O’ZBEKISTON RESPUBLIKASI XALQ TA`LIM VAZIRLIGI

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MATEMATIKA O’QITISH METODIKASI KAFEDRASI

Himoya qilishga ruxsat beraman
Fizika – matematika fakulteti dekani
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“____” ______________________ 2015 yil

Fizika – matematika fakulteti
B 5110100 Matematika o‘qitish metodikasi ta’lim yo’nalishi bo’yicha bakalavr darajasini olish uchun

“Cheva va Menelay teoremalari hamda ular yordamida masalalar yechish” mavzusidagi

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BMI “Matematika o’qitish metodikasi” kafedrasi yig’ilishi qarori bilan

(Qaror № __ “_31_” ____may____ 2015 yil) himoyaga tavsiya etildi

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Jizzax – 2015

M U N DA R I J A

Kirish

Masalaning qo'yilishi. Tekislikda uchta to'g'ri chiziqning bir nuqtada kesishishi ya'ni konkurentlik masalasi juda qiziq geometrik muammolardan biridir. Odatda maktab kursidagi geometriya kitoblarda uchburchak bissektrisa yoki balandliklari yoki medianalarining konkurentligi haqidagi tasdiqlar isbotsiz keltiriladi. Shuning uchun mazkur malakaviy bitiruv ishi uchburchak kesmalarining bir nuqtada kesishishi uchun yetarli va zaruriy shartlarini topish masalasiga bag'ishlanadi.

Masalaning dolzarbligi. Bugungi kunda respublikamizda matematika faniga, ayniqsa matematik olimpiadalarga katta e'tibor berilmoqda. Umuman olgan respublika, viloyat va xalqaro bosqich olimpiadalarida yosh matematiklarimiz ko'rsatayotgan natijalar yomon emas. Bir qator chet el matematika musobaqalarida berilgan geometriya bo'yicha misollar konkurentlik masalalariga bag'ishlangan. Masalan, Ipak yo'li (Silk Road) matematika olimpiadasida (2011 yil), Xalqaro matematika olimpiadasi (IMO, 2009 yil), Sank-Peterburgdagi musobaqada (2008-2009 yillar), Xalqaro matematika olimpiadalarining tanlov savollarida (IMO Sharlist). Biz qarayotgan Cheva va Menelay teoremasi, uning analoglaridan juda qiziqarli masalalar berilgan. Biroq bugungi kunda mazkur teorema bo'yicha o'zbek tilidagi adabiyotlar juda kam. Shuning uchun malakaviy bitiruv ishi mavzusi bugungi kundagi dolzarb mavzulardan biri hisoblanadi.

Ishning maqsadi va vazifalari. Malakaviy bitiruv ishining maqsadi va vazifalari olimpiada bilan shug'ullanuvchi o'quvchilarga va ularning ustozlariga Cheva va Menelay teoremasi hamda undan kelib chiquvchi natijalar haqida ma'lumot berish hamda ushbu teoremaning qo'llanilishiga doir namunaviy misollar yechib ko'rsatishdan iboratdir.

Ilmiy tatqiqot metodlari. Elementar geometriyaning planimetriya bo'limida ishlatiladigan teoremalar va usullar.

Ishning ilmiy ahamiyati. Ish referativ xarakterga ega.

Ishning amaliy ahamiyati. Mazkur bitiruv ishida qaralayotgan teorema uchburchakda bir uchdan chiqib, ikkinchi uchi shu uch qarshisidagi tomonda yotadigan kesmalar (chevianalar)ning uchburchak tomonlaridan ajratadigan kesmalari yoki shu uchga uning tomonlar va cheviana hosil qilgan burchaklarga ko'ra chevianalarning konkurent bo'lishi yoki bo'lmasligini oldindan aytib berishga imkon tug'diradi.

Ishning tuzilishi. Ish kirish qismi, 2 bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat.

I BOB.CHEVA VA MENELAY TEOREMASI HAMDA UNDAN KELIB CHIQADIGAN BA'ZI NATIJALAR.

Faraz qilamiz, bizga ΔABC berilgan bo'lsin.

Ta'rif. Uchburchakning biror uchi bilan qarama-qarshi tomonidagi biror nuqtani tutashtiruvchi kesma cheviana deyiladi.

Demak, ABC uchburchakda X, Y, Z nuqtalar mos ravishda BC, CA, AB tomonlarga tegishli bo'lsa, u holda AX, BY, CZ kesmalar chevianalar bo'ladi.

Ta'rif. Agar uchta to'g'ri chiziq (kesma) bitta nuqtadan o'tsa, ular konkurent deyiladi.

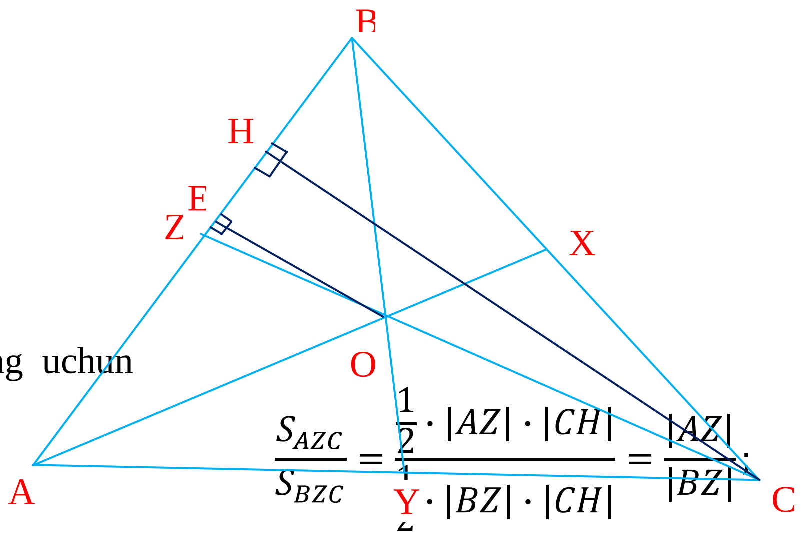
Cheviana atamasi italyan matematigi Jovanni Chevaning ismi sharafiga qo'yilgan bo'lib, u 1678-yilda quyidagi muhim teoremani chop ettirgan.

Teorema.(Cheva) Agar ABC uchburchakda AX, BY va CZ chevianalar konkurent bo'lsa, u holda

$$\frac{|BX|}{|XC|} \cdot \frac{|CY|}{|YA|} \cdot \frac{|AZ|}{|ZB|} = 1$$

munosabat o'rinli bo'ladi.

Isbot. Faraz qilaylik AX, BY, CZ kesmalar O nuqtada kesishsin. ΔAZC va ΔBZC lar umumiy CH balandlikka ega.



Shuning uchun

$$\frac{S_{AZC}}{S_{BZC}} = \frac{\frac{1}{2}\cdot|AZ|\cdot|CH|}{\frac{1}{2}\cdot|BZ|\cdot|CH|} = \frac{|AZ|}{|BZ|}.$$

Xuddi shunday ΔAZO va ΔBZO lar umumiy OE balandlikka ega. Demak,

$$\frac{S_{AZO}}{S_{BZO}} = \frac{\frac{1}{2}\cdot|AZ|\cdot|OE|}{\frac{1}{2}\cdot|BZ|\cdot|OE|} = \frac{|AZ|}{|BZ|}.$$

Demak, bu yerdan

$$\frac{|AZ|}{|BZ|} = \frac{S_{AZC}}{S_{BZC}} = \frac{S_{AZO}}{S_{BZO}} \qquad (1)$$

munosabatga ega bo'lamiz. Biz proporsiyaning quyidagi xossasidan foydalanamiz:

Lemma. $\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$. $\quad (b \neq d)$

Isbot. $\frac{a}{b} = \frac{c}{d} = \lambda$ $(*)$ bo'lsin. Biz $\frac{a-c}{b-d} = \lambda$ ni isbotlaymiz. Haqiqatdan ham $(*)$ dan $a = \lambda b,\ c = \lambda d$ deb olsak:

$$\frac{a-c}{b-d} = \frac{\lambda b - \lambda d}{b-d} = \frac{\lambda(b-d)}{b-d} = \lambda$$

Bu lemmani (1) munosabat uchun qo'llasak,

$$\frac{|AZ|}{|BZ|} = \frac{S_{AZC}}{S_{BZC}} = \frac{S_{AZO}}{S_{BZO}} = \frac{S_{AZC} - S_{AZO}}{S_{BZC} - S_{BZO}} = \frac{S_{AOC}}{S_{BOC}}$$

Tenglikka ega bo'lamiz.

Aynan shu yo'l bilan

$$\frac{|BX|}{|XC|} = \frac{S_{AOB}}{S_{AOC}} \quad va \quad \frac{|CY|}{|YA|} = \frac{S_{BOC}}{S_{AOB}}$$

ekanligi ko'rsatiladi. U holda

$$\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ|}{|ZB|} = \frac{S_{AOB}}{S_{AOC}}\cdot\frac{S_{BOC}}{S_{AOB}}\cdot\frac{S_{AOC}}{S_{BOC}} = 1$$

tenglik kelib chiqadi. Shuni isbotlash kerak edi.

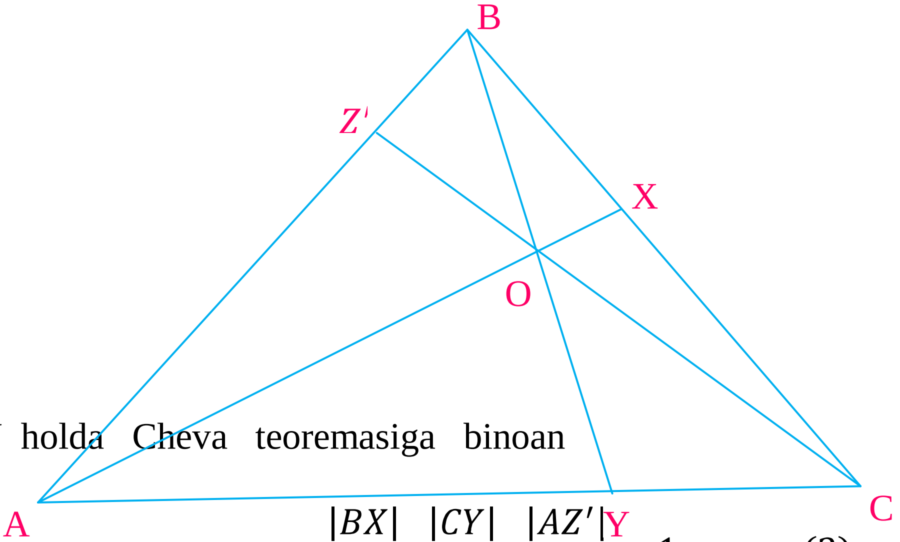
Bu teoremaga teskari teorema ham o'rinlidir.

Teorema.(Cheva teskari teoremasi). Agar ABC ucburchakda AX, BY, CZ chevianalar konkurent bo'lsa, u holda

$$\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ|}{|ZB|}=1 \qquad (2)$$

tenglik bajariladi.

Isbot. Faraz qilamiz, ΔABC da AX va BY chevianalar O nuqtada kesishsin .CO to'g'ri chiziq AB tomonni Z' nuqtada kessin.



U holda Cheva teoremasiga binoan

$$\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ'|}{|Z'B|}=1 \qquad (3)$$

bo'ladi. (2) va (3) dan

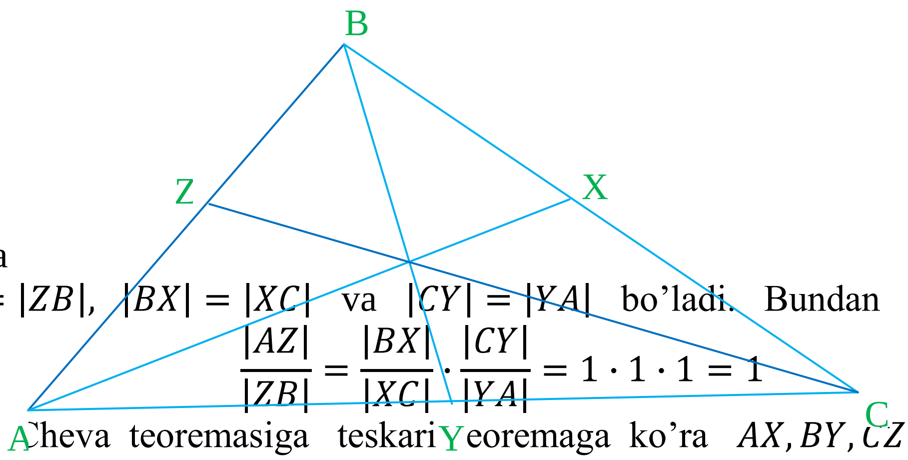
$$\frac{|AZ|}{|ZB|}=\frac{|AZ'|}{|Z'B|}$$

kelib chiqadi. Bu yerdan esa $Z=Z'$ ekanligi ya'ni $O\in CZ$ ekanligi hosil bo'ladi. Teorema isbotlandi. ▲

Cheva teoremasidan quyidagi ajoyib natijalar kelib chiqadi.

Natija. Uchburchak medianalari bir nuqtada kesishadi.

Isbot. ΔABC da AX, BY, CZ lar mediana bo'sin.



U holda

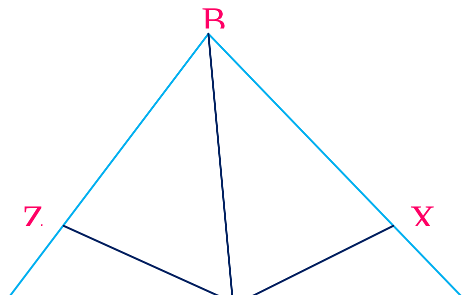
$|AZ|=|ZB|,\ |BX|=|XC|$ va $|CY|=|YA|$ bo'ladi. Bundan

$$\frac{|AZ|}{|ZB|}=\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}=1\cdot1\cdot1=1$$

Demak, Cheva teoremasiga teskari teoremaga ko'ra AX, BY, CZ lar konkurentdir . ▲

Natija 2. Uchburchak bissektrisalari bir nuqtada kesishadi.

Isbot. AX, BY, CZ kesmalar ΔABC ning bissektrisalari bo'lsin.



Bissektrisaning xossasiga ko'ra

$$\frac{|BX|}{|AB|}=\frac{|XC|}{|AC|}$$

U holda

$$\frac{|BX|}{|XC|}=\frac{|AB|}{|AC|}$$

Xuddi shunday

$$\frac{|CY|}{|YA|}=\frac{|BC|}{|AB|} \quad va \quad \frac{|AZ|}{|BZ|}=\frac{|AC|}{|BC|}$$

tengliklarni isbotlash mumkin. Bu yerdan

$$\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ|}{|BZ|}=\frac{|AB|}{|AC|}\cdot\frac{|BC|}{|AB|}\cdot\frac{|AC|}{|BC|}=1$$

kelib chiqadi. Cheva teoremasiga teskari teoremadan ega, AX, BY va CZ larning konkurentligini hosil qilamiz. ▲

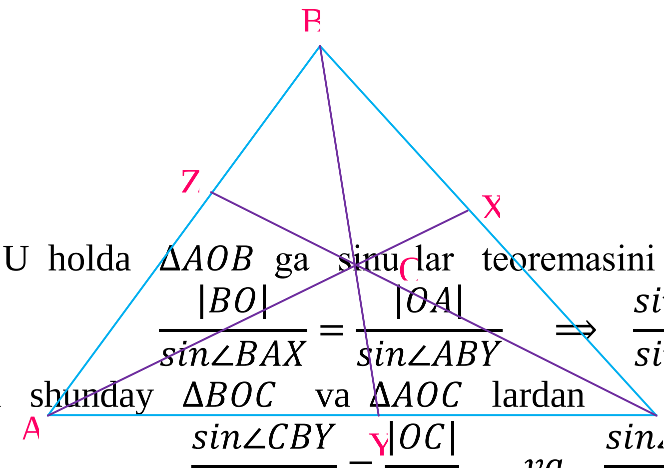
Shuni aytib o'tish joizki Cheva teoremasi nafaqat kesmalar, balki burchaklar bo'yicha ham qo'llanadi. Quyida Cheva teoremasining burchaklar bo'yicha analogi berilgan.

Teorema. ΔABC da AX, BY, CZ chevianalar konkurent bo'lishi uchun

$$\frac{sin\angle BAX}{sin\angle CAX}\cdot\frac{sin\angle CBY}{sin\angle ABY}\cdot\frac{sin\angle ACZ}{sin\angle BCZ}=1 \qquad (5)$$

bo'lishi yetarli va zarur.

Isbot. Zarurligi. Faraz qilamiz, AX, BY va CZ chevianalar O nuqtada kesishsin.



U holda ΔAOB ga sinuslar teoremasini qo'llasak,

$$\frac{|BO|}{sin\angle BAX}=\frac{|OA|}{sin\angle ABY} \Rightarrow \frac{sin\angle BAX}{sin\angle ABY}=\frac{|BO|}{|OA|}$$

Xuddi shunday ΔBOC va ΔAOC lardan

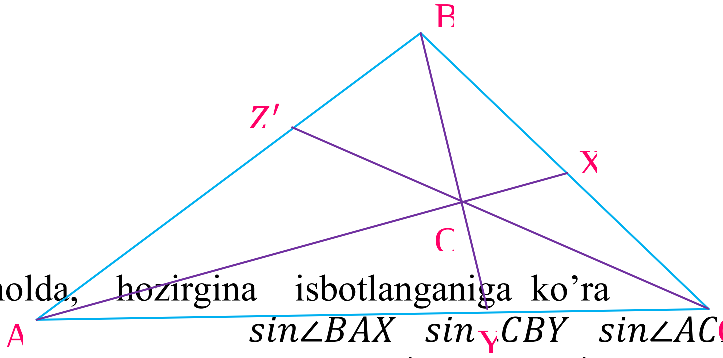
$$\frac{sin\angle CBY}{sin\angle BCZ}=\frac{|OC|}{|BO|} \quad va \quad \frac{sin\angle ACZ}{sin\angle CAX}=\frac{|AO|}{|CO|}$$

munosabatlarni hosil qilamiz. Bularni hadlab ko'paytirib

$$\frac{sin\angle BAX}{sin\angle CAX}\cdot\frac{sin\angle CBY}{sin\angle ABY}\cdot\frac{sin\angle ACZ}{sin\angle BCZ}=\frac{|OA|\cdot|BO|\cdot|OC|}{|OA|\cdot|BO|\cdot|OC|}=1$$

Shu isbotlash kerak edi.

Yetarliligi. Faraz qilamiz, AX va BY lar O nuqtada kesishsin. CO kesma AB ni Z' nuqtada kessin.



U holda, hozirgina isbotlanganiga ko’ra

$$\frac{sin\angle BAX}{sin\angle CAX}\cdot\frac{sin\angle CBY}{sin\angle ABY}\cdot\frac{sin\angle ACZ'}{sin\angle BCZ'}=1$$

bo’lishi kerak. Teorema shartiga ko’ra esa

$$\frac{sin\angle BAX}{sin\angle CAX}\cdot\frac{sin\angle CBY}{sin\angle ABY}\cdot\frac{sin\angle ACZ}{sin\angle BCZ}=1$$

Bu ikki tenglikdan

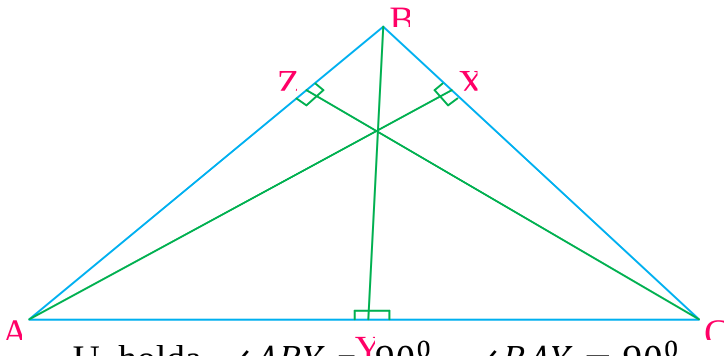
$$\frac{sin\angle ACZ'}{sin\angle BCZ'}=\frac{sin\angle ACZ}{sin\angle BCZ}$$

kelib chiqadi. Oxirgi munosabatdan $\angle ACZ'=\angle ACZ$ va $\angle BCZ'=\angle BCZ$ bo’lgandagika o’rinli. Demak, $Z=Z'$ va AX,BY,CZ konkurent.

Ushbu teoremadan quyidagi natija kelib chiqadi.

Natija. O’tkir burchakli uchburchakning balandliklari konkurentdir.

Isbot. AX,BY va CZ lar ΔABC ning balandliklari bo’lsin.



U holda, $\angle ABY=90^0-\angle BAY=90^0-\angle BAC$ va demak

$$sin\angle ABY=\sin(90^0-\angle BAC)=cos\angle BAC$$

va xuddi shunday

$$sin\angle CBY=\sin(90^0-\angle BCA)=cos\angle BCA.$$

Bu yerdan

$$\frac{sin\angle CBY}{sin\angle ABY}=\frac{cos\angle BCA}{cos\angle BAC}$$

Xuddi shunday

$$\frac{sin\angle BAX}{sin\angle CAX}=\frac{cos\angle ABC}{cos\angle BCA}\qquad va\qquad \frac{sin\angle ACZ}{sin\angle BCZ}=\frac{cos\angle BAC}{cos\angle ABC}$$

ekanligi ko’rsatiladi. Bu tengliklarni hadlab ko’paytirsak,

$$\frac{sin\angle CBY}{sin\angle ABY}\cdot\frac{sin\angle BAX}{sin\angle CAX}\cdot\frac{sin\angle ACZ}{sin\angle BCZ}=\frac{cos\angle BCA}{cos\angle BAC}\cdot\frac{cos\angle ABC}{cos\angle BCA}\cdot\frac{cos\angle BAC}{cos\angle ABC}=1$$

kelib chiqadi. Teorema 3 ga ko'ra esa bu AX, BY, CZ chevianalarning konkurentligini bildiradi. Teorema isbotlandi. ▲

Burchaklar bo'yicha Cheva teoremasi quyidagi umumlashmaga ega.

Teorema(uchburchak uchun umumlashgan Cheva teoremasi). Faraz qilamiz, X, Y, Z nuqtalar ΔABC ning mos ravishda BC, AC, AB tomonlari yotgan to'g'ri chiziqlardan olingan bo'lsin. U holda AX, BY, CZ kesmalar yotgan to'g'ri chiziqlar konkurent bo'lishi uchun yoki

a) $\frac{|BX|}{|XC|} \cdot \frac{|CY|}{|YA|} \cdot \frac{|AZ|}{|ZB|} = 1$

yoki

b) $\frac{sin\angle BAX}{sin\angle CAX} \cdot \frac{sin\angle CBY}{sin\angle ABY} \cdot \frac{sin\angle ACZ}{sin\angle BCZ} = 1$

Munosabatlar bajarilishi yetarli va zarur.

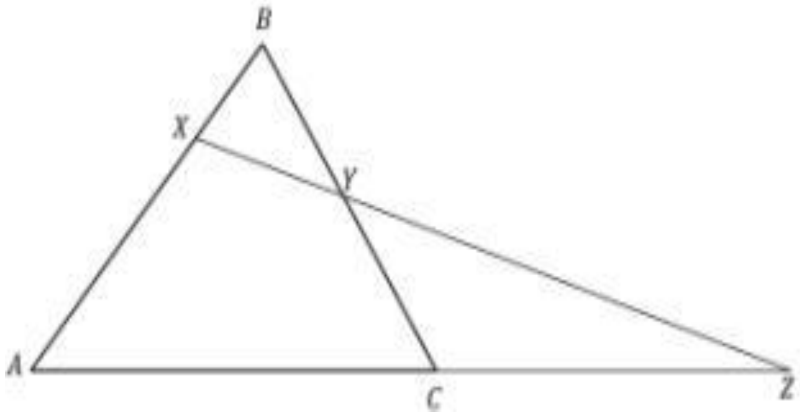
Ushbu teoremaning isboti oldingilaridan uncha katta farq qilgamagani uchun buni isbotlab o'tirmaymiz. Ushbu teoremadan quyidagi natijalar kelib chiqadi.

Natija. Ixtiyoriy uchburchakning balandliklari bir nuqtada kesishadi.

Natija. Faraz qilamiz, ΔABC va $\Delta A_1B_1C_1$ lar teng bo'lmagan uchburchaklar bo'lib, $AB \parallel A_1B_1$, $AC \parallel A_1C_1$ va $BC \parallel B_1C_1$ bo'lsin. U holda AA_1, BB_1 va CC_1 to'g'ri chiziqlar konkurent bo'ladi.

Menelay teoremasi. Agar ΔABC ning tonomlaridan yoki tomonlarini davomidan X,Y,Z nuqtalarni : X nuqta- AB da,Y- BC da, Z- CA da yotsa,u holda bu nuqtalar quyidagi shartni bajarsa ular bir to`gri chiziqda yotadi.

$$\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1. (**)$$



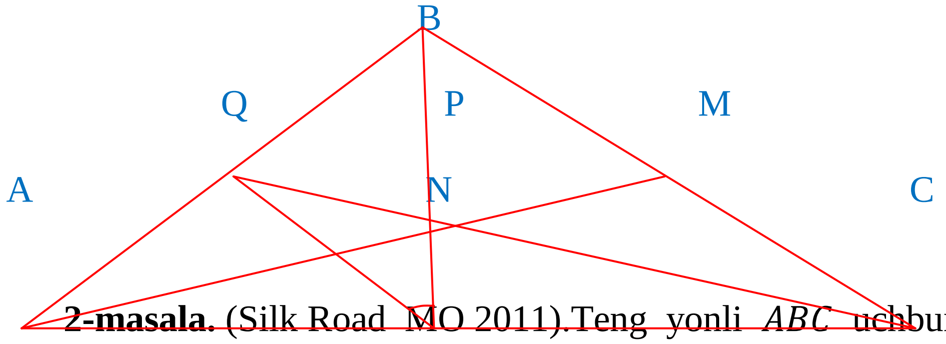
II-Bob. Cheva va Menelay teoremasining qo'llanishiga doir masalalar

1-masala. .(Mathematical Reflections, №2, 2007) ABC uchburchakning AM medianasi bilan BN bissektrisasi P nuqtada kesishadi. CP va AB to'g'ri chiziqlar Q nuqtada kesishadi. BNQ uchburchak teng yonli ekanligini isbotlang.

Isbot: AM, BN va CQ chevianalar uchun Cheva teoremasini qo'llaymiz:

$$\frac{AQ}{BQ}\cdot\frac{BM}{MC}\cdot\frac{CN}{NA}=1 \qquad (1)$$

AM mediana ekanligidan $BM=MC$ tenglik o'rinli demak, $\frac{BM}{MC}=1.$ U holda (1)ga ko'ra $\frac{AQ}{BQ}\cdot\frac{CN}{NA}=1$ bundan $\frac{AQ}{BQ}=\frac{NA}{CN}$ va natijada Fales teoremasiga ko'ra $QN \parallel BC.$ BN kesma QN va BC ni kesayotgani uchun $\Rightarrow \angle NBC=\angle BNQ.$ BN –bissektrisa. $\Rightarrow \angle QBN=\angle NBC. \Rightarrow \angle QBN=\angle QNB=\angle NBC$ ya'ni $\angle QNB=\angle QBN$ u holda BNQ teng yonli. Uchburchak bo'ladi. Shuni isbotlash kerak edi.▲



2-masala. (Silk Road MO 2011).Teng yonli ABC uchburchakda $\angle ACB>90^0$. AC tomonni C nuqtadan davom ettirib, shu davomda $\angle KBC=\angle ABC$ tenglikni qanoatlantiradigan K nuqta olingan. $\angle BKC$ va $\angle ACB$ burchaklar bissektrisalari S nuqtada, AB va KS to'g'ri chiziqlar L nuqtada, BS va CL to'g'ri chiziqlar esa M nuqtada kesishsin. KM to'g'ri chiziq BC tomonning o'rtasidan o'tishini isbotlang.

Isbot: KM to'g'ri chiziq BC ning o'rtasidan o'tishini isbotlash uchun $\frac{BK}{CK}=\frac{sin\angle MKC}{sin\angle MKB}$ ni isbotlasak yetarli. $CAB=\angle ABC=\alpha$ bo'lsin. $\Rightarrow \angle KBC=\alpha.$ $\Rightarrow \angle AKB=\pi-3\alpha \Rightarrow \angle AKL=\angle LKB=90^0-\frac{3\alpha}{2}.$ Bizga ma'lumki uchburchakning bitta ichki va ikkita tashqi bissektrisalari bir nuqtada kesishadi. ΔBKC da SB kesma tashqi bissektrisadir. $\Rightarrow \angle CBS=90^0-\frac{\alpha}{2}\Rightarrow$ $\angle LBS=90^0-\frac{3\alpha}{2}\Rightarrow \angle AKS=\angle ABS\Rightarrow AKBS$ to'rtburchakkka tashqi aylana chizish mumkin. ΔBKC da M nuqta uchun Cheva teoremasini qo'llaymiz:

$$\frac{sin\angle BCM}{sin\angle MCK}\cdot\frac{sin\angle MKC}{sin\angle MKB}\cdot\frac{sin\angle MBK}{sin\angle MBC}=1\Rightarrow$$

$$\Rightarrow\frac{sin\angle MKC}{sin\angle MKB}=\frac{sin\angle MKC}{sin\angle BCM}\cdot\frac{sin\angle MBC}{sin\angle MBK} \qquad (1)$$

$$\frac{sin\angle MCK}{sin\angle BCM}=\frac{\sin(\pi-\angle ACL)}{sin\angle BCL}=\frac{\sin\angle ACL}{sin\angle BCL}=\frac{AL}{BL}=\frac{AK}{BK}=\frac{sin2\alpha}{sin\alpha}\Rightarrow$$

$$\Rightarrow\frac{sin\angle MCK}{sin\angle BCM}=\frac{sin2\alpha}{sin\alpha} \qquad (2)$$

$$\frac{sin\angle MBC}{sin\angle MBK}=\frac{\sin(90^0-\frac{3\alpha}{2}+\alpha)}{\sin(90^0-\frac{3\alpha}{2}+2\alpha)}=\frac{\sin(90^0-\frac{\alpha}{2})}{\sin(90^0+\frac{\alpha}{2})}=\frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=1\Rightarrow$$

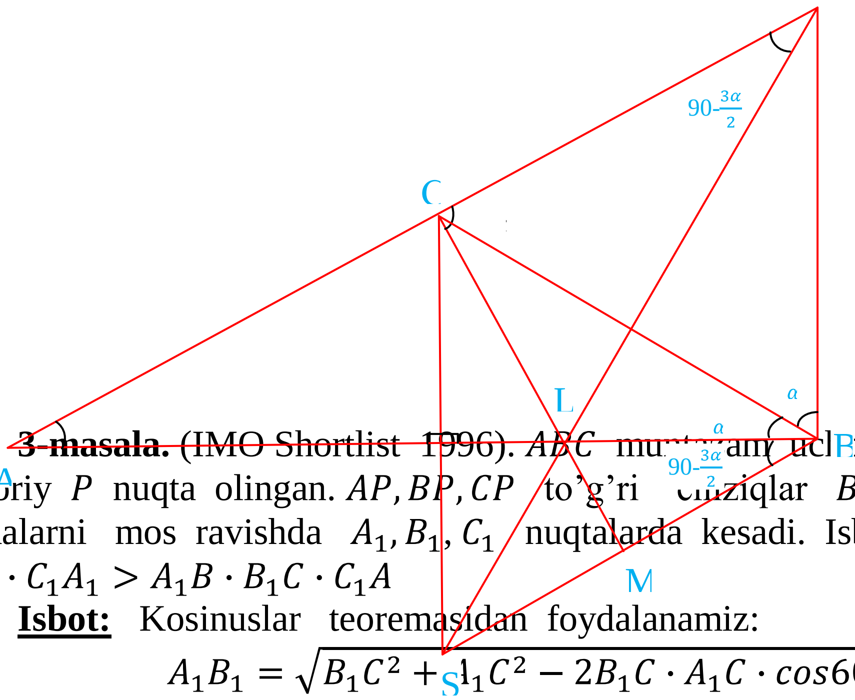
$$\Rightarrow \frac{sin\angle MBC}{sin\angle MBK} = 1 \qquad (3)$$

(2) va (3) larni (1) ga qo'yamiz: $\frac{sin\angle MKC}{sin\angle MKB} = \frac{sin\angle MCK}{sin\angle BCM} \cdot \frac{sin\angle MBC}{sin\angle MBK} = \frac{sin2\alpha}{sin\alpha} \Rightarrow$

$$\Rightarrow \frac{sin\angle MKC}{sin\angle MKB} = \frac{sin2\alpha}{sin\alpha} \qquad (4)$$

Bizga ma'lumki, $\frac{BK}{KC} = \frac{sin2\alpha}{sin\alpha}$ (5).

(4) va (5) ga ko'ra $\frac{BK}{KC} = \frac{sin\angle MKC}{sin\angle MKB} \Rightarrow KM$ to'g'ri chiziq BC kesmaning o'rtasidan o'tadi. $\Rightarrow$ isbotlandi.▲



3-masala. (IMO Shortlist 1996). ABC muntazam uchburchakning ichida ixtiyoriy P nuqta olingan. AP, BP, CP to'g'ri chiziqlar BC, CA, AB kesmalarni mos ravishda A_1, B_1, C_1 nuqtalarda kesadi. Isbotlang: $A_1B_1 \cdot B_1C_1 \cdot C_1A_1 > A_1B \cdot B_1C \cdot C_1A$

<u>Isbot:</u> Kosinuslar teoremasidan foydalanamiz:

$$A_1B_1 = \sqrt{B_1C^2 + A_1C^2 - 2B_1C \cdot A_1C \cdot cos60^0} =$$

$$= \sqrt{B_1C^2 + A_1C^2 - 2B_1C \cdot A_1C \cdot \frac{1}{2}} =$$

$$= \sqrt{B_1C^2 + A_1C^2 - B_1C \cdot A_1C} \Rightarrow$$

$$\Rightarrow A_1B_1 = \sqrt{B_1C^2 + A_1C^2 - B_1C \cdot A_1C}.$$

Bizga ma'lumki, $(B_1C - A_1C)^2 \geq 0 \Rightarrow$

$$\Rightarrow B_1C^2 - 2B_1C \cdot A_1C + A_1C^2 \geq 0 \Rightarrow$$

$$\Rightarrow B_1C^2 - B_1C \cdot A_1C + A_1C^2 \geq B_1C \cdot A_1C \Rightarrow$$

$$\Rightarrow A_1B_1 = \sqrt{B_1C^2 - B_1C \cdot A_1C + A_1C^2} \geq \sqrt{B_1C \cdot A_1C} \Rightarrow$$

$$\Rightarrow A_1B_1 \geq \sqrt{B_1C \cdot A_1C} \qquad (1)$$

Xuddi shunday yo'l bilan quyidagi tengsizliklarni hosil qilamiz:

$$B_1C_1 \geq \sqrt{AC_1 \cdot AB_1} \qquad (2)$$

$$C_1A_1 \geq \sqrt{BC_1 \cdot A_1B} \qquad (3)$$

(1), (2), (3) tengsizliklarni ko'paytirib yuborsak,

$$A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq \sqrt{B_1C \cdot A_1C \cdot AC_1 \cdot AB_1 \cdot BC_1 \cdot A_1B} \qquad (4)$$

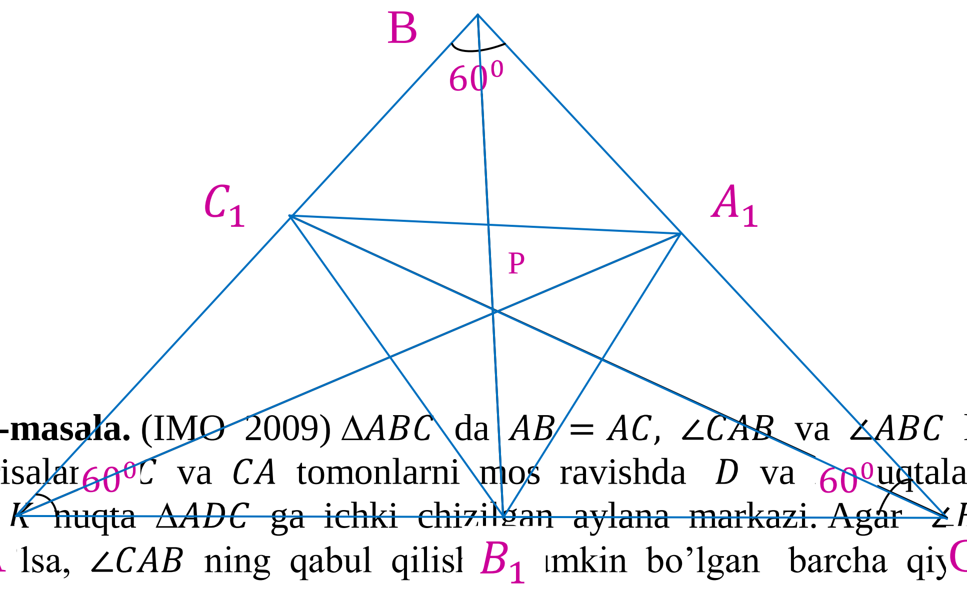
ga ega bo'lamiz.

Cheva teoremasiga ko'ra $A_1B \cdot BC_1 \cdot AC_1 = AB_1 \cdot BC_1 \cdot A_1C \Rightarrow$

$\Rightarrow$ (4) ga ko'ra: $A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq \sqrt{B_1C \cdot A_1C \cdot AC_1 \cdot AB_1 \cdot BC_1 \cdot A_1B} =$

$$= \sqrt{(B_1C \cdot A_1B \cdot AC_1)^2} = B_1C \cdot A_1B \cdot AC_1 \Rightarrow$$

$A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq B_1C \cdot A_1B \cdot AC_1$ shuni isbotlash kerak edi.▲



4-masala. (IMO 2009) ΔABC da $AB = AC$, $\angle CAB$ va $\angle ABC$ larning bissektrisalari BC va CA tomonlarni mos ravishda D va E nuqtalarda kesadi. K nuqta ΔADC ga ichki chizilgan aylana markazi. Agar $\angle BEK = 45^0$ bo'lsa, $\angle CAB$ ning qabul qilishi mumkin bo'lgan barcha qiymatlarini toping.

Yechish: $\angle BAC = \alpha$ bo'lsin. $\Rightarrow \angle BAD = \frac{\alpha}{2} \Rightarrow \angle ABC = \angle ACB = 90^0 - \frac{\alpha}{2}$

$$\Rightarrow \angle ABE = \angle EBC = \angle ACK = \angle BCK = 45^0 - \frac{\alpha}{4} \Rightarrow$$

$$\angle AEB = \angle CBE + \angle ECB = 45^0 - \frac{\alpha}{4} + 90^0 - \frac{\alpha}{2} = 135^0 - \frac{3\alpha}{4} \Rightarrow$$

$\Rightarrow \angle KEC = \frac{3\alpha}{4}$, chunki shartga ko'ra $\angle BEK = 45^0$ $AD \cap BE = I$ bo'lsin. K va I nuqtalarning ikkalasi ham $\angle ACB$ ning bissektrisasida yotadi. $\Rightarrow$ $C, K. I$ $-$kollinear. $\Rightarrow \angle CIE = 180^0 - 45^0 - \frac{3\alpha}{4} - 45^0 + \frac{\alpha}{4} = 90^0 - \frac{\alpha}{2} \Rightarrow$

$$\Rightarrow \angle CIE = 90^0 - \frac{\alpha}{2} \Rightarrow \angle CID = 45^0 + \frac{\alpha}{4}.$$

$DIEC$ to'rtburchakda K nuqta uchun Cheva teoremasidan foydalanamiz:

$$\frac{sin45^0}{sin45^0} \cdot \frac{\sin(45^0 + \frac{\alpha}{4})}{cos\frac{\alpha}{2}} \cdot \frac{sin45^0}{sin\frac{3\alpha}{4}} \cdot \frac{\sin(45^0 - \frac{\alpha}{4})}{\sin(45^0 - \frac{\alpha}{4})} = 1$$

Bundan esa

$$\frac{\sin(45^0+\frac{\alpha}{4})}{cos\frac{\alpha}{2}} \cdot \frac{sin45^0}{sin\frac{3\alpha}{4}} = 1 \text{ va } \quad \sin\left(45^0 + \frac{\alpha}{4}\right) \cdot sin45^0 = sin\frac{3\alpha}{4} \cdot cos\frac{\alpha}{2}$$

Endi sinuslar ko'paytmasini yig'indisiga keltirsak,

$$\frac{1}{2}\left(cos\frac{\alpha}{4} - cos\left(90^0 + \frac{\alpha}{4}\right)\right) = \frac{1}{2}\left(sin\frac{5\alpha}{4} + sin\frac{\alpha}{4}\right)$$

Ni hosil qilamiz. Bundan

$$cos\frac{\alpha}{4}+sin\frac{\alpha}{4}=sin\frac{5\alpha}{4}+sin\frac{\alpha}{4}\Rightarrow cos\frac{\alpha}{4}=sin\frac{5\alpha}{4}$$

Kelib chiqadi. Buni α ga nisbatan tenglama deb qarasak

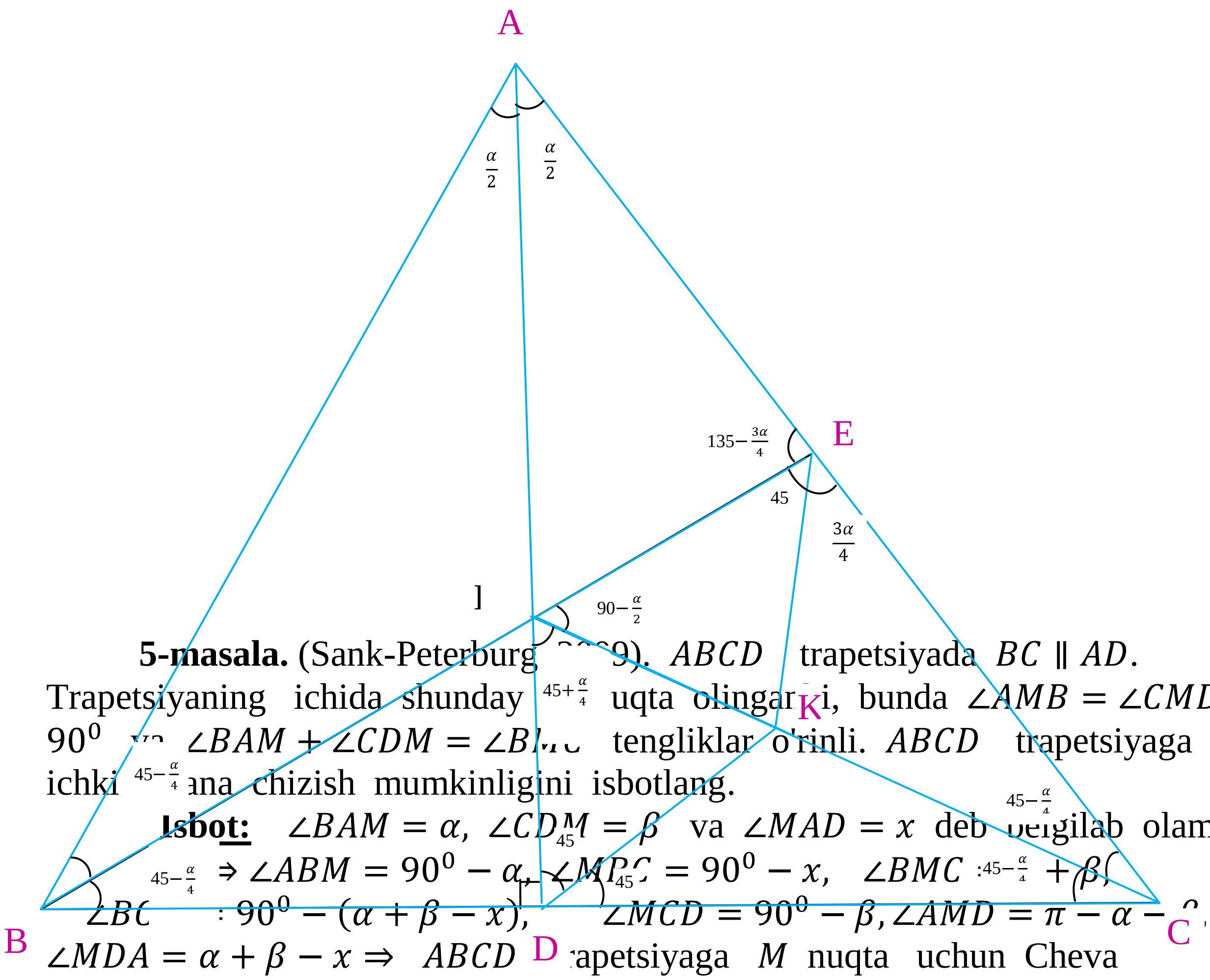
$$sin\left(90^0-\frac{\alpha}{4}\right)=sin\frac{5\alpha}{4}.$$

U holda quyidagi ikkita hol bo'lishi mumkin:

1) $\frac{5\alpha}{4}=90^0-\frac{\alpha}{4}\Rightarrow\frac{6\alpha}{4}=90^0\ \Rightarrow\frac{3\alpha}{2}=90^0\Rightarrow\ \alpha=60^0$

2) $\frac{5\alpha}{4}+90^0-\frac{\alpha}{4}=180^0\ \ \Rightarrow\ \alpha=90^0$

Bu qiymatlarning ikkalasi ham masala shartini qanoatlantiradi.
Javob: 60^0 va 90^0 ▲



5-masala. (Sank-Peterburg [illegible]9). $ABCD$ trapetsiyada $BC \parallel AD$. Trapetsiyaning ichida shunday [illegible] uqta olingan[illegible]i, bunda $\angle AMB=\angle CMD=90^0$ [illegible] $\angle BAM+\angle CDM=\angle B$[illegible] tengliklar o'rinli. $ABCD$ trapetsiyaga ichki [illegible]ana chizish mumkinligini isbotlang.

<u>Isbot:</u> $\angle BAM=\alpha,\ \angle CDM=\beta$ va $\angle MAD=x$ deb belgilab olamiz. [illegible] $\Rightarrow\angle ABM=90^0-\alpha,\ \angle MDC=90^0-x,\ \ \angle BMC$ [illegible] $+\beta,$ $\angle BC$[illegible] $=90^0-(\alpha+\beta-x),\ \ \angle MCD=90^0-\beta,\angle AMD=\pi-\alpha-$[illegible] $\angle MDA=\alpha+\beta-x\Rightarrow\ \ ABCD$ [illegible]apetsiyaga M nuqta uchun Cheva teoremasini qo'llaymiz:

$$\frac{sin\alpha}{sinx}\cdot\frac{\sin(90^0-\alpha)}{\sin(90^0-x)}\cdot\frac{sin\big(90^0-(\alpha+\beta-x)\big)}{\sin(90^0-\beta)}\cdot\frac{\sin\ \beta}{\sin(\alpha+\beta-x)}=1$$

Bu yerda quyidagi ekvivalent munosabatlar zanjiri kelib chiqadi.

$$\Leftrightarrow\frac{sin\alpha}{sinx}\cdot\frac{\cos\alpha}{\cos x}\cdot\frac{\cos(\alpha+\beta-x)}{\cos\beta}\cdot\frac{\sin\ \beta}{\sin(\alpha+\beta-x)}=1\Leftrightarrow$$

$$\Leftrightarrow\frac{\sin(\alpha+\beta-x)}{\sin\ \beta}\cdot\frac{\cos\beta}{\cos(\alpha+\beta-x)}\cdot\frac{\cos x}{\cos\alpha}\cdot\frac{sin\alpha}{sinx}=1\Leftrightarrow$$

$$\Leftrightarrow \frac{\sin\,(\alpha+\beta-x)}{\cos\,(\alpha+\beta-x)}\cdot\frac{\cos x}{sinx}=\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\sin\,\beta}{\cos\,\beta}\Leftrightarrow$$

$$\Leftrightarrow \frac{\sin\,(\alpha+\beta-x)\cos x}{\cos\,(\alpha+\beta-x)\,sinx}=\frac{\cos\alpha\sin\,\beta}{\sin\alpha\cos\,\beta}\Leftrightarrow$$

$$\frac{\frac{1}{2}(\sin(\alpha+\beta)+\sin\,(\alpha+\beta-2x))}{\frac{1}{2}(\sin(\alpha+\beta)-\sin\,(\alpha+\beta-2x))}=\frac{\cos\alpha\sin\,\beta}{\sin\alpha\cos\,\beta}\Leftrightarrow$$

$$\Leftrightarrow \frac{\sin(\alpha+\beta)+\sin\,(\alpha+\beta-2x)}{\sin(\alpha+\beta)-\sin\,(\alpha+\beta-2x)}=\frac{\frac{1}{2}(\sin(\alpha+\beta)-\sin\,(\alpha-\beta))}{\frac{1}{2}(\sin(\alpha+\beta)+\sin\,(\alpha-\beta))}\Leftrightarrow$$

$$\Leftrightarrow \frac{\sin(\alpha+\beta)+\sin\,(\alpha+\beta-2x)}{\sin(\alpha+\beta)-\sin\,(\alpha+\beta-2x)}=\frac{(\sin(\alpha+\beta)-\sin\,(\alpha-\beta))}{(\sin(\alpha+\beta)+\sin\,(\alpha-\beta))}\Rightarrow$$

$$\Leftrightarrow sin^2(\alpha+\beta)+\sin(\alpha-\beta)sin(\alpha+\beta)+sin(\alpha-\beta)\sin(\alpha+\beta-2x)+$$
$$+\sin(\alpha-\beta)sin(\alpha+\beta-2x)=sin^2(\alpha+\beta)-\sin(\alpha-\beta)sin(\alpha+\beta)-$$
$$-sin(\alpha+\beta)\sin(\alpha+\beta-2x)+sin(\alpha-\beta)\sin(\alpha+\beta-2x)\Leftrightarrow$$
$$\Leftrightarrow 2\sin(\alpha-\beta)sin(\alpha+\beta)+2sin(\alpha+\beta)\sin(\alpha+\beta-2x)=0\Leftrightarrow$$
$$\Leftrightarrow 2sin(\alpha+\beta)[(\sin(\alpha-\beta)+\sin(\alpha+\beta-2x)]=0$$

$\alpha+\beta>0$ va $\alpha+\beta<\pi$ ekanligidan $\sin(\alpha+\beta)\neq 0$. U holda

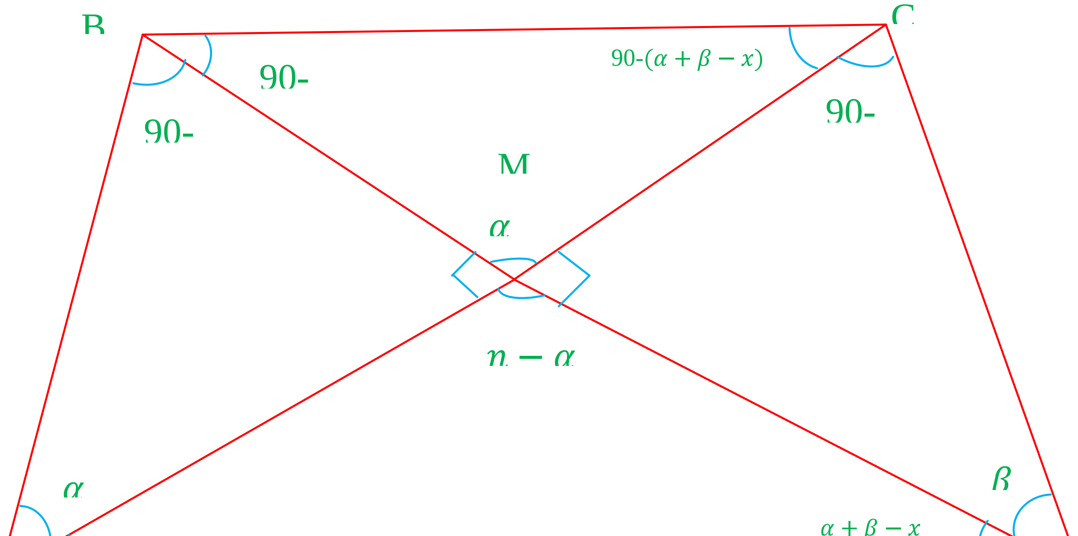
$$\sin(\alpha-\beta)+\sin(\alpha+\beta-2x)=0$$

va bu yerdan

$$2sin\frac{\alpha-\beta+\alpha+\beta-2x}{2}cos\frac{\alpha-\beta-\alpha-\beta+2x}{2}=0$$

ya’ni

$2\sin(\alpha-x)\cos(x-\beta)=0.$ U holda $sin(\alpha-x)=0$ yoki $cos(x-\beta)=0.$
$cos(x-\beta)=0$ bo’lsa, $x=\beta+\frac{\pi}{2}$ bo’ladi. Bu yerdan $x>\frac{\pi}{2}$. Ammo $x<90^0\ \Rightarrow$
ziddiyat. Demak, $cos(x-\beta)\neq 0$ ekan. Agar $sin(\alpha-x)=0.$ bo’lsa $\alpha-x=0$ yoki $\alpha-x=\pi.$ $\alpha<\pi\ \Rightarrow\alpha-x\neq\pi\Rightarrow\ \alpha-x=0$ bo’ladi.
Demak, $\alpha=x\Rightarrow$
AM,BM,CM,DM kesmalar mos ravishda $\angle BAD,\angle ABC,\angle BCD,\angle CDA$ larning bissektrisalari. To’rtburchakning bissektrisalari bir nuqtada, ya’ni M nuqtada kesishyapti $\Rightarrow$ unga ichki aylana chizish mumkin. ▲



6-masala. (Sank-Peterburg 2008). Aylanaga ichki chizilgan $ABCD$ to'rtburchak berilgan. AB va DC nurlar E nuqtada, AC va BD kesmalar esa F nuqtada kesishadi. EF nurda shunday P nuqta olinganki, bunda $\angle BPE = \angle CPE$ tenglik o'rinli. Isbotlang: $\angle APB = \angle DPC$.

<u>Isbot:</u> Shartga ko'ra $\angle BPE = \angle CPE \Rightarrow \angle APB = \angle DPC$ ni isbotlash uchun $\angle APF = \angle DPF$ ni isbotlash yetarli. $\angle ADB = \alpha,\ \angle CAD = \beta,\ \angle BDP = x$ $\angle CAP = \varphi,\ \angle APF = y,\ \angle DPF = z,\ \angle BAP = n$ deb belgilashlar olamiz.

$\Rightarrow y = z$ ekanligini isbotlaymiz. $ABCD$ siklik $\Rightarrow \angle PDC = n + u - x$,

$\angle DBC = \beta,\ \angle BCA = \alpha$.

$$\frac{AB}{CD} = \frac{AB}{AD}\cdot\frac{AD}{CD} = \frac{sin\alpha}{sin\angle ABD}\cdot\frac{sin\angle ACD}{sin\beta} = \frac{sin\alpha}{sin\beta} \Rightarrow \frac{AB}{CD} = \frac{sin\alpha}{sin\beta} \qquad (1)$$

$$\frac{AP}{AE} = \frac{sin\angle BEP}{siny},\quad \frac{PD}{ED} = \frac{sin\angle CEP}{sinz} \Rightarrow \frac{AP}{PD}\cdot\frac{ED}{AE} = \frac{sin\angle BEP}{sin\angle CEP}\cdot\frac{sinz}{siny}$$

$\Delta AED \backsim \Delta BEC \ \Rightarrow \frac{ED}{AE} = \frac{BE}{CE} \Rightarrow \frac{AP}{PD}\cdot\frac{BE}{CE} = \frac{sin\angle BEP}{sin\angle CEP}\cdot\frac{sinz}{siny} \qquad (2)$

ΔAED da F nuqta uchun Cheva teoremasini qo'llaymiz:

$$\frac{sin\beta}{\sin(n+\varphi)}\cdot\frac{sin\angle BEP}{sin\angle CEP}\cdot\frac{\sin(n+\varphi)}{sin\alpha} = 1 \Rightarrow \frac{sin\angle BEP}{sin\angle CEP} = \frac{sin\alpha}{sin\beta}$$

buni (2) ga qo'yamiz: $\frac{AP}{PD}\cdot\frac{BE}{CE} = \frac{sin\alpha}{sin\beta}\cdot\frac{sinz}{siny}$ (1) ga ko'ra $\frac{CD}{AB} = \frac{sin\beta}{sin\alpha} \Rightarrow$

$$\Rightarrow \frac{AP}{PD}\cdot\frac{BE}{CE}\cdot\frac{CD}{AB} = \frac{sin\alpha}{sin\beta}\cdot\frac{sinz}{siny}\cdot\frac{sin\beta}{sin\alpha} = \frac{sinz}{siny} \qquad \Rightarrow \frac{AP}{PD}\cdot\frac{BE}{CE}\cdot\frac{CD}{AB} = \frac{sinz}{siny} \quad (3)$$

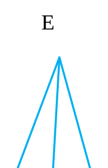
$$\frac{AP}{PD} = \frac{S_{\Delta AEF}}{S_{\Delta AFD}} = \frac{\frac{1}{2}AE\cdot EF\cdot sin\angle BEP}{\frac{1}{2}ED\cdot EF\cdot sin\angle CEP} = \frac{AE}{ED}\cdot\frac{sin\angle BEP}{sin\angle CEP} = \frac{EC}{EB}\cdot\frac{sin\alpha}{sin\beta} \Rightarrow$$

$\Rightarrow \frac{AP}{PD} = \frac{EC}{EB}\cdot\frac{sin\alpha}{sin\beta},\ \frac{CD}{AB} = \frac{sin\beta}{sin\alpha}$ ni (3) ga qo'yamiz:

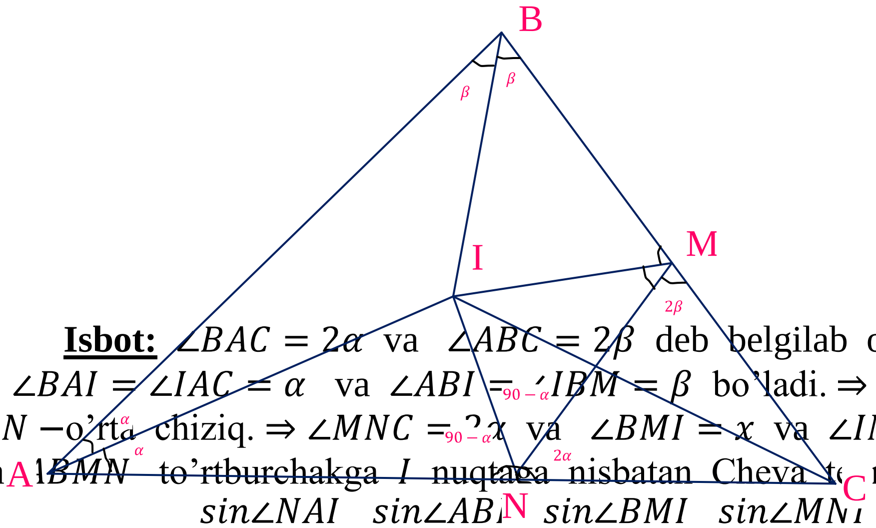
$$\frac{AP}{PD}\cdot\frac{BE}{CE}\cdot\frac{CD}{AB} = \frac{sinz}{siny} \Leftrightarrow \frac{EC}{EB}\cdot\frac{sin\alpha}{sin\beta}\cdot\frac{BE}{CE}\cdot\frac{sin\beta}{sin\alpha} = \frac{sinz}{siny} \Rightarrow \frac{sinz}{siny} = 1 \Rightarrow$$

$\Rightarrow siny = sinz \Rightarrow y = z$ yoki $y + z = \pi$. $y + z = \pi$ bo'laolmaydi. $\Rightarrow y = z \Rightarrow$

$\Rightarrow \angle APB = \angle CPD. \Rightarrow$ isbotlandi.▲



7-masala. ABC uchburchakda I nuqta ichki chizilgan aylana markazi. M va N nuqtalar mos ravishda BC va AC tomonlarning o'rtalari. Agar $\angle AIN = 90^0$ bo'lsa, isbotlang: $\angle BIM = 90^0$.



<u>**Isbot:**</u> $\angle BAC = 2\alpha$ va $\angle ABC = 2\beta$ deb belgilab olamiz.
$\Rightarrow \angle BAI = \angle IAC = \alpha$ va $\angle ABI = \angle IBM = \beta$ bo'ladi. $\Rightarrow \angle ANI = 90^0 - \alpha$, MN −o'rta chiziq. $\Rightarrow \angle MNC = 2\alpha$ va $\angle BMI = x$ va $\angle IMN = y$ deb olamiz va ΔBMN to'rtburchakga I nuqtaga nisbatan Cheva teoremasini qo'llaymiz:

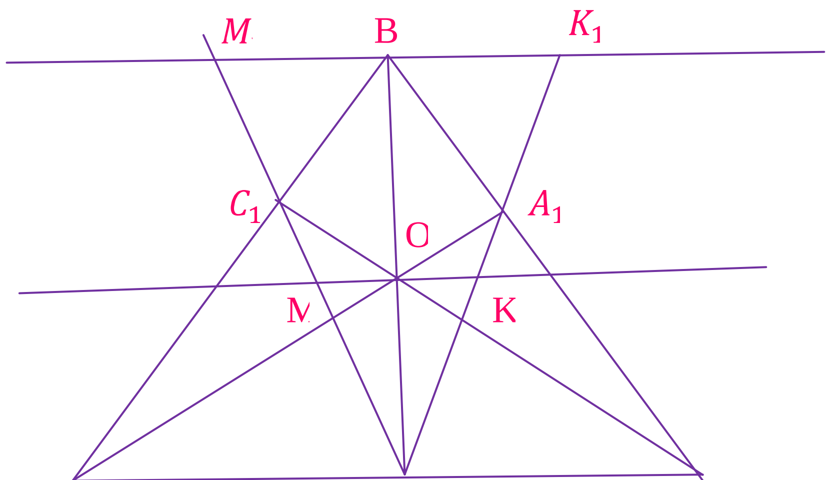
$$\frac{sin\angle NAI}{sin\angle BAI} \cdot \frac{sin\angle ABI}{sin\angle IBM} \cdot \frac{sin\angle BMI}{sin\angle IMN} \cdot \frac{sin\angle MNI}{sin\angle INA} = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{sin\alpha}{sin\alpha} \cdot \frac{sin\beta}{sin\beta} \cdot \frac{sinx}{siny} \cdot \frac{\sin(90^0-\alpha)}{\sin(90^0-\alpha)} = 1 \Rightarrow$$

$$\Rightarrow \frac{sinx}{siny} = 1 \quad \Rightarrow sinx = siny \ \Rightarrow x = y \quad \text{yoki} \quad x + y = \pi.$$

Ammo, $y + x + 2\beta = \pi$ ekanligidan $x + y < \pi$ bo'ladi. $\Rightarrow x = y \Rightarrow$
$\Rightarrow y + x + 2\beta = 2x + 2\beta = \pi \ \Rightarrow x + \beta = 90^0 \Rightarrow \angle BIM = 180^0 - (x + \beta) =$
$= 180^0 - 90^0 = 90^0 \Rightarrow \angle BIM = 90^0 \Rightarrow$ isbotlandi.▲

8-masala. ABC uchburchakning AB, BC, CA tomonlaridan mos ravishda C_1, A_1, B_1 nuqtalar shunday olinganki, bunda AA_1, BB_1, CC_1 to'g'ri chiziqlar O nuqtada kesishadi. O nuqtadan AC to'gri chiziqqa parallel qilib o'tkazilgan to'g'ri chiziq A_1B_1 va B_1C_1 larni mos ravishda K va M nuqtalarda kesib o'tadi. $OK = OM$ tenglik o'rinli kanligini ko'rsating.



Isbot: B nuqtadan AC ga parallel to'g'ri chiziqlar o'tkazamiz. B_1A_1 va B_1C_1 nurlar bu to'g'ri chiziqni mos ravishda K_1 va M_1 nuqtalarda kesib o'tsin. $OK = OM$ ni isbotlash uchun $BK_1 = BM_1$ tenglikni isbotlasak yetarli.

$$\Delta BM_1C_1 \backsim \Delta AB_1C_1 \Rightarrow \frac{AC_1}{C_1B} = \frac{AB_1}{BM_1} \qquad (1)$$

$$\Delta BA_1K_1 \backsim \Delta B_1A_1C \Rightarrow \frac{BA_1}{A_1C} = \frac{BK_1}{B_1C} \qquad (2)$$

(1) va (2) tengsizliklarni ko'paytirib yuboramiz:

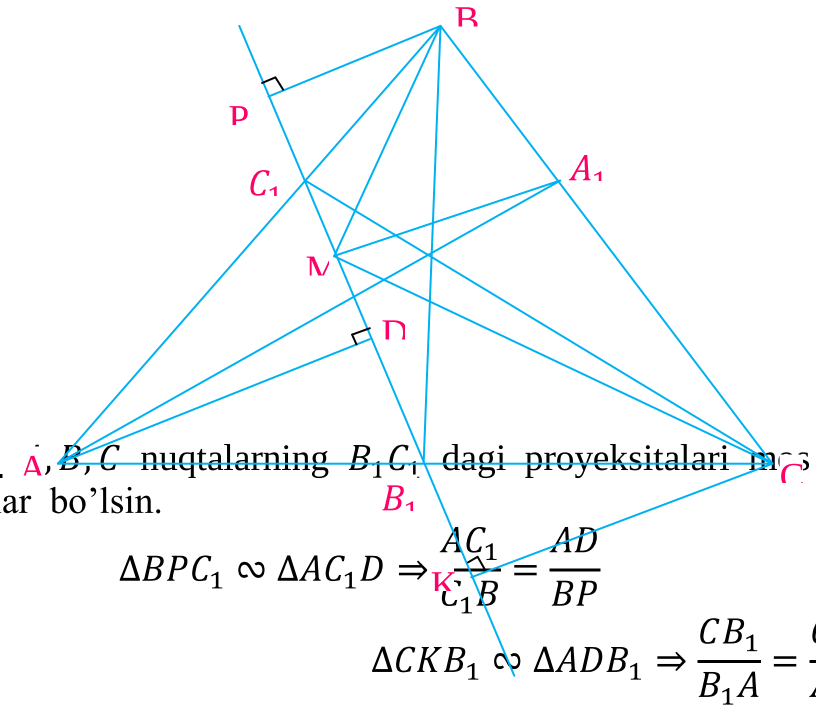
$$\frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} = \frac{AB_1}{BM_1} \cdot \frac{BK_1}{B_1C} \Rightarrow \frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} \cdot \frac{B_1C}{AB_1} = \frac{BK_1}{BM_1} \qquad (*)$$

ΔABC da O nuqta uchun Cheva teoremasiga ko'ra:

$\frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} = 1$ $(*)$ da ko'ra

$$\frac{BK_1}{BM_1} = \frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} = 1 \Rightarrow \frac{BK_1}{BM_1} = 1 \Rightarrow BK_1 = BM_1 \Rightarrow \text{isbotlandi.} \blacktriangle$$

9-masala. ABC uchburchakning AB, BC, CA tomonlaridan mos ravishda C_1, A_1, B_1 nuqtalar shunday olinganki, bunda AA_1, BB_1, CC_1 kesmalar bir nuqtada kesishadi. A_1 ning B_1C_1 dagi proyeksiyasi M nuqta bo'lsa, isbotlang: MA_1 kesma $\angle BMC$ ning bissektrisasi bo'ladi.



Isbot: A, B, C nuqtalarning B_1C_1 dagi proyeksitalari mos ravishda D, P, K nuqtalar bo'lsin.

$$\Delta BPC_1 \backsim \Delta AC_1D \Rightarrow \frac{AC_1}{C_1B} = \frac{AD}{BP} \qquad (1)$$

$$\Delta CKB_1 \backsim \Delta ADB_1 \Rightarrow \frac{CB_1}{B_1A} = \frac{CK}{AD} \qquad (2)$$

(1) va (2) ni bir-biriga ko'paytirib yuboramiz:

$$\frac{AC_1}{C_1B} \cdot \frac{CB_1}{B_1A} = \frac{AD}{BP} \cdot \frac{CK}{AD} = \frac{CK}{BP} \Rightarrow \qquad \frac{CK}{BP} = \frac{AC_1}{C_1B} \cdot \frac{CB_1}{B_1A} \qquad (3)$$

ABC uchburchak uchun Cheva teoremasini qo'llaymiz:

$$\frac{AC_1}{C_1B}\cdot\frac{BA_1}{A_1C}\cdot\frac{CB_1}{B_1A}=1\Rightarrow\frac{AC_1}{C_1B}\cdot\frac{CB_1}{B_1A}=\frac{A_1C}{BA_1}\qquad(4)$$

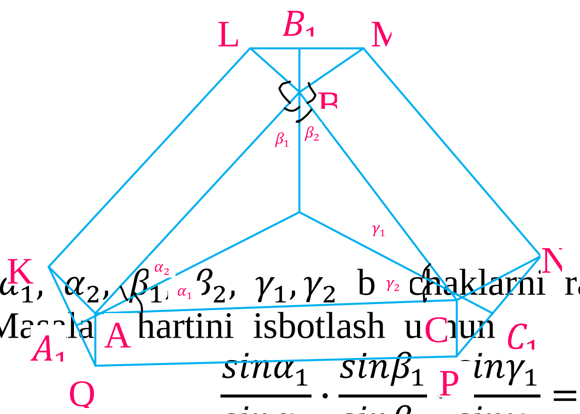
(3) va (4) ga ko'ra: $\frac{CK}{BP}=\frac{A_1C}{BA_1}$ $\quad(*)$

Bizga ma'lumki, $\frac{A_1C}{BA_1}=\frac{KM}{MP}$ tenglik o'rinli. $\Rightarrow (*)$ ga ko'ra

$\frac{CK}{BP}=\frac{KM}{MP}$ bo'ladi. $\Rightarrow \Delta BPM \backsim \Delta CKM \Rightarrow \angle BMP=\angle CMK \Rightarrow$

$\Rightarrow 90^0-\angle BMP=90^0-\angle CMK \Rightarrow \angle BMA_1=\angle CMA_1 \Rightarrow$ isbotlandi.▲

10-masala. ABC uchburchakning tomonlariga tashqi ravishda $ABLK, BCNM, CAQP$ to'g'ri to'rtburchaklar yasalgan. A, B va C nuqtalardan mos ravishda KQ, LM va NP to'g'ri chiziqlar o'tkazilgan perpendikulyarlar bir nuqtada kesishini isbotlang.



Isbot: $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ burchaklarni rasmdagidek qilib belgilaymiz. Masala shartini isbotlash uchun

$$\frac{sin\alpha_1}{sin\alpha_2}\cdot\frac{sin\beta_1}{sin\beta_2}\cdot\frac{sin\gamma_1}{sin\gamma_2}=1$$

ni isbotlasak yetarli. (Cheva teoremasiga ko'ra) B ning ML dagi proyeksiyasi B_1 , A ning KQ dagi proyeksiyasi A_1, C ning NQ dagi proyeksiyasi C_1 nuqta bo'lsin $\angle KAB=90^0\Rightarrow$

$\Rightarrow \angle KAA_1=90^0-\alpha_2\Rightarrow\angle AKA_1=\alpha_2.\quad \angle QAC=90^0\Rightarrow\angle QAA_1=90^0-\alpha_1\Rightarrow$

$$\Rightarrow\angle AQA_1=\alpha_1.\quad\Rightarrow\frac{AK}{AQ}=\frac{sin\alpha_1}{sin\alpha_2}\qquad(1)$$

Xuddi shunday yo'l bilan $\frac{MB}{BL}=\frac{sin\beta_1}{sin\beta_2}$ (2) va $\frac{CP}{CN}=\frac{sin\gamma_1}{sin\gamma_2}$ (3) larni isbotlash mumkin.

(1), (2) va (3) larni ko'paytirib yuboramiz:

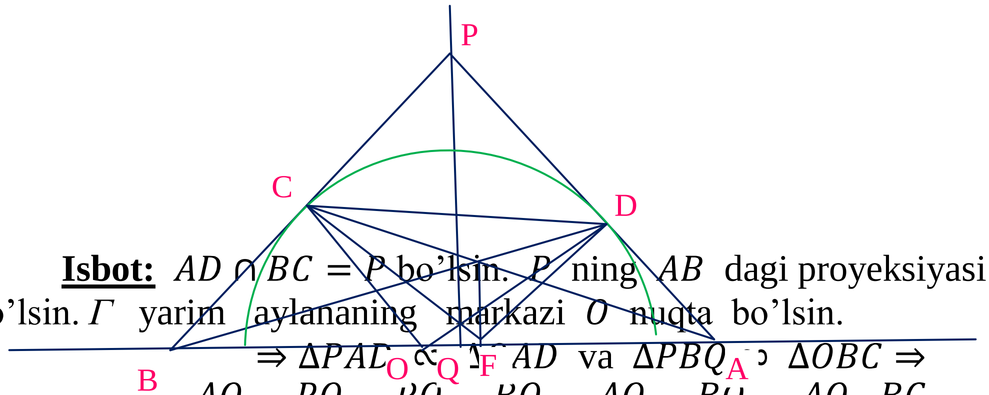
$$\frac{sin\alpha_1}{sin\alpha_2}\cdot\frac{sin\beta_1}{sin\beta_2}\cdot\frac{sin\gamma_1}{sin\gamma_2}=\frac{AK}{AQ}\cdot\frac{MB}{BL}\cdot\frac{CP}{CN}\qquad(4)$$

$AK=BL,\qquad BM=CN,$

$$CP=AQ\Rightarrow\frac{AK}{BL}\cdot\frac{BM}{CN}\cdot\frac{CP}{AQ}=\frac{AK}{AQ}\cdot\frac{MB}{BL}\cdot\frac{CP}{CN}=1\quad\Rightarrow\quad(4)\ \ ga$$

ko'ra $\frac{sin\alpha_1}{sin\alpha_2}\cdot\frac{sin\beta_1}{sin\beta_2}\cdot\frac{sin\gamma_1}{sin\gamma_2}=1\Rightarrow$ isbotlandi.▲

11-masala. (IMO Short List 1994). l to'g'ri chiziqning bir tomoniga Γ yarim aylana chizilgan. C va D nuqtalar Γ yarim aylanada yotadi. C va D nuqtalardan Γ yarim aylanaga o'tkazilgan urinmalar l to'g'ri chiziqni mos ravishda B va A nuqtalarda kesib o'tadi. $AC\cap BD=E$. l to'g'ri chiziqdan shunday F nuqta olinganki, bunda EF to'g'ri chiziq $\angle CFD$ ning bissektrisasini o'z ichiga olishini isbotlang.



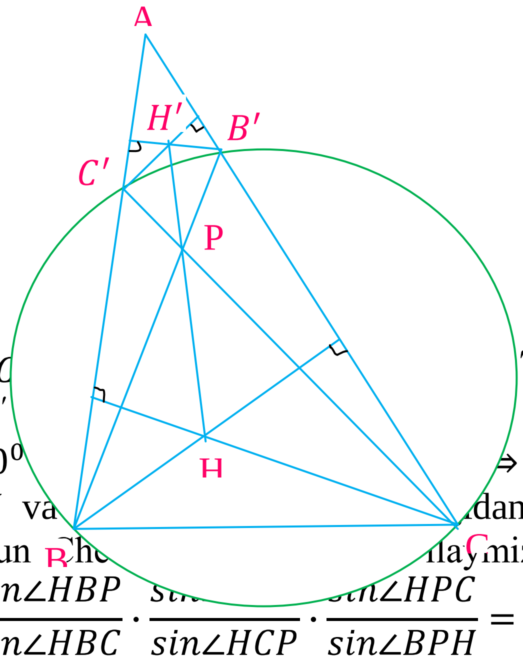
Isbot: $AD \cap BC = P$ bo'lsin. P ning AB dagi proyeksiyasi Q nuqta bo'lsin. Γ yarim aylananing markazi O nuqta bo'lsin.

$$\Rightarrow \Delta PAQ \sim \Delta OAD \text{ va } \Delta PBQ \sim \Delta OBC \Rightarrow$$

$$\Rightarrow \frac{AQ}{AD} = \frac{PQ}{OD} = \frac{PQ}{OC} = \frac{BQ}{BC} \Rightarrow \frac{AQ}{AD} = \frac{BQ}{BC} \Rightarrow \frac{AQ}{BQ} \cdot \frac{BC}{AD} = 1.$$

$PC = PD$ ekanligidan $\frac{AQ}{BQ} \cdot \frac{BC}{PC} \cdot \frac{PQ}{AD} = 1 \Rightarrow$ Cheva teoremasiga ko'ra $AC,\ BD$ va PQ to'gri chiziqlar bir nuqtada kesishadi. $\Rightarrow E$ nuqta PQ to'gri chiziqda yotadi. $\Rightarrow Q = F$. $\angle PQO = \angle PDO = 90^0$ ekanligidan P, C, O, Q, D nuqtalar bitta aylanada yotadi. $F = Q$ ekanligidan C, O, F, D, P nuqtalar bitta aylanada yotadi. $\Rightarrow \angle EFC = \angle PDC = \angle PCD = \angle EFD \Rightarrow \angle EFC = \angle EFD \Rightarrow$ isbotlandi.▲

12-masala. (IMO Shot List 1995) ABC uchburchak berilgan. B va C nuqtalardan o'tuvchi aylana AB va AC tomonlarni mos ravishda C' va B' nuqtalarda kesib o'tadi. H va H' nuqtalar mos ravishda ABC va $AB'C'$ uchburchakning ortomarkazlari. BB', CC' va HH' to'g'ri chiziqlar bir nuqtada kesishini isbotlang.



Isbot: $\angle BC$ … $'H' \Rightarrow \angle BC'C + \angle AC'H' =$
$\angle BB'C + \angle AB'H'$ …

$\Rightarrow 180^0$ … $\Rightarrow \angle PC'H' = \angle PB'H'$.

$\angle C'BH = \angle B'CH$ va … dan $\Rightarrow \angle PBH = \angle PCH$. ΔBPC ga H nuqta uchun Che… laymiz:

$$\frac{sin\angle HBP}{sin\angle HBC} \cdot \frac{sin\ldots}{sin\angle HCP} \cdot \frac{sin\angle HPC}{sin\angle BPH} = 1 \Rightarrow \frac{sin\angle BPH}{sin\angle HCB}$$

$$= \frac{sin\angle HBP}{sin\angle HBC} \cdot \frac{sin\angle HCB}{sin\angle HCP} \qquad (1)$$

$\Rightarrow \angle PBH = \angle PCH$ ekanligidan $sin\angle HBP = sin\angle HCP \Rightarrow$ (1) ga ko'ra

$$\frac{sin\angle BPH}{sin\angle HCB} = \frac{sin\angle HCB}{sin\angle HBC} \qquad (2)$$

$\Delta B'PC'$ ga H' nuqta uchun Cheva teoremasini qo'llaymiz:

$$\frac{sin\angle PB'H'}{sin\angle H'B'C'} \cdot \frac{sin\angle B'C'H'}{sin\angle H'C'P} \cdot \frac{sin\angle H'PC'}{sin\angle H'PB'} = 1 \qquad (3)$$

$\angle PC'H' = \angle PB'H'$ ekanligidan $sin\angle H'C'P = sin\angle PB'H' \Rightarrow$ (3) ga ko'ra

$$\frac{sin\angle B'C'H'}{sin\angle H'B'C'} \cdot \frac{sin\angle H'PC'}{sin\angle H'PB'} = 1 \quad \Rightarrow \frac{sin\angle B'PH'}{sin\angle H'PC'} = \frac{sin\angle B'C'H'}{sin\angle H'B'C'} \qquad (4)$$

$\Delta H'B'C'$ ΔHBC ekanligini ko'rish qiyin emas. $\Rightarrow \angle H'B'C' = \angle HBC$ va $\angle H'C'B' = \angle HCB \Rightarrow$

$$\Rightarrow \frac{sin\angle B'C'H'}{sin\angle H'B'C'} = \frac{sin\angle BCH}{sin\angle HBC} \qquad (5)$$

(2), (4) va (5) ga ko'ra

$$\frac{sin\angle B'PH'}{sin\angle H'PC'} = \frac{sin\angle BPH}{sin\angle HPC} \Rightarrow \frac{\sin(\angle B'PC' - \angle H'PC')}{sin\angle H'PC'} = \frac{\sin(\angle BPC - \angle HPC)}{sin\angle HPC} \Rightarrow$$

$$\Rightarrow \frac{\sin\angle B'PC' cos\angle H'PC' - \cos\angle B'PC' sin\angle H'PC'}{sin\angle H'PC'}$$

$$= \frac{\sin\angle BPC cos\angle HPC - \cos\angle BPC sin\angle HPC}{sin\angle HPC} \Rightarrow$$

$$\Rightarrow \sin\angle B'PC' \cdot ctg\angle H'PC' - \cos\angle B'PC' = \sin\angle BPH ctg\angle HPC - \cos\angle BPC \quad (*)$$

$\angle B'PC' = \angle BPC$ ekanligidan (*) ga ko'ra

$$ctg\angle H'PC' = ctg\angle HPC \Rightarrow \angle H'PC' = \angle HPC$$

$\Rightarrow H', P, H$ nuqtalar collinear. $\Rightarrow HH'$ to'g'ri chiziqlar ham P nuqtadan ya'ni BB' va CC' larning kesishish nuqtasidan o'tadi. $\Rightarrow BB', CC'$ va HH' to'g'ri chiziqlar bir nuqtada kesishadi. ▲

13-masala. Uchburchak bissektrisalari bir nuqtada kesishishini isbotlang.

ΔABC uchburchak berilgan. AA_1, BB_1, CC_1 -lar uchburchakning bissektrissalari. AA_1, BB_1, CC_1bissektrissalarning bir nuqtada kesishishini isbotlang.

Isbot:

1-usul: (Cheva teoremasini ishlatmasdan)

1)Dastlab burchak bissektrissasi haqidagi teormani isbotlaymiz.

Teorema. Bissektrissada yotgan nuqtalar burchak tomonlaridan bir xil uzoqlikda yotadi.

(Teskari teorema) tomonlaridan bir xil uzoqlikda nuqtalar bissektrissada yotadi.

I.$\angle BAC$ berilgan, AM – bissektrissa $\angle BAC$, AM– bisse ktrissadagi M-proizvovnalnaya nuqta.

$$M \in AM$$

(M,K) = (M,L) ekanligini isbot qiling.

Isbot

1) Qo`shimcha yasashlar kiritamiz.AB va AC nurga MK va ML perprndikulyarlar o`tkazamiz. (рис. 3).

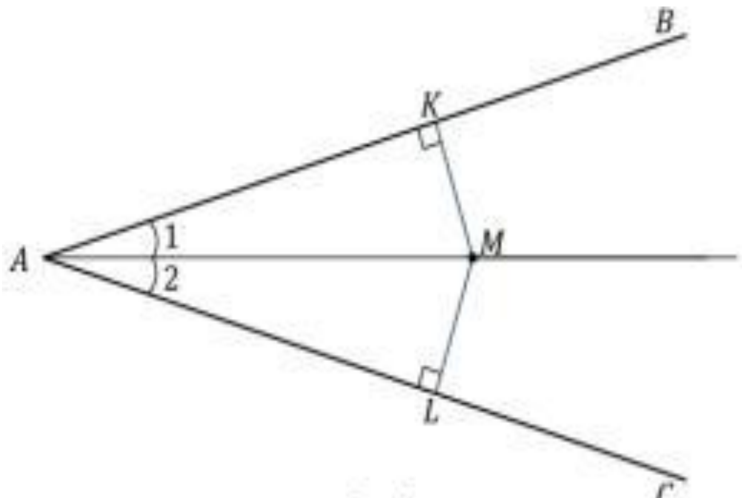


Рис. 3

2)ΔAMK va ΔAML to`g`ri burchakli uchburchaklarga qaraymiz ($\angle AKM = \angle ALM = 90°$, bunda $MK \perp AB$ va $ML \perp AC$). AM - umumiy gipotenuza.

AM - $\angle BAC$ ning bissektrissasi. Bundan$\angle 1 = \angle 2$

Demak, $\Delta AMK = \Delta AML$ to`gri burchakli uchburchaklar o`tkir burchagi va gipotenuzasiga ko`ra teng.

Demak, MK = ML tenguchburchak larning mos elementlari sifatida, ya`ni$(MK) = (ML)$.

II. $\angle BAC$ berilgan . M nuqta $\angle BAC$ burchakning ichki soxasida yotadi. , $K \in AB$, $L \in AC$, $MK \perp AB$, $ML \perp AC$, $c(M,K) = c(M,L)$.

$\angle BAC$ burchakning AM – bissektrissasi ekanligini isbotlang.

AM –umumiy gipotenuza;

$(M,K) = (M,L)$ shartga ko`ra $MK = ML$.

To`g`ri burchakli uchburchaklarning , katetlari va gipotenuzasi ten bo`lganidan $\Delta AMK = \Delta AML$.

Demak, teng uchburchalarning elementlari ham teng ligidan $\angle 1 = \angle 2$. va AM – $\angle BAC$ ning bissektrissasi ekanligidan $\angle 1 = \angle 2$.

Isbotlandi.

2) Endi bu teoremani isbotlashni davom etamiz. Endi **uchburchakning bissektrissalari bir nuqtada kesishishini** isbotlaymiz.

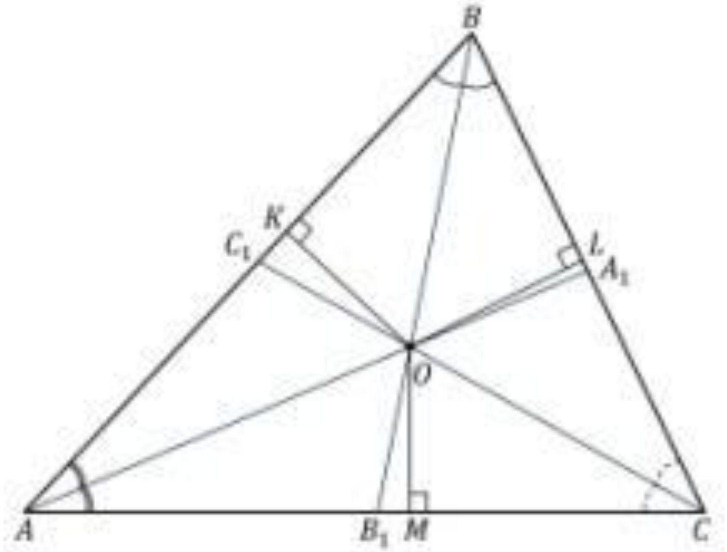


Рис. 4

1) ΔABC ka qaraymiz. AA_1 va BB_1bissektrissalari kesishgan nuqtani Odeb belgilaymiz. AA_1vaBB_1bissektrissalarkesishishadi, bundan

$\angle BAA_1 + \angle ABB_1 < 180°$.

Qo`shimcha yasashlar kiritamiz: $OK \perp AB$, $OL \perp BC$, $OM \perp CA$ larni o`tkazamiz (рис. 4).

1) Teorema isbotiga ko`ra$OK = OM$ и $OK = OL$ (AA_1vaBB_1 –ΔABCning bissektrissasi). $OM = OL$, O nuqta $\angle ACB$ ning tomonlaridan bir xil uzoqlikda yotuvchi nuqtasi, demak O nuqta bu $\angle ACB$ ning CC_1bissektrissasida yotadi.

Ko`rinib turibdiki, ΔABC – AA_1, BB_1, CC_1 – bissektrissalari bir O nuqtada kesishadi.

Isbotlandi.

II usul (Cheva teoremasidan foydalanib).

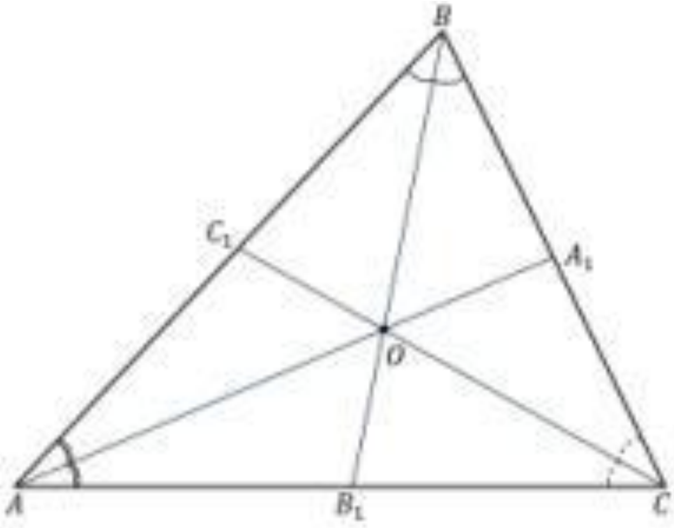


Рис. 5

1) Uchburchakning bissektrissasi qarshisidagi tomonini kesmalarga ajratadi va ular o`zining yopishgan tomoni bilan nisbatlari teng. Ya`ni:

AA_1 –ΔABC ning bissektrissasi ekanligidan:

$$\frac{BA_1}{AB} = \frac{A_1C}{AC}; \ \frac{BA_1}{A_1C} = \frac{AB}{AC}. \qquad (1)$$

BB_1 –ΔABCning bissektrissasi ekanligidan:

$$\frac{CB_1}{BC} = \frac{B_1A}{AB}; \ \frac{CB_1}{B_1A} = \frac{BC}{AB}. \qquad (2)$$

CC_1 – ΔABCning bissektrissasi ekanligidan:

$$\frac{AC_1}{AC} = \frac{C_1B}{BC}; \ \frac{AC_1}{C_1B} = \frac{AC}{BC}. \qquad (3)$$

2) (3), (1) va (2) larni kopaytirishdan quyidagiga ega bo`lamiz:

$$\frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} = \frac{AC}{BC} \cdot \frac{AB}{AC} \cdot \frac{BC}{AB} = 1.$$

Buyerda Cheva teoremasiga ko`ra , ΔABC ning AA_1, BB_1, CC_1 - bissektrissalari bir nuqtada kesishadi. – O nuqtada.

14-masala. Uchburchakning medianalari bir nuqtada kesishadi, kesishish nuqtasida uchidan boshlab **2:1** nisbatda bo`ladi.

$\Delta ABC - da\ AA_1, BB_1, CC_1$ medianalari berilgan.

1) ΔABC ningAA_1, BB_1 vaCC_1 medianalari O nuqtada kesishishini isbotlang.

2) $\frac{AO}{A_1O} = \frac{BO}{B_1O} = \frac{CO}{C_1O} = \frac{2}{1}$.

<u>**Isbot:**</u> I usul (Cheva va Menelay teoremalaridan foydalanmasdan)

1) ΔABC uchburchakka qaraymiz.AA_1vaBB_1medianalari kesishgan nuqtani O bilan belgilaymiz. AA_1va BB_1medianalari kesishadi, shuning uchun $\angle BAA_1 + \angle ABB_1 < 180°$.

Qo`shimcha yasashlar o`tkazamiz,A_1B_1 kesmani o`tkazamiz.(рис.

AA_1va$BB_1 - \Delta ABC$medianasi bo`lgani uchun, bundan $A_1$va$B_1$ nuqta $BC$va$AC$ tomonlarining teng ikkiga bo`luvchi nuqtasi hisoblanadi., ya`ni:$BA_1 = A_1C$, $AB_1 = B_1C$.

Bu yerda, uchburchakning o`rtachizig`i (uchburchakning o`rta chizig`i bu, uning tomonlarining o`rtasini tutashtiruvchi kesma) $A_1B_1$kesma $\Delta ABC$ uchburchakning o`rta chizig`i hisoblanadi.

Uchburchakning o`rta chizigi uning bir tomoniga parallel va uning yarmiga tengligidan $A_1B_1 \parallel AB и A_1B_1 = \frac{1}{2}AB$ bo`ladi.

2) ΔAOB vaΔA_1OB_1uchburchaklarga qaraymiz.

$\angle 1 = \angle 2$ichki almashinuvchi burchaklar,ularABvaA_1B_1 parallel to`g`ri chiziqlarga ($AB \parallel A_1B_1$ isbotga ko`ra) $AA_1$ kesuvchi o`tkazishdan hosil bo`lgan;

$\angle 3 = \angle 4$ ichki almashinuvchi burchaklar, ularABvaA_1B_1 parallel to`g`ri chiziqlarga ($AB \parallel A_1B_1$ isbotga ko`ra) $BB_1$ kesuvchi o`tkazishdan hosil bo`lgan;

Demak , ikkiburchagigako`ra$\Delta AOB \sim \Delta A_1OB_1$,va unig tomonlari o`zaro prorortsional..

Shunday qilib , k –o`xshashlik koeffisenti:

$$k = \frac{AO}{A_1O} = \frac{BO}{B_1O} = \frac{AB}{A_1B_1}.$$

Isbotga ko`ra$A_1B_1 = \frac{1}{2}AB$; $AB = 2A_1B_1$, shuning uchun$AO = 2A_1O$, $BO = 2B_1O$.Buning natijasida, ΔABC uchburchakning AA_1va BB_1 medianalari

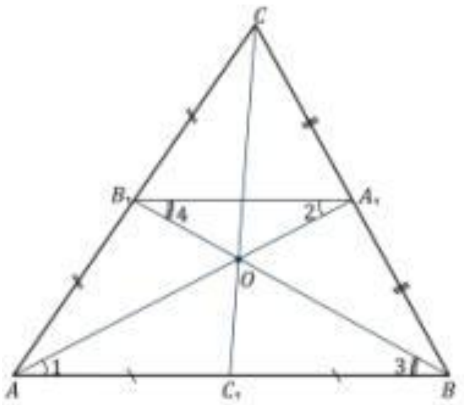


Рис. 6

Onuqtada kesishadi va kesishish nuqtasida uchidan boshlab $2:1$ nisbatda bo`linadi.

3) Bundan ko`rinib turibdiki, $BB_1$ va $CC_1$medianalar kesishgan nuqta uni uchidan boshlab 2:1 nisbatda bo`ladi va ular ham O nuqtada kesishadi.

Isbotlandi.

II usul(Cheva va Menelay teoremasidan foydalanib).

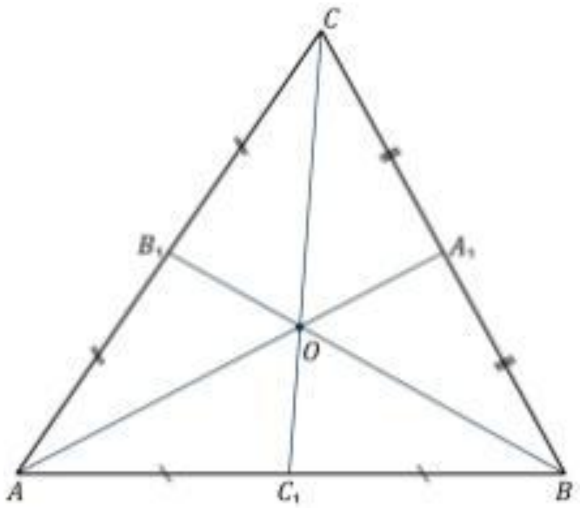


Рис. 7

1) Shartga ko`ra AA_1, BB_1, CC_1 –ΔABC uchburchakning medianalari, bundan $BA_1 = A_1C, AB_1 = B_1C, AC_1 = C_1B$,shuning uchun:

$$\frac{AB_1}{B_1C} \cdot \frac{CA_1}{A_1B} \cdot \frac{BC_1}{C_1A} = \frac{AB_1}{AB_1} \cdot \frac{CA_1}{CA_1} \cdot \frac{BC_1}{BC_1} = \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} = 1.$$

Demak,

$$\frac{AB_1}{B_1C} \cdot \frac{CA_1}{A_1B} \cdot \frac{BC_1}{C_1A} = 1.$$

Bu yerda Cheva va Menelay teormasiga ko`ra ΔABC uchburchakning AA_1, BB_1, CC_1medianalari bir nuqtada kesishadi – O nuqtada.

2) ΔACC_1uchburchakka qaraymiz.

BB_1 to`gri chiziq $\Delta ACC_1$uchburchakning ikki tomonini kesadi. $(BB_1 \cap AC = B_1,\ B_1 \cap CC_1 = O)$ va $(AC_1$ –nuqta, $BB_1 \cap AC_1 = B)$, demak ,Menelay teoremasiga ko`ra:

$$\frac{AB_1}{B_1C} \cdot \frac{CO}{OC_1} \cdot \frac{C_1B}{BA} = 1;\ \frac{AB_1}{AB_1} \cdot \frac{CO}{OC_1} \cdot \frac{C_1B}{AC_1 + C_1B} = 1;$$
$$\frac{1}{1} \cdot \frac{CO}{OC_1} \cdot \frac{1}{2} = 1; \quad \frac{CO}{OC_1} \cdot \frac{1}{2} = 1.$$

Demak ,

$$\frac{CO}{OC_1} = \frac{2}{1}.$$

3) Menelay teoremasiga ko`ra ΔBAA_1 uchburchakning CC_1 kesuvchisi, bundan:

$$\frac{AO}{OA_1} = \frac{2}{1}; \ \frac{BO}{OB_1} = \frac{2}{1}.$$

Kelib chiqadi.

Demak, ΔABC uchburchakning medianalari bir nuqtada kesishadi va kesishgan nuqtasida uchidan boshlab 2:1 nibatdabo`ladi.

15-masala. ΔABC uchburchakning BC tomonida N nuqta olingan, $NC = 3BN$. AC tomonini davom etirrishdan shunday M nuqta,$MA = AC$olingan.

MN to`g`ri chiziq AB tomonini Fnuqtada kesadi.$\frac{BF}{FA}$nisbatni toping.

Berilgan: ΔABC, $N \in BC$, $NC = 3BN$, CA –nur, $M \in CA$, $MA = AC$, $MN \cap AB = F$.

$$\frac{BF}{FA}$$

nisbatni toping.

<u>**Isbot:**</u> I usul (Menelay teoremasidan foydalanmasdan).

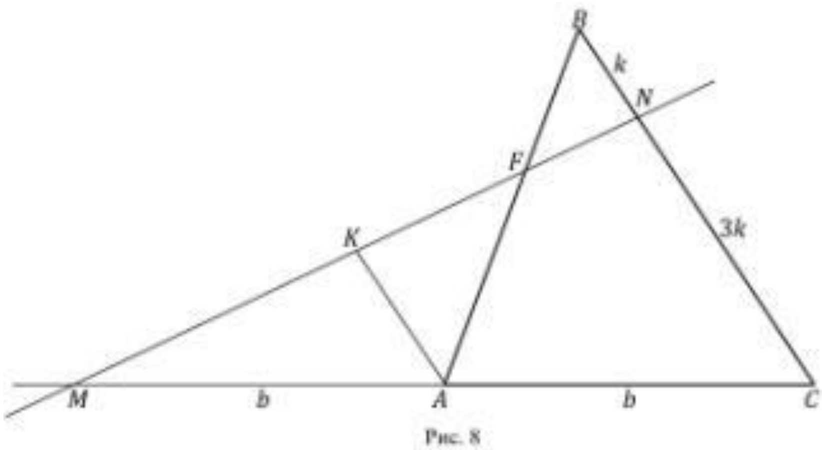


Рис. 8

Qo`shimcha yasashlar yasaymiz:$AK \parallel BC$kesmalarni o`tkazamiz.(рис 8).

$BN = k$ bo`lsin, u holda shartga ko`ra ($NC = 3BN$).$NC = 3k$;$AC ==$ $b$bo`lsin, u holda shartga ko`ra($MA = AC$).$MA = b$.

1) ΔMKA va ΔMNC uchburchaklarga qaraymiz.

$\angle AMK = \angle CMN$ –ΔMKA va ΔMNC uchburchakning umumiy burchagi;

$\angle MKA = \angle MNC$burchaklar, AKvaBCparallel to`gri chiziqlar ning kesishishidan hosil bo`lgan. ($AK \parallel BC$ qo`shimch yasashlarga ko`ra) MN kesuvchi, $K \in MN$.

$\Delta MKA \sim \Delta MNC$ uchburchaklar ikki burchagiga ko`ra o`xshash.

demak, k_1 –o`xshashlik koeffisenti:

$$k_1 = \frac{MA}{MC} = \frac{AK}{NC}; \ \frac{MA}{MA + AC} = \frac{AK}{NC}; \ \frac{b}{2b} = \frac{AK}{3k}; \ \frac{AK}{3k} = \frac{1}{2}; 2AK = 3k.$$

demak,

$$AK = \frac{3k}{2}$$

2) ΔBFN va ΔAFK uchburchaklarga qaraymiz.

$\angle BFN = \angle AFK$ vertikal burchaklar;

$\angle KAF = \angle NBF$ icki almashinuvchi burchaklar, AKvaBC ikki to`g`ri chiziqning kesishishidan($AK \parallel BC$ qo`shimcha yasashlarga ko`ra) MNkesuvchi, $K \in MN, F \in MN$.

Demak , $\Delta BFN \sim \Delta AFK$ uchburchaklar ikki burchagiga ko`ra o`xshash.

Demak , k_2 – o`xshashlik koeffisenti:

$$k_2 = \frac{BF}{FA} = \frac{BN}{AK}; \ \frac{BF}{FA} = \frac{k}{AK}.$$

Isbotga ko`ra:

$$AK = \frac{3k}{2},$$

Bundan biz:

$$\frac{BF}{FA} = \frac{k}{\frac{3k}{2}} = \frac{2k}{3k} = \frac{2}{3}.$$

$\frac{BF}{FA} = \frac{2}{3}$ ekanligiga ega bo`lamiz.

Javob: $\frac{BF}{FA} = \frac{2}{3}$.

IIusul (Menelay teoremasidan foydalanib).

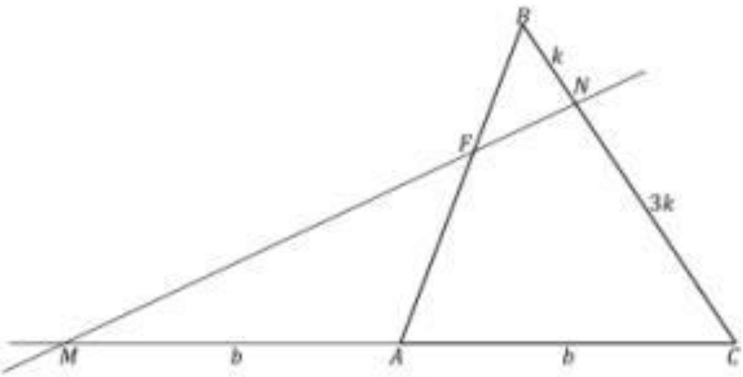


Рис. 9

$BN = k$ bo`lsin, u holda shartga ko`ra ($NC = 3BN$): $NC = 3k$; $AC == b$ bo`lsin, u holda shartga ko`ra ($MA = AC$): $MA = b$.

MNtog`richiziq$\Delta ABC$uchburchakning ikki tomonini kesadi($MN \cap AB = F$, $MN \cap BC == N$)va uchunchisini davom ettirganda ($CA$ –nur, $MN \cap CA = M$),demak Menelay teoremasiga ko`ra:

$$\frac{CN}{NB} \cdot \frac{BF}{FA} \cdot \frac{AM}{MC} = 1; \; \frac{3k}{k} \cdot \frac{BF}{FA} \cdot \frac{b}{2b} = 1; \; \frac{BF}{FA} \cdot \frac{3}{2} = 1.$$

Demak ,

$$\frac{BF}{FA} = \frac{2}{3}.$$

$$\frac{BF}{FA} = \frac{2}{3}.$$

16-masala. $AD - \Delta ABC$ uchburchakning medianasi.AD medianada K nuqta olingan, u $\frac{AK}{KD} = \frac{3}{1}$ nisbatda.$BK$to`g`richiziqΔABCuchburchakni ikkita ΔABPvaΔCBP uchburchakka bo`ladi., $BK \cap AC = P$. $\frac{S_{\Delta ABP}}{S_{\Delta CBP}}$ nisbatni toping.

<u>Isbot:</u> Iusul (Menelay teoremasidan foydalanmasdan).

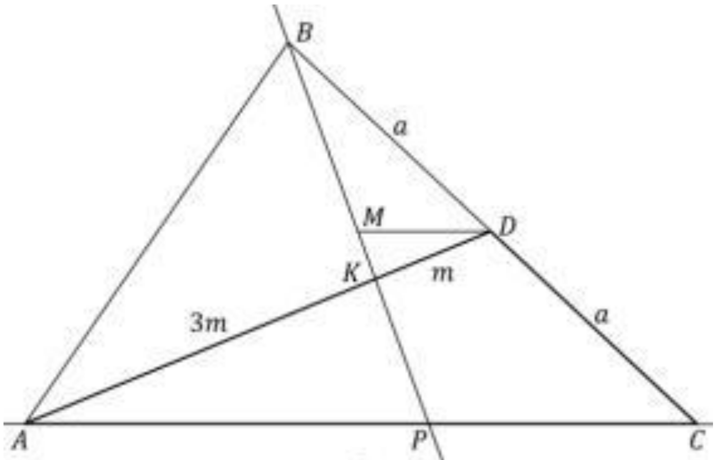


Рис. 12

Qo'shimcha yasashlar kiritamiz: $MD \parallel AC$ kesmalarni o`tkazamiz (рис.12).

$BD = a$ berilgan bo`lsin, u holda shartga ko`ra($AD - \Delta ABC$ uchburchakning medianasi):$CD = a$; berilgan bo`lsin u holda shartga ko`ra: $KD = m$ berilgan bo`lsin ,u holda shartga ko`ra, ($\frac{AK}{KD} = \frac{3}{1}$):$AK = 3KD = 3m$.

1) ΔABP va ΔCBP uchburchaklarga qaraymiz. AP asosi va PC bir to`g`ri chiziqda yotadi.($AC$to`g`ri chiziqda), B uchi umumiy. Shuning uchun bu uchburchaklarning balandliki h ham umumiy bo`ladi. Demak,

$$\frac{S_{\Delta ABP}}{S_{\Delta CBP}} = \frac{AP}{PC}.$$

2) ΔBPC va ΔBMD uchburchaklarga qaraymiz.

$\angle PBC = \angle MBD$ –ΔBPC va ΔBMD uchburchaklar uchun ummumiy burchak;

$\angle BDM = \angle BCP$burchaklarMDva$AC$to`g`ri chiziqlarning kesishishidan hosil bo`lgan burchaklar. ($MD \parallel AC$ qo`shimcha yasashlarga ko`ra) BC - kesuvchi, $D \in BC$.

$\Delta BPC \sim \Delta BMD$ ikki burchagiga ko`ra.

Demak,k_1 –o`xshashlik koeffisenti:

$$k_1 = \frac{BC}{BD} = \frac{PC}{MD}; \ \frac{BD + CD}{BD} = \frac{PC}{MD}; \ \frac{2a}{a} = \frac{PC}{MD}; \ \frac{PC}{MD} = \frac{2}{1}.$$

Bundan:

$$PC = 2MD.$$

3) ΔAKP va ΔDKM uchburchaklarga qaraymiz.

$\angle AKP = \angle DKM$ vertikal burchaklar;

MD va ACparallel tog`ri chiziqlarning kesishishidan hosil bolgan $\angle APK = \angle DMK$ almashinuvchi burchaklar ($MD \parallel AC$ qo`shimcha yasashlarga ko`ra) BPkesuvchi, $K \in BP$.

$\Delta AKP \sim \Delta DKM$ ikki burchagiga ko`ra.

Demak,k_2 –o`xshashlik koeffisenti:

$$k_2 = \frac{AK}{DK} = \frac{AP}{DM}; \ \frac{3m}{m} = \frac{AP}{DM}; \ \frac{AP}{DM} = \frac{3}{1}; AP = 3DM.$$

Bundan isbotga ko`ra:

$$PC = 2MD,$$

Bundan :

$$\frac{AP}{PC} = \frac{3MD}{2MD} = \frac{3}{2}.$$

$$\frac{AP}{PC} = \frac{3}{2}.$$

Kelib chiqadi.

4) demak,

$$\frac{S_{\Delta ABP}}{S_{\Delta CBP}} = \frac{AP}{PC} = \frac{3}{2}.$$

Javob : $\frac{S_{\Delta ABP}}{S_{\Delta CBP}} = \frac{3}{2}$.

II usul (Menelay teoremasiga ko`ra).

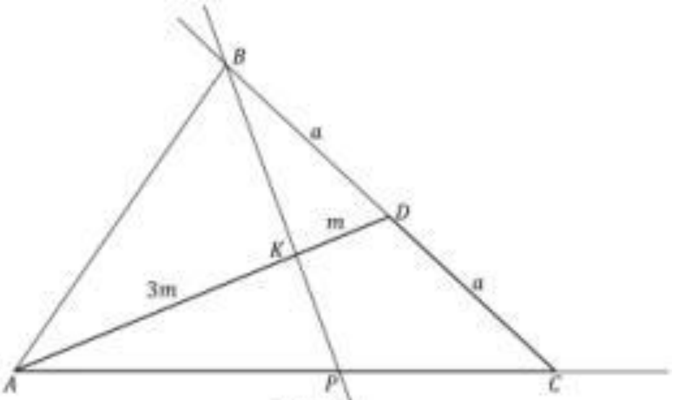


Рис. 13

$BD = a$ bo`lsin,u holda shartga ko`ra ($AD - \Delta ABC$ uchburchak medianasi): $CD = a$ bo`lsin; $KD = m$, u holda shartga ko`ra ($\frac{AK}{KD} = \frac{3}{1}$): $AK = 3KD = 3m$.

1) ΔABP va ΔCBP uchburchaklarga qaraymiz. Asosi AP va PC bir to`g`ri chiziqda yotadi. (AC to`gri chiziqda), B uchi umumiy. Shuning uchun bu uchburchaklarning balandligi h ham umumiy, demak,

$$\frac{S_{\Delta ABP}}{S_{\Delta CBP}} = \frac{AP}{PC}.$$

2) BP to`g`ri chiziq ΔADC uchburchakning ikki tomonini kesadi ($BP \cap AC = P$, $BP \cap AD == K$) va uchunchisini davomi(CD –nur, $BP \cap CD = B$), demak, menelay teoremasiga ko`ra:

$$\frac{AP}{PC}\cdot\frac{CB}{BD}\cdot\frac{DK}{KA}=1;\ \frac{AP}{PC}\cdot\frac{CD+BD}{BD}\cdot\frac{DK}{KA}=1;\ \frac{AP}{PC}\cdot\frac{2a}{a}\cdot\frac{m}{3m}=1;\ \frac{AP}{PC}\cdot\frac{2}{3}=1.$$

Demak ,

$$\frac{AP}{PC}=\frac{3}{2}.$$

3) Bundan ,

$$\frac{S_{\Delta ABP}}{S_{\Delta CBP}}=\frac{AP}{PC}=\frac{3}{2}.$$

XULOSA

Mazkur bitiruv malakaviy ishi Cheva va Menelay teoremasi undan kelib chiqadigan ba'zi natijalarning isbotlarini keltirib chiqarishga qaratilgan. Bitiruv malakaviy ishining I-bobida uchburchaklar kesmalarining konkurent bo'lishi uchun yetarli va zaruriy shartlar keltirilgan va Cheva hamda Menelay teoremasidan kelib chiqadigan muhim natijalar ham isbotlab ko'rsatilgan.

II-bobda esa teorema yordamida yechiladigan bir nechta olimpiada masalalari berilgan. Ishda olingan natijalar va usullar geometriyada muhim tushunchalardan biri bo'lgan uchburchak kesmalarining konkurent bo'lishi masalasi hal qilishda qo'llaniladi. Shuningdek ushbu ishdan olimpiadalarga tayyorgarlik ko'rayotgan maktab va litsey o'quvchilari ham foydalanishi mumkin.

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