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KO`P O`ZGARUVCHINING FUNKSIYASINI LIMITI VA ULARNI HISOBLASH

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M U N D A R I J A

KIRISH

Bir o`zgaruvchinig funksiyasini limitini hisoblash masalasi ancha oson masala hisoblanadi, chunki unda o`zgaruvchi miqdor nuqtaga ko`pi bilam ikkita yonalish bo`ylab yaqinlashadi**.** Ko`p o`zgaruvchining funksiyasi bo`lgan holda esa o`zgaruvchi nuqtaga cheksiz ko`p yo`nalishlar bo`ylab yaqinlashadi, Shu sababli kop` o`zgaruvching funksiyasini hisoblash masalasi ancha qiyin masala. Ushbu Bitiruv malakaviy ishida karrali limitlarni eng qulay yo`l hisoblangan bir o`zgaruvchiga keltirib hisoblash usuli bilan hisoblash, yani limitni har bir o`zgaruvchi bo`yicha alohida ketma-ket hisoblash usuli o`rganilgan. Bunday limitga takroriy limit deyiladi. Takroriy limitlar hamma vaqt ham limitga teng bo`lavermaydi. Masalan qo`yidagi funksiyani qaraylik

$$f(x,y)=\frac{x^2y}{x^4+y^2}.$$

Bu funksiya (x,y) nuqta $(0,0)$ nuqtaga shu $(0,0)$ nuqtadan o`tuvchi istalgan to`gri chiziq bo`ylab intilganda ham funksiya limiti 0 ga teng, lekin $(0,0)$ nuqtada funksiya limiti mavjud emas.

Karrali limitlarning mavjudligidan takroriy limitlarni mavjudligi ham kelib chiqavermaydi. Bunga ham misol qurib ko`rsatilgan.

Bitiruv malakaviy ishida karrali integralni hisoblash masalasiga bag`ishlangan teoremalar keltirilgan: Agar 1) $(x_1,x_2)\to(x_1^0,\ x_2^0)$ da $f(x_1,x_2)$ funksiyaning karrali limiti mavjud:

$$\lim_{\substack{x_1\to x_1^0\\ x_2\to x_2^0}} f(x_1,x_2)=b,$$

2) Har doim tayinlangan x_1 da quyidagi

$$\lim_{x_1\to x_1^0} f(x_1,x_2)=\varphi(x_1)$$

limit mavjud bo`lsa, u holda

$$\lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1, x_2)$$

takroriy limit ham mavjud bo`lib,

$$\lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1, x_2) = b$$

bo`ladi.

Ushbu bitiruv malakaviy ishi mavzusini yo`ritishda [4, 5, 6, 7] adabiyotlardan va www. Ziyonet. uz. saytidan unumli foydalandik .

1-§. Funksiya limiti va Limitga ega bo`lgan funksiyalarning xossalari

Chekli limitga ega bolgan ko`p o`zgaruvchili funksiyalar ham chekli limitga ega bir o`zgaruvchili funksiyalarning xossalariga o`xshash xossalarga ega. Shuni etiborga olib, biz quyida chekli limitga ega bo`lgan ko`p o`zgaruvchili funksiyalarning xossalarini keltiramiz.

Biror $M \subset R^m$ to`plamda $f(x)$ funksiya berilgan bo`lib, $a(a \in R^m)$

nuqta shu M to`plamning limit nuqtasi bo`lsin.

1^0. Agar

$$\lim_{x \to a} f(x) = b$$

mavjud bo`ib, $b > p$ $(b < q)$ bo`lsa, a nuqtaning etarli kichik atrofidagi $x \in M$ $(x \neq a)$ nuqtalarda $f(x) > p$ $(f(x) < q)$ bo`ladi. xususan, $b \neq 0$ bo`lsa, u holda a nuqtaning etalicha kichik atrofida $f(x) \neq 0$ bo`ladi.

2^0. Agar

$$\lim_{x \to a} f(x) = b$$

mavjud bo`lsa, $a$ nuqtaning etarli kichik atrofidagi $x \in M$ $(x \neq a)$ nuqtalarda $f(x)$ funksiya chegaralangan bo`ladi.

Endi M da ikkita $f_1(x)$ va $f_2(x)$ funksiyalar berilgan bo`lsin.

3^0. Agar

$$\lim_{x\to a} f_1(x) = b_1, \qquad \lim_{x\to a} f_2(x) = b_2$$

bo`lib, $a$ nuqtaning $U_\delta(a)$ atrofidagi barcha $x$ nuqtalarda $(x \in M \cap U_\delta(a))$ $f_1(x) \leq f_2(x)$ bo`lsa, u holda $b_1 \leq b_2$ bo`ladi.

4^0. Agar a nuqtaning $U_\delta(a)$ atrofidagi $x \in M \cap U_\delta(a)$ nuqtalarda

$$f_1(x) \leq f(x) \leq f_2(x)$$

bo`lib, $x \to a$ da $f_1(x)$ va $f_2(x)$ funksiyalar limitga ega hamda

$$\lim_{x\to a} f_1(x) = \lim_{x\to a} f_2(x) = b$$

bo`lsa, u holda $f(x)$ funksiya ham limitga ega va

$$\lim_{x\to a} f(x) = b$$

bo`ladi.

5^0. Agar $x \to a$ da $f_1(x)$ va $f_2(x)$ funksiyalar limitga ega bo`lsa, $f_1(x) \pm f_2(x)$ funksiya ham limitga ega bo`ladi va

$$\lim_{x\to a}[f_1(x) \pm f_2(x)] = \lim_{x\to a} f_1(x) \pm \lim_{x\to a} f_2(x)$$

6^0. Agar $x \to a$ da $f_1(x)$ va $f_2(x)$ funksiyalar limitga ega bo`lsa, $f_1(x) \cdot f_2(x)$ funksiya ham limitga ega bo`ladi va

$$\lim_{x\to a}[f_1(x) \cdot f_2(x)] = \lim_{x\to a} f_1(x) \cdot \lim_{x\to a} f_2(x)$$

7^0. Agar $x \to a$ da $f_1(x)$ va $f_2(x)$ funksiyalar limitga ega bo`lib, $\lim_{x\to a} f_2(x) \neq 0$ bo`lsa, $\frac{f_1(x)}{f_2(x)}$ hamlimitga ega bo`ladi va

$$\lim_{x\to a}\frac{f_1(x)}{f_2(x)}=\frac{\lim\limits_{x\to a} f_1(x)}{\lim\limits_{x\to a} f_2(x)}$$

2-§. Takroriy limitlar va karrali limitlar

Biz yuqorida $f(x)=f(x_1,x_2,\dots,x_m)$ funksiyaning $a=(a_1,\,a_2,\dots,a_m)$ nuqtadagi limiti

$$\lim_{x\to a} f(x)=b \quad \Big(\lim_{\substack{x_1\to a_1\\ \dots\\ x_m\to a_m}} f(x_1,x_2,\dots,x_m)=b\Big)$$

bilan tanishdik. Demak, funksiyaning limiti, uning argumentlari $x_1,x_2,\dots,x_m$ larning bir yo`la, mos ravishda $a_1,\ a_2,\dots,a_m$ sonlarga intilgandagi limitidan iboratdir.

Ko`p o`zgaruvchili funksiyalar uchun (ulargagina xos bo`lgan) boshqa formadagi limit tushunchasini ham kiritish mumkin.

$f(x_1,x_2,\dots,x_m)$ funksiya $M\subset R^m$ to`plamda berilgan bo`lib, $a=(a_1,\,a_2,\dots,a_m)$ nuqta M to`plamning limit nuqtasi bo`lsin. Bu funksiyaning $x_1\to a_1$ dagi limiti

$$\lim_{x_1\to a_1} f(x_1,x_2,\dots,x_m)$$

ni qaraylik. Ravshanki, bu limit, birinchidan bir o`zgaruvchili funksiya limitining o`zginasi, ikkinchidan u $x_2,x_3,\dots,x_m$ o`zgaruvchilarga bog`liq:

$$\lim_{x_1\to a_1} f(x_1,x_2,\dots,x_m)=\varphi_1(x_2,x_3,\dots,x_m).$$

so`ng $\varphi_1(x_2, x_3, \dots, x_m)$ funksiyaning $x_2 \to a_2$ dagi limiti

$$\lim_{x_2 \to a_2} \varphi_1(x_2, x_3, \dots, x_m) = \varphi_2(x_3, x_4, \dots, x_m)$$

ni qaraylik.

Yuqoridagidek birin-ketin $x_3 \to a_3$, $x_4 \to a_4$, … , $x_m \to a_m$ da limitga o`tib

$$\lim_{x_m \to a_m} \lim_{x_{m-1} \to a_{m-1}} \dots \lim_{x_2 \to a_1} f(x_1, x_2, \dots, x_m)$$

ni hosil qilamiz. Bu limit $f(x_1, x_2, \dots, x_m)$ $funksiyaning\ takroriy$ $limiti$ deb ataladi.

Demak, funksiyaning takroriy limiti, uning argumentlari $x_1, x_2, \dots, x_m$ larining har birining birin-ketin mos ravishda $a_1, a_2, \dots, a_m$ sonlarga intilgandagi limitidan iborat.

Xuddi yuqoridagidek, $f(x_1, x_2, \dots, x_m)$ funksiyaning $x_{i_1}, x_{i_2}, \dots, x_{i_k}$ argumentlari mos ravishda $a_{i_1}, a_{i_2}, \dots, a_{i_k}$ larga intilgandagi takroriy limiti

$$\lim_{x_{i_k} \to a_{i_k}} \dots \lim_{x_{i_1} \to a_{i_1}} f(x_1, x_2, \dots, x_m)$$

ni ham qarash mumkin.

Shuni ham aytish kerakki, $f(x_1, x_2, \dots, x_m)$ funksiya argumentlari $x_1, x_2, \dots, x_m$ lar mos ravishda $a_1, a_2, \dots, a_m$ sonlarga turli tartibda intlganda funksiyaning turli takroriy limitlari hosil bo`ladi.

Misollar. 1. ushbu

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1 \cdot x_2}{\sqrt{x_1^2 + x_2^2}}, & agar\ x_1^2 + x_2^2 > 0\ bo`lsa, \\ 0 \quad , & agar\ x_1^2 + x_2^2 = 0\ bo`lsa \end{cases}$$

funksiyaning takroriy limiti topilsin.

$$\lim_{x_1\to 0} f(x_1,x_2) = \lim_{x_1\to 0} \frac{x_1 \cdot x_2}{\sqrt{x_1^2 + x_2^2}} = 0, \lim_{x_2\to 0}\lim_{x_1\to 0} f(x_1,x_2) = 0.$$

shuningdek ,

$$\lim_{x_2\to 0} f(x_1,x_2) = \lim_{x_2\to 0} \frac{x_1 \cdot x_2}{\sqrt{x_1^2 + x_2^2}} = 0, \lim_{x_1\to 0}\lim_{x_2\to 0} f(x_1,x_2) = 0.$$

Demak, berilgan funksiyaning takroriy limitlari mavjud va ular bir-biriga teng bo`lib, bu takroriy limitlar funksiyaning (karrali) limitiga teng bo`ladi.

2. Ushbu

$$f(x_1,x_2) = \begin{cases} \dfrac{2x_1 - x_2}{x_1 + 3x_2}, & agar\, x_1 + 3x_2 \neq 0 \text{ } bo`lsa, \\ 0 & , agar\, x_1 + 3x_2 = 0 \text{ } bo`lsa \end{cases}$$

funksiyani qaraylik. Bu funksiyaning takroriy limiti quyidagicha:

$$\lim_{x_1\to 0} \frac{2x_1 - x_2}{x_1 + 3x_2} = -\frac{1}{3}, \qquad \lim_{x_2\to 0}\lim_{x_1\to 0} \frac{2x_1 - x_2}{x_1 + 3x_2} = -\frac{1}{3},$$

$$\lim_{x_2\to 0} \frac{2x_1 - x_2}{x_1 + 3x_2} = 2, \qquad \lim_{x_1\to 0}\lim_{x_2\to 0} \frac{2x_1 - x_2}{x_1 + 3x_2} = 2.$$

Demak, berilgan funksiyaning takroriy limitlari mavjud bo`lib, ularning biri $-\frac{1}{3}$ ga, ikkinchisi esa 2 ga teng.

Biroq $x = (x_1,x_2) \to (0,0)$ da $f(x_1,x_2)$ funksiyaning (karrali) limiti mavjud emas. Chunki $(0,0)$ nuqtaga intiluvchi ikkita

$$x^{(n)} = \left(\frac{1}{n},\frac{1}{n}\right) \to (0,0),$$

$$x^{(n)} = \left(\frac{5}{n},\frac{4}{n}\right) \to (0,0)$$

ketma-ketliklar olinsa, ular uchun mos ravishda

$$f\left(\frac{1}{n},\frac{1}{n}\right)=\frac{1}{4}\to\frac{1}{4}, \quad f\left(\frac{5}{n},\frac{4}{n}\right)=\frac{6}{17}\to\frac{6}{17}$$

bo`ladi. bu esa $(x_1,x_2)\to(0,0)$ da berilgan funksiyaning (karrali) limiti mavjud emasligini bildiradi.

3. Ushbu

$$f(x,y)=\begin{cases} y+x\sin\frac{1}{y}, & y\neq 0,\\ 0, & y=0\end{cases}$$

funksiyaning (0,0) nuqtadagi takroriy va karrali limitlarini hisoblang.

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y)=\lim_{y\to 0}\lim_{x\to 0}\left(y+x\lim\frac{1}{y}\right)=\lim_{y\to 0} y=0$$ takroriy limit mavjud.

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y)=\lim_{x\to 0}\left(x\lim_{y\to 0}\sin\frac{1}{y}\right)$$ takroriy limit mavjud emas, lekin

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$$

karrali limit mavjud va u nolga teng. Haqiqatdan ham,

$$0 \le |f(x,y) - 0| = \left| y + x\sin\frac{1}{y} \right| \le |x| + |y| \quad (x \ne 0) \Rightarrow \lim_{\substack{x\to 0 \\ y\to 0}} f(x,y) = 0 .$$

Yuqorida keltirilgan misollardan ko`rinadiki, funksiyaning biror nuqtada karrali limitining mavjud bo`lishidan, uning shu nuqtada takroriy limitining mavjud bo`lishi va aksincha, funksiyaning biror nuqtada takroriy limitlarining mavjud bo`lishidan, uning shu nuqtada karrali limitining mavjud bo`lishi kelib chiqavermas ekan. Undan tashqari funksiyaning takroriy limitlari bir-biriga har doim teng bo`lavermas ekan.

Biz quyida funksiyaning karrali va takroriy limitlari orasidagi bog`lanish hamda ularning malum shartlarda o`zaro tengligi haqida teorema isbotlaymiz.

$f(x_1,x_2)$ funksiya $M = \{(x_1,x_2) \in R^2 \colon |x_1 - x_1^0| < a_1,$

$|x_2 - x_2^0| < a_2\}$ to`plamda berilgan bo`lsin.

1-Teorema. Agar 1) $(x_1,x_2) \to (x_1^0,\ x_2^0)$ da $f(x_1,x_2)$ funksiyaning karrali limiti mavjud:

$$\lim_{\substack{x_1\to x_1^0 \\ x_2\to x_2^0}} f(x_1,x_2) = b,$$

2) Har doim tayinlangan x_1 da quyidagi

$$\lim_{x_1\to x_1^0} f(x_1,x_2) = \varphi(x_1)$$

limit mavjud bo`lsa, u holda

$$\lim_{x_1\to x_1^0} \lim_{x_2\to x_2^0} f(x_1,x_2)$$

takroriy limit ham mavjud bo`lib,

$$\lim_{x_1\to x_1^0} \lim_{x_2\to x_2^0} f(x_1,x_2) = b$$

bo`ladi.

isbot. $f(x_1,x_2)$ funksiya $(x_1,x_2) \to (x_1^0,\ x_2^0)$ da karrali

$$\lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0}} f(x_1,x_2) = b$$

limitga ega bo`lsin. Limitning tarifiga ko`ra, $\forall \varepsilon > 0$ son olinganda ham, shunday $\delta > 0$ topiladiki, ushbu

$$\{(x_1,x_2) \in R^2 : |x_1 - x_1^0| < \delta,\ \ |x_2 - x_2^0| < \delta\} \subset M$$

to`plamning barcha (x_1,x_2) nuqtalari uchun

$$|f(x_1,x_2) - b| < \varepsilon \tag{1}$$

bo`ladi. Endi teoremaning 2) shartini etiborga olib, $x_1$ o`zgaruvchining $|x_1 - x_1^0| < \delta$ tengsizlikni qanoatlantiradigan qiymatini tayinlab, $x_2 - x_2^0$ da (1) tengsizlikda limitga o`tib

$$|\varphi(x_1) - b| \le \varepsilon$$

ni toping. Demak, $\forall \varepsilon > 0$ son olinganda ham, shunday $\delta > 0$ topiladiki , $|x_1 - x_1^0| < \delta$ bo`lganda $|f(x_1,x_2) - b| \le \varepsilon$ bo`ladi.

Bu esa

$$\lim_{x_1 \to x_1^0} \varphi(x_1) = b$$

bo`lishini bildiradi. Keyingi munosabatdan

$$\lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1,x_2) = b$$

bo`lishi kelib chiqadi. Teorema isbot bo`ldi .

2-**Teorema.** Agar 1) $(x_1,x_2) \to (x_1^0,\ x_2^0)$ da $f(x_1,x_2)$ funksiyaning karrali limiti mavjud:

$$\lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0}} f(x_1, x_2) = b,$$

2) har bir tayinlangan x_2 da quyidagi

$$\lim_{x_1 \to x_1^0} f(x_1, x_2) = \varphi(x_2)$$

limit mavjud bo`lsa, u holda

$$\lim_{x_2 \to x_2^0} \lim_{x_1 \to x_1^0} f(x_1, x_2)$$

takroriy limit ham mavjud bo`lib,

$$\lim_{x_2 \to x_2^0} \lim_{x_1 \to x_1^0} f(x_1, x_2) = b$$

bo`ladi.

Natija. Agar bir vaqtda yuqoridagi ikki teoremalarning shartlari bajarilsa, u holda

$$\lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0}} f(x_1, x_2) = \lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1, x_2) = \lim_{x_2 \to x_2^0} \lim_{x_1 \to x_1^0} f(x_1, x_2)$$

bo`ladi.

Biz ikki o`zgaruvchili funksiyaning karrali va takroriy limitlari orasidagi bog`lanishni ifodalovchi teoremalarni keltirdik.

Xuddi yuqoridagidek $f(x_1, x_2, \dots, x_m)$ funksiyaning $x_{i_1}, x_{i_2}, \dots, x_{i_k}$ o`zgaruvchilari bo`yicha

$$\lim_{\substack{x_{i_1} \to x_{i_1}^0 \\ x_{i_2} \to x_{i_2}^0 \\ \dots\dots \\ x_{i_k} \to x_{i_k}^0}} f(x_1, x_2, \dots, x_m)$$

karrali hamda

$$\lim_{x_{i_1} \to x_{i_1}^0} \lim_{x_{i_2} \to x_{i_2}^0} \dots \lim_{x_{i_k} \to x_{i_k}^0} f(x_1, x_2, \dots, x_m)$$

takroriy limitlari va ular orasidagi bog`lanishni qarash mumkin.

1-**misol.** Quyidagi funksiyaning $(x,y)=(0,0)$ nuqtadagi limitini hisoblang

$$f(x,y)=\frac{2+x-y}{1+2x^2+3y^2}.$$

Yechish.

$$\lim_{(x,y)\to(0,0)}\frac{2+x-y}{1+2x^2+3y^2}=\frac{2+0-0}{1+0+0}=2/$$

2-misol. Quyidagi

$$\lim_{\substack{x\to\infty\\ y\to\infty}}\left(x^{20}+y^{20}\right)e^{-\left(x^2+y^2\right)}$$

limitni hisoblang.

Yechish: Quyidagi oddiy tengsizliklardan foydalanamiz

$$0<\left(x^{20}+y^{20}\right)e^{-\left(x^2+y^2\right)}=\frac{x^{20}}{e^{x^2+y^2}}+\frac{y^{20}}{e^{x^2+y^2}}<\frac{x^{20}}{e^{x^2}}+\frac{y^{20}}{e^{y^2}}.$$

$$0\le\lim_{\substack{x\to\infty\\ y\to\infty}}\left(x^{20}+y^{20}\right)e^{-\left(x^2+y^2\right)}\le\lim_{\substack{x\to\infty\\ y\to\infty}}\left(\frac{x^{20}}{e^{x^2}}+\frac{y^{20}}{e^{y^2}}\right)=0.$$

Demak,

$$\lim_{\substack{x\to\infty\\ y\to\infty}} \left(x^{20}+y^{20}\right)e^{-\left(x^2+y^2\right)}=0$$

3– misol. Quyidagi

$$\lim_{\substack{x\to\infty\\ y\to\infty}} \frac{x^2+y^2}{x^4+y^4}$$

limitni hisoblang $\left(x\neq 0;\ \ y\neq 0\right)$.

Yechish:

$$0\langle \frac{x^2+y^2}{x^4+y^4}=\frac{x^2}{x^4+y^4}+\frac{y^2}{x^4+y^4}\leq\frac{x^2}{x^4}+\frac{y^2}{y^4}=\frac{1}{x^2}+\frac{1}{y^2}$$

$\lim\limits_{\substack{x\to+\infty\\ y\to+\infty}}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=0$ dan $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \frac{x^2+y^2}{x^4+y^4}=0$.

Ba`zibir hollarda $x=a+r\cos\varphi,\quad y=b+r\sin\varphi$ almashtirish

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$$

karrali limitni hisoblashni osonlashtiradi. Bu yerda

$$f(x,y)=f(a+r\cos\varphi,\ \ b+r\sin\varphi)=F(r,\varphi)$$

bo`lib,

$$\lim_{\substack{x\to a\\ y\to b}} f(x,y)=A\Leftrightarrow \lim_{r\to 0} F(r,\varphi)=A$$

4-misol. Quyidagi

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{x^4 y^5}}{(x^2+y^2)^2}$$

karrali limitni hisoblang.

Yechish: $x = a + r\cos\varphi$, $y = b + r\sin\varphi$ almashtirishdan foydalamiz, bunda *a=0; b=0* va $x\to 0\ \ y\to 0\ \ da\ \ r\to 0$;

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{x^4 y^5}}{(x^2+y^2)^2} = \lim_{r\to 0} \frac{\sqrt{(r\cos\varphi)^4\cdot(r\sin\varphi)^5}}{(r^2\cos^2\varphi + r^2\sin^2\varphi)^2} =$$

$$= \lim_{r\to 0} \frac{r^4\sqrt{r}\cdot\sqrt{\cos^4\varphi\sin^5\varphi}}{r^4} = \lim_{r\to 0}\sqrt{r}\cdot\sqrt{\cos^4\varphi\sin^5\varphi} = 0,$$

chunki

$$\sqrt{\cos^4\varphi\sin^{35}\varphi}$$

chegaralangan .

5-misol. Agar $f(x,y) = \frac{x^y}{1+x^y}$ bo`lsa, quyidagi takroriy limitlarni aniqlang.

$$\lim_{x\to+\infty}\left(\lim_{y\to+0} f(x,y)\right) \quad \text{va} \quad \lim_{y\to+0}\left(\lim_{x\to+\infty} f(x,y)\right).$$

Yechish: x ni o`zgarmas desak $y>0$ da $x^y$ $y-$ ning funksiyasi sifatida uzliksiz bo`ladi, shu sababli

$$\lim_{y\to+0} x^y = 1$$

bo`ladi;

y ning o`zgarmas ($y>0$) qiymatida, x ning barcha $x>0$ qiymatida x^y x ning funksiyasi sifatida uzluksizligidan.

$$\lim_{y\to+\infty} x^y = +\infty$$

bo’ladi

$$\lim_{x\to+\infty}\left(\lim_{y\to+0}\frac{x^y}{1+x^y}\right)=\lim_{x\to+\infty}\frac{1}{1+1}=\lim_{x\to+\infty}\frac{1}{2}=\frac{1}{2},$$

$$\lim_{y\to+0}\left(\lim_{x\to+\infty}\frac{x^y}{1+x^y}\right)=\lim_{y\to+0}\left(\lim_{x\to+\infty}\frac{1}{\frac{1}{x^y}+1}\right)=\lim_{x\to+0}1=1\quad.$$

6-misol. Quyidagi karrali limitni hisoblang

$$\lim_{\substack{x\to\infty\\ y\to3}}\left(1+\frac{1}{x}\right)^{\frac{x^2}{x+y}}=\lim_{\substack{x\to\infty\\ y\to3}}\left(\left(1+\frac{1}{x}\right)^x\right)^{\frac{x}{x+y}}=e^1=e$$

7-misol. Karrali limitni hisoblang

$$\lim_{\substack{x\to+\infty \\ y\to+\infty}} \left(\frac{xy}{x^2+y^2} \right)^{x^2}.$$

Yechish: Ixtiyoriy x va y lar uchun $\left| \frac{xy}{x^2+y^2} \right| \le \frac{1}{2}$ tengsizlik o`rinli. Bundan kelib chiqadiki

$$\lim_{\substack{x\to+\infty \\ y\to+\infty}} \left(\frac{xy}{x^2+y^2} \right)^{x^2} = 0$$

8-misol. Limitni hisoblang

$$\lim_{\substack{x\to 0 \\ y\to 0}} \left(x^2+y^2\right)^{x^2y^2}.$$

Yechish:

$$1 = \lim_{\substack{x\to 0 \\ y\to 0}} (x^2y^2)^{x^2y^2} \le \lim_{\substack{x\to 0 \\ y\to 0}} \left(x^2+y^2\right)^{x^2y^2} \le$$

$$\le \lim_{\substack{x\to 0 \\ y\to 0}} (x^2+y^2)^{x^2+y^2} = 1 \quad .$$

Demak, $\lim_{\substack{x\to 0 \\ y\to 0}} \left(x^2+y^2\right)^{x^2y^2} = 1.$

9- misol.

$$\lim_{\substack{x\to+\infty \\ y\to+0}} \left(1+\frac{2}{x}\right)^{\frac{x^2}{x+y}} = \lim_{\substack{x\to+\infty \\ y\to+0}} \left(\left(1+\frac{2}{x}\right)^{x}\right)^{\frac{x}{x+y}} = (e^2)^1 = e^2$$

10- misol.

$$\lim_{\substack{x\to 0 \\ y\to 0}} \frac{\sin(x^4y^2)}{(x^2+y^2)^2} = \lim_{\substack{x\to 0 \\ y\to 0}} \frac{\sin(x^4y^2)}{x^4y^2}\,\frac{x^4y^2}{\left(x^2+y^2\right)^2} = 1\times 0 = 0$$

11-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y) = \sin\frac{\pi x}{2x+y};\; x_0 = \infty,\; y_0 = \infty\,.$$

Yechish:

$$\lim_{x\to\infty}\lim_{y\to\infty} \sin\frac{\pi x}{2x+y} = \lim_{x\to\infty} 0 = 0\,.$$

$$\lim_{y\to\infty}\lim_{x\to\infty} \sin\frac{\pi x}{2x+y} = \lim_{y\to\infty} \sin\tfrac{\pi}{2} = 1$$

12-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y)=\frac{x^2+xy+y^2}{x^2-xy+y^2};\, x_0=y_0=0\,.$$

Yechish:

$$\lim_{x\to 0}\lim_{y\to 0}\frac{x^2+xy+y^2}{x^2-xy+y^2}=\lim_{x\to 0}1=1,$$

$$\lim_{y\to 0}\lim_{x\to 0}\frac{x^2+xy+y^2}{x^2-xy+y^2}=\lim_{y\to 0}1=1.$$

13-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y)=\frac{\sin(x+y)}{2x+3y};\, x_0=y_0=0$$

Yechish:

$$\lim_{x\to 0}\lim_{y\to 0}\frac{\sin(x+y)}{2x+3y}=\lim_{x\to 0}\frac{\sin x}{2x}=\frac{1}{2},$$

$$\lim_{y\to 0}\lim_{x\to 0}\frac{\sin(x+y)}{2x+3y}=\lim_{y\to 0}\frac{\sin y}{3y}=\frac{1}{3}.$$

15-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y)=\frac{\cos x-\cos y}{x^2+y^2};\,x_0=y_0=0\ .$$

Yechish:

$$\lim_{x\to0}\lim_{y\to0}\frac{\cos x-\cos y}{x^2+y^2}=\lim_{x\to0}\frac{\cos x-1}{x^2}=-\frac{1}{2},$$

$$\lim_{y\to0}\lim_{x\to0}\frac{\cos x-\cos y}{x^2+y^2}=\lim_{y\to0}\frac{1-\cos y}{y^2}=\frac{1}{2}.$$

16-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y)=\frac{x^2+y^2}{x^2+y^4};\,x_0=y_0=0.$$

Yechish:

$$\lim_{y\to0}\lim_{x\to0}\frac{x^2+y^2}{x^2+y^4}=\lim_{y\to0}\frac{1}{y^2}=\infty,$$

$$\lim_{x\to0}\lim_{y\to0}\frac{x^2+y^2}{x^2+y^4}=\lim_{x\to0}1=1.$$

17-misol. Quyidagi funksiyaning berilgan nuqtada takroriy limitlarini hisoblang

$$f(x,y)=\sin\frac{x^{y}}{1+x^{v}}\,;\, x_0=+\infty,\, y_0=+0\,.$$

Yechish:

$$\lim_{y\to+0}\lim_{x\to+\infty}\sin\frac{x^{y}}{1+x^{y}}=\lim_{y\to+0}\sin 1=\sin 1,$$

$$\lim_{x\to+\infty}\lim_{y\to+0}\sin\frac{x^{y}}{1+x^{y}}=\lim_{x\to+\infty}\sin\frac{1}{2}=\sin\tfrac{1}{2}\,.$$

18-misol. $f(x,y)=\dfrac{x^2}{|x|+|y|}$ funksiya O(0,0) nuqtada cheksiz kichik bo`lishi isbotlansin.

Yechish: Buning uchun biz qo`yidagi tengsizlikdan foydalanamiz:

$$|f(x,y)|=\left|\frac{x^2}{|x|+|y|}\right|\le|x|\,.$$

$|x|$ - cheksiz kichik miqdor, demak $f(x,y)$ funksiya ham cheksiz kichik bo`ladi.

19-misol. $f(x,y)=\sin(x+y)\cdot\ln(x^2+y^2)$ funksiya O(0,0) nuqtada cheksiz kichik bo`lishi isbotlansin.

Yechish: Buning uchun biz qo`yidagi tengsizlikdan foydalanamiz:

$$|f(x,y)|=\left|\frac{\sin(x+y)}{x+y}(x+y)\ln(x^2+y^2)\right|\le$$

$$\le\left|(|x|+|y|)\ln(x^2+y^2)\right|\le\sqrt{|x|+|y|}\,.$$

$\sqrt{|x|+|y|} \to 0$, yani cheksiz kichik miqdor, demak $f(x,y)$ ham cheksiz kichik.

20-misol. $f(x,y)=\dfrac{x^2y}{x^4+y^2}$ funksiya quyidagi xossalarga ega ekanligi isbotlansin.

a) (x,y) nuqta $(0,0)$ nuqtaga shu $(0,0)$ nuqtadan o\`tuvchi istalgan to\`gri chiziq bo\`ylab intilganda ham funksiya limiti 0 ga teng.

b) $(0,0)$ nuqtada funksiya limiti mavjud emas.

Yechim a). $(0,0)$ nuqtadan o\`tuvchi istalgan bir to\`g\`ri chiziq $y=kx$ ko\`rinishdagi formula bilan beriladi. Bundan foydalanib

$$f(x,kx)=\frac{kx}{x^2+k^2}$$

Formulaga ega bo\`lmiz. Bu ixtiyoriy k larda x nolga intilganda nolga intiladi.

Yechim b). Agar biz $y=x^2$ deb olsak natijada

$$f(x,x^2)=\frac{x^4}{2x^4}=\frac{1}{2}$$

Formulaga ega bo\`lmiz. Demak $y=x^2$ chiziq bo\`ylab funksiya limiti 1/2 ga teng ekan. Bundan esa funksiyaning $(0,0)$ nuqtada uzluksiz ekanligi kelib chiqadi.

3-§. Ko`p o`zgaruvchili funksiyaning uzluksizligi va tekis uzluksizligi

Funksiyaning uzluksizligi tushunchasi limit tushunchasi bilan chambarchas bog`liqdi. Uzluksizlik tushunchasiga limit orqali ta`rif beriladi.

$y = f(x_1, x_2, ..., x_n)$ funksiya $G \subset R^n$ to`plamda berilgan bo`lib, $a = (a_1, a_2, ..., a_n) \in G$ nuqta G to`plamning limit nuqtasi bo`lsin. Quyidagi belgilashlarni kiritamiz:

$$\Delta f(a) = f(a_1 + \Delta x_1, ..., a_n + \Delta x_n) - f(a_1, ..., a_n)$$

funksiyaning a nuqtadagi **to`liq orttirmasi**;

$$\Delta x_k f(a) = f(a_1, ..., a_{k-1}, a_k + \Delta x_k, a_{k+1}, ..., a_n) - f(a_1, ..., a_n)$$

funksiyaning a nuqtadagi $\boldsymbol{x_k}$ **o`zgaruvchi bo`yicha xususiy orttirmasi** (k=1,n).

1-Ta`rif ([4],[7], [8]). *Agar* $\lim\limits_{x \to a} f(x) = f(a)$ $\left(yoki \lim\limits_{\substack{\Delta x_1 \to 0 \\ \cdots \\ \Delta x_m \to 0}} \Delta f(a) = 0 \right)$

bo`lsa , unda f(x) funksiya a ***nuqtada uzluksiz*** *deb ataladi.*

**2-Ta`rif ([4],[7], [8]).** *Agar* $\lim\limits_{\Delta x_k \to 0} \Delta_{x_k} f(a) = 0$ $\left(k = \overline{1, n}\right)$ *bo`lsa,* $y = f(x_1, x_2, ..., x_n)$ *funksiya* $a = (a_1, a_2, ..., a_n) \in G$ *nuqtada* $\boldsymbol{x_k}$

o`zgaruvchi bo`yicha uzluksiz *deb ataladi. Odatda funksiyaning bunday uzluksizligi uning* ***xususiy uzluksizligi*** *deb ataladi.*

3-Ta`rif (Geyne ta`rifi, [4],[7], [8] adabiyotlarga qarang). *Agar G to'plamning limit nuqtalaridan tuzilgan a ga (*a$\in$M*) intiluvchi har qanday* $\{x^{(n)}\}$ *ketma – ketlik olinganda ham mos* $\{f(x^{(n)})\}$ *ketma – ketlik hamma vaqt f(a) ga intilsa, f(x) funksiya* ***a nuqtada uzluksiz*** *deb ataladi.*

4-Ta`rif (Koshi ta`rifi, [4],[7], [8] adabiyotlarga qarang). *Agar* $\forall\varepsilon>0$ *son olinganda ham* $\exists\delta>0$ *topilsaki,* $\rho(x,a)<\delta$ *tengsizlikni qanoatlantiruvchi barcha* $x\in G$ *nuqtalarda*

$$|f(x)-f(a)|<\varepsilon$$

tengsizlik bajarilsa, unda ***f(x) funksiya a nuqtada uzluksiz*** *deb ataladi.*

5-Ta`rif. ([4],[7], [8])** *Agar* $y=f(x_1,x_2,...,x_n)$ *funksiya G to`plamning har bir nuqtasida uzluksiz bo`lsa, bu funksiya shu* ***to`plamda uzluksiz* *deyiladi.*

21-misol. Ushbu

$$f(x,y)=ax+by+c \quad (a\in R,\quad b\in R,\quad c\in R)$$

funksiyaning R^2 da uzluksiz bo`lishini ko`rsating.

Yechish: $\forall\varepsilon>0$ sonni olamiz. Unga ko`ra $\delta=\dfrac{\varepsilon}{2d}$ deyilsa, u holda

$$\rho\left((x, y),(x_0, y_0)\right)=\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$$

tengsizlikni qanoatlantiruvchi, $\forall (x_0 y_0)\in R^2$ nuqtalarda

$$\begin{aligned}
&|f(x, y)-f(x_0, y_0)|=|ax+by+c-(ax_0+by+c)|= \\
&=|a(x-x_0)+b(y-y_0)|\leq \\
&\leq |a||x-x_0|+|b||y-y_0| \left[\begin{array}{l} d=\max(|a|,|b|), \\ |x-x_0|<\sqrt{(x-x_0)^2+(y-y_0)^2}, \\ |y-y_0|<\sqrt{(x-x_0)^2+(y-y_0)^2}. \end{array}\right] \\
&\leq 2d\cdot\sqrt{(x-x_0)^2+(y-y_0)^2}<2\delta=\varepsilon.
\end{aligned}$$

tengsizlik o`rinli bo`ladi. Bu esa Koshi ta`rifiga ko`ra *f(x,y)* funksiyaning $\forall (x_0, y_0)$ nuqtada uzluksiz bo`lishini bildiradi.

22 – misol. Ushbu

$$f(x, y)=\frac{y}{x^2+y^2+5}$$

funksiyaning ixtiyoriy $(x_0, y_0)\in R^2$ nuqtada uzluksiz bo`lishini ko`rsating.

Yechish: (x_0, y_0) nuqtaga $\Delta x,$ Δy orttirmalar berib, funksiyaning to`liq orttirmasini topamiz:

$$\Delta f(x_0, y_0)=f(x_0+\Delta x,\ y_0+\Delta y)-f(x_0, y_0)=\frac{y_0+\Delta y}{(x_0+\Delta x)^2+(y_0+\Delta y)^2+5}-\frac{y_0}{x_0^2+y_0^2+5}=$$

$$=\frac{(y_0+\Delta y)(x_0^2+y_0^2+5)-y_0\left[(x_0+\Delta x)^2+(y_0+\Delta y)^2+5\right]}{\left[(x_0+\Delta x)^2+(y_0+\Delta y)^2+5\right](x_0^2+y_0^2+5)}=$$

bu tengliklardan

$$\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \Delta f(x_0, y_0) = 0$$

bo`lishi kelib chiqadi. 1-tarifdan berilgan funksiya $(x_0, y_0)$ nuqtada uzliksiz bo`ladi.

3-Teorema([7],[8]). *Agar* $y = f(x_1, x_2, ..., x_n)$ *funksiya* $a = (a_1, a_2, ..., a_n) \in G$ *nuqtada uzluksiz bo`lsa, funksiya shu nuqtada har bir o`zgaruvchisi bo`yicha ham uzluksiz bo`ladi.*

23 – misol. Ushbu

$$f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, x^2+y^2 \neq 0, \\ 0, \quad x^2+y^2 = 0 \end{cases}$$

funksiya (0,0) nuqtada har bir o`zgaruvchi bo`yicha xususiy uzluksiz , lekin shu nuqtada bir yo`la uzluksiz emas, bu nuqtada hatto limitga ega emas.

Yechish: Oldin x o`zgaruvchi bo`yicha uzluksizligini ko`rsatamiz.

Agar $y \neq 0$ va $x \to x_0 \neq 0$ bo`lsa,

$$\lim_{x\to x_0} f(x,y) = \lim_{x\to x_0} \frac{2xy}{x^2+y^2} = \frac{2x_0 y}{x_0^2+y^2} = f(x_0, y).$$

Agar *y=0* va $x \to x_0 \neq 0$ bo'lsa

$$\lim_{x\to x_0} f(x,0) = \lim_{x\to x_0} \frac{2x\cdot 0}{x^2+0} = \lim_{x\to x_0} 0 = 0 = f(x_0,0)$$

Endi $y = 0$ va $x \to x_0 = 0$ desak,

$$\lim_{x\to x_0} f(x,0)=0=f(0,0)$$

bo`ladi.

Demak, $\lim\limits_{x\to x_0} f(x,y)=f(x_0,y)$.

Bu berilgan $f(x,y)$ funksiyaning x o`zgaruvchisi bo`yicha xususiy uzluksiz bo`lishini ko`rsatadi. Xuddi shu usulda funksiyaning y o`zgaruvchisi bo`yicha ham uzluksizligi ko`rsatiladi.

Berilgan funksiya (0,0) nuqtada ikkala o`zgaruvchi bo'yicha bir yo`la uzluksiz emas, chunki $x\to 0,\quad y\to 0$ da

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)=\lim_{\substack{x\to 0\\ y\to 0}}\frac{2xy}{x^2+y^2}$$

mavjud emas.

Aytaylik, (x,y) nuqta (0,0) nuqtaga tekislikdagi $y=kx$ to`g`ri chiziq bo`yicha intilsin.

$$\lim_{\substack{x\to 0\\ y\to 0\\ (y=kx)}}\frac{2xy}{x^2+y^2}=\lim_{x\to 0}\frac{2x\cdot kx}{x^2+(kx)^2}=\lim_{x\to 0}\frac{2kx^2}{x^2+(kx)^2}=\frac{2k}{1+k^2}$$

bo`ladi. Demak, ($x$,y) nuqta turli to`g`ri chiziqlar bo`yicha (0,0) ga intilganda limitning qiymati turlicha bo`ladi. Bu hol esa qaralayotgan limitning mavjud emasligini bildiradi.

2 – usul. Yuqoridagi funksiyani (0,0) nuqtada uzluksiz emasligini, ya`ni (0,0) nuqtada limit mavjud emasligini boshqacha yo`l bilan ko`rsatamiz. (0,0) nuqtaga intiluvchi ikkita ketma – ketlikni qaraylik:

$$\left(x_n, y_n\right) = \left(\frac{1}{n}, \frac{1}{n}\right) \to (0,0),$$

$$\left(x_n^{'}, y_n^{'}\right) = \left(\frac{2}{n}, \frac{1}{n}\right) \to (0,0).$$

Bu ketma – ketliklarda

$$f\left(x_n, y_n\right) = \frac{2 \cdot \frac{1}{n} \cdot \frac{1}{n}}{\frac{1}{n^2} + \frac{1}{n^2}} = 1 \to 1$$

$$f\left(x_n^{'}, y_n^{'}\right) = \frac{2 \cdot \frac{2}{n} \cdot \frac{1}{n}}{\frac{4}{n^2} + \frac{1}{n^2}} = \frac{4}{5} \to \frac{4}{5}$$

bo\`ladi va berilgan funksiyaning karrali limitning mavjud emasligini va (0,0) nuqtada funksiya qiymati 0 bo\`lish sharti bajarilmadi. Demak, $f(x, y)$ funksiya (0,0) nuqtada uzluksiz emas.

**6-Ta\`rif**. Agar $a$ nuqtada $f(x)$ funksiyaning limiti mavjud bo\`lmas yoki cheksiz bo\`lsa yoki mavjud bo\`lib shu nuqtadagi qiymatiga teng bo\`lmasa, u holda $f(x)$ funksiya a nuqtada uzilishga ega deyiladi.

24 – misol.

$$f(x, y) = \frac{x + y}{x^3 + y^3}$$

funksiyani uzluksizlikka tekshiring.

Yechish: Kasrning surati va maxraji (x, y) o\`zgaruvchining funksiyasi sifatida uzluksiz. Funksiya maxraji $x^3 + y^3$ nol bo\`lgan nuqtalarda uzilishga ega bo`ladi. $x^3+y^3=0$ tenglamani y *argument*ga nisbatan yechib, $y=-x$ ni topamiz. Funksiya $y=-x$ nuqtalarda uzilishga ega.

Endi, $x_0\neq 0$, $y_0\neq 0$ va $x_0+y_0=0$. Unda

$$\lim_{\substack{x\to x_0\\ y\to y_0}}\frac{x+y}{x^3+y^3}=\lim_{\substack{x\to x_0\\ y\to y_0}}\frac{1}{x^2-xy+y^2}=\frac{1}{x_0^{\,2}-x_0y_0+y_0^{\,2}}$$

Bundan kelib chiqadiki, $x_0\neq 0$, da $y=-x$ to`g`richiziq nuqtalari tuzatish mumkin bo`lgan uzilish nuqtalari to`plami bo`ladi.

Quyidagi

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{x+y}{x^3+y^3}=\lim_{\substack{x\to 0\\ y\to 0}}\frac{1}{x^2-xy+y^2}=+\infty$$

funksiya (0,0) nuqtada uzilishga ega.

7-Ta`rif. ([4],[7], [8] adabiyotlarga qarang*).*** Agar ixtiyoriy $\forall\varepsilon>0$ uchun shunday bir $\exists\delta=\delta(\varepsilon)>0$ topilib, $G$ to`plamning $\rho(x^{`},x^{``})<\delta$ tengsizlikni qanoatlantiruvchi ixtiyoriy $\forall x'$ va x'' nuqtalarida $|f(x'')-f(x')|<\varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiya G **to`plamda tekis uzluksiz funksiya deb ataladi.

25 – misol. Ushbu

$$f(x,y)=ax+by+c$$

funksiyaning

$$M=\{(x,y)\in R^2:|x|<+\infty,\ |y|<+\infty,\ a\in R,\ b\in R,\ c\in R\}$$

to`plamda tekis uzluksiz bo`lishini ko`rsating.

Yechish: $(x_1, y_1) \in G$ va $(x_2, y_2) \in G$ nuqtalar uchun quyidagiga ega bo`lamiz

$$|f(x_1,y_1) - f(x_2,y_2)| = |ax_1 + by_1 + c - (ax_2 + by_2 + c)| =$$
$$= |a(x_1 - x_2) + b(y_1 - y_2)| \le |a| \cdot |x_1 - x_2| + |b| \cdot |y_1 - y_2|$$

$\forall \varepsilon > 0$ sonni olib, unga ko`ra olinadigan $\delta > 0$ so<u>nda</u>

$$|x_1 - x_2| < \delta, \ |y_1 - y_2| < \delta, \quad \left(\delta = \frac{\varepsilon}{2d}, \quad d = \max(|a|, \ |b|) \right)$$

shart bajarilganda

$$|f(x_1,y_1) - f(x_2,y_2)| \le d\,(|x_1 - x_2| + |y_1 - y_2|) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

bo`lib, 7 – ta`rifdan berilgan funksiyaning G to`plamda tekis uzluksizligi kelib chiqadi.

4-Teorema (Kantor teoremasi [4], [7], [8]). Agar $y = f(x_1, x_2, ..., x_n)$ funksiya chegaralangan yopiq $G \subset R^n$ to`plamda uzluksiz bo`lsa, u holda funksiya shu to`plamda tekis uzluksiz bo`ladi.

XULOSA

Shunday qilib ko`p o`garuvching fuksiyasini nuqtadagi limiti bir o`zgaruvchining funksiyasini limitidan farq qilib, unda o`zgaruvchi nuqtaga cheksiz ko`p yo`nalaishlar bo`ylab yaqinlashar ekan. Bir o`garuvchida faqat ikkita yo`nalish bo`ylab yaqinlashadi. Shu sababli ko`p o`zgaruvchining funksiyasini limitini hisoblash bir qancha qiyinchiliklarga ega masaladi. Ko`p o`zgaruvchining funksiyasini limitini bir o`zgaruvchiga olib kelib, takroriy limit sifatida hisoblash eng qulay yo`llardan biri ekan. Lekin hamma vaqt ham bu usul qo`l kelavermaydi. Biz yuqorida qurib ko`rsatgan misollarimizdan ko`rinadiki takroriy limitlarning mavjudligidan karrali limitning mavjudligi kelib chiqavermaydi va aksincha**.**

1-Teorema. Agar 1) $(x_1, x_2) \to (x_1^0,\ x_2^0)$ da $f(x_1, x_2)$ funksiyaning karrali limiti mavjud:

$$\lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0}} f(x_1, x_2) = b,$$

2) Har doim tayinlangan x_1 da quyidagi

$$\lim_{x_1 \to x_1^0} f(x_1, x_2) = \varphi(x_1)$$

limit mavjud bo`lsa, u holda

$$\lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1, x_2)$$

takroriy limit ham mavjud bo`lib,

$$\lim_{x_1 \to x_1^0} \lim_{x_2 \to x_2^0} f(x_1, x_2) = b$$

bo`ladi.

Funksiyaning uzluksizligi tushunchasi limit tushunchasi bilan chambarchas bog`liqdi. Uzluksizlik tushunchasiga limit orqali ta`rif beriladi.

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