O‘ZBEKISTON RESPUBLIKASI OLIY VA
O‘RTA MAXSUS TA’LIM VAZIRLIGI

O‘RTA MAXSUS, KASB-HUNAR TA’LIMI MARKAZI

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ALGEBRA VA MATEMATIK ANALIZ ASOSLARI

II qism

Akademik litseylar uchun darslik

7- nashri

„O‘QITUVCHI“ NASHRIYOT-MATBAA IJODIY UYI
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O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi O'rta maxsus, kasb-hunar ta'limi markazi tomonidan akademik litseylar uchun darslik sifatida tavsiya etilgan va undan kasb-hunar kollejlari o'quvchilari ham foydalanishlari mumkin.

O'zbekiston Respublikasida xizmat ko'rsatgan
Xalq ta'limi xodimi **H. A. NASIMOV**ning
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SO‘ZBOSHI

Ushbu «Algebra va matematik analiz asoslari (II qism)» darsligi shu nomli kitob I qismining uzviy davomi bo‘lib, akademik litseylar va kasb-hunar kollejlari uchun mo‘ljallangan hamda shu fan bo‘yicha akademik litseylar va kasb-hunar kollejlari o‘quv rejasiga asosan aniq fanlar yo‘nalishi, tabiiy fanlar yo‘nalishi, shuningdek, matematika umumta’lim fani sifatida o‘rganiladigan guruhlarining algebra va matematik analiz asoslari kursining o‘quv dasturidagi barcha materiallarni o‘z ichiga oladi.

Mualliflarning SamDU qoshidagi akademik litseyda to‘plagan ish tajribalari asosida yaratilgan va o‘n bobdan iborat bo‘lgan ushbu darslikda quyidagi mavzular yoritilgan:

- Trigonometrik funksiyalar.
- Nostandart tenglamalar, tengsizliklar va sistemalar.
- Sonli ketma-ketliklar va ularning limiti.
- Funksiyaning limiti va uzluksizligi.
- Hosila.
- Integral.
- Differensial tenglamalar.
- Kombinatorika elementlari.
- Ehtimollik nazariyasi va matematik statistika elementlari.
- Chiziqli algebra elementlari.

Har bir bob paragraflarga, paragraflar esa bandlarga bo‘lingan.

Darslikning yaratilish jarayonida o‘zlarining qimmatli maslahatlarini ayamagan SamDU qoshidagi akademik litseyning oliy toifali matematika o‘qituvchilari B. I. Usmonov va Z. A. Pashayevga, shuningdek, kitob qo‘lyozmasini kompyuterda sahifalagan I. Nasimov va A. Siddiqovga o‘z minnatdorchiligimizni bildiramiz.

Ayniqsa, kitobni diqqat bilan o‘qib chiqib, uning sifatini yaxshilash bo‘yicha qimmatli maslahatlar bergan Samarqand viloyati Ishtixon tumanidagi 21- o‘rta maktabning oliy toifali matematika o‘qituvchisi, O‘zbekiston Respublikasida xizmat ko‘rsatgan Xalq ta’limi xodimi A. A. Nasimovga samimiy tashakkur izhor etishni o‘z burchimiz deb bilamiz.

Mualliflar

I BOB

TRIGONOMETRIK FUNKSIYALAR

1-§. Sonli argumentning trigonometrik funksiyalari

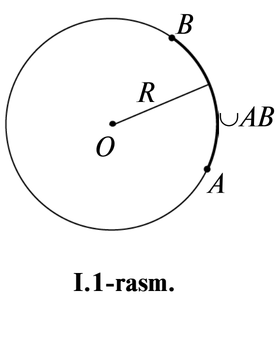
1. Burchaklar va yoylar. Markazi O nuqtada bo'lgan R radiusli aylanadagi A va B nuqtalar uni ikki qismga – *yoylar*ga ajratadi (I.1-rasm). Yoyning A va B nuqtalari *yoyning uchlari*, qolgan nuqtalari esa *yoyning ichki nuqtalari* deyiladi.

Uchlari A va B nuqtalar bo'lgan yoy uning faqat uchlarini ko'rsatish orqali $\cup AB$ ko'rinishda yoki uchlari A va B nuqtalar bo'lgan yoylarni bir-biridan farqlash uchun yoyning uchlari va yoyning biror ichki K nuqtasini ko'rsatish orqali $\cup AKB$ ko'rinishda belgilanadi (I.2-rasm).

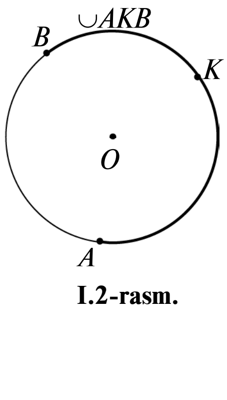
Agar AB kesma aylananing diametri bo'lsa, AB yoy *yarim aylana* deyiladi. Agar AB kesma aylananing diametri bo'lmasa va AB yoyning har qanday ichki nuqtasini aylananing markazi bilan tutashtiruvchi kesma AB kesmani kesib o'tsa (kesib o'tmasa), AB yoy *yarim aylanadan kichik* (mos ravishda *yarim aylanadan katta*) deyiladi.

Aylananing markazidan chiquvchi va berilgan yoyni kesib o'tuvchi barcha nurlardan tashkil topgan yassi burchakni *berilgan yoyga mos markaziy burchak*, berilgan yoyni esa shu *markaziy burchakka mos yoy* deb ataymiz (I.3-rasm).

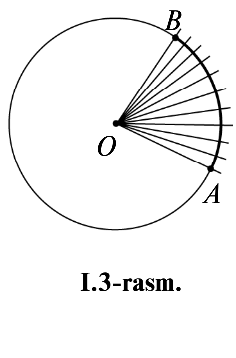
Yoy uzunligi metr (m) va uning ulushlarida, shuningdek, fut, duym, angstrem, mikronlarda ham o'lchanadi (1 fut = =12 duym $\approx$ 30,479 sm, 1 angstrem = $1 \cdot 10^{-8}$ sm, 1 mikron= =$1 \cdot 10^{-3}$ mm).



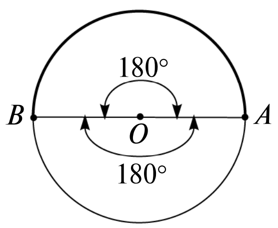
I.1-rasm.



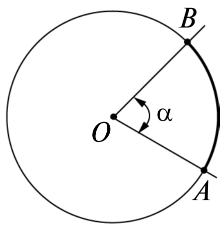
I.2-rasm.



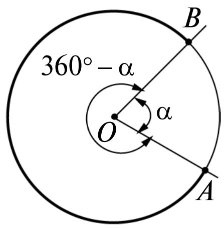
I.3-rasm.



I.4-rasm.



I.5-rasm.



I.6-rasm.

Geometriya kursidan ma'lumki, markaziy burchakning gradus o'lchovi va yoyning gradus o'lchovi quyidagicha aniqlanadi:

1) yarim aylanaga mos markaziy burchak 180° ga teng (I.4-rasm);

2) yarim aylanadan kichik AB yoyga mos markaziy burchakning gradus o'lchovi OA va OB nurlar hosil qilgan odatdagi burchakning gradus o'lchoviga teng (bu yerda O — aylana markazi, I.5-rasm);

3) yarim aylanadan katta yoyga mos markaziy burchakning gradus o'lchovi $360° - \alpha$ ga teng, bu yerda α — to'ldiruvchi burchak (yarim aylanadan kichik yoyga mos markaziy burchak)ning gradus o'lchovi (I.6-rasm).

4) yoyning gradus o'lchovi shu yoyga mos markaziy burchakning gradus o'lchoviga teng (I.7-rasm).

Burchak va yoylarning burchak kattaligini o'lchashda gradusning ulushlaridan ham foydalanishga to'g'ri keladi. Gradus va uning ayrim ulushlari orasidagi bog'lanishlarni keltiramiz:

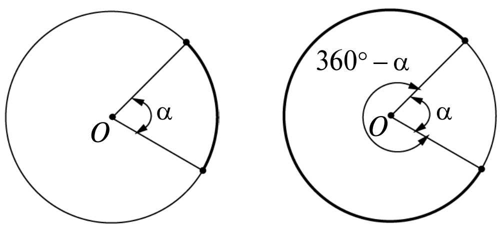
$$1° = 60' \text{ (minut, daqiqa)}, \quad 1' = 60'' \text{ (sekund, soniya)}.$$

Buyuk o'zbek olimi Mirzo Ulug'bek o'z asarlarida sekundning $\frac{1}{60}$ ulushi solisa(tersiy)dan ham foydalangan. Uning asarlarida

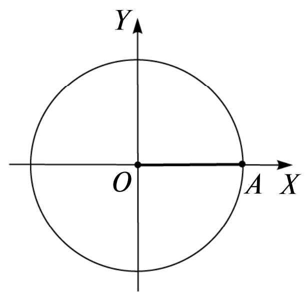
$$1 \text{ daraja} = 60 \text{ daqiqa}, \quad 1 \text{ daqiqa} = 60 \text{ soniya},$$

$$1 \text{ soniya} = 60 \text{ solisa(tersiy)}$$

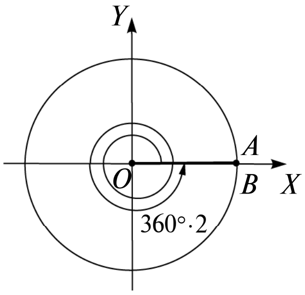
ekanligi keltiriladi.



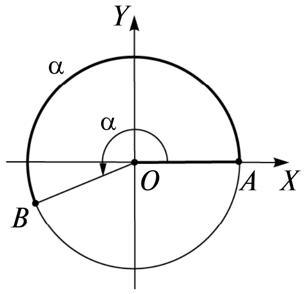
I.7-rasm.



I.8-rasm.



I.9-rasm.



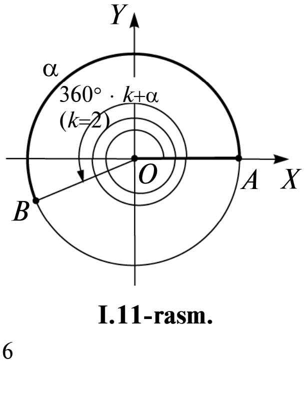
I.10-rasm.

Tekislikda to‘g‘ri burchakli *XOY* Dekart koordinatalari sistemasi kiritilgan bo‘lsin. Markazi koordinatalar boshida bo‘lgan *R* radiusli aylanani qaraymiz (I.8-rasm). Bu aylana *OX* o‘qning musbat yarim o‘qini *A* nuqtada kessin. *OA* radius *boshlang‘ich radius*, *A* nuqta esa *boshlang‘ich nuqta* deb ataladi.

OX o‘qning musbat yarim o‘qini koordinatalar boshi (qo‘zg‘almas nuqta) atrofida musbat yo‘nalish(soat strelkasining harakat yo‘nalishiga qarama-qarshi yo‘nalish)da va manfiy yo‘nalish (soat strelkasining harakat yo‘nalishi)da istalgancha uzluksiz siljitish (harakatlantirish) mumkin deb hisoblaymiz.

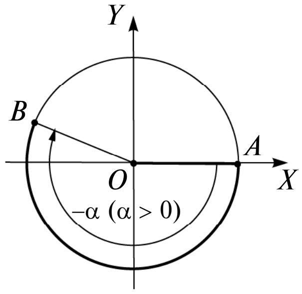
OX musbat yarim o‘q qo‘zg‘almas *O* nuqta atrofida musbat yo‘nalishda siljitilsa, *OA* radius biror *OB* radiusga o‘tadi. Agar *OA* va *OB* radiuslar ustma-ust tushsa (I.9-rasm), siljitish natijasida *A* nuqta aylanani bir yoki bir necha marta to‘liq aylanib chiqqan bo‘ladi. Bu holda biz boshlang‘ich tomoni *OA* va oxirgi tomoni *OB* bo‘lgan aylanish burchagiga ega bo‘lamiz. Uning gradus o‘lchovi $360° \cdot k$ ga teng, bu yerda k – aylanishlar soni.

Agar *OA* va *OB* radiuslar ustma-ust tushmasa, *A* nuqta aylanani to‘liq aylanib chiqmagan yoki aylanani bir yoki bir necha marta aylanib chiqib, yana *AB* yoyni bosib o‘tgan bo‘ladi. Bu holda boshlang‘ich tomoni *OA* va oxirgi tomoni *OB* bo‘lgan burish burchagiga ega bo‘lamiz. Bu burish burchagining gradus o‘lchovi quyidagicha aniqlanadi:

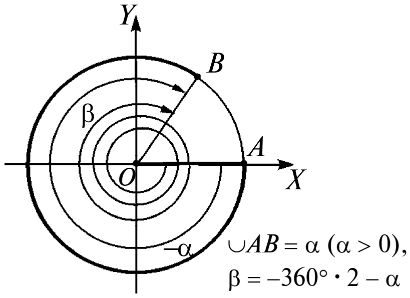


I.11-rasm.

1) *A* nuqta aylanani to‘liq aylanib chiqmagan bo‘lsa (I.10-rasm), burish burchagining gradus o‘lchovi *AB* yoyning gradus o‘lchoviga teng;



I.12-rasm.



I.13-rasm.

2) A nuqta aylanani k ($k \in N$) marta aylanib chiqib, yana AB yoyni bosib o'tgan bo'lsa (I.11-rasm), burish burchagining gradus o'lchovi $360° \cdot k + \alpha$ ga teng, bu yerda α — shu AB yoyning gradus o'lchovi.

Endi OX musbat yarim o'qni qo'zg'almas O nuqta atrofida manfiy yo'nalishda siljitamiz. Xuddi yuqoridagi kabi mulohazalar yuritib, gradus o'lchovlari $-360° \cdot k$ (k — aylanishlar soni) bo'lgan aylanish burchaklariga hamda gradus o'lchovlari $-360° \cdot k - \alpha$ (bu yerda $k \in \{0;\ 1;\ 2;\ 3;\ ...\}$) ga teng bo'lgan burish burchaklariga ega bo'lamiz (I.12- va I.13-rasmlar).

Gradus o'lchovi 0° ga teng burchakni ham qaraymiz. Bu burchak boshlang'ich nuqta o'z o'rnida harakatsiz turgan holatga mos keladi. Shu sababli uni burish burchagi sifatida ham, aylanish burchagi sifatida ham qarash mumkin.

Mashqlar

1.1. 1) Markaziy burchak 18° ga teng. Gradus o'lchovi shu markaziy burchakning gradus o'lchovidan $3\frac{1}{2}$ marta katta bo'lgan markaziy burchakka mos yoyning gradus o'lchovini toping;

2) Gradus o'lchovi 54° li yoyga mos markaziy burchakning gradus o'lchovidan 3 marta kichik bo'lgan markaziy burchakning gradus o'lchovini toping.

1.2. 1) 3600° li burchak aylanish burchagi bo'la oladimi?
2) –3600° li burchak 0° li burchakka tengmi?
3) –542° li burish burchagida nechta aylanish burchagi bor?

1.3. 1) Markazi koordinatalar boshida bo‘lgan $R = 3$ sm radiusli aylana chizing. Boshlang‘ich radiusni 540° buring va hosil bo‘lgan yoy uzunligini toping;

2) Markazi koordinatalar boshida bo‘lgan $R = 3$ sm radiusli aylana chizing. Boshlang‘ich radiusni –270° buring va hosil bo‘lgan yoyning uzunligini toping.

2. Burchak va yoylarning radian o‘lchovi. Koordinatali aylana. Burchak va yoylarning burchak kattaliklarini o‘lchashning yana bir sistemasi — *radian o‘lchovi sistemasi* bilan tanishamiz.

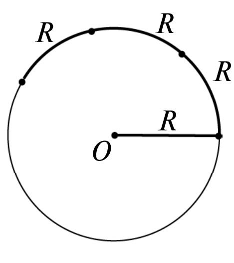
R radiusli aylanani qaraylik (I.14-rasm). Uzunligi $2\pi R$ bo‘lgan bu aylanada umumiy ichki nuqtaga ega bo‘lmagan va har birining uzunligi *R* ga teng bo‘lgan 2π ta yoy mavjud. Bu yoylardan har birining, shuningdek ularga mos har bir markaziy burchakning burchak kattaligi $\frac{360°}{2\pi} = \frac{180°}{\pi}$ ga tengdir. Demak, uzunligi aylana radiusiga teng yoyning va unga mos markaziy burchakning burchak kattaligi aylana radiusiga bog‘liq emas. Shu sababli, uzunligi aylana radiusiga teng bo‘lgan yoyning burchak kattaligini shu aylana yoylarini o‘lchashda o‘lchov birligi sifatida, unga mos markaziy burchak kattaligini esa burchaklarni o‘lchashda o‘lchov birligi sifatida olish mumkin.

Uzunligi aylana radiusiga teng yoy *1 radianli yoy*, unga mos markaziy burchak esa *1 radianli burchak* deyiladi (I.15-rasm).

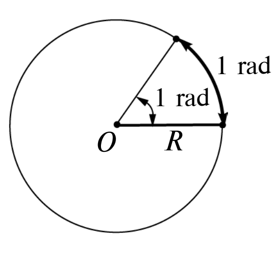
Yuqoridagi mulohazalardan quyidagi bog‘lanishlarni olamiz:

$$1 \text{ radian} = \frac{180°}{\pi}, \quad 1° = \frac{\pi}{180} \text{ radian}.$$

Bu ikki tenglik yordamida radian o‘lchovidan gradus o‘lchoviga o‘tish va gradus o‘lchovidan radian o‘lchoviga o‘tish formulalari hosil bo‘ladi:



I.14-rasm.



I.15-rasm.

$$a = \left(\frac{180a}{\pi}\right)^{\circ}, \qquad \alpha^{\circ} = \frac{\pi \cdot \alpha}{180}.$$

1-misol. $120°$ ni radianlarda, $\frac{3\pi}{4}$ va 5 (rad)larni esa graduslarda ifodalang.

Yechish. $\alpha^{\circ} = \frac{\pi \cdot \alpha}{180}$ (rad) formulaga ko'ra

$$120^{\circ} = \frac{\pi \cdot 120}{180}(\text{rad}) = \frac{2\pi}{3}$$

tenglikni, $a_{\text{rad}} = \left(\frac{180a}{\pi}\right)^{\circ}$ formulaga ko'ra

$$\frac{3\pi}{4} = \left(\frac{180 \cdot \frac{3\pi}{4}}{\pi}\right)^{\circ} = 135^{\circ} \quad \text{va} \quad 5 = \left(\frac{180 \cdot 5}{\pi}\right)^{\circ} = \left(\frac{900}{\pi}\right)^{\circ}$$

tengliklarni hosil qilamiz.

2-misol. Radiusi $R = 5$ (uzun. birl.) bo'lgan aylananing uzunligi $l = 10$ (uzun. birl.)ga teng yoyini graduslarda va radianlarda ifodalang.

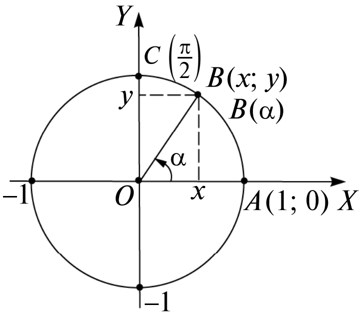
Yechish. $l = 10$ (uzun. birl.) $= \frac{10}{5}$ (rad) $= 2$ (rad) va

$l = 2\ (\text{rad}) = \left(\frac{180 \cdot 2}{\pi}\right)^{\circ} = \left(\frac{360}{\pi}\right)^{\circ} \approx 114^{\circ}19'52''$ tengliklarga egamiz.

3-misol. Agar radiusi $R = 4$ bo'lgan doiraviy sektorning yoyi 3 rad ga teng bo'lsa, shu sektorning S yuzini toping.

Yechish. Yoyi π rad ga teng doiraviy sektor (yarim doira)ning yuzi $\frac{\pi \cdot R^2}{2}$ (bu yerda R — radius) ga teng bo'lgani uchun, yoyi 1 rad bo'lgan doiraviy sektorning yuzi $\frac{R^2}{2}$ ga, yoyi a rad ga teng bo'lgan doiraviy sektorning yuzi esa $a \cdot \frac{R^2}{2}$ ga teng. Shu sababli $S = 3 \cdot \frac{4^2}{2} = 24$ kv. birlik.

Eslatma. 5 ning graduslarda ifodalangan aniq qiymati $\left(\frac{900}{\pi}\right)^{\circ}$ ga teng. Uning graduslarda ifodalangan taqribiy qiymatini hosil qilish uchun π ni uning kerakli aniqlikdagi taqribiy qiymati bilan almashtirish kerak bo'ladi. Masalan, $\pi \approx 3$ deb olinsa, $5 = \left(\frac{900}{\pi}\right)^{\circ} \approx \left(\frac{900}{3}\right)^{\circ} = 300^{\circ}$ ga ega bo'lamiz. Xuddi shu kabi, $1^{\circ} = \frac{\pi}{180^{\circ}}$ rad $\approx 0{,}017$ (rad), $1\ (\text{rad}) = \left(\frac{180}{\pi}\right)^{\circ} \approx$ $\approx 57^{\circ}17'44''$ munosabatlar hosil qilinadi.



I.16-rasm.

Tekislikda XOY Dekart koordinatalari sistemasi kiritilgan bo'lsin. Markazi koordinatalar boshi O (0; 0) da bo'lgan $R = 1$ radiusli aylananing A(1;0) nuqtasini *boshlang'ich nuqta*, OA radiusini esa *boshlang'ich radius* deb ataymiz (I.16-rasm) va shu aylanada koordinatalar sistemasini quyidagi tartibda kiritamiz.

Boshlang'ich nuqta A(1; 0) ni yangi koordinatalar sistemasining koordinatalar boshi (sanoq boshi) sifatida olamiz. Uning yangi koordinatalar sistemasidagi koordinatasi 0 ga teng. Boshlang'ich radiusni O(0; 0) nuqta atrofida α radianli burchakka buramiz (bu yerda va bundan keyin aylanish burchagi burish burchagining xususiy holi sifatida qaraladi). Natijada A nuqta aylananing biror $B(x;\ y)$ nuqtasiga o'tadi (I.16-rasm). $B(x;\ y)$ nuqtaning yangi koordinatasi (aylanadagi koordinatasi) α ga teng deb qabul qilamiz va $B(\alpha)$ ko'rinishda belgilaymiz.

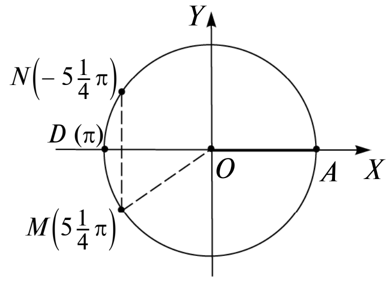
Masalan, C (0; 1) nuqta boshlang'ich radiusni O (0; 0) nuqta atrofida $\frac{\pi}{2}$ rad burchakka burishdan hosil qilinadi. Shu sababli, uning yangi koordinatalar sistemasidagi koordinatasi $\frac{\pi}{2}$ ga tengdir (I.16-rasm).

Aylananing har bir nuqtasi aylanadagi koordinatalar sistemasida cheksiz ko'p koordinatalarga ega, chunki boshlang'ich radiusni O(0; 0) nuqta atrofida α, $\alpha \pm 2\pi$, $\alpha \pm 4\pi$, ..., ya'ni $\alpha \pm 2k\pi$, $k \in Z$ burchaklarga burish natijasida boshlang'ich nuqta aylananing ayni bir B nuqtasiga o'tadi va $\alpha \pm 2k\pi$, $k \in Z$ sonlarning har biri B nuqtaning koordinatasi (aylanadagi koordinatasi!) bo'ladi.

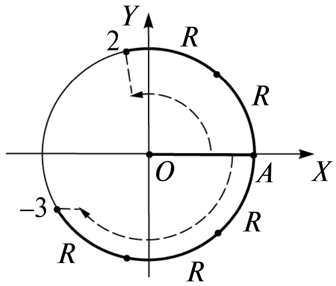
Yuqoridagi usul bilan koordinatalar sistemasi kiritilgan birlik aylana *koordinatali aylana* (yoki *koordinatalar aylanasi*) deb ataladi.

4-misol. Koordinatali aylanada $M\left(5\frac{1}{4}\pi\right)$, $N\left(-5\frac{1}{4}\pi\right)$ nuqtalarni belgilang.

Yechish. 1) $5\frac{1}{4}\pi = 2\cdot 2\pi + \left(\pi + \frac{\pi}{4}\right)$ bo'lgani uchun $M\left(5\frac{1}{4}\pi\right)$ va $M_1\left(\pi + \frac{\pi}{4}\right)$ nuqtalar koordinatali aylanada ustma-



I.17-rasm.



I.18-rasm.

ust tushadi. $D(\pi)$ nuqtani (I.17-rasm) musbat yo'nalish bo'yicha $\frac{\pi}{4} = 45^\circ$ burchakka burib, $M\left(5\frac{1}{4}\pi\right)$ nuqtani hosil qilamiz;

2) N va M nuqtalar AD diametrga nisbatan simmetrik nuqtalar bo'lgani uchun M nuqtani shu diametrga nisbatan simmetrik almashtirib, $N\left(-5\frac{1}{4}\pi\right)$ nuqtani hosil qilamiz (I.17-rasm).

5 - m i s o l . Koordinatali aylanada 2 va –3 sonlarini belgilang.

Y e c h i s h . 2 sonining koordinatali aylanadagi tasviri (koordinatasi 2 ga teng bo'lgan nuqta)ni topish uchun uzunligi 1 radian (aylana radiusi)ga teng bo'lgan yoyni boshlang'ich A nuqtadan boshlab, musbat yo'nalishda ketma-ket ikki marta qo'yamiz (I.18-rasm).

–3 sonining koordinatali aylanadagi tasvirini topish uchun uzunligi 1 radianga teng bo'lgan yoyni boshlang'ich A nuqtadan boshlab, manfiy yo'nalishda ketma-ket uch marta qo'yish yetarli (I.18-rasm).

M a s h q l a r

1.4. Aylana radiusi $R = 10$ sm, yoyi l (sm), yoki a (rad), yoki α° birliklarning birida berilgan. Yoy qolgan ikki birlikda ifodalansin:

1) $l = 1;\ 2;\ 5;\ 10;\ 20;\ 30;$

2) $\alpha = 2^\circ;\ 10^\circ;\ 10^\circ 30';\ 60^\circ;\ 90^\circ;\ 180^\circ;\ 350^\circ;\ -30^\circ;\ -45^\circ;$

3) $\alpha = 360^\circ;\ 540^\circ;\ 700^\circ;\ 720^\circ 30';\ 750{,}5^\circ;\ 1000{,}5^\circ;\ -450^\circ;\ -660^\circ;$

4) $\alpha = 2;\ 5;\ 10;\ 20\pi;\ 50{,}5\pi;\ -5;\ -\pi;\ -5\pi$ (rad).

1.5. Quyidagi nuqtalar birlik aylanada belgilansin hamda ularga aylana markaziga, gorizontal va vertikal diametrlarga nisbatan simmetrik joylashgan nuqtalar topilsin:

A $(\pi/8)$, B $(2\pi/3)$, C $(5\pi/8)$, D $(36°)$, E $(220°)$, F $(-75°)$, G (4), H (-5).

1.6. Muntazam sakkizburchakning ikki qo‘shni tomoni orasidagi burchagini graduslar va radianlarda ifodalang.

1.7. Aylana radiusi $R = 6$ dm. Yoylar $\alpha°$ kattalikda berilgan. Ularni radianlarda ifodalang va mos sektorlarning yuzini toping:

$\alpha = 12°;\ 15°;\ 22°30';\ 24°;\ 30°;\ 45°;\ 60°;\ 72°;\ 90°;\ 120°;\ 180°;\ 225°;$

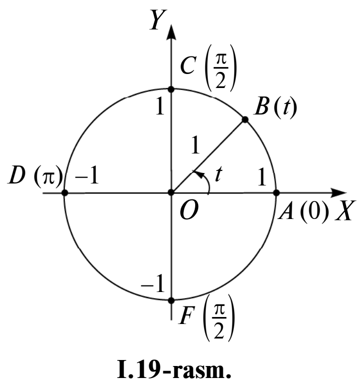
$270°;\ 315°;\ 330°.$

1.8. Jism $\omega = \frac{\pi}{6}$ rad/s burchak tezlik bilan aylanmoqda. U $t = 10$ s da qanday burchakka buriladi? 2 min da-chi?

3. Sonli argumentning sinusi, kosinusi, tangensi va kotangensi. Tekislikda XOY Dekart koordinatalar sistemasi kiritilgan va t haqiqiy son berilgan bo‘lsin. t haqiqiy songa koordinatali aylananing koordinatasi t ga teng bo‘lgan $B(t)$ nuqtasini mos qo‘yamiz (I.19-rasm).

$B(t)$ nuqtaning abssissasi *t sonning kosinusi*, ordinatasi esa *t sonning sinusi* deyiladi va mos ravishda $\cos t$, $\sin t$ orqali belgilanadi.

$B(t)$ nuqta ordinatasining shu nuqta abssissasiga nisbati (agar bu nisbat mavjud bo‘lsa) *t sonning tangensi* deyiladi va $\mathrm{tg}\,t$ orqali belgilanadi.



I.19-rasm.

$B(t)$ nuqta abssissasining shu nuqta ordinatasiga nisbati (agar bu nisbat mavjud bo‘lsa) *t sonning kotangensi* deyiladi va $\mathrm{ctg}\,t$ orqali belgilanadi.

Sonning sinusi, kosinusi, tangensi va kotangensi tushunchalarining aniqlanishidan ko‘rinadiki,

$$\mathrm{tg}\,t = \frac{\sin t}{\cos t} \quad (\cos t \neq 0), \qquad (1)$$

$$\text{ctg}t = \frac{\cos t}{\sin t} \quad (\sin t \neq 0) \tag{2}$$

munosabatlar o‘rinli va koordinatali aylananing $B(t)$ nuqtasi XOY koordinatalar sistemasidagi $B(\cos t;\ \sin t)$ nuqta bilan ustma-ust tushadi. $B(\cos t;\ \sin t)$ nuqta birlik aylanada yotgani sababli, uning koordinatalari shu birlik aylana tenglamasi $x^2 + y^2 = 1$ ni qanoatlantiradi:

$$\cos^2 t + \sin^2 t = 1. \tag{3}$$

Sonning sinusi va kosinusi tushunchalarining aniqlanishidan ko‘rinadiki, ixtiyoriy t haqiqiy son uchun $B(\cos t;\ \sin t)$ nuqta birlik aylanada yotadi. Shu sababli, (3) tenglik t ning har qanday haqiqiy qiymatida o‘rinli.

1 - m i s o l. $0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ sonlarining sinusi, kosinusi, tangensi va kotangensini toping.

Y e c h i s h. $0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ sonlariga koordinatali aylananing $A\ (0)$, $C\left(\frac{\pi}{2}\right)$, $D(\pi)$, $F\left(\frac{3\pi}{2}\right)$ nuqtalari mos keladi (I.19-rasm). Bu nuqtalar XOY koordinatalar sistemasida mos ravishda quyidagi koordinatalarga ega: $A\ (1;\ 0)$, $C\ (0;\ 1)$, $D\ (-1;\ 0)$, $F\ (0;\ -1)$.

Sonning sinusi, kosinusi, tangensi va kotangensi tushunchalarining aniqlanishiga ko‘ra, quyidagi tengliklarga ega bo‘lamiz:

$\cos 0 = 1$; $\cos\frac{\pi}{2} = 0$; $\cos \pi = -1$; $\cos\frac{3\pi}{2} = 0$;

$\sin 0 = 1$; $\sin\frac{\pi}{2} = 1$; $\sin \pi = 0$; $\sin\frac{3\pi}{2} = -1$;

$\text{tg}\ 0 = 0$; $\text{tg}\frac{\pi}{2}$ — mavjud emas; $\text{tg}\ \pi = 0$; $\text{tg}\frac{3\pi}{2}$ — mavjud emas;

$\text{ctg}\ 0$ — mavjud emas; $\text{ctg}\frac{\pi}{2} = 0$; $\text{ctg}\ \pi$ — mavjud emas; $\text{ctg}\frac{3\pi}{2} = 0$.

2 - m i s o l. $\sin\frac{\pi}{4}$, $\cos\frac{\pi}{4}$, $\text{tg}\frac{\pi}{4}$, $\text{ctg}\frac{\pi}{4}$ larni hisoblang.

Y e c h i s h. Koordinatali aylanada $B\left(\frac{\pi}{4}\right)$ nuqtani yasaymiz (I.20-rasm) va bu nuqtaning XOY koordinatalar tekisligidagi koordinatalarini aniqlaymiz.

OBC teng yonli to‘g‘ri burchakli uchburchakda $OB^2 = OC^2 + BC^2 = 2BC^2$ bo‘lgani uchun $2BC^2 = 1$ yoki $BC = \frac{\sqrt{2}}{2}$ ga ega bo‘lamiz. $B\left(\frac{\pi}{4}\right)$ nuqtaning abssissasi ham, ordinatasi ham musbatdir. Demak, $B\left(\frac{\pi}{4}\right)$ nuqta $B\left(\frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2}\right)$ nuqta bilan ustma-ust tushadi. sinα, cosα, tgα, ctgα larning aniqlanishiga ko‘ra,

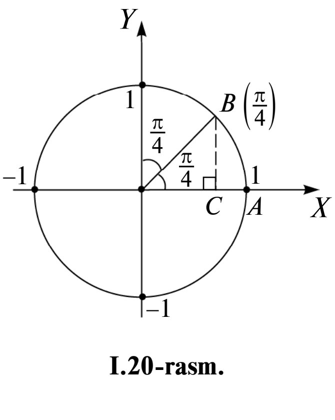
$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2},\quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2},\quad \mathrm{tg}\frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1,\quad \mathrm{ctg}\frac{\pi}{4} = 1$$

tengliklarga ega bo‘lamiz.

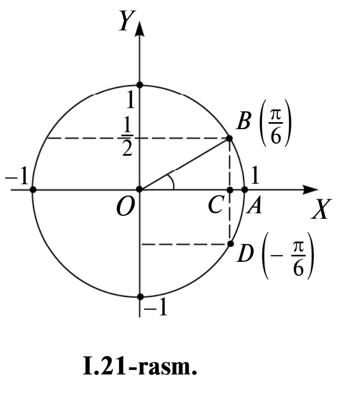
3-misol. $\frac{\pi}{6}$ va $-\frac{\pi}{6}$ ning sinusi, kosinusi, tangensi va kotangensini toping.

Yechish. $B\left(\frac{\pi}{6}\right)$ nuqtani yasaymiz (I.21-rasm) va bu nuqtaning dekart koordinatalarini topamiz. $B\left(\frac{\pi}{6}\right)$ nuqtaning dekart koordinatalari musbat sonlardir. OBC to‘g‘ri burchakli uchburchakda $BC = \frac{1}{2}OB = \frac{1}{2}\cdot 1 = \frac{1}{2}$ bo‘lgani uchun Pifagor teoremasiga ko‘ra $OC = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ bo‘ladi. Demak, $B\left(\frac{\pi}{6}\right)$ nuqta $B\left(\frac{\sqrt{3}}{2};\ \frac{1}{2}\right)$ nuqta bilan ustma-ust tushadi. Son argumentning sinusi, kosinusi, tangensi va kotangensining aniqlanishiga ko‘ra

$$\sin\frac{\pi}{6} = \frac{1}{2},\quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2},\quad \mathrm{tg}\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}},\quad \mathrm{ctg}\frac{\pi}{6} = \sqrt{3}.$$



I.20-rasm.



I.21-rasm.

$D\left(-\frac{\pi}{6}\right)$ va $B\left(\frac{\pi}{6}\right)$ nuqtalar OX o‘qqa nisbatan simmetrik bo‘lgani uchun $D\left(-\frac{\pi}{6}\right)$ nuqta $D\left(\frac{\sqrt{3}}{2};\ -\frac{1}{2}\right)$ nuqta bilan ustma-ust tushadi. Shu sababli

$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \qquad \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},$$

$$\operatorname{tg}\left(-\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}, \qquad \operatorname{ctg}\left(-\frac{\pi}{6}\right) = -\sqrt{3}.$$

$y = \sin t$, $y = \cos t$, $y = \operatorname{tg} t$ va $y = \operatorname{ctg} t$ formulalar bilan aniqlangan funksiyalar *asosiy trigonometrik funksiyalar* deyiladi. Ularning ayrim asosiy xossalarini keltiramiz.

1°. $y = \sin t$ *funksiya chegaralangan funksiya va barcha* $t \in R$ *lar uchun* $|\sin t| \le 1$ *munosabat o‘rinli.*

I s b o t . Biror $t \in R$ uchun $|\sin t| > 1$ bo‘lsin. U holda $\left|\sin^2 t\right| = |\sin t|^2 > 1$ bo‘lgani uchun $\sin^2 t + \cos^2 t = |\sin t|^2 + |\cos t|^2 \ge |\sin t|^2 + 0 = |\sin t|^2 > 1$, ya’ni $\sin^2 t + \cos^2 t > 1$ tengsizlikka ega bo‘lamiz. Bu esa (3) ga ziddir.

Demak, barcha $t \in R$ sonlar uchun $|\sin t| \le 1$ munosabat o‘rinli va $\sin t$ funksiya chegaralangan funksiyadir.

2°. $y = \cos t$ *funksiya chegaralangan va barcha* $t \in R$ *lar uchun* $|\cos t| \le 1$ *munosabat o‘rinli.*

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	-1
$\operatorname{tg}\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Mavjud emas	0	Mavjud emas	0
$\operatorname{ctg}\alpha$	Mavjud emas	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Mavjud emas	0	Mavjud emas

I s b o t . $\forall t \in R$ da $0 \le \sin^2 t \le 1$ bo'lgani uchun $\cos^2 t = 1 - \sin^2 t \le 1$ bo'ladi. Oxirgi tengsizlikdan, $\forall t \in R$ da $|\cos t| \le 1$ ekani ko'rinadi. Demak, $\cos t$ funksiya chegaralangan funksiya va $\forall t \in R$ da $|\cos t| \le 1$.

1°, 2°- xossalardan, $y = \sin x$ va $y = \cos x$ funksiyalardan har birining qiymatlar sohasi $[-1;\ 1]$ kesmadan iborat ekanligi kelib chiqadi.

Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari jadvalini keltiramiz:

M a s h q l a r

1.9. Ushbu sonli qiymatlarga kosinus teng bo'la oladimi? Sinuschi?

1) 0,562; 2) –0,562; 3) 1,002; 4) –1,002;

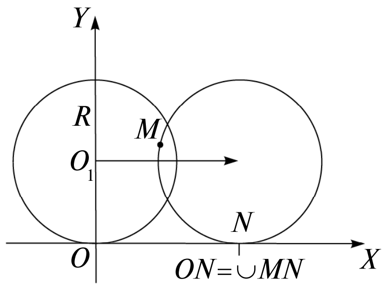
5) $\frac{a}{\sqrt{a^2-b^2}}$; 6) $\frac{a}{\sqrt{a^2+b^2}}$, $a > 0,\ b > 1$; 7) $\frac{\sqrt[3]{3}}{\sqrt{3}}$; 8) π;

9) $\frac{3\frac{1}{7}}{\pi}$; 10) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}-1}$; 11) $\sqrt{8}-\sqrt{2}$; 12) $\frac{1}{2}\left(a+\frac{1}{a}\right)$, $a > 1$.

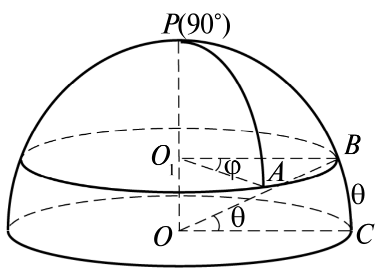
1.10. 1) Agar $\alpha = 0°;\ 90°;\ \pi;\ \frac{\pi}{4};\ 1{,}5\pi$ bo'lsa, $\sin\alpha + \cos\alpha$ va $2(1 - \cos\alpha)$ ni toping;

2) $y = \sin^2\alpha + 2\cos^2\alpha$ ifoda qabul qiladigan eng katta qiymatni toping.

1.11. R radiusli halqa OX o'qining musbat yo'nalishi bo'yicha yumalab bormoqda (I.22-rasm). O'qning 1 birlik kesma uzunligi R ga teng. Harakat boshida aylananing M nuqtasi O nuqtada turgan bo'lsin.



I.22-rasm.



I.23-rasm.

1) Agar M nuqta α rad ga burilsa, aylananing O_1 markazi qanchaga siljiydi?

2) O_1 markaz ($x = 3$; $y = 1$) nuqtaga kelishi uchun M nuqta qancha burilishi kerak?

3) O_1 nuqta 5 birlik/s tezlik bilan siljimoqda. M nuqtaning burchak tezligini toping.

4) O_1 nuqta sekundiga R masofaga siljisa, M nuqtaning t momentdagi o'rnining koordinatalarini toping.

1.12. Geografik kengligi θ ga teng bo'lgan parallelda geografik uzunliklarining farqi φ ga teng bo'lgan ikki A va B nuqta olingan (I.23-rasm). Yer shari radiusi R ga teng. $\cup AB = l$ ni toping.

1.13. A nuqtaga 120° burchak ostida qo'yilgan 10 N va 12 N kattalikdagi ikki kuchning teng ta'sir etuvchisini toping.

1.14. Daryo qirg'og'idagi tepalikdan shu qirg'oq gorizontal yo'nalishga nisbatan 30°, narigi qirg'oq 15° burchak ostida ko'rinadi. Daryoning kengligi 100 m. Tepalikning balandligi va uning uchidan daryo qirg'og'igacha bo'lgan masofani toping.

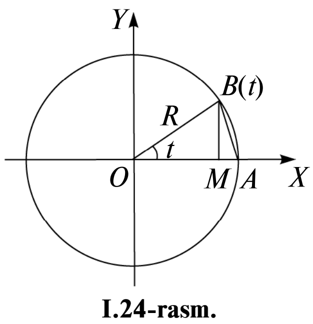
1.15. Yer radiusi R ga teng. Shimoliy yarim sharda geografik uzunligi λ ga, kengligi φ ga teng bo'lgan B nuqta olingan. Toping:

1) B nuqtadan ekvator tekisligigacha bo'lgan masofa;

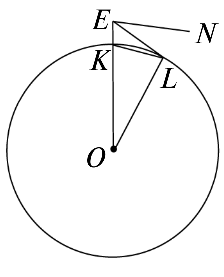
2) B nuqtaning ekvator tekisligidagi proyeksiyasining koordinatalari (abssissalar o'qi ekvator bilan nolinchi meridian kesishuvidagi nuqta ustidan o'tadi). Hisoblashlarni $R = 6367$ km, $\lambda = 30°$ va $\varphi = 60°$ uchun ham bajaring.

1.16. ΔBOA da $OA = OB = R$, $MA = R(1 - \cos t)$, $BM = R\sin t$, $OM = R\cos t$ (I.24-rasm). Isbot qiling:

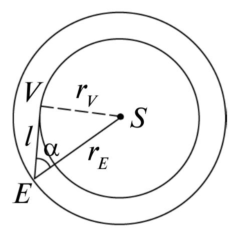
$$\sin t = \pm\sqrt{(1-\cos t)(1+\cos t)}; \quad AB = R\sqrt{2(1-\cos t)}.$$



I.24-rasm.



I.25-rasm.



I.26-rasm.

1.17. Teng yonli AOB uchburchak yuzi 64, $MA=8$ (I.24-rasm). AB –?

1.18. Yer sathidan $EK = h$ (I.25-rasm) balandlikda joylashgan E kuzatuv punktidan gorizont chizig'idagi L nuqta gorizontal yo'nalishga nisbatan $\angle NEL = \alpha$ burchak ostida ko'rinadi (Abu Rayhon Beruniyning «Qonuni Ma'sudiy» asaridan). Agar $h \approx 3$ km va $R \approx 6367$ km bo'lsa, α ni toping.

1.19. I.26-rasmda V Venera va E Yer orbitalari aylana ko'rinishida tasvirlangan, Yerning S Quyoshdan uzoqligi $r_E = 149500000$ km.

Oddiy kuzatishda Venera Quyoshga nisbatan $\alpha \approx 46°$ burchak ostida chetlashgan ko'rinadi. Bu chetlanish ko'pi bilan qancha bo'lishi mumkin?

1) Veneraning Quyoshdan r_V uzoqligini hisoblang.

2) Venera sutkalik harakati davomida Quyoshdan α qadar ortda qolishi mumkin. U holda u kechasi ko'rinadi. Aksincha, α qadar oldin o'tgan bo'lsa, ertalab, Quyosh chiqmasdan oldin ko'rinadi. Nima uchun, tushuntiring.

1.20. I.24-rasmda tasvirlangan koordinatali aylanada $\cup AB = t$.

1) t, $360° + t$, $360° - t$, $-360° + t$, $2\pi k + t$, $k \in Z$ yoylarga mos nuqtalar ustma-ust tushadimi? Agar ular ustma-ust tushsa, bu nuqtalarga mos trigonometrik funksiyalar o'rtasida qanday bog'lanishlar mavjud bo'ladi? Misollar keltiring. Shu ishni $\pi k + t$, $k \in Z$ va $\frac{\pi}{2} + t$, $k \in Z$ nuqtalar uchun takrorlang;

2) yuqoridagi ishni $B(t)$ nuqtaga O markazga nisbatan simmetrik bo'lgan $E(\pi + t)$ nuqtaga nisbatan ham bajaring.

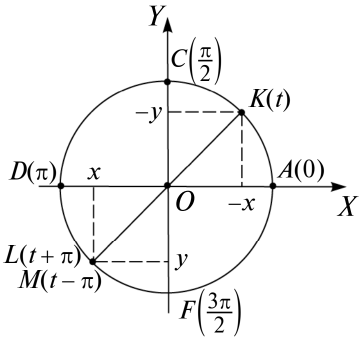
4. Trigonometrik funksiyalarning davriyligi. Trigonometrik funksiyalarning davriyligi haqidagi teoremalarni keltiramiz.

1-teorema. $\cos t$ *va* $\sin t$ *funksiyalarning har biri davriy funksiya va ularning asosiy davri* 2π *ga teng.*

Isbot. Ixtiyoriy $t \in R$ son uchun $K(t)$, $L(t + 2\pi)$, $M(t - 2\pi)$ nuqtalar koordinatali aylanada ustma-ust tushadi. Shu sababli ularning Dekart koordinatalari bir xil:

$$x = \cos t = \cos(t - 2\pi) = \cos(t + 2\pi),$$
$$y = \sin t = \sin(t - 2\pi) = \sin(t + 2\pi). \qquad (1)$$

Demak, $\cos t$ va $\sin t$ funksiyalar davriy funksiyalar va 2π soni ulardan har birining biror davridir. 2π soni ulardan har biri uchun asosiy davr bo'lishligini ko'rsatamiz.



I.27-rasm.

$0 < T_1 < 2\pi$ soni $\cos t$ ning davri deb faraz qilaylik. U holda, masalan, $t = 0$ da $\cos 0 = \cos(0 + T_1) = 1$, ya'ni $\cos T_1 = 1$ bo'lishi kerak. Koordinatali aylanada abssissasi 1 ga teng bo'lgan faqat bitta (1; 0) nuqta mavjud va unga $t = 2\pi k$, $k \in Z$ sonlari mos keladi. T_1 son esa bu sonlar orasida mavjud emas. Demak, farazimiz noto'g'ri, kosinus funksiyaning asosiy davri 2π sonidan iborat. Shu kabi, masalan, $t = \frac{\pi}{2}$ da $\sin\frac{\pi}{2} = \sin\left(\frac{\pi}{2} + T_1\right) = 1$ tenglikni qanoatlantiradigan va 2π dan kichik bo'lgan T_1 musbat son yo'q. Demak, $T = 2\pi$ soni sinus funksiyaning asosiy davri.

2-teorema. *$\operatorname{tg} t$ davriy funksiya va uning asosiy davri π ga teng.*

Isbot. $t \neq \frac{\pi}{2} + \pi k$, $k \in Z$ bo'lsin. $K(t)$, $L(t + \pi)$, $M(t - \pi)$ nuqtalarni qaraymiz. $L(t + \pi)$, $M(t - \pi)$ nuqtalar ayni bir xil Dekart koordinatalariga ega, ya'ni ular ustma-ust tushadi. Shu nuqtalarning umumiy abssissasi x, umumiy ordinatasi esa y bo'lsin (I.27-rasm). U holda, $\operatorname{tg}(t + \pi) = \operatorname{tg}(t - \pi) = \frac{y}{x}$ bo'ladi. $K(t)$ va $L(t + \pi)$ nuqtalar diametral qarama-qarshi nuqtalar bo'lgani uchun $K(t)$ nuqtaning abssissasi $-x$ ga, ordinatasi esa $-y$ ga tengdir (I.27-rasm). Shu sababli,

$$\operatorname{tg} t = \frac{-y}{-x} = \frac{y}{x} = \operatorname{tg}(t + \pi) = \operatorname{tg}(t - \pi).$$

Demak, $\operatorname{tg} t$ funksiya davriy funksiya va $t = \pi$ soni uning biror davridir. Bu son $\operatorname{tg} t$ ning asosiy davri ekanini ko'rsatamiz. T son $\operatorname{tg} t$ ning asosiy davri, ya'ni barcha $t \neq \frac{\pi}{2} + \pi k$, $k \in Z$ sonlari uchun $\operatorname{tg}(t + T) = \operatorname{tg} t$ tenglik o'rinli bo'lsin. Oxirgi tenglik $t = 0$ da ham bajariladi: $\operatorname{tg} T = 0$. Bu yerdan $T = \pi k$, $k \in Z$ ekanini ko'ramiz. Shunday qilib, $\operatorname{tg} t$ ning asosiy davri πk, $k \in Z$ sonlari orasidagi eng kichik musbat son, ya'ni π sonidir. Demak, $T = \pi$.

3-teorema. ctg t *davriy funksiya va uning asosiy davri* π *ga teng* (mustaqil isbotlang).

Mashqlar

1.21. $y = 1 - \cos t$ funksiyaning davriyligini isbot qiling va asosiy davrini toping.

1.22. 1) $f(x) = \sin x$, $g(x) = \frac{1}{x}$, $x \neq 0$ bo'lsa, $g \circ f$ kompozitsiya davriy funksiya bo'la oladimi? $f \circ g$ -chi? Agar shunday bo'lsa, davrini toping.

2) Agar $\frac{5}{3}$ va $\frac{2}{7}$ sonlari f funksiyaning davrlari bo'lsa, $\frac{17}{21}$ soni ham uning davri bo'lishini isbot qiling.

3) $y = \cos(\alpha x)$ ning asosiy davrini toping.

4) $y = \sqrt{\cos x}$ ning aniqlanish sohasini toping va davriylikka tekshiring.

5)Quyidagi funksiyalarning davriy emasligini isbot qiling:

$$y = \cos x^2; \quad y = \sin\sqrt{|x|}; \quad y = \sin x^3;$$

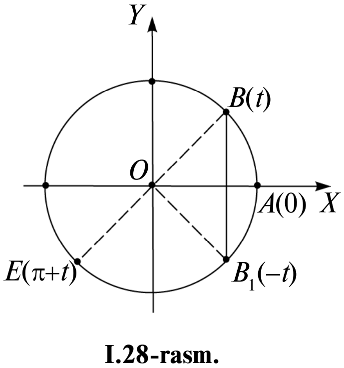
$$y = \sin x + \cos(x\sqrt{3}).$$

1.23. Funksiyalarning davrini toping:

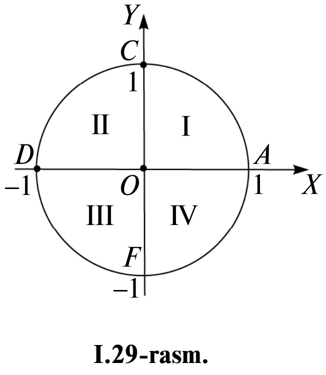
1) $y = \sin 5x + \cos 4x$; 2) $y = \cos x + 2\sin 4{,}9x$;

3) $y = \sqrt{\sin 4{,}3x - \sin 10x + 3}$.

5. Sinus va kosinus funksiyalarning xossalari. Sinus va kosinus funksiyalarning xossalari bilan tanishishni davom ettiramiz.



I.28-rasm.



I.29-rasm.

1) $\sin t$ funksiya argumentning $t = \pi k,\ k\in Z$ qiymatlaridagina, $\cos t$ funksiya esa argumentning $t = \frac{\pi}{2} + \pi k,\ k\in Z$ qiymatlaridagina nolga aylanadi. Haqiqatan, koordinatali aylanada faqat ikki $A(0) = A(1;\ 0)$ va $D(\pi) = D(-1;\ 0)$ nuqtaning ordinatasi nolga teng, ya'ni $y = \sin t = 0$ (I.27-rasm). Bu nuqtalarga $2\pi k,\ k\in Z$ va $\pi + 2\pi k,\ k\in Z$ sonlar to'plamlari mos. Bu ikkala to'plamni bitta $\{\pi k,\ k\in Z\}$ to'plamga birlashtirib yozamiz. Shu kabi koordinatali aylanada faqat ikki $C\left(\frac{\pi}{2}\right) = C(0;\ 1)$ va $F\left(\frac{3\pi}{2}\right) = F(0;\ -1)$ nuqta abssissasi nolga teng (I.27-rasm), ya'ni $x = \cos t = 0$. Bu nuqtalarga $\frac{\pi}{2} + 2\pi k$, $\frac{3\pi}{2} + 2\pi k = \frac{\pi}{2} + (2k+1)\pi$, $k\in Z$ sonlar to'plamlari yoki $\left\{\frac{\pi}{2} + \pi,\ k \in Z\right\}$ to'plam mos;

2) $\cos t$ — juft funksiya, $\sin t$ — toq funksiya. Haqiqatan, $B(t)$ va $B_1(-t)$ nuqtalar abssissalar o'qiga nisbatan simmetrik joylashganligidan (I.28-rasm) ularning abssissalari teng, ordinatalari esa faqat ishoralari bilan farq qiladi. Demak, $\cos(-t) = \cos t$, ya'ni $\cos t$ juft funksiya, $\sin(-t) = -\sin t$, ya'ni $\sin t$ toq funksiya;

3) agar $B(t)$ nuqta koordinatali aylana bo'ylab π qadar siljitilsa, $\cos t$ va $\sin t$ funksiyalar o'z ishoralarini o'zgartiradi:

$$\cos(t + \pi) = -\cos t; \tag{1}$$

$$\sin(t + \pi) = -\sin t. \tag{2}$$

Haqiqatan, $B(t)$ va $E(\pi + t)$ nuqtalar koordinatalar boshiga nisbatan simmetrik joylashganligidan (I.28-rasm) ularning koordinatalari qarama-qarshi ishorali bo'ladi;

4) A (1; 0), C (0; 1), D (–1; 0), F (0; –1) nuqtalar koordinatali aylanani to'rt chorakka ajratadi (I.29-rasm). Agar $A(0)$ nuqta A dan C gacha siljitilsa, A nuqta abssissasi 1 dan 0 gacha kamayadi, ordinatasi esa 0 dan 1 gacha o'sadi. Demak, $0 \le t \le \frac{\pi}{2}$ oraliqda (I chorakda) $\sin t$ funksiya nomanfiy va 0 dan 1 gacha o'sadi, $\cos t$ ham nomanfiy, lekin 1 dan 0 gacha kamayadi. Qolgan choraklarda ham shu kabi ma'lumotlarni to'plab, quyidagi jadvalni tuzamiz:

Funksiya	$0 < t < \frac{\pi}{2}$	$\frac{\pi}{2} < t < \pi$	$\pi < t < \frac{3\pi}{2}$	$\frac{3\pi}{2} < t < 2\pi$
$\sin t$	musbat, 0 dan 1 gacha o‘sadi	musbat, 1 dan 0 gacha kamayadi	musbat, 0 dan –1 gacha kamayadi	musbat, –1 dan 0 gacha o‘sadi
$\cos t$	musbat, 0 dan 1 gacha o‘sadi	musbat, 1 dan 0 gacha kamayadi	musbat, 0 dan –1 gacha kamayadi	musbat, –1 dan 0 gacha o‘sadi

Mashqlar

1.24. $1 - \cos t$ funksiya (Mirzo Ulug‘bek bu funksiyani sahm t funksiya deb atagan) ishoralarining saqlanish oraliqlarini, nollarini, juft-toqligini aniqlang, $180° \pm \alpha$ yoy sahmining $1 + \cos\alpha$ ga tengligini tekshiring, α yoy sahmi $1 - \cos\alpha$ ga teng.

1.25. $\sin t$, $\cos t$ va $1 - \cos t$ mos ravishda:

1) $\frac{2\sqrt{10}}{7}$; $\frac{3}{7}$; $\frac{4}{7}$ ga teng bo‘lishi mumkinmi?

2) $-\frac{a}{\sqrt{a^2+b^2}}$; $\frac{b}{\sqrt{a^2+b^2}}$; $1-\frac{b}{\sqrt{a^2+b^2}}$ ga-chi ?

1.26. Quyida t ning qiymatlari ko‘rsatilgan. $B(t)$ nuqta qaysi chorakda joylashgan, $\sin t$, $\cos t$ lar qanday ishoraga ega bo‘ladi?

1) $\frac{5}{4}\pi$; 2) $\frac{4}{5}\pi$; 3) $\frac{\pi}{7}$; 4) $\frac{2\pi}{3}$; 5) 3; 6) 3,13; 7) $1{,}7\pi$; 8) $1{,}78\pi$; 9) $-1{,}78\pi$; 10) $-2{,}8$; 11) -4; 12) $-1{,}31$; 13) 49600; 14) $356°$; 15) $247°36'42''$; 16) $34680°$; 17) $-674°$; 18) $-107°13'55''$.

1.27. Ayniyatlarni isbot qiling:

1) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\cos x \sin x}$, bunda $\sin x \neq 0$, $\cos x \neq 0$;

2) $\frac{\sin 30°}{\cos 30°} + \frac{\cos 30°}{\sin 30°} = \frac{4}{\sqrt{3}}$; 3) $\sin^2 x \cdot \cos^2 x + \cos^2 x + \sin^4 x = 1$;

4) $\sin^2 x - \cos^2 x = \sin^4 x - \cos^4 x$;

5) $\sin^2 x - \sin^2 x\cos^2 x - \cos^4 x = 1 - 2\cos^2 x$;

6) $\cos^2 x + \sin^2 x \cdot \cos^4 x - \sin^6 x = 1 - 2\sin^4 x$;

7) $6(\sin^4 x + \cos^4 x) - 4(\cos^6 x + \sin^6 x) = 2$.

1.28. $\sin x - \cos x = m$ bo‘lsin. $\sin x$ va $\cos x$ ni hisoblamay, quyidagilarni toping:

1) $\sin^3 x - \cos^3 x$; 2) $\sin^4 x - \cos^4 x$.

1.29. Tenglamalarni yeching:

1) $\sin 10x = 0$; 2) $\cos 5x = 0$; 3) $\sin\frac{x}{3} = 0$; 4) $\cos\frac{x}{5} = 0$;

5) $\sin\left(2x - \frac{\pi}{6}\right) = 0$; 6) $\sin\left(6x + \frac{\pi}{4}\right) = 0$; 7) $\cos\left(\frac{x}{6} - \frac{\pi}{2}\right) = 0$;

8) $\sin\left(\frac{x}{4} + \frac{\pi}{8}\right) = 0$.

1.30. Ifodalarni soddalashtiring:

1) $2\cos(\pi + x) + 3\cos(-x) + \cos(\pi - x)$;

2) $\sin(\pi + x) - 2\sin(\pi - x) - 3\sin(-x)$;

3) $4\cos(-x) + 5\sin(\pi + x) - 2\sin(\pi - x) - 6\cos(\pi + x)$.

1.31. a) Quyidagi funksiyalarni juft-toqlikka tekshiring:

1) $\sin^9 x$; 2) $\cos^9 x$; 3) $\sin^8 x$; 4) $5\cos^5 x + 6\cos^4 x$;

5) $3\sin^3 x - 2\sin^2 x$; 6) $3\sin^3 x + 4\cos^5 x$; 7) $\frac{4\sin^2 x + \cos^5 x + 2}{\sin^5 x}$.

b) Ifodalarning ishoralarini aniqlang:

1) $\sin\frac{6}{5}\pi \cdot \cos\frac{7}{4}\pi$; 2) $\sin\frac{3}{4}\pi \cdot \sin\frac{7}{3}\pi \cdot \cos\frac{7}{4}\pi$;

3) $\sin 0{,}9 \ \cos(-1) \ \cos 4$.

1.32. Sinus va kosinus funksiyalar qaysi choraklarda bir xil ishoraga ega?

1.33. Agar:

1) $\cos t = 3\sin t$; 2) $\cos t = \sin^2 t$; 3) $\sin t = 2\cos^3 t$; 4) $\cos t = \sin^4 t$ bo‘lsa, t qaysi chorakka tegishli bo‘ladi?

1.34. Agar: 1) t burchak ikkinchi chorakka tegishli va $\cos t = -\frac{2}{3}$ bo‘lsa, $\sin t$ nimaga teng bo‘ladi?

2) t burchak uchinchi chorakka tegishli va $\sin t = -\frac{4}{5}$ bo‘lsa, $\cos t$ nimaga teng bo‘ladi?

1.35. Qaysi biri katta:

1) $\sin 45°$ yoki $\sin\frac{\pi}{3}$; 2) $\cos 45°$ yoki $\cos\frac{\pi}{3}$; 3) $\sin 50°$ yoki $\cos 50°$?

1.36. Ifodalarning qiymatlarini hisoblang:

1) $\sin 240°$; 2) $\cos 240°$; 3) $\sin\frac{9\pi}{6}$; 4) $\sin\left(-\frac{11\pi}{3}\right)$;

5) $\cos\left(-\frac{7\pi}{3}\right)$; 6) $\cos^2\left(-\frac{13\pi}{3}\right)+\sin^2\left(-\frac{11\pi}{6}\right)$;

7) $\cos\pi\cdot\cos 90^{\circ}-\frac{3\cos(-180)^{\circ}}{\cos 180^{\circ}}$;

8) $\cos\pi\cdot\sin\frac{3\pi}{2}+\frac{\sin\frac{5\pi}{2}}{\cos 0}-\frac{1}{\cos\left(-\frac{5\pi}{4}\right)}$; 9) $\frac{\cos^2 30^{\circ}}{\sin^2 30^{\circ}}-\frac{\sin^2 45^{\circ}}{\sin^2 45^{\circ}}$.

1.37. $f(x) = 6\sin 4x - 3\cos 4x$ funksiyaning $f(0)$, $f\left(\frac{\pi}{2}\right)$, $f\left(-\frac{\pi}{4}\right)$ qiymatlarini hisoblang.

1.38. Ayirmalarning ishoralarini aniqlang:

1) $\sin 38^{\circ} - \sin 40^{\circ}$; 2) $\cos 51^{\circ} - \mathrm{sos} 21^{\circ}$; 3) $\sin\frac{\pi}{9}-\sin\frac{\pi}{4}$;

4) $\sin 48^{\circ} - \sin 52^{\circ}$; 5) $\cos\frac{\pi}{9}-\cos\frac{\pi}{4}$; 6) $\sin 132^{\circ} - \sin 152^{\circ}$;

7) $\cos\frac{\pi}{10}-\cos\frac{\pi}{20}$; 8) $\sin\frac{\pi}{10}-\sin\frac{\pi}{20}$; 9) $\sin 12^{\circ} - \cos 732^{\circ}$.

1.39. Funksiyalarning o‘sish va kamayish oraliqlarini toping:

1) $y=\sin\frac{x}{3}$; 2) $y=\cos\frac{x}{3}$; 3) $y = \sin 5x$; 4) $y = \cos 5x$;

5) $y=\sin\left(x+\frac{\pi}{3}\right)$; 6) $y=\cos\left(x+\frac{\pi}{3}\right)$; 7) $y=4\sin\left(x-\frac{\pi}{3}\right)$;

8) $y=\sin\left(\frac{x}{3}+3\right)$; 9) $y = \cos^2 x$; 10) $y=\sin^2\frac{x}{2}$;

11) $y=-3\cos^4\frac{x}{3}$; 12) $y=\cos(5x+60^{\circ})$; 13) $y=\sin(2x-60^{\circ})$.

1.40. Funksiyalarning o‘sish va kamayish oraliqlarini toping:

1) $y = 2\sin 3x - 3\cos 2x$; 2) $y=\frac{1}{2\cos x}$; 3) $y=\frac{1}{\cos(2x+1)}$;

4) $y=\sqrt{|\sin x|}$; 5) $y = \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x$;

6) $y = \cos 2x + \sin^2 x$; 7) $y = \cos x + \sin 2x$;

8) $y=\sqrt{3}\sin\left(\frac{1}{2}x+\frac{\pi}{4}\right)$.

1.41. Funksiyalarni o‘sish va kamayishga tekshiring:

1) $y = \sin^2 x + 1$; 2) $y = \cos 3x - \cos 5x + 6x$;

3) $y = (\cos 2x + 3)(1 - 4\sin x)$; 4) $y=\cos\frac{1}{2}x-\sin\frac{1}{2}x+3x-7$.

1.42. Funksiya x ning qaysi qiymatlarida aniqlanmagan?

1) $y = \frac{1}{\sin x}$; 2) $y = \frac{1}{\cos x}$; 3) $y = \frac{\cos x}{1-\sin x}$;

4) $y = 0{,}8\sin^2 x - 0{,}75$; 5) $y = \frac{2\sin x+1}{2\cos x-1}$.

1.43. Quyidagi funksiyalarning eng kichik musbat davrini toping:

1) $y = \cos 4x$; 2) $y = 10\cos 0{,}5x$;

3) $y = 2\cos\left(3x - \frac{\pi}{6}\right)$; 4) $y = 9\cos(\omega t + \varphi),\ \omega \neq 0$;

5) $y = -30\sin\left(5x + \frac{\pi}{4}\right)$; 6) $y = \frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6} - 2x\right)$;

7) $y = \frac{5}{6}\sin\left(4\pi x + \frac{\pi}{3}\right)$; 8) $y = A\sin(\omega t + \varphi),\ A \in R,\ \omega \neq 0$;

9) $y = 10\cos 4x - 8\sin\left(5x + \frac{\pi}{4}\right)$; 10) $y = -10\cos\frac{x}{2} + 4\sin\frac{x}{2}$;

11) $y = 5\cos x \sin 2x$; 12) $y = \frac{\sin x-2}{\cos x+2}$.

1.44. Funksiyalarning aniqlanish sohasini toping, maksimal va minimal qiymatlarini hisoblang; agar x ning qiymati $\pi/6$ dan $\pi/3$ gacha ortsa, funksiya qanday o‘zgaradi?

1) $y = \sin\left(\frac{\pi}{6} - x\right)$; 2) $y = \cos(45° - x)$.

6. Tangens va kotangens funksiyalarning xossalari.

1) *Tangens va kotangens davriy funksiyalardir va ularning asosiy davri* $T = \pi$ (4- band, 2, 3- teoremalar);

2) tg$t$ va ctg$t$ — toq funksiyalar. Haqiqatan, $\cos\alpha \neq 0$ da

$\mathrm{tg}(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\mathrm{tg}t$ ga, $\sin t \neq 0$ da $\mathrm{ctg}(-t) = -\mathrm{ctg}t$ ga ega bo‘lamiz. Tangens va kotangenslarning davri π ga teng va ular toq funksiyalar bo‘lgani uchun $\mathrm{tg}(\pi - t) = \mathrm{tg}(\pi + (-t)) = \mathrm{tg}(-t) = -\mathrm{tg}t$, $\mathrm{ctg}(\pi + (-t)) = -\mathrm{ctg}t$, ya’ni

$$\mathrm{tg}(\pi - t) = -\mathrm{tg}t, \quad (1)$$

$$\mathrm{ctg}(\pi - t) = -\mathrm{ctg}t. \quad (2)$$

tengliklar o‘rinli bo‘ladi;

3) $\left(0;\ \frac{\pi}{2}\right)$ *oraliqda* tg$t$ *funksiya* 0 *dan* $+\infty$ *gacha o‘sadi,* ctg$t$ *esa* $+\infty$ *dan 0 gacha kamayadi.*

Haqiqatan, $\left(0;\ \frac{\pi}{2}\right)$ da $\sin t$ va $\cos t$ musbat, sinus o‘suvchi, kosinus kamayuvchi, demak, tg$t$ o‘suvchi, xususan, $t = 0$ da

$\mathrm{tg}0=\frac{\sin t}{\cos t}=\frac{0}{1}=0$ bo‘ladi, t burchak $\frac{\pi}{2}$ ga yaqinlashganda sin$t$ qiymati 1 gacha o‘sadi, cos$t$ esa 0 gacha kamayadi, natijada $\frac{\sin t}{\cos t}=\mathrm{tg}t$ funksiya $+\infty$ gacha o‘sadi, aksincha $\frac{\cos t}{\sin t}=\mathrm{ctg}t$ funksiya 0 gacha kamayadi.

Olingan xulosalar hamda tangens va kotangensning toq funksiyaligidan foydalanib, $\left(-\frac{\pi}{2};\ 0\right)$ oraliqda ularning manfiy ekanligini, tg$t$ funksiyaning $-\infty$ dan 0 gacha o‘sishini hamda ctg t ning 0 dan $-\infty$ gacha kamayishini aniqlaymiz. Uchinchi va to‘rtinchi choraklardagi holatlarini aniqlashda ularning xossalari $T=\pi$ davr bilan takrorlanishidan foydalanamiz. Xususan, $\left(\pi;\ \frac{3\pi}{2}\right)$ dagi holat $\left(0;\ \frac{\pi}{2}\right)$ dagiga, $\left(\frac{\pi}{2};\ \pi\right)$ dagi holat $\left(-\frac{\pi}{2};\ 0\right)$ dagiga o‘xshash.

4) *t ning* tg$t$, ctg$t$ *funksiyalar aniqlangan qiymatlarida quyidagi ayniyatlar o‘rinli:*

$$\mathrm{tg}t\,\mathrm{ctg}t=1,\ t\neq\frac{k\pi}{2},\ k\in Z, \tag{3}$$

$$1+\mathrm{tg}^2t=\frac{1}{\cos^2 t},\ t\neq\frac{\pi}{2}+k\pi,\ k\in Z, \tag{4}$$

$$1+\mathrm{ctg}^2t=\frac{1}{\sin^2 t},\ t\neq k\pi,\ k\in Z. \tag{5}$$

(3) ayniyat $\mathrm{tg}t=\frac{\sin t}{\cos t}$ va $\mathrm{ctg}t=\frac{\cos t}{\sin t}$ tengliklarni ko‘paytirish orqali, (4) va (5) ayniyatlar esa $\sin^2t+\cos^2t=1$ tenglikning har ikkala qismini avval $\cos^2t$ ga, so‘ng $\sin^2t$ ga bo‘lish orqali hosil bo‘ladi.

M i s o l. Agar $\mathrm{tg}t=-\frac{2}{3}$ va $t\in\left(\frac{\pi}{2};\ \pi\right)$ bo‘lsa, sin$t$, cos$t$, ctg$t$ ning qiymatini topamiz.

Y e c h i s h. II chorakda $\sin t>0$, $\cos t<0$, u holda $\mathrm{ctg}t<0$. (3) ayniyat bo‘yicha $\mathrm{ctg}t=-\frac{3}{2}$; (4) ayniyat bo‘yicha:

$\cos^2t=\frac{1}{1+\left(-\frac{2}{3}\right)^2}=\frac{9}{13}$, $\cos t=-\frac{3}{\sqrt{13}}=-\frac{3\sqrt{13}}{13}$. Shu kabi (5) bo‘yicha $\sin t=\frac{2\sqrt{13}}{13}$ ni topamiz.

Mashqlar

1.45. Ifodalarning qiymatini toping:

1) $(a\text{tg}30°)^2 - (b\text{ctg}45°)^2$; 2) $\left(a\text{ctg}\frac{\pi}{4}\right)^3 - \left(b\text{ctg}\frac{\pi}{6}\right)^3$;

3) $a^2\text{ctg}^2\frac{\pi}{4} + b^2\text{ctg}^2\frac{\pi}{6} - c^2\text{ctg}^2\frac{\pi}{3}$;

4) $a\sin\pi\text{tg}1{,}2\pi - b\cos1{,}5\pi + c\text{tg}\pi\sin1{,}8\pi$;

5) $a^2\left(1-\cos\frac{\pi}{2}\right)^2 + b^2\cos^2(-31{,}1\pi)\text{ctg}\frac{\pi}{2}$; 6) $a^2\text{tg}^3\frac{\pi}{3} - b^2\text{ctg}^2\frac{\pi}{3}$.

1.46. Quyidagi qiymatni asosiy trigonometrik funksiyalarning qaysi biri qabul qila oladi:

1) $\frac{a^2+1}{2a},\ a>0$; 2) $\frac{2a^2+1}{3a},\ a>0$; 3) $\frac{a^4+2a^2+1}{4a^2},\ a>0$;

4) $\frac{(a+b)^2}{4ab},\ a>0,\ b>0,\ a\neq b$; 5) $\frac{2\sqrt{ab}}{a+b},\ a>0,\ b>0,\ a\neq b$.

1.47. Ayniyatlarni isbot qiling:

1) $\frac{\cos x}{\text{ctg}x} = \sin x$; 2) $\cos x\,\text{tg}x = \sin x$; 3) $1 = \sin^2 x + \frac{\sin^2 x}{\text{tg}^2 x}$;

4) $\frac{\cos^2 x}{\text{ctg}^2 x} + \cos^2 x = 1$; 5) $(1 - \cos^2 x)(1 + \text{ctg}^2 x) = 1$;

6) $\cos^2 x\,\text{tg}^2 x + \cos^2 x = 1$; 7) $(1 + \text{tg}^2 x)(1 - \sin^2 x) = 1$;

8) $(\text{tg}x+1)^2 + (\text{tg}x-1)^2 = \frac{2}{\cos^2 x}$; 9) $\frac{1}{1+\text{ctg}^2 x}\cdot\frac{1+\text{tg}^2 x}{\text{tg}^2 x} = 1$;

10) $(1 - \cos\alpha + \sin\alpha)^2 = 2(1 - \cos\alpha)(1 + \sin\alpha)$;

11) $\frac{1}{\sin^2\alpha\sin^2\beta} - \frac{\text{ctg}^2\alpha}{\sin^2\beta} - \text{ctg}^2\beta = 1$;

12) $\text{ctg}^2\alpha\,\text{ctg}^2\beta + \text{ctg}^2\alpha + \frac{1}{\sin^2\beta} = \frac{1}{\sin^2\alpha\sin^2\beta}$.

1.48. $\sin\alpha = \frac{2ab}{a^2+b^2},\ a>0,\ b>0,\ 0\le\alpha\le\frac{\pi}{2}$ bo‘lsa, $\cos\alpha$ va $\text{tg}\alpha$ ni toping.

1.49. $\text{ctg}\alpha = -1$ ekani ma’lum. $\frac{8\sin\alpha - 6\cos\alpha}{4\cos\alpha - 3\sin\alpha}$ kasrning qiymatini toping.

1.50. $\cos\alpha = -0,5$, $90° \le \alpha \le 180°$ bo‘lsa, $\sin\alpha$, $\text{tg}\alpha$ va $\text{ctg}\alpha$ ni toping.

1.51. $\sin\alpha = -\frac{2}{3}$, $\frac{3\pi}{2} < \alpha < 2\pi$ bo‘lsa, $\cos\alpha$, $\text{tg}\alpha$, $\text{ctg}\alpha$ ni toping.

1.52. Ifodalarni soddalashtiring:

1) $\text{ctg}^2\alpha - \cos^2\alpha + \cos^2\alpha\,\text{ctg}^2\alpha$; 2) $\cos^2\alpha + \sin^2\alpha\,\text{tg}^2\alpha - \text{tg}^2\alpha$;

3) $\sin^2\alpha - \frac{1}{1+\text{ctg}^2\alpha}$; 4) $\cos^2\alpha - \frac{1}{1+\text{tg}^2\alpha}$.

1.53. Barcha trigonometrik funksiyalarni

1) $\sin\alpha$; 2) $\cos\alpha$; 3) $\text{tg}\alpha$; 4) $\text{ctg}\alpha$ orqali ifodalang.

1.54. Isbot qiling:

1) $\text{tg}\alpha + \text{ctg}\alpha \ge 2$, bunda $\text{tg}\alpha > 0$;

2) $\frac{(\sin x - \cos x)^2 - 1}{\text{tg}x - \sin x\cos x} = -2\text{ctg}^2 x$;

3) $\frac{\sin^2 x}{\text{tg}^3 x} - \frac{\cos^2 x}{\text{ctg}^3 x} + \frac{1}{\sin x\cos x} = 2\text{tg}x$; 4) $\sin^3 x - \sin^6 x \le \frac{1}{4}$.

1.55. Agar $\sin x + \cos x = 1,5$ bo‘lsa, quyidagilarni hisoblang:

1) $\text{tg}^2 x + \text{ctg}^2 x$; 2) $\text{tg}^3 x + \text{ctg}^3 x$; 3) $\text{tg}^4 x + \text{ctg}^4 x$.

1.56. Agar $\cos^6 x + \sin^6 x = q$ bo‘lsa, $\cos^4 x + \sin^4 x$ ni toping.

1.57. $y = \frac{\sin x}{x}$ funksiya $\left(0;\ \frac{\pi}{2}\right)$ oraliqda kamayuvchi funksiya ekanligini isbot qiling.

1.58. $y = \frac{\text{tg}x}{x}$ funksiya $\left(0;\ \frac{\pi}{2}\right)$ oraliqda o‘suvchi funksiya ekanligini isbot qiling.

1.59. Ayirmalar ishorasini aniqlang:

1) $\text{tg}164° - \text{tg}165°$; 2) $\text{tg}379° - \text{tg}10°$; 3) $\text{ctg}187° - \text{ctg}6°$;

4) $\text{tg}\left(5\frac{1}{7}\pi\right) - \text{tg}\left(5\frac{1}{6}\pi\right)$; 5) $\text{ctg}\frac{\pi}{6} - \text{tg}\frac{\pi}{6}$.

1.60. $x \in (\pi;\ 2\pi)$ oraliqda quyidagi funksiyalarning monoton o‘sish va monoton kamayish oraliqlarini aniqlang:

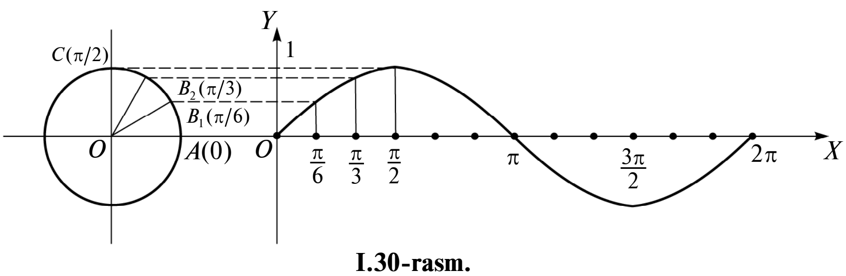
1) $y = \text{tg}x$; 2) $y = \text{ctg}x$; 3) $y = \frac{1}{1+\text{ctg}^2 x}$; 4) $y = \frac{1}{1+\text{tg}^2 x}$; 5) $y = \text{ctg}^4 x$.

1.61. $y = \sqrt{\operatorname{tg}(\sin x)}$ funksiyaning aniqlanish sohasini toping, bu funksiya qabul qiladigan eng kichik va eng katta qiymatlarni hisoblang, agar x ning qiymati $\frac{\pi}{6}$ dan $\frac{\pi}{3}$ gacha ortsa, y funksiya qanday o'zgaradi?

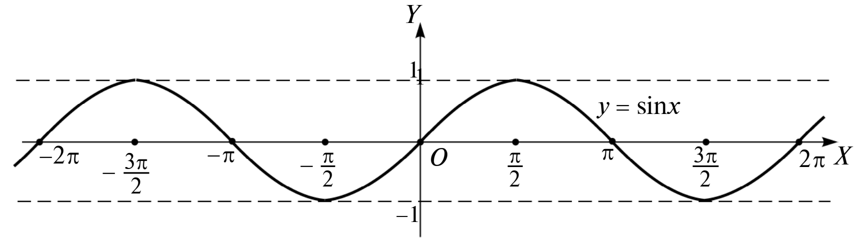
2-§. Trigonometrik funksiyalarning grafiklari

1. Sinus va kosinus funksiyalarning grafigi. $y = \sin x$ funksiya grafigi *sinusoida*, $y = \cos x$ funksiyaning grafigi esa *kosinusoida* deb ataladi. Ularni yasashda trigonometrik funksiyalarning xossalaridan foydalanamiz.

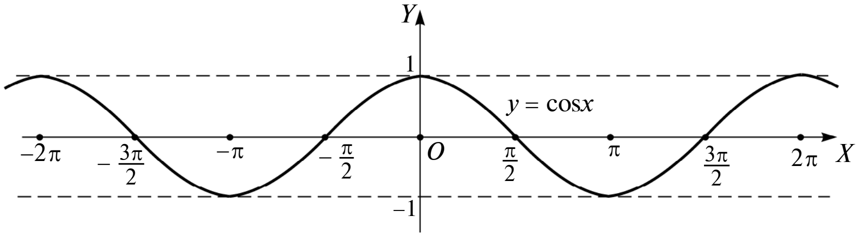
$\sin x$ — davriy funksiya va uning asosiy davri $T = 2\pi$ bo'lgani uchun, OX o'qida uzunligi 2π ga teng bo'lgan biror oraliqni, masalan, $[-\pi; \pi]$ oraliqni ajratamiz (I.30-rasm) va unda grafikning mos qismini yasaymiz. Agar sinusning toq funksiya ekani e'tiborga olinsa, $[-\pi; \pi]$ oraliqning yarmi $[0; \pi]$ bilan chegaralanish, $\sin x = \sin(\pi - x)$ ekani, ya'ni x va $\pi - x$ nuqtalar $\frac{\pi}{2}$ ga nisbatan simmetrik joylashganliklari ham nazarda tutilsa, $[0; \frac{\pi}{2}]$ oraliq bilan chegaralanish yetarli. Shu oraliqda yasalgan qismi $x = \frac{\pi}{2}$ to'g'ri chiziqqa nisbatan simmetrik akslantirilsa, grafikning $[\frac{\pi}{2}; \pi]$ dagi qismi hosil qilinadi, natijada grafikning $[0; \pi]$ dagi qismi chizilgan bo'ladi. Bu qism $(0; 0)$ koordinatalar boshiga nisbatan simmetrik akslantirilsa, $[-\pi; \pi]$ oraliqdagi qismi hosil bo'ladi. Endi uni 2π davr bilan son o'qi bo'yicha davom ettirish qoldi. Grafikni $[0; \frac{\pi}{2}]$ oraliqda geometrik yasash uchun koordinatali aylananing I choragini (AC yoyni, I.30-rasm)



I.30-rasm.



I.31-rasm.



I.32-rasm.

B_1, B_2, ... nuqtalar bilan teng bo‘laklarga ajratamiz. OX o‘qining shu oralig‘i ham shuncha teng bo‘lakka ajratiladi. Agar aylanadagi bo‘linish nuqtalaridan OX o‘qiga parallel va OX o‘qidagi bo‘linish nuqtalardan OY o‘qiga parallel to‘g‘ri chiziqlar o‘tkazsak, ularning kesishish nuqtalari izlanayotgan sinusoidada yotgan bo‘ladi. Nuqtalar ustidan uzluksiz chiziq chizamiz. U sinusoidaning eskizi bo‘ladi.

$y = \cos x$ kosinusoidani ham yuqorida ko‘rsatilgan tartibda yasash mumkin. Funksiyaning asosiy davri $T = 2\pi$. Demak, grafikni uzunligi 2π ga teng biror oraliqda, masalan, $[-\pi; \pi]$ oraliqda yasash, so‘ng uni son o‘qi bo‘yicha 2π davr bilan ikki tomonga davom ettirish kerak. $\cos x$ juft funksiya bo‘lganidan bu oraliqning $[0; \pi]$ qismini, $\cos(\pi - x) = -\cos x$ munosabatga ko‘ra esa yanada kichik $[0; \frac{\pi}{2}]$ oraliqni tanlaymiz. Unda yasalgan grafik Ox o‘qidagi $x = \frac{\pi}{2}$ nuqtaga nisbatan simmetrik almashtirilsa, grafikning $x = \pi$ gacha qismi hosil bo‘ladi. Bu qism ordinatalar o‘qiga nisbatan simmetrik almashtirilsa, grafikning $[-\pi; \pi]$ dagi qismi hosil qilinadi. Grafikning $[0; \frac{\pi}{2}]$ dagi qismi yuqorida sinusoidani yasashda ko‘rsatilgandek hosil qilinadi. Lekin bunda grafikdagi nuqta ordinatasi koordinatali aylanada unga mos nuqta abssissasiga teng bo‘lishi kerak.

Kosinusoidani yasashning boshqa yo‘li sinusoidani $\frac{\pi}{2}$ qadar chapga parallel ko‘chirishdan iborat.

I.31, I.32-rasmlarda mos ravishda sinusoida va kosinusoida tasvirlangan.

Mashqlar

1.62. $y = \cos x$ funksiya grafigi (I.32-rasm) bo‘yicha shu funksiyaning monotonlik oraliqlarini ko‘rsating.

1.63. Funksiyalarning grafiklarini yasang:

1) $y = 3\cos x$; 2) $y = -2\sin x$; 3) $y = |\cos x|$;

4) $y = \cos|x|$; 5) $y = \sin\left(x - \frac{\pi}{6}\right)$; 6) $y = \sin\left|x - \frac{\pi}{6}\right|$;

7) $y = [\cos x]$; 8) $y = \{\cos x\}$; 9) $y = 3\sin\left\{x + \frac{\pi}{4}\right\}$.

1.64. Grafiklarni chizing:

1) $|y| = \cos x$; 2) $|y| = \cos|x|$; 3) $|y| = |\sin x|$;

4) $|y| = \left|\cos\left|x - \frac{\pi}{6}\right|\right|$; 5) $y = 1 - \cos|x|$.

1.65. Funksiyalarning aniqlanish sohasini, qiymatlari sohalarini, o‘sish, kamayish va ishoralarining saqlanish oraliqlarini, maksimum va minimum nuqtalarini ko‘rsating, grafiklarini yasang:

1) $y = \cos x + 3$; 2) $y = 2\sin\left(\frac{\pi}{2} - 2x\right)$;

3) $y = 3\cos(\pi - 2x)$; 4) $y = \left|\cos x + \frac{3}{2}\right|$; 5) $y = -(1 - \cos x)$.

1.66. 1) $y = \sin x$ funksiyaning grafigini yasang. Sinusoidani siljitish yordamida kosinusoidani hosil qiling;

2) sinusoidani 2 birlik yuqoriga va 1 birlik chapga surish natijasida qanday funksiyaning grafigi hosil bo‘ladi? Shu funksiyaning xossalarini uning grafigi bo‘yicha aniqlang.

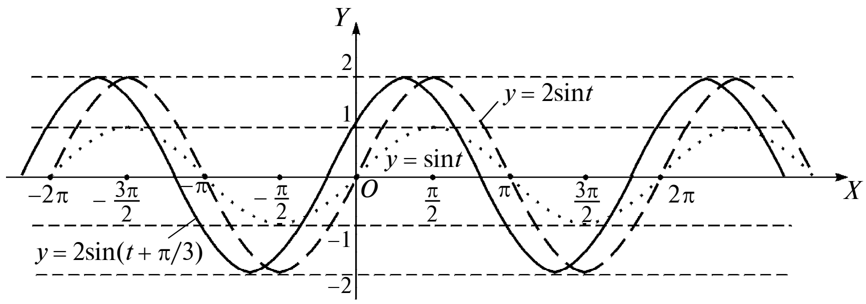
1.67. Quyidagi funksiyalarni juft-toqlikka, davriylikka tekshiring va grafiklarini yasang:

1) $y = \sin 3x$; 2) $y = \cos 3x$; 3) $y = \frac{1}{3}\sin x$;

4) $y = \frac{1}{3}\cos x$; 5) $y = 3\sin x$; 6) $y = 3\cos x$;

7) $y = \sin x + 3$; 8) $y = \cos x - 3$; 9) $y = 3\sin 0{,}5x + 1$;

10) $y = 2\cos 3x - 1$.



I.33-rasm.

2. Sinusoidal tebranishlar. Tebranma harakat trigonometrik funksiyalar orqali ifodalanadi. Matematik mayatnikning harakat tenglamasi, o'zgaruvchan elektr toki kuchi yoki kuchlanishining o'zgarish qonuniyatlari bunga misol bo'la oladi. Eng sodda tebranma harakat *sinusoidal* (yoki *garmonik*) tebranishlardir.

Biror nuqta radiusi A ga teng aylana bo'yicha ω rad/s burchak tezlik bilan harakat qilayotgan bo'lsin. Nuqta t s da ωt radianga teng yoy chizadi. Agar aylananing markazi koordinatalar boshida joylashtirilgan va $t = 0$ vaqt momentida nuqta biror $B_0(\alpha)$ nuqtada turgan bo'lsa, t vaqtdan so'ng u $B(\omega t + \alpha)$ ga keladi. B nuqta koordinatalari:

$$x = A\cos(\omega t + \alpha) \tag{1}$$

va

$$u = A\sin(\omega t + \alpha). \tag{2}$$

Bunga qaraganda B nuqtaning t ga bog'liq ravishda harakati davomida uning x va y koordinatalari OX va OY o'qlari bo'yicha ko'pi bilan $|A|$ qadar oldinga-keyinga siljiydi, tebranadi va o'tilgan masofa (1) va (2) munosabatlardagi sinus va kosinus qiymatiga bog'liq bo'ladi. Bu harakat sinusoidal tebranishdir. (1) va (2) tenglikdagi A son tebranishning qulochini ifodalaydi va tebranish *amplitudasi* deyiladi, ω esa 2π vaqt birligi ichidagi to'liq tebranishlar soni bo'lib, *burchak chastotasi* deyiladi. α son nuqtaning aylanadagi boshlang'ich o'rni, ya'ni *boshlang'ich faza*. (1) va (2) funksiyalarning asosiy davri $T = \frac{2\pi}{\omega}$ (isbot qiling!).

(1) yoki (2) garmonik tebranishlar grafigini sinusoidadan foydalanib yasash maqsadida (2) funksiya ifodasini $y = A\sin\omega\left(t + \frac{\alpha}{\omega}\right)$ ko'rinishda yozamiz. Bunga qaraganda grafikni yasash uchun $\sin t$

sinusoidani OX o‘qi bo‘yicha A koeffitsiyent bilan cho‘zish, OY o‘qi bo‘yicha ω koeffitsiyent bilan qisish va koordinatalar boshini $L\left(-\frac{\alpha}{\omega};\ 0\right)$ nuqtaga akslantiruvchi parallel ko‘chirishni bajarish kerak.

M i s o l. $y = 2\sin\left(t + \frac{\pi}{3}\right)$ funksiya grafigini yasaymiz.

Y e c h i s h. $y = \sin t$ sinusoidani OY o‘qi yo‘nalishida 2 marta cho‘zishni va Ot o‘qi bo‘yicha $\frac{\pi}{3}$ qadar chapga parallel ko‘chirishni bajaramiz (I.33-rasm).

M a s h q l a r

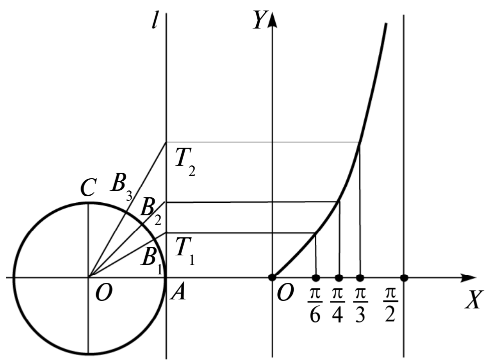
1.68. Formulalar bilan berilgan garmonik tebranishlarning amplitudasi, davri, boshlang‘ich fazasini toping va grafigini yasang:

1) $y = 3\sin\left(2t + \frac{\pi}{2}\right)$; 2) $y = 0{,}8\sin\pi\left(t + \frac{\pi}{3}\right)$; 3) $y = 2{,}5\sin(0{,}5t + 1)$;

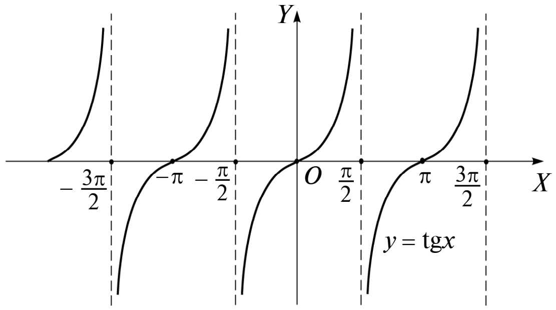
4) $y = 3{,}5\sin\left(0{,}5\pi t - \frac{\pi}{6}\right)$; 5) $y = 2\pi\sin 4t$; 6) $y = 3\sin(2t - 1)$.

3. Tangens va kotangens funksiyalarning grafigi. tg$x$ toq funksiya, davri $T = \pi$ bo‘lganidan uning grafigini $\left[0;\ \frac{\pi}{2}\right]$ oraliqda yasash, so‘ng uni koordinatalar boshiga nisbatan simmetrik akslantirish va abssissalar o‘qi bo‘yicha πk, $k \in Z$ lar qadar surish kerak bo‘ladi.

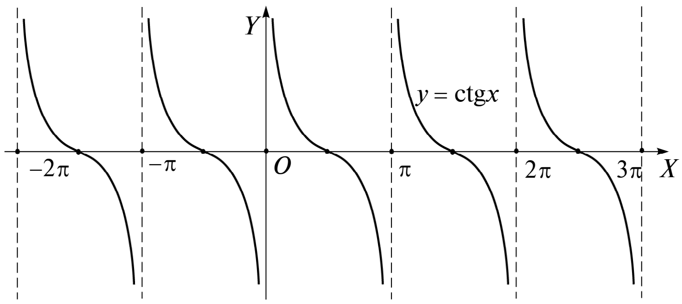
Markazi O_1 $(-1;\ 0)$ nuqtada bo‘lgan birlik aylananing $\cup AC = \frac{\pi}{2}$ yoyi teng uzoqlikda olingan B_1, B_2,... nuqtalar bilan bir necha teng bo‘lakka bo‘lingan bo‘lsin (I.34-rasm). Bu nuqtalar va O_1 nuqtadan o‘tkazilgan O_1B_1, O_1B_2, ... to‘g‘ri chiziqlar Al tangenslar chizig‘i bilan T_1, T_2, ...



I.34-rasm.



I.35-rasm.



I.36-rasm.

nuqtalarda kesishsin. Chizmaga qaraganda $AT_1 = \text{tg}\frac{\pi}{6}$, $AT_2 = \text{tg}\frac{\pi}{3}$ va hokazo. T_1, T_2, ... nuqtalardan OX o'qiga parallel va $x = \frac{\pi}{6}$, $\frac{\pi}{3}$, ... nuqtalardan OY o'qiga parallel o'tkazilgan to'g'ri chiziqlarning kesishish nuqtalari belgilanadi. Ular ustidan o'tadigan egri chiziq tg$x$ funksiyaning grafigi (*tangensoida*) bo'ladi.

Grafik $x = \frac{\pi}{2}$ to'g'ri chiziqqa tomon yaqinlashganida yuqoriga cheksiz ko'tariladi. Endi koordinatalar boshiga nisbatan markaziy simmetriya, so'ng abssissalar o'qi bo'yicha πk, $k \in Z$ davrlar bilan parallel ko'chirishlarni bajarish grafikning kattaroq oraliqdagi davomini beradi (I.35-rasm).

ctg$x$ funksiyaning grafigi (*kotangensoida*) ham shu kabi yasaladi (I.36-rasm).

Mashqlar

1.69. Quyidagi funksiyalarning xossalarini tekshiring va grafiklarini yasang:

1) $y = \text{ctg}2x$; 2) $y = \text{ctg}\left(2x - \frac{\pi}{3}\right)$; 3) $y = \text{ctg}\frac{x}{2}$;

4) $y = 2\text{ctg}x$; 5) $y = |\text{ctg}2x|$; 6) $y = \frac{1}{2}\text{tg}2x$;

7) $y = |\text{tg}2x|$; 8) $y = \text{tg}2\left(x + \frac{\pi}{4}\right)$; 9) $y = \text{ctg}|x|$;

10) $y = \text{tg}\left|x - \frac{\pi}{6}\right|$; 11) $y = [\text{ctg}x]$; 12) $y = \{\text{ctg}x\}$;

13) $y = \left[\text{tg}\frac{x}{2}\right]$.

3-§. Qo'shish formulalari

1. Ikki burchak ayirmasining va yig'indisining kosinusi va sinusi. Chizmada (I.37- rasm) $\angle BOA = \alpha$, $\angle COA = \beta$, $\varphi = \beta - \alpha$, $BD \perp CE$, $CQ \perp OB$, $DE = BF$, $DB = EF = OF - OE = \cos\alpha - \cos\beta$, $QB = OB - OQ = 1 - \cos\varphi$, $CQ = \sin\varphi$, $CD = CE - BF = \sin\beta - \sin\alpha$, CDB va CQB to'g'ri burchakli uchburchaklar umumiy CB gipotenuzaga ega. Pifagor teoremasi bo'yicha:

$$BC^2 = CQ^2 + QB^2 = CD^2 + DB^2$$

yoki

$$\sin^2\varphi + (1 - \cos\varphi)^2 = (\sin\beta - \sin\alpha)^2 + (\cos\alpha - \cos\beta)^2,$$

$$\sin^2\varphi + 1 - 2\cos\varphi + \cos^2\varphi =$$

$$= \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha + \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta,$$

$$(\sin^2\varphi + \cos^2\varphi) + 1 - 2\cos\varphi = (\sin^2\beta + \cos^2\beta) + (\cos^2\alpha + \sin^2\alpha) -$$

$$- 2(\sin\alpha\sin\beta - \cos\alpha\cos\beta),\ 2 - 2\text{sos}\varphi = 2 - 2(\sin\alpha\sin\beta -$$

$$-\cos\alpha\cos\beta)$$

yoki

$$\cos(\beta - \alpha) = \cos\alpha\cos\beta + \sin\alpha\sin\beta. \qquad (1)$$

(1) munosabat bo'yicha va funksiyalarning xossalaridan foydalanib, yana boshqa formulalarni topish mumkin:

$$\cos(\alpha + \beta) = \cos(\alpha - (-\beta)) =$$

$$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta),$$

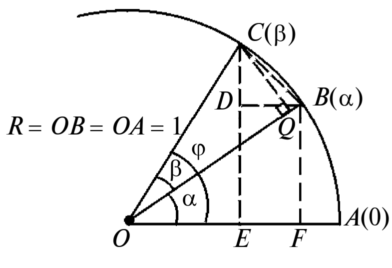
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta. \quad (2)$$

Xususan:

a) $\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\frac{\pi}{2}\cos\alpha +$

$+\sin\frac{\pi}{2}\sin\alpha = 0\cdot\cos\alpha +$

$+1\cdot\sin\alpha = \sin\alpha$,



I.37-rasm.

$$\cos\left(\frac{\pi}{2}+\alpha\right)=\cos\frac{\pi}{2}\cos\alpha-\sin\frac{\pi}{2}\sin\alpha=0\cdot\cos\alpha-1\cdot\sin\alpha=-\sin\alpha\,;$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha\,; \qquad (3)$$

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha\,. \qquad (4)$$

b) $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right)=\cos\alpha\,,$

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}+\alpha\right)\right)=\cos(-\alpha)=\cos\alpha\,.$

Demak,

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha\,; \qquad (5)$$

$$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha\,. \qquad (6)$$

$\alpha\pm\beta$ burchak sinusi uchun formulalar yuqorida topilgan formulalardan foydalanib chiqariladi:

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)=$$

$$=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta+\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

yoki

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta. \qquad (7)$$

Agar (7) formuladagi β o‘rniga $-\beta$ qo‘yilsa, natijada:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta. \qquad (8)$$

M i s o l. $\cos150^\circ$ va $\sin150^\circ$ ni topamiz.

Y e c h i s h. (4) va (6) formulalar bo‘yicha:

$$\cos150^\circ=\cos(90^\circ+60^\circ)=-\sin60^\circ=-\frac{\sqrt{3}}{2}\,;$$

$$\sin150^\circ=\sin(90^\circ+60^\circ)=\cos60^\circ=\frac{1}{2}\,.$$

M a s h q l a r

1.70. Hisoblang: 1) $\sin\frac{\pi}{12}$; 2) $\cos\frac{4\pi}{3}$; 3) $\cos\frac{5\pi}{4}$; 4) $\sin\frac{4\pi}{3}$.

1.71. Agar $\sin x = \frac{3}{8}$; $\sin t = \frac{4}{9}$; $0 < x < \frac{\pi}{2}$; $\frac{\pi}{2} < t < \pi$ bo‘lsa, quyidagilarni toping:

1) $\sin(x - t)$; 2) $\sin(x + t)$; 3) $\cos(x - t)$; 4) $\cos(x + t)$.

1.72. Agar $\cos x = -0{,}8$; $\sin y = 0{,}4$; $\pi < x < \frac{3\pi}{2}$; $\frac{\pi}{2} < y < \pi$ bo‘lsa, quyidagilarni toping:

1) $\cos(x + y)$; 2) $\cos(x - y)$; 3) $\sin(x + y)$; 4) $\sin(x - y)$.

1.73. Ifodalarni soddalashtiring:

1) $\cos(x + t)\sin(x - t) + \sin(x + t)\cos(x - t)$;

2) $\cos(\alpha + \beta)\cos(\alpha - \beta) - \sin(\alpha + \beta)\sin(\alpha - \beta)$;

3) $\cos(45^\circ + \alpha)\cos(45^\circ - \alpha) + \sin(45^\circ + \alpha)\sin(45^\circ - \alpha)$;

4) $\frac{\cos(\alpha-\beta)-\sin\alpha\sin\beta}{\sin(\alpha-\beta)+\sin\alpha\cos\beta}$; 5) $\frac{\sin(\beta+\alpha)-2\sin\alpha\cos\beta}{\cos(\alpha+\beta)+2\sin\alpha\sin\beta}$;

6) $\frac{\cos(\alpha+\beta)+\sin\alpha\sin\beta}{\cos(\alpha+\beta)-\cos\alpha\cos\beta}$; 7) $\frac{\cos(\alpha-\beta)-\sin\alpha\sin\beta}{\cos(\alpha-\beta)-\cos\alpha\cos\beta}$.

1.74. Ayniyatlarni isbot qiling:

1) $\cos(45^\circ - \alpha) = \frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$; 2) $\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} = \mathrm{ctg}\alpha\,\mathrm{ctg}\beta + 1$;

3) $\sin(\alpha - \beta)\sin(\alpha + \beta) = \cos^2\beta - \cos^2\alpha$;

4) $\sin 2x\cos x + \cos 2x\sin x = \sin 3x$;

5) $\sin(\alpha + \beta) + \cos(\alpha - \beta) = (\sin\alpha + \cos\alpha)(\sin\beta + \cos\beta)$;

6) $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\gamma)}{\cos\beta\cos\gamma} + \frac{\sin(\gamma-\alpha)}{\cos\gamma\cos\alpha} = 0$.

1.75. Funksiyalarning juft-toqligi va davriyligini tekshiring hamda grafiklarini yasang:

1) $y = \cos x\cos\frac{x}{2} + \sin x\sin\frac{x}{2}$; 2) $y = \sin x\cos\frac{x}{3} - \sin\frac{x}{3}\cos x$.

2. Ikki burchak yig‘indisi va ayirmasining tangensi va kotangensi. 1-banddagi formulalardan foydalanamiz. Buning uchun $\cos(\alpha + \beta) \neq 0$, ya’ni $\alpha + \beta \neq \frac{\pi}{2} + \pi k$, $k \in Z$ va $\cos\alpha \neq 0$, $\cos\beta \neq 0$ bo‘lishi, ya’ni α va β lar $\frac{\pi}{2} + \pi k$, $k \in Z$ ga teng bo‘lmasligi kerak. Shu shartlardan quyidagilarga ega bo‘lamiz:

$$\operatorname{tg}(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{1-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}}=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}.$$

Bundan:

$$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}. \tag{1}$$

Xuddi shunday,

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}. \tag{2}$$

Quyidagi formulalar ham shu kabi hosil qilinadi:

$$\operatorname{ctg}(\alpha+\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta-1}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}, \tag{3}$$

$$\operatorname{ctg}(\alpha-\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta+1}{\operatorname{ctg}\alpha-\operatorname{ctg}\beta}. \tag{4}$$

M i s o l. $\operatorname{ctg}\frac{7\pi}{12}$ ni hisoblaymiz.

Y e c h i s h. $\operatorname{ctg}\frac{7\pi}{12}=\operatorname{ctg}\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\dfrac{\operatorname{ctg}\frac{\pi}{3}\cdot\operatorname{ctg}\frac{\pi}{4}-1}{\operatorname{ctg}\frac{\pi}{3}+\operatorname{ctg}\frac{\pi}{4}}=\dfrac{\frac{\sqrt{3}}{3}\cdot 1-1}{\frac{\sqrt{3}}{3}+1}=\dfrac{\sqrt{3}-3}{\sqrt{3}+3}=\dfrac{1+\sqrt{3}}{1-\sqrt{3}}.$

Mashqlar

1.76. Agar $\sin(2\alpha+\beta)=2\sin\beta$ bo‘lsa, $\operatorname{tg}(\alpha+\beta)=3\operatorname{tg}\alpha$ bo‘lishini isbot qiling, bunda $\beta\neq k\pi$, $k\in Z$.

1.77. (1)–(4) formulalarning chap qismlarini uning o‘ng qismlaridan hosil qiling.

1.78. Hisoblang:

1) $\operatorname{tg}75°$; 2) $\operatorname{ctg}\frac{5\pi}{12}$; 3) $\operatorname{ctg}105°$; 4) $\operatorname{tg}15°$; 5) $\operatorname{ctg}15°$.

1.79. Berilgan: 1) $\operatorname{tg}x=1{,}5$, $\operatorname{tg}y=-0{,}5$; 2) $\operatorname{ctg}x=1{,}5$, $\operatorname{ctg}y=-0{,}5$. Toping: 1) $\operatorname{tg}(x-y)$; 2) $\operatorname{tg}(x+y)$; 3) $\operatorname{ctg}(x-y)$; 4) $\operatorname{ctg}(x+y)$.

1.80. Berilgan: $\operatorname{tg}\alpha=\frac{1}{3}$, $\operatorname{ctg}\beta=\frac{4}{5}$, $\operatorname{tg}\gamma=1$. Hisoblang:

1) $\operatorname{tg}(\alpha+\beta+\gamma)$; 2) $\operatorname{ctg}(\alpha+\beta+\gamma)$; 3) $\operatorname{tg}(\alpha+\beta-\gamma)$.

1.81. Hisoblang: 1) $\frac{\text{tg}26^\circ+\text{tg}34^\circ}{1-\text{tg}26^\circ\text{tg}34^\circ}$; 2) $\frac{\text{ctg}\frac{\pi}{5}\text{ctg}\frac{9\pi}{20}+1}{\text{ctg}\frac{\pi}{5}-\text{ctg}\frac{9\pi}{20}}$.

1.82. Ayniyatlarni isbot qiling:

1) $\text{tg}\left(\frac{\pi}{4}-x\right)=\frac{1-\text{tg}x}{1+\text{tg}x}$; 2) $\text{ctg}\left(\frac{\pi}{4}-x\right)=\frac{\text{ctg}x-1}{\text{ctg}x+1}$; 3) $\text{ctg}2\alpha=\frac{\text{ctg}^2\alpha-1}{2\text{ctg}\alpha}$;

4) $\frac{\text{ctg}\left(\frac{\pi}{3}+\alpha\right)\cdot\text{ctg}\left(\frac{\pi}{3}-\alpha\right)-1}{\text{ctg}\left(\frac{\pi}{3}+\alpha\right)+\text{ctg}\left(\frac{\pi}{3}-\alpha\right)}=-\frac{\sqrt{3}}{3}$; 5) $\frac{1}{\text{ctg}\alpha\cdot\text{ctg}\beta-1}-\frac{\text{tg}(\alpha+\beta)}{\text{tg}\alpha+\text{tg}\beta}=-1$

1.83. Ifodalarni soddalashtiring:

1) $\frac{\text{ctg}(\alpha-\beta)\cdot\text{ctg}\beta-1}{\text{ctg}\beta+\text{ctg}(\alpha-\beta)}$; 2) $\frac{\text{tg}(\alpha-\beta)-\text{tg}\alpha}{1+\text{tg}(\alpha-\beta)\cdot\text{tg}\alpha}$;

3) $\frac{\text{tg}5x-\text{tg}2x}{1+\text{tg}5x\cdot\text{tg}2x}$; 4) $\frac{\text{ctg}5x\cdot\text{ctg}4x+1}{\text{ctg}5x-\text{ctg}4x}$.

1.84. Ikki egri chiziq orasidagi burchak shu chiziqlarning kesishish (umumiy) nuqtasidan o‘tkazilgan ikki urinma orasidagi burchakka teng. Quyida ko‘rsatilgan funksiyalar grafiklari orasidagi burchak tangensini toping:

1) $y=x^2$ va $y=\sqrt{x}$; 2) $y=\frac{6}{x^2+1}$ va $y=x^2$;

3) $y=3x^2+4x-6$ va $y=x^2+x+3$; 4) $y=x^2$ va $y=-\frac{2}{x}$.

1.85. ABC uchburchakda $\text{tg}A:\text{tg}B:\text{tg}C=1:2:3$. Shu burchaklarning tangenslari va sinuslarini toping.

3. Keltirish formulalari. Oldingi bandlarda $\pi-\alpha$, $\pi+\alpha$, $\frac{\pi}{2}-\alpha$, $\frac{\pi}{2}+\alpha$ burchaklar sinusi, kosinusi, tangensi, kotangensi uchun formulalar chiqarilgan edi. Ulardan hamda ikki burchak yig‘indisi va ayirmasi formulalaridan foydalanib, $\frac{3\pi}{2}\pm\alpha$, $2\pi\pm\alpha$ burchaklar uchun formulalarni chiqara olamiz. Bu formulalar bir burchak funksiyasini boshqa burchak funksiyalari orqali ifodalashga, xususan, o‘tmas burchak funksiyalarini o‘tkir burchak funksiyalariga keltirishga imkon beradi. Masalan,

$$\cos\left(\frac{3}{2}\pi+\alpha\right)=\cos\left(\frac{\pi}{2}+(\pi+\alpha)\right)=-\sin(\pi+\alpha)=\sin\alpha. \quad (1)$$

Shu kabi,

$$\sin\left(\frac{3}{2}\pi+\alpha\right)=-\cos\alpha\,; \tag{2}$$

$$\operatorname{tg}\left(\frac{3}{2}\pi+\alpha\right)=\frac{\sin\left(\frac{3}{2}\pi+\alpha\right)}{\cos\left(\frac{3}{2}\pi+\alpha\right)}=\frac{-\cos\alpha}{\sin\alpha}=-\operatorname{ctg}\alpha\,; \tag{3}$$

$$\operatorname{ctg}\left(\frac{3}{2}\pi+\alpha\right)=-\operatorname{tg}\alpha\,. \tag{4}$$

Keltirish formulalari ko‘p, ularni esda saqlash maqsadida ushbu *mnemonik qoida*dan ham foydalanamiz (yunoncha *mnemonikon* — ko‘p qoidalar majmuasini yodda saqlashni yengillashtiruvchi usul):

1) *agar argument* $2\pi \pm \alpha$ *ko‘rinishda bo‘lsa, trigonometrik funksiyaning nomi o‘zgarmaydi;*

2) *agar argument* $\frac{\pi}{2} \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$ *ko‘rinishda bo‘lsa, funksiyaning nomi o‘zgaradi (sinus kosinusga va aksincha, tangens kotangensga va aksincha);*

3) *berilgan trigonometrik funksiya argumenti qaysi chorakda yotgan bo‘lsa, funksiyaning o‘sha chorakdagi ishorasi izlanayotgan funksiya oldiga qo‘yiladi.*

Keltirish formulalarini quyidagi jadval ko‘rinishida umumlashtiramiz:

	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
$\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$
$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$	$\cos\alpha$
$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$
$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$

M i s o l. a) $\cos(15\pi+\alpha)$; b) $\operatorname{tg}(\pi+\alpha)$ ifodalarni o‘tkir burchak trigonometrik funksiya ko‘rinishiga keltiramiz, $0<\alpha<\frac{\pi}{2}$.

Y e c h i s h. a) $\cos(7\cdot 2\pi+\pi+\alpha)=\cos(\pi+\alpha)$. Bunda $\pi+\alpha$ burchak, demak, $15\pi+\alpha$ burchak ham, uchinchi chorakka qarashli. Bu chorakda kosinusning ishorasi manfiy, hosil bo‘ladigan funksiyaning nomi kosinusligicha qoladi. Demak, $\cos(15\pi+\alpha)=-\cos\alpha$;

b) uchinchi chorakda tangens musbat. Natijada $\operatorname{tg}(\pi+\alpha)=\operatorname{tg}\alpha$ hosil bo‘ladi.

Mashqlar

1.86. Bir necha keltirish formulasini geometrik usulda isbotlang.

1.87. Ifodaning qiymatini toping:

1) $\sin 1080°$; 2) $\cos 1080°$; 3) $\mathrm{tg}\,1080°$;

4) $\mathrm{ctg}\,1080°$; 5) 1080° li yoy sahmini; 6) $\sin\left(7\frac{5}{6}\pi\right)$;

7) $\cos\left(-\frac{49}{6}\pi\right)$; 8) $\mathrm{tg}\left(-\frac{29}{8}\pi\right)$; 9) $\mathrm{ctg}\left(-\frac{32}{3}\pi\right)$.

1.88. Ifodalarni soddalashtiring:

1) $\sin\left(-\frac{55\pi}{3}\right)\cos\left(\frac{51\pi}{4}\right)-\sin\left(\frac{73\pi}{3}\right)\cos\left(\frac{101\pi}{4}\right)$; 2) $\dfrac{\mathrm{tg}\left(-\frac{32\pi}{3}\right)\mathrm{tg}\left(\frac{47\pi}{4}\right)}{1-\mathrm{ctg}\left(\frac{17\pi}{6}\right)\mathrm{ctg}\left(\frac{21\pi}{4}\right)}$.

1.89. Ayniyatlarni isbot qiling:

1) $$\frac{2\sin\left(\frac{5\pi}{2}+\alpha\right)\cos(16\pi+\beta)}{\cos(\alpha-\beta)+2\cos\left(\frac{5\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}+\beta\right)}=\sqrt{\frac{1+\mathrm{tg}^2(\pi+\alpha+\beta)}{1-\mathrm{tg}^2(\pi-\alpha+\beta)}};$$

2) $$\frac{\sin\left(\frac{9\pi}{2}+\alpha\right)+\cos\left(\frac{11\pi}{2}+\alpha\right)}{2(\cos(4\pi+\alpha)+\sin\alpha)}\cdot(\sin(5\pi+\alpha)+\cos(9\pi-\alpha))=-\frac{1}{2};$$

3) $$\frac{\sin(11\pi-x)\cos\left(\frac{7\pi}{2}+x\right)\mathrm{tg}\left(x-\frac{19\pi}{2}\right)}{\cos\left(\frac{19\pi}{2}-x\right)\cos\left(\frac{15\pi}{2}-x\right)\mathrm{tg}(x-11\pi)}=-\mathrm{ctg}^2x.$$

4. Ikkilangan va uchlangan argumentning trigonometrik funksiyalari. Agar $\alpha+\beta$ burchak trigonometrik funksiyalari formulalarida $\alpha=\beta$ deyilsa, 2α burchak trigonometrik funksiyalari formulalari hosil qilinadi. Ular 2α argument funksiyasini α argument funksiyasi orqali ifodalashga imkon beradi:

$$\sin 2\alpha=2\sin\alpha\cos\alpha; \quad (1) \qquad \cos 2\alpha=\cos^2\alpha-\sin^2\alpha; \quad (2)$$

$$\mathrm{tg}\,2\alpha=\frac{2\,\mathrm{tg}\,\alpha}{1-\mathrm{tg}^2\alpha}; \quad (3) \qquad \mathrm{ctg}\,2\alpha=\frac{\mathrm{ctg}^2\alpha-1}{2\,\mathrm{ctg}\,\alpha}. \quad (4)$$

Aksincha, α argument funksiyasini 2α funksiyasi orqali ham berish mumkin. Chunonchi, $1=\sin^2\alpha+\cos^2\alpha$ ayniyat va (2) formula bo‘yicha $1+\cos 2\alpha=2\cos^2\alpha$ va $1-\cos 2\alpha=2\sin^2\alpha$ yoki

$$\cos 2\alpha = 2\cos^2\alpha - 1 \tag{5}$$

va

$$\cos 2\alpha = 1 - 2\sin^2\alpha \tag{6}$$

hosil qilinadi. (5) va (6) formulalarni quyidagi ko‘rinishda ham yozish mumkin:

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}; \tag{7}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}. \tag{8}$$

Agar $\cos\alpha \neq 0$ bo‘lsa, (1) tenglikning o‘ng qismini $\sin^2\alpha +$ $+\cos^2\alpha$ ga, ya’ni 1 ga, so‘ng surat va maxrajni $\cos^2\alpha$ ga bo‘lsak, quyidagini hosil qilamiz:

$$\sin 2\alpha = \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha} = \frac{2\cdot\frac{\sin\alpha\cos\alpha}{\cos^2 a}}{\frac{\sin^2\alpha+\cos^2\alpha}{\cos^2 a}},$$

bundan:

$$\sin 2\alpha = \frac{2\mathrm{tg}\alpha}{1+\mathrm{tg}^2\alpha}. \tag{9}$$

Shu kabi:

$$\cos\alpha \neq 0 \text{ да } \cos 2\alpha = \frac{1-\mathrm{tg}^2\alpha}{1+\mathrm{tg}^2\alpha}. \tag{10}$$

Shuningdek, $\mathrm{ctg}2\alpha = \frac{1}{\mathrm{tg}2\alpha}$ va (3) formula bo‘yicha:

$$\mathrm{ctg}2\alpha = \frac{1-\mathrm{tg}^2\alpha}{2\mathrm{tg}\alpha},\ a \neq \frac{\pi k}{2},\ k \in Z. \tag{11}$$

Uchlangan argument 3α ning trigonometrik funksiyalarini yuqorida topilgan formulalardan foydalanib topish mumkin. Masalan,

$$\sin 3\alpha = \sin(2\alpha + \alpha) = \sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha =$$

$$= 2\sin\alpha\cos^2\alpha + (1 - 2\sin^2\alpha)\sin\alpha = \sin\alpha(2(\cos^2\alpha - \sin^2\alpha) + 1) =$$

$$= \sin\alpha(2(1 - 2\sin^2\alpha) + 1) = \sin\alpha(3 - 4\sin^2\alpha),$$

$$\sin 3\alpha = \sin\alpha(3 - 4\sin^2\alpha). \tag{12}$$

Shu kabi: $\cos 3\alpha = \cos\alpha(4\cos^2\alpha - 3)$.

Mashqlar

1.90. $\sin\alpha = -0{,}83$, $\pi < \alpha < \frac{3\pi}{2}$ bo‘yicha $\sin 2\alpha$, $\cos 2\alpha$, $\operatorname{tg} 2\alpha$ ni toping.

1.91. $\cos\alpha = -0{,}4$, $\sin\alpha < 0$ bo‘yicha $\sin 2\alpha$, $\cos 2\alpha$, $\operatorname{tg} 2\alpha$ ni toping.

1.92. $\operatorname{tg}\alpha = -3$ bo‘yicha $\operatorname{ctg} 2\alpha$ ni toping.

1.93. Agar $0 < \alpha < \frac{\pi}{2}$ bo‘lsa, $\sin 2\alpha < 2\sin\alpha$ bo‘lishini isbot qiling.

1.94. $\operatorname{ctg}\alpha = -1{,}2$, $\frac{\pi}{2} < \alpha < \pi$ bo‘yicha $\sin 3\alpha$, $\cos 3\alpha$, $\cos 4\alpha$, $\operatorname{tg} 4\alpha$ ni toping.

1.95. Agar $\operatorname{tg}\alpha = 0{,}3$, $\operatorname{tg}\beta = 0{,}4$ bo‘lsa, $\operatorname{tg}(2\alpha - \beta)$ ni toping.

1.96. Ayniyatlarni isbot qiling:

1) $\dfrac{\cos 2t + 1 - \cos^2 t}{\cos\left(\frac{5\pi}{2} + 2t\right)} = -\dfrac{1}{2}\operatorname{ctg} t$; 2) $\dfrac{2\sin 2t - \sin 4t}{\sin 4t + 2\sin 2t} = \operatorname{tg}^2 t$;

3) $\dfrac{\sin 2t + \operatorname{tg} 2t}{\operatorname{tg} 2t} = 2\cos^2 t$; 4) $\dfrac{1 - 2\cos^2 2t}{\frac{1}{2}\sin 4t} = \operatorname{tg} 2t - \operatorname{ctg} t$;

5) $\dfrac{\cos t + \operatorname{ctg} t}{\operatorname{ctg} t} = 1 + \sin 2t$; 6) $\dfrac{2\cos^2 t - 1}{2\operatorname{ctg}\left(\frac{\pi}{4} - t\right)\sin^2\left(\frac{\pi}{4} - t\right)} = 1$;

7) $1 + \cos t = \dfrac{\sin t + \operatorname{tg} t}{\operatorname{tg} t}$; 8) $\dfrac{2\sin 4t(1 - \operatorname{tg}^2 2t)}{1 + \operatorname{ctg}^2\left(\frac{\pi}{2} + 2t\right)} = \sin 8t$;

9) $\dfrac{\cos 2t + 5\cos 3t + \cos 4t}{\sin 2t + 5\sin 3t + \sin 4t} = \operatorname{ctg} 3t$; 10) $\operatorname{tg}55^\circ \operatorname{tg}65^\circ \operatorname{tg}75^\circ = \operatorname{tg}85^\circ$.

1.97. Ifodalarni soddalashtiring:

1) $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$;

2) $\sin\left(\alpha - \frac{5\pi}{2}\right)\cos\alpha - \sin^2(5\pi - \alpha)\sin^2\alpha - \cos^2(5\pi - \alpha)\cos^2\left(\frac{11\pi}{2} + \alpha\right)$;

3) $2\operatorname{ctg}\left(\frac{13\pi}{2} - 2\alpha\right) + \dfrac{2\sin(17\pi - \alpha)}{\sin\left(\frac{15\pi}{2} + \alpha\right) + \operatorname{tg}\alpha\sin(-\alpha)}$;

4) $\dfrac{2\cos\alpha - \sin 2\alpha}{\sin^2 3\alpha - \sin\alpha + \cos^2 3\alpha}$; 5) $\dfrac{\sin^4 2t + 2\cos 2t\sin 2t - \cos^4 2t}{2\cos^2 2t - 1}$;

6) $1 + 2\cos 2\alpha + 2\cos 4\alpha + 2\cos 6\alpha$;

7) $\cos\left(\frac{7\pi}{2}-2\alpha\right)\mathrm{tg}(5\pi-\alpha)+\sin\left(\frac{17\pi}{2}+2\alpha\right)$; 8) $\frac{(1+\mathrm{tg}\,2\alpha)\cos\left(\frac{\pi}{4}+2\alpha\right)}{1-\mathrm{tg}\,2\alpha}$.

5. Yarim argumentning trigonometrik funksiyalari. Bu formulalar oldingi bandda berilgan (4)—(11) formulalardagi α o‘rniga $\frac{\alpha}{2}$ ni qo‘yish orqali hosil qilinadi. Jumladan,(7),(8) formulalar bo‘yicha $\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$, $\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$ yoki

$$\left|\cos\frac{\alpha}{2}\right|=\sqrt{\frac{1+\cos\alpha}{2}}; \tag{1}$$

$$\left|\sin\frac{\alpha}{2}\right|=\sqrt{\frac{1-\cos\alpha}{2}}. \tag{2}$$

Agar (2) tenglik (1) ga hadma-had bo‘linsa:

$$\left|\mathrm{tg}\frac{\alpha}{2}\right|=\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \tag{3}$$

tenglik hosil bo‘ladi. $\mathrm{ctg}\alpha=\frac{1}{\mathrm{tg}\alpha}$ bo‘lgani uchun

$$\left|\mathrm{ctg}\frac{\alpha}{2}\right|=\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}. \tag{4}$$

tenglik ham o‘rinlidir.

(1)—(4) formulalar trigonometrik funksiyalar qiymatlarining modulini topishga imkon beradi. Ularning ishoralari esa $\frac{\alpha}{2}$ argumentning qaysi chorakka tegishli ekaniga bog‘liq.

M i s o l. $\sin\alpha=\frac{\sqrt{5}}{3}$, $\frac{\pi}{2}\le\alpha\le\pi$ ekani ma’lum. $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $\mathrm{tg}\frac{\alpha}{2}$ ni topamiz.

Y e c h i s h. Shartdan foydalanib $\frac{\pi}{4}\le\frac{\alpha}{2}\le\frac{\pi}{2}$ bo‘lishini aniqlaymiz. Bu oraliqda barcha trigonometrik funksiyalar musbat. Yuqorida topilgan formulalardan foydalanamiz. Oldin $\cos\alpha$ ni topaylik:

$$\cos\alpha=\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{5}{9}}=-\frac{2}{3}.$$

U holda:

$$\cos\frac{\alpha}{2}=\sqrt{\frac{1+\left(-\frac{2}{3}\right)}{2}}=\frac{1}{\sqrt{6}},\ \sin\frac{\alpha}{2}=\sqrt{\frac{1-\left(-\frac{2}{3}\right)}{2}}=\sqrt{\frac{5}{6}},\ \mathrm{tg}\frac{\alpha}{2}=\sqrt{5}.$$

Yarim argumentning tangensi uchun yana bir formula hosil qilish maqsadida $\operatorname{tg}\frac{\alpha}{2}=\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$ tenglikning o‘ng qismidagi kasr surat va maxrajini $2\sin\frac{\alpha}{2}$ ga ko‘paytiramiz:

$$\operatorname{tg}\frac{\alpha}{2}=\frac{2\sin^2\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}}=\frac{2\cdot\frac{1-\cos\alpha}{2}}{\sin\left(2\cdot\frac{\alpha}{2}\right)}=\frac{1-\cos\alpha}{\sin\alpha},$$

$$\operatorname{tg}\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}. \qquad (5)$$

Agar surat va maxraj $2\cos\frac{\alpha}{2}$ ga ko‘paytirilsa,

$$\operatorname{tg}\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}. \qquad (6)$$

(5) va (6) formulalar bo‘yicha:

$$\operatorname{ctg}\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}. \qquad (7)$$

Mashqlar

1.98. Berilgan: 1) $\alpha=\frac{\pi}{6}$; 2) $\alpha=\frac{\pi}{4}$; 3) $\cos\alpha=-0{,}4$, $\frac{\pi}{2}<\alpha<\pi$; 4) $\operatorname{ctg}\alpha=4$, $\pi<\alpha<\frac{3}{2}\pi$; 5) $\sin\alpha=0{,}8$; $450°<\alpha<540°$.

Toping: $\sin\frac{\alpha}{2}$; $\cos\frac{\alpha}{2}$; $\operatorname{tg}\frac{\alpha}{2}$.

1.99. $\sin15°$, $\cos15°$, $\sin18°$, $\cos18°$, $\sin12°$, $\cos12°$ lar hisoblansin.

1.100. Ifodalarni soddalashtiring:

1) $\sqrt{\frac{1-\cos6\alpha}{2}}$; 2) $\sqrt{1+\cos10x}$; 3) $2\cos^2\frac{\alpha}{2}-\cos\alpha$;

4) $\frac{1+\cos4\alpha}{\sin4\alpha}$; 5) $\sqrt{\frac{\sin\left(\frac{7\pi}{2}-\alpha\right)+1}{\sin\left(\frac{5\pi}{2}+\alpha\right)+1}}$.

1.101. Ayniyatlarni isbot qiling:

1) $1-\cos\left(\frac{\pi}{2}+\alpha\right)=2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$;

2) $1+\cos\left(\frac{5\pi}{2}+\alpha\right)=2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$; 3) $\dfrac{\mathrm{tg}\left(\pi-\frac{\alpha}{2}\right)-\mathrm{ctg}\frac{\alpha}{2}}{\mathrm{tg}\frac{\alpha}{2}+\mathrm{ctg}\left(\pi-\frac{\alpha}{2}\right)}=-\cos\alpha$;

4) $\mathrm{tg}\left(\pi-\frac{\alpha}{2}\right)+\mathrm{ctg}\frac{\alpha}{2}=2\mathrm{ctg}\alpha$; 5) $\sin^4\alpha+\cos^4\alpha=\frac{3+\cos 4\alpha}{4}$;

6) $\cos^4\alpha-\sin^4\alpha=\cos 2\alpha$.

6. Trigonometrik funksiyalarni yarim argument tangensi orqali ifodalash. $\sin\alpha$ va $\cos\alpha$ ni $\mathrm{tg}\frac{\alpha}{2}$ orqali ifodalashda $\sin\alpha=$ $=2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$, $\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$ va $\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}=1$ formulalardan foydalanamiz. $\sin\alpha=\dfrac{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$ tenglikka egamiz. Bu tenglikdagi kasrning surat va maxrajini $\cos^2\frac{\alpha}{2}\neq 0$ ga bo‘lib,

$$\sin\alpha=\frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}} \tag{1}$$

tenglikni hosil qilamiz. Xuddi shu kabi, $\cos\alpha=\dfrac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$ tenglik yordamida quyidagi tenglik hosil qilinadi:

$$\cos\alpha=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}. \tag{2}$$

$\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni $\mathrm{tg}\frac{\alpha}{2}$ orqali ifodalash uchun (1) ni (2) ga va aksincha, (2) ni (1) ga hadma-had bo‘lish yetarli. Natijada quyidagi tengliklarga ega bo‘lamiz:

$$\mathrm{tg}\alpha=\frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}^2\frac{\alpha}{2}}, \tag{3}$$

$$\mathrm{ctg}\alpha=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{2\,\mathrm{tg}\frac{\alpha}{2}}. \tag{4}$$

M i s o l. Agar $\mathrm{tg}\frac{\alpha}{2}=-\frac{2}{3}$ bo‘lsa, $\frac{2+3\cos\alpha}{4-5\sin\alpha}$ ni hisoblang.

Y e c h i s h . (1) va (2) formulalarga ko‘ra,

$$\sin\alpha = \frac{2\cdot\left(-\frac{2}{3}\right)}{1+\left(-\frac{2}{3}\right)^2} = -\frac{12}{13}, \quad \cos\alpha = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{5}{13}.$$

Bundan $\frac{2+3\cos\alpha}{4-5\sin\alpha} = \frac{2+\frac{15}{13}}{4+\frac{60}{13}} = \frac{41}{112}$.

M a s h q l a r

1.102. Ifodani soddalashtiring:

1) $\frac{\mathrm{tg}\alpha\cdot\cos 2\alpha}{1+\mathrm{tg}^2\alpha}$; 2) $\frac{\cos 2\alpha-\cos 2\alpha\cdot\mathrm{tg}^2\alpha}{1+\mathrm{tg}^2\alpha}$.

1.103. $\mathrm{tg}\frac{x}{2} = \frac{1}{2}$ bo‘lsa, $\sin x$, $\cos x$, $\mathrm{tg}x$, $\mathrm{ctg}x$ ni toping.

1.104. $\sin x + \cos x = \frac{1}{5}$ bo‘lsa, $\mathrm{tg}\frac{x}{2}$ ni toping.

1.105. Ayniyatni isbotlang:

1) $\frac{\cos\alpha}{1-\sin\alpha} = \frac{1+\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}\frac{\alpha}{2}}$; 2) $\frac{\cos\alpha}{1+\sin\alpha} = \frac{\mathrm{ctg}\frac{\alpha}{2}-1}{\mathrm{ctg}\frac{\alpha}{2}+1}$;

3) $\left(\frac{\mathrm{tg}\frac{\alpha}{2}+1}{\mathrm{tg}\frac{\alpha}{2}-1}\right)^2 = \frac{1+\sin\alpha}{1-\sin\alpha}$; 4) $\frac{1+\sin\alpha+\cos\alpha}{1+\sin\alpha-\cos\alpha} = \mathrm{ctg}\frac{\alpha}{2}$;

5) $\cos^2\frac{\alpha}{2}\left(1+\mathrm{tg}\frac{\alpha}{2}\right)^2 = 1+\sin\alpha$; 6) $\sin^2\frac{\alpha}{2}\left(\mathrm{ctg}\frac{\alpha}{2}-1\right)^2 = 1-\sin\alpha$.

7. Trigonometrik funksiyalar yig‘indisini ko‘paytmaga va ko‘paytmasini yig‘indiga aylantirish. Ikki burchak yig‘indisi va ayirmasi sinusi munosabatlarini hadma-had qo‘shaylik:

$$\begin{array}{c} \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \hline \sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta, \end{array}$$

bundan:

$$\sin\alpha\cos\beta = \frac{1}{2}\big(\sin(\alpha+\beta)+\sin(\alpha-\beta)\big). \qquad (1)$$

Shu kabi ikki burchak kosinusi yig‘indisi va ayirmasi munosabatlarini hadma-had qo‘shsak va ayirsak, quyidagi formulalar hosil bo‘ladi:

$$\cos\alpha\cos\beta = \frac{1}{2}\big(\cos(\alpha+\beta)+\cos(\alpha-\beta)\big), \qquad (2)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big). \qquad (3)$$

Trigonometrik funksiyalar ko‘paytmasini yig‘indi yoki ayirma ko‘rinishiga keltirish maqsadida $\alpha+\beta=u$, $\alpha-\beta=v$ deb olamiz. Bulardan $\alpha=\frac{u+v}{2}$, $\beta=\frac{u-v}{2}$ larni topib, (1) formulaga qo‘ysak, natijada:

$$\sin u+\sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2}. \qquad (4)$$

(4) formulada v ni $-v$ ga almashtirsak,

$$\sin u-\sin v = 2\sin\frac{u-v}{2}\cos\frac{u+v}{2}. \qquad (5)$$

(2) va (3) formulalar bo‘yicha quyidagi tengliklar hosil bo‘ladi:

$$\cos u+\cos v = 2\cos\frac{u+v}{2}\cos\frac{u-v}{2}, \qquad (6)$$

$$\cos u-\cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2}. \qquad (7)$$

1 - m i s o l. $\cos 45^\circ+\cos 15^\circ$ ni hisoblaymiz.

Y e c h i s h. (6) formula bo‘yicha:

$$\cos 45^{\circ}+\cos 15^{\circ} = 2\cos\frac{45^{\circ}+15^{\circ}}{2}\cos\frac{45^{\circ}-15^{\circ}}{2} =$$

$$= 2\cos 60^{\circ}\cos 30^{\circ} = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}.$$

Tangens va kotangensga taalluqli formulalarni chiqaraylik:

$$\mathrm{tg}u+\mathrm{tg}v = \frac{\sin u}{\cos u}+\frac{\sin v}{\cos v} = \frac{\sin u\cos v+\cos u\sin v}{\cos u\cos v} = \frac{\sin(u+v)}{\cos u\cos v},$$

bundan

$$\mathrm{tg}u+\mathrm{tg}v = \frac{\sin(u+v)}{\cos u\cos v},\ u,\ v\neq\frac{\pi}{2}+\pi k,\ k\in Z. \qquad (8)$$

Quyidagi formulalar ham shu tartibda keltirib chiqariladi:

$$\mathrm{tg}u-\mathrm{tg}v = \frac{\sin(u-v)}{\cos u\cos v},\ u,\ v\neq\frac{\pi}{2}+\pi k,\ k\in Z, \qquad (9)$$

$$\operatorname{ctg}u + \operatorname{ctg}v = \frac{\sin(u+v)}{\sin u \sin v},\ u,\ v \neq \pi k,\ k \in Z, \qquad (10)$$

$$\operatorname{ctg}u - \operatorname{ctg}v = \frac{\sin(u-v)}{\sin u \sin v},\ u,\ v \neq \pi k,\ k \in Z. \qquad (11)$$

2-misol. Agar $u + v + w = \pi$ bo‘lsa, $\operatorname{ctg}u + \operatorname{ctg}v - \operatorname{tg}w = -\operatorname{ctg}u\operatorname{ctg}v\operatorname{tg}w$ bo‘lishini isbot qilamiz.

Yechish.

$$\operatorname{ctg}u+\operatorname{ctg}v-\operatorname{tg}w=\operatorname{ctg}u+\operatorname{ctg}v-\operatorname{tg}(\pi-(u+v))=\operatorname{ctg}u+\operatorname{ctg}v+\operatorname{tg}(u+v)=$$

$$=\frac{\sin(u+v)}{\sin u\sin v}+\frac{\sin(u+v)}{\cos(u+v)}=\frac{\sin(u+v)(\cos(u+v)+\sin u\sin v)}{\sin u\sin v\cos(u+v)}=\frac{\sin(u+v)\cos u\cos v}{\sin u\sin v\cos(u+v)}=$$

$$=\operatorname{ctg}u\operatorname{ctg}v\operatorname{tg}(u+v)=\operatorname{ctg}u\operatorname{ctg}v\operatorname{tg}(\pi-w)=-\operatorname{ctg}u\operatorname{ctg}v\operatorname{tg}w$$

Mashqlar

1.106. Trigonometrik funksiyalar yig‘indisi ko‘rinishida yozing:

1) $2\sin22^\circ\cos12^\circ$; 2) $\sin x\sin(x-1)$;

3) $4\sin35^\circ\cos25^\circ\sin15^\circ$; 4) $8\cos3^\circ\cos6^\circ\cos12^\circ\cos24^\circ$.

1.107. Hisoblang:

1) $\cos80^\circ\cos40^\circ\cos20^\circ$; 2) $\operatorname{tg}35^\circ\operatorname{tg}55^\circ$.

3) $\frac{2\cos80^\circ-\cos20^\circ}{\sin10^\circ}$; 4) $\cos20^\circ\cos40^\circ\cos80^\circ$;

5) $\cos9^\circ\cos27^\circ\cos63^\circ\cos81^\circ$; 6) $\sin20^\circ\sin40^\circ\sin60^\circ\sin80^\circ$.

1.108. Ko‘paytma yoki ko‘paytmalar ko‘rinishida tasvirlang:

1) $\cos43^\circ + \cos37^\circ$; 2) $\sin76^\circ - \sin26^\circ$;

3) $\sin57^\circ - \cos64^\circ$; 4) $\sin18^\circ + \cos15^\circ$;

5) $\sin\frac{\pi}{5}-\cos\frac{\pi}{7}$; 6) $\cos6x + \cos4x$;

7) $\cos^2\alpha - \cos^2\beta$; 8) $\cos\alpha - \cos\beta$;

9) $\operatorname{ctg}^2\alpha - \operatorname{ctg}^2\beta$; 10) $\cos x+\cos3x+\cos5x+\cos7x$;

11) $\sin x+\sin2x+\sin3x+\sin4x$;

12) $\sin20^\circ + \sin10^\circ + \sin30^\circ$;

13) $\cos\alpha+\cos\beta+\sin\frac{\alpha+\beta}{2}$; 14) $\sin2x-\cos x-\sin5x$;

15) $\cos\left(\frac{\pi}{10}+x\right)-\cos\left(\frac{\pi}{10}-x\right)-\sin x$;

16) $\sin(5\alpha + \beta) + \sin(3\alpha + \beta) + \sin2\alpha$;

17) $\sqrt{\text{ctg}\alpha + \cos\alpha} + \sqrt{\text{ctg}\alpha - \cos\alpha},\ 0 < \alpha < \frac{\pi}{2}$;

18) $1 - \text{ctg}\alpha$; 19) $1 + \text{ctg}\alpha$;

20) $1 - \text{tg}\alpha$; 21) $1 - \sin(\pi - x) + \cos(\pi - x)$;

22) $\text{ctg}\alpha + \text{ctg}2\alpha - \text{tg}3\alpha$.

1.109. Ayniyatni isbot qiling:

1) $\frac{\sin(t+s)-\sin(t-s)}{\sin(t+s)+\sin(t-s)} = \text{ctg}t\,\text{ctg}s$;

2) $2(1+\sin t\sin s+\cos t\cos s) = 4\cos^2\frac{t-s}{2}$.

3) $(\sin3t + \sin5t)^2 + (\cos3t + \cos5t)^2 = 4\cos^2t$;

4) $\text{tg}20° + \text{tg}40° + \text{tg}80° - \text{tg}60° = 8\cos50°$;

5) $\text{ctg}^6 20^{\circ} - 9\text{ctg}^4 20^{\circ} + 11\text{ctg}^2 20^{\circ} = \frac{1}{3}$;

6) $\cos 9°\cos 27°\cos 63°\cos 81° + \cos 12°\cos 24°\cos 48°\cos 96° = 0$;

7) $\sin\alpha + \sin2\alpha + \sin3\alpha + ... + \sin n\alpha = \dfrac{\sin\frac{n\alpha}{2}\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$;

8) $\cos\alpha + \cos2\alpha + \cos3\alpha + ... + \cos n\alpha = \dfrac{\sin\frac{n\alpha}{2}\cos\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$;

9) Agar $\sin\alpha + \sin\beta = 2\sin(\alpha + \beta)$ va $\frac{\alpha+\beta}{2} \neq k\pi$ bo‘lsa, $\text{tg}\frac{\alpha}{2} + \text{tg}\frac{\beta}{2} = \frac{1}{3}$ bo‘ladi;

10) Agar $0 \le x \le \frac{\pi}{2}$ bo‘lsa, $\sqrt{1+\sin x} - \sqrt{1-\sin x} = 2\sin\frac{x}{2}$ bo‘ladi.

1.110. Yig‘indini hisoblang:

1) $\cos\alpha + 2\cos2\alpha + 3\cos3\alpha + ... + n\cos n\alpha$;

2) $\frac{1}{\cos\alpha\cos2\alpha} + \frac{1}{\cos2\alpha\cos3\alpha} + ... + \frac{1}{\cos9\alpha\cos10\alpha}$;

3) $\sin\alpha - \sin2\alpha + \sin3\alpha - ... + (-1)^n \sin n\alpha$.

8. Garmonik tebranishlarni qo‘shish. 1) $y = A\sin(\omega t + \alpha)$ garmonik tebranishni ikki garmonik tebranish yig‘indisi ko‘rinishida yozishda $\sin(x + z) = \sin x\cos z + \cos x\sin z$ formuladan foydalanamiz:

$$y = A\sin(\omega t + \alpha) = A\cos\alpha\sin\omega t + A\sin\alpha\cos\omega t,$$

bunga $C_1 = A\cos\alpha$, $C_2 = A\sin\alpha$ belgilash kiritsak:

$$y = A\sin(\omega t + \alpha) = C_1\sin\omega t + C_2\cos\omega t, \quad A = \sqrt{C_1^2 + C_2^2}\,; \qquad (1)$$

2) ikkita garmonik tebranishning $C_1\sin\omega t + C_2\cos\omega t$ yig‘indisini $A\left(\frac{C_1}{A}\sin\omega t + \frac{C_2}{A}\cos\omega t\right)$ ko‘rinishda yozib olaylik. $\left(\frac{C_1}{A}\right)^2 + \left(\frac{C_2}{A}\right)^2 = 1$ bo‘lgani uchun $\frac{C_1}{A} = \cos\alpha$, $\frac{C_2}{A} = \sin\alpha$ tengliklar o‘rinli bo‘ladigan $\alpha \in [0;\ 2\pi]$ son mavjuddir.

Bu yerdan ikkita garmonik tebranishning $C_1\sin\omega t + C_2\cos\omega t$ yig‘indisini $C_1\sin\omega t + C_2\cos\omega t = A\sin(\omega t + \alpha)$ ko‘rinishda yozish mumkinligi va garmonik tebranishlarning yig‘indisi ham garmonik tebranish bo‘lishini ko‘ramiz;

3) bir xil ω chastotali ikki $C_1\sin\omega t + C_2\cos\omega t$ va $a\sin\omega t + b\cos\omega t$ garmonik tebranish yig‘indisi $(C_1 + a)\sin\omega t + (C_2 + b)\cos\omega t$ bo‘ladi, bunda ω – yig‘indining chastotasi, A – amplitudasi, $A = \sqrt{(C_1 + a)^2 + (C_2 + b)^2}$.

Yig‘indining amplitudasi va boshlang‘ich fazasini qo‘shiluvchi tebranishlar amplitudalari va boshlang‘ich fazalari orqali ifodalaylik:

$C_1 = A_1\cos\alpha_1$, $C_2 = A_2\cos\alpha_2$, $a = A_2\cos\alpha_2$, $b = A_2\sin\alpha_2$ bo‘lsin. U holda:

$$A^2 = (C_1 + a)^2 + (C_2 + b)^2 = (A_1\cos\alpha_1 + A_2\cos\alpha_2)^2 +$$
$$+(A_1\sin\alpha_1 + A_2\sin\alpha_2)^2 = A_1^2\cos^2\alpha_1 + 2A_1A_2\cos\alpha_1\cos\alpha_2 +$$
$$+A_2^2\cos^2\alpha_2 + A_1^2\sin^2\alpha_1 + 2A_1A_2\sin\alpha_1\sin\alpha_2 + A_2^2\sin^2\alpha_2 =$$
$$= A_1^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2) + A_2^2 \qquad (2)$$

Yig‘indi tebranishning α boshlang‘ich fazasi quyidagi tengliklardan biri bo‘yicha topiladi:

$$\cos\alpha = \frac{C_1 + a}{A} = \frac{1}{A}\left(A_1\cos\alpha_1 + A_2\cos\alpha_2\right), \qquad (3)$$

$$\sin\alpha = \frac{C_2+b}{A} = \frac{1}{A}\left(A_1\sin\alpha_1 + A_2\sin\alpha_2\right), \qquad (4)$$

$$\text{tg}\alpha = \frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2}. \qquad (5)$$

Har xil chastotali garmonik tebranishlarni qo‘shish masalalari oliy matematika kurslarida qaraladi.

M i s o l. $y = 5\sin 9x + \sqrt{75}\cos 9x$ funksiyaning eng katta va eng kichik qiymatlarini toping.

Y e c h i s h. Funksiyani $y = A\sin(\omega t + \alpha)$ ko‘rinishda tasvirlaymiz:

$$y = \sqrt{5^2 + (\sqrt{75})^2} \cdot \left(\frac{5}{\sqrt{5^2+(\sqrt{75})^2}}\sin 9x + \frac{75}{\sqrt{5^2+(\sqrt{75})^2}}\cos 9x\right) =$$

$$= 10 \cdot \left(\frac{1}{2}\sin 9x + \frac{\sqrt{3}}{2}\cos 9x\right) = 10\sin\left(9x + \frac{\pi}{3}\right).$$

Bu yerdan, $-10 \le 10\sin\left(9x + \frac{\pi}{3}\right) \le 10$ tengsizlikka ega bo‘lamiz. $y\left(\frac{7\pi}{27}\right) = -10$, $y\left(\frac{\pi}{54}\right) = 10$ tengliklar o‘rinli va barcha $x \in R$ larda $-10 \le y(x) \le 10$ bo‘lgani uchun $y(x)$ funksiyaning eng kichik qiymati –10 ga, eng katta qiymati esa 10 ga teng bo‘ladi.

Mashqlar

1.111. Ifodalarni $A\sin(\omega t + \alpha)$ ko‘rinishga keltiring:

1) $3\sin 5t + 4\cos 5t$; 2) $11\sin\left(3t - \frac{\pi}{6}\right) + \sqrt{23}\cos\left(3t - \frac{\pi}{6}\right)$;

3) $12\sin\left(2t + \frac{\pi}{3}\right) + 5\cos\left(2t + \frac{\pi}{3}\right)$.

1.112. Quyida berilgan funksiyalarning eng katta va eng kichik qiymatlarini toping:

1) $60\sin 3x - 11\cos 3x$; 2) $-12\sin 4x + 5\cos 4x$;

3) $2\sin x + \sqrt{5}\cos x$; 4) $4\sin\left(3x - \frac{\pi}{4}\right) - 3\cos\left(3x - \frac{\pi}{4}\right)$.

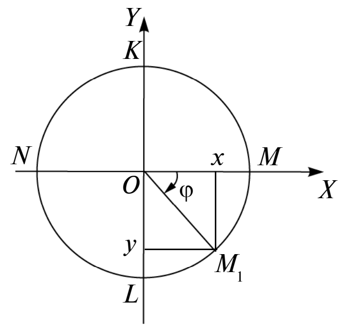
1.113. Garmonik tebranishlar yig‘indisini toping:

1) $y = 4\sin 3t$ va $y = 5\sin\left(3t - \frac{\pi}{3}\right)$;

2) $y = 6\sin\left(2t + \frac{\pi}{4}\right)$ va $y = \sin\left(2t - \frac{\pi}{4}\right)$;

3) $y = 2\sin\left(5t - \frac{\pi}{3}\right)$ va $y = 5\sin\left(5t + \frac{\pi}{3}\right)$;

4) $y = 3\sin 6t$ va $y = 2\sin\left(6t - \frac{\pi}{6}\right)$.



I.38-rasm.

1.114. O'zgaruvchan tok tarmog'ida I tok kuchi va t vaqt orasidagi bog'lanish $I = I_m \sin(\omega t + \varphi)$ orqali beriladi, bunda I_m — tok kuchining amplitudaviy qiymati, ω — tebranishning doimiy chastotasi, φ — tok kuchi bilan kuchlanishning fazaviy farqi. Agar $I_m = 3$, $\omega = 600$, $\varphi = 1{,}2$ bo'lsa, boshlang'ich fazadagi tok kuchining maksimal qiymatini va sekundiga davrlar sonini toping.

1.115. Boshlang'ich t_0 vaqt momentida $R = |OM|$ radiusli aylanadagi M nuqta o'zi turgan qism aylana bo'ylab ω burchak tezlik bilan harakat qilmoqda (I.38-rasm). U t vaqtda $\varphi = \angle MOM_1$ burchakka burilgan, $\varphi = \omega t$ va $M_1(x; y)$ nuqtaga yetgan. Harakat davomida OX o'qidagi x nuqta MN diametr bo'yicha, OY o'qidagi y nuqta KL diametr bo'yicha borib-qaytadi va bu harakat garmonik tebranish qonuniyati bo'yicha o'tadi.

1) Koordinatalari shakli o'zgaruvchan to'g'ri burchakli uchburchak OxM_1 ning katetlari ekanidan foydalanib, x va y nuqtalarning garmonik tebranishlari tenglamalarini tuzing;

2) bunday tenglamalarni $\varphi_0 \neq 0$ va burilish burchagi $\varphi + \varphi_0$ ga teng bo'lgan hol uchun ham tuzing;

3) $R = 10$ sm, $\varphi_0 = 0$, $\varphi = 30°$; $60°$; $90°$; $180°$; $270°$; $450°$ da jism qanchaga siljiydi?

4) $\omega = 2\pi/T$ va $t = 10$ s bo'lsa, bir to'liq tebranish uchun ketadigan T vaqtni hisoblang, φ ning qiymatlarini 3- savoldan oling.

4- §. Trigonometrik tenglamalar va tengsizliklar

Noma'lum son faqat trigonometrik funksiyalarning argumenti sifatida qatnashgan tenglama (tengsizlik) *trigonometrik tenglama* (*trigonometrik tengsizlik*) deyiladi.

$\sin\alpha = m$, $\cos\alpha = m$, $\mathrm{tg}\alpha = m$, $\mathrm{ctg}\alpha = m$ ko‘rinishdagi tenglamalar *eng sodda trigonometrik tenglamalardir.* Bu tenglamalarda tenglik belgisi tengsizlik belgisi bilan almashtirilsa, *eng sodda trigonometrik tengsizliklar* hosil bo‘ladi.

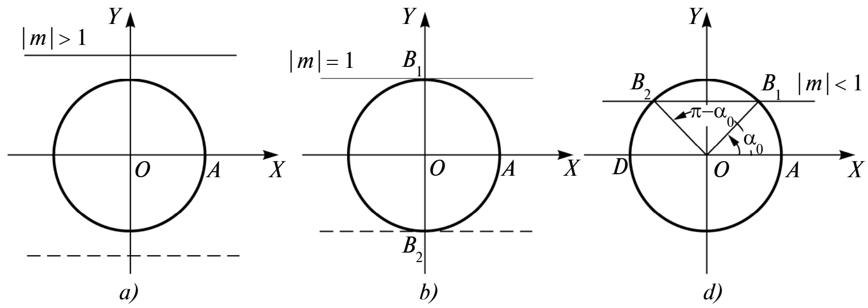
Odatda trigonometrik tenglamalarni (tengsizliklarni) yechish bitta yoki bir nechta eng sodda trigonometrik tenglamalarni (tengsizliklarni) yechishga keltiriladi.

1. $\sin\alpha = m$ ko‘rinishdagi eng sodda tenglama. Arksinus. $\sin\alpha = m$ tenglamani yechish birlik aylanadagi shunday $B(\alpha)$ nuqtani topishdan iboratki, uning $y = \sin\alpha$ ordinatasi m ga teng bo‘lishi kerak. Buning uchun gorizontal diametrga parallel bo‘lgan $y = m$ to‘g‘ri chiziq bilan birlik aylananing kesishish nuqtalarini topish kerak. Uch hol bo‘lishi mumkin:

a) agar $|m| > 1$ bo‘lsa, $y = m$ to‘g‘ri chiziq aylanani kesmay, undan yuqori yoki quyidan o‘tadi (I.39-*a* rasm). Demak, bu holda tenglama yechimga ega emas;

b) agar $|m| = 1$ bo‘lsa, to‘g‘ri chiziq aylanaga yo yuqoridagi $B_1\left(\frac{\pi}{2}\right)$ nuqtada yoki quyidagi $B_2\left(-\frac{\pi}{2}\right)$ nuqtada urinib o‘tadi (I.39-*b* rasm). Bu holda tenglama yagona ildizga ega: $\alpha = \frac{\pi}{2}$ yoki $\alpha = -\frac{\pi}{2}$. Agar funksiyaning $T = 2\pi$ asosiy davri ham e’tiborga olinsa, yechimni $\alpha = \frac{\pi}{2} + 2\pi k,\ k \in Z$ $\left(\alpha = -\frac{\pi}{2} + 2\pi k,\ k \in Z\right)$ ko‘rinishda yozish mumkin;

d) $|m| < 1$ bo‘lsa, $y = m$ to‘g‘ri chiziq aylanani $B_1(\alpha_0)$ va $B_2(\pi - \alpha_0)$ nuqtalarda kesadi(1.39-*d* rasm). Demak, tenglamaning yechimi shu nuqtalarning koordinatalari bo‘lgan barcha sonlar to‘plamlarining birlashmasi bo‘ladi:



I.39-rasm.

$$\{\alpha_0 + 2k\pi,\ k\in Z\}\cup\{\pi - \alpha_0 + 2k\pi,\ k\in Z\}.$$

Yechimni $x = \alpha_0 + 2k\pi,\ k\in Z;\ x = \pi - \alpha_0 + 2k\pi,\ k\in Z$ ko'rinishda ham yozish mumkin.

Yechimning geometrik tahlilida $y = m$ to'g'ri chiziq bilan sinusoidaning kesishish nuqtasi haqida ham gapirilishi mumkin.

1 - m i s o l. $\sin\alpha = \frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Y e c h i s h. $y = \frac{\sqrt{3}}{2}$ $(y < 1)$ to'g'ri chiziq koordinatali aylanani $B_1\left(\frac{\pi}{3}\right)$ va $B_2\left(\frac{2\pi}{3}\right)$ nuqtalarda kesadi (I.39-*d* rasm). B_1 nuqta barcha $\frac{\pi}{3} + 2k\pi,\ k\in Z$ sonlar to'plamiga, B_2 nuqta esa barcha $\frac{2\pi}{3} + 2k\pi,\ k\in Z$ ko'rinishdagi sonlar to'plamiga mos. Barcha yechimlar to'plamini $\alpha = \frac{\pi}{3} + 2k\pi,\ k\in Z;\ \alpha = \frac{2\pi}{3} + 2k\pi,\ k\in Z$ yoki $\left\{\frac{\pi}{3} + 2k\pi,\ k\in Z\right\}\cup\left\{\frac{2\pi}{3} + 2k\pi,\ k\in Z\right\}$ ko'rinishda yozish mumkin.

2 - m i s o l. a) $\sin\alpha = 1$; b) $\sin\alpha = -1$; d) $\sin\alpha = 0$ tenglamalarni yechamiz.

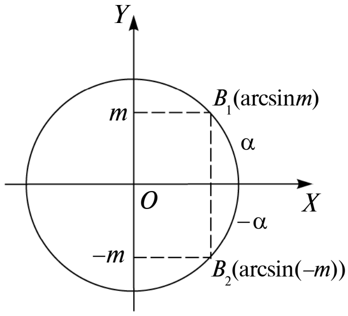
Y e c h i s h. a) Koordinatali aylanada faqat bitta $B_1\left(\frac{\pi}{2}\right)$ nuqtaning ordinatasi 1 ga teng (I.39-*b* rasm). Y e c h i m: $\alpha = \frac{\pi}{2} + 2\pi k,\ k\in Z$;

b) $B_2\left(-\frac{\pi}{2}\right) = B_2(0;\ -1)$ nuqta bo'yicha $\alpha = -\frac{\pi}{2} + 2\pi k,\ k\in Z$;

d) ordinatasi 0 bo'lgan nuqta ikkita: $A(0)$ va $D(\pi)$ (I.39-*d* rasm). A nuqtaga $2k\pi,\ k\in Z$, D nuqtaga esa $\pi + 2k\pi,\ k\in Z$ sonlar mos keladi.

J a v o b: $\alpha = 2k\pi,\ k\in Z;\ \alpha = \pi + 2k\pi,\ k\in Z.$

$|m|\le 1$ da $y = m$ to'g'ri chiziq va o'ng yarim birlik aylana yagona umumiy nuqtaga ega bo'ladi. Shu sababli $\sin\alpha = m$ $(|\,m\,| \le 1)$ tenglama $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ oraliqqa tegishli bo'lgan yagona x_0 yechimga ega. $\sin\alpha = m$ tenglamani qanoatlantiruvchi $\alpha_0 \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ soni m sonning *arksinusi* deyiladi va $\arcsin m$ orqali belgilanadi. Ta'rifga ko'ra



I.40-rasm.

$$\sin(\arcsin m) = m \qquad (1)$$

va

$$-\frac{\pi}{2} \le \arcsin m \le \frac{\pi}{2} \qquad (2)$$

bo‘ladi. Aksincha, $\sin\alpha = m$ va $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$ bo‘lsa, $\alpha = \arcsin m$ bo‘ladi.

3-misol. a) $\arcsin\frac{\sqrt{3}}{2}$; b) $\arcsin\left(-\frac{1}{2}\right)$; d) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ ifodalarni hisoblaymiz.

Yechish. a) $\sin x = \frac{\sqrt{3}}{2}$ bo‘yicha $x_1 = \frac{\pi}{3}$, $x_2 = \frac{2\pi}{3}$. Arksinusning ta’rifi bo‘yicha $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ bo‘lishi kerak. Bu shartga $x_1 = \frac{\pi}{3}$ to‘g‘ri keladi. Demak, $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

b) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $-\frac{\pi}{2} \le -\frac{\pi}{6} \le \frac{\pi}{2}$ bo‘lgani uchun $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ bo‘ladi.

d) $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $-\frac{\pi}{2} \le -\frac{\pi}{3} \le \frac{\pi}{2}$. Demak, $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

I.40-rasmdan $y = m$ va $\alpha = \arcsin m$ sonlari orasidagi bog‘lanish ayon bo‘ladi. Chizmada $\alpha = \arcsin m$ va $-\alpha = \arcsin(-m)$. Demak,

$$\arcsin(-m) = -\arcsin m. \qquad (3)$$

Shunday qilib, $|m| \le 1$ bo‘lgan holda $\sin\alpha = m$ tenglamaning α yechimi $\{\arcsin m + 2k\pi,\ k\in Z\} \cup \{\pi - \arcsin m + 2k\pi,\ k\in Z\}$ to‘plamlar birlashmasi ko‘rinishida yoki $\alpha = \arcsin m + 2k\pi$, $k\in Z$; $\alpha = \pi - \arcsin m + 2k\pi$, $k\in Z$ ko‘rinishda yoki bu keyingi ikki formulani birlashtirib,

$$\alpha = (-1)^k \arcsin m + k\pi,\ k\in Z \qquad (4)$$

ko‘rinishda yozish mumkin.

4-misol. a) $\sin\alpha = \frac{1}{7}$; b) $\sin\alpha = -\frac{1}{9}$ tenglamalarni yechamiz.

Yechish. a) $\sin\alpha = \frac{1}{7}$ tenglama yechimini (4) formula bo‘yicha $\alpha = (-1)^k \arcsin\frac{1}{7} + k\pi$, $k \in Z$ ko‘rinishda yozamiz;

b) (3) munosabatga ko‘ra $\arcsin\left(-\frac{1}{9}\right)=\arcsin\frac{1}{9}$.

Y e c h i m : $\left\{-\arcsin\frac{1}{9}+2k\pi,\ k\in Z\right\}\cup\left\{\pi+\arcsin\frac{1}{9}+2k\pi,\ k\in Z\right\}$ yoki $\alpha=(-1)^{k+1}\arcsin\frac{1}{9}+k\pi,\ k\in Z$ ekanligi kelib chiqadi.

Mashqlar

1.116. Tenglamalarni yeching va grafik yordamida tushuntiring:

1) $\sin x=-0{,}5$; 2) $\sin x=-0{,}75$; 3) $\sin x=0{,}2$; 4) $\sin x=\frac{7}{8}$; 5) $\sin x=\frac{\sqrt{2}}{2}$; 6) $\sqrt{3}\sin x+\frac{3}{2}=0$; 7) $5\sin x-7=0$; 8) $6\sin x-2=0$.

1.117. Tenglamalarni yeching:

1) $4\sin^2 x-1=0$; 2) $-2\sin^2 x+\sin x+1=0$;

3) $3\sin^2 x-4\sin x-0{,}75=0$; 4) $\sqrt{3}\sin x-2\sin^2 x=0$.

1.118. Qiymatini toping:

1) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$; 2) $\arcsin\left(\frac{\sqrt{2}}{2}\right)$; 3) $\arcsin 0{,}5$.

1.119. Hisoblang:

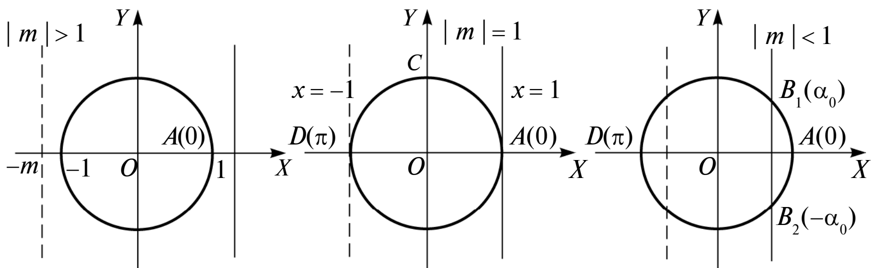
1) $\arcsin(\sin 30^\circ)$; 2) $\arcsin\left(\sin\frac{\pi}{12}\right)$;

3) $\arcsin(\sin 2)$; 4) $\arcsin(\sin 10)$.

1.120. $\arcsin\alpha$ quyidagi qiymatlarni qabul qila oladimi?

1) $\frac{\pi}{3}$; 2) -3π; 3) $-\frac{\pi}{6}$; 4) $\frac{\pi}{4}$; 5) $-\frac{\pi}{4}$; 6) $\sqrt{3}$; 7) $-\pi$; 8) $4\sqrt{5}$.

2. $\cos\alpha=m$ ko‘rinishdagi eng sodda tenglama. Arkkosinus. Koordinatali aylanada olingan har qaysi $B(\alpha)$ nuqtaning



I.41-rasm.

abssissasi $x = \cos\alpha$ ga teng. Shunga ko‘ra berilgan m bo‘yicha $\cos\alpha = m$ tenglamani yechish nuqtaning $x = m$ abssissasi bo‘yicha unga mos $\alpha = \alpha_0$ yoy kattaligini topishdan iborat. Uch holni qaraymiz:

1-hol. $|m| > 1$ da $x = m$ vertikal to‘g‘ri chiziq aylanani kesmaydi (I.41-*a* rasm). Bu holda tenglama yechimga ega emas. Masalan, $\cos\alpha = 2{,}8$ tenglama yechimga ega emas, chunki $m = 2{,}8 > 1$.

2-hol. Agar $|m| = 1$ bo‘lsa, to‘g‘ri chiziq aylanani faqat bir nuqtada, ya’ni yo $A(1;\ 0)$ nuqtada, yoki $D(-1;\ 0)$ nuqtada kesadi (I.41-*b* rasm). A nuqtaning aylana bo‘yicha koordinatasi $\alpha = 2\pi k$, $k \in Z$. Shunga ko‘ra $\cos\alpha = 1$ ning yechimi $\alpha = 2\pi k$, $k \in Z$ sonlar to‘plami bo‘ladi. $D(-1;\ 0) = D(\pi + 2\pi k)$ ekani e’tiborga olinsa, $\cos\alpha = -1$ ning yechimi $\alpha = \pi + 2\pi k$ sonlar to‘plami bo‘ladi.

3-hol. $|m| < 1$ bo‘lsa, $x = m$ to‘g‘ri chiziq aylanani ikki nuqtada kesadi (I.41-*d* rasm). Ulardan biri $B_1(\alpha_0)$ nuqta $0 \le \alpha_0 \le \pi$ yuqori yarim aylanada joylashadi. α_0 son m sonning *arkkosinusi* deyiladi va $\alpha_0 = \arccos m$ orqali belgilanadi. Ta’rifga ko‘ra $\cos\alpha = \cos(\arccos m) = m$ va $0 \le \arccos m \le \pi$ bo‘ladi.

Shu kabi $B_2(-\alpha_0)$ nuqta uchun: $\cos(-\alpha_0) = \cos\alpha_0 = m$. Bundan $-\alpha_0 = \arccos m$ yoki $\alpha_0 = -\arccos m$. Demak, $|m| < 1$, $k \in Z$ da $\cos\alpha = m$ tenglamaning yechimi $\{\arccos m + 2\pi k,\ k \in Z\} \cup \{-\arccos m + 2\pi k,\ k \in Z\}$ sonlarto‘plamlari birlashmasi bo‘ladi. Uni

$$\{\pm\arccos m + 2\pi k,\ k \in Z\} \qquad (1)$$

yoki

$$\pm\arccos m + 2\pi k,\ k \in Z \qquad (2)$$

ko‘rinishda ham yozish mumkin. I.42-rasmdan, OY o‘qiga nisbatan simmetrik joylashgan $B_1(\arccos m) = B_1(\alpha)$ va $B_2(\arccos(-m)) = B_2(\pi - \alpha)$ nuqtalar bo‘yicha $\alpha = \arccos m$ va $\pi - \alpha = \arccos(-m)$ bo‘lishini aniqlaymiz. Undan:

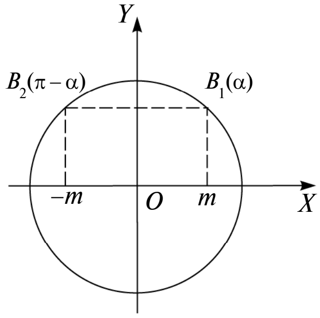
$$\arccos(-m) = \pi - \arccos m \qquad (3)$$

hosil qilinadi, bunda $0 \le \alpha \le \pi$.

1-misol. $\cos\alpha = \frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Yechish. $\cos\frac{\pi}{6} = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ bo‘ladi. Demak, $x = \frac{\sqrt{3}}{2}$ to‘g‘ri chiziq koordinatali aylanani $B_1\left(\arccos\frac{\sqrt{3}}{2}\right) = B_1\left(\frac{\pi}{6}\right)$ nuqtada va abssissalar o‘qiga nisbatan B_1 ga simmetrik joylashgan

$B_2\left(-\arccos\frac{\sqrt{3}}{2}\right)=B_2\left(-\frac{\pi}{6}\right)$ nuqtada kesadi. Yechim B_1 nuqta bo‘yicha $\frac{\pi}{6}+2\pi k,\ k\in Z$ sonlar to‘plami va B_2 nuqta bo‘yicha $-\frac{\pi}{6}+2\pi k,\ k\in Z$ sonlar to‘plami birlashmasi bo‘ladi:



I.42-rasm.

$\cos\alpha=\frac{\sqrt{3}}{2};\ \left\{\frac{\pi}{6}+2k\pi,\ k\in Z\right\}\cup$

$\cup\left\{-\frac{\pi}{6}+2k\pi,\ k\in Z\right\}$ yoki

$$\alpha=\pm\frac{\pi}{6}+2k\pi,\ k\in Z.$$

2 - m i s o l. $\arccos\left(-\frac{1}{2}\right)$ ni hisoblang.

Y e c h i s h. (3) formulaga ko‘ra, quyidagini topamiz:

$$\arccos\left(-\frac{1}{2}\right)=\pi-\arccos\frac{1}{2}=\pi-\frac{\pi}{3}=\frac{2\pi}{3}.$$

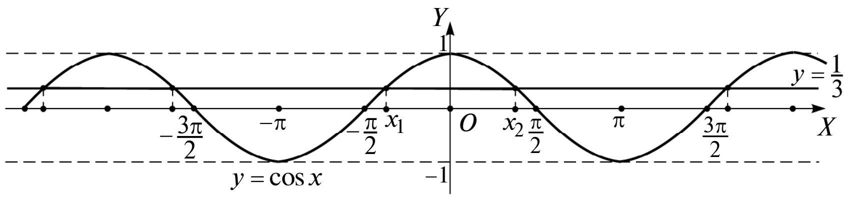
3 - m i s o l. $\cos x=-\frac{3}{7}$ tenglamani yeching.

Y e c h i s h. $x=\pm\arccos\left(-\frac{3}{7}\right)+2\pi k,\ k\in Z$ ga egamiz. (3) ga ko‘ra $x=\pm\left(\pi-\arccos\frac{3}{7}\right)+2\pi k,\ k\in Z$ bo‘ladi.

4 - m i s o l. $\cos x=\frac{1}{3}$ tenglamani $y=\cos x$ funksiya grafigi yordamida yeching.

Y e c h i s h. Ayni bir XOY koordinatalar sistemasida $y=\cos x$ va $y=\frac{1}{3}$ funksiyalar grafiklarini yasaymiz (I.43-rasm).

Bu grafiklar cheksiz ko‘p nuqtalarda kesishadi. $y=\cos x$ funksiya davri 2π bo‘lgan davriy funksiya bo‘lgani uchun berilgan



I.43-rasm.

tenglamaning $[-\pi;\ \pi]$ kesmadagi barcha yechimlarini topish va qolgan yechimlarni shu yechimlar orqali aniqlash mumkin.

$[-\pi;\ \pi]$ oraliqda $y = \cos x$ funksiya grafigi $y = \frac{1}{3}$ funksiya grafigi bilan ikkita kesishish nuqtasiga ega. Kesishish nuqtalarining $x_1 = -\arccos\frac{1}{3}$, $x_2 = \arccos\frac{1}{3}$ abssissalari berilgan tenglamaning $[-\pi;\ \pi]$ dagi barcha yechimlaridir. Shu sababli barcha yechimlar quyidagicha aniqlanadi: $x = \pm\arccos\frac{1}{3} + 2\pi k,\ k \in Z$.

5 - m i s o l. $\arccos(\cos 53°)$ ni toping.

Y e c h i s h. $\arccos(\cos m) = m$, $(0 \le m \le \pi)$ ayniyatdan foydalanamiz. $53^{\circ} = \frac{53\pi}{180}$ va $0 < \frac{53\pi}{180} < \pi$ bo‘lgani uchun bu ayniyatga ko‘ra

$$\arccos(\cos 53^{\circ}) = \arccos\left(\cos\frac{53\pi}{180}\right) = \frac{53\pi}{180}.$$

Mashqlar

1.121. Tenglamani $y = \cos x$ funksiya grafigi yordamida yeching:

1) $\cos x = 0$; 2) $\cos x = 0{,}5$; 3) $\cos x = -\frac{2}{9}$;

4) $\cos x = -\frac{\sqrt{2}}{2}$; 5) $\cos x = 2{,}4$; 6) $2\cos x + \sqrt{3} = 0$.

1.122. Ifodaning qiymatini toping:

1) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$; 2) $\arccos(-0{,}5)$; 3) $\arccos(\cos 30°)$;
4) $\arccos(\cos(-30°))$; 5) $\arccos(\sin 30°)$; 6) $\arccos(\cos 2)$;
7) $\arccos(\cos(-2))$; 8) $\arccos(\sin 2)$; 9) $\arccos(\sin(-2))$;
10) $\arccos(\cos 88)$; 11) $\arccos(\sin 86)$.

1.123. Tengliklarning to‘g‘riligini tekshiring:

1) $\arccos x = -\arcsin x$; 2) $-\arccos x = \pi + \arccos x$.

1.124. Ifodaning qiymatini toping:

1) $\cos\left(\arccos\frac{\sqrt{3}}{2} - \arcsin\frac{\sqrt{3}}{2}\right)$; 2) $\sin\left(\arccos\frac{1}{2} + \arcsin\frac{\sqrt{3}}{2}\right)$.

1.125. Tenglamani yeching:

1) $\cos^2 x - 3 = 0$; 2) $\cos 2x = \left(-\frac{\sqrt{2}}{2}\right)$; 3) $6\cos^2 x + 3 = 0$;

4) $3\cos^2 x - 5 = 0$; 5) $2\cos^2 x - 1 = 0$; 6) $4\cos^2 x - 1 = 0$.

1.126. Tenglamani yeching:

1) $\cos^2 x - 2\cos x = 0$; 2) $2\cos^2 x - \cos x = 0$;

3) $2\cos^2 x - \cos x - 1 = 0$; 4) $2\cos^2 x - 3\cos x + 1 = 0$.

3. $\mathrm{tg}\alpha = m$ va $\mathrm{ctg}\alpha = m$ ko'rinishdagi eng sodda tenglamalar. Arktangens va arkkotangens. Koordinatali aylananing har bir $B(\alpha)$ nuqtasi Dekart koordinatalar sistemasidagi biror $B\ (x,\ y)$ nuqta bilan ustma-ust tushishini va $x = \cos\alpha$, $y = \sin\alpha$ ekanini bilamiz. Shunga ko'ra, noma'lum α qatnashayotgan $\mathrm{tg}\alpha = m$ yoki $\frac{\sin\alpha}{\cos\alpha} = m$ tenglamaning barcha yechimlarini koordinatali aylana bilan $\frac{y}{x} = m$, ya'ni $y = mx$ to'g'ri chiziqning kesishish nuqtalari yordamida aniqlash mumkin. m ning har qanday qiymatida $y = mx$ to'g'ri chiziq aylanani $O\ (0;\ 0)$ nuqtaga nisbatan simmetrik bo'lgan B_1 va B_2 nuqtalarda kesadi (I.44-rasm). Ulardan biri $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ o'ng yarim aylanada yotadi. Bu nuqta $B_1(\alpha_0)$ bo'lsin. Ikkinchi nuqta $B_2(\alpha_0 + \pi)$ bo'ladi. Demak, $\mathrm{tg}\alpha = m$ tenglamaning barcha yechimlari to'plami $\alpha = \alpha_0 + 2k\pi$, $k \in Z$ va $\alpha = (\alpha_0 + \pi) + 2k\pi$, $k \in Z$ sonlar to'plamlari birlashmasidan iborat. Barcha yechimlar

$$\alpha = \alpha_0 + k\pi,\ k \in Z \tag{1}$$

formula bilan aniqlanadi.

m sonning *arktangensi* deb $\left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$ oraliqda yotadigan shunday α songa aytiladiki, uning uchun $\mathrm{tg}\alpha = m$ bo'ladi. m sonning arktangensi $\alpha = \mathrm{arctg}\,m$ orqali belgilanadi. Ta'rifga asosan, har qanday m son uchun quyidagi munosabatlar o'rinli bo'ladi:

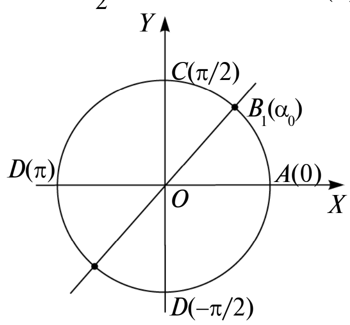
$$\mathrm{tg}(\mathrm{arctg}\,m) = m,\ -\frac{\pi}{2} < \mathrm{arctg}\,m < \frac{\pi}{2}. \tag{2}$$

Aksincha, $\mathrm{tg}\alpha = m$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ bo'lsa, $\alpha = \mathrm{arctg}\,m$ bo'ladi.

Yuqoridagi shartlardan va tangens toq funksiyaligidan $\mathrm{tg}(-\alpha) = -\mathrm{tg}\alpha = -m$ bo'lgani uchun quyidagi tenglik o'rinli bo'ladi:

$$\mathrm{arctg}(-m) = -\mathrm{arctg}\,m \tag{3}$$

Arkkotangens tushunchasi ham shu kabi kiritiladi.



I.44-rasm.

m sonning *arkkotangensi* deb $(0;\ \pi)$ oraliqda yotadigan shunday α songa aytiladiki, uning uchun $\operatorname{ctg}\alpha = m$ bo'ladi. *m* sonning arkkotangensi $\alpha = \operatorname{arcctg} m$ orqali belgilanadi. Uning uchun quyidagi tenglik o'rinli:

$$\operatorname{arcctg}(-m) = \pi - \operatorname{arcctg} m. \qquad (4)$$

1 - m i s o l. a) $\operatorname{tg} x = -\sqrt{3}$; b) $\operatorname{ctg} x = -\sqrt{3}$ tenglamalarni yechamiz.

Y e c h i s h. a) $\operatorname{tg}\left(-\frac{\pi}{3}\right) = -\sqrt{3}$, demak, $x = -\frac{\pi}{3} + \pi k,\ k \in Z$.

b) $\operatorname{ctg}\frac{5\pi}{6} = -\sqrt{3}$, demak, $x = \frac{5\pi}{6} + \pi k,\ k \in Z$.

2 - m i s o l. a) $\operatorname{arctg}\left(-\sqrt{3}\right)$; b) $\operatorname{arcctg}\left(-\sqrt{3}\right)$ sonlarni topamiz.

Y e c h i s h. a) (3) formula bo'yicha $\operatorname{arctg}\left(-\sqrt{3}\right) =$ $= -\operatorname{arctg}\sqrt{3} = -\frac{\pi}{3}$;

b) (4) formula bo'yicha $\operatorname{arcctg}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Mashqlar

1.127. Tenglamani yeching (grafikdan ham foydalaning):

1) $\operatorname{tg} x = -\frac{\sqrt{3}}{2}$; 2) $\operatorname{ctg} x = -\sqrt{3}$; 3) $\operatorname{ctg} x = 0{,}2$.

1.128. Ifodaning qiymatini toping:

1) $\operatorname{arcctg}\left(-\frac{\sqrt{3}}{2}\right)$; 2) $\operatorname{arcctg} 1$; 3) $\operatorname{arctg}(-1)$; 4) $\operatorname{arctg} 0$;

5) $\operatorname{arcctg} 0$.

1.129. Hisoblang:

1) $\operatorname{tg}\left(\arcsin\frac{\sqrt{3}}{2}\right)$; 2) $\operatorname{ctg}(\arcsin 0{,}5)$;

3) $\operatorname{tg}(\arccos 0{,}5)$; 4) $\operatorname{ctg}(\operatorname{arctg}(-1))$;

5) $\operatorname{tg}\left(\operatorname{arcctg}\left(-\frac{2}{3}\right)\right)$; 6) $\sin\left(\operatorname{arcctg}(\sqrt{3})\right)$;

7) $\cos\left(\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)\right)$; 8) $\cos(\operatorname{arcctg}(-0{,}8))$.

1.130. $\operatorname{arcctg} x = \frac{\pi}{2} - \operatorname{arctg} x$ tenglikning to'g'riligini tekshiring.

1.131. Hisoblang:

1) $\sin\left(\operatorname{arctg}\frac{\sqrt{3}}{2}-\operatorname{arcctg}\frac{\sqrt{3}}{2}\right)$; 2) $\operatorname{tg}\left(\arcsin\frac{\sqrt{2}}{2}-\operatorname{arctg}\sqrt{3}\right)$.

1.132. arctg$x$ quyidagi qiymatlarni qabul qila oladimi? arcctg$x$-chi:

1) 0; 2) $-0{,}01$; 3) $-\pi$; 4) $\pi/2$;

5) $3\pi/2$; 6) $\sqrt{2}$; 7) -1; 8) π?

4. Tenglamalarni yechishning asosiy usullari. Trigonometrik tenglama noma'lum argumentning trigonometrik funksiyalariga nisbatan

$$R(z)=a_0z^n+a_1z^{n-1}+\ldots+a_{n-1}z+a_n=0 \qquad (1)$$

ko'rinishdagi algebraik tenglamaga keltirilishi mumkin, bunda z orqali $\sin\lambda x$, $\cos\lambda x$, $\operatorname{tg}\lambda x$, $\operatorname{ctg}\lambda x$ funksiyalardan biri ifodalangan. Algebraik tenglama kabi (1) trigonometrik tenglamalarni yechishda yangi noma'lum kiritish, ko'paytuvchilarga ajratish va hokazo usullar qo'llaniladi. Jarayon eng sodda trigonometrik tenglamalardan birini yechishgacha boradi. Trigonometrik tenglamalarni yechishda asosan quyidagi hollar uchraydi:

1) $R(f(x))=0$ tenglamada R trigonometrik funksiya belgisi ostida x ga bog'liq bo'lgan $f(x)$ ifoda turibdi. $f(x)=z$ almashtirish orqali tenglama eng sodda $R(z)=0$ trigonometrik tenglamalardan biriga keltirilishi mumkin. Uning $z=z_i$ ildizlari birma-bir $f(x)=z$ ga qo'yiladi va x ning qiymatlari topiladi.

1 - m i s o l. $\sin\left(10x+\frac{\pi}{8}\right)=\frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Y e c h i s h . Misolimizda $f(x)=10x+\frac{\pi}{8}$. Tenglamaga $10x+\frac{\pi}{8}=z$ almashtirish kiritsak, $\sin z=\frac{\sqrt{3}}{2}$ tenglama hosil bo'ladi. Uning yechimi: $z=(-1)^k\frac{\pi}{3}+k\pi,\ k\in Z$. Bu $10x+\frac{\pi}{8}=z$ ga qo'yiladi va javob topiladi:

$$x=\frac{1}{10}\left(-\frac{\pi}{8}+(-1)^k\frac{\pi}{3}+k\pi\right),\ k\in Z.$$

2 - m i s o l. $\operatorname{tg}\left(x^2+6x+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$ tenglamani yechamiz.

Y e c h i s h . $z=x^2+6x+\frac{\pi}{6}$ almashtirish kiritamiz. Tenglama $\operatorname{tg}z=\frac{\sqrt{3}}{3}$ ko'rinishga keladi. Undan $z=\frac{\pi}{6}+k\pi,\ k\in Z$ ni topamiz.

U holda $x^2+6x+\frac{\pi}{6}=\frac{\pi}{6}+k\pi,\ k\in Z$ yoki $x^2+6x-k\pi=0,\ k\in Z$. Kvadrat tenglamaning ildizlari $x=-3\pm\sqrt{9+k\pi},\ k\in Z$, bunda $9+k\pi\geq 0$ yoki $k\geq-\frac{9}{\pi}=-2{,}86...$, ya'ni $k=-2;\ -1;\ 0;\ 1;...$.

J a v o b : $x=-3\pm\sqrt{9+k\pi},\ k\in Z,\ k\geq-2$.

2) $\sin x=\sin\alpha$, $\cos x=\cos\alpha$ va $\mathrm{tg}x=\mathrm{tg}\alpha$ tenglamalar. Bu tenglamalar mos ravishda $x=(-1)^k\alpha+k\pi,\ k\in Z,\ x=\pm\alpha+2n\pi,\ n\in Z,\ x=\alpha+m\pi,\ m\in Z$ formulalar yordamida yechilishi mumkin.

3 - m i s o l . $\cos(5x-45^\circ)=\cos(2x+60^\circ)$ tenglamani yeching.

Y e c h i s h . $5x-45^\circ=\pm(2x+60^\circ)+360^\circ k,\ k\in Z$ tenglamalarni yechamiz. $5x-45^\circ=+(2x+60^\circ)+360^\circ k,\ k\in Z$ tenglikdan $x=35^\circ+120^\circ k,\ k\in Z$ yechimlar guruhini, $5x-45^\circ=-(2x+60^\circ)+360^\circ k$, $k\in Z$ tenglikdan esa $x=\frac{1}{7}\left(-15^\circ+360^\circ k\right),\ k\in Z$ yechimlar guruhini topamiz.

Shunday qilib, $x=35^\circ+120^\circ k,\ k\in Z;\ x=\frac{1}{7}\left(-15^\circ+360^\circ k\right)$, $k\in Z$.

4 - m i s o l . $\sin x^2=\sin(6x-5)$ tenglamani yechamiz.

Y e c h i s h . $x^2=(-1)^k(6x-5)+k\pi,\ k\in Z$ tenglama hosil bo'ladi.

Agar k juft bo'lsa, ya'ni $k=2n,\ n\in Z$ da $x^2=6x-5+2n\pi,\ n\in Z$ kvadrat tenglama kelib chiqadi. Uning yechimi

$$x_{1,2}=3\pm\sqrt{9-(5-2n\pi)},\ n\in Z,\ n\geq\left[-\frac{2}{\pi}\right].$$

Agar k toq bo'lsa, ya'ni $k=2m+1,\ m\in Z$ da $x^2=-6x+5+(2m+1)\pi,\ m\in Z$ ko'rinishda bo'ladi va bundan

$$x_{1,2}=-3\pm\sqrt{9+(5+2(m+1)\pi)},\ m\in Z,\ m\geq\left[-\frac{14+\pi}{2\pi}\right].$$

3) $f(R(x))=0$ tenglamada R trigonometrik funksiya boshqa f funksiya belgisi ostida turadi. $R(x)=z$ almashtirish masalani $f(z)=0$ tenglamani yechishga keltiradi. Bu tenglamaning $z_1,\ z_2,\ ...$ ildizlari bo'yicha $R(x)=z_1,\ R(x)=z_2,\ ...$ tenglamalar majmuasini hosil qilamiz. Uni yechish bilan masala hal qilinadi.

5 - m i s o l . $\sin^2x+3\sin x+1{,}25=0$ tenglamani yechamiz.

Y e c h i s h . $\sin x = z$ almashtirish natijasida $z^2 + 3z + 1{,}25 = 0$ kvadrat tenglama hosil bo'ladi. Uning ildizlari $z_1 = -5$, $z_2 = -1$. $\sin x = -5$ tenglama yechimga ega emas. $\sin x = -1$ tenglama $x = -90° + 360°k$, $k \in Z$ yechimlarga ega.

4) Ba'zan berilgan tenglamani *ko'paytuvchilarga ajratish* usulidan trigonometrik funksiyalar yig'indisini ko'paytma ko'rinishiga keltirishda foydalaniladi.

6 - m i s o l . $2\cos x - 2\sin 2x - 2\sqrt{2}\sin x + \sqrt{2} = 0$ tenglamani yechamiz.

Y e c h i s h . $\sin 2x = 2\sin x \cos x$ almashtirish tenglamani $2\cos x - 4\sin x\cos x - 2\sqrt{2}\sin x + \sqrt{2} = 0$ ko'rinishga keltiradi. Uning chap qismini ko'paytuvchilarga ajratamiz:

$$2\cos x(1 - 2\sin x) + \sqrt{2}(1 - 2\sin x) = 0,$$
$$(1 - 2\sin x)(2\cos x + \sqrt{2}) = 0,$$

bundan:
$$\begin{cases} 1 - 2\sin x = 0, \\ 2\cos x + \sqrt{2} = 0, \end{cases} \Rightarrow \begin{cases} \sin x = 0{,}5, \\ \cos x = \frac{-\sqrt{2}}{2}. \end{cases}$$

J a v o b : $\left\{(-1)^k 30° + 180°k,\ k \in Z\right\} \cup \left\{\pm 135° + 360°k,\ k \in Z\right\}$.

7 - m i s o l . $\sqrt{1 + \frac{1}{2}\sin x} = \cos x$ tenglamani yeching.

Y e c h i s h . Bu tenglama $\begin{cases} \cos x \ge 0, \\ 1 + \frac{1}{2}\sin x = \cos^2 x \end{cases}$ yoki $\begin{cases} \cos x \ge 0, \\ \sin x\left(\sin x + \frac{1}{2}\right) = 0 \end{cases}$ tenglamalar sistemasiga teng kuchlidir (VI bob, 7-§; 1-band). $\begin{cases} \cos x \ge 0, \\ \sin x\left(\sin x + \frac{1}{2}\right) = 0, \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} \cos x \ge 0, \\ \sin x = 0; \end{cases} \\ \begin{cases} \cos x \ge 0, \\ \sin x = -\frac{1}{2} \end{cases} \end{array}\right.$ bo'lgani uchun berilgan tenglamaning barcha yechimlari $x = 2k\pi$, $k \in Z$ va $x = -\frac{\pi}{6} + 2k\pi$, $k \in Z$ formulalar bilan aniqlanadi.

J a v o b : $\{2k\pi,\ k \in Z\} \cup \left\{-\frac{\pi}{6} + 2k\pi,\ k \in Z\right\}$.

M a s h q l a r

1.133. Tenglamani yeching:

1) $\sin 10x = -\frac{\sqrt{3}}{2}$; 2) $\cos 10x = \frac{\sqrt{3}}{2}$;

3) $\mathrm{tg} 10x = \sqrt{3}$; 4) $\mathrm{ctg} 10x = \frac{\sqrt{3}}{3}$;

5) $\sin(6x - 60^\circ) = -1$; 6) $\cos(4x + 30^\circ) = 0$;

7) $\mathrm{tg}(5x - 45^\circ) = 0$; 8) $\sin^2 \frac{3}{x} = \frac{3}{4}$;

9) $\cos^2(2x - 45^\circ) = \frac{1}{4}$; 10) $\mathrm{tg}^2\left(6x - \frac{\pi}{4}\right) = 3$;

11) $\sin^2\left(7x - \frac{\pi}{6}\right) = 3$; 12) $\cos^2\left(4x + \frac{\pi}{3}\right) = -\frac{1}{4}$;

13) $\mathrm{tg}^2\left(5x - \frac{\pi}{3}\right) = -1$; 14) $\sin(4x^2) = 0{,}5$;

15) $\cos^2 6x^2 = 0{,}25$; 16) $\mathrm{tg}\frac{5}{x} = -1$.

1.134. Tenglamani yeching:

1) $\sin 4x \cos 3x \,\mathrm{tg} 8x = 0$; 2) $\cos 4x = -\cos 5x$;

3) $\mathrm{tg} 5x = -\mathrm{tg}\frac{x}{3}$; 4) $\sin 11x = -\sin 15x$;

5) $\cos 4x = \cos x$; 6) $\mathrm{tg} 3x = -\mathrm{ctg} 5x$;

7) $\sin\frac{x}{3} = \cos\frac{x}{4}$; 8) $\mathrm{tg}(5\pi - x) = -\mathrm{ctg}\left(2x + \frac{\pi}{6}\right)$;

9) $\sin\frac{x}{6} = \sin\frac{4}{x}$; 10) $\sin\left(x^2 - \frac{\pi}{4}\right) = -\cos x^3$;

11) $\mathrm{ctg} 7 = -\mathrm{ctg}\frac{3}{x}$; 12) $\mathrm{ctg}\sqrt{x} = \mathrm{tg} 2x$;

13) $\cos^2 2x + 3\cos 2x + 2 = 0$; 14) $\mathrm{tg}^2 5x - 3\mathrm{tg} 5x - 4 = 0$;

15) $\sin x^2 = -\sin 3x^2$; 16) $\sin^2 x + \sin^2 2x + 2 = 0$;

17) $\sqrt{2}(\cos^3 x + \sin^3 x) = \sin 2x$.

5. Xususiy usullar. 1) Agar tenglama tarkibida har xil trigonometrik funksiyalar qatnashsa, ularni bir ismli funksiyaga keltirish, so‘ngra almashtirishlarni bajarish kerak.

1 - m i s o l . $3\sin^2 x + 4\sin x + 2\cos^2 x - 7 = 0$ tenglamani yechamiz.

Y e c h i s h . $\cos^2 x = 1 - \sin^2 x$ almashtirish berilgan tenglamani $3\sin^2 x + 4\sin x + 2 - 2\sin^2 x - 7 = 0$ yoki $\sin^2 x + 4\sin x - 5 = 0$ ko'rinishga keltiradi. Oxirgi tenglamadan $\sin x = z$ almashtirish bajarsak, $z^2 + 4z - 5 = 0$ kvadrat tenglama hosil bo'ladi. Bu kvadrat tenglama $z_1 = -5$, $z_2 = 1$ ildizlarga ega. $z = \sin x$ ekanligini e'tiborga olsak, $\sin x = -5$ va $\sin x = 1$ tenglamalar hosil bo'ladi. Ularning birinchisi yechimga ega emas, ikkinchisi esa $x = \frac{\pi}{2} + 2k\pi,\ k \in Z$ yechimlarga ega.

2) Chap qismi sin$x$ va cos$x$ ga nisbatan ratsional funksiya bo'lgan $R(\sin x, \cos x) = 0$ tenglama. Oldingi bandlarda ko'rsatib o'tilganidek, u va v ga nisbatan *ratsional funksiya* deb, qiymatlari u va v larni qo'shish, ko'paytirish va bo'lish orqali hosil bo'ladigan funksiyaga aytiladi. $R(\sin x, \cos x) = 0$ tenglamada:

a) agar sin$x$ (yoki cos$x$) faqat juft daraja bilan qatnashayotgan bo'lsa, $\cos x = u$ (mos ravishda $\sin x = u$) almashtirish bajariladi;

b) agar bir vaqtda sin$x$ ifoda $-\sin x$ ga, cos$x$ esa $-\cos x$ ga almashtirilganda $R(\sin x; \cos x)$ funksiya o'zgarmasa, ya'ni $R(\sin x; \cos x) = R(-\sin x; -\cos x)$ bo'lsa, $\mathrm{tg}u = z$ almashtirish bajariladi.

2 - m i s o l . $\cos^4 x + 3\sin x - \sin^4 x - 2 = 0$ tenglamani yechamiz.

Y e c h i s h . cos$x$ funksiya faqat juft daraja bilan qatnashmoqda. $\cos^4 x = (1 - \sin^2 x)^2 = 1 - 2\sin^2 x + \sin^4 x$ bo'lganidan tenglama faqat sinusga bog'liq: $2\sin^2 x - 3\sin x + 1 = 0$; endi bu tenglama $\sin x = u$ almashtirish bilan $2u^2 - 3u + 1 = 0$ ko'rinishga keladi. Buning ildizlari: $u_1 = \frac{1}{2}$, $u_2 = 1$. Shu tariqa masala $\sin x = \frac{1}{2}$ va $\sin x = 1$ eng sodda trigonometrik tenglamalarni yechishga keladi. Bu tenglamalar berilgan tenglamaning barcha yechimlarini beradi:

$$x = (-1)^k \frac{\pi}{6} + \pi k,\ k \in Z;\ x = \frac{\pi}{2} + 2\pi k,\ k \in Z.$$

3 - m i s o l . $\sin^2 x + 2\sin 2x + 5\cos^2 x - 4 = 0$ tenglamani yechamiz.

Y e c h i s h . Tenglamani $\sin^2 x + 4\sin x \cos x + 5\cos^2 x - 4 = 0$ ko'rinishda yozib olaylik. Bu tenglamani x ning cos$x$ ni nolga aylantiradigan hech qanday qiymati qanoatlantirmaydi, chunki $\cos x = 0$ bo'lganda $\sin^2 x = 1$ bo'lib, tenglamadan $-3 = 0$ noto'g'ri tenglik hosil bo'ladi. Bundan tashqari, sin$x$ va cos$x$ oldidagi ishoralar bir vaqtda o'zgartirilganda, tenglikning chap qismi o'zgarmaydi. Demak, $\mathrm{tg}x = u$ almashtirishni bajarish mumkin.

Tenglamaning ikkala qismini $\cos^2 x$ ga bo‘lamiz:

$$\mathrm{tg}^2 x + 4\,\mathrm{tg}x + 5 - \frac{4}{\cos^2 x} = 0.$$

$\frac{1}{\cos^2 x} = 1 + \mathrm{tg}^2 x$ bo‘lgani uchun

$$3\mathrm{tg}^2 x - 4\mathrm{tg}x - 1 = 0$$

tenglama hosil bo‘ladi. Bu tenglamada $\mathrm{tg}x = t$ almashtirish bajarsak, $3t^2 - 4t - 1 = 0$ tenglamaga ega bo‘lamiz. Bu kvadrat tenglama $\frac{2\pm\sqrt{7}}{3}$ ildizlarga ega. Topilgan ildizlar yordamida berilgan tenglamaning barcha ildizlarini aniqlaymiz:

$$x = \mathrm{arctg}\frac{2-\sqrt{7}}{3} + \pi k,\ k \in Z;\ x = \mathrm{arctg}\frac{2+\sqrt{7}}{3} + \pi k,\ k \in Z.$$

3) $R(\sin x;\ \cos x) = 0$ tenglamaning chap qismi sinus va kosinusga nisbatan bir jinsli funksiya, ya’ni, agar $\sin x$ va $\cos x$ bir vaqtda biror λ ga ko‘paytirilsa, tenglamaning chap qismi λ^n ga ko‘paytirilgan bo‘ladi: $R(\lambda\sin x;\ \lambda\cos x) = \lambda^n R(\sin x;\ \cos x)$, bunda n — funksiyaning bir jinslilik darajasi, o‘zgarmas miqdor. Bu holda tenglikning ikkala qismi $\cos^n x$ ga bo‘linadi va $\mathrm{tg}x = u$ almashtirish bajariladi. Agar tenglikning barcha hadlari $\cos^m x$ ga bo‘linadigan bo‘lsa, u holda $\cos^m x$ qavsdan tashqari chiqarilsa, berilgan tenglama ikki tenglamaga ajraladi.

4 - m i s o l. $9\cos^6 x - 4\sin^3 x\cos^3 x = 0$ tenglamani yechamiz.

Y e c h i s h. Tenglamaning barcha hadlari $\cos^3 x$ ga bo‘linadi. $\cos^3 x$ ni qavsdan tashqariga chiqaramiz:

$$\cos^3 x(9\cos^3 x - 4\sin^3 x) = 0 \Rightarrow \left[\begin{array}{l} \cos^3 x = 0, \\ 9\cos^3 x - 4\sin^3 x = 0. \end{array}\right.$$

$\cos x = 0$ tenglama izlanayotgan yechimning bir turkumini beradi: $x = \frac{\pi}{2} + \pi k,\ k \in Z$. Ikkinchi tenglama $\cos x$ va $\sin x$ ga nisbatan bir jinsli. Uning ikkala qismini $\cos^3 x$ ga bo‘lamiz ($\cos x \neq$ $\neq 0$, ya’ni $x \neq \pm\frac{\pi}{2} + \pi k,\ k \in Z$ hol qaralyapti). Natijada: $9 - 4\mathrm{tg}^3 x =$ $=0$ tenglamaga ega bo‘lamiz. Bu tenglama yechimning yana bir turkumini beradi: $x = \mathrm{arctg}\sqrt[3]{\frac{9}{4}} + \pi k,\ k \in Z$.

Ba’zan oddiy almashtirishlar tenglamani unga teng kuchli bir jinsli tenglamaga keltirishi mumkin. Masalan, $\cos^2 x - 6\sin x\cos x = 4$ ning o‘ng qismini $\sin^2 x + \cos^2 x$ ga (ya’ni 1 ga) ko‘paytirish tenglamani $\cos^2 x - 6\sin x\cos x = 4(\cos^2 x + \sin^2 x)$ yoki $3\cos^2 x + 6\sin x\cos x + 4\sin^2 x = 0$ bir jinsli tenglamaga aylantiradi. 3- misolda ham shunday yo'l tutish mumkin edi.

4) Agar trigonometrik tenglamada x dan boshqa yana $2x$, $3x$ va hokazo argumentning ko'p karrali trigonometrik funksiyalari ham qatnashayotgan bo'lsa, ular ikkilangan, uchlangan argument trigonometrik funksiyalari yordamida faqat bir argumentga bog'liq trigonometrik funksiya orqali ifodalanishi mumkin.

5 - m i s o l . $\sin 3x\sin x - \sin^2 2x + \sin x - 0{,}25 = 0$ tenglamani yechamiz.

Y e c h i s h . $\sin 2\varphi = 2\sin\varphi\cos\varphi$, $\sin 3\varphi = 3\sin\varphi - 4\sin^3\varphi$, $\cos^2\varphi = 1 - \sin^2\varphi$ formulalardan foydalanib, tenglamani ushbu ko'rinishga keltiramiz:

$$3\sin^2 x - 4\sin^4 x - 4\sin^2 x\cdot(1 - \sin^2 x) + \sin x - 0{,}25 = 0$$

yoki ixchamlashtirishlardan so'ng: $\sin^2 x - \sin x + 0{,}25 = 0$.

Y e c h i m i : $x = (-1)^k\cdot 30° + 180°k$, $k\in Z$.

6 - m i s o l . $\cos 3t\sin 6t - \cos 4t\sin 5t = 0$ tenglamani yechamiz.

Y e c h i s h . Karrali argument trigonometrik funksiyalari formulalaridan foydalansak, ifoda ancha murakkab ko'rinishga keladi. Bu o'rinda ko'paytmani yig'indiga aylantirish formulalaridan kelib chiqadigan quyidagi tengliklardan foydalanish qulay:

$$\cos 3t\sin 6t = \frac{1}{2}\sin 9t + \frac{1}{2}\sin 3t,$$

$$\cos 4t\sin 5t = \frac{1}{2}\sin 9t + \frac{1}{2}\sin t.$$

Bu ifodalar berilgan tenglamaga tatbiq etilsa va shakl almashtirishlar bajarilsa, $\sin 3t - \sin t = 0$ yoki $2\sin t\cos 2t = 0$ tenglama hosil bo'ladi. Uning yechimlari $t = \pi k$; $t = \frac{\pi}{4} + \frac{\pi k}{2}$, $k\in Z$ sonlardan iborat. Bu sonlar berilgan tenglamaning barcha yechimlaridir.

5) $a\sin x + b\cos x = c$ ko'rinishdagi tenglamalarni yechishning eng qulay usuli yordamchi burchak kiritish usulidir.

Agar $c = 0$ bo'lsa, yechish usuli bizga tanish bo'lgan bir jinsli tenglama hosil bo'ladi.

$c \neq 0$, $a^2 + b^2 \neq 0$ bo'lsin. Tenglamaning ikkala tomonini ham $\sqrt{a^2 + b^2}$ ga bo'lamiz:

$$\frac{a}{\sqrt{a^2+b^2}}\cos x + \frac{b}{\sqrt{a^2+b^2}}\sin x = \frac{c}{\sqrt{a^2+b^2}}.$$

$$\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2+\left(\frac{b}{\sqrt{a^2+b^2}}\right)^2=1$$ bo‘lgani uchun $\frac{a}{\sqrt{a^2+b^2}}=\sin\varphi$

va $\frac{b}{\sqrt{a^2+b^2}}=\cos\varphi$ tengliklar o‘rinli bo‘ladigan φ son mavjuddir.

Bu yerda,

$$\cos x\sin\varphi+\sin x\cos\varphi=\frac{c}{\sqrt{a^2+b^2}} \text{ yoki } \sin(x+\varphi)=\frac{c}{\sqrt{a^2+b^2}}$$

tenglama hosil bo‘ladi. Hosil bo‘lgan bu tenglama $\frac{c}{\sqrt{a^2+b^2}}\le 1$ bo‘lgandagina yechimga ega: $x=-\varphi+(-1)^n\arcsin\frac{c}{\sqrt{a^2+b^2}}+n\pi,\ n\in Z$.

7 - m i s o l. $\sqrt{3}\cos x+\sin x=2$ tenglamani yechamiz.

Y e c h i s h. Tenglamaning ikkala tomonini $\sqrt{(\sqrt{3})^2+1^2}=2$ ga bo‘lsak, $\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1$ hosil bo‘ladi. $\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}$, $\frac{1}{2}=\cos\frac{\pi}{3}$ bo‘lganligi uchun

$$\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x=1\Rightarrow\sin\left(x+\frac{\pi}{3}\right)=1\Rightarrow$$

$$\Rightarrow x+\frac{\pi}{3}=\frac{\pi}{2}+2k\pi\Rightarrow x=\frac{\pi}{6}+2k\pi,\ k\in Z.$$

8 - m i s o l. $2\sin x+3\cos x=4$ tenglamani yechamiz.

Y e c h i s h. $\frac{4}{\sqrt{2^2+3^2}}>1$ bo‘lgani uchun tenglama yechimga ega emas.

6) Ba’zi trigonometrik tenglamalar chap yoki o‘ng tomonini baholash yo‘li bilan oson yechiladi.

9 - m i s o l. $\cos 2x+\cos 3x+\cos 4x=3$ tenglamani yechamiz.

Y e c h i s h. Tenglamaning chap tomonidagi yig‘indi $\cos 2x=1$, $\cos 3x=1$, $\cos 4x=1$ tengliklar bir vaqtda bajarilgandagina 3 ga teng bo‘ladi. Bu tengliklar bir vaqtda bajarila olmaydi. Demak, tenglama yechimga ega emas.

1 0 - m i s o l. $1-\cos^2 3x+\sin^2 2x=0$ tenglamani yechamiz.

Y e c h i s h. Tenglamani quyidagicha yozib olamiz: $\sin^2 2x+\sin^2 3x=0$. Bundan, $\sin^2 2x=\sin^2 3x=0$ sistema hosil bo‘ladi.

$\sin 2x = 0$ tenglama $x = \pi k,\ x = \frac{\pi}{2} + \pi n$ ildizlarga ega. $x = \frac{\pi}{2} + \pi n$, $n \in Z$ sonlari $\sin^2 3x = 0$ tenglamani qanoatlantirmaydi. $x = \pi k$ ildiz esa $\sin^2 3x = 0$ tenglamani qanoatlantiradi. Demak, berilgan tenglama $x = \pi k,\ k \in Z$ ildizlarga ega.

7) $P(\sin x \pm \cos x, \sin x \cos x) = 0$ ko'rinishdagi tenglamalar (bu yerda P bilan $\sin x \pm \cos x$ ga nisbatan ratsional funksiya belgilangan). Bu kabi tenglamalar $\sin x \pm \cos x = t$ almashtirish yo'li bilan yechiladi.

11-misol. $\sin x + \cos x = 1 - 2\sin x \cos x$ tenglamani yechamiz.

Yechish. $\sin x + \cos x = t$ almashtirish kiritsak, $\sin^2 x + 2\sin x \cos x + \cos^2 x = t^2$ yoki $2\sin x \cos x = t^2 - 1$ bo'ladi va tenglama $t = 1 - (t^2 - 1)$ ko'rinishga keladi. Bu tenglamaning $t_1 = 1$; $t_2 = -2$ ildizlari yordamida $\sin x + \cos x = 1$; $\cos x + \sin x = -2$ tenglamalarni hosil qilamiz.

$\sin x + \cos x = 1$ tenglama $x = (-1)^k \frac{\pi}{4} - \frac{\pi}{4} + k\pi,\ k \in Z$ ildizlarga ega.

$\cos x + \sin x = -2$ tenglama esa yechimga ega emas. Demak, berilgan tenglama $x = (-1)^k \frac{\pi}{4} - \frac{\pi}{4} + k\pi,\ k \in Z$ ildizlarga ega.

Mashqlar

1.135. Tenglamani yeching:

1) $2\sin^2 x + \cos^2 x - 2 = 0$;
2) $2\sin^2 x + \cos x = 0$;
3) $\sin x \cos x = 0$;
4) $\sin^2 x + \sin x - \cos^2 x + 1 = 0$;
5) $\sin x \cos x = \frac{\sqrt{3}}{4}$;
6) $\cos^2 2x + \sin 2x = 2$.

1.136. Bir jinsli va unga keltiriladigan tenglamani yeching:

1) $\sin^2 x - 2\sin x \cos x + \cos^2 x = 0$;
2) $7\cos^2 x - 3\sin^2 x = 0$;
3) $\cos^2 2x - 10\sin 2x \cos 2x + 21\sin^2 2x = 0$;
4) $8\sin^2 x - \cos^2 x = 0$;
5) $\cos^2 x - 2\cos 2x - 4\sin^2 x = 0$;
6) $\sin^2 3x + 7\cos^2 3x = 6\sin 3x \cos 3x$;
7) $\cos^6 x + \cos^4 x \sin^2 x = \cos^3 x \sin^3 x + \cos^2 x \sin^4 x$;
8) $2\sin^4 x - 6\sin^3 x \cos x - 23\cos^2 x \sin^2 x = 0$;

9) $10\cos^2\frac{x}{2}-3\sin x-5=0$; 11) $\sin^6 x+\cos^6 x=\frac{1}{4}$;

10) $\cos^4 x-\sin^4 x=2\sin^2 x$; 12) $\sin^8 x+\cos^8 x=\cos^2 2x$.

1.137. O‘rniga qo‘yish usulidan foydalanib yeching:

1) $\cos^2 x+1=2\cos x$;

2) $3\cos^2 x\sin x+1=3\cos^2 x+\sin x$;

3) $6\cos^3 x+6\sin^2 x-3\cos x-3=0$;

4) $5\sin^2 x\cos x+6\cos^2 x-10\cos x+6=0$;

5) $1+\sin^2 2x=(1-\cos^2 x)^2$.

1.138. Tenglamalarni yeching:

1) $\cos 2x+\cos x=0$; 2) $\cos 3x=2\cos 2x-1$;

3) $2\cos^2 x=4\sin x\cos x-1$; 4) $\cos^2 x-3\sin x\cos x=-1$;

5) $\left(\sin x-\frac{1}{\sin x}\right)^2+\left(\cos x-\frac{1}{\cos x}\right)^2=1$;

6) $2\cos\left(\frac{\pi}{2}+\sqrt{x}\right)+1=0$;

7) $(\cos 5x+\cos 7x)^2=(\sin 5x+\sin 7x)^2$;

8) $\sin^2 4x-\cos^2 x=2\sin 4x\cos^4 x$;

9) $\sin^3 2x-5\sin^2 2x+4=0$.

1.139. Tenglamani $\sin x+\cos x=t$ almashtirish yordamida yeching:

1) $2(\sin x+\cos x)+\sin 2x+1=0$; 2) $\sin x+\cos x=1+\frac{\sin 2x}{2}$;

3) $\sin^4 2x+\cos^4 2x=\sin 2x\cos 2x$.

1.140. Tenglamani baholash usuli bilan yeching:

1) $2\sin^8 x+3\cos^8 x=5$; 2) $(\cos 2x-\cos 4x)^2=4+\cos^2 3x$;

3) $\sin 3x+\cos 2x+2=0$.

1.141. Tenglamani yordamchi burchak kiritish usuli bilan yeching:

1) $12\cos x-5\sin x=-13$; 2) $\sin x+\cos x=\sqrt{2}$;

3) $\sqrt{3}\sin x-\cos x=1$.

6. Universal almashtirish. 3-§ ning 4-bandidagi (9) va (10) formulalardan foydalanib, $x\neq(2k+1)\pi$, $k\in Z$ qiymatlar uchun quyidagi munosabatlarni hosil qilamiz:

$$\sin x = \frac{2\,\mathrm{tg}\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}}; \qquad (1)$$

$$\cos x = \frac{1-\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}}. \qquad (2)$$

Agar $R(\sin x;\cos x) = 0$ tenglamada (1) va (2) almashtirishlar bajarilib, $\mathrm{tg}\frac{x}{2} = z$ o‘rniga qo‘yish tatbiq etilsa, chap tomoni z ga nisbatan ratsional funksiya bo‘lgan tenglama hosil bo‘ladi:

$$R\left(\frac{2z}{1+z^2};\ \frac{1-z^2}{1+z^2}\right) = 0. \qquad (3)$$

(3) tenglama ildizlarini (agar bu ildizlar mavjud bo‘lsa) birma-bir $\mathrm{tg}\frac{x}{2} = z$ ga qo‘yib, x ning izlanayotgan qiymatlari topiladi. $\mathrm{tg}\alpha$ ifoda $\alpha = \frac{\pi}{2} + k\pi,\ k \in Z$ qiymatlarda aniqlanmaganligi sababli x ning $x = (2k + 1)\pi,\ k\in Z$ qiymatlari alohida tekshiriladi.

M i s o l . $3\sin t - \cos t = 3$ tenglamani yechamiz.

Y e c h i s h . (1) va (2) formulalardan foydalanib, $\sin t$ va $\cos t$ ni $\mathrm{tg}\frac{t}{2}$ orqali ifodalaymiz, so‘ng $\mathrm{tg}\frac{t}{2} = z$ almashtirishni kiritamiz. Natijada, $\frac{6z}{1+z^2} = \frac{2z^2+4}{1+z^2}$ ratsional tenglama hosil bo‘ladi. Uning ildizlari $z_1 = 1,\ z_2 = 2$. Ularni ketma-ket $\mathrm{tg}\frac{t}{2} = z$ ga qo‘yamiz. $\mathrm{tg}\frac{t}{2} = 1$ tenglamadan $t = \frac{\pi}{2} + 2\pi k,\ k \in Z$ ni, $\mathrm{tg}\frac{t}{2} = 2$ tenglamadan esa $t = 2\mathrm{arctg}2 + 2\pi k,\ k\in Z$ ni hosil qilamiz.

M a s h q l a r

1.142. $a\sin x + b\cos x = c$ tenglamani yeching. Tenglama a, b, c larga nisbatan qanday shartlarda yechimga ega bo‘ladi?

1.143. Tenglamalarni yeching:

1) $4\sin x - 7\cos x = 7$; 2) $\frac{1-\cos x}{1+\sin x} = \frac{1}{2}$;

3) $\sqrt{3}\cos x-\sin x=1$; 4) $2\sin\left(x+\frac{\pi}{3}\right)-\sin\left(x-\frac{\pi}{3}\right)=\sqrt{2}$;

5) $\cos x-\cos(\pi-2x)-2=0$; 6) $2\sin 6x=\sqrt{2(1-\cos 4x)}$;

7) $\operatorname{ctg}\left(2x+\frac{\pi}{4}\right)-\operatorname{ctg}\left(2x-\frac{\pi}{4}\right)=0$; 8) $\sin x\cos x\sin 2x=\frac{1}{8}$;

9) $\sin(x+20°)+\sin(2x+40°)+\sin(3x+60°)=0$;

10) $\sqrt{3}\cos^2 x+(\sqrt{3}-1)\sin x\cos x-\sin^2 x=0$;

11) $\sqrt{3}\cos x+\sin x=0$;

12) $\sqrt{3+2(2\sin x-\cos 2x)}+\sqrt{3-2(2\sin x+\cos 2x)}=2$;

1.144. Grafik yordamida taqribiy yeching:

1) $\sin x=x-0,5$; 2) $\sin x=x+1$; 3) $x=1+0,5\sin x$.

7. Trigonometrik tenglamalar sistemasi. Trigonometrik tenglamalar sistemasini yechishda yuqorida ko‘rilgan tenglamalarni yechish usullaridan va trigonometrik formulalardan foydalaniladi.

Quyida tenglamalar sistemasini yechishning o‘ziga xos xususiyatlari bilan tanishamiz.

1 - m i s o l. $\begin{cases}5\sin x=\sin y,\\ 3\cos x=2-\cos y\end{cases}$ tenglamalar sistemasini yechamiz.

Y e c h i s h . Berilgan sistemani $\begin{cases}5\sin x=\sin y,\\ 2-3\cos x=\cos y\end{cases}$ ko‘rinishda yozib olamiz. Bu sistemaning kvadratga ko‘tarilgan tenglamalarini hadma-had qo‘shsak,

$$25\sin^2 x+4-12\cos x+9\cos^2 x=1$$

yoki

$$16\cos^2 x+12\cos x-28=0$$

tenglama hosil bo‘ladi. Hosil bo‘lgan bu tenglamani $\cos x$ ga nisbatan yechib, $\cos x=1$, $\cos x=-\frac{7}{4}$ tenglamalarga ega bo‘lamiz. $\cos x=-\frac{7}{4}$ tenglama yechimga ega emas. Demak, $\cos x=1$ bo‘lishi zarur. $\cos x=1$, ya’ni $x=2\pi n$, $n\in Z$ da $\sin x=0$ bo‘lgani uchun

$\begin{cases} 5\cdot 0=\sin y, \\ 2-3\cdot 1=\cos y \end{cases}$ sistemaga ega bo‘lamiz. Bu sistemadan, $y = (2k+1)\pi$, $k \in Z$ ekanligi topiladi.

J a v o b : $x = 2\pi n$, $n \in Z$, $y = (2k+1)\pi$, $k \in Z$.

2 - m i s o l. $\begin{cases} \sin x+\sin y=\frac{4}{3}, \\ x+y=\frac{\pi}{3} \end{cases}$ tenglamalar sistemasini yechamiz.

Y e c h i s h :

$$\begin{cases} \sin x+\sin y=\frac{4}{3}, \\ x+y=\frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}=\frac{4}{3}, \\ x+y=\frac{\pi}{3} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2\sin\frac{\pi}{6}\cos\frac{x-y}{2}=\frac{4}{3}, \\ x+y=\frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} \cos\frac{x-y}{2}=\frac{4}{3}, \\ x+y=\frac{\pi}{3}. \end{cases}$$

$\left|\cos\frac{x-y}{2}\right| \le 1$ shart bajarilmaganligi uchun sistema yechimga ega emas.

M a s h q l a r

1.145. Trigonometrik tenglamalar sistemasini yeching:

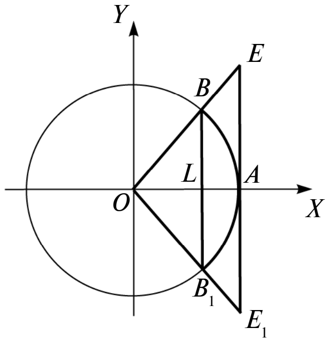
1) $\begin{cases} \sin x+\sin y=1, \\ x+y=\frac{\pi}{3}; \end{cases}$ 2) $\begin{cases} 4\sin x\cos y=1, \\ 3\,\mathrm{tg}x-\mathrm{tg}y=0; \end{cases}$

3) $\begin{cases} 4\cos x\cos y=3, \\ 4\sin x\sin y=-1; \end{cases}$ 4) $\begin{cases} \cos x+\cos y=-1{,}6, \\ x+y=\frac{2\pi}{3}. \end{cases}$

8. Trigonometrik tengsizliklarni isbotlash. Trigonometrik tengsizliklarni isbotlash masalasi ba’zan algebraik tengsizliklarni isbotlash masalasiga keltiriladi.

1 - m i s o l. $4\cos x - 3\sin x \le 5$ tengsizlikni isbot qilamiz.

Y e c h i s h . $\mathrm{tg}\frac{x}{2}=z$, $x \ne \pi + 2\pi k$, $k \in Z$ universal o‘rniga qo‘yish tengsizlikni quyidagi ko‘rinishga keltiradi:



I.45-rasm.

$$\frac{4(1-z^2)}{1+z^2}-\frac{6z}{1+z^2}\le 5\Rightarrow 9z^2+.$$

$$+6z+1\ge 0\Rightarrow(3z+1)^2\ge 0 \qquad (1)$$

(1) tengsizlik z ning har qanday qiymatida o‘rinli. Demak, berilgan tengsizlik barcha $x\ne\pi+2\pi k,\ k\in Z$ larda bajariladi. Tekshirish tengsizlikning $x=\pi+2\pi k,\ k\in Z$ uchun ham o‘rinli ekanini ko‘rsatadi.

2 - m i s o l . ABC uchburchakda $\operatorname{tg}^2\frac{A}{2}+\operatorname{tg}^2\frac{B}{2}+\operatorname{tg}^2\frac{C}{2}\ge 1$ tengsizlikning bajarilishini isbot qilamiz.

I s b o t . ABC uchburchakning A, B va C burchaklari uchun

$$\left(\operatorname{tg}\frac{A}{2}-\operatorname{tg}\frac{B}{2}\right)^2+\left(\operatorname{tg}\frac{A}{2}-\operatorname{tg}\frac{C}{2}\right)^2+\left(\operatorname{tg}\frac{B}{2}-\operatorname{tg}\frac{C}{2}\right)^2\ge 0$$

yoki

$$\operatorname{tg}^2\frac{A}{2}+\operatorname{tg}^2\frac{B}{2}+\operatorname{tg}^2\frac{C}{2}\ge\operatorname{tg}\frac{A}{2}\operatorname{tg}\frac{B}{2}+\operatorname{tg}\frac{A}{2}\operatorname{tg}\frac{C}{2}+\operatorname{tg}\frac{B}{2}\operatorname{tg}\frac{C}{2}$$

munosabat o‘rinli ekanligi ravshan. Bu yerda geometriya kursida ma’lum bo‘lgan

$$\operatorname{tg}\frac{A}{2}\operatorname{tg}\frac{B}{2}=\frac{p-c}{p},\quad \operatorname{tg}\frac{A}{2}\operatorname{tg}\frac{C}{2}=\frac{p-b}{p}\ \text{va}\ \operatorname{tg}\frac{B}{2}\operatorname{tg}\frac{C}{2}=\frac{p-a}{p}$$

tengliklardan foydalansak (bu yerda a, b, c — uchburchakning mos ravishda A, B, C burchaklari qarshisidagi tomonlar, $p=\frac{a+b+c}{2}$), isbotlanishi kerak bo‘lgan tengsizlikni hosil qilamiz.

3 - m i s o l . $0<\alpha<\pi$ bo‘lsin. U holda $\sin\alpha<\alpha<\operatorname{tg}\alpha$ bo‘lishini isbot qilamiz.

I s b o t . Birlik aylanada (I.45-rasm) B_1OB va OEE_1 uchburchaklarni yasaymiz, $\cup B_1AB=2\alpha$, $\cup AB=\alpha$ bo‘lsin. 1-§ ning 1-bandida keltirilgan mulohazalarga ko‘ra $BL<\cup AB<AE$ bo‘ladi. Bundan

$$\sin\alpha<\alpha<\operatorname{tg}\alpha \qquad (1)$$

bo‘ladi.

Mashqlar

1.146. Tengsizlikni isbotlang:

1) $\mathrm{ctg}2\alpha < 0{,}5\mathrm{ctg}\alpha,\ 0 < \alpha < \pi$;

2) $\sin^3\alpha - \sin^6\alpha \le 0{,}25$;

3) $-\sqrt{3} \le \frac{3\sin\alpha}{2+\cos\alpha} \le \sqrt{3}$;

4) $\cos A\cos B\cos C \le \frac{1}{8},\ A + B + C = 180°$;

5) $\cos(\cos x) > 0$, bunda $x \in R$;

6) $\cos x - \sin x - \cos 2x > 0$;

7) $\sqrt{5-2\sin x} \ge 6\sin x - 1$;

8) aniqlanish sohasidan olingan ixtiyoriy x da $|\mathrm{tg}x + \mathrm{ctg}x| \ge 2$;

9) $|3\sin x - 4\cos x| \le 5$;

10) α va β o‘tkir burchaklar uchun $\sin(\alpha + \beta) < 2\sin\alpha + \sin\beta$;

11) agar α va β o‘tkir burchaklar uchun $\sin(\alpha + \beta) = 2\sin\alpha$ o‘rinli bo‘lsa, u holda $\alpha < \beta$;

12) α ning barcha qiymatlarida $\sin\alpha\cos\alpha \le 0{,}5$ bo‘ladi;

13) $\frac{1}{\sin^2\alpha} + \frac{1}{\cos^2\alpha} \ge 4$; 14) $\frac{1}{\sin^4\alpha} + \frac{1}{\cos^4\alpha} \ge 8$.

9. Eng sodda trigonometrik tengsizliklarni yechish. $\sin x > m$, $\cos x > m$, $\mathrm{tg}x > m$, $\mathrm{ctg}x > m$ kabi ko‘rinishdagi tengsizliklarni yechishda koordinatali aylanadan yoki trigonometrik funksiyalarning grafiklaridan foydalanamiz.

1-misol. a) $\sin\alpha > 0$; b) $\sin\alpha > m$, $1 \le m < 1$; в) $\sin\alpha < m$ tengsizliklarni yechamiz.

Yechish. a) $\sin\alpha > 0$ ning yechimlar to‘plami sinusoidaning abssissalar o‘qidan yuqorida joylashgan bo‘laklari bilan aniqlanadi (I.46-*a* rasm). Bu bo‘laklardan biri abssissalar o‘qining $(0;\ \pi)$ oralig‘iga, qolganlari undan $2\pi k$, $k \in Z$ uzoqliklarda joylashgan oraliqlarga mos keladi. Demak, $2\pi k < \alpha < (2k + 1)\pi$, $k \in Z$ ko‘rinishdagi oraliqlarda yotuvchi α sonlargina yechim bo‘ladi.

b) $\sin\alpha > m$ tengsizlikni yechamiz, bunda $-1 \le m < 1$. Birlik aylananing ordinatalari m dan katta bo‘lgan nuqtalari $y = m$ to‘g‘ri chiziqdan yuqorida joylashadi. Ular MBN yoyni hosil qiladi

(I.46-*b* rasm). Bu yoyga $M(\alpha_0)$ va $N(\pi-\alpha_0)$ nuqtalar kirmaydi. Shunday qilib, $\sin\alpha > m$ tengsizlikning yechimi $(\alpha_0;\ \pi-\alpha_0)$ interval yordamida aniqlanadi. $\alpha_0=\arcsin m$ va $y=\sin x$ funksiya davriy funksiya bo‘lgani uchun berilgan tengsizlikning barcha yechimlar to‘plamini

$$\bigcup_{k\in Z}(\arcsin m+2k\pi;\ \pi-\arcsin m+2k\pi)$$

yoki

$$\arcsin m+2k\pi<\alpha<\pi-\arcsin m+2k\pi,\ k\in Z$$

ko‘rinishda yozamiz.

$\sin\alpha>m$ tengsizlik $m\geq 1$ da bajarilmaydi, $m<-1$ da esa barcha α larda bajariladi.

d) $\sin\alpha<m$ tengsizlikni yechish $\alpha=-z$ o‘rniga qo‘yish orqali yuqorida qaralgan holga keladi: $\sin z>-m$. Uning barcha yechimlarini yozamiz:

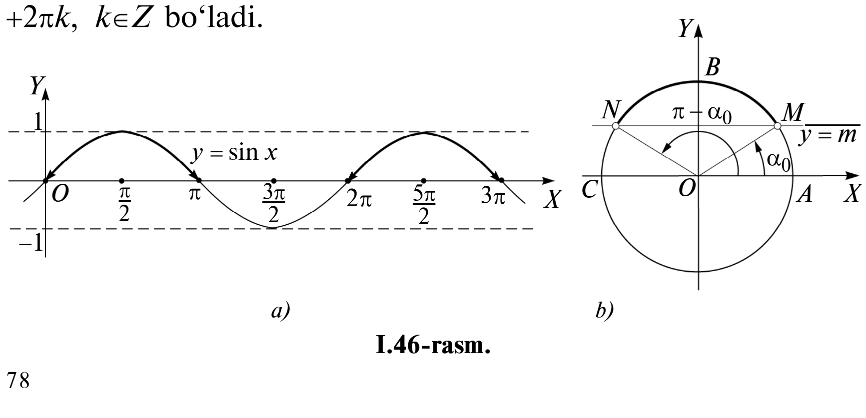
$$\arcsin(-m)+2\pi k<z<\pi-\arcsin(-m)+2\pi k,\ k\in Z.$$

$\arcsin(-m)=-\arcsin m$ va $z=-\alpha$ bo‘lgani uchun berilgan tengsizlikning barcha yechimlari quyidagicha bo‘ladi:

$$-\pi-\arcsin m+2k\pi<\alpha<\arcsin m+2k\pi,\ k\in Z.$$

2-misol. a) $\cos\alpha>m$; b) $\cos\alpha<m$ tengsizliklarni yechamiz.

Yechish. a) $m\geq 1$ da tengsizlik yechimga ega emas, $m<-1$ da esa α ning barcha qiymatlari tengsizlikni qanoatlantiradi. Biz $-1\leq m<1$ bo‘lgan holni qaraymiz. I.41-*d* rasmga qaralganda $m<\cos\alpha\leq 1$ ga B_2AB_1 yoy mos keladi, bunda $B_1(\alpha_0)$ va $B_2(-\alpha_0)$ lar $x=m$ to‘g‘ri chiziq bilan koordinatali aylananing kesishish nuqtalari, $A(0)$ —hisob boshi nuqtasi. Demak, $\cos\alpha>m$ tengsizlikning yechimi $-\alpha_0<\alpha<\alpha_0$ yoki $-\arccos m<\alpha<\arccos m$, yoki funksiya davri e’tiborga olinsa, $-\arccos m+2\pi k<\alpha<\arccos m+$ $+2\pi k,\ k\in Z$ bo‘ladi.



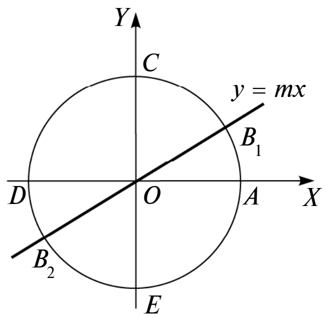
a) *b)*

I.46-rasm.

b) $\cos\alpha < m$ tengsizligini yechish $\alpha = \pi - z$ almashtirish orqali yuqorida qaralgan tengsizlikka keltiriladi: $\cos z > -m$. Bundan $-\arccos(-m) + 2k\pi < z < \arccos(-m) + 2k\pi$, $k \in Z$ ni topamiz. $z = \pi - \alpha$ va $\arccos(-m) = \pi - \arccos m$ bo'lgani uchun

$$\arccos m + 2k\pi < \alpha < 2(k+1)\pi - \arccos m, \quad k \in Z$$

bo'ladi.



I.47-rasm.

3 - m i s o l . $\text{tg}\alpha < m$ va $\text{tg}\alpha > m$ tengsizliklar yechimini topamiz.

Y e c h i s h . arctg*m* ta'rifidan foydalanamiz (I.47-rasm). $B_1(\alpha_0)$ nuqta EAC yarim aylanani EAB_1 va B_1C yoylarga ajratadi, bunda $E\left(-\frac{\pi}{2}\right)$ va $C\left(\frac{\pi}{2}\right)$. Undan E, B_1, C nuqtalar chiqariladi. EAB_1 yoyda $\text{tg}\alpha < m$, B_1C yoyda esa $\text{tg}\alpha > m$ tengsizlik bajariladi. Demak, $\text{tg}\alpha < m$ tengsizlikning yechimi

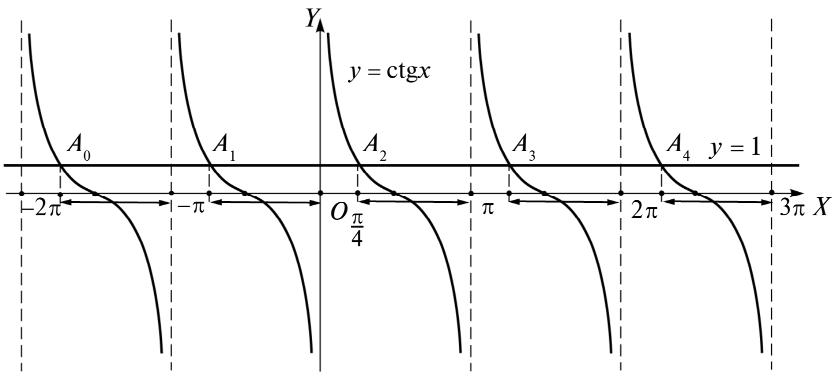
$$-\frac{\pi}{2} + k\pi < \alpha < \text{arctg}\, m + k\pi, \quad k \in Z,$$

$\text{tg}\alpha > m$ tengsizlik yechimi esa

$$\text{arctg}\, m + k\pi < \alpha < \frac{\pi}{2} + k\pi, \quad k \in Z$$

bo'ladi.

Shu kabi $\text{ctg}\alpha < m$, $\text{ctg}\alpha > m$ tengsizliklar yechimi mos ravishda $\text{arcctg}\, m + k\pi < \alpha < \pi + k\pi$, $k \in Z$ va $k\pi < \alpha < \text{arcctg}\, m + k\pi$, $k \in Z$ bo'ladi.



I.48-rasm.

4 - m i s o l. $\mathrm{ctg}x \le 1$ tengsizlikni yechamiz.

Y e c h i s h. $y = 1$ to‘g‘ri chiziq $y = \mathrm{ctg}x$ kotangensoidani cheksiz ko‘p A_0, A_1, A_2, ... nuqtalarda kesadi (I.48-rasm). Hosil bo‘ladigan oraliqlardan biri $\left[\frac{\pi}{4};\ \pi\right]$. Kotangensning davrini ham e’tiborga olib, yechimni $\frac{\pi}{4} + \pi k < x < \pi + \pi k,\ k \in Z$ ko‘rinishda yozamiz.

Mashqlar

1.147. Tengsizlikni yeching:

1) $\sin x < 0{,}5$, bunda $x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$; 2) $\sin x \le 1$;

3) $\sin x < \frac{\sqrt{3}}{2},\ x \in [-\pi;\ \pi]$; 4) $\sin x \ge -\frac{\sqrt{3}}{2},\ x \in [0;\ 2\pi]$;

5) $\sin x > \frac{\sqrt{2}}{2},\ x \in [-\pi;\ \pi]$; 6) $\sin x \le -\frac{\sqrt{2}}{2},\ x \in [0;\ 2\pi]$;

7) $\sin x > -\frac{\sqrt{2}}{2},\ x \in [0;\ 2\pi]$.

1.148. Tengsizlikni yeching:

1) $\cos x \le \frac{1}{2},\ x \in [0;\ 2\pi]$; 2) $\cos x > \frac{1}{2},\ x \in [0;\ 2\pi]$;

3) $\cos x \ge \frac{\sqrt{3}}{2},\ x \in [-\pi;\ \pi]$; 4) $\cos x < \frac{\sqrt{3}}{2},\ x \in [-\pi;\ \pi]$;

5) $\cos x \ge \frac{\sqrt{3}}{2},\ x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$; 6) $\cos x < \frac{\sqrt{2}}{2},\ x \in [0;\ \pi]$;

7) $\cos x \le -\frac{\sqrt{2}}{2},\ x \in [0;\ 2\pi]$; 8) $\cos x \le -\frac{\sqrt{3}}{2},\ x \in [-\pi;\ \pi]$.

1.149. Tengsizlikni yeching:

1) $\mathrm{tg}x < \sqrt{3},\ x \in \left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$; 2) $\mathrm{tg}x \ge \sqrt{3},\ x \in \left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$;

3) $\mathrm{tg}x < 1,\ x \in [0;\ \pi]$; 4) $\mathrm{tg}x \ge \frac{\sqrt{3}}{3},\ x \in \left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$;

5) $\mathrm{ctg}x < 1,\ x \in \left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$; 6) $\mathrm{ctg}x \ge -1,\ x \in [0;\ \pi]$;

7) $\mathrm{tg}x < 2$; 8) $\mathrm{tg}2x > 2$; 9) $\mathrm{ctg}x < -1$.

1.150. Tengsizlikni yeching:

1) $\sqrt{3}\cos 2x + 1 > 0,\ x \in [0;\ \pi]$;

2) $\sqrt{3}\sin 2x + 1 \ge 0,\ x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$;

3) $-\frac{\sqrt{2}}{2} < \cos x < \frac{\sqrt{2}}{2}$, $x \in [0;\ 2\pi]$;

4) $0 < \sin x < \frac{\sqrt{2}}{2}$, $x \in [-\pi;\ \pi]$;

5) $\sin^2 x > \frac{1}{4}$, $x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$;

6) $4\sin^2 x - 1 \le 0$, $x \in [0;\ 2\pi]$.

1.151. Tengsizlikni yeching:

1) $\sin x > -0{,}80$, $x \in \left[\frac{\pi}{2};\ \frac{3\pi}{2}\right]$;

2) $\sin x \le -0{,}80$, $x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$;

3) $\cos x \ge 0{,}6$, $x \in [0;\ \pi]$;

4) $\cos x < 0{,}7$, $x \in [\pi;\ 2\pi]$.

10. Trigonometrik tengsizliklarni intervallar usuli bilan yechish. $f(t) > 0$ yoki $f(t) < 0$ trigonometrik tengsizliklarni yechishda intervallar usulidan foydalanamiz. Shu maqsadda oldin $f(t)$ funksiyaning T_0 asosiy davri, $f(t) = 0$ tenglamaning $[0;\ T_0)$ oraliqda yotgan ildizlari va uzilish nuqtalari topiladi. Ular $[0;\ T_0)$ oraliqni bir necha intervalga ajratadi. Sinash nuqtalari usuli qo'llanilib, funksiyaning intervallardagi ishoralari aniqlanadi. Funksiyaning xossalaridan, jumladan, juft-toqligidan foydalanish ishni osonlashtiradi.

1 - m i s o l. $f(\alpha) = \cos 2\alpha - \cos 3\alpha < 0$ tengsizlikni yechamiz.

Y e c h i s h. 1) $\cos 2\alpha$ ning davri: $\cos(2\alpha + 2\pi) = \cos 2(\alpha + T_1)$, bundan $2\alpha + 2\pi = 2(\alpha + T_1)$, $T_1 = \pi$; shu kabi $\cos 3\alpha$ ning davri $T_2 = \frac{2\pi}{3}$. Bu sonlarning eng kichik umumiy bo'linuvchisi , ya'ni $T_0 = 2\pi$ soni $f(x)$ funksiyaning asosiy davri bo'ladi;

2) $f(\alpha) = 0$ tenglama ildizlari $2\alpha = \pm 3\alpha + 2\pi k$, $k \in Z$ munosabat bo'yicha aniqlanadi. Bizga ular ichidan $(0;\ T_0)$ oraliqda yotganlarini aniqlash yetarli, qolganlari T_0 davr bilan takrorlanadi. Oraliqning $\alpha = 0$ chap uchida $f(0) = 0$, ya'ni $f(x) < 0$ tengsizlik bajarilmaydi. Demak, oraliqning chap uchi ochiq qoladi. Oraliqning ichida yotgan ildizlarni topamiz. Shu maqsadda munosabatdagi k ga ketma-ket 0, 1, 2, ... qiymatlar berish va α ning qiymatlari ichidan $(0;\ 2\pi)$ intervalda yotganlarini ajratish kerak. Ular: $\frac{2\pi}{5}$, $\frac{4\pi}{5}$, $\frac{6\pi}{5}$, $\frac{8\pi}{5}$.

3) f funksiya son o‘qida uzluksiz;

4) $(0;\ 2\pi)$ oraliq $\left(0;\ \frac{2\pi}{5}\right]$, $\left[\frac{2\pi}{5};\ \frac{4\pi}{5}\right]$, $\left[\frac{4\pi}{5};\ \frac{6\pi}{5}\right]$, $\left[\frac{6\pi}{5};\ \frac{8\pi}{5}\right]$, $\left[\frac{8\pi}{5};\ 2\pi\right)$ intervallarga ajraladi;

5) $\left(0;\ \frac{2\pi}{5}\right]$ oraliqdan sinash nuqtasi sifatida $\frac{\pi}{3}$ ni olaylik. Unda $f\left(\frac{\pi}{3}\right) = \cos\frac{2\pi}{3} - \cos\frac{3\pi}{3} = -\frac{1}{2} + 1 > 0$. Demak, bu oraliqda berilgan tengsizlik bajarilmaydi. Tekshirish tengsizlik $\left[\frac{2\pi}{5};\ \frac{4\pi}{5}\right]$, $\left[\frac{6\pi}{5};\ \frac{8\pi}{5}\right]$ oraliqlarda bajarilishini ko‘rsatadi. Yechim ushbu oraliqlar birlashmasidan iborat:

$$\left(\frac{2\pi}{5} + 2k\pi;\ \frac{4\pi}{5} + 2k\pi\right) \cup \left(\frac{6\pi}{5} + 2k\pi;\ \frac{8\pi}{5} + 2k\pi\right),\ k \in Z.$$

2 - m i s o l. $f(\alpha) = \mathrm{tg}\alpha - \mathrm{tg}\frac{\alpha}{3} > 0$ tengsizligini yechamiz.

Y e c h i s h. 1) $\mathrm{tg}\alpha$ ning davri $T_1 = \pi$, $\mathrm{tg}\frac{\alpha}{3}$ ning davri $T_2 = 3\pi$. T_1 va T_2 ning eng kichik umumiy bo‘linuvchisi, ya’ni f ning asosiy davri $T_0 = 3\pi$. Tengsizlikning $[0;\ 3\pi]$ oraliqdagi yechimini topish yetarli. Qolganlari son o‘qida 3π davr bilan takrorlanadi;

2) $\mathrm{tg}\alpha - \mathrm{tg}\frac{\alpha}{3} = 0$ tenglamaning ildizlari: $\alpha = 3\pi k,\ k \in Z$. Ulardan $[0;\ 3\pi]$ oraliqda yotgani 0 va 3π;

3) f funksiya $\cos\alpha = 0$ va $\cos\frac{\alpha}{3} = 0$ da, ya’ni $\alpha = \frac{\pi}{2} + \pi k$, $k \in Z$ nuqtalarda uzilishga ega. Shu jumladan $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$ nuqtalar qaralayotgan $[0;\ 3\pi]$ oraliqda joylashgan;

4) topilgan nuqtalar $[0;\ 3\pi]$ oraliqni $\left[0;\ \frac{\pi}{2}\right]$, $\left[\frac{\pi}{2};\ \frac{3\pi}{2}\right]$, $\left[\frac{3\pi}{2};\ \frac{5\pi}{2}\right]$, $\left[\frac{5\pi}{2};\ 3\pi\right]$ qismlarga ajratadi;

5) sinash nuqtalari yordamida berilgan tengsizlik yechimi ushbu intervallardan tuzilganligini aniqlaymiz:

$$\left(3\pi k;\ \frac{\pi}{2} + 3\pi k\right),\ \left(\frac{3\pi}{2} + 3\pi k;\ \frac{5\pi}{2} + 3\pi k\right),\ k \in Z.$$

f — toq funksiya. Shunga ko‘ra hisoblashlarni $[0;\ 3\pi]$ da emas, balki $\left[-\frac{3\pi}{2};\ \frac{3\pi}{2}\right]$ da bajarish ma’qul. Haqiqatan, $f(\alpha)>0$ tengsizlik $\left[0;\ \frac{\pi}{2}\right]$ da bajarilsa, $\left[-\frac{\pi}{2};\ 0\right]$ da $f(\alpha)<0$ tengsizlik bajariladi.

Y e c h i s h: $\left(-\frac{3\pi}{2}+3\pi k;\ -\frac{\pi}{2}+3\pi k\right)$, $\left(3\pi k;\ \frac{\pi}{2}+3\pi k;\ \right)$, $k\in Z$.

M a s h q l a r

1.152. Tengsizliklarni yeching:

1) $\sin 3x<-\cos 3x$; 2) $\sin 3x<\cos 3x$; 3) $\sin 3x\cos 3x>0$;
4) $\cos 3x \operatorname{tg} 3x>0$; 5) $\sin 2x+\operatorname{tg} 2x<0$; 6) $\operatorname{tg} 2x>\sqrt{3}$;
7) $\operatorname{ctg}\frac{2\pi x}{3(x-2)}<1$; 8) $\cos x\cos 3x<\cos 2x\cos 4x$;
9) $\sin^4 x-\cos^4 x<\frac{1}{2}$; 10) $(1-\operatorname{ctg} x)\sin^2 x>1$;
11) $3\sin 2x-1>\sin x+\cos x$.

11. Trigonometrik funksiya qiymatini taqribiy hisoblash.

$0<x<\frac{\pi}{2}$ da $\sin x<x<\operatorname{tg} x$ bo‘lishini bilamiz. Ikkinchi tomondan $\sin\frac{x}{2}=\sqrt{\frac{1-\cos x}{2}}$ ekanligidan $\cos x=1-2\sin^2\frac{x}{2}>1-2\cdot\left(\frac{x}{2}\right)^2=$ $=1-\frac{x^2}{2}$ ni olamiz. Bu tengsizlik va $\operatorname{tg} x=\frac{\sin x}{\cos x}$ munosabatdan foydalanib, ixchamlashtirishlardan so‘ng ushbu qo‘sh tengsizlik hosil bo‘ladi:

$$x-\frac{x^3}{2}<\sin x<x<\operatorname{tg} x. \qquad (1)$$

1 - m i s o l. sin0,05 qiymatini 0,01 gacha aniqlikda topamiz.

Y e c h i s h. (1) bo‘yicha:

$$0,05-\frac{0,000125}{2}<\sin 0,05<0,05;\ \ 0,0499<\sin 0,05<0,05,$$

bundan $\sin 0,05x\approx\frac{0,05+0,0499}{2}=0,050.$

Yuqori aniqlik talab qilingan hollarda ushbu formulalardan foydalanish mumkin (isboti oliy matematika kursida o‘rganiladi):

$$\sin x=x-\frac{x^3}{3!}-\frac{x^5}{5!}-\ldots; \qquad (2)$$

$$\cos x = 1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \ldots . \quad (3)$$

2 - m i s o l. a) sin0,15; b) cos0,15 ni 10^{-4} gacha aniqlikda topamiz.

Y e c h i s h. Talab etilayotgan aniqlikka erishish uchun (2) va (3) formulalarni qanday darajali hadgacha olishni bilish kerak:

$$\frac{0,15^3}{3!} = 0,00056 > 0,0001, \quad \frac{0,15^4}{4!} = 0,000021 < 0,001.$$

Demak, (2) formula beshinchi darajali hadgacha, (3) formula esa to‘rtinchi darajali hadgacha olinishi yetarli. Hisoblashlarga o‘tamiz:

a) $\sin 0,15 \approx 0,15 - \frac{0,15^3}{3!} - \frac{0,15^5}{5!} \approx 0,14948;$

b) $\cos 0,15 \approx 1 - \frac{0,15^2}{2!} - \frac{0,15^4}{4!} \approx 0,98876.$

M a s h q l a r

1.153. Ifodaning qiymatini 0,0001 gacha aniqlikda toping:

1) sin0,24; 2) cos0,18; 3) tg0,15;

4) sin31°; 5) cos23°; 6) tg16°.

5-§. Teskari trigonometrik funksiyalar

1. Arkfunksiyalar va ularning asosiy xossalari. Ushbu

$$y = \sin x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2} \quad (1)$$

funksiyani qaraymiz.

x argumentning qiymatlari $-\frac{\pi}{2}$ dan $\frac{\pi}{2}$ gacha o‘sib borganda y ning qiymatlari –1 dan 1 gacha o‘sib borishi va [–1; 1] kesmani to‘ldirishi bizga ma’lum (I.31-rasm). Bu yerdan, y ning [–1; 1] kesmadagi har bir qiymatiga $\sin x = y, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ shartlarni qanoatlantiruvchi birgina x sonni, ya’ni $x = \arcsin y$ sonni mos qo‘yish mumkinligi kelib chiqadi.

Har bir $y \in [-1; 1]$ songa uning arksinusini mos qo‘yib, quyidagi funksiyaga ega bo‘lamiz:

$$x = \arcsin y, \ -1 \le y \le 1. \qquad (2)$$

x va y o‘zgaruvchilarning (1) shartni qanoatlantiruvchi har qanday qiymatlari jufti (2) shartni ham qanoatlantiradi va aksincha, shu juftlik uchun (2) shart bajarilsa, u holda x va y uchun (1) shart ham bajariladi (isbotlang). Demak, (1) va (2) funksiyalar o‘zaro teskari funksiyalardir.

Odatda funksiyaning argumenti x bilan, funksiyaning o‘zi esa y bilan belgilangani uchun (2) da x ni y bilan, y ni esa x bilan almashtirib, quyidagi funksiyaga ega bo‘lamiz:

$$y = \arcsin x, \ -1 \le x \le 1. \qquad (3)$$

$y = \arcsin x$ funksiya $y = \sin x$, $x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ funksiyaga teskari funksiya bo‘lgani uchun uning ayrim xossalarini $y = \sin x$, $x \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ funksiyaning xossalaridan hosil qilish mumkin.

1°. $y = \arcsin x$ *funksiyaning aniqlanish sohasi* $[-1;\ 1]$ *kesmadan iborat*, chunki $y = \sin x$ funksiyaning qiymatlari sohasi $[-1;\ 1]$ kesmadan iborat.

2°. $y = \arcsin x$ *funksiyaning qiymatlari sohasi* $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ *kesmadan iborat*, chunki $y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ funksiyaning aniqlanish sohasi $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ kesmadan iborat.

E s l a t m a . $y = \sin x$ funksiya $(-\infty;\ +\infty)$ oraliqda teskarilanuvchi emas, chunki har qanday $y \in [-1;\ 1]$ songa $\sin x = y$ shartni qanoatlantiruvchi cheksiz ko‘p $x \in (-\infty;\ +\infty)$ sonlar mos keladi. $y = \sin x$ funksiyaning teskarilanuvchi bo‘lishligini ta’minlash uchun uning aniqlanish sohasi sun’iy ravishda toraytiriladi va aniqlanish sohasi sifatida $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ kesma olinadi.

3°. $y = \arcsin x$ *funksiya* $[-1;\ 1]$ *kesmada o‘sadi*, chunki $y = \sin x$ funksiya $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ kesmada o‘suvchi funksiyadir.

4°. $y = \arcsin x$ *funksiya toq funksiya*, ya’ni $\arcsin(-x) = -\arcsin x$ tenglik barcha $x \in [-1;\ 1]$ lar uchun o‘rinli (qarang, 4- § ning 1-bandi).

5°. $y = \arcsin x$ *funksiya davriy funksiya emas.*

Haqiqatan ham, $y = \arcsin x$ funksiya davriy funksiya va $T \neq 0$ son $y = \arcsin x$ funksiyaning biror davri, ya’ni barcha $x \in [-1;\ 1]$

lar uchun $\arcsin(x+T) = \arcsin x$ tenglik bajariladi deb faraz qilaylik. U holda bu tenglikdan $x=0\in[-1;\,1]$ da $\arcsin T=0$ ga ega bo'lamiz. Demak, $T=0$. Bu esa $T\neq 0$ ekanligiga zid. Shunday qilib, $y=\arcsin x$ funksiya davriy funksiya emas.

$y = \arcsin x$ funksiya $y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ funksiyaga teskari funksiya bo'lgani uchun, uning grafigini $y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ funksiya grafigini $y=x$ to'g'ri chiziqqa nisbatan simmetrik almashtirish bilan hosil qilish mumkin (I.49-*a* rasm).

Tarixiy ma'lumot

Mirzo Ulug'bek o'zining «Zij» asarida sinusni *jayb*, $1-\cos x$ ni *sahm* (x), arksinusni esa *jaybi ma'kus* (teskari sinus) deb atagan. Bu funksiyalarni doiraviy segmentda tasvirlagan (I.49-*b* rasm, hozirgi yozuvlar bizniki) va ularning ayrim xossalaridan foydalangan.

$y = \cos x$, $0 \le x \le \pi$ funksiya teskarilanuvchi va unga teskari funksiya $y=\arccos x$, $-1 \le x \le 1$ funksiyadan iborat ekanligi xuddi yuqoridagi kabi mulohazalar yuritib hosil qilinadi.

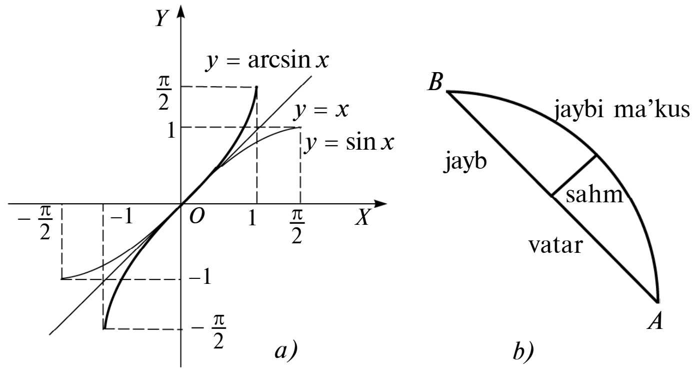
$y=\arccos x$ funksiyaning asosiy xossalarini keltiramiz:

1°. $y=\arccos x$ *funksiyaning aniqlanish sohasi* $[-1;\,1]$ *kesmadan iborat.*

2°. $y=\arccos x$ *funksiyaning qiymatlari sohasi* $[0;\,\pi]$ *kesmadan iborat.*

3°. $y=\arccos x$ funksiya $[-1;\,1]$ *kesmada kamayadi.*

4°. $y=\arccos x$ *funksiya juft funksiya ham emas, toq funksiya ham emas* (4-§, 2-band.).

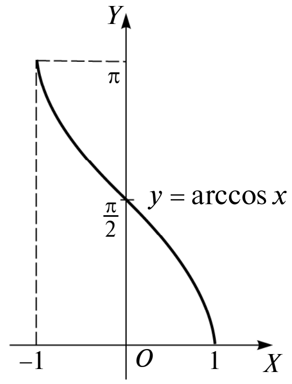


I.49-rasm.

5°. $y = \arccos x$ *funksiya davriy emas.*

$y = \arccos x$ funksiyaning grafigi $y = \cos x$, $0 \le x \le \pi$ funksiya grafigini $y = x$ to‘g‘ri chiziqqa nisbatan simmetrik almashtirish bilan hosil qilinadi (I.50-rasm).

$y = \arcsin x$ va $y = \arccos x$ funksiyalarning asosiy xossalarini jadvalda keltiramiz:



I.50-rasm.

Xossalar	**Funksiya**	
	$y = \arcsin x$	$y = \arccos x$
Aniqlanish sohasi	$[-1; 1]$	$[-1; 1]$
Qiymatlar sohasi	$\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$	$[0; \pi]$
Monotonligi	$[-1; 1]$ oraliqda o‘suvchi	$[-1; 1]$ oraliqda kamayuvchi
Juft-toqligi	toq funksiya	juft emas, toq emas
Davriyligi	davriy emas	davriy emas

$y = \operatorname{arctg} x$, $x \in R$ funksiya $y = \operatorname{tg} x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ funksiyaga, $y = \operatorname{arcctg} x$, $x \in R$ funksiya esa $y = \operatorname{ctg} x$, $0 < x < \pi$ funksiyaga teskari funksiyadir. Ularning asosiy xossalarini jadval ko‘rinishida keltiramiz:

Xossalar	**Funksiya**	
	$y = \operatorname{arctg} x$	$y = \operatorname{arcctg} x$
Aniqlanish sohasi	$(-\infty; +\infty)$	$(-\infty; +\infty)$
Qiymatlar sohasi	$\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$	$(0; \pi)$
Monotonligi	$(-\infty; +\infty)$ oraliqda o‘suvchi	$(-\infty; +\infty)$ oraliqda kamayuvchi
Juft-toqligi	toq funksiya	juft emas, toq emas
Davriyligi	davriy emas	davriy emas

$y = \text{arctg}x$ va $y = \text{arcctg}x$ funksiyalarning grafiklari mos ravishda I.51-rasm va I.52-rasmlarda tasvirlangan.

$y = \arcsin x$, $y = \arccos x$, $y = \text{arctg}x$ va $y = \text{arcctg}x$ funksiyalar *teskari trigonometrik funksiyalar* deb ataladi. Boshqa funksiyalar kabi, bu funksiyalarning ham ba'zi nuqtalardagi aniq qiymatlarini, masalan, $\arcsin 1 = \frac{\pi}{2}$, $\arccos 1 = 0$, $\text{arctg}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$, $\text{arcctg}\sqrt{3} = \frac{\pi}{6}$ ekanligini ko'rsatish mumkin. Umumiy holda esa turli hisoblash vositalari (grafiklar, jadvallar, kalkulatorlar va h.k.) yordamida bu funksiyalarning taqribiy qiymatlari kerakli aniqlikda hisoblanadi.

1-misol. Kalkulator yordamida $\arcsin 0{,}5773 \approx 0{,}615$, $\arccos 0{,}5773 \approx 0{,}836$ ekanligini topish mumkin.

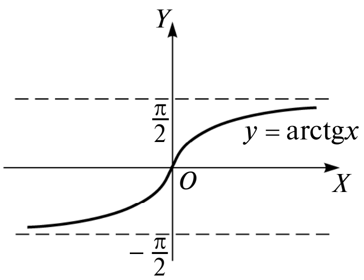
2-misol. $y = \arcsin(x^2 + 1)$ funksiyaning aniqlanish sohasini va qiymatlari sohasini topamiz.

Yechish. $y = \arcsin x$ funksiyaning aniqlanish sohasi $[-1; 1]$ kesmadan iborat. Shu sababli, $y = \arcsin(x^2 + 1)$ funksiya x ning $-1 \le x^2 + 1 \le 1$ shart bajariladigan barcha qiymatlaridagina aniqlangandir. $-1 \le x^2 + 1 \le 1$ tengsizlik $x = 0$ bo'lganda va faqat shu holda bajariladi.

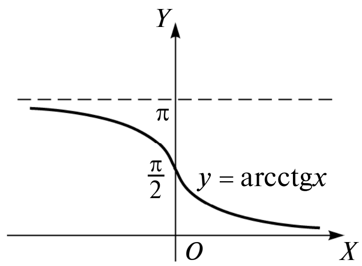
$x = 0$ bo'lganda, $y(0) = \arcsin(0^2 + 1) = \arcsin 1 = \frac{\pi}{2}$. Shunday qilib, berilgan funksiyaning aniqlanish sohasi $\{0\}$ to'plamdan, qiymatlar sohasi esa $\left\{\frac{\pi}{2}\right\}$ to'plamdan iborat.

3-misol. $y = \frac{\arccos(x^2+1)}{x}$ funksiyaning aniqlanish sohasi va qiymatlari sohasini topamiz.

Yechish. Bu funksiya x ning $x \ne 0$ va $-1 \le x^2 + 1 \le 1$ shartlarni qanoatlantiradigan barcha qiymatlaridagina aniqlangan.



I.51-rasm.



I.52-rasm.

x ning $x \neq 0$, $-1 \leq x^2 + 1 \leq 1$ shartlar bir vaqtda o‘rinli bo‘ladigan qiymati mavjud emas. Shunday qilib, berilgan funksiyaning aniqlanish sohasi, shuningdek, qiymatlar sohasi ham bo‘sh to‘plamdir.

4 - m i s o l. $y = \arcsin x$, $-\frac{1}{2} \leq x < 0$ funksiyaning aniqlanish sohasini va qiymatlar sohasini topamiz.

Y e c h i s h. Funksiyaning berilishidan ko‘rinadiki, uning aniqlanish sohasi $\left(-\frac{1}{2};\ 0\right]$ oraliqdan, qiymatlar sohasi esa $y = \arcsin x$ funksiya $[-1;\ 1]$ kesmada o‘suvchi va $y\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$, $y(0) = 0$ bo‘lgani uchun $\left[-\frac{\pi}{6};\ 0\right)$ oraliqdan iborat bo‘ladi.

Mashqlar

1.154. a) Quyidagi funksiyaning aniqlanish sohasini toping:

1) $y = \arcsin(x+1)$; 2) $y = \arccos\frac{1+4x}{3}$; 3) $y = \arccos\frac{x-5}{7}$.

b) $y = \mathrm{arcctg}^2 x - 3$ funksiya son o‘qi bo‘yicha chegaralanganmi?

d) $y = 1 - \cos x$ (sahm)ga teskari funksiyani ta’riflang, ifodasini yozing, aniqlanish va o‘zgarish sohalarini toping, monotonlikka tekshiring, grafigini yasang.

1.155. Jadvaldagi bo‘sh kataklarni to‘ldiring (hisoblashlarni EHM yoki mikrokalkulatordan foydalanib bajaring):

x	0,7				
$\arcsin x$		0,85 (rad)			
$\arccos x$			0,9 (rad)		
$\mathrm{arctg}\, x$				$-\pi/6$ (rad)	
$\mathrm{arcctg}\, x$					0,3

2. Arkfunksiyalar qatnashgan ayrim ayniyatlar. Teskari trigonometrik funksiyalarga berilgan ta’riflarga ko‘ra, masalan, $y = \sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ va $x = \arcsin y$, $-1 \leq y \leq 1$ bir ma’noli munosabatlardir. Agar $y = \sin x$ tenglikda x o‘rniga $\arcsin y$ qo‘yilsa, ushbu ayniyat hosil bo‘ladi:

$$\sin(\arcsin y) = y,\ -1 \leq y \leq 1. \tag{1}$$

Shu tariqa quyidagi ayniyatlarni ham olish mumkin:

$$\cos(\arccos y) = y, \ -1 \leq y \leq 1, \qquad (2)$$

$$\mathrm{tg}(\mathrm{arctg} y) = y, \ -\infty < y < +\infty, \qquad (3)$$

$$\mathrm{ctg}(\mathrm{arcctg} y) = y, \ -\infty < y < +\infty. \qquad (4)$$

Agar $x = \arcsin y$ tenglikda y o‘rniga $\sin x$ qo‘yilsa:

$$\arcsin(\sin x) = x, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}. \qquad (5)$$

Shu kabi:

$$\arccos(\cos x) = x, \ 0 \leq x \leq \pi, \qquad (6)$$

$$\mathrm{arctg}(\mathrm{tg} x) = x, \ -\frac{\pi}{2} < x < \frac{\pi}{2}, \qquad (7)$$

$$\mathrm{arcctg}(\mathrm{ctg} x) = x, \ 0 < x < \pi. \qquad (8)$$

1 - m i s o l. $\arcsin x + \arccos x = \frac{\pi}{2}$ (bu yerda, $|x| \leq 1$) ayniyatni isbot qilamiz.

I s b o t. $|x| \leq 1$ bo‘lsin. U holda $\arcsin x$ va $\frac{\pi}{2} - \arccos x$ ifodalar ma’noga ega. $\arcsin x$, $\frac{\pi}{2} - \arccos x$ sonlarining har biri $y = \sin x$ funksiyaning o‘sish oraliqlaridan biriga, xususan, $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ oraliqqa tegishli va $\sin(\arcsin x) = x$, $\sin\left(\frac{\pi}{2} - \arccos x\right) = x$ tengliklar o‘rinli. Shu sababli $\arcsin x = \frac{\pi}{2} - \arccos x$.

2 - m i s o l. $\arcsin(\sin x)$ ifodani hisoblaymiz.

Y e c h i s h. Har qanday $x \in R$ son uchun $\sin x \in [-1; 1]$ bo‘lgani sababli $\arcsin(\sin x)$ ifoda barcha $x \in R$ sonlar uchun ma’noga ega va $\arcsin a$ ning ta’rifiga ko‘ra, $\arcsin(\sin x) \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$.

$\arcsin(\sin x) = y$ bo‘lsin. U holda $\sin y = \sin x$, $y \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ shartlar bajariladi. $\sin y = \sin x$ bo‘lgani uchun, $x = (-1)^k y + k\pi$ yoki $y = \frac{x - k\pi}{(-1)^k} = \frac{(-1)(k\pi - x)}{(-1)^k} = (-1)^{1-k}(k\pi - x)$ (bu yerda $k \in Z$) bo‘ladi. Bu yerdan ko‘rinadiki, $|y| \leq \frac{\pi}{2}$ bo‘lishi uchun $|k\pi - x| \leq \frac{\pi}{2}$ bo‘lishi yetarlidir.

Shunday qilib, $\arcsin(\sin x) = (-1)^{1-k}(k - \pi)$, bu yerda k son $|k\pi - x| \leq \frac{\pi}{2}$ tengsizlikni qanoatlantiradigan butun son.

Mashqlar

1.156. Ayniyatni isbotlang:

1) $\cos(\arcsin x) = \sqrt{1-x^2}$;

2) $\arccos x = \operatorname{arcctg} \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$;

3) $\operatorname{arcctg} x = \arccos \frac{x}{\sqrt{1+x^2}}$;

4) $\operatorname{arctg} x = \arcsin \frac{x}{\sqrt{1+x^2}}, \quad -\infty < x < +\infty$;

5) $2\arcsin x = \arcsin\left(2x\sqrt{1-x^2}\right), \quad 0 \le x \le \frac{\pi}{2}$;

6) $\operatorname{arctg}(\operatorname{tg} x) = x - k\pi, \quad \pi - \frac{\pi}{2} < x < k\pi + \frac{\pi}{2}$;

7) $\cos(\operatorname{arctg} x) = \frac{1}{\sqrt{1+x^2}}$; 8) $\sin(\operatorname{arctg} x) = \frac{1}{\sqrt{1-x^2}}$.

1.157. Ifodaning qiymatini toping:

1) $\operatorname{arctg}(\operatorname{tg}3)$; 2) $\arcsin(\sin 4)$;

3) $\arccos\left(\cos\frac{\pi}{3}\right)$; 4) $\operatorname{arcctg}\left(\operatorname{ctg}\frac{\pi}{6}\right)$;

5) $\arccos\left[\sin\left(-\frac{\pi}{8}\right)\right]$; 6) $\arcsin\left(\cos\frac{30}{7}\pi\right)$;

7) $\operatorname{arctg}\left(\operatorname{ctg}\frac{3\pi}{7}\right)$;

8) $2\left[\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right] + 3\arccos\left(-\frac{1}{2}\right) - \operatorname{arcctg}1$;

9) $\cos(2\arccos x)$; 10) $\cos(3\arccos x)$;

11) $\cos\left(\operatorname{arctg}\left(-\frac{5}{24}\right)\right)$; 12) $\arcsin(\sin 100)$;

13) $\sin(3\arcsin x)$.

3. Teskari trigonometrik funksiyalar qatnashgan tenglamalar va tengsizliklar. Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechishda teng argumentlarda bir xil ismli trigonometrik funksiyalarning qiymatlari ham teng bo‘lishidan, ya’ni trigonometrik funksiyalarning bir qiymatlilik xossasidan foydalaniladi.

Ko'pchilik hollarda arkfunksiyalar ko'rinishida berilgan teng argumentlarning bir xil ismli trigonometrik funksiyalarini tenglashtirib, berilgan tenglamaga nisbatan soddaroq tenglama (masalan, algebraik tenglama) hosil qilish mumkin bo'ladi. Hosil qilingan tenglama berilgan tenglamaga umuman olganda teng kuchli emas, chunki bir xil ismli trigonometrik funksiya qiymatlarining tengligidan shu funksiya argumentlarining tengligi kelib chiqmaydi.

1 - m i s o l. $\arcsin\frac{3}{5}x+\arcsin\frac{4}{5}x=\arcsin x$ tenglamani yechamiz.

Y e c h i s h. Tenglamaning aniqlanish sohasi x ning $\left|\frac{3}{5}x\right|\le 1$, $\left|\frac{4}{5}x\right|\le 1$, $|x| \le 1$ tengsizliklar bir vaqtda bajariladigan qiymatlari to'plami $\{x: |x| \le 1\}$ dan iborat.

Berilgan tenglama chap va o'ng tomonlarining sinuslarini tenglashtiramiz:

$$\sin\left(\arcsin\tfrac{3}{5}x+\arcsin\tfrac{4}{5}x\right)=\sin(\arcsin x).$$

Yig'indining sinusi formulasidan va $\sin(\arcsin\alpha) = \alpha$, $\cos(\arcsin\alpha)=\sqrt{1-\alpha^2}$ (bu yerda $|\alpha| \le 1$) ayniyatlardan foydalanib,

$$\tfrac{3}{5}x\cdot\sqrt{1-\tfrac{16}{25}x^2}+\tfrac{4}{5}x\cdot\sqrt{1-\tfrac{9}{25}x^2}=x.$$

tenglamaga ega bo'lamiz. Bu tenglama $x_1 = 0$, $x_{2,3} = \pm 1$ ildizlarga ega. Ularning har birini tenglamaga bevosita qo'yib ko'rib, bu ildizlar tenglamani qanoatlantirishini ko'ramiz. Masalan, $x = 1$ uchun

$$\arcsin\tfrac{3}{5}+\arcsin\tfrac{4}{5}=\arcsin\tfrac{3}{5}+\arccos\tfrac{3}{5}=\tfrac{\pi}{2}.$$

2 - m i s o l. $(\arcsin x)^3 + (\arccos x)^3 = \pi^3$ tenglamani yechamiz.

Y e c h i s h. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ ayniyatdan foydalanib va $\arcsin x+\arccos x=\frac{\pi}{2}$ ekanligini nazarda tutib, quyidagi tenglamaga ega bo'lamiz:

$$\left(\frac{\pi}{2}\right)^3-3\cdot\frac{\pi}{2}\cdot\arcsin x\cdot\arccos x=\pi^3$$

yoki

$$\arcsin x\cdot\arccos x=-\frac{7}{12}\pi^2.$$

$y=\arccos x$, $\left(|y|\le\frac{\pi}{2}\right)$ desak, $y^2-\frac{\pi}{2}y-\frac{7\pi^2}{12}=0$ kvadrat tenglama hosil bo‘ladi. Bu kvadrat tenglamaning ildizlari absolut qiymati jihatidan $\frac{\pi}{2}$ dan katta bo‘lgani uchun berilgan tenglama yechimga ega emas.

3 - m i s o l. $2\arcsin^2x-5\arcsin x+2=0$ tenglamani yechamiz.

Y e c h i s h. $\arcsin x=z$ almashtirish berilgan tenglamani $2z^2-5z+2=0$ kvadrat tenglamaga keltiradi. Uning ildizlari $z_1=0{,}5$, $z_2=2$. Lekin z_2 ildiz $-\frac{\pi}{2}\le\arcsin x\le\frac{\pi}{2}$ bo‘lish shartini qanoatlantirmaydi. z_1 ildiz bo‘yicha $\arcsin x=\frac{1}{2}$ tenglamani tuzamiz. Bu tenglamaning yechimi $x=\sin 0{,}5$.

Endi arkfunksiyalar qatnashgan tengsizliklarga doir misollar qaraymiz.

4 - m i s o l. $\mathrm{arctg}^2x-4\,\mathrm{arctg}x+3>0$ tengsizlikni yechamiz.

Y e c h i s h. $\mathrm{arctg}x=y$ deb olsak, berilgan tengsizlik quyidagi ko‘rinishni oladi:

$$y^2-4y+3>0.$$

Bu tengsizlik $y<1$ yoki $y>3$ bo‘lganda bajariladi. Eski o‘zgaruvchiga qaytib, $\mathrm{arctg}x<1$ va $\mathrm{arctg}x>3$ tengsizliklarga ega bo‘lamiz. $\mathrm{arctg}x<1$ tengsizlik $x\in(-\infty;\ \mathrm{tg}1)$ yechimlarga, $\mathrm{arctg}x>3$ tengsizlik esa $x\in(\mathrm{tg}3;\ +\infty)$ yechimlarga ega.

J a v o b: $(-\infty;\ \mathrm{tg}1)\cup(\mathrm{tg}3;\ +\infty)$.

5 - m i s o l. $\arcsin x>\arccos x$ tengsizlikni yeching.

Y e c h i s h. Tengsizlikning aniqlanish sohasi $[-1;\ 1]$ kesmadan iborat.

$x<0$ bo‘lganda $\arcsin x<0$, $\arccos x>0$ bo‘lgani uchun berilgan tengsizlik manfiy yechimlarga ega emas. $0\le x\le 1$ bo‘lsin. U holda $\arcsin x\in\left[0;\ \frac{\pi}{2}\right]$ va $\arccos x\in\left[0;\ \frac{\pi}{2}\right]$ bo‘ladi. $\left[0;\ \frac{\pi}{2}\right]$ oraliqda $\sin x$ funksiya o‘suvchi bo‘lgani uchun, $x\in[0;\ 1]$ bo‘lganda berilgan tengsizlik sin(arcsin$x$) > sin(arccos$x$) tengsizlikka teng kuchlidir. Bu yerdan $x > \sqrt{1-x^2}$ tengsizlikni hosil qilamiz. $x > \sqrt{1-x^2}$ tengsizlikni [0; 1] oraliqda yechib, $x \in \left(\frac{\sqrt{2}}{2};\ 1\right]$ ni olamiz.

Mashqlar

1.158. Tenglamani yeching:

1) $\arcsin^2 x - \frac{3\pi}{2}\arcsin x + \frac{5\pi^2}{4} = 0$;

2) $\arccos^2 x - \frac{2\pi}{3}\arccos x + \frac{\pi^2}{12} = 0$;

3) $\mathrm{arctg}^2 x - \frac{7\pi}{12}\mathrm{arctg} x + \frac{\pi^2}{12} = 0$;

4) $\arcsin\left(x^2 - 5x + \frac{7}{4}\right) = -\frac{\pi}{6}$;

5) $\arcsin\left(x^2 - 3x + \frac{\pi}{4}\right) = \frac{\pi}{2}$;

6) $\arcsin x + \arcsin x\sqrt{3} = \frac{\pi}{2}$;

7) sin(2arcsin$x$)+ cos(2arcsin$x$)=1;

8) $\sin\left(\frac{1}{5}\arccos x\right) = 1$.

1.159. Tengsizlikni yeching:

1) $-\frac{\pi}{4} \le \arcsin(x^2 - 4) \le \frac{\pi}{3}$;

2) $\frac{5}{4} \le \mathrm{arctg}^2\, 3x - 2\mathrm{arctg} 3x \le 3$;

3) arccos$x$ < arcsin$x$; 4) arctg$x$ < arcctg$x$.

1.160. 1) Agar:

$$\mathrm{tg}\alpha = \frac{5+\sqrt{x}}{2},\ \mathrm{tg}\beta = \frac{5-\sqrt{x}}{2},\ \alpha + \beta = 45°$$

bo‘lsa, x ni toping;

2) ABC uchburchakda:

$$\angle A = \alpha,\ \angle B = \beta,\ \angle C = \gamma,\ \alpha : \beta : \gamma = 1 : 2 : 3,\ BC = 2\sqrt{3}$$

bo‘lsa, uning perimetrini toping.

I bob bo'yicha topshiriq

1—6- topshiriqlar hamma uchun, 7—8- topshiriqlar variantlar bo'yicha bajariladi. Hisoblashlarda EHM, mikrokalkulator va jadvaldan foydalanish ma'qul.

T o p s h i r i q n i n g m a z m u n i. Jism O nuqtadan (I.53-rasm) gorizontga nisbatan α burchak ostida v_0 boshlang'ich tezlik bilan otilgan. Tashqi kuchlar ta'sir etmaganda u $s = v_0t$ to'g'ri chiziqli tekis harakat qilgan va t vaqtdan so'ng biror $B(x, y')$ nuqtaga kelgan bo'lar edi, nuqta koordinatalari $x = v_0t \cos\alpha$, $y' = v_0 t \sin\alpha$. Lekin jism Yerning tortish kuchi ta'siri ostida harakat chizig'idan chetga chiqadi va t vaqtdan so'ng $\frac{gt^2}{2}$ qadar quyida joylashgan $A(x, y)$ nuqtaga keladi (havo qarshiligi hisobga olinmagan holda):

$$x = v_0t\cos\alpha; \qquad (1)$$

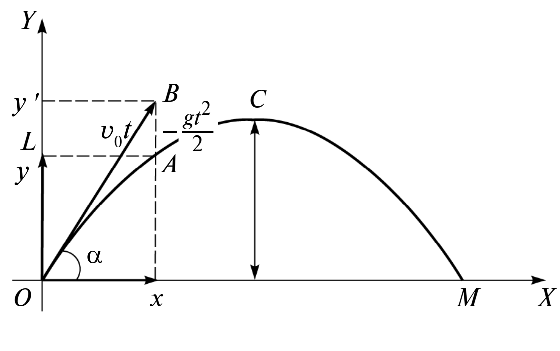
$$y = v_0t\sin\alpha - \frac{gt^2}{2}. \qquad 2)$$

Qolgan vaqt momentlarida ham harakat shu tarzda davom etadi.

(2) munosabat bosh had koeffitsiyenti ishorasi manfiy, ozod hadi nolga teng bo'lgan kvadrat uchhad. Demak, jism parabolik trayektoriya bo'yicha harakat qiladi.

Shu kabi, v_0 boshlang'ich tezlikning tashkil etuvchilari $v_x = v_0\cos\alpha$, $v_y = v_0t \sin\alpha$. Lekin gravitatsiya ta'siri ostida jismning t vaqt paytidagi v tezligi

$$v_x = v_0\cos\alpha, \qquad (3)$$



I.53-rasm.

$$v_y = v_0 \sin\alpha - gt \qquad (4)$$

tashkil etuvchilarga (koordinata o‘qlaridagi proeksiyalarga) ega bo‘ladi.

T o p i n g:

1) jismning t vaqt momentidagi v tezligi;

2) jismning $y = f(x)$ ko‘rinishdagi harakat tenglamasi;

3) qancha vaqtdan so‘ng jismning eng yuqoriga (C nuqta) ko‘tarilishi, $t = t_C$;

4) eng yuqori qanday balandlikkacha ko‘tarilishi (y_C);

5) jism borib tushadigan masofa (x_M);

6) jism eng uzoqqa borib tushishi uchun u O nuqtadan gorizontga nisbatan qanday α burchak ostida otilishi kerakligi;

7) v_0 = 10 km/s tezlik va gorizontga nisbatan α = 10° + 5°k, $k = \overline{0;\ 30}$ burchak ostida otilgan jism qanday balandlikkacha ko‘tariladi va qanday masofaga borib tushadi?

8) qanday α burchak ostida otilgan jism s = 1 + 0,1k (km), $k = \overline{0;\ 30}$ uzoqlikka borib tushadi?

9) ikkinchi jism H = 0,5 + 0,05k (km) balandlikdagi L nuqtadan o‘ng tomonga qarab y = 1 + 0,05k (km/s) tezlik bilan to‘g‘ri chiziqli tekis harakat qilib bormoqda. O nuqtadagi birinchi jism gorizontga nisbatan qanday burchak ostida otilsa, u ikkinchi jismga borib tegadi va bu qancha vaqtdan so‘ng sodir bo1adi? $k = \overline{0;\ 30}$.

K o ‘ r s a t m a: 1) $v = \sqrt{v_x^2 + v_y^2}$. (3) va (4) munosabatlardan foydalanilsa, natijada $v = \sqrt{v_0^2 - 2g \cdot \left(v_0 t \sin\alpha - \frac{gt^2}{2} \right)}$;

2) (1) munosabat bo‘yicha $t = \frac{x}{v_0 \cos\alpha}$ ni topib, (2) ga qo‘ysak:

$$y = x\mathrm{tg}\alpha - \frac{g}{2v_0^2 \cos^2\alpha} \cdot x^2;$$

3) harakat boshida $v = v_0$ bo‘lgan tezlik harakat davomida kamayib borib, C nuqtada nolga teng bo‘ladi: $0 = v_0 \sin\alpha - gt$, bundan $t_C = \frac{v_0 \sin\alpha}{g}$;

4) (2) parabola C uchining ordinatasi ifodasidan foydalanilsa,

$$y_C = -\frac{v_0^2 \sin^2 \alpha}{4 \cdot \left(-\frac{g}{2}\right)} = \frac{v_0^2 \sin^2 \alpha}{2g};$$

5) (1) munosabat va t_C qiymatidan foydalanilsa,

$$s = |OM| = 2x_C = 2v_0 \cdot \frac{v_0 \sin^2 \alpha}{g} \cdot \cos\alpha = \frac{v_0^2 \sin 2\alpha}{g};$$

6) oldingi savol bo‘yicha topilgan $s = \frac{v_0^2 \sin 2\alpha}{g}$ munosabatga qaraganda s masofa $\sin 2\alpha = 1$, ya’ni $\alpha = 45°$ da eng katta qiymatga erishadi.