambda_0 - \lambda & 1 & \dots & 0 \\ 0 & \lambda_0 - \lambda & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_0 - \lambda \end{Vmatrix}$$

ko'rinishda bo'lib, u λ - matritsadan iborat.

Bu λ - matritsani normal diagonal shaklga keltiraylik.

$J_0 - \lambda E$ matritsaning tuzilishiga qarab, $D_n(\lambda) = (\lambda_0 - \lambda)^n$ (Bunda $D_n(\lambda)$ ning bosh koeffitsiyenti 1 ga teng bo'lishi uchun $(\lambda_0 - \lambda)^n = (-1)^n (\lambda_0 - \lambda)^n$ o'rniga $(\lambda_0 - \lambda)^n$ ni oldik) ekanini topamiz.

Endi, $J_0 - \lambda E$ da 1 - ustun va n - yo'lni o'chirsak ushbu $(n-1)$ - tartibli minor hosil bo'ladi:

$$\begin{vmatrix} 1 & 0 & \ldots & 0 & 0 \\ \lambda_0 - \lambda & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & \lambda_0 - \lambda & 1 \end{vmatrix} = 1$$

Bundan barcha $(n-1)$tartibli minorlarning eng katta umumiy bo'luvchisi 1 ga teng ya'ni $D_{n-1}(\lambda) = 1$ degan natija kelib chiqadi.

$D_{n-1}(\lambda)$ ko'phad $D_{n-2}(\lambda)$ ko'phadga $D_{n-2}(\lambda)$ esa $D_{n-3}(\lambda)$ ko'phadga va hokazo bo'linishi lozimligidan: $D_{n-1}(\lambda) = D_{n-2}(\lambda) = \ldots = D_2(\lambda) = D_1(\lambda) = 1$

Demak,

$$E_1(\lambda) = E_2(\lambda) = \ldots = E_{n-1}(\lambda) = 1, E_n(\lambda) = (\lambda - \lambda_0)^n$$

bo'lib, $J_0 - \lambda E$ matritsa ushbu normal diagonal shaklga keltiriladi:

$$\begin{Vmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & 0 & (\lambda - \lambda_0)^n \end{Vmatrix} \quad (2.2.1)$$

Bunda quyidagini ta'kidlab o'tish lozim: $J_0 - \lambda E$ matritsaning yo'llari va ustunlariga nisbatan ckekli son marta elementar almashtirishlarni qo'llanish orqali ham bu matritsani normal diagonal shaklga ya'ni (2.2.1) shaklga keltirishimiz mumkin.

Jordan matritsasining xarakteristik matritsasi

2.2.1-misol. Ushbu Jordan matritsasini olaylik:

$$J = \begin{Vmatrix} 4 & 1 & 0 & & & \\ 0 & 4 & 1 & & & \\ 0 & 0 & 4 & & & \\ & & & 2 & 1 & \\ & & & 0 & 2 & \\ & & & & & 5 \end{Vmatrix}$$

Bu matritsa quyidagi bitta 3 - tartibli bitta 2 - tartibli va bitta 1 - tartibli kataklardan tuzilgan:

$$J_1 = \begin{Vmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{Vmatrix}, \quad J_1 = \begin{Vmatrix} 2 & 1 \\ 0 & 2 \end{Vmatrix}, \quad J_3 = \begin{Vmatrix} 5 \end{Vmatrix}$$

J matritsaning xarakteristik matritsasi.

$$J-\lambda E=\begin{Vmatrix} 4-\lambda & 1 & 0 & & & \\ 0 & 4-\lambda & 1 & & & \\ 0 & 0 & 4-\lambda & & & \\ & & & 2-\lambda & 1 & \\ & & & 0 & 2-\lambda & \\ & & & & & 5-\lambda \end{Vmatrix}$$

ko'rinishga ega ekanligi ravshan. Bundagi kataklar ushbu matritsalarni ifodalaydi:

$$J_1-\lambda E_1=\begin{Vmatrix} 4-\lambda & 1 & 0 \\ 0 & 4-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{Vmatrix}=\begin{Vmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{Vmatrix}-\lambda\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}$$

$$J_2-\lambda E_2=\begin{Vmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{Vmatrix}=\begin{Vmatrix} 2 & 1 \\ 0 & 2 \end{Vmatrix}-\lambda\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$$

$$J_3-\lambda E_3=\|5-\lambda\|=\|5\|-\lambda\|1\|$$

Demak, E_1 birlik matritsaning tartibi J_1 ning tartibiga E_2 ning tartibi J_2 ning tartibiga va E_3 ning tartibi J_3 ning tartibiga teng.

Shu 2.2.1-misolga asoslanib Jordan matritsasining xarakteristik matritsasini umumiy ko'rinishda quyidagicha yozamiz:

$$J-\lambda E=\begin{Vmatrix} J_1-\lambda E_1 & & \\ & J_2-\lambda E_2 & \\ & & J_s-\lambda E_s \end{Vmatrix} \qquad (2.2.2)$$

bunda E_1 birlik matritsaning tartibi J_i katakning tartibiga, ya'ni m_i ga teng.

$J-\lambda E$ matritsaning normal diagonal shaklini topish uchun ushbu lemmani isbotlaymiz.

Lemma. $f_1(\lambda), f_2(\lambda),..., f_n(\lambda)$ ko'phadlarning har ikkitasi o'zaro tub bo'lsa

$$F(\lambda)=\begin{Vmatrix} f_1(\lambda) & 0 & ... & 0 \\ 0 & f_2(\lambda) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & & f_n(\lambda) \end{Vmatrix} \text{ va } \Phi(\lambda)=\begin{Vmatrix} 1 & 0 & ... & & 0 & 0 \\ 0 & 1 & ... & & 0 & 0 \\ 0 & 0 & ... & & 1 & 0 \\ 0 & 0 & ... & 0 & & \Phi_n(\lambda) \end{Vmatrix}$$

matritsalar ekvivalent bo'ladi, bunda

$$\varphi_n(\lambda)= f_1(\lambda)f_2(\lambda)... f_n(\lambda)$$

Isbot: $f_1(\lambda)$ va $f_2(\lambda)$ o'zaro tub bo'lgani sababli shunday $q_1(\lambda)$ va $q_2(\lambda)$ ko'phadlar mavjud bo'lib,

$$f_1(\lambda)q_1(\lambda)+ f_2(\lambda)q_2(\lambda)=1 \quad (2.2.3)$$

tenglik bajariladi.

$F(\lambda)$ matritsaning 1 - ustunini $q_1(\lambda)$ ga qarshi ko'paytirib 2- ustuniga va 2-yo'lini $q_2(\lambda)$ ga ko'paytirib 1 - yo'liga qo'shamiz. Bu vaqtda, (2.2.3) tenglikka asosan,

$$\begin{Vmatrix} f_1(\lambda) & 0 & ... & 0 \\ 0 & f_2(\lambda) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & & f_n(\lambda) \end{Vmatrix} \quad (2.2.4)$$

hosil bo'ladi. Bu (2.2.4) matritsaning 1 - va 2 - ustunlarini almashtiramiz va hosil bo'lgan yangi matritsada 1 - ustunni $-f_2(\lambda)$ ga ko'paytirib 2- ustunga qo'shamiz va shuningdek 1 - yo'lni $-f_2(\lambda)$ ga ko'paytirib 2 - yo'lga qo'shamiz; undan keyin 2

- ustunni -1 ga ko'paytiramiz va natijada ushbu matritsani hosil qilamiz:

$$\begin{Vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \varphi_2(\lambda) & 0 & \dots & 0 \\ 0 & 0 & f_3(\lambda) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & f_n(\lambda) \end{Vmatrix}$$

bunda $\varphi_2(\lambda) = f_1(\lambda) f_2(\lambda)$. $\varphi_2(\lambda)$ va $f_3(\lambda)$ ham o'zaro tub bo'lganligi sababli yuqoridagi mulohazani 2 - va 3 - yo'llarga nisbatan takrorlab,

$$\begin{Vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \varphi_3(\lambda) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & f_n(\lambda) \end{Vmatrix}$$

matritsani hosil qilamiz bunda

$$\varphi_3(\lambda) = \varphi_2(\lambda) f_3(\lambda) = f_1(\lambda) f_2(\lambda) f_3(\lambda)$$

Yana $\varphi_3(\lambda)$ va $f_4(\lambda)$ ning o'zaro tubligidan foydalanib, 3 - va 4 - ustunlar va yo'llar bilan ish ko'ramiz va hokazo. Bu protsesni oxirigacha davom ettirsak, xuddi $\Phi(\lambda)$ matritsa hosil bo'ladi.

$F(\lambda)$ matritsadan $\Phi(\lambda)$ matritsaga elementar almashtirishlar yordamida o'tganimiz uchun, bu ikkita λ - matritsa-ekvivalent matritsalardir.

Eslatma: Diagonal matritsada bosh diagonalning istalgan ikkita i - va j - elementlarini bir-biri bilan almashtirish bu matritsada elementar almashtirishlar bajarish bilan erishiladi. Haqiqatan, buning uchun matritsaning i - va j - yo'llarini va shuningdek, i - va j - ustunlarini almashtirish kifoya. Endi $J-\lambda E$ matritsani normal diagonal shaklga keltirish masalasiga o'tamiz.

Agar (2.2.2) matritsaning bitta $J_i-\lambda E_i$ katagidan o'tuvchi yo'llar va ustunlargagina tegishli elementar almashtirishlarni bajarsak, bu katak (2.2.1) ga o'xshash ko'rinishni oladi va boshqa hamma kataklar o'zgarmay qoladi.

Agar shu protsesni hamma kataklar uchun takrorlab chiqsak, $J-\lambda E$ matritsa ushbu

$$\left\| \begin{array}{cccccccccccc} 1 & 0 & \dots & 0 & & & & & & & & \\ 0 & 1 & \dots & 0 & & & & & & & & \\ \dots & \dots & \dots & \dots & & & & & & & & \\ 0 & 0 & \dots & (\lambda-\lambda_1)m_1 & & & & & & & & \\ & & & & 1 & 0 & \dots & 0 & & & & \\ & & & & 0 & 1 & \dots & 0 & & & & \\ & & & & \dots & \dots & \dots & \dots & & & & \\ & & & & 0 & 0 & \dots & (\lambda-\lambda_2)m_2 & & & & \\ & & & & & & & & 1 & 0 & \dots & 1 \\ & & & & & & & & 0 & 1 & \dots & 0 \\ & & & & & & & & \dots & \dots & \dots & \dots \\ & & & & & & & & 0 & 0 & \dots & (\lambda-\lambda_s)m_s \end{array} \right\|$$

diagonal matritsaga o'tadi, bunda yozilmagan elementlarning hammasi nolga teng. $\lambda_1,\lambda_2,\dots\lambda_s$ sonlar orasida bir-biriga tenglari mavjud bo'lishi mumkin.

(2.2.5) matritsada bosh diagonal elementlari bir-biri bilan shunday almashtirilgan bo'lib, buning natijasida quyidagilar bajarilgan deb faraz eta olamiz.

Birinchidan, o'zaro teng λ_i lar ushbu qatorlar bo'yicha joylashgan

$$\begin{matrix} \lambda_1, \lambda_2, \dots \lambda_k \\ \lambda_{k+1}, \lambda_{k+2}, \dots \lambda_l \\ \dots\dots\dots\dots\dots\dots\dots . \\ \lambda_{r+1}, \lambda_{r+2}, \dots \lambda_s \end{matrix} \qquad (2.2.6)$$

Shunday qilib, bitta qatordagi λ_i lar o'zaro teng bo'lib, turli qatordagilar esa har xildir.

Ikkinchidan, $k \geq l - k \geq .... \geq s - r$ ya'ni har bir qatordagi λ_i larning soni keyingi qatordagi λ_i lar sonidan kichik emas.

Uchinchidan, (2.2.6) qatordagi λ_i lar shunday joylashganki, ularga oid kataklarning tartiblari ushbu tengsizliklarni qanoatlantiradi

$$\begin{matrix} m_1 \geq m_2 \geq \dots \geq m_k \\ m_{k+1} \geq m_{k+2} \geq \dots \geq m_l \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ m_{r+1} \geq m_{r+2} \geq \dots \geq m_s \end{matrix} \qquad (2.2.7)$$

(2.2.5) matritsaning bosh diagonalidagi 1 dan farqli hamma

$$(\lambda - \lambda_1)_1^m, (\lambda - \lambda_2)_2^m, \dots. (\lambda - \lambda_s)_s^m$$

elementlari quyidagi tartibda yozib chiqamiz

$$\begin{matrix} (\lambda - \lambda_k)^{m_1}, & (\lambda - \lambda_k)^{m_2}, & \dots & (\lambda - \lambda_k)^{m_k} \\ (\lambda - \lambda_i)^{m_{k+1}} & (\lambda - \lambda_e)^{m_{k+2}} & \dots & (\lambda - \lambda_e)^{m_e} \\ \dots & \dots & \dots & \dots \\ (\lambda - \lambda_s)^{m_{r+1}} & (\lambda - \lambda_s)^{m_{r+2}} & \dots & (\lambda - \lambda_s)^{m_s} \end{matrix} \qquad (2.2.8)$$

chunki, $\lambda_1 = \lambda_2 = \dots = \lambda_k$, $\lambda_{k+1} = \lambda_{k+2} = \dots = \lambda_l$ va hokazo.

(2.2.8) jadvalning 1-ustunidagi darajalarni o’zaro, 2-ustundagi darajalarni o’zaro va hokazo ko’paytirib chiqsak,

$$(\lambda-\lambda_k)^{m_1}(\lambda-\lambda_e)^{m_{k+1}}....(\lambda-\lambda_s)^{m_{r+1}}$$

ko’paytma

$$(\lambda-\lambda_k)^{m_2}(\lambda-\lambda_e)^{m_{k+2}}....(\lambda-\lambda_s)^{m_{r+2}}$$

ko’paytmaga bo’linadi ((2.2.7) ga asosan).

Shuningdek, ikkinchi ko’paytma uchinchi ustundagi darajalar ko’paytmasiga bo’linadi va hokazo.

Bu yerda shuni aytib o’tish lozimki, agar, masalan, $k>l-k$ bo’lsa, u holda (2.2.8) jadvalning $(l-k)$ dan keyingi ustunlarida $\lambda-\lambda_e$ ning darajalari bo’lmaydi, biz o’sha joylarda $(\lambda-\lambda_e)^0=1$ turadi deb shartlashib olamiz.

(2.2.8) jadvalning 1-ustunidagi darajalar ko’paytmasini $E_n(\lambda)$ bilan, 2-ustundagi darajalar ko’paytmasini $E_{n-k+1}(\lambda)$ bilan belgilaylik. Yuqoridagi shartga muvofiq masalan, k-ustunda $\lambda-\lambda_e$ ayirmaning darajasi bo’lmasa biz u joyda $(\lambda-\lambda_e)^0=1$ turadi deb hisoblaymiz.

Yuqorida tushuntirganimizdek

$$E_n(\lambda), E_{n-1}(\lambda), ..., E_{n-k+1}(\lambda) \quad (2.2.9)$$

ko’paytmalarning har qaysisi o’zidan keyingisiga bo’linadi.

Endi (2.2.5) matritsada har qaysi o’zidan keyingisiga bo’linadi.

Endi (2.2.5) matritsada bosh diagonal elementlarini o’zaro shunday almashtiraylikki, buning natijasida hosil bo’ladigan matritsaning bosh diagonalida avval hamma 1 lar joylashsin, undan keyin (2.2.8) jadvalning k -ustunidagi darajalar, so'ngra $(k-1)$ - ustunidagi darajalar va hokazo, eng keyin 1-ustunidagi darajalar joylashsin.

Bunday matritsada (2.2.8) jadvalning k - ustunidagi darajarini o'z ichiga olgan katak bilan ish ko'raylik.

$\lambda_k, \lambda_e, \ldots \lambda_s$ sonlar har xil bo'lgani uchun k -ustundagi hamma darajalar o'zaro tub hadlardan iborat (k -ustunda bu sonlarning ba'zilari bo'lmasligi ham mumkin).

Shu sababli mana shu aytilgan katakdan o'tuvchi satr va ustunlarga nisbatan yuqoridagi lemmani qo'llasak bu katak

$$\begin{Vmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & E_{n-k+1}(\lambda) \end{Vmatrix}$$

ko'rinishni oladi.

Ana shu mulohazani ko'rilayotgan matritsada (2.2.8) jadvalning har bir ustunidagi darajalarinigina o'z ichiga olgan katak uchun takrorlab chiqamiz.

Bu almashtirishlardan keyin hosil qilingan matritsaning bosh diagonalidagi elementlarini o'zaro almashtirib eng oxirida quyidagi matritsaga kelamiz

$$\begin{Vmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & \ldots & & & & & \\ & & & 1 & & & & \\ & & & & E_{n=k+1}(\lambda) & & & \\ & & & & & \ldots & & \\ & & & & & & E_{n-1}(\lambda) & \\ & & & & & & & E_n(\lambda) \end{Vmatrix}$$

Bu xuddi (2.2.2) Jordan matrisaning normal diagonal shakli.

<u>2.2.2-misol:</u>

Ushbu 15-tartibli xarkteristik matritsani normal diagonal shaklga keltiraylik.

$$\begin{Vmatrix}
2-\lambda & 1 & 0 & & & & & & & & & & & & \\
0 & 2-\lambda & 1 & & & & & & & & & & & & \\
0 & 0 & 2-\lambda & & & & & & & & & & & & \\
& & & 2-\lambda & & & & & & & & & & & \\
& & & & 5-\lambda & 1 & & & & & & & & & \\
& & & & 0 & 5-\lambda & & & & & & & & & \\
& & & & & & 5-\lambda & 1 & 0 & & & & & & \\
& & & & & & 0 & 5-\lambda & 1 & & & & & & \\
& & & & & & 0 & 0 & 5-\lambda & & & & & & \\
& & & & & & & & & -3-\lambda & 1 & 0 & 0 & & \\
& & & & & & & & & 0 & -3-\lambda & 1 & 0 & & \\
& & & & & & & & & 0 & 0 & -3-\lambda & 1 & & \\
& & & & & & & & & 0 & 0 & 0 & -3-\lambda & &
\end{Vmatrix}$$

Normal diagonal matritsani hosil qilish uchun (2.2.8) jadvalni tuzish kerak.

Berilgan matritsada kataklarning eng ko'pchiligi 2 ga oiddir bu kataklar 3-,2-va 1-tartibli. Undan keyingi o'rinda 5 ga oid kataklar turadi ular 3-va 2-tartibli.

Nihoyat (-3) ga oid bitta 4-tartibli katak bor. Demak bu matritsa uchun jadval ushbu k o’rinishda bo’ladi

$$\begin{array}{l} (\lambda-2)^3, (\lambda-2)^2, \lambda-2 \\ (\lambda-5)^3, (\lambda-5)^2 \\ (\lambda+3)^4 \end{array}$$

Ustunlar bo’yicha ko’paytirib, (2.2.9) ko’paytmalarni tuzamiz

$$\begin{array}{l} E_n(\lambda) = E_{15}(\lambda) = (\lambda-2)^3 (\lambda-5)^5 (\lambda+3)^4 \\ E_{n-1}(\lambda) = E_{14}(\lambda) = (\lambda-2)^2 (\lambda-5) \\ E_{n-1}(\lambda) = E_{n-1}(\lambda) = \lambda-2 \end{array}$$

(Bunda $n=15$ va $k=3$ bo’lgani uchun oxirgi ko’paytma $E_{n-k+1}(\lambda) = E_{n-2}(\lambda) = E_{13}(\lambda)$ dir.).

Shunday qilib, 15-tartibli normal diagonal matritsa quyidagidan iborat

$$\begin{Vmatrix} 1 &&&&&&&&&&&&&& \\ & 1 &&&&&&&&&&&&& \\ && 1 &&&&&&&&&&&& \\ &&& 1 &&&&&&&&&&& \\ &&&& 1 &&&&&&&&&& \\ &&&&& 1 &&&&&&&&& \\ &&&&&& 1 &&&&&&&& \\ &&&&&&& 1 &&&&&&& \\ &&&&&&&& 1 &&&&&& \\ &&&&&&&&& 1 &&&&& \\ &&&&&&&&&& 1 &&&& \\ &&&&&&&&&&& 1 &&& \\ &&&&&&&&&&&& E_{13}(\lambda) && \\ &&&&&&&&&&&&& E_{14}(\lambda) & \\ &&&&&&&&&&&&&& E_{15}(\lambda) \end{Vmatrix}$$

2.3 Ixtiyoriy matritsani Jordan matritsasiga keltirish.

Bu rejada kompleks sonlar maydonidagi n-tartibli ixtiyoriy A matritsa bilan o’xshash Jordan matritsasini topish masalasini ko’ramiz.Buning uchun avval λ - matritsalarga doir quyidagi lemmalarni isbotlaymiz.

2.3.1-lemma: n-darajali

$$F(\lambda)=A_0\lambda^n+A_1\lambda^{n-1}+....+A_{n-1}\lambda+A_n$$

λ - matritsa va birinchi darajali

$$A-\lambda E$$

λ matritsa uchun

$$F(\lambda)=(A-\lambda E)Q(\lambda)+R \quad (10)$$

tenglikni qanoatlantiruvchi $Q(\lambda)$ va o’zgarmas R matritsalar mavjuddir.

Isbot: Ushbu

$$F(\lambda)+(A-\lambda E)A_0\lambda^{n-1}=F(\lambda)-A_0\lambda^n+AA_0\lambda^{n-1}$$

matritsaning darajasi $n-1$ dan yuqori emas.

Agar

$$F(\lambda)+(A-\lambda E)A_0\lambda^{n-1}=A^1{}_0\lambda^{n-1}+A^1{}_1\lambda^{n-2}+...+A^1_{n-1}$$

desak, u holda

$$F(\lambda)+(A-\lambda E)A\lambda^{n-1}+(A-\lambda E)A^1{}_0\lambda^{n-2}=F(\lambda)-(A-\lambda E)(A_0\lambda^{n-1}+A^1{}_0\lambda^{n-2})$$

matritsaning darajasi $(n-2)$ dan yuqori bo'lmaydi. Bu protsesni davom ettirib

$$F(\lambda)+(A-\lambda E)(A_0\lambda^{n-1}+A^1{}_0\lambda^{n-2}+\ldots+A_0^{(n-2)}\lambda+A_0^{(n-1)})=R$$

ko'rinishdagi R o'zgarmas matritsaga kelamiz.

Buni

$$F(\lambda)=(A-\lambda E)(-A_0\lambda^{n-1}+A^1{}_0\lambda^{n-2}-\ldots-A_0^{(n-2)}\lambda-A_0^{(n-1)})+R$$

ko'rinishda yozib qavs ichidagi $n-1$ darajali matritsani $Q_1(\lambda)$ bilan belgilasak xuddi (2.3.10) tenglik hosil bo'ladi.

Shunga o'xshsh

$$F(\lambda)=Q_1(\lambda)(A-\lambda E)+R_1 \quad (2.3.11)$$

tenglikni qanoatlantiruvchi $Q_1(\lambda)$ va o'zgarmas R_1 matritsaning mavjudligini isbotlanadi.

2.3.2-lemma. Ikkita n -tartibli A va B o'zgarmas matritsalar bir-biriga o'xshash bo'lishi uchun ularning $A-\lambda E$ va $B-\lambda E$ xarakteristik matritsalari ekvivalent bo'lishi zarur va yetarli.

Isbot: I. A va B matritsalar o'xshash bo'lsa, xosmas C matritsa mavjid bo'lib, $B=C^{-1}AC$ tenglik bajariladi. Bunga asosan: $B-\lambda C=C^{-1}AC-\lambda E=C^{-1}(A-\lambda E)C$

xosmas C matritsa teskarilanuvchi matritsani ifodalaganligi sababli

$A-\lambda E$ va $B-\lambda E$ va matritsalar uchun shartlar bajariladi. Demak, bu matritsalar ekvivalent ekan.

II .Endi $A-\lambda E$ va $B-\lambda E$ matritsalar ekvivalent desak, $P(\lambda)$ va $Q(\lambda)$ teskarilanuvchi matritsalar mavjud bo'lib

$$B-\lambda E = P(\lambda)(A-\lambda E)Q(\lambda) \quad (2.3.12)$$

tenglik bajariladi.

(2.3.10) va (2.3.11) tengliklarga asosan:

$$\begin{gathered} P(\lambda)=(B-\lambda E)P_1(\lambda)+P_0 \\ Q(\lambda)=Q_1(\lambda)(B-\lambda E)+Q_0 \end{gathered}, \quad (2.3.13)$$

bunda P_0 va Q_0 - o'zgarmas matritsalar (2.3.13) ifodalarni (2.3.12) tenglikka qo'yamiz:

$$\begin{gathered} B-\lambda E=[(B-\lambda E)P_1(\lambda)+P_0](A-\lambda E)[Q(\lambda)(B-\lambda E)+Q_0]=(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_1(\lambda) \\ (B-\lambda E)++(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_0+P_0(A-\lambda E)Q_1(\lambda)(B-\lambda E)+P_0(A-\lambda E)Q_0 \end{gathered}$$

(2.3.14)

Bu tenglikda $P(A-\lambda E)Q_0$ ifodani chap tomonga o'tkazib o'ng tomonga qolgan yig'indini $S(\lambda)$ bilan belgilaymiz. Bu vaqtda (2.3.14) tenglik

$$B-\lambda E-P_0(A-\lambda E)Q_0=S(\lambda) \quad (2.3.15)$$

ko'rinishini oladi, bunda

$$S(\lambda)=(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_1(\lambda)(B-\lambda E)+(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_0+P_0(A-\lambda E)Q_1(\lambda)(-\lambda E)$$

(2.3.13) tengliklarning ikkinchisini nazarda tutib (2.3.16) yig'indining avvalgi ikkita qo'shiluvchisini quyidagicha yozish mumkin.

$$(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_1(\lambda)(B-\lambda E)+(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_0=(B-\lambda E)P_1(\lambda)(A-\lambda E)$$
$$[Q_1(\lambda)(B-\lambda E)+Q_0]=(B-\lambda E)P_1(\lambda)(B-\lambda E)Q(\lambda)$$

Endi (2.3.16) yig'indining uchinchi qo'shiluvchisiga

$$(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_1(\lambda)(A-\lambda E)$$

ifodani qo'shamiz va ayiramiz .

Buning natijasida $S(\lambda)$ yig'indining ko'rinishi bunday bo'ladi:

$$S(\lambda)=(B-\lambda E)P_1(\lambda)(A-\lambda E)Q(\lambda)+P(\lambda)(A-\lambda E)Q_1(\lambda)(B-\lambda E)-$$
$$-(B-\lambda E)P_1(\lambda)(A-\lambda E)Q_1(\lambda)(B-\lambda E)$$

(2.3.12) tenglikdan hosil qilingan

$$(A-\lambda E)Q(\lambda)=P^{-1}(\lambda)(B-\lambda E)$$
$$P(\lambda)(A-\lambda E)=Q^{-1}(\lambda)(B-\lambda E)$$

ifodalarni (2.3.17) ga qo'yib,ushbuga ega bo'lamiz:

$$S(\lambda)=(B-\lambda E)[P_1(\lambda)P^{-1}(\lambda)+Q^{-1}(\lambda)Q_1(\lambda)+P_1(\lambda)(A-\lambda E)Q_1(\lambda)](B-\lambda E)$$

Kvadrat qavs ichidagi yig'indi λ ga nisbatan $T(\lambda)$ ko'phad. Bu ko'phadning darajasini m deylik.

U holda $S(\lambda)=(B-\lambda E)T(\lambda)(B-\lambda E)$ ko'phadning darajasi $m+2$ bo'ladi

Demak, $m\geq 0$ ekanini nazarda tutsak, $S(\lambda)$ ning darajasini 2 dan kam bo'lmasligi kerak. Ammo, (2.3.15) tenglikka asosan, $S(\lambda)$ ning darajasi 1 dan yuqori bo'la olmaydi.

Bu ziddiyat faqat $T(\lambda)=0$ shartdagina yo'qotilishi mumkin.Bu vaqtda

$S(\lambda)=0$ bo'lib, (2.3.15) tenglikdan

$$B-\lambda E=P_0(A-\lambda E)Q_0$$

yoki

$$B-\lambda E=P_0AQ_0-\lambda P_0Q_0$$

tenglik hosil bo'ladi.Demak,matritsali ko'phadlarning teng bo'lish shartiga asosan:

$$P_0Q_0=E \quad (2.3.18)$$

va

$$B=P_0AQ_0 \quad (2.3.19)$$

(2.3.18) tenglikdan P_0 va Q_0 matritsalarning xosmas matritsalar bo'lish ko'rinadi va $P_0=Q_0^{-1}$ ekani hosil bo'ladi. Shu sababli, (2.3.19) tenglik $B=Q_0^{-1}AQ_0$ ko'rinishini oladi. Bu esa A va B matritsalarning o'xshashligini ifodalaydi.

Endi bu rejaning asosiy masalasiga kelamiz.

Dastlab quyidagi eslatmaga e'tibor beraylik: (2.3.8) jadvaldagi har bir $(\lambda-\lambda_t)^{m_i}$ daraja $J-\lambda E$ matritsaning λ_t soniga oid m_i - tartibli katagini xarakterlaydi. Shunday qilib, (2.3.8) jadvaldagi hamma darajalar $J-\lambda E$ matritsani xarakterlaydi. (2.3.8) jadval bilan aniqlangan kataklarni $J-\lambda E$ matritsada joylashtirish tartibi ixtiyoriydir,chunki bu kataklarni turlicha tartibda joylashtirish bilan hosil qilingan hamma $J-\lambda E$ ko'rinishdagi matritsalar bir-biriga ekvivalent.

(2.3.8) jadvaldagiga asoslanib, $J-\lambda E$ matritsani tuzgandan keyin J matritsani topish yengil.

2.3.1-misol:

$$(\lambda-8)^2, \lambda-8$$
$$(\lambda+1)$$
$$(\lambda-11)$$

jadval bo’yicha avval

$$\begin{Vmatrix} 8-\lambda & 1 & & & \\ 0 & 8-\lambda & & & \\ & & 8-\lambda & & \\ & & & -1-\lambda & \\ & & & & 11-\lambda \end{Vmatrix}$$

matritsani tuzib,so’ngra ushbu Jordan matritsasini hosil qilamiz.

$$\begin{Vmatrix} 8 & 1 & & & \\ 0 & 8 & & & \\ & & 8 & & \\ & & & -1 & \\ & & & & 11 \end{Vmatrix}.$$

<u>1-teorema:</u> Kompleks sonlar maydonidagi har bir n-tartibli o’zgarmas A matritsadagi o’xshash bo’lgan J Jordan matritsasi mavjud.

Isbot: A matritsaning $A-\lambda E$ xarakteristik matritsasini olib, ma’lum qoida bilan uning $D_n(\lambda), D_{n-1}(\lambda), \ldots, D_1(\lambda)$ ko’phadlarini tuzamiz va ularga asoslanib,

$$E_1(\lambda), E_2(\lambda), \ldots, E_n(\lambda) \quad (2.3.20)$$

ko’paytuvchilarni hosil qilamiz.

U holda $A-\lambda E$ matritsaning normal diogonal shakli quyidagicha bo’lishi ma’lum:

$$\begin{Vmatrix} E_1(\lambda) & 0 & \dots & 0 \\ 0 & E_2(\lambda) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & E_n(\lambda) \end{Vmatrix} \quad (2.3.21)$$

(2.3.20) ko'phadlarning 1 dan farqlilari

$$E_{n-k+1}(\lambda), E_{n-k+2}(\lambda), \dots, E_n(\lambda) \quad (2.3.22)$$

bo'lsin. Agar (2.3.22) ko'phadlarni $(\lambda-\lambda_i)$ larning darajalari bo'yicha ko'paytuvchilarga yoysak:

$$\begin{gathered} E_n(\lambda)=(\lambda-\lambda_1)^{\alpha_1}(\lambda-\lambda_2)^{\alpha_2}\dots\dots(\lambda-\lambda_e)_e^{\alpha} \\ E_{n-1}(\lambda)=(\lambda-\lambda_1)^{\beta_1}(\lambda-\lambda_2)^{\beta_2}\dots\dots(\lambda-\lambda_e)^{\beta_\lambda} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ E_{n-k+1}(\lambda)=(\lambda-\lambda_1)^{\tau_1}(\lambda-\lambda_2)^{\tau_2}\dots\dots(\lambda-\lambda_e)^{\tau_e} \end{gathered} \quad (2.3.23)$$

bo'ladi, bunda $\lambda_1, \lambda_2, \dots\lambda_e$ ildizlar har xil va (2.3.22) ko'phadlarning har biri o'zidan avvalgisiga bo'lingani uchun: $\alpha_i \geq \beta_i \geq \dots \geq \gamma_i \geq 0$ (2.3.23) yoyilmalardagi darajalarni ustunlar bo'yicha yozib chiqsak, quyidagi jadval hosil bo'ladi.

$$\begin{gathered} (\lambda-\lambda_1)^{\alpha_1}(\lambda-\lambda_1)^{\beta_1}\dots\dots(\lambda-\lambda_1)^{\gamma_1} \\ (\lambda-\lambda_2)^{\alpha_2}(\lambda-\lambda_2)^{\beta_2}\dots\dots(\lambda-\lambda_e)^{\gamma_2} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (\lambda-\lambda_e)^{\alpha_e}(\lambda-\lambda_e)^{\beta_e}\dots\dots(\lambda-\lambda_e)^{\gamma_e} \end{gathered} \quad (2.3.24)$$

Bu jadvalga asosan, $J-\lambda E$ matritsani tuzamiz.Uning normal diogonal formasi xuddi (2.3.21) matritsadan iborat.

Shunday qilib, $A-\lambda E$ va $J-\lambda E$ matritsaning har qaysisi (2.3.21) matritsa bilan ekvivalent bo'ladi. Demak, 2.3.2-lemmaga asosan, A bilan J o'xshash matritsalardir.

Berilgan A matritsaga o'xshash J Jordan matritsasini topish – bu A matritsani Jordan matritsasiga keltirish deyiladi.

$\lambda_1, \lambda_2, \dots\lambda_e$ Sonlar $|A-\lambda E|$ xarakteristik ko'phadning ildizlarini ifodalaydi.

Umuman,bu ko'phadning ildizlari karrali.

Isbot qilinganlardan quyidagi xulosa kelib chiqadi: har bir A o'zgarmas matritsa kataklarning joylashish tartibi aniqligida Jordan matritsasiga birdan – bir usul bilan keltiriladi, ya'ni A ning turlicha Jordan matritsalari bir – biridan faqat kataklarning joylanish tartibi bilangina farq qiladi.

2.3.1 – teorama. A matritsaning J Jordan matritsasi diagonal matritsadan iborat bo'lishi uchun (2.3.24) jadvaldagi ayirmalarning hammasi 1 – darajali bo'lishi zarur va yetarli.

Isbot

I (2.3.24) jadvaldagi hamma ayirmalar birinchi darajali bo'lsin:

$$\begin{array}{l} \lambda-\lambda_1, \lambda-\lambda_1, \ldots. \\ \lambda-\lambda_2, \lambda-\lambda_2, \ldots. \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \lambda-\lambda_e, \lambda-\lambda_e, \ldots. \end{array}$$

U holda

$$J-\lambda E=\left\|\begin{matrix} \lambda_1-\lambda & & & \\ & \lambda_2-\lambda & & \\ & & \ddots & \\ & & & \lambda_e-\lambda \end{matrix}\right\| \quad (2.3.25)$$

va, demak,

$$J=\left\|\begin{matrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_e \end{matrix}\right\| \quad (2.3.26)$$

II Aksincha, J matritsaning shakli (2.3.26) dan iborat bo'lsa, $J-\lambda E$ matritsaning shakli (2.3.25) dan iborat bo'ladi. Bu esa (2.3.24) jadvaldagi ayirmalarning 1 – darajali ekanini ko'rsatadi.

Quyidagi matritsalarni Jordan matritsalariga keltiraylik.

2.3.1-misol:

$$\begin{Vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{Vmatrix}$$

Xarakteristik ko'phad tuzib, uni birinchi darajali ko'paytuvchilarga ajratamiz

$$\begin{Vmatrix} 1-\lambda & 1 & -1 \\ 1 & 1-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{Vmatrix} = (1-\lambda)^2(2-\lambda)+1+(1-\lambda)-(2-\lambda)=(1-\lambda)^2(2-\lambda)$$

Demak, $D_3(\lambda)=(1-\lambda)^2(2-\lambda)$

Ushbu

$$\begin{vmatrix} 1 & -1 \\ 1-\lambda & 1 \end{vmatrix} = 2-\lambda, \quad \begin{vmatrix} 1 & 1-\lambda \\ 1 & 0 \end{vmatrix} = \lambda-1$$

2-tartibli minorlar o'zaro tub bo'lgani sababli $D_2(\lambda)=1$

U holda $D_1(\lambda)=1$

Endi

$$E_3(\lambda)=\frac{D_3(\lambda)}{D_2(\lambda)}=(1-\lambda)^2(2-\lambda)$$
$$E_2(\lambda)=\frac{D_2(\lambda)}{D_1(\lambda)}=1$$
$$E_1(\lambda)=D_1(\lambda)=1$$

(2.3.24) jadval ushbu ko'rinishga ega bo'ladi:

$$\begin{matrix} (1-\lambda)^2 \\ (\lambda-2) \end{matrix}.$$

Shunday qilib, $J-\lambda E$ matritsa 1 ga oid 2 – tartibli va 2 ga oid 1 – tartibli kataklardan tuziladi.

Shuning uchun J matritsaning shakli bunday bo'ladi.

$$\begin{Vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{Vmatrix}$$

<u>2.3.2-misol</u>:

$$\begin{Vmatrix} 2 & -2 & 1 \\ 3 & -5 & 3 \\ 6 & -12 & 7 \end{Vmatrix}$$

Bunga asosan

$$\begin{Vmatrix} 2-\lambda & -2 & 1 \\ 3 & -5-\lambda & 3 \\ 6 & -12 & 7-\lambda \end{Vmatrix} = -(1-\lambda)^2(2-\lambda)$$

Demak, $D_3(\lambda) = (1-\lambda)^2(2-\lambda)$

2-tartibli minorlar:

$$(\lambda-1)(\lambda+4); \ \ 12(\lambda-1); \ \ 6(\lambda-1); \ \ 3(\lambda-1);$$

$$(\lambda-1)(\lambda-8); \ -3(\lambda-1); \ (\lambda-1); \ 2(\lambda-1); \ (\lambda-1)^2$$

bo'lib, bundan $D_2(\lambda) = \lambda - 1$.

Birinchi tartibli minorlar uchun:

$$D_1(\lambda) = 1$$

Shunday qilib,

$$E_3(\lambda) = \frac{D_3(\lambda)}{D_2(\lambda)} = (\lambda-1)(\lambda-2)$$

$$E_2(\lambda) = \frac{D_2(\lambda)}{D_1(\lambda)} = \lambda - 1$$

$$E_1(\lambda) = D_1(\lambda) = 1$$

(2.3.24) jadvalning ko'rinishi

$$(\lambda-1), \ (\lambda-1),$$

$$(\lambda-2)$$

Shu sababli

2.3.3 – teorema R unitar fazoda har bir A chiziqli almashtirishning istalgan

$$\overline{e_1}\,\overline{e_2},\ldots\overline{e_n} \quad (2.3.27)$$

bazisdagi A matritsasi uchun shunday

$$\overline{f_1}, \overline{f_2}, \ldots \overline{f_n} \quad (2.3.28)$$

bazis mavjudki, bu bazisda A almashtirishning matritsasi Jordan matritsasidan iborat.

Isbot: 1 – teoremaga asosan A almashtirishning (2.3.27) bazisdagi A matritsasi bilan o'xshash J matritsa mavjud. Demak xosmas C matritsa uchun

$J = C^{-1}AC$

Tenglik bajariladi, bunda

$$C = \begin{Vmatrix} C_{11} & C_{12} & \ldots & C_{1n} \\ C_{21} & C_{22} & \ldots & C_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ C_{n1} & C_{n2} & \ldots & C_{nn} \end{Vmatrix}$$

deylik. Agar

$$\begin{aligned} &\overline{f_1} = c_{11}\overline{e_1} + c_{11}\overline{e_2} + \ldots + c_{n1}\overline{e_n} \\ &\overline{f_2} = c_{12}\overline{e_1} + c_{22}\overline{e_2} + \ldots + c_{n2}\overline{e_n} \\ &.. \\ &\overline{f_n} = c_{1n}\overline{e_1} + c_{2n}\overline{e_2} + \ldots + c_{nn}\overline{e_n} \end{aligned}$$

bazisni olsak, u holda J matritsa A almashtirishning shu (2.3.28) bazisdagi matritsasi bo'ladi.

2.3.3 – misol

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$A-\lambda E=\begin{pmatrix}1-\lambda & 1 & 0 & 0\\ 0 & 1-\lambda & 0 & 0\\ 0 & 0 & 4-\lambda & 4\\ 0 & 0 & -1 & -\lambda\end{pmatrix}$$

4- tartibli minori

$$\begin{vmatrix}1-\lambda & 1 & 0 & 0\\ 0 & 1-\lambda & 0 & 0\\ 0 & 0 & 4-\lambda & 4\\ 0 & 0 & -1 & -\lambda\end{vmatrix}=(1-\lambda)(-1)^{1+1}\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 4-\lambda & 4\\ 0 & -1 & -\lambda\end{vmatrix}=(1-\lambda)\cdot((1-\lambda)(4-\lambda)\cdot(-\lambda)+4(1-\lambda))=$$

$$(1-\lambda)\cdot\left((1-\lambda)\left(\lambda^2-4\lambda+4\right)\right)=(1-\lambda)^2(\lambda-2)^2=(1-\lambda)^2(\lambda-2)^2$$

$$D_4(\lambda)=(1-\lambda)^2(\lambda-2)^2$$

3 – tartibli minorlari.

$$\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 4-\lambda & 4\\ 0 & -1 & -\lambda\end{vmatrix}=(1-\lambda)\cdot(4-\lambda)(-\lambda)+4(1-\lambda)=(1-\lambda)\left(\lambda^2-4\lambda+4\right)=(1-\lambda)(\lambda-2)^2$$

$$\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 4-\lambda & 4\\ 0 & -1 & -\lambda\end{vmatrix}=0,\quad \begin{vmatrix}0 & 1-\lambda & 0\\ 0 & 0 & 4\\ 0 & 0 & -\lambda\end{vmatrix}=0,\quad \begin{vmatrix}0 & 1-\lambda & 0\\ 0 & 0 & 4-\lambda\\ 0 & 0 & -1\end{vmatrix}=0$$

$$\begin{vmatrix}1 & 0 & 0\\ 0 & 4-\lambda & 4\\ 0 & -1 & -\lambda\end{vmatrix}=-\lambda(4-\lambda)+4=-4\lambda+\lambda^2+4=(\lambda-2)^2$$

$$\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 4-\lambda & 4\\ 0 & -1 & -\lambda\end{vmatrix}=(1-\lambda)(4-\lambda)(-\lambda)+4(1-\lambda)=(1-\lambda)\left(\lambda^2-4\lambda+4\right)=(1-\lambda)(\lambda-2)^2$$

$$\begin{vmatrix}1-\lambda & 1 & 0\\ 0 & 0 & 4\\ 0 & 0 & -\lambda\end{vmatrix}=0\quad \begin{vmatrix}1-\lambda & 1 & 0\\ 0 & 0 & 4-\lambda\\ 0 & 0 & -\lambda\end{vmatrix}=0.$$

$$\begin{vmatrix}1 & 0 & 0\\ 1-\lambda & 0 & 0\\ 0 & -1 & -\lambda\end{vmatrix}=0\quad \begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 0 & 0\\ 0 & -1 & -\lambda\end{vmatrix}=0\quad \begin{vmatrix}1-\lambda & 1 & 0\\ 0 & 1-\lambda & 0\\ 0 & 0 & -\lambda\end{vmatrix}=-\lambda(1-\lambda)^2$$

$$\begin{vmatrix}1-\lambda & 1 & 0\\ 0 & 1-\lambda & 0\\ 0 & 0 & -1\end{vmatrix}=-(1-\lambda)^2\quad \begin{vmatrix}1 & 0 & 0\\ 1-\lambda & 0 & 0\\ 0 & 4-\lambda & 4\end{vmatrix}=0\quad \begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 0 & 0\\ 0 & 4-\lambda & 4\end{vmatrix}=0$$

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 4 \end{vmatrix} = 4(1-\lambda)^2 \quad \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (1-\lambda)^2(4-\lambda) \quad D_3(\lambda)=1$$

2 – tartibli minorlari

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 \quad \begin{vmatrix} 0 & 1-\lambda \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 1 & 0 \\ 1-\lambda & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 4-\lambda \end{vmatrix} = (1-\lambda)^2(4-\lambda) \quad \begin{vmatrix} 0 & 4-\lambda \\ 0 & -1 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 4-\lambda & 4 \end{vmatrix} = 0$$

$$\begin{vmatrix} 4-\lambda & 4 \\ -1 & -\lambda \end{vmatrix} = (-\lambda)(4-\lambda)+4 = \lambda^2 - 4\lambda + 4 = (\lambda-2)^2 \quad D_2(\lambda)=1$$

Bizga ko’rinib turibdiki 1 – tartibli minorlarning EKUBi 1 ga teng ya’ni

$$D_1(\lambda)=1 \quad D_0(\lambda)=1 \quad D_0(\lambda)$$

doim 1 ga teng deb olinadi.

$$E_1(\lambda)=\frac{D_1(\lambda)}{D_0(\lambda)}=\frac{1}{1}=1 \quad E_2(\lambda)=\frac{D_2(\lambda)}{D_1(\lambda)}=\frac{1}{1}=1 \; E_3(\lambda)=\frac{D_3(\lambda)}{D_2(\lambda)}=\frac{1}{1}=1$$

$$E_4(\lambda)=\frac{D_4(\lambda)}{D_3(\lambda)}=\frac{(\lambda-1)^2(\lambda-2)^2}{1}=(\lambda-1)^2(\lambda-2)^2$$

$$\begin{aligned} E_4(\lambda) &= (\lambda-\lambda_1)^{\alpha_1}(\lambda-\lambda_2)^{\alpha_2} = (\lambda-1)^2(\lambda-2)^2 \\ E_3(\lambda) &= (\lambda-\lambda_1)^{\beta_1}(\lambda-\lambda_2)^{\beta_2} = (\lambda-1)^0(\lambda-2)^0 \\ E_4(\lambda) &= (\lambda-\lambda_1)^{\gamma_1}(\lambda-\lambda_2)^{\gamma_2} = (\lambda-1)^0(\lambda-2)^0 \\ E_4(\lambda) &= (\lambda-\lambda_1)^{\xi_1}(\lambda-\lambda_2)^{\xi_2} = (\lambda-1)^0(\lambda-2)^0 \end{aligned}$$

$$\beta_1=0,\ \beta_2=0,\ \gamma_1=0,\ \gamma_2=0,\ \xi_1=0,\ \xi_2=0,\ \gamma_1=2,\ \gamma_2=2.$$

$$\begin{matrix} (\lambda-1)^2,(\lambda-1)^0,(\lambda-1)^0,(\lambda-1)^0 \\ (\lambda-2)^2,(\lambda-2)^0,(\lambda-2)^0,(\lambda-2)^0 \end{matrix} \Rightarrow \begin{matrix} (\lambda-1)^2,1,1,1 \\ (\lambda-2)^2,1,1,1 \end{matrix}$$

Shu sababli

$$J_A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \quad J_A = \begin{pmatrix} J_1 & \\ & J_2 \end{pmatrix}$$

bu yerda

$$J_1=\begin{pmatrix}1&1\\0&1\end{pmatrix}\quad J_2=\begin{pmatrix}2&1\\0&2\end{pmatrix}$$

<u>2.3.4-misol:</u>

$$A=\begin{pmatrix}1&0&0&0\\0&2&0&1\\0&0&2&19\\0&1&1&1\end{pmatrix}\quad A-\lambda E=\begin{pmatrix}1-\lambda&0&0&0\\0&2-\lambda&0&1\\0&0&2-\lambda&19\\0&1&1&1-\lambda\end{pmatrix}$$

$$A-\lambda E=\begin{pmatrix}1-\lambda&0&0&0\\0&2-\lambda&0&1\\0&0&2-\lambda&19\\0&1&1&1-\lambda\end{pmatrix}$$

4 – tartibli minori:

$$(1-\lambda)(2-\lambda)^2(1-\lambda)-(2-\lambda)-19(2-\lambda)=(1-\lambda)(2-\lambda)\left(2-3\lambda+\lambda^2-1-1\right)=$$
$$=(1-\lambda)(2-\lambda)\left(\lambda^2-3\lambda-18\right)(1-\lambda)$$

$$\begin{pmatrix}1-\lambda&0&0&0\\0&2-\lambda&0&1\\0&0&2-\lambda&19\\0&1&1&1-\lambda\end{pmatrix}=(-1)^{1+1}\begin{vmatrix}2-\lambda&0&1\\0&2-\lambda&19\\1&1&1-\lambda\end{vmatrix}$$

$$A-\lambda E=\begin{pmatrix}1-\lambda&0&0&0\\0&2-\lambda&0&1\\0&0&2-\lambda&19\\0&1&1&1-\lambda\end{pmatrix}$$

3 – tartibli minorlari.

$$\begin{vmatrix}2-\lambda&0&1\\0&2-\lambda&19\\1&1&1-\lambda\end{vmatrix}=(2-\lambda)^2(1-\lambda)-(2-\lambda)-19(2-\lambda)=(2-\lambda)\left((1-\lambda)(2-\lambda)-1-19\right)=$$
$$=(2-\lambda)\left(2-\lambda-2\lambda+\lambda^2-1-19\right)=(2-\lambda)\left(\lambda^2-3\lambda-18\right)$$

$$\begin{vmatrix}0&0&1\\0&2-\lambda&19\\0&1&1-\lambda\end{vmatrix}=0\quad\begin{vmatrix}0&2-\lambda&1\\0&0&19\\0&1&1-\lambda\end{vmatrix}=0\quad\begin{vmatrix}0&2-\lambda&0\\0&0&2-\lambda\\0&0&1\end{vmatrix}=0\quad\begin{vmatrix}0&0&0\\0&2-\lambda&19\\1&1&1-\lambda\end{vmatrix}=0$$

$$\begin{vmatrix}1-\lambda&0&1\\0&2-\lambda&19\\0&1&1-\lambda\end{vmatrix}=(1-\lambda)^2(2-\lambda)-19(1-\lambda)=(1-\lambda)\left(2-3\lambda+\lambda^2-19\right)=(1-\lambda)\left(\lambda^2-3\lambda-17\right)$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 0 & 19 \\ 0 & 1 & 1-\lambda \end{vmatrix} = -19(1-\lambda) \quad \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda \\ 0 & 1 & 1 \end{vmatrix} = -(2-\lambda)(1-\lambda)$$

$$\begin{vmatrix} 0 & 0 & 0 \\ 2-\lambda & 0 & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = -(1-\lambda)$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2(2-\lambda)-(1-\lambda)=(1-\lambda)(\lambda^2-3\lambda+1)$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & 1 \end{vmatrix} = (1-\lambda)(2-\lambda) \quad \begin{vmatrix} 0 & 0 & 0 \\ 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 19 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2-\lambda & 19 \end{vmatrix} = -(2-\lambda)(1-\lambda) \quad \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 19 \end{vmatrix} = 19(1-\lambda)(2-\lambda)$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)^2 \quad D_3(\lambda)=1$$

2 – tartibli minorlar

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) \quad \begin{vmatrix} 0 & 2-\lambda \\ 1 & 1 \end{vmatrix} = -(2-\lambda) \quad \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 2-\lambda \\ 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 \\ 2-\lambda & 19 \end{vmatrix} = -(2-\lambda) \quad \begin{vmatrix} 2-\lambda & 19 \\ 1 & 1-\lambda \end{vmatrix} = \lambda^2-3\lambda-17 \quad \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 1-\lambda & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 1-\lambda & 0 \\ 0 & 1 \end{vmatrix} = 1-\lambda$$

$$\begin{vmatrix} 0 & 0 \\ 2-\lambda & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 2-\lambda & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 0 & 2-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 1 \\ 0 & 19 \end{vmatrix} = 0 \quad \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^2$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 0 & 19 \end{vmatrix} = 19(2-\lambda) \quad \begin{vmatrix} 0 & 2-\lambda \\ 0 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 19 \\ 0 & 1-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 19 \\ 1 & 1-\lambda \end{vmatrix} = -19$$

$$D_2(\lambda)=1, \ D_1(\lambda)=1, \ D_0(\lambda)=1$$

$$E_1(\lambda)=\frac{D_1(\lambda)}{D_0(\lambda)}=\frac{1}{1}=1, \ E_1(\lambda)=1$$

$$E_2(\lambda)=\frac{D_2(\lambda)}{D_1(\lambda)}=\frac{1}{1}=1, \ E_2(\lambda)=1$$

$$E_3(\lambda)=\frac{D_3(\lambda)}{D_2(\lambda)}=\frac{1}{1}=1,\ \ E_3(\lambda)=1$$

$$E_4(\lambda)=\frac{D_4(\lambda)}{D_3(\lambda)}=\frac{(\lambda-1)(\lambda-2)(\lambda+3)(\lambda-6)}{1}=(\lambda-1)(\lambda-2)(\lambda+3)(\lambda-6)$$

$$E_4(\lambda)=(\lambda-1)(\lambda-2)(\lambda+3)(\lambda-6)$$

$$E_4(\lambda)=(\lambda-\lambda_1)^{\alpha_1}(\lambda-\lambda_2)^{\alpha_2}(\lambda-\lambda_3)^{\alpha_3}(\lambda-\lambda_4)^{\alpha_4}=(\lambda-1)(\lambda-1)(\lambda+3)(\lambda-6)$$
$$E_3(\lambda)=(\lambda-\lambda_1)^{\beta_1}(\lambda-\lambda_2)^{\beta_2}(\lambda-\lambda_3)^{\beta_3}(\lambda-\lambda_4)^{\beta_4}=(\lambda-1)^0(\lambda-2)^0(\lambda-6)^0(\lambda+3)^0$$
$$E_4(\lambda)=(\lambda-\lambda_1)^{\gamma_1}(\lambda-\lambda_2)^{\gamma_2}(\lambda-\lambda_3)^{\gamma_3}(\lambda-\lambda_4)^{\gamma_4}=(\lambda-1)^0(\lambda-2)^0(\lambda-6)^0(\lambda+3)^0$$
$$E_4(\lambda)=(\lambda-\lambda_1)^{\xi_1}(\lambda-\lambda_2)^{\xi_2}(\lambda-\lambda_3)^{\xi_3}(\lambda-\lambda_4)^{\xi_4}=(\lambda-1)^0(\lambda-2)^0(\lambda-6)^0(\lambda+3)^0$$

$$\alpha_1=\alpha_2=\alpha_3=\alpha_4=1,\ \ \beta_1=\beta_2=\beta_3=\beta_4=\gamma_1=\gamma_2=\gamma_3=\gamma_4=\xi_1=\xi_2=\xi_3=\xi_4=0,\ \ \lambda_1=1$$

$$\lambda_2=2\quad \lambda_3=6\quad \lambda_4=-3$$

$$\begin{matrix}(\lambda-1);\\(\lambda-2);\\(\lambda-6);\\(\lambda+3);\end{matrix}\quad A=\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&6&0\\0&0&0&-3\end{pmatrix}=\begin{pmatrix}J_1&0&0&0\\0&J_2&0&0\\0&0&J_3&0\\0&0&0&J_4\end{pmatrix}\quad J_1=\|1\|\quad J_2=\|2\|\quad J_3=\|6\|\quad J_4=\|-3\|$$

2.3.5 – misol.

$$A=\begin{pmatrix}1&2&5&3\\0&1&4&4\\0&0&1&4\\0&0&0&1\end{pmatrix}$$

$$A-\lambda E=\begin{pmatrix}1&2&5&3\\0&1&4&4\\0&0&1&1\\0&0&0&1\end{pmatrix}-\lambda\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1&2&5&3\\0&1&4&4\\0&0&1&1\\0&0&0&1\end{pmatrix}-\begin{pmatrix}\lambda&0&0&0\\0&\lambda&0&0\\0&0&\lambda&0\\0&0&0&\lambda\end{pmatrix}=$$
$$=\begin{pmatrix}1-\lambda&2&5&3\\0&1-\lambda&4&4\\0&0&1-\lambda&1\\0&0&0&1-\lambda\end{pmatrix}$$

4 – tartibli minorlar

$$\begin{vmatrix} 1-\lambda & 2 & 5 & 3 \\ 0 & 1-\lambda & 4 & 4 \\ 0 & 0 & 1-\lambda & 1 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 1-\lambda & 4 & 4 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((1-\lambda)^3\right) = (1-\lambda)^4$$

3 – tartibli minorlar

$$A-\lambda E = \begin{pmatrix} 1-\lambda & 2 & 5 & 3 \\ 0 & 1-\lambda & 4 & 4 \\ 0 & 0 & 1-\lambda & 1 \\ 0 & 0 & 0 & 1-\lambda \end{pmatrix} \quad \begin{vmatrix} 1-\lambda & 4 & 4 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 \quad \begin{vmatrix} 0 & 4 & 4 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 1-\lambda & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 1-\lambda & 4 \\ 0 & 0 & 1-\lambda \\ 0 & 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 2 & 5 & 3 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 2(1-\lambda)^2$$

$$\begin{vmatrix} 1-\lambda & 5 & 3 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 \quad \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 1-\lambda & 2 & 5 \\ 0 & 0 & 1-\lambda \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 5 & 3 \\ 1-\lambda & 4 & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 8(1-\lambda)-(1-\lambda)^2\cdot 5 = (1-\lambda)(8-(1-\lambda)\cdot 5) = (1-\lambda)(8-5+5\lambda) = (1-\lambda)(3+5\lambda)$$

$$\begin{vmatrix} 1-\lambda & 5 & 3 \\ 0 & 4 & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 4(1-\lambda)^2 \quad \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 \quad \begin{vmatrix} 1-\lambda & 2 & 5 \\ 0 & 1-\lambda & 4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 5 & 3 \\ 1-\lambda & 4 & 4 \\ 0 & 1-\lambda & 1 \end{vmatrix} = 8+3(1-\lambda)^2-8(1-\lambda)-5(1-\lambda) = 8+3-6\lambda-3\lambda^2-8+8\lambda-5+5\lambda =$$

$$= -3\lambda^2+7\lambda-2 = -\left(3\lambda^2-7\lambda+2\right)$$

$$\begin{vmatrix} 1-\lambda & 5 & 3 \\ 0 & 4 & 4 \\ 0 & 1-\lambda & 1 \end{vmatrix} = 4(1-\lambda)-4(1-\lambda)^2 = 4(1-\lambda)(1-1+\lambda) = 4\lambda(1-\lambda)$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & 4 \\ 0 & 0 & 1 \end{vmatrix} = (1-\lambda)^2 \quad \begin{vmatrix} 1-\lambda & 2 & 5 \\ 0 & 1-\lambda & 4 \\ 0 & 0 & 1 \end{vmatrix} = (1-\lambda)^3 \quad D_3(\lambda) = 1$$

2 – tartibli minorlari.

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 \quad \begin{vmatrix} 0 & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 1-\lambda \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 4 & 4 \\ 0 & 1-\lambda \end{vmatrix} = 4(1-\lambda) \quad \begin{vmatrix} 1-\lambda & 4 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2$$

$$\begin{vmatrix}1-\lambda & 4\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}4 & 4\\ 1-\lambda & 1\end{vmatrix}=4-4+4\lambda=4\lambda,\ \begin{vmatrix}1-\lambda & 4\\ 0 & 1\end{vmatrix}=1-\lambda,\ \begin{vmatrix}1-\lambda & 4\\ 0 & 1-\lambda\end{vmatrix}=(1-\lambda)^2,$$

$$\begin{vmatrix}1-\lambda & 2\\ 0 & 1-\lambda\end{vmatrix}=(1-\lambda)^2,\ \begin{vmatrix}1-\lambda & 5\\ 0 & 4\end{vmatrix}=4(1-\lambda),\ \begin{vmatrix}1-\lambda & 3\\ 0 & 4\end{vmatrix}=4(1-\lambda),\ \begin{vmatrix}2 & 3\\ 1-\lambda & 4\end{vmatrix}=8-3+3\lambda=5+3\lambda,$$

$$\begin{vmatrix}2 & 5\\ 1-\lambda & 4\end{vmatrix}=8-5+5\lambda=3-5\lambda,\ \begin{vmatrix}5 & 3\\ 4 & 4\end{vmatrix}=20-12=8,\ \begin{vmatrix}0 & 1-\lambda\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}0 & 1\\ 0 & 1-\lambda\end{vmatrix}=0,\ \begin{vmatrix}1-\lambda & 2\\ 0 & 0\end{vmatrix}=0,$$

$$\begin{vmatrix}2 & 5\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}1-\lambda & 5\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}1-\lambda & 3\\ 0 & 1-\lambda\end{vmatrix}=(1-\lambda)^2,\ \begin{vmatrix}2 & 3\\ 0 & 1-\lambda\end{vmatrix}=2(1-\lambda),\ \begin{vmatrix}5 & 3\\ 0 & 1-\lambda\end{vmatrix}=5(1-\lambda),$$

$$\begin{vmatrix}1-\lambda & 2\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}1-\lambda & 5\\ 0 & 1-\lambda\end{vmatrix}=(1-\lambda)^2,\ \begin{vmatrix}1-\lambda & 3\\ 0 & 1\end{vmatrix}=1-\lambda,\ \begin{vmatrix}2 & 5\\ 0 & 1-\lambda\end{vmatrix}=2(1-\lambda),\ \begin{vmatrix}2 & 3\\ 0 & 1\end{vmatrix}=2,$$

$$\begin{vmatrix}5 & 3\\ 1-\lambda & 1\end{vmatrix}=2+3\lambda,\ \begin{vmatrix}0 & 1-\lambda\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}0 & 4\\ 0 & 0\end{vmatrix}=0,\ \begin{vmatrix}2 & 3\\ 0 & 1\end{vmatrix}=2,\ \begin{vmatrix}0 & 4\\ 0 & 1-\lambda\end{vmatrix}=0,$$

$$D_2(\lambda)=1,\ D_1(\lambda)=1,\ D_0(\lambda)=1,$$

$$E_1(\lambda)=\frac{D_1(\lambda)}{D_0(\lambda)}=\frac{1}{1}=1,\ E_1(\lambda)=1,$$

$$E_2(\lambda)=\frac{D_2(\lambda)}{D_1(\lambda)}=\frac{1}{1}=1,\ E_2(\lambda)=1,$$

$$E_3(\lambda)=\frac{D_3(\lambda)}{D_2(\lambda)}=\frac{1}{1}=1,\ E_3(\lambda)=1,$$

$$E_4(\lambda)=\frac{D_4(\lambda)}{D_3(\lambda)}=\frac{(\lambda-1)^4}{1}=(\lambda-1)^4,\ E_4(\lambda)=(\lambda-1)^4,$$

$$\begin{aligned}E_4(\lambda)&=(\lambda-\lambda_1)^{\alpha_1}=(\lambda-1)^4\\ E_3(\lambda)&=(\lambda-\lambda_1)^{\beta_1}=(\lambda-1)^0\\ E_4(\lambda)&=(\lambda-\lambda_1)^{\gamma_1}=(\lambda-1)^0\\ E_4(\lambda)&=(\lambda-\lambda_1)^{\xi_1}=(\lambda-1)^0\end{aligned}$$

$$\alpha_1=4,\ \beta_1=\gamma_1=\xi_1=0,\ \lambda_1=1\ \ (\lambda-1)^4\quad A=\begin{pmatrix}1&1&0&0\\0&1&1&0\\0&0&1&1\\0&0&0&1\end{pmatrix}=(J_1)\quad J_1=\begin{vmatrix}1&1&0&0\\0&1&1&0\\0&0&1&1\\0&0&0&1\end{vmatrix}$$

Xulosa.

II-bob asosiy qismdan iborat bo’lib, bu bobda matritsaning Jordan shakli va kvadrat matritsalar to’plamida aniqlangan funksiyalar haqida ma’lumotlar berilgan va misollar tushuntirilgan. Bu bobda matritsaning Jordan shakli: Jordan matritsasi, Jordan katagining xarakteristik matritsasi, Jordan matritsasining xarakteristik matritsasi, ixtiyoriy matritsani Jordan matritsasiga keltirish haqida ta’riflar, teorema va isbotlari berilgan va misollar orqali tushuntirilgan.

Mavzuga oid misollar ishlab ko’rsatilgan.

Xotima

I bobda asosiy tushunchalar ifodalangan bo’lib, unda matritsalar halqasi: matritsa haqida, matritsani elementar almashtirishlar ,pog’onali matritsa, matritsalar ustida amallar, Elementar matritsalar, xos va xosmas matritsalar, matritsaviy tenglamalar, kvadrat matritsa determinanti haqida asosiy tushunchalar, ta’riflar va teoremalar berilgan va misollar orqali tushuntirilgan.

Matritsali funksiyalar haqida asosiy tushunchalarga to’xtalib o’tilgan.

Xususan xarakteristik ko’phadning tarifi, kvadrat matritsaning xos qiymatlari haqida asosiy tushunchalar misollar orqali tushuntirilgan.

Kvadrat matritsalar to’plamida aniqlangan funksiyalar haqida tushuntirib berilgan.

II bob asosiy qismdan iborat bo’lib, bu bobda matritsaning Jordan shakli va kvadrat matritsalar to’plamida aniqlangan funksiyalar haqida ma’lumotlar berilgan va misollar tushuntirilgan. Bu bobda matritsaning Jordan shakli: Jordan matritsasi, Jordan katagining xarakteristik matritsasi, Jordan matritsasining xarakteristik matritsasi, ixtiyoriy matritsani Jordan matritsasiga keltirish haqida ta’riflar, teorema va isbotlari berilgan va misollar orqali tushuntirilgan. Mavzuga oid misollar ishlab ko’rsatilgan.

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