O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

Farg‘ona politexnika instituti

«muxandis - iqtisod» fakulteti

«Oliy matematika» kafedrasi

Elementar matematikaning
«Ko‘rsatkichli va logarifmik tenglama, tengsizliklar»
bo‘limi bo‘yicha

uslubiy qo‘llanma

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Institut Uslubiy
Kengashida tasdiqlangan
«__» __________ 2011 y.
№ __ bayonnoma

Farg‘ona - Texnika – 2011y.

Ushbu uslubiy qo‘llanma oliy o‘quv yurtiga kiruvchilar va tayyorlov kursi tinglovchilari uchun mo‘ljallangan bo‘lib,unda logarifmik va ko‘rsatkichli tenglama hamda tengsizliklarni yechish uslubi ularning turlariga qarab bayon qilingan. Undan oliy o‘quv yurtiga o‘qishga kirishda mustaqil tayyorlanuvchilar, shuningdek maktabda matematika to‘garaklarini tashkil qilishda ham bu qo‘llanmadan keng foydalanishi mumkin.

Uslubiy qo‘llanma «Oliy matematika kafedrasi» yig‘ilishida (bayon №)
muhokama qilindi.

Uslubiy qo‘llanma muxandis - iqtisod fakulteti uslubiy komissiyasi
tomonidan tasdiqlangan.(Bayon №-)

Mualliflar: dots. A. Abdurazoqov.

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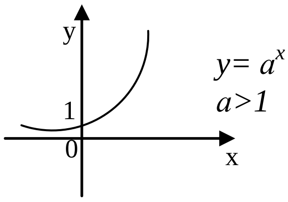
Muxarrir: doc. M. Mamajonov

Ko‘rsatkichli va logarifmik tenglama va tengsizliklar.

1. $y= a^x$ ko‘rinishdagi funkciya *ko‘rsatkichli funkciya* deyiladi. Bunda $a\neq 1$, $a>0$.

2. $a>1$ bo‘lsa, $y= a^x$ funkciya quyidagi xossalarga ega bo‘ladi:

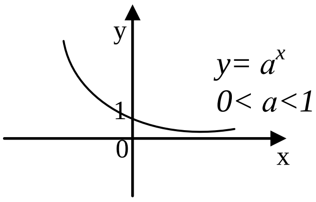
a) D(y)=R;
b) $E(y)=R_+$;
v) funkciya o‘sadi;
g) $x=0$ da $y=1$;
d) $x>0$ da $a^x>1$;
e) $x<0$ da $0< a^x<1$.



1-shakl.

3. $y= a^x$ funkciya $0< a<1$ da quyidagi xossalarga ega bo‘ladi:

a) D(y)=R;
b) $E(y)=R_+$;
*v)*funkciya kamayadi;
g) $x=0$ da $y=1$;
d) $x>0$ da $0< a^x<1$;
e) $x<0$ da $a^x>1$.

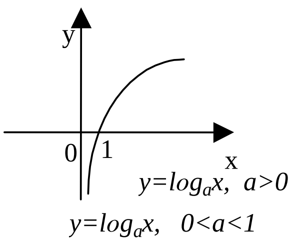


2-shakl.

Ta’rif: Musbat *b* sonining *a* asosga ko‘ra logarifmi deb *b* ni hosil qilish uchun *a* ni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi. Bunda $a\neq 1, a>0$.
v sonining *a* asosga ko‘ra logarifmi odatda $\log_a b$ ko‘rinishda yoziladi.
Ta’rif: Asosi 10 dan iborat bo‘lgan logarifmlarni *o‘nli logarifmlar* deyiladi. Ularni odatda lgb ko‘rinishda yoziladi, bunda *b* ixtiyoriy musbat son.
Ta’rif: Asosi *e* sonidan iborat logarifmni *natural logarifm yoki Neper logarifmi* deyiladi. Ularni odatda lnb ko‘rinishda yoziladi, bunda *b* ixtiyoriy musbat son.
$y=a^x$ ko‘rsatkichli funkciya monoton funkciya bo‘lib, u teskarilanuv-chidir.
$y=a^x$ funkciya grafigini $y=x$ to‘g‘ri chizig‘iga nisbatan simmetrik akslantirsak, $y=\log_a x$ funkciya grafigini hosil qilamiz.

4. $a>1$ bo‘lsa, logarifmik funkciya quyidagi xossalarga ega bo‘ladi:

a) $D(y)=R_+$;
b) E(y)=R;
v) funkciya o‘sadi;
g) $x=0$ da $\log_a x=0$;
d) $0<x<1$ da $\log_a x<0$;
e) $x>1$ da $\log_a x>0$ (3-shakl).



3-шакл.

5. $y=\log_a x$ funkciya $0<x<1$ da quyidagi xossalarga ega:

a) $D(y)=R_+$;

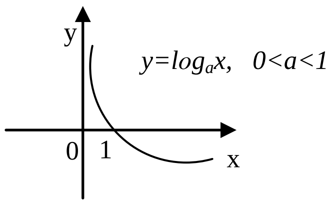
b) $E(y)=R$;

v) funkciya kamayadi;

g) *x=0 da* $\log_a x=0$;

d) $0<x<1$ *da* $\log_a x>0$;

e) $x>1$ *da* $\log_a x<0$ *(4-shakl).*



4- shakl.

Logarifmlar quyidagi asosiy xossalarga ega:

1. $\log_a(x\cdot y)=\log_a x+\log_a y;$
2. $\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y;$
3. $\log_a y^k=k\cdot\log_a b;$
4. $\log_a y=\frac{\log_c y}{\log_c a}; \qquad (c\neq 1;\ c>0);$
5. $\log_{a^k} y=\frac{1}{k}\log_a y, \qquad k\neq 0;$
6. $\log_{a^m} y^n=\frac{n}{m}\log_a y, \quad m\neq 0;$
7. $\log_a a=1;$
8. $\log_a 1=0;$
9. $x^{\log_a y}=y^{\log_a x}$, bu yerda *x,y* lar musbat haqiqiy sonlar.
10. $a^{\log_a b}=b$
11. $\log_a y=\log_{a^n} y^n$
12. $\log_a y=\frac{1}{\log_y a}.$

Yuqoridagi barcha formulalar o‘nli va natural logarifmlar uchun ham o‘rinli bo‘laveradi.

1. Ko‘rsatkichli tenglama.

Ta’rif: Noma’lum o‘zgaruvchi daraja ko‘rsatkichida ishtirok etgan tenglamaga *ko‘rsatkichli tenglama* deyiladi.

Sodda ko‘rsatkichli tenglama $a^x=b$ ko‘rinishda yoziladi, bunda $a>0,\ b>0,\ a\neq 1$. Bu tenglamaning ikkala qismini *a* asosga ko‘ra logarifmlab, $x=\log_a b$ ni topamiz.

1- misol. $5^{2x-1}=y^{3-x}$ tenglamani yeching.

Yechish. $2x-1=(3-x)\log_5 y$ *yoki* $x=\frac{1+3\log_5 y}{2+\log_5 y}$ ni hosil qilamiz.

Quyida ko‘rsatkichli tenglamalarning maxsus turlarini qaraymiz:

1.1. $\alpha a^{2x}+\beta a^{x}+\gamma=0$ $(a>0,\ a\neq 1)$ ko‘rinishdagi tenglama

Bunday tenglama $a^x=t_1$ va $a^x=t_2$ tenglamalar sistemasini yechishga keltiriladi. Bunda $t_1,\ t_2$ $\alpha a^{2x}+\beta a^{x}+\gamma=0$ tenglamaning ildizlari.

2- misol. $8^{\frac{2}{x}}-2^{3\left(1+\frac{1}{x}\right)}+12=0$ tenglamani yeching.

Yechish: Daraja ko‘rsatkichining xossalaridan foydalanib,

$$\left(8^{\frac{1}{x}}\right)^2-2^{3\left(1+\frac{1}{x}\right)}+12=0 \quad ёки \quad \left(8^{\frac{1}{x}}\right)^2-8\cdot 8^{\frac{1}{x}}+12=0$$

tenglamani hosil qilamiz.

$t=8^{\frac{1}{x}}$ almashtirish bajarib, t^2-8t+10=0 ni olamiz. Uni yechib, t_1=2 t_2=6 ni olamiz.

U holda dastlabki tenglama $\begin{cases} 8^{\frac{1}{x}}=2 \\ 8^{\frac{1}{x}}=6 \end{cases}$ sistemaga ekvivalent bo‘ladi. Bu sistemani 8 asosga ko‘ra logarifmlab,

$$\begin{cases} \frac{1}{x}=\log_8 2 \\ \frac{1}{x}=\log_8 6 \end{cases} \Leftrightarrow \begin{cases} x=\frac{1}{\log_8 2} \\ x=\frac{1}{\log_8 6} \end{cases} \Leftrightarrow \begin{cases} x=\log_2 8 \\ x=\log_6 8 \end{cases} \Leftrightarrow \begin{cases} x=3 \\ x=3\log_6 2 \end{cases}$$

ni hosil qilamiz.

1.2. $\alpha a^{2x}+\beta(ab)^x+\gamma b^{2x}=0$ ko‘rinishdagi tenglama

Bunday tenglama $t=\frac{a^x}{b^x}$ almashtirish yordamida kvadrat tenglamaga keladi.

3- misol. $9^x+6^x=2^{2x+1}$ tenglamani yeching.

Yechish: Daraja ko‘rsatkichining xossalaridan foydalanib, quyidagi $3^{2x}+2^x\cdot 3^x=2\cdot 2^{2x}$ tenglamani hosil qilamiz. Bu tenglamaning ikkala qismini 2^{2x} ga bo‘lib, $t=\left(\frac{3}{2}\right)^x$ belgilash kiritib, $t^2+t=2$ kvadrat tenglamani hosil qilamiz. Uni yechib, $t_1=1,$ $t_2=-2$ ildizlarni topamiz.

U holda $\begin{cases} \left(\frac{3}{2}\right)^x=1 \\ \left(\frac{3}{2}\right)^x=-2 \end{cases}$ sistema hosil bo‘ladi, uning 1-tenglamasini $\frac{3}{2}$ asos bo‘yicha logarifmlab, $x=0$ ekanini topamiz.Sistemaning 2-tenglamasi yechimga ega emas, chunki $\left(\frac{3}{2}\right)^x$ x ning har qanday qiymatlarida noldan katta.

Javob: $x=0.$

1.3. Ko‘paytuvchilarga yoyish bilan yechiladigan tenglamalar

4- misol. $25\cdot2^x-10^x+5^x=25$ tenglamani yeching.
Yechish: Qo‘shiluvchilarni quyidagicha guruxlaymiz: $25\cdot2^x-25+5^x-2^x\cdot5^x=0$ $25(2^x-1)+5^x(1-2^x)=0$, umumiy ko‘paytuvchini qavsdan tashqariga chiqarsak, $(25-5^x)(2^x-1)=0$ tenglamani olamiz. Bundan esa $5^x=25$ yoki $2^x=1$ tenglamalarni olamiz. Bu tenglamalardan $x=2$ yoki $x=0$ yechimni topamiz.
Javob: $x=\{0;2\}$

1.4. $a(x)^{b(x)}=1$ ko‘rinishdagi tenglama

Bu tenglama $\begin{cases} a(x)=1 \\ b(x)=0 \end{cases}$ sistemaga teng kuchli bo‘ladi.

5- misol. $|x-5|^{x^2-5x+6}=1$ tenglamani yeching.

Yechish: Berilgan tenglama $\begin{cases} x-5=1 \\ x^2-5x+6=0 \end{cases}$ sistemaga teng kuchli.

Sistemaning 1- tenglamasi $x_1=6$ ildizga, ikkinchisi esa $x_2=2$, $x_3=3$ ildizlarga ega. Dastlabki tenglama ildizlari $\{2;3;6\}$ sonlar bo‘ladi.

Mustaqil yechish uchun misollar.

1. $4^x-5\cdot2^x=24$ (3)
2. $2^x\cdot3^{x^2}=6$ $(1;-1-\log_3 2)$
3. $6^{x+1}+6^{x+2}=2^{x+1}+2^{x+2}+2^{x+3}$ (-1)
4. $\left(\frac{2}{3}\right)^x\cdot\left(\frac{9}{8}\right)^x=\frac{27}{64}$ (3)
5. $4^{x+1,5}+2^{x+2}=4$ (-1)
6. $2^{x^2-6x-2,5}=16\sqrt{2}$ (-1;7)
7. $3^{x^2}\cdot4^{x^2}=12^{-3}\cdot12^{6-2x}$ (-3;1)
8. $(10^{5-x})^{(6-x)}=100$ (4;7)
9. $5^x+25\cdot5^{-x}=26$ (0;2)
10. $4^x+8=9\cdot2^x$ (0;3)

2. Logarifmik tenglamalar

Ta'rif. Noma'lum o'zgaruvchi logarifm belgisi ostida qatnashgan tenglama *logarifmik tenglama* deyiladi.
Logarifmik tenglamalarni yechish logarifmlarning quyidagi xossalariga asoslanadi:
$\log_a x=b(a>0,\ a\neq 1,\ b>0)$, logarifmik tenglama $x=a^b$, yagona yechimga ega, ya'ni $b=\log_a a^b$.
Logarifmik tenglamalarni yechishda logarifm belgisi ostidagi ifoda faqat musbat qiymatlar qabul qilish, logarifm asosidagi ifoda musbat va birdan farqli bo'lishi kerakligini hisobga olish kerak bo'ladi.
Quyida logarifmik tenglamalarni yechish usullarini keltiramiz.
1- misol $\log_2 x+\log_4 x+\log_8 x=11$ tenglamani yeching.
Yechish: Quyidagicha shakl almashtiramiz:

$$\log_2 x+\log_{2^2} x+\log_{2^3} x=11$$

Logarifmning xossalaridan foydalanib,
$\log_2 x+\frac{1}{2}\log_2 x+\frac{1}{3}\log_2 x=11$ yoki $\frac{11}{6}\log_2 x=11$ yoki $\log_2 x=6$ tenglamani hosil qilamiz. Bu tenglama $\begin{cases} x=2^6 \\ x>0 \end{cases}$ sistemaga teng kuchli

Javob: $x=64$
2- misol. $\lg(5-x)+2\lg\sqrt{3-x}=1$ tenglamani yeching.
Yechish: Berilgan tenglamaning qabul qiladigan qiymatlari (QQQ) sohasini belgilaymiz:
$\begin{cases} 5-x>0 \\ 3-x>0 \end{cases} => x<3.$ logarifmlarning asosiy xossalaridan foydalanib, dastlabki tenglamani quyidagicha yozamiz:

$$\lg(5-x)+2\cdot\frac{1}{2}\lg(3-x)=\lg 10$$

$$\lg(5-x)(3-x)=\lg 10$$

$$(5-x)(3-x)=10$$

$x^2-8x+5=0$, bu tenglama $x_1=4+\sqrt{11}$, $x_2=4-\sqrt{11}$ ildizlarga ega. $x_1>3$ demak u berilgan tenglamaning ildizi bo'lmaydi, chunki QQQ sohasi $x<3$.
Ikkinchi ildiz QQQ sohasiga mos , ya'ni $4-\sqrt{11}<4-\sqrt{9}=1$.
Javob: $x=4-\sqrt{11}$.
3- misol. $3\lg(x^2)-\lg^2(-x)=9$ tenglamani yeching
Yechish: Avvalo berilgan tenglamaning QQQ sohasini aniqlaymiz. $-x>0$ yoki $x<0$. Bundan esa $\sqrt{x^2}=|x|=-x$

$$3\cdot 2\lg\sqrt{x^2}-\lg^2(-x)-9=0$$

$\lg^2(-x)-6\lg(-x)+9=0$

$(\lg(-x)-3)^2=0$

$\lg(-x)=3$

$(-x)=10^3$

$x=-1000$

Javob: $x=-1000$

4- misol: $\log_{\frac{1}{5}} x+\log_4 x=1$ tenglamani yeching.

Yechish: $\frac{4}{5}$ yangi asosga o‘tib, quyidagi tenglamani hosil qilamiz.

$\frac{\log_{\frac{4}{5}} x}{\log_{\frac{4}{5}}\frac{1}{5}}+\frac{\log_{\frac{4}{5}} x}{\log_{\frac{4}{5}} 4}=1$ tenglamaning chap qismini umumiy maxrajga keltirib,

$\frac{\log_{\frac{4}{5}} x\left(\log_{\frac{4}{5}}\frac{1}{5}+\log_{\frac{4}{5}} 4\right)}{\log_{\frac{4}{5}}\frac{1}{5}\cdot\log_{\frac{4}{5}} 4}=1$ ni hosil qilamiz. $\log_{\frac{4}{5}}\frac{1}{5}+\log_{\frac{4}{5}} 4=\log_{\frac{4}{5}}\frac{4}{5}=1$ ekanini e’tiborga olsak, oxirgi tenglama quyidagi ko‘rinishni oladi:

$\log_{\frac{4}{5}} x=\log_{\frac{4}{5}}\frac{1}{5}\cdot\log_{\frac{4}{5}} 4$, bundan esa $x=\frac{4}{5}^{\log_{\frac{4}{5}}\frac{1}{5}\cdot\log_{\frac{4}{5}} 4}=\left(\frac{1}{5}\right)\log_{\frac{4}{5}} 4$ bo‘ladi.

Javob: $x=(0{,}2)\cdot\log_{0,8} 4$

Mustaqil yechish uchun misollar

1. $\log_2 x+\frac{4}{\log_x 2}=5$ (2)

2. $\sqrt{\lg x}=\lg\sqrt{x}$ $(1{,}10^4)$

3. $\log_5^2 x-\log_{\sqrt{5}} x-3=0$ (0,2;125)

4. $2-\lg(2x-1)=\lg(x-9)$ (13)

5. $\log_2^2 x-3\log_2 2x+5=0$ (2;4)

6. $\frac{\lg(2x-5)}{\lg(3x^2-39)}=\frac{1}{2}$ (4;16)

7. $x+\log_2(12-2^x)=5$ (2;3)

8. $\log_3(3^x - 8) = 2 - x$ (2)

9. $\log_{\sqrt{3}} x - \frac{1}{\log_x 3} = 1$ (3)

10. $\log_8(x-2) - \frac{1}{2}\log_8(x-3) = \frac{1}{3}$ (4)

3. Ko‘rsatkichli - logarifmik tenglamalar

Bir vaqtda ko‘rsatkichli va logarifmik tenglamalarni yechish usullarini qo‘llash kerak bo‘ladigan ba’zi misollarni ko‘ramiz.

1- misol. $3^{\log_3^2 x} + x^{\log_3 x} = 162$ tenglamani yeching.

Yechish: Asosiy logarifmik ayniyatga asosan:

$x = 3^{\log_3 x}$,

$3^{\log_3^2 x} + \left(3^{\log_3 x}\right)^{\log_3 x} = 162$ bundan $2 \cdot 3^{\log_3^2 x} = 162$, yoki $3^{\log_3^2 x} = 81 <=> \log_3^2 x = 4$

oxirgi tenglama $\begin{cases} \log_3 x = -2 \\ x > 0 \end{cases}$ va $\begin{cases} \log_3 x = 2 \\ x > 0 \end{cases}$ sistemalarga teng kuchli.

Ularni yechib, $x_1 = 3^{-2} = \frac{1}{9}$; $x_2 = 3^2 = 9.$ ekanini topamiz

Javob: $x = \left\{\frac{1}{9}; 9\right\}$

2- misol. $x^{\lg 2x} = 5$ tenglamani yeching.

Yechish: Berilgan tenglamani 10 asosga ko‘ra logarifmlaymiz.

$\lg 2x \cdot \lg x = \lg 5,$ *t=lgx* belgilash kiritamiz. $t^2 + (\lg 2)t - \lg 5 = 0$ buni yechamiz:

$D = \lg^2 2 + 4\lg 5 = \lg^2 2 + 4(\lg 10 - \lg 2) = \lg^2 2 - 4\lg 2 + 4 = (\lg 2 - 2)^2$

u holda berilgan tenglama quyidagi ildizlarga ega bo‘ladi.

$t_1 = -1,$ va $t_2 = 1 - \lg 2 = \lg 5$ bundan esa $\begin{cases} \lg x = -1 \\ \lg x = \lg 5 \end{cases}$ yoki $\begin{cases} x = \frac{1}{10} \\ x = 5 \end{cases}$ ni olamiz

Javob: $\left\{\frac{1}{10}; 5\right\}.$

Mustaqil yechish uchun misollar

1. $x^{1+\lg x}=100$ $\left(\frac{1}{100};10\right)$

2. $\lg(x^{\lg x})=1$ (0,1;10)

3. $x^{\lg x}=10000$ (0,01;100)

4. $x^{\log_5(x-1)}=25$ $\left(\frac{1}{6};25\right)$

5. $x^{\log_3(3x)}=9$ $\left(\frac{1}{9};3\right)$

6. $x^{\log_5(x-2)}=125$ (0,2;125)

7. $x^{\lg x-2}=1000$ $\left(\frac{1}{10};1000\right)$

8. $x^{1-\frac{\lg x}{4}}=10$ (100)

9. $x^{\lg x-3}=0{,}01$ (10;100)

10. $x^{\lg x}=100x$ $\left(\frac{1}{10};100\right)$

4. Ko‘rsatkichli va logarifmik tenglamalar sistemasi

Logarifmik tenglamalarni o‘z ichiga olgan sistemalarni yechishda ham algebraik tenglamalar sistemasini yechishdagi usullardan foydalaniladi.

1- misol. $\begin{cases} x^{\log_3 y}+2y^{\log_3 x}=27 \\ \log_3 y-\log_3 x=1 \end{cases}$ sistemani yeching.

Yechish: Asosiy logarifmik ayniyatni hisobga olsak,
$y^{\log_3 x}=3^{\log_3 y\cdot\log_3 x}=x^{\log_3 y}$ ekanidan

$\begin{cases} 3\cdot x^{\log_3 y}=27 \\ \log_3\left(\frac{y}{x}\right)=1 \end{cases} \Leftrightarrow \begin{cases} x^{\log_3 y}=9 \\ \frac{y}{x}=3 \end{cases}$ ni topamiz

Sistemaning 1-tenglamasini 3 asosga ko‘ra logarifmlaymiz:

$$\begin{cases}\log_3 x\cdot\log_3 y=2\\ y=3x\end{cases} <=> \begin{cases}\log_3 x\cdot\log_3(3x)=2\\ y=3x\end{cases} <=> \begin{cases}\log_3 x(1+\log_3 x)-2=0\\ y=3x\end{cases} <=>$$

$$<=> \begin{cases}\log_3^2 x+\log_3 x-2=0\\ y=3x\end{cases}$$ ni

hosil qilamiz. Uning 1- tenglamasini *log₃x* ga nisbatan kvadrat tenglama qilib yechsak, $\log_3 x=1$ yoki $x_1=3$

$\log_3 x=-2$ yoki $x_2=\frac{1}{9}$ ni hosil qilamiz.

$y_1=3\cdot x_1=9$ va $y_2=3\cdot x_2=3\cdot\frac{1}{9}=\frac{1}{3}$

Javob: $\left\{(3;9),\left(\frac{1}{9};\frac{1}{3}\right)\right\}$

2-misol. $\begin{cases}5^{\sqrt[3]{x}}\cdot 2^{\sqrt{y}}=200\\ 5^{2\sqrt[3]{x}}+2^{2\sqrt{y}}=689\end{cases}$ sistemani yeching.

Yechish: $u=5^{\sqrt[3]{x}}$, $\vartheta=2^{\sqrt{y}}\,(u>0,\vartheta>0)$ belgilash kiritamiz. U holda berilgan sistema quyidagi ko‘rinishga keladi:

$$\begin{cases}u\cdot\vartheta=200, & (1)\\ u^2+\vartheta^2=689, & (2)\end{cases}$$

(1) tenglamani har ikki qismini 2 ga ko‘paytirib , (2) tenglamaga qo‘shsak, $u^2+2u\vartheta+\vartheta^2=1089 <=> (u+\vartheta)^2=1089$ ni olamiz. u>0, ϑ>0 bo‘lgani uchun u+ϑ>0 ,ya’ni $u+\vartheta=\sqrt{1089}=33$ bo‘ladi. Shunday qilib, (1) va (2) tenglamalar sistemasi quyidagi simmetrik sistemaga teng kuchli bo‘ladi:

$$\begin{cases}u\cdot\vartheta=200\\ u+\vartheta=33\end{cases}$$

$t^2-33t+200=0$ xarakteristik tenglama tuzamiz. Uning ildizlari $t_1=8$ va $t_2=25$ bo‘ladi. Bundan esa

$\begin{cases}u=25\\ \vartheta=8\end{cases}$ va $\begin{cases}u=8\\ \vartheta=25\end{cases}$ bo‘ladi. *u* va ϑ larning ifodalarini o‘rniga qo‘yib, quyidagi sistemalarni hosil qilamiz:

$\begin{cases}5^{\sqrt[3]{x}}=25\\ 2^{\sqrt{y}}=8\end{cases}$ va $\begin{cases}5^{\sqrt[3]{x}}=8\\ 2^{\sqrt{y}}=25\end{cases}$. Ularni yechamiz:

1). $$\begin{cases}5^{\sqrt[3]{x}}=25\\ 2^{\sqrt{y}}=8\end{cases} <=> \begin{cases}\sqrt[3]{x}=2\\ \sqrt{y}=3\end{cases} <=> \begin{cases}x=8\\ y=9,\end{cases}$$

2). $$\begin{cases}5^{\sqrt[3]{x}}=8\\ 2^{\sqrt{y}}=25\end{cases} <=> \begin{cases}\sqrt[3]{x}=\log_5 8\\ \sqrt{y}=\log_2 25\end{cases} <=> \begin{cases}x=(\log_5 8)^3\\ y=(\log_2 25)^2\end{cases}$$

Javob: $\{(8;9);(\log_5^3 8;\log_2^2 25)\}$

Mustaqil yechish uchun misollar

1. $\begin{cases} 6^{3y^2-4y} = \dfrac{1}{6} \\ \dfrac{x}{y} = 5 \end{cases}$ $\left\{\left(\dfrac{5}{3};\dfrac{1}{3}\right);(5;1)\right\}$

2. $\begin{cases} \log_2(x^2+y^2) = 5 \\ 2\log_4 x + \log_2 y = 4 \end{cases}$ $\{4;4\}$

3. $\begin{cases} x - y = 2 \\ 3^x \cdot 2^y = 324 \end{cases}$ $\{4;2\}$

4. $\begin{cases} 4^x \cdot 3^y = 36 \\ y - x = 1 \end{cases}$ $\{1;2\}$

5. $\begin{cases} \log_{\sqrt{2}}(x-y) = 2 \\ \log_2 x - 4 = \log_2 3 - \log_2 y \end{cases}$ $\{8;6\}$

6. $\begin{cases} \log_{\sqrt{2}}(y-x) = 4 \\ 3^x \cdot 2^y = 576 \end{cases}$ $\{2;6\}$

7. $\begin{cases} 3^{\log_3}(x-y) = 1 \\ \log_3(2x-1) + \log_3 y = 1 \end{cases}$ $\{2;1\}$

8. $\begin{cases} 2^{\log_2(3x-4)} = 8 \\ \log_9(x^2-y^2) - \log_3(x+y) = 0{,}5 \end{cases}$ $\{4;1\}$

9. $\begin{cases} 4^x \cdot 2^x = 16 \\ \log_{\sqrt{3}}(x+y) = 2 \end{cases}$ $\left\{\dfrac{4}{3};\dfrac{5}{3}\right\}$

10. $\begin{cases} y = 1 + \log_4 x \\ x^y = 4^6 \end{cases}$ $\left\{(16;3);\left(\dfrac{1}{64};-2\right)\right\}$

5. Ko‘rsatkichli va logarifmik tengsizliklar

Ko‘rsatkichli va logarifmik tengsizliklarni yechish ko‘rsatkichli va logarifmik funkciyalarning monotonlik xossasiga asoslangan . Agar *a>1* bo‘lsa, $a^{f(x)} < a^{g(x)}$ ko‘rsatkichli tengsizlik $f(x) < g(x)$ tengsizlikka, agar *0<a<1* bo‘lsa, $f(x) > g(x)$ bo‘lgan tengsizlikka tengkuchli bo‘ladi. Shunga o‘xshash *a>1* bo‘lsa, $\log_a f(x) < \log_a g(x)$ logarifmik tengsizlik $\begin{cases} f(x) > 0 \\ f(x) < g(x) \end{cases}$ tengsizlikka, agar *0<a<1* bo‘lsa, u holda $\begin{cases} g(x) > 0 \\ f(x) > g(x) \end{cases}$ tengsizlikka tengkuchli bo‘ladi.

1-misol. $\log_{0,3} \log_6 \frac{x^2+x}{x+4} < 0$ tengsizlikni yeching .

Yechish: $0 < 0,3 < 1$ va $\log_{0,3} 1 = 0$ ekanidan $\log_6 \frac{x^2+x}{x+4} > 1$ tengsizlikni hosil qilamiz. Oxirgi tengsizlikdan $6 > 1$ va $\log_6 6 = 1$ ekanini e’tiborga olsak, quyidagi tengsizlikni olamiz:

$\frac{x^2+x}{x+4} > 6$, uni yechamiz.

$$\frac{x^2+x}{x+4} > 6 <=> \frac{x^2-5x-24}{x+4} <=> \frac{(x-8)(x+3)}{x+4} > 0 <=> (x-8)(x+3)(x+4) > 0$$

Oraliqlar usuli yordamida bu tengsizlikni yechib, $x \in (-4;-3) \cup (8;+\infty)$ ekanini topamiz.

2-misol. $\log_{2x}(x^2 - 5x + 6) < 1$ tengsizlikni yeching.

Yechish: *a)* Aytaylik $2x > 1$ yoki $x > \frac{1}{2}$ bo‘lsin, u holda berilgan sistema quyidagi sistemaga tengkuchli bo‘ladi:

$$\begin{cases} x^2 - 5x + 6 < 2x \\ x^2 - 5x + 6 > 0 \end{cases} \text{ yoki } \begin{cases} (x-1)(x-6) < 0 \\ (x-2)(x-3) > 0 \end{cases}$$

1-tengsizlikning yechimi $x \in (1;6)$, 2-tengsizlik $x \in (-\infty;2) \cup (3;+\infty)$ yechimga ega bo‘ladi. Bu yechimlardan umumiy yechimni topamiz: $x \in (1;2) \cup (3;6) \in \left(\frac{1}{2};+\infty\right)$

b) Aytaylik $0 < 2x < 1$, ya’ni $0 < x < \frac{1}{2}$ bo‘lsin, u holda berilgan tengsizlik $x^2 - 5x + 6 > 2x$ tengsizlikka tengkuchli bo‘ladi. Uni yechib, $x \in (-\infty;1) \cup (6;+\infty)$ ni topamiz. Bu to‘plamni $x \in \left(0;\frac{1}{2}\right)$ to‘plam bilan kesib, $x \in \left(0;\frac{1}{2}\right)$ ni topamiz.

a) va *b)* qismlarda topilgan yechimlarni birlashtirib, $x \in \left(0;\frac{1}{2}\right) \cup (1;2) \cup (3;6)$ yechimni topamiz.

3-misol. $4^{-x+0.5} - 7 \cdot 2^{-x} - 4 < 0$ tengsizlikni yeching.

Yechish: Berilgan tengsizlikni quyidagicha yozamiz

$2 \cdot \left(2^{-x}\right)^2 - 7 \cdot 2^{-x} - 4 < 0,\quad t = 2^{-x}$ belgilash kiritib,

$\begin{cases} t > 0 \\ 2t^2 - 7t - 4 < 0 \end{cases}$ sistemani olamiz. Uni yechamiz.

$\begin{cases} t > 0 \\ 2t^2 - 7t - 4 < 0 \end{cases} \Leftrightarrow \begin{cases} t > 0 \\ 2(t - 0{,}5)(t - 4) < 0 \end{cases}$, $\mathrm{t} > 0$ u holda $2(\mathrm{t} + 0{,}5) > 0,$ demak oxirgi sistema $\begin{cases} t > 0 \\ t - 4 < 0 \end{cases}$ sistemaga tengkuchli, ya’ni

$\begin{cases} t > 0 \\ t < 4 \end{cases} \Leftrightarrow 0 < t < 4$

Yuqorida kiritilgan belgilashni hisobga olib, $2^{-x} < 2^2 \Leftrightarrow -x < 2 \Leftrightarrow x > -2$ ni hosil qilamiz.

Javob: $x \in (-2;+\infty)$

4- misol. $\log_3(4^x + 1) + \log_{4^x+1} 3 > 2{,}5$ tengsizlikni yeching.

Yechish: $\log_{(4^x+1)} 3 = \dfrac{1}{\log_3(4^x + 1)}$ ekanini e’tiborga olsak, berilgan tengsizlikni quyidagicha yozish mumkin.

$\log_3(4^x + 1) + \dfrac{1}{\log_3(4^x + 1)} - 2{,}5 < 0$, bunda $y = \log_3(4^x + 1)$ deb belgilasak,

$y + \dfrac{1}{y} - 2{,}5 < 0$ tengsizlikni hosil qilamiz. Uni yechamiz.

$y(y^2 - 2{,}5y + 1) > 0 \Leftrightarrow y(y - 2)(y - \frac{1}{2}) > 0$ Oraliqlar usulidan foydalanib, $y \in (0;\frac{1}{2}) \cup (2;+\infty)$ ekanini topamiz. Shunday qilib, berilgan tengsizlik quyidagi sistemaga tengkuchli.

$$\begin{cases} 0 < \log_3(4^x + 1) < \frac{1}{2} \\ \log_3(4^x + 1) > 2 \end{cases} \Leftrightarrow \begin{cases} 1 < 4^x + 1 < 3^{\frac{1}{2}} \\ 4^x + 1 > 9 \end{cases} \Leftrightarrow \begin{cases} 0 < 4^x < \sqrt{3} - 1 \\ 4^x > 8 \end{cases} \Leftrightarrow \begin{cases} x < \log_4(\sqrt{3} - 1) \\ x > \log_4 8 = \frac{3}{2}\log_2 2 = 1{,}5. \end{cases}$$

Javob: $x \in (-\infty; \log_4(\sqrt{3}-1)) \cup (1,5;+\infty)$

Mustaqil yechish uchun misollar

1. $\log_8(x^2-4x-3)<1$ $\qquad$ $x \in (-1;1) \cup (-3;5)$

2. $2^{x^2+3x} > 8 \cdot 2^x$ $\qquad$ $x \in (-\infty;-3) \cup (1;+\infty)$

3. $\log_{\frac{1}{4}} \frac{36-x}{x} \le -\frac{1}{2}$ $\qquad$ $x \in (0;12)$

4. $\left(\frac{2}{3}\right)^{x^2+x-3} \le \frac{8}{27}$ $\qquad$ $x \in (-\infty;-3] \cup [2;+\infty)$

5. $3{,}5^{x^2-6x} \ge 1$ $\qquad$ $x \in (-\infty;0] \cup [6;+\infty)$

6. $\log_2(x^2-x-19{,}5) \ge 1$ $\qquad$ $x \in (-\infty;-4) \cup (5;+\infty)$

7. $4^{x-1} \le \left(\frac{1}{2}\right)^{x-2}$ $\qquad$ $x \in \left(-\infty;-\frac{4}{3}\right]$

8. $(x-5)\log_2^2(x+8) \le 8$ $\qquad$ $x \in (-8;5)$

9. $\log_{0,5}(x-5) \ge \log_{0,5}(3x-1)$ $\qquad$ $x \in (5;+\infty)$

10. $\frac{0{,}2^{x-0,5}}{\sqrt{5}} > 5 \cdot 0{,}04^{x-1}$ $\qquad$ $x \in (3;+\infty)$

6. Kombinaciyalashgan tenglamalar

1-misol. $\frac{1}{2}+16^{\sin x}=\frac{6}{16^{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}}$ tenglamani yeching.

Yechish: $16^{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}=16^{\frac{1+\cos\left(x+\frac{\pi}{2}\right)}{2}}=4^{1-\sin x}=\frac{4}{4^{\sin x}}$, u holda berilgan tenglamani

$\frac{1}{2}+16^{\sin x}=\frac{3\cdot 4^{\sin x}}{2}$ deb yozish mumkin. Avvalo bu tenglamani ko‘rsatkichli deb qaraymiz. $u=4^{\sin x}$ yangi o‘zgaruvchini kiritamiz. U holda

$\frac{1}{2}+u^2=\frac{3u}{2}$ yoki $2u^2-3u+1=0$ ni hosil qilamiz. Uni yechib,

$u_1=1,\quad u_2=\frac{1}{2}$ ni topamiz. U holda

1) $4^{\sin x}=1$, $\sin x=0$ 2) $4^{\sin x}=\frac{1}{2}$, $\sin x=-\frac{1}{2}$ larga ega bo‘lamiz.

Ularni yechamiz:

$\sin x=0$

$x=\pi k,$ ёки $x=(-1)^{n+1}\cdot\frac{\pi}{6}+\pi$

Javob: $x=\pi k$; $x=(-1)^{n+1}\cdot\frac{\pi}{6}+\pi.$

2- misol. $(tgx)^{sinx}=(ctg)^{cosx}$ tenglamani yeching.

Yechish: Bu tenglamani ko‘rsatkichli-darajali tenglama deb qaraymiz.

$(tgx)^{sinx}=(tg)^{-cosx}$, (1)

$\begin{cases} tgx<0, \\ tgx\neq -1; \end{cases}$ $tgx=-1,$ $tgx=0;$ $\begin{cases} tgx<0, \\ tgx\neq 1; \end{cases}$ $tgx=1,$

Agar $tgx<0$ bo‘lsa, $tgx\neq -1$, u holda (1) ga ko‘ra $sinx=-cosx$, yoki $tgx=-1$ hosil bo‘ladi. Bu esa $tgx\neq -1$ shart bilan birgalikda emas.

Agar $tgx=-1$ bo‘lsa, u holda $|\sin x|=|\cos x|=\frac{\sqrt{2}}{2}$.Bu esa (1)da manfiy son irracional darajaga ko‘tarilgan bo‘lib, u ma’noga ega emas.

Agar $tgx=0$ bo‘lsa, $x=\pi n$. U holda $sinx=0$. Demak (1) tenglamaning chap qismi 0^0 bo‘lib, u ma’noga ega bo‘lmaydi. Agar $tgx>0$ bo‘lsa, u holda $tgx\neq 1$ bo‘lib, (1)dan $sinx=-cosx$ yoki $tgx=-1$ hosil bo‘lib, $tgx>0$ shartga zid bo‘ladi. Nihoyat $tgx=1$ bo‘lsa, u holda (1)dan $1^{sinx}=1^{-cosx}$ yoki $1=1$. Demak (1) tenglama $tgx=1$ ga keladi, undan esa $x=\frac{\pi}{4}+\pi k$ yechimni olamiz.

Javob: $x=\frac{\pi}{4}+\pi k$.

Foydalanilgan adabiyotlar

1. В.А. Протасеня и другие. Руководство к решению задач по математике Минск–1991г, «Вышэйшая школа»
2. R.Xaitov E.O. Qoraboev, M. Tojiboev. Matematikadan metodik ko‘rsatma. Farg‘ona – 1991y.
3. N.K Mamadaliyev. Matematikadan metadik qo‘llanma I,II qism. Toshkent – 1992y.
4. В.Б Лидский ва бошыалар. Задачи по элементарной математики. «Наука», Москва – 1970 г.
5. П. Т. Дыбов. ва бошыалар. Сборник задач по математике. «Высшая школа», Москва – 1983 г.
6. Е. К. Васенкова, Методические указания по математике. Москва –1990 г.
7. N.K. Mamadaliev «Matematika» Toshkent - 2002 y.

Mundarija